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ABSTRACT

Stock and Treasury bond comovement, volatilities, and their relations to their price valuations and fundamentals change stochastically over time, both in magnitude and direction. These stochastic changes are explained by a general equilibrium model in which agents learn about composite economic and inflation regimes. We estimate our model using both fundamentals and asset prices, and find that inflation news signal either positive or negative future real economic growth depending on the times, thereby affecting the direction of stock/bond comovement. The learning dynamics generate strong non-linearities between volatilities and price valuations. We find empirical support for numerous predictions of the model.

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At the onset of the 2008 financial crisis, stock return volatility skyrocketed to new record levels as stock prices plunged. At the same time, the price of Treasury bonds shot up, as investors dumped risky stocks and purchased safe Treasuries. The correlation between stocks and Treasury bonds turned strongly negative. This behavior of stocks and bonds in the Great Recession stands in sharp contrast with their behavior during the equally severe 1981-82 recession, when investors dumped both stocks *and* Treasuries, and their correlation was strongly positive (see Panel A of Figure 1).¹ Indeed, over the years several properties of the relation between stock and bond prices and their volatilities have changed. For instance, Treasury bonds' volatilities and their yields were positively related in the 1980s, while they have been negatively related in the last decade (see Panel B of Figure 1). Similarly, and perhaps even more puzzlingly, while the stock market volatility is mostly negatively related to its price-earnings ratio, it occurs at times that volatility increases when prices increase, as for instance in the late 1990s (see Panel C of Figure 1).² In this paper we show that all of these stochastic changes in the relation between stock and bond prices, volatilities, and cross-covariance, are in fact interconnected, and generated by market participants' variation in their beliefs about economic and inflation regimes.

We study an endowment economy in which the drift rates of real earnings growth, real consumption, and inflation follow a joint regime-switching model. Market participants cannot observe the current regime and thus must learn about it by observing real fundamental growth, inflation, and other signals. Investors also suffer from some degree of “money illusion,” that is, they partly discount future real cash flows using a nominal stochastic discount factor. This latter assumption embedded in our learning-based model allows us to study the relative importance of the “proxy hypothesis” of Fama (1981) and the “money illusion hypothesis” of Modigliani and Cohn (1979) as competing explanations of the joint comovement of stocks and bonds. We obtain closed-form formulas for stock and Treasury bond prices, their volatilities and cross-covariances. Because we, as econometricians, do not have full information about the signals used by market participants to form their beliefs, we exploit our analytical formulas and estimate the time series of beliefs from fundamentals as well as prices, volatilities, and covariances.

Our empirical results suggest that while some degree of money illusion is important to fit

¹See also Baele, Bekaert, and Inghelbrecht (2011) and Campbell, Sunderam, and Viceira (2009).

²Most asset pricing models predicts a negative relation between aggregate volatility and price valuations. For instance, both the habit formation model of Campbell and Cochrane (1999) [see Figures 3 and 5] and the long-run risk model of Bansal and Yaron (2004) [see Section II.C.3] imply a negative relation between valuation ratios and volatility (see also Beeler and Campbell (2012)). Using data from Bob Shiller's web site and daily returns from 1926 to 2008 from CRSP we find that correlation between volatility and the price/earnings ratio turned positive also in the late 1920s, in the postwar period, and in the late 1950s.

asset prices, the bulk of the (co)variation over time of stocks and Treasury bonds is chiefly explained by the learning dynamics. The latter also generates a strong time variation of the conditional relation between return volatilities and price valuations. Indeed, our estimated model provides a coherent quantitative explanation behind the dynamic nature of stock and bond prices, volatilities and comovement.

Consider first the time varying comovement between stocks and bonds returns shown in Panel A of Figure 1. Our estimated model suggests that in the early 1980s, investors faced large uncertainty about whether the U.S. would enter a persistent *stagflation* regime. Any CPI reading above market expectations was then taken as an indication that the U.S. was transiting into such regime, which brings about low growth and high inflation. The former makes stock prices decline, the latter makes long-term yields increase, and thus bond prices decrease. Thus, data-driven fluctuations in investors' beliefs about a stagflation regime make the prices of stocks and Treasuries move in the same direction, and increase the volatility of both. In the last decade the opposite is happening. The market now fears a *deflationary* regime, which is accompanied by zero or negative inflation and low growth. In this case, CPI news above expectations are good news for the economy, as investors interpret them as signals that the bad deflationary regime could be averted. Stock markets rise at higher-than-expected CPI, and Treasury yields increase in expectation of higher inflation. Thus, data-driven beliefs about entering a deflationary regime push the prices of stocks and Treasuries in opposite directions. That is, the same economic mechanism – *the signaling role of inflation* – generates diametrically opposite implications about the comovement of stock and bonds, depending on market beliefs about the current regime.

This time-varying signaling role of inflation not only brings about a strong time-variation in the joint behavior of stocks and bonds, but it also helps explain other phenomena as well. Consider for instance the time varying relation between yields and bond return volatility, documented in Panel B of Figure 1. In late 1970s - early 1980s, higher inflation realizations brought about an increase in the beliefs of transiting to a high inflation regime. The increase in expected inflation pushed up yields while the increase in uncertainty pushed up volatility. In the 1990s, inflation uncertainty subsided, and agents's beliefs settled for middle regimes, again leading to a positive correlation between yields and volatility, both of them lower. As discussed in the previous paragraph, the new millennium, in contrast, brought about a renewed uncertainty about the inflation regime, this time on whether a deflationary regime may occur. Such uncertainty increases bond return volatility, while yields decrease because of lower expected inflation, leading to a negative relation between bond volatility and yields.

Indeed, a similar economic mechanism explains the strong time variation in the relation between stock return volatility and the P/E ratio. For instance, according to our estimates, the positive relation between volatility and prices in the late 1990s was due to an increase in investors' beliefs about the U.S. entering into a sustained high growth regime. These beliefs increased both the P/E ratio, as they increased expected cash flows, and volatility, as they increased the uncertainty on whether this transition to a high growth state was true or not.

Our model not only provides a unified economic framework explaining several facts about the dynamic relations between stock and bond prices, volatilities, and correlations, but it also generates additional testable predictions, which we quantify by running Monte Carlo simulations. For instance, the time varying signaling role of inflation predicts that the covariance between stocks and bonds should be related to expected inflation, and especially to "extreme inflation" probabilities, that is, the probability of very high and very low inflation. Using data from the Survey of Professional Forecasters (SPF) we find strong evidence about both channels: Stock-bond covariance is higher when expected inflation is higher, when the probability of high future inflation is higher, and when the probability of a low/negative inflation is lower. Similarly, we find evidence that inflation uncertainty is also related to the comovement of stocks and bonds, as is earnings uncertainty once it is interacted with the extreme inflation probabilities, as predicted by the model. Finally, the regime switching model with learning also predicts that the relation between stock and bond covariances are non-linearly related to asset prices. We find evidence of such non-linearities in the data.

Similar empirical results hold for bond and stock return volatility. For the former, consistently with the model's main mechanism, we find that bond return volatility is related to expected earnings, to extreme inflation probabilities, and to inflation and earnings uncertainty, although the statistical significance of the latter depend on proxies. Intriguingly, and consistently with the model's prediction, we find that bond return volatility is non-linearly related to the long-term yield, being higher when the long-term yield is both high or low. This result is consistent with agents' uncertainty about entering either the hyper-inflation regime or the deflationary regime, as explained earlier. In either case, bond return volatility is high because uncertainty is high, but the level of long-term yields are at their opposite extremes. This insight may also explain the weak evidence in favor of a linear relation between volatility and bond yields (see e.g. Collin-Dufresne and Goldstein (2002)).

Finally, we also find empirical support for the model's prediction that stock return volatility should be higher when expected earnings are lower, when extreme inflation probabilities are higher, and when earnings uncertainty is higher, although in all these cases, the R^2 is

relatively low, in the range of 10% to 20%. Interestingly, however, Monte Carlo simulations of the model indeed show that it is extremely hard to forecast stock return volatility, using either fundamentals or prices. For instance, even in simulations when there is no noise in the volatility estimate, the model produces a modest $R^2 = 12\%$ when we consider a linear regression of volatility onto bond yields and log P/E ratios. The R^2 in simulations increases considerably when non-linear terms are added, to an adjusted $R^2 = 34\%$, but it is far from perfect. The empirical data show a similar pattern, namely, that a linear regression of integrated stock return volatility on long-term yields and log P/E ratio gives insignificant coefficients, while once non-linear terms are included, they are all strongly significant. These nonlinearities are naturally generated by the learning model we propose in this paper.

Our paper is related to numerous strands of literature. First, it is related to the previous literature about Bayesian learning and asset prices, and especially David (1997), Veronesi (1999), Veronesi (2000), and David (2008). Compared to these articles, we consider a much richer environment to investigate the joint dynamics of stocks and Treasury bonds. Through a careful estimation of the model, we identify one specific novel mechanism, the time-varying signaling role of inflation, as the main driver of the co-movement of stocks and bonds, for which we provide additional empirical evidence. Uncertainty about inflation also features in Piazzesi and Schneider (2006), who embed learning dynamics in a recursive utility framework, and show that high inflation shocks signal bad news for consumption growth, which in turn explain a positive term structure of interest rates. In our model, however, inflation may signal bad times or good times, depending on current beliefs about composite regimes.

Our paper is also related to the literature on consumption-based models to explain the dynamics of stock and/or bonds and their second moments (e.g. Campbell and Cochrane (1999), Bansal and Yaron (2004), Wachter (2006).) These articles do not examine the joint dynamics of stocks and bonds. Brandt and Wang (2003) and Bekaert and Grenadier (2010) use habit formation models to study stocks and bonds, but they do not focus on the dynamics of their conditional covariance, and, in particular, what economic mechanism is responsible for its sign changes. Our paper offers a novel explanation, the time varying signaling role of inflation, which we find support for in the data. Our model also produces additional implications on the ever changing relation between yields, price-earnings ratios, and conditional volatilities which we test in the data.

Our empirical investigation of time varying second moments also connects our paper to the vast empirical literature on time varying volatility, of which we do not attempt a survey, but refer to Anderson, Bollerslev, and Diebold (2010) for recent advances and references,

and to Engle and Rangel (2008) and Ludvigson and Ng (2007) for recent contributions on the relation between stock return volatility and fundamentals. Unlike this literature, we introduce and estimate a general equilibrium model to obtain new predictions about the joint dynamics of stocks and bonds, and their relation to fundamentals and price valuations.

Finally, our work relates to the literature on money illusion and asset prices. Our modeling device is borrowed from Basak and Yan (2010), who show that money illusion can explain several patterns of stock and bond returns. Our model with learning about composite regimes, however, highlights an amplification mechanism that explains several additional facts about the dynamics of second moments. Piazzesi and Schneider (2008) show that money illusion can explain why nominal interest rates and the price-dividend ratio on housing are sometimes positively correlated (as in late 1970s/early 1980s) and sometimes negatively correlated (as during 2000's housing boom). While our model also generates dynamic variation in correlations, the channel is different, ours being due to learning about composite regimes, which in turn induces the time varying signaling role of inflation.

The paper develops as follows. Section I describes our general equilibrium model and our main asset pricing formulas. Section II discusses our empirical methodology, while Section III describes the empirical results. Section IV tests the model's main predictions about the comovement of stocks and bonds, while Section V focuses on the volatility of stocks and bonds. Section VI contains an out-of-sample forecast exercise, and Section VII concludes. An appendix contains the proofs of the propositions, as well as details of our empirical methodology.

I. Structure of the Model

We consider a standard endowment economy populated by consumers/investors who are endowed with constant relative risk aversion (CRRA) preferences

$$U(C, t) = e^{-\rho t} \frac{C^{1-\gamma}}{1-\gamma} \quad (1)$$

γ is the coefficient of relative risk aversion, and ρ is the parameter of time preference.

There are three fundamental variables: the representative agent's real consumption C_t , aggregate real earnings E_t , and the nominal price of aggregate consumption Q_t . Their stochastic variation is described by the joint diffusion processes

$$\frac{dC_t}{C_t} = \kappa_t dt + \sigma_C dW_t, \quad (2)$$

$$\frac{dE_t}{E_t} = \theta_t dt + \sigma_E dW_t, \quad (3)$$

$$\frac{dQ_t}{Q_t} = \beta_t dt + \sigma_Q dW_t, \quad (4)$$

where $W_t = (W_{1t}, W_{2t}, W_{3t}, W_{4t})'$ is a four-dimensional vector of independent Brownian processes, and σ_i , for $i = C, Q$, and E , are 1×4 constant vectors, known by investors.

Investors do not observe the drift rates κ_t , θ_t , and β_t , whose joint dynamics are described below, but learn about them by observing realized consumption growth dC_t/C_t , inflation dQ_t/Q_t and earnings growth dE_t/E_t . Investors also observe an unbiased signal, S_t , on earnings' drift:

$$\frac{dS_t}{S_t} = \theta_t dt + \sigma_S dW_t, \quad (5)$$

where σ_S is known by investors. To streamline the notation let $X_t = (C_t, Q_t, E_t, S_t)'$, which has the drift vector $\nu_t = (\kappa_t, \beta_t, \theta_t, \theta_t)'$, and volatility matrix $\Sigma = (\sigma'_C, \sigma'_Q, \sigma'_E, \sigma'_S)'$.

