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ABSTRACT

Much recent research has focused on the development and analysis of extensions of the New Keynesian framework that model labor market frictions and unemployment explicitly. The present paper describes some of the essential ingredients and properties of those models, and their implications for monetary policy.

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An online appendix is available at:
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1 Introduction

The existence of involuntary unemployment has long been recognized as one of the main ills of modern industrialized economies. And the rise in unemployment that invariably accompanies all economic downturns is, arguably, one of the main reasons why cyclical fluctuations are generally viewed as undesirable.

Despite the central role of unemployment in the policy debate, that variable has been—at least until recently—conspicuously absent from the new generation of models that have become the workhorse for the analysis of monetary policy, inflation and the business cycle, and which are generally referred to as New Keynesian.¹ That absence may be justified on the grounds that explaining unemployment and its variations has never been the focus of that literature, so there was no need to model that phenomenon explicitly. But this could be interpreted as suggesting that there is no independent role for unemployment—as distinguished, say, from measures of output or employment—as a determinant of inflation (or other macro variables) or as a variable that central banks should be concerned about and even respond to in a systematic way. In other words, under the previous view, unemployment and the frictions underlying it are not essential for understanding fluctuations in nominal and real variables, nor a key ingredient in the design of monetary policy.²

¹The reader can find a textbook exposition of the New Keynesian model in Walsh (2003), Woodford (2003), and Galí (2008). An early version and analysis of the baseline New Keynesian model can be found in Yun (1996), who used a discrete-time version of the staggered price-setting model originally developed in Calvo (1983). King and Wolman (1996) provided a detailed analysis of the steady state and dynamic properties of the model. Goodfriend and King (1997), Rotemberg and Woodford (1999) and Clarida, Galí and Gertler (1999) were among the first to conduct a normative policy analysis using that framework.

²The term "unemployment" cannot be found in the index of Walsh (2003) or Woodford

On the other hand, understanding the determinants of unemployment and the nature of its fluctuations has been at the heart of a parallel literature, one that has built on search and matching models in the Diamond-Mortensen-Pissarides tradition.³ Since the influential work of Hall (2005) and Shimer (2005), pointing to the difficulties of a calibrated version of such a model to account for the size of observed fluctuations in unemployment and other labor market variables, that literature has taken a more quantitative turn and sparked the interest of mainstream macroeconomists. Yet, and at least until recently, the models used in that literature have been purely *real*, and hence they had nothing to say about the role of monetary policy, either as a source of unemployment fluctuations, or as a tool to stabilize those fluctuations.⁴

Over the past few years, however, a growing number of researchers have turned their attention towards the development and analysis of frameworks that combine elements from the two traditions described above. The typical framework in this literature combines the nominal rigidities and consequent monetary non-neutralities of New Keynesian models with the real frictions in labor markets that are characteristic of the search and matching models. To the extent of my knowledge, Chéron and Langot (2000) were the first to bring together nominal rigidities and labor market frictions, showing how the resulting framework could generate both a Beveridge curve (a negative

(2003), two textbooks providing a modern treatment of monetary economics. In Galí (2008) I briefly mention "unemployment" in the concluding chapter, but only in reference to the recent extensions of the New Keynesian model discussed in the present paper.

³Early contributions to the current vintage of search and matching models include Diamond (1982 a,b), Mortensen (1982 a,b) and Pissarides (1984). See Pissarides (2000) for a comprehensive exposition of the search and matching approach.

⁴Incidentally, it is worth pointing out that standard RBC models share the shortcomings of both paradigms: they neither can explain involuntary unemployment nor have any role for monetary policy.

correlation between vacancies and unemployment) and a Phillips curve (a negative correlation between inflation and unemployment) in the presence of both technology and monetary shocks. Subsequently, Walsh (2003, 2005) and Trigari (2009) analyzed the impact of embedding labor market frictions into the basic New Keynesian model with sticky prices but flexible wages, with a focus on the size and persistence of the effects of monetary policy shocks.

More recent contributions have extended that work in two dimensions. First, they have relaxed the assumption of flexible wages, and introduced different forms of nominal and real wage rigidity. The work of Trigari (2006) and Christoffel and Linzert (2005) fall into that category. Secondly, the focus of analysis has gradually turned to normative issues, and more specifically, to the implications of labor market frictions and unemployment for the design of monetary policy. Thus, the work of Blanchard and Galí (2010) (in a model with real wage rigidities) and Thomas (2008) (under nominal wage rigidities) provides an explicit analysis of the optimal monetary policy in the context of a simple New Keynesian model with labor market frictions.⁵ As argued below, and perhaps not surprisingly, those two extensions are not unrelated: the presence of wage rigidities has important implications, not only for the macroeconomic effects of different shocks, but also for the relative desirability of alternative policies.

While still in its infancy, the abovementioned literature has already provided some insights of interest and has laid the ground for a possible "evolution" of the estimated DSGE models currently used for policy analysis, one that would introduce labor market frictions and unemployment explicitly in

⁵See also the analysis in Arseneau and Chugh (2008) in a model with flexible prices and quadratic costs of nominal wage adjustment.

the full-fledged monetary models of the kind originally developed by Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2008). The recent work of Gertler, Sala and Trigari (2008) provides an excellent illustration of the progress being made in that direction.

The objective of the present paper is twofold. First, to describe some of the essential ingredients of a model that combines labor market frictions and nominal rigidities. And, secondly, to illustrate how such a model can be used to address questions of interest pertaining to the interaction between labor market frictions and nominal rigidities. Two broad questions are emphasized in the analysis below:

- What is the role of labor market frictions in shaping the economy's response to aggregate shocks?
- What are the implications of those frictions for the design of monetary policy? In particular, should central banks pay attention to unemployment when setting interest rates?

In order to address those questions, I develop an extension of the New Keynesian model that allows for labor market frictions and unemployment. The model is highly stylized, combining elements found in existing papers, but abstracting from ingredients that (in my view) are not essential given the purpose at hand. Relative to the relevant literature, the main novelty of the framework developed here lies in the introduction of variable labor market participation. That feature is meant to overcome the surprising contrast between the importance given by the New Keynesian literature to the elasticity of labor supply (e.g. as a determinant of the persistence of the effects of monetary policy shocks) and the assumption of a fully inelastic

labor supply found almost invariably in existing models with labor market frictions. In the latter, changes in unemployment match one-for-one those in employment (with the opposite sign), so there is no information contained in measures of unemployment that is not revealed by observing employment.

Several lessons emerge from the analysis below, which are summarized next in the form of bullet points.

- Quantitatively realistic labor market frictions are likely to have, by themselves, a limited effect on the economy's equilibrium dynamics. Instead, their main role is "to make room" for wage rigidities, with the latter leading to inefficient responses to shocks and significant trade-offs for monetary policy.
- When combined with a realistic Taylor-type rule, the introduction of price rigidities in a model with labor market frictions has a limited impact on the economy's equilibrium response to real shocks (though, of course, it is sufficient to make monetary policy non-neutral).
- If the conditions that guarantee the efficiency of the steady state are assumed, the optimal policy under flexible wages (i.e. wages subject to period-by-period Nash bargaining) is one of strict inflation targeting, which requires that the price level be stabilized at all times. If, instead, nominal wages are bargained over and readjusted infrequently, the optimal policy involves moderate deviations from price stability and can be approximated well by a simple interest rate rule that responds to price inflation with a coefficient of about 1.5.
- Deviations in the unemployment rate from its efficient level are generally a source of welfare losses above and beyond those generated by

fluctuations in the output or employment gaps. An optimized simple interest rate rule calls for a systematic (though relatively weak) stabilizing policy response to inefficient fluctuations in unemployment.

The paper is organized as follows. Section 2 presents some evidence on the cyclical behavior of labor market variables and inflation, as well as a simple structural interpretation of their fluctuations. Section 3 develops a baseline model with labor market frictions and price rigidities, allowing for two alternative wage-setting environments (flexible and sticky wages). Section 4 discusses the properties of a calibrated version of the model, focusing on the implied responses to monetary and technology shocks. Section 5 presents the welfare criterion associated with the model under the assumption of an efficient steady state, and discusses the responses to a technology shock under the optimal monetary policy and the optimal simple rule. Section 6 discusses possible model extensions. Section 7 concludes. References and discussion of the relevant literature can generally be found at the end of each section.

2 Evidence on the Cyclical Behavior of Labor Market Variables and Inflation

This section summarizes the cyclical properties of employment, the labor force, the unemployment rate, the real wage and inflation in the postwar U.S. economy. I use quarterly data corresponding to the sample period 1948Q1-2008Q4 and drawn from the HAVER database. GDP is taken to be the benchmark cyclical indicator. As a wage measure I use hourly compensation in the nonfarm business sector. The GDP deflator is the price level used to compute inflation and the real wage. Employment, the labor force, and GDP

are normalized by working age population and, together with the real wage, are expressed in natural logarithms. All variables are detrended using a band-pass filter that seeks to preserve fluctuations with a periodicity between 6 and 32 quarters.

The first panel of Table 1 reports two key unconditional second moments for the cyclical component of each variable: its standard deviation relative to GDP and its correlation with GDP. Many of the facts reported here are well known but are summarized here as a reminder. Thus, note that employment is substantially more volatile than the labor force, with unemployment lying somewhere in between. The real wage is also shown to be substantially less volatile than GDP. Turning to the correlation with GDP, we see that both employment and the labor force are procyclical, though the latter only moderately so (their respective correlations are 0.83 and 0.30). The unemployment rate is highly countercyclical, with a correlation with GDP close to -0.9 . Price inflation is mildly procyclical, but the real wage is essentially acyclical.

In addition to the unconditional statistics just summarized, Table 1 also reports conditional statistics based on a decomposition of each variable into "technology-driven" and "demand-driven" components. The decomposition is based on a partially-identified VAR with five variables: (log) labor productivity, (log) employment, the unemployment rate, price inflation and the average price markup. The latter is computed as the difference between (log) labor productivity and the (log) real wage.⁶ Following the strategy proposed in Galí (1999), I identify technology shocks as the only source of

⁶The baseline results discussed below are based on a specification of the VAR with (log) employment in first differences and the unemployment rate detrended using a second order polynomial of time. The main findings are robust to an alternative specification with employment detrended in log-levels.

the unit root in labor productivity. The structural VAR contains four additional shocks that are left unidentified, and referred to loosely as "demand" shocks. I define the "demand" component of each variable of interest as the sum of its components associated with each of those four shocks.⁷

The second and third panels in Table 1 report some statistics of interest for the demand and technology components of a number of variables, computed after detrending the estimated components with a band-pass filter analogous to the one applied earlier to the raw data. Note that the conditional second moments associated with the demand-driven component are very similar to the unconditional second moments. This is not surprising once one realizes that non-technology shocks account for the bulk of the volatility of the cyclical component of all variables (statistics not shown here). The only exception lies in the strong negative correlation between the real wage and GDP conditional on demand shocks, which contrasts with the near zero unconditional correlation between the same variables.

The conditional statistics associated with the technology-driven components are shown in the third panel of Table 1. Note that the labor force is now largely acyclical and the real wage mildly procyclical, both of which contrast with the corresponding unconditional statistics. Also, while the technology components of employment and the unemployment rate are shown to be procyclical and countercyclical, as measured by the corresponding correlation with GDP, a look at the estimated dynamic responses of those variables to a technology shock reveals a more complex pattern. Figure 1 displays the estimated responses to a favorable technology shock, i.e. one which is shown to increase output and labor productivity permanently. Note that

⁷The reader is referred to Galí (1999) for a detailed description of the econometric approach.

output hardly changes in the short run, with its response building up only gradually over time. On the other hand, employment declines on impact in response to that shock, and only gradually reverts back to its initial level. A similar result can be found in Galí (1999), Basu, Fernald and Kimball (2006), Francis and Ramey (2005), and Galí and Rabanal (2004), among others, using alternative VAR specifications (and with a focus on hours rather than employment).⁸ The previous authors have argued that such estimated responses to a technology shock are at odds with the predictions of a standard calibrated real business cycle model, which would call for a simultaneous upward adjustment of output and employment in response to a technology improvement. The existence of short-run demand constraints, possibly resulting from the interaction of nominal rigidities and a not-fully-accommodating monetary policy, has been posited as an explanation for that evidence.

Figure 1 also provides evidence on the response of variables other than output and employment to a positive technology shock. In particular we see that the labor force declines slightly but permanently after that shock. That decline in the labor force can only offset partially the larger fall the large drop in employment, thus leading to a persistent increase in the unemployment rate, which is only reverted after six quarters. Similar evidence of a short run rise in unemployment in response to a positive supply shock can also be found Blanchard and Quah (1989) and, more recently, by Barnichon (2008). The latter author argues that such evidence implies a rejection of a central prediction of the standard search and matching model, though it can be

⁸The previous evidence is not uncontroversial. For a critical perspective on that evidence see Christiano, Eichenbaum and Vigfusson (2003) and Chari, Kehoe and McGrattan (2008).

accounted for once that model is extended to allow for nominal rigidities and a suitable monetary policy rule.

Next I explore whether a model that combines nominal rigidities and labor market frictions can account for different aspects of the evidence just described.

3 A Model with Nominal Rigidities and Labor Market Frictions

3.1 Households

I assume a large number of identical households. Each household is made up of a continuum of members represented by the unit interval. There is assumed to be full consumption risk sharing within each household.⁹ The household seeks to maximize the objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{1}$$

where $\beta \in [0, 1]$ is the discount factor, $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$ is an index of the quantities consumed of the different types of final goods, and L_t is an index of the total effort or time that household members allocate to labor market activities. More specifically, I define L_t as

$$L_t = N_t + \psi U_t \tag{2}$$

where N_t and U_t denote, respectively, the fraction of household members who are employed and unemployed (and looking for a job).¹⁰ Parameter $\psi \in$

⁹Merz (1995) was the first to adopt a the assumption of a representative "large" household with a conventional utility function in the context of a search model.

¹⁰I focus on variations in labor input at the extensive margin, and abstract from possible variations over time in hours per worker (or effort per worker). Even though the latter

$[0, 1]$ represents the marginal disutility generated by an unemployed member relative to an employed one. Non-participation in the labor market generates no disutility to the household. Note that the labor force (or participation rate) is given by $N_t + U_t \equiv F_t$. The following constraints must be satisfied for all t : $C_t(i) \geq 0$, all $i \in [0, 1]$, $0 \leq N_t + U_t \leq 1$, $U_t \geq 0$ and $N_t \geq 0$.

The household's period utility is assumed to take the form

$$U(C_t, L_t) \equiv \log C_t - \frac{\chi}{1 + \varphi} L_t^{1+\varphi} \quad (3)$$

and where the disutility implied by labor market activities can be interpreted as resulting from foregone leisure and/or consumption of home produced goods. Note that by setting $\psi = 0$ the resulting utility function specializes to one commonly used in monetary models of the business cycle. That specification is consistent with a balanced growth path and involves a direct parametrization of the Frisch labor supply elasticity, which is given by $1/\varphi$. On the other hand, if $\varphi = 0$ is assumed, we can interpret the term $\chi N_t + \chi\psi U_t$ as the sum of the disutilities of labor market activities of all household members, with work and unemployment generating, respectively, individual disutilities of χ and $\chi\psi$ (with no disutility generated by non-participation).¹¹ Note also that the chosen specification differs from the one generally used in the search and matching literature, where the marginal rate of substitution is assumed to be constant, thus implying a fully inelastic labor supply above a certain threshold wage.

Employment evolves over time according to

$$N_t = (1 - \delta)N_{t-1} + x_t U_t^0 \quad (4)$$

display non trivial cyclical movements in the data, its introduction seems unnecessary to convey the basic points made below. See Trigari (2009) and Thomas (2008), among others, for examples of related models that allow for variation in (disutility-generating) hours per worker.

¹¹See, e.g., Shimer (2009).

where δ is a constant separation rate, x_t is the job finding rate, and U_t^0 is the fraction of household members who are unemployed (and looking for a job) at the beginning of period t . Note that $U_t = (1 - x_t)U_t^0$.¹²

The household faces a sequence of budget constraints given by

$$\int_0^1 P_t(i)C_t(i)di + Q_tB_t \leq B_{t-1} + \int_0^1 W_t(j)N_t(j)dj + \Pi_t$$

where $P_t(i)$ is the price of good i , $W_t(j)$ is the nominal wage paid by firm j , B_t represents purchases of one-period bonds (at a price Q_t), and Π_t is a lump-sum component of income (which may include, among other items, dividends from ownership of firms or lump-sum taxes). The above sequence of period budget constraints is supplemented with a solvency condition which prevents the household from engaging in Ponzi schemes.

Optimal demand for each good takes the familiar form:

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \quad (5)$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ denotes the price index for final goods. Note also that (5) implies that total consumption expenditures can be written as $\int_0^1 P_t(i)C_t(i) di = P_tC_t$.

The intertemporal optimality condition is given by

$$Q_t = \beta E_t \left\{ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\} \quad (6)$$

In the model with frictionless, perfectly competitive labor markets the household would determine how much labor to supply, taking as given the

¹²Note that (4) implies that current hires become productive in the same period. This is the timing assumed in Blanchard and Galí (2009) and consistent with the bulk of the business cycle literature, where employment is assumed to be a non-predetermined variable. In contrast, most search and matching models assume it takes one period for a new hire to become productive, thus making employment predetermined, and preventing it from responding contemporaneously to shocks.

(single) market wage. The wage would adjust so that all the labor supplied is employed, implying the absence of involuntary unemployment. Thus, we would have $L_t = N_t$ for all t , and under the assumed preferences, an intratemporal optimality condition would hold, equating the real wage to the marginal rate of substitution, $W_t/P_t = \chi C_t N_t^\varphi$, and implicitly determining the quantity of labor supplied. The present model departs from that Walrasian benchmark in an important respect: the wage does not "automatically" adjust to guarantee that all the labor supplied is employed. Instead, the wage is bargained bilaterally between individual workers and firms in order to split the surplus generated by existing employment relations. Employment is then the result of the aggregation of firms' hiring decisions, given the wage protocol. In other words, employment is demand determined, with the households' participation decision influencing employment only indirectly, through its impact on wages and on hiring costs.

