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THE MACROECONOMIC EFFECTS OF HOUSING WEALTH, HOUSING FINANCE, AND LIMITED RISK-SHARING IN GENERAL EQUILIBRIUM

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ABSTRACT

This paper studies the role of time-varying risk premia as a channel for generating and propagating fluctuations in housing markets, aggregate quantities, and consumption and wealth heterogeneity. We study a two-sector general equilibrium model of housing and non-housing production where heterogeneous households face limited opportunities to insure against aggregate and idiosyncratic risks. The model generates large variability in the national house price-rent ratio, both because it fluctuates endogenously with the state of the economy and because it rises in response to a relaxation of credit constraints and decline in housing transaction costs (financial market liberalization). These factors, together with a rise in foreign ownership of U.S. debt calibrated to match the actual increase over the period 2000-2006, generate fluctuations in the model price-rent ratio that explains a large fraction of the increase in the national price-rent ratio observed in U.S. data over this period. The model also predicts a sharp decline in home prices starting in 2007, driven by the economic contraction and by a presumed reversal of the financial market liberalization. Fluctuations in the model's price-rent ratio are driven by changing risk premia, which fluctuate endogenously in response to cyclical shocks, the financial market liberalization, and its subsequent reversal. By contrast, we show that the inflow of foreign money into domestic bond markets plays a small role in driving home prices, despite its large depressing influence on interest rates. Finally, the model implies that procyclical increases in equilibrium price-rent ratios reflect rational expectations of lower future housing returns, not higher future rents.

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1 Introduction

Residential real estate is a large and volatile component of household wealth. Moreover, volatility in housing wealth is often accompanied by large swings in house prices relative to housing fundamentals. For example, Figure 1 shows that national house price-rent ratios climbed to unusual heights by the end of 2006, but have since exhibited sharp declines.[1]

This paper studies the role of time-varying risk premia as a channel for generating and propagating fluctuations in housing markets, aggregate quantities, and consumption and wealth heterogeneity. Existing macroeconomic models of housing production have studied home price fluctuations in models where risk premia are held constant or not modeled (see literature review below). In the present model, risk premia vary endogenously with fluctuations in housing wealth and housing finance, both of which influence opportunities for risk-sharing. To what extent can episodes of national house price appreciation be attributed to a liberalization in housing finance, such as declines in collateral constraints or reductions in the costs of borrowing and conducting transactions? How do movements in house prices affect expectations about future housing fundamentals and future home price appreciation? To what extent do changes in housing wealth and housing finance affect output and investment, risk premia in housing and equity markets, measures of cross-sectional risk-sharing, and life-cycle patterns in wealth accumulation and savings?

In this paper we address these questions by studying a two-sector general equilibrium model of housing and non-housing production, where heterogenous households face limited risk-sharing opportunities as a result of incomplete financial markets. This macroeconomic model is sufficiently general as to account for the endogenous interactions among financial and housing wealth, output and investment, interest rates, consumption and wealth inequality and, especially, risk premia in both housing and equity assets.

A house in our model is a residential durable asset that provides utility to the household, is illiquid (expensive to trade), and can be used as collateral in debt obligations. The model economy is populated by a large number of overlapping generations of households who receive utility from both housing and nonhousing consumption and who face a stochastic life-cycle earnings profile. We introduce market incompleteness by modeling heterogeneous agents who face idiosyncratic and aggregate risks against which they cannot perfectly insure, and by imposing collateralized borrowing constraints on households.

[1] "Rent" here refers to the flow value of housing services, i.e., the dividend generated by the housing asset. We discuss this below.

Within the context of this model, we focus our theoretical investigation on the macroeconomic consequences of three systemic changes in housing finance with an emphasis on how these factors affect risk premia in housing markets. First, we investigate the impact of changes in housing collateral requirements. Second, we investigate the impact of changes in housing transactions costs. Third, we investigate the impact of an influx of foreign capital into the domestic bond market. We argue below that all three factors fluctuate over time and changed markedly during or preceding the period of rapid home price appreciation from 2000-2006. In particular, this period was marked by a widespread relaxation of collateralized borrowing constraints and declining housing transactions costs, a combination we refer to hereafter as *financial market liberalization.* The period was also marked by a sustained depression of long-term interest rates that coincided with a vast inflow of capital from foreign governmental holders into U.S. bond markets. In the aftermath of the credit crisis that began in 2007, the erosion in credit standards and transactions costs has been reversed.[2] We use our framework as a laboratory for studying the impact of fluctuations in either direction of these features of housing finance. The main contribution of the framework is to demonstrate the potential theoretical importance of time-varying risk premia as a channel for transmitting the effects of such fluctuations to housing and equity market prices, as well as to aggregate quantities and consumption and wealth heterogeneity.

We summarize the model's main implications as follows.

House prices relative to measures of fundamental value are volatile. The model generates substantial variability in the national house price-rent ratio, both because it fluctuates procyclically with the state of the economy, and because it rises in response to a relaxation of credit constraints and decline in housing transaction costs. When we combine a financial market liberalization with an inflow of foreign capital into the domestic bond market calibrated to match the rise in foreign ownership of U.S. Treasury and agency debt over the period 2000-2006, these factors together generate fluctuations in the model price-rent ratio that explain 70 percent of the increase in national price index-rent ratios observed in two different U.S. data sources on house prices over this period, and 45 percent of those from a third data source. The model also predicts a sharp decline in home prices starting in 2007, driven by the economic contraction and by a presumed reversal of the financial market liberalization (but not the foreign capital inflow).

A financial market liberalization drives price-rent ratios up because it drives

[2]Some analysts have argued that, since the credit crisis, borrowing restrictions and credit constraints have become even more stringent than historical norms in the pre-boom period (e.g., Streitfeld (2009)).

risk premia down. The main impetus for rising price-rent ratios in the model is the simultaneous occurrence of positive economic shocks and a financial market liberalization, phenomena that generate an endogenous decline in risk premia on housing and equity assets. A financial market liberalization reduces risk premia for two reasons, both of which are related to the ability of heterogeneous households to insure against aggregate and idiosyncratic risks. First, lower collateral requirements directly increase access to credit, which acts as a buffer against unexpected income declines. Second, lower transactions costs reduce the expense of obtaining the collateral required to increase borrowing capacity and provide insurance. These factors lead to an increase in risk-sharing, or a decrease in the cross-sectional variance of marginal utility.

It is important to note that the rise in price-rent ratios caused by a financial market liberalization must be attributed to a decline in risk premia and not to a fall in interest rates. Indeed, the very changes in housing finance that accompany a financial market liberalization reduce precautionary saving and drive the endogenous interest rate up, rather than down. With no accompanying movement in aggregate risk, this endogenous rise in the interest rate would lead to a housing bust rather than boom. It follows that price-rent ratios rise after a financial market liberalization because the decline in risk premia more than offsets the rise in equilibrium interest rates. These findings underscore the crucial role of foreign capital in maintaining low interest rates during a financial market liberalization. Without an infusion of foreign capital, any period of looser collateral requirements and lower housing transactions costs (such as that which characterized the period of rapid home price appreciation from 2000-2006) would be accompanied by an increase in equilibrium interest rates, as households endogenously respond to the improved risk-sharing opportunities afforded by a financial market liberalization by reducing precautionary saving.

Foreign purchases of U.S. bonds play a central role in lower interest rates but a small role in housing booms. The model implies that a rise in foreign purchases of domestic bonds, equal in magnitude to those observed in the data from 2000-2006, leads to a quantitatively large decline in the equilibrium real interest rate. In partial equilibrium analyses where risk premia are held fixed, a decline in the interest rate of this magnitude would be sufficient–by itself–to explain the rise in price-rent ratios observed from 2000-2006. But we show that, in general equilibrium, borrowed sums from the rest of the world can play at most a limited role in asset booms, despite their large depressing influence on interest rates. Foreign purchases of U.S. bonds crowd domestic savers out of the safe bond market, exposing them to greater systematic risk in equity and housing markets. In response, risk premia on housing and equity assets rise, substantially offsetting the effects of lower interest rates and limiting the impact of foreign capital inflows on home prices.

Procyclical increases in equilibrium price-rent ratios reflect rational expectations of lower future returns, not higher future rents. It is commonly assumed that increases in national house-price rent ratios reflect an expected increase in future housing fundamentals, such as rental growth. In partial equilibrium analyses where discount rates are held constant, this is the only outcome possible (e.g., Sinai and Souleles (2005), Campbell and Cocco (2007)). This reasoning, however, ignores the general equilibrium response of both residential investment and discount rates to economic growth. In the model here, positive economic shocks stimulate greater housing demand and greater residential investment. Under plausible parameterizations, the latter can lead to an equilibrium *decline* in future rental growth as the housing stock rises. It follows that high price-rent ratios in expansions must entirely reflect expectations of future house price depreciation (lower discount rates), driven in the model by falling risk premia as collateral values and risk-sharing opportunities rise with the economy.

Financial market liberalization plus foreign capital leads to a shift in the composition of wealth towards housing, increases financial wealth inequality, but reduces housing and consumption inequality. A financial market liberalization plus an inflow of foreign capital into the domestic bond market leads households of all ages and incomes to shift the composition of their wealth towards housing, consistent with observed changes in household-level data from 2000 to 2007. These factors also have implications for inequality. We show that a financial market liberalization and foreign capital infusion reduce consumption and housing wealth inequality, but increase financial wealth inequality.

The paper is organized as follows. The next subsection briefly discusses related literature. Section 2 describes recent changes in the three key aspects of housing finance discussed above: collateral constraints, housing transactions costs, and foreign capital in U.S. debt markets. Section 3 presents the theoretical model. Section 4 presents our main findings, including benchmark business cycle and financial market statistics. Here we show that the model generates forecastable variation in equity and housing returns, and a sizable equity premium and Sharpe ratio simultaneously with a plausible degree of variability in aggregate consumption. Section 5 concludes.

1.1 Related Literature

Our paper is related to a growing body of literature in finance that studies the asset pricing implications of incomplete markets models. The focus of this literature, however, is typically on the equity market implications of pure exchange economies with exogenous endowments, with no role for housing or the production side of the economy.[3] Storesletten, Telmer, and Yaron (2007), Gomes and Michaelides (2008), and Favilukis (2013) explicitly model the production side of the economy, but focus on single-sector economies without housing.

Within the incomplete markets environment, our work is related to several important papers that study questions related to housing and/or consumer durables more generally. We are aware of only one other paper (at the time of the first draft of this paper) that solves for equilibrium asset prices in a model where the portfolio choice problem involves three assets (housing, stocks and bonds). Piazzesi and Schneider (2008) do so, as here. Other papers typically either do not model production (instead studying a pure exchange economy), and/or the portfolio choice problem underlying asset allocation between a risky and a risk-free asset, or are analyses of partial equilibrium environments. See for example, the general equilibrium exchange-economy analyses that embed bond, stock and housing markets of Ríos-Rull and Sánchez-Marcos (2006), Lustig and Van Nieuwerburgh (2007, 2008), and the partial equilibrium analyses of Peterson (2006), Ortalo-Magné and Rady (2006), and Corbae and Quintin (2009). We add to this literature by considering each of these general equilibrium features. We add to Piazzesi and Schneider (2008) by modeling the production side, in two sectors.

Other researchers have studied the role of incomplete markets in housing decisions in models without aggregate risk. Fernández-Villaverde and Krueger (2005) study how consumption over the life-cycle is influenced by consumer durables in an incomplete markets model with production, but limit their focus to equilibria in which prices, wages and interest rates are constant over time. Kiyotaki, Michaelides, and Nikolov (2008) study a life-cycle model with housing and non-housing production, but focus their analysis on the perfect foresight equilibria of an economy without aggregate risk and an exogenous interest rate.

Iacoviello and Pavan (2009) study the role of housing and debt for the volatility of the aggregate economy in an incomplete markets model with aggregate risk but with a single production and single saving technology. Because there is no risk-free asset, their model is

[3] See for example Aiyagari and Gertler (1991), Telmer (1993), Lucas (1994), Heaton and Lucas (1996), Basak and Cuoco (1998), Luttmer (1999)), for a study of single sector exchange economies, or Lustig and Van Nieuwerburgh (2005) for a two-sector exchange economy model.

silent about the role of risk premia in the economy. Campbell and Hercowitz (2006) also study the effects of changing collateral constraints in a general equilibrium model that combines collateralized household debt with heterogeneity of time preference as an explanation for the "Great Moderation" in macroeconomic volatility. This model contains aggregate risk but the only security traded is one-period collateralized debt, thus this setup is also silent on the role of risk premia in aggregate fluctuations. The importance of aggregate risk and fluctuating risk premia is the central focus of our paper and its main theoretical contribution. To the best of our knowledge, this paper is the first to investigate the role of time-varying risk premia as a primary channel for generating and propagating fluctuations in housing and equity markets, aggregate quantities, and risk-sharing in a general equilibrium macro-production model of housing and non-housing output.

Outside of the incomplete markets environment, a strand of the macroeconomic literature studies housing behavior in a two-sector, general equilibrium business cycle framework either with production (e.g., Davis and Heathcote (2005), Kahn (2008)) or without production (e.g., Piazzesi, Schneider, and Tuzel (2007)). The focus in these papers is on environments with complete markets for idiosyncratic risks and a representative agent representation. Kahn (2008) studies long-term trends in house prices and output in a two-sector representative agent production economy and shows that such movements can generate house prices that are substantially more volatile than output. But this model abstracts from heterogeneity and financial frictions, both of which lie at the heart of movements in risk premia in our framework. We argue here that fluctuations in housing risk premia are essential for understanding the large observed boom-bust patterns in aggregate price-rent ratios, which cannot be readily attributed empirically to either sharp swings in expected rental growth rates or expected real interest rates. All of these representative agent models are silent on questions involving how housing wealth is affects and is affected by risk-sharing, inequality, and age and income heterogeneity.

It is important to note that our paper does not address the question of *why* credit market conditions changed so markedly in recent decades (we discuss this in the conclusion). It is widely understood that the financial market liberalization we study was preceded by a number of revolutionary changes in housing finance, notably by the rise in securitization. These changes initially decreased the risk of individual home mortgages and home equity loans, allowing for a more efficient allocation of risk and, some have argued, making it optimal for lending contracts to feature lower collateral requirements and housing transactions fees (e.g. Green and Wachter (2008); Piskorski and Tchistyi (2011); Strongin, O'Neill, Himmelberg, Hindian, and Lawson (2009)). As these researchers note, however, these initially risk-reducing changes in housing finance were accompanied by government deregulation of financial institutions that ultimately increased risk, by permitting such institutions to alter the composition of their assets towards more high-risk securities, by permitting higher leverage ratios, and by presiding over the spread of complex financial holding companies that replaced the long-standing separation between investment bank, commercial bank and insurance company. Industry analysis suggests that the market's subsequent revised expectation upward of the riskiness of the underlying mortgage assets since 2007 has led to a reversal in collateral requirements and transactions fees. It is precisely these changes in credit conditions that are the focus of this study.

2 Changes in Housing Finance

A detailed documentation of changes in the three key aspects of housing finance we study, collateral constraints, housing transactions costs, and foreign capital in U.S. debt markets, is given in the Appendix. Here we summarize this evidence as follows. There was a widespread relaxation of underwriting standards in the U.S. mortgage market during the period leading up to the credit crisis of 2007. By the end of 2006, households routinely bought homes with 100% financing using a piggyback second mortgage or home equity loan. Industry analysts indicate that maximum loan-to-value (LTV) ratios for combined (first and second) mortgages have since returned to more normal levels of no greater than 75-80% of the appraised value of the home. There was also a significant decline in transactions costs for buying homes and for home equity extraction: pecuniary costs (such as mortgage and home equity closing costs) fell by up to 90%, but non-pecuniary costs also declined. In the aftermath of the credit crisis, these costs have increased. Favilukis, Kohn, Ludvigson, and Van Nieuwerburgh (2013) provide an extensive discussion of the evidence for these changes.

The period was also characterized by a secular decline in real interest rates that coincided with a surge in foreign ownership of U.S. Treasury and Agency securities. The real annual interest rate on the 10-year Treasury bond fell from 3.87% at the start of 2000 to 2.04% by the end of 2006, while the 10-year Treasury Inflation Protected (TIPS) rate fell from 4.32% to 2.25% over this period. Real rates fell further to all time lows during the housing bust. The real 10-year Treasury bond rate declined from 2.04% to -0.04% from the end of 2006 to end of 2012, while the TIPS rate declined from 2.25% to -0.76%. At the same time, foreign ownership of U.S. Treasuries (T-bonds and T-notes) increased from 13.5% of marketable Treasuries outstanding in 1984 to 61% of marketable Treasuries by 2008. By June 2012, foreign holdings represented 52.5% of marketable Treasuries, driven by a large increase in foreign purchases between 2008 and 2012 and an even larger increase in the supply of marketable Treasuries. But foreign holdings of long-term and short-term U.S. Treasury and Agency debt as a fraction of GDP continued to increase in the 2008 to 2012 period, from 31% to 40.6% of GDP by 2012. By pushing real interest rates lower, the rise in foreign capital has been directly linked to the surge in mortgage originations over this period (e.g., Strongin, O'Neill, Himmelberg, Hindian, and Lawson (2009)). Economic policymakers, such as Federal Reserve Chairman Ben Bernanke, have also emphasized the role of foreign capital in driving interest rates lower and in fueling house price inflation.[4]

It is important to emphasize that, while foreign ownership of U.S. Treasuries surged from 2000-2007, there was no corresponding increase in Treasury *supply* over this period. The fraction of marketable Treasuries relative to GDP was stable between 1999 and 2007 at around 30%.

We consider one specification of the model in which we introduce foreign demand for domestic bonds into the market clearing condition, referred to hereafter as *foreign capital*. This foreign capital is modeled as owned by governmental holders who place all of their funds in domestic riskless bonds. We do this for two reasons. First, by the end of 2008, Foreign Official Institutions (FOI) held 70% of all foreign holdings of U.S. Treasuries. Moreover, as explained in Kohn (2002), government entities have specific regulatory and reserve currency motives for holding U.S. Treasuries and face both legal and political restrictions on the type of assets that can be held, forcing them into safe securities. As of June 2010, the bond market portfolio composition of FOI consists of U.S. Treasuries (78%) and Agency mortgage backed securities (MBS) and U.S. Agency debt (19.5%). They hold only a tiny position in risky corporate debt of any kind (2.5%). Second, Krishnamurthy and Vissing-Jorgensen (2007) show that demand for U.S. Treasury securities by governmental holders is completely inelastic, implying that when these holders receive funds to invest they buy safe U.S. securities such as Treasuries or Agencies, regardless of their price relative to other U.S. assets. U.S. Agency MBS and U.S. Agency debt are pools of conforming mortgages, guaranteed by the Government Sponsored Enterprises (GSEs), and the corporate bonds the GSEs issue to finance their portfolio investment (mostly in Agency MBS), respectively. The safe mort-

[4] For example, see remarks by then Governor Ben S. Bernanke at the Sandridge Lecture, Virginia Association of Economics, Richmond, Virginia, March 10, 2005, and by Chairman Bernanke, at the International Monetary Conference, Barcelona, Spain (via satellite), June 3, 2008.

gages our model features–we abstract from default risk–resemble well the Agency MBS in the real world, in practice treated as equivalent to Treasuries. The equivalence of Treasury and Agency securities was made formal by the Conservatorship of Fannie Mae and Freddie Mac in September 2008. In summary, evidence suggests that foreign governmental holders have very deep pockets and will pay whatever price necessary to push non-governmental holders out of the safe U.S. bond market when their demand is not met with an equal increase in supply.

3 The Model

3.1 Firms

The production side of the economy consists of two sectors. One sector produces the non-housing consumption good, and the other sector produces the housing good. We refer to the first as the "consumption sector" and the second as the "housing sector." Time is discrete and each period corresponds to a year. In each period, a representative firm in each sector chooses labor (which it rents) and investment in capital (which it owns) to maximize the value of the firm to its owners.

3.1.1 Consumption Sector

Denote output in the consumption sector as

$$Y_{C,t} \equiv K_{C,t}^{\alpha} \left(Z_{C,t} N_{C,t}\right)^{1-\alpha} \tag{1}$$

where $Z_{C,t}$ is the stochastic productivity level at time t, K_C is the capital stock in the consumption sector, α is the share of capital, and N_C is the quantity of labor input in the consumption sector. Let I_C denote investment in the consumption sector. The firm's capital stock $K_{C,t}$ accumulates over time subject to proportional, quadratic adjustment costs, $\varphi\left(\frac{I_{C,t}}{K_{C,t}} - \delta\right)^2 K_{C,t}$, modeled as a deduction from the earnings of the firm. The dividends to shareholders are equal to

$$D_{C,t} = Y_{C,t} - w_t N_{C,t} - I_{C,t} - \varphi\left(\frac{I_{C,t}}{K_{C,t}} - \delta\right)^2 K_{C,t},$$

where w_t is the wage rate (equal across sectors in equilibrium). The firm maximizes the present discounted value $V_{C,t}$ of a stream of earnings:

$$V_{C,t} = \max_{N_{C,t}, I_{C,t}} E_t \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} D_{C,t+k}, \tag{2}$$

where $\frac{\beta^k \Lambda_{t+k}}{\Lambda_t}$ is a stochastic discount factor discussed below. The evolution equation for the firm's capital stock is

$$K_{C,t+1} = (1-\delta) K_{C,t} + I_{C,t},$$

where δ is the depreciation rate of the capital stock.

The firm does not issue new shares and finances its capital stock entirely through retained earnings.

3.1.2 Housing Sector

The housing firm's problem is analogous to the problem solved by the representative firm in the consumption sector, except that housing production utilizes an additional fixed factor of production, $\mathcal{L}_t$, representing a combination of land and government permits for residential construction.[5] Denote output in the residential housing sector as

$$Y_{H,t} = (Z_{H,t} \mathcal{L}_t)^{1-\phi} \left(K_{H,t}^{\nu} Z_{H,t}^{1-\nu} N_{H,t}^{1-\nu} \right)^{\phi}, \tag{3}$$

$Y_{H,t}$ represents construction of new housing (residential investment), $1-\phi$ is the share of land/permits in housing production, and ν is the share of capital in the construction component $\left(K_{H,t}^{\nu} Z_{H,t}^{1-\nu} N_{H,t}^{1-\nu} \right)$ of housing production. Variables denoted with an "H" subscript are defined exactly as above for the consumption sector, but now pertain to the housing sector, e.g., $Z_{H,t}$ denotes the stochastic productivity level in the housing sector.

Following Davis and Heathcote (2005), we assume that a constant quantity L of new land/permits suitable for residential development is available each period. We assume that this constant supply L of land/permits is made available for residential construction by the government who rents the land/permits to home developers at the competitive rental rate equal to the marginal product of $\mathcal{L}_t$.[6] The proceeds from land rentals are used by the government to finance (wasteful) government spending G_t. When a house is sold, the government issues a transferable lease for the land/permits in perpetuity at no charge to the homeowner. The assumption is that the buyer of the home is the effective owner, even though (by eminent domain) the government retains the legal right to the land/permits.

The dividends to shareholders in the housing sector are denoted

$$D_{H,t} = p_t^H Y_{H,t} - p_t^L \mathcal{L}_t - w_t N_{H,t} - I_{H,t} - \varphi \left(\frac{I_{H,t}}{K_{H,t}} - \delta \right)^2 K_{H,t},$$

[5] Glaeser, Gyourko, and Saks (2005) argue that the increasing value of land for residential development is tied to government-issued construction permits, rather than to the acreage itself.

[6] It is important to note that it is the *flow* of land/permits that is presumed to be a fixed, constant amount each period, not the stock of these.

where p_t^H is the relative price of one unit of housing in units of the non-housing consumption good and p_t^L is the price of land/permits. Note that p_t^H is the time t price of a unit of housing of fixed quality and quantity.

The housing firm maximizes

$$V_{H,t} = \max_{N_{H,t}, I_{H,t}} E_t \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} \left(D_{H,t+k} \right), \tag{4}$$

Capital in the housing sector evolves:

$$K_{H,t+1} = (1-\delta) K_{H,t} + I_{H,t}.$$

Note that $Y_{H,t}$ represents residential construction; thus the law of motion for the aggregate residential housing stock H_t is

$$H_{t+1} = (1-\delta_H) H_t + Y_{H,t},$$

where δ_H denotes the depreciation rate of the housing stock.

The shocks $Z_{C,t}$ and $Z_{H,t}$ are sources of aggregate risk in the economy. The presence of aggregate risk (correlated countercyclically with idiosyncratic risk) is crucial for generating risk premia in housing and equity markets that are not only high on average, but also significantly time-varying (Mankiw (1986); Krueger and Lustig (2010)). $Z_{C,t}$ and $Z_{H,t}$ are calibrated to follow two-state Markov chain, as described in the Appendix. In addition, with $Z_{C,t}$ labor augmenting, and $Z_{H,t}$ labor and land augmenting, as written in (1) and (3), we may allow for balanced (deterministic) growth in each productivity level in an economy where land/permits L and labor supply N are non-growing. Under this assumption, the price of land grows deterministically at the same rate as technology and the rest of the aggregate economy.[7]

3.2 Risky Asset Returns

The firms' values $V_{H,t}$ and $V_{C,t}$ are the *cum* dividend values, measured before the dividend is paid out. The *cum* dividend returns to shareholders in the housing sector and the consumption sector are defined, respectively, as

$$R_{Y_H,t+1} = \frac{V_{H,t+1}}{(V_{H,t} - D_{H,t})} \qquad R_{Y_C,t+1} = \frac{V_{C,t+1}}{(V_{C,t} - D_{C,t})}.$$

[7] This assumption is essentially the same as the one made in Davis and Heathcote (2005), where land in their model was presumed to grow at the same rate as the population. Our model has no population growth, so the analogous assumption is that land is not growing.