The drift vector ν_t follows a continuous-time Markov chain with n *composite* regimes and generator matrix Λ . Each composite regime is a vector collecting the drifts of fundamentals $\nu^i = (\kappa^i, \beta^i, \theta^i, \theta^i)$, for $i = 1, \dots, n$. The probability to move from regime i to j in the infinitesimal time interval dt is

$$\lambda_{ij} dt = \text{prob}(\nu_{t+dt} = \nu^j | \nu_t = \nu^i), \quad \text{for } i \neq j, \quad \lambda_{ii} = - \sum_{j \neq i} \lambda_{ij}.$$

Denote investors' subjective probability of regime i as

$$\pi_{it} = \text{prob}(\nu_t = \nu^i | \mathcal{F}_t).$$

Given an initial condition $\pi_0 = \hat{\pi}$ with $\sum_{i=1}^n \hat{\pi}_i = 1$ and $0 \leq \hat{\pi}_i \leq 1$ for all i , from Wonham (1964) the probabilities $\pi_t = (\pi_{1t}, \dots, \pi_{nt})'$ follow the Vector Autoregressive process

$$d\pi_t = \Lambda' \pi_t dt + \Sigma_\pi(\pi_t) d\tilde{W}_t, \quad (6)$$

where $\Sigma_\pi(\pi_t)$ is a $(n \times 4)$ matrix, with i -th row given by

$$\sigma_i(\pi_t) = \pi_{it} [\nu^i - \bar{\nu}(\pi_t)]' \Sigma'^{-1}, \quad (7)$$

$$\bar{\nu}(\pi_t) = \sum_{i=1}^n \pi_{it} \nu^i = \mathbb{E} \left(\frac{dX_t}{X_t} | \mathcal{F}_t \right). \quad (8)$$

In (6), \tilde{W}_t is a 4×1 vector of Brownian motions defined by normalized expectation errors:

$$d\tilde{W}_t = \Sigma^{-1} \left[\frac{dX_t}{X_t} - \mathbb{E} \left(\frac{dX_t}{X_t} | \mathcal{F}_t \right) \right] = \Sigma^{-1} (\nu_t - \bar{\nu}(\pi_t)) dt + dW_t. \quad (9)$$

From this last equation, we can rewrite the process for fundamentals as

$$\frac{dX_t}{X_t} = \bar{\nu}(\pi_t) dt + \Sigma d\tilde{W}_t \quad (10)$$

The vector processes in (10) and (6) fully characterize the economic environment, now conditional on agents information \mathcal{F}_t . Note that the same 4×1 vector of Brownian motions drive both fundamentals and agents' beliefs. In particular, the quadratic diffusion term of (6) is endogenously due to Bayes law: Intuitively, when investors' current (prior) beliefs are spiked around one regime, then the diffusion $\Sigma_\pi(\pi_t) \approx 0$, that is, from Bayes rule we need large news ($d\tilde{W}$) to move the posterior probabilities. Vice versa, if current (prior) beliefs display large uncertainty (e.g. π_t uniform across regimes), then $\Sigma_\pi(\pi_t)$ is large, and even small news move the posterior probabilities. This variation in the sensitivity of posterior probabilities to news, and its dependence on current beliefs, is the key feature of the model.³

A. Money Illusion

Given CRRA preferences the real state price density is given by the marginal utility of consumption $e^{-\rho t} C_t^{-\gamma}$. Thus, the real and nominal stochastic discount factors (SDF) are:

$$\begin{aligned} \text{Real SDF} &= e^{-\rho(\tau-t)} \left(\frac{C_\tau}{C_t} \right)^{-\gamma}; \\ \text{Nominal SDF} &= e^{-\rho(\tau-t)} \left(\frac{C_\tau}{C_t} \right)^{-\gamma} \left(\frac{Q_t}{Q_\tau} \right) \end{aligned}$$

Rational investors use the real SDF to discount real quantities, while agents suffering from money illusion use the nominal SDF to discount real quantities. We follow Basak and Yan (2010) and assume the mixed SDF

$$\frac{M_\tau}{M_t} = e^{-\rho(\tau-t)} \left(\frac{C_\tau}{C_t} \right)^{-\gamma} \left(\frac{Q_t}{Q_\tau} \right)^\delta \quad (11)$$

³Alternative learning models with an exogenous time-variation of signal's precision would also generate a time varying variance of state variables. Such model would require assumptions about the exogenous time variation in signal precision, while in our model such variation is endogenous from the learning dynamics. We verify that the beliefs stemming from our model conform to evidence from Surveys in Section III.A

where $\delta \in [0, 1]$. The two extremes $\delta = 0$ and $\delta = 1$ correspond to the rational and fully illusioned investor, respectively. When $0 < \delta < 1$, we obtain an intermediate case in which the representative agent suffers from incomplete money illusion, in the sense that inflation only partly affects the real stochastic discount factor. We estimate the illusion parameter δ along with the other parameters in Section III.

Given (11), the price of any asset with real payoff D_τ is obtained by the pricing formula

$$P_t = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(\tau-t)} \frac{M_\tau}{M_t} D_\tau d\tau \right]$$

where the expectation is taken with respect to the agents' information set.

B. Stock and Bond Prices

From (11) the stochastic discount factor follows the process

$$\frac{dM_t}{M_t} = -r_{f,t} dt - \sigma_M d\tilde{W}_t, \quad (12)$$

where the market prices of risks are constant

$$\sigma_M = \gamma \sigma_C + \delta \sigma_Q \quad (13)$$

and the real rate is given by $r_{f,t} = \sum_{i=1}^n \pi_{it} k^i$ with

$$k^i = \rho + \gamma \kappa^i + \delta \beta^i - \frac{1}{2} \gamma (\gamma + 1) \sigma_C \sigma'_C - \frac{1}{2} \delta (\delta + 1) \sigma_Q \sigma'_Q - \gamma \delta \sigma_Q \sigma'_C \quad (14)$$

Exploiting (12), we obtain the following proposition:

Proposition 1. (a) The P/E ratio at time t is

$$\frac{P_t}{E_t}(\pi_t) = \sum_{i=1}^n G_i \pi_{it} \quad (15)$$

where the constants G_i are given in closed form in equation (34) in the Appendix.

(b) The price of a nominal zero-coupon bond at time t with time to maturity τ is

$$B_t(\pi_t, \tau) = \sum_{i=1}^n \pi_{it} B_i(\tau), \quad (16)$$

where $B_i(\tau)$ are functions of τ and are given in closed form in equation (37) in the Appendix.

In (a), we refer to each constant G_i as the *conditional P/E ratio*, as it represents investors' P/E valuation conditional on regime ν^i . As in the classic Gordon growth model, we find the conditional P/E ratio G_i depends on earnings' drift rate θ^i , the conditional real rate k^i , the equity premium $\sigma_M \sigma'_E$, as well as the payout ratio, which we assume constant. Since investors do not observe the regime ν^i , they weight each conditional P/E ratio G_i by its conditional probability π_{it} thereby obtaining (15).

Similarly, in (b) the bond price is a weighted average of the nominal bond prices that would prevail in each regime ν^i , $B_i(\tau)$, which we refer to as the *conditional bond price*. Again, since investors do not actually observe the current regime, they price the bond as a weighted average. Notwithstanding the regime shifts in drift rates, all asset prices follow continuous paths, a result of the continuous updating of beliefs.

Let the nominal log stock price be given by

$$\log(P_t^N) = \log(Q_t P_t) = \log(Q_t) + \log(E_t) + \log\left(\sum_{i=1}^n \pi_{it} G_i\right) \quad (17)$$

From Ito's Lemma, we obtain the following proposition:

Proposition 2. (a) The volatility of nominal stock returns is:⁴

$$\sigma^N(\pi_t) = \sigma_Q + \sigma_E + \frac{\sum_{i=1}^n G_i \pi_{it} (\nu^i - \bar{\nu}(\pi_t))' (\Sigma')^{-1}}{P/E(\pi_t)}. \quad (18)$$

(b) The volatility of nominal bond returns is

$$\sigma^B(\pi_t, \tau) = \frac{\sum_{i=1}^n B_i(\tau) \pi_{it} (\nu^i - \bar{\nu}(\pi_t))' (\Sigma')^{-1}}{B(\tau)}. \quad (19)$$

Intuitively, from the nominal price (17) there are three sources of variation in returns: shocks to fundamentals (Q_t and E_t) and shocks to beliefs ($\sum_{i=1}^n \pi_{it} G_i$). The volatility of stock returns in (18) reflects these different sources of variation. In particular, the last term in (18) is a learning-based, time-varying endogenous component that depends on beliefs π_{it} . To provide an intuition for (18), suppose that at some time t there are two regimes i and j

⁴With some abuse of terminology, we refer to the diffusion of a return process as its volatility. Equations (18) and (19) show the diffusion vectors of the return processes. Strictly speaking, the “volatility” of stock returns, for instance, is the scalar given by $\sqrt{\sigma^N(\pi) \sigma^N(\pi)'}.$

for which $\pi_{jt} \approx (1 - \pi_{it})$. In this case, we can rewrite (18) as

$$\sigma^N(\pi_t) \approx \sigma_E + \sigma_Q + \frac{(G_i - G_j) \pi_{it} (1 - \pi_{it}) (\nu^i - \nu^j)' (\Sigma')^{-1}}{P/E(\pi_t)} \quad (20)$$

Expression (20) shows that if the conditional P/E ratios in the two regimes are similar to each other, $G_i \approx G_j$, then the last term is close to zero, even when investors have a large uncertainty on which regime holds, i.e. even if $\pi_{it} \approx \pi_{jt} \approx 0.5$. Vice versa, if G_i and G_j are very different from each other, even mild uncertainty on the regimes may generate a high learning-induced volatility, especially if $P/E(\pi_t)$ is small. This special case highlights that volatility depends on a complex interaction of uncertainty across regimes (the term $\pi_{it}(1 - \pi_{it})$ in (20)) and market participants' price valuations of those regimes (the term $(G_i - G_j)$ in (20)). The volatility dynamics are more complex when investors give positive probability to many regimes, but the general intuition is similar.

The form of the bond's volatility in (19) is similar to the stock's volatility in (18), except that the conditional P/E ratio G_i in any regime i is replaced by the conditional bond price $B_i(\tau)$ in that regime. In addition, there is no exogenous fundamental component.

The two diffusion terms (18) and (19) are the heart of this paper. From these, we can easily derive the covariances between stocks and bonds of different maturities as

$$\text{Cov} \left(\frac{dP_t^N(\pi_t)}{P_t^N(\pi_t)}, \frac{dB_t(\pi_t, \tau)}{B_t(\pi_t, \tau)} \right) = \sigma^N(\pi_t) \sigma^B(\pi_t, \tau)' \quad (21)$$

While it is hard to place a sign on the stock-bond covariance in general, we can do so at points of time when only two of the n regimes have positive probability. In particular, we are interested in finding intuitive sufficient conditions for the covariance to be positive or negative. Such conditions will be useful to interpret the empirical results in Section II.

Proposition 3. Let there be 2 regimes i and j such that at date t $\pi_{it} + \pi_{jt} = 1$. Then the covariance between stocks and bonds can be decomposed as

$$\begin{aligned} \text{Cov} \left(\frac{dP^N}{P^N}, \frac{dB(\pi_t, \tau)}{B(\pi_t, \tau)} \right) &= \frac{(B_i(\tau) - B_j(\tau)) \pi_{it} (1 - \pi_{it}) [(\theta_i - \theta_j) + (\beta_i - \beta_j)]}{B(\pi_t, \tau)} \\ &\quad + \frac{(B_i(\tau) - B_j(\tau)) (G_i - G_j) (\pi_{it} (1 - \pi_{it}))^2 \times c_0}{B(\pi_t, \tau) \times P/E(\pi_t)} \end{aligned} \quad (22)$$

and $c_0 = (\nu_i - \nu_j) (\Sigma \Sigma')^{-1} (\nu_i - \nu_j)' > 0$ is a constant.

The first component of the stock-bond return covariance in equation (22) captures the covariance between bond returns and the fundamental volatility of stock returns, namely, $\sigma^B(\pi, \tau) (\sigma_E + \sigma_Q)'$. The second component in (22) captures instead the learning effect. To further the intuition assume without loss of generality that the growth rate in regime j is smaller than in regime i : $\theta_j < \theta_i$. Assume also that, quite naturally, the conditional price/earnings ratio is smaller in the regime with lowest growth: $G_j < G_i$. The following corollary provides a further characterization:

Corollary 1. Let $\theta_j < \theta_i$ and $G_j < G_i$. The covariance between stocks and bonds returns is

- (a) negative if $\beta_j < \beta_i$ and $B_j(\tau) > B_i(\tau)$;
- (b) positive if $\beta_i < \beta_j < \beta_i + (\theta_i - \theta_j)$ and $B_j(\tau) < B_i(\tau)$.

Condition (a) in Corollary 1 assumes that the low growth regime j corresponds to low inflation and, quite naturally, the nominal bond price is thus higher in such a regime, $B_j(\tau) > B_i(\tau)$. In this case, both covariance terms in (22) are negative. The first term is negative because negative shocks to earnings or inflation not only decrease directly the stock price (as $P_t^N = E_t Q_t \left(\sum_{j=1}^n \pi_{jt} G_j \right)$), but they also increase the probability to be in regime j (low inflation), which in turn pushes up the bond price, as $B_j(\tau) > B_i(\tau)$. The second term is negative because an increase in the probability of being in regime j also reduces the P/E ratio itself (as $G_j < G_i$), exactly when the bond price increases (as $B_j(\tau) > B_i(\tau)$).

Condition (b) assumes the low growth regime j now corresponds to a high inflation regime, $\beta_j > \beta_i$ – although not too high – as well as again that the bond price in high inflation regime is lower than in the low inflation regime, $B_j(\tau) < B_i(\tau)$. Under these conditions, both covariance terms in (22) are positive. The intuition is similar as in case (a): a negative shock to earnings decreases the stock price (directly because earnings are lower and indirectly because P/E is also lower), but it increases the probability to be in regime j which now has high inflation. This change in belief push down bond prices, and generates a positive covariance. Compared to case (a), though, there is a little difference in intuition in what pertains to inflation shocks, which explains why β_j has to be smaller than the upper bound $\beta_i + (\theta_i - \theta_j)$. Indeed, a positive shock to inflation has the direct effect of pushing the nominal stock price P_t^N up (through Q_t), but also to push the nominal bond price *down*, as investors increase the probability to be in a high inflation state. An upper bound on β_j limits the impact of this inflation learning effect and thus ensures a positive covariance.

The case with multiple regimes is more complex, and recall that (a) and (b) are only intuitive sufficient conditions, but not necessary. The next section contains a numerical example that puts together numerous effects in still a simplified setting. These results are useful to understand the intuition of the empirical results in later sections.

C. A Numerical Example

The formulas in Propositions 1, 2, and 3 highlight that the regime shift model with learning generates strongly non-linear relations between stocks and bonds prices, their volatilities, and their cross-covariances, which can also flip signs depending on conditions. Before turning to the estimation, however, we illustrate such effects within a simplified example.

Let there be two real growth regimes with $\theta^L < \theta^H$, $\kappa^L < \kappa^H$, and three inflation regimes $\beta^L < \beta^M < \beta^H$. Suppose that recessions occur either in high inflation regimes (e.g. the 1980s) or in low/negative inflation regime (e.g. the Great Depression). That is, there are only three *composite* regimes, paired as follows $\nu^1 = (\theta^L, \kappa^L, \beta^L)$, $\nu^2 = (\theta^H, \kappa^H, \beta^M)$ and $\nu^3 = (\theta^L, \kappa^L, \beta^H)$. To yield simple interpretable closed-form pricing formulas, assume here that there are no regime shifts, i.e. $\Lambda = 0$. It follows that the conditional P/E ratios are:

$$G_1 = \frac{c}{K + \gamma\kappa^L + \delta\beta^L - \theta^L}; \quad G_2 = \frac{c}{K + \gamma\kappa^H + \delta\beta^M - \theta^H}; \quad G_3 = \frac{c}{K + \gamma\kappa^L + \delta\beta^H - \theta^L}$$

where $K = \rho - \frac{1}{2}\gamma(\gamma + 1)\sigma_C\sigma'_C - \frac{1}{2}\delta(\delta + 1)\sigma_Q\sigma'_Q - \gamma\delta\sigma_Q\sigma'_C + (\gamma\sigma_C + \delta\sigma_Q)\sigma'_E$, and c is the dividend/earnings payout ratio.