3.2 Firms

As in much of the literature on nominal rigidities and labor market frictions, I assume a model with a two-sector structure. Firms in the final goods sector do not use labor as an input, but are subject to nominal rigidities in the form of restrictions to the frequency of their price-setting decisions. On the other hand, firms in the intermediate goods sector take the price of the good they produce as given, use labor as an input (subject to hiring costs), and engage in wage bargaining with its workers. That modelling strategy, originally proposed in Walsh (2005), has the advantage of getting around the difficulties associated with having price setting decisions and

wage bargaining concentrated in the same firms.¹³

3.2.1 Final Goods

I assume a continuum of monopolistically competitive firms indexed by $i \in [0, 1]$, each producing a differentiated final good. All firms have access to an identical technology

$$Y_t(i) = X_t(i)$$

where $X_t(i)$ is the quantity of the (single) intermediate good used by firm i as an input.

Under *flexible prices* each firm would set the price of its good optimally each period, subject to a demand schedule with constant price elasticity ϵ .¹⁴ Profit maximization thus implies the familiar price-setting condition:

$$P_t(i) = \mathcal{M}^p(1 - \tau)P_t^I$$

where P_t^I is the price of the intermediate good, $\mathcal{M}^p \equiv \frac{\epsilon}{\epsilon-1}$ is the optimal or desired (gross) markup and τ is a subsidy on the purchases of intermediate goods. Note that $(1 - \tau)P_t^I$ is the nominal marginal cost facing the final goods firm. Since all firms choose the same price it follows that

$$P_t = \mathcal{M}^p(1 - \tau)P_t^I$$

for all t .

Instead of flexible prices, I assume in much of what follows a price-setting environment as in Calvo (1983), with each firm being able to adjust its price each period only with probability $1 - \theta_p$. That probability is independent

¹³See Kuester (2007) and Thomas (2008b) for an analysis of a version of the model where price setters are subject to labor market frictions.

¹⁴As discussed below, this requires that the demand of final goods coming from intermediate goods firms (in order to pay for their hiring costs), has the same price elasticity as the demand originating in households.

across firms and independent of the time elapsed since the last price adjustment. Thus, parameter $\theta_p \in [0, 1]$ also represents the fraction of firms that keep their prices unchanged in any given period and can thus be interpreted as an index of price rigidities.

All firms adjusting their price in any given period choose the same price, denoted by P_t^* , since they face an identical problem. The (log-linearized) optimal price setting condition in this environment is given by¹⁵

$$p_t^* = \mu^p + (1 - \beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k (E_t\{p_{t+k}^I\} - \tau) \quad (7)$$

where lower case letters denote the logs of the original variables, $\mu^p \equiv \log \mathcal{M}^p$. Thus, firms that adjust their price in any given period, choose a (log) price that is equal to the desired (log) markup over a weighted average of current and (expected) future (log) marginal costs, with the weights being a function of both the discount factor β and the Calvo parameter θ_p .

By combining (7) with the (log-linearized) law of motion for the aggregate price level given by¹⁶

$$p_t = \theta_p p_{t-1} + (1 - \theta_p) p_t^* \quad (8)$$

one can derive the inflation equation

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p \widehat{\mu}_t^p \quad (9)$$

where $\pi_t^p \equiv p_t - p_{t-1}$ is price inflation, $\widehat{\mu}_t^p \equiv \mu_t^p - \mu^p = p_t - (p_t^I - \tau) - \mu^p$ denotes the deviation of the (log) average price markup from its desired (and steady state) value, and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}$. Equation (9) makes clear that

¹⁵See, e.g. Galí (2008, chapter 3), for details of the derivation.

¹⁶Equation (8) can be derived by log-linearizing the expression for the aggregate price level P_t around a zero inflation steady state, and using the fact that a fraction $1 - \theta_p$ of firms set the same price P_t^* , while the price index for the remaining fraction that keep their price unchanged is P_{t-1} , since they are drawn randomly from the universe of firms.

whatever is the influence of labor market frictions and wage-setting practices on the dynamics of price inflation it must necessarily work through their impact on firms' markups, since variations in price inflation are the result of misalignments between current and desired price markups.

3.2.2 Intermediate Goods

The intermediate good is produced by a continuum of identical, perfectly competitive firms, represented by the unit interval and indexed by $j \in [0, 1]$. All such firms have access to a production function

$$Y_t^I(j) = A_t N_t(j)^{1-\alpha}$$

Variable A_t represents the state of technology, which is assumed to be common across firms and to vary exogenously over time. More precisely, I assume that $a_t \equiv \log A_t$ follows an $AR(1)$ process with autoregressive coefficient ρ_a and variance σ_a^2 .

Employment at firm j evolves according to

$$N_t(j) = (1 - \delta)N_{t-1}(j) + H_t(j) \tag{10}$$

where $\delta \in (0, 1)$ is an exogenous separation rate, and $H_t(j)$ represents the measure of workers hired by firm j in period t . Note that new hires start working in the period they are hired. That timing assumption, which follows Blanchard and Galí (2009), deviates from the standard one in the search and matching literature (which requires a one period lag before a hired worker becomes productive), but is consistent with conventional business cycle models, where employment is not a predetermined variable.

Labor Market Frictions. Following Blanchard and Galí (2009), I introduce labor market frictions in the form of a cost per hire, represented

by G_t and defined in terms of the bundle of final goods. That cost is assumed to be exogenous to each individual firm.

Though G_t is taken as given by each individual firm, it is natural to think of it as depending on aggregate factors. One natural such determinant is the degree of tightness in the labor market, which can be approximated by the job finding rate $x_t \equiv H_t/U_t^0$, i.e. the ratio of aggregate hires, $H_t \equiv \int_0^1 H_t(j) dj$, to the size of the unemployment pool at the beginning of the period, U_t^0 . More specifically, I assume¹⁷

$$\begin{aligned} G_t &= G(x_t) \\ &= \Gamma x_t^\gamma \end{aligned}$$

Relation to the matching function approach. The above formulation is equivalent to the matching function approach adopted by the search literature. Under the latter, firms and workers match according to a function $M(V_t, U_t^0)$ where V_t represents the number of aggregate vacancies, and where a firm can post vacancies at a unit cost Γ . Under the assumption of homogeneity of degree one in the matching function, the fraction of posted vacancies that get filled within the period is given by $M(V_t, U_t^0)/V_t \equiv q(V_t/U_t^0)$, where $q' < 0$. On the other hand, the job finding rate is given by $x_t = M(V_t, U_t^0)/U_t^0 \equiv p(V_t/U_t^0)$ where $p' > 0$. It follows that a fraction $q(p^{-1}(x_t))$ of vacancies posted are filled with the resulting cost per hire being given by $G_t = \Gamma/q(p^{-1}(x_t))$, which is increasing in x_t . In particular, under the assumption of a Cobb-Douglas matching function $M(V_t, U_t^0) = V_t^\varsigma U_t^{01-\varsigma}$ we

¹⁷Instead, Blanchard and Galí (2009) assume a hiring cost of the form $A_t \Gamma x_t^\gamma$. Though at the possible cost of less realism, that formulation has the advantage of preserving the homogeneity of the efficiency conditions with respect to the technology shock A_t , leading to an constrained-efficient allocations with a constant employment, which is a convenient benchmark.

have $G_t = \Gamma x_t^{\frac{1-\varsigma}{\varsigma}}$, which coincides with the above specification of the cost function, for $\gamma \equiv \frac{1-\varsigma}{\varsigma}$.

In the presence of labor market frictions, wages (and, as a result, employment) may differ across firms, since they cannot be automatically arbitrated out by workers switching from low to high wage firms. I make this explicit by using the subindex j to refer to the wage and other variables that are potentially firm-specific. Given a wage $W_t(j)$, the optimal hiring policy of firm j is described by the condition

$$MRPN_t(j) = \frac{W_t(j)}{P_t} + G_t - (1 - \delta) E_t \{ \Lambda_{t,t+1} G_{t+1} \} \quad (11)$$

where $MRPN_t(j) \equiv (P_t^I/P_t) (1-\alpha)A_t N_t(j)^{-\alpha}$ is the marginal revenue product of labor (expressed in terms of final goods) and $\Lambda_{t,t+k} \equiv \beta^k (C_t/C_{t+k})$ is the stochastic discount factor for k -period ahead (real) payoffs.¹⁸ In words, each period the firm hires workers up to the point where the marginal revenue product of labor equals the cost of a marginal worker. The latter, represented by the right hand side of (11), has three components: (i) the real wage $W_t(j)/P_t$, (ii) the hiring cost G_t , and (iii) the discounted savings in future hiring costs that result from having to hire $(1 - \delta)$ fewer workers the following period. Equivalently, and solving (11) forward, we have:

$$G_t = E_t \left\{ \sum_{k=0}^{\infty} \Lambda_{t,t+k} (1 - \delta)^k \left(MRPN_{t+k}(j) - \frac{W_{t+k}(j)}{P_{t+k}} \right) \right\}$$

i.e. the hiring cost must equate the (expected) surplus generated by the (marginal) worker.¹⁹

¹⁸Note that intermediate good firms are perfectly competitive and thus take the price P_t^I as given.

¹⁹Implicitly it is assumed that the firm is always doing some positive hiring. This will be the case if exogenous separations are large enough and shocks are small enough.

For notational convenience it is useful to define the *net* hiring cost as $B_t \equiv G_t - (1 - \delta)E_t \{\Lambda_{t,t+1} G_{t+1}\}$. Thus, one can rewrite (11) more compactly as:

$$MRPN_t(j) = \frac{W_t(j)}{P_t} + B_t \quad (12)$$

The previous optimality condition can be used to derive an expression for the (log) average price markup in the final goods sector, which was shown above to be the driving force of inflation. Using $n_t \simeq \int_0^1 n_t(j) dj$ and $w_t \simeq \int_0^1 w_t(j) dj$ as approximate measures of (log) aggregate employment and the (log) average nominal wage around a symmetric steady state, log-linearization of (12) and subsequent integration over all firms yields the following expression for the average markup in the final goods sector:²⁰

$$\widehat{\mu}_t^p = (a_t - \alpha \widehat{n}_t) - [(1 - \Phi) \widehat{\omega}_t + \Phi \widehat{b}_t] \quad (13)$$

where $\omega_t \equiv w_t - p_t$ is the average (log) real wage, and $\Phi \equiv \frac{B}{(W/P)+B}$ measures the importance of (non-wage) hiring costs relative to the wage. Also, note for future reference that

$$\widehat{b}_t = \frac{1}{1 - \beta(1 - \delta)} \widehat{g}_t - \frac{\beta(1 - \delta)}{1 - \beta(1 - \delta)} (E_t \{\widehat{g}_{t+1}\} - \widehat{r}_t) \quad (14)$$

where $\widehat{g}_t = \gamma \widehat{x}_t$ and where r_t denotes the real return on a riskless one-period bond.²¹

Finally, note that (12) also implies

$$\alpha (n_t(j) - n_t) = -(1 - \Phi) (\omega_t(j) - \omega_t) \quad (15)$$

²⁰Under the assumption that $\frac{P^I}{P}$, N , $\frac{W/P}{A}$ and $\frac{B}{A}$ have well defined steady states, the previous equation will also hold in log-levels (with an added constant term), and hence will be consistent with non-stationary technology.

²¹The price of a one-period riskless real bond is given by $\exp\{-r_t\} = E_t \{\Lambda_{t,t+1}\}$. Log-linearizing around a steady state we have

$$\widehat{r}_t \equiv r_t - \rho \simeq -E_t \{\widehat{\lambda}_{t,t+1}\}$$

where $\rho \equiv -\log \beta$ and $\lambda_{t,t+1} \equiv \log \Lambda_{t,t+1}$.

i.e. the relative demand for labor by any given firm depends exclusively on its relative wage, with the corresponding elasticity being given by $-(1 - \Phi)/\alpha$. Note that this is a consequence of the hiring cost being common to all firms and independent of each firm's hiring and employment levels.²²

3.2.3 A Brief Detour: Labor Market Frictions and Inflation Dynamics

Empirical assessments of the price setting block of the New Keynesian model have often focused on inflation equation (9) and made use of the fact that, in the absence of labor market frictions, the average price markup (or, equivalently, the real marginal cost, with the sign reversed) is given by

$$\begin{aligned}\widehat{\mu}_t^p &= (a_t - \alpha \widehat{n}_t) - \widehat{\omega}_t \\ &= -\widehat{s}_t^n\end{aligned}$$

where $\widehat{s}_t^n \equiv \widehat{\omega}_t - (\widehat{y}_t - \widehat{n}_t)$ is the (log) labor income share, expressed as a deviation from its mean. The latter variable is readily available for most industrialized countries and can thus be used to construct a measure of the average markup, which can in turn serve as the basis for any empirical evaluation of (9).²³

The analysis above implies that in the presence of labor market frictions

$$\begin{aligned}\widehat{\mu}_t^p &= (a_t - \alpha \widehat{n}_t) - [(1 - \Phi) \widehat{\omega}_t + \Phi \widehat{b}_t] \\ &= -\widehat{s}_t^n - \Phi (\widehat{b}_t - \widehat{\omega}_t)\end{aligned}$$

²²The assumption of a decreasing returns technology is required in order for wage differentials across firm to be consistent with equilibrium, given the assumption of price taking behavior (otherwise only the firm with the lowest wage would not be priced out of the market). As an alternative, Thomas (2008) assumes a constant returns technology, but combines it with the assumption of firm-specific convex vacancy posting costs, in the form of management utility losses.

²³See Galí and Gertler (1999), Galí, Gertler and López-Salido (2001) and Sbordone (2002) for early applications of that approach.

Thus, the resulting empirical inflation equation may be written as

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \lambda_p \left(\widehat{s}_t^n + \Phi (\widehat{b}_t - \widehat{\omega}_t) \right) \quad (16)$$

Given (11) and the fact that $\widehat{g}_t = \gamma \widehat{x}_t$ it follows that in the presence of labor market frictions the measure of the average markup takes the form of a "corrected" labor income share, where the correction involves information on the current and future job finding rate.

In a recent paper, Krause, López-Salido and Lubik (2008) revisit the empirical evidence on inflation dynamics using an equation similar to (16), together with data on the job finding rate to construct a modified markup series. They conclude that the impact of labor market frictions on the driving variable of inflation is rather limited. To some extent this is something one could anticipate for, as discussed below, under a realistic calibration of hiring costs, $\frac{B}{W/P} = (0.045)(1 - \beta(1 - \delta)) \simeq 0.006$, implying too small a coefficient Φ to make a significant difference in the markup measure, at least in the absence of implausibly large fluctuations in net hiring costs relative to wages.

3.3 Monetary Policy

Under the model's baseline specification, monetary policy is assumed to be described by a simple Taylor-type interest rate rule represented by

$$i_t = \rho + \phi_\pi \pi_t^p + \phi_y \widehat{y}_t + v_t \quad (17)$$

where $i_t \equiv -\log Q_t$ is the yield on a one-period nominally riskless bond, $\rho \equiv -\log \beta$ is the household's discount rate, and v_t is an exogenous policy shifter, which is assumed to follow an $AR(1)$ process with autoregressive coefficient ρ_v and variance σ_v^2 .

Following Taylor (1993, 1999), I take a properly calibrated version of the previous rule as a rough approximation to actual monetary policy in the U.S. Much of the recent literature on nominal rigidities and labor market frictions has also adopted an interest rate rule similar to (17), even though some details may differ across papers.²⁴ Even though (17) is used as a baseline specification of monetary policy, I also consider alternative specifications of the policy rule when I turn to the normative analysis in Section 6.

Next I turn to a description of wage determination.

3.4 Labor Market Frictions and Wage Determination

I consider two alternative assumptions regarding wage setting: flexible wages and sticky wages. Under flexible wages, all wages are renegotiated and (potentially) adjusted every period. Under sticky wages only a constant fraction of firms can adjust their nominal wages in any given period. In both cases, the wage is determined according to a Nash bargaining protocol, with constant shares of the total surplus associated with each existing employment relation accruing to the worker (or his household) and the firm, respectively.

In contrast with the existing monetary models with labor market frictions, the framework below lies in its explicit (albeit stylized) modelling of the participation decision. This is possible through the introduction of a (utility) cost to labor market participation, which the household must trade-off against the probability and benefits resulting from becoming employed.²⁵

²⁴Thus, Walsh (2005), Faia (2008) and Trigari (2009) include the lagged nominal rate in the rule as a source of inertia, but impose that the shock be serially uncorrelated. In addition, Walsh (2005) also assumes no systematic response to output, whereas Faia (2008) also includes unemployment as an argument of the rule. Chéron and Langot (2000) and Walsh (2003) are an exception in that they assume an exogenous process for the money supply, a less appealing specification from the point of view of realism.

²⁵My approach here generalizes the one used by Shimer (2008) in the context of a real search and matching model.

Next I show, for both the flexible and sticky wage environments, how the surplus is split between households and firms as a function of the wage. In all cases, workers are assumed to act in a way consistent with maximization of the utility of their household, as specified in (1) and (3) (as opposed to maximization of their hypothetical "individual" utility).

