THE CROSS-SECTION AND TIME-SERIES OF STOCK AND BOND RETURNS

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ABSTRACT

We propose an arbitrage-free stochastic discount factor (SDF) model that jointly prices the cross-section of returns on portfolios of stocks sorted on book-to-market dimension, the cross-section of government bonds sorted by maturity, the dynamics of bond yields, and time series variation in expected stock and bond returns. Its pricing factors are motivated by a decomposition of the pricing kernel into a permanent and a transitory component. Shocks to the transitory component govern the level of the term structure of interest rates and price the cross-section of bond returns. Shocks to the permanent component govern the dividend yield and price the average equity returns. Third, shocks to the relative contribution of the transitory component to the conditional variance of the SDF govern the Cochrane-Piazzesi (2005, CP) factor, a strong predictor of future bond returns. These shocks price the cross-section of book-to-market sorted stock portfolios. Because the CP factor is a strong predictor of economic activity one- to two-years ahead, positive shocks to CP signal improving economic conditions, leading to a positive price of risk. Value stocks are riskier and carry a return premium because they are more exposed to such shocks.

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As long as some investors have access to both stock and bond markets, the absence of arbitrage opportunities imposes cross-market restrictions on the stochastic discount factor, henceforth SDF. Despite tremendous progress in the separate modeling of SDFs for bond markets and stock markets, the cross-market restrictions are typically not imposed. As a result, the state-of-the art term structure model does not price stocks and the state-of-the-art equity pricing model does not price bonds. We propose a parsimonious no-arbitrage SDF model that exploits these cross-market restrictions to learn about the pricing of risk across stock and bond markets.

A decomposition of the pricing kernel into a permanent and a transitory component is a useful device for understanding which risk factors are necessary to achieve consistent risk pricing in stock and bond markets. Alvarez and Jermann (2005) (AJ), Hansen, Heaton, and Li (2008) and Hansen and Scheinkman (2009) show that any SDF can be decomposed in a permanent component and a transitory component. AJ conclude that the permanent component accounts for almost all of the variation in the SDF. They reach this conclusion because the equity risk premium, their empirical proxy for the maximum risk premium in the economy, is so much higher in the data than the 30-year bond risk premium, their empirical proxy for the infinite-horizon bond risk premium. One minus the ratio of the risk premium on the infinite-horizon bond and the maximum risk premium is a lower bound on the contribution of the permanent component to the overall variance of the SDF. Thus, their paper shows that a necessary ingredient for any SDF model is a large permanent component (on average), and it shows that the innovations to this permanent component are naturally linked to the pricing of equity.[1] We identify these shocks as shocks to the dividend-price (DP) ratio on the aggregate stock market and show that exposure to DP shocks help us match average equity risk premia.

A second necessary ingredient of any SDF model is a transitory component.[2] In a model without transitory component, the term structure of interest rates is flat and bond excess returns are zero at all maturities. Clearly, such models are at odds with the fact that bond yields and bond returns depend on maturity and fluctuate over time. We identify transitory shocks as shocks to the level of the term structure, which is the most important source of variation in bond yields. We confirm Cochrane and Piazzesi (2008)'s finding that cross-sectional variation in returns across bond portfolios sorted by maturity can be captured by differential exposure to the level factor.

There is a third necessary feature of the SDF, which is that the relative contribution of the permanent and transitory innovations to the pricing kernel must vary over time. AJ study the ratio of the *unconditional* variance of the permanent component to the unconditional variance of the SDF in terms of the average bond risk premium relative to the average equity risk premium.

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[1] AJ show that if the pricing kernel has only a transitory component, the long-term bond is the asset with the highest risk premium. To measure the importance of the permanent component, they study both stocks and bonds and conclude that most of the variation is driven by the permanent component. This suggests that shocks to stock prices are a prime candidate to capture permanent shocks to the pricing kernel.

[2] The transitory component of the pricing kernel equals the inverse of the return on an infinite-maturity bond.

Since the bond risk premium and the equity risk premium both vary over time, it seems natural to consider the *conditional* variance ratio in addition. We model risk premia on bonds as a linear function of the Cochrane and Piazzesi (2005, 2008) factor (CP), a powerful predictor of future bond returns. We model risk premia on stocks as a linear function of the dividend-price ratio of the aggregate stock market, following a large stock return predictability literature. In principle, CP and DP could each drive fluctuations in the relative importance of the transitory and permanent components of the SDF. Our estimation results instead imply that the CP factor accounts for almost all of the action in the conditional variance ratio. When CP rises, so does the importance of the transitory component of the SDF; the conditional variance of the permanent component relative to the total conditional variance falls. The CP factor is much less persistent than the dividend yield DP; most of its variation occurs at business-cycle frequencies. The conditional variance ratio inherits this business-cycle frequency variation from CP.

Introducing shocks that drive the relative contribution of the transitory component of the SDF turns out to be very useful for understanding the link between stock and bond pricing. Exposure to innovations in the CP factor, which itself predicts future bond returns, helps explain the value premium puzzle in the stock market. Neither shocks to the transitory component (level shocks) nor shocks to the permanent component (DP shocks) can help explain why high book-to-market (value) stocks have higher returns than low book-to-market (growth) stocks because book-to-market decile portfolios have similar exposures to both. However, value stocks have a large positive exposure to shocks to the CP factor whereas growth stocks have much lower or even zero exposure, see Figure 1. Given the positive risk price of CP shocks we estimate, this differential exposure results in a value spread.

[Figure 1 about here.]

Why is the price of CP risk positive and why are value stocks riskier than growth stocks? We show that the CP factor is not only a strong forecaster of future bond returns but also of future economic activity. A high CP factor predicts stronger economic activity, as measured by the Chicago Fed National Activity Index (CFNAI), one to two years later, see Figure 2. The R-squared peaks at a horizon of 20 months at 15%; it gradually increases before and gradually declines afterwards. Since high CP readings signal better economic prospects, they coincide with low marginal utility growth for an average investor with a preference for early resolution of uncertainty, and hence the price of CP risk ought to be positive. This is what we find. This result also implies that value returns are high exactly when economic activity is expected to increase. Since these are times of low marginal utility growth, this positive covariance makes value stocks riskier, and explains why value stocks earn higher average returns than growth stocks.

[Figure 2 about here.]

The above argument thus suggests a model with three priced sources of risk: the DP factor to capture shocks to the permanent component, the level factor to capture shocks to the transitory component, and the CP factor to capture shocks to the contribution of the permanent component to the variance of the SDF. We use these three factors as the key state variables in an otherwise standard affine asset pricing model. First, we estimate the dynamics of the state vector and of the nominal short-term interest rate by matching the time series of yields (forward rates) of various maturities. Second, we estimate the dynamics of the risk prices by matching time series properties of expected excess stock and bond returns. In particular, the price of level risk depends on the CP factor as in Cochrane and Piazzesi (2008) and the price of DP risk depends on DP. Third, we choose the average risk prices on our three factors by matching the cross-section of average returns on the aggregate stock market, the decile book-to-market portfolios, and five maturity-sorted bond portfolios. Our model generates a mean absolute pricing error of only 0.40% per year on these 16 portfolios. The pricing errors show no pattern along the book-to-market nor the bond maturity dimensions. In summary, our affine model with three priced risk factors simultaneously accounts for the cross-section of average stock returns on the aggregate market and the decile book-to-market returns, the cross-section of maturity-sorted bond portfolios, the dynamics of expected stock and bond returns, and the dynamics of bond yields.

The rest of the paper is organized as follows. After discussing related literature in Section 1, Section 2 sets up an affine asset pricing model and shows how to decompose its SDF into a permanent and a transitory component. Section 3.1 estimates the affine asset pricing model and discusses the implications for the conditional variance ratio. Section 4 discusses the robustness of our results. Specifically, we study an extension where the CP factor also predicts stock returns, we study several other sets of test assets, and examine sub-samples. We find results that are robust. Notably, our model is able to account for the risk premia on the corporate bond portfolios; the average pricing error across the original 16 test assets and the additional 4 credit risk portfolios is 0.49% per year. Section 5 concludes.

1   Related Literature

This paper relates to several strands of the literature. The last twenty years have seen dramatic improvements in economists' understanding of what determines differences in yields (e.g., Cox, Ingersoll, and Ross (1985), Duffie and Kan (1996), Dai and Singleton (2000, 2002), Duffee (2002), Ang and Piazzesi (2003), Ang, Bekaert, and Wei (2008), and Cochrane and Piazzesi (2008)) and returns on bonds (Campbell and Shiller (1991), Cochrane and Piazzesi (2005) and Ludvigson and Ng (2009)), as well as what determines heterogeneity in stock returns which differ by characteristics such as size and book-to-market value (e.g., Fama and French (1992, 1993)). Yet, these two

literatures have developed largely separately and employ largely different asset pricing factors. This is curious from the perspective of a no-arbitrage model. As long as some investors have access to both markets, stock and bond prices ought to equal the expected present discounted value of their future cash-flows, discounted by the *same* stochastic discount factor. This paper contributes to both literatures and helps to bridge the gap between them. It speaks to a large empirical literature and a small but fast-growing theoretical literature.

On the empirical side, the nominal short rate or the yield spread is routinely used either as a predictor of the aggregate return on the stock market or as a conditioning variable in an estimation of a conditional beta model of the cross-section of stock returns. Chen, Roll, and Ross (1986) were the first to study the connection between stock returns and bond yields. Ferson and Harvey (1991) study stock and bond returns' sensitivity to aggregate state variables, among which the slope of the yield curve. They conclude that time variation in equity risk premia is important for understanding the cross-sectional variation in size and industry equity portfolios, and that interest rate risk premia are important for understanding the cross-sectional variation in bond return portfolios. Similarly, Fama and French (1993) find that three factors (market, size, and book-to-market) account for the cross-sectional variation in stock returns and that two bond factors (the excess return on a long-term bond over the short rate and a default spread) explain the variation in government and corporate bond returns. However, all of their stock portfolios load in the same way on their term structure factors. Ang and Bekaert (2007) find some predictability of nominal short rates for future aggregate stock returns. Brennan, Wang, and Xia (2004) write down an intertemporal-CAPM model where the real rate, expected inflation, and the Sharpe ratio move around the investment opportunity set. They show that this model prices the cross-section of stocks. Similarly, Petkova (2006) studies the connection between the Fama-French factors and innovations in state variables such as the default spread, the dividend-price ratio, the yield spread, and the short rate. In contrast to this literature, our focus is on the joint pricing of stock and bond returns, the dynamics of bond yields, as well as their link with the permanent and transitory components of the SDF. Baker and Wurgler (2007) show that government bonds comove most strongly with "bond-like stocks," which are stocks of large, mature, low-volatility, profitable, dividend-paying firms that are neither high growth nor distressed. They propose a common sentiment indicator driving stock and bond returns. Finally, Lustig, Van Nieuwerburgh, and Verdelhan (2008) price both nominal bond yields and the aggregate stock market return in a no-arbitrage model in order to measure the wealth-consumption ratio in the data; they do not study the cross-section of bond nor stock returns.

On the theory side, several representative-agent models have been developed that are successful in accounting for many of the features of both stocks and bonds. Examples are the external habit model of Campbell and Cochrane (1999), whose implications for bonds were studied by Wachter (2006) and whose implications for the cross-section of stocks were studied *separately* by Menzly,

Santos, and Veronesi (2004) and Santos and Veronesi (2006). Likewise, the implications of the long-run risk model of Bansal and Yaron (2004) for the term structure of interest rates were studied by Piazzesi and Schneider (2006) and Bansal and Shaliastovich (2007), while Bansal, Dittmar, and Lundblad (2005) and Bansal, Dittmar, and Kiku (2007) *separately* study the implications for the cross-section of equity portfolios. A small but growing literature models stock and bond returns jointly. Examples are the affine models of Bekaert, Engstrom, and Grenadier (2005), Bekaert, Engstrom, and Xing (2008), and the linear-quadratic model of Campbell, Sunderam, and Viceira (2008) all of which explore the relationship between aggregate stock and bond markets. Lettau and Wachter (2009) and Gabaix (2009) additionally study the cross-section of stock returns. The former is a model with common shocks to the risk premium in stock and bond markets, while the latter is a time-varying rare disasters model.

Finally, there is substantial recent interest in understanding the temporal composition of risk in asset prices, including Cochrane and Hansen (1992), Bansal and Lehman (1997), Hansen, Heaton, and Li (2008), Hansen and Scheinkman (2009), Borovicka, Hansen, Hendricks, and Scheinkman (2009), Alvarez and Jermann (2004, 2005), Martin (2008), Backus, Routledge, and Zin (2008), and Backus, Chernov, and Martin (2009).

2   Decomposing Affine Models for Stocks and Bonds

We develop an affine valuation model to price both stocks and bonds. The next section shows how a relatively minor change in setup relative to the canonical term structure literature allows us to price stocks in addition to bonds. The exercise leads to a model with consistent risk pricing across stocks and bonds. To pave the way, this section sets up a generic affine pricing model. We show how to decompose the variation in its stochastic discount factor into a part that is transitory and a part that reflects permanent shocks. This decomposition will prove to be very useful in understanding the link between stock and bond returns.

2.1   Setup

Let P_t be the price of a risky asset and D_{t+1} its (stochastic) cash-flow. Then the *stochastic discount factor* (SDF) $SDF_{t+1} > 0$ makes $P_t = E_t[SDF_{t+1}(P_{t+1} + D_{t+1})]$. The stochastic discount factor SDF_{t+1} is the ratio of the *pricing kernels* at times $t+1$ and t:

$$SDF_{t+1} = \frac{M_{t+1}}{M_t}.$$

Our SDF takes the same form as in the term structure literature:

$$SDF_{t+1} = \exp\left(-y_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t - \Lambda_t'\varepsilon_{t+1}\right), \tag{1}$$

where y_t is the short-term interest rate, ε_{t+1} is a $N \times 1$ vector of shocks to the $N \times 1$ vector of state variables X_t, and where Λ_t is the $N \times 1$ vector of market prices of risk associated with these shocks at time t. As in the canonical affine model, the short rate in (2) and the prices of risk in (3) are affine in the state vector X_t, and the state vector in (4) follows a first-order vector-autoregression with intercept γ_0, companion matrix Γ, and conditionally normally, i.i.d. distributed innovations, $\varepsilon_t \sim \mathcal{N}(0, \Sigma)$:

$$y_t = \delta_0 + \delta_1'X_t, \tag{2}$$
$$\Lambda_t = \Lambda_0 + \Lambda_1 X_t, \tag{3}$$
$$X_{t+1} = \gamma_0 + \Gamma X_t + \varepsilon_{t+1}. \tag{4}$$

The stochastic discount factor, the short rate, and all other objects in the paper such as yields and returns are nominal. Lowercase letters denote natural logarithms: $sdf_t = \log(SDF_t)$ and $m_t = \log(M_t)$.

First, this model (1)-(4) implies an affine term structure of bond yields of different maturities (Duffie and Kan (1996) and Duffee (2002)). The price of a nominal bond of maturity τ is exponentially affine in the state variables X:

$$P_t(\tau) = \exp\{A(\tau) + B'(\tau)X_t\}. \tag{5}$$

By no-arbitrage, we have:

$$
\begin{aligned}
P_t(\tau) &= E_t\left[SDF_{t+1}P_{t+1}(\tau-1)\right] \\
&= \exp\{-\delta_0 - \delta_1'X_t + A(\tau-1) + B'(\tau-1)\gamma_0 + \\
&\quad B'(\tau-1)\Gamma X_t - \Lambda_t'\Sigma B(\tau-1) + \frac{1}{2}B'(\tau-1)\Sigma B(\tau-1)\},
\end{aligned}
$$

which implies that $A(\tau)$ and $B(\tau)$ follow from the recursions:

$$A(\tau) = -\delta_0 + A(\tau-1) + B'(\tau-1)\gamma_0 - \Lambda_0'\Sigma B(\tau-1) + \frac{1}{2}B'(\tau-1)\Sigma B(\tau-1), \tag{6}$$
$$B(\tau) = -\delta_1 + (\Gamma - \Sigma\Lambda_1)' B(\tau-1), \tag{7}$$

initiated at $A(0) = 0$ and $B(0) = 0_{1 \times N}$. Bond yields and bond prices are related through: $y_t(\tau) = -\log(P_t(\tau))/\tau$. Finally, the short rate $y_t = -A(1) - B(1)'X_t$, which recovers equation (2).