To simplify further, assume $\kappa^L \approx \kappa^H$, so the main difference among the conditional P/E ratios is due to inflation and expected earnings growth. Even if both regimes 1 and 3 have low real earnings growth θ^L , note that $G_3 < G_1$: Because of money illusion, the conditional P/E ratio during stagflation is lower than during low inflation, although real growth is the same. Assume further than the high growth rate θ^H is sufficiently high such that the conditional P/E ratio is highest in this regime. We thus have the ordering $G_2 > G_1 > G_3$.

Similarly, conditional bond prices are

$$B_1(\tau) = e^{-(\gamma\kappa^L + (1+\delta)\beta^L + J)\tau}; \quad B_2(\tau) = e^{-(\gamma\kappa^H + (1+\delta)\beta^M + J)\tau}; \quad B_3(\tau) = e^{-(\gamma\kappa^L + (1+\delta)\beta^H + J)\tau}$$

where $J = \rho - \frac{\gamma}{2}(1 + \gamma)\sigma_C\sigma'_C - (1 + \rho)\gamma\sigma_C\sigma'_Q - \frac{(1+\rho)(2+\rho)}{2}\sigma_Q\sigma'_Q$. In this case, if $\kappa^L \approx \kappa^H$, then $B_1(\tau) > B_2(\tau) > B_3(\tau)$, as higher inflation lowers the conditional bond price.

The rankings of conditional P/E ratios and conditional bond prices are different in the

three regimes. In particular, if there is uncertainty between boom and recession, it matters greatly whether the recession is accompanied by high inflation or low inflation. In the former case, we have $G_2 > G_3$ and $B_2(\tau) > B_3(\tau)$. Thus, from Corollary 1 we should expect a positive correlation between bond and stock returns. In contrast, if there is uncertainty between boom and recession, but the latter is accompanied by low inflation, we have $G_2 > G_1$ and $B_2(\tau) < B_1(\tau)$, and thus Corollary 1 implies a negative correlation.

With only three regimes we effectively have a two factor model, as the three probabilities must sum to one. The low dimensionality of the economy in this example allows us to illustrate its properties by plotting three dimensional surfaces on the probability simplex, which we do in Panel A - F of Figure 2. In each panel, we label the corners of the simplex according to their characteristics: High Growth/Medium Inflation (HG, MI), Low Growth/High Inflation (LG, HI), and Low Growth/Low Inflation (LG, LI).

Panel A plots the P/E ratio, which is highest in the (HG, MI) regime, the lowest in the (LG, HI) regime, and somewhat in the middle in the (LG, LI) regime. As mentioned, the difference in P/E ratio in the two Low-Growth regimes is due to money illusion. Panel B plots the yield of the 5-year zero coupon bond, which is increasing in expected inflation, and thus is highest in (LG, HI), intermediate in (HG, MI), and lowest in (LG, LI) regime.

Panel C shows the volatility of stock returns. As it can be seen, the volatility is the smallest at the corners of the simplex. However, in real data, such corners are typically not reached as learning takes time and regime changes make the posterior beliefs move away from the corners. Return volatility is highest when there is uncertainty between regimes (HG, MI) and (LG, HI) than when there is uncertainty between any other two pairs. The reason is that the conditional P/E ratios are most apart from each other for the former two regimes than any other pair of regimes, and thus the result follows from the discussion after equation (20). Comparing the shape of volatility in Panel C with the P/E ratio in Panel A, we see that as we move to the (HG, MI) regime, the P/E ratio increases and volatility decreases. That is, in this example, booms are correlated with low volatility. However, we also note that there are areas in which both the P/E and volatility increase, if good news on economic growth are correlated with an increase in uncertainty, for instance.

Panel D shows the volatility bond returns. Once again, we see large non-linearities compared to the yield in panel B. In particular, while lower yields are correlated with lower volatility in general, we also see areas in which higher bond return volatility occurs when yield are quite low, especially around the (LG, LI) corner. If beliefs hover around this corner, we should observe a positive relation between bond return volatility and yields.

Finally, Panels E and F report the covariance and correlation between stock and bond returns. Focusing the discussion on Panel F for simplicity, the correlation is clearly positive in the area closer to (LG, HI) corner. That is, if beliefs are moving around a period of low growth and high inflation, stocks and bonds are moving in the same direction. As explained, positive shocks to inflation increase expected inflation (which decrease bond prices) and decrease expected future earnings (which decrease stock prices.) In sharp contrast, the correlation turns negative around the edge between (HG, MI) and (LG, LI) corners. As explained in Corollary 1, the reason is that in this case positive inflation shocks, which decrease bond prices, are signals that the dreaded (LG, LI) regime may be averted, which increase stock prices. The correlation between stocks and bonds is then negative.

Figure 3 plots the second moments described in Panels C to F against the log P/E ratios and 5-year yield in Panels A and B. As it can be seen, even in this simple setting with only three composite regimes, the relation between volatilities, covariances and asset prices is strongly non-linear. For instance, in Panel A stock return volatility may be increasing or decreasing in log P/E as well as in long-term yields. Covariances and correlations in Panels C and D are also non-linearly related to prices and yields, and they turn negative for low yields and medium high log P/E. Indeed, from Panel F of Figure 2 the negative correlation occurs for probabilities between (HG, MI) and (LG, LI): The first regime has high P/E and medium yields, while the second regime has low P/E and low yields, yielding the result. These non-linear relations imply, for instance, that simple linear regression of second moments on asset prices would not capture all of the information that the latter contain.

II. Estimation Methodology

A. Data

Aggregate earnings for the economy are approximated as the operating earnings of S&P 500 firms, and these data are obtained from Standard and Poor's.⁵ Similarly, the aggregate P/E ratio is estimated as the S&P 500 index at the end of quarter divided by operating earnings. We use the Consumer Price Index (CPI), obtained from the Federal Reserve Bank of St. Louis, as our inflation series, which is also used to discount nominal earnings. We

⁵Operating earnings typically exclude certain expense or income items that are nonrecurring or unusual in nature, such as restructuring charges and capital gains/losses on unusual asset sales, and are hence used in industry to assess the long term fundamentals of firms. For example, I/B/E/S uses this concept of earnings in analyst surveys. Before 1988, only 4-quarter moving averages of earnings are available from Standard and Poor. For consistency and to deal with seasonalities, we compute 4-quarter moving average of earnings also for the remaining 1988-2010 period. Such earnings are used in the construction of the P/E ratio. Finally, we winsorize the real earnings growth data at the 1% level to reduce the impact of extreme observations, as observed in the recent crisis.

fix the dividend/earnings payout ratio to $c = 0.5$. The time series of daily stock returns is obtained from the Center for Research in Security Prices (CRSP), and the time series of daily zero-coupon yields are from Gurkaynak, Sack, and Wright (2007). Realized volatilities of stocks and bonds are estimated as squared average of daily returns in any given quarter. Returns are not demeaned, although the demeaning of the series does not significantly affect our results. BlueChip forecasts of aggregate earnings growth are from Buraschi and Welhan (2013). All other survey data are from the Survey of Professional Forecasters (SPF) from the Federal Reserve Bank of St. Louis.

One important issue is the definition of real consumption growth to use as fundamental variable in the estimation procedure. While historically the “risk premium” puzzle literature concentrated on aggregate real non-durable consumption and services obtained from the National Income and Products Accounts (NIPA) from the Bureau of Economic Analysis, the more recent literature has questioned whether these consumption data are in fact appropriate for asset pricing calculations. The skepticism stems from numerous sources: First, NIPA quarterly data have been shown to be “managed” and smoothed using numerous filters, which substantially dampens its volatility and increases its autocorrelation (see Savov (2011)). Second, aggregate consumption may poorly reflect the consumption of the representative investor in asset pricing models. For instance, low stock market participation hints at the necessity to use stock-holder consumption in asset pricing tests (see e.g. Vissing-Jorgensen(2002), Malloy, Moskowitz, and Vissing-Jorgensen (2009)). Finally, perceived consumption volatility may be higher than the realized volatility of NIPA consumption due to fat tails or potential crashes (Barro (2006), Weitzman (2007)).

In this paper, we do not take a stand on this issue and rather proceed through a different route. Namely, we explicitly take into account that we, as econometricians, cannot observe the true consumption of the representative investor. However, we can exploit NIPA consumption growth as a noisy *signal* (for us) about the true consumption growth of the representative investor. More formally, keeping the continuous time notation for convenience, we treat the NIPA consumption growth $\left(\frac{d\widehat{C}_t}{\widehat{C}_t}\right)$ as a signal for the true unobservable (to us) consumption growth (dC_t/C_t) , a signal that we model simply as a linear regression:

$$\left(\frac{d\widehat{C}_t}{\widehat{C}_t}\right) = \alpha_0 + \alpha_1 \left(\frac{dC_t}{C_t}\right) + \sigma_N dW_{N,t} \quad (23)$$

where $dW_{N,t}$ is a Brownian motion uncorrelated with other stochastic variables. Because the representative agent’s consumption growth dC_t/C_t affects the state price density and

thus is reflected in asset prices, the estimation procedure described in Section II.B allows us to estimate the coefficients α_0 , α_1 and the signal noise σ_N . Clearly, the characteristics of the consumption process that we estimate will also reflect the preferences we employ in the model, but given the widespread use of CRRA utility in asset pricing, this choice offers a solid benchmark to evaluate the empirical results, and assess any potential bias.

Our sample runs from 1958 to 2010. Our filtering process is started by using the stationary beliefs implied by the parameter values at the starting date. Since investors' beliefs at the initial date are likely influenced more strongly by most recent data received, as is standard in Bayesian econometric methods, we use a burn-in period of 8 quarters and thus report all results for the sub-sample from 1960-2010. Daily Treasury yields are only available since 1961Q3, and thus the sample for second moments of bonds only runs from 1962 to 2010. Finally, the Survey of Professional Forecasters' dataset starts in 1968Q4, although some variables are available for a shorter sample.

B. Estimation Methodology

We use both fundamental and financial variables to estimate the model's parameters and the time series of investors' beliefs over fundamental composite regimes. For fundamentals, we use inflation, real earnings growth, and real NIPA consumption growth. For financial variables, we employ the time series of the S&P500 P/E ratios, the 3-month Treasury bill rate, the 1- and 5-year Treasury bond yields, as well as, the stock return volatility, the 1- and 5-year bond return volatility, and their cross-covariances. Thus, inference is based on three fundamental time-series and nine financial time series. We employ a Simulated Method of Moments (SMM) method for inference, as described next.

Let Ψ denote the set of structural parameters in the fundamental processes of consumption, inflation, earnings, signals, and NIPA consumption in equations (2) to (5), and (23). We denote by \mathcal{L} the likelihood function for the fundamentals data observed at discrete points of time (quarterly), which we compute by simulating several sample paths of the state variables in small discrete subintervals using the Euler discretization scheme [see e.g. Brandt and Santa-Clara (2002)].

The discretization to small subintervals approximate our continuous time specification, which is required to obtain closed form formulae for the second moments of stocks and bonds. The latter are key input to our empirical strategy. Indeed, we then use the pricing formulae for the P/E ratio and Treasury bond prices in Proposition 1, their volatilities in equations (18) and (19), and covariances in (21) to generate model-determined moments. Let $\{e(t)\}$

denote the errors of the pricing and volatility variables, and define $\epsilon(t) = (e(t)', \frac{\partial \mathcal{L}}{\partial \Psi}(t)')'$, where the second term is the score of the likelihood function of fundamentals with respect to Ψ . We minimize the usual SMM objective, given in (49) in the Appendix. The details are in the Appendix.

It is worth emphasizing three key aspects of our choice of the SMM method of estimation. First, a simulation-based approach is necessary in our case since the likelihood function for the fundamental data observed at discrete points in time is not available in closed-form. Moreover, simulations allow us to construct the likelihood function of the data that we actually observe, such as the four-quarter moving average of earnings growth that we have available. By simulating four-quarter moving averages of earnings growth when we build the likelihood function, we can address concerns related to time aggregation, for instance. Second, our SMM approach allows us to account for the fact that the econometrician observes only three fundamental variables, while investors in addition observe their true consumption and signals about earnings, and hence update their beliefs about fundamental drifts based on a finer information filtration. The simulation procedure ensures that we respect the dynamics of beliefs that are implied by investors' filter, explicitly given in (6), that we use observable fundamental shocks (inflation, earnings, and consumption) as the main drivers of such beliefs, and that we thus formulate the marginal likelihood of the data as perceived by the econometrician. We note that this SMM procedure implies that beliefs are mainly driven by observable fundamental shocks, providing an economic interpretation to our state variables ("beliefs"), as it will be apparent in Section III.A.

Finally, the SMM simulation approach easily deal with the strong non-linearities embedded in the Bayes updating formula for beliefs (equation (6)) and in the conditional second moments of asset prices (equations (18), (19), and (21)). This capability allows to combine information in asset price and volatility moments with the information in fundamental data so that the extracted investors' beliefs are potentially quite different from estimation methods that rely only on fundamental information [see, for example, Hamilton (1989)].

B. Number of Regimes

Before we discuss our empirical results, we comment on our methodology to choose a proper number of regimes. As is well known, formal tests on the number of regimes are difficult and lack power (see e.g. discussion in Hamilton (2008)). Moreover, such tests only rely on fundamental variables, and not on asset prices, and thus they are not suited to deal for instance with "Peso Problems", that is, the fact that asset prices depend on potential regimes that may not occur in sample. Given our interest in estimating the model using both

fundamentals and asset prices, we must take this problem into account. Because our SMM estimation combines in one vector both scores from the likelihood function and pricing errors from financial variables, we follow Gray (1996) and Bansal and Zhou (2002) and make use of the GMM-based χ^2 criterion to determine an appropriate number of *composite* regimes. We settle on six composite regimes, whose parameters are discussed in the next section.

III. Estimation Results

Table 1 contains the estimates of our model. Panel A reports the preferences parameters. Both the time discount coefficient $\rho = 1.965\%$ and the risk aversion coefficient $\gamma = 10.56$ are not dissimilar from findings in the recent literature. The money illusion coefficient is $\delta = 0.8084$, which highlights that nominal quantities directly impact the real stochastic discount factor. Indeed, from (14) we obtain that the real rate of interest is related to expected inflation, a finding consistent with e.g. Evans (1998). In subsection IV.C we discuss the results when we do not impose money illusion.

Panel B reports the six *composite* regimes. Our estimates reveal that three composite regimes have very low, in fact negative, earnings growth ($\theta^1 = -5.18\%$), two regimes have medium earnings growth ($\theta^2 = 3.26\%$), and one regime has very high earnings growth ($\theta^3 = 5.41\%$). We denote these three growth rates as LG, MG, and HG, respectively. Panel B also shows that negative earnings growth is accompanied by either medium (MI) or high (HI) inflation ($\beta^3 = 4.67\%$ or $\beta^4 = 10.19\%$), or (close to) zero inflation (ZI) $\beta^1 = 0.43\%$. In contrast, low inflation (LI) $\beta^2 = 2.53\%$ occurs only during booming periods (MG and HG).