3.4.1 The Case of Flexible Wages

Under this scenario each firm negotiates every period with its workers over their individual compensation. The value accruing to the representative household from a member employed at firm j , expressed in terms of final goods, is given by:

$$\mathcal{V}_t^N(j) = \frac{W_t(j)}{P_t} - MRS_t + E_t \{ \Lambda_{t,t+1} ((1 - \delta)\mathcal{V}_{t+1}^N(j) + \delta\mathcal{V}_{t+1}^U) \}$$

where $MRS_t \equiv \chi C_t L_t^\varphi$ is the household's marginal rate of substitution between consumption and labor market effort (or, equivalently, the marginal disutility of labor market effort, expressed in terms of the final goods bundle), and \mathcal{V}_t^U is the value generated by a member who is unemployed at the beginning of period t .²⁶ The latter is given by

$$\mathcal{V}_t^U = x_t \int_0^1 \frac{H_t(z)}{H_t} \mathcal{V}_t^N(z) dz + (1 - x_t) (-\psi MRS_t + E_t \{ \Lambda_{t,t+1} \mathcal{V}_{t+1}^U \})$$

The value associated with non-participation is normalized to zero. Under the assumption of an interior allocation with positive non-participation, the household must be indifferent between sending an additional member to the labor market or not. Thus, it must be the case that $\mathcal{V}_t^U = 0$ for all t . The

²⁶Note that in defining below the surplus relative to the value of an unemployed person at the beginning of the period I am implicitly assuming that if no wage agreement is reached the worker always has a chance to join the pool of the unemployed and look for a job in the same period.

latter condition in turn implies:

$$\psi MRS_t = \frac{x_t}{1-x_t} \int_0^1 \frac{H_t(z)}{H_t} \mathcal{S}_t^H(z) dz \quad (18)$$

where $\mathcal{S}_t^H(j) \equiv \mathcal{V}_t^N(j) - \mathcal{V}_t^U(j) = \mathcal{V}_t^N(j)$ denotes the surplus accruing to the household from an established employment relation at firm j .²⁷

Thus we have:

$$\mathcal{S}_t^H(j) = \frac{W_t(j)}{P_t} - MRS_t + (1-\delta) E_t \{ \Lambda_{t,t+1} \mathcal{S}_{t+1}^H(j) \} \quad (19)$$

On the other hand, the surplus from an existing employment relation accruing to firm j is given by

$$\mathcal{S}_t^F(j) = MRPN_t(j) - \frac{W_t(j)}{P_t} + (1-\delta) E_t \{ \Lambda_{t,t+1} \mathcal{S}_{t+1}^F(j) \} \quad (20)$$

Note that under the maintained assumption that the firm is maximizing profits, it follows from (11) and (20) that $\mathcal{S}_t^F(j) = G_t$ for all $j \in [0, 1]$ and t . In words, the surplus that a profit maximizing firm gets from an existing employment relation equals the hiring cost (which is also the cost of replacing a current worker by a new one, and thus what a firm "saves" from maintaining an existing relation).

The reservation wage for a worker employed at firm j is the minimum wage consistent with a non-negative surplus. It is given by

$$\Omega_t^H(j) = MRS_t - (1-\delta) E_t \{ \Lambda_{t,t+1} \mathcal{S}_{t+1}^H(j) \}$$

The corresponding reservation wage for the firm, i.e. the wage consistent with a non-negative surplus for the firm is:

$$\Omega_t^F(j) = MRPN_t(j) + (1-\delta) E_t \{ \Lambda_{t,t+1} \mathcal{S}_{t+1}^F(j) \}$$

²⁷Note that under the assumption that $\psi = 0$, there would be no cost associated with remaining unemployed so, to the extent the surplus from employment $\mathcal{S}_t^H(j)$ was positive, there would be full participation, so that $U_t = 1 - N_t$ for all t .

The *bargaining set* at firm j in period t is defined by the range of wage levels consistent with a non-negative surplus for both the firm and the worker, and thus corresponds to the interval $[\Omega_t^H(j), \Omega_t^F(j)]$. Note that the size of the bargaining set is given by

$$\begin{aligned}\Omega_t^F(j) - \Omega_t^H(j) &= \mathcal{S}_t^F(j) + \mathcal{S}_t^H(j) \\ &\geq G_t\end{aligned}$$

In other words, the presence of labor market frictions in the form of hiring costs guarantees the existence, in equilibrium, of a non-trivial bargaining set and, as a consequence, room for bargaining between firms and workers. As emphasized by Hall (2005) any wage that lies within the bargaining set is consistent with a privately efficient employment relation, i.e. one that neither the worker nor the firm has an incentive to terminate.

Until the work of Hall (2005) and Shimer (2005), the search and matching literature has generally relied on the assumption of *period-by-period Nash bargaining* between workers and firms as a "selection rule" to determine the prevailing wage. This has also been the case for the more recent vintage of models with sticky prices, when no wage rigidities are assumed (see, e.g. Walsh (2003, 2005) and Trigari (2005)). In what follows, I take the assumption of period-by-period Nash bargaining as the one defining the *flexible wage* economy, leaving a discussion of alternative for the next subsection.

Period-by-period Nash bargaining implies that the firm and each of its workers determine the wage in period t by solving the problem

$$\max_{W_t(j)} \mathcal{S}_t^H(j)^{1-\xi} \mathcal{S}_t^F(j)^\xi$$

subject to (19) and (20), and where $\xi \in (0, 1)$ denotes the relative bargaining power of firms vis a vis workers.

The solution to that problem implies the following constant share rule:

$$\xi \mathcal{S}_t^H(j) = (1 - \xi) \mathcal{S}_t^F(j)$$

The associated (Nash) wage is thus given by

$$\begin{aligned} \frac{W_t(j)}{P_t} &= \xi \Omega_t^H(j) + (1 - \xi) \Omega_t^F(j) \\ &= \xi MRS_t + (1 - \xi) MRPN_t(j) \end{aligned} \quad (21)$$

Using (12) to substitute for $MRPN_t(j)$ we confirm that the wage is common to all firms and, as a result, so will be employment, the hiring rate, and the marginal revenue product. Thus, we can henceforth omit the j index in what follows and write the Nash wage as

$$\frac{W_t}{P_t} = \xi MRS_t + (1 - \xi) MRPN_t \quad (22)$$

which combined with (11) (evaluated at the symmetric equilibrium) implies

$$G_t - (1 - \delta) E_t \{\Lambda_{t,t+1} G_{t+1}\} = \xi (MRPN_t - MRS_t) \quad (23)$$

Finally, note that under Nash bargaining the participation condition (18) can be rewritten as²⁸

$$\xi^\psi MRS_t = (1 - \xi) \frac{x_t}{1 - x_t} G_t \quad (24)$$

3.4.2 The Case of Sticky Wages

The flexibility of wages implied by the assumption of period-by-period Nash bargaining made in the previous subsection stands in conflict with the empirical evidence. More specifically, equation (22) implies that the nominal wage of all workers should experience continuous adjustments in response

²⁸As before, (24) is only needed when $\psi > 0$, so that $N_t \neq L_t$.

to changes in the price level, consumption, employment, productivity and any other variable that may affect the marginal rate of substitution or the marginal revenue product of firms. By contrast, the evidence based on observation of individual wages point to substantial nominal wage rigidities. Thus, Taylor's (1999) survey of the evidence concludes that the average frequency of wage changes is about one year. Evidence of similar (and even stronger) nominal wage rigidities can be found in more recent studies using U.S. micro data (e.g. Barattieri, Basu and Gottschalk (2009)) as well as micro data and surveys from many European countries (European Central Bank (2009)).

Motivated by that evidence, and by the difficulties of calibrated search and matching models with *flexible wages* to account for the observed volatility of unemployment or the "excess smoothness" of the real wage relative to labor productivity and GDP, many researchers have introduced different forms of wage rigidities in models with labor market frictions. As argued by Hall (2005), those frictions "make room" for such rigid wages, since they imply a non-trivial wage bargaining set consistent with privately efficient employment relations. In Hall's words, that property "...provides a full answer to the condemnation of sticky wage models in Robert Barro (1977), for invoking an inefficiency that intelligent actors could easily avoid."

Perhaps not surprisingly given the indeterminacy inherent to the existence of a bargaining set, the range of proposals to model wage rigidities in the literature is broad. Thus, some authors introduce real wage rigidities (in either real or monetary models) by postulating an "ad-hoc" real wage schedule which implies (potentially) continuous adjustment of all wages, though one that is smoother than that implied by period-by-period Nash bargaining (see, e.g. Hall (2005), Blanchard and Galí (2007, 2010), and Christoffel and

Linzert (2005)). An alternative approach to modelling wage rigidities assumes staggered wage setting, so that only a fraction of workers are allowed to bargain over and adjust their wage in any given period. In that case, each individual wage remains unchanged for several periods, either in real terms (Gertler and Trigari (2009)) or, more realistically, in nominal terms (as in Bodart et al. (2006), Gertler, Sala and Trigari (2008) and Thomas (2008)).

Here I follow the last group of authors and introduce wage rigidities in the form of staggered nominal wage setting à la Calvo. More specifically, I assume that the *nominal* wages paid by a given firm to its employees are renegotiated (and likely reset) with probability $1 - \theta_w$ each period, independently of the time elapsed since the last adjustment at that firm. The newly set wage is determined through Nash bargaining between each individual worker and the firm. Once the nominal wage is set, it remains unchanged until a new opportunity for resetting the wage arises. As a result, in any given period the wage (both real and nominal) will generally deviate from the flexible Nash wage derived in the previous subsection. Yet, and to the extent that shocks are not too large, the wage will remain within the relevant bargaining set and will thus be privately efficient to maintain the corresponding employment relation.

Most importantly, I assume that workers hired between renegotiation periods are paid the *average* wage prevailing at the firm. Thus, the average wage will have an influence on the firm's hiring and employment levels. Yet, I assume that the number of workers is large enough that neither the firm nor the worker bargaining over the wage internalize the impact that their choice will have on the average wage. In a symmetric equilibrium all workers will get the same wage, which ex-post will be equal to the average.²⁹ It is

²⁹This assumption simplifies the subsequent analysis considerably.

important to stress that the previous assumption is not an innocuous one. If new hires could negotiate their wage freely at the time of being hired, the existence of long spells with unchanged nominal wages for incumbent workers would have no direct impact on the hiring decisions and, as a result, on output and employment, as emphasized by and Pissarides (2009). The empirical evidence on the relevance of wage stickiness for new hires remains controversial. Some authors have provided evidence pointing to greater wage flexibility for new hires (see. e.g. Haefke et al. (2008), and the references in Pissarides (2009)), while others reject the existence of any significant differences between new hires and incumbent workers (e.g. Gertler and Trigari (2009) and Galuscak et al. (2008)).³⁰

An immediate consequence of the staggering assumption is that wages will generally differ across firms, and so will employment and output. That dispersion in the allocation of workers across otherwise identical firms, coupled with the assumption of decreasing returns, is inefficient from a social viewpoint, a point further discussed below in the context of the normative analysis of the model.³¹

Next I derive the basic equations describing the surpluses accruing to households and firms from existing employment relations, as a preliminary step to the analysis of wage determination as the outcome of a Nash bargain.

Let $\mathcal{V}_{t+k|t}^N$ denote the value accruing to a household in period $t+k$ from the employment of a member at a firm that last reset its wage in period t .

³⁰See Section 6 for a brief discussion of an extension by Brodard et al. (2007) allowing for differential flexibility between incumbents and new hires.

³¹The inefficiencies resulting from staggered nominal wage setting were already stressed in Erceg et al. (2000), in the context of a model without labor market frictions. Wage staggering in Thomas (2008) leads to an aggregate inefficiency as a result of the convexity of vacancy posting costs at the level of each firm. Here the inefficiency results from the presence of decreasing returns to labor.

Under the assumption made above we have:

$$\mathcal{V}_{t+k|t}^N = \frac{W_t^*}{P_{t+k}} - MRS_{t+k} + E_{t+k} \left\{ \Lambda_{t+k,t+k+1} \left[(1-\delta)(\theta_w \mathcal{V}_{t+k+1|t}^N + (1-\theta_w) \mathcal{V}_{t+k+1|t+k+1}^N) + \delta V_{t+k+1}^U \right] \right\} \quad (25)$$

for $k = 0, 1, 2, 3, \dots$ where W_t^* denotes the nominal wage newly set in period t .³² Note that the last term on the right hand side of (25) reflects the fact that the continuation value depends on whether wages are readjusted or not in the following period.

On the other hand, the value accruing to a household in period t from a member who is unemployed (but part of the labor force) at the beginning of period t is given by:

$$\mathcal{V}_t^U = x_t \int_0^1 \left(\frac{H_t(z)}{H_t} \right) \mathcal{V}_t^N(z) dz + (1-x_t) (-\psi MRS_t + E_t \{ \Lambda_{t,t+1} V_{t+1}^U \})$$

Again, optimal participation implies $\mathcal{V}_t^U = 0$ for all t . As a result

$$\mathcal{S}_{t+k|t}^H = \frac{W_t^*}{P_{t+k}} - MRS_{t+k} + (1-\delta) E_{t+k} \left\{ \Lambda_{t+k,t+k+1} (\theta_w \mathcal{S}_{t+k+1|t}^H + (1-\theta_w) \mathcal{S}_{t+k+1|t+k+1}^H) \right\} \quad (26)$$

and

$$\psi MRS_t = \frac{x_t}{1-x_t} \int_0^1 \left(\frac{H_t(z)}{H_t} \right) \mathcal{S}_t^H(z) dz \quad (27)$$

Iterating (26) forward and evaluating the resulting expression at $k = 0$ one can determine the household surplus from an employment relation at a firm whose wages are currently being reset:

$$\begin{aligned} \mathcal{S}_{t|t}^H &= E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k} \left(\frac{W_t^*}{P_{t+k}} - MRS_{t+k} \right) \right\} \\ &\quad + (1-\theta_w)(1-\delta) E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k+1} \mathcal{S}_{t+k+1|t+k+1}^H \right\} \quad (28) \end{aligned}$$

³²Note that even though newly set wages can in principle differ across workers and firms, ex-post all individual wages set in any given period will be identical. That justifies the omission of firm or worker indexes in W_t^*

On the other hand, the period $t + k$ surplus accruing to a firm that last renegotiated its wages in period t , resulting from a marginal employment relation, is given by

$$\mathcal{S}_{t+k|t}^F = MRP N_{t+k|t} - \frac{W_t^*}{P_{t+k}} + (1-\delta)E_{t+k} \left\{ \Lambda_{t+k,t+k+1} (\theta_w \mathcal{S}_{t+k+1|t}^F + (1-\theta_w) \mathcal{S}_{t+k+1|t+k+1}^F) \right\} \quad (29)$$

for $k = 0, 1, 2, 3, \dots$, where $MRPN_{t+k|t} \equiv \frac{P_{t+k}^I}{P_{t+k}} (1-\alpha) A_{t+k} N_{t+k|t}^{-\alpha}$ is the firm's marginal revenue product of labor, and $N_{t+k|t}$ its employment level.

Note, for future reference, that when combined with the optimal choice of employment by the firm at each point in time (as described by (11)), (29) implies:

$$\mathcal{S}_{t+k|t}^F = G_{t+k}$$

for all t and k . In words, the surplus accruing to the firm is always equal to the current hiring cost, independently of how long the wage has remained unchanged.

Iterating (29) forward and evaluating the resulting expression at $k = 0$ yields

$$\begin{aligned} \mathcal{S}_{t|t}^F &= E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k} \left(MRP N_{t+k|t} - \frac{W_t^*}{P_{t+k}} \right) \right\} \\ &\quad + (1-\theta_w)(1-\delta)E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k+1} \mathcal{S}_{t+k+1|t+k+1}^F \right\} \end{aligned} \quad (30)$$

In the present environment, the Nash bargained wage at a firm that resets nominal wages in period t is given by the solution to

$$\max_{W_t^*} (\mathcal{S}_{t|t}^H)^{1-\xi} (\mathcal{S}_{t|t}^F)^\xi$$

subject to (28) and (30). The implied sharing rule is given by

$$\xi \mathcal{S}_{t|t}^H = (1-\xi) \mathcal{S}_{t|t}^F \quad (31)$$

which combined with (28) and (30) requires that the nominal wage newly set in period t satisfy the condition:

$$E_t \left\{ \sum_{k=0}^{\infty} ((1-\delta)\theta_w)^k \Lambda_{t,t+k} \left(\frac{W_t^*}{P_{t+k}} - \Omega_{t+k|t}^{tar} \right) \right\} = 0 \quad (32)$$

where

$$\Omega_{t+k|t}^{tar} \equiv \xi MRS_{t+k} + (1-\xi) MRPN_{t+k|t} \quad (33)$$

can be interpreted as the k -period ahead *target* real wage. Note that the expression for the latter corresponds to that of the relevant Nash wage under *flexible wages*, as derived in the previous subsection (see equation (21)).

Log-linearizing the wage setting rule (32) around a zero inflation steady state we obtain:

$$w_t^* = (1 - \beta(1 - \delta)\theta_w) E_t \sum_{k=0}^{\infty} (\beta(1 - \delta)\theta_w)^k E_t \{ \omega_{t+k|t}^{tar} + p_{t+k} \} \quad (34)$$

where $\omega_{t+k|t}^{tar} \equiv \log \Omega_{t+k|t}^{tar}$. In words, the nominal wage set through Nash bargaining corresponds to a weighted average of the current and expected future *target* nominal wages relevant to the firm that is resetting wages. The weights decline geometrically with the horizon, at a rate which is a function of the degree of wage stickiness and the separation rate, since both those factors determine the expected duration of the newly set wage.

Next I rewrite the above expression in terms of *average* target wages. Log-linearizing (33) around a symmetric steady state we have

$$\widehat{\omega}_{t+k|t}^{tar} = \Upsilon (\widehat{c}_{t+k} + \widehat{\varphi}_{t+k}) + (1 - \Upsilon) (-\widehat{\mu}_{t+k}^p + a_{t+k} - \alpha \widehat{n}_{t+k|t}) \quad (35)$$

where $\Upsilon \equiv \frac{\xi MRS}{W/P}$. Let ω_t^{tar} denote the (log) *average* target wage, defined as the current target wage for a (hypothetical) firm whose employment matched average employment. Formally,

$$\widehat{\omega}_t^{tar} \equiv \Upsilon (\widehat{c}_t + \widehat{\varphi}_t) + (1 - \Upsilon) (-\widehat{\mu}_t^p + a_t - \alpha \widehat{n}_t) \quad (36)$$

Note that one can interpret $\widehat{\omega}_t^{tar}$ as the Nash bargained wage that would be observed in a flexible wage environment, conditional on the levels of consumption and (average) marginal revenue product generated by the equilibrium allocation under sticky wages.