Second, for each asset j with gross return R^j, a no-arbitrage condition must hold:

$$E_t \left[SDF_{t+1} R^j_{t+1} \right] = 1,$$

Log returns can always be written as the sum of expected and unexpected returns: $r^j_{t+1} = E_t[r^j_{t+1}] + \eta^j_{t+1}$. Unexpected log returns η^j_{t+1} are assumed to be conditionally normally distributed. We denote the covariance matrix between shocks to returns and shocks to the state variables by Σ_{Xj}. We define log excess returns to include a Jensen adjustment:

$$rx^j_{t+1} \equiv r^j_{t+1} - y_t + \frac{1}{2} V[\eta^j_{t+1}].$$

The no-arbitrage condition then implies:

$$E_t \left[rx^j_{t+1} \right] = -Cov_t \left[rx^j_{t+1}, sdf_{t+1} \right] = Cov \left[\eta^j_{t+1}, \varepsilon'_{t+1} \right] \Lambda_t \equiv \Sigma_{Xj} \left(\Lambda_0 + \Lambda_1 X_t \right). \qquad (8)$$

Unconditional expected excess returns are computed by taking the unconditional expectation of (8):

$$E \left[rx^j_{t+1} \right] = \Sigma_{Xj} \left(\Lambda_0 + \Lambda_1 E\left[X_t \right] \right) \equiv \Sigma_{Xj} \hat{\Lambda}_0. \qquad (9)$$

2.2   Variance Decomposition of the SDF

Hansen, Heaton, and Li (2008) and Hansen and Scheinkman (2009) show that -under mild regularity conditions- any pricing kernel can be decomposed as:

$$M_t = \beta^t M^e_t M^P_t,$$

in which M^P_t is a martingale, $E_t[M^P_{t+1}] = M^P_t$, and M^e_t solves the eigenfunction problem:

$$E_t \left[\frac{M_{t+1}}{M_t} \frac{1}{M^e_{t+1}} \right] = \beta \frac{1}{M^e_t}. \qquad (10)$$

They also show that the martingale component of the pricing kernel determines the pricing of cash flows in the long run. Such decompositions are therefore helpful in understanding asset pricing properties across different horizons. The following proposition shows how to decompose the SDF of the canonical affine valuation model.

Proposition 1. *The stochastic discount factor of the affine model can be decomposed as:*

$$\frac{M_{t+1}^e}{M_t^e} = \exp\left\{-B_\infty'\gamma_0 + B_\infty'\left(I - \Gamma\right)X_t - B_\infty'\varepsilon_{t+1}\right\}, \tag{11}$$

$$\frac{M_{t+1}^P}{M_t^P} = \exp\left\{-\frac{1}{2}\left(\Lambda_t - B_\infty\right)'\Sigma\left(\Lambda_t - B_\infty\right) - \left(\Lambda_t - B_\infty\right)'\varepsilon_{t+1}\right\}, \tag{12}$$

with constant β given by:

$$\beta = \exp\left\{-\delta_0 + \left(\gamma_0' - \Lambda_0'\Sigma\right)B_\infty + \frac{1}{2}B_\infty'\Sigma B_\infty\right\}.$$

The proof, which is relegated to Appendix A, solves the eigenfunction problem in (10). We also provide an alternative proof based on the Alvarez and Jermann (2005) methodology (AJ). While the decompositions of Hansen and Scheinkman (2009) and AJ do not coincide in general, they do in affine valuation models. Following the language in AJ, we refer to the martingale component M_t^P as the *permanent* component and to the dominant pricing component for bonds $M_t^T \equiv \beta^t M_t^e$ as the *transitory* component of the pricing kernel.[3]

The next section shows how the decomposition of the SDF in a transitory and a permanent component is useful in identifying important candidate pricing factors for our empirical SDF. AJ link the variance decomposition of the SDF into permanent and transitory components to the maximum risk premium across all assets and the risk premium on a long-horizon bond. Similar to the Hansen and Jagannathan (1991) bounds, the variance ratio allows us to learn about the properties of the SDF from studying stock and bond returns directly.

We compute the same variance decomposition for affine valuation models. To this end, we define the transposed *risk-neutral companion matrix* $\Theta = \left(\Gamma - \Sigma\Lambda_1\right)'$. Using this definition, equation (7) simplifies to $B(\tau) = -\delta_1 + \Theta B(\tau - 1)$. This recursion has the following solution:

$$B(\tau) = -(I - \Theta^\tau)(I - \Theta)^{-1}\delta_1.$$

The object B_∞, which measures the sensitivity of the infinite horizon bond to the state variables X, is given by

$$B_\infty \equiv \lim_{\tau \to \infty} B(\tau) = -(I - \Theta)^{-1}\delta_1. \tag{13}$$

With this decomposition in hand, we can calculate what fraction of the conditional variance of the stochastic discount factor comes from the permanent and from the transitory components.

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[3]While both AJ and Hansen, Heaton, and Li (2008) use the terminology "transient or transitory component," this label might be somewhat misleading in the sense that this pricing component is the dominant term in pricing long-horizon bonds. We thank Lars Hansen for pointing this out to us.

The variance of the log stochastic discount factor is given by:

$$\text{var}_t \left(\log SDF_{t+1} \right) = \Lambda_t' \Sigma \Lambda_t.$$

Likewise, for the permanent component, we have:

$$\text{var}_t \left(\log M_{t+1}^P / M_t^P \right) = \left(\Lambda_t - B_\infty \right)' \Sigma \left(\Lambda_t - B_\infty \right).$$

Define the fraction of the conditional variance of the log SDF that comes from the permanent component by ω_t:

$$\omega_t \ \equiv \ \frac{\text{var}_t \left(\log M_{t+1}^P / M_t^P \right)}{\text{var}_t \left(\log SDF_{t+1} \right)}.$$

It follows that:

$$
\begin{aligned}
\omega_t \ &= \ \frac{\left(\Lambda_t - B_\infty \right)' \Sigma \left(\Lambda_t - B_\infty \right)}{\Lambda_t' \Sigma \Lambda_t} \\
&= \ 1 - \frac{B_\infty' \Sigma \Lambda_t - \frac{1}{2} B_\infty' \Sigma B_\infty}{\frac{1}{2} \Lambda_t' \Sigma \Lambda_t} \\
&= \ 1 - \frac{E_t[r_{t+1}^b(\infty) - y_t]}{\max_j E_t[r_{t+1}^j - y_t]}.
\end{aligned}
\tag{14}
$$

2.3 Link with Stock and Bond Pricing

The conditional variance ratio in (14) is one minus the ratio of the expected excess log return on a bond of infinite maturity (excluding Jensen adjustment) to the highest attainable expected excess log return on any risky asset in the economy (excluding Jensen adjustment).[4] Intuitively, this ratio is one minus the ratio of the long *bond risk premium* over the *maximum risk premium*. Fluctuations in the conditional variance of the SDF arising from the permanent (transitory) component become less (more) important when the long bond risk premium increases relative to the maximum risk premium.

Variation in the transitory component of the SDF is tightly linked to the bond market. In particular, the transitory component equals the inverse of the gross return on an infinite maturity bond:

$$\frac{M_{t+1}^T}{M_t^T} = \left(R_{t+1}(\infty) \right)^{-1}.$$

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[4]From the no-arbitrage equation and the concavity of the log, it follows that $\frac{1}{2}\text{var}_t\left(sdf_{t+1}\right) = \frac{1}{2}\Lambda_t'\Sigma\Lambda_t \geq E_t[r_{t+1}^j - y_t]$ for any risky asset j. This equation holds with equality for an asset j whose return is perfectly conditionally correlated with the *sdf* and has conditional volatility equal to the condition volatility of *sdf*. Such an asset always exists in an economy where markets are complete and where leverage is allowed.

Hence, shocks to the transitory component are shocks to the long-horizon bond return. Without a transitory component, excess bond returns are zero and the term structure of interest rates is constant. To see this in the context of our affine model, it suffices to set $\delta_1 = 0$. It follows that $B(\tau)$ and hence the bond risk premium $B(\tau)\Sigma\Lambda_t$ are zero at any maturity τ. Because also $B_\infty = 0$, $\omega_t = 1$ in equation (14) and all variation in the SDF comes from permanent shocks. Another example of an economy without transitory shocks is the standard consumption-based asset pricing model, which features CRRA preferences and i.i.d. consumption growth. In that model, all bond risk premia are zero and all variation in the SDF reflects permanent shocks. In reality, bond yields are not constant and bond returns are not zero; they fluctuate over time. Hence, variation in bond risk premia suggests not only the existence of a transitory component in the SDF, but also time variation in the conditional variance of that transitory component.

AJ argue that variation in the permanent component of the SDF is naturally linked to the stock market. They show that in a world without permanent shocks to the SDF, the highest risk premium in the economy is the (infinite maturity) bond risk premium. They then argue that a model with only transitory shocks is counterfactual because various empirical measures of the long-horizon bond risk premium are small relative to various measures of the maximum risk premium; the *unconditional* variance ratio $E[\omega_t]$ is close to one. Their proxies for the maximum risk premium are the return on the value-weighted stock market portfolio, a levered-up version of the stock market return, or a fixed-weight portfolio formed from ten size-decile stock portfolios. Hence, shocks to the aggregate stock market return are a natural candidate for shocks to the permanent component of the SDF. The model with only a transitory component arises as a special case of our model when $\Lambda_1 = 0$ and $\Lambda_0 = B_\infty$. Since the last entry of B_∞ is zero, it implies that the price of DP risk must be zero, which is exactly the price or risk we associate with permanent shocks.

A large literature on stock and bond return predictability shows that equity and bond risk premia vary over time. For example, a variable such as the log dividend price ratio (DP) on the aggregate stock market is a good predictor of future excess stock returns, and a variable such as the Cochrane and Piazzesi factor (Cochrane and Piazzesi 2005), henceforth CP factor, is a good predictor of future excess bond returns. Variation in equity risk premia suggests time variation in the conditional variance of the permanent component, while variation in bond risk premia suggests time variation in the conditional variance of the transitory component. Importantly, stock and bond risk premia are far from perfectly correlated with each other. While the findings of AJ show that the SDF must have a large permanent component *on average*, the return predictability evidence shows that the conditional variance ratio ω_t cannot be constant. Understanding what drives the variation in ω_t is one of the key questions of this paper.

This analysis suggests that any successful asset pricing model must have three essential features:

(1) shocks to the permanent component of the SDF, (2) shocks to the transitory component of the SDF, and (3) shocks to the relative importance of the permanent and transitory components. While these insights are general, in the next section we specialize our analysis to the context of the affine asset pricing model. In that model, shocks to the dividend yield, henceforth DP factor, capture shocks to stock returns and hence to the permanent component. Shocks to the level of the term structure of interest rates capture shocks to bond returns, and hence to the transitory component of the SDF. Finally, shocks to the CP factor capture shocks to the bond risk premium and shocks to the DP factor capture shocks to the equity risk premium. One, or a combination, of these two shocks thus captures shocks to the relative contribution of the permanent and transitory components ω_t. Thus, our three key state variables, the DP factor, the level factor, and the CP factor, follow naturally from the preceding discussion.

Appendix B gives an example of a consumption-based model asset pricing model that is nested by our more general affine framework. It is the minimal setup that has the three essential features. Aggregate consumption has a transitory (autoregressive) component and a permanent (random walk) component. The transitory component is homoscedastic while the variance of the permanent component's innovations follows an autoregressive process. All three innovations are independent. The conditional variance of the permanent component, which can be interpreted as capturing time-varying economic uncertainty, governs the variation in the ratio ω_t. When economic uncertainty decreases, say in periods with good prospects for economic activity, more of the conditional variance of the SDF comes from the transitory shocks.

3   Estimation

This section shows that an affine valuation model with three priced sources of risk connected to the permanent component, the transitory component, and the relative contribution of each is sufficiently general to account for the bulk of variation in (1) the cross-section of average returns on stock portfolios and bond portfolios, (2) the dynamics of stock and bond risk premia, and (3) the dynamics of bond yields. The result is a parsimonious model with consistent pricing of risk across stock and bond markets.

3.1   Estimation Strategy

We start by laying out our strategy for estimating the parameters of the model in equations (1)-(4). We partition the set of parameters in three subsets, which are estimated in three steps.

One set of parameters governs the dynamics of bond yields. It consists of 16 parameters in the risk-neutral companion matrix Θ and 4 parameters in the vector δ_1. Following Cochrane and Piazzesi (2008), we estimate these parameters to match the time-series of demeaned yields on

1-year bonds and forward rates of maturities 2, 3, 4, and 5 years.

The second set of parameters governs the dynamics of the market prices of risk in the matrix Λ_1. For parsimony, we only estimate 2 such parameters freely. They are chosen to match the time variation in aggregate stock and bond risk premia, respectively. In particular, we match the coefficients in the predictability regression of aggregate stock market excess return on the DP factor and in the predictability regression of the excess return on an equally-weighted portfolio of one- to five-year bonds on the CP factor. All individual stock portfolios have expected returns that are affine in the DP factor and all individual bond portfolios have expected returns that are affine in the CP factor. This part of the estimation requires estimates of the residuals, which we obtain by estimating the VAR model in (4) by OLS.

The third, and last, set of parameters governs average prices of risk in Λ_0. As motivated above, we estimate three parameters: the price of DP risk, the price of level risk, and the price of CP risk. They are chosen to match the unconditional average excess return on the aggregate stock market, on ten book-to-market sorted stock portfolios, and on five maturity-sorted bond portfolios. In a robustness analysis below, we study alternative choices of equity portfolios as well as corporate bond portfolios.

While only three state variables have shocks that are priced sources of risk, the state vector contains two additional elements: a slope and a curvature factor. They are the second and third principal components of forward rates. While their prices of risk are zero, they are useful for adequately describing expectations of future bond yields. In sum, our state vector contains five "yield" variables in the following order: the CP factor, the level factor, the slope factor, the curvature factor, and the log dividend yield on the aggregate stock market portfolio DP. Their dynamics are described by equation (4).

We impose a restriction that the dividend yield does not forecast future bond yields, beyond the current bond yields themselves: $\Gamma_{(1:4,5)} = 0_{4\times1}$. Without this restriction, the model implies that the dividend yield is an affine function of any five yields. This implication of the model is easily rejected in the data. Finally, we construct the CP factor following Cochrane and Piazzesi (2005).[5]

The next sections present the estimation results and further details on the estimation. We discuss the results in order of importance, while the description above is the order in which we estimate the parameters.

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[5]We use monthly Fama-Bliss yield data for nominal government bonds of maturities one- through five-years. These data are available from June 1953 until December 2008. We construct one- through five-year forward rates from the one- through five-year bond prices. We then regress the equally-weighted average of the one-year excess return on bonds of maturities of two, three, four, and five years on a constant, the one-year yield, and the two-through five-year forward rates. The yields are one-period lagged relative to the return on the left-hand side. The CP factor is the fitted value of this predictive regression. The R^2 of this regression in our sample of monthly data is 20.4%, roughly twice that of the fiver-year minus one-year yield spread, another well-known bond return predictor.

3.2 The Cross-Section of Unconditional Expected Returns

We first describe the estimation of the average price of risk parameters in $\hat{\Lambda}_0$. More precisely, we estimate the average price of CP risk (first element), the average price of level risk (second element), and the average price of DP risk (fifth element of $\hat{\Lambda}_0$). We do so in order to minimize the root mean-squared pricing errors on a cross-section of J stock and bond portfolios.

Let $E\left[rx_{t+1}\right]$ be the $J \times 1$ vector of average log excess returns (including a Jensen adjustment). Let $\Sigma_{X(i)J}$ be the $J \times 1$ vector of covariances between the ith shock to the state vector and unexpected returns η on all of the J assets, then the no-arbitrage conditions in equation (9) imply that

$$E\left[rx_{t+1}\right] = \Sigma_{X(1,2,5)J}\hat{\Lambda}_{0(1,2,5)}.$$

The three prices of risk that minimize the equally-weighted average of squared pricing errors are found by regressing the $J \times 1$ average excess returns on the $J \times 3$ covariances.

Our test assets are the ten value-weighted portfolios sorted on their book-to-market ratio from Fama and French (1992), the value-weighted stock market return from CRSP (NYSE, Amex, and Nasdaq), and five bond portfolios with maturities 1, 2, 5, 7, and 10 years from CRSP. The data are monthly from June 1952 until December 2008. In order to form unexpected returns η, we regress each stock portfolio's log excess return on the lagged DP factor and each bond portfolio's log excess return on the lagged CP factor. Unexpected returns are the residual from these regressions; see Section 3.3. Innovations to the state vector follow from equation-by-equation OLS estimation of the VAR model in (4).

The first column of Table 1 shows the expected excess returns, expressed in percent per year, on our 16 test assets we wish to explain. They are the pricing errors resulting from a model where all prices of risk in $\hat{\Lambda}_0$ are zero, i.e., from a risk-neutral SDF model (*RN SDF*). Average excess returns on bonds are between 1.1 and 2.0% per year and increase in maturity. The aggregate excess stock market return is 6.0%, and the risk premia on the book-to-market portfolios range from 5.0% (BM1, growth stocks) to 10.1% (BM10, value stocks), implying a value premium of 5.15% per year.

The results from our model are in the second column of Table 1. The top panel shows the pricing errors. Our model succeeds in reducing the mean absolute pricing errors (MAPE) on the 16 stock and bond portfolios to a mere 40 basis points per year. The model succeeds in eliminating the value spread: The spread between the extreme portfolios is only 25bp per year. We also match the market equity risk premium and the average bond risk premium. Pricing errors on the stock and bond portfolios are an order of magnitude lower than in the first column and substantially below those in several benchmark models we discuss below. In sum, our three-factor pricing model is able to account for the bulk of the cross-sectional variation in stock and bond returns with a single set of market price of risk estimates.

