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ABSTRACT

Much attention has been devoted to the quantification of the gains from trade. In this paper our goal is to quantify the gains from openness, which includes trade as well as other ways in which countries interact. We focus on trade and multinational production (MP), which in 2007 was almost twice as large as trade flows. We present and calibrate a model where countries interact through trade as well as MP, and then quantify the overall gains from openness and the role of both of these channels in generating those gains. The model captures several dimensions of the complex interaction between trade and MP: trade and MP are competing ways to serve a foreign market; MP relies on imports of intermediate goods from the home country; and trade and MP are intimately linked when multinationals' foreign affiliates export part of their output. The calibrated model implies that while the gains from trade are around twice the gains calculated in trade-only models, the gains from MP are a bit lower than those calculated in MP-only models.

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1 Introduction

There is an extensive literature on the gains that countries derive from interacting with each other. The attention has focused on quantifying the gains from single mechanisms in isolation, especially trade in goods (e.g., Eaton and Kortum) and to a lesser extent Foreign Direct Investment (FDI) or multinational production (MP) (e.g., Ramondo, 2008, McGrattan and Prescott, 2009).[1] Much less attention has been given to the quantitative implications of the interaction between trade and MP. In this paper we construct and calibrate a general equilibrium model to evaluate the gains from openness to trade and to MP. Because of the rich interactions between trade and MP in our model, we find higher gains from trade than in existing models with only trade, while our computed gains from MP are slightly lower than those in models with only MP.

We build on the Ricardian model of international trade developed by Eaton and Kortum (2002). Our main innovation is to incorporate MP into the model by allowing a country's technologies to be used for production abroad. The model has tradable intermediate goods and non-tradable consumption goods, as in Alvarez and Lucas (2007). For non-tradable goods, serving a foreign market can only be done through MP, but for tradable goods we have to consider the choice between exports and MP.[2] Trade flows are affected by iceberg-type costs that may vary across country pairs. To avoid these costs, or to benefit from lower costs abroad, firms producing tradable goods may prefer to serve a foreign market through MP rather than exports. We assume that MP entails some efficiency losses that may vary across country pairs. Further, we allow for the possibility that multinationals' foreign affiliates rely, at least partially, on imported inputs from their home country; in our empirical approach, we think of this as "intra-firm" trade.[3] Our set-up also allows firms to use a third country as a "bridge", or export platform, to serve a particular market; we refer to this as bridge MP (or simply BMP). For

[1]Multinational production measures the sales of foreign affiliates of multinational firms. This is arguably at least as important as trade: for example, in 2007 total worldwide MP was almost twice as high as total world exports (UNCTAD's World Investment Report, 2009).

[2]A significant part of MP flows is in non-tradable goods. Around 50% of the value of production by US affiliates of foreign multinationals is in sectors other than manufacturing, agriculture and mining (own calculations from Bureau of Economic Analysis). Additionally, according to the UNCTAD (2009), in 2007, Foreign Direct Investment stocks in the service sector represented more than 60% of the total stock in developed countries.

[3]The empirical evidence points to significant intra-firm trade flows related to multinational activities (Hanson, Mataloni, and Slaughter, 2003; Bernard, Jensen, and Schott, 2005; and Alfaro and Charlton, 2007). According to our own calculations using data from the Bureau of Economic Analysis, "intra-firm" imports from their headquarters represent more than 7.5% of total gross production done by foreign affiliates of American multinationals.

example, a firm from country i producing a tradable good can serve country n by doing MP in country l, and shipping it to country n. This entails MP costs associated with the pair $\{i, l\}$, and also trade costs associated with the pair $\{l, n\}$.[4]

Our model captures several dimensions of the complex interaction between trade and MP. First, as in models of "horizontal" FDI such as Horstmann and Markusen (1992), Brainard (1993), Markusen and Venables (2000), and Helpman, Melitz and Yeaple (2004), trade and MP are competing ways to serve a foreign market.[5] This implies that an increase in trade costs generates smaller welfare losses as MP partially replaces the decline in trade. Second, as in the models of "vertical" FDI, like in Helpman (1984, 1985), and more recently Keller and Yeaple (2009), the reliance by foreign affiliates on imports of home-country inputs implies that MP boosts trade and trade facilitates MP.[6] This complementarity between trade and MP implies that an increase in trade costs leads to larger welfare losses through an indirect negative impact on MP. Finally, complementarity between trade and MP also arises in our model due to the presence of BMP: since BMP flows entail both trade and MP flows, an increase in trade costs decreases MP associated with BMP and generates larger losses.

The existence of these forces of substitutability and complementarity between trade and MP implies that models with only trade and models with only MP may generate biased estimates of the gains from trade and MP.[7] If complementarity forces dominate, for example, the gains from trade calculated in trade-only models will be lower than those in our model, which takes appropriate account of such forces by calculating the gains from trade as the increase in real income as we move from a situation with only MP to the actual equilibrium with both trade and MP. Similarly, the gains from MP calculated in MP-only models may be biased as well.[8] An

[4] Foreign subsidiaries of multinationals often sell a sizable part of their output outside of the host country of production: around 30% of total sales of US affiliates in Europe are made outside the host country of production (Blonigen, 2005).

[5] Studies using firm-level data find evidence of such substitutability between trade and MP when considering narrow product lines (see Belderbos and Sleuwaegen, 1988; Head and Ries, 2001; Barba-Navaretti and Venables, 2004; and Head and Ries, 2004). For example, increased presence of Japanese auto-makers in the U.S. accompanies a decline in automobile exports from Japan (Head and Ries, 2001).

[6] Several studies find that higher FDI leads to an increase in exports of parts and supplies from the home country to foreign affiliates (see Belderbos and Sleuwaegen, 1988; Blonigen, 2001; Head and Ries, 2001; Barba-Navaretti and Venables, 2004; Head, Ries and Spencer, 2004).

[7] Recent attempts to compute the gains from trade in trade-only models are Eaton and Kortum (2002), Alvarez and Lucas (2007), Fieler (2009), and Waugh (2009). See also Arkolakis, Costinot and Rodríguez-Clare (2009).

[8] Recent papers estimating gains from MP in MP-only models are Ramondo (2008), McGrattan and Precott (2009), and Burstein and Monge-Naranjo (2009).

important goal in this paper is to gauge the strength of substitutability and complementarity forces and then to explore their effect on the gains from trade and MP.

We calibrate the model using data on bilateral trade and MP flows for a set of OECD countries, as well as data on intra-firm trade flows for U.S. multinationals and foreign multinationals operating in U.S. Trade data alone, however, cannot identify the parameter that determines the strength of comparative advantage (i.e., the parameter θ in Eaton and Kortum, 2002). Thus, we appeal to the model's implications for the long run real income growth rate to calibrate this parameter.[9] Importantly, growth is driven by the same mechanism that generates the gains from openness in the static model, namely the aggregate economies of scale associated with the fact that a larger population is linked to a higher stock of non-rival ideas. This is why calibrating the comparative advantage parameter so that the model's implied growth rate matches the one we observe in the data is a key part of the quantitative exercise.

The calibrated model entails strong aggregate economies of scale that lead to potentially very large gains from openness. For example, in a world of one hundred symmetric countries, moving from complete isolation to *frictionless* trade and MP would imply an increase in the real wage of 150% in each country. The presence of high costs of trade and MP, however, imply that the gains from openness are much lower than this. For example, our calibrated model implies that the average gains from openness in a set of five small OECD countries (i.e., Finland, Norway, Denmark, Greece and Portugal) are only 12%.[10] Still, these gains from openness are more than three times larger than the gains from trade calculated in trade-only models, which for these five small countries are (on average) 4%.

Our results suggest that while the gains from trade are underestimated in quantitative exercises performed with trade-only models, the gains from MP calculated with MP-only models are overestimated. The gains from trade calculated with our model are roughly twice as large as the gains calculated in trade-only models (7.3% against 4%, on average, for the five small countries mentioned above). This is because trade facilitates MP by allowing multinationals' foreign affiliates to import inputs from their home country. Since MP can be seen as a channel for international technology diffusion (as it allows a country's technologies to be used for

[9] Although the model we present is static, in the Appendix we show that the equilibrium of the static model can be seen as the steady state equilibrium of a dynamic model where productivity evolves according to an exogenous "research" process. This dynamic model exhibits "semi-endogenous" growth as in Jones (1995) and Kortum (1997), and is closely related to Eaton and Kortum (2001).

[10] Each of these countries has roughly 1% of the OECD's total estimated equipped labor force.

production in other countries), our model captures the idea that trade facilitates technology diffusion.[11] In contrast to this result for the gains from trade, the gains from MP calculated in our calibrated model are slightly lower than the gains computed in MP-only models (5.6% vs. 6.3% on average for the five small countries mentioned above). This is because the substitutability forces associated with the fact that trade and MP are competing ways of serving a foreign market dominate the complementarity forces created by BMP.

Our model is in principle consistent with the notion that the reallocation of production to foreign countries by U.S. multinationals could depress domestic wages. This is because outward MP could lead to a decline in U.S. exports, worsening its terms of trade.[12] But there is a countervailing force: outward MP also generates a demand for exports of inputs to foreign subsidiaries.[13] Our calibrated model shows that these two forces roughly balance each other for the U.S. and hence outward MP has basically no net effect on the U.S. real wage.

The models that come closest to the one we present here are Garetto (2009) and Irarrazabal, Moxnes and Opromolla (2009).[14] Garetto develops a model in which multinationals from the rich country produce intermediate goods in low wage locations and then ship those goods back home for final assembly and consumption (there is no BMP). Garetto's model entails an extreme type of complementarity between trade and MP: without trade there would be no MP. Irarrazabal, Moxnes and Opromolla (2009) introduce intra-firm trade into Helpman, Melitz and Yeaple's (2004) "proximity-concentration tradeoff" model of trade and MP to explain the high correlation observed between these two flows across country pairs. The model does not allow for multinationals' foreign affiliates to export their production (there is no BMP). Consistent with our results, Irarrazabal et al. find gains from MP that are smaller that the gains that would be computed in models with only MP. Again, this is because of the forces of substitutability between trade and MP. On the other hand, the absence of BMP implies that the gains from MP computed by Irarrazabal et al. are significantly lower than the ones we calculate using our model.

[11]Yet, our model does not incorporate any causal link whereby trade or MP enhance international knowledge spillovers. The large literature on this topic is surveyed in Keller (2004).

[12]Similar ideas have been presented in relation to the debate about off-shoring by rich countries, see Samuelson (2004), Rodríguez-Clare (2009). See also the empirical work on the effect of outward FDI on employment in the U.S. by Harrison and McMilan (2008) and in Germany by Becker and Muendler (2009).

[13]This mechanism is similar to the one in Irarrazabal, Moxnes and Opromolla (2009).

[14]Another related paper is Rodríguez-Clare (2008), which explores the interactions between trade and diffusion in model that is also based on Eaton and Kortum (2002) and Eaton and Kortum (2006).

2 The Model

We extend Eaton and Kortum's (2002) model of trade to incorporate MP. Our model is Ricardian with a continuum of tradable intermediate goods and non-tradable final goods, produced under constant-returns-to-scale. We adopt the probabilistic representation of technologies as first introduced by Eaton and Kortum (2002), but we enrich it to incorporate MP. We embed the model into a general equilibrium framework similar to the one in Alvarez and Lucas (2007). All proofs are in the Appendix.

2.1 The Closed Economy

To introduce the notation and main features of our model, consider first a closed economy with L units of labor. A representative agent consumes a continuum of final goods indexed by $u \in [0,1]$ in quantities $q_f(u)$. Preferences over final goods are CES with elasticity $\sigma_f > 0$. Final goods are produced with labor and a continuum of intermediate goods indexed by $v \in [0,1]$. Formally, intermediate goods in quantities $q_g(v)$ are aggregated into a *composite intermediate good* via a CES production function with elasticity $\sigma_g > 0$. We denote the total quantity produced of this composite intermediate good as Q. The composite intermediate good and labor are used to produce final goods via Cobb-Douglas technologies with varying productivity levels,

$$q_f(u) = z_f(u) L_f(u)^{\alpha} Q_f(u)^{1-\alpha}. \tag{1}$$

The variables $L_f(u)$ and $Q_f(u)$ denote the quantity of labor and the composite intermediate good used in the production of final good u, respectively, and $z_f(u)$ is a productivity parameter. Similarly, intermediate goods are produced according to

$$q_g(v) = z_g(v) L_g(v)^{\beta} Q_g(v)^{1-\beta}. \tag{2}$$

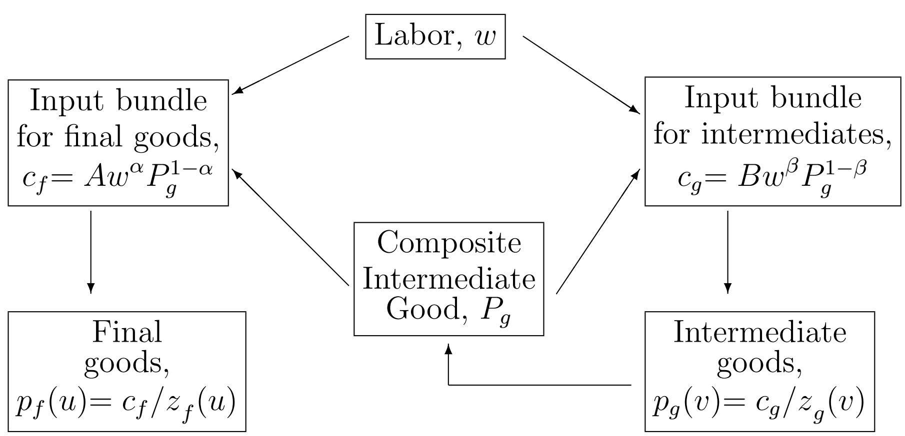
Resource constraints are

$$\begin{aligned} \int_0^1 L_f(u)du + \int_0^1 L_g(v)dv &= L, \\ \int_0^1 Q_f(u)du + \int_0^1 Q_g(v)dv &= Q. \end{aligned}$$

To complete the description of the environment in the closed economy, the productivity parameters $z_f(u)$ and $z_g(v)$ are random variables drawn independently from a Fréchet distribution with parameters T and $\theta > \max\{1, \sigma - 1\}$, $F(z) = \exp\left(-Tz^{-\theta}\right)$, for $z > 0$.

To describe the competitive equilibrium for this economy it is convenient to introduce the notion of an *input bundle for the production of final goods*, and an *input bundle for the production of intermediate goods*, both of which are produced via Cobb-Douglas production functions with labor and the composite intermediate good, and used to produce final and intermediate goods, as specified in (1) and (2), respectively. The unit cost of the input bundle for final goods is $c_f = Aw^{\alpha}P_g^{1-\alpha}$, and the unit cost of the input bundle for intermediate goods is $c_g = Bw^{\beta}P_g^{1-\beta}$, where w and P_g are the wage and the price of the composite intermediate good, respectively, and A and B are constants that depend on α and β, respectively. In a competitive equilibrium prices of final goods are given by $p_f(u) = c_f/z_f(u)$, and prices of intermediate goods are given by $p_g(v) = c_g/z_g(v)$. In turn, the aggregate price for intermediates is $P_g = \left(\int_0^1 p_g(v)^{1-\sigma_g}dv\right)^{1/(1-\sigma_g)}$. Figure 1 illustrates the cost structure in the closed economy.

Figure 1: Cost Structure in the Closed Economy



The characterization of the equilibrium follows closely the analysis in Eaton and Kortum (2002) and Alvarez and Lucas (2007), so we omit the details. Suffice it to say here that the equilibrium real wage is given by

$$\frac{w}{P_f} = \widetilde{\gamma} \cdot T^{\frac{1+\eta}{\theta}}, \tag{3}$$

where $P_f = \left(\int_0^1 p_f(u)^{1-\sigma_f}du\right)^{1/(1-\sigma_f)}$ is the price index for final goods, $\eta \equiv (1-\alpha)/\beta$, and $\widetilde{\gamma}$ is a positive constant.[15]

[15]In Eaton and Kortum (2002) the real wage is proportional to $T^{1/\beta\theta}$ while in Alvarez and Lucas (2007) the real wage is proportional to $T^{\eta/\theta}$. The difference with Eaton and Kortum's result arises because of the presence

2.2 The World Economy

Now consider a set of countries indexed by $i \in \{1, ..., I\}$ with preferences and technologies as described above. Country i has L_i units of labor. Each country i has a technology to produce each final good and each intermediate good, at home or abroad. These technologies are described by the vectors $\mathbf{z}_{fi}(u) \equiv \{z_{f1i}(u), ..., z_{fIi}(u)\}$ and $\mathbf{z}_{gi}(v) \equiv \{z_{g1i}(v), ..., z_{gIi}(v)\}$. When a country i produces in another country $l \neq i$, we say that there is multinational production or MP by country i in country l. Sometimes, we also say that MP in country l is carried out by country i "multinationals". The corresponding productivity parameter in this case is $z_{fli}(u)$, or $z_{gli}(v)$. We adopt the convention that the subscript n denotes the destination country, l the country of production, and i the country where the technology originates. Note that if $z_{fli}(u) = z_{gli}(v) = 0$ whenever $l \neq i$, for all $u, v \in [0, 1]$, our model collapses to the Alvarez and Lucas (2007) version of Eaton and Kortum (2002) model of trade with no MP.

Intermediate goods are tradable but final goods are not. Trade is subject to iceberg-type costs: $d_{nl} \geq 1$ units of any good must be shipped from country l for one unit to arrive in country n. We assume that $d_{nn} = 1$ for all n and the triangle inequality holds: $d_{nl} \leq d_{nj} d_{jl}$ for all n, l, j. Similarly, MP incurs an iceberg-type efficiency loss of $h_{sli} \geq 1$ associated with using an idea from i to produce in l, with $h_{sii} = 1$ for all i, for $s = f, g$. Thus, whereas national production of final good u in country l entails unit cost $c_{fl}/z_{fll}(u)$, MP of final good u by i in l entails unit cost $c_{fl} h_{fli}/z_{fli}(u)$. Similarly, whereas national production of intermediate good v in l has unit cost $c_{gl}/z_{gll}(v)$, MP of intermediate good v by i in l entails unit cost $c_{gli}/z_{gli}(v)$. The unit cost c_{gli} differs from $c_{gl} h_{gli}$ because we assume that MP in intermediate goods requires the use of what we call a *multinational input bundle for the production of intermediate goods*. In particular, we assume that the multinational input bundle combines the national input bundle from the home country (i.e., the country where the technology originates) and the host country (i.e., the country where production takes place). The home country national input bundle must be shipped to the host country of production, and this implies paying the corresponding transportation cost. The unit cost of the home country national input bundle used in MP by country i in country l is then $c_{gi} d_{li}$. The host country national input bundle has unit cost c_{gl}, but MP in intermediates incurs an efficiency loss of $h_{gli} \geq 1$, so the unit cost of the host country

of non-tradable goods whereas the difference with Alvarez and Lucas arises because in our model T also affects the productivity of final goods. The proof of Proposition 1 in the Appendix contains the derivation of this result.

national input used in MP by i in l is $c_{gl}h_{gli}$. Combining the costs of home and host country national input bundles into a CES aggregator, the unit cost of the multinational input bundle for intermediates produced by i in l is

$$c_{gli} = \left[(1-a)\left(c_{gl}h_{gli}\right)^{1-\xi} + a\left(c_{gi}d_{li}\right)^{1-\xi}\right]^{\frac{1}{1-\xi}}, \tag{4}$$

where $a \in [0,1]$ and $\xi > 1$. Note that $c_{gii} = c_{gi}$. Moreover, if $a = 0$, then $c_{gli} = c_{gl}h_{gli}$. The parameter ξ indicates the degree of substitutability between the national input bundles from the home and host countries.