Combining (35) and (36) with (15)

$$\widehat{\omega}_{t+k|t}^{tar} = \widehat{\omega}_{t+k}^{tar} + (1 - \Upsilon)(1 - \Phi) (w_t^* - w_{t+k}) \quad (37)$$

Substituting (37) into (34), and after some algebraic manipulation we can derive the difference equation

$$w_t^* = \beta(1-\delta)\theta_w E_t\{w_{t+1}^*\} - \frac{1 - \beta(1-\delta)\theta_w}{1 - (1-\Upsilon)(1-\Phi)} (\widehat{\omega}_t - \widehat{\omega}_t^{tar}) + (1 - \beta(1-\delta)\theta_w) w_t \quad (38)$$

The law of motion for the (log) average wage $w_t \equiv \int_0^1 w_t(j) dj$ is given by

$$w_t = \theta_w w_{t-1} + (1 - \theta_w) w_t^* \quad (39)$$

Combining (38) and (39), one can derive the following wage inflation equation:

$$\pi_t^w = \beta(1-\delta)E_t\{\pi_{t+1}^w\} - \lambda_w(\widehat{\omega}_t - \widehat{\omega}_t^{tar}) \quad (40)$$

where $\lambda_w \equiv \frac{(1-\beta(1-\delta)\theta_w)(1-\theta_w)}{\theta_w(1-(1-\Upsilon)(1-\Phi))}$. Note that the driving variable behind fluctuations in wage inflation is *the wage gap* $\omega_t - \omega_t^{tar}$, defined as the deviation between the average wage and the average *target* wage.³³

Finally, and as shown in Appendix 4, the optimal participation condition (27) can be approximated around the zero inflation steady state as follows:

$$\widehat{c}_t + \widehat{\varphi}_t = \frac{1}{1-x} \widehat{x}_t + \widehat{g}_t - \Xi \pi_t^w \quad (41)$$

³³Thomas (2008) derives a similar representation for wage inflation, in the context of a slightly different model with efficient hours choice, convex vacancy posting costs, and constant returns.

where $\Xi \equiv \frac{\xi(W/P)}{(1-\xi)G} \frac{\theta_w}{(1-\theta_w)(1-\beta(1-\delta)\theta_w)}$. Note that under flexible wages $\theta_w = 0$, implying $\Xi = 0$. The left hand side of (41) measures the cost of labor market participation (through joining the pool of unemployed at the beginning of the period), while the right hand is the expected reward from that participation, both expressed as log deviations from their steady state values. That reward is increasing in the job finding rate and in the size of current hiring costs (since workers with newly set wages will generate a surplus proportional to that variable), and decreasing in wage inflation (since the latter is positively related to the gap between the newly set wage and the average wage, with the latter being the one that is relevant to the participation decision).

Sustainability of the fixed wage. Both the firm and the worker will find it efficient to maintain an existing employment relation as long as their respective surpluses are positive. Thus, for a worker and firm that last reset the wage in period t , this will be the case as long as the nominal wage W_t^* remains within the bargaining set bounded by the reservation wages of the firm and the worker.

Formally, we require

$$W_t^* \in [\underline{W}_{t+k|t}, \overline{W}_{t+k|t}]$$

where

$$\underline{W}_{t+k|t} \equiv P_{t+k} \left(MRS_{t+k} - (1-\delta)E_{t+k} \left\{ \Lambda_{t+k,t+k+1} (\theta_w \mathcal{S}_{t+k+1|t}^H + (1-\theta_w) \mathcal{S}_{t+k+1|t+k+1}^H) \right\} \right)$$

and

$$\overline{W}_{t+k|t} \equiv P_{t+k} \left(MRP N_{t+k|t} + (1-\delta) E_{t+k} \left\{ \Lambda_{t+k,t+k+1} G_{t+k+1} \right\} \right)$$

Note that in the zero inflation steady state we have $W^* = P (\xi \underline{W} + (1-\xi) \overline{W})$, so that the newly set wage lies within the bargaining set. Thus, the

probability that the wage of any firm remains within that set outside the steady state will be larger the more stable are prices and consumption, employment, unemployment and technology (the variables underlying MRS_t and $MRPN_{t+k|t}$). This will be the case, in turn, if shocks are "sufficiently small," an assumption that I maintain in what follows. Notice, however, that given the Calvo structure, which implies that there are *some* wages that remained unchanged for arbitrarily long periods, it will be unavoidable that a small fraction of firms violate that condition in finite time (which would call for terminating the relationship or, more plausibly, violating the exogenous Calvo constraint on the timing of wage adjustments). Gertler and Trigari (2009) and Thomas (2009) conduct simulations of related models and conclude that, for plausible calibrations of the wage rigidity parameter and shocks of empirically plausible size, the typical wage has a very small probability of falling outside the bargaining set before it gets to be readjusted. On those grounds, and following the literature, in my analysis I ignore that possibility, thus assuming that no wage ever hits the boundaries of the bargaining set.³⁴

3.4.3 Relation to the New Keynesian Wage Inflation Equation.

Equation (40) has a structure analogous to the wage inflation equation that arises in the New Keynesian model with staggered nominal wage setting, as originally developed by Erceg, Henderson and Levin (2000; EHL, henceforth). In the latter, each household is specialized in supplying a differentiated type of labor service, whose demand has a constant elasticity ϵ_w . In any given period it is allowed to reset the corresponding nominal wage

³⁴See Galí and van Rens (2009) for a model in which wages are adjusted only when they hit the boundaries of the bargaining set.

unilaterally with a constant probability $1 - \theta_w$. The implied (log-linearized) optimal wage setting rule in the EHL model takes the form

$$w_t^* = \mu^w + (1 - \beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \{ mrs_{t+k|t} + p_{t+k} \} \quad (42)$$

where $\mu^w \equiv \log \frac{\epsilon_w}{\epsilon_w - 1}$ is the desired (log) wage markup of the real wage over the marginal rate of substitution (i.e. the one prevailing in the absence of wage rigidities). The previous optimal wage setting rule can be contrasted with (34), the one prevailing under staggered wage setting with Nash bargaining.

The wage inflation equation that results from combining the log-linearized optimal wage setting rule (42) with a law of motion for the average wage identical to (39) can be written as

$$\pi_t^w = \beta E_t \{ \pi_{t+1}^w \} - \lambda_{ehl} (\widehat{\omega}_t - \widehat{mrs}_t) \quad (43)$$

where mrs_t is the average (log) marginal rate of substitution between consumption and hours, and λ_{ehl} is a coefficient that is inversely related to the degree of wage stickiness θ_w . In particular, under the specification of preferences used in the model above with $\psi = 0$, we have $\widehat{mrs}_t = \widehat{c}_t + \varphi \widehat{n}_t$ and $\lambda_{ehl} \equiv (1 - \beta\theta_w)(1 - \theta_w) / (\theta_w(1 + \epsilon_w\varphi))$.³⁵

Three main differences with respect to (40)) are worth pointing out. First, the "effective" discount factor is smaller in the model with frictions, since it incorporates the probability of termination of each relationship (and thus of the associated wage), whereas in the EHL model the wage applies to the same group of workers throughout its duration, not to a specific relation that may be subject to termination. Secondly, the implicit target wage in the

³⁵See Galí (2009) for a discussion of the relation between the New Keynesian Wage inflation equation and the original Phillips curve.

EHL model is given by the average marginal rate of substitution (augmented with a constant desired wage markup), whereas in the model with frictions the target wage is also a function of the marginal revenue product of labor, since that variable also influences the total surplus to be split through the wage negotiation. Finally, the difference in the coefficient on the wage gap between the two formulations captures the different adjustments needed to express the wage inflation equation in terms of average variables: the average marginal rate of substitution in the EHL model, and the average marginal revenue product of labor in the present model. Note that under the special parameter configuration $\delta = 0$ and $\xi = 1$, the form of the wage inflation equation of the present model matches exactly that of the EHL model.

3.5 Aggregate Demand and Output

Under the assumption that hiring costs take the form of a bundle of final goods given by the same CES function as the one defining the consumption index, the demand for each final good will be given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} (C_t + G_t H_t)$, where $H_t \equiv \int_0^1 H_t(j) dj$ denotes aggregate hires. Thus, the implied constancy of the price elasticity of demand justifying the constant desired markup $\mathcal{M}^p \equiv \frac{\epsilon}{\epsilon-1}$ assumed above.

Letting aggregate output be given by $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ it can be easily checked that the aggregate goods market clearing condition may be written as

$$Y_t = C_t + G_t H_t \tag{44}$$

Hence, aggregate demand has two components. The first component is consumption, which evolves according to the Euler equation (6). The second component is the demand for final goods originating in firms' hiring activities.

Turning to the supply side, one can derive the following aggregate relation between final goods and intermediate input

$$\begin{aligned} X_t &\equiv \int_0^1 X_t(i) \, dj \\ &= Y_t \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \, di \end{aligned} \quad (45)$$

where the term $D_t^p \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \, di \geq 1$ captures the inefficiency resulting from dispersion in the quantities produced and consumed of the different final goods, which is itself a consequence of the price dispersion caused by staggered price setting.

On the other hand, the total supply of intermediate goods is given by

$$\begin{aligned} X_t &= \int_0^1 Y_t^I(j) \, dj \\ &= A_t N_t^{1-\alpha} \int_0^1 \left(\frac{N_t(j)}{N_t} \right)^{1-\alpha} \, dj \end{aligned} \quad (46)$$

where the term $D_t^w \equiv 1 / \int_0^1 \left(\frac{N_t(j)}{N_t} \right)^{1-\alpha} \, dj \geq 1$ captures the inefficiency resulting from dispersion in the allocation of labor across firms due to the staggering of wages, combined with the assumption of decreasing returns ($\alpha > 0$).

As shown in Appendix 1, in a neighborhood of the zero inflation steady state we have $D_t^p \simeq 1$ and $D_t^w \simeq 1$ up to a first order approximation. Thus, combining (45) and (46) we obtain the approximate aggregate production relation:

$$Y_t = A_t N_t^{1-\alpha} \quad (47)$$

For the sake of convenience, Appendix 3 collects all the model's (log) linearized equilibrium conditions, as derived in the previous sections. Next

I use those equilibrium conditions to characterize the behavior of a calibrated version of my model economy.

4 Equilibrium Dynamics: The Effects of Monetary Policy and Technology Shocks

This section presents the equilibrium responses of several variables of interest to the model's exogenous shocks—monetary policy and technology—and discusses how those responses are affected by nominal rigidities and labor market frictions. As a preliminary step I discuss the model's steady state, which is partly the basis for the calibration.

4.1 Steady State and Calibration

The model's steady state is independent of the degree of price and wage rigidities, and of the monetary policy rule. For simplicity, I assume a steady state with zero inflation and no secular growth. I normalize the level of technology in the steady state to be $A = 1$. Notice that in that steady state there are no relative price distortions so $D^p = D^w = 1$. Thus, the goods market clearing condition, evaluated at the steady state, can be written as

$$N^{1-\alpha} = C + \delta N \Gamma x^\gamma \quad (48)$$

Evaluating (23) at the steady state we have

$$(1 - \beta(1 - \delta)) \Gamma x^\gamma = \xi \left(\frac{1 - \alpha}{\mathcal{M}^p(1 - \tau)} N^{-\alpha} - \chi CL^\varphi \right) \quad (49)$$

Finally, the steady state participation condition requires

$$(1 - x)\xi\psi\chi CL^\varphi = (1 - \xi) \Gamma x^{1+\gamma} \quad (50)$$

The remaining steady state conditions include:

$$xU = (1 - x)\delta N \quad (51)$$

$$L = N + \psi U \tag{52}$$

In order to calibrate the model I adopt the following strategy. First, I pin down the steady state employment rate, participation rate and job finding rate using observed average values in the postwar U.S. economy. This leads to the choice of $N = 0.59$ and $L = 0.62$, which in turn imply $U = 0.03$. Note that the implied unemployment rate as a fraction of the labor force—the conventional definition—is then close to five percent. ($0.03/0.62 \simeq 0.048$). Following Blanchard and Galí (2010), I set the steady state value for the (quarterly) job finding rate x to 0.7. The implied separation rate is thus $\delta = (x/1 - x)U/N \simeq 0.12$. Following convention I set $\alpha = 1/3$ and $\beta = 0.99$. Parameter φ is the inverse of the Frisch labor supply elasticity, a more controversial parameter due to the conflict between micro and macro evidence. I set $\varphi = 5$ in the baseline calibration, which corresponds to a Frisch elasticity of 0.2.

The baseline values for the parameters determining the degree of price and wage stickiness are set to imply average durations of one year in both cases, i.e. $\theta_p = 0.75$ and $\theta_w = 0.75$. This is roughly consistent with micro-economic evidence on wage and price setting.³⁶

Using the equivalence with the matching function approach discussed above and using estimates of the latter I set $\gamma = 1$. I also assume $\mathcal{M}^p(1-\tau) = 1$, so that the subsidy fully offsets the distortionary effects of final goods firms market power, one of the conditions for an efficient steady state. Following Hagedorn and Manovskii (2008) and Shimer (2009), who rely on the evidence reported in Silva and Toledo (2009), I take the average cost of hiring a worker to be 4.5% of the quarterly wage, i.e. $G = 0.045 (W/P)$. Accordingly, the

³⁶See, e.g. Nakamura and Steinsson (2008) and Basu and Gottschalk (2009) for recent U.S. micro evidence on price and wage rigidities, respectively.

share of hiring costs in GDP is $\Theta = \delta NG/Y = (0.045)\delta S^n$, where S^n is the labor income share. Setting the latter to $2/3$ we have $\Theta = 0.0014$, i.e. slightly above one-tenth of a percentage point of GDP. It follows that $\Gamma = G/x^\gamma = \Theta/(N^\alpha x^\gamma \delta) = 0.02$.

This leaves me with three free (though related) parameters, the firm's share in the Nash bargain (ξ), the weight of unemployment in the the disutility of labor market effort (ψ), and the parameter scaling that disutility (χ). Given the value for one of these parameters, I can determine the remaining two by combining (48), (50) and (49). Given the choice of $\gamma = 1$ above, perhaps a natural benchmark setting for ξ is 0.5, which—as shown below—would be the value consistent with an efficient steady state and is often assumed in the literature. Yet, that configuration implies $\psi = 0.041$, a weight on unemployment which is arguably unrealistically small if one takes into consideration not only the time allocated to job search activities by the unemployed, but also the psychological costs of unemployment.³⁷ Thus, and as an alternative parameter configuration I choose $\xi = 0.05$, which is associated with $\psi = 0.82$, possibly a more plausible value. As discussed below, the choice of a value in that range has significantly different, and more plausible, implications for the economy's response to a monetary policy shock. The implied settings for χ corresponding to the two calibrations are 15.5 and 12.3, respectively.

³⁷Thus, if the disutility of the unemployed (relative to the nonparticipant) results exclusively from the time allocated to job search activities and we take the standard workweek for the employed to be of 40 hours, that calibration would that the unemployed 1.6 hours a week are allocated to job search activities. This is somewhat below the 2.5 hours per week of job search observed in time use surveys, as discussed in Krueger and Mueller (2008). The latter paper also provides survey-based evidence of subjective well-being, showing that unemployed individuals in the U.S. report considerably lower life satisfaction than than the employed. Under literal interpretation of the model that evidence would call for a ψ value above unity.

Finally, I calibrate the coefficients in the interest rate rule in a way consistent with the specification in Taylor (1993), i.e. I set $\phi_\pi = 1.5$ and $\phi_y = 0.5/4 = 0.0125$ (the latter adjustment being justified by Taylor’s use of annualized inflation rate vs. quarter-to-quarter inflation here). That calibration is generally viewed a reasonable approximation to monetary policy in U.S., at least over the past three decades.

4.2 The Effects of Monetary Policy and Technology Shocks

Figure 2a displays the dynamic responses of six macro variables (output, unemployment, employment, labor force, inflation and the real wage) to an exogenous monetary policy shock, under the baseline assumption of $\xi = 0.5$, which is consistent with an efficient steady state. More specifically, disturbance v_t in the interest rate rule is assumed to rise by 0.25 percentage points, and to die out gradually according to an AR(1) process with an autoregressive coefficient $\rho_v = 0.5$. Note that, in the absence of an endogenous component in the rule, such an experiment would be associated with a one percentage point increase in the (annualized) interest rate,

Though the estimated VAR model discussed in section 2 did not specifically seek to identify monetary shocks, to the extent that those shocks and other demand shocks generate similar patterns among the variables considered, we can use the estimated conditional moments associated with demand shocks as a rough benchmark when evaluating the model’s response to a monetary policy shock.

Figure 2a shows that both output and employment decline in response to the tightening of monetary policy, due to the contraction in consumption (not shown) resulting from the interest rate hike. Note also that the labor force increases by nearly 5 percent, driving up the unemployment rate by

about 5 percentage points. In light of the evidence presented in section 2, both responses seem implausibly large and, in the case of the labor force, it appears to go in the wrong direction. Note also that price inflation is procyclical, in a way consistent with the evidence. The procyclical response of the real wage is, on the other hand, at odds with the estimated negative correlation with output conditional on demand shocks.

Figure 2b displays the corresponding responses to a technology shock. The latter takes the form of a one percent increase in a_t , which dies out gradually according to an AR(1) process with an autoregressive coefficient of 0.9. Note that, in a way consistent with the estimated impulse responses shown in Figure 1, output rises and inflation declines, as one would expect from a positive technology shock. Note also that the real wage rises gradually in the short run, a natural consequence of the existence of nominal wage rigidities. Furthermore, and in contrast with the standard search and matching model, employment declines and unemployment increases in response to the same positive technology shock. This is consistent with the evidence presented in Section 2 and in the literature referred therein. As was the case with monetary shocks, however, the rise in unemployment is largely driven by the increase in the labor force, which is far more volatile than employment and comoves negatively with the latter variable. This is in contrast with an estimated correlation (conditional on demand shocks) between the labor force and employment of 0.85.

A possible reason for the unrealistically large fluctuations in the labor force and unemployment shown in Figures 2a and 2b is the low value of parameter ψ (about 0.04) associated with the calibration underlying those figures. Such a low value penalizes little fluctuations in those variables, given employment. Figures 3a and 3b show the model's implied responses

to monetary and technology shocks under the alternative calibration, with $\psi = 0.82$ and $\xi = 0.05$. As the figures make clear, now the labor force experiences much smaller variations, and comoves positively with employment. The latter's movements are the dominant force behind the variations in unemployment, in a way consistent with the evidence. The response of the remaining variables is not qualitatively affected. Thus, the only variable whose response is at odds with the evidence in section 2 is the real wage, which responds procyclically to a monetary shock in the model, while displaying a negative correlation with output conditional on "demand" shocks in the data. That discrepancy could be due, however, to the presence of shocks other than technology shocks or monetary shocks (e.g. fiscal policy or labor supply shocks) that may be responsible for the negative correlation picked up by the partially identified VAR discussed in section 2. Given the previous findings, and unless otherwise noted, I stick to this alternative calibration in the remainder of the paper.