Notable in Panel B is that the drift rates of the representative agent's consumption (dC_t/C_t) are equal to each other, $\kappa^i = 2.04\%$ across the six regimes. Left unconstrained, the SMM procedure led to very small differences in consumption drifts across regimes, visible only at the 3rd decimal point, and thus we opted to constrain them to be equal to each other. That is, the representative agent's true consumption growth is well described by a simple i.i.d. process with no regime changes. While this finding contrasts with some mild predictability observed in NIPA consumption, it is instead consistent with recent empirical evidence that uses alternative proxies for consumption growth: For instance, Savov (2011) finds that "garbage" consumption growth displays no serial correlation at the annual frequency. Similarly, Malloy, Moskowitz, and Vissing-Jorgensen (2009) stockholder's consumption growth displays much weaker serial correlation than NIPA consumption.

Panel C of Table 1 reports the diffusion matrix of the fundamental variables. Earnings' volatility, at 10.5% per year, is far more volatile than inflation, at 3.31%. The two series are

instantaneously negatively correlated (correlation = -56%). The additional signal on earnings drift is slightly less informative than earnings growth itself, as it has a larger volatility (12%). Including the signal in the estimation enables us to better calibrate investors' belief response to earnings shocks, which affects all asset volatilities.

We estimate the volatility of the representative agent's real consumption growth at 6.34%, which is much higher than the NIPA consumption volatility during our sample (= 1.22%). Recall the relation between NIPA consumption and true consumption growth of the representative agent (unobservable to the econometrician) is given by equation (23) whose estimated regression coefficient $\alpha_1 = 0.18$ suggests that NIPA consumption growth is far smoother than the representative agent's consumption growth. To put our estimated volatility in perspective, Savov (2011) reports a volatility of "garbage consumption" of 2.88%, while Malloy, Moskowitz, and Vissing-Jorgensen (2009) find stockholders' consumption growth has 6.05% volatility (and 18.5% for the top 30% of the distribution). Recent literature also suggests to consider additional sources of consumption, such as durables (volatility 5.56%, e.g. Yogo (2006)) or luxury goods (volatility ranging between 6% and 36%, e.g. Ait-Sahalia, Parker, and Yogo (2004)), together with non-homothetic preferences. Finally, fat tails in the consumption distribution, as in Barro (2006) and Weitzman (2007) would also lead to an effective higher volatility (dispersion) of consumption growth.

Panel D of Table 1 reports the infinitesimal generator matrix Λ . In the interest of parameter parsimony, we restrict Λ to depend only six jump intensities $0 = \lambda_0 < \lambda_1 < \dots < \lambda_5$. The position of the λ_i in Λ was decided after a first estimation round with unconstrained parameters. Using the scores of the likelihood function and the errors of the price and volatility variables, we evaluate the SMM objective function, which serves as an omnibus test statistic [see for example Gray (1996) and Bansal and Zhou (2002)]. The overall SMM objective function value, which has a chi-squared distribution with 19 degrees of freedom, is 22.991, implying a p -value of 23.77%, so we fail to reject our model.⁶

B. The Dynamics of Beliefs

Figure 4 plots the time series of the conditional beliefs for each composite regime. The left-hand side panels report the growth regimes, while the right-hand side panels report the medium and high growth regimes. The visible patterns are in line with historical events: The low growth probabilities spike at around NBER recession bands, although in different

⁶The on-line appendix compares our 6 composite regime model with a 4-regime model with two inflation and two real earnings growth regimes. The 4-regime model is rejected (p -value = 2.77%). A 5-regime model similar to our 6-regime model but without the zero-inflation regime is also rejected (p -value = 4.19%).

panels depending on the time period. In the 1980s, the low-growth spike occurs in the high inflation and medium inflation regimes (Panels A and C), while in the last decade they occur in the zero-inflation regime (Panel E). The most common regime is the medium-growth, low inflation regime (Panel B). The very high growth regime, in Panel F, only saw a mild increase in probability in the late 1990s, and then again between the dot-com bubble crash and the next recession. The posterior probability of this high growth regime never exceeded 25%.

While the beliefs extracted from fundamentals and financial variables are reasonable and line up with historical events, we also compare them to the mean probability assessments from the Survey of Professional Forecasters (SPF). Long time series of such SPF probabilities are available only for a few series, one of which is inflation, specifically the growth rate of GDP deflator (GNP deflator before 1992), and another is the probability of a recession. The Appendix discusses the construction of such series, but in a nutshell, SPF also asks forecasters to provide their probability assessment that the variable of interest (e.g. inflation) will be in given intervals in following year. SPF then aggregates the data and provides the average probabilities across forecasters for each interval. Panels A to D of Figure 5 compare the marginal probabilities of each inflation regime (HI, MI, LI, ZI) from our model (black lines) with the SPF probabilities (grey lines). Keeping in mind that the SPF probabilities are about next year's inflation and not of inflation regimes, it is still reassuring to see that the correlations across the four panels range between 79% to 49%.⁷ Panel E reports instead the SPF probability to be in a recession next quarter together with the marginal probability of low growth (LG) from the model. Once again, the two series have a high correlation of 48%.

Panel A of Figure 6 plots realized inflation during the quarter (grey line) together with model's expectation (black solid line). As can be seen, the model's expected inflation was low in the early part of the sample, grew steadily up to early 1980s, move downward again in the 1990s, and it dipped close to zero occasionally in the last decade. The plot also reveals, however, that the model's expectation does not match the high CPI inflation of the late 1970s/early 1980s, but it is instead closer to the SPF consensus forecast of the GDP deflator (dashed line), the only survey-based inflation forecast from SPF available for the long sample. Panel A of Table 2 shows that a regression of CPI inflation on model's expected inflation gives an $R^2 = 52\%$, demonstrating that the model captures the main variation in

⁷There is by necessity some arbitrariness in the definition of the inflation ranges corresponding to High, Medium, Low, and Zero inflation in SPF probabilities. In Figure 5 we defined cutoffs for each inflation range as the middle point between the estimated inflation regimes, resulting in intervals $(-\infty, 1.47), (1.47, 3.59), (3.59, 7.43), (7.43, \infty)$. Small changes to these intervals do not change the results, and in fact often increase the correlations with model probabilities. For instance, a High Inflation cutoff of 8% instead of 7.43% increases the correlation in Panels A and B to 80% and 66%, respectively.

average inflation. Panel B shows that the model’s expected inflation is highly correlated with both the SPF GDP deflator expectation (93%) and the SPF CPI inflation expectation (89%), where the latter is only available from 1981.Q3.

Panel B of Figure 6 plots realized real earnings growth (grey line) together with the model’s expected earnings (black solid line). For comparison, we also plot the SPF real GDP growth (dashed line). The model’s expected earnings is much smoother than realized earnings, but a regression of realized earnings on model’s expected earnings still provides a $R^2 = 25\%$, as shown in Panel A of Table 2. Panel B of the Table shows that the model’s expected earnings is positively correlated with other proxies from surveys, such as SPF real GDP, SPF real corporate profits, BlueChip earnings growth, although such correlations are lower than in the case of inflation due to the higher volatility of real earnings.

B. The Dynamics of Asset Prices

We now discuss the model’s implications for the dynamics of asset prices. First, Panel B of Table 1 reports the model-implied conditional P/E ratios and bond yields across the six composite regimes. Recall that these are prices conditional on knowing which regime is currently in force. For instance, if investors knew for certainty that the regime was medium growth, low-inflation (MG,LI), then the P/E ratio would be 16.33, the 3-month yield 4.57% and the long-term yield 4.64%, demonstrating a rather flat term structure. As intuition would have it, higher growth generates a higher P/E ratio. The three low-growth (LG) regimes have different P/E ratios depending on the inflation regime, with higher inflation implying a lower P/E ratio. The difference is mainly due to money illusion, as the effective real rate increases with higher inflation regime, which pushes down the P/E ratio (a small effect also comes from different transition probabilities). The conditional bond yields also have an intuitive pattern, with higher yields in higher-inflation regimes. The slope of the term structure is negative in the high-inflation regime, positive in the zero-inflation regime, and otherwise only mildly positive or essentially flat.

In terms of dynamics, Panel C of Figure 6 shows that the model’s implied P/E ratio tracks well the realized P/E ratio, with the exception of the extreme valuations in the dot-com period. Indeed, as in the data, the model’s beliefs (in Figure 4) imply a model P/E ratio in the 15 - 17 range in 1960s/early 1970s, a much lower P/E around 10 in the late 1970s/early 1980s, and a steady increase to a P/E around 20 - 25 in the late 1990s, with an abrupt drop and bounce back both after the dot-com bubble and in the 2008 crisis. A regression of the realized P/E onto the model P/E reveals an R^2 of 53%, showing that the belief variations captures much of the variation in P/E ratios. Similarly, Panels A and B of

Figure 7 show that the model’s implied 3-month and 5-year yield track well their realized counterparts, with the exception of the 5-year yield missing the extremely high yields during the early 1980s. Regressions of realized yields on model-generated yields produces R^2 of 69% and 67% for 3-month and 5-year yields, respectively, showing that the beliefs dynamics in Figure 4 generate most of the variation in yields through the bond pricing formula (16).

Panels D of Figure 6, and C and D of Figure 7 show that the model-implied second moments (black lines) track well the data counterparts (grey line). Indeed, regressions of realized second moments onto the model’s counterparts gives R^2 of 36%, 51% and 38% for stock return volatility (ex-crash), the 5-year bond return volatility, and the covariance between stocks and bonds, respectively. We focus on the economics of these dynamics in the next section, but we notice here that the beliefs dynamics in Figure 4 not only are able to capture economically significant amounts of the variation in fundamentals (inflation and earnings) and prices (P/E ratios and yields), but also the variation in volatilities and covariances. That is, our model whose five state-variables (beliefs) are mainly driven by fundamental shocks (earnings, inflation, and consumption) contemporaneously capture the variation of eleven time series.⁸

IV. The Time Varying Signaling Role of Inflation

Our model provides a relatively simple economic explanation for the dramatic time variation of the conditional covariance between stocks and bonds documented in Panel A of Figure 1: Inflation news are positive or negative signals of future economic growth depending on investors current beliefs about the underlying composite regime.

To better quantify this mechanism and obtain predictions on empirically observable quantities, we perform two simulation exercises: First, we simulate 5000 years of quarterly data from the model using the parameter estimates in Table 1 and we use them to provide a number of implications about the dynamics of conditional moments and prices. Second, we also simulate 100 samples of 200 quarterly data each – the length of our sample – to glean the effect of small sample on the statistics. We then compare the results of simulations to the empirical data. In particular, in each of Tables 3 to 6, Panel A contains the results of the simulations for each parameter of interest, both the “population value” from the single run of 5000 years of data (the stand-alone coefficient) and the [5%-95%] percentiles from the 100

⁸Figures 6 and 7 only report eight series. In addition, we fit the model to the 1-year bond volatility, the covariance between stocks and 1-year bond returns, and the covariance across bonds. Results are not reported for brevity, but are similar to the 5-year bond returns. The model explains the dynamics of only eleven time series, and not twelve that we use, as real consumption growth turned out to be i.i.d.

simulated samples. Panel B reports the model’s predictions in the specific 1960-2010 sample, that is, it reports the results of the analysis where model quantities are used but fitted to the sample. This exercise is informative as it provides precise model’s predictions for the sample actually analyzed in the data. Finally, Panel C reports the results in empirical data.

A. Stock-Bond Covariance versus Fundamentals

We first consider the relation between the conditional covariance of stock and bond returns, and a number of fundamental-based explanatory variables suggested by our model. More specifically, we run in simulations and in the data the contemporaneous regression

$$(\text{Covariance})_t = b_0 + b_1 \mathbf{X}_t + \epsilon_t \quad (24)$$

where \mathbf{X}_t is a vector of explanatory variables, discussed below, and “(Covariance) $_t$ ” is either given by our theoretical formula in equation (21) (Panels A and B of Table 3), or is computed from daily stock and bond returns over the quarter (Panel C of Table 3). All the results in this section use the 5-year bond return to compute the covariance, although very similar results also hold for the 1-year bond.

1. Model’s Implications

The time-varying signaling role of inflation mechanism discussed earlier suggests that expected inflation should be an important driver of the time variation in the conditional covariance, and that expected real earnings growth should not have much of an impact. We thus run regression (24) with \mathbf{X}_t given by expected inflation and expected real earnings growth. Column (1) of Panel A of Table 3 shows that indeed in simulations expected inflation and expected earnings growth explain 46% of the variation in the stock-bond covariance (between 20% and 85% in 90% of the short samples). In fact, expected inflation alone explains over 45% of the stock-bond covariance while expected earnings only explain 6% (results not reported for brevity). The dominant effect of expected inflation is also evident in small samples: while the coefficient on expected inflation is always positive (in 90% of samples), the coefficient on expected earnings is both positive and negative. Such simulation results are in line with the earlier intuition about the time-varying signaling role of inflation.

To dig deeper, column (2) reports the results of regression (24) when the explanatory variables are “extreme probabilities of inflation”, that is, the probability of high inflation $P_{HI,t}$ and the probability of zero inflation $P_{ZI,t}$. These two probabilities by themselves explain 40% of the variation in the conditional covariance (between 20% and 92% in small

samples), and their opposite signs highlight that in the model much of the variation in conditional covariance has to do with extreme regimes. That is, an increase in P_{HI} increases the covariance while an increase in P_{ZI} decreases it to make it more negative.

Finally, in our model, uncertainty about fundamentals plays an important role. In the regressions, we define uncertainty as the posterior variance of inflation or earnings growth:

$$\text{UncInf}_t = E_t [(\beta - E_t(\beta))^2] = \sum_{i=1}^n \pi_{it} (\beta^i - \bar{\beta}_t)^2 \quad (25)$$

$$\text{UncEarn}_t = E_t [(\theta - E_t(\theta))^2] = \sum_{i=1}^n \pi_{it} (\theta^i - \bar{\theta}_t)^2 \quad (26)$$

As we see in column (3), inflation and earnings uncertainty have a high explanatory power for the conditional covariance. Inflation uncertainty enters positively and is the strongest driver of covariance as it explains over 72% of the variation by itself (result not reported). Earnings uncertainty, by contrast, on average decreases (i.e. makes it more negative) the covariance between stocks and bonds and does not explain as much of its variation. A similar result is also visible in the simple example in Section I.C. As discussed, from Panel E of Figure 2 uncertainty between (HG, MI) and (LG, LI) makes the covariance more negative, while the uncertainty between (HG, MI) and (LG, HI) makes the covariance more positive. However, covariance is also increasing in inflation uncertainty when the latter is between (LG, HI) and (LG, LI). Thus, on average, in the example, inflation uncertainty tends to increase covariance, while earnings uncertainty has more of an ambiguous effect.