Finally, we assume that the productivity vectors $\mathbf{z}_{fi}(u)$ and $\mathbf{z}_{gi}(v)$ for each good are random variables that are drawn independently across goods and countries from a multivariate Fréchet distribution with parameters $(T_{1i}, T_{2i}, ..., T_{Ii})$, $\theta > \max\{1, \sigma - 1\}$, and $\rho \in [0,1)$,[16]

$$F_i(\mathbf{z}_{si}) = \exp\left[-\left(\sum_l \left(T_{li}\left(z_{sli}\right)^{-\theta}\right)^{\frac{1}{1-\rho}}\right)^{1-\rho}\right]. \tag{5}$$

Note that

$$\lim_{x\to\infty} F_i(x, x, ..., z_{sli}, ..., x) = \exp\left[-T_{li}z_{sli}^{-\theta}\right],$$

so that the marginal distributions are Fréchet. The parameter ρ determines the degree of correlation among the elements of $\mathbf{z}_{si}$: if $\rho = 0$, productivity levels are uncorrelated across production locations, while in the limit as $\rho \to 1$ they are perfectly correlated, so that productivity is independent of the production location (i.e., $z_{sii} = z_{sli}$, for all l).

2.3 Equilibrium Analysis

Since final goods are identical except for their productivity parameters (i.e., they enter preferences symmetrically), we follow Alvarez and Lucas (2007) and drop index u, labeling final goods by $Z_f \equiv (\mathbf{z}_{f1}, ..., \mathbf{z}_{fI})$. Similarly, we label intermediate goods by $Z_g \equiv (\mathbf{z}_{g1}, ..., \mathbf{z}_{gI})$. The unit cost of a final good Z_f in country n produced with a technology from country i is $c_{fn}h_{ni}/z_{fni}$, while the unit cost of an intermediate good Z_g in country n produced in country l with a technology from country i is $c_{gli}d_{nl}/z_{gli}$.

In a competitive equilibrium the price of final good Z_f in country n is simply the minimum unit cost at which this good can be obtained, $p_{fn}(Z_f) = \min_i c_{fn}h_{fni}/z_{fni}$. Similarly, the price

[16] This distribution is discussed in footnote 14, Eaton and Kortum (2002).

of intermediate good Z_g in country n is $p_{gn}(Z_g) = \min_{i,l} c_{gli} d_{nl}/z_{gli}$. Note that if $l = i$, then the intermediate good is exported from i to n while if $i \neq l = n$ there is MP from i to n. Finally, if $i \neq l$ and $l \neq n$ then country l is used as an export platform by country i to serve country n. We say that in this case there is "bridge MP", or simply BMP, by country i in country l.

Recall that in Eaton and Kortum (2002) the allocation of expenditures across exporters is elegantly characterized by simple formulas of the technology parameters, unit costs, and trade costs. Thanks to the assumption that technologies are distributed according to the multivariate Fréchet distribution, this property extends in a natural way in our model to the allocation of expenditures across technology sources and production locations.

Lemma 1 *(a) The shares of expenditure by country n on final and intermediate goods produced with country i technologies are, respectively,*

$$\phi_{fni} = \frac{\Phi_{fni}}{\Phi_{fn}} \text{ and } \phi_{gni} = \frac{\Phi_{gni}}{\Phi_{gn}},$$

where

$$\Phi_{fni} \equiv T_{ni} \left(c_{fn} h_{fni}\right)^{-\theta}, \Phi_{gni} \equiv \left(\sum_l \left(T_{li} \left(c_{gli} d_{nl}\right)^{-\theta} \right)^{\frac{1}{1-\rho}} \right)^{1-\rho}, \text{ and } \Phi_{sn} \equiv \sum_i \Phi_{sni}, \text{ for } s = f, g;$$

(b) Of the total expenditure by country n on intermediate goods produced with country i technologies, the share that is spent on goods produced in country l is

$$\pi_{gni,l} = \left(\frac{T_{li} \left(c_{gli} d_{nl}\right)^{-\theta}}{\Phi_{gni}} \right)^{\frac{1}{1-\rho}}.$$

It is easy to show that the price index in country n for final goods ($s = f$) and intermediates ($s = g$) is given by

$$P_{sn} = \gamma_s \Phi_{sn}^{-1/\theta}, \tag{6}$$

where $\gamma_s \equiv \Gamma(1 + (1 - \sigma_s)/\theta)^{1/1-\sigma}$, and $\Gamma(\cdot)$ is the Gamma function.[17] Since Φ_{fn} and Φ_{gn} are functions of the unit costs, $(c_{f1}, ..., c_{fn})$ and $(c_{g1}, ..., c_{gn})$, which in turn are a function of wages and the price indices $(P_{g1}, ..., P_{gn})$, the set of equations associated with (6), for $s = g$ and $n = \{1, ..., I\}$, implicitly determines P_{gn} as a function of $\mathbf{w} = (w_1, ..., w_I)$. In vector notation, this defines the function $P_g(\mathbf{w})$: $I \to I$ (see Alvarez and Lucas, 2007). Together with $P_g(\mathbf{w})$,

[17]This follows just as in Eaton and Kortum (2002) given that the price distributions in country n of intermediate and final goods are $G_{gn}(p) = 1 - \exp\left(-\Phi_{gn} p^\theta\right)$ and $G_{fn}(p) = 1 - \exp\left(-\Phi_{fn} p^\theta\right)$, respectively.

equation (6) for $s = f$ also defines a function $P_f(\mathbf{w})$ that determines the price index for final goods as a function of wages.

We next use the results of Lemma 1 to characterize trade and MP flows and to close the model with the trade balance conditions.[18] The total expenditure on final goods by country n is equal to the country's total income, $w_n L_n$. We refer to the total value of final goods produced in n with country i technologies as the value of MP in final goods by i in n, denoted by Y_{fni}. Part (b) of Lemma 1 implies that ϕ_{sni} and $\pi_{sni,l}$ not only represent the share of goods purchased by country n produced with different technologies and in different production locations, but also expenditure shares. Thus,

$$Y_{fni} = \phi_{fni} w_n L_n.$$

Note that $\sum_i Y_{fni} = w_n L_n \sum_i \phi_{fni} = w_n L_n$.

Since total expenditure on intermediates by country n is $P_{gn}Q_n$, the value of MP in intermediates by country i in country l to serve country n is $\phi_{gni}\pi_{gni,l}P_{gn}Q_n$. Thus, MP in intermediates by i in l is

$$Y_{gli} = \sum_n \phi_{gni}\pi_{gni,l}P_{gn}Q_n. \tag{7}$$

Total imports by country n from l are given by the sum of intermediate goods produced in country l with technologies from any other country, $\sum_i \phi_{gni}\pi_{gni,l}P_{gn}Q_n$, plus the imports of country l's input bundle for intermediates used by country l's multinationals operating in country n. For concreteness, we refer to the first type of trade as "arms-length" and the second type of trade as "intra-firm". To compute intra-firm trade flows, let ω_{nl} be the cost share of the home country input bundle for the production of intermediates in country n by multinationals from country l. From equation (4), $\omega_{nl} = a\left(c_{gl}d_{nl}/c_{gnl}\right)^{1-\xi}$. The value of imports of the input bundle for intermediates by n from l associated with MP by l in n is $\omega_{nl}Y_{gnl}$. Total imports by country n from $l \neq n$ are then given by the sum of arms-length trade and intra-firm trade,

$$X_{nl} = \sum_i \phi_{gni}\pi_{gni,l}P_{gn}Q_n + \omega_{nl}Y_{gnl}. \tag{8}$$

For country n , aggregate imports are $\sum_{l\neq n} X_{nl}$, while aggregate exports are $\sum_{l\neq n} X_{ln}$. The trade balance condition for country n is then

$$\sum_{l\neq n} X_{nl} = \sum_{l\neq n} X_{ln}. \tag{9}$$

[18]The trade balance conditions are the appropriate equilibrium conditions given that MP entails no profits under perfect competition.

As in Alvarez and Lucas (2007), the total expenditure on the composite intermediate good is proportional to the country's total income. Formally,

Lemma 2 $P_{gn}Q_n = \eta w_n L_n$, *for all* n.

Since the terms ϕ_{gni}, $\pi_{gni,l}$, and ω_{li} are functions of $\mathbf{w}$, the trade balance conditions in (9) constitute a system of I equations in $\mathbf{w}$. This system of equations together with some normalization of wages yields an equilibrium wage vector $\mathbf{w}$. The functions $P_g(\mathbf{w})$ and $P_f(\mathbf{w})$ defined above then determine the price indices for intermediate and final goods in all countries.

2.4 Gravity Equations

We now now show that arms-length trade and MP flows satisfy modified gravity equations. Given equation (8), arms-length exports from l to n are given by

$$\widehat{X}_{nl} \equiv X_{nl} - \omega_{nl} Y_{gnl} = \sum_i \phi_{gni} \pi_{gni,l} P_{gn} Q_n.$$

For the special case with $\rho = 0$ these trade flows satisfy a gravity equation similar to that in Eaton and Kortum (2002) except that the technology parameters determining the location of the productivity distributions in each country are "augmented" by the possibility of MP. Formally, let $T'_{gl} \equiv \sum_i T_{li} h_{gli}^{-\theta}$ be an augmented technology parameter for the production of intermediate goods in country l that takes into account the possibility of using technologies from other countries appropriately discounted by the efficiency losses h_{gli}. Applying Lemma 1 it can be shown that arms-length trade flows from l to n are

$$\widehat{X}_{nl} = \frac{T'_{gl} \left(c_{gl} d_{nl}\right)^{-\theta}}{\sum_k T'_{gk} \left(c_{gk} d_{nk}\right)^{-\theta}} \eta w_n L_n. \tag{10}$$

This implies that country $l's$ normalized import share in country n depends only on the trade cost d_{nl}, and the price indices P_{gn} and P_{gl},

$$\frac{\widehat{X}_{nl}/w_n L_n}{\widehat{X}_{ll}/w_l L_l} = \left(\frac{d_{nl} P_{gl}}{P_{gn}}\right)^{-\theta}. \tag{11}$$

This is exactly like in Eaton and Kortum (2002) -see their equation (12). In the gravity literature, d_{nl} is referred to as the "bilateral resistance" term while P_{gl} and P_{gn} are "multilateral resistance" terms.

Overall trade flows will not satisfy a gravity equation because arms-length and intra-firm trade flows are subject to two different elasticities with respect to trade costs: the Eaton and Kortum elasticity, θ, and the elasticity $\xi - 1$ that indicates the degree of substitution between Home and local input bundles for MP. Gravity relations do not hold even when considering only arms-length trade flows in the general case with $\rho > 0$. The reason is that there are I^2 ways to produce any intermediate good, resulting from the combination of I source countries and I production locations. The productivity parameters $z_{g1i}, z_{g2i}, ..., z_{gIi}$ associated with source country i are positively correlated (since $\rho > 0$) whereas the productivity parameters $z_{gl1}, z_{gl2}, ..., z_{glI}$ associated with production location l are uncorrelated (by assumption of independence across the vectors $\mathbf{z}_{gi}$, for $i = 1, 2, ..., I$). The different degrees of correlation among the elements of the columns and rows of the Z_g matrix makes it generally impossible to express all the determinants of bilateral trade and MP flows in a bilateral resistance term together with multilateral resistance terms, as in equation (11).[19]

In the case with $\rho = 0$ MP flows also satisfy a gravity-like relationship. Using equation (7) and some manipulation, we have

$$Y_{gli} = \frac{T_{li}(\gamma_g c_{gli})^{-\theta}}{P_{gl}^{-\theta}} \Psi_{gl}, \tag{12}$$

where

$$\Psi_{gl} \equiv \sum_n \left(\frac{d_{nl} P_{gl}}{P_{gn}} \right)^{-\theta} \eta w_n L_n$$

can be interpreted as country $l's$ market potential. The term $T_{li}(\gamma_g c_{gli})^{-\theta}/P_{gl}^{-\theta}$ captures the "relative competitiveness" of i technologies in country l.[20] We can then write

$$\frac{Y_{gli}/\Psi_{gl}}{Y_{gii}/\Psi_{gi}} = \left(\frac{\widetilde{h}_{gli} P_{gi}}{P_{gl}} \right)^{-\theta}, \tag{13}$$

where $\widetilde{h}_{gli}^{-\theta} = (T_{li} c_{gli}^{-\theta} / T_{ii} c_{gii}^{-\theta})$ is an average relative cost of producing in country l rather than in country i with country i's technologies. The term $\widetilde{h}_{gli}$ in equation (13) plays the analogous role of the bilateral resistance term $d_{nl}^{-\theta}$ in equation (11).

[19] One exception is when $T_{li} h_{li}^{-\theta}$ is "separable" in the sense that it can be written as the product of a source and a destination-specific terms: $T_{li} h_{gli}^{-\theta} = \kappa_l \mu_i$, for all l, i. In this case we obtain an expression similar to (10) but with T'_{gl} substituted by $T''_{gl} = \kappa_l^{1/(1-\rho)}$, and θ substituted by $\theta/(1-\rho)$. The reason why this works is that the distribution of $(\widetilde{z}_{g1}, \widetilde{z}_{g2}, ..., \widetilde{z}_{gI})$, for $\widetilde{z}_{gl} \equiv \max_i \{z_{gli}/c_{gli}\}$, is also a multivariate Fréchet with parameters θ and ρ.

[20] Rearranging (11), we have $X_{nl} = \frac{X_{ll}}{w_l L_l} \left(\frac{d_{nl} P_{gl}}{P_{gn}} \right)^{-\theta} w_n L_n$. Adding up over n and using trade balance, $\sum_n X_{nl} = \eta w_l L_l$, we see that $\Psi_{gl} = \frac{\eta w_l L_l}{X_{ll}/\eta w_l L_l}$: larger and more open countries have a higher market potential.

2.5 Gains from trade, MP, and openness

In this paper we are particularly interested in quantifying the country-level gains from trade, MP, and openness. We first establish some terminology.

The gains from openness for country n (GO_n) are given by the proportional change in country $n's$ real wage, w_n/P_{fn}, as we move from a counterfactual equilibrium characterized by isolation, which attains when trade and MP costs are infinite ($d_{nl}, h_{sli} \to \infty$ for $n \neq l$, $l \neq i$, and $s = f, g$), to the actual equilibrium.

The gains from trade for country n (GT_n) are given by the proportional change in w_n/P_{fn} as we move from the counterfactual equilibrium with MP but no trade (actual h_{sli} for all l, i and $s = f, g$ but $d_{nl} \to \infty$ for $n \neq l$) to the actual equilibrium.

Similarly, the gains from MP for country n (GMP_n) are given by the proportional change in w_n/P_{fn} as we move from the counterfactual equilibrium with trade but no MP for all countries (actual d_{nl} for all n, l but $h_{sli} \to \infty$ for all l, i and $s = f, g$) to the actual equilibrium. The gains from MP can be decomposed into those that arise from MP in intermediates, GMP_{gn}, and those that arise from MP in final goods, GMP_{fn}, with $GMP_n = GMP_{fn} \times GMP_{gn}$.

We are interested in comparing GT_n and GMP_n with the gains that would be computed in models with only trade and models with only MP. We refer to these gains as GT_n^* and GMP_n^*, respectively. Formally, GT_n^* (GMP_n^*) is the proportional change in country $n's$ real wage as we move from a counterfactual equilibrium characterized by isolation to an equilibrium with no MP (trade) but with the same trade (MP) flows as in the actual equilibrium.[21] The following lemma establishes that GT_n^* and GMP_n^* can be calculated as simple formulas from trade and MP shares, respectively.

Lemma 3 *The gains from trade and the gains from MP in trade-only and MP-only models can be directly calculated from trade and MP shares as follows:*

$$GT_n^* = \left(\frac{X_{nn}}{\eta w_n L_n}\right)^{-\eta/\theta}, \tag{14}$$

$$GMP_{gn}^* = \left(\frac{Y_{gnn}}{\eta w_n L_n}\right)^{-\eta/\theta}, \tag{15}$$

$$GMP_{fn}^* = \left(\frac{Y_{fnn}}{w_n L_n}\right)^{-1/\theta}, \tag{16}$$

[21] Of course, the trade (MP) costs necessary to yield the same trade (MP) flows as in the actual equilibrium may be different in a trade-only (MP-only) model than in our model with trade and MP.

with the total gains from MP given by $GMP_n^* = GMP_{gn}^* \cdot GMP_{fn}^*$.

The formula for the gains from trade as a function of normalized trade flows in equation (14) is very similar to Eaton and Kortum (2002) (see their equation 15) and exactly the same as the one that applies in Alvarez and Lucas (2007). The formulas for gains from MP in intermediates in equation (15) and in final goods in equation (16) are analogous to the expressions for the gains from trade.

One of the main points of this paper is that GT_n can be higher or lower than GT_n^* because of the forces of substitutability and complementarity present in our model. If $GT_n > GT_n^*$ then we say that trade is MP-complement: the gains from trade are higher than the ones that would be computed in trade-only models because trade also leads to gains by facilitating MP. On the contrary, if $GT_n < GT_n^*$ then we say that trade is MP-substitute: the gains from trade are lower than the ones that would be computed in trade-only models because trade decreases the gains from MP. If $GT_n = GT_n^*$ then we say that trade is MP-independent.

Analogously, if $GMP_n < GMP_n^*$ then we say that MP is trade-substitute while if $GMP_n > GMP_n^*$ ($GMP_n = GMP_n^*$) then we say that MP is trade-complement (trade-independent).

2.6 Three special cases

Before presenting the calibration of the full model in the next section, it is instructive to consider three special cases for which we can derive analytical results: (1) the case with $a = \rho = 0$, which implies that trade is MP-independent, (2) the case of symmetric countries, and (3) the case of a rich and a poor country with $a = 0$ and frictionless trade.

2.6.1 $a = \rho = 0$

The following proposition establishes that if $a = \rho = 0$ then trade is MP-independent. In this case, models with only trade can be safely used to compute gains from trade and this can be done using a simple formula that expresses the gains from trade as a function of trade shares.