4.3 The Role of Labor Market Frictions

In order to ascertain the role played by the presence of labor market frictions in shaping the economy's response to different shocks, I compare the model's implied responses to those shocks in the presence or not of such frictions. A perfectly competitive labor market is assumed in the case of no frictions. In both cases I maintain the assumption of flexible wages, i.e. period-by-period Nash bargaining.

Figures 4a and 4b display the economy's response to a monetary policy and a technology shock, respectively. Note that, in most cases the difference is quantitatively very small. Qualitatively, the only significant difference lies in the non-zero unemployment response to either shock in the presence

of frictions, whereas in their absence a perfectly competitive labor market guarantees that there is no unemployment, implying that its response to shocks is flat at zero, as shown in the figure. The variations in unemployment generated by the introduction of frictions are, however, very small for both shocks. This result is reminiscent of the so-called Shimer puzzle, i.e. the finding of too small a volatility of unemployment implied by a calibrated (real) search and matching framework with flexible wages and driven by technology shocks (Shimer (2005)).

The finding of a small role of labor market frictions in the response to monetary policy shocks contrasts somewhat with the conclusions from a related analysis in Walsh (2005). More precisely, Walsh shows that the introduction of labor market frictions has consequences on the pattern of the response of output and inflation to a monetary policy shock roughly equivalent to a substantial increase in the degree of price rigidities³⁸ in an otherwise standard New Keynesian model with Walrasian labor markets. In practice, it leads to a significantly more sluggish response of inflation and a larger and more persistent response of output. A possible explanation for the discrepancy between Walsh's results and those found here lies in the fact that his model with labor market frictions assumes a constant marginal disutility from work, whereas his New Keynesian model introduces (with no apparent justification) a different utility function with an increasing marginal disutility of work. The latter feature will generally make wages and hence marginal costs more sensitive to variations in activity, thus leading to a larger response of prices in the short run, and a more dampened output response.³⁹

³⁸Corresponding to a increase in the Calvo parameter θ_p from 0.5 to 0.85, which is equivalent to raising the average duration of prices from two to more than six quarters.

³⁹A similar discrepancy arises vis a vis Trigari (2009) in her comparison of the responses

4.4 The Role of Price Stickiness

How does the introduction of sticky prices affect, qualitatively and quantitatively, the response of unemployment and other variables to aggregate shocks? In order to address this question I analyze the response to monetary and technology shocks of two versions of the model economy developed above, with the only difference among them is the presence or not of staggered price setting in the final goods sector. In both cases I maintain the assumption of full wage flexibility.

Figures 5a and 5b display the corresponding impulse response functions. First, and not surprisingly, we see that the introduction of price stickiness has a significant impact on the economy's response to a monetary policy shock (Figure 5a). Thus, under flexible prices no real variable is affected by the shock, and only inflation declines in response to the tightening of policy. In contrast, once a realistic degree of price stickiness is allowed for, the model implies a decline in output, employment and the labor force, with a rise in the unemployment rate (after a tiny one period decline). Inflation and the real wage also decline, as expected.

The impact of price stickiness on the response to a positive technology shock (Figure 5b) appears to be much more limited. In particular, the effect on the size of the output response—more muted under sticky prices—is hardly discernible. The difference is sufficient, however, to account for a sign reversal in the response of employment, from positive to negative, though quantitatively the size of the employment adjustment is very small

of a search model and a New Keynesian model to a monetary policy shock. Thus, in Trigari's search model labor adjustment takes place along two margins, hours per worker and employment, whereas in her New Keynesian model only hours per worker are allowed to vary. As argued by Trigari herself, that difference makes the elasticity of marginal cost to output larger in the New Keynesian model, which accounts for the weaker and less persistent response of output in the latter case.

in both cases. Combined with a small influence (in the same direction) on the response of the labor force, the impact of price stickiness on the response of unemployment to the technology shock is almost negligible.⁴⁰ The only sizable impact of price stickiness appears to be on the response of the real wage, which declines considerably as a result of the large rise in the markup of final goods firms that results from their failure to lower prices to match the decline in the price of intermediate goods. This is reflected in a muted rise in the marginal revenue product of intermediate goods firms and, as a result, on the wage.

4.5 The Role of Wage Stickiness

Finally, I turn to an examination of the role played by wage stickiness in shaping the responses of the economy with labor market frictions to monetary and technology shocks. Figures 6a and 6b display, respectively, the simulated responses to those shocks. For each type of shock, responses under two alternative calibrations are displayed. The solid line corresponds to an economy with flexible prices ($\theta_w = 0$), whereas the starred line assumes $\theta_w = 0.75$, implying an average duration of wages of one year. In both cases prices are assumed to be sticky.

As Figure 6a makes clear, the presence of sticky wages strengthens substantially the effects of a monetary policy shock on economic activity. In particular, the decline in output and employment is roughly twice as large as in the case of flexible wages. Since the response of the labor force is hardly affected, the resulting increase in unemployment is also much larger.

⁴⁰See Andrés et al. (2006) for a similar exercise in a model with endogenous capital accumulation, price indexation, endogenous match destruction. Their findings point to a stronger role for price rigidities in accounting for the volatility of vacancies relative to unemployment, but no so much for the volatility of unemployment itself, which goes down slightly when stronger price rigidities are assumed.

In addition, and not surprisingly, we see how the average real wage shows a much smoother response in the presence of staggered contracts, leading to less downward pressure on marginal costs and, as a result, a smaller decline in inflation.

The impact of wage stickiness on the responses to a technology shock is also substantial, as shown in Figure 6b. In particular, the negative response of employment is now larger, and that of the labor force (slightly) smaller. This is sufficient for the response of the unemployment rate to switch its sign, and thus to rise in response to a positive technology shock. Once again, that implication contrasts with the prediction of real models with labor market frictions (e.g. Shimer (2005)), but is consistent with the evidence presented in Section 2.

Note also that the introduction of sticky wages dampens the response of the real wage even further in the short run, driving closer to the near-zero short run response uncovered by the empirical evidence in Section 2.

As discussed above, the presence of labor market frictions, by itself, does not appear to have much impact on the economy's response to shocks. The indirect impact is, however, more substantial to the extent that it justifies the presence of sticky wages in equilibrium.

Having looked at some of the positive predictions of the model under alternative sets of assumptions, I turn next to its normative implications.

5 Labor Market Frictions, Nominal Rigidities and Monetary Policy Design

I start this section by describing the constrained-efficient allocation, and then turn my attention to the optimal design of monetary policy in the

presence of labor market frictions and nominal rigidities. Ultimately, the purpose of the analysis is to shed light on how the existence of unemployment and wage rigidities should influence the conduct of monetary policy.

5.1 The Social Planner's Problem

The social planner maximizes the representative household's utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right)$$

subject to the resource constraint

$$C_t + \Gamma x_t^\gamma H_t = A_t N_t^{1-\alpha}$$

and the definitions

$$\begin{aligned} L_t &= N_t + \psi U_t \\ H_t &= N_t - (1 - \delta) N_{t-1} \\ x_t &= \frac{H_t}{U_t / (1 - x_t)} \end{aligned}$$

In contrast with firms and households, the social planner internalizes the impact of its hiring and participation decisions on the job finding rate x_t and, hence, of the hiring cost. The optimality conditions characterizing the resulting constrained-efficient allocation are given by

$$MRS_t = MPN_t - (1 + \gamma) (G_t - (1 - \delta) E_t \{ \Lambda_{t,t+1} G_{t+1} \}) \quad (53)$$

and

$$\psi MRS_t = \gamma \frac{x_t}{1 - x_t} G_t \quad (54)$$

where $MPN_t \equiv (1 - \alpha) A_t N_t^{-\alpha}$ is the marginal product of labor and, as above, $MRS_t \equiv \chi C_t L_t^\varphi$ is the marginal disutility of labor market effort, expressed in terms of the final consumption bundle.

5.1.1 The Efficient Steady State

Evaluated at the steady state, the previous two efficiency conditions take the form:

$$(1 + \gamma)(1 - \beta(1 - \delta)) \Gamma x^\gamma = (1 - \alpha)N^{-\alpha} - \chi CL^\varphi \quad (55)$$

$$(1 - x)\psi\chi CL^\varphi = \gamma\Gamma x^{1+\gamma} \quad (56)$$

By comparing (55)-(56) with the corresponding steady state conditions of the decentralized economy (49)-(50), it is easy to see that the latter's steady state will be efficient whenever

$$\mathcal{M}^p(1 - \tau) \quad (57)$$

and

$$\xi(1 + \gamma) = 1 \quad (58)$$

In words, condition (57) requires that the subsidy on the purchases of intermediate goods should exactly offset the impact of firms' market power, as reflected in the desired gross markup \mathcal{M}^p . Condition (58) is a version of the Hosios condition similar to the one derived in Blanchard and Galí (2010). It involves an inverse relation between firms' relative bargaining power, ξ , and the elasticity of hiring costs, γ . That inverse relation captures the negative externality (in the form of larger hiring costs) caused by firms' hiring decisions, and the positive externality resulting from higher participation (in the form of reduced hiring costs). The stronger are these externalities (corresponding to a larger γ), the lower is the relative bargaining power of firms (the smaller ξ) consistent with an efficient allocation, since the implied higher wages would induce fewer hires and more participation.

5.2 Optimal Monetary Policy

For simplicity, and throughout this section, I maintain the assumption of a constrained-efficient steady state, i.e. conditions (57) and (58) are assumed to hold. The assumption of an efficient steady state is often made in the literature on optimal monetary policy, for in that case the latter focuses exclusively on offsetting (or at least alleviating) the consequences of inefficient fluctuations in response to shocks.⁴¹ Like before, I consider the two scenarios of flexible and sticky wages in turn.

5.2.1 The Case of Flexible Wages

Under the assumption of period-by-period Nash bargaining of wages analyzed in Section 4.1, it is easy to check that the optimal monetary policy corresponds to a strategy of strict inflation targeting, i.e. full stabilization of the price level. To see this, note from (9) that under that policy the markup of final goods firms will remain constant and equal to the desired level, i.e. $P_t/P_t^I = \mathcal{M}^p(1 - \tau)$, for all t . Combined with assumption (57), it follows that $MRPN_t = MPN_t = (1 - \alpha)A_tN_t^{-\alpha}$ for all t . Thus, and imposing (58), one can easily check that equilibrium conditions (23) and (24) match exactly the efficiency conditions (53) and (54). In other words, the resulting equilibrium allocation is efficient.

Intuitively, under assumptions (57) and (58), the equilibrium of an economy in which both prices and (Nash bargained) wages are flexible involves a constrained-efficient allocation. Under flexible wages, a monetary policy that succeeds in fully stabilizing the price level replicates that natural allocation, and is thus optimal. That policy can be implemented with the assumed

⁴¹See Woodford (2003) and Galí (2008) for a discussion of these issues in the context of the New Keynesian model without frictions.

interest rate rule by choosing an arbitrarily large coefficient ϕ_π . That environment is thus characterized by what Blanchard and Galí (2007) refer to as "the divine coincidence," i.e. the absence of a trade-off between inflation stabilization and the attainment of an efficient allocation: one implies the other.

The previous finding hinges on the efficiency of the flexible price equilibrium allocation, guaranteed by assumptions (57)-(58). Faia (2009) analyzes the optimal policy in a related model (i.e. one with labor market frictions, sticky prices, and flexible wages), while relaxing the assumption of efficiency of the flexible price allocation. She shows that in that case it is optimal for the central bank to deviate from a policy of strict inflation targeting, though the size of the deviations implied by her calibrated model are quantitatively small.

5.2.2 The Case of Sticky Wages

As is well known from the analysis of Erceg et al. (2001) and others, when both prices and wages are sticky it is generally impossible for the central bank to replicate the constrained-efficient equilibrium allocation, which under assumptions (57)-(58) corresponds to the equilibrium allocation in the absence of nominal rigidities (the *natural* allocation, for short), as discussed above. The intuition behind that result is straightforward: in response to real shocks the real wage will generally adjust in the equilibrium with flexible prices and wages, and that adjustment will be necessary to support the resulting (constrained-efficient) allocation. Of course, any adjustment of the real wage requires some variation in either the price level or the nominal wage. But in the presence of sticky prices and wages such variations will occur only in response to deviations of average price markups and/or aver-

age real wages from their natural counterparts (see equations (9) and (40)), from which it follows that the natural (and efficient, under my assumptions) allocation will not be attainable.

In order to determine the optimal policy in that context I start by deriving a second order approximation to the representative household's utility losses caused by deviations from the constrained efficient allocation due to the presence of nominal rigidities. In so doing I restrict myself to the case of small fluctuations around the efficient steady state. As derived in Appendix 4, the loss function takes the following form (expressed in terms of the consumption-equivalent loss, as a fraction of GDP):

$$\mathbb{L} \equiv \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\epsilon}{\lambda_p} (\pi_t^p)^2 + \frac{(1-\Phi)^2(1-\alpha)}{\alpha\lambda_w^*} (\pi_t^w)^2 + \frac{(1+\varphi)(1-\Omega)N}{(1-\alpha)L} \left(\tilde{y}_t + \frac{(1-\alpha)\psi U}{N} \tilde{u}_t \right)^2 \right) \quad (59)$$

where $\tilde{y}_t \equiv y_t - y_t^n$ and $\tilde{u}_t \equiv u_t - u_t^n$ are, respectively, the output and unemployment gaps relative to their natural counterparts (where the latter are defined as their equilibrium values under flexible prices and wages); $\lambda_w^* \equiv (1-\theta_w)(1-\beta\theta_w)/\theta_w$ is inversely related to the degree of wage rigidities θ_w ; and $1-\Omega \equiv \frac{MRS}{MPN} = 1 - \frac{B(1+\gamma)}{MPN}$ is the steady state gap between the marginal rate of substitution and the marginal product of labor resulting from the existence of labor market frictions. Note that in the absence of labor market frictions and under flexible wages $\lambda_w^* \rightarrow \infty$, $\Omega = 0$, $U = 0$ and $N/L = 1$, so the previous loss function collapses to the one familiar from the basic New Keynesian model.⁴²

The presence of labor market frictions has two implications for the welfare criterion. First, to the extent that they are accompanied by staggered

⁴²See the expression in p. 81 in Galí (2008), under $\sigma = 1$.

nominal wage setting, fluctuations in wage inflation will generate welfare losses due to the implied dispersion in wages and the resulting losses from an inefficient allocation of labor across firms.⁴³ Note that here the size of the welfare losses resulting from any given departure from wage stability is (i) increasing in $1 - \Phi$ (which measures the weight of wages in the total cost of employing a new worker), (ii) decreasing in the degree of diminishing returns to labor α (for the latter dampens the extent of employment dispersion caused by any given level of wage dispersion) and (iii) increasing in the degree of wage stickiness θ_w (which determines the degree of wage dispersion caused by a given deviation from zero wage inflation).

Secondly, and to the extent that $\psi > 0$, the welfare criterion above points to a specific role for unemployment gap fluctuations as a source of welfare losses, beyond that associated with variations in the output gap (or the employment gap, which by construction is proportional to output gap). That role is related to the fact that unemployment is a component of effective labor market effort, and that fluctuations in the latter (relative to its efficient benchmark) generate disutility. The importance of unemployment fluctuations is thus increasing in ψ and U , which determine the weight of unemployment in the total disutility from market effort.

The equilibrium allocation under the optimal monetary policy can be determined by minimizing (59) subject to the log-linearized equilibrium conditions listed in Appendix 2 (excluding the Taylor rule). Figure 7 displays the equilibrium responses to a technology shock of the same variables considered earlier, under the optimal policy. For the sake of comparison it also

⁴³By contrast, in the monopoly union model of Erceg et al. (2000) the welfare losses from wage inflation are a consequence of the distorted allocation of employment across labor types *within each firm*, resulting from dispersion in their wages caused by staggered wage setting.

displays the corresponding responses under the Taylor rule used above. The simulation is based on a calibration with stickiness in both prices and wages. Note that the optimal response implies some deviation from price stability. In particular it requires a temporary decline in inflation, which makes it possible for the real wage to adjust upward with a smaller upward adjustment of nominal wages.⁴⁴ It also allows for a stronger accommodation of the increase in productivity, as reflected in the larger positive response of output. In accordance, employment is allowed to rise, and unemployment to decline. Note also that the optimal policy is associated with a smaller decline in inflation than the Taylor rule. Despite the greater price stability, the cumulative response of the real wage is stronger under the optimal policy, which requires positive wage inflation (not shown) in contrast with the wage deflation associated with the equilibrium under the Taylor rule.

Is there a simple interest rate rule that the central bank could follow that would improve on the assumed Taylor rule? In order to answer that question I compute the optimal rule among the class of interest rate rules of the form:

$$i_t = \rho + \phi_p \pi_t^p + \phi_y \hat{y}_t + \phi_w \pi_t^w + \phi_u u_t$$

where I have added wage inflation and the unemployment rate as arguments, relative to the conventional Taylor rule. The coefficients that minimize the households welfare loss, determined by iterating over all possible configurations, are $\phi_p = 1.51$, $\phi_y = -0.10$, $\phi_w = 0.01$, and $\phi_u = -0.025$. Figure 8 summarizes the dynamic response of the economy under that optimal simple rule, and compares it to the corresponding responses under the fully optimal policy. As the figure makes clear the differences between the two

⁴⁴See Thomas (2008) for a related result in the context of a similar model.

are practically negligible. Note that relative to the standard Taylor rule, the optimized simple rule calls for further accommodation of supply-driven output variation and also puts some weight on stabilization of unemployment. Interestingly, the optimal coefficient on price inflation is very close to 1.5, the value often assumed in standard calibrations of the Taylor rule (following Taylor (1993)). Perhaps more surprisingly, the weight on wage inflation is close to zero. This is in contrast with the findings in Erceg et al. (2000), where stabilization of wage inflation emerges as a highly desirable policy from a welfare viewpoint.⁴⁵ On the other hand, the desirability of a systematic policy response to unemployment fluctuations is in line with the findings on optimal simple rules in Blanchard and Galí (2009) and Faia (2009).