The previous intuition suggests there should be a differential effect of earnings uncertainty when it occurs during high inflationary periods or during low inflationary periods. Column (4) of Panel A of Table 3 shows the result of regression (24) when we interact earnings uncertainty “UncEarn” with the extreme inflation probabilities $P_{HI,t}$ and $P_{ZI,t}$. As expected, the coefficient on $\text{UncEarn} \times P_{ZI,t}$ is strongly negative, while the coefficient of $\text{UncEarn} \times P_{HI,t}$ is positive, although its sign is not uniform in the [5%, 95%] interval. The R^2 for the long simulation increases to 84%, but importantly, the 5% lowest R^2 moves from 52% to 75% when we replace earnings uncertainty with its two components, showing the importance of distinguishing between high and low inflation periods..

Panel B of Table 3 shows that similar results as in Panel A hold in the model for the specific sample 1960-2010. In particular, column (1) shows that according to the model expected inflation and expected earnings should explain 74% of the variation on the conditional covariance between stocks and bonds, which is even higher than the population value

in Panel A, but still within the 90% bounds of the small samples distribution. This difference is likely due to the large increase in inflation in the mid 1970s as well as the deflationary bouts in the last decade, events that in this sample seem to be driving the stock-bond covariance by a great deal. Indeed, consistent with this interpretation, column (2) shows that the extreme inflation probabilities explain 72% of the variation in the conditional covariance between stocks and bonds, against 40% in population. Even in this shorter sample and consistent with the mechanism of the model, the extreme probabilities have opposite signs. Interestingly, column (3) shows that in the 1960 - 2010 sample, inflation and earnings uncertainty do not explain as much of the variation of the conditional covariance as fundamentals do (65% R^2 using uncertainty versus 72-74% using fundamentals). However, column (4) of Panel B also highlights that decomposing earnings uncertainty between high inflation and low inflation periods does generate an increase of the role of uncertainty, as the coefficient of the interaction $\text{UncEarn} \times P_{ZI,t}$ is significantly negative, and the adjusted R^2 jumps to 85%.

2. Empirical Tests

Moving to the empirical tests, Panel C of Table 3 runs the same regressions (24) but on empirical proxies for the various explanatory variables. Specifically, in columns (1a) to (1c) expected inflation is proxied by the SPF consensus forecasts of GDP deflator, which is the expected inflation series with the longest sample. For real earnings growth, we use the SPF consensus forecasts of real corporate profits (column 1a), real GDP (column 1b), or the probability of a boom (column 1c). Across all columns (1a) - (1c), expected inflation is significant (t-stats between 2.8 - 3.0), expected earnings – however proxied – is not significant, and overall R^2 's range between 19% and 24%. Column (1d) reports the regression results when we use the model's expectations as explanatory variables. Again expected inflation is significant while expected real earnings growth is not. The R^2 in this case is 26%.

Column (2a) - (2c) in Panel C test whether “extreme inflation probabilities” can explain the variation of the conditional covariance between stocks and bonds. Columns (2a) and (2b) proxy for the extreme probabilities using the SPF probabilities for high inflation and zero inflation in the following year ($t + 1$) and in the current year (t), respectively,⁹ while column (2c) uses model's fitted beliefs. As predicted by the model, the results show the two extreme probabilities have opposite signs, they are mostly significant and explain between 22% and 30% of the variation in the conditional covariance.

To test for the effect of uncertainty on the conditional covariance, we need to find suitable

⁹Inflation probabilities for the following year ($t + 1$) are available only after 1981.Q2. Before then, we still use the probability of the current year (t) instead of ($t + 1$).

proxies for uncertainty. While it is customary to use the dispersion in forecasters' expectations to proxy for uncertainty, such dispersion better proxies for difference of opinions, which we do not have in our model. In contrast, the availability of inflation probabilities from SPF allows us to directly compute a proxy for inflation uncertainty as the conditional variance of future inflation. In particular, as discussed in the appendix, from the original SPF data we compute the probabilities that inflation will lie in unit intervals from -4% to 16% (which are the extreme boundaries over the overall sample). In subsection III.A we aggregated such probabilities to compare them to the model-implied beliefs (see Figure 5). Here, we can use such probabilities to compute the conditional variance

$$\text{UncInf}_t = V_t[I] = \sum_{j=1}^n p_{j,t} (I^j - E_t[I])^2 \quad (27)$$

where I^j are midpoints of inflation unit intervals. One benefit of this procedure is that such proxy of inflation has the same units as the conditional variance used in simulations (see Equation (25)), and thus coefficients are directly comparable.

A similar procedure can be exploited to compute a proxy for real economic growth uncertainty starting from the probability of a recession. We note that because in this case SPF measures only the probability p_t of a decline in real GDP, without specifying the exact amount of the decline (or the increase in case of growth), we rely on the following proxy:¹⁰

$$\text{UncEarn}_t = p_t(1 - p_t) \quad (28)$$

Column (3a) uses these SPF-based uncertainty measures as proxies for inflation and earnings uncertainty. In addition, we also use in column (3b) the model-implied earnings and inflation uncertainty, already used in Panel B of the same table. In either case, inflation uncertainty is positive, significant, and with a similar coefficient as predicted by the model in Panels A and B. In contrast, earnings uncertainty has a negative coefficient and is insignificant. Moreover, with an R^2 of 13% and 18%, variation in uncertainty is important to explain the variation in stock and bond covariances, although not as important as fundamentals, consistently with the result in Panel B.

In column (4a) and (4b) we finally check whether the lack of power of earnings growth uncertainty is due to the mixing of its positive and negative impact on covariance depending

¹⁰We note that (28) is proportional to the conditional variance of GDP growth in the case of two regimes ("decline" and "no decline"), as $V_t[\theta] = \sum_{i=1}^2 p_{it} (\theta^i - E_t[\theta])^2 = p_{1t}(1 - p_{1t})(\theta^1 - \theta^2)^2$.

on whether we are in high inflation or low inflation periods. As was the case in column (4) of Panels A and B, we find in column (4a) that the interaction $\text{UncEarn} \times P_{HI,t}$ is significantly positive, $\text{UncEarn} \times P_{ZI,t}$ is significantly negative, and the adjusted R^2 increases to 37%. Using model probabilities in column (4b) yields similar, albeit weaker results. These results are consistent with the economic model proposed in this paper.

B. Stock-Bond Covariance versus Stock and Bond Prices

While the previous subsection looked at the direct impact of fundamentals to explain the time variation of the conditional covariance between stock and bonds returns, motivated by the example in Subsection I.C and especially Figure 3, in this subsection, we test the model’s predictions about the relation between conditional covariance, the P/E ratio and the 5-year yields. Indeed, insofar as the 5-year yield contains information about expected future inflation – an implication of our model – then it should explain the time series variation in conditional covariance between stocks and bonds as well.

We start by looking at simulated data. Column (1) of Panel A of Table 4 shows the regression results in simulations of model-implied conditional covariance on the 5-year yield and log P/E ratio. As can be seen, consistently with the time varying signaling role of inflation, the yield and the log P/E ratio explain about 41% of the time variation in the conditional covariance. Moreover, most of the predicting power comes from the 5-year yield, and indeed, in small samples, the coefficient on the 5-year yield is always positive, while the coefficient on log P/E has ambiguous sign. The adjusted R^2 increases to 46% when we include some non-linear terms, such as the square of the 5-year yield and log P/E ratio, and their product. Non-linear terms are to be expected given the results in Section I.C. Interestingly, in small samples, the [5% – 95%] intervals spans zero for all parameters, implying that the signs of the non-linear terms can be positive or negative. This implies that depending on the “type of sample,” different non-linear effects are picked up by the regression. However, non-linear terms are important. In fact, across the small samples, the 5% lowest *adjusted* R^2 more than doubles, moving from 16% to 34%, when we add non-linear terms, which demonstrates that even if across samples coefficients have different signs, they are both statistically and economically important in each sample.

In the previous section we documented that the conditional stock/bond covariance depends on inflation and earnings uncertainty. We use here the variance of stock returns as an alternative proxy of uncertainty. The simulation results are in column (3) of Panel A. In the long sample, the regression coefficient on the variance is positive, although the R^2 is just 35%. Indeed, in small samples, the coefficient on the variance is both positive and negative,

and the R^2 goes from 0 to 92%. That is, depending on the sample, everything can happen. The results are different if we interact the variance of stock returns with the probability to be in the high inflation regime or low inflation regime (column 4). In this case, consistently with the results of previous section, the interaction $\text{Stock Vol} \times P_{HI,t}$ is positive and the interaction $\text{Stock Vol} \times P_{ZI,t}$ is negative (although “weakly” in small samples). Important, with the interaction terms, the R^2 increases from 35% to 75% in the long simulations, and in small samples, the 5% lowest R^2 increases from 0% to 62%. This latter result underscores the need to control for high or low inflation to increase the explanatory power of stock variance.

Focusing now on the realized 1960-2010 sample, column (1) of Panel B shows that the fitted model predicts a strong relation between conditional covariance and the 5-year yield (t-stat 6.19), while also the log P/E is significant (t-stat 2.70). Column (2) also shows that in this sample, non-linear terms marginally increase the fit of the conditional covariance, with the squared log P/E ratio significant and the adjusted R^2 increasing from 67% to 70%. Interestingly, column (3) shows that in this 1960-2010 sample we should expect an unconditional negative relation between conditional covariance and stock return variance, although the R^2 is just 25%. However, interacting the stock volatility with extreme inflation probabilities increases the R^2 considerably to 77% (column 4), with both interaction terms $\text{Stock Vol} \times P_{HI,t}$ and $\text{Stock Vol} \times P_{ZI,t}$ significant and with opposite signs.

The empirical tests in Panel C largely confirm the predictions of the model. In column (1), the 5-year yield is strongly significant as an explanatory variable of the conditional covariance, highlighting that when expected inflation is high, and thus the yield is high, the covariance between stocks and bonds is also high. A drop in expected inflation that decreases the long term yields moves the conditional covariance lower, possibly negative as it was the case around the last two recessions. In addition, as predicted by the model, including non-linear terms increases the (adjusted) R^2 of the regression, and the squared log P/E enters significantly. We also note that with $R^2 = 47\%$, the yield and log P/E ratio have the highest explanatory power for conditional covariance than any of the fundamentals.

In column (3) we find that unconditionally, the covariance of bonds and stocks is negatively related to stock return variance. Similar results have been interpreted as evidence of “flight to quality,” according to which in times of troubles, investors dump stocks and buy Treasuries (see Connolly, Sun, and Stivers (2005)). However, such evidence is also consistent with the time varying signaling role of inflation. Indeed, consistently with the latter explanation, columns (4a) and (4b) show that the interaction of variance with the extreme inflation probabilities yield a substantial increase in the adjusted R^2 (from 20% in column

(3) to 38% and 27% in columns (4a) and (4b), respectively), with regression coefficients of opposite signs. The two economic explanations are clearly not competing with each other, but our model highlights that “flight-to-quality” must assume that Treasury bonds are in fact safe assets, which may not during stagflationary periods.

Panels C and D of Figure 8 highlight the extent of the non-linearities in the relation between conditional covariance and correlation of stocks and bonds and the long-term yield and log P/E in the data. In particular, the panels plot two dimensional Gaussian kernel regressions of realized covariances (or correlation) on log P/E and the 5-year yield. The plots show that the covariance and correlation turn negative for low-medium values of the 5-year yield and medium-high price/earning ratios, which, interestingly, was a characteristic of the simple example discussed in Section I.C (see Panels C and D in Figure 3).

In summary, the data strongly support the model’s predictions about the dynamic relation between the stock/bond covariance and both fundamentals and asset prices. We now discuss the impact of money illusion on the performance of the model.

C. Money Illusion versus Inflation Signals

Our results shed some light on the distinct roles that two competing hypothesis, the “money illusion hypothesis” of Modigliani and Cohn (1979) and the “proxy hypothesis” of Fama (1981), play in explaining the comovement of stocks and Treasury bonds. First, money illusion on its own tends to generate a positive covariance between stocks and bonds. Indeed, positive inflation shocks increase expected inflation, which lower bond prices, and also increase the effective discount rate, which decreases stock prices. Such a mechanism would explain the strong comovement of stocks and bonds in the late 1970s. However, the events in the last decade, which witness a strong negative covariance between stocks and bonds, show that the real-rate effect must be a small component of the covariation in stock and bond prices. Why then do we estimate a relatively high and significant coefficient of money illusion $\delta = 0.8084$ (see Table 1)? There are two reasons: First, money illusion implies a lower P/E ratio and a higher long-term yield when expected inflation is high, thereby helping the model match the level of prices and yields in the late 1970s. The effect of money illusion on the level of prices is shown in Panels A and B of Figure 9, which plots the fitted P/E ratio and yields with and without money illusion (we re-estimate the model in the latter case).

Second, Panel C of the figure also shows that the model with money illusion actually produces a stronger pattern of *negative* covariance during the last decade compared to the

model without money illusion, a finding that may appear puzzling at first given that money illusion per se' tends to generate a positive covariance between stocks and bonds. The reason is that money illusion generates different real rates across regimes, which increases the differences across conditional bond prices. As a consequence, money illusion has the indirect effect of increasing the impact of learning on bond prices, thereby generating a stronger positive covariance in the 1970s, and a stronger negative covariance in the last decade. Indeed, the volatility of bond returns is also higher compared to the case of no money illusion during the late 1970s and the last two recessions, as shown in Panel D. Thus, perhaps paradoxically, the money illusion not only helps match the level of bonds and stock prices in the 1970s, and their higher *positive* covariance, but also match the higher *negative* covariance in the last decade, as it works as an amplifier of the Bayesian learning effects.

V. Volatility, Price Valuations, and Fundamentals

In this section we show that the same mechanism that produces time variation in conditional covariance of stock and bond returns has also strong implications for the relation between return volatilities, and their relation to price valuations (yields and log P/E ratios).

A. Bond Volatility

As in earlier sections, we run both in simulations and in the data regressions of the type

$$(\text{Bond Volatility})_t = b_0 + b_1 \mathbf{X}_t + \epsilon_t \quad (29)$$

where \mathbf{X}_t is a vector of explanatory variables, and “(Bond Volatility) $_t$ ” is either given by our theoretical formula $\sqrt{\sigma^B(\pi_t, \tau)\sigma^B(\pi_t, \tau)'}$ in equation (19) (Panels A and B of Table 5), or is computed from daily bond returns over the quarter (Panel C). All the results in this section pertain to the 5-year bond, although very similar results also hold for the 1-year bond.

Panel A of Table 5 reports the simulation results. The explanatory variables we use are the same we described in Sections IV.A and IV.B. Column (1) shows that expected inflation and expected earnings have an impact on the volatility of long-term bond, albeit with opposite signs, with a “population” R^2 of 51%. Interestingly, expected earnings seems to have most of the impact, as in small sample its coefficient is always negative in the 5%-95% range of the distribution, and R^2 range between 27% and 84%. This result is consistent with a business cycle effect on bond volatility, that becomes higher when expected earnings are lower. While column (2) shows the extreme probabilities explain a large part of the variation in volatility, we also see in column (3) that the main drivers of bond volatility are

inflation and earnings uncertainty. Of the two, inflation uncertainty is the most important, as it explains over 86% of the variation on its on (result not reported).