We define $V^e_{j,t} = V_{j,t} - D_{j,t}$ for $j = H, C$ to be the *ex* dividend value of the firm.[8]

3.3 Individuals

The economy is populated by A overlapping generations of individuals, indexed by $a = 1, ..., A$, with a continuum of individuals born each period. There are two types of individuals: A small minority are *bequesters* (those who have a bequest motive in their value functions), while the others are *non-bequesters* (those who do not have a bequest motive); each will be described below. Individuals live through two stages of life, a working stage and a retirement stage. Adult age begins at age 21, so a equals this effective age minus 20. Agents live for a maximum of $A = 80$ (100 years). Workers live from age 21 ($a = 1$) to 65 ($a = 45$) and then retire. Retired workers die with an age-dependent probability calibrated from life expectancy data. The probability that an agent is alive at age $a + 1$ conditional on being alive at age a is denoted $\pi_{a+1|a}$. Upon death, any remaining net worth of an individual is transferred to a newborn who replaces her. Non-bequesters leave only accidental bequests, while bequesters leave deliberate bequests. (In practice, accidental bequests are unintentional and will therefore be quite small.) We assume that newborns who receive a deliberate bequest are themselves born into the world with a bequest motive, while those who receive only accidental bequests have no bequest motive. Thus bequesters form dynasties and the fraction of each type in the economy remains constant over time.[9]

Both bequesters and non-bequester individuals have an intraperiod utility function given by

$$U(C^i_{a,t}, H^i_{a,t}) = \frac{\widetilde{C}_{a,t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \qquad \widetilde{C}_{a,t} = \left(C^i_{a,t}\right)^{\chi} \left(H^i_{a,t}\right)^{1-\chi},$$

where $C_{a,t}$ is non-housing consumption of an individual of age a, and $H_{a,t}$ is the stock of housing, σ^{-1} is the coefficient of relative risk aversion, χ is the share of non-housing consumption in utility. Implicit in this specification is the assumption that the service flow from houses is proportional to the stock $H_{a,t}$. The distinction between bequesters and non-bequesters is that the former receive additional utility from their net worth holdings at the

[8] Using the *ex* dividend value of the firm the return reduces to the more familiar *ex* dividend definition: $R^e_{j,t+1} = \frac{V^e_{j,t+1}+D_{j,t+1}}{V^e_{j,t}}$.

[9] The collateral constraint (Equation 11 below) implies that net worth is non-negative. If household i dies in period t then his net worth at death is equivalent to the amount inherited by the newborn who replaces the dead individual. We allow the newborn to make an optimal portfolio choice for how this wealth is allocated in the first period of life.

time of death. This additional utility appears in the value function, described below.

Non-bequesters maximize the value function

$$\begin{aligned} V_a(\mu_t, Z_t, Z^i_{a,t}, W^i_{a,t}, H^i_{a,t}) &= \max_{H^i_{a+1,t+1}, \theta^i_{a+1,t+1} B^i_{a+1,t+1}} \{U(C^i_{a,t}, H^i_{a,t}) \\ &\quad + \beta \pi_{a+1|a} E_t[V_{a+1}(\mu_{t+1}, Z_{t+1}, Z^i_{a,t+1}, W^i_{a+1,t+1}, H^i_{a+1,t+1})]\}. \end{aligned} \tag{5}$$

Bequesters maximize an alternative value function taking the form

$$\begin{aligned} V_a(\mu_t, Z_t, Z^i_{a,t}, W^i_{a,t}, H^i_{a,t}) &= \max_{H^i_{a+1,t+1}, \theta^i_{a+1,t+1} B^i_{a+1,t+1}} \{U(C^i_{a,t}, H^i_{a,t}) \\ &\quad + \beta \pi_{a+1|a} E_t[V_{a+1}(\mu_{t+1}, Z_{t+1}, Z^i_{a,t+1}, W^i_{a+1,t+1}, H^i_{a+1,t+1})]\} \\ &\quad + \beta \left(1 - \pi_{a+1|a}\right) E_t \left[\xi \frac{\left(W^i_{a+1,t+1} + p^H_{t+1} H^i_{a+1,t+1}\right)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} \right]. \end{aligned} \tag{6}$$

Recalling that $\pi_{a+1|a}$ is the probability of being alive next year given an individual is alive this year, equation (6) says that bequesters receive additional utility as a function of their net worth in the year that they die. The parameter ξ governs the strength of the bequest motive.

Financial market trade is limited to a one-period riskless bond and to risky capital, where the latter is restricted to be a mutual fund of equity in the housing and consumption sectors.[10] The mutual fund is a value-weighted portfolio with return

$$R_{K,t+1} = \frac{V^e_{H,t}}{V^e_{H,t} + V^e_{C,t}} R_{Y_H,t+1} + \frac{V^e_{C,t}}{V^e_{H,t} + V^e_{C,t}} R_{Y_C,t+1}. \tag{7}$$

The gross bond return is denoted $R_{f,t} = \frac{1}{q_{t-1}}$, where q_{t-1} is the bond price known at time $t-1$.

Individuals are heterogeneous in their labor productivity. To denote this heterogeneity, we index individuals i. Before retirement households supply labor inelastically. The stochastic process for individual income for workers is

$$Y^i_{a,t} = w_t L^i_{a,t},$$

where $L^i_{a,t}$ is the individual's labor endowment (hours times an individual-specific productivity factor), and w_t is the aggregate wage per unit of productivity. Labor productivity is

[10] Notice that this is a mutual fund that owns equity in the consumption producing firm and in the residential development firm (housing). It is *not* a mutual fund that owns the residential housing stock.

specified by a deterministic age-specific profile, G_a, and an individual shock Z_t^i:

$$\begin{aligned} L_{a,t}^i &= G_a Z_t^i \\ \log\left(Z_t^i\right) &= \log\left(Z_{t-1}^i\right) + \epsilon_t^i, \qquad \epsilon_t^i \sim i.i.d.\left(0, \sigma_t^2\right), \end{aligned}$$

where G_a is a deterministic function of age capturing a hump-shaped profile in life-cycle earnings and $\epsilon_{a,t}^i$ is a stochastic i.i.d. shock to individual earnings. To capture countercyclical variation in idiosyncratic risk of the type documented by Storesletten, Telmer, and Yaron (2004), we use a two-state specification for the variance of idiosyncratic earnings shocks:

$$\sigma_t^2 = \begin{cases} \sigma_E^2 & \text{if } Z_{C,t} \geq E\left(Z_{C,t}\right) \\ \sigma_R^2 & \text{if } Z_{C,t} < E\left(Z_{C,t}\right) \end{cases}, \qquad \sigma_R^2 > \sigma_E^2 \tag{8}$$

This specification implies that the variance of idiosyncratic labor earnings is higher in "recessions" ($Z_{C,t} \leq E\left(Z_{C,t}\right)$) than in "expansions" ($Z_{C,t} \geq E\left(Z_{C,t}\right)$). The former is denoted with an "R" subscript, the latter with an "E" subscript. The counter-cyclical increase in income dispersion is an important contributor to the equity risk premium in our model (see Krueger and Lustig (2010)). Finally, labor earnings are taxed at rate τ in order to finance social security retirement income.

At age a, agents enter the period with wealth invested in bonds, B_a^i, and shares θ_a^i of risky capital. The total number of shares outstanding of the risky asset is normalized to unity. We rule out short-sales in the risky asset,

$$\theta_{a,t}^i \geq 0. \tag{9}$$

An individual who chooses to invest in the mutual fund pays a fixed, per-period participation cost, $F_{K,t}$, following evidence in Vissing-Jorgensen (2002).

We assume that the housing owned by each individual requires maintenance expenses $p_t^H H_{a,t}^i \delta_H$, where δ_H is the rate of depreciation of the aggregate housing stock. At time t, households may choose to change the quantity of housing consumed at time $t+1$ by selling their current house for $p_t^H H_{a,t}^i$ and buying a new house for $p_t^H H_{a,t+1}^i$. Because houses are illiquid, it is expensive to change housing consumption. An individual who chooses to change housing consumption pays a transaction cost $F_{H,t}^i$. These costs contain a fixed component and a variable component proportional to the value of the house. These costs encompass any expense associated with changing housing consumption regardless of how it is financed, e.g., moving costs (both pecuniary and non-pecuniary).

An important time-varying component of the transactions cost in illiquid housing (one that varied significantly over the boom-bust episode) is the cost directly associated with housing finance, specifically the borrowing costs incurred for loans backed by housing collateral. We use direct evidence to calibrate a transactions cost per dollar borrowed, given by $F^i_{B,t} = \lambda |B^i_{a+1,t+1}|$, whenever $B^i_{a+1,t+1} < 0$, which represents a borrowing position in the risk-free asset. The parameter λ controls the magnitude of these borrowing costs as a fraction of the amount borrowed.

Denote the sum of the per period equity participation cost, housing transaction cost and borrowing costs for individual i as

$$F^i_t \equiv F_{K,t} + F^i_{H,t} + F_{B,t}.$$

Define the individual's gross financial wealth at time t as

$$W^i_{a,t} \equiv \theta^i_{a,t} \left(V^e_{C,t} + V^e_{H,t} + D_{C,t} + D_{H,t} \right) + B^i_{a,t}.$$

The budget constraint for an agent of age a who is not retired is

$$\begin{aligned} C^i_{a,t} + p^H_t \delta_H H^i_{a,t} + B^i_{a+1,t+1} q_t + \theta^i_{a+1,t+1} \left(V^e_{C,t} + V^e_{H,t} \right) \leq \; & W^i_{a,t} + (1-\tau) w_t L^i_{a,t} \\ & + p^H_t \left(H^i_{a,t} - H^i_{a+1,t+1} \right) - F^i_t \end{aligned} \tag{10}$$

where τ is a social security tax rate and

$$\begin{aligned} F^i_{H,t} &= \begin{cases} 0, & H^i_{a+1,t+1} = H^i_{a,t} \\ \psi_0 + \psi_1 p^H_t H^i_{a,t}, & H^i_{a+1,t+1} \neq H^i_{a,t} \end{cases}. \\ F_{K,t} &= \begin{cases} 0 & \text{if } \theta^i_{a+1,t+1} = 0 \\ \overline{F} & \text{if } \theta^i_{a+1,t+1} > 0 \end{cases}. \\ F^i_{B,t} &= \begin{cases} 0 & \text{if } B^i_{a+1,t+1} > 0 \\ \lambda \cdot |B^i_{a+1,t+1}| & \text{if } B^i_{a+1,t+1} < 0 \end{cases}. \end{aligned}$$

F_B is a cost that implies borrowers pay a higher interest rate than lenders receive. $F^i_{H,t}$ is the housing transactions cost which contains both a fixed and variable component and depends on age only through $H^i_{a,t}$. Equation (10) says that the amount spent on non-housing consumption, on housing maintenance, and on bond and equity purchases must be less than or equal to the sum of the individual's gross financial wealth and after-tax labor income, less the cost of purchasing any additional housing, less all asset market transactions costs.

An additional important constraint in the model is

$$-B^i_{a+1,t+1} \leq (1 - \varpi) \, p^H_t H^i_{a,t+1}, \qquad \forall a, t \tag{11}$$

Equation (11) is the collateral constraint, where $0 \leq \varpi \leq 1$. It says that households may borrow no more than a fraction $(1-\varpi)$ of the value of housing, implying that they must post collateral equal to a fraction ϖ of the value of the house. This constraint can be thought of as a down-payment constraint for new home purchases, but it also applies to any borrowing against home equity, not just to first lien mortgages. It should be emphasized that $1-\varpi$ gives the *maximum* combined (first and additional mortgages) LTV ratio. This will differ from the *average* LTV ratio because not everyone borrows up to the credit limit. Notice that if the price p_t^H of the house rises and nothing else changes, the individual can finance a greater level of consumption of both housing and nonhousing goods and services.[11]

Let Z_{ar}^i denote the value of the stochastic component of individual labor productivity, $Z_{a,t}^i$, during the last year of working life. Each period, retired workers receive a government pension $PE_{a,t}^i = Z_{ar}^i X_t$ where $X_t = \tau \frac{N^W}{N^R}$ is the pension determined by a pay-as-you-go system, and N^W and N^R are the numbers of working age and retired households.[12] For agents who have reached retirement age, the budget constraint is identical to that for workers (10) except that wage income $(1-\tau) w_t L_{a,t}^i$ is replaced by pension income $PE_{a,t}^i$.

Let $Z_t \equiv (Z_{C,t}, Z_{H,t})'$ denote the aggregate shocks. The state of the economy is a pair, (Z, μ), where μ is a measure defined over $\mathcal{S} = (\mathcal{A} \times \mathcal{Z} \times \mathcal{W} \times \mathcal{H})$, where $\mathcal{A} = \{1, 2, ...A\}$ is the set of ages, where $\mathcal{Z}$ is the set of all possible idiosyncratic shocks, where $\mathcal{W}$ is the set of all possible beginning-of-period financial wealth realizations, and where $\mathcal{H}$ is the set of all possible beginning-of-period housing wealth realizations. That is, μ is a distribution of agents across ages, idiosyncratic shocks, financial and housing wealth. The presence of aggregate shocks implies that μ evolves stochastically over time. We specify a law of motion, Γ, for μ,

$$\mu_{t+1} = \Gamma(\mu_t, Z_t, Z_{t+1}).$$

[11] Borrowing takes place using one-period debt. Thus, an individual's borrowing capacity fluctuates period-by-period with the value of the house. Having long-term fixed rate debt is more realistic, but computationally intractable in our setup.

[12] The decomposition of the population into workers and retirees is determined from life-expectancy tables as follows. Let X denote the total number of people born each period. (In practice this is calibrated to be a large number in order to approximate a continuum.) Then $N^W = 45 \cdot X$ is the total number of workers. Next, from life expectancy tables, if the probability of dying at age $a > 45$ is denoted p_a then $N^R = \sum_{a=46}^{80} (1-p_a) X$ is the total number of retired persons.

3.4 Stochastic Discount Factor

The stochastic discount factor (SDF), $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$, appears in the dynamic value maximization problem (2) and (4) undertaken by each representative firm. As a consequence of our incomplete markets setting, a question arises about how to model $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$. For example, the intertemporal marginal rates of substitution (MRS) of any shareholder in this setting is a valid stochastic discount factor. Much of the existing literature has avoided this ambiguity by assuming that firms rent capital from households on a period-by-period basis, thereby solving a series of static optimization problems. Since the problem is static, the question of discounting is then mute. In this static case, however, one needs to impose some other form of exogenous shock, for example stochastic depreciation in the rented capital stocks (e.g., Storesletten, Telmer, and Yaron (2007), Gomes and Michaelides (2008)), in order to make the volatility of the equity return realistic. Here we instead keep depreciation deterministic and model dynamic firms that own capital and face adjustment costs when changing their capital stocks, requiring us to take a stand on the SDF. We do this for several reasons. First, in our own experimentation we found that the amount of stochastic depreciation required to achieve reasonable levels of stock market volatility produced excessive volatility in investment. Second, it is difficult to know what amount of stochastic depreciation, if any, is reasonable. Third, an economy populated entirely of static firms is unrealistic. In the real world, firms own their own capital stocks and must think dynamically about shareholder value.

For these reasons, we assume that the representative firm in each sector solves the dynamic problem presented above and discount future profits using a weighted average of the individual shareholders' intertemporal marginal rate of substitution implied by the first-order condition for optimal consumption choice, where the weights, $\theta^i_{a,t}$, correspond to the shareholder's proportional ownership in the firm. Let $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$ denote this weighted average. For non-bequesters, the marginal rate of substitution is simply the MRS in non-housing consumption,

$$\begin{aligned}\frac{\beta\partial U/\partial C^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}} &= \beta\left(\frac{C^i_{a+1,t+1}}{C^i_{a,t}}\right)^{-\frac{1}{\sigma}}\left[\frac{\left(\frac{H^i_{a+1,t+1}}{C^i_{a+1,t+1}}\right)^{(1-\chi)\left(1-\frac{1}{\sigma}\right)}}{\left(\frac{H^i_{a,t}}{C^i_{a,t}}\right)^{(1-\chi)\left(1-\frac{1}{\sigma}\right)}}\right] \\ &= \frac{\beta\partial V_{a+1}\left(\cdot\right)/\partial W^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}},\end{aligned}$$

where $V_{a+1}\left(\cdot\right) \equiv V_a(\mu_t, Z_t, Z^i_{a,t}, W^i_{a,t}, H^i_{a,t})$, and the last equality above follows from the envelope theorem. For bequesters, there is additional randomness in the marginal rate of substitution created by the probability of death. Thus we write the bequesters MRS as

$$\frac{\beta \partial V_{a+1}\left(\cdot\right)/\partial W^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}} = \begin{cases} \frac{\beta \partial U/\partial C^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}} & \text{with } prob = \pi_{a+1|a} \\ \frac{\beta E_t\left[\xi\left(NW^i_{a+1,t+1}\right)^{-\frac{1}{\sigma}}\right]}{\partial U/\partial C^i_{a,t}} & \text{with } prob = 1-\pi_{a+1|a} \end{cases}.$$

Recalling that the total number of shares in the risky portfolio is normalized to unity, we therefore model the stochastic discount factor as

$$\frac{\beta \Lambda_{t+1}}{\Lambda_t} \equiv \int_{\mathcal{S}} \theta^i_{a+1,t+1} \frac{\beta \partial V_{a+1}\left(\cdot\right)/\partial W^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}} d\mu, \tag{12}$$

where $\frac{\beta \partial V_{a+1}/\partial W^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}}$ takes the appropriate value for each individual, as described above.

Since we weight each individual's MRS by its proportional ownership (and since short-sales in the risky asset are prohibited), only those households who have taken a positive position in the risky asset (shareholders) will receive non-zero weight in the SDF.

Although this specification of the stochastic discount factor leads to an equilibrium that depends on the control of the firm being fixed according to the proportional ownership structure described above, it is not necessarily quantitatively sensitive to this assumption on ownership control. For example, Carceles Poveda and Coen-Pirani (2009) show that, given the firm's objective of value maximization, the equilibrium allocations in their incomplete markets models are invariant to the choice of stochastic discount factor within the set that includes the MRS of any household (or any weighted average of these) for whom the Euler equation for the risky asset return is satisfied. They show in addition that the equilibrium allocations of such economies are the same as the allocations obtained in otherwise identical economies with "static" firms that rent capital from households on a period-by-period basis. They also prove this for a case with adjustment costs.[13] We have checked that our results are not affected by the following variants of the SDF above: (i) equally weighting the MRS of shareholders (gives proportionally more weight to small stakeholders), (ii) weighting the MRS of shareholders by the squares of their ownership stakes, $\left(\theta^i_{a+1,t+1}\right)^2$, (gives proportionally more weight to big stakeholders), (iii) using the MRS of the largest shareholder. This completes the description of the model economy. We now turn to the definition of housing and equity returns.

[13] Our calibration of adjustment costs implies that they quantitatively small, amounting to less than one percent of investment per year.

3.5 Housing and Equity Returns

To understand the model's implications for housing returns, note that the first-order condition for optimal housing choice with non-binding borrowing constraints and no transactions costs is

$$U_{C^i_{a,t}} = \frac{1}{p^H_t}\beta E_t\left[U_{C^i_{a+1,t+1}}\left(\frac{U_{H^i_{a+1,t+1}}}{U_{C^i_{a+1,t+1}}} + p^H_{t+1}\left(1-\delta_H\right)\right)\right], \tag{13}$$

where the partial derivative $\frac{\partial U}{\partial C^i_{a,t}}$ is written $U_{C^i_{a,t}}$, and analogously for $U_{C^i_{a+1,t+1}}$ and $U_{H^i_{a+1,t+1}}$. Each individual's housing return is given by $\left(U_{H^i_{a+1,t+1}}/U_{C^i_{a+1,t+1}} + p^H_{t+1}\left(1-\delta_H\right)\right)/p^H_t$ where $U_{H^i_{a+1,t+1}}/U_{C^i_{a+1,t+1}}$ is a measure of fundamental value, the service flow value generated by the housing asset.[14] In a competitive equilibrium, $U_{H^i_{a+1,t+1}}/U_{C^i_{a+1,t+1}}$ is equal to the relative price of housing services. For brevity, we refer to this quantity hereafter as "rent," but it should be kept in mind that it is actually a measure of the flow dividend from the housing asset for owner-occupied housing.[15]

To obtain the model's implications for a national housing return, computed from an aggregate house price index combined with an aggregate housing service flow index (also quantities readily observable in aggregate data), we form an aggregate (across households) measure of the individual housing service flows and refer to it as "national rent," denoted $\mathcal{R}_{t+1}$. In the model, p^H_t is the price of a unit of housing stock, which holds fixed the composition of housing (quality, square footage, etc.) over time. It is the same for everyone, thus it the model-based national house price index, akin to a repeat-sale index in the data. We combine $\mathcal{R}_{t+1}$ with the national house-price index p^H_{t+1} to compute a corresponding

[14] Binding borrowing constraints change (13) from an equality to an inequality and transactions costs add additional terms to the price term $p^H_{t+1}\left(1-\delta_H\right)$, but neither of these change the definition of the housing service flow or the definition of the individual return.

[15] The addition of an explicit rental market would make the numerical solution intractable, given the existing complexity. It is however unclear whether the presence of a rental market would dampen or amplify the endogenous boom-bust in our defined aggregate price-rent ratio, and the results are likely to depend on the modeling details. For example, during the boom, young renters would want to become owners, which would increase the demand for owned housing and decrease the demand for rented housing. Absent immediate adjustment of the total housing stock and with costly conversion of houses from rental to ownership type, the relative demand shift would induce the price-rent ratio to increase, possibly beyond that in the benchmark model. Similarly, the reversal of the FML would create a desire for owners to rent, but the fixed total housing supply, the relative scarcity of rental housing inherited from the boom years, and costly conversion all could prompt a larger fall in the national price-rent ratio than in the benchmark model, at least initially. Fully understanding the effect of a rental market in the model economy is interesting, but would substantially complicate the numerical computation and is left for future research.

national housing index return $R_{H,t+1}$:

$$R_{H,t+1} \equiv \frac{p_{t+1}^{H}\left(1-\delta_{H}\right)+\mathcal{R}_{t+1}}{p_{t}^{H}}, \tag{14}$$

$$\mathcal{R}_{t+1} \equiv \int_{\mathcal{S}} \frac{U_{H_{a+1,t+1}^{i}}}{U_{C_{a+1,t+1}^{i}}} d\mu. \tag{15}$$

We refer to $p_{t+1}^{H}/\mathcal{R}_{t+1}$, as the national "price-rent" ratio for brevity. We also compute the standard deviation of the return on the housing index return (14), $Std\left[R_{H,t+1}\right]$, and the ratio of the time-series mean excess return on this index, divided by the standard deviation,

$$SR\left[R_{H}\right] \equiv \frac{E\left[R_{H,t+1}-R_{f,t+1}\right]}{Std\left[R_{H,t+1}-R_{f,t+1}\right]}.$$

This latter quantity is denoted "$SR\left[R_{H}\right]$" to recall the familiar "Sharpe Ratio" concept in finance, but it should be emphasized that the corresponding measure here does not represent an actual risk-return tradeoff that a household could earn, because it ignores the effects of housing transactions costs and binding borrowing constraints.[16] These measures are, however, comparable to analogous objects constructed from aggregate data using national house price indexes and national rent or housing service flow indexes, as discussed below and at length in the Appendix.

We compare our model results with four different measures of single-family residential price-rent ratios and associated housing index returns: a measure based on the Flow of Funds, *FoF*, a measure based on the Freddie Mac Conventional Mortgage House Price index, *Freddie Mac*, and a measure based on the *Core Logic* house price index, *CL*. Each of these are combined with a measure of an aggregate housing service flow expenditure estimate or index to compute "rent" and a national house price-rent ratio and return on the national housing index. The Appendix details our construction of these variables from data. Aggregate rent is always some measure of the housing service flow. A complicating factor is that these measures are aggregates of both rent for renters and imputed rent for owner-occupiers. But census data show that two-thirds of housing is owner-occupied, so that most of what is in these measures is an imputed service flow for owner-occupiers. Moreover, the correlation between "rent of primary residence" for renters and owners equivalent rent is extremely high (94% between 2000 and 2012), so whether we compute the price-rent ratio for renters or for owners we get the same facts in terms of run-up, volatility, and comovements.

[16] In addition, in this case the statistic pertains to the return on an aggregate house price index, which is not a tradeable asset. Thus, what is denoted "SR" here is just another aggregate statistic that can be compared across model and data, not representative of a true risk-return tradeoff.

As explained in the Appendix, we do not attempt to match our model to the *levels* of the price-rent ratios, which are unidentified from the data, instead focusing on the changes in these ratios over time.

In addition to these statistics based on national home price and housing service flow aggregates, in our model we compute housing return statistics at the *individual* level. These individual statistics are denoted with an "N" subscript. This is done in the model by taking the time-series mean and standard deviation of the *individual housing return*, defined

$$R^i_{H,t+1} \equiv \left(\frac{U_{H^i_{a+1,t+1}}}{U_{C^i_{a+1,t+1}}} + p^H_{t+1} (1 - \delta_H) \right) / p^H_t . \tag{16}$$

The mean and standard deviation of the individual housing return for individual i are

$$R^i_H \equiv \frac{1}{T_H} \sum_{t=0}^{T_H} R^i_{H,t+1}, \tag{17}$$

$$Std\left[R^i_H\right] \equiv \sqrt{\frac{1}{T_H} \sum_{t=0}^{T_H} \left(R^i_{H,t+1} - R^i_H\right)^2} \tag{18}$$

where T_H is the number of years the household is alive. We then compute the ratio, for each i, of the mean of the individual housing return to the standard deviation of that return, $R^i_H / Std\left[R^i_H\right]$, and then take the average across households of these to report $SR_N\left[R_H\right]$

$$SR_N\left[R_H\right] \equiv \frac{1}{N} \sum_{i=1}^{N} \frac{R^i_H}{Std\left[R^i_H\right]}, \tag{19}$$

where N is the total number of households.[17] We analogously report the cross-sectional average of the mean and standard deviation of the individual housing returns, $E_N\left[R_H\right] \equiv \frac{1}{N}\sum_{i=1}^{N} E\left[R^i_H\right]$ and $Std_N\left[R_H\right] = \frac{1}{N}\sum_{i=1}^{N} Std\left[R^i_H\right]$.