Proposition 1 *Assume* $a = \rho = 0$. *Then trade is MP-independent in the sense that* $GT_n = GT_n^*$. *Moreover,* $GO_n = GT_n^* \times GMP_n^*$.

To understand this result, recall that our model captures two opposite forces affecting the relationship between trade and MP. First, trade tends to be MP-complement because of the need to import home-country intermediate goods by multinationals' foreign subsidiaries. Second, trade tends to be MP-substitute because trade and MP are alternative ways to serve a particular market. The first force is not present if $a = 0$ because in this case foreign subsidiaries do not demand home-country intermediate goods. The second force is not present if $\rho = 0$ because with no correlation across productivities in different locations, there is a sense in which there is no longer *a technology* that can be used in different countries. This Proposition implies that if $a = \rho = 0$ then it would be valid to use the trade-only model to compute gains from trade. Moreover, as the last part of the Proposition establishes, one can jointly use the trade-only and MP-only models to compute the overall gains from openness since $GO_n = GT_n^* \times GMP_n^*$.

In contrast to the result that trade is MP-independent, parameters $a = \rho = 0$ do not imply that MP is trade-independent. The following lemma establishes the relationship between GMP_n and GMP_n^* for this case:

Lemma 4 *Let* $\widetilde{X_{nn}/\eta w_n L_n}$ *be the domestic demand share in the counterfactual equilibrium with trade but no MP. Then*

$$GMP_n = GMP_n^* \left(\frac{X_{nn}/\eta w_n L_n}{\widetilde{X_{nn}/\eta w_n L_n}} \right)^{-\eta/\theta}.$$

Two simple examples help to illustrate this result. In both examples there are two countries labeled North (N) and South (S), with $T_{NN} = T_{SN} = T_N$ and $T_{NS} = T_{SS} = T_S$. The first example has $T_N > 0$ but $T_S = 0$. The equilibrium in this case entails MP by North in South but no MP by South in North. Since South has no technologies of its own, there would be no trade in the counterfactual equilibrium with no MP, hence $\widetilde{X_{NN}/\eta w_N L_N} = 1$. But $X_{NN}/\eta w_N L_N < 1$ in the actual equilibrium. This implies that $GMP_N > GMP_N^*$, so MP is trade-complement for North. This example captures the gains from BMP for North, which can satisfy domestic demand at a lower cost by using its superior technologies to produce in South. In the second example we have frictionless trade and both regions are identical except that South is small: $T_N/L_N = T_S/L_S$ and $L_S < L_N$. It is easy to show that in this case the domestic demand share for South increases as we move from the counterfactual equilibrium with no MP to the actual equilibrium with MP ($X_{SS}/\eta w_S L_S > \widetilde{X_{SS}/\eta w_S L_S}$). This implies that $GMP_S < GMP_S^*$, so MP is trade-substitute for South: as South becomes more productive thanks to MP, it effectively becomes larger and the gains from trade decline.

2.6.2 Symmetry

The symmetric case can be solved analytically, yet the basic intuition regarding the role of the various parameters carries to the general case with asymmetric countries. We derive intuitive formulas for the gains from trade, MP and openness, and then explore the conditions under which trade (MP) behaves as substitute or complement for MP (trade). We are also interested in differentiating between the complementarity that arises from the possibility of doing BMP and the one that arises from the use of the home country's input bundle in multinational activities.

Symmetry entails $L_i = L$ for all i, $T_{li} = T$, for all l, i, and $d_{nl} = d$ and $h_{fnl} = h_{gnl} = h$ for all $l \neq n$. In equilibrium, wages, costs, and prices are equalized across countries, $w_n = w$, $c_n = c$, and $P_{sn} = P_s$, for $s = g, f$ and all n. Thus, the cost of the multinational input bundle collapses to $c_{gli} = m \cdot c_g$, for all $l \neq i$, with $m \equiv \left[(1-a)h^{1-\xi} + ad^{1-\xi}\right]^{\frac{1}{1-\xi}}$, and $c_{gll} = c_g$ for all l. The share of spending on the home input bundle done by MP is simply $\omega = a(d/m)^{1-\xi}$.

The equilibrium is characterized as follows (see the Appendix for formal derivations). In the case of final goods the situation is straightforward: a country uses some of its own technologies to serve domestic consumers through local production, and also to serve foreign consumers through MP. For intermediate goods, there is trade, MP, and BMP: countries use some of their own technologies to produce at home to serve domestic and foreign consumers (through exports), and they use some of their technologies for MP whose output is sold to local consumers (MP), sent back home or sold to third markets (BMP).[22] There is also trade associated with the import of the home country input bundle for MP.

The following proposition shows how access to foreign ideas through trade and MP increases a country's real wage.

Proposition 2 *Under symmetry we have*

$$GO = \left[1 + (I-1)h^{-\theta}\right]^{1/\theta} \cdot \left[\Delta_0 + (I-1)\Delta_1\right]^{\eta/\theta}, \tag{17}$$

$$GT = \frac{GO}{\lim_{d\to\infty} GO} = \left[\frac{\Delta_0 + (I-1)\Delta_1}{1 + (I-1)\widetilde{m}^{-\theta}}\right]^{\eta/\theta}, \tag{18}$$

$$GMP = \frac{GO}{\lim_{h\to\infty} GO} = \left[1 + (I-1)h^{-\theta}\right]^{1/\theta} \times \left[\frac{\Delta_0 + (I-1)\Delta_1}{1 + (I-1)d^{-\theta}}\right]^{\eta/\theta}, \tag{19}$$

[22] The assumption that technologies are draws from a multivariate Fréchet distribution with $\rho \in [0, 1)$ implies that there is some BMP even with symmetric countries; BMP vanishes only when $\rho \to 1$.

where

$$\Delta_0 \equiv \left(1+(I-1)(md)^{-\frac{\theta}{1-\rho}}\right)^{1-\rho}, \tag{20}$$

$$\Delta_1 \equiv \left(d^{-\frac{\theta}{1-\rho}}+m^{-\frac{\theta}{1-\rho}}+(I-2)(md)^{-\frac{\theta}{1-\rho}}\right)^{1-\rho}, \tag{21}$$

$$\widetilde{m} \equiv \lim_{d\to\infty} m = (1-a)^{\frac{1}{1-\xi}}\, h. \tag{22}$$

The expression for the gains from openness in equation (17) indicates that a country that opens up to both trade and MP in the intermediate goods' sector benefits from using its own technologies at home and abroad, captured by the term Δ_0, and $I-1$ foreign technologies, captured by the term Δ_1. When domestic technologies are used (the term Δ_0), production can be carried out in $I-1$ foreign locations through MP at the cost m, and then goods shipped back home at the cost d. Hence, technologies are "fully" discounted by $(md)^{-\theta/(1-\rho)}$. In turn, foreign technologies can be accessed by importing goods in which case they are discounted by $d^{-\theta/(1-\rho)}$ (first term in Δ_1), by doing MP in which case they are discounted by $m^{-\theta/(1-\rho)}$ (second term in Δ_1), and by doing BMP in $I-2$ different locations in which case the full discount $(md)^{-\theta/(1-\rho)}$ applies (third term in Δ_1). The term in the first bracket in equation (17) captures the gains from accessing $(I-1)$ foreign technologies through MP in the final goods' sector, at a discount of $h^{-\theta}$.

It is clear that the gains from openness decrease with h as well as d: the higher trade or MP costs, the lower the gains from openness. Additionally, the parameter ρ appears in GO in association with intermediate goods: as ρ indicates the correlation between technology draws for a given source country across different production locations, it matters only when both trade and MP are allowed. As one would expect, GO decreases with ρ. In the case where $\rho \to 1$ (so that $z_{sli} = z_{sji}$ for all l, j and $s = g, f$), BMP in intermediate goods vanishes. Furthermore, in this case, trade and MP do not overlap: if $d > h$, there is only MP (and trade associated with MP, i.e., imports of the home country input bundle), but no other trade of individual intermediate goods; in contrast, if $h > d$, there is only trade but no MP (see the Appendix).

The expression for GT in equation (18) indicates that a country that opens up to trade benefits through specialization according to Ricardian comparative advantage (which here takes into account trade flows associated with BMP) and from the fact that trade facilitates MP by allowing multinational affiliates to import inputs from their home country. The following proposition describes parameter configurations under which trade is MP-complement or MP-substitute.

Proposition 3 *Assume countries are symmetric. (a) If $\rho = 0$ and $a > 0$, trade is MP-complement, and if $\rho > 0$ and $a = 0$, trade is MP-substitute. (b) Assume $a, \rho > 0$. If $\xi \rightarrow 1$, trade is MP-complement, while if $h < d$ and $\xi \rightarrow \infty$ then trade is MP-substitute.*

To gain some intuition for these results, start from the case with $a = \rho = 0$, for which we know from Proposition 1 that trade is MP-independent. As ρ increases above zero, the positive correlation between productivity draws across locations generates substitutability. Alternatively, as a increases above zero, the demand for home-country inputs by multinationals introduces complementarity. If both $\rho > 0$ and $a > 0$ then we need to consider the parameter ξ. If ξ is close to 1, the low elasticity of substitution between home and host country inputs for MP generates no gains from MP if trade is not possible. Hence, trade is MP-complement. Conversely, if ξ is high then only the cheapest input bundle is used for MP; if $h < d$ then trade does not contribute to decrease MP costs. This implies that trade is MP-substitute.

Turning to the gains from MP, the first term of the RHS in (19) captures the gains associated with final goods, whereas the second term captures the gains associated with intermediate goods. For intermediates, the gains from MP are affected by the substitutability between trade and MP that arises for $\rho > 0$.

Proposition 4 *Assume countries are symmetric. If $\rho = 0$, MP is trade-independent while if $\rho > 0$ MP is trade-substitute.*

We emphasize two implications of this proposition. First, the value of a does not affect whether MP is trade-independent or trade-substitute. This is because while trade facilitates MP by reducing the unit cost of the multinational input bundle ($m < \widetilde{m}$ if $a > 0$), MP does not facilitate trade; MP only adds a competing alternative to trade in serving other markets. Second, the result that MP is trade-independent for $\rho > 0$ is consistent with Lemma 4: under symmetry we have $\widetilde{X}_{nn}/\eta w_n L_n = X_{nn}/\eta w_n L_n$. This is because in this case MP affects all countries equally and therefore has no effect on trade shares.

2.6.3 Two countries with $a = 0$ and frictionless trade

This special case shows that the rich country can experience losses from MP. We consider two countries labeled North (N) and South (S), with $T_{NN} = T_{SN} = T_N$, $T_{NS} = T_{SS} = T_S$, and $T_N/L_N > T_S/L_S$. This last feature implies that wages will tend to be higher in North than in

South. We assume that MP generates no demand for home-country intermediate goods ($a = 0$) and that trade is frictionless ($d_{NS} = d_{SN} = 1$). Our main result for this case is established in the following proposition:

Proposition 5 *There exists $\rho^* \in [0, 1)$ such that North gains from frictionless MP in intermediate goods ($GMP_{gN} > 1$) for $\rho \in [0, \rho^*)$ while it loses for $\rho \in (\rho^*, 1)$ ($GMP_{gN} < 1$).*

The reason why MP can have a negative impact on the rich country is that outward MP effectively reduces the demand for a country's exports, worsening its terms of trade.[23] But this relies on there being strong substitutability between trade and MP as alternative ways of serving foreign markets, hence the need for a high correlation parameter ρ for this to be a dominant effect. Note that here we have assumed $a = 0$ – in general, with $a > 0$, outward MP would generate an increased demand for home-country inputs, and this would make it less likely for the rich country to lose from MP (see Irarrazabal et. al., 2009, and also Becker and Muendler, 2009).[24]

3 Calibration

3.1 Data Description

We restrict our analysis to the set of nineteen OECD countries considered by Eaton and Kortum (2002): Australia, Austria, Belgium, Canada, Denmark, Spain, Finland, France, United Kingdom, Germany, Greece, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Sweden, United States. Except when mentioned otherwise, all the data is averaged over the period 1990-2002. We use STAN data on manufacturing trade flows from country i to country n as the empirical counterpart for trade in intermediates in the model, X_{ni}. We use UNCTAD data on the gross value of production for multinational affiliates from i in n as the empirical counterpart of bilateral MP flows in the model, $Y_{ni} \equiv Y_{fni} + Y_{gni}$. We normalize bilateral trade flows by the importer's total expenditure on intermediate goods, and bilateral MP flows by total expenditure on final goods in the host country.

[23] A similar negative terms of trade effect has been noted in regards to off-shoring by rich countries (see Samuelson, 2004, and Rodríguez-Clare, 2009).

[24] It is also important to note that in our model outward MP generates no profits, since there is perfect competition. Such profits would lead to additional gains from MP for rich countries as in Burstein and Monge-Naranjo (2009).

Apart from the bilateral trade and MP data, our quantitative model should also be consistent with moments regarding the importance of intra-firm trade by multinationals and the importance of MP in final goods relative to all MP. The necessary data to compute these moments are available from the Bureau of Economic Analysis (BEA) for the U.S., but not systematically for other countries. We also use data from the BEA to compute a measure of BMP for foreign affiliates of U.S. multinationals and for U.S. affiliates of foreign multinationals. The Appendix presents more detail on the data and some summary statistics.

3.2 Calibration Procedure

Trade and MP costs. We reduce the number of parameters to calibrate by assuming that bilateral trade and MP costs in the intermediate goods' sector are a function of distance and whether countries share a border and language, respectively,

$$d_{ni} = 1 + (\delta_0^d + \delta_{dist}^d dist_{ni}) \times (\delta_{border}^d)^{b_{ni}} \times (\delta_{language}^d)^{l_{ni}}, \tag{23}$$

$$h_{gni} = 1 + (\delta_0^h + \delta_{dist}^h dist_{ni}) \times (\delta_{border}^h)^{b_{ni}} \times (\delta_{language}^h)^{l_{ni}}, \tag{24}$$

for all $n \neq i$, with $d_{nn} = 1$ and $h_{gnn} = 1$. The variable $dist_{ni}$ is the distance between i and n. The variable b_{ni} (l_{ni}) equals one if countries share a border (a language), and zero otherwise.[25] Hence, if $\delta_{border} < 1$ countries that share a border have lower iceberg costs. Similarly, if $\delta_{language} < 1$, countries speaking the same language have lower iceberg costs. From this cost specification, we need to calibrate a set of eight parameters,

$$\Upsilon = \{\delta_0^d, \delta_0^h, \delta_{dist}^d, \delta_{dist}^h, \delta_{border}^d, \delta_{border}^h, \delta_{language}^d, \delta_{language}^h\}.$$

We further assume that MP costs in the final good sector are proportional to the ones in the intermediate good sector,

$$h_{fni} = \max\left[1, \mu \cdot h_{gni}\right]. \tag{25}$$

Parameters α, β, and ξ. We set the labor share in the intermediate goods' sector, β, to 0.5, and the labor share in the final sector, α, to 0.75, as calibrated by Alvarez and Lucas (2007).[26] This implies $\eta \equiv (1-\alpha)/\beta = 0.5$.

[25] Data on bilateral distance, common border, and common language is from the Centre d'Etudes Prospectives et Informations internationales (CEPII).

[26] They calibrate the parameters α and β to match the fraction of U.S. employment in the non-tradable sector and the share of value added in gross manufacturing output, respectively. Jones (2009) also uses $\beta = 0.5$.

For the parameter ξ which captures the degree of complementarity between home and host country inputs for MP, we appeal to estimates from the labor literature. Becker and Muendler (2009) estimate cross-wage elasticities of labor demand for German multinationals across multiple production locations. Their results suggest a value of approximately $\xi = 1.5$.[27,28]

Fréchet Parameters. The parameters that characterize the multivariate Fréchet distribution are θ, ρ, and the matrix $\{T_{li}\}$. Since we cannot recover the parameter θ from a gravity regression due to the presence of MP, we appeal to the model's implications for the growth rate of real income. As we show in the Appendix, the static model presented above is fully consistent with a dynamic model where the productivity matrices Z_g and Z_f evolve according to an exogenous "research" process whereby the arrival of ideas is proportional to the workforce. This dynamic model exhibits "semi-endogenous" growth as in Jones (1995) and Kortum (1997), and is closely related to Eaton and Kortum (2001). Importantly, growth is driven by the same forces that generate the gains from openness in the static model, namely the aggregate economies of scale associated with the fact that a larger population is linked to a higher stock of non-rival ideas. It then seems natural to calibrate θ to match the growth rate of real income per worker observed in the data.

Growth rates in the steady state are the same for all countries, and not affected by openness. This implies that the growth rate for the open economy is the same as the one for the closed economy. From equation (3), the growth rate of the real wage in the closed economy is

$$g = \left[\frac{1+\eta}{\theta}\right] g_T, \tag{26}$$

where g_T is the growth rate of the parameter T. In our multi-country model, there is no single parameter T but rather a matrix $\{T_{li}\}$. But, since all these T_{li}'s grow at a common rate g_T, the growth rate of the real wage for the open economy is the same as the one in equation (26) for the closed economy. Since we assume that the arrival of ideas is proportional to the workforce, in the long run g_T is equal to the growth rate of labor, which we assume common across countries. However, as argued in Jones (2002), the last decades have been characterized by an increase in

[27] Becker and Muendler estimate that the effect of a 1% increase in German wages on the demand for labor by multinationals in other countries of Western Europe is 1.2. Since the average share of these multinationals' wage bill allocated to German workers is 62%, the implied elasticity of substitution is 1.94. They also estimate that the elasticity of German multinationals labor demand in Germany to wages in Western Europe is 0.2.. Given that the average share of these multinationals' wage bill allocated to Western European workers is 15%, the implied elasticity of substitution is 1.3. The average of these two elasticities is close to 1.5.

[28] This is also close to the elasticity of substitution between skilled and unskilled workers, which Katz and Murphy (1992) estimate at 1.4.

the share of employment in R&D. Since the faster increase in R&D employment would surely affect the growth rate of T, it is important to take this into account in the calibration. Following Jones (2002), we use the growth rate of R&D employment over the last decades in the five top R&D-performing countries as our measure of the growth rate of T.[29] For the period 1950-1993, Jones (2002) calculates this growth rate to be 4.8%, so we set $g_T = 0.048$.

Although g in our model stands for the growth rate of real wages or real GDP per capita, in the calibration we need to consider the role of physical and human capital accumulation. Thus, we use the growth rate of TFP as a calibration target for g. Based on Jones (2002), we set $g = 0.01$. Using (26) and $\eta = 0.5$, $g_T = 0.048$ and $g = 0.01$ imply $\theta = 7.2$.[30] This value is remarkably close to the central values of θ estimated by Eaton and Kortum (2002), Alvarez and Lucas (2007), and Simonovska and Waugh (2009) following different methodologies and sets of countries (the respective values are 8.3, 6.7, and 7.5).