Given the relatively small values of the coefficients on variables other than price inflation in the optimized interest rate rule, a rule of the form $i_t = \rho + 1.5\pi_t^p$ leads to technology shock responses (not shown) that are similar to those generated by the optimized one. That rule can be interpreted as capturing the notion of *flexible* inflation targeting, whereby central banks seek to attain a pre-specified inflation target only gradually ("in the medium term," using the language of the ECB), as opposed to the *strict* inflation targeting that is optimal in environments in which price stickiness is the only nominal distortion.

The previous findings are consistent, at least in a qualitative sense, with the existing literature on optimal monetary policy in environments with labor market frictions and wage rigidities, despite the differences in modelling

⁴⁵The structure of the present model and the associated inefficiencies resulting from wage dispersion lead to a coefficient on wage volatility in the loss function that is about one-third the size of the coefficient on price inflation. That ranking is reversed for standard calibrations of the Erceg et al (2000) model.

details. This is the case, in particular, for Blanchard and Galí (2010) (in a model with real wage rigidities) and Thomas (2008) (in a model with staggered nominal wage setting like the present one).

6 Possible Extensions

As argued in the introduction, it is not the goal of the present paper to offer an exhaustive analysis of existing models of monetary policy and unemployment. Instead, I have developed and analyzed a relatively streamlined model, but one which in my view contains the key ingredients to illustrate the consequences of the coexistence of nominal rigidities and labor market frictions. The model is, however, sufficiently flexible to be able to accommodate many extensions that can already be found in the literature. A list of some of those extensions, with a brief description of ways to introduce them, but without any further analysis, follows.

Real wage rigidities and wage indexation. As emphasized by Blanchard and Galí (2007, 2009) the presence of real wage rigidities may have implications for the optimal design of monetary policy that are likely to differ from the ones generated by a model with nominal wage rigidities only (like the one emphasized here). Among other things, in the presence of real wage rigidities, the policymaker cannot use price inflation to facilitate the adjustment of real wages. A simple way to introduce real wage rigidities would be to allow for (possibly partial) wage indexation to contemporaneous wage inflation between wage renegotiations. Formally, one can assume:

$$W_{t+k|t} = W_{t+k-1|t} (P_{t+k}/P_{t+k-1})^\varsigma$$

for $k = 1, 2, 3, \dots$ and $W_{t|t} = W_t^*$, and where $W_{t+k|t}$ is the nominal wage in period $t+k$ for an employment relationship whose wage was last renegotiated

in period t . Note that parameter $\zeta \in [0, 1]$ measures the degree of indexation. An alternative specification, often used in the New Keynesian literature (e.g. Smets and Wouters (2007)) and adopted by Gertler, Sala and Trigari (2008), assumes instead indexation to past inflation. Formally,

$$W_{t+k|t} = W_{t+k-1|t} (P_{t+k-1}/P_{t+k-2})^\zeta$$

for $k = 1, 2, 3, \dots$. In the latter case, even with full indexation, price inflation can still be used to speed up the adjustment of real wage to shocks that warrant such an adjustment, due to the lags in indexation.

Greater wage flexibility for new hires. As discussed above, a number of authors (Haefke et al. (2007), Pissarides (2008), Carneiro et al. (2008)) have argued that while the wages of incumbent workers display some clear rigidities, the latter may not have allocative consequences (to the extent they remain within the bargaining set) since the wage that determines hiring decision is the wage of new hires, which is likely to be more flexible, according to some evidence. Even though that evidence remains controversial and has been disputed in some quarters (see references above), it may be of interest to see how such differential flexibility can be introduced in the model, and to explore its positive and normative implications. A tractable and flexible way of introducing that feature, proposed in Bodart et al (2006), involves the assumption that new hires at a firm are paid either the average wage (with probability η) or a freely negotiated wage (with probability $1 - \eta$). Parameter η is thus an index of the degree of relative wage flexibility for new hires. That assumption would require a change in the equation describing the value of unemployment, since the probability of bargaining over wage at the time of being hired would now be $1 - \theta_w \eta$, instead of $1 - \theta_w$. One could then quantify the extent to which the responses to shocks and

the optimal policy vary with η .

Smaller wealth effects. The analysis above has relied on a specification of utility with wealth effects of labor supply that are likely to be implausibly large. That could explain the unusual unrealistic behavior of the labor force under some of the calibrations discussed above. One way to get around that problem is to assume the following alternative specification of the utility function, originally proposed in Galí (2010):⁴⁶

$$U(C_t, L_t) \equiv \Theta_t \log C_t - \frac{\chi}{1 + \varphi} L_t^{1+\varphi}$$

where $\Theta_t \equiv \bar{C}_t/Z_t$, \bar{C}_t is aggregate consumption (taken as given by each individual household), and

$$Z_t = Z_{t-1}^\vartheta \bar{C}_t^{1-\vartheta}$$

and $\vartheta \in [0, 1]$. In that case the marginal rate of substitution between consumption and market effort is given (in logs) by

$$mrs_t = z_t + \varphi l_t$$

where $z_t = (1 - \vartheta)c_t + \vartheta z_{t-1}$. Thus, changes in consumption will have an arbitrarily small effect on the short-run supply for market effort, if ϑ is close to unity. Given that the gap between z_t and c_t is stationary (even when c_t displays a linear trend or a unit root), the previous specification of utility will still be consistent with a balanced growth path.

Other demand shocks. The analysis of optimal monetary policy above assumes the economy faces only a technology shock (naturally, the monetary policy shock is turned off for the purposes of that exercise). How the

⁴⁶See Jaimovich and Rebelo (2008) for an alternative specification of utility in the same spirit.

policy implications may vary once a shock other than technology is introduced seems worthy of investigation. In particular, it may be the case that in that scenario the optimal policy will attach a greater weight to output stabilization.⁴⁷

7 Conclusions

Over the past few years a growing number of researchers have turned their attention towards the development and analysis of extensions of the New Keynesian framework that model unemployment explicitly. The present paper has described some of the essential ingredients and properties of those models, and their implications for monetary policy.

The analysis of a calibrated version of the model developed here suggests that labor market frictions are unlikely, either by themselves or through their interaction with sticky prices, to have large effects on the equilibrium response to shocks, in an economy with nominal rigidities and a monetary policy described by a simple Taylor type rule. In that respect, perhaps the most important contribution of those frictions lies in their ability to reconcile the presence of wage rigidities with privately efficient employment relations. The presence of those nominal wage rigidities has, on the other hand, important consequences for the economy's response to shocks as well as for the optimal design of monetary policy. Thus, in the model developed above, the optimal policy allows for significant deviations from price stability, in order to facilitate the adjustment of real wages to real shocks. Furthermore, the outcome of that policy can be approximated by means of a simple interest rate rule that responds to both price inflation and the unemployment rate.

⁴⁷Sveen and Weinke (2008) make a forceful case for the importance of demand shocks in accounting for labor market dynamics.

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Table 1. Cyclical Properties

	<i>Unconditional</i>		<i>Demand</i>		<i>Technology</i>	
	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x, y)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x, y)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x, y)$
<i>Employment</i>	0.60	0.83	0.59	0.92	0.90	0.51
<i>Labor force</i>	0.20	0.30	0.20	0.31	0.39	0.02
<i>Unemployment rate</i>	0.49	-0.90	0.50	-0.93	0.62	-0.76
<i>Real Wage</i>	0.44	0.07	0.32	-0.78	0.27	0.27
<i>Inflation</i>	0.19	0.27	0.18	0.37	0.27	0.60

Figure 1. Estimated Effects of Technology Shocks

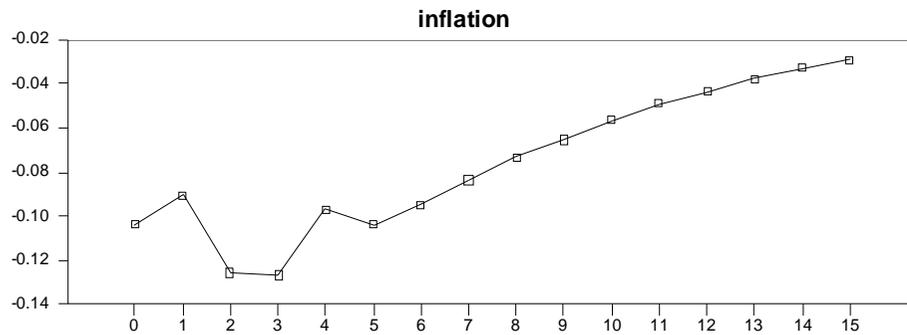
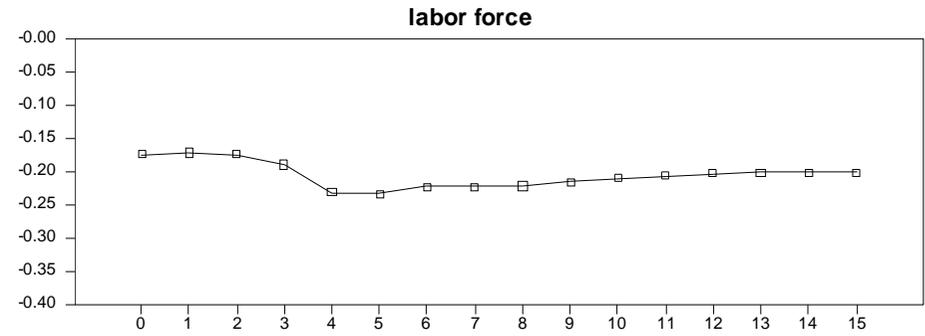
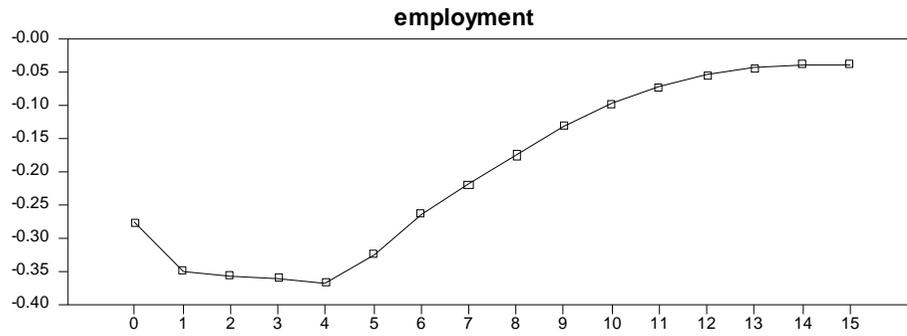
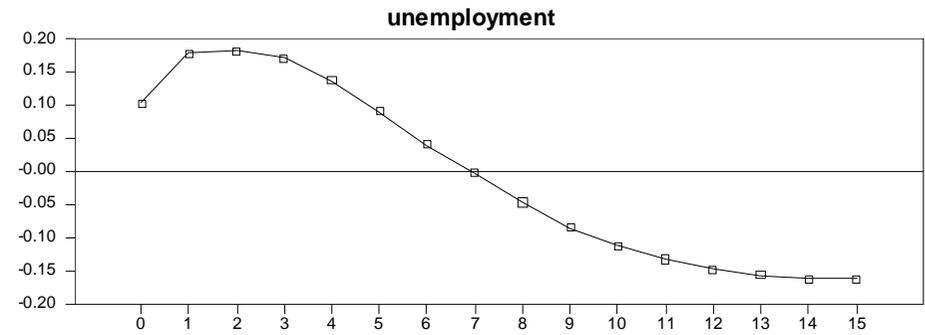
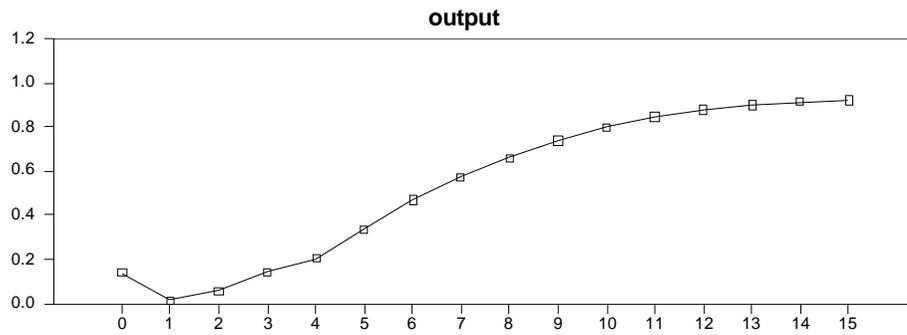


Figure 2a. The Effects of Monetary Policy Shocks: Sticky Wages ($\xi=0.5$)

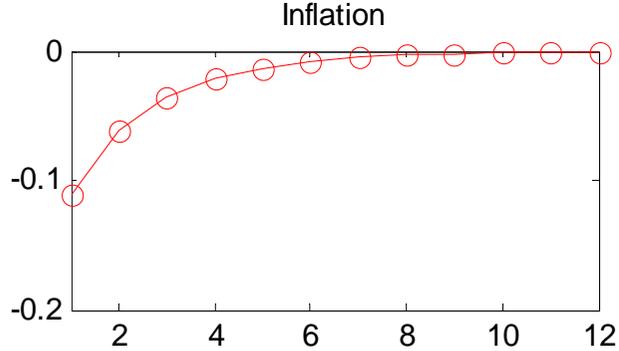
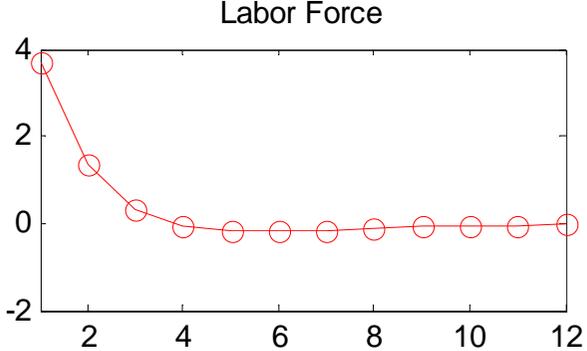
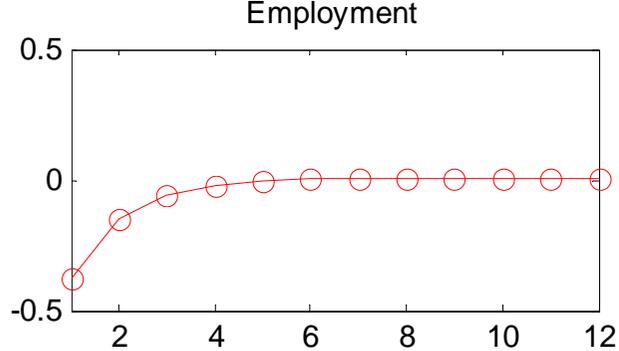
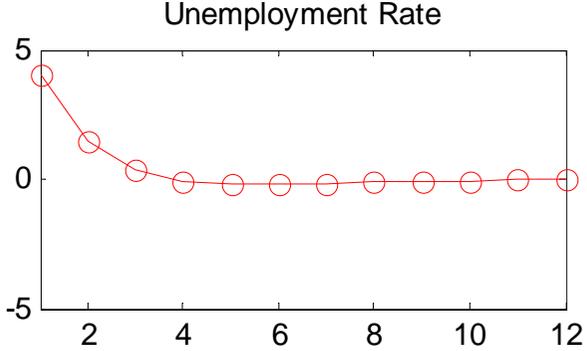
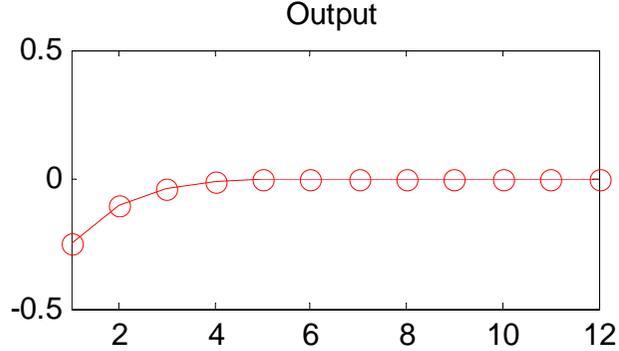


Figure 2b. The Effects of Technology Shocks: Sticky Wages ($\xi=0.5$)

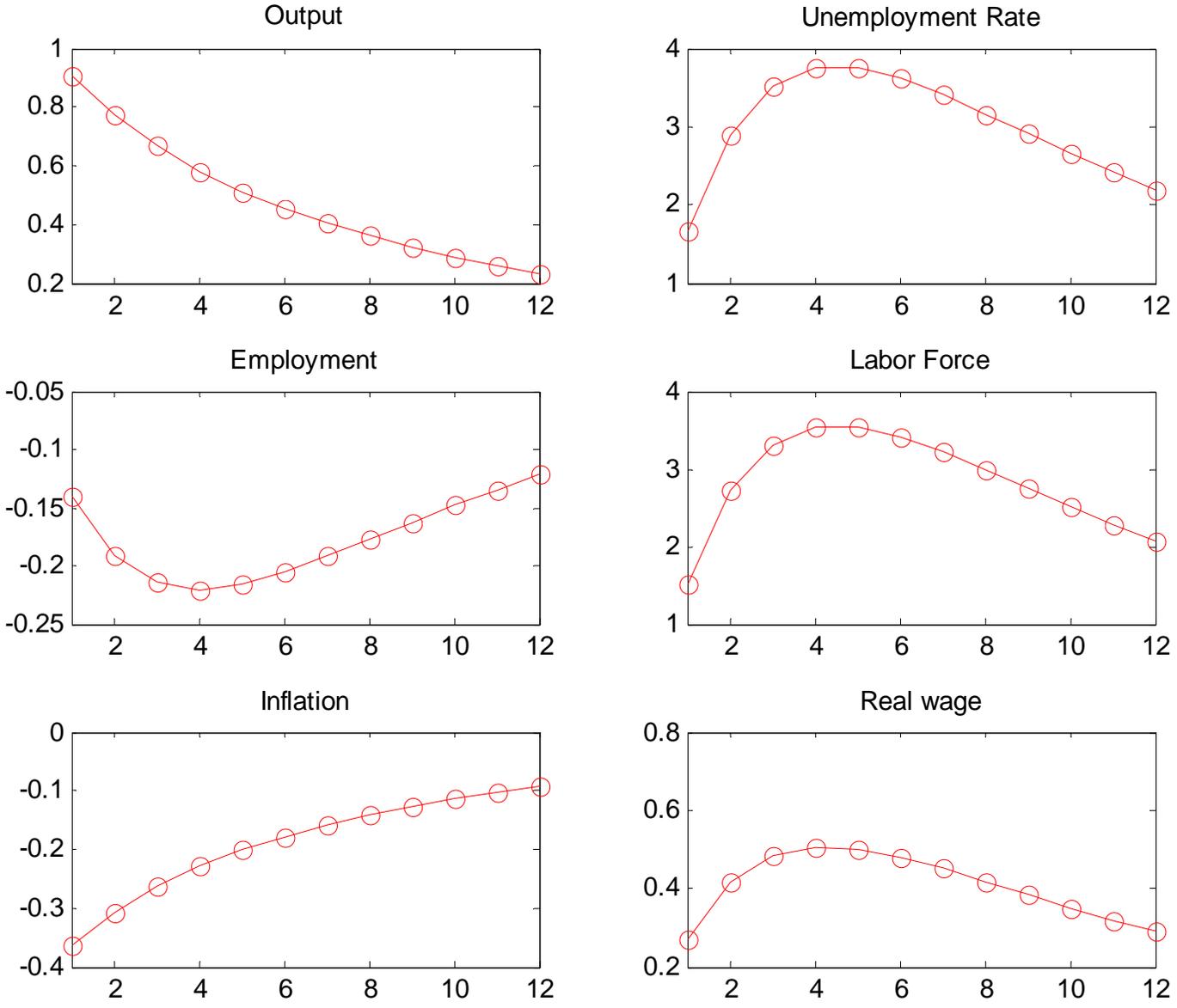


Figure 3a. The Effects of Monetary Policy Shocks: Sticky Wages ($\xi=0.05$)