Although $E_N\left[R_H\right]$ and $SR_N\left[R_H\right]$ are straightforward to compute in the model, corresponding historical data are far more limited and are not of the same quality as the data on national house price and rental indexes discussed above. Flavin and Yamashita (2002) report returns based on a relatively small sample of 1,817 Survey of Consumer Finance households for the period from 1968-1992. But the housing service flow is not measured in these data and so the authors are forced to make strong assumptions on the rental yield (a 5% constant rental yield is assumed for all households). The capital gain component is also

[17] As noted above, this measure, like $SR\left[R_H\right]$, does not represent an actual risk-return tradeoff a household could earn, since again it ignores the effects of housing transactions costs and binding borrowing constraints.

based on self-reported house value estimates, which are expected to have large measurement error. More recently, Landvoigt, Piazzesi, and Schneider (2013) obtain high quality housing transaction data at the individual level and measure capital gains in individual returns. But the sample is just for San Diego and only for the period 1999-2007. The service flow and rental yield are not part of these data. Lacking comprehensive, high quality national data on individual housing returns, we don't report any empirical counterparts to model-based statistics $E_N[R_H]$, $Std_N[R_H]$, and $SR_N[R_H]$, but we note that the findings in both of these papers suggest that individual housing returns contain a large idiosyncratic component that makes them more volatile than national house price indexes. We discuss this further below in the Robustness section when we add stochastic individual housing depreciation shocks to our benchmark model.

Returning to the model, the risky capital return $R_{K,t}$ in (7) is the return on a value-weighted portfolio of assets. This is not the same as the return on equity, which is a levered claim on the assets. To obtain an equity return, $R_{E,t}$, the return on assets, $R_{K,t}$, must be adjusted for leverage:

$$R_{E,t} \equiv R_{f,t} + (1 + B/E)(R_{K,t} - R_{f,t}),$$

where B/E is the fixed debt-equity ratio and where $R_{K,t}$ is the portfolio return for risky capital given in (7).[18] Note that this calculation explicitly assumes that corporate debt in the model is exogenous, and held in fixed proportion to the value of the firm. (There is no financing decision.) For the results reported below, we set $B/E = 2/3$ to match aggregate debt-equity ratios computed in Benninga and Protopapadakis (1990). As above we define the statistic $SR[R_E]$ as $E[R_{E,t+1} - R_{f,t+1}]/Std[R_{E,t+1} - R_{f,t+1}]$.

3.6 Equilibrium

An equilibrium is defined as a set of prices (bond prices, wages, risky asset returns) given by time-invariant functions $q_t = q(\mu_t, Z_t)$, $p_t^H = p^H(\mu_t, Z_t)$, $w_t = w(\mu_t, Z_t)$, and $R_{K,t} = R_K(\mu_t, Z_t)$, respectively, a set of cohort-specific value functions and decision rules for each individual i, $\left\{V_a, H^i_{a+1,t+1}, \theta^i_{a+1,t+1} B^i_{a+1,t+1}\right\}_{a=1}^{A}$ and a law of motion for μ, $\mu_{t+1} = \Gamma(\mu_t, Z_t, Z_{t+1})$ such that:

1. Households optimize. Non-bequesters maximize (5) subject to (10), (11), if the individual of working age, and subject to (11) and the analogous versions of (10) (using

[18]The cost of capital R_K is a portfolio weighted average of the return on debt R_f and the return on equity R_e: $R_K = aR_f + (1-a)R_e$, where $a \equiv \frac{B}{B+E}$.

pension income in place of wage income), if the individual is retired. Bequesters maximize (6) subject to (10), (11), if the individual of working age, and subject to (11) and the analogous versions of (10) (using pension income in place of wage income), if the individual is retired.

2. Firm's maximize value: $V_{C,t}$ satisfies (2), $V_{H,t}$ satisfies (4).

3. The price of land/permits p_t^L satisfies $p_t^L = (1-\phi)\, p_t^H Z_{H,t}^{1-\nu\phi} \mathcal{L}_t^{-\phi} \left(K_{H,t}^{\nu} N_{H,t}^{1-\nu}\right)^{\phi}$.

4. Land/permits supply equals land/permits demand: $L = \mathcal{L}_t$.

5. Wages $w_t = w(\mu_t, Z_t)$ satisfy

$$w_t = (1-\alpha)\, Z_{C,t}^{1-\alpha} K_{C,t}^{\alpha} N_{C,t}^{-\alpha} \tag{20}$$

$$w_t = (1-\nu)\, \phi p_t^H Z_{H,t}^{1-\nu\phi} \mathcal{L}_t^{1-\phi} K_{H,t}^{\nu\phi} N_{H,t}^{\phi(1-\nu)-1}. \tag{21}$$

6. The housing market clears: $p_t^H = p^H(\mu_t, Z_t)$ is such that

$$Y_{H,t} = \int_{\mathcal{S}} \left(H_{a,t+1}^i - H_{a,t}^i (1-\delta_H)\right) d\mu. \tag{22}$$

7. The bond market clears: $q_t = q(\mu_t, Z_t)$ is such that

$$\int_{\mathcal{S}} B_{a,t}^i d\mu + B_t^F = 0, \tag{23}$$

where $B_t^F \geq 0$ is an exogenous supply of foreign capital discussed below.

8. The risky asset market clears:

$$1 = \int_{\mathcal{S}} \theta_{a,t}^i d\mu. \tag{24}$$

9. The labor market clears:

$$N_t \equiv N_{C,t} + N_{H,t} = \int_{\mathcal{S}} L_{a,t}^i d\mu. \tag{25}$$

10. The social security tax rate is set so that total taxes equal total retirement benefits:

$$\tau N_t w_t = \int_{\mathcal{S}} PE_{a,t}^i d\mu, \tag{26}$$

11. Government revenue from land/permit rentals equals total government spending, G_t:

$$p_t^L \mathcal{L}_t = G_t$$

12. The presumed law of motion for the state space $\mu_{t+1} = \Gamma(\mu_t, Z_t, Z_{t+1})$ is consistent with individual behavior.

Equations (20), (21) and (25) determine the $N_{C,t}$ and therefore determine the allocation of labor across sectors:

$$(1-\alpha) Z_{C,t}^{1-\alpha} K_{C,t}^{\alpha} N_{C,t}^{-\alpha} = (1-\nu) \phi p_t^H Z_{H,t}^{1-\nu\phi} \mathcal{L}_t^{1-\phi} K_{H,t}^{\nu\phi} (N_t - N_{C,t})^{\phi(1-\nu)-1}. \tag{27}$$

Also, the aggregate resource constraint for the economy must take into account the housing and risky capital market transactions/participation costs and the wasteful government spending, which reduce consumption, the adjustment costs in productive capital, which reduce firm profits, and the change in net foreign capital in the bond market, which finances domestic consumption and investment. Thus, non-housing output equals non-housing consumption (inclusive of costs F_t) plus government spending plus aggregate investment (gross of adjustment costs) less the net change in the value of foreign capital:

$$\begin{aligned} Y_{C,t} = C_t + F_t + G_t + \left(I_{C,t} + \phi_C\left(\frac{I_{C,t}}{K_{C,t}}\right)K_{C,t}\right) + \left(I_{H,t} + \phi_H\left(\frac{I_{H,t}}{K_{H,t}}\right)K_{H,t}\right) \\ - \left(B_{t+1}^F q(\mu_t, Z_t) - B_t^F\right) \end{aligned} \tag{28}$$

where C_t and F_t are aggregate quantities defined as[19]

$$C_t \equiv \int_{\mathcal{S}} C_{a,t}^i d\mu \qquad F_t \equiv \int_{\mathcal{S}} F_t^i d\mu. \tag{29}$$

We seek a bounded-rationality equilibrium. The state space agents face is infinite dimensional. To solve the model, it is necessary to approximate the infinite dimensional object μ with a finite dimensional object. The appendix explains the solution procedure and how we specify a finite dimensional vector to represent the law of motion for μ. We also present the results of various numerical checks, designed to evaluate the degree of departure along specific dimensions from the fully rational equilibrium.

3.7 Model Calibration

The model's parameters and their numerical calibration are reported in Table 1. A detailed explanation of this calibration is given in the Appendix. The calibration corresponds to four alternative parameterizations.

[19] Note that (28) simply results from aggregating the budget constraints across all households, imposing all market clearing conditions, and using the definitions of dividends as equal to firm revenue minus costs.

Model 1 is our benchmark calibration, with "normal" collateral requirements and housing transactions costs calibrated to roughly match the data prior to the housing boom of 2000-2006. Model 1 has $\varpi = 0.25$ and borrowing costs λ set to match direct estimates of the percentage of amount borrowed lending costs in the year 2000, equal to $\lambda = 5.5\%$ of the amount borrowed. A detailed justification of this value is given in the Appendix section on Changes in Housing Finance.

Model 1B is identical to Model 1 except that lower borrowing transactions costs $\lambda = 3.5\%$ of the amount borrowed. This decline is calibrated to match evidence of a decline in costs from 2000-2006 (see the Appendix).

Model 2 is identical to Model 1B except that it has undergone a complete financial market liberalization, with lower borrowing transactions costs *and* lower collateral requirements. The Appendix also provides a detailed discussion of the evidence for changes in both collateral requirements and housing transactions costs. Based on this evidence, the down-payment declines from $\varpi = 25\%$ in Model 1 and Model 1B, to $\varpi = 1\%$ in Model 2. Borrowing costs remain $\lambda = 3.5\%$ as in Model 1B. Comparisons between Model 1B and Model 2 therefore isolate the effects of changing collateral constraints on the housing market.

In both Model 1 and Model 2, trade in the risk-free asset is entirely conducted between domestic residents: $B_t^F = 0$. The *Model 3* calibration is identical to that of Model 2 except that we add an exogenous foreign demand for the risk-free bond: $B_t^F > 0$ equal to 18% of average total output, $\overline{Y}$, an amount that is approximately equal to the rise in foreign ownership of U.S. Treasuries and agency debt over the period 2000-2008.

Finally, a *Model 4* calibration, studied below in a simulated transition, uses the collateral requirements and transactions costs of Model 1 ($\varpi = 25\%$ and $\lambda = 5.5\%$) but keeps the foreign flows $B_t^F > 0$ as in Model 3.

The share of land/permits in the housing production function is set to 10%, to match evidence used in Davis and Heathcote (2005), requiring $\phi = 0.9$. The technology shocks Z_C and Z_H are assumed to follow two-state independent Markov chains. Their calibration, as well as that of the idiosyncratic productivity shocks, is described in the Appendix.

4 Results

This section presents some of the model's main implications. Much of our analysis consists of a comparison of stochastic steady states across Models, 1, 2 and 3.[20] We also study a dynamic transition path for house prices and national price-rent ratios designed to mimic the state of the economy and housing market conditions over the period 2000-2009. We start by presenting a set of benchmark business cycle and life-cycle results.

4.1 Benchmark Results

4.1.1 Business Cycle Variables

Table 2 presents benchmark results for Hodrick-Prescott (Hodrick and Prescott (1997), HP) detrended aggregate quantities. Panel A of Table 2 presents business cycle moments from U.S. annual data over the period 1953 to 2012. Panels B through E of Table 2 presents simulated data to summarize the implications for these same moments for the Models 1, 1B, 2 and 3. We report statistics for total output, or $GDP \equiv Y_C + p^H Y_H + C_H$, for non-housing consumption (inclusive of expenditures on financial services), equal to $C + F$, for housing consumption $C_{H,t}$, defined as price per unit of housing services times quantity of housing or $C_{H,t} \equiv \mathcal{R}_t H_t$, for total (housing and non-housing) consumption $C_T = C + F + C_H$, for non-housing investment (inclusive of adjustment costs) I, for residential investment $p_t^H Y_{H,t}$ and for total investment $I_T = I + p^H Y_H$. The recorded statistics are very similar across all models, so we mainly discuss them with reference to Model 1.

Table 2 shows that, in both the model and the data, consumption is less volatile than GDP. The standard deviation of total aggregate consumption divided by the standard deviation of GDP is 0.85 in Model 1, comparable to the 0.633 value found in the data. The level of GDP volatility in the model is very close to that in the data. Total investment is more volatile than output, both in the model and in the data, but the model produces too little relative volatility: the ratio of the standard deviation of investment to that of output is 1.43 in Model 1 but is 2.9 in the data. One simple way to increase investment volatility in the model is to reduce adjustment costs for changing capital. Unfortunately, this drives equity market volatility to an unrealistically low level. This trade-off is a common problem in production-based asset pricing models (e.g., see Jermann (1998); Boldrin,

[20] With all shocks in the model set to zero, the portfolio choice problem is indeterminant since all assets earn the risk-free return. Thus, there is no deterministic steady state in this model. We define stochastic steady state as the average equilibrium allocation over a very long simulated sample path.

Christiano, and Fisher (2001)). Alternatives that could potentially circumvent this tradeoff are to increase volatility of investment by adding stochastic depreciation in capital as in Storesletten, Telmer, and Yaron (2007) and Gomes and Michaelides (2008), or by adding investment specific technology shocks. On the other hand, the model does a good job of matching the relative volatility of residential investment to output: in the data the ratio of these volatilities is 4.65, while it is 4 in Model 1. Finally, both in the model and the data, residential investment is less correlated with output than is consumption and total investment.

4.1.2 Correlations of House and Stock Prices with Real Activity

Table 3 presents correlations of house and stock prices with real activity. The housing price index is procyclical, both in the data and in the model. Table 3 shows that the correlation between HP-detrended GDP and HP-detrended house prices range from 0.42 to 0.52 in the data depending on the sample and data source, and from 0.88 to 0.93 in our various models. In the data, the correlation between GDP and the price-rent ratio ranges from 0.46 to 0.56, whereas in the model correlations range from 0.44 to 0.7. And the correlation between the national price-rent ratio and price-dividend ratio ranges from 0.23 to 0.48 in the data, while the model has these correlations between 0.40 and 0.86.

Thus the correlation between the price-rent ratio and price-dividend ratio on stocks and between the price-rent ratio and the cycle is weaker in the data than it is in the model, but within a reasonable and (in some cases) overlapping range. The correlation between the price-rent ratio and GDP is roughly correct. So while the model does overstate the correlation between GDP and between the stock market and housing, it does not imply that these variables are perfectly correlated or nearly so. One possible explanation for why the model overstates the correlation between house prices and GDP is that it may be missing movements in risk or risk aversion that are uncorrelated with real activity. Greenwald, Lettau, and Ludvigson (2013) argue that such movements are an empirically important component of stock market behavior. It's possible that similar forces–not captured by the models explored here–are at work in housing markets.

The correlation between residential investment and GDP in the model is quite high (50-67%) and not that far from the data (77-87%) (column 3 of Table 3). Note that we could increase the correlation between the productivity shocks $Z_{C,t}$ and $Z_{H,t}$ shocks to exactly match the correlation between residential investment and GDP, but this would worsen the model's implications for the correlation between the price-dividend and price-rent ratio.

In each of these models, the one-period lagged value of residential investment and GDP have a statistically significant and positive correlation of 0.08, 0.10, and 0.14, respectively. Davis and Heathcote (2005) have noted that real business cycle models with housing have difficulty delivering a positive correlation between one-period lagged residential investment and GDP. The model here produces such a positive correlation, but the magnitude is lower than that found in historical data (where this correlation is 0.57). Both in the data and the model correlations with residential investment at greater lags are statistically zero.

The model also produces a strong positive correlation between land price and total investment, equal to 42% in Model 1, 48% in Model 2 and 49% in Model 3, consistent with evidence in Liu, Wang, and Zha (2011) that these variables are positively correlated.

Many models with housing have difficulty matching the relative volatility of house prices to GDP volatility. For example, Davis and Heathcote (2005) report that the ratio of standard deviations of these HP filtered quantities is 0.52 in their model, whereas it is well above one in the data. We computed the standard deviation of our HP filtered aggregate house price relative to HP filtered GDP. The ratio of these standard deviations is 1.59, 1.46, and 1.79 in Models 1, 2, and 3, respectively. The corresponding numbers in the data from 1953-2012 are 2.32 using the FoF measure of housing wealth. From 1975-2012 the ratio of these standard deviations is 2.65 using the FoF index and 2.06 using the Freddie Mac index. Thus the model producing a volatile house price index relative to economic fundamentals, consistent with the data.

4.1.3 Life Cycle Age-Income Profiles

Turning to individual-level implications, Figure 2 presents the age and income distribution of wealth, both in the model and in the historical data as given by the Survey of Consumer Finance (SCF). The figure shows wealth, by age, divided by average wealth across all households, for three income groups (low, medium and high earners). In both the model and the data, financial wealth is hump-shaped over the life-cycle, and is slightly negative or close to zero early in life when households borrow to finance home purchases. As agents age, wealth accumulates. In the data, financial (nonhousing) wealth peaks between 60 and 70 years old (depending on the income level). In the model, the peak for all three income groups is 65 years. For most individuals who are not bequesters, financial wealth is drawn down after retirement until death. Households in the model continue to hold some net worth in the final years of life to insure against the possibility of living long into old age. A similar observation holds in the data. For low and medium earners, the model gets the average amount of wealth about right, but it somewhat under-predicts the wealth of high earners early in the life-cycle.

The right-hand panels in Figure 2 plot the age distribution of housing wealth. Up to age 65, the model produces about the right level of housing wealth for each income group, as compared to the data. In the data, however, housing wealth peaks around age 60 for high earners and age 67 for low and medium earners. By contrast, in the model housing wealth remains high until death. In the absence of an explicit rental market, owning a home is the only way to generate housing consumption, an argument in the utility function. For this reason, agents in the model continue to maintain a high level of housing wealth later in life even as they drawn down financial wealth.

We now turn to results that focus on how key variables in the model are influenced by a financial market liberalization and foreign capital influx.

4.2 Portfolio Shares

What is the effect of a financial market liberalization and foreign capital influx on the optimal portfolio decisions of individuals? Table 4 exhibits the age and income distribution of housing wealth relative to total net worth, both over time in the SCF data and in Models, 1, 1B, 2 and 3. The benchmark model captures an empirical stylized fact emphasized by Fernández-Villaverde and Krueger (2005), namely that young households hold most of their wealth in consumer durables (primarily housing) and very little in financial assets. Houses are 64% of the value of net worth in the data in 2001 for young individuals (35 years an under); the analogous figures in Model 1 is 65%. The model does a good job of matching the housing wealth share averaged across all households as well, and of the "old" (individuals 35 year or older).

Double sorting on age and net worth, the data imply that young poor individuals and young medium-wealth individuals hold a much larger fraction of their wealth as housing, something the model replicates well. For example, the young medium-wealth households in the data have housing wealth that is between 1.26 and 2 times net worth in data from 2001-2007, and 1.95 in 2010. In the model, young medium-wealth households have housing wealth that is between 2.0 and 2.6 across the various models. By contrast, young rich households have a much smaller ratio of housing wealth to net worth, both in the model and in the data.

By comparing the stochastic steady states of Model 1 and Model 3, we see that the model also predicts that a financial market liberalization plus an inflow of foreign capital leads households of all ages and income groups to shift the composition of their wealth towards housing. A corresponding increase occurs in the data from 2001-2010 for all these groups. This occurs in the model because the combination of lower interest rates, lower collateral constraints, and lower housing transactions costs in Model 3 makes possible greater housing investment by the young, whose incomes are growing and who rely on borrowing to expand their housing consumption. Table 4, Panel B, shows that the housing wealth-total wealth ratio rises by 7% for the young between Model 1 and Model 3, and by about 40% for the young-poor and young-middle-wealth individuals. But the decline in housing transactions costs also has important effects on the asset allocation of net savers (primarily older, higher income individuals), consistent with the findings of Stokey (2009) who shows that such costs can have large effects on portfolio decisions. Here, a decline in housing transactions costs makes housing relatively less risky as compared to equity, which causes even unconstrained individuals to shift the composition of their wealth towards housing.

4.3 Asset Pricing

4.3.1 Return Moments

Table 5 presents asset pricing implications of the model, for the calibrations represented by Models 1, 1B, 2 and 3. The statistics reported are averages over 1000 periods. We first discuss the implications of the benchmark Model 1 and then move on to discuss how the statistics change with a financial market liberalization and inflow of foreign money.

The benchmark model roughly matches the volatility of the risk-free rate and only slightly overstates the historical mean return for the risk-free rate. The model produces a sizable equity return of 5% per annum (annual equity premium of roughly 3%). These values are lower than the 8.3% per annum (6.9% per annum equity premium), but the model produces an annual Sharpe ratio (defined here as the mean excess return divided by its standard deviation) of 0.40 (panel B), compared to 0.38 in the data. Two factors related to the cyclicality of the cross-sectional distribution of consumption contribute to the model's high average Sharpe ratio. First, idiosyncratic income risk is countercyclical. Second, house prices and therefore collateral values are procyclical, making borrowing constraints countercyclical. These factors mean that insurance/risk-sharing opportunities are reduced when households need them most–in recessions–resulting in a high risk premium and Sharpe ratio.

Turning to the implications for housing assets, the average house price index return in Model 1 is 12.25% per annum; the standard deviation of the housing index return in the model is 6.55% per annum. The housing index return Sharpe ratio for Model 1 is 1.82.

These are within range of the data for aggregate housing price index returns: the average house price index return in the data ranges from 9.23 to 10.83%; the standard deviation of this return ranges from 5.6 to 7%, and the house price index Sharpe ratio ranges from 1.57 to 0.98. Note that the standard deviation of individual housing returns, $Std_N [R_H]$, which equals 9.03% annually in Model 1, is higher than the standard deviation of the housing index return, which is 6.55% in Model 1. Thus the model goes in the right direction for matching evidence that individual housing returns are more volatile than aggregate index returns (Flavin and Yamashita (2002), Landvoigt, Piazzesi, and Schneider (2013)), though it still doesn't go far enough as we discuss further below.

Financial Market Liberalization and the Housing Boom We now analyze how these statistics in the model are affected by financial market liberalization. Table 5 shows that both the equity premium and the equity Sharpe ratio fall in an economy that has undergone a financial market liberalization. The equity premium and equity Sharpe ratio fall from Model 1 to Model 1B and again from Model 1B to Model 2 (Panel B). Thus both components of the financial market liberalization–the lower housing financing costs of Model 1B as compared to Model 1 and the lower collateral requirements of Model 2 as compared to Model 1B–reduce measures of risk in equity markets. But these forces lower the risk premium and Sharpe ratio on the housing index return even more. The housing risk premium is lower by 11% percent in Model 1B compared to Model 1, and is lower by another 23% in Model 2 compared to Model 1B. This decline in the riskiness of both housing and equity assets reflects the greater amount of risk-sharing possible after a financial market liberalization. The housing price index Sharpe ratio declines 23% from Model 1 to Model 3, more than the equity Sharpe ratio which falls 15%. This occurs because there is an additional factor pushing down the housing risk premium that is inoperative for the equity market: a financial market liberalization is accompanied by a decline in transactions costs for housing but not for equity (or the risk-free asset).

Column 9 of Table 5 shows the change in the price-rent ratio in the data between 2000 and 2006 (Panel A) and the percentage change relative to Model 1 in the model (Panel B). In the model, the national price-rent ratio $p^H/\mathcal{R}$ is 8% higher in Model 1B than it is in the benchmark Model 1, but it is 26% higher in Model 2 than it is in Model 1. This says that 30% of the increase in the price-rent ratio from the financial market liberalization comes from the decline in transactions costs; the remaining 70% is attributable to the relaxation of credit standards. Together, these results isolate the effect of a financial market liberalization, since they are a comparison of stochastic steady states only. (Below we study a dynamic transition that includes economic shocks.)

At the same time, Table 5 column 5 shows that a financial market liberalization by itself leads to an *increase* in equilibrium interest rates. The endogenous risk-free interest rate is 2.63% per annum Model 2, whereas it is 2.12% in Model 1. This occurs because the relaxation of borrowing constraints and housing transactions costs reduces precautionary savings, as households endogenously respond to the improved risk-sharing/insurance opportunities afforded by financial market liberalization. It follows that the increase in price-rent ratios following a financial market liberalization must be entirely attributed to the decline in the housing risk premium, which more than offsets the rise in equilibrium interest rates.[21]

The Role of Foreign Capital in the Housing Boom Model 3 adds to Model 2 an inflow of foreign capital calibrated to match the increase in foreign ownership of U.S. Treasuries and U.S. agency debt over the period 2000-2006. Table 5 shows that such an increase has a large downward impact on the equilibrium interest rate, which falls from 2.63% in Model 2 to 1.93% in Model 3. The last column of Table 5 shows that the average price-rent ratio is 26.8% higher in the stochastic steady state of Model 3 than in it is in the benchmark Model 1. For comparison, from 2000-2006 the price-rent ratio increased 31% according to the Flow of Funds measure, 32% according to the Freddie Mac measure, and 48.9% according to the Core Logic measure. Notice that the vast majority of the rise in the price-rent ratio over the benchmark Model 1 comes not from the foreign-capital-driven lower interest rates, but rather from the financial market liberalization: the price-rent ratio is only 1 percentage point higher in Model 3 than it is in Model 2. This represents less than 4 percent of the total change from Model 1 to Model 3. The reason foreign flows have such a small influence on the national house price-rent ratio has to do with the endogenous response of the housing (and equity) risk premium to an increase in foreign demand for the safe asset. Foreign purchases reduce the effective supply of the safe asset to domestic households and make investing in both equity and housing assets more *risky*. Domestic savers are crowded out of the bond market by foreign governmental holders who are willing to hold the safe asset at any price, forcing domestic residents as a whole to take a leveraged position in the risky

[21] Note also that because we are comparing stochastic steady states across Models 1, 1B, 2, and 3, the long-run annualized values of rental growth are the same across all three models because it is pinned down by the steady state growth of technology, which is the same in each model, assumed to be two percent. Thus none of the differences in price-rent ratio across the models is attributable to differences in expected rental growth rates.

assets. Thus both housing and equity risk premia rise from Model 2 to Model 3. As a result of the foreign flows, domestic investors become more exposed to systematic risk, resulting in a higher equity Sharpe ratio and greater volatility of the stochastic discount factor, $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$, as we move from Model 2 to Model 3.