The bottom panel of the table shows the point estimates for $\hat{\Lambda}_0$. We estimate a positive price of CP risk, while the price of level factor risk and DP risk are negative. We also calculate (asymptotic) standard errors on the Λ_0 estimates using GMM with an identity weighting matrix. The standard errors are 33.73 for the CP factor price (88.06), 9.36 for the level factor price (-23.98), and 1.43 for the DP factor price (-1.98). Hence, the first two risk prices are statistically different from zero (with t-stats of 2.6 and -2.6), whereas the last one is not (t-stat of -1.4). The signs on the price of level and DP risk estimates are as expected. First, a positive shock to the level factor leads to a drop in bond prices and bond returns. A negative shock to bond returns is a positive shock to the transitory component of the SDF. Positive shocks to the SDF increase the representative agent's marginal utility of consumption, and, hence, they carry negative risk prices. Likewise, a positive shock to the DP factor leads to a drop in stock prices and stock returns. It is a positive shock to the permanent component of the SDF, which again should carry a negative risk price. We return at length to the CP factor and its positive risk price below.

To help us understand the separate roles of each of the three risk factors in accounting for the risk premia on these stock and bond portfolios, we switch on only one risk factor and set the other risk prices to zero. Column 3 of Table 1 minimizes the pricing errors on the same 16 test assets but only allows for a non-zero price of level risk (Column *Level*). This is the bond pricing model proposed by Cochrane and Piazzesi (2008). The cross-section of average bond returns is well described by differences in exposure to the level factor. Long-horizon bonds have returns that are more sensitive to interest rate shocks than short-horizon bonds; a familiar duration argument. However, this bond SDF is unable to jointly explain the cross-section of stock and bond returns; the MAPE is 2.68%. All pricing errors on the stock portfolios are large and positive, there is a 6% value spread, and all pricing errors on the bond portfolios are large and negative. Clearly, exposure to the level factor alone does not help to understand the high equity risk premium nor the value risk premium. Value and growth stocks have similar exposure to the level factor. The reason that this model does not do better pricing the bond portfolios is that the excess returns on stock portfolios are larger in magnitude and therefore receive most attention in the optimization. Consequently, the estimation concentrates its efforts on reducing the pricing errors of stocks.

To illustrate that this bond SDF *is* able to price the cross-section of bonds, we estimate the same model by minimizing only the bond pricing errors (the first five moments in the table). The fourth column of Table 1 (*Level - only bonds*) confirms the Cochrane and Piazzesi (2008) results that the bond pricing errors are small. However, the overall MAPE increases to 4.82%. The canonical bond pricing model offers one important ingredient for the joint pricing of stocks and bonds: bonds' heterogeneous exposure to the level factor. However, this ingredient does not help to account for equity returns. In the language of our model, equity portfolios display no interesting heterogeneity in their exposure to shocks to the transitory component of the SDF. A model with

only transitory shocks cannot jointly explain stock and bond returns.

Another natural candidate is the canonical equity pricing model: the capital asset pricing model (CAPM). The only non-zero price of risk is the one corresponding to the dividend yield.[6] The fifth column of Table 1 (DP) reports pricing errors for the CAPM. This model is again unable to jointly price stock and bond returns. The MAPE is 1.50%. One valuable feature is that the aggregate market portfolio is priced well and the pricing errors of book-to-market portfolio returns go through zero. This means that the model gets the common level in all stock portfolio returns right. However, the pattern of pricing errors still contains the entire value spread. Pricing errors on bond portfolios are sizeable as well and are all positive. In the language of our model, neither book-to-market nor bond portfolios display interesting heterogeneity in their exposure to shocks to the permanent component of the SDF. Models with only permanent shocks, such as the canonical CAPM or consumption-CAPM, cannot jointly explain stock and bond returns.

[Table 1 about here.]

So, while the level factor helps to explain the cross-sectional variation in average bond returns and the DP factor helps to explain the level of equity risk premia, neither factor is able to explain why value stocks have much higher risk premia than growth stocks. The sixth column of Table 1 indeed shows that having both the level and the DP factor priced does not materially improve the pricing errors and leaves the value premium puzzle in tact.

This is where the CP factor comes in. Figure 3 decomposes each asset's risk premium into its three components: risk compensation for exposure to the CP factor, the level factor, and the DP factor. The top panel is for the market portfolio (first bar) and the five bond portfolios (next five bars). The bottom panel shows the decomposition for the 10 book-to-market portfolios, ordered from growth (left bars) to value (right bars). This bottom panel shows that all ten book-to-market portfolios have about equal exposure to both DP and level shocks. The spread between value and growth risk premia entirely reflects differential compensation for CP risk. Value stocks have a large and positive exposure to CP shocks of while growth stocks have a low or even negative exposure to CP shocks. The differential exposure between the tenth and first book-to-market portfolio is 0.0053 and is statistically different from zero. Multiplying by the market price of CP risk of 88.06, this delivers a value premium of 0.46% per month or 5.58% per year.[7] That is, the CP factor's contribution to the risk premia (more than) fully accounts for the value premium.

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[6]We also considered a model in which the dividend yield is replaced by the aggregate stock market return. The fit for the unconditional expected returns is very similar. However, such a model does not capture the predictability of stock returns, making the current model a parsimonious alternative that can capture both the conditional and unconditional price of aggregate stock market risk.

[7]Another way to understand the 5.58% value premium is as the product of a CP factor beta for the value-minus-growth portfolio and a risk premium for CP risk. The former is 74.27 and the latter is 0.0063 per month or 0.0752 per year. Hence, the risk premium for CP risk is 0.075% per year. Since the factors are close to orthogonal to each other, the overall risk premium is approximately the sum of three such terms.

Given the monotonically increasing pattern in exposures of the book-to-market portfolios to CP shocks, a positive price of CP risk estimate is what allows the model to match the value premium. In the language of our model, book-to-market portfolios do display interesting heterogeneity in their exposure to shocks that affect the relative contribution of the permanent versus transitory component to the overall variance of the SDF.

The key economic questions that are left unanswered at this point are: what economic source of risk do CP shocks capture, and why do value stocks have higher exposure to these shocks than growth stocks? Below we argue that an increase in CP signals an improvement in the business cycle going forward. This implies that CP innovations should command a positive price of risk, as for instance in a model where the representative agent has a preference for early resolution of uncertainty. Value stocks' returns are more sensitive to CP shocks, resulting in the value premium.

Turning to the top panel, the first bar makes clear that the aggregate stock market's risk premium entirely reflects compensation for DP risk. For the five bond portfolios matters are more intricate. Risk premia are positive and increasing in maturity due to their exposure to level risk. The exposure to level shocks is negative and the price of level risk is negative, resulting in a positive contribution to the risk premium. This is the duration effect mentioned above. But bonds also have a negative exposure to CP shocks. Being a proxy for the risk premium in bond markets, positive shocks to CP lower bond prices and returns. This effect is larger the longer the maturity of the bond. Given the positive price of CP risk, this exposure translates into an increasingly negative contribution to the risk premium. The sum of the level and CP contributions delivers the observed pattern of risk premia that increase in maturity.

To highlight the challenge in jointly pricing stocks and bonds, Appendix C develops a stark model where (1) the CP factor is a perfect univariate pricing factor for the book-to-market portfolios (it absorbs all cross-sectional variation), (2) the level factor is a perfect univariate pricing factor for the bond portfolios, and (3) the CP and the level factor are uncorrelated. This model will generally fail to price the stock and bond portfolios jointly because the bond portfolios are exposed to the CP factor, while the stock portfolios are not exposed to the level factor. Given these facts, a joint pricing model only works if the exposures of maturity-sorted bond portfolios to CP are linear in maturity, with a similar (absolute) slope as the level exposures. The appendix underscores the challenges in finding a model with consistent risk prices across stocks and bonds, or put differently the challenge of going from univariate to multivariate pricing models.

[Figure 3 about here.]

3.3 Time Series of Conditional Expected Returns

The second block of parameters governs the dynamics of expected excess stock and bond returns. These are the parameters in the matrix Λ_1, which determine how market prices of risk vary with the

state variables (recall equation 8). First, we model expected excess stock returns on the aggregate stock market as well as on the decile book-to-market portfolios as an affine function of the log dividend-price ratio on the aggregate stock market DP:

$$rx_{t+1}^j = rx_0 + \xi_s^j DP_t + \eta_{t+1}^j, \tag{15}$$

where j denote the stock portfolios. This specification follows a long tradition in the stock return predictability literature; see for instance Fama and French (1988), Cochrane (2006), Lettau and Van Nieuwerburgh (2008), and Binsbergen and Koijen (2009). To generate stock return predictability, we make the market price of DP risk dependent on the DP factor. We choose $\Lambda_{1(5,5)}$ to exactly match the return predictability coefficient ξ_s^j for the aggregate stock market ($j = m$). In the model, this coefficient is $\Sigma_{X(5)m}\Lambda_{1(5,5)}$. Setting it equal to the estimate from (15) gives $\Lambda_{1(5,5)} = \Sigma_{X(5)m}^{-1}\xi_s^m$. Our point estimate is negative signalling that an increase in DP leads to higher equity risk premia because the price of DP risk is negative and a higher DP makes it more negative.

While the model matches the predictability of aggregate stock market returns by the dividend yield exactly, it also *implies* a predictive coefficient for the ten book-to-market portfolios. Figure 9 juxtaposes the implied predictive coefficient and the one the freely estimated from equation (15) for each of the decile book-to-market portfolios. It shows that the two sets of coefficients line up well.

[Figure 4 about here.]

Second, we impose that the CP ratio does not predict future stock returns. This restriction amounts to:

$$\Lambda_{1(5,1)} = -\Lambda_{1(2,1)}\frac{\Sigma_{X(2)m}}{\Sigma_{X(5)m}}.$$

Hence, $\Lambda_{1(5,1)}$ is not a free parameter, but follows from the estimated covariances and the estimated risk price coefficient $\Lambda_{1(2,1)}$ to which we turn next. In our sample, we find only weak evidence for one-month-ahead return predictability of the market portfolio or of any of the 10 book-to-market portfolios by the CP factor, beyond that captured by DP. Nevertheless, we consider such an extension in Section 4.1.

Third, we assume that excess bond returns are predicted by the CP factor. Cochrane and Piazzesi (2005) show that this factor is a strong forecaster of bond returns. Expected excess bond returns are given by:

$$rx_{t+1}^j = rx_0 + \xi_b^j CP_t + \eta_{t+1}^j, \tag{16}$$

where j denote the bond portfolios. To generate bond return predictability, we make the market price of level risk dependent on the CP factor, following Cochrane and Piazzesi (2008). We choose $\Lambda_{1(2,1)}$ so as to exactly match the return predictability coefficient in an equation that regresses the

monthly excess return on an equally-weighted portfolio of our five CRSP bond portfolios ($j = b$) on the lagged CP factor. Call that predictability coefficient ξ_b^b. Then $\Lambda_{1(2,1)}$ is found as the solution of:

$$\xi_b^b = \Sigma_{X(2)b}\Lambda_{1(2,1)} + \Sigma_{X(5)b}\Lambda_{1(5,1)} = \Lambda_{1(2,1)}\left(\Sigma_{X(2)b} - \Sigma_{X(5)b}\frac{\Sigma_{X(2)m}}{\Sigma_{X(5)m}}\right),$$

where the last line substitutes in the restriction on $\Lambda_{1(5,1)}$ from above. The parameter $\Lambda_{1(2,1)}$ is estimated to be negative, consistent with the results in Cochrane and Piazzesi (2008). A negative sign means that an increase in CP leads to higher bond risk premia because the price of level risk is negative and a higher CP makes it more negative.

Our model also matches the forecasting power of CP for excess bond returns measured over an *annual* holding period, as reported by Cochrane and Piazzesi (2005). To show this, we simulate 5,000 time series of our model of 678 observations, which is the length of our monthly sample. For each of the samples, we construct annual returns of 2-year,..., 5-year bonds in excess of the 1-year return. We construct the CP factor for each of the samples and regress the bond returns on the CP factor. This delivers 5,000 predictive coefficients for each maturity. Figure 5 reports the predictive coefficients, averaged across the 5,000 sample paths, alongside the estimates in the data. The model closely replicates the predictability the predictability of long-term bond returns. The simulated slope coefficients are within one standard deviation from the observed slope coefficients. The model simulation also does a good job matching the R^2 of these predictability regressions; they are about one standard deviation away from the observed ones.

[Figure 5 about here.]

Finally, because bond yields do not depend on the lagged DP factor, excess bond returns are not predictable by the lagged dividend yield in the model. A regression of excess bond returns on lagged DP and lagged CP shows a coefficient on lagged DP, which is statistically and economically indistinguishable from zero. In summary, we estimate the following matrix Λ_1:

$$\Lambda_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -732.91 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 14.16 & 0 & 0 & 0 & -4.78 \end{bmatrix}.$$

With this Λ_1 matrix in hand, we can form $\Lambda_0 = \hat{\Lambda}_0 - \Lambda_1 E[X]$.

Figure 6 shows risk premia (log expected excess returns) on the aggregate stock market and on the equally-weighted bond portfolio. A salient differences between equity and bond risk premia is their persistence. Equity risk premia have a monthly autocorrelation of 0.992, which amounts to

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a half-life of 83 months (7 years), while bond risk premia have a monthly autocorrelation of 0.821, which amounts to a half-life of 3 months. Hence, there are *two frequencies* in risk premia: the bond risk premium is a business cycle frequency variable whereas the equity risk premium moves at much lower frequencies . Together with the low correlation between bond and stock risk premia (0.04 in our sample), this implies that two variables are needed to describe risk premia. Our model matches the time series of expected stock and bond returns (save the average pricing error reported in Table 1) and hence accounts for these two frequencies in equity and bond risk premia.[8]

[Figure 6 about here.]

3.4 Term Structure of Interest Rates

The final set of parameters to discuss are the parameters in the risk-neutral companion matrix Θ and in the dynamics of the short rate δ_1. Following the term structure literature (e.g., Duffee (2002) and especially Cochrane and Piazzesi (2008)), we estimate these parameters to match the demeaned one-year bond yield and the demeaned two-year through five-year forwards rates. In the model, yields and therefore forward rates are affine in the state vector; see equation (5). In the data, yields and forward rates are from the Fama-Bliss data set for June 1952 until December 2008.

Since the dividend yield dynamics are completely decoupled from the bond yields dynamics, we can estimate the first 4×4 block of Θ and the first four elements of δ, for a total of 20 parameters.[9] We choose them so as to minimize the mean squared pricing errors between the observed and the model-implied forwards via a non-linear least squares optimization. The model's SDF does a nice job matching demeaned forward rates of maturities one- through five-years. The annualized standard deviation of the pricing errors of annual forwards equal 12bp, 22bp, 26bp, 35bp, and 39bp for the one- through five year forwards, respectively. The implied *yield* pricing errors are 12bp, 7bp, 6bp, 9bp, and 3bp per year for the one- through five bond yields.[10] Figure 7 shows the implied yields in model alongside the yields in the data. In sum, the model can account for the term structure of interest rates.

[Figure 7 about here.]

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[8]Lochstoer (2009) develops a habit model with two goods. The conditional volatility of luxury good consumption is cyclical while the conditional volatility of relative price changes of luxury goods is highly persistent. His model generates two frequency in equity risk premia. We connect the cyclical bond risk premium to the cross-sectional variation in stock returns.

[9]Recall that we impose that $\Gamma_{(1:4,5)} = 0_{4 \times 1}$. The parameters in the last row of Γ cannot be identified off bond yields. We estimate these parameters from an OLS regression of the dividend yield on the four lagged term structure factors and the lagged dividend yield itself.

[10]The parameter δ_0 in (2) is chosen to minimize the difference between average forward rates in the model and in the data. The resulting parameter is 0.0040. This parameter plays no role in the rest of our analysis.

A key object of interest that follows from the parameter estimates in this block is B_∞, defined in equation (13). It measures the sensitivity of the bond return to shocks to the state variables for a bond of infinite maturity. By the same token, it controls how shocks to the state variables affect the transitory and permanent components of the SDF in equations (11) and (12). This, in turn, affects the conditional variance ratio ω_t in equation (14) to which we turn next. It is worth pointing out that the AJ decomposition of the SDF is only valid when the dynamics of the state under the risk-neutral measure are stationary. Otherwise, bond yields would not be finite at infinite maturity and the regularity conditions in AJ would be violated. Our estimates imply a largest eigenvalue for Θ strictly less than one, so that the decomposition is well-defined.

3.5   Decomposition of the SDF

With all estimates for the affine valuation model in hand and the reassurance that the estimated model prices average returns in the cross-section of stocks and bonds and captures the dynamics of expected returns and bond yields, we can study the decomposition of the stochastic discount factor into its transitory and permanent components.

The top panel of Figure 8 shows the ratio ω_t of the conditional variance of the permanent component of the stochastic discount factor to the conditional variance of the (entire) stochastic discount factor; see equation (14). We recover the main result of Alvarez and Jermann (2005) that almost all the *unconditional* variation in the SDF comes from the permanent component. The ratio is about 1 on average.[11] However, the figure also shows that there is substantial variation over time in the *conditional* variance ratio. It fluctuates between 0.92 and 1.11 in our estimated affine valuation model.