From column (4), the simulations show that although the 5-year yield and log P/E ratio are useful to explain the variation in conditional volatilities, with a population R^2 of 37% they are far from perfect. Column (5) underscores the importance of non-linearities, as the 5% lower threshold of the adjusted R^2 moves from 9% with only linear elements to 32% with non-linear terms. These results are consistent with the intuition in Section I.C.

Panel B reports the same regression results but on model-implied quantities that are fitted to the specific 1960-2010 sample. As in simulations, expected earnings is a strongly significant explanatory variable (t-stat. of -6.56), as is though also expected inflation itself (t-stat of 4.63). In the 1960-2010 fitted data, the expected earnings and inflation explain about 77% of the conditional variation in bond volatility. We also see in column (2) that the probability of very high inflation P_{HI} is also strongly significant, (t-stat of 8.22), but the probability of zero inflation P_{ZI} is not. Also in this realized sample, uncertainty strongly affects bond return volatility (column (3)). Interestingly, we see in column (4) that while the log P/E ratio is a significant explanatory variable for bond return volatility (t-stat. of -3.32) with an R^2 of 53%, including non-linear terms does not increase much the fit. Indeed, all coefficients result insignificant and the adjusted R^2 increases modestly to 57%.

Turning to the empirical data in Panel C, and using the same proxies discussed in Sections IV.A and IV.B, we find that expected earnings is a significant explanatory variable for the conditional volatility of bond returns (columns 1b - 1d), except when the proxy is the consensus forecasts of real corporate profits (column 1a). R^2 's are in the area of 16% to 42%, the latter when model expectations are used. Extreme inflation probabilities are also significant (columns (2a) - (2c)), but P_{HI} is only marginally significant when it is proxied by SPF inflation probabilities. R^2 s are between 22% and 44%, the latter again when model beliefs are used. A similar result holds for uncertainty: earnings uncertainty is significant using SPF beliefs, and inflation uncertainty is significant using model's beliefs. R^2 are 24% and 45%, respectively.

Finally, column (4) shows only that bond volatility is only mildly related to the 5-year yield and the log P/E ratio when only linear regressors are used, but column (5) shows that the adjusted R^2 increases to almost 50% using non-linear regressors. These non-linear relations are predicted by the model. Especially noteworthy is the significant positive coefficient on " $Y(5)^2$ ", which shows that volatility of bond returns is high both when yields are high and when they are low, in contrast to classic models such as Cox, Ingersoll, and Ross (1985).

Indeed, as shown in Panel C of Figure 7, bond return volatility was certainly high in the mid 1970s, when yields were high, but the volatility was also high in the last decade, when yields were very low, a pattern that is consistent with the intuition of our model, as well as the empirical findings about conditional covariance in Section IV. Panel A of Figure 10 plots the 5-year rolling correlation of bond return volatility and yields, both in the data and fitted from the model, and show that the two series track well each other (correlation = 67%), showing that the model is consistent with this time variation. The non-linear relation between bond return volatility with respect to the long-term yield and the log P/E ratio is also visible in Panel B of Figure 8, which plots the result of a bivariate kernel regression of bond volatility on log P/E and long-term yield. From the figure, bond volatility dips to its lowest points for medium levels of log P/E and long-term yields.

B. Stock Volatility

As in earlier sections, we run both in simulations and in the data regressions of the type

$$(\text{Stock Volatility})_t = b_0 + b_1 \mathbf{X}_t + \epsilon_t \quad (30)$$

where \mathbf{X}_t is a vector of explanatory variables, and “(Stock Volatility) $_t$ ” is either given by our theoretical formula $\sqrt{\sigma^N(\pi_t, \tau) \sigma^N(\pi_t, \tau)'}$ in equation (18) (Panels A and B of Table 6), or is computed from daily stock returns over the quarter (Panel C).

Panel A of Table 6 shows the results in simulations. As is well known, volatility is very hard to explain in the data, and thus what is interesting of Panel A is not only what the model predicts about the relation between volatility and fundamentals, but also how difficult it is to explain volatility, notwithstanding our use of exact volatility formulas and no measurement errors. Indeed, column (1) shows that expected earnings and expected inflation are important drivers of volatility, but they only explain 22% of volatility in population (and the R^2 is as low as 7% with 95% probability in small samples). Similarly, extreme probabilities do not explain much of stock return volatility. Instead, inflation and earnings uncertainty explain 59% of volatility, although the [5%-95%] small sample interval for R^2 is very wide, between 22% and 97%.

Most interestingly, column (4) of Panel A shows that a linear regression of stock volatility on long-term yield and log P/E ratio does not explain much of the volatility in population (only 12%), with the 90% interval for R^2 between 4% and 72% in small samples. However, column (5) shows that non-linear terms are quite important: the population adjusted R^2 almost triples to 34%, and even in small samples, the 5% lowest adjusted R^2 increases to 21%

from 4% (and the highest reaches 94%). That is, we should find a special role of non-linear terms of yields and log P/E ratios as explanatory variables of stock return volatility.

Panel B of Table 6 shows that similar results pertain to the special 1960 - 2010 sample: In particular, the fitted model shows that volatility is explained by expected inflation and earnings (t-stats. of -3.10 and -5.00, respectively) and extreme probabilities P_{HI} and P_{ZI} (t-stats. of 2.21 and 6.55, respectively). Interestingly, in this 1960-2010 sample inflation and earnings uncertainty have low explanatory power for stock return volatility, even in the fitted model, as only earnings uncertainty is significant (t-stat 2.35) and the R^2 is just 26%. Column (4) shows that even the fitted model shows that a linear regression of volatility on the long-term yield and log P/E gives a low R^2 (= 6%) but that non-linear terms are very important in sample, as the adjusted R^2 increases to 35% and many non-linear terms are significant.

Panel C reports the same regression results in the empirical data. Consistently with previous literature, and the model, we find that return volatility is higher when expected earnings is lower (t-stats ranging between -3.56 to -2.15, R^2 ranging between 12% and 33%). Lower expected inflation also seems to have some explanatory power, consistently with Panel B, but only for two of the proxies. Extreme probabilities (column (2)) also explain the variation, although to a less of an extent than expected earnings growth. Interesting, the three SPF probabilities P_{HI}, P_{ZI} and P_{Boom} explain together 26% of the conditional variation of return volatility (result not reported). In column (3), earnings uncertainty enters strongly significantly (t-stats = 3.66) but with still a low $R^2 = 17\%$.

Finally, we see that a linear regression of volatility on the long-term yield and log P/E ratio results in insignificant coefficients and $R^2 = 0\%$, but that the inclusion of non-linear terms increases the fit to $R^2 = 17\%$ and most non-linear terms are significant. This result is in line with the predictions of the model. Although the R^2 is smaller than in the simulations, we should recall that simulations use our formula for instantaneous volatility, and not a noisy proxy such as the integrated volatility from daily observations.¹¹ Panel B of Figure 10 plots the 5-year rolling correlation of stock return volatility and the P/E ratio, both in the data and fitted from the model. The correlation between the two series is 36%. Most importantly, the figure shows that the model is capable of generating a time varying relation between volatility and log P/E, as shown in Section I.C. Indeed, the non-linear relation between return volatility with respect to the long-term yield and the log P/E ratio is also visible in Panel A of Figure 8, which plots the fitted volatility from a kernel regression of stock

¹¹Using realized volatility in simulations, $R^2 = 19\%$ in population and between 8% - 48% in small samples.

volatility on $\log P/E$ and long-term yield. The figure suggests in fact a significantly lower volatility for intermediate values of both $\log P/E$ and long-term yield, and higher volatility for the extremes of both.

VI. Out-of-Sample Forecasts

In this final section we show that our model not only provides a unified economic framework to understand the variation in the comovement of stocks and bonds, and their volatilities, but its quantitative nature also help predict future variation in volatilities and covariances. This feature of the model is important as it shows that the learning dynamics implied by our regime-shift model captures the variation in asset prices and volatilities at the proper frequency. One important problem in performing out-of-sample forecasts, however, is that we cannot implement a rolling estimation of our regime shift model, as our SMM procedure is very time consuming. We settle to re-estimate the model every five years, starting from 1984 (which is the middle point in our 1958 - 2010 sample). We keep throughout our six-regimes. For every set of parameters estimated at time t and given the current values of beliefs π_t , we then simulate the beliefs π_s , $s > t$ and use our closed-form formulas for volatilities and covariances, in expressions (18), (19), and (21), to determine model-forecasts. More specifically, for a given asset A , the optimal forecast of volatility between quarters T_1 and T_2 given the information that investors have at time t is

$$V^*(T_1, T_2; t) = \sqrt{E \left[\int_{T_1}^{T_2} \sigma^A(\pi_s) \sigma^A(\pi_s)' ds | \mathcal{F}_t \right]}. \quad (31)$$

Similarly, the optimal forecast of covariance of returns of assets A and B is given by

$$C^*(T_1, T_2; t) = E \left[\int_{T_1}^{T_2} \sigma^A(\pi_s) \sigma^B(\pi_s)' ds | \mathcal{F}_t \right]. \quad (32)$$

Table 7 provide out of sample forecasts of the model volatilities and covariances at three different horizons, 2-quarters, 4 quarters, and 8-quarters ahead. The table reports the R^2 of realized volatilities over forecasted volatilities, as well as the forecasts' mean absolute errors (MAE). In order to have a solid benchmark of comparison, columns 3 and 4 report the same statistics from a similar out-of-sample linear regression using a number of explanatory variables put forward in the literature, namely, the NBER recession indicator, lagged stock return when negative, the P/E ratio, the 3-month yield, the 5-year yield, the volatility of inflation and earnings, estimated from simple GARCH models.¹² Columns 6 and 7 also add

¹²We also added some quadratic terms in the benchmark specification but with no improvement.

lagged volatility or covariance in the group of controls. Finally, to compare quantitatively the model’s forecasts against the benchmarks we also report the t-statistics obtained from Diebold - Mariano tests (see Diebold and Mariano (1995)), which formally tests whether absolute errors under the model are lower or higher than under the benchmark alternatives.

Concentrating on the 4-quarter horizon, we see that our consumption-based model performs rather well compared to the benchmarks, as it produces a higher R^2 and lower MAE for most of the cases analyzed. In particular, the model performs especially well in forecasting the stock return volatility, the 5-year bond return volatility, and the covariance between stocks and 5-year bond returns. The R^2 are higher than the alternatives and the MAE are smaller. Interestingly, while the model does not fare as well as the benchmark for the 1-year bond return volatility and the covariance between 1-year bond and stocks under the MAE metric (DM tests significantly positive), we still do find higher adjusted R^2 under the model than under the alternatives. That is, the model may not capture the exact levels of the 1-year bond return volatilities / covariance with stocks, but with an out-of-sample $R^2 = 35\%$ the model seems to captures well their time variation.

VII. Conclusion

We show that the time varying comovement of stocks and bonds over the years can be mainly attributed to the changes over time of inflation shocks as signals about future real economic growth. The recessionary periods in the 1980s led to large positive comovement in stocks and bonds, while the reverse was true in the last decade. These changes in pattern across the two time periods are consistent with positive inflation shocks signaling a transition toward a stagflationary regime in the early 1980s, while signaling the avoidance of a deflationary regime in the last decade. In the first case, such shocks are bad news for the economy, while in the latter they are good news for the economy.

We propose a regime switching model with learning in which CRRA agents are affected by money illusion to explain the observed variations in the data. The key to the model is that agents learn about “composite regimes”, that is, each regime has a special pairing between inflation, earnings growth, and consumption growth. Indeed, our estimate of the model parameters reveal that the two extreme inflation regimes, very high and very low inflation, only occur with negative economic growth in the data. Thus, as investors increase the probability to move to either a very high or very low inflation regime, they also naturally increase the probability to move to a regime with negative economic growth. The fitted model is indeed capable of replicating the time variation in conditional covariance over time, and its relation to asset prices and fundamentals.

The main mechanism identified by the model provides several additional testable implications, such as that the covariance between stocks and bonds should be related to expected inflation, to the probabilities of extreme inflation (either very low or very high), to uncertainty, non-linearly to long-term yields and log P/E ratios. Using Survey of Professional Forecaster's data, we find considerable evidence in support of the predictions of the model.

In addition, the learning dynamics has numerous implications for the dynamics of the volatility of bonds and stock returns. In particular, the model shows that bond return volatility is non-linearly related to its long-term yield, a fact that implies that sometimes volatility increases with yields but sometimes decreases with yields. Similarly, the model shows that even in simulations and without estimation noise, it is hard to explain stock return volatility. Indeed, the model shows that the volatility of stock returns is non-linearly related to both the log P/E ratio and long-term yield. This implies that return volatility may be at times positively related to the log P/E ratio, as agents learn about a potentially very high growth rate of the economy. Empirical evidence support the predictions of the model.

Appendix

A. Proofs of Propositions

Proposition 1 (a) The P/E ratio at time t is

$$\frac{P_t}{E_t}(\pi_t) = \sum_{j=1}^n G_j \pi_{jt} \equiv G \cdot \pi_t, \quad (33)$$

where for each $j = 1, \dots, n$ the constant G_j is given by

$$G_j = c \mathbb{E} \left[\int_t^\infty \frac{M_s}{M_t} \frac{E_s}{E_t} ds \middle| \nu_t = \nu^j, \mathcal{F}_t \right] \quad (34)$$

where c is the payout ratio. In addition, the vector $G = (G_1, \dots, G_n)$ satisfies $G = c A^{-1} \cdot \mathbf{1}_n$,

$$A = \text{Diag}(k^1 - \theta^1 + \sigma_M \sigma'_E, \dots, k^n - \theta^n + \sigma_M \sigma'_E) - \Lambda. \quad (35)$$

(b) The price of a nominal zero-coupon bond at time t with maturity τ is

$$B_t(\pi_t, \tau) = \sum_{i=1}^n \pi_{it} B_i(\tau), \quad (36)$$

where the $n \times 1$ vector valued function $B(\tau)$ with element $B_i(\tau) = \mathbb{E} \left(\frac{M_{t+\tau}}{M_t} \cdot \frac{Q_t}{Q_{t+\tau}} \middle| \nu_t = \nu^i \right)$ is

$$B(\tau) = \Omega e^{\omega \tau} \Omega^{-1} \mathbf{1}_n. \quad (37)$$

In (37), Ω and ω denote the matrix of eigenvectors and the vector of eigenvalues, respectively, of the matrix $\hat{\Lambda} = \Lambda - \text{Diag}(r^1, r^2, \dots, r^n)$, where each $r^i = k^i + \beta^i - \sigma_M \sigma'_Q - \sigma_Q \sigma'_Q$, is the nominal rate that would obtain in the i^{th} regime, were regimes observable. In addition, $e^{\omega \tau}$ denotes the diagonal matrix with $e^{\omega_i \tau}$ in its (i, i) position.