Despite the increase in risk premia resulting from the foreign capital inflow, the housing risk premium is still lower in Model 3 than in the baseline Model 1 because the decline from Model 1 to Model 2 more than offsets the rise from Model 2 to Model 3. Still, the rise from Model 2 to Model 3 means that the endogenous response of risk premia to foreign purchases of U.S. government bonds substantially limits the extent to which foreign capital inflows can influence home prices. These findings underscore the importance of general equilibrium effects on risk premia for understanding the role of foreign capital inflows in a housing boom. In partial equilibrium models of the housing market (e.g., Titman (1982)), or in small open-economy models without aggregate risk (e.g., Kiyotaki, Michaelides, and Nikolov (2008)), the risk premium is held exogenously fixed. As a consequence, a decline in the interest rate equal in magnitude to that generated by the large influx of foreign money considered here, would be sufficient–by itself–to explain the rise in price-rent ratios observed from 2000-2006. In general equilibrium this is not possible because a foreign capital inflow causes the endogenous risk premium to rise at the same time that it causes interest rates to fall, substantially offsetting the effect of lower interest rates on home prices.

4.3.2 Transition Dynamics: Housing Boom to Bust

Above we studied the effects of housing finance by comparing stochastic steady states. The steady state differences between models show long-run changes only and do not account for business cycle fluctuations. In this section we study a dynamic transition path for house prices and price-rent ratios, in response to a series of shocks designed to mimic both the state of the economy and housing market conditions over the period 2000-2012.[22] We assume that, at time 0 (taken to be the year 2000), the economy begins in the stochastic steady state of Model 1. In 2001, the economy undergoes an unanticipated shift to Model 3 (financial market liberalization and foreign holdings of U.S. bonds equal to 18% of GDP), at which

[22] Ideally, we would study such a path after solving a larger framework that specified a probability law over parameters corresponding to the different models (1 through 3) defined above. Unfortunately, solving such a specification in the existing model would be computationally infeasible. We therefore pursue the simpler strategy described above.

time the policy functions and beliefs of Model 3 are applied.[23] The adjustment to the new stochastic steady state of model 3 is then traced out over the seven year period from 2001 to 2006, as the state variables evolve. Starting in 2007 and continuing through 2012, the economy is presumed to undergo a surprise reversal of the financial market liberalization but not the foreign capital inflow, and as such unexpectedly shifts to a new state in which all the parameters of Model 1 again apply except those governing the foreign capital inflow, which we assume remains equal to 18% of GDP annually, as in Model 3. This hybrid of Models 1 and 3 is referred to as *Model 4*.

In addition, we feed in a specific sequence of aggregate shocks designed to mimic the business cycle over this period. The aggregate technology shock processes Z_C and Z_H follow Markov chains, with two possible values for each shock, "low" and "high" (see the Appendix). Denote these possibilities with the subscripts "l" and "h":

$$Z_C = \{Z_{Cl}, Z_{Ch}\}, \ Z_H = \{Z_{Hl}, Z_{Hh}\}.$$

As the general economy began to decline in 2000, construction relative to GDP in U.S. data continued to expand, and did so in every quarter until the end of 2005. Thus, the recession of 2001 was a nonhousing recession. Starting in 2006, construction relative to GDP fell and has done so in every quarter through 2009. Thus, in contrast to the 2001 recession, housing led the recession of 2007-2009. To capture these cyclical dynamics, we feed in the following sequence of shocks for the period 2000-2009: $\{Z_{Cl}, Z_{Hh}\}_{t=2000}$, $\{Z_{Cl}, Z_{Hh}\}_{t=2001}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2002}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2003}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2004}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2005}$, $\{Z_{Ch}, Z_{Hl}\}_{t=2006}$, $\{Z_{Cl}, Z_{Hl}\}_{t=2007}$, $\{Z_{Cl}, Z_{Hl}\}_{t=2008}$, $\{Z_{Cl}, Z_{Hl}\}_{t=2009}$. Although the transition is designed to focus on the boom from 2000-2006 and recession-associated bust from 2007-2009, we extend the graph out to 2012 by assuming that the 2009 shocks continue through 2012. This appears reasonable since residential investment to GDP in fact falls every year starting in 2006 up to and including 2011, and only modestly recovers in 2012 to a very weak level by historical standards (where it is 2.44%, up from 2.25% but far below the level in any year from 1947-2007 when construction is never below 3.2% of GDP).[24]

[23] Along the transition path, foreign holdings of bonds are increased linearly from 0% to 18% of GDP from 2000 to 2006 and held constant at 18% from 2006 to 2012.

[24] One aspect of the economic environment that did change after 2009 and that we abstract from in the transition post-2009 is that there was an increase in the supply of Treasury debt as a fraction of marketable Treasuries outstanding in 2009 that occurred as part of the American Recovery and Reinvestment Act of 2009. But while this lead outstanding debt/GDP to rise from 2009-2012, so did foreign holdings. In fact, the latter rise by even more over this period (the fraction of foreign holdings/GDP rises from 30% to 40%).

Figure 3 shows the transition dynamics of the price-rent ratio, $p_t^H/\mathcal{R}_t$, (right scale) are such that it rises by 22% over the period 2000-2006,boosted by economic growth, the financial market liberalization, and lower interest rates. House prices themselves (left scale) rise 27.9% from 2000-2006. The increase in $p_t^H/\mathcal{R}_t$ from 2000-2006 is smaller than the increase in p_t^H because, in the model, rents increase modestly over this period in response to positive economic shocks, up 5%. In the data, using the price of Shelter relative to CPI ex-shelter we find that rents rose 3.0% from January 2000-June 2006, so the model is roughly consistent with the modest increases in rent over this period. In the economic contraction over the period from 2007 to 2012, the model generates a decline of 23.4% in the price-rent ratio and a decline of more than 15% in home prices p_t^H, driven by the economic contraction and by a presumed reversal of the financial market liberalization.

The second panel of Figure 3 shows that the price of land/permits p_t^L for the model rises and falls over the transition with the price of housing. The expansion not only drives a construction and housing boom; it also raises the price of the fixed factor of housing production by 23% from 2000-2006. Land/permits prices subsequently fall along with house prices from 2007 to 2012 by 33%, as the economy contracts and collateral constraints and transactions costs revert to previously higher levels.

The third panel of Figure 3 shows the change in the price-rent ratio in the model under several model-based counterfactual scenarios, by holding fixed different features of the changing economy the transition was designed to imitate. The line marked "FML, reversal, flows" is identical to the full baseline transition reported in Panel A, which assumes business cycle variation described above, a financial market liberalization (FML, move from Model 1 to Model 3) and increase in foreign flows starting in 2001 and continuing through 2006, and a reversal of the FML but not of the foreign flows starting in 2007. We now compare this line to various counterfactuals, all of which maintain the presumed business cycle variation.

The line marked "No reversal, no flows" is a hypothetical transition in which there is a FML starting in 2001, but no foreign flows into the safe bond market at any time and no reversal of the FML in 2007 (credit constraints remain lax and transactions costs remain low, as in Model 3). We can see that this case generates 100% of the run-up in the price-rent ratio generated by the baseline transition, but captures virtually none of the bust. This reinforces the point that, despite its depressing influence on interest rates, the vast influx of foreign

The transition simply assumes a constant flow of foreign funds as a fraction of GDP after 2009, but we are slightly understating the effect of these flows for those last three years. As explained below, these flows have little impact on the transition dynamics of home prices.

funds into safe assets has little effect on home values in the general equilibrium of the model. It also shows that a reversal of the financial market liberalization is crucial for generating a housing bust: negative productivity shocks alone are not enough. The line marked "No FML reversal, flows" shows a transition without a FML reversal but adds the constant capital flows that are part of the benchmark transition. This line is by definition identical to the baseline transition until 2007, then looks much like the counterfactual without a FML reversal and without foreign flows after 2007. As just noted, this is to be expected because the flows have little effect on price-rent ratio. The line marked "Business cycle only" is a transition produced by assuming that the only source of fluctuation in the economy are the changes in the productivity shocks described above. Under this scenario there is no FML, no reversal of the FML, and no foreign flows (the economy stays in Model 1). It is clear that this line generates a much smaller boom in the price-rent ratio, and a much smaller bust. This reinforces the conclusion that business cycle variation alone–even in a model with incomplete markets and idiosyncratic risk–is not enough to generate sizable fluctuations in the price-rent ratio. The "Business cycle only" line may be compared with a model in which all idiosyncratic shocks are set to zero, so that the only source of risk in the economy are the aggregate productivity shocks. The line marked "No idiosyncratic risk" is a transition in such an economy, where idiosyncratic risk is turned off and only business cycle fluctuations exist. This case generates negligible changes in the price-rent ratio, much smaller than even the "Business cycle only" economy with idiosyncratic risk. Aggregate risk alone is not enough to generate fluctuations in risk-sharing and risk-premia. Since changes in risk premia are what's required to generate large movements in the price-rent ratio, we conclude that the financial market liberalization is indispensable for generating the observed dynamics in price-rent ratios.[25]

4.3.3 Cyclical Dynamics of Housing: What Do Changes in House Price-Rent Ratios Forecast?

In this section we ask to what extent cyclical changes in the price-rent ratio in the model reflect changing expectations of future rental growth rates, changing expectations of future home price appreciation, or both. Notice that, in the model, 100% of the variability in the log

[25]Two cases that are complementary to the no idiosyncratic risk are those without any aggregate risk, and those with both aggregate and idiosyncratic risk but low risk aversion. The presence of aggregate risk is crucial for idiosyncratic risk to have an important effect on risk premia, as is a sufficient amount of averison to any risks that are present.

price-rent ratio is attributable to variation in the expected present discounted value of future rental growth rates. This variability can itself be divided into two parts: that attributable to variation in expected future rents, and that attributable to variation in expected future housing returns (discount rates). We look within each model at the relation between purely cyclical changes in price-rent ratios and subsequent movements in housing return and rents. The left panels of Table 6 show regression results (coefficient, t-stat, R^2) from both the model and the data for predicting long-horizon future housing returns and long-horizon future rental growth rates using today's price-rent ratio.

As Table 6 shows, a high house price-rent ratio forecasts *lower* future housing returns, or future home price depreciation. When home prices are high relative to fundamental value, this forecasts a decline in future home prices, not quicker growth in fundamentals. This aspect of the model is consistent with empirical evidence in the bottom left panels of Table 6 (see also Campbell, Davis, Gallin, and Martin (2010), Cochrane (2011)). In the model this occurs in part because high price-rent ratios in an expansion forecast lower future *excess* returns to housing assets, driven by a lower housing risk premium. The housing risk premium falls as the economy grows for two reasons. First, economic growth reduces (but does not eliminate) uninsurable idiosyncratic income risk via (8). Second, the endogenous increase in house prices raises collateral values and relaxes borrowing restrictions, affording households more insurance against remaining income risk. Both factors push down the risk premium on the house price index return.

Table 6 also shows that high price-rent ratios forecast lower future rental growth, though not as strongly as they forecast lower future returns to housing. It is often suggested that increases in price-rent ratios reflect an expected increase in rental growth. For example, in a partial equilibrium setting where discount rates are constant, higher house prices relative to fundamentals can only be generated by higher implicit rental growth rates in the future (Sinai and Souleles (2005), Campbell and Cocco (2007)). The partial equilibrium setting, however, ignores the endogenous response of both discount rates and residential investment to economic growth. In general equilibrium, positive economic shocks can simultaneously drive discount rates down and residential investment up, leading high price-rent ratios to reflect an expected decline in rental growth. As the housing supply expands, the cost of future housing services (rent) is forecast to be lower. It follows that high price-rent ratios in expansions must entirely reflect expectations of future home price depreciation (lower future returns). Although future rental growth is expected to be lower, price-rent ratios still rise in response to positive economic shocks because the expected decline in future housing returns more than offsets the expected fall in future rental growth.[26]

For completeness, Table 6 also reports predictability results for equity returns. In model generated data, both the raw equity return and the excess return are forecastable by the price-dividend ratio over long horizons, consistent with evidence from U.S. stock market returns.[27] High price-dividend ratios forecast low future equity returns and low excess returns (low equity risk premia) over horizons ranging from 1 to 30 years. Compared to the data, the model produces about the right amount of forecastability in excess equity returns, but produces too much forecastability of dividend growth. This is not surprising since, unlike an endowment/exchange economy where dividends are set exogenously, in the model here both profits and the value of the firm respond endogenously to aggregate shocks.[28]

4.4 Risk Sharing and Inequality

In the limited risk-sharing environment here, risk premia are driven by the amount of risk-sharing/insurance possible in the economy. To evaluate the model's implications for direct risk-sharing measures and inequality, Table 7 presents four types of measures of inequality or risk-sharing: (i) the Gini coefficient for net worth, housing wealth, and financial wealth, (ii) the cross-sectional standard deviation in the individual consumption share in aggregate consumption, (iii) the Gini coefficient of consumption, and (iv) the cross-sectional standard deviation of the individual marginal rates of substitution, $\frac{\beta \partial U/\partial C^i_{a+1,t+1}}{\partial U/\partial C^i_{a,t}}$. The first of these are measures of inequality in wealth; the next two are measures of inequality in the numeraire consumption good. The last is a measure of risk-sharing. Under perfect risk-sharing (complete markets) individuals equate their marginal rates of substitution state by state. Thus, the cross-sectional standard deviation of the marginal rates of substitution is a quantitative measure of market incompleteness, with higher values indicating less risk-sharing.

Table 7 shows that the model does a good job of matching overall wealth inequality. For example, the Gini coefficient for total wealth (net worth) in the data is 80.31 in 2001 in the data and it is 75.56 in Model 1. Relative to the data, the model somewhat overstates inequality in financial wealth (Gini equal to 121 in Model 1 versus 94 in 2001 data), and

[26] Predictable variation in housing returns must therefore account for more than 100 percent of the variability in price-rent ratios.

[27] See, for example, the summary evidence in Cochrane (2005), Chapter 20, Lettau and Ludvigson (2010), and Lettau and Van Nieuwerburgh (2008).

[28] For this same reason, the model also produces too much predictability in raw returns driven by too much predictability in interest rates. Positive economic shocks increase consumption but not as much as income, thus saving and the capital stock rise, pushing down expected rates of return to saving and interest rates.

somewhat understates inequality in housing (Gini equal to 46 in Model 1 versus 63 in 2001 data).[29] But the model is broadly consistent with the facts: large inequality in financial wealth and much smaller degrees of inequality in housing wealth. In the data, financial wealth inequality rises sharply by 15 points from 2001 to 2010, and this is exactly matched by the 15 point increase in financial wealth inequality in our model from Model 1 to Model 3.

Table 7 shows that the decline in risk premia from Model 1 to Model 2 (documented above) coincides with an increase in risk-sharing and a decline in consumption and housing wealth inequality. Risk-sharing improves both because a financial liberalization directly increases access to credit, and because lower transactions costs reduce the expense of acquiring additional collateral, which increases borrowing capacity that can be used to insure against shocks. Both factors allow heterogeneous households to insure more of their risks. These same measures of risk-sharing and consumption inequality fall further from Model 2 to Model 3, a comparison that isolates the influence of the foreign capital inflow. Note from Table 5 that equity risk premia rise from Model 2 to Model 3. This increase in equity risk premia occurs despite the greater risk sharing. To understand this result, it must be kept in mind that not everyone is a shareholder. The cross-sectional standard deviation of the MRS of shareholders rises from Model 2 to Model 3 (not shown), even though it declines slightly for the entire population, (Table 7 column one).[30] It follows that the small decline in the cross-sectional standard deviation of the marginal rates of substitution observed from Model 2 to Model 3 reflects a tradeoff: foreign flows make borrowing less costly for non-equity holders and improve their risk-sharing, but equity holders are forced into a leveraged position and exposed to more systematic risk. The cross-sectional standard deviation of marginal rates of substitution across all individuals reflects a tradeoff between these two groups can therefore go up or down, depending on which group's experience dominates.

What about wealth inequality? A financial market liberalization and foreign demand for the risk-free asset have reinforcing effects on financial wealth inequality: financial wealth inequality increases from Model 1 to Model 2, and again from Model 2 to Model 3. These

[29] Note that the Gini index can exceed one when negative values for financial wealth (negative net worth for people with debt).

[30] Younger individuals in particular hold few assets other than what is needed to satisfy a borrowing constraint. Many of these individuals hold so little equity so that their marginal rates of substitutions are not given weight in the shareholder SDF used to compute stock returns. At the same time these individuals are net borrowers and are better able to smooth consumption in the presence of foreign purchases of the safe asset, which increases the availability of funds and drives down the interest rate.

changes in wealth in the model may be compared to those in recent data. In the Survey of Consumer Finances (SCF), the Gini index for financial wealth rises by 14.9 percent between 2001 and 2010. In the model, the Gini for financial wealth increases by almost exactly the same amount: 15.1%. Of that increase, 7.3 percentage points comes from a decline in transactions costs, 5.3 percentage points from a decline in the cumulative loan-to-value limits, and 2.5 percentage points from the foreign demand for safe assets. In addition, both in the model and in the data, the Gini index for housing wealth in the SCF data declines slightly from 2001 to 2010, while in the model it also falls slightly from Model 1 to Model 3.

Why does a financial market liberalization have an upward effects on financial wealth inequality while reducing consumption and housing inequality? A financial market liberalization relaxes financial frictions, making it easier to borrow against home equity and making it less costly to transact. This improves risk-sharing and reduces consumption inequality and housing inequality. But financial wealth inequality rises because as domestic borrowers (mostly young individuals) take advantage of lower collateral requirements and transactions costs to increase current consumption, their net worth position deteriorates. At the same time, domestic savers as a whole are forced to shift the composition of their wealth toward risky securities as a result of the foreign capital inflow. They therefore earn a higher rate of return on the risky asset and on their savings, as compared to Model 2, which drives their wealth more positive. These changes mean that inequality in consumption and housing wealth declines from Model 1 to Model 3, while inequality in financial wealth increases. This feature of the model is consistent with changes in the data from 2001-2010 presented in Table 7, and with evidence that wealth inequality has risen more than consumption inequality in recent decades.[31]

4.5 Robustness

This section discusses robustness of our results to different assumptions for the model calibration. Tables 8 and 9 present the model's implications for the same statistics reported in Table 2 (business cycle statistics) and Table 5 (asset pricing moments) for a number of different calibrations of the model. For ease of comparison, the first panel of each table reproduces our benchmark results. Panel A of Table 8 reproduces the results from Model

[31] Krueger and Perri (2006) and Heathcote, Perri, and Violante (2009) study income and consumption inequality directly, and show that consumption inequality has risen less than income inequality. Their results for saving and income inequality suggest that wealth inequality has risen more than consumption inequality over time.

1 reported in Table 2. Panel A of Table 9 reproduces the results from both Model 1 and Model 3 that are reported in Table 5.

As a first change to the calibration, we study a model with a higher land share of production, equal to 25% instead of 10% . Overall the results are very similar to the benchmark model, with business cycle statistics almost identical (Table 8). One interesting difference shown in Table 9 is that the housing risk premium falls by less, percentage-wise, when moving from Model 1 to Model 3 than it does in the benchmark case with land share equal to 10%. For this reason the aggregate price-rent ratio rises by slightly less from Model 1 to Model 3 (22% rather than 26.6%). Although a higher land share makes housing returns more volatile, the greater inelasticity of housing also makes these returns less correlated with individuals' housing-consumption expenditure shares, and therefore their marginal rates of substitution. The result is a smaller decline in risk premia and lesser increase in the price-rent form Model 1 to Model 3.

Next we solve a case of our model with idiosyncratic housing depreciation shocks in order to assess the potential impact of idiosyncratic house price risk. Idiosyncratic housing risk could help the model better match evidence for a large idiosyncratic component in individual housing return volatility (Flavin and Yamashita (2002), Landvoigt, Piazzesi, and Schneider (2013)). For this case, we change the individual's house deprecation rate, δ_H, to be stochastic, equal to 0 with probability 0.5 and 5% with probability 0.5. The mean is the same as the benchmark value, 2.5%. This amount of idiosyncratic risk causes swings of 100% in either direction around estimates of the mean, which implies substantial idiosyncratic housing risk. Importantly, these shocks have the potential to affect the volatility of individual housing returns $Std_N\left[R_H\right]$ and the $SR_N\left[R_H^{\cdot}\right]$ statistic, as computed in (19). The aggregate business cycle moments for this case are very similar to the benchmark case without idiosyncratic depreciation shocks (Table 8 Panel C). Table 9 Panel C shows that the additional idiosyncratic volatility in house prices does make the average (across households) individual housing return more volatile: $Std_N\left[R_H\right] = 9.05$ in Model 1 without depreciation shocks, but $Std\left[R_H^i\right] = 9.32$ with depreciation shocks. This results in a slight decline in the $SR_N\left[R_H^{\cdot}\right]$ statistic relative to the benchmark model (column 11). The decline is small, however, in part because there is also a rise in the average housing return, displayed both in the mean national house price index return, and in the cross-sectional mean of individual housing returns $E_N\left[R_H\right]$. Housing risk premia rise along with the added idiosyncratic risk, so $SR_N\left[R_H^{\cdot}\right]$ declines only slightly. Thus, while depreciation shocks add risk to individual housing returns, they generate only a modest amount of additional variability in housing returns. Individual housing returns are more volatile than returns on the house price index, but these mechanisms are not enough in the model to capture the magnitude of estimated idiosyncratic volatility in housing returns found in some local housing markets (e.g., the markets of San Diego county Landvoigt, Piazzesi, and Schneider (2013)).[32]

As a third variation on our model, we solve a case with a form of idiosyncratic shocks more akin to "unemployment" shocks, modeled as rare but very large declines in income. The modeling approach follows Krusell and Smith (1999), whereby "unemployment" is characterized by an event that happens rarely but results in a large decline in income. Income shocks are therefore asymmetric, in the sense that good shocks are smaller but more frequent, while bad shocks are infrequent and very bad. The exact parametrization is such that, in expansions, income grows by roughly 1% per year 95 percent of the time, but falls by 21% per year five percent of the time. In recessions, income grows by 2.8% per year 95% of the time, but can fall 39% five percent of the time. Mean labor income growth is unchanged from the benchmark model. The greater volatility in recessions generates countercyclical idiosyncratic income risk. Note that the "unemployment" states do not imply that income declines all the way to zero; this is a simple way of capturing some basic exogenous level of insurance against employment shocks. Table 8, Panel D shows that, as for the case with depreciation shocks, this case has implications for real business cycle moments that are very similar to the benchmark Model 1. But there are some notable differences for housing and equity assets. Table 9, Panel D shows that much more severe down-side risk generates slightly higher equity and housing index return risk premia, and the increase in the price-rent ratio from Model 1 to Model 3 is less (18%) than in the benchmark model (27%). The more extreme idiosyncratic income risk of this case generates more precautionary saving in both Model 1 and Model 3, so that the same financial market liberalization leads to less of an increase in borrowing, less of an increase in risk-sharing, and less of an impact on asset prices.

5 Conclusion

In this paper we have studied the macroeconomic and household-level consequences of fluctuations in housing wealth and housing finance. The framework studied here endogenizes the interaction among financial and housing wealth, output and investment, rates of return

[32] Landvoigt, Piazzesi, and Schneider (2013) estimate a standard deviation of the idiosyncratic component of individual housing returns for households in San Diego to be close to 9% and argue that total volatility is likely to be on the order of 14%.

and risk premia in both housing and equity assets, and consumption and wealth inequality. We have focused much of our investigation on studying the macroeconomic impact of systemic changes in housing finance that were a key characteristic of housing markets during the housing boom period from 2000-2006 and its aftermath. The main contribution of this analysis is to illustrate the potential role of time-varying housing risk premia that occur in general equilibrium environments in transmitting the effects of economic shocks that shift risk-sharing opportunities to housing and equity markets, as well as to macroeconomic activity. This channel is absent in previous macroeconomic theories of housing production, where risk premia are either held fixed or not modeled at all.

The model here implies that national house price-rent ratios may fluctuate considerably in response to a financial market liberalization, as well as in response to movements in the aggregate economy. A fundamental result of the paper is that these factors influence households' opportunities for risk-sharing, and it is through this mechanism that they influence home prices. In a simulated transition for the period 2000-2012, the model captures a large fraction of the run-up observed in U.S. national house price-rent ratios from 2000-2006 and predicts a sharp decline in housing markets starting in 2007. The general equilibrium environment is important for understanding some features of these results. For example, the model implies that the rise in price-rent ratios over the boom must be attributed to a decline in risk premia not interest rates, and that procyclical increases in national house price-rent ratios must reflect lower future housing returns rather than higher future rents, a finding that is difficult to comprehend without taking into account the endogenous response of interest rates, residential investment and discount rates to changes in borrowing terms and economic shocks.

A financial market liberalization increases house prices because it drives risk premia in both the housing and equity market down and shifts the composition of wealth for all age and income groups towards housing. These changes, along with economic shocks, are the largest drivers of volatility in the model price-rent ratio. By contrast, borrowed funds from the rest of the world–while having a large depressing effect on interest rates–were found to play a limited role in generating asset booms. This latter result runs contrary to the perception that, by driving interest rates lower, the vast inflow of foreign money into U.S. bond markets from 2000 to 2006 was a major factor in the housing boom.[33]

Although the theoretical framework studied here generates a large boom-bust pattern in

[33] This perception has been voiced by policymakers, academics, and industry analysts. See for example, Bernanke (2005, 2008), and Stiglitz (2010).

home prices comparable to recent data, it has no role for a bubble: all of the variability in the model's price-rent ratio is attributable to variability in the (boundedly rational) expected present discounted value of future rents. An important part of this variability is attributable to the changes in housing finance we have studied. But the model does not explain all features of the data, nor does it explain 100% boom-bust in housing from 2000-2009. It generates too high a correlation between home values and GDP and too little idiosyncratic house return volatility, and abstracts from important real-world features of housing markets such as rental markets and long-term home mortgages. This leaves room for much important additional work, especially empirical, in determining the relative role of various economic factors in driving home values over the business cycle.

The model takes no stand on whether the changes in housing finance we've documented can be characterized as a rational response to economic conditions and/or regulatory changes. Focusing on features of the recent housing boom, Piskorski and Tchistyi (2011) study the mortgage contracting problem in a partial equilibrium setting with stochastic (exogenous) home price appreciation. They find that many elements of the housing boom, such as the relaxation of credit limits, the subsidization of risky (subprime) borrowers, and the clustering of defaults among riskier borrowers, can be explained as the outcome of an optimal dynamic mortgage contracting problem in which both borrowers and lenders are fully rational. Combining the partial-equilibrium mortgage contracting problem with the general equilibrium model of limited risk-sharing is a formidable challenge for future research.