The dynamic model derived in the Appendix implies that $T_{li} = \lambda_i$, where λ_i represents the stock of ideas in country i and is proportional to L_i. We allow λ_i/L_i to differ across countries and assume that it varies directly with the share of R&D employment observed in the data (an average over the nineties taken from the World Development Indicators). Thus, for example, since the share of R&D employment in the U.S. is 0.9% and in Greece it is 0.3%, we assume that λ/L is three times higher in the U.S. than in Greece.

Finally, regarding the parameter ρ, which indicates the degree of correlation of technology draws across production locations, we choose to fix it to a central value of $\rho = 0.5$ and also explore the alternative with $\rho = 0$ (no correlation). We chose not to include this parameter in our calibration procedure below as its effect on trade and MP shares is rather indirect. All our results, particularly the results on gains in the next section, are robust to different values for ρ.[31]

Country sizes. We choose to calibrate the vector $\mathbf{L} \equiv \{L_1, ..., L_{19}\}$ rather than using data on total employment because our model abstracts from physical and human capital as well as differences in production efficiency not captured in λ_i/L_i. Thus, we set $\mathbf{L}$ so that the implied real GDP in the model, $w_n L_n / P_{fn}$, matches the one we observe in the data (real GDP -PPP

[29] These countries are France, West Germany, Japan, the United States and the United Kingdom.

[30] An alternative approach to calibrate θ is to target the elasticity of TFP to population across countries while controlling for the effects of trade, institutions and geography, as in Alcala and Ciccone (2004). Depending on the specification, these authors find an elasticity ranging from 1/6 to 1/4.5, a range which encompasses an elasticity of 1/4.8 as implied by $\theta = 7.2$.

[31] We also experienced with $\rho = 0.9$ obtaining similar results (not shown).

adjusted- from the Penn World Tables 6.2, average over the nineties).

Table 1 summarizes the values and definitions for parameters calibrated independently of the algorithm presented next.

Parameter	Value	Source	Definition
α	0.5	Alvarez and Lucas (2007)	labor share in final sector
β	0.75	Alvarez and Lucas (2007)	labor share in intermediate sector
ξ	1.5	Becker and Muendler (2009)	elasticity of substitution in eq. (4)
λ_i/L_i	in Table 10	share of R&D employment	per capita stock of ideas in country i
θ	7.2	eq. (26) with $g = 0.01$, $g_T = 0.048$	variability parameter in M.V. Fréchet
ρ	$\{0, 0.5\}$	–	correlation parameter in M.V. Fréchet

Table 1: Parameter Values.

Calibration Algorithm. For a given value for ρ, the values for θ, ξ, β, α, a set of parameters $[\,\boldsymbol{\Upsilon}, \mathbf{L}, a, \mu]$, and the data matrices for bilateral distance, common border and common language, we compute the equilibrium and generate a simulated data set with 361 observations (one for each country-pair, including the domestic pairs), for the following variables: trade shares, MP shares and real GDP levels. Additionally, the model generates bilateral "intra-firm" trade to MP ratios, $\omega_{li}Y_{gli}/\left(Y_{gli}+Y_{fli}\right)$, MP in the intermediate goods sector as a share of total MP, $Y_{gli}/\left(Y_{gli}+Y_{fli}\right)$, and BMP as a share of total MP, $\sum_{n\neq l}\phi_{gni}\pi_{gni,l}X_{gn}/(Y_{gli}+Y_{fli})$. We compute averages of these three variables across country pairs where the United States are either the home or host country (i.e., $i = US$ or $l = US$).

Let R_H^2 be a measure of the explanatory power of the model for bilateral trade shares and MP shares, respectively, given by

$$R_H^2 = 1 - \frac{\sum_{n,i;n\neq i}\left[H_{ni}^{data} - H_{ni}^{model}\right]^2}{\sum_{n,i;n\neq i}(H_{ni}^{data})^2}, \tag{27}$$

where H stands for either trade shares, $X_{ni}/(\eta w_n L_n)$, or MP shares, $Y_{ni}/(w_n L_n)$. The algorithm used to compute the model's equilibrium extends the one in Alvarez and Lucas (2007). Given the vector $\boldsymbol{\Upsilon}$, the parameters a and μ and the vector $\mathbf{L}$ are chosen so that the model matches the the moments computed above for the importance of intra-firm trade and MP in intermediate goods as well as real GDP levels in the data. The parameters in $\boldsymbol{\Upsilon}$ are then chosen to minimize the loss function $(1 - R_{\widetilde{X}}^2) + (1 - R_{\widetilde{Y}}^2)$.

3.3 Results

Table 2 reports the calibrated parameters while Table 3 summarizes statistics from the data and calibrated model. We report the vector **L** in the Appendix.

	$\rho = \mathbf{0.5}$		$\rho = \mathbf{0}$	
Cost Parameters:	trade	MP	trade	MP
Distance δ_{dist}	0.059	0.063	0.065	0.067
Common Border δ_{border}	0.75	0.90	0.74	0.97
Common Language $\delta_{language}$	0.68	0.63	0.73	0.64
Constant δ_0	0.50	0.64	0.63	0.95
Average Costs	1.81	1.97	1.97	2.30
Standard Deviation of Costs	0.22	0.22	(0.23)	(0.22)
[min, max]	[1.27, 2.65]	[1.37, 2.87]	[1.34, 2.89]	[1.60, 3.26]
"Intra-firm" trade parameter a	0.15		0.14	
MP cost parameter for final sector μ	1.25		1.07	

Table 2: Calibrated Parameters.

For both calibrations, the effect of distance on trade and MP costs is similar: a 10% increase in distance between a country-pair increases costs by almost 2% for both trade and MP.[32] Both calibrations suggest that a common border decreases trade costs by more than MP costs ($\delta^d_{border} < \delta^h_{border}$), while the opposite is true for country-pairs with a common language ($\delta^d_{language} > \delta^h_{language}$). These calibrated "gravity" parameters translate into MP costs in the tradable sector that are almost 10% (20%) higher on average than trade costs, for $\rho = 0.5$ ($\rho = 0$). Moreover, the high correlation between bilateral trade and MP shares in the data of 0.71 is reflected in a very high correlation between trade and MP costs of 0.99 (0.98 when $\rho = 0$).

The next two tables illustrate how well the model matches the patterns in the data along several dimensions. Table 3 reports statistics from the data and the model's equilibrium at the calibrated parameters. For bilateral trade and MP shares, we report mean, standard deviation and correlation coefficient. We also show the average BMP share implied by the model for U.S. affiliates abroad and U.S. affiliates of foreign multinationals.

While the average bilateral trade and MP shares generated by the model are similar to the ones in the data, the correlation between the two flows is slightly higher in the model.

[32] This is computed for country-pairs that do not share a border or a language and that are separated by the average distance across all country pairs.

	Data	Model	
		$\rho = 0.5$	$\rho = 0$
Bilateral trade share			
average	0.019	0.018	0.018
standard deviation	0.035	0.03	0.03
Bilateral MP share			
average	0.022	0.018	0.017
standard deviation	0.045	0.031	0.030
Correlation Coefficient bilateral trade and MP shares	0.70	0.82	0.85
Average "BMP" by US in l (as share of MP by US in l)	0.30	0.03	0.13
Average "BMP" by i in US (as share of MP by i in US)	0.05	0.003	0.025

Table 3: Summary Statistics. Data and Calibrated Model.

Regarding BMP, our model is reassuringly consistent with the data in the sense that the share of BMP for U.S. affiliates abroad is much higher than the share of BMP for U.S. affiliates of foreign multinationals. This is what we would expect since the U.S. is the largest country in our sample and has the (second) highest research intensity (see table 10 in the Appendix), discouraging the use of the U.S. as an export platform. For both $\rho = 0$ and $\rho = 0.5$, however, our model fails to generate as much BMP as observed in the data.

Table 4 shows the measure of the model's explanatory power in equation (27), for bilateral trade and MP separately. Additionally, it presents correlations between magnitudes in the model and data for bilateral trade and MP shares across country-pairs, as well as correlations for aggregate exports, imports, outward MP and inward MP, as shares of GDP of the source and receiving country, respectively.

Both R^2's and correlation coefficients for bilateral trade and MP shares are high, indicating that the model captures fairly well the observed bilateral patterns of these two flows. When we express total exports and total imports as shares of GDP the correlations between model and data are still high. Correlations are lower but still fairly positive when we compute total outward and inward MP as shares of GDP. The model performs poorly in capturing the level of outward and inward MP shares for the largest countries in the sample (i.e., Germany, Japan, and the United States). Table 11 and scatters in the Appendix shows the actual and simulated data for these four variables for each country against country size.[33]

[33]One could, of course, allow for more degrees of freedom in the calibration of trade and MP costs. For

	$\rho = \mathbf{0.5}$	$\rho = \mathbf{0}$
Model's $\mathbf{R^2}$		
bilateral trade shares	0.76	0.75
bilateral MP shares	0.65	0.64
Correlations between model and data		
bilateral trade shares	0.83	0.83
bilateral MP shares	0.76	0.75
total exports shares	0.80	0.77
total imports shares	0.81	0.82
total outward MP shares	0.43	0.43
total inward MP shares	0.32	0.33

Bilateral MP = gross value of production for affiliates from country i in l; Total Outward MP = total gross value of production for foreign affiliates from country i; Total Inward MP = total gross value of production for foreign affiliates in country l.

Table 4: Model's Goodness of Fit.

Gravity estimates of θ. We can explore the bias of estimates of the parameter θ from gravity equations using our simulated data with $\rho = 0.5$. We showed in section 2.4 that, in general, the parameter θ is not the elasticity of trade flows to trade costs and cannot be recovered from running a gravity equation for trade. This is due to the existence of intra-firm trade ($a > 0$) and the fact that ρ might be different from zero. We use our simulated data to estimate θ from the gravity equation for normalized trade flows in Eaton and Kortum (2002), namely

$$\frac{X_{ni}/X_{nn}}{X_{ii}/X_{ii}} = \left(\frac{d_{ni}P_{gi}}{P_{gn}}\right)^{-\theta}.$$

Using non-linear least square, $\widetilde{\theta} = 8.21$ (s.e. 0.02), very close to the Eaton and Kortum's estimate of 8.28 but significantly different from the true value of θ of 7.2. The upward bias in $\widetilde{\theta}$ arises because $\rho > 0$ and implies that MP is trade-substitute. This leads to an additional channel (beside the standard one associated with θ in trade-only models) through which higher trade costs decrease trade flows, i.e. MP replaces trade. As expected, when we use the calibrated data with $\rho = 0$, $\widetilde{\theta} = 7.01$ (s.e. 0.01), much closer to the true value of θ.

example, one could have country specific effects determining inward MP costs. Smaller inward MP costs for the large countries, for example, would allow the model to better match the observed inward MP flows as shares of GDP. We have refrained from pursuing this to keep the calibration as simple as possible.

4 Gains from Openness, Trade, and Multinational Production

4.1 Gains in Trade-Only and MP-Only Models

We first compute the gains from trade and MP in trade-only and MP-only models, denoted by GT_n^* and GMP_n^*, respectively. As shown in Lemma 3, these gains can be computed directly using the data on bilateral trade and MP shares, and the calibrated values for $\theta = 7.2$ and $\eta = 0.5$.[34] As shown in Proposition 1, if we impose $a = \rho = 0$ then the gains from trade in our model are equal to the gains from trade computed in a trade-only model (i.e., $GT_n = GT_n^*$). Moreover, the gains from openness in our model would then be $GO_n^* \equiv GT_n^* \cdot GMP_n^*$. Table 5 shows these gains calculated directly from the data.

The gains from openness tend to be more than twice as large as the gains from trade. For Canada, Germany and the United States, for example, GO_n^* is at least three times larger than GT_n^*. The gains from MP are higher than the gains from trade for almost all countries because MP flows tend to be larger than trade flows (in the data). When we restrict our attention to the intermediate goods sector, the gains from MP are generally smaller than the gains from trade.

4.2 Gains in the calibrated model

The calculations under independence shown above miss the potential gains coming from the interactions between trade and MP. We explore the effect of such interactions in this section by computing the different gains in the calibrated version of our model. Table 6 shows the calculated gains from openness, trade, and MP, averaged across the nineteen OECD countries in our sample, for the calibrated version of the model with $\rho = 0.5$ and $\rho = 0$.

The variables GT_n^* and GMP_n^* are calculated as indicated in Lemma 3, but using trade and MP shares implied by the calibrated model. Using the simulated as opposed to the actual data delivers almost the same average results because the model matches fairly accurately the average bilateral trade and MP shares. But, at the country level, using the simulated or the actual data does make a difference because our calibrated model does not perfectly match the

[34] For GT_n^*, we calculate $X_{nn}/(\eta w_n Y_n)$ from the bilateral trade data we described above as $1 - \sum_{i \neq n} X_{ni}/(\eta w_n Y_n)$. We use analogous procedure for MP. (See table 12 in the appendix) But, calculating GMP_{fn}^* from the data requires the amount of MP done in final goods. We assume that the share of MP in the intermediate good sector in each country is 0.48 as the one observed for the US.

	GO_n^*	GT_n^*	GMP_n^*	GMP_{gn}^*	GMP_{fn}^*	$L_n/\sum_k L_k$
New Zealand	1.06	1.02	1.04	1.02	1.02	0.6
Finland	1.07	1.03	1.04	1.02	1.02	0.8
Norway	1.03	1.01	1.03	1.01	1.01	0.9
Denmark	1.03	1.01	1.02	1.01	1.01	1.0
Portugal	1.14	1.04	1.09	1.05	1.04	1.1
Greece	1.04	1.03	1.01	1.004	1.005	1.1
Austria	1.10	1.06	1.04	1.02	1.02	1.3
Sweden	1.06	1.01	1.05	1.03	1.03	1.4
Belgium	1.23	1.14	1.08	1.04	1.04	1.5
Netherlands	1.18	1.09	1.09	1.05	1.04	2.3
Australia	1.06	1.02	1.05	1.02	1.02	2.9
Spain	1.07	1.03	1.04	1.02	1.02	4.1
Canada	1.12	1.04	1.08	1.04	1.04	4.2
Great Britain	1.10	1.04	1.06	1.03	1.03	6.8
France	1.06	1.03	1.03	1.01	1.02	7.0
Italy	1.04	1.02	1.02	1.01	1.01	7.2
Germany	1.07	1.02	1.04	1.02	1.02	9.4
Japan	1.03	1.02	1.01	1.004	1.004	15
United States	1.04	1.01	1.03	1.01	1.01	32
Average	1.08	1.035	1.045	1.022	1.021	5.3

Countries are sorted by (simulated) size.

Table 5: Gains from trade and MP according to trade-only and MP-only models.

data.[35] Meaningful comparisons between GT_n with GT_n^*, and between GMP_n with GMP_n^*, must then be performed using the simulated data from the calibrated model. We show results averaged across countries for $\rho = 0$ and $\rho = 0.5$ in Tables 6 and results by country for $\rho = 0.5$ in Table 7 (results by country for $\rho = 0$ are in Table 13 in the Appendix).

	GO_n	GT_n	GT_n^*	GMP_n	GMP_n^*
Average			$\rho = 0.5$		
All sectors	1.079	1.050	1.030	1.035	1.04
Intermediate good sector	1.056	1.050	1.030	1.013	1.019
Final good sector	1.021	-	-	1.021	1.021
			$\rho = 0$		
All sectors	1.066	1.041	1.029	1.038	1.039
Intermediate good sector	1.043	1.041	1.029	1.016	1.018
Final good sector	1.021	-	-	1.021	1.021

Table 6: Gains from Openness, Trade, and MP. Calibrated Model.

The implied average gains from openness are around $7 - 8\%$. These gains are more than twice as large as the average gains from openness coming from a trade-only model, $GT_n^* = 3\%$, or an MP-only model, $GMP_n^* = 4\%$. On average, more than two thirds of the gains from openness are from trade and MP in the intermediate goods sector ($GO_{gn} = 4 - 6\%$), while the remaining one third of the total gains are from MP in the final goods sector ($GO_{fn} = 2\%$).

The calibrated model implies that trade is MP-complement since on average $GT_n > GT_n^*$. Adding trade enhances MP by facilitating "intra-firm" trade and reducing the unit costs of MP: the average unit cost for the multinational input bundle decreases by 50% with respect to the scenario with only MP but not trade. For the group of the 6 smallest countries in our sample (i.e., New Zealand, Finland, Norway, Denmark, Portugal and Greece), the calibrated model with $\rho = 0.5$ yields average gains from trade of 7% while the average GT_n^* is 4%.

Turning to MP, the calibrated model implies that, on average, MP is trade-substitute since $GMP_n = 3.5\% < 4\% = GMP_n^*$. As suggested by the analytical results under symmetry, the complementarity forces associated with BMP are not strong enough to overcome the substitutability arising from the fact that MP adds a competing alternative to trade in serving foreign markets. Still, the substitutability here is quite weak, and disappears as we consider lower values of ρ —for $\rho = 0$ MP is practically trade-independent.

[35]Recall that we calibrated only 8 parameters determining the trade and MP costs as a function of bilateral geographic and language variables.

These results imply that while trade-only models tend to underestimate the gains from trade by a significant amount, MP-only models tend to overestimate the gains from MP by a small amount. Another interesting result is that the gains from trade are much larger than the gains from MP in the intermediate goods sector. Thus, once we have trade, adding the possibility of doing MP in intermediates does not generate large gains, but when we have only MP, allowing for trade generates significant extra gains.[36]

Table 7 shows gains by country for the model calibrated with $\rho = 0.5$. Not surprisingly, the gains from openness are larger for smaller countries: the correlation coefficient between L_n and GO_n is -0.65. For all countries, trade behaves as MP-complement, $GT_n > GT_n^*$, while MP behaves as a mild trade-substitute, $GMP_n < GMP_n^*$. For a small country like Denmark, which represents around 1% of OECD(19)'s size (measured by simulated L), the gains from openness of around 12% more than double the gains calculated using trade-only models, $GT_n^* = 4.6\%$, or MP-only models, $GMP_n^* = 5.6\%$. The gains from trade for Denmark are much higher than those calculated with a trade-only model, $GT_n = 7.6\% > 4.6\% = GT_n^*$, while the gains from MP are slightly lower than those calculated with a MP-only model, $GMP_n \approx 5\% < 5.6\% = GMP_n^*$.

It is noteworthy that Japan and the United States, the two largest countries in our sample, gain nothing from MP. In fact, if we restrict our attention to the gains from MP in the intermediate good sector, then both Japan and the U.S. actually lose from MP (i.e., $GMP_{gn} < 1$). Intuitively, by doing outward MP these countries reallocate production from home to foreign countries, in effect sharing their superior technologies with the rest of the world and worsening their terms of trade (see Proposition 5). In principle, as explained in Section 2.6.3, there are three forces that could counteract this negative effect: first, gains from inward MP, second, gains from BMP, and third, increased demand for home production of inputs by foreign affiliates of multinational firms. In the calibrated model these forces are not strong enough and hence the net effect of MP can be negative. It is important to caution, however, that the calibrated model fails to generate the high inward MP flows observed in the data for the largest countries in our sample (i.e., Japan and the U.S.). Hence, our measures of the gains from MP for these countries are significantly underestimated.[37]

[36]When we calibrate the model with trade and MP fixing $\rho = 0.9$ (not shown), we still get that trade is MP-complement, and MP is a mild trade-substitute.