The bottom panel shows what key variable that drives this time variation in ω_t: the CP factor. The two time series have a correlation of -99.2%. Hence, we find that the CP factor is the most important driver of the conditional variance ratio. In particular, positive shocks to CP indicate an increasing importance of the transitory component of the SDF. The DP factor -which drives all variation in equity risk premia and is more persistent than CP- has a much lower correlation with the variance ratio. Because CP has a half-life of three months, it is a variable that moves at business-cycle frequency. As a result, the relative variance ratio ω_t is also a business-cycle frequency variable, inheriting its cyclical dynamics from CP.

[Figure 8 about here.]

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[11]Unlike AJ, we do not rely on a direct measurement of long-horizon (20- or 30-year) bond yields or forward rates for this inference. Given potential convenience yield or liquidity effects at the long end of the term structure, this is arguably an advantage.

3.6 Interpreting the CP Factor and the Value Premium

The remaining economic questions are why the estimated price of CP risk is positive and why value stocks are riskier than growth stocks.

We consider the following predictive regression in which we forecast future economic activity, measured by the Chicago Fed National Activity Index ($CFNAI$),[12] using the current CP:

$$CFNAI_{t+k} = c_k + \beta_k CP_t + \varepsilon_{t+k}, \tag{17}$$

where k is the forecast horizon expressed in months. The regressions are estimated by OLS and we calculate Newey-West standard errors with $k-1$ lags. The sample runs from March 1967 until December 2008 because that is when the $CFNAI$ is available.

Figure 2 shows the coefficient β_k in the top panel, its t-statistic in the middle panel, and the regression R-squared in the bottom panel. The forecast horizon k is displayed on the horizontal axis and runs from 1 to 36 months. The key finding is the strong predictability of the CP factor for future economic activity 12- to 24-months ahead. All three statistics display a hump-shaped pattern, gradually increasing until approximately 20 months and gradually declining afterwards. The maximum t-statistic is about 4.5, which corresponds to an R-squared value of just under 15%. The results suggest that CP is a strong predictor of future economic activity. Increases in CP signal higher economic activity in the future. The positive relationship between CP and better economic prospects explains why the price of CP risk is positive: innovations to CP are good news and lower the marginal utility of wealth for investors. This finding also explains why value stocks are riskier than growth stocks. We showed that value stocks have a higher exposure to CP shocks than growth stocks. This implies that value returns are high, exactly when economic activity is expected to increase, i.e., when the marginal value of wealth for investors is low. This makes value stocks riskier than growth stocks and results in the value premium.

4 Robustness and Extensions

This section considers several robustness checks. First, we consider an extension of the benchmark model where expected stock returns depend not only of the log dividend yield, but also on the CP factor. Second, we use a different weighting matrix in the market price of risk estimation. Third, we compare our results to alternative pricing models. Fourth, we study additional stock and bond portfolios. Fifth, we do a sub-sample analysis. Sixth, we calculate the maximum Sharpe ratio the

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[12]The CFNAI is a weighted average of 85 existing monthly indicators of national economic activity. It is constructed to have an average value of zero and a standard deviation of one. Since economic activity tends toward trend growth rate over time, a positive index reading corresponds to growth above trend and a negative index reading corresponds to growth below trend.

model admits with portfolio constraints.

4.1 Stock Returns Also Predicted by CP

In the benchmark model, expected (log) excess returns on stocks depend only on the (log) dividend yield DP_t. In a first extension, we specify expected stock returns as a linear function of both DP_t and CP_t, modifying equation (15) to:

$$rx_{t+1}^j = rx_0 + \xi_{s1}^j DP_t + \xi_{s2}^j CP_t + \eta_{t+1}^j, \tag{18}$$

where j denotes the aggregate stock market portfolio ($j = m$) as well as the decile book-to-market portfolios. The reason for the extension is threefold. First, Cochrane and Piazzesi (2005) show evidence of predictability of the aggregate stock market return by CP_t. Second, we find some evidence for predictability of book-to-market portfolio returns by the CP factor, especially for the value portfolio. Third, we want to investigate the sensitivity of our earlier result that the conditional variance ratio ω_t is almost exclusively driven by CP_t to the specification of the equity risk premium.[13]

We choose the market price of risk parameters $\Lambda_{1(5,5)}$ and $\Lambda_{1(5,1)}$ to exactly match the return predictability coefficients ξ_{s1}^m and ξ_{s2}^m, respectively. The risk price $\Lambda_{1(2,1)}$ continues to match the bond return predictability coefficient ξ_b^b. All three risk price estimates are close to those in the benchmark model.[14] Figure shows the predictability coefficients in data and model for the ten book-to-market portfolios. These are over-identifying restrictions of the model. They show that the model does a good job replicating the predictability in the data. The one exception is that the extreme value portfolio (BM10) has stronger predictability in the data than in the model. Allowing the market price of CP risk to depend on the CP factor, i.e., a non-zero $\Lambda_{1(1,5)}$, would allow us to capture this differential predictability of value versus growth stocks. For brevity and parsimony, we do not pursue such an extension here.

[Figure 9 about here.]

More importantly, the unconditional pricing errors on the sixteen stock and bond portfolios are essentially unchanged from our benchmark results in Table 1. The MAPE remains 40 basis

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[13]There is no empirical evidence for the dividend yield predicting future bond returns, so we do not pursue the reverse exercise.

[14]As before, we have $\Lambda_{1(5,5)} = \Sigma_{X(5)m}^{-1} \xi_{s1}^m$. The other two risk prices are found by solving

$$\begin{bmatrix} \xi_{s2}^m \\ \xi_b^b \end{bmatrix} = \begin{bmatrix} \Sigma_{X(2)m} & \Sigma_{X(5)m} \\ \Sigma_{X(2)b} & \Sigma_{X(5)b} \end{bmatrix} \begin{bmatrix} \Lambda_{1(2,1)} \\ \Lambda_{1(5,1)} \end{bmatrix}.$$

We obtain the point estimates $\Lambda_{1(5,5)} = -4.678$, $\Lambda_{1(5,1)} = -40.969$, and $\Lambda_{1(2,1)} = -693.502$.

points per year and the price of risk parameters in $\hat{\Lambda}_0$ remain virtually unchanged. The conditional variance ratio in (14) also remains unchanged. CP remains the only driver of ω_t, with a correlation of -99.1%. (The DP ratio has a correlation with ω_t of -11.4%.) Hence, our finding that the CP factor is the driver of the relative importance of the transitory versus permanent component of the pricing kernel does not hinge on whether stock returns are predictable by the CP factor or not.

4.2   Weighted Least-Squares

Our cross-sectional estimation results in Table 1 assume a GMM weighting matrix equal to the identity matrix. This is equivalent to minimizing the root mean-squared pricing error across the 16 test assets. The estimation devotes equal attention to each pricing error, an leads to a RMSE of 50.6bp per year. A natural alternative to the identity weighting matrix is to give more weight to the assets that are more precisely measured. We use the inverse covariance matrix of excess returns, as in Hansen and Jagannathan (1997). This amounts to weighting the bond pricing errors more heavily than the stock portfolio pricing errors in our context. When using the Hansen-Jagannathan distance matrix, we find a MAPE of 56bp per year compared to 40bp per year. However, the weighted RMSE drops from 50.6bp to 25.6bp per year. The reason for the improvement in RMSE is that the pricing errors on the bonds decrease substantially, from a MAPE of 35bp to 14bp per year. Finally, the price of risk estimates in $\hat{\Lambda}_0$ are comparable to those in the benchmark case. The price of CP risk remains positive and is estimated to be somewhat lower than in the benchmark case, at 59.68 (with a t-statistic of 4.3). The market price of level risk remains statistically negative (-18.58 with t-statistic of -2.7), and the price of DP risk remains negative (-2.38) and turns marginally significant (t-statistic of 1.89). We conclude that our results are similar when we use a different weighting matrix.

4.3   Alternative Pricing Models

In Section 3.2, we compare our SDF model to the a SDF model that has been designed to price bonds, as proposed by Cochrane and Piazzesi (2008), and the CAPM, the most basic model for pricing stocks. It might also be interesting to also compare the model to the three-factor model of Fama and French (1992), which offers a better-performing alternative to the CAPM for pricing the cross-section of stocks. We ask how well it prices the cross-section of book-to-market stocks and government bonds over our monthly sample from June 1952 until December 2008. We use the market return (MKT), the size (SMB), and the value factor (HML) as pricing factors and price the same 16 test assets. The SDF of the Fama-French model takes the same form as equation (1) but the innovations ε^X are the innovations in MKT, SMB, and HML.[15] The last column of

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[15]As for all other returns, innovations to these factors are formed by regressing the MKT, SMB, and HML returns on lagged DP; see equation (15).

Table 1 contains the pricing errors in the Fama-French models. The MAPE is 57 basis points per year, which is somewhat higher than the 40 basis points of our model in the second column. The slightly worse fit in the last column is due to higher pricing errors on the bond portfolios. This is consistent with the findings in Fama and French (1993) who introduce additional pricing factors beyond MKT, SMB, and HML to price bonds. Our results suggest that three factors suffice. In unreported results, we find that the difference between the MAPE of our model and the Fama-French model increases when we weight the 16 Euler equation errors by the inverse of their variance as opposed to equally. The reason is that our model fits the bond return moments better.

It is important to reiterate that the focus of this paper is not simply on pricing unconditional expected returns. Our SDF simultaneously matches the predictability evidence in equity and bond markets, and fits the dynamics of bond yields. Even if we extend the Fama and French model with the bond factors proposed in Fama and French (1993), the resulting five-factor model would be insufficiently rich to match the conditional moments and the term structure dynamics.

4.4 Other Test Assets

Given that we found a unified SDF that does a good job pricing the cross-section and time-series of book-to-market sorted stock and maturity-sorted bond returns, a natural question that arises is whether the same SDF model also prices other stock or bond portfolios. In addition, studying more test assets allows us to address the Lewellen, Shanken, and Nagel (2009) critique. They argue that explanatory power of many risk-based models for the cross-section of (size and) value stocks may be poorly summarized by the cross-sectional R^2. One of their proposed remedies is to use more test assets in the evaluation of asset pricing models. Our benchmark results address this criticism already by adding maturity-sorted government bond portfolios to the cross-section of book-to-market stock portfolios. In addition, we now study several other sets of test assets. We start by adding corporate bond portfolios. Then we study replacing ten decile book-to-market portfolios by ten size decile portfolios, 25 size and book-to-market portfolios, and ten earnings-price portfolios.

4.4.1 Adding Corporate Bond Portfolios

One asset class that deserves particular attention is corporate bonds. After all, at the firm level, stocks and corporate bonds are both claims on the firm's cash flows albeit with different priority structure. We ask whether, at the aggregate level, our SDF model is able to price portfolios or corporate bonds sorted by ratings class. Fama and French (1993) also include a set of corporate bond portfolios in their analysis but end up concluding that a separate credit risk factor is needed to price these portfolios. Instead, we find that the same three factors we used so far are able to price the cross-section of corporate bond portfolios.

We use data from Citi's Yield Book for four investment-grade portfolios: AAA, AA, A, and BBB. Return data for these portfolios are available monthly from January 1980 until December 2008. Their annualized excess returns are listed in the first column of Table 2. For the sample 1980-2008, there is almost no spread in average returns between the four corporate bond portfolios. This lack of spread is almost entirely due to the credit crisis in the Fall of 2008. For example, in October 2008, BBB bonds suffered a -10.7% return (a seven-standard deviation drop) whereas AAA bonds only suffered a -1.5% drop. For the sample up to December 2006, there is a 60bp spread per year between BBB and AAA. Even though there is not much of a spread in the full sample, there is a lot of action in the time series of these corporate bond returns, and therefore in the conditional covariance with the stochastic discount factor. This makes it non-trivial for the model to generate no spread between the corporate bond returns.

In a first exercise, we calculate Euler equations errors for these four portfolios, using our SDF model presented in Section 3.2. That is, we do not re-estimate the market price of risk parameters $\hat{\Lambda}_0$, but simply calculate the pricing errors for the corporate bond portfolios. The resulting annualized pricing errors are listed in the second column of Table 2. The model does a reasonable job pricing the corporate bonds: pricing errors are about 1.3% per year, compared to excess returns of about 3.5% per year. The mean absolute pricing error among all twenty test assets (ten BM portfolios, the market portfolio, five Treasury bond portfolios, and four corporate bond portfolios) is 58 basis points per year, compared to 40 basis points per year without the corporate bond portfolios.

Equally interesting is to re-estimate the market price of risk parameters of the SDF model when the corporate bond portfolios are included in the set of test assets. We do not allow for additional priced factors; the CP, level, and DP factors remain the only three priced risk factors. The third column of Table 2 shows that the corporate bond pricing errors are down to below 60 basis points per year on average. The overall MAPE on all 20 assets is 49 basis points per year, a mere 9 basis points above the MAPE when corporate bonds were not considered and 9 basis points less than when the corporate bonds were not included in the estimation. There is no monotone pattern in the pricing errors on the corporate bonds, suggesting our model is successful in generating no spread between the excess returns. Finally, comparing Columns 2 and 3, the point estimates for the market prices of risk Λ_0 in Panel B are very similar for the models with or without corporate bonds. The last column reports results for the Fama-French three-factor model. Its pricing errors are higher than in our three-factor model; the MAPE is 77 basis points. Average pricing errors on the corporate bond portfolios are about 1.05% per year, and monotonically declining in credit quality.

[Table 2 about here.]

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4.4.2 Different Stock Portfolios

Table 3 shows three exercises where we replace the ten book-to-market sorted portfolios by other sets of stock portfolios. In the first three columns we use ten market capitalization-sorted portfolios alongside the bond portfolios and the market. The first column shows the risk premia to be explained (risk neutral SDF). Small firms (S1) have about 3.5% higher risk premia than large stocks (S10). Our model in the second column manages to bring the overall mean absolute pricing error down from 5.8% per year to 0.52% per year, comparable to the 40 basis points we obtained with the book-to-market portfolios. This MAPE is comparable to that in the Fama-French model in the third column. The Fama-French model does better eliminating the spread between small and large stocks, whereas our model does better pricing the bond portfolios alongside the size portfolios. The next three columns use ten earnings-price-sorted stock portfolios and are organized the same way. High earnings-price portfolios have average risk premia that are 4.4% higher per year than low earnings-price portfolios. Our model reduces this spread in risk premia to less than 1% per year, while continuing to price the bonds reasonably well. The MAPE is 78 basis points per year compared to 91 in the Fama-French model. The last three columns use the five-by-five market capitalization- and book-to-market-sorted portfolios. Our three-factor model manages to bring the overall mean absolute pricing error down from 7.29% per year to 1.37% per year. This is again comparable to the three-factor Fama-French model. Relative to the FF model, ours reduces the pricing errors on the S1B1 portfolio but makes a larger error on the S1B4 and S1B5 portfolios. In all size quintiles but the first, our model is successful at eliminating most of the value spread, just like the FF model. Finally, the market price of risk estimates $\hat{\Lambda}_0$ for the three additional sets of test assets are similar to those we found for the book-to-market portfolios in Table 1.

[Table 3 about here.]

4.5 Subsample Analysis

We investigate the robustness of our main result in Table 1 by studying two subsamples. If we start the analysis in 1963, an often-used starting point for cross-sectional equity analysis (e.g., Fama and French (1993)), we find very similar results. The left columns of Table 4 shows a MAPE of 40bp per year, identical to what we found for the full sample. Our model improves relative to the Fama-French three-factor model, which has a pricing error of 72bp. There are no monotone patterns in the pricing errors on bonds or book-to-market decile portfolios. In the right columns, we investigate the results in the second half of our sample, 1980-2008. Mean absolute pricing errors rise to 59 basis points but the risk premia to be explained in this subsample are higher as well. In this subsample, the MAPE under the Fama-French model is 106 basis points. Panel B of Table 4 shows that the price of risk estimates are similar to the ones from the benchmark estimation.

These subsample results use yield factors which are estimated over the entire sample. In unreported results, we have re-estimated the state vector (e.g., the CP factor) over the sub-sample in question. The results are similar.

[Table 4 about here.]