Future work could also address the role of regional heterogeneity in house price-rent ratios. The framework in this paper provides a model of the national price-rent ratio. But other researchers have emphasized that price-rent ratios varied widely across the U.S. during the boom-bust period (e.g., Gyourko, Mayer, and Sinai (2006)). An extension of the model here could account for this heterogeneity, at least in part, if different regions were differentially exposed to the financial market liberalization, perhaps because of differences in demographics that implied some regions were more affected by the changes in credit constraints and mortgage transactions costs than others. Mian and Sufi (2009) provide evidence of the existence of such regional heterogeneity. For example, they find that zip codes with a high prevalence of subprime debt experienced an unprecedented relative growth in mortgage credit from 2002 to 2005 despite sharply declining relative (and in some cases absolute) income growth in those zip codes.

Appendix

This appendix provides a detailed description of changes in housing finance, describes how we calibrate the stochastic shock processes in the model as well as all other parameters, describes the historical data we use to measure house price-rent ratios and returns, and describes our numerical solution strategy.

Changes in Housing Finance

This section documents the empirical evidence for changes in three features of housing finance. First are changes in collateralized borrowing requirements, broadly defined. Collateralized borrowing constraints can take the form of an explicit down payment requirement for new home purchases, but they also apply to home equity borrowing. Recent data suggests that down payment requirements for a range of mortgage categories declined during or preceding the period of rapid home price appreciation from 2000 to 2006. Loan-to-value (LTV) ratios on subprime loans rose from 79% to 86% over the period 2001-2005, while debt-income ratios rose (Demyanyk and Hemert (2008)). For the top 50 percent of leveraged homeowners, the average down payment on securitized subprime and Alt-A loans went from 14% in 2000:Q1 to 2.7% in 2006:Q2 (Geanakoplos (2011)). Other reports suggest that the increase LTV ratios for prime mortgages was even greater, with one industry analysis finding that LTV ratios for conforming first and second mortgages rose from 60.4% in 2002 to 75.2% in 2006.[34] These changes coincided with a surge in borrowing against existing home equity between 2002 and 2006 (Mian and Sufi (2009)).

By the end of 2006 households routinely were able to buy homes with 100% or higher financing using a piggyback second mortgage or home equity loan. The fraction of households with second liens rose dramatically during the boom. For subprime loans, that fraction rose from 3% in 2002 to 30% by then end of 2006; for Alt-A loans it rose from 3% to 44%.[35] In

[34] Source: UBS, April 16, 2007 Lunch and Learn, "How Did We Get Here and What Lies Ahead," Thomas Zimmerman, page 5.

[35] An indirect indicator of the prevalence of the use of second mortgages is the fraction of first liens with LTV exactly equal to 80%. This fraction rose substantially between 2002 and 2006, as shown by Krainer, LeRoy, and Munpyung (2009). They also show that the fraction of FRMs with LTV greater than 80% decreased from 22% to 6% over this period. Their hypothesis is that mortgage lending underwent a shift from a practice of achieving greater home-buyer leverage by simply increasing the LTV on the first lien (common prior to the housing boom), to a practice of achieving such greater leverage by combining an exactly 80% LTV first lien with a second lien taken out simultaneously (common during the housing boom). In short, during the

addition, second or third liens were often the way in which existing home owners tapped into their home equity, often several quarters after they took out the original mortgage. This equity extraction through second liens is in addition to extraction via cash-out refinancing, another innovation of the boom which became increasingly prevalent. Lee, Mayer, and Tracy (2011) show that second lien balances grew from about $200 billion at the start of 2002 to over $1 trillion by the end of 2007. It also shows that the prevalence of second mortgages rose in every U.S. region from below 10% at the start of the boom (bit higher in coastal cyclical markets) to around 40% in 2006 (except for the Midwest declining region which peaks at a 20% share).

More generally, there was a widespread relaxation of underwriting standards in the U.S. mortgage market during the period leading up to the credit crisis of 2007, which provide a back-door means of reducing collateral requirements for home purchases. The loosening of standards can be observed in the marked rise in simultaneous second-lien mortgages and in no-documentation or low-documentation loans.[36] By the end of 2006 households routinely bought homes with 100% financing using a piggyback second mortgage or home equity loan. See also Mian and Sufi (2009). Loans for 125% of the home value were even available if the borrower used the top 25% to pay off existing debt. Industry analysts indicate that LTV ratios for combined (first and second) mortgages have since returned to more normal levels of no greater than 75-80% of the appraised value of the home. We assess the impact of these changes collectively by modeling them as a reduction in collateralized borrowing constraints and subsequent rise.

Second in our study of housing finance are transactions costs. The period of rapid home price appreciation was marked by a decline in the cost of conducting housing transactions; houses, in effect, became more liquid. Closing costs for mortgages, mortgage refinancing, and home equity extraction all fell sharply in the years during and preceding the housing boom that ended in 2006. The Federal Housing Financing Board reports monthly data on mortgage and mortgage refinancing closing costs (based on a survey of the largest lenders). Closing costs on first mortgages and mortgage refinancings combined for Freddie Mac 30-year conforming mortgages declined 40% from the end of 2000 to end of 2006. These costs

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housing boom high LTV ratios were achieved by taking out "piggyback" second mortgages rather than by loading all leverage onto the first lien, as was previous practice. Consistent with this hypothesis, Krainer, LeRoy, and Munpyung (2009) find that the default rate on first lien mortgages with exactly 80% LTV ratios was higher than that on first lien mortgages that had either 79% or 81% LTV ratios.

[36] FDIC Outlook: Breaking New Ground in U.S. Mortgage Lending, December 18, 2006. <http://www.fdic.gov/bank/analytical/regional/ro20062q/na/2006_summer04.html#10A>.

began moving back up in the aftermath of the credit crisis of 2007/2008. From 2007 to 2009, closing costs on Freddie Mac 30-year conforming mortgages surged back up 56%.

The most specific estimates in the reduction in borrowing costs are from Berndt, Hollifield, and Sandas (2010). They study subprime mortgage loans originated by New Century Financial Corp. Total broker compensation decreased form 5.0% of the loan amount in 1997 to 4.3% in 2000 to 2.8% in 2006. Broker compensation includes direct fees and the yield spread premium, a fee the broker receives for steering the borrower towards a specific mortgage product. They also show that fees decreased after controlling for other loan characteristics. Corroborating evidence comes from a measure of total amount of dollars spent on real-estate related financial services divided by total dollars in real estate loans made. A reduction in that fraction signifies that these services become cheaper per unit. The data available on fees are "Financial service charges, fees, and commissions" from National Income and Product Accounts (NIPA) table 2.4.5. While this measure of fees includes all financial fees and is therefore a bit too broad, a substantial portion of the fees that are earned in this period are known to be real estate related. No finer breakdown of fees is available. For the denominator, the FDIC has information on real estate loans for all FDIC-insured commercial and savings banks. Fees per real estate dollar lent rise until 2000 and then fall from 7.3% in 2000 to 5.1% in 2006.

Finally, transactions costs associated with home equity extraction declined significantly and coincided with a surge of 350% in mortgage equity withdrawal rates from 2000-2006.[37] Kennedy and Greenspan (2007) compiled data on closing costs for home equity loans (HEL) and home equity lines of credit (HELOC) from periodic releases of the Home Equity Survey Report, published by the American Bankers Association. The data indicate that these costs trended down significantly: for HELOCs, they were 76% lower in 2004 than they were in 1988. For closed-end HELs, the costs declined 41% from 1998 to 2004. The surveys indicate that non-pecuniary costs, in the form of required documentation, time lapsed from loan application to loan closing, and familiarity with available opportunities for refinancing and home-equity extraction, also declined substantially. Mortgage closing costs for first and second (home equity) mortgages, home equity lines of credit, and refinancing eroded considerably in the period during or preceding the housing boom, by 90% in some cases. Although some of these costs began to decline in the late 1980s and early 1990s, industry analysts report that there was a delay in public recognition. Mortgage servicers only gradually im-

[37] Figures based on updated estimates provided by James Kennedy of the mortgage analysis in Kennedy and Greenspan (2005).

plemented marketing tools designed to inform customers of lower costs for refinancing and home equity withdrawal. Likewise, news that borrowers could expect a reduction in financial documentation and shortened time periods from application to approval and from approval to closing also spread slowly (Peristiani, Bennett, Monsen, Peach, and Raiff (1997)).

Taken together, we use the above evidence and estimates to calibrate the lending cost parameter, λ. The data collected by Berndt, Hollifield, and Sandas (2010) and the aggregate data on fees per dollar of real estate loan speak directly to the value of this parameter. As a compromise between the level and changes of these two measures of fees between 2000 and 2006, we choose to model a decline in fees from $\lambda = 5.5\%$ to $\lambda = 3.5\%$ of the amount lent when we lower housing financing costs.

In summary, the decline in both transaction costs and collateral constraints that we study in the model is designed to capture the broader empirical phenomenon that subprime mortgages, second mortgages, and home equity lines of credit all became much more widely available between 2000 and 2006. For example, subprime constituted less that 10% of all mortgages in 2000, but it accounted for 40% of all originations in 2006.

Third in our study of housing finance are foreign purchases of U.S. assets. A key development in the housing market in recent years is the secular decline in interest rates, which coincided with a surge in foreign ownership of U.S. bonds. Figure A.1 shows that both 30-year FRMs and the 10-year Treasury bond yield have trended downward, with mortgage rates declining from around 18 percent in the early 1980s to below 3.5 percent by the end of 2012. This was not merely attributable to a decline in inflation: the real annual interest rate on the ten-year Treasury bond fell from 3.6% in December 1999 to 0.93% in June 2006 using the consumer price index as a measure of inflation. Alternatively, the 10-year TIPS yield declined from 4.32% to 2.53% over this same period, or 180bp. The 10yr TIPS rate reached a prior low of 1.64% in September 2005, which represents a decline of 270bp, the same decline observed for the 10-year Treasury from December 1999 to June 2006. In the post-crisis period, both measures of the 10-year real rate continue to decline and are in negative territory at the end of our sample. At the same time, foreign ownership of U.S. Treasuries (T-bonds and T-notes) increased from \$118 billion in 1984, or 13.5% of marketable Treasuries outstanding, to \$2.2 trillion in 2008, or 61% of marketable Treasuries (Figure A.2, Panel A). Dollar holdings continue to rise to \$4.7 trillion in 2012, but the share declines to 52.5%. Foreign holdings of long-term U.S. Agency and Government Sponsored Enterprise-backed agency securities quintupled between 2000 and 2008, rising from \$261 billion to \$1.46 trillion, or from 7% to 21% of total agency debt. Agency holdings fall to \$991 billion or

14% of the amount outstanding by 2012. Foreign holdings of long-term and short-term U.S. Treasury and Agency debt as a fraction of GDP doubled from 14.6% to 29.3% over the period 2000-2008 (Figure A.2, Panel B). Over this period, the fraction of marketable Treasuries relative to GDP was stable between 2000 and 2008 at around 31%. In the post-2008 period, foreign holdings continue to rise to 40.6% of GDP by 2012. Over the 2008-2012 period, the supply of marketable Treasuries rises to 67% of GDP, so that foreign holdings fall as a share of Treasuries outstanding but rise relative to GDP. We study a model where foreign purchases of U.S. debt equal 18% of GDP, which is close to the observed average for the period 1974-2012.

Calibration of Shocks

The aggregate technology shock processes Z_C and Z_H are calibrated following a two-state Markov chain, with two possible values for each shock, $\{Z_C = Z_{Cl}, Z_C = Z_{Ch}\}$, $\{Z_H = Z_{Hl}, Z_H = Z_{Hh}\}$, implying four possible combinations:

$$\begin{aligned} Z_C &= Z_{Cl,} \qquad Z_H = Z_{Hl} \\ Z_C &= Z_{Ch,} \qquad Z_H = Z_{Hl} \\ Z_C &= Z_{Cl,} \qquad Z_H = Z_{Hh} \\ Z_C &= Z_{Ch,} \qquad Z_H = Z_{Hh.} \end{aligned}$$

Each shock is modeled as,

$$\begin{aligned} Z_{Cl} &= 1 - e_C, \qquad Z_{Ch} = 1 + e_C \\ Z_{Hl} &= 1 - e_H, \qquad Z_{Ch} = 1 + e_H, \end{aligned}$$

where the volatilities of e_C and e_H are calibrated to match the volatilities of GDP and residential investment in the data.

We assume that Z_C and Z_H are independent of one another. Let $\mathbf{P}^C$ be the transition matrix for Z_C and $\mathbf{P}^H$ be the transition matrix for Z_H. The full transition matrix equals

$$\mathbf{P} = \begin{bmatrix} p_{ll}^H \mathbf{P}^C & p_{lh}^H \mathbf{P}^C \\ p_{hl}^H \mathbf{P}^C & p_{hh}^H \mathbf{P}^C \end{bmatrix},$$

where

$$\mathbf{P}^H = \begin{bmatrix} p_{ll}^H & p_{lh}^H \\ p_{hl}^H & p_{hh}^H \end{bmatrix} = \begin{bmatrix} p_{ll}^H & 1 - p_{ll}^H \\ 1 - p_{hh}^H & p_{hh}^H \end{bmatrix},$$

and where we assume $\mathbf{P}^C$, defined analogously, equals $\mathbf{P}^H$. We calibrate values for the matrices as

$$\begin{aligned}
\mathbf{P}^C &= \begin{bmatrix} .60 & .40 \\ .25 & .75 \end{bmatrix} \\
\mathbf{P}^H &= \begin{bmatrix} .60 & .40 \\ .25 & .75 \end{bmatrix} => \\
\mathbf{P} &= \begin{bmatrix} .36 & .24 & .24 & .16 \\ .15 & .45 & .10 & .30 \\ .15 & .10 & .45 & .30 \\ .0625 & .1875 & .1875 & .5625 \end{bmatrix}.
\end{aligned}$$

With these parameter values, we match the average length of expansions divided by the average length of recessions (equal to 5.7 in NBER data from over the period 1945-2001). We define a recession as the event $\{Z_{Cl,}, Z_{Hl,}\}$, so that the probability of staying in a recession is $p_{ll}^H p_{ll}^C = 0.36$, implying that a recession persists on average for $1/(1-.36) = 1.56$ years. We define an expansion as either the event $\{Z_{Ch,} Z_{Hl,}\}$ or $\{Z_{Cl}, Z_{Hh}\}$ or $\{Z_{Ch,} Z_{Hh,}\}$. Thus, there are four possible states (one recession, three expansion). The average amount of time spent in each state is given by the stationary distribution (4×1) vector $\boldsymbol{\pi}$, where

$$\mathbf{P}\boldsymbol{\pi} = \boldsymbol{\pi}.$$

That is, $\boldsymbol{\pi}$ is the eigenvector for $\mathbf{P}$ with corresponding eigenvalue equal to 1. The first element of $\boldsymbol{\pi}$, denoted π_1, multiplies the probabilities in $\mathbf{P}$ for transitioning to any of the four states tomorrow conditional on being in a recession state today. π_1 therefore gives the average amount of time spent in the recession state, while π_2, π_3, and π_4 give the average amount of time spent in the other three (expansion) states. Given the matrix $\mathbf{P}$ above, the solution for $\boldsymbol{\pi}$ is

$$\boldsymbol{\pi} = \begin{pmatrix} 0.1479 \\ 0.2367 \\ 0.2367 \\ 0.3787 \end{pmatrix}.$$

This implies the chain spends 14.79% of the time in a recession state and 85.21% of the time in expansion states, so the average length of expansions relative to that of recessions is $85.21/(14.79) = 5.76$ years.

Idiosyncratic income shocks follow the first order Markov process $\ln\left(Z_{a,t}^i\right) = \ln\left(Z_{a-1,t-1}^i\right) + \epsilon_{a,t}^i$. We directly calibrate the specification in levels:

$$Z_{a,t}^i = Z_{a,t-1}^i \left(1 + E_{a,t}^i\right).$$

The regime-switching conditional variance in the unit root process in idiosyncratic earnings is calibrated following Storesletten, Telmer, and Yaron (2007) to match their estimates from the Panel Study of Income Dynamics. These correspond to $\sigma_E = 0.0768$, and $\sigma_R = 0.1296$ in (8). Thus, in our benchmark models, $E_{a,t}^i$ takes on one of two values in each aggregate state:

$$\begin{aligned}
E_{a,t}^i &= \begin{cases} 0.0768 & \text{with } \Pr = 0.5 \\ -0.0768 & \text{with } \Pr = 0.5 \end{cases}, \qquad \text{if } Z_{C,t} \geq E\left(Z_{C,t}\right) \\
E_{a,t}^i &= \begin{cases} 0.1296 & \text{with } \Pr = 0.5 \\ -0.1296 & \text{with } \Pr = 0.5 \end{cases}, \qquad \text{if } Z_{C,t} < E\left(Z_{C,t}\right).
\end{aligned}$$

Thus, $E\left(Z_{a,t}^i / Z_{a,t-1}^i\right) = 1$. For the case of the model designed to mimic unemployment shocks (see text), $E_{a,t}^i$ takes on:

$$\begin{aligned}
E_{a,t}^i &= \begin{cases} 0.112 & \text{with } \Pr = 0.95 \\ -0.21 & \text{with } \Pr = 0.05 \end{cases}, \qquad \text{if } Z_{C,t} \geq E\left(Z_{C,t}\right) \\
E_{a,t}^i &= \begin{cases} 0.028 & \text{with } \Pr = 0.95 \\ -0.39 & \text{with } \Pr = 0.05 \end{cases}, \qquad \text{if } Z_{C,t} < E\left(Z_{C,t}\right) \\
\sigma_R &> \sigma_E.
\end{aligned}$$

Where again, $E\left(Z_{a,t}^i / Z_{a,t-1}^i\right) = 1$.

Calibration of Parameters

Parameters pertaining to the firms' decisions are set as follows. The capital depreciation rate, δ, is set to 0.12, which corresponds to the average Bureau of Economic Analysis (BEA) depreciation rates for equipment and structures. The housing depreciation rate δ_H, is set to 0.025 following Tuzel (2009). Following Kydland and Prescott (1982) and Hansen (1985), the capital share for the non-housing sector is set to $\alpha = 0.36$. For the residential investment sector, the value of the capital share in production is taken from a BEA study of gross product originating, by industry. The study finds that the capital share in the construction sector ranges from 29.4% and 31.0% over the period 1992-1996. We therefore set the capital share in the housing sector to $\nu = 0.30$.[38] The adjustment costs for capital in both sectors are assumed to be the same quadratic function of the investment to capital-ratio, $\varphi\left(\frac{I}{K} - \delta\right)^2$, where the constant φ is chosen to represent a tradeoff between the desire to match aggregate investment volatility simultaneously with the volatility of asset returns. Under this calibration, firms pay a cost only for net new investment; there is no cost to replace depreciated capital. This implies that the total adjustment cost $\varphi\left(\frac{I}{K} - \delta\right)^2 K_t$ under our calibration is quite small: on average less than one percent of investment, I_t. The fixed quantity of land/permits available each period, L, is set to a level that permits the model to approximately match the housing investment-GDP ratio. In post-war data this ratio is 4.8%; under our calibration of L, the ratio ranges from 5% to 6.2% across Model, 1, 2 and 3.

Parameters of the individual's problem are set as follows. The survival probability $\pi_{a+1|a} = 1$ for $a + 1 \leq 65$. For $a + 1 > 65$, we set $\pi_{a+1|a}$ equal to the fraction of households over 65 born in a particular year alive at age $a + 1$, as measured by the U.S. Census Bureau. From these numbers, we obtain the stationary age distribution in the model, and use it to match the average earnings over the life-cycle, G_a, to that observed from the Survey of Consumer Finances. Risk aversion is set to $\sigma^{-1} = 8$, to help the model match the high Sharpe ratio for equity observed in the data. Low values for this parameter imply unrealistically low risk premia and Sharpe ratios. The static elasticity of substitution between C and H is set to unity (Cobb-Douglas utility), following evidence in Davis and Ortalo-Magne (2010) that expenditure shares on housing are approximately constant over time and across U.S. metropolitan statistical areas. The weight, χ on C in the utility function is set to 0.70, corresponding to a housing expenditure share of 0.30.

We choose the time discount parameter β and the bequest parameters ς (fraction of bequesters) and ξ (strength of bequest motive) jointly to match the average level of interest rates and the degree of wealth inequality in the data. For this we set $\beta = 0.7$, $\zeta = 0.10$ and $\xi = 10^{16}$. While $\beta = 0.7$ is low compared to models where no agents have a bequest motive, note that the saving rate in this economy is equivalent to one with a much higher β because of the bequest motives. In fact, a low β for most of the population combined with a strong bequest motive for a small fraction of the population allows us to roughly match the high

[38] From the November 1997 SURVEY OF CURRENT BUSINESS, "Gross Product by Industry, 1947–96, "by Sherlene K.S. Lum and Robert E. Yuskavage. http://www.bea.gov/scb/account_articles/national/1197gpo/maintext.htm

Gross Product Originating is equal to gross domestic income, whose components can be grouped into categories that approximate shares of labor and capital. Under a Cobb-Douglas production function, these equal shares of capital and labor in output.

level of wealth inequality in the data. In order to match wealth inequality and the average interest rates, the bequest parameters (conditional on β) are not separately identifiable and are admittedly arbitrary. We assume that the bequest motive is passed from parents to offspring so that a dynasty with a bequest motive never switches to one without, and vice versa.

The other parameters of the individual's problem are less precisely pinned down from empirical observation. The costs of stock market participation could include non-pecuniary costs as well as explicit transactions fees. Vissing-Jorgensen (2002) finds support for the presence of a fixed, per period participation cost, but not for the hypothesis of variable costs. She estimates the size of these costs and finds that they are small, less than 50 dollars per year in year 2000 dollars. These findings motivate our calibration of these costs so that they are no greater than 1% of per capita, average consumption, denoted $\overline{C}^i$ in Table 1.

We are aware of no publicly available time series on collateral requirements for mortgages and home equity loans. However, our own conversations with government economists and industry analysts who follow the housing sector indicated that, prior to the housing boom that ended in 2006, the combined LTV for first and second conventional mortgages (mortgages without mortgage insurance) was rarely if ever allowed to exceed 75 to 80% of the appraised value of the home. In addition, home equity lines of credit were not widely available until relatively recently (McCarthy and Steindel (2007)). By contrast, during the boom years households routinely bought homes with 100% financing using a piggyback second or home equity loan. Our Model 1 sets the maximum combined LTV (first and second mortgages) to be 75%, corresponding to $\varpi = 25\%$. In Model 2, we lower this to $\varpi = 1\%$. It should be emphasized that $1 - \varpi$ gives the *maximum* combined (first and second mortgage) LTV ratio. This will differ from the *average* LTV ratio because not everyone borrows up to the credit limit.

The fixed and variable "moving" component of the housing transactions costs are governed by the parameters ψ_0 and ψ_1. These costs are more comprehensive than the costs of buying and selling existing homes. They include costs of any change in housing consumption, such as home improvements and additions, as well as non-pecuniary psychological costs. To anchor the baseline level of these costs, in Model 1 we set fixed costs ψ_0 and variable costs ψ_1 to match the average number of years individuals in the model go without changing housing consumption equal to the average length of residency (in years) for home owners in the Survey of Consumer Finances across the 1989-2001 waves of the survey. In the equilibrium of our model, this amount corresponds to a value for ψ_0 that is approximately 3.2% of annual per capita consumption, and a value for ψ_1 that is approximately 5.5% of the value of the house $p_t^H H_{a,t}^i$. These costs are maintained between M1, M2 and M3.

As discussed above in the Appendix section on Changes in Housing Finance, we use direct estimates to calibrate the lending cost parameter, λ. As a compromise between the two different measures of fees and their reduction between 2000 and 2006, we set $\lambda = 5.5\%$ in M1 and reduce it to 3.5% of the amount lent in Models 1B, 2, and 3. Given the comprehensive (and therefore unobservable) nature of transactions costs in the model, the calibration of the Model 1B, 2 and 3 decline in costs is intended to be conservative compared to the larger percentage decline in observable costs associated with mortgage contracts, mortgage refinancing, and home equity extraction.

Finally, we calibrate foreign ownership of U.S. debt, B_t^F, by targeting a value for foreign bond holdings relative to GDP. Specifically, when we add foreign capital to the economy in Model 3, we experiment with several constant values for $B_t^F \equiv B^F$ until the model solution implies a value equal to 18% of average total output, $\overline{Y}$, an amount that is approximately equal to the rise in foreign ownership of U.S. Treasuries and agency debt over the period 2000-2008. Figure 4, Panel B shows that, as of the middle of 2008, foreign holdings of long-term Treasuries alone represent 15% of GDP. Higher values are obtained if one includes foreign holdings of U.S. agency debt and/or short-term Treasuries. Depending on how many of these categories are included, the fraction of foreign holdings in 2008 ranges from 15-30%.

Aggregate House Price and Return Data

We construct data on house prices relative to measures of fundamental value in the housing market, or the service flow from housing. As explained in the main text, for simplicity we refer to the latter as "rent" even though what we really measure is the housing service flow, where two-thirds of the housing stock is owner-occupied. Our first measure of house prices uses aggregate housing wealth for the household sector from the Flow of Funds (FoF) (which includes the part of private business wealth which is residential real estate wealth) and housing consumption from the National Income and Products Accounts. The price-rent ratio is the ratio of housing wealth in the fourth quarter of the year divided by housing service flow expenditures summed over the year. The return is constructed as housing wealth in the fourth quarter plus housing consumption over the year divided by housing wealth in the fourth quarter of the preceding year. We subtract CPI inflation to express the return in real terms and population growth in order to correct for the growth in housing quantities that is attributable solely to population growth. (Since the return is based on a price times quantity, it grows mechanically with the population. In the model, population growth is zero.) The advantage of this housing return series is that it is for residential real estate and for the entire population. The disadvantages are that it is not a per-share return (it has the growth in the housing stock in it, which we only partially control for by subtracting population growth), it is not an investable asset return, and it does not control for quality changes in the housing stock. There is also substantial measurement error in how the Flow of Funds imputes market prices to value the housing stock as well as in how the BEA imputes housing services consumption for owners. These errors, however, may be more likely to affect the level of the price-rent ratio more than the change in the ratio.