[37]This can be easily seen by comparing GMP_n^* calculated with the observed data (in table 5) and simulated data (in table 7): while GMP_{US}^* calculated from the data is 3%, the model's calibration delivers 0.1%. See Table 11 in the Appendix.

	GO_n	GT_n	GT_n^*	GMP_n	GMP_n^*	GMP_{gn}	$L_n/\sum_k L_k$
New Zealand	1.10	1.06	1.02	1.07	1.08	1.037	0.6
Finland	1.09	1.06	1.05	1.03	1.03	1.007	0.8
Norway	1.11	1.07	1.04	1.04	1.05	1.014	0.9
Denmark	1.12	1.08	1.05	1.05	1.06	1.016	1.0
Portugal	1.15	1.08	1.03	1.08	1.09	1.033	1.1
Greece	1.13	1.07	1.03	1.08	1.09	1.033	1.1
Austria	1.15	1.09	1.05	1.07	1.08	1.022	1.3
Sweden	1.08	1.05	1.04	1.03	1.03	1.009	1.4
Belgium	1.16	1.10	1.07	1.06	1.07	1.016	1.5
Netherlands	1.08	1.06	1.04	1.03	1.04	1.012	2.3
Australia	1.02	1.01	1.01	1.01	1.01	1.005	2.9
Spain	1.05	1.04	1.02	1.02	1.03	1.010	4.1
Canada	1.10	1.07	1.04	1.05	1.06	1.021	4.2
Great Britain	1.05	1.03	1.02	1.02	1.02	1.009	6.8
France	1.04	1.03	1.02	1.01	1.01	1.002	7.0
Italy	1.04	1.03	1.02	1.02	1.02	1.008	7.2
Germany	1.03	1.02	1.02	1.005	1.008	1.001	9.4
Japan	1.003	1.002	1.002	1.000	1.001	1.000	14.5
United States	1.005	1.005	1.005	1.000	1.001	0.999	32.0
Average	1.079	1.050	1.030	1.035	1.041	1.01	5.26

Countries are sorted by (simulated) size.

Table 7: Gains from Openness, Trade, and MP, by country. Calibration with $\rho = 0.5$.

Finally, it is interesting to explore how changes in trade and MP costs affect a country's real income level. For this analysis we focus on New Zealand, a small and relatively isolated country with average bilateral trade and MP costs that are higher than the average for our sample: average inward trade (MP) costs are 2.32 (2.49) versus an average of 1.77 (1.91) in our sample. We performed a simple experiment to quantify the effect on New Zealand if its bilateral inward and outward trade and MP costs became equal to those of Canada or Belgium, two "centrally" located countries. We compute the percentage change in the real wage for New Zealand of moving from the equilibrium in the calibrated model to one of three counterfactual scenarios: (1) a situation with the trade costs equal to those of Canada or Belgium; (2) a situation with the MP costs equal to those of Canada or Belgium; and (3) a situation with both the trade and MP costs equal to those of Canada or Belgium.

New Zealand's iceberg-type costs as in:	Canada	Belgium
	% change in real wage	
Trade	16%	14%
MP	38%	22%
Trade and MP	54%	35%

We use the calibrated version of the model with $\rho = 0.5$. Change in real wage for New Zealand of moving from the calibrated level of trade and MP costs to: (1) a situation where trade costs are as the ones calibrated for Canada (Belgium); (2) a situation where MP costs are as the ones calibrated for Canada (Belgium); (3) a situation where both trade and MP costs are as the ones calibrated for Canada (Belgium).

Table 8: Gains from Openness, Trade, and MP: the case of New Zealand.

The potential gains for New Zealand of having its bilateral trade and MP costs decline to the levels prevailing in Canada or Belgium are large. Table 8 shows that if trade costs were changed to the level of Canada, New Zealand's real wage would increase by 16%, while doing the same for MP costs would increase its real wage by 38%. The gains of simultaneously changing trade and MP costs to Canadian levels would increase the real wage in New Zealand by 54%. These gains for New Zealand would mainly come from having cheaper access to U.S. technologies through MP in both tradable and non-tradable goods. Table 8 shows that New Zealand also would experience significant gains if its trade and MP costs declined to the levels prevailing in Belgium, but not as much as if they declined to the levels prevailing in Canada. Overall, the gains computed in this experiment are quite large compared to the gains from trade and MP for New Zealand in 7. This result is consistent with Eaton and Kortum's (2002) finding that the gains from trade relative to autarky are small relative to the gains of removing existing trade costs towards a frictionless world.

5 Final Remarks

It seems reasonable to think that countries, specially small ones, benefit greatly from their interaction with the rest of the world. Whereas much attention has been devoted to trade as the main channel for such benefits, we argue in this paper for the need to broaden the scope of the investigation to other channels. We have taken a step in this direction by developing and calibrating a multi-country general-equilibrium Ricardian model of trade and MP. An important consideration in building this model has been to allow for forces that make trade and MP substitutes as well as forces that make them complements, as the empirical evidence suggests. The calibration reveals that the gains from openness are much higher than the gains from trade, and also higher than the gains from MP. On net, trade behaves as complement for MP, while MP behaves as a mild substitute for trade. As a result, the gains from trade calculated as the increase in real income from a situation with only MP to the (calibrated) situation with trade and MP are more than twice as high as those calculated in models with only trade, as in Eaton and Kortum (2002) and Alvarez and Lucas (2007). This is because our model captures the indirect gains from trade associated with its role in facilitating MP. Meanwhile, the gains from MP calculated with our model are just mildly lower than the ones calculated in MP-only models.

We have focused on trade and MP as the only channels through which countries gain from openness, but clearly they are not the only channels through which these gains are generated. In particular, the direct diffusion of ideas across countries could be as important as trade and MP. For example, countries may benefit from Japan's superior technology in producing automobiles by importing cars from Japan, by having Japanese firms produce cars domestically through MP, or by the diffusion of Japanese technologies to domestic firms. Evaluating the role of this type of diffusion in generating gains from openness is an important issue that we leave for future research.

A final remark concerns the assumption that research efforts in our framework are exogenous. How would the results change if this assumption were relaxed? In the simplest version of the model with only trade, as Eaton and Kortum (2001), trade does not affect countries' research intensity (i.e., the share of the labor force devoted to research). The reason is that although trade expands the market for ideas, it also increases competition, and these two effects exactly balance out. Thus, the gains from trade would not be affected by having endogenous research efforts. But, this result can change when we allow for MP because now countries with a comparative advantage in innovation can specialize in research and use their superior technologies for MP abroad. A comparative advantage in innovation would be reflected in higher ratios of λ_n to L_n. Such countries would naturally have a trade deficit that would be paid for by the repatriation of profits made abroad through MP. This is related to the explanations for the imbalance in the U.S. external accounts in Hausmann and Sturtzeneger (2006) and McGrattan and Prescott (2009). Again, this is an important topic that we leave for future research.

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A Proofs

Proof of Lemma 1. Consider first the case of intermediate goods and let $p_{gni} \equiv \min_l c_{gli} d_{nl}/z_{gli}$. The probability that p_{gni} is lower than p is

$$G_{gni}(p) \;=\; 1 - \Pr\left(z_{gli} \leq c_{gli} d_{nl}/p \text{ for all } l\right),$$

that under the assumption that z_{gli} are draws from the multivariate Frèchet in equation (5) is given by

$$\begin{aligned} G_{gni}(p) &= 1 - \exp\left[-\left(\sum_l \left(T_{li}\left(c_{gli} d_{nl}/p\right)^{-\theta}\right)^{\frac{1}{1-\rho}}\right)^{1-\rho}\right] \\ &= 1 - \exp\left[-\Phi_{gni} p^{\theta}\right]. \end{aligned}$$

Since $G_{gni}(p)$ is independent across i, then the reasoning in Eaton and Kortum (2002) can be immediately applied to show that country n will buy goods produced with country i's technologies for a measure of goods equal to $\phi_{gni} = \Phi_{gni}/\sum_j \Phi_{gnj}$. The corresponding result for final goods is derived simply by letting $d_{nl} \to \infty$ for $n \neq l$.

Of the goods purchased by country n that are produced with country i technologies, what is the share of these goods that are produced in country l? This is equal to the probability that, for a specific good, country l is the cheapest location for i to produce for n with its technology. This is equivalent to $c_{gli} d_{nl}/z_{gli} \leq c_{gji} d_{nj}/z_{gji}$, or $z_{gji} \leq z_{gli}(c_{gji} d_{nj})/(c_{gli} d_{nl})$ for all $j \neq l$. Without loss of generality, assume that $l = 1$. The probability that $z_{gji} \leq a_{ni,j} z_{g1i}$ for all $j \neq 1$ where $a_{ni,j} \equiv (c_{gji} d_{nj})/(c_{g1i} d_{n1})$ is given by $\int_0^\infty F_1(z, a_{ni,2} z, ..., a_{ni,I} z) dz$. But

$$F_1(z, a_{ni,2} z, ..., a_{ni,I} z) = \left(\left(c_{g1i} d_{n1}\right)^{\frac{1}{\theta}} \Phi_{ni}\right)^{1-\frac{1}{1-\rho}} T_{1i}^{\frac{1}{1-\rho}} \theta z^{-\theta-1} \exp\left[-\left(c_{g1i} d_{n1}\right)^{\frac{1}{\theta}} \Phi_{ni} z^{-\theta}\right],$$

and

$$\int_0^\infty \theta c_{g1i} d_{n1} \Phi_{ni} z^{-\theta-1} \exp\left[-c_{g1i} d_{n1} \Phi_{ni} z^{-\theta}\right] dz = 1.$$

This implies that

$$\int_0^\infty F_1(z, a_{ni,2} z, ..., a_{ni,I} z) dz = \frac{\left(T_{1i}\left(c_{g1i} d_{n1}\right)^{-\theta}\right)^{\frac{1}{1-\rho}}}{\sum_j \left(T_{ji}\left(c_{ji} d_{nj}\right)^{-\theta}\right)^{\frac{1}{1-\rho}}},$$

and hence, of the goods that country n buys that are produced with country i technologies, the share that are produced in country l is $\pi_{ni,l}$.

The previous results relate to shares of goods while we are interested in expenditure shares. Just as in Eaton and Kortum (2002), however, the price distribution of the goods that country n buys is independent of the production location and is also independent of the origin of the technology with which the good is produced. This implies that all the adjustment is on the "extensive margin" and that the share of goods that country n buys from country l that are produced with country i technologies is also the share of the total expenditure by country n that is allocated to those goods. To see this, focus on intermediate goods and condition on market n and technologies from country i. The probability that $p_{gni} \leq p$ and that l is the least cost production location to reach n is the probability that $d_{nl}c_{gli}/z_{gli} \leq p$ and $d_{nj}c_{gji}/z_{gji} \geq d_{nl}c_{gli}/z_{gli}$ for all j, or $z_{gli} \geq d_{nl}c_{gli}/p$ and $z_{gji} \leq z_{gli}(d_{nj}c_{gji})/(d_{nl}c_{gli})$ for all j. Without loss of generality, assume that $l = 1$ and again let $a_{ni,j} \equiv (d_{nj}c_{gji})/(d_{n1}c_{g1i})$. We want to compute $\int_{d_{n1}c_{g1i}/p}^{\infty} F_1(z, a_{ni,2}z, ..., a_{ni,I}z)dz$. But simple math establishes that

$$\int_{d_{n1}c_{g1i}/p}^{\infty} F_1(z, a_{ni,2}z, ..., a_{ni,I}z)dz = \left[(d_{n1}c_{g1i})^{\theta}\,\Phi_{gni}\right]^{-\frac{1}{1-\rho}} T_1^{\frac{1}{1-\rho}} \left[1 - \exp\left(-\Phi_{gni}p^{\theta}\right)\right].$$

The distribution of prices in market n conditional on the good on the good produced in 1 with technology i, we need to divide by $\pi_{ni,1}\phi_{gni}$. This yields a probability equal to

$$G_{gni}(p) = 1 - \exp\left(-\Phi_{gni}p^{\theta}\right).$$

Since this does not depend on 1, it implies that for market n and conditioning on country i technologies, the distribution of p that actually are produced in l is the same for $l = 1, 2, ..., I$. But independence across i allows us to apply the results from Eaton and Kortum (2002) to establish that the distribution of prices for goods that n actually buys from i is $G_{gn}(p) = 1 - \exp\left(-\Phi_{gn}p^{\theta}\right)$ for all i. This implies that the average price of goods is the same irrespective of where they are produced and irrespective of the origin of the technology. The proof for final goods follows immediately from independence across i's.□

Proof of Lemma 2. First note that P_nQ_n is the total cost of the intermediate goods *used in production* in country n. We first calculate the total cost of the intermediate goods *produced* in country n. This is $w_nL_{gn} + P_{gn}Q_{gn}$, plus the intra-firm imports of foreign multinationals located in n, $\sum_{i\neq n}\omega_{ni}Y_{gni}$, minus the exports of the domestic input bundle for intermediates to country $n's$ subsidiaries abroad, $\sum_{i\neq n}\omega_{in}Y_{gin}$. Hence, the total cost of intermediate goods produced in country n is

$$w_nL_{gn} + P_{gn}Q_{gn} + \sum_{i\neq n}\omega_{ni}Y_{gni} - \sum_{i\neq n}\omega_{in}Y_{gin}.$$

In equilibrium, this must be equal to the value of intermediate goods produced in country n. Hence,

$$w_n L_{gn} + P_{gn} Q_{gn} + \sum_{i \neq n} \omega_{ni} Y_{gni} - \sum_{i \neq n} \omega_{in} Y_{gin} = \sum_i Y_{gni}. \tag{28}$$

But,

$$\begin{aligned} \sum_i Y_{gni} &= \sum_i \sum_j \phi_{gji} \pi_{gji,n} P_{gj} Q_j \\ &= \sum_i \phi_{gni} \pi_{gni,n} P_{gn} Q_n + \sum_{j \neq n} \sum_i \phi_{gji} \pi_{gji,n} P_{gj} Q_j \\ &= \sum_i \phi_{gni} \pi_{gni,n} P_{gn} Q_n + \sum_{j \neq n} \left(X_{jn} - \omega_{jn} Y_{gjn} \right). \end{aligned}$$

Substituting into equation (28) and simplifying we get

$$w_n L_{gn} + P_{gn} Q_{gn} + \sum_{i \neq n} \omega_{ni} Y_{gni} = \sum_i \phi_{gni} \pi_{gni,n} P_{gn} Q_n + \sum_{i \neq n} X_{in}.$$

Using the trade balance condition in equation (9) to substitute $\sum_{i \neq n} X_{in}$ for $\sum_{i \neq n} X_{ni}$, using equation (8), and simplifying, yields

$$w_n L_{gn} + P_{gn} Q_{gn} = \left(\sum_i \sum_j \phi_{gnj} \pi_{gnj,i} \right) P_{gn} Q_n.$$

But, $\sum_i \sum_j \phi_{gnj} \pi_{gnj,i} = \sum_j \phi_{gnj} \sum_i \pi_{gnj,i} = 1$, hence,

$$w_n L_{gn} + P_{gn} Q_{gn} = P_{gn} Q_n. \tag{29}$$

We know that

$$\frac{L_{fn}}{Q_{fn}} = \left(\frac{\alpha}{1-\alpha} \right) \frac{P_{gn}}{w_n}, \tag{30}$$

and

$$\frac{L_{gn}}{Q_{gn}} = \left(\frac{\beta}{1-\beta} \right) \frac{P_{gn}}{w_n}. \tag{31}$$

Plugging equation (31) into (29), we get

$$\left(\frac{\beta}{1-\beta} \right) P_{gn} Q_{gn} + P_{gn} Q_{gn} = P_{gn} Q_n,$$

from which it is straightforward that $Q_{gn} = (1-\beta) Q_n$, and combined it with $Q_{fn} + Q_{gn} = Q_n$, we have

$$Q_{fn} = \beta Q_n. \tag{32}$$

Plugging $Q_{gn} = (1-\beta) Q_n$ back into equation (29), we get

$$w_n L_{gn} = \beta P_{gn} Q_n,$$

and using $L_{gn} + L_{fn} = L_n$, we have

$$w_n (L_n - L_{fn}) = \beta P_{gn} Q_n. \tag{33}$$

From equations (30) and (32), we get

$$w_n L_{fn} = \left(\frac{\alpha}{1-\alpha}\right) \beta P_{gn} Q_n.$$

Using equation (33), we then have $L_{fn} = \left(\frac{\alpha}{1-\alpha}\right)(L_n - L_{fn})$, and hence $L_{fn} = \alpha L_n$. Plugging into equation (33), we finally get $(1-\alpha) w_n L_n = \beta P_{gn} Q_n$, or $P_{gn} Q_n = \eta w_n L_n$.□

Proof of Lemma 3. A trade-only model is obtained from our model with $h_{fli}, h_{gli} \to \infty$ for all $l \neq i$. In this case it is easy to show from Lemma 1 that trade flows satisfy the Eaton and Kortum gravity equation,

$$X_{nl} = \frac{T_{ll} (c_{gl} d_{nl})^{-\theta}}{\sum_k T_{kk} (c_{gk} d_{nk})^{-\theta}} \eta w_n L_n.$$

But Lemma 1 implies that $\sum_k T_{kk} (c_{gk} d_{nk})^{-\theta} = \gamma_g^{\theta} P_{gn}^{-\theta}$ and hence, using

$$c_{gn} = B w_n^{\beta} P_{gn}^{1-\beta}, \tag{34}$$

we have

$$\frac{w_n}{P_{gn}} = (\gamma_g B)^{-1/\beta} T_{nn}^{1/\beta\theta} \left(\frac{X_{nn}}{\eta w_n L_n}\right)^{-1/\beta\theta}. \tag{35}$$

Lemma 1 implies that $P_{fn} = \gamma_f T_{nn}^{-1/\theta} c_{fn}$, and together with

$$c_{fn} = A w_n^{\alpha} P_{gn}^{1-\alpha} \tag{36}$$

implies that $w_n / P_{fn} = (\gamma_f A)^{-1} T_{nn}^{1/\theta} (w_n / P_{gn})^{1-\alpha}$. Using equation (35), we finally get

$$w_n / P_{fn} = T_{nn}^{(1+\eta)/\theta} \left(\frac{X_{nn}}{\eta w_n L_n}\right)^{-\eta/\theta},$$

where $\widetilde{\gamma} \equiv (\gamma_f A)^{-1} (\gamma_g B)^{-\eta}$. This establishes the result for the real wage in the closed economy in equation (3) and it also shows that the gains from trade in a trade-only economy are given by equation (14).