4.6   Constraining the Maximum Sharpe Ratio on Bonds

A common problem with affine term structure models is that they imply high Sharpe ratios, arising from unconstrained bond portfolios.[16] Our model is no exception because it contains an affine term structure model as a building block. Given our four-factor term structure model, there exists a portfolio of four bonds that is perfectly correlated with the maximum risk premium achievable with bonds.[17] The bond portfolio that achieves the maximum bond risk premium contains very large long and short positions (on the order of 10^6), and has an annualized Sharpe ratio of 2.6. Because this magnitude Sharpe ratio might seem too high, we consider a *constrained* maximum Sharpe ratio which imposes limits on the portfolio positions. Since the problem is situated in the bond positions, we can safely ignore the stock portfolios, and compute the constrained maximum *bond* risk premium. In particular, we search for the vector of portfolio weights on one-, two-, three-, and four-year bonds that maximizes the correlation between the resulting portfolio return and the SDF, subject to constraints on the portfolio weights. Our constraints are of the form $-\alpha \leq w_j \leq 1$, and we study $\alpha = 0$, .50, and 1. Panel B of Table 5 shows that the resulting constrained maximum Sharpe ratios are .77 (.28), 0.94 (.17), and .94 (.17) annualized, respectively (time series standard deviations in parentheses). These constrained Sharpe ratios are much more reasonable than the unconstrained one. In fact, they are not much higher than the Sharpe ratios that can be achieved with individual bonds themselves. As Panel A shows, the model does a good job matching the Sharpe ratios on these individual bonds. We conclude that our model estimates imply reasonable Sharpe ratios on reasonably restricted bond portfolios.

[Table 5 about here.]

5   Conclusion

This paper argues that any successful asset pricing model must have a stochastic discount factor with three properties: a large permanent component, a smaller but non-zero transitory component, and variation in the relative importance of the permanent and transitory components. We

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[16]We thank Greg Duffee for pointing this out to us.

[17]The stock market block does not cause additional problems. It turns out that essentially all of the maximum risk premium (96% of it) can be achieved by combining bonds alone. That maximum *bond* risk premium is $\Lambda'_{t(1:4)}\Sigma_{(1:4,1:4)}\Lambda_{t(1:4)}$ in the model.

set up and estimate an affine asset pricing model that has all three features. A first factor, the level factor, captures variation in the transitory component of the stochastic discount factor. This factor is instrumental in pricing the cross-section of average returns on bond portfolios sorted by maturity. Without a transitory component, the term structure of interest rates would be flat and bond excess returns would be zero. A second factor, the dividend yield, captures variation in the permanent component of the SDF. This factor allows us to match the mean equity risk premium on the aggregate stock market. Without the permanent component, the highest risk premium in the economy would be the one on a long-term bond, a prediction belied by the higher equity risk premium. A third factor, the Cochrane-Piazzesi factor, captures variation in the relative contribution of the two components to the variance of the pricing kernel. Since CP is a business cycle frequency variable, so is the relative variance ratio. Without this factor the model's predicted return for high book-to-market (value) stocks would be the same as for low book-to-market (growth) stocks. We find that value stocks have a large positive exposure to CP shocks while growth stocks have a small negative exposure. This differential exposure together with the estimated price of risk allows the model to quantitatively account for the value premium. Intuitively, value firms have higher exposure to business cycle risk, as captured by the relative importance of the transitory component of the SDF. The result of our exercise is a relatively standard looking and parsimoniously-specified no-arbitrage asset pricing model that can account for cross-sectional differences in average stock and bond returns, the times series of expected stock and bond returns, and the dynamics of bond yields, with a common set of risk prices.

In future work we plan to study equilibrium asset pricing models that can quantitatively generate the links between returns on book-to-market sorted stock portfolios and maturity-sorted bond portfolios that we have documented here. Such models should connect the CP factor to specific sources of macroeconomic risk, such as a business cycle frequency component in the conditional expectation or the conditional variance of aggregate consumption growth.

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A   Decomposition of the Affine SDF

A.1   Proof based on Hansen and Scheinkman (2009)

We solve the eigenfunction problem:

$$E\left[\frac{M_{t+1}}{M_t}e\left(X_{t+1}\right)\right] = \beta e\left(X_t\right). \tag{19}$$

We conjecture and then verify that the solution is exponentially linear in the state variables, for some constant vector c:

$$e\left(X_t\right) = \exp\left(c'X_t\right).$$

Plugging in this conjecture leads to:

$$
\begin{aligned}
E\left[\frac{M_{t+1}}{M_t}e\left(X_{t+1}\right)\right] &= E\left[\exp\left(-\delta_0 - \delta_1'X_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t - \Lambda_t'\varepsilon_{t+1} + c'\gamma_0 + c'\Gamma X_t + c'\varepsilon_{t+1}\right)\right] \\
&= \exp\left(-\delta_0 - \delta_1'X_t + \frac{1}{2}c'\Sigma c - \Lambda_t'\Sigma c + c'\gamma_0 + c'\Gamma X_t\right) \\
&= \beta\exp\left(c'X_t\right),
\end{aligned}
$$

were the last line implicitly defines:

$$
\begin{aligned}
\beta &= \exp\left\{-\delta_0 + \frac{1}{2}c'\Sigma c + \left(\gamma_0' - \Lambda_0'\Sigma\right)c\right\} \\
c' &= -\delta_1' - c'\left(\Sigma\Lambda_1 - \Gamma\right).
\end{aligned}
$$

The solution for c is given by:

$$c = -\left(I + \left(\Sigma\Lambda_1 - \Gamma\right)'\right)^{-1}\delta_1 = B_\infty,$$

where the last equality follows from the definition of B_∞ in the main text. Thus, we can rewrite the expression for β as:

$$\beta = \exp\left\{-\delta_0 + \left(\gamma_0' - \Lambda_0'\Sigma\right)B_\infty + \frac{1}{2}B_\infty'\Sigma B_\infty\right\}$$

This establishes that $e(X_t) = \exp\left(B_\infty'X_t\right)$. We define $M_t^e \equiv \frac{1}{e(X_t)} = \exp\left\{-B_\infty'X_t\right\}$, so that

$$\frac{M_{t+1}^e}{M_t^e} = \exp\left\{-B_\infty'\gamma_0 + B_\infty'(I-\Gamma)X_t - B_\infty'\varepsilon_{t+1}\right\}.$$

We define the *transitory* component of the pricing kernel as $M_t^T \equiv \beta^t M_t^e$, so that the transitory component of the SDF equals:

$$\frac{M_{t+1}^T}{M_t^T} = \beta\frac{M_{t+1}^e}{M_t^e}.$$

Given equation (19), we can define the *permanent* or martingale component of the SDF, $\frac{M_{t+1}^P}{M_t^P}$, as the component that has unit expectation by construction:

$$
\begin{aligned}
\frac{M_{t+1}^P}{M_t^P} &\equiv \beta^{-1} \frac{M_{t+1}}{M_t} \frac{e(X_{t+1})}{e(X_t)} \\
&= \beta^{-1} \exp \left\{ -\delta_0 - \delta_1' X_t - \frac{1}{2} \Lambda_t' \Sigma \Lambda_t - \Lambda_t' \varepsilon_{t+1} + B_\infty' \gamma_0 + B_\infty' \left(\Gamma - I \right) X_t + B_\infty' \varepsilon_{t+1} \right\} \\
&= \exp \left\{ \Lambda_0' \Sigma B_\infty - \frac{1}{2} B_\infty' \Sigma B_\infty - \delta_1' X_t - \frac{1}{2} \Lambda_t' \Sigma \Lambda_t - \left(\Lambda_t - B_\infty \right)' \varepsilon_{t+1} + B_\infty' \left(\Gamma - I \right) X_t \right\} \\
&= \exp \left\{ -\frac{1}{2} \left(\Lambda_t - B_\infty \right)' \Sigma \left(\Lambda_t - B_\infty \right) - \left(\Lambda_t - B_\infty \right)' \varepsilon_{t+1} \right\}
\end{aligned}
$$

A.2   Proof based on Alvarez and Jermann (2005)

We start from the definition of the transitory component of the pricing kernel, which is defined in AJ as:

$$
M_t^T = \lim_{\tau \to \infty} \frac{\beta^{t+\tau}}{P_t(\tau)}.
$$

Using the fact that the term structure of bond yields is affine in our model, we have

$$
M_t^T = \lim_{\tau \to \infty} \beta^{t+\tau} \exp \left(-A(\tau) - B(\tau)' X_t \right).
$$

The constant β is chosen in order to satisfy Assumption 1 in Alvarez and Jermann (2005):

$$
0 < \lim_{\tau \to \infty} \frac{P_t(\tau)}{\beta^\tau} < \infty, \tag{20}
$$

Because of the affine term structure of our model and the stationarity of the state X under the risk-neutral measure, the limit $\lim_{\tau \to \infty} B(\tau) = B_\infty < \infty$ is finite. Taking limits on both sides of equation (6), we get

$$
\begin{aligned}
\lim_{\tau \to \infty} A(\tau) - A(\tau - 1) &= \lim_{\tau \to \infty} \left(\gamma_0' - \Lambda_0' \Sigma \right) B(\tau - 1) - \delta_0 + \lim_{\tau \to \infty} \frac{1}{2} B(\tau - 1)' \Sigma B(\tau - 1) \\
&= -\delta_0 + \left(\gamma_0' - \Lambda_0' \Sigma \right) B_\infty + \frac{1}{2} B_\infty' \Sigma B_\infty.
\end{aligned}
$$

This shows that the limit of $A(\tau) - A(\tau - 1)$ is finite, so that $A(\tau)$ grows at a linear rate in the limit. We choose the constant β to offset the growth in $A(\tau)$ as τ becomes very large. Setting

$$
\beta = \exp \left(-\delta_0 + \left(\gamma_0' - \Lambda_0' \Sigma \right) B_\infty + \frac{1}{2} B_\infty' \Sigma B_\infty \right), \tag{21}
$$

guarantees that condition (20) is satisfied. Intuitively, without the β term, $\lim_{\tau \to \infty} A(\tau) = -\infty$ and $\lim_{\tau \to \infty} P_t(\tau) = 0$. Hence, condition (20) is a regularity condition imposing that bond yields remain finite at infinite maturity.

We can now write the transitory component of the SDF as:

$$
\begin{aligned}
\frac{M_{t+1}^T}{M_t^T} &= \beta \exp\left(-B_\infty' X_{t+1} + B_\infty' X_t\right), \\
&= \beta \exp\left(-B_\infty'\left(\gamma_0 + \Gamma X_t + \varepsilon_{t+1}\right) + B_\infty' X_t\right), \\
&= \beta \exp\left(-B_\infty'\gamma_0 + B_\infty'\left(I - \Gamma\right) X_t - B_\infty'\varepsilon_{t+1}\right).
\end{aligned}
$$

With the transitory component in hand, we can back out the permanent component of the SDF via:

$$
\frac{M_{t+1}^P}{M_t^P} = \frac{M_{t+1}}{M_t}\left(\frac{M_{t+1}^T}{M_t^T}\right)^{-1}.
$$

Plugging in the expressions for the SDF and for its transitory component, we get:

$$
\begin{aligned}
\frac{M_{t+1}^P}{M_t^P} &= \exp\left(-y_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t - \Lambda_t'\varepsilon_{t+1}\right) \times \\
&\quad \beta^{-1}\exp\left(B_\infty'\gamma_0 - B_\infty'\left(I - \Gamma\right) X_t + B_\infty'\varepsilon_{t+1}\right),
\end{aligned}
$$

which simplifies to

$$
\frac{M_{t+1}^P}{M_t^P} = \beta^{-1}\exp\left(-\delta_0 + B_\infty'\gamma_0 - \left[B_\infty'\left(I - \Gamma\right) + \delta_1'\right] X_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t - \left(\Lambda_t' - B_\infty'\right)\varepsilon_{t+1}\right).
$$

We need to verify that the permanent component is a martingale, i.e., that $E_t\left[M_{t+1}^P/M_t^P\right] = 1$. Plugging in for β from (21), we get:

$$
\frac{M_{t+1}^P}{M_t^P} = \exp\left(\Lambda_0'\Sigma B_\infty - \frac{1}{2}B_\infty'\Sigma B_\infty - \left[B_\infty'\left(I - \Gamma\right) + \delta_1'\right] X_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t - \left(\Lambda_t' - B_\infty'\right)\varepsilon_{t+1}\right)
$$

Taking conditional expectations, we get:

$$
E_t\left[\frac{M_{t+1}^P}{M_t^P}\right] = \exp\left(-\left[B_\infty'\left(I - \Gamma\right) + \delta_1' + B_\infty'\Sigma\Lambda_1\right] X_t\right).
$$

But the term in brackets is zero because:

$$
B_\infty' = -\delta_1' + B_\infty'(\Gamma - \Sigma\Lambda_1).
$$

This can be seen to be true by taking limits as $\tau \to \infty$ on both sides of equation (7).

B Example of Consumption-Based Model

In this appendix we explore a consumption-based asset pricing model that fits into the more general framework developed in Section 2. It is meant as an example that may help to develop economic intuition for the more general model. It is the simplest structural model we could conceive that has the three necessary ingredients outlined in Section 3.3.

B.1 Setup

Endowment This is an endowment economy with a continuum of identical atomless agents. The endowment process, which must equal consumption in equilibrium, contains a transitory and a permanent component:

$$C_t = C_t^T C_t^P, \text{ or } c_t = c_t^T + c_t^P,$$

where uppercase letters denote levels and lowercase letters denote logs. This is a standard Beveridge-Nelson decomposition of log consumption into a cyclical component c_t^T and a random walk component c_t^P. All quantities are expressed in nominal terms, as in the main text.

We make the following assumptions on the two components of consumption:

$$c_{t+1}^T = \mu_c + \rho c_t^T + \sigma \varepsilon_{t+1}, \tag{22}$$

$$c_{t+1}^P = c_t^P - \frac{1}{2}s_t^2 + s_t \eta_{t+1}, \tag{23}$$

$$s_{t+1}^2 - \bar{s}^2 = \nu(s_t^2 - \bar{s}^2) + \sigma_w w_{t+1}, \tag{24}$$

where $(\varepsilon_{t+1}, \eta_{t+1}, w_{t+1})$ are standard normal i.i.d. random variables. The transitory component is a homoscedastic AR(1) process in logs and the permanent component is a random walk in levels ($E_t[C_{t+1}^P] = C_t^P$) with heteroscedastic innovations. The conditional variance of the permanent consumption shocks follows itself an AR(1) process. One could think of the latter process as capturing time-varying economic uncertainty about the long-run component of consumption. This time variation could be operating at business-cycle frequency if ν is not too high.

We note that an extension of this model could replace equation (22) by

$$c_{t+1}^T = \mu_c + \rho c_t^T + \psi(s_t^2 - \bar{s}^2) + \sigma \varepsilon_{t+1}.$$

The only difference is that, in this version, s_t^2 would forecast future consumption growth. The derivations would proceed in analogous manner.

Preferences The agents have constant relative risk aversion (CRRA) preferences with risk aversion parameter γ and time discount factor β. The log inter-temporal marginal rate of substitution, or $sdf =$

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$\log(SDF)$, is given by:

$$
\begin{aligned}
sdf_{t+1} &= \log(\beta) - \gamma\Delta c_{t+1}, \\
&= \log(\beta) - \gamma\mu_c + \gamma(1-\rho)c_t^T + \frac{\gamma}{2}s_t^2 - \gamma\sigma\varepsilon_{t+1} - \gamma s_t\eta_{t+1}, \tag{25}
\end{aligned}
$$

where we substituted in consumption growth implied by equations (22)-(23).

B.2 Term Structure of Interest Rates

Like the general model in the main text, this model implies an exponentially-affine term structure of interest rates, where the price of a bond of maturity τ is given by:

$$
P_t(\tau) = \exp\left\{A(\tau) + B(\tau)c_t^T + C(\tau)\left(s_t^2 - \bar{s}^2\right)\right\}.
$$

The corresponding bond yield is $y_t(\tau) = -\log(P_t(\tau))/\tau$. To see this, we guess this form and verify it by satisfying the Euler equation for the bond.