Our other two measures of housing returns start from a base-period price-rent ratio. We then roll this P/R ratio forward with two different repeat-sales house price indices and with the BLS rental price index (seasonally adjusted). Since the level of the price-rent ratio is indeterminate (given by the ratio of two indexes), we normalize the level of the series by assuming that the 1975 price-rent ratio is the same as that of the FoF price-rent ratio in 1975. The return is the price index plus the rent divided by the price index at the end of the previous year. We subtract CPI inflation to express the return in real terms. For the base period, we choose the first quarter in which the price index is available. The latter is common across the three housing measures and reflects the shelter component of the CPI. Shelter includes utilities. Using the CPI component for rent that excludes utilities delivers similar results. The two price indices are the Freddie Mac House Price Index ("Freddie Mac") and the Core Logic national house price index series SFC ("Core Logic"). Specifically, we use the price index in the last month of the quarter and the shelter index in the last month of the quarter to construct a quarterly price to rent ratio. The annual return on housing is calculated based on the price-rent ratio and the rental index in the fourth quarter of the year and those in the fourth quarter of the previous year. That is, the return assumes no reinvestment within the year. We deflate nominal returns with CPI inflation. For the period 1976-2012, the Freddie Mac return averages 10.0% with volatility of 5.4% while the Core Logic return is 9.2% on average with volatility of 7.0%. The two return series have a correlation of 96% over this sample. The Core Logic return series has a correlation of 96% with the FoF measure while the Freddie Mac series has a 92% correlation with the FoF measure.

Repeat-sale indices control for quality of housing and only use traded housing prices. The Freddie Mac series is based on conforming mortgages, which excludes large loans above the conforming loan limit and non-prime loans such as subprime, Alt-A, and other exotic mortgages that became increasingly popular during the 2000-2006 boom. It also is a per-share return (no quantities). However, it is based on a very large database, which allows for more precise measurement of the repeat-sale price index. Core Logic has both prime and non-prime coverage, national coverage, and starting in 1976. For these reasons we prefer to the Core Logic series as the most comprehensive repeat-sale index.

We note that the average aggregate housing return implied by these series differs from (and is somewhat higher than) other estimates in the literature, for example those in Piazzesi, Schneider, and Tuzel (2007) (PST). Several immediate differences in our definitions of returns are (i) the PST definition subtracts off depreciation as a maintenance cost whereas ours does not, (ii) the PST measure subtracts off an estimate of property taxes whereas ours does not, and (iii) the PST measure housing wealth using the BEA value of fixed assets, structures category, and then gross it up with the average annual land share. By contrast, in our FoF measure of the housing return, we measure housing wealth as the value of land plus structures. (iii) Our sample includes the housing boom, whereas the sample in PST ends before the boom in 2001.

Our measure of housing returns assumes that, if the house is rented out, the maintenance (equal to depreciation) and after-deduction property taxes are being charged as part of the rent, so that they should not be subtracted again when measuring the per-period housing return. The PST definition assumes instead that rent as measured is a source of income from which maintenance (depreciation) and after-deduction property taxes should be subtracted. Since we do not really know what is included in measured rent or imputed rent for owners, both seem like defensible concepts. Our approach in model and data is consistent in that we do not subtract maintenance from the return in the model either (the model has no property taxes).

We have investigated several alternative ways of constructing gross housing returns. Instead of the NIPA measure of the value of residential land and structures just discussed, we have explored measuring housing wealth as the sum of the value of all structures and the value of land based on data provided by the Lincoln Institute. House prices are benchmarked against three different house price indices (Case-Shiller-Weiss, Federal Housing Financing Agency, and Decennial Census). All three measures result in return series very similar to ours. We also study measuring housing wealth as the current cost net stock of private residential real estate from the BEA Fixed Asset Tables, as in PST. We divide this series by 1 minus the (time-varying) land share (available from the Lincoln Institute) to include the value of land. The corresponding annual gross housing return is 1% lower than the series based on the Lincoln Institute housing wealth numbers, with very similar volatility.

It is important to note that the *level* of the average price-rent ratio in the data is not identified. For Freddie Mac and CL, the price-rent ratio cannot be inferred at all, since both price in the numerator and rent in the denominator are given by indexes. For FoF, we observe the stock of housing wealth and the flow of housing services from NIPA, where the latter is a measure of housing expenses for renters aggregated with an imputed rent measure for owner-occupiers. Although both the wealth and housing services are in dollar units, it is notoriously difficult to impute rents for owner-occupiers from the rental data of non-homeowners, a potentially serious problem since owners represent two-thirds of the population. Moreover, because owners are on average wealthier than non-homeowners, the NIPA imputed rent measure for owner-occupiers is likely to be biased down, implying that the level of the price-rent ratio is likely to be biased up and the average housing return biased down. For this reason, we do not attempt to match our model to the levels of the price-rent ratios and housing returns in the data, instead focusing on the changes in these ratios over time.

Numerical Solution Procedure

The numerical solution strategy consists of solving the individual's problem taking as given her beliefs about the evolution of the aggregate state variables. With this solution in hand, the economy is simulated for many individuals and the simulation is used to compute the equilibrium evolution of the aggregate state variables, given the assumed beliefs. If the equilibrium evolution differs from the beliefs individuals had about that evolution, a new set of beliefs are assumed and the process is repeated. Individuals' expectations are rational once this process converges and individual beliefs coincide with the resulting equilibrium evolution. One important note: we have no results on uniqueness. We are unaware of any such results in the literature concerning models with the degree of complexity considered here, as is typically the case.

The state of the economy is a pair, (Z_t, μ_t), where μ_t is a measure defined over

$$\mathcal{S} = (\mathcal{A} \times \mathcal{Z} \times \mathcal{W} \times \mathcal{H}),$$

where $\mathcal{A} = \{1, 2, ...A\}$ is the set of ages, where $\mathcal{Z}$ is the set of all possible idiosyncratic shocks, where $\mathcal{W}$ is the set of all possible beginning-of-period financial wealth realizations, and where $\mathcal{H}$ is the set of all possible beginning-of-period housing wealth realizations. That is, μ_t is a distribution of agents across ages, idiosyncratic shocks, financial, and housing wealth. Given a finite dimensional vector to approximate μ_t, and a vector of individual state variables

$$\mu_t^i = (Z_t^i, W_t^i, H_t^i),$$

the individual's problem is solved using dynamic programming.

An important step in the numerical strategy is approximating the joint distribution of individuals, μ_t, with a finite dimensional object. The resulting approximation, or "bounded rationality" equilibrium has been used elsewhere to solve overlapping generations models with heterogenous agents and aggregate risk, including Krusell and Smith (1998a); Ríos-Rull and Sánchez-Marcos (2006); Storesletten, Telmer, and Yaron (2007); Gomes and Michaelides (2008); Favilukis (2013), among others. For our application, we approximate this space with a vector of aggregate state variables given by

$$\mu_t^{AG} = (Z_t, K_t, S_t, H_t, p_t^H, q_t),$$

where

$$K_t = K_{C,t} + K_{H,t}$$

and

$$S_t = \frac{K_{C,t}}{K_{C,t} + K_{H,t}}.$$

The state variables are the observable aggregate technology shocks, the first moment of the aggregate capital stock, the share of aggregate capital used in production of the consumption good, the aggregate stock of housing, and the relative house price and bond price, respectively. The bond and the house price are natural state variables because the joint distribution of all individuals only matters for the individual's problem in so far as it affects asset prices. Note that knowledge of K_t and S_t is tantamount to knowledge of $K_{C,t}$ and $K_{H,t}$ separately, and vice versa ($K_{C,t} = K_t S_t$; $K_{H,t} = K_t(1 - S_t)$).

Because of the large number of state variables and because the problem requires that prices in two asset markets (housing and bond) must be determined by clearing markets every period, the proposed problem is highly numerically intensive. To make the problem tractable, we obviate the need to solve the dynamic programming problem of firms numerically by instead solving analytically for a recursive solution to value function taking the form $V(K_t) = Q_t K_t$, where Q_t (Tobin's q) is a recursive function. We discuss this below.

In order to solve the individual's dynamic programming problem, the individual must know μ_{t+1}^{AG} and μ_{t+1}^i as a function of μ_t^{AG} and μ_t^i and aggregate shocks Z_{t+1}. Here we show that this can be achieved by specifying individuals' beliefs for the laws of motion of four quantities:

A1 K_{t+1},

A2 p_{t+1}^{H},

A3 q_{t+1}, and

A4 $[\frac{\beta_{t+1}\Lambda_{t+1}}{\Lambda_t}(Q_{C,t+1} - Q_{H,t+1})]$, where $Q_{C,t+1} \equiv V_{C,t+1}/K_{C,t+1}$ and analogously for $Q_{H,t+1}$.

Let $\frac{\beta_{t+1}\Lambda_{t+1}}{\Lambda_t} \equiv M_{t+1}$. The beliefs are approximated by a linear function of the aggregate state variables as follows:

$$\varkappa_{t+1} = A^{(n)}\left(Z_t, Z_{t+1}\right) \times \widetilde{\varkappa}_t, \tag{30}$$

where $A^{(n)}\left(Z_t, Z_{t+1}\right)$ is a 4×5 matrix that depends on the aggregate shocks Z_t, and $Z_{t+1}$and where

$$\begin{aligned} \varkappa_{t+1} &\equiv \left[K_{t+1}, p_{t+1}^{H}, q_{t+1}, [M_{t+1}(Q_{C,t+1} - Q_{H,t+1})]\right]', \\ \widetilde{\varkappa}_t &\equiv \left[K_t, p_t^{H}, q_t, S_t, H_t\right]'. \end{aligned}$$

We initialize the law of motion (30) with a guess for the matrix $A^{(n)}\left(Z_t, Z_{t+1}\right)$, given by $A^{(0)}\left(Z_t, Z_{t+1}\right)$. The initial guess is updated in an iterative procedure (described below) to insure that individuals' beliefs are consistent with the resulting equilibrium.

Given (30), individuals can form expectations of μ_{t+1}^{AG} and μ_{t+1}^{i} as a function of μ_t^{AG} and μ_t^{i} and aggregate shocks Z_{t+1}. To see this, we employ the following equilibrium relation (as shown below) linking the investment-capital ratios of the two production sectors:

$$\frac{I_{H,t}}{K_{H,t}} = \frac{I_{C,t}}{K_{C,t}} + \frac{1}{2\varphi}E_t\left[M_{t+1}(Q_{C,t+1} - Q_{H,t+1})\right]. \tag{31}$$

Moreover, note that $E_t\left[M_{t+1}(Q_{C,t+1} - Q_{H,t+1})\right]$ can be computed from (30) by integrating the 4th equation over the possible values of Z_{t+1} given $\widetilde{\varkappa}_t$ and Z_t.

Equation (31) is derived by noting that the consumption firm solves a problem taking the form

$$V\left(K_{C,t}\right) = \max_{I_{C,t}, N_{C,t}} Z_{C,t}^{1-\alpha} K_{Ct}^{\alpha} N_{C,t}^{1-\alpha} - w_t N_{C,t} - I_{C,t} - \varphi\left(\frac{I_{C,t}}{K_{C,t}} - \delta\right)^2 + E_t\left[M_{t+1} V\left(K_{C,t+1}\right)\right].$$

The first-order condition for optimal labor choice implies $N_{C,t} = \left(\frac{Z_{C,t}^{1-\alpha}(1-\alpha)}{w_t}\right)^{1/\alpha} K_{C,t}$. Substituting this expression into $V(K_{C,t})$, the optimization problem may be written

$$\begin{aligned} V(K_{C,t}) &= \max_{I_t} X_{C,t}K_{C,t} - I_{C,t} - \varphi\left(\frac{I_{C,t}}{K_{C,t}} - \delta\right)^2 K_{C,t} + E_t\left[M_{t+1}V(K_{C,t+1})\right] \quad (32) \\ s.t. \qquad K_{C,t+1} &= (1-\delta)K_{C,t} + I_{C,t} \end{aligned}$$

where

$$X_{C,t} \equiv \alpha\left(\frac{Z_{C,t}^{1-\alpha}}{w_t}(1-\alpha)\right)^{(1-\alpha)/\alpha} Z_{C,t}$$

is a function of aggregate variables over which the firm has no control.

The housing firms solves

$$\begin{aligned} V(K_{H,t}) = \max_{I_{H,t},N_{H,t}} & p_t^H Z_{H,t}^{1-\nu\phi}(\mathcal{L}_t)^{1-\phi}\left(K_{H,t}^{\nu}N_{H,t}^{1-\nu}\right)^{\phi} - w_t N_{H,t} - I_{H,t} - p_t^L \mathcal{L}_t \\ & -\varphi\left(\frac{I_{H,t}}{K_{H,t}} - \delta\right)^2 + E_t\left[M_{t+1}V(K_{H,t+1})\right]. \end{aligned} \quad (33)$$

The first-order conditions for optimal labor and land/permits choice for the housing firm imply that $N_{H,t} = k_N K_{H,t}$, $\mathcal{L}_t = k_L K_{H,t}$, where

$$\begin{aligned} k_N &= \left(k_1^{\phi}k_2^{1-\phi}\right)^{1/\nu\phi} \\ k_L &= \left(k_1^{\phi(1-\nu)}k_2^{1-\phi(1-\nu)}\right)^{1/\phi\nu} \\ k_1 &= p_t^H Z_{H,t}^{1-\nu\phi}\phi(1-\nu)/w_t \\ k_2 &= p_t^H Z_{H,t}^{1-\nu\phi}(1-\phi)/p_t^L. \end{aligned}$$

Substituting this expression into $V(K_{H,t})$, the optimization problem may be written

$$\begin{aligned} V(K_{H,t}) &= \max_{I_t} X_{Ht}K_{H,t} - I_{H,t} - \varphi\left(\frac{I_{H,t}}{K_{H,t}} - \delta\right)^2 K_{H,t} + E_t\left[M_{t+1}V(K_{H,t+1})\right] \quad (34) \\ s.t. \qquad K_{H,t+1} &= (1-\delta)K_{H,t} + I_{H,t} \end{aligned}$$

where

$$X_{H,t} = p_t^H Z_{H,t}^{1-\nu\phi}\phi\nu k_N^{(1-\nu)\phi}k_L^{1-\phi}.$$

Let s index the sector as either consumption, C, or housing, H. We now guess and verify that for each firm, $V(K_{s,t+1})$, for $s = C, H$ takes the form

$$V(K_{s,t+1}) = Q_{s,t+1}K_{s,t+1}, \qquad s = C, H \quad (35)$$

where $Q_{s,t+1}$ depends on aggregate state variables but is not a function of the firm's capital stock $K_{s,t+1}$ or investment $I_{s,t}$. Plugging (35) into (32) we obtain

$$V(K_{s,t}) = \max_{I_t} X_{s,t}K_{s,t} - I_t - \varphi\left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)^2 K_{s,t} + E_t\left[M_{t+1}Q_{s,t+1}\right]\left[(1-\delta)K_{s,t} + I_{s,t}\right]. \quad (36)$$

The first-order conditions for the maximization (36) imply

$$\frac{I_{s,t}}{K_{s,t}} = \delta + \frac{E_t\left[M_{t+1}Q_{s,t+1}\right] - 1}{2\varphi}. \quad (37)$$

Substituting (37) into (36) we verify that $V(K_{s,t})$ takes the form $Q_{s,t}K_{s,t}$:

$$\begin{aligned} V(K_{s,t}) \equiv\ & Q_{s,t}K_{s,t} = X_{s,t}K_{s,t} - \left(\delta + \frac{E_t\left[M_{t+1}Q_{s,t+1}\right]-1}{2\varphi}\right)K_{s,t} - \varphi\left(\frac{E_t\left[M_{t+1}Q_{s,t+1}\right]-1}{2\varphi}\right)^2 K_{s,t} \\ & + (1-\delta)\left(E_t\left[M_{t+1}Q_{s,t+1}\right]\right)K_{s,t} + E_t\left[M_{t+1}Q_{s,t+1}\right]\left(\delta + \frac{E_t\left[M_{t+1}Q_{s,t+1}\right]-1}{2\varphi}\right)K_{s,t}. \end{aligned}$$

Rearranging terms, it can be shown that $Q_{s,t}$ is a recursion:

$$Q_{s,t} = X_{s,t} + (1-\delta) + 2\varphi\left(\frac{E_t\left[M_{t+1}Q_{s,t+1}\right]-1}{2\varphi}\right) + \varphi\left(\frac{E_t\left[M_{t+1}Q_{s,t+1}\right]-1}{2\varphi}\right)^2. \quad (38)$$

Since $Q_{s,t}$ is a function only of $X_{s,t}$ and the expected discounted value of $Q_{s,t+1}$, it does not depend on the firm's own $K_{s,t+1}$ or $I_{s,t}$. Hence we verify that $V(K_{s,t}) = Q_{s,t}K_{s,t}$. Although $Q_{s,t}$ does not depend on the firm's individual $K_{s,t+1}$ or $I_{s,t}$, in equilibrium it will be related to the firm's investment-capital ratio via:

$$Q_{s,t} = X_{s,t} + (1-\delta) + \left[2\varphi\left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)\right] + \varphi\left(\frac{I_{s,t}}{K_{s,t}} - \delta\right)^2, \quad (39)$$

as can be verified by plugging (37) into (38). Note that (37) holds for the two representative firms of each sector, i.e., $Q_{C,t}$ and $Q_{H,t}$, thus we obtain (31) above.

With (39), it is straightforward to show how individuals can form expectations of μ_{t+1}^{AG} and μ_{t+1}^{i} as a function of μ_t^{AG} and μ_t^i and aggregate shocks Z_{t+1}. Given a grid of values for K_t and S_t individuals can solve for $K_{C,t}$ and $K_{H,t}$ from $K_{C,t} = K_tS_t$ and $K_{H,t} = K_t(1-S_t)$. Combining this with beliefs about K_{t+1} from (30), individuals can solve for $I_t \equiv I_{C,t} + I_{H,t}$ from $K_{t+1} = (1-\delta)K_t + I_t$. Given I_t and beliefs about $\left[\frac{\beta^k \Lambda_{t+k}}{\Lambda_t}(Q_{C,t+1} - Q_{H,t+1})\right]$ from (30), individuals can solve for $I_{C,t}$ and $I_{H,t}$ from (31). Given $I_{H,t}$ and the accumulation equation $K_{H,t+1} = (1-\delta)K_{H,t} + I_{H,t}$, individuals can solve for $K_{H,t+1}$. Given $I_{C,t}$ individuals can solve for $K_{C,t+1}$ using the accumulation equation $K_{C,t+1} = (1-\delta)K_{C,t} + I_{C,t}$. Using $K_{H,t+1}$ and $K_{C,t+1}$, individuals can solve for S_{t+1}. Given a grid of values for H_t, H_{t+1} can be computed from $H_{t+1} = (1 - \delta_H) H_t + Y_{H,t}$, where $Y_{H,t} = Z_{H,t}^{1-\nu\phi} (\mathcal{L}_t)^{1-\phi} \left(K_{H,t}^{\nu} N_{H,t}^{1-\nu}\right)^{\phi}$ is obtained from knowledge of $Z_{H,t}$, $K_{H,t}$ (observable today), from the equilibrium condition $\mathcal{L}_t = L$, and by combining (25) and (27) to obtain the decomposition of N_t into $N_{C,t}$ and $N_{H,t}$. Equation (30) can be used directly to obtain beliefs about q_{t+1} and p_{t+1}^H.

To solve the dynamic programming problem individuals also need to know the equity values $V_{C,t}$ and $V_{H,t}$. But these come from knowledge of $Q_{s,t}$ (using (39)) and $K_{s,t}$ via $V_{s,t} = Q_{s,t} K_{s,t}$ for $s = C, H$. Values for dividends in each sector are computed from

$$\begin{aligned} D_{C,t} &= Y_{C,t} - I_{C,t} - w_t N_{C,t} - \phi_C \left(\frac{I_{C,t}}{K_{C,t}}\right) K_{C,t}, \\ D_{H,t} &= p_t^H Y_{H,t} - I_{H,t} - p_t^L \mathcal{L}_t - w_t N_{H,t} - \phi_H \left(\frac{I_{H,t}}{K_{H,t}}\right) K_{H,t} \end{aligned}$$

and from

$$w_t = (1 - \alpha) Z_{C,t}^{1-\alpha} K_{C,t}^{\alpha} N_{C,t}^{-\alpha} = (1 - \nu) \phi p_t^H Z_{H,t}^{1-\nu\phi} \mathcal{L}_t^{1-\phi} K_{H,t}^{\nu\phi} N_{H,t}^{\phi(1-\nu)-1}$$

and by again combining (25) and (27) to obtain the decomposition of N_t into $N_{C,t}$ and $N_{H,t}$. Finally, the evolution of the aggregate technology shocks Z_{t+1} is given by the first-order Markov chain described above; hence agents can compute the possible values of Z_{t+1} as a function of Z_t.

Values for $\mu_{t+1}^i = (Z_{t+1}^i, W_{t+1}^i, H_{t+1}^i)$ are given from all of the above in combination with the first order Markov process for idiosyncratic income $\log\left(Z_{a,t}^i\right) = \log\left(Z_{a-1,t-1}^i\right) + \epsilon_{a,t}^i$. Note that H_{t+1}^i is a choice variable, while $W_{t+1}^i = \theta_t^i (V_{C,t+1} + V_{H,t+1} + D_{C,t+1} + D_{H,t+1}) + B_{t+1}^i$ requires knowing $V_{s,t+1} = Q_{s,t+1} K_{s,t+1}$ and $D_{s,t+1}$, $s = C, H$ conditional on Z_{t+1}.These in turn depend on $I_{s,t+1}$, $s = C, H$ and may be computed in the manner described above by rolling forward one period both the equation for beliefs (30) and accumulation equations for $K_{C,t+1}$, and $K_{H,t+1}$.

The individual's problem, as approximated above, may be summarized as follows (where we drop age subscripts when no confusion arises). The problem is illustrated for the non-bequesters; the problem for the bequesters is analogous using (6) in place of (5).

$$V_{a,t}\left(\mu_t^{AG}, \mu_t^i\right) = \max_{H_{t+1}^i, \theta_{t+1}^i, B_{t+1}^i} U(C_t^i, H_t^i) + \beta \pi_i E_t [V_{a+1,t+1}\left(\mu_{t+1}^{AG}, \mu_{t+1}^i\right)] \tag{40}$$

The above problem is solved subject to (10), (11) if the individual of working age, and subject to the analogous versions of (10), (11), (using pension income in place of wage income), if the individual is retired. The problem is also solved subject to an evolution equation for the state space:

$$\mu_{t+1}^{AG} = \Gamma^{(n)}(\mu_t^{AG}, Z_{t+1}).$$

$\Gamma^{(n)}$ is the system of forecasting equations that is obtained by stacking all the beliefs from (30) and accumulation equations into a single system. This step requires us to make an initial guess for $A^{(0)}$ in equations (A1)-(A4). This dynamic programming problem is quite complex numerically because of a large number of state variables but is otherwise straightforward. Its implementation is described below.

We next simulate the economy for a large number of individuals using the policy functions from the dynamic programming problem. Using data from the simulation, we calculate (A1)-(A4) as linear functions of $\widetilde{\varkappa}_t$ and the initial guess $A^{(0)}$. In particular, for every Z_t and Z_{t+1} combination we regress (A1)-(A4) on K_t, S_t, H_t, p_t^H, and q_t. This is used to calculate a new $A^{(n)} = A^{(1)}$ which is used to re-solve for the entire equilibrium. We continue repeating this procedure, updating the sequence $\left\{A^{(n)}\right\}$, $n = 0, 1, 2, ...$ until (1) the coefficients in $A^{(n)}$ between successive iterations is arbitrarily small, (2) the regressions have high R^2 statistics, and (3) the equilibrium is invariant to the inclusion of additional state variables such as additional lags and/or higher order moments of the cross-sectional wealth and housing distribution. We discuss numerical accuracy below.

During the simulation step, an additional numerical complication is that two markets (the housing and bond market) must clear each period. This makes p_t^H and q_t convenient state variables: the individual's policy functions are a response to a menu of prices p_t^H and q_t, Given values for $Y_{H,t}$, $H_{a+1,t+1}^i$, $H_{a,t}^i$, $B_{a,t}^i$ and B_t^F form the simulation, and given the menu of prices p_t^H and q_t and the beliefs (30), we then choose values for p_{t+1}^H and q_{t+1} that clear markets in $t+1$. The initial allocations of wealth and housing are set arbitrarily to insure that prices in the initial period of the simulation, p_1^H and q_1, clear markets. However, these values are not used since each simulation includes an initial burn-in period of 150 years that we discard for the final results.

The procedure just described requires a numerical solution to the individual's problem, a simulation using that solution for a large number of agents, and then a repetition (many times) of this procedure using the updated coefficients in $A^{(n)}$. The continuum of individuals born each period in this solution step is approximated by a number large enough to insure that the mean and volatility of aggregate variables is not affected by idiosyncratic shocks. We check this by simulating the model for successively larger numbers of individuals in each age cohort and checking whether the mean and volatility of aggregate variables changes. In addition, we have solved particular cases of the model for different numbers of agents. For numbers ranging from a total of 2,500 to 40,000 agents in the population we found no significant differences in the aggregate allocations we report. But, because of the high numbers of iterations required for convergence, larger numbers of agents drastically increase the computational burden and solving time. Due to the number of cases with different parameter configurations we solve, we therefore use 2,500 agents in this iterative-simulation part of the model solution algorithm. We can, however, use a much larger number of agents for the aggregate statistics we report as output of the model, since, once the model is solved, computing these requires only a one-time simulation. Doing so insures that our reported statistics are as free as possible of any small amount of remaining idiosyncratic (income and death) risk. We therefore generate data for 40,000 agents when we perform the one-time simulation used to report aggregate statistics for the figures and tables of the paper. We could not readily increase the number of agents in the one-time simulation beyond 40,000 because attempts to do so exceeded the available memory on a workstation computer.