A similar procedure leads to the formula for the gains from MP in an MP-only model, which obtains from our model in the limit as $a = 0$ and $d_{nl} \to \infty$ for all $n \neq l$. In particular, Lemma 1 implies that $Y_{gli} = T_{li}(\gamma_g c_{gl} h_{gli}/P_{gl})^{-\theta} \eta w_l L_l$. Together with equation (34) and $\sum_k T_{ki} \left(c_{gk} h_{ki}\right)^{-\theta} = \gamma_g^{\theta} P_{gl}^{-\theta}$ (from Lemma 1), we get

$$\frac{w_n}{P_{gn}} = (\gamma_g B)^{-1/\beta} T_{nn}^{1/\beta\theta} \left(\frac{Y_{gnn}}{\eta w_n L_n}\right)^{-1/\beta\theta}. \tag{37}$$

But Lemma 1 also implies that $P_{fn} = \gamma_f \left(T'_{fn}\right)^{-1/\theta} c_{fn}$. Together with equations (36) and (35), we get

$$\frac{w_n}{P_{fn}} = \widetilde{\gamma} \left(T'_{fn}\right)^{1/\theta} T_{nn}^{\eta/\theta} \left(\frac{Y_{fnn}}{w_n L_n}\right)^{-1/\theta} \left(\frac{Y_{gnn}}{\eta w_n L_n}\right)^{-\eta/\theta}. \tag{38}$$

Finally, again from Lemma 1, we know that

$$Y_{fni} = \frac{T_{ni} \left(c_{fn} h_{fni}\right)^{-\theta}}{\Phi_{fn}} w_n L_n,$$

and hence

$$\frac{Y_{fnn}}{w_n L_n} = \frac{T_{nn} c_{fn}^{-\theta}}{\Phi_{fn}} = \frac{T_{nn} c_{fn}^{-\theta}}{\sum_i T_{ni} \left(c_{fn} h_{fni}\right)^{-\theta}} = \frac{T_{nn}}{T'_{fn}}. \tag{39}$$

Plugging equation (39) into (38), we finally get

$$\frac{w_n}{P_{fn}} = \widetilde{\gamma} T_{nn}^{(1+\eta)/\theta} \left(\frac{Y_{fnn}}{w_n L_n}\right)^{-1/\theta} \left(\frac{Y_{gnn}}{\eta w_n L_n}\right)^{-\eta/\theta}.$$

This immediately establishes the rest of the results of Lemma 3.□

Proof of Proposition 1. From equation (7) and the results of Lemma 1, we have

$$Y_{gli} = \gamma_g^{-\theta} \frac{T_{li} c_{gli}^{-\theta}}{P_{gl}^{-\theta}} \cdot \sum_n \left(\frac{d_{nl} P_{gl}}{P_{gn}}\right)^{-\theta} \eta w_n L_n.$$

Using $\Psi_{gl} \equiv \sum_n \left(\frac{d_{nl} P_{gl}}{P_{gn}}\right)^{-\theta} \eta w_n L_n$,

$$Y_{gll} = \gamma_g^{-\theta} \left(c_{gl}/P_{gl}\right)^{-\theta} T_{ll} \Psi_{gl}. \tag{40}$$

Using $c_{gl} = B w_l^{\beta} P_{gl}^{1-\beta}$, this implies

$$w_l/P_{gl} = \left(\gamma_g^{-\theta} B^{-\theta} T_{ll}\right)^{1/\beta\theta} \left(Y_{gll}/\Psi_{gl}\right)^{-1/\beta\theta}. \tag{41}$$

Letting $T'_{fn} \equiv \sum_i T_{ni} h_{fni}^{-\theta}$, from equation (6) and $c_{fn} = A w_n^{\alpha} P_{gn}^{1-\alpha}$, we have $P_{fn} = \gamma_f \left(T'_{fn}\right)^{-1/\theta} A w_n^{\alpha} P_{gn}^{1-\alpha}$. This implies that $w_n/P_{fn} = (\gamma_f A)^{-1} T_{fn}'^{1/\theta} \left(w_n/P_{gn}\right)^{1-\alpha}$. Using equation (41), we then get

$$w_n/P_{fn} = (\gamma_f A)^{-1} T_{fn}'^{1/\theta} \left(\gamma_g^{-\theta} B^{-\theta} T_{nn}\right)^{\eta/\theta} \left(Y_{gnn}/\Psi_{gn}\right)^{-\eta/\theta}. \tag{42}$$

But from equation (11) we can write $\Psi_{gl} = [(\eta w_l L_l)/X_{ll}] \sum_n X_{nl}$. Under trade balance, this implies

$$\Psi_{gn} = \frac{\eta w_n L_n}{X_{nn}} \eta w_n L_n. \tag{43}$$

Plugging into equation (42), the real wage is then

$$w_l/P_{fl} = \widetilde{\gamma} T_{ll}^{\eta/\theta} T_{fl}'^{1/\theta} \left(\frac{Y_{gll}}{\eta w_l L_l} \frac{X_{ll}}{\eta w_l L_l}\right)^{-\eta/\theta}. \tag{44}$$

Using equations (39) and (44), we finally have

$$\frac{w_l}{P_{fl}} = (\gamma_f A)^{-1} (\gamma_g B)^{-\eta} T_{ll}^{(1+\eta)/\theta} \left(\frac{Y_{fll}}{w_l L_l}\right)^{-1/\theta} \left(\frac{Y_{gll}}{\eta w_l L_l} \frac{X_{ll}}{\eta w_l L_l}\right)^{-\eta/\theta}. \tag{45}$$

We now show that

$$\frac{Y_{gll}}{\eta w_l L_l} = \frac{T_{ll}}{T'_{gl}}. \tag{46}$$

From equation (40) we have

$$\frac{Y_{gll}}{\eta w_l L_l} = \frac{\gamma_g^{-\theta} \left(c_{gl}/P_{gl}\right)^{-\theta} T_{ll} \Psi_{gl}}{\eta w_l L_l}.$$

Using equation (6) for $s = g$ and $a = \rho = 0$, equation (43), and $T'_{gk} = \sum_i T_{ki} h_{gki}^{-\theta}$, this implies

$$\frac{Y_{gll}}{\eta w_l L_l} = \frac{T_{ll}}{T'_{gl}} \frac{T'_{gl} c_{gl}^{-\theta}}{\sum_k T'_{gk} (c_{gk} d_{lk})^{-\theta}} \frac{\eta w_l L_l}{X_{ll}} = \frac{T_{ll}}{T'_{gl}}.$$

The results in equations (39) and (46) implies that

$$\frac{w_l}{P_{fl}} = (\gamma_f A)^{-1} (\gamma_g B)^{-\eta} T_{ll}^{(1+\eta)/\theta} \left(\frac{T_{ll}}{T'_{fl}}\right)^{-1/\theta} \left(\frac{T_{ll}}{T'_{gl}} \frac{X_{ll}}{\eta w_l L_l}\right)^{-\eta/\theta}. \tag{47}$$

This result immediately implies that

$$\begin{aligned} GO_n &\equiv \frac{w_n/P_{fn}}{\lim_{h_{gli},h_{fli},d_{nl}\to\infty} w_n/P_{fn}} \\ &= \left(\frac{Y_{fnn}}{w_n L_n}\right)^{-1/\theta} \cdot \left(\frac{Y_{gnn}}{\eta w_n L_n}\right)^{-\eta/\theta} \cdot \left(\frac{X_{nn}}{\eta w_n L_n}\right)^{-\eta/\theta} \\ &= GMP^*_{fn} \cdot GMP^*_{gn} \cdot GT^*_n. \end{aligned}$$

(Note that the limit $h_{gli}, h_{fli}, d_{nl} \to \infty$ is taken for all $l \neq i$ and $n \neq l$ -the same applies for all limits below).

We can also use the result on real wages to compute the gains from trade:

$$GT_n \equiv \frac{w_n/P_{fn}}{\lim_{d_{nl}\to\infty} w_n/P_{fn}} = \left(\frac{X_{nn}}{\eta w_n L_n}\right)^{-\eta/\theta} = GT_n^*.$$

□

Proof of Lemma 4. First, it is easy to show that $GMP_{fn} = GMP_{fn}^*$ for $\rho = a = 0$. Given that neither trade flows nor MP flows in intermediate goods depend on MP in final goods we immediately get, using the results from Proposition 1, that

$$GMP_{fn} \equiv \frac{w_n/P_{fn}}{\lim_{h_{fli}\to\infty} w_n/P_{fn}} = \left(\frac{T_{nn}}{T'_{fn}}\right)^{-1/\theta} = \left(\frac{Y_{fnn}}{w_n L_n}\right)^{-1/\theta} = GMP_{fn}^*.$$

The gains from MP in intermediates GMP_{gn} is determined by $GMP_{gn}^* = \left(\frac{Y_{gnn}}{\eta w_n L_n}\right)^{-\eta/\theta}$ together with the way in which $X_{nn}/\eta w_n L_n$ changes as we take $h_{gli} \to \infty$. Let $\widetilde{X_{nn}/\eta w_n L_n}$ be the domestic demand share for the counterfactual equilibrium with $h_{gli} \to \infty$. Then

$$GMP_{gn} \equiv \frac{w_n/P_{fn}}{\lim_{h_{gli}\to\infty} w_n/P_{fn}} = \left(\frac{Y_{gnn}}{\eta w_n L_n}\right)^{-\eta/\theta} \left(\frac{\widetilde{X_{nn}/\eta w_n L_n}}{X_{nn}/\eta w_n L_n}\right)^{-\eta/\theta}.$$

Since the second term on the RHS is in general not equal to one, this implies that $GMP_{gn} \neq GMP_{gn}^*$, and hence $GMP_n \neq GMP_n^*$.[38] □

Characterization of Symmetric Equilibrium

Under symmetry, we can derive explicit expressions for trade and MP shares as well as for the real wage. Using the results of Lemma 1, MP in final goods from any other country as a share of a country's total income is given by

$$\widetilde{Y}_f \equiv \frac{Y_{fni}}{w_n L_n} = \frac{h^{-\theta}}{1+(I-1)h^{-\theta}},$$

for $i \neq n$, while for $i = n$,

$$\frac{Y_{fnn}}{w_n L_n} = \frac{1}{1+(I-1)h^{-\theta}}. \tag{48}$$

[38]Note that if we consider the limit $h_{gli} \to \infty$ but compute the trade flows with $T_{ll} = T'_{gl}$ then the domestic demand share is the same as the one that prevails in the actual equilibrium, $X_{nn}/\eta w_n L_n$.

Turning to MP in intermediate goods, we have

$$\widetilde{Y}_g \equiv \frac{Y_{gni}}{\eta w_n L_n} = \widetilde{Y}_{g1} + \widetilde{Y}_{gB,0} + \widetilde{Y}_{gB,1},$$

for $i \neq n$. The term $\widetilde{Y}_{g1}$ captures MP for goods destined to stay in the domestic market, $\widetilde{Y}_{gB,0}$ is MP for goods that go back to the country where the technology originates, and $\widetilde{Y}_{gB,1}$ is MP for goods that go to a third market. Both $\widetilde{Y}_{gB,0}$ and $\widetilde{Y}_{gB,1}$ take place through BMP. The respective formulas are

$$\widetilde{Y}_{g1} = \frac{\Delta_1^{-\rho/(1-\rho)} m^{-\theta/(1-\rho)}}{\Delta_0 + (I-1)\Delta_1},\ \widetilde{Y}_{gB,0} = \frac{\Delta_0^{-\rho/(1-\rho)} (md)^{-\theta/(1-\rho)}}{\Delta_0 + (I-1)\Delta_1},\ \widetilde{Y}_{gB,1} = \frac{(I-2)\Delta_1^{-\rho/(1-\rho)} (md)^{-\theta/(1-\rho)}}{\Delta_0 + (I-1)\Delta_1}.$$

This implies that

$$\frac{Y_{gnn}}{\eta w_n L_n} = \frac{\Delta_0^{-\rho/(1-\rho)} + (I-1)\Delta_1^{-\rho/(1-\rho)} d^{-\theta/(1-\rho)}}{\Delta_0 + (I-1)\Delta_1}. \tag{49}$$

The equilibrium trade share is given by:

$$\widetilde{X} \equiv X_{nl}/\eta w_n L_n = \widetilde{X}_{0,B} + \widetilde{X}_1 + \widetilde{X}_{1,B} + \omega \widetilde{Y}_g,$$

for $l \neq n$. The term $\widetilde{X}_{0,B}$ captures the imports of goods produced abroad (in l) with the importer's (country n) own technologies through BMP; the term $\widetilde{X}_1$ is the standard component associated with imports from a country that used that country's technology for production (country l uses its technologies to export to n); the term $\widetilde{X}_{1,B}$ captures imports of goods produced with country l technologies in countries other than l (BMP); and the term $\omega\widetilde{Y}_g$ captures imports of the input bundle from l for domestic operations of country l multinationals. The formulas for $\widetilde{X}_{0,B}$ and $\widetilde{X}_{1,B}$ are the same as the formulas for $\widetilde{Y}_{gB,0}$ and $\widetilde{Y}_{gB,1}$, respectively, while $\widetilde{X}_1 = \widetilde{Y}_{g1}(d/m)^{-\theta/(1-\rho)}$. This implies that

$$\frac{X_{nn}}{\eta w_n L_n} = \frac{\Delta_0^{-\rho/(1-\rho)} + (I-1)\Delta_1^{-\rho/(1-\rho)} m^{-\theta/(1-\rho)}}{\Delta_0 + (I-1)\Delta_1}. \tag{50}$$

It is easy to see from these results that the total value of BMP as a share of total MP is

$$BMP = \frac{\widetilde{Y}_{gB,0} + \widetilde{Y}_{gB,1}}{\widetilde{Y}_f + \widetilde{Y}_g}.$$

Consider the limit as $\rho \to 1$, so that technology draws are the same across production locations. In this case, $BMP \to 0$. Further, when $h > d$, $\widetilde{Y}_g \to 0$ and there is only trade, $\widetilde{X} =$

$d^{-\theta}/(1+(I-1)d^{-\theta})$. On the contrary, when $h<d$, trade is just associated with MP flows, $\widetilde{X}=\omega m^{-\theta}/(1+(I-1)m^{-\theta})=\omega\widetilde{Y}_g$.

Proof of Proposition 2. We want to compute GO, GT, and GMP under symmetry. We start by computing the real wage when there is trade and MP, and under isolation. We know that $w_i=w$, for all i, and that the price index for intermediate goods collapses to

$$P_g=\gamma\Phi_g^{-1/\theta}=(\gamma B)^{1/\beta}\cdot[\Delta_0+(I-1)\Delta_1]^{-1/\beta\theta}\cdot T^{-1/\beta\theta}\cdot w.$$

The price index for final goods is

$$P_f=\gamma\Phi^{-1/\theta}=\gamma\left[1+(I-1)h^{-\theta}\right]^{-1/\theta}\cdot T^{-1/\theta}\cdot Aw^{\alpha}P_g^{1-\alpha}.$$

Using the result for P_g above, the real wage is

$$\frac{w}{P_f}=\widetilde{\gamma}^{-1}\left[1+(I-1)h^{-\theta}\right]^{1/\theta}\cdot[\Delta_0+(I-1)\Delta_1]^{\eta/\theta}\,T^{(1+\eta)/\theta}, \tag{51}$$

where $\widetilde{\gamma}\equiv(\gamma A)(\gamma B)^{\eta}$. The real wage under isolation is obtained by letting $d\to\infty$ and $h\to\infty$ in equation (51),

$$\left(\frac{w}{P_f}\right)^{ISOL}=\widetilde{\gamma}^{-1}\cdot T^{(1+\eta)/\theta}.$$

Thus,

$$GO\equiv\frac{w/P_f}{(w/P_f)^{ISOL}}=\left[1+(I-1)h^{-\theta}\right]^{1/\theta}[\Delta_0+(I-1)\Delta_1]^{\eta/\theta}.$$

To calculate GT, we need to calculate the real wage when there is only MP. By letting $d\to\infty$ in equation (51), the real wage with only MP is

$$\left(\frac{w}{P_f}\right)^{MP}=\widetilde{\gamma}^{-1}\cdot\left[1+(I-1)h^{-\theta}\right]^{1/\theta}\cdot\left[1+(I-1)\widetilde{m}^{-\theta}\right]^{\eta/\theta}\cdot T^{(1+\eta)/\theta},$$

where$\widetilde{m}\equiv\lim_{d\to\infty}m=(1-a)^{\frac{1}{1-\xi}}\,h$. Hence,

$$GT\equiv\frac{w/P_f}{(w/P_f)^{MP}}=\left[\frac{\Delta_0+(I-1)\Delta_1}{1+(I-1)\widetilde{m}^{-\theta}}\right]^{\eta/\theta}.$$

Similarly, by letting $h\to\infty$ in equation (51), the real wage when there is only trade is

$$\left(\frac{w}{P_f}\right)^{T}=\widetilde{\gamma}^{-1}\cdot\left[1+(I-1)d^{-\theta}\right]^{\eta/\theta}\cdot T^{(1+\eta)/\theta},$$

and hence

$$GMP \equiv \frac{w/P_f}{(w/P_f)^T} = \left[1 + (I-1)h^{-\theta}\right]^{1/\theta} \left[\frac{\Delta_0 + (I-1)\Delta_1}{1 + (I-1)d^{-\theta}}\right]^{\eta/\theta}.$$

It is easy to check that the first term on the RHS is GMP_f while the second term is GMP_g.□

Proof of Proposition 3. From Proposition 2, Lemma 3, and equation (50) we have

$$GT = \left[\frac{1 + (I-1)\widetilde{m}^{-\theta}}{\Delta_0 + (I-1)\Delta_1}\right]^{-\eta/\theta},$$

and

$$GT^* = \left[\frac{\Delta_0^{-\rho/(1-\rho)} + (I-1)\Delta_1^{-\rho/(1-\rho)} m^{-\theta/(1-\rho)}}{\Delta_0 + (I-1)\Delta_1}\right]^{-\eta/\theta}.$$

Denote $B^* \equiv \Delta_0^{-\rho/(1-\rho)} + (I-1)\Delta_1^{-\rho/(1-\rho)} m^{-\theta/(1-\rho)}$, and $B' \equiv 1 + (I-1)\widetilde{m}^{-\theta}$.

(a) For $\rho = 0$, $B^* = 1 + (I-1)m^{-\theta}$. But $a > 0$ implies $\widetilde{m} > m$ and hence $\widetilde{m}^{-\theta} < m^{-\theta}$, so $1 + (I-1)\widetilde{m}^{-\theta} < 1 + (I-1)m^{-\theta}$, and $B^* > B'$, then $GT > GT^*$ and trade is MP-complement. For $\rho > 0$ and $a = 0$, the sign of $B^* - B$ is the same as the sign of $BB \equiv \Delta_0^{-\rho/(1-\rho)} + (I-1)\Delta_1^{-\rho/(1-\rho)} m^{-\theta/(1-\rho)} - 1 - (I-1)m^{-\theta}$. But,

$$BB = \left(1 + (I-1)(md)^{-\theta/(1-\rho)}\right)^{-\rho} - 1 + (I-1)m^{-\theta}\left[\left((d/m)^{-\theta/(1-\rho)} + 1 + (I-2)d^{-\theta/(1-\rho)}\right)^{-\rho} - 1\right].$$

This is negative if $\rho > 0$.