Proof of Proposition 1. The proof of part (a) follows from an extension of the proof in Veronesi (2010), where $D_t = c E_t$, and it is contained in the on-line appendix for completeness. To prove part (b), consider the process of the nominal stochastic discount factor $N_t = M_t/Q_t$

$$\frac{dN_t}{N_t} = -r_t dt - \sigma_N dW$$

where $r_t = \kappa_t + \beta_t - \sigma_M \sigma'_Q - \sigma_Q \sigma'_Q$ and $\sigma_N = \sigma_M + \sigma_Q$. From the process for N_t we find

$$N_{t+\tau} = N_t \exp \left[\int_t^{t+\tau} -r_u - \frac{1}{2} \sigma_N \sigma'_N du - \sigma_N (W_{t+\tau} - W_t) \right]$$

Hence, the bond at time t with maturity τ is

$$B(\pi_t, \tau) = \mathbb{E} \left[\frac{N_{t+\tau}}{N_t} | \mathcal{F}_t \right] = \sum_{i=1}^n \mathbb{E} \left[\frac{N_{t+\tau}}{N_t} | \nu_t = \nu^i \right] \pi_{it}$$

Consider now a small interval Δ and define $B_i(\tau) = \mathbb{E} \left[\frac{N_{t+\tau}}{N_t} | \nu_t = \nu^i \right]$. We have:

$$\begin{aligned} B_i(\tau) &= \mathbb{E} \left[\frac{N_{t+\tau}}{N_t} | \nu_t = \nu^i \right] = \mathbb{E} \left[\left(\frac{N_{t+\Delta}}{N_t} \right) \left(\frac{N_{t+\tau}}{N_{t+\Delta}} \right) | \nu_t = \nu^i \right] \\ &= \mathbb{E} \left[\left(e^{(-r^i - \frac{1}{2} \sigma_N \sigma'_N) \Delta - \sigma_N (W_{t+\Delta} - W_t)} \right) \left(\frac{N_{t+\tau}}{N_{t+\Delta}} \right) | \nu_t = \nu^i \right] \\ &= e^{-r^i \Delta} \mathbb{E} \left[\frac{N_{t+\tau}}{N_{t+\Delta}} | \nu_t = \nu^i \right] \\ &= e^{-r^i \Delta} \left\{ (1 + \lambda_{ii} \Delta) B_i(\tau - \Delta) + \sum_{j \neq i} \lambda_{ij} \Delta B_j(\tau - \Delta) \right\} \\ &= e^{-r^i \Delta} \left\{ B_i(\tau - \Delta) + \sum_{j=1}^n \lambda_{ij} \Delta B_j(\tau - \Delta) \right\} \end{aligned}$$

Rearranging, we obtain that

$$\frac{B_i(\tau) - B_i(\tau - \Delta)}{\Delta} = \frac{e^{-r^i \Delta} - 1}{\Delta} B_i(\tau - \Delta) + e^{-r^i \Delta} \left\{ \sum_{j=1}^n \lambda_{ij} B_j(\tau - \Delta) \right\}$$

Taking the limit as $\Delta \rightarrow 0$, and rearranging

$$B'_i(\tau) = (\lambda_{ii} - r^i) B_i(\tau) + \sum_{j \neq i} \lambda_{ij} B_j(\tau)$$

In vector form $B'(\tau) = \hat{\Lambda} B(\tau)$ whose solution is (37). ■

Proof of Proposition 2. Follows from Ito's Lemma applied to the prices in Proposition 1.

■

Proof of Proposition 3. With only two regimes with positive probability, the diffusions

are

$$\sigma^N(\pi_t) = \sigma_E + \sigma_Q + \frac{(G_i - G_j) \pi_{it} (1 - \pi_{it}) (\nu^i - \nu^j)' (\Sigma')^{-1}}{P/E(\pi_t)} \quad (38)$$

$$\sigma^B(\pi_t, \tau) = \frac{(B_i(\tau) - B_j(\tau)) \pi_{it} (1 - \pi_{it}) (\nu^i - \nu^j)' (\Sigma')^{-1}}{B(\pi_t, \tau)} \quad (39)$$

Recalling that we can write $\sigma_E = \iota_E \Sigma$ and $\sigma_Q = \iota_Q \Sigma$, where ι_E and ι_Q are rows of the identity matrix corresponding to the earnings and inflation regimes. The result then follows directly from the covariance formula $Cov\left(\frac{dP^N}{P^N}, \frac{dB(\pi_t, \tau)}{B(\pi_t, \tau)}\right) = \sigma^N(\pi_t) \sigma^B(\pi_t, \tau)'$. Q.E.D.

B. SMM Estimation of the Regime Switching Model

The SMM procedure is similar to that in David (2008), but it is expanded to take into account the unique features of our model. In particular, (a) the model uses stock and bond volatilities, and covariances, as moments in the estimation; (b) there is a difference between the information sets of the econometrician and the investors, stemming from the observation of the additional signal about earnings' drift and their true consumption, compared to the econometrician's observation of NIPA consumption; (c) we address the issue of time aggregation in observed fundamentals, as Standard and Poor's provides the aggregate four quarter moving average of earnings, as opposed to the quarterly earnings. Here we make adjustments to the simulated likelihood function to account for the aggregation; (d) we impose long run equilibrium in the stock market by imposing equality in the long run growth rates of earnings and consumption; (e) we use only five parameters for the entire generator matrix to reduce the number of estimated parameters, as shown in Table 1. We follow a 2-step estimation methodology. In the first step we estimate using the full set of generator elements, and in the second step we group the elements according to their estimated values and reestimate the model with equality constraints estimated for generator elements within each group.

Our procedure carefully distinguishes between the information sets of investors and the econometrician. Investors beliefs evolve as they observe the vector X_t as in the belief process in (6). The econometrician's information set differs by a) not observing the signal process, and b) observing the smoothed consumption process (23).

Since the econometrician does not observe all fundamentals, we denote $Y_t = (\hat{C}_t, Q_t, E_t)'$, with

$$\frac{dY_t}{Y_t} = \varrho_t dt + \Sigma_3 d\hat{W}_t, \quad (40)$$

where $\hat{W}_t = (W_{1t}, W_{2t}, W_{3t}, W_{4t}, W_{Nt})'$ is the vector of shocks augmented by the noise in the

observed consumption process. $\frac{dY_t}{Y_t}$ is to be interpreted as “element-by-element” division, $\varrho_t = (\alpha_0 + \alpha_1 \kappa_t, \beta_t, \theta_t)'$, and $\hat{\Sigma}_3 = ((\alpha_1 \sigma_C, \sigma_N)', (\sigma_E, 0)', (\sigma_Q, 0)')'$. Since fundamentals are stationary in growth rates, we can write

$$dy_t = [\bar{\varrho}(\pi_t) - \frac{1}{2}(\alpha_1^2 \sigma_C \sigma_C' + \sigma_N^2, \sigma_Q \sigma_Q', \sigma_E \sigma_E')]dt + \hat{\Sigma}_3 d\hat{W}_t, \quad (41)$$

The specification of the system is completed with the belief dynamics in (6), which depends on the state vector X_t observed by investors.

The econometrician has data series $\{y_{t_1}, y_{t_2}, \dots, y_{t_K}\}$. Let Ψ be the set of parameters determining the fundamental processes of the model. We start by specifying the likelihood function over data on fundamentals observed discretely using the procedure in the SML methodology of Brandt and Santa-Clara (2002). Adapting their notation, let

$$\mathcal{L}(\Psi) \equiv p(y_{t_1}, \dots, y_{t_K}; \Psi) = p(\pi_{t_0}; \Psi) \prod_{k=1}^K p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi),$$

where $p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi)$ is the marginal density of fundamentals at time t_{k+1} conditional on investors' beliefs at time t_k . Since $\{\pi_{t_k}\}$ for $k = 1, \dots, K$ is not observed by the econometrician, we maximize

$$E[\mathcal{L}(\Psi)] = \int \dots \int \mathcal{L}(\Psi) f(\pi_{t_1}, \pi_{t_2}, \dots, \pi_{t_K}) d\pi_{t_1}, d\pi_{t_2}, \dots, d\pi_{t_K}, \quad (42)$$

where the expectation is over all continuous sample paths for the fundamentals, \tilde{y}_t , such that $\tilde{y}_{t_k} = y_{t_k}$, $k = 1, \dots, K$. In general, along each path, the sequence of beliefs $\{\pi_{t_k}\}$ will be different.

As a first step, we need to calculate $p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi)$. Following Brandt and Santa-Clara (2002), we simulate paths of the state variables over smaller discrete units of time using the Euler discretization scheme:

$$\tilde{y}_{t+h} - \tilde{y}_t = [\bar{\varrho}(\pi_t) - \frac{1}{2}(\alpha_1^2 \sigma_C \sigma_C' + \sigma_N^2, \sigma_Q \sigma_Q', \sigma_E \sigma_E')]h + \hat{\Sigma}_3 \sqrt{h} \tilde{\epsilon}_t; \quad (43)$$

$$\pi_{t+h} - \pi_t = \mu(\pi_t)h + \sigma(\pi_t)\sqrt{h}\tilde{\epsilon}_t \quad (44)$$

where $\tilde{\epsilon}$ is a 5×1 vector of standard normal variables, and $h = 1/M$ is the discretization interval. The Euler scheme implies that the density of the 3×1 fundamental growth vector y_t over h is trivariate normal.

We approximate $p(\cdot|\cdot)$ with the density $p_M(\cdot|\cdot)$, which obtains when the state variables are discretized over M subintervals. Since the drift and volatility coefficients of the state variables in (6) and (41) are infinitely differentiable, and $\Sigma\Sigma'$ is positive definite, Lemma 1 in Brandt and Santa-Clara (2002) implies that $p_M(\cdot|\cdot) \rightarrow p(\cdot|\cdot)$ as $M \rightarrow \infty$.

First consider the case where earnings are observed quarterly. The Chapman-Kolmogorov equation implies that the density over the interval (t_k, t_{k+1}) with M subintervals satisfies $p_M(y_{t_{k+1}} - y_{t_k}, t_{k+1}|\pi_{t_k}, t_k; \Psi) =$

$$\int \int \phi\left(y_{t_{k+1}} - y; \varrho(\pi)h, \hat{\Sigma}_3 \hat{\Sigma}_3' h; \Psi\right) \times p_M(y - y_{t_k}, \pi, m, t_k + (M-1)h|\pi_{t_k}, t_k) d\pi dy, \quad (45)$$

where $\phi(y; \text{mean, variance})$, denotes a trivariate normal density, and y denotes the simulated growth rate of the state vector Y after $M-1$ subintervals. We discuss below how we will calculate $\phi(\cdot)$ when the observed earnings growth rate is not a quarterly growth rate, but instead the growth rate of the four quarter moving average of earnings.

Now $p_M(\cdot|\cdot)$ can be approximated by simulating L paths of the state variables in the interval $(t_k, t_k + (M-1)h)$ and computing the average

$$\hat{p}_M(y_{t_{k+1}} - y_{t_k}, t_{k+1}|\pi_{t_k}, t_k; \Psi) = \frac{1}{L} \sum_{l=1}^L \phi\left(y_{t_{k+1}} - y^{(l)}; \varrho(\pi^{(l)})h, \hat{\Sigma}_3 \hat{\Sigma}_3' h; \Psi\right). \quad (46)$$

The Strong Law of Large Numbers (SLLN) implies that $\hat{p}_M \rightarrow p_M$ as $L \rightarrow \infty$.

To compute the expectation in (42), we simulate S paths of the system (43) to (44) “through” the full time series of fundamentals. Each path is started with an initial belief, $\pi_{t_0} = \pi^*$, the stationary beliefs implied by the generator matrix Λ . In each time interval (t_k, t_{k+1}) we simulate $(M-1)$ successive values of $\tilde{y}_t^{(s)}$ using the discrete scheme in (43), and set $\tilde{y}_{t_k}^{(s)} = y_{t_k}$. The results in the paper use $M = 90$ for quarterly data, so that shocks are approximated at roughly a daily frequency. We approximate the expected likelihood as

$$\hat{\mathcal{L}}^{(S)}(\Psi) = \frac{1}{S} \sum_{s=1}^S \prod_{k=0}^{K-1} \hat{p}_M(y_{t_{k+1}}^{(s)} - y_{t_k}^{(s)}, t_{k+1}|\pi_{t_k}^{(s)}, t_k; \Psi), \quad (47)$$

where $\hat{p}_M(\cdot|\cdot)$ is the density approximated in (46). The SLLN implies that $\hat{\mathcal{L}}^{(S)}(\Psi) \rightarrow E[\mathcal{L}(\Psi)]$ as $S \rightarrow \infty$. We often report $\bar{\pi}_{t_k} = 1/S \sum_{s=1}^S \pi_{t_k}^{(s)}$, which is the econometrician’s expectation of investors’ belief at t_k .

We now return to the issue of density of observed fundamentals $\phi(\cdot)$ in (45). Since

S&P provides the four quarter moving average of earnings, our observed vector contains the quarterly growth rates of consumption and inflation, but the growth rate of four-quarter moving average of earnings. Our simulated system of observables in (43) instead computes the quarterly growth rate of all fundamentals when aggregated over all the subintervals. To deal with the aggregation of earnings, we instead compute

$$\phi \left(\hat{c}_{t_{k+1}} - c, q_{t_{k+1}} - q, \hat{e}_{t_{k+1}} - 1/4(e + e_{t_k}^{(s)} + e_{t_{k-1}}^{(s)} + e_{t_{k-2}}^{(s)}) \right), \quad (48)$$

where $e_{t_k}^{(s)}$ is the model's simulated quarterly earnings growth rate in the interval ending at time t_k along the s -th sample path in the previous paragraph, and c, q , and e denote the simulated growth rate for the period ending at t_{k+1} after $(M - 1)$ subintervals.¹³

The likelihood function identifies only the parameters of the fundamental processes, which we denote as Ψ . To identify the preference parameters, which we collectively call Φ , we need to augment the likelihood function with some moments of asset prices. Let $\epsilon(t) = (e(t)', \frac{\partial \mathcal{L}_1}{\partial \Psi}(t)')'$ where the second term is the score of the likelihood function of fundamentals with respect to Ψ . An additional advantage of including pricing errors is that prices depend on beliefs, and as opposed to fundamentals, are forward looking. This helps us extract beliefs and estimate all the parameters with some forward looking information. We now discuss the pricing errors.

From Proposition 1, we can compute the time series of model-implied price-earnings ratios and bond yields at the discrete data points t_k , $k = 1, \dots, K$ as

$$\widehat{P/E}_{t_k} = C \cdot \bar{\pi}_{t_k}, \quad \hat{i}_{t_k}(\tau) = -\frac{1}{\tau} \log(B(\tau) \cdot \bar{\pi}_{t_k}).$$

We note that the constants C s and the functions $B(\tau)$ both depend on the parameters of the fundamental processes, Ψ . Hence, we let the pricing errors be denoted

$$e_{t_k}^P = \left(\widehat{P/E}_{t_k} - P/E_{t_k}, \hat{i}_{t_k}(0.25) - i_{t_k}(0.25), \hat{i}_{t_k}(1) - i_{t_k}(1), \hat{i}_{t_k}(5) - i_{t_k}(5) \right).$$

¹³We log linearize the model's growth rate of the moving average. In particular the first order approximation of the growth rate is

$$\log \left[\frac{\exp(w + z + y + x) + \exp(y + z + w) + \exp(z + w) + \exp(w)}{\exp(y + z + w) + \exp(z + w) + \exp(w) + 1} \right] \simeq \frac{1}{4}(w + z + y + x).$$

For the subset of the earnings data, where we have the quarterly growth rates available, the approximation leads to growth rate very close to the growth rate of the moving average.