Numerical Solution to Individual's Dynamic Programming Problem

We now describe how the individual's dynamic programming problem is solved.

First we choose grids for the continuous variables in the state space. That is we pick a set of values for W^i, H^i, K, H, S, p^H, and q. Because of the large number of state variables, it is necessary to limit the number of grid points for some of the state variables given memory/storage limitations. We found that having a larger number of grid points for the individual state variables was far more important than for the aggregate state variables, in terms of the effect it had on the resulting allocations. Thus we use a small number of grid points for the aggregate state variables but compensate by judiciously choosing the grid point locations after an extensive trial and error experimentation designed to use only those points that lie in the immediate region where the state variables ultimately reside in the computed equilibria. As such, a larger number of grid points for the aggregate state variables was found to produce very similar results to those reported using only a small number of points. We pick 35 points for W^i, 16 points for H^i, and four points for K, H, S, p^H, and q. The grid for W^i starts at the borrowing constraint and ends far above the maximum wealth reached in simulation. This grid is very dense around typical values of financial wealth and is sparser for high values. The housing grid is constructed in the same way.

Given the grids for the state variables, we solve the individual's problem by value function iteration, starting for the oldest (age A) individual and solving backwards. The oldest individual's value function for the period after death is zero for all levels of wealth and housing (alternately it could correspond to an exogenously specified bequest motive). Hence the value function in the final period of life is given by $V_A = \max_{H^i_{t+1},\theta^i_{t+1},B^i_{t+1}} U(C^i_A, H^i_A)$

subject to the constraints above for (40). Given V_A (calculated for every point on the state space), we then use this function to solve the problem for a younger individual (aged $A-1$). We continue iterating backwards until we have solved the youngest individual's (age 1) problem. We use piecewise cubic splines (Fortran methods PCHIM and CHFEV) to interpolate points on the value function. Any points that violate a constraint are assigned a large negative value.

Numerical Checks

This section presents some numerical checks designed to quantify departures from a fully rational equilibrium. At the end of this section, we discuss an important caution about these tests, namely that their appropriateness for our bounded rationality equilibrium is open to question, and there is no consensus on the acceptable degree of departure from full rationality.

Table A.1 begins with standard R^2 statistics. The one-step-ahead R^2 statistics for the four equations (A1)-(A4) are reported in column 1 Table A.1, with the lowest being 0.996 for the Q forecasting equation (A4). These statistics are all quite high and suggest a high degree of accuracy.

These R^2 statistics amount to a one-period-ahead test of the forecasting equations. Although most studies using the Krusell and Smith (1998b) approach report just these one-period-ahead tests, Den Haan (2010) argues that even very accurate one-period ahead forecasts can result in inaccurate multi-period forecasts as the errors build up over time.

Krusell and Smith (1998b) suggest also looking at forecasting errors several periods ahead. In particular, for a given sequence of aggregate shocks from t to $t+k$, they suggest using the forecasting equations (A1)-(A4) to iteratively forecast the state variables k periods ahead. This forecast would use actual state variables at t, the aggregate shocks from t to $t+k$, and equations (A1)-(A4), but it would not use actual state variable realizations between t and $t+k$. The forecasted state variables at $t+k$ are then compared to the realized state variables from simulating the actual model (with heterogenous agents) for the same sequence of shocks. Den Haan (2010) extends this idea by simulating the model over the full sample (in our case 1,750 periods), then taking the same sequence of aggregate shocks and using the forecasting equations to simulate the state variables for the same sample. Note that since this is done for a large sample (furthermore, the first 250 periods are thrown out), this is tantamount to Krusell and Smith (1998b)'s suggestion for a very large (or infinite) k. We will refer to this test as $k = \infty$ for short. Indeed, we have also confirmed that as k rises in Krusell and Smith (1998b)'s test, the forecast errors approach the errors in Den Haan (2010)'s test. (This is also consistent with results reported in Den Haan (2010)).

For each state variable x_t and forecast $\widehat{x}_t$, Den Haan (2010) suggests reporting the standard deviation of the forecast error $\sigma(x_t - \widehat{x}_t)$ scaled by the standard deviation of the state variable $\sigma(x_t)$. Note that this measure is equivalent to reporting the R^2, because the R^2 is defined as $1 - \frac{\sigma(x_t - \widehat{x}_t)^2}{\sigma(x_t)^2}$. This definition of R^2 works for any k, including the one period ahead R^2 discussed above, and reported in column 1 of Table A.1. In column 2 we report the R^2 statistics from the procedure suggested by Den Haan (2010) ($k = \infty$). These R^2 statistics are somewhat lower, but still relatively high, with the lowest occurring for the equation for the aggregate capital (A1), equal to 0.94.

Note that the results above are all based on in-sample calculations because the forecasting coefficients, and the forecast errors are computed on time-series with the same sequence of aggregate shocks. Den Haan (2010) suggests doing the same experiments on simulated data with an alternative sequence of aggregate shocks. To do so, we start with the coefficients computed from simulating our actual model. We then simulate an alternative sequence of aggregate shocks to construct a time-series of forecasted state variables $\widehat{x}_t$ using these forecast coefficients (A1-A4). We use the same alternative sequence of aggregate shocks to simulate our actual model with 40,000 agents to construct a time-series of state variables x_t. We report these errors in column 3. The R^2 statistics do fall some, but are still relatively close to column 3. The lowest is again for the capital equation, now equal to 0.917 when using out-of-sample data.

Because we are computing a bounded rationality equilibrium, it is expected that forecasts are imperfect. It is not surprising therefore that the R^2 are not unity. There is no accepted cutoff in the literature. For example Den Haan (2010) writes "accuracy tests can also be too strong in the sense that solutions that are close to the true solution in most important aspects are still rejected by the accuracy test."

The forecasting tests discussed above check how close our bounded rational equilibrium is to a fully rational equilibrium, along the forecasting dimensions described. As another test of numerical accuracy, one can check whether the Euler equation errors are close to zero. This test is inappropriate for most of the agents in our model, since these agents are constrained often over their lifetime, and by definition their Euler equations do not hold with equality. (Recall that households face fixed stock market participation costs, moving costs, a collateral constraint, and a wedge between the lending and borrowing rate.) We can, however, identify a subset of agents in our model who are likely to be unconstrained, namely the wealthy bequesters who have a strong preference for saving. For these agents, the stock market participation and moving costs are relatively small, while the collateral constraint is rarely binding. The wedge between borrowing and lending rates may still be binding, in which case households will invest 100% of their portfolio in the stock market, but will not lever up. Under these conditions, it can be shown that the Euler equation should hold for these households with the equity return, as well as with their net portfolio return (but not for the risk free rate). These Euler equation errors are reported in Table A.2. They are all sufficiently small.

We have also experimented extensively with grid sizes to confirm that our results are not sensitive to grid size. Unfortunately, due to the size of the state space, we are unable to simultaneously raise all grid sizes. However, we have resolved the model with (i) the grids for individual financial wealth and housing both doubled in size from 35 and 16 to 70 and 32 respectively, (ii) the grids for aggregate capital and aggregate housing both doubled in size from 4 to 8, (iii) the grids for housing prices and interest rates both doubled in size from 4 to 8. In all cases the aggregate quantities and prices look very similar to those reported in the text.

We close this section by noting an important caution about these numerical checks. The model we are solving has a bounded rationality equilibrium, while the numerical checks are aimed at evaluating whether the model solution is consistent with the fully rational one. This incongruity between the numerical checks and the model environment is compounded by the complexity of the framework: when a very large number of agents face an infinite-dimensional state space, the fully rational equilibrium is not computable and the degree of departure from the fully rational equilibrium is unknowable. The fully rational equilibrium may not be a reasonable one with which to compare a model. The cost in terms of the agent's objectives of computing the fully rational policy could in principle be infinite, so that no expenditure of resources on computing better policies would be economically optimal for an agent. Although the numerical checks conducted here suggest that–for the aspects of the model evaluated by the checks–our equilibrium is close to what would be implied by a fully rational one, the fundamental question of how closely our equilibrium policies and prices correspond to those of the fully rational one cannot ultimately be answered. We simply conclude with a caution. As we address issues of contemporary economic importance, we would do well to acknowledge the enormous complexity of real-world problems economic players face, and the possibility that the fully rational outcome is an unattainable theoretical construct appropriate only in unrealistically simplistic environments.

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Figure 1: Price-Rent Ratios in the Data

The figure compares three measures of the price-rent ratio. The first measure ("Flow of Funds") is the ratio of residential real estate wealth of the household sector from the Flow of Funds to aggregate housing services consumption from NIPA. The second measure ("Freddie Mac") is the ratio of the Freddie Mac Conventional Mortgage Home Price Index for purchases to the Bureau of Labor Statistics's price index of shelter (which measures rent of renters and imputed rent of owners). The third series ("Core Logic") is the ratio of the Core Logic national house price index (SFC) to the Bureau of Labor Statistics's price index of shelter. The data are quarterly from 1970.Q1 until 2012.Q4 (or whenever first available). All price-rent series are normalized to a value of 100 in 2000.Q4.

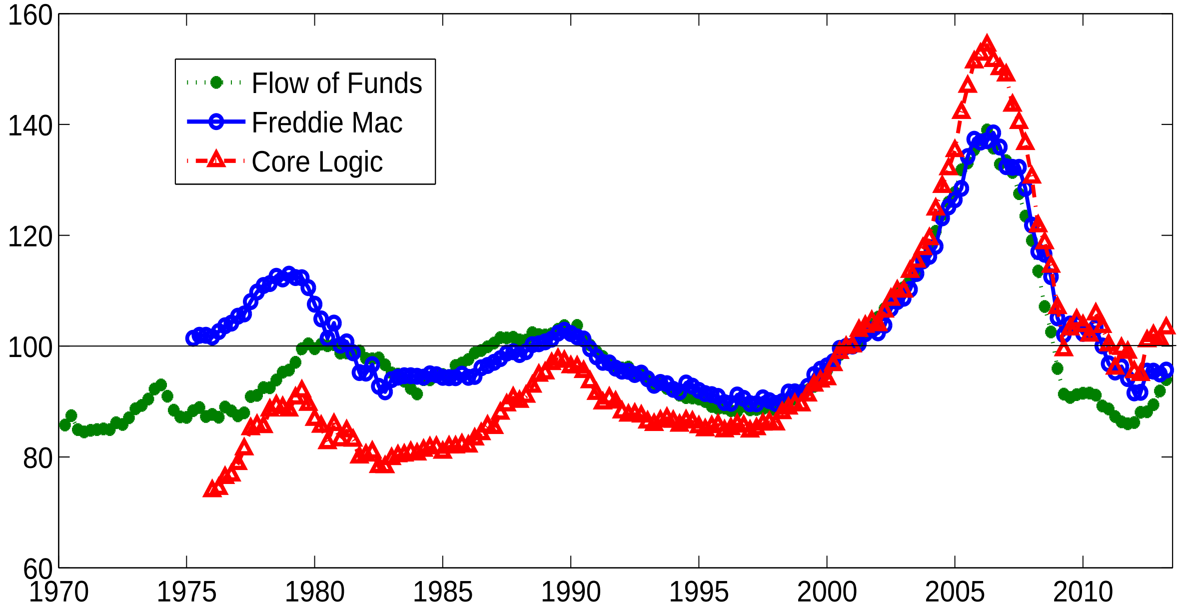


Figure 2: Wealth by Age and Income in Model and Data

The figure plots total wealth ("Wealth") by age in the left columns and housing wealth ("Housing") by age in the right columns. The top panels are for the Data, the middle panels for Model 1, and the bottom panels for Model 2. We use all ten waves of the Survey of Consumer Finance (1983-2010, every 3 years). We construct housing wealth as the sum of primary housing and other property. We construct total wealth as the sum of housing wealth and net financial wealth. Net financial wealth is the sum of all other assets (bank accounts, bonds, IRA, stocks, mutual funds, other financial wealth, private business wealth, and cars) minus all liabilities (credit card debt, home loans, mortgage on primary home, mortgage on other properties, and other debt). We express wealth on a per capital basis by taking into account the household size, using the Oxford equivalence scale for income. For each age between 22 and 81, we construct average total wealth and housing wealth using the SCF weights. To make information in the different waves comparable to each other and to the model, we divide housing wealth and total wealth in a given wave by average net worth (the sum of housing wealth and net financial wealth) across all respondents for that wave. We do the same in the model. The Low Earner label refers to those in the bottom 25% of the income distribution, where income is wage plus private business income. The Medium Earner group refers to the 25-75 percentile of the income distribution, and the High Earner is the top 25%. The model computations are obtained from a 1,500 year simulation. The "Model 1" is the model with normal collateral constraints and borrowing costs; "Model 2" reports on the model with looser collateral constraints and lower borrowing costs. In particular, the down-payment goes from 25% to 1% and the borrowing premium goes from 5.5 to 3.5%.

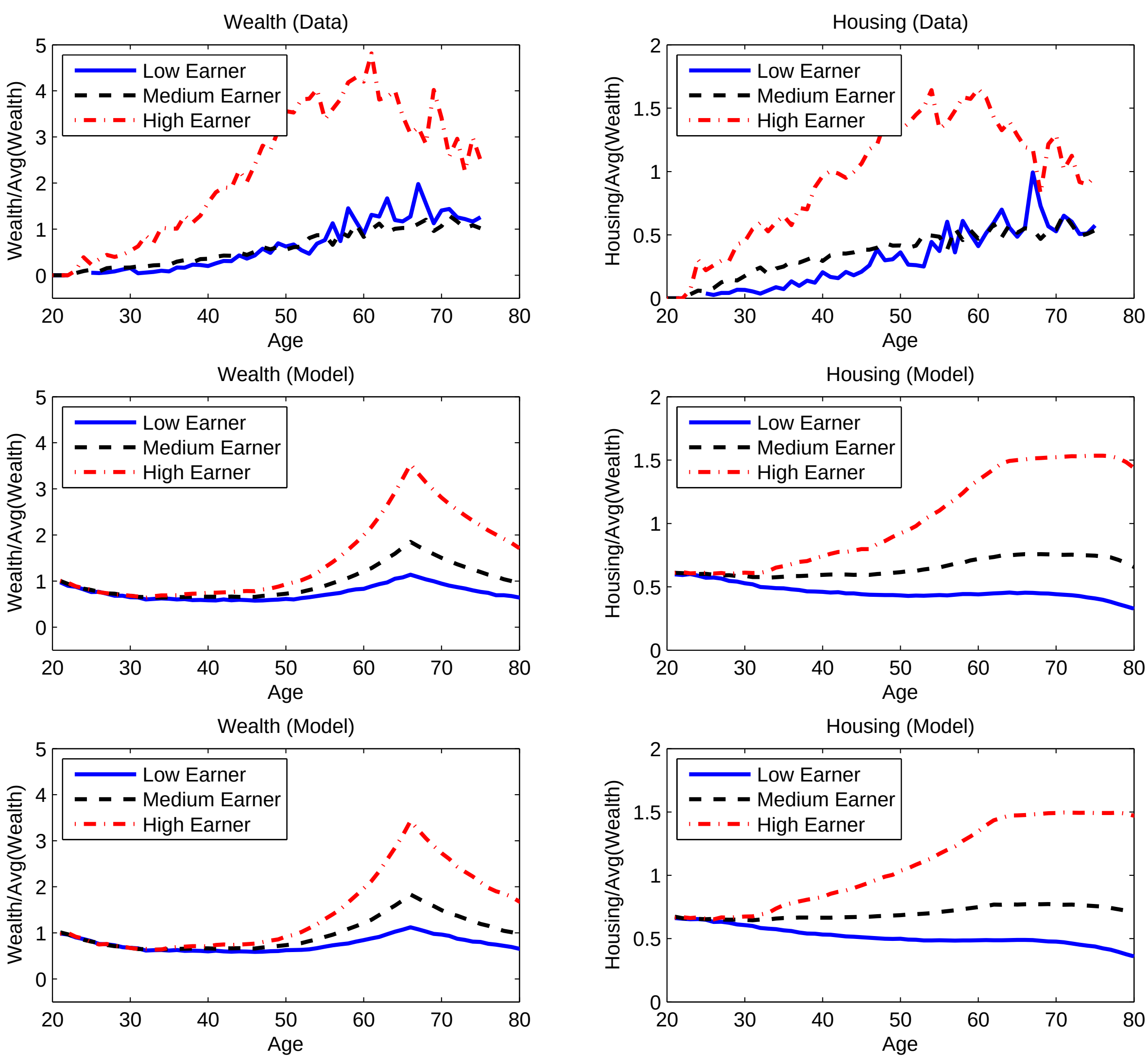


Figure 3: Transition Dynamics in Model: Price-Rent Ratio, House Price, and Land Price

The top panel of the figure plots the house price p^H, plotted against the left axis (solid line, circles), and the price-rent ratio $p^H/\mathcal{R}$, plotted against the right axis (dashed line, crosses) for a transition generated from the model. The middle panel plots the price of land generated from the same transition exercise (solid line, stars). The bottom panel plots the price-rent ratio dynamics in a few counter-factual scenarios. The path begins in the year 2000 in the stochastic steady state of Model 1, the model with tight borrowing constraints and high borrowing costs. In 2001, the world undergoes an unanticipated change to Model 3, the model with looser borrowing constraints, lower borrowing costs, and foreign holdings of U.S. bonds equal to 18% of GDP. The figure traces the first 6 years of the transition from the stochastic steady state of Model 1 to the stochastic steady state of Model 3. Along the transition path, agents use the policy functions from Model 3 evaluated at state variables that begin at the stochastic steady state values of Model 1, and gradually adjust to their stochastic steady state values of Model 3. Along the transition path, foreign holdings of U.S. bonds increase linearly from 0% in 2000 to 18% of GDP by 2006, and remain constant thereafter. In 2007, the world unexpectedly changes to Model 4. Model 4 is the same as Model 1 but with foreign holdings of U.S. bonds equal to 18% of GDP, as in Model 3 ("Reversal of FML in 2007"). The transition path is drawn for a particular sequence of aggregate productivity shocks in the housing and non-housing sectors, as explained in the text.

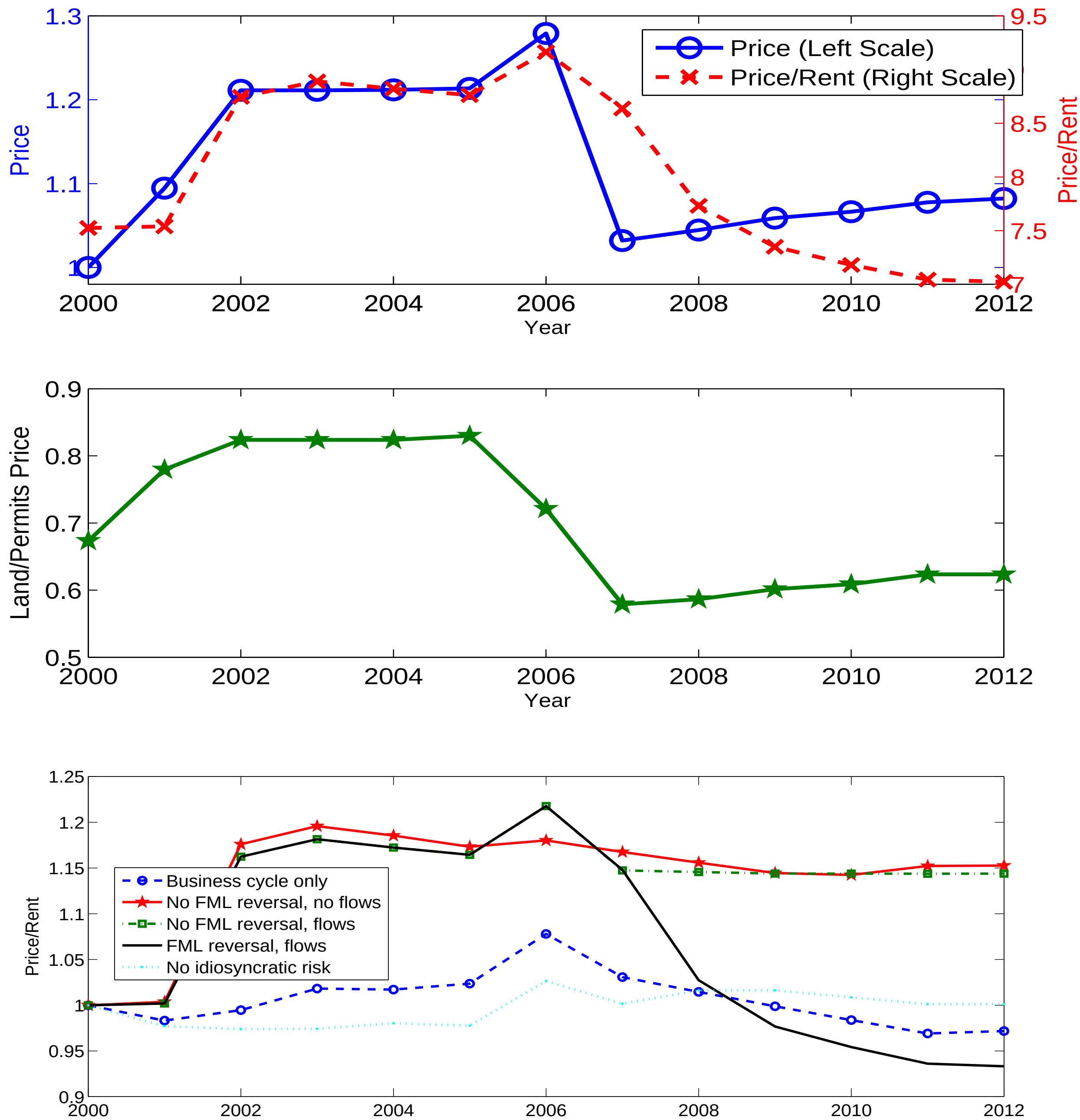


Table 1: Calibration

This table reports the parameter values of our model. The baseline "Model 1" is the model with normal collateral constraints and borrowing costs; "Model 2" reports on the model with looser collateral constraints and lower borrowing costs. In particular, the down-payment goes from 25% to 1% and the borrowing premium goes from 5.5 to 3.5%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 18% of GDP. The model is simulated for $N = 40,000$ agents.

	Parameter	Description	Baseline, Model 1	Model 2	Model 3
		Production			
1	φ	cap. adjustment cost	4		
2	δ	deprec., K_C, K_H	12% p.a.		
3	δ_H	depreciation, H	2.5% p.a.		
4	α	capital share, Y_C	0.36		
5	ν	capital share, Y_H	0.30		
6	ϕ	non-land share, Y_H	0.9		
		Preferences			
7	σ^{-1}	risk aversion	8		
8	χ	weight on C	0.70		
9	β	time disc factor	0.70		
10	ζ	fraction of bequesters	0.10		
11	ξ	strength of bequest	10^{16}		
		Demographics and Income			
12	G_a	age earnings profile	SCF		
13	$\pi_{a+1\|a}$	survival prob	mortality tables		
14	σ_E	st. dev ind earnings, E	0.0768		
15	σ_R	st. dev ind earnings, R	0.1298		
		Transactions Costs			
16	$\overline{F}$	participation cost, K	$\approx 1\%\ \overline{C}^i$		
17	ψ_0	fixed trans cost, H	$\approx 3.2\%\ \overline{C}^i$		
18	ψ_1	variable trans cost, H	$\approx 5.5\%\ p_t^H H^i$		
19	ϖ	collateral constr	25%	1%	1%
20	λ	borrowing cost	5.5%	3.5%	3.5%
		Foreign Supply			
21	B^F	foreign capital	0	0	$18\%\ \overline{Y}$

Table 2: Real Business Cycle Moments

Panel A denotes business cycle statistics in annual post-war U.S. data (1953-2012). The data combine information from NIPA Tables 1.1.5, 2.1, and 2.3.5. Output ($Y = Y_C + p^H Y_H + C_H$) is gross domestic product minus net exports minus government expenditures. Total consumption (C_T) is total private sector consumption (housing and non-housing). Housing consumption ($C_H = \mathcal{R} * H$) is consumption of housing services. Non-housing consumption (C) is total private sector consumption minus housing services. Housing investment ($p^H Y_H$) is residential investment. Non-housing investment (I) is the sum of private sector non-residential structures, equipment and software, and changes in inventory. Total investment is denoted I_T (residential and non-housing). For each series in the data, we first deflate by the disposable personal income deflator, We then construct the trend with a Hodrick-Prescott (1980) filter with parameter $\lambda = 100$. Finally, we construct detrended data as the log difference between the raw data and the HP trend, multiplied by 100. The standard deviation (first column), correlation with GDP (second column), and the first-order autocorrelation are all based on these detrended series. The autocorrelation AC is a one-year correlation in data and model. The share of GDP (fourth column) is based on the raw data. Panel B denotes the same statistics for the Model 1 with normal transaction costs and costs of borrowing. Panel C reports on Model 1B with lower costs of borrowing: the borrowing premium goes from 5.5 to 3.5%. Panel D reports on Model 2 which has looser collateral constraints than Model 1B and the same borrowing costs: the down-payment goes from 25% to 1%. Panel E reports on Model 3 which has the same borrowing costs and collateral constraints as Model 2 but has foreign capital of 18% of GDP.