(b) For $a > 0$, $\lim_{\xi \to 1} \widetilde{m} \to \infty$. Thus $\lim_{\xi \to 1} GT = GO_g$. Thus, for trade to be MP-complement when $\xi \to 1$, we need to show that $GO_g > GT^*$. But this is equivalent to $G \equiv \Delta_0^{-\rho/(1-\rho)} + (I-1)\Delta_1^{-\rho/(1-\rho)} m^{-\theta/(1-\rho)} > 1$. Using the definitions for Δ_0 and Δ_1, we have

$$G = \left(1 + (I-1)(md)^{-\theta/(1-\rho)}\right)^{-\rho} + (I-1)\left(d^{-\theta/(1-\rho)} m^{\theta/(1-\rho)} + 1 + (I-2)d^{-\theta/(1-\rho)}\right)^{-\rho} m^{-\theta}.$$

For $\rho = 0$, $G = 1 + (I-1)m^{-\theta} > 1$, so $GO_g > GT^*$. For $0 < \rho < 1$ and $d = 1$, $G = \left(1 + (I-1)m^{-\theta/(1-\rho)}\right)^{1-\rho} > 1$. Since G is increasing in d, it follows that $G > 1$ for all d.

Now consider again the case with $a > 0$ and let $\xi \to \infty$. We want to show that if $h < d$ then trade is MP-substitute, or $GT < GT^*$. We have $\lim_{\xi \to \infty} \widetilde{m} = h$ and $\lim_{\xi \to \infty} m = \min[h, d] = h$. Then $GT < GT^*$ in the limit when $\xi \to \infty$ is equivalent to

$$1 + (I-1)h^{-\theta} > \left[1 + (I-1)(hd)^{-\theta/(1-\rho)}\right]^{-\rho} + (I-1)h^{-\theta/(1-\rho)}\left[d^{-\theta/(1-\rho)} + h^{-\theta/(1-\rho)} + (I-2)(hd)^{-\theta/(1-\rho)}\right]^{-\rho}.$$

The first term on the RHS of this inequality is smaller than one, so it is sufficient to show that

$$h^{-\theta} > h^{-\theta/(1-\rho)}\left[d^{-\theta/(1-\rho)} + h^{-\theta/(1-\rho)} + (I-2)(hd)^{-\theta/(1-\rho)}\right]^{-\rho}.$$

If $d \to \infty$ then the RHS is $h^{-\theta}$. Since the RHS is decreasing in d, then it must be lower than $h^{-\theta}$ for any finite d.$\square$

Proof of Proposition 4. From Proposition 2, Lemma 3, and equation (49), we have

$$GMP_g = \left[\frac{1+(I-1)d^{-\theta}}{\Delta_0 + (I-1)\Delta_1}\right]^{-\eta/\theta},$$

and

$$GMP_g^* = \left[\frac{\Delta_0^{1-1/(1-\rho)} + (I-1)\Delta_1^{1-1/(1-\rho)} d^{-\theta/(1-\rho)}}{\Delta_0 + (I-1)\Delta_1}\right]^{-\eta/\theta}.$$

It is obvious that $GMP_g^* = GMP_g$ for $\rho = 0$. We want to show that $GMP_g^* > GMP_g$ for $0 < \rho < 1$. But $GMP_g^* > GMP_g$ is equivalent to

$$1+(I-1)d^{-\theta} > \left(1+(I-1)(md)^{-\theta/(1-\rho)}\right)^{-\rho} + (I-1)\left(1+(m/d)^{-\theta/(1-\rho)} + (I-2)m^{-\theta/(1-\rho)}\right)^{-\rho} d^{-\theta}.$$

This is clearly true for $0 < \rho < 1$.$\square$

Proof of Proposition 5. Lemma 1 implies that under frictionless trade we have a share $\phi_S = \frac{\Phi_S}{\Phi_N + \Phi_S}$ of expenditure on intermediate goods in each country goes to goods produced with South technologies, where

$$\begin{aligned}\Phi_N &\equiv T_N \left(c_{gN}^{-\theta/(1-\rho)} + (hc_{gS})^{-\theta/(1-\rho)}\right)^{1-\rho},\\ \Phi_S &\equiv T_S \left((hc_{gN})^{-\theta/(1-\rho)} + c_{gS}^{-\theta/(1-\rho)}\right)^{1-\rho}.\end{aligned}$$

On the other hand, a share π_{SS} (π_{SN}) of intermediates produced with South (North) technologies are produced in the South, where

$$\begin{aligned}\pi_{SS} &= \frac{c_{gS}^{-\theta/(1-\rho)}}{(hc_{gN})^{-\theta/(1-\rho)} + c_{gS}^{-\theta/(1-\rho)}},\\ \pi_{SN} &= \frac{(hc_{gS})^{-\theta/(1-\rho)}}{c_{gN}^{-\theta/(1-\rho)} + (hc_{gS})^{-\theta/(1-\rho)}}.\end{aligned}$$

Trade balance condition then implies

$$(\phi_S \pi_{SS} + \phi_N \pi_{SN})\, \eta w_N L_N = (1 - \phi_S \pi_{SS} - \phi_N \pi_{SN}) \eta w_S L_S.$$

No MP ($h \to \infty$) implies $\pi_{SN} = 0$, $\Phi_N = T_N c_{gN}^{-\theta}$ and $\Phi_S = T_S c_{gS}^{-\theta}$, so the trade balance condition implies (just as in Alvarez and Lucas, 2007)

$$w_N / w_S = \nu^{1/(1+\theta\beta)},$$

where $\nu \equiv \frac{T_N/L_N}{T_S/L_S}$. The real wage in North is then

$$\lim_{h\to\infty} \frac{w_N}{P_{fN}} = T_N{}^{1/\theta} \left(T_N + T_S \nu^{\lambda}\right)^{\eta/\theta}.$$

where $\lambda \equiv \theta\beta/(1+\theta\beta)$. As one would expect, this does not depend on ρ (since there is no MP).

Now consider the case with frictionless MP ($h_g = 1$ -but still $h_f \to \infty$). The trade balance condition now implies

$$w_N/w_S = \delta \equiv (L_S/L_N)^{1/(1+\theta\beta/(1-\rho))},$$

while the final goods price index in North is

$$P_{fN} = T_N^{-1/\theta} (T_N + T_S)^{-\eta/\theta} \left(1 + \delta^{\theta\beta/(1-\rho)}\right)^{-(1-\rho)\eta/\theta} w_N.$$

Hence,

$$w_N/P_{fN} = \frac{1}{(1 + \delta^{\theta\beta/(1-\rho)})^{-(1-\rho)\eta/\theta}} T_N{}^{1/\theta} (T_N + T_S)^{\eta/\theta}$$

The question of whether $GMP_N \gtreqless 1$ is equivalent to

$$\begin{aligned} \lim_{h\to 1} \frac{w_N}{P_{fN}} &= \left(1 + \delta^{\theta\beta/(1-\rho)}\right)^{(1-\rho)\eta/\theta} (T_N + T_S)^{\eta/\theta} \\ &\gtreqless \left(T_N + T_S \nu^{\theta\beta}\right)^{\eta/\theta} = \lim_{h\to\infty} \frac{w_N}{P_{fN}}. \end{aligned}$$

This is equivalent to

$$f(\rho) \equiv \left(1 + l^{\theta\beta/(1-\rho+\theta\beta)}\right)^{(1-\rho)} (\nu + l) - v - l\nu^{\lambda} \gtreqless 0,$$

where $l \equiv L_S/L_N$. Note that

$$f(0) = l^{-\lambda} + \nu/l + 1 - (\nu/l)^{\lambda}.$$

Since $0 < \lambda < 1$ then $\nu/l + 1 > (\nu/l)^{\lambda}$ for any ν/l, so $f(0) > 0$. On the other hand,

$$\lim_{\rho\to 1} f(\rho) = -v - l\nu^{\lambda} < 0.$$

Moreover, it is easy to show that $f'(\rho) < 0$ for $\rho \in]0,1[$, implying that there is a $\rho^* \in]0,1[$ such that $GMP_N > 1$ for $\rho < \rho^*$ and $GMP_N < 1$ for $\rho > \rho^*$.□

B The Dynamic Model

Assume that an idea from country i has productivity q_{li} in country l. Assume that the vector $\mathbf{q}_i = (q_{1i}, ..., q_{Ii})$ is drawn from the following multivariate distribution:

$$H(\mathbf{q}_i) = 1 - \left(\sum_{l=1}^{I} (q_{li}/\underline{\varepsilon})^{-\frac{\theta}{1-\rho}} \right)^{1-\rho}$$

with $\sum_{l=1}^{I} q_{li}^{-\theta/(1-\rho)} < \underline{\varepsilon}^{-\theta/(1-\rho)}$ for $\rho \in (0,1)$ and $\theta > 1$. Note that the marginal distribution of $q_{li} \geq \underline{\varepsilon}$ for any li is $1 - (q_{li}/\underline{\varepsilon})^{-\theta}$, so we can think of $H(\cdot)$ as a multivariate Pareto distribution.

Research is modeled as the creation of ideas, although for simplicity here we assume that this is exogenous. In particular, we assume that there is an instantaneous (and constant) rate of arrival $2\overline{\varepsilon}\zeta_i$ of new ideas per person in country i. The parameter ζ_i varies across countries and captures differences in "research" productivity across countries, while $\overline{\varepsilon}$ is a common parameter that will be normalized below. Ideas are specific to goods, and the good to which an idea applies can be an intermediate good or a final good with equal probability. If the idea applies to an intermediate (final) good the identity of the good is drawn from a uniform distribution in $v \in [0,1]$ ($u \in [0,1]$). This implies that at time t there is a probability $\overline{\varepsilon}\zeta_i L_i(t)$ of drawing an idea for any particular (intermediate or final) good. The arrival of ideas is then a Poisson process with rate function $\overline{\varepsilon}\zeta_i L_i(t)$, so the number of ideas that have arrived for a particular good by time t is distributed Poisson with rate $\overline{\varepsilon}\lambda_i(t)$, where $\lambda_i(t) \equiv \int_0^t \zeta_i L_i(t) ds$. (From here onwards, we suppress the time index.)

The technology frontier for country i is the upper envelope of all the vectors $\mathbf{q}_i$. That is, letting Ω_{li} denote the set of all q_{li} associated with ideas existing in country i at a certain point in time, then the technology frontier for country i is $\mathbf{z}_i \equiv (\max\{q_{1i} \in \Omega_{1i}\}, ..., \max\{q_{Ii} \in \Omega_{Ii}\})$. This is distributed according to

$$\begin{aligned} F_i(z_i) &= \Pr(Z_{1i} \leq z_{1i}, ..., Z_{Ii} \leq z_{Ii}) = \sum_{k=0}^{\infty} \frac{e^{-\overline{\varepsilon}\lambda_i} (\overline{\varepsilon}\lambda_i)^k}{k!} H(\mathbf{z}_i)^k \\ &= e^{-\overline{\varepsilon}\lambda_i} \sum_{k=0}^{\infty} \frac{[\overline{\varepsilon}\lambda_i H(\mathbf{z}_i)]^k}{k!} = e^{-\overline{\varepsilon}\lambda_i (1 - H(\mathbf{z}_i))} \\ &= \exp\left[-\overline{\varepsilon}\underline{\varepsilon}^{\theta} \lambda_i \left(\sum_{l=1}^{I} z_{li}^{-\frac{\theta}{1-\rho}} \right)^{1-\rho} \right] \end{aligned}$$

for $\sum_{l=1}^{I} z_{li}^{-\theta/(1-\rho)} < \underline{\varepsilon}^{-\theta/(1-\rho)}$. Letting $T_{li} \equiv \lambda_i$, setting $\overline{\varepsilon}\underline{\varepsilon}^{\theta} = 1$, and taking the limit as $\underline{\varepsilon}^{\theta} \to 0$, then we get the multivariate Fréchet distribution in (5).

Assuming that $L_i(t)$ grows at the constant rate g_L (that we assume common across countries), in steady state $\lambda_i(t) = \zeta_i L_i(t)/g_L$, so $\lambda_i(t)$ and hence $T_{li}(t)$ grow at rate g_L for all l, i. This implies that the static equilibrium described in section 2.3 is replicated at all dates, and that the real wage in all countries is increasing at rate $g = \frac{1}{\theta}(1+\eta)g_L$.

C Data

The UNCTAD measure of MP includes both local sales in n and exports to any other country, including the home country i. The number of observations drops to 219 country-pairs for which we have available data. For a detailed description of the UNCTAD MP data see Ramondo (2008).

Total expenditure on intermediate goods in the model is $\eta w_n L_n$, while in the data we compute a measure of total expenditures on manufacturing from all the countries in our sample. This measure is computed as gross production in manufacturing in country n, plus total imports of manufacturing goods into country n from the remaining countries in the sample, minus total manufacturing exports from country n to the rest of the world. Data on these three variables for each country are from the STAN database (an average over the period 1990-2002). Total expenditure on final goods in the model is $w_n L_n$ while in the data we use GDP for country n plus total imports into country n from the remaining eighteen OECD countries in the sample, minus total exports from country n to the rest of the world. Data on GDP is from the World Development Indicators, in current dollars, and total exports and imports are from Feenstra and Lipsey (2005).

We use intra-firm imports by multinationals' foreign affiliates from their home country as the empirical counterpart for imports of the national input bundle from the home country for MP, normalized by gross production of affiliates from i in n, $\omega_{ni} Y_{gni} / \left(Y_{gni} + Y_{fni}\right)$.We combine data on intra-firm exports from U.S. parent companies to their affiliates abroad with data on imports done by foreign affiliates located in U.S. from their parent firms, an average over the period 1990-2003.

For the empirical counterpart of the bilateral share of MP in intermediate goods,$Y_{gni} / \left(Y_{gni} + Y_{fni}\right)$, we use data on gross production of affiliates from country i in n in the manufacturing sector as share of total gross production for affiliates of multinational firms from i in n. The relevant data on bilateral MP in manufacturing is also for $i = U.S.$ or $n = U.S.$, an average over the period 1999-2003.

We are able to compute BMP when the U.S. is the source or the destination country, again as an average over the period 1999-2003. The BEA divides total sales of American affiliates produced in country l into sales to the local market, to the US, and to third foreign markets. This is the empirical counterpart for $\sum_{n \neq l} \phi_{gni} \pi_{gni,l} X_{gn} / (Y_{gli} + Y_{fli})$, from $i = US$ in a country

l belonging to the OECD(19). We average out across l's, and obtain an average bilateral BMP share for the US affiliates in the OECD(19). A similar procedure yields the average bilateral BMP share for US affiliates of foreign multinationals.

Bilateral distance is the distance in kilometers between the largest cities in the two countries. Common language is a dummy equal to one if both countries have the same official language or more than 20% of the population share the same language (even if it is not the official one). Common border is equal to one if two countries share a border.

D Summary Statistics

	Mean	Standard Deviation	Observations
Distance (in km)	6,006	6,099	342
Common Language	0.11	0.31	342
Common Border	0.09	0.28	342
bilateral trade share	0.019	0.035	342
bilateral MP share	0.022	0.043	219
bilateral intra-firm share†	0.074	0.072	34
bilateral MP share in manufacturing†	0.48	.13	33

†: from/to the United States.

Table 9: Summary Statistics. Data.

	R&D employment (% of total employment)	Real GDP pw (as share of U.S.)	L_n (as share of U.S.)	λ_n (as share of U.S.)
	data	data	model	model
Australia	0.68	0.80	0.09	0.07
Austria	0.49	0.80	0.04	0.02
Belgium	0.67	0.89	0.05	0.04
Canada	0.62	0.80	0.13	0.10
Denmark	0.61	0.77	0.03	0.02
Spain	0.36	0.70	0.13	0.05
Finland	1.22	0.71	0.03	0.04
France	0.62	0.79	0.22	0.16
Great Britain	0.53	0.70	0.21	0.13
Germany	0.60	0.75	0.29	0.21
Greece	0.28	0.56	0.04	0.01
Italy	0.29	0.88	0.23	0.08
Japan	0.95	0.65	0.45	0.51
Netherlands	0.51	0.82	0.07	0.04
Norway	0.77	0.84	0.03	0.02
New Zealand	0.45	0.64	0.02	0.01
Portugal	0.29	0.53	0.03	0.01
Sweden	0.83	0.71	0.04	0.04
United States	0.85	1.00	1.00	1.00
Average	0.61	0.75	0.16	0.14

Table 10: Data and Model's Variables.

E Gains from Openness: Calibration with $\rho = 0$

as % of GDP	Exports	Imports	MP out	MP in	Exports†	MP out	MP in
	Data				Model's calibration ($\rho = 0.5$)		
Australia	0.05	0.10	0.10	0.28	0.04	0.03	0.08
Austria	0.18	0.24	0.13	0.26	0.26	0.13	0.45
Belgium	0.45	0.48	0.22	0.46	0.32	0.24	0.42
Canada	0.23	0.22	0.26	0.45	0.20	0.09	0.35
Denmark	0.23	0.19	0.17	0.12	0.24	0.11	0.34
Spain	0.11	0.15	0.03	0.23	0.14	0.06	0.18
Finland	0.22	0.16	0.48	0.24	0.24	0.23	0.22
France	0.13	0.14	0.18	0.20	0.15	0.13	0.08
Great Britain	0.12	0.15	0.32	0.34	0.12	0.10	0.16
Germany	0.16	0.13	0.29	0.28	0.12	0.13	0.06
Greece	0.09	0.17	0.01	0.06	0.16	0.04	0.48
Italy	0.12	0.12	0.07	0.14	0.11	0.04	0.13
Japan	0.05	0.02	0.16	0.06	0.01	0.03	0.005
Netherlands	0.32	0.27	1.00	0.50	0.20	0.11	0.24
Norway	0.11	0.17	0.18	0.17	0.23	0.14	0.31
New Zealand	0.14	0.18	0.04	0.25	0.10	0.03	0.47
Portugal	0.18	0.26	0.04	0.51	0.19	0.05	0.51
Sweden	0.23	0.19	0.36	0.32	0.21	0.15	0.21
United States	0.04	0.05	0.16	0.18	0.03	0.08	0.01
Average	0.17	0.18	0.22	0.27	0.16	0.10	0.25

†: in the model, total exports = total imports by country. MP out is total gross value of production for foreign affiliates from country i; MP in is total gross value of production for foreign affiliates in country l.

Table 11: Trade and MP shares. Data and Model.