Proof. Guess and verify

$$
\begin{aligned}
P_t(\tau+1) &= E_t[\exp\{sdf_{t+1} + \log(P_{t+1}(\tau))\}] \\
&= E_t[\exp\{sdf_{t+1} + A(\tau) + B(\tau)c_{t+1}^T + C(\tau)\left(s_t^2 - \bar{s}^2\right)\}] \\
&= \exp\{\log(\beta) - \gamma\mu_c + \frac{\gamma}{2}s_t^2 + A(\tau) + [\gamma(1-\rho) + B(\tau)\rho]c_t^T + \nu C(\tau)\left(s_t^2 - \bar{s}^2\right)\} \times \\
&\quad E_t\left[\exp\left\{-\gamma s_t\eta_{t+1} + [B(\tau) - \gamma]\sigma\varepsilon_{t+1} + C(\tau)\sigma_w w_{t+1}\right\}\right]
\end{aligned}
$$

The expectation is over a log-normal random variable. The rest follows from recognizing that:

$$
\log P_t(\tau+1) = A(\tau+1) + B(\tau+1)c_t^T + C(\tau+1)\left(s_t^2 - \bar{s}^2\right),
$$

and matching up coefficients. The coefficients $A(\tau)$, $B(\tau)$, and $C(\tau)$ satisfy the following recursions:

$$
\begin{aligned}
A(\tau+1) &= A(\tau) + \log(\beta) - \gamma\mu_c + .5\gamma(1+\gamma)\bar{s}^2 + .5\left[B(\tau) - \gamma\right]^2\sigma^2 + .5C(\tau)^2\sigma_w^2, \tag{26} \\
B(\tau+1) &= \rho B(\tau) + \gamma(1-\rho), \tag{27} \\
C(\tau+1) &= \nu C(\tau) + .5\gamma(1+\gamma), \tag{28}
\end{aligned}
$$

where the recursion is initialized at $A(0) = 0$, $B(0) = 0$, and $C(0) = 0$. □

The short rate y_t equals

$$
\begin{aligned}
y_t &= -A(1) - B(1)c_t^T - C(1)\left(s_t^2 - \bar{s}^2\right), \\
&= -\log(\beta) + \gamma\mu_c - .5\left[\gamma(1+\gamma)\bar{s}^2 + [B_\infty - \gamma]^2\sigma^2 + C_\infty^2\sigma_w^2\right] - \gamma(1-\rho)c_t^T - .5\gamma(1+\gamma)\left(s_t^2 - \bar{s}^2\right).
\end{aligned}
$$

The one-period log return on a bond of maturity τ, $r^b(\tau)$, is given by:

$$
\begin{aligned}
r^b_{t+1}(\tau) &= p_{t+1}(\tau - 1) - p_t(\tau), \\
&= [A(\tau - 1) - A(\tau)] + [B(\tau - 1)\rho - B(\tau)]\, c^T_t + [C(\tau - 1)\nu - C(\tau)]\left(s^2_t - \bar{s}^2\right) + \\
&\quad B(\tau - 1)\sigma\varepsilon_{t+1} + C(\tau - 1)\sigma_w w_{t+1}.
\end{aligned}
$$

The one-period log expected excess return on a bond of maturity τ is given by:

$$
\begin{aligned}
E_t\left[r^b_{t+1}(\tau) - y_t\right] &= [A(\tau - 1) - A(\tau) + A(1)] + [B(\tau - 1)\rho - B(\tau) + B(1)]\, c^T_t \\
&\quad + [C(\tau - 1)\nu - C(\tau) + C(1)]\left(s^2_t - \bar{s}^2\right), \\
&= B(\tau - 1)\gamma\sigma^2 - \frac{1}{2}B(\tau - 1)^2\sigma^2 - \frac{1}{2}C(\tau - 1)^2\sigma^2_w. \tag{29}
\end{aligned}
$$

The Jensen term, which is one-half the conditional variance of the bond return, equals:

$$
\frac{1}{2}V_t\left[r^b_{t+1}(\tau)\right] = \frac{1}{2}B(\tau - 1)^2\sigma^2 + \frac{1}{2}C(\tau - 1)^2\sigma^2_w.
$$

The bond risk premium, the expected excess return on a real bond of maturity τ, including a Jensen adjustment, is given by

$$
E_t[rx^b_{t+1}(\tau)] = -Cov_t[sdf_{t+1}, r^b_{t+1}(\tau)] = B(\tau - 1)\gamma\sigma^2 > 0. \tag{30}
$$

The bond risk premium is higher, the higher the risk aversion coefficient γ, the higher the volatility of the cyclical consumption component σ, and the lower the persistence of the cyclical consumption component ρ ($\partial B(\tau)/\partial\rho = -\gamma\tau\rho^{\tau-1} < 0$ for $\rho > 0$). The bond risk premium does not depend on the properties of the permanent component of consumption. Note that the bond risk premium is always positive in this model.

The recursions for B and C have closed-form solutions. Taking the limit as the maturity of the bond τ goes to infinity, delivers

$$
B_\infty \equiv \lim_{\tau\to\infty} B(\tau) = \frac{\gamma(1 - \rho)}{1 - \rho} = \gamma, \tag{31}
$$

$$
C_\infty \equiv \lim_{\tau\to\infty} C(\tau) = .5\frac{\gamma(1 + \gamma)}{1 - \nu}, \tag{32}
$$

B.3   Decomposing the SDF

As in the general model, the pricing kernel can be decomposed into a permanent and a transitory component:

$$
M_t = M^T_t M^P_t = \beta^t \left(C^T_t\right)^{-\gamma}\left(C^P_t\right)^{-\gamma}.
$$

This decomposition implies that the log stochastic discount factor satisfies:

$$sdf_{t+1} = sdf_{t+1}^T + sdf_{t+1}^P.$$

Following the procedure outlined in Appendix A, we can solve for the transitory component sdf^T by exploiting that:

$$
\begin{aligned}
sdf_{t+1}^T &\equiv -r_{t+1}^b(\infty), \\
&= .5\left[\gamma(1+\gamma)\bar{s}^2 + [B_\infty - \gamma]^2\sigma^2 + C_\infty^2\sigma_w^2\right] + \\
&\quad \gamma(1-\rho)c_t^T + .5\gamma(1+\gamma)\left(s_t^2 - \bar{s}^2\right) - B_\infty\sigma\varepsilon_{t+1} - C_\infty\sigma_w w_{t+1},
\end{aligned}
$$

where the second equality follows from the definition of the bond return, evaluated at $\tau = \infty$, and from using the recursions (26)-(28).

The permanent component sdf^P can be found from:

$$
\begin{aligned}
sdf_{t+1}^P &\equiv sdf_{t+1} - sdf_{t+1}^T, \\
&= \log(\beta) - \gamma\mu_c - .5\left[\gamma^2\bar{s}^2 + [B_\infty - \gamma]^2\sigma^2 + C_\infty^2\sigma_w^2\right] \\
&\quad -.5\gamma^2\left(s_t^2 - \bar{s}^2\right) + [B_\infty - \gamma]\sigma\varepsilon_{t+1} - \gamma s_t\eta_{t+1} + C_\infty\sigma_w w_{t+1},
\end{aligned}
$$

Note that because $B_\infty = \gamma$, the permanent component of the SDF does not depend on the transitory consumption shock ε. Also note that the innovation to the conditional variance of the permanent component of consumption growth, w_{t+1}, contributes positively to sdf_{t+1}^P and negatively to sdf_{t+1}^T.

The conditional variance of the transitory component of the log SDF is:

$$V_t\left[sdf_{t+1}^T\right] = B_\infty^2\sigma^2 + C_\infty^2\sigma_w^2.$$

The conditional variance of the permanent component of the log SDF is:

$$V_t\left[sdf_{t+1}^P\right] = [B_\infty - \gamma]^2\sigma^2 + \gamma^2 s_t^2 + C_\infty^2\sigma_w^2.$$

The conditional variance of the entire log SDF is :

$$V_t\left[sdf_{t+1}\right] = \gamma^2\sigma^2 + \gamma^2 s_t^2.$$

The fraction of the conditional variance of the log SDF that comes from the permanent component, ω_t, in equation (14), in this consumption example is:

$$
\begin{aligned}
\omega_t &= \frac{\frac{1}{2}\left([B_\infty - \gamma]^2\sigma^2 + \gamma^2 s_t^2 + C_\infty^2\sigma_w^2\right)}{\frac{1}{2}\left(\gamma^2\sigma^2 + \gamma^2 s_t^2\right)}, \\
&= 1 - \frac{B_\infty\gamma\sigma^2 - \frac{1}{2}B_\infty^2\sigma^2 - \frac{1}{2}C_\infty^2\sigma_w^2}{\frac{1}{2}\left(\gamma^2\sigma^2 + \gamma^2 s_t^2\right)}. \quad (33)
\end{aligned}
$$

The conditional variance ratio equals one minus the ratio of the expected excess return on a bond of infinite-maturity (see equation 29) divided by the maximum expected excess return in the economy. Depending on parameters, this ratio can be greater or smaller than one on average.

Finally, the ratio varies over time because the economic uncertainty variable s_t^2 varies over time: ω_t increases in s_t^2. When economic uncertainty decreases (and the bond risk premium is positive), more of the conditional variance of the SDF comes from the transitory shocks, and less comes from the permanent shocks. To the extent that economic uncertainty decreases when economic activity is expected to pick up, the model predicts that the contribution from the transitory component increases when economic prospects improve. This is consistent with what we find in the data: a higher CP is associated with better economic prospects and with a higher importance of the transitory component of the SDF.

C How Pricing Stocks and Bonds Jointly Can Go Wrong

Consider two factors F_t^i, $i = 1, 2$, with innovations η_{t+1}^i. We normalize $\sigma\left(\eta_{t+1}^i\right) = 1$. Let $cov\left(\eta_{t+1}^1, \eta_{t+1}^2\right) = \rho = corr\left(\eta_{t+1}^1, \eta_{t+1}^2\right)$. We also have two cross-sections of test assets with excess, geometric returns r_{t+1}^{ki}, $i = 1, 2$ and $k = 1, ..., K_i$, with innovations ε_{t+1}^{ki}. We assume that these returns include the Jensen's correction term. Suppose that both cross-sections exhibit a one-factor pricing structure:

$$E\left(r_{t+1}^{ki}\right) = cov\left(\varepsilon_{t+1}^{ki}, \eta_{t+1}^i\right)\lambda_i, \ i = 1, 2.$$

The first factor perfectly prices the first set of test assets, whereas the second factor prices the second set of test assets. We show below that this does *not* imply that there exists a single SDF that prices both sets of assets.

Consider the following model of unexpected returns for both sets of test assets:

$$\varepsilon_{t+1}^{k1} = E\left(r_{t+1}^{k1}\right)\eta_{t+1}^1,$$
$$\varepsilon_{t+1}^{k2} = E\left(r_{t+1}^{k2}\right)\eta_{t+1}^2 + \alpha_{2k}\eta_{t+1}^3,$$

with $cov\left(\eta_{t+1}^2, \eta_{t+1}^3\right) = 0$. Unexpected returns on the first set of test assets are completely governed by innovations to the first factor, whereas unexpected returns on the second set of test assets contain a component $\alpha_{2k}\eta_{t+1}^3$ that is orthogonal to the component governed by innovations to the second factor. These η^3 shocks are not priced (by assumption). We assume that they are correlated with the η^1 shocks: $cov\left(\eta_{t+1}^1, \eta_{t+1}^3\right) \neq 0$.

This structure implies:

$$cov\left(\varepsilon_{t+1}^{ki}, \eta_{t+1}^i\right) = E\left(r_{t+1}^{ki}\right)var\left(\eta_{t+1}^i\right) = E\left(r_{t+1}^{ki}\right),$$

and hence $\lambda_i = 1$, $i = 1, 2$. Then we have:

$$cov\left(\varepsilon_{t+1}^{k1}, \eta_{t+1}^1\right) = E\left(r_{t+1}^{k1}\right), \qquad cov\left(\varepsilon_{t+1}^{k1}, \eta_{t+1}^2\right) = E\left(r_{t+1}^{k1}\right)\rho,$$

$$cov\left(\varepsilon_{t+1}^{k2}, \eta_{t+1}^1\right) = \left(r_{t+1}^{k2}\right)\rho + \alpha_{2k}cov\left(\eta_{t+1}^1, \eta_{t+1}^3\right), \quad cov\left(\varepsilon_{t+1}^{k2}, \eta_{t+1}^2\right) = E\left(r_{t+1}^{k2}\right).$$

The main point is that, if α_{2k} is not proportional to $E\left(r_{t+1}^{k2}\right)$, then there exist no Λ_1 and Λ_2 such that:

$$E\left(r_{t+1}^{ki}\right) = cov\left(\varepsilon_{t+1}^{ki}, \eta_{t+1}^1\right)\Lambda_1 + cov\left(\varepsilon_{t+1}^{ki}, \eta_{t+1}^2\right)\Lambda_2.$$

On the other hand, if there is proportionality and $\alpha_{2k} = \alpha E\left(r_{t+1}^{k2}\right)$, then we have:

$$cov\left(\varepsilon_{t+1}^{k2}, \eta_{t+1}^1\right) = E\left(r_{t+1}^{k2}\right)\left(\rho + \alpha cov\left(\eta_{t+1}^1, \eta_{t+1}^3\right)\right) = E\left(r_{t+1}^{k2}\right)\xi,$$

and Λ_1 and Λ_2 are given by:

$$\Lambda_1 = \frac{1 - \rho}{1 - \xi\rho}, \text{and } \Lambda_2 = \frac{1 - \xi}{1 - \xi\rho}.$$

This setup is satisfied approximately in our model, where the first set of test assets are the book-to-market portfolios, η^1 are CP innovations, the second set of test assets are the bond portfolios, and η^2 are level innovations. Unexpected bond returns contain a component η^3 that is uncorrelated with level innovations, but that is correlated with CP innovations. Unexpected book-to-market portfolio returns, in contrast, are largely uncorrelated with level innovations. The result above illustrates that consistent risk pricing is possible because unexpected bond returns' exposure to CP shocks has a proportionality structure. This can also be seen in the top panel of Figure 3.

Table 1: Unified SDF Model for Stocks and Bonds - Pricing Errors

Panel A of this table reports pricing errors on 10 book-to-market sorted stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. They are expressed in percent per year. Each column corresponds to a different stochastic discount factor (SDF) model. The first column contains the risk-neutral SDF and therefore reports the average pricing errors that are to be explained. The second column presents our SDF model with three priced factors (Our Model). The third column presents the results for a bond pricing model, where only the level factor is priced (Level). In the fourth column, we only use the bond returns as moments to estimate the same SDF as in the third column (Level-only bonds). The SDF model of the fifth column only allows the innovations to the dividend-price ratio on the aggregate market portfolio to be priced, and therefore is a CAPM-like model (DP). The sixth column allows for both the prices of DP and level risk to be non-zero (Level + DP). The last column refers to the three factor model of Fama and French (1992). The last row of Panel A reports the mean absolute pricing error across all 16 securities (MAPE). Panel B reports the estimates of the prices of risk. The first six columns report market prices of risk $\hat{\Lambda}_0$ for (a subset) of the following pricing factors : ε^{CP} (CP), ε^L (Level), and ε^{DP} (DP). In the last column, the pricing factors are the innovations in the excess market return (MKT), in the size factor (SMB), and in the value factor (HML), where innovations are computed as the residuals of a regression of these factors on the lagged dividend-price ratio on the market. The data are monthly from June 1952 through December 2008.

	RN SDF	**Our Model**	Level	Level-only bonds	DP	Level + DP		FF
				Panel A: Pricing Errors (in % per year)				
1-yr	1.11	-0.43	-0.41	0.69	1.00	0.68		0.91
2-yr	1.31	-0.55	-1.62	0.50	1.15	0.53		0.97
5-yr	1.69	-0.19	-4.10	0.09	1.43	0.19		1.08
7-yr	1.99	0.38	-4.82	0.11	1.61	0.15		1.22
10-yr	1.62	0.17	-6.02	-0.49	1.09	-0.54		0.56
Market	6.00	-0.74	4.22	5.51	-1.55	-0.06		-0.06
BM1	4.97	0.02	3.38	4.53	-3.28	-3.15		0.53
BM2	5.86	-0.36	3.79	5.29	-1.85	-1.85		0.07
BM3	6.65	0.04	4.52	6.06	-0.83	-0.86		0.03
BM4	6.37	-0.06	4.39	5.82	-0.99	-0.99		-0.84
BM5	7.30	0.79	5.39	6.77	0.55	0.52		-0.17
BM6	7.35	0.25	5.08	6.72	0.40	0.31		-0.30
BM7	7.32	-1.17	5.13	6.72	0.76	0.66		-1.01
BM8	9.10	0.28	7.20	8.58	2.34	2.32		0.05
BM9	9.69	0.70	7.94	9.20	2.76	2.78		0.95
BM10	10.12	0.27	9.25	9.88	2.35	2.61		0.41
MAPE	**5.53**	**0.40**	**4.83**	**4.81**	**1.50**	**1.23**		**0.57**

				Panel B: Prices of Risk Estimates				
CP	0	88.06	0	0	0	0	MKT	5.08
Level	0	-23.98	-42.22	-11.69	0	-9.18	SMB	-4.76
DP	0	-1.98	0	0	-3.46	-3.26	HML	6.93

Table 2: Unified SDF Model for Stocks, Treasuries, and Corporate Bonds

Panel A of this table reports pricing errors on 10 book-to-market-sorted stock portfolios, the value-weighted market portfolio, five Treasury bond portfolios of maturities 1, 2, 5, 7, and 10 years, and four corporate bond portfolios sorted by S&P credit rating (AAA, AA, A, and BBB). They are expressed in percent per year. Each column corresponds to a different SDF model, as described in the text. The last row reports the mean absolute pricing error across all 20 securities (MAPE). Panel B reports the estimates of the prices of risk $\tilde{\Lambda}_0$. The estimation period for stocks and Treasury bonds is June 1952 through December 2008, while the corporate bond portfolio data are available only from January 1980 until December 2008.