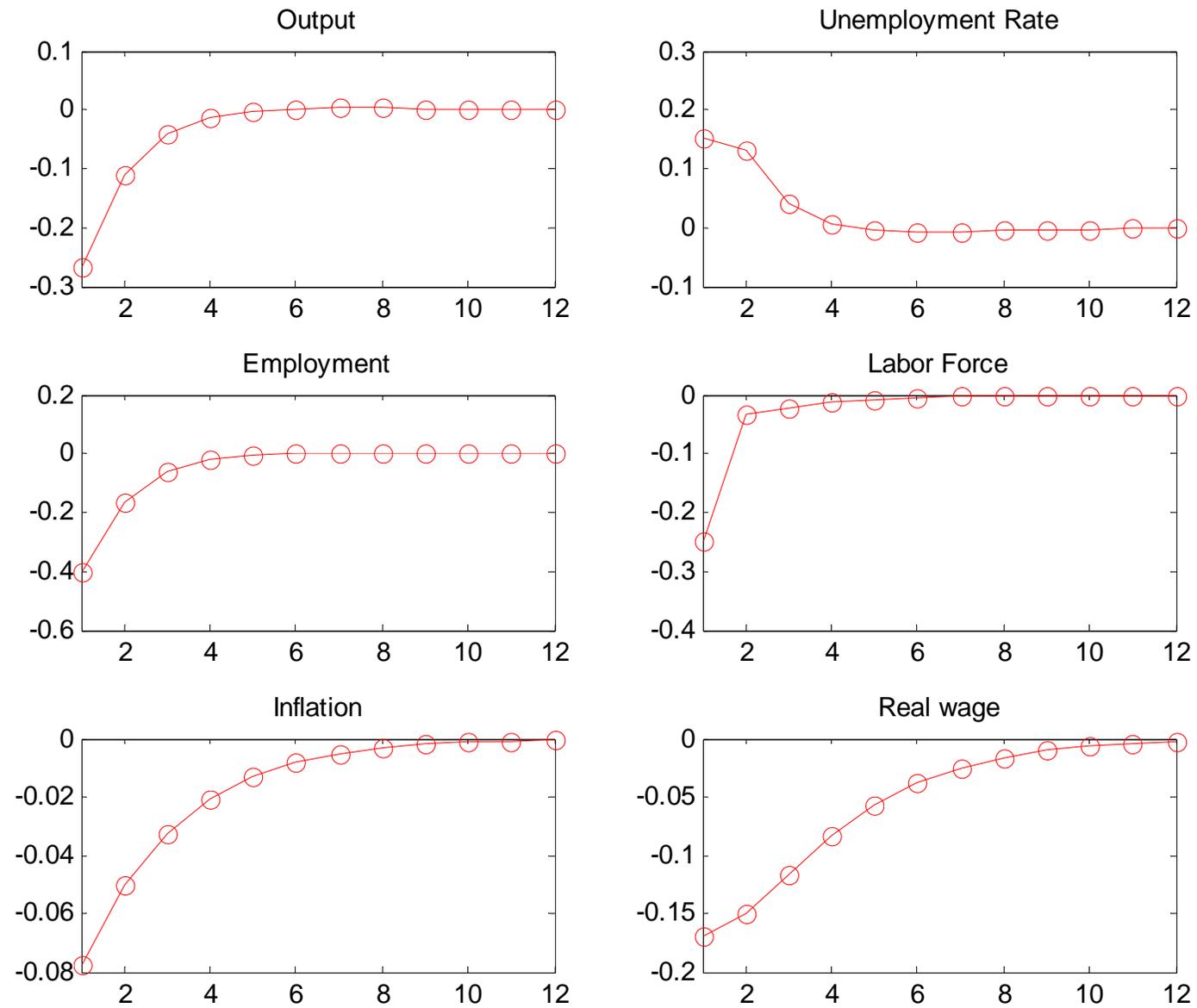


Figure 3b. The Effects of Technology Shocks: Sticky Wages ($\xi=0.05$)

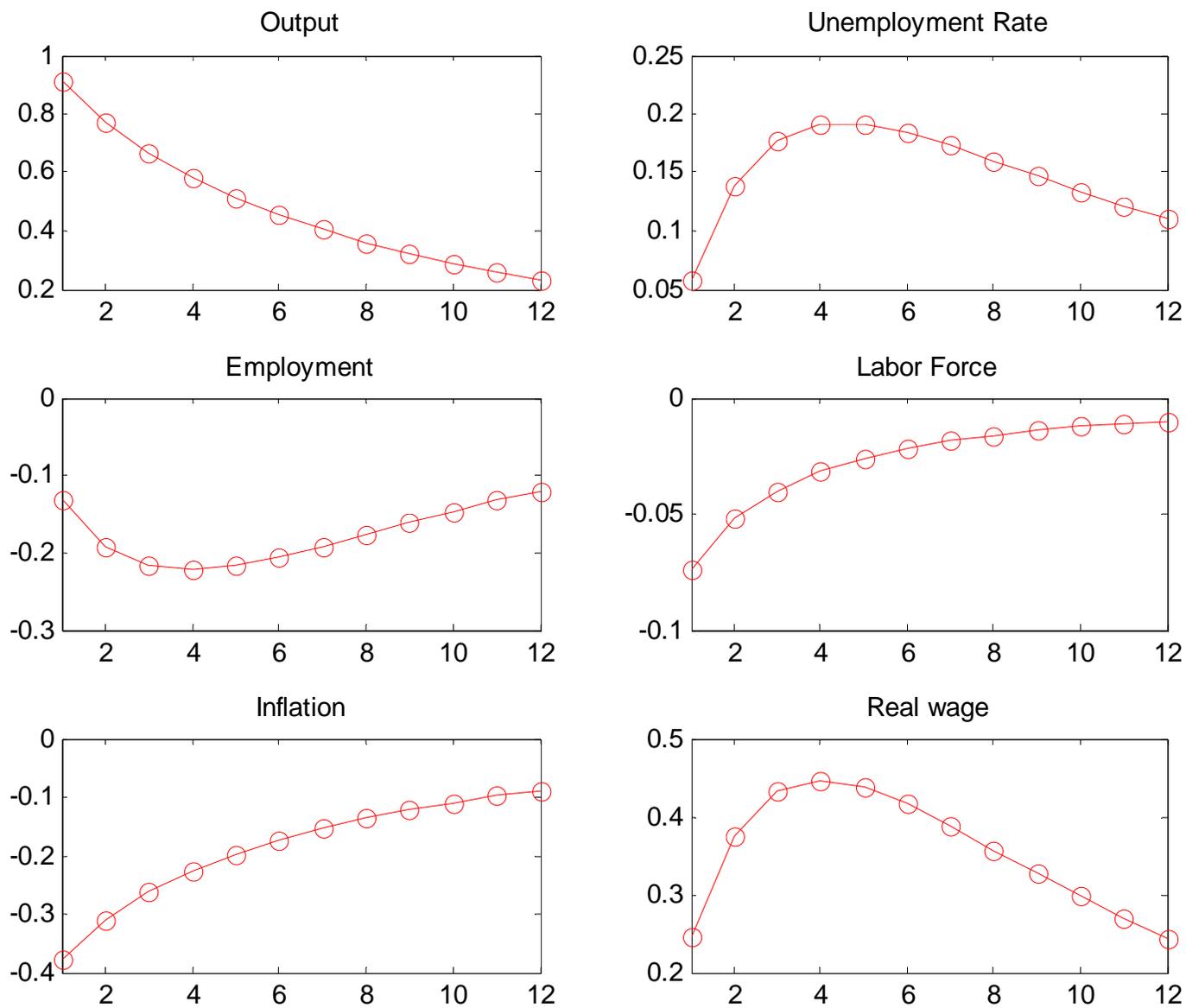


Figure 4a. The Role of Labor Market Frictions
Flexible Wages, Monetary Policy Shock