	Domestic Trade shares			Domestic MP shares		
	data	model $\rho = 0.5$	model $\rho = 0$	data	model $\rho = 0.5$	model $\rho = 0$
Australia	0.80	0.92	0.94	0.71	0.92	0.94
Austria	0.45	0.47	0.44	0.74	0.55	0.53
Belgium	0.16	0.37	0.36	0.54	0.58	0.53
Canada	0.60	0.61	0.61	0.55	0.65	0.65
Denmark	0.85	0.52	0.51	0.88	0.66	0.66
Spain	0.67	0.73	0.75	0.77	0.82	0.85
Finland	0.66	0.51	0.54	0.76	0.78	0.77
France	0.70	0.71	0.75	0.80	0.92	0.93
Great Britain	0.55	0.75	0.79	0.66	0.84	0.85
Germany	0.74	0.76	0.79	0.72	0.94	0.95
Greece	0.63	0.68	0.63	0.94	0.52	0.56
Italy	0.75	0.79	0.81	0.86	0.87	0.90
Japan	0.72	0.97	0.98	0.94	0.995	0.995
Netherlands	0.31	0.59	0.61	0.50	0.76	0.78
Norway	0.90	0.53	0.53	0.83	0.69	0.69
New Zealand	0.75	0.80	0.79	0.75	0.53	0.55
Portugal	0.55	0.63	0.57	0.49	0.49	0.52
Sweden	0.92	0.59	0.62	0.68	0.79	0.79
United States	0.88	0.94	0.94	0.82	0.99	0.99
Average	0.66	0.68	0.68	0.73	0.75	0.76

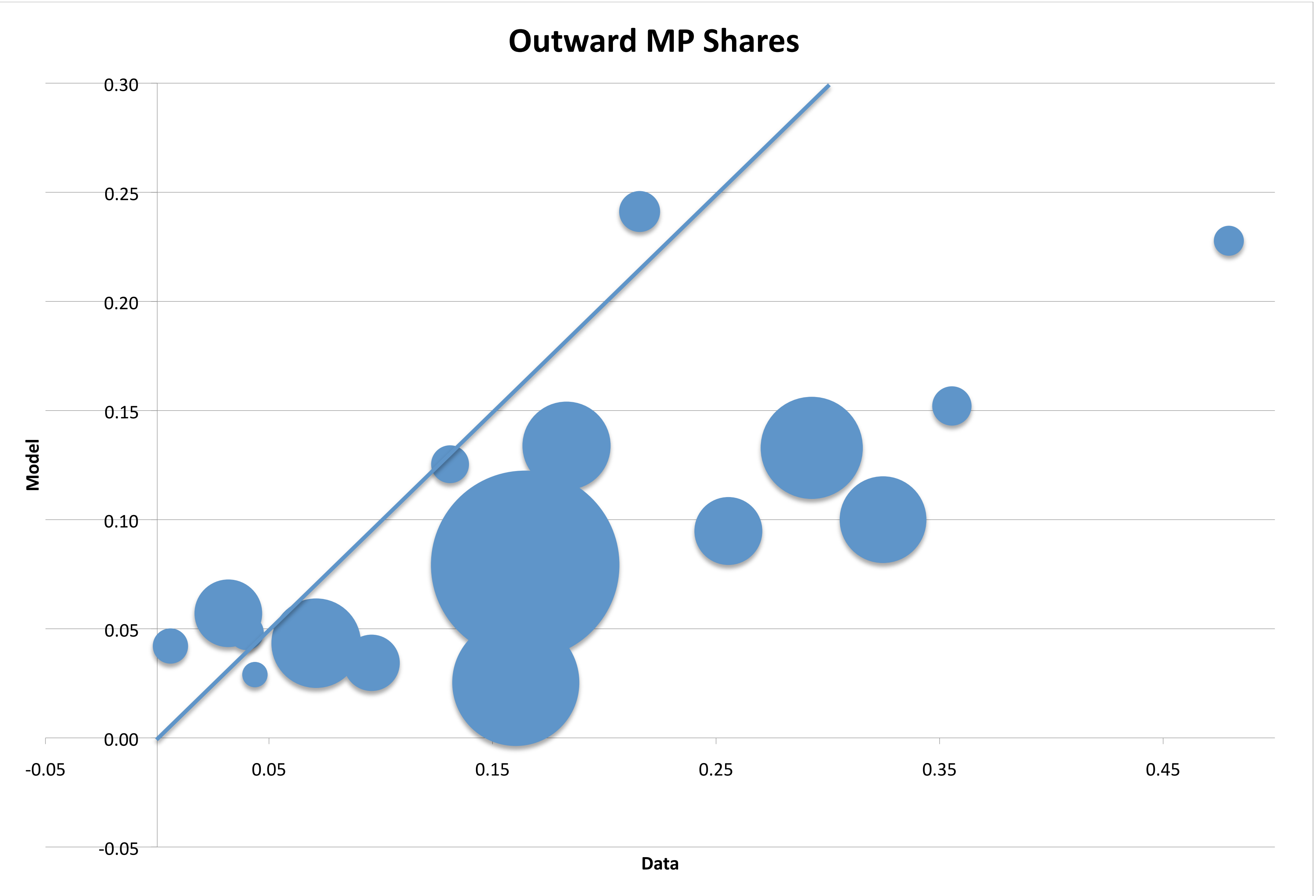
Trade domestic shares are for manufacturing. MP domestic shares are for all sectors. Domestic shares are normalized by country's total mfg. expenditure (gross value of production in mfg. minus total mfg. exports plus mfg. imports from the countries in the sample). MP shares are normalized by country's GDP.

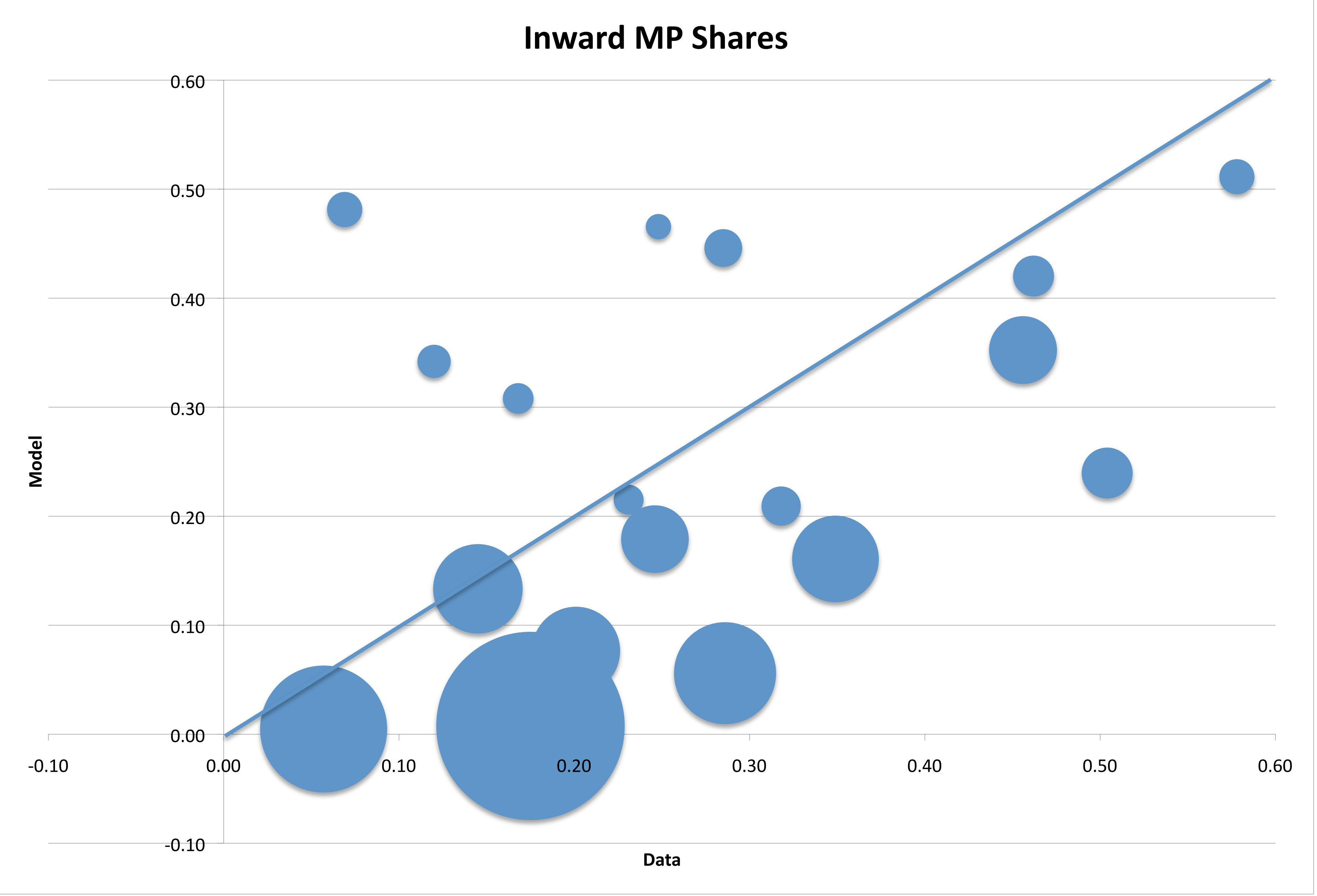
Table 12: Domestic Trade and MP shares. Data and Model.

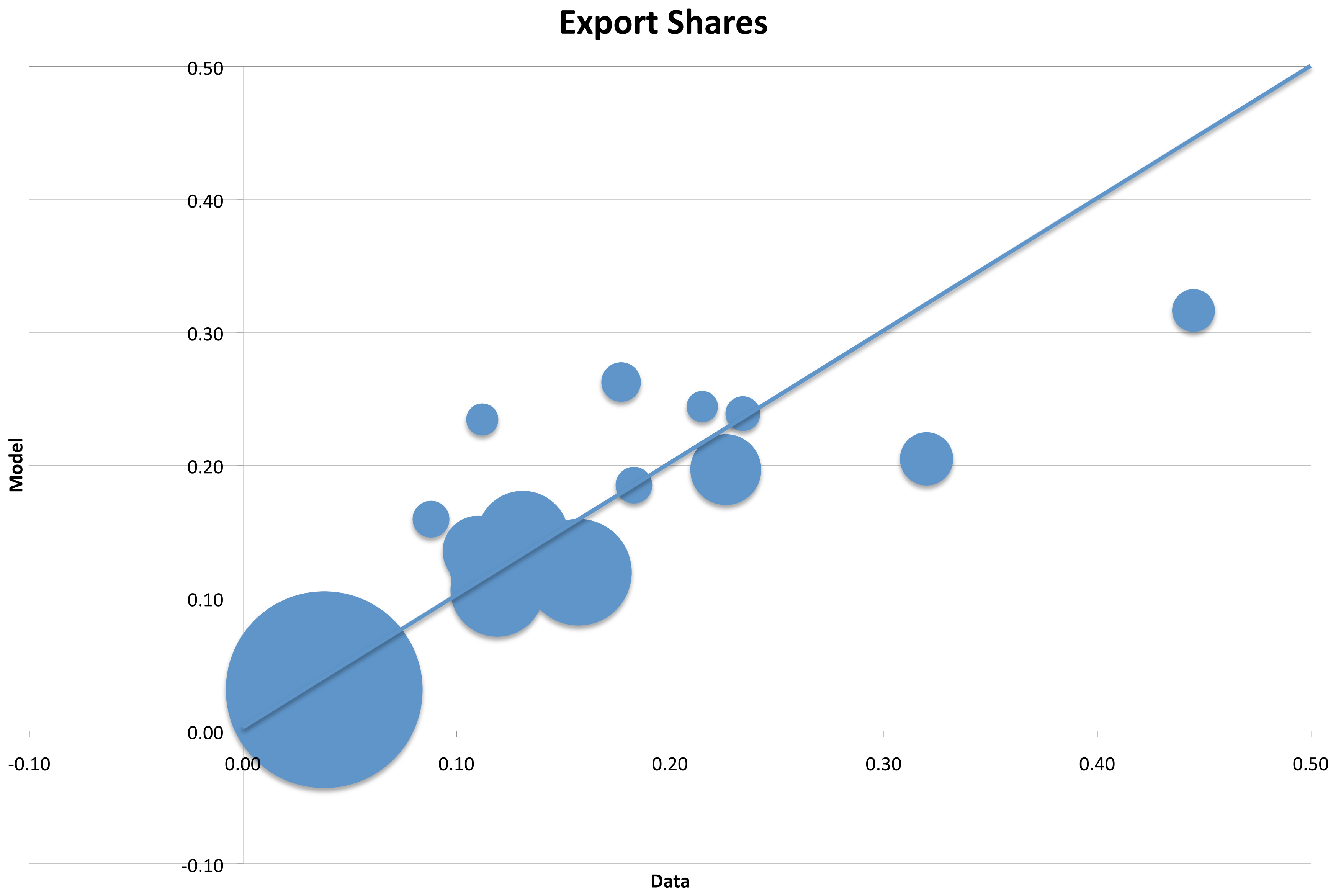
	GO_n	GT_n	GT_n^*	GMP_n	GMP_n^*	GMP_{gn}	$L_n/\sum_k L_k$
New Zealand	1.088	1.038	1.017	1.073	1.077	1.0275	0.6
Finland	1.078	1.056	1.044	1.036	1.036	1.0168	0.8
Norway	1.091	1.060	1.045	1.048	1.051	1.0212	0.9
Denmark	1.099	1.064	1.048	1.053	1.056	1.0228	1.0
Portugal	1.119	1.065	1.040	1.080	1.086	1.0318	1.1
Greece	1.104	1.055	1.033	1.072	1.077	1.0288	1.1
Austria	1.135	1.082	1.059	1.077	1.084	1.0319	1.3
Sweden	1.063	1.045	1.034	1.031	1.032	1.0144	1.4
Belgium	1.147	1.096	1.074	1.076	1.082	1.0325	1.5
Netherlands	1.067	1.046	1.035	1.034	1.034	1.0153	2.4
Australia	1.013	1.007	1.004	1.009	1.009	1.0038	2.9
Spain	1.040	1.027	1.020	1.021	1.022	1.0097	4.1
Canada	1.087	1.053	1.035	1.053	1.057	1.0225	4.2
Great Britain	1.037	1.023	1.016	1.021	1.022	1.0095	6.8
France	1.030	1.024	1.020	1.011	1.010	1.0057	7.0
Italy	1.028	1.019	1.014	1.015	1.015	1.0068	7.3
Germany	1.023	1.019	1.016	1.008	1.007	1.0045	9.4
Japan	1.002	1.002	1.002	1.001	1.001	1.0004	14.4
United States	1.005	1.004	1.004	1.002	1.001	1.001	31.8
Average	1.066	1.041	1.029	1.038	1.040	1.016	5.3

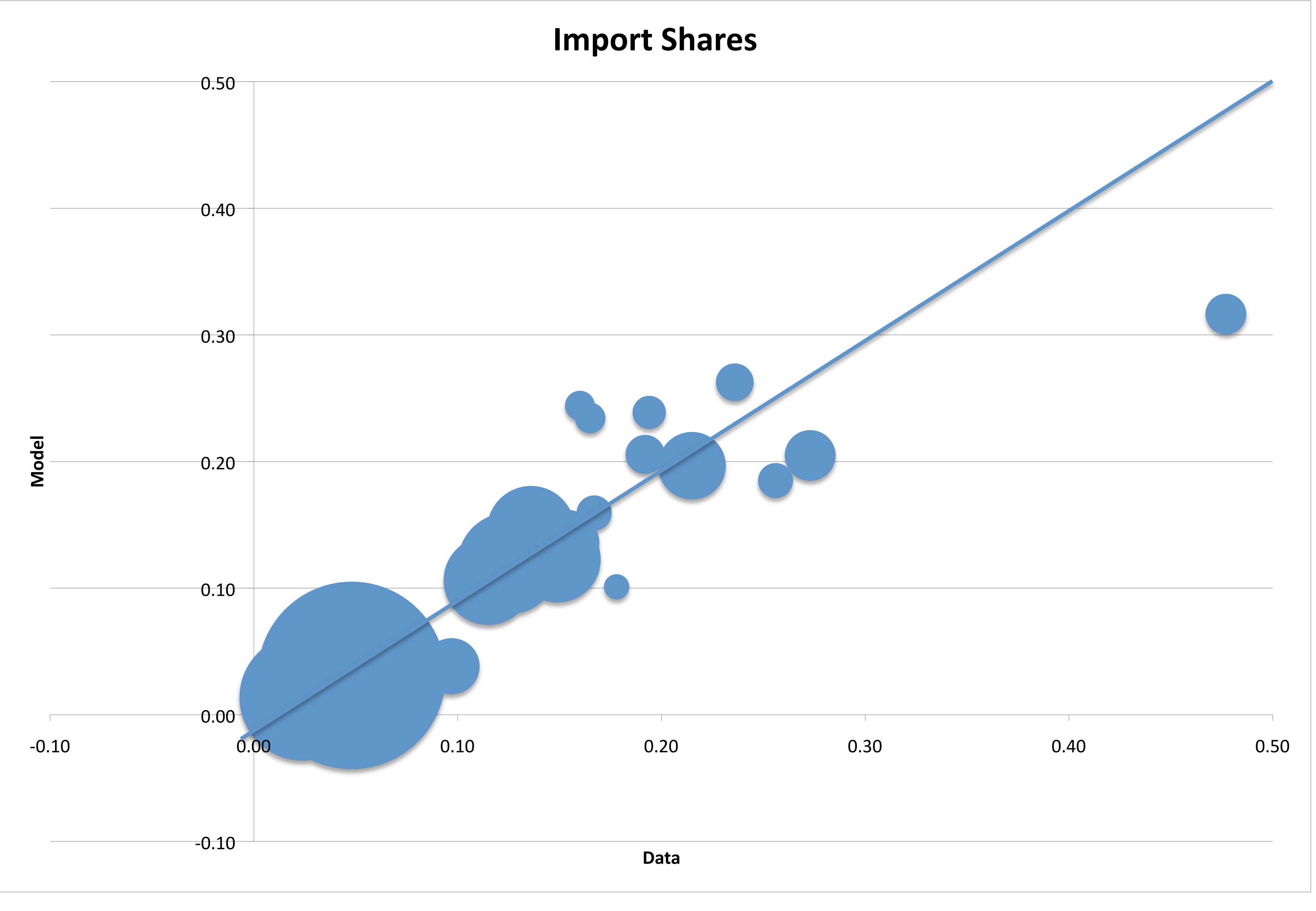
Countries are sorted by (simulated) size.

Table 13: Gains from Openness, Trade, and MP. Calibration with $\rho = 0$.


Outward MP Shares
Model
Data
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.05
0.05
0.15
0.25
0.35
0.45


Inward MP Shares
Model
Data
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.10
0.00
0.10
0.20
0.30
0.40
0.50
0.60


Export Shares
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
Model
-0.10
0.00
0.10
0.20
0.30
0.40
0.50
Data


Import Shares
Model
Data
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.10
0.00
0.10
0.20
0.30
0.40
0.50