	RN SDF	Our Model not re-estimated	Our Model re-estimated	FF SDF
		Panel A: Pricing Errors (% per year)		
1-yr	1.11	-0.44	-0.20	0.84
2-yr	1.31	-0.56	-0.19	0.81
5-yr	1.69	-0.19	0.35	0.68
7-yr	1.99	0.39	0.96	0.66
10-yr	1.62	0.17	0.77	-0.02
Market	6.00	-0.75	-0.81	0.18
BM1	4.97	0.02	-0.32	0.79
BM2	5.86	-0.34	-0.46	0.18
BM3	6.65	0.04	-0.01	-0.15
BM4	6.37	-0.05	-0.12	-1.10
BM5	7.30	0.78	0.78	-0.78
BM6	7.35	0.26	0.32	-0.65
BM7	7.32	-1.16	-0.93	-1.63
BM8	9.10	0.28	0.51	-0.32
BM9	9.69	0.69	0.91	1.10
BM10	10.12	0.27	0.42	1.26
Credit1	3.47	-1.60	-0.59	1.17
Credit2	3.47	-1.00	-0.09	1.09
Credit3	3.46	-1.16	-0.26	1.01
Credit4	3.48	-1.53	-0.71	0.90
MAPE	**5.12**	**0.58**	**0.49**	**0.77**
		Panel B: Prices of Risk Estimates		
CP/Market		88.16	79.61	6.36
Level/SMB		-24.03	-18.84	-10.88
DP/HML		-1.96	-2.15	8.08

Table 3: Other Stock Portfolios - Pricing Errors

This table reports robustness with respect to different stock market portfolios, listed in the first row. Panel A of this table reports pricing errors (in % per year) on various stock portfolios, the value-weighted market portfolio, and five bond portfolios of maturities 1, 2, 5, 7, and 10 years. Each column corresponds to a different stochastic discount factor (SDF) model. The first column contains the risk-neutral SDF. The second column presents our SDF model with three priced factors (Our). The third column refers to the three factor model of Fama and French (FF). The last row of Panel A reports the mean absolute pricing error across all securities (MAPE). Panel B reports the estimates of the prices of risk. The data are monthly from June 1952 through December 2008.

	10 Size Portfolios				10 Earnings-Price Portfolios				25 Size and Value Portfolios		
Assets	RN SDF	Our	FF	Assets	RN SDF	Our	FF	Assets	RN SDF	Our	FF
Panel A: Pricing Errors (in % per year)											
1-yr	1.11	0.13	0.85		1.11	-0.31	1.04		1.11	-0.96	0.96
2-yr	1.31	0.05	0.95		1.31	-0.41	1.25		1.31	-0.99	1.12
5-yr	1.69	0.21	1.26		1.69	-0.07	1.73		1.69	-0.13	1.49
7-yr	1.99	0.56	1.64		1.99	0.51	2.13		1.99	0.82	1.83
10-yr	1.62	0.15	0.75		1.62	0.31	1.55		1.62	0.89	1.14
S1	8.97	1.59	-0.25	EP1	3.30	0.80	0.41	S1B1	2.87	-3.36	-5.01
S2	8.70	0.28	0.22	EP2	2.14	-1.07	-1.02	S1B2	9.10	-0.12	0.36
S3	9.14	0.89	0.28	EP3	3.25	-0.60	-0.20	S1B3	9.32	0.55	0.43
S4	8.55	0.46	0.27	EP4	2.63	-1.84	-0.99	S1B4	11.94	4.27	2.51
S5	8.65	0.21	0.42	EP5	3.01	-1.68	-1.03	S1B5	12.99	3.83	1.74
S6	8.01	-0.04	-0.18	EP6	4.77	-0.15	0.09	S2B1	4.74	-1.52	-1.72
S7	7.95	-0.24	0.04	EP7	4.71	-0.35	-0.03	S2B2	8.25	0.51	-0.04
S8	7.45	-0.96	-0.53	EP8	5.04	0.18	-0.42	S2B3	10.60	2.20	1.67
S9	6.85	-0.60	-0.72	EP9	6.45	0.59	0.36	S2B4	11.09	1.65	1.13
S10	5.46	-0.97	0.32	EP10	7.74	1.75	0.34	S2B5	12.32	2.51	0.54
								S3B1	5.84	-1.51	0.44
								S3B2	8.65	0.49	0.80
								S3B3	9.25	0.44	0.35
								S3B4	10.17	0.30	0.36
								S3B5	11.61	1.28	0.24
								S4B1	6.41	0.02	1.77
								S4B2	6.81	-1.14	-0.87
								S4B3	8.89	-0.39	0.05
								S4B4	9.49	-2.25	-0.01
								S4B5	9.56	-2.00	-1.76
								S5B1	5.42	1.14	2.19
								S5B2	6.22	0.59	0.38
								S5B3	6.64	0.24	0.10
								S5B4	6.60	-2.85	-2.08
								S5B5	7.59	-2.90	-2.62
Market	6.00	-1.05	-0.13		6.00	1.89	1.93		6.00	-0.66	0.27
MAPE	**5.84**	**0.52**	**0.55**		**3.55**	**0.78**	**0.91**		**7.29**	**1.37**	**1.16**
Panel B: Market Prices of Risk											
CP/Market		48.21	6.22			81.22	2.19			134.73	4.04
Level/SMB		-15.59	1.25			-22.84	4.19			-29.83	2.16
Market/HML		-2.56	17.48			-0.83	5.37			-1.45	8.82

Table 4: Other Sample periods - Pricing Errors

This table reports robustness with respect to different sample periods. It is otherwise identical to Table 1. The data are monthly from January 1963 through December 2008 in the left columns and from December 1978 until December 2008 in the right columns.

	1963-2008			1980-2008		
	RN kernel	Our kernel	FF kernel	RN kernel	Our kernel	FF kernel
Panel A: Pricing Errors (in % per year)						
1-yr	1.18	-0.36	0.95	1.55	0.10	1.22
2-yr	1.34	-0.66	0.97	2.09	-0.17	1.40
5-yr	1.89	0.03	1.23	3.16	0.11	1.64
7-yr	2.27	0.45	1.47	3.82	0.18	1.70
10-yr	1.98	0.05	0.85	3.88	0.03	1.19
Market	4.73	-0.90	-0.19	5.80	-1.14	0.54
BM1	3.40	-0.18	0.75	4.43	-1.27	0.28
BM2	4.90	-0.07	0.22	6.74	0.69	0.80
BM3	5.51	0.01	0.11	7.10	0.32	-0.12
BM4	5.54	0.63	-0.85	7.59	1.46	-0.38
BM5	5.25	0.03	-1.25	6.57	0.63	-1.60
BM6	6.29	0.18	-0.43	6.50	-0.28	-1.07
BM7	7.10	-0.74	-0.30	7.62	-0.42	-1.04
BM8	7.74	-0.63	-0.24	7.01	-1.32	-1.01
BM9	8.90	0.69	0.87	8.90	0.46	0.64
BM10	9.99	0.87	0.80	9.79	0.85	2.36
MAPE	**4.88**	**0.40**	**0.72**	**5.78**	**0.59**	**1.06**
Panel B: Market Prices of Risk						
CP/Market		61.39	4.19		41.12	5.40
Level/SMB		-24.83	-2.23		-24.90	-12.03
DP/HML		-0.83	6.73		-1.35	4.40

Table 5: Maximum Sharpe Ratio on Bonds

This table reports annualized Sharpe ratios (SR) on bonds in model and data. Panel A shows Sharpe ratios on the CRSP bond portfolios of maturities one- through ten-years. The left columns report the time-series average and standard deviation (in parentheses) in the model, while the two most right columns report the same moments in the data. Sharpe ratios are annualized by multiplying the monthly Sharpe ratios by $\sqrt{12}$. We use the CRSP bond portfolios to form the moments in the data, and form conditional Sharpe ratios by taking into account that conditional expected returns depend on lagged CP. The data are from June 1952 until December 2008. Panel B reports model-implied annualized Sharpe ratios on unconstrained and constrained bond portfolios. These portfolios are formed by combining bonds of maturities one- through four-years (any four bonds would do). The unconstrained portfolio achieves the maximum Sharpe ratio in the economy. The last three rows report Sharpe ratios for constrained bond portfolios, where the constraints are of the form $-\alpha \leq w_j \leq 1$, and we study $\alpha = 0, .5$, and 1.

	Model		Data	
	mean	st.dev.	mean	st.dev.
Panel A: SR on individual bonds				
1-yr	0.81	(0.28)	0.71	(0.28)
2-yr	0.71	(0.31)	0.48	(0.32)
5-yr	0.45	(0.33)	0.33	(0.30)
7-yr	0.33	(0.33)	0.33	(0.29)
10-yr	0.24	(0.32)	0.22	(0.30)
Panel B: SR on bond portfolios				
unconstr.	2.62	(0.07)		
$\alpha = 0.0$	0.77	(0.28)		
$\alpha = 0.5$	0.94	(0.17)		
$\alpha = 1.0$	0.94	(0.17)		

Figure 1: Exposure of value and growth portfolio returns to CP innovations

The figure shows the covariance of innovations to CP and innovations to returns on 10 portfolios sorted on the BM ratio. Portfolio 1 is the extreme growth portfolio; portfolio 10 the extreme value portfolio. Innovations are constructed as described in Section 3.1.
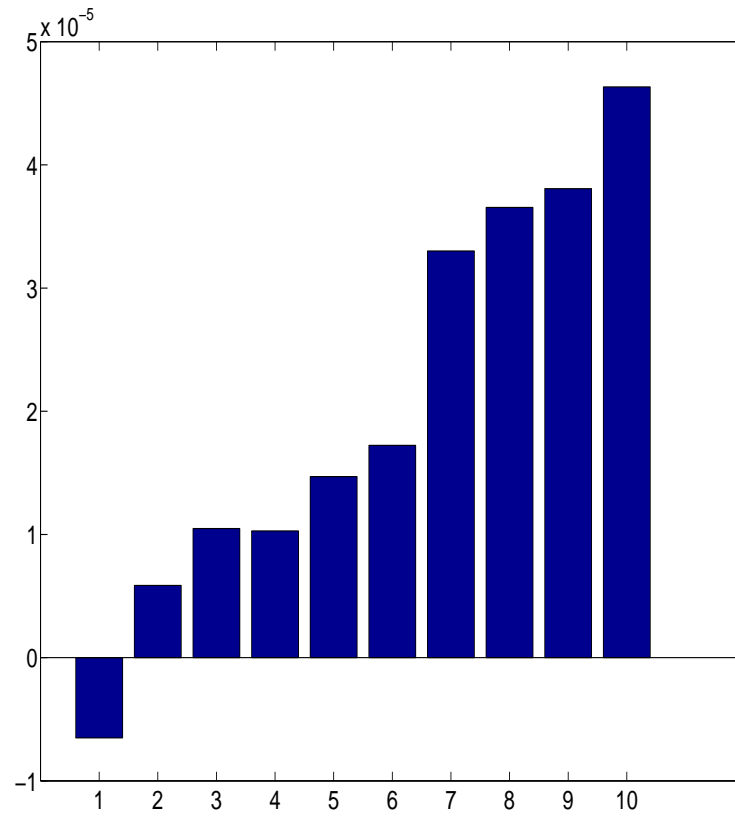
Figure 2: Economic activity and the CP Factor

The top panel displays the predictive coefficient in (17), the middle panel the t-statistic, and the bottom panel the corresponding R-squared value. We consider $k = 1, \ldots, 36$ lags and the standard errors are computed using Newey-West standard errors with $k - 1$ lags. The sample is for March 1967 until December 2008.
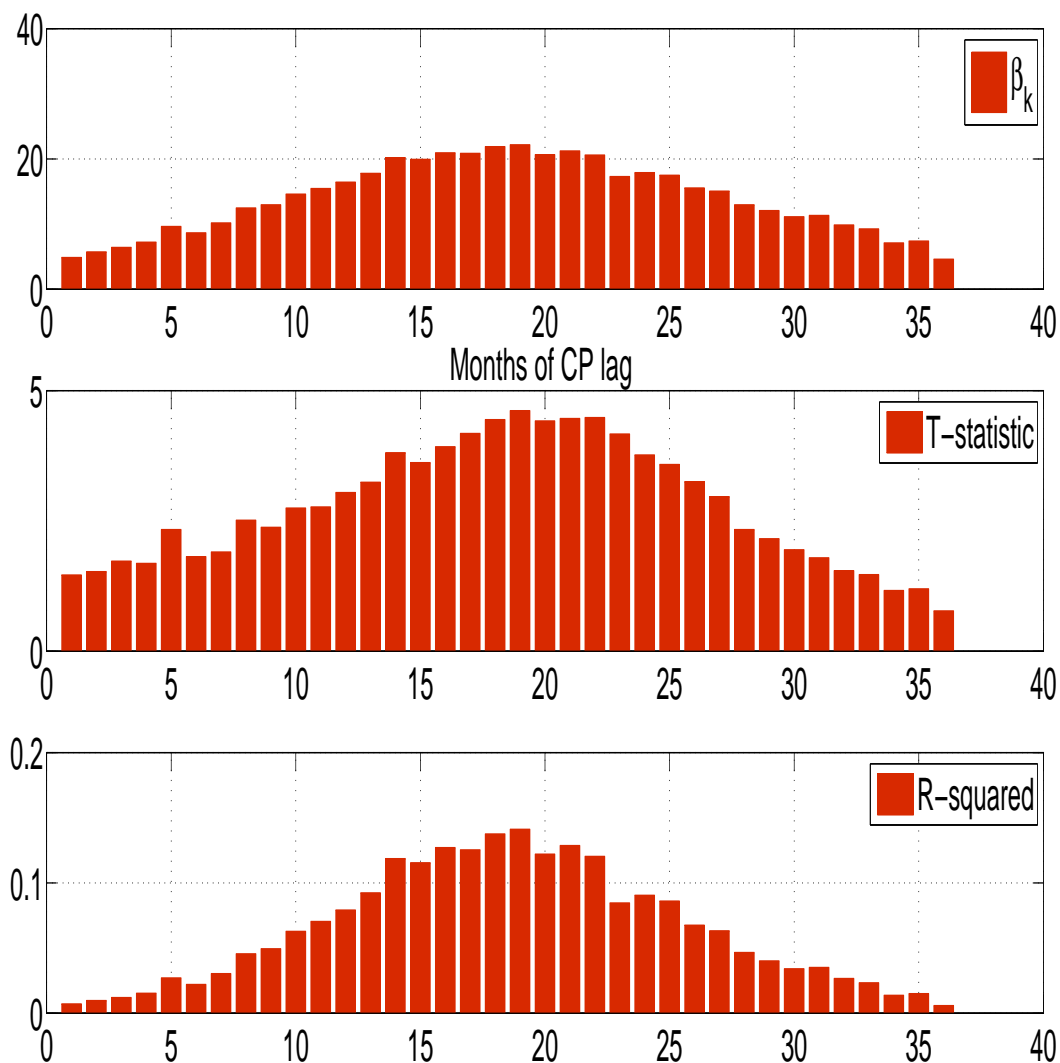
Figure 3: Exposure of Portfolio Excess Returns to Priced Innovations

The figure plots the risk premium decomposition into risk compensation for exposure to the CP factor, the level factor, and the DP factor. The top panel is for the market portfolio (first set of three bars) and the five bond portfolios (last five sets of bars) whereas the bottom panel is for the book-to-market decile portfolios. The three bars for each asset are computed as $\Sigma'_{XR}\tilde{\Lambda}_0$, which is a five-by-one vector containing three non-zero elements. The data are monthly from June 1952 until December 2008.
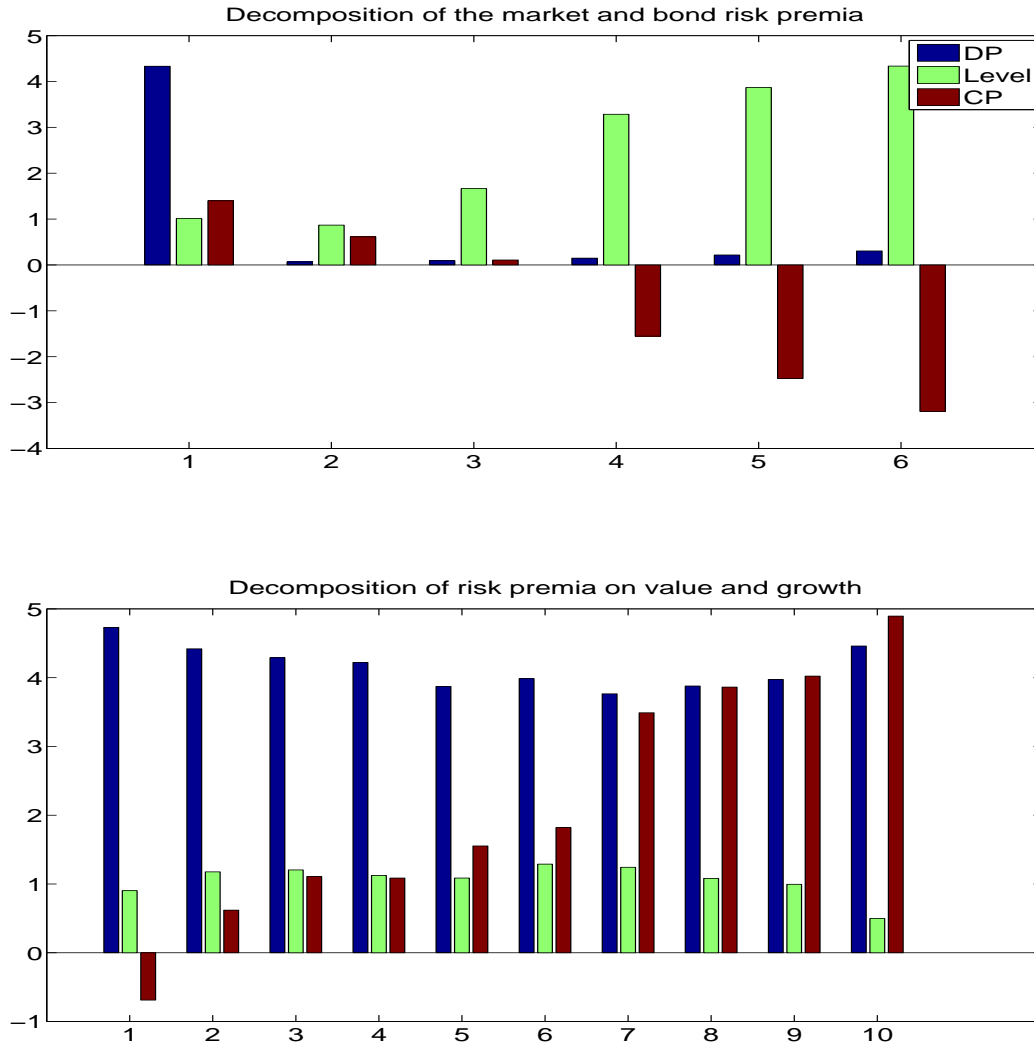
Figure 4: Implied Predictive Coefficients of the Ten Book-to-Market Portfolios