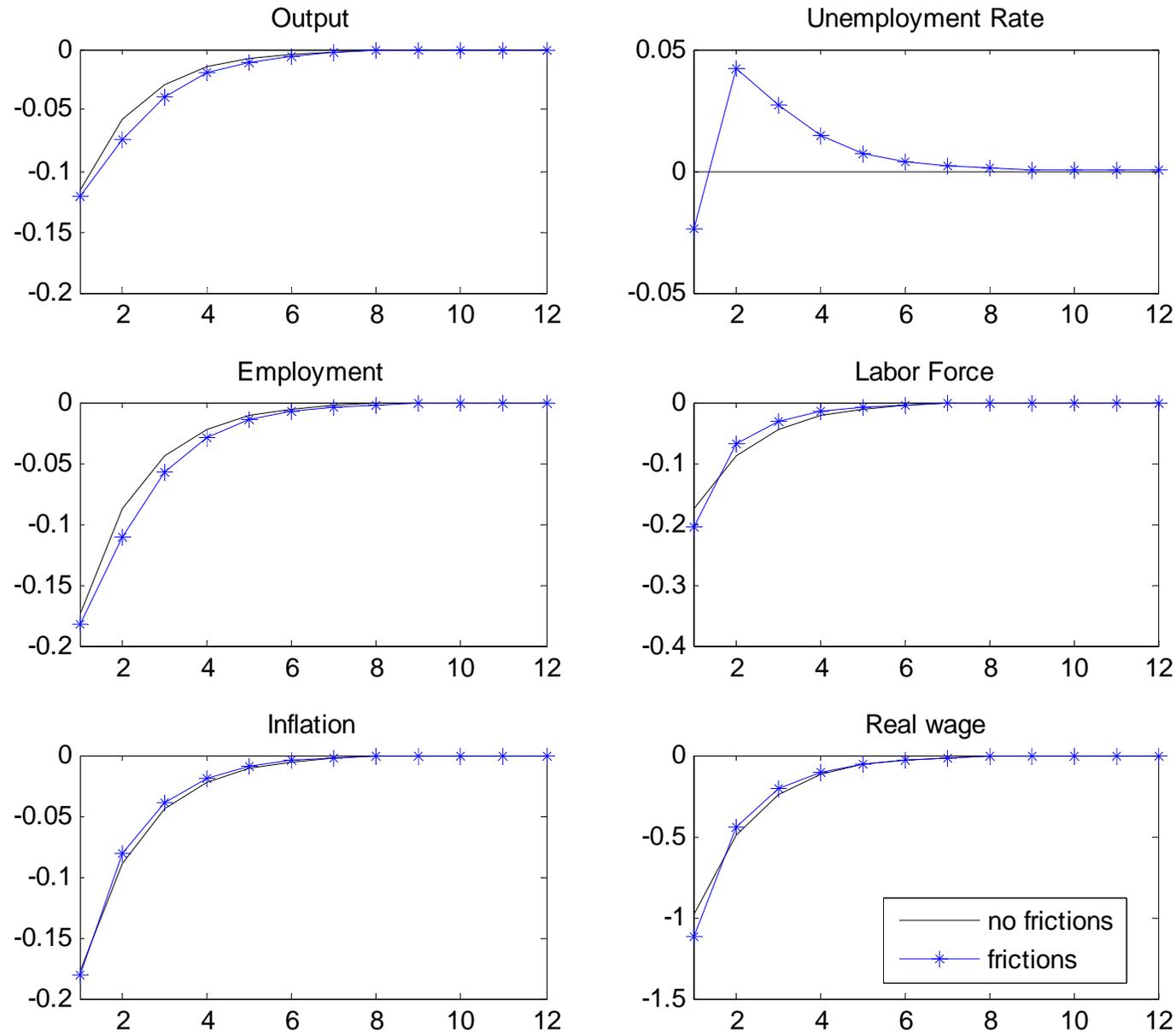


Figure 4b. The Role of Labor Market Frictions
Flexible Wages, Technology Shock

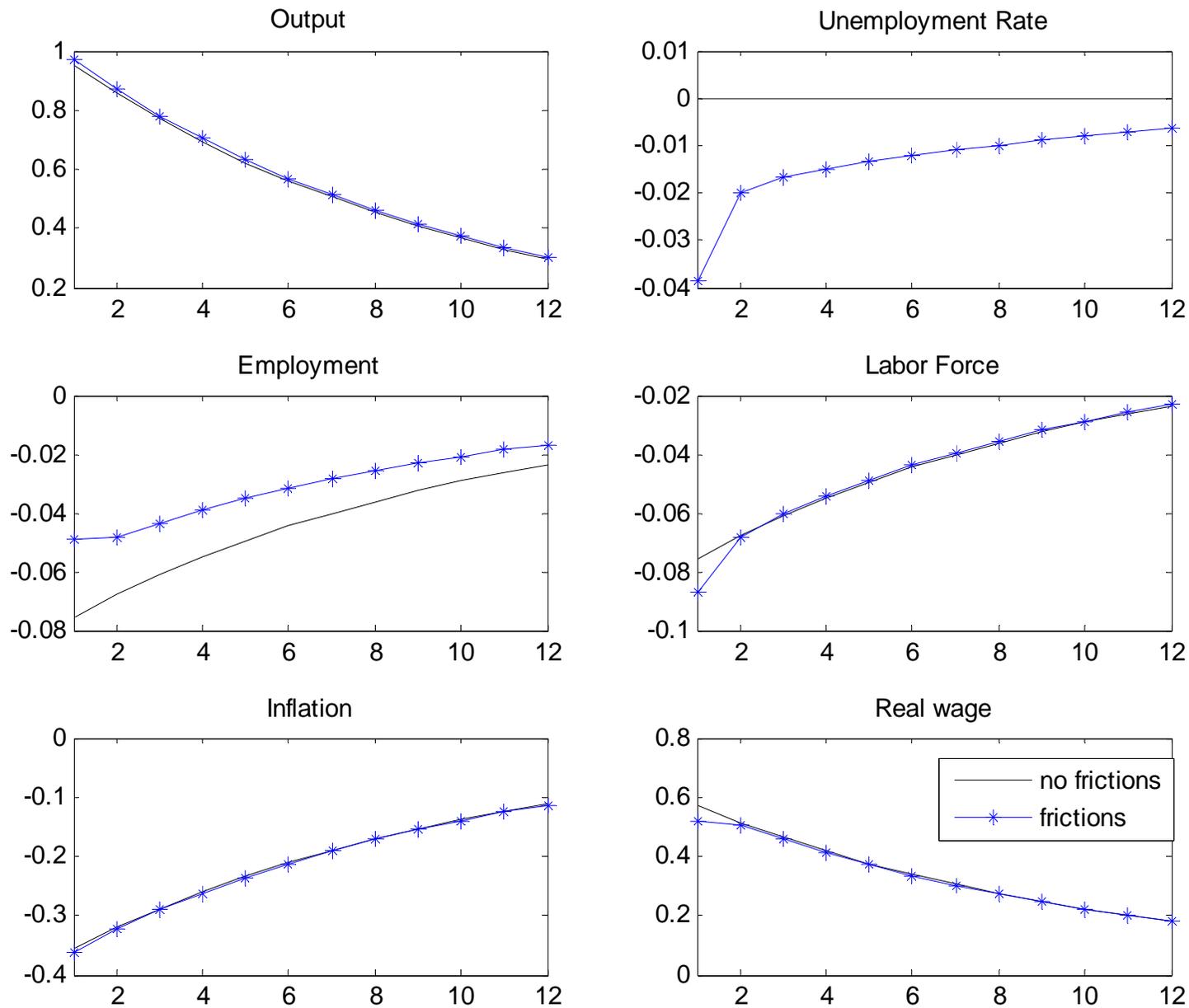


Figure 5a. The Role of Price Stickiness
Flexible Wages, Monetary Policy Shock

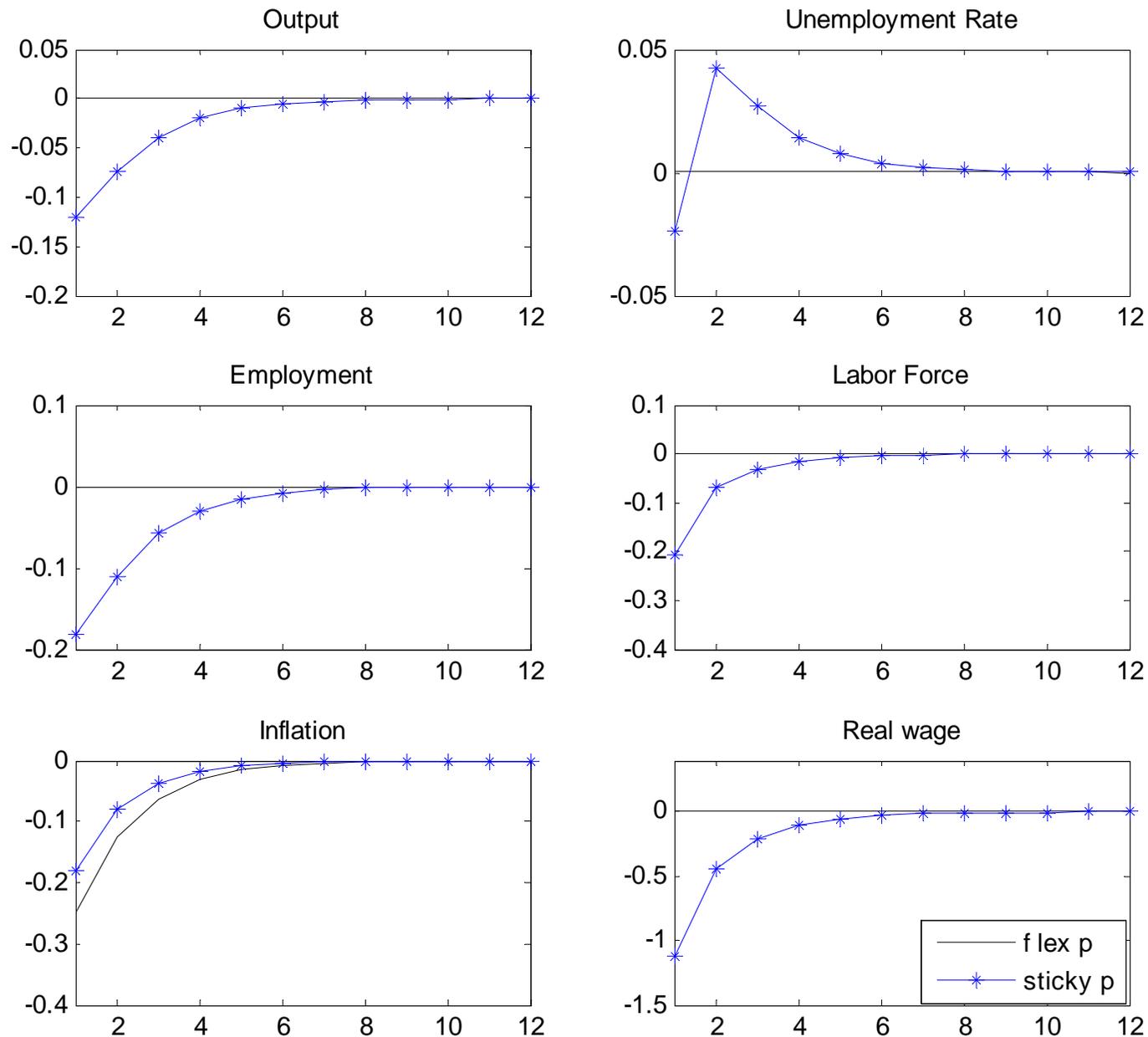


Figure 5b. The Role of Price Stickiness
Flexible Wages, Technology Shock

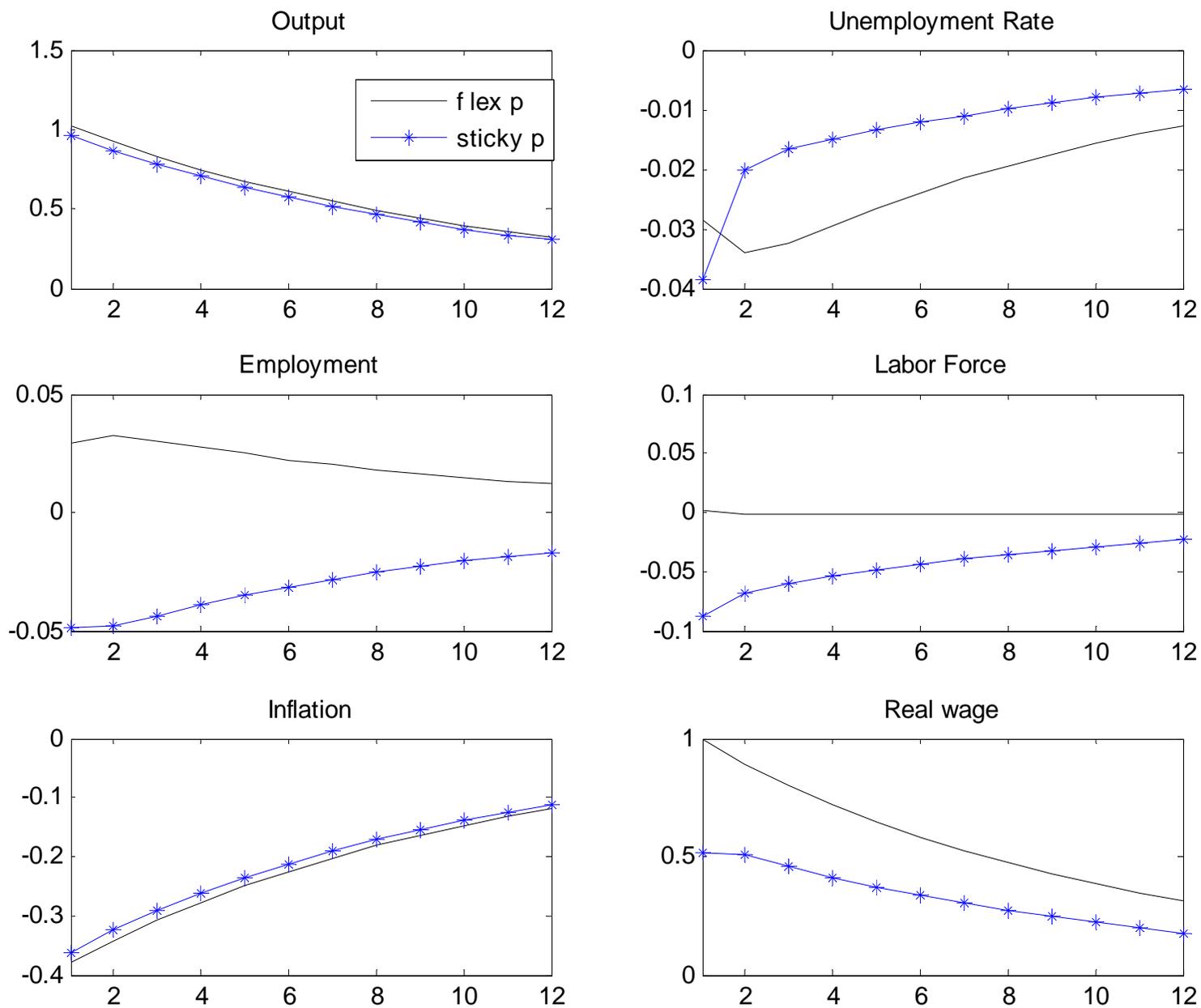


Figure 6a. The Role of Wage Stickiness

Sticky Prices, Monetary Policy Shock

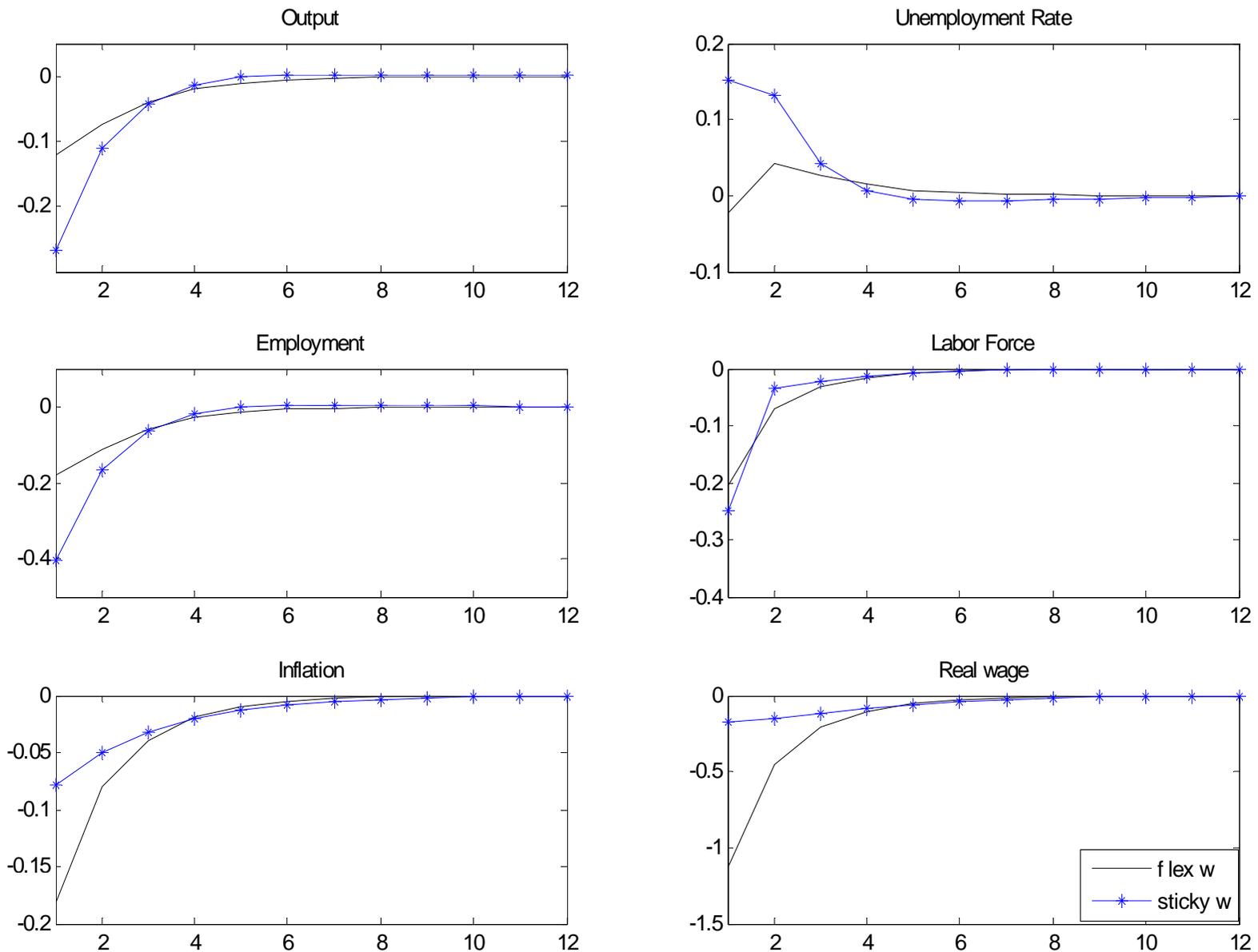


Figure 6b. The Role of Wage Stickiness
Sticky Prices, Technology Shock

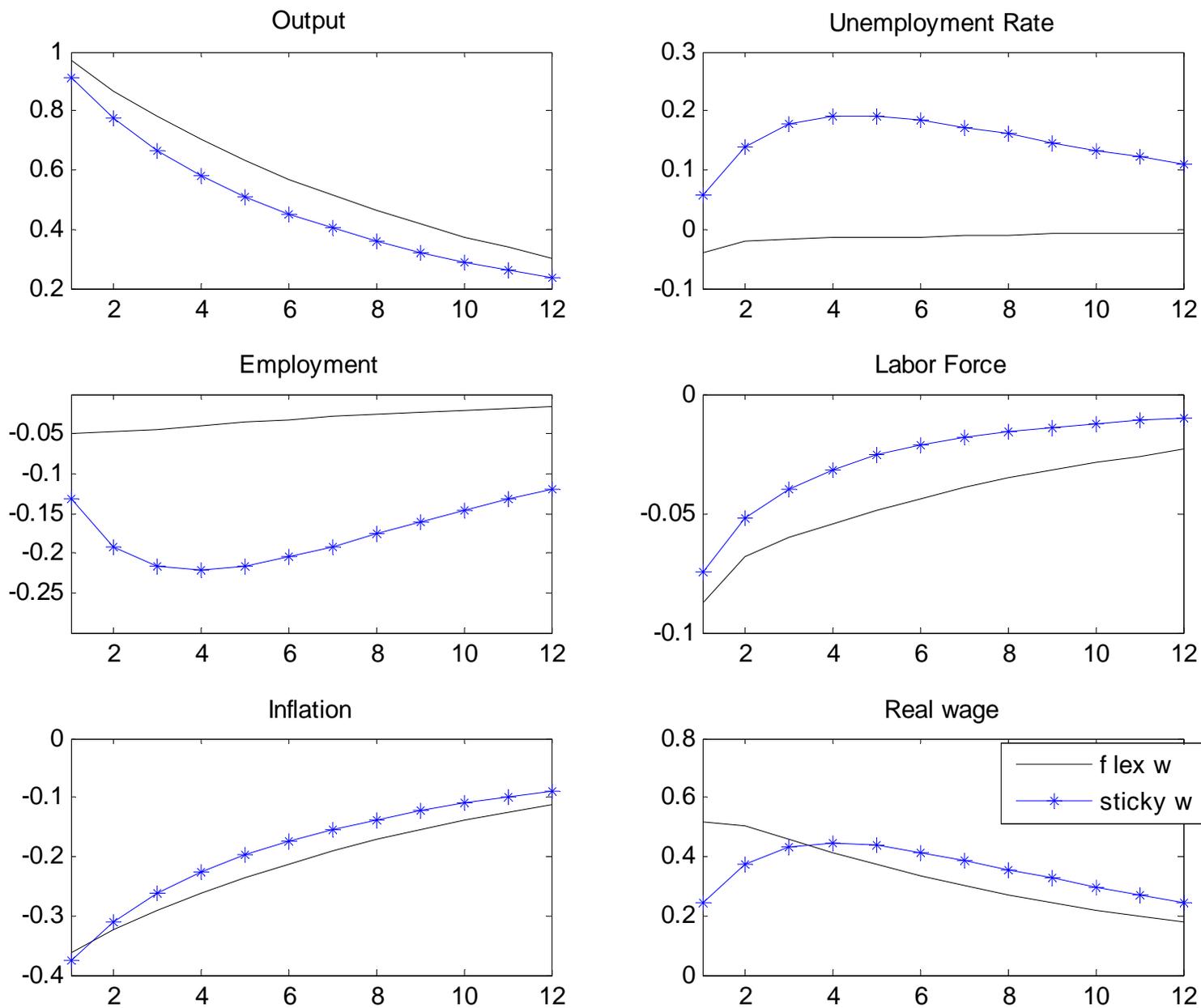


Figure 7. Monetary Policy Design: Optimal vs. Taylor
Sticky Prices and Wages, Technology Shock

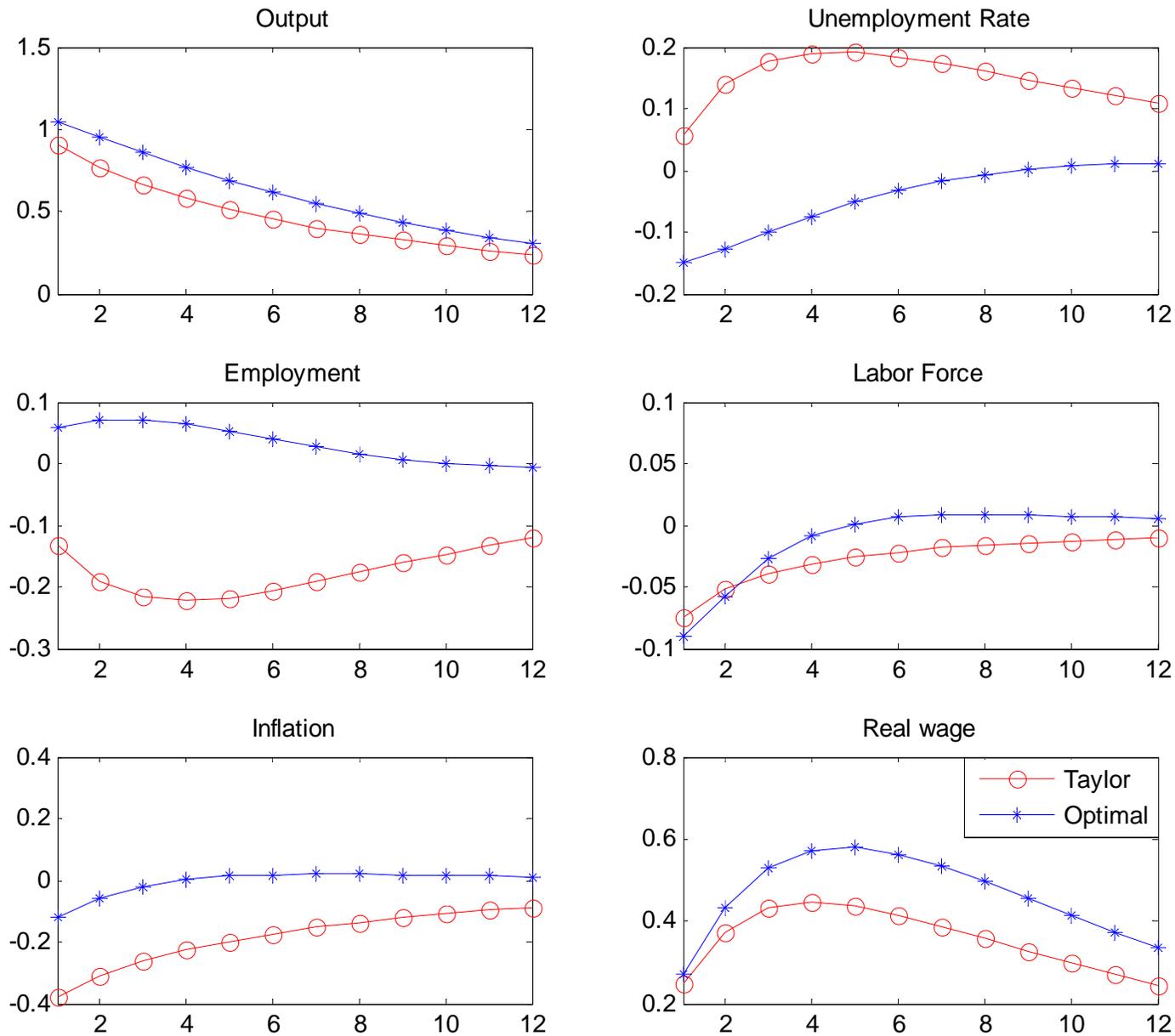


Figure 8. Monetary Policy Design: Optimal vs. Optimal Simple
Sticky Prices and Wages, Technology Shock