	Panel A: Data (1953-2012)			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	3.00	1.00	0.51	1.00
C_T	1.90	0.92	0.65	0.80
C	2.14	0.92	0.63	0.66
C_H	1.45	0.55	0.71	0.14
I_T	8.84	0.93	0.40	0.20
I	9.07	0.82	0.33	0.14
$p^H Y_H$	13.95	0.77	0.60	0.06
	Panel B: Model 1			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.95	1.00	0.18	1.00
C_T	2.50	0.99	0.20	0.71
C	2.24	0.99	0.16	0.46
C_H	3.00	0.98	0.25	0.26
I_T	4.21	0.98	0.17	0.28
I	3.78	0.94	0.18	0.23
$p^H Y_H$	11.73	0.57	0.14	0.05
	Panel C: Model 1B			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	3.00	1.00	0.20	1.00
C_T	2.59	0.98	0.24	0.71
C	2.32	0.98	0.21	0.46
C_H	3.11	0.98	0.28	0.25
I_T	4.22	0.96	0.15	0.29
I	3.64	0.92	0.14	0.23
$p^H Y_H$	12.44	0.58	0.13	0.05

Real Business Cycle Moments (continued)

	Panel D: Model 2			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.95	1.00	0.17	1.00
C_T	2.50	0.97	0.17	0.70
C	2.27	0.96	0.14	0.47
C_H	2.98	0.97	0.20	0.23
I_T	4.37	0.95	0.18	0.29
I	3.81	0.94	0.20	0.23
$p^H Y_H$	12.95	0.50	0.14	0.06
	Panel E: Model 3			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	3.08	1.00	0.19	1.00
C_T	2.92	0.97	0.20	0.70
C	2.74	0.96	0.17	0.47
C_H	3.34	0.97	0.26	0.23
I_T	5.66	0.98	0.17	0.30
I	4.87	0.94	0.18	0.24
$p^H Y_H$	14.11	0.67	0.14	0.06

Table 3: Correlations House Prices and Real Activity

The table reports the correlations between house prices p^H, house price-rent ratios $p^H/\mathcal{R}$, and residential investment $p^H Y_H$ with GDP Y. It also reports the correlation of house price-to-rent ratios with the price-dividend ratio on stocks in the last column. Panel A is for the data. The house price and price-rent ratio are measured three different ways. In the first row (Data 1), the housing price is the aggregate value of residential real estate wealth in the fourth quarter of the year (Flow of Funds). The price-rent ratio divides this housing wealth by the consumption of housing services summed over the four quarters of the year (NIPA). In Data 2, the housing price is the repeat-sale Core Logic National House Price Index (series SFD). The price-rent ratio divides this price by the rental price index for shelter (BLS). It assumes a price rent ratio in 1975.Q4, equal to the one in Data 1. The price and price-rent ratio values in a given year are the fourth quarter values. The annual price indices, GDP, and residential investment are first deflated by the disposable personal income price deflator and then expressed as log deviations from their Hodrick-Prescott trend. Panel B is for the Model. Model 1 has benchmark collateral constraints and costs of borrowing. Model 1B has lower costs of borrowing: the borrowing premium goes from 5.5 to 3.5%, but the same collateral constraints as Model 1. Model 2 has looser collateral constraints than Model 1B and the same borrowing costs as in Model 1B: the down-payment goes from 25% to 1% . Model 3 which has the same borrowing costs and collateral constraints as Model 2 but has foreign capital of 18% of GDP.

Correlations	(Y, p^H)	$(Y, p^H/\mathcal{R})$	$(Y, p^H Y_H)$	$(p^H/\mathcal{R}, P/D)$
		Panel A: Data		
Data 1 (1953-2012)	0.42	0.46	0.77	0.28
Data 1 (1975-2012)	0.52	0.56	0.87	0.23
Data 2 (1975-2012)	0.48	0.44	0.87	0.48
		Panel B: Model		
Model 1	0.88	0.44	0.57	0.63
Model 1B	0.90	0.48	0.58	0.40
Model 2	0.92	0.53	0.50	0.82
Model 3	0.93	0.70	0.67	0.86

Table 4: Housing Wealth Relative to Total Wealth

The first column reports average housing wealth divided by average net worth. The second column reports average housing wealth of the young (head of household is aged 35 or less) divided by average net worth of the young. The third column reports average housing wealth of the old divided by average net worth of the old. The fourth (fifth) [sixth] column reports average housing wealth of the low (medium) [high] net worth households divided by average net worth of the low (medium) [high] net worth households. Low (medium) [high] net worth households are those in the bottom 25% (middle 50%) [top 25%] of the net worth distribution, relative to the cross-sectional net worth distribution at each age. The data in Panel A are from the Survey of Consumer Finance for 2001-2010. We exclude households with negative net worth. Panel B is for the model. Model 1 has benchmark collateral constraints and costs of borrowing. Model 1B has lower costs of borrowing: the borrowing premium goes from 5.5 to 3.5%, but the same collateral constraints as Model 1. Model 2 has looser collateral constraints than Model 1B and the same borrowing costs as in Model 1B: the down-payment goes from 25% to 1% . Model 3 has the same borrowing costs and collateral constraints as Model 2, but has foreign capital of 18% of GDP. In the model, housing wealth is $P_H * H$ and total wealth is $W + P_H * H$.

	Panel A: Data (SCF)					
	All	Young	Old	Poor	Med	Rich
2001	0.42	0.64	0.41	1.13	0.76	0.35
2004	0.52	0.98	0.49	1.41	0.87	0.43
2007	0.51	0.89	0.49	1.35	0.95	0.42
2010	0.49	0.92	0.48	1.78	0.95	0.41
	Y/P	Y/M	Y/R	O/P	O/M	O/R
2001	1.74	1.26	0.52	1.11	0.73	0.34
2004	2.82	1.70	0.83	1.36	0.83	0.41
2007	1.75	2.02	0.69	1.33	0.89	0.40
2010	4.52	1.95	0.71	1.70	0.91	0.39
	Panel B: Model					
	All	Young	Old	Poor	Med	Rich
Model 1	0.56	0.65	0.54	1.56	1.13	0.31
Model 1B	0.58	0.65	0.56	1.67	1.24	0.30
Model 2	0.60	0.69	0.58	1.78	1.28	0.32
Model 3	0.61	0.72	0.59	1.71	1.27	0.32
	Y/P	Y/M	Y/R	O/P	O/M	O/R
Model 1	2.09	2.01	0.38	1.47	1.03	0.28
Model 1B	2.17	2.15	0.37	1.58	1.14	0.27
Model 2	2.63	2.61	0.40	1.66	1.14	0.29
Model 3	2.44	2.52	0.41	1.61	1.13	0.30

Table 5: Return Moments

The table reports the mean and standard deviation of the equity index, a levered claim to physical capital (Columns 1 and 2), the mean and standard deviation of the return on the housing index (Columns 3 and 4), the mean and standard deviation of the risk-free rate (Columns 5 and 6), and the ratio of the average excess return (in excess of the riskfree rate) on the equity index and the housing index, divided by the standard deviation of the return (Columns 7 and 8). Column (9) reports the change in the price-rent ratio, measured as the percentage change between 2000 and 2006 in the data and the percentage change relative to Model 1 in the model. Columns (10) and (11) report the average (across households) of individual housing returns and Sharpe ratios. The individual Sharpe ratio on housing is defined as the ratio of the individual excess housing return and the standard deviation of the individual return. Panel A reports the data. The housing index return and price-rent ratio are measured three different ways. In the first row (Data 1), the housing return is the aggregate value of residential real estate wealth in the fourth quarter of the year (Flow of Funds) plus the consumption of housing services summed over the four quarters of the year (NIPA) divided by the value of residential real estate in the fourth quarter of the preceding year. We subtract CPI inflation to express the return in real terms and population growth in order to correct for the growth in housing quantities due to population growth. In Data 2 (Data 3), the housing return uses the repeat-sale Core Logic National (Freddie Mac) House Price Index and the BLS rental price index for shelter. It assumes a price rent ratio in 1975 equal to the one in Data 1 in 1975. We subtract realized CPI inflation from realized housing returns to form monthly real housing returns. We construct annual real housing returns by compounding monthly real housing returns over the year. The equity index return in the data is measured as the CRSP value-weighted stock return. We subtract realized annual CPI inflation from realized annual stock returns between 1953 and 2012 to form real annual stock returns. The risk-free rate is measured as the yield on a one-year government bond at the start of the year minus the realized inflation rate over the course of the year. The data are from the Fama-Bliss data set and available from 1953 until 2012. There are no good data for the individual housing returns for our sample. Panel B is for the model. The leverage ratio (debt divided by equity) we use in the model is 2/3: $R_E = R_f + (1 + B/E)(R_K - R_f)$, where R_K is the return on physical capital.

	(1) $E[R_E]$	(2) $Std[R_E]$	(3) $E[R_H]$	(4) $Std[R_H]$	(5) $E[R_f]$	(6) $Std[R_f]$	(7) $SR[R_E]$	(8) $SR[R_H]$	(9) $\Delta p^H/\mathcal{R}$	(10) $E_N[R_H]$	(11) $Std_N[R_H]$	(12) $SR_N[R_H]$
Panel A: Data												
Data 1 (53-12)	8.26	18.73	10.83	5.60	1.38	2.56	0.38	1.57	31.1%			
Data 1 (76-12)	8.72	16.83	10.44	6.58	1.87	2.83	0.42	1.19	31.1%			
Data 2 (76-12)	8.72	16.83	9.23	6.99	1.87	2.83	0.42	0.98	48.9%			
Data 3 (76-12)	8.72	16.83	10.04	5.35	1.87	2.83	0.42	1.39	32.3%			
Panel B: Model												
Model 1	5.00	7.90	12.25	6.55	2.12	3.05	0.40	1.82	−−	10.29	9.03	1.14
Model 1B	4.78	7.73	11.33	6.63	2.25	2.88	0.37	1.61	8.0%	9.22	8.68	1.06
Model 2	5.06	7.88	9.64	5.91	2.63	3.05	0.34	1.41	25.8%	7.07	7.04	1.00
Model 3	5.27	10.02	9.67	7.61	1.93	4.05	0.37	1.23	26.8%	7.91	7.85	1.01

Table 6: Predictability

Panel A reports the coefficients, t-stats, and R^2 of real return and real dividend growth predictability regressions. The return regression specification is: $\frac{1}{k}\sum_{j=1}^{k} r_{t+j}^{i} = \alpha + \kappa^{r} pd_t^i + \varepsilon_{t+k}$, where k is the horizon in years, r^i is the log housing return (left panel) or log stock return (right panel), and pd_t^i is the log price-rent ratio (left panel) or price-dividend ratio on equity (right panel). The dividend growth predictability specification is similar: $\frac{1}{k}\sum_{j=1}^{k} \Delta d_{t+j}^{i} = \alpha + \kappa^{d} pd_t^i + \varepsilon_{t+k}$, where Δd^i is the log rental growth rate (left panel) or log dividend growth rate on equity (right panel). Panel B reports the the coefficients, t-stats, and R^2 of *excess* return predictability regressions. The return regression specification is: $\frac{1}{k}\sum_{j=1}^{k} r_{t+j}^{i,e} = \alpha + \kappa^{r,e} pd_t^i + \varepsilon_{t+k}$, where k is the horizon in years, $r^{i,e}$ is the log real housing return in excess of a real short-term bond yield (left panel) or the log real stock return in excess of a real short-term bond yield (right panel), and pd_t^i is the log price-rent ratio (left panel) or price-dividend ratio on equity (right panel). In the model, we use the return on physical capital for the real return on equity and the return on the one-year bond as the real bond yield. The model objects are obtained from a 1750-year simulation, where the first 250 periods are discarded as burn-in. The model is the benchmark Model 1. The housing return is the aggregate housing return defined in the main text. In the data, we use the CRSP value-weighted stock return, annual data for 1953-2012. The housing return in the data is based on the annual Flow of Funds data for 1953-2012 (Data 1). We subtract CPI inflation to obtain the real returns and real dividend or rental growth rates. The real bond yield is the 1-year Fama-Bliss yield in excess of CPI inflation.

Panel A: Raw Returns and Dividends/Rents

	Housing - Model 1							Equity - Model 1					
k	κ^r	t-stat	R^2	κ^d	t-stat	R^2	k	κ^r	t-stat	R^2	κ^d	t-stat	R^2
1	−0.86	−19.22	18.90	−0.21	−6.97	2.71	1	−0.11	−14.69	12.58	0.40	21.72	25.09
2	−0.59	−22.17	27.49	−0.19	−10.20	6.37	2	−0.08	−15.38	16.94	0.28	24.88	36.60
3	−0.45	−25.06	34.67	−0.17	−12.56	9.85	3	−0.06	−16.60	20.97	0.22	26.90	42.60
5	−0.28	−29.28	40.97	−0.11	−13.79	12.57	5	−0.04	−18.32	22.73	0.13	27.18	41.77
10	−0.15	−36.56	52.19	−0.05	−14.26	13.88	10	−0.02	−22.38	29.31	0.07	28.07	47.45
20	−0.08	−44.06	59.72	−0.03	−15.74	15.84	20	−0.01	−26.87	39.02	0.04	35.25	58.02
30	−0.05	−43.04	60.31	−0.02	−14.89	15.60	30	−0.01	−23.49	38.49	0.03	32.72	63.55
	Housing - Data							Equity - Data					
k	κ^r	t-stat	R^2	κ^d	t-stat	R^2	k	κ^r	t-stat	R^2	κ^d	t-stat	R^2
1	-0.04	-0.6	1.1	0.00	0.3	0.1	1	-0.10	-2.4	5.2	-0.02	-0.5	0.2
2	-0.11	-3.0	11.0	-0.00	-0.2	0.0	2	-0.09	-2.4	8.8	-0.00	-0.1	0.0
3	-0.16	-5.6	27.2	0.00	0.0	0.0	3	-0.08	-3.5	12.4	0.01	0.3	0.1
5	-0.20	-5.9	55.2	-0.01	-0.4	0.7	5	-0.07	-5.4	16.6	0.02	1.3	2.3

Panel B: Excess Returns

	Housing - Model 1			Equity - Model 1			
k	$\kappa^{r,e}$	t-stat	R^2	k	$\kappa^{r,e}$	t-stat	R^2
1	−0.67	−12.2	11.3	1	−0.11	−13.5	11.8
2	−0.21	−7.3	4.9	2	−0.04	−8.5	5.4
3	−0.14	−6.1	4.0	3	−0.02	−6.5	4.1
5	−0.10	−5.6	3.7	5	−0.02	−4.6	2.6
10	−0.07	−6.3	5.2	10	−0.01	−3.9	2.9
20	−0.04	−7.1	6.7	20	−0.01	−3.6	2.5
30	−0.03	−7.0	6.9	30	−0.00	−2.4	1.4
	Housing - Data			Equity - Data			
k	$\kappa^{r,e}$	t-stat	R^2	k	$\kappa^{r,e}$	t-stat	R^2
1	-0.02	-0.2	0.2	1	-0.10	-2.3	5.2
2	-0.10	-1.4	6.5	2	-0.09	-2.1	8.4
3	-0.17	-4.4	19.9	3	-0.07	-2.8	11.6
5	-0.21	-10.1	44.1	5	-0.06	-3.8	14.2

Table 7: Inequality

This table reports various measures of cross-sectional risk sharing: the cross-sectional standard deviation of the consumption share $C^i_{T,a,t}/C_{T,t}$, the cross-sectional standard deviation of the intertemporal marginal rate of substitution, and the Gini coefficients of consumption, total wealth, housing wealth, and financial wealth. We simulate the model for $N = 2400$ households and for $T = 1150$ periods (the first 150 years are burn-in and discarded). The data are from the Survey of Consumer Finances. All numbers are multiplied by 100.

	Panel A: Data		
	Gini wealth	Gini housing	Gini fin.
2001	80.31	62.63	93.99
2004	80.47	64.26	100.28
2007	80.65	62.42	102.62
2010	83.75	61.77	108.87
	Panel B: Model		
	Gini wealth	Gini housing	Gini fin.
Model 1	75.63	45.89	120.95
Model 1B	76.16	44.61	128.25
Model 2	76.49	42.88	133.56
Model 3	75.16	41.91	136.06
	CS stdev IMRS	CS stdev cons share	Gini cons
Model 1	73.52	74.88	37.98
Model 1B	73.00	74.63	37.67
Model 2	72.19	74.34	37.74
Model 3	71.63	74.14	37.08

Table 8: Robustness: Real Business Cycle Moments

The table reports the same moments as Table 2. Panel A repeats the benchmark results. Panel B considers an alternative economy with a higher land share of 25% instead of 10%. Panel C contains results for a model where housing depreciation contains an additive idiosyncratic component which is 0% or 5% with equal probability. Panel D contains results for a model with idiosyncratic unemployment risk: In expansions, idiosyncratic income is 0.0112 with probability 0.95 and -0.21 with probability 0.05, while in recessions it is 0.0208 with probability 0.95 and -0.39 with probability 0.05.

	Panel A: Benchmark (Model 1)			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.95	1.00	0.18	1.00
C_T	2.50	0.98	0.20	0.71
C	2.23	0.98	0.16	0.46
C_H	3.03	0.97	0.26	0.26
I_T	4.25	0.97	0.17	0.28
I	3.84	0.93	0.18	0.23
$p^H Y_H$	11.79	0.56	0.14	0.05
	Panel B: Model with Higher Land Share (Model 1)			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.96	1.00	0.19	1.00
C_T	2.55	0.98	0.24	0.71
C	2.29	0.98	0.21	0.45
C_H	3.06	0.97	0.28	0.25
I_T	4.13	0.96	0.15	0.28
I	3.80	0.93	0.12	0.23
$p^H Y_H$	10.53	0.59	0.13	0.05
	Panel C: Model with Depreciation Shocks (Model 1)			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.95	1.00	0.18	1.00
C_T	2.50	0.98	0.19	0.71
C	2.25	0.98	0.15	0.46
C_H	2.99	0.98	0.23	0.25
I_T	4.24	0.97	0.16	0.28
I	3.79	0.93	0.18	0.23
$p^H Y_H$	12.37	0.56	0.13	0.05
	Panel D: Model with Idiosyncratic Unemployment Risk (Model 1)			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.95	1.00	0.16	1.00
C_T	2.67	0.98	0.15	0.72
C	2.51	0.98	0.14	0.46
C_H	2.99	0.98	0.17	0.25
I_T	3.92	0.95	0.18	0.28
I	3.29	0.92	0.19	0.23
$p^H Y_H$	13.22	0.54	0.13	0.05

Table 9: Robustness: Return Moments

The table reports the same moments as Table 5. Panel A repeats the benchmark results. Panel B considers an alternative economy with a higher land share of 25% instead of 10%. Panel C contains results for a model where housing depreciation contains an additive idiosyncratic component which is 0% or 5% with equal probability. Panel D contains results for a model with idiosyncratic unemployment risk: In expansions, idiosyncratic income is 0.0112 with probability 0.95 and -0.21 with probability 0.05, while in recessions it is 0.0208 with probability 0.95 and -0.39 with probability 0.05.

	(1) $E[R_E]$	(2) $Std[R_E]$	(3) $E[R_H]$	(4) $Std[R_H]$	(5) $E[R_f]$	(6) $Std[R_f]$	(7) $SR[R_E]$	(8) $SR[R_H]$	(9) $\Delta p^H/\mathcal{R}$	(10) $E_N[R_H]$	(11) $Std_N[R_H]$	(12) $SR_N[R_H]$
Panel A: Benchmark Model												
Model 1	5.00	7.90	12.25	6.55	2.12	3.05	0.40	1.82	$--$	10.29	9.03	1.14
Model 3	5.27	10.02	9.67	7.61	1.93	4.05	0.37	1.23	26.8%	7.91	7.85	1.01
Panel B: Model with Higher Land Share												
Model 1	4.86	8.22	12.41	6.43	1.95	2.96	0.39	1.92	$--$	10.61	9.08	1.17
Model 3	5.06	10.10	9.70	7.60	1.53	4.10	0.38	1.23	22.1%	8.43	8.47	1.00
Panel C: Model with Depreciation Shocks												
Model 1	4.99	7.89	12.32	6.48	2.09	3.10	0.40	1.86	$--$	10.38	9.32	1.11
Model 3	5.26	10.04	10.03	7.99	1.74	3.98	0.39	1.25	23.6%	8.42	8.74	0.96
Panel D: Model with Idiosyncratic Unemployment Risk												
Model 1	5.07	6.92	13.05	6.55	2.13	2.61	0.46	1.97	$--$	11.12	8.91	1.25
Model 3	5.38	8.83	11.15	8.11	1.83	3.62	0.44	1.38	17.7%	9.24	8.79	1.05

Figure A.1: Fixed-rate Mortgage Rate and Ten-Year Constant Maturity Treasury Rate

The solid line plots the 30-year Fixed-Rate Mortgage rate (FRM); the dashed line plots the ten-year Constant Maturity Treasury Yield (CMT). The FRM data are from Freddie Mac's Primary Mortgage Market Survey. They are average contract rates on conventional conforming 30-year fixed-rate mortgages. The CMT yield data are from the St.-Louis Federal Reserve Bank (FRED II). The data are monthly from January 1971 until December 2012.

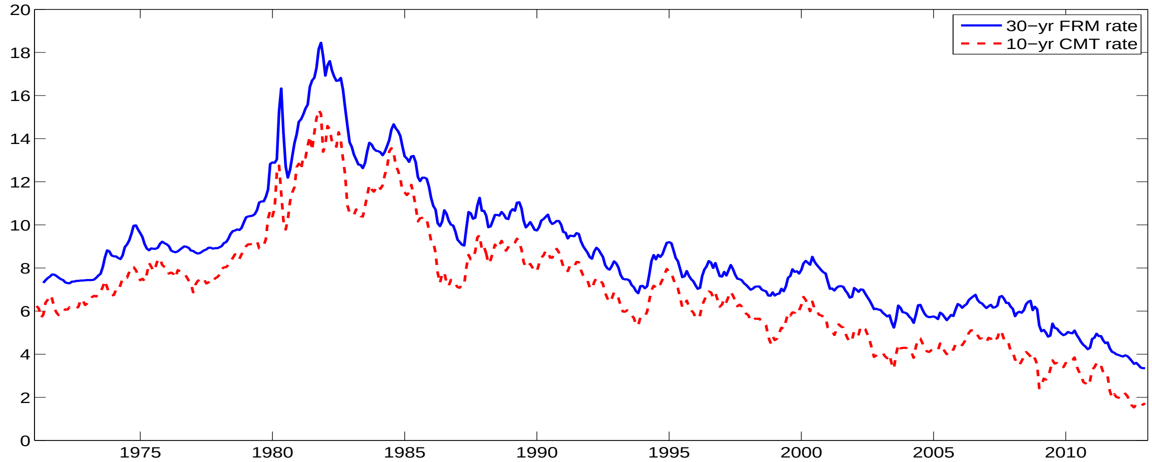
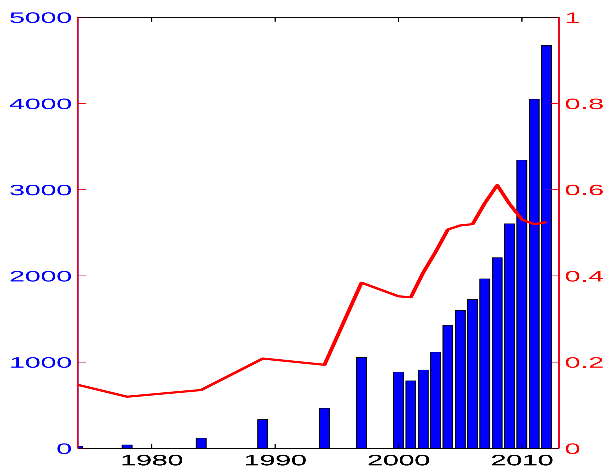


Figure A.2: Foreign Holdings of US Treasuries

Panel A plots foreign holdings of U.S. Treasuries. The bars, measured against the left axis, plot foreign holdings of long-term U.S. Treasury securities (T-notes, and T-bonds). It excludes (short-term) T-bills, measured in millions of nominal U.S. dollars. The solid line, measured against the right axis, plots those same holdings as a percent of total marketable U.S. Treasuries. Marketable U.S. Treasuries are available from the Office of Public Debt, and are measured as total marketable held by the public less T-bills. Panel B plots foreign holdings of U.S. Treasury securities (T-bills, T-notes, and T-bonds) and the sum of U.S. treasuries and U.S. Agency debt (e.g., debt issued by Freddie Mac and Fannie Mae), relative to GDP. The first two series report only long-term debt holdings, while the other two series add in short-term debt holdings. Since no short-term debt holdings are available before 2002, we assume that total holdings grow at the same rate as long-term holdings before 2002. The foreign holdings data from the Treasury International Capital System of the U.S. Department of the Treasury. The foreign holdings data are available for December 1974, 1978, 1984, 1989, 1994, 1997, March 2000, and annually for June 2002 through June 2012. Nominal GDP is from the National Income and Product Accounts, Table 1.1.5, line 1.

Panel A: Foreign Holdings of U.S. Treasuries



Panel B: Foreign Holdings Relative to GDP

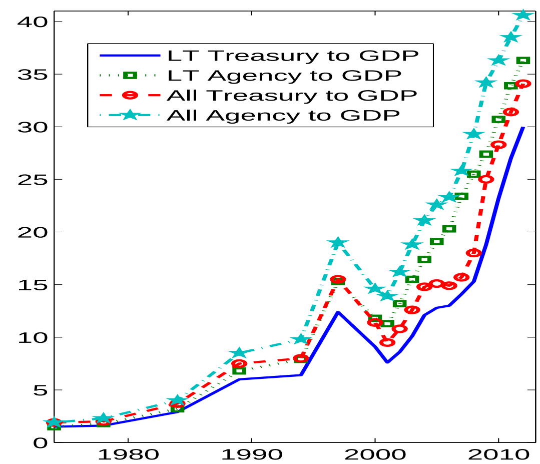


Table A.1: Numerical Checks

This table describes the accuracy of our forecasting equations. For various horizons and simulations (described in the text), we report the R^2, defined as $1 - \frac{\sigma(x_t - \overline{x}_t)^2}{\sigma(x_t)^2}$ where x_t is the actual variable and $\widehat{x}_t$ is its forecast. The simulation can be done with the same sequence of aggregate shocks as the simulation used to compute the original coefficients (in-sample) or with a new sequence of aggregate shocks (out-of-sample). The R^2 statistics are computed either one period ahead ($k = 1$), or using the full sample procedure described in Den Haan (2010) ($K = \infty$).

R^2 in Forecasting Equation			
In/Out of Sample	In	In	Out
k (periods ahead)	1	∞	∞
A1	0.997	0.942	0.917
A2	0.999	0.997	0.996
A3	1.000	1.000	1.000
A4	0.996	0.973	0.970

Table A.2: Euler equation errors

This table reports the Euler equation errors for bequesters whose portfolio weight in bonds is between 0% and 100% (this includes most of the bequesters). We report the errors for models 1, 2, and 3 for the return on equity, and the net return on the portfolio.

Model	1	2	3
$E[M * R^e - 1]$	-0.1%	0.0%	0.2%
$E[M * R^p - 1]$	-0.2%	-0.2%	0.0%