The figure plots the predictive coefficients implied by the SDF model and the ones implied by running a predictive regression in the data. The returns are on the ten book-to-market portfolios and the log dividend yield on the aggregate stock market is used to predict future returns. The data run from June 1952 until December 2008.
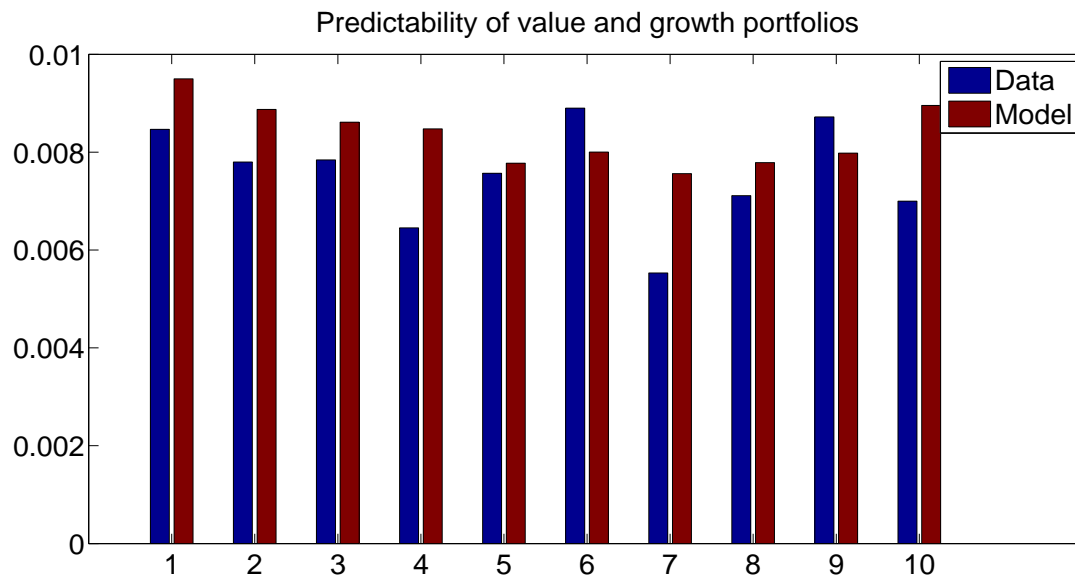

Predictability of value and growth portfolios

Figure 5: Implied Predictive Coefficients of Bond Returns

The figure plots the coefficients in model and data obtained from predictive regressions of annual holding period returns on bonds of two-year, three-year, four-year, and five-year maturity in excess of one-year bond returns on the one-year lagged CP factor. To compute the predictive coefficients in the model, we simulate 5,000 samples of the same length as our data and estimate the CP factor and the predictive regressions in each of these samples. We report the average, across 5,000 samples, predictive coefficient. The data run from June 1952 until December 2008.
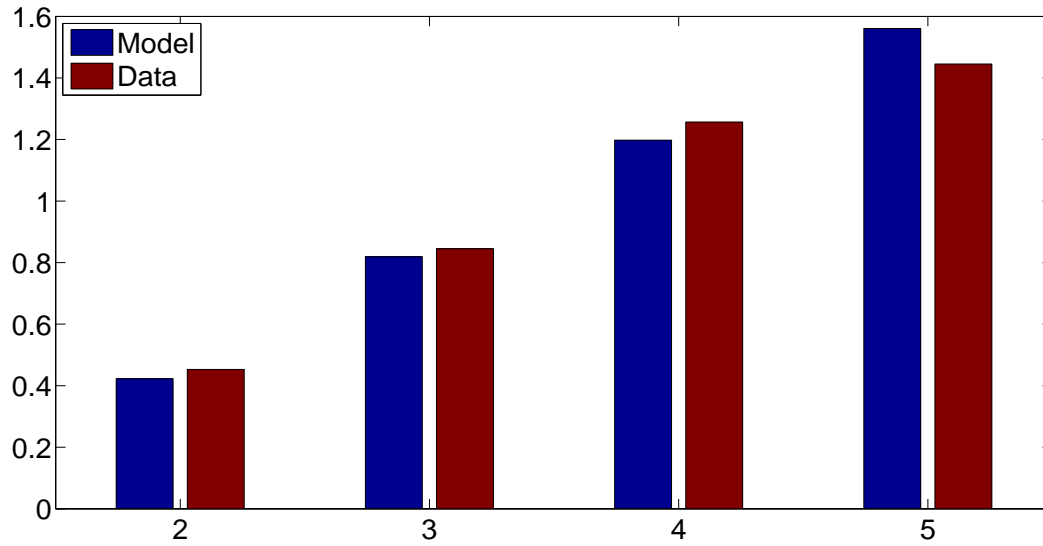
Figure 6: Expected Excess Returns on Stocks and Bonds

The figure plots the expected log excess return on the aggregate stock market (solid line) and the expected log excess return on an equally-weighted portfolio of bond returns (dashed line). Expected excess stock returns are modeled as a linear function of the log dividend-price ratio on the aggregate stock market. Expected excess bond returns are modeled as a linear function of Cochrane and Piazzesi's CP factor. The excess returns are annualized by multiplication by 12. The data run from June 1952 until December 2008.
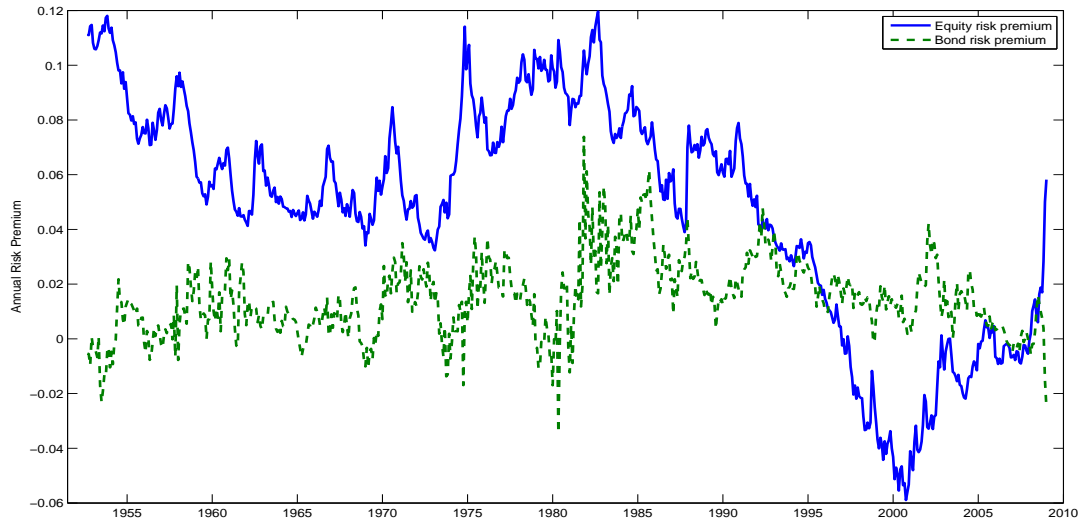
Figure 7: SDF Model-Implied Bond Yields

This figure plots annual yields on nominal bonds of maturities 1- through 5 years as implied by stochastic discount factor model estimated in Section 3.4. The top panel is for the 1-year bond yield, the bottom panel for the 5-year bond yield. The data are Fama-Bliss yields on nominal bonds of maturities 1- through 5 years for June 1952 until December 2008.
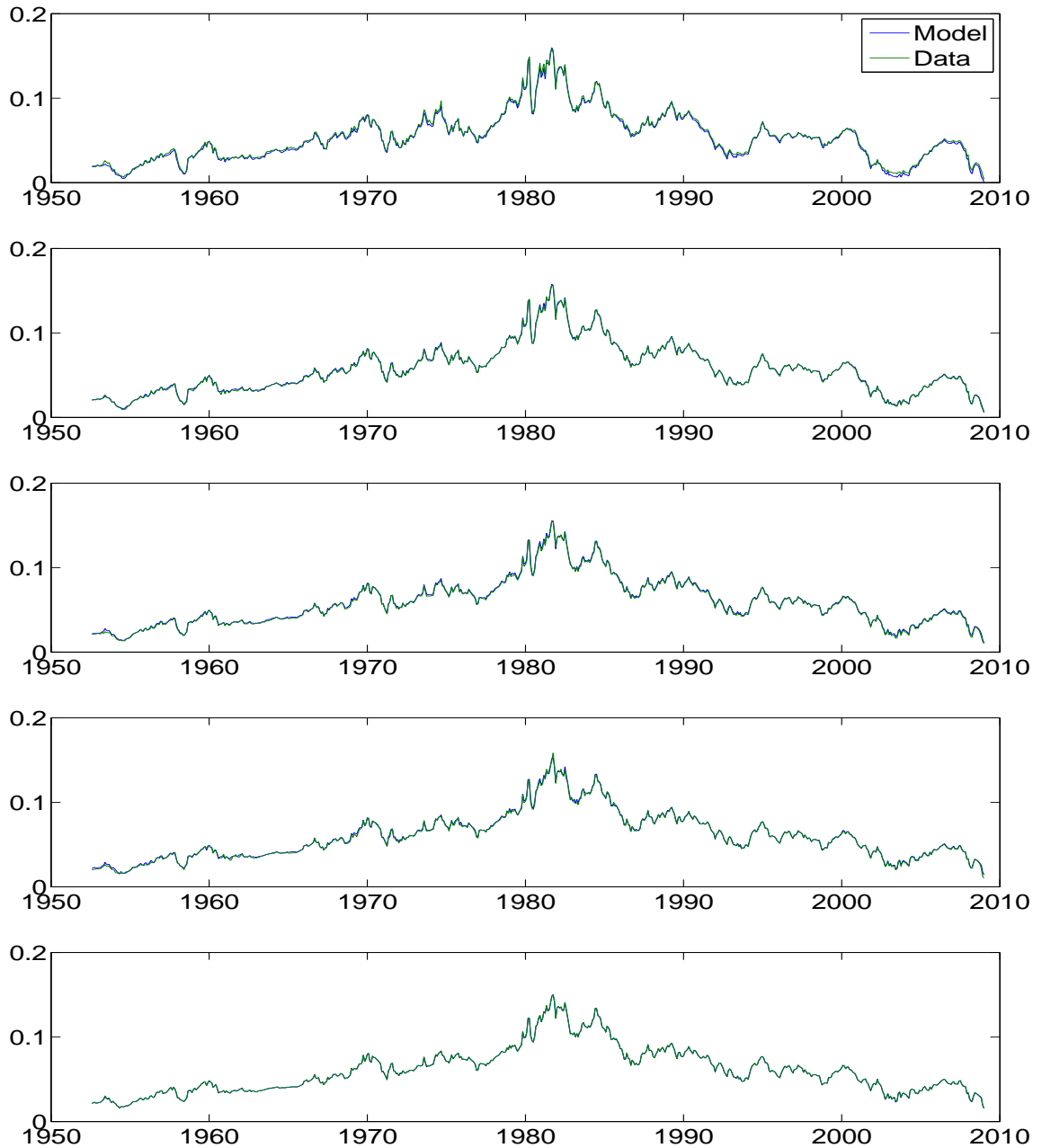
Figure 8: SDF Decomposition and the CP Factor

The top panel plots the ratio ω_t of the conditional variance of the permanent component of the stochastic discount factor to the conditional variance of the (entire) stochastic discount factor; see equation (14). The bottom panel plots the CP factor, constructed as in Cochrane and Piazzesi (2005). The two series have a correlation of -0.99. The sample is for July 1952 until December 2008.

Contribution of Permanent Component to Conditional Variance of SDF
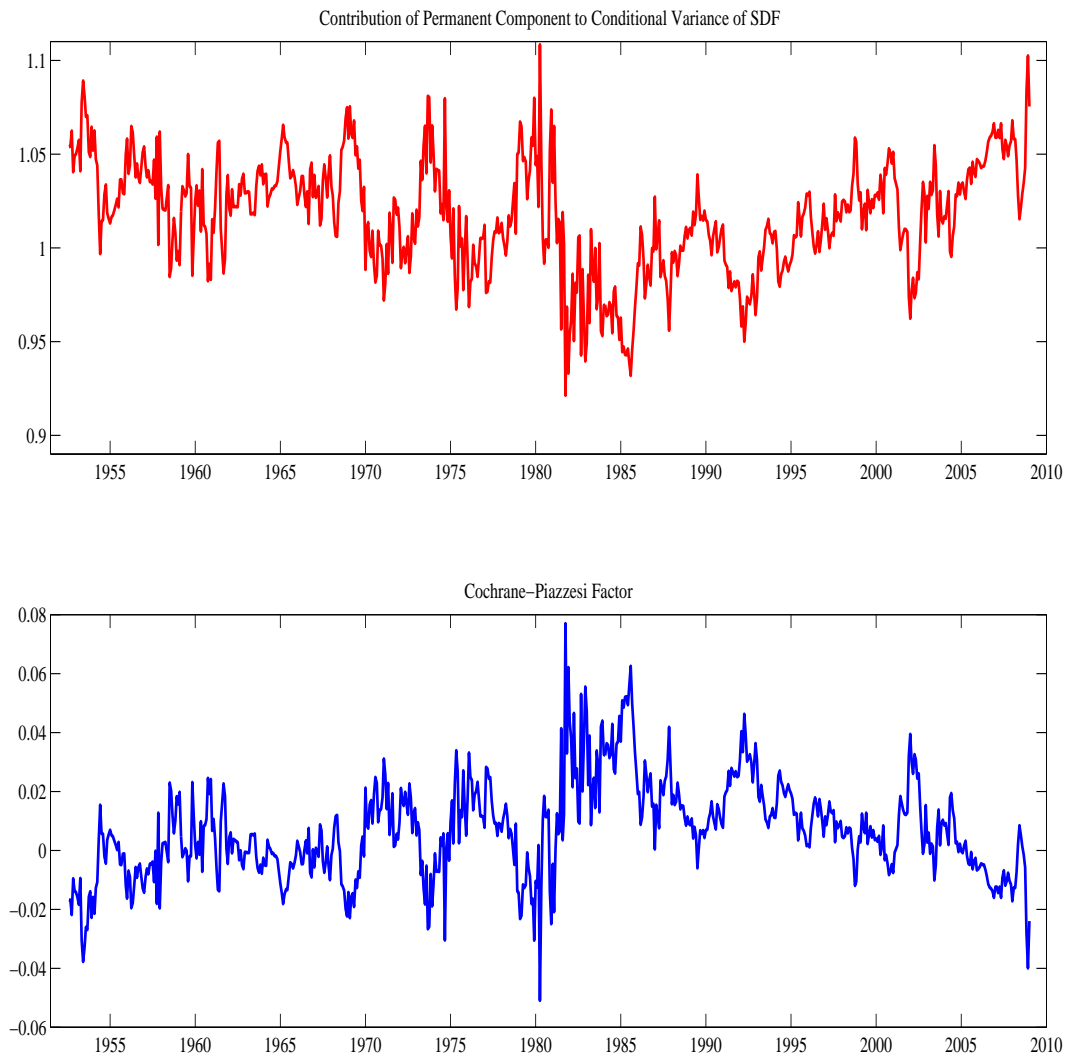
Cochrane–Piazzesi Factor

Figure 9: Implied Predictive Coefficients of the Ten Book-to-Market Portfolios

The figure plots the predictive coefficients implied by the SDF model and the ones implied by running a predictive regression in the data. The returns are for the ten book-to-market portfolios; they are regressed on the lagged log dividend yield on the aggregate stock market DP and the lagged CP factor. The left panel plots the coefficients on DP from the multi-variate regression while the right panel plots the coefficients on CP. The data run from June 1952 until December 2008.