Also note that since the pricing formulas are linear in beliefs, $1/S \sum_{s=1}^S C \cdot \pi_{t_k}^{(s)} = C \cdot \bar{\pi}_{t_k}$ (and similarly for the bond yields) and no information is lost by simply evaluating the errors at the econometrician's conditional mean of beliefs. We similarly formulate the volatility errors as

$$e_{t_k}^V = \left(\widehat{\sigma_{t_k}^N} - \sigma_{t_k}^N, \widehat{\sigma_{t_k}^B(1)} - \sigma_{t_k}^B(1), \widehat{\sigma_{t_k}^B(5)} - \sigma_{t_k}^B(5) \right),$$

where the model-implied nominal stock volatility is obtained from the derived expression $\sigma^N(\pi)$ in (18) and averaged over the simulations as $\widehat{\sigma_{t_k}^N} = \frac{1}{S} \sum_{s=1}^S \sigma^N(\pi_{t_k}^{(s)})$. Similarly, the model-implied nominal bond volatility is obtained from the derived expression $\sigma^B(\pi, \tau)$ in (19) and averaged over the simulations as $\widehat{\sigma_{t_k}^B(\tau)} = \frac{1}{S} \sum_{s=1}^S \sigma^B(\pi_{t_k}^{(s)}, \tau)$, for $\tau = 1, 5$. Additionally, we construct similar covariance errors as

$$e_{t_k}^C = \left(\widehat{\sigma_{t_k}^N \sigma_{t_k}^B(1)} - \sigma_{t_k}^N \sigma_{t_k}^B(1), \widehat{\sigma_{t_k}^N \sigma_{t_k}^B(5)} - \sigma_{t_k}^N \sigma_{t_k}^B(5), \widehat{\sigma_{t_k}^B(1) \sigma_{t_k}^B(5)} - \sigma_{t_k}^B(1) \sigma_{t_k}^B(5) \right),$$

Finally, we construct the Sharpe ratio errors at each date as follows: using the model's market price of risk and estimated stock volatility at each date, we construct the Sharpe ratio at each date as $(\sigma_M \widehat{\sigma_{t_k}^N})' / \sqrt{\widehat{\sigma_{t_k}^N} \widehat{\sigma_{t_k}^N}}$, and we take its difference from an empirical unconditional estimate of 0.25, which is close to the Sharpe ratio on the S&P 500 index for our sample from 1958 to 2010.

We now form the GMM objective:

$$c = \left(\frac{1}{T} \sum_{t=1}^T \epsilon_t \right)' \cdot \Omega^{-1} \cdot \left(\frac{1}{T} \sum_{t=1}^T \epsilon_t \right). \quad (49)$$

The number of scores in $\frac{\partial \log(\hat{\mathcal{L}})}{\partial \Psi}$ equals the number of parameters driving the fundamental processes in Ψ . In addition, we have three preference parameters, and ten additional moments (three prices, three volatilities, three covariances, and the Sharpe Ratio). Finally, we have 11 equality constraints, ten for the elements of the generator matrix, and one for the long run equilibrium condition for consumption and earnings growth rates. Overall, therefore, the statistic c in (49) has a chi-squared distribution with 19 degrees of freedom. We correct the variance covariance matrix for autocorrelation and heteroskedasticity using the Newey-West method using a lag length of $q = 8$.

The asymptotic distribution of the constrained GMM estimator satisfies

$$\sqrt{T}(\hat{\theta} - \theta_0) \sim N[\mathbf{0}, B^{-1/2} M B^{-1/2}],$$

where $M = I - B^{-1/2} A' (A B^{-1} A')^{-1} A B^{-1/2}$, $A = \nabla_{\theta} a(\theta_0)$, $B = G' \Omega^{-1} G$, and $G = E[\nabla_{\theta} g(z_t, \theta_0)]$. $a(\theta_0)$ is the $2 \times k$ vector of constraints on the parameters, and $g(z_t, \theta_0)$ is the vector of moment conditions using data point z_t . The estimate of G as

$$G_T = \begin{bmatrix} \frac{1}{T} \sum_{t=1}^T \frac{\partial e'}{\partial \Phi} & \frac{1}{T} \sum_{t=1}^T \frac{\partial e'}{\partial \Psi} \\ \mathbf{0} & \frac{1}{T} \sum_{t=1}^T \left[\frac{\partial \xi_1}{\partial \Psi}(t)' \frac{\partial \xi_1}{\partial \Psi}(t) \right] \end{bmatrix} \quad (50)$$

C. Survey of Professional Forecasters Probability

For a limited number of variables, the Survey of Professional Forecasters (SPF) reports forecasters' probability assessment that a given variable will be in predefined intervals (bins) in the next quarter or year (depending on the variable). For GNP/GDP deflator variables, such series are available from 1968:Q4. We obtain our four inflation beliefs series in Figure 5 by proceeding in two steps: First, we use simple linear interpolation to convert the available data into probabilities on unit intervals from -4% to 16%, which represent the minimum and maximum bins available in the overall sample.¹⁴ Second, we then aggregate probabilities from the unit bins into probabilities for coarser intervals to match the number of regimes. More specifically, we take the middle point between our inflation regime estimates in Table 1, and use such middle points to define four adjacent intervals: $(-\infty, 1.47)$, $(1.47, 3.59)$, $(3.59, 7.43)$, $(7.43, \infty)$.

¹⁴We also used a more elaborate Gaussian fitting for each quarter, and results were very similar.

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Table 1
Parameter Estimates

Panel A: Preference Parameters										
	ρ	γ	δ		ρ	γ	δ			
Coeff.	0.01965	10.56	0.8084	st. err.	0.002	.028	0.007			
Panel B: Composite Regimes and Conditional Prices										
	Coefficients (%)			standard errors (%)			Conditional Prices			
	Earn.	Infl.	Cons.	Earn.	Infl.	Cons.	P/E	y(0.25)	y(5)	
(MG,LI)	3.26	2.53	2.04	0.84	0.10	0.05	16.33	4.57%	4.64%	
(LG, MI)	-5.18	4.67	2.04	2.97	0.30	0.05	11.40	8.52%	8.48%	
(MG,MI)	3.26	4.67	2.04	0.84	0.30	0.05	11.58	8.46%	8.69%	
(LG,HI)	-5.18	10.19	2.04	2.97	2.71	0.05	9.57	18.03%	12.71%	
(HG,LI)	5.41	2.53	2.04	2.91	0.10	0.05	35.90	4.58%	4.75%	
(LG,ZI)	-5.18	0.43	2.04	2.97	0.88	0.05	13.45	0.89%	2.32%	
Panel C: Diffusion Matrix and NIPA Consumption Signal										
	Coefficients (%)				standard errors (%)					
	α_0	α_1	σ_N		α_0	α_1	σ_N			
Earnings	10.50	0	0	0	0.26					
Inflation	-1.86	2.74	0	0	0.21	0.11				
Consumption	3.54	0	5.26	0	0.37		0.22			
Signal	0	-6.77	0	9.80		8.46		118.54		
Coeff.	α_0	α_1	σ_N	st. err	α_0	α_1	σ_N			
	0.0168	0.1801	0.37%		-	0.27	0.20%			
Panel D: Infinitesimal Generator										
	(MG,LI)	(LG, MI)	(MG,MI)	(LG,HI)	(HG,LI)	(LG,ZI)	St.Prob.		coeff.	st. error
(MG,LI)		λ_1	λ_2	0	0	λ_2	60.80%	λ_1	0.30%	0.08%
(LG, MI)	λ_3		λ_5	λ_4	λ_2	λ_2	5.707%	λ_2	1.96%	6.90%
(MG,MI)	0	λ_3		λ_2	0	0	18.60%	λ_3	12.24%	0.10%
(LG,HI)	λ_3	λ_4	0		0	0	4.469%	λ_4	14.79%	1.31%
(HG,LI)	λ_1	0	λ_2	0		0	4.945%	λ_5	23.74%	0.55%
(LG,ZI)	λ_5	0	0	0	0		5.477%			

Notes: Simulated Methods of Moments (SMM) estimates of the regime-switching model's parameters. The methodology combines the scores of the (simulated) likelihood function from fundamentals (real earnings, inflation, and NIPA consumption) with pricing errors from financial variables (S&P500 index P/E ratio, 3-months and 5-year Treasury yields, the quarterly empirical volatilities of S&P 500 index, the 1-year and the 5-year bonds, and the quarterly empirical covariance between the S&P 500 returns, the 1-year and 5-year bonds.) The last three columns of Panel B also report the conditional P/E ratios and conditional yields across the six composite regimes. The data sample is 1958 - 2010, except for volatilities and covariances whose sample is 1962 - 2010.

Table 2
Model Fit and Data

Panel A: Model's Fit to Fundamentals and Financial Variables					
	constant	slope	t(constant)	t(slope)	R^2
Inflation	-0.71	2.00	-4.62	10.78	0.52
Earnings Growth	-1.99	5.32	-3.11	6.31	0.25
P/E ratios	-5.14	1.35	-1.41	5.59	0.53
3m Yield	-1.76	1.15	-2.26	8.39	0.69
5y Yield	-2.05	1.41	-1.71	6.10	0.67
Stock Volatility	-0.02	1.15	-0.54	3.70	0.30
Stock Volatility (ex crash)	-0.03	1.16	-0.61	3.73	0.36
1-Y Bond Volatility	0.00	0.52	3.34	8.08	0.57
5-Y Bond Volatility	0.01	0.90	1.10	4.87	0.51
Stock - 1Y Bond Covariance	0.00	0.25	-0.32	5.72	0.34
Stock - 5Y Bond Covariance	0.00	0.47	-0.14	4.98	0.38

Panel B: Correlations across Measures of Expected Fundamentals (%)				
	Inflation	Model Exp. Inf.	SPF PGDP	SPF CPI
Inflation	100	73	69	39
Model Exp. Inf.	73	100	93	89
SPF PGDP	69	93	100	98
SPF CPI	39	89	98	100

	Earnings	Model Exp. Earn.	Blue Chip	SPF RGDP	SPF RCP
Earnings	100	49	63	52	54
Model Exp. Earn.	49	100	51	34	29
Blue Chip	63	51	100	66	65
SPF RGDP	52	34	66	100	61
SPF RCP	54	29	65	61	100

Notes: Panel A reports the results of the regressions

$$\begin{aligned}
 (\text{Fundamentals}) &= b_0 + b_1 E_t[\text{Fundamentals}] + \epsilon_t \\
 (\text{Financial Variable})_t^{Data} &= b_0 + b_1 (\text{Financial Variable})_t^{Model} + \epsilon_t
 \end{aligned}$$

where “Fundamentals” is either real earnings growth or inflation rate, and financial variables are identified in each row. In these regressions, both expected fundamentals and model-implied financial variables are conditional on the fitted beliefs. The sample is 1960 - 2010, except for volatilities and covariances, whose sample is 1962-2010. All t-statistics are Newey-West adjusted for heteroskedasticity and autocorrelation using 12 lags. Panel B reports the cross-correlation among measures of expected fundamentals, where the correlation is computed in the longest available sample: Inflation, Model Exp. Inf., Earnings, and Model Exp. Earnings from 1958Q1 to 2010Q4, SPF PGDP, SPF RGDP and SPF RCP from 1968Q3 to 2010Q4, SPF CPI from 1981Q2 to 2010Q4, Blue Chip from 1984Q1 to 2010Q4.

Table 3

Stock-Bond Covariance versus Fundamentals

	Panel A. Model: Simulations				Panel B. Model: Fitted			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Const	-0.71 [-2.40, 0.40]	0.13 [-0.07,0.38]	-.09 [-0.22,0.02]	-0.15 [-0.31, 0.04]	-1.44 (-5.92)	0.11 (2.21)	-0.03 (-0.60)	-0.16 (-5.38)
ExpInf	0.28 [0.02, 0.73]				0.40 (7.31)			
ExpEarn	-0.01 [-.014, .011]				0.08 (2.37)			
P_{HI}		2.27 [1.40, 15.85]				2.85 (7.17)		
P_{ZI}		-0.83 [-5.48 -0.30]				-2.25 (-3.87)		
UncInf			34.28 [18.90, 41.89]	25.36 [-2.84,36.47]			32.24 (6.05)	21.82 (6.98)
UncEarn			-4.12 [-6.90,-0.48]				-5.77 (-2.71)	
UncEarn $\times P_{HI}$				1.56 [-14.61, 156.73]				-2.77 (-1.03))
UncEarn $\times P_{ZI}$				-11.31 [-33.45, -6.21]				-18.18 (-5.18)
Adj R ²	0.46 [0.20, 0.85]	0.40 [0.20, 0.92]	0.81 [0.52, 0.93]	0.84 [0.75, 0.95]	0.74	0.72	0.65	0.85

	Panel C. Data										
	(1a)	(1b)	(1c)	(1d)	(2a)	(2b)	(2c)	(3a)	(3b)	(4a)	(4b)
	Exp	Exp	Exp	Model	Beliefs: SPF(t+1)	Beliefs: SPF(t)	Beliefs: Model	Beliefs: SPF(t+1)	Beliefs: Model	Beliefs: SPF(t+1)	Beliefs: Model
Const	-0.34 (-1.94)	0.43 (-1.64)	-0.97 (-1.70)	-0.62 (-2.79)	0.14 (2.25)	0.08 (1.08)	0.06 (1.07)	-0.16 (-1.53)	-0.04 (-0.71)	-0.41 (-2.38)	-0.14 (-1.57)
ExpInf	0.10 (2.99)	0.10 (2.80)	0.11 (2.94)	0.18 (3.83)							
ExpEarn	0.00 (1.24)	0.04 (1.04)	0.01 (1.43)	0.02 (0.91)							
P_{HI}					0.22 (1.71)	0.29 (2.11)	1.03 (5.09)				
P_{ZI}					-3.30 (-4.75)	-1.99 (-4.28)	-1.11 (-4.54)				
UncInf								22.89 (3.00)	12.84 (2.57)	10.59 (2.88)	14.87 (2.80)
UncEarn								-1.12 (-0.83)	-1.93 (-1.21)		
UncEarn $\times P_{HI}$										0.01 (2.00)	-7.37 (-1.51)
UncEarn $\times P_{ZI}$										-0.21 (-4.10)	-8.10 (-3.56)
Adj R ²	0.19	0.20	0.24	0.26	0.30	0.22	0.23	0.13	0.18	0.37	0.29

Notes: Regression of $(\text{Covariance})_t = b_0 + \mathbf{b}_1 \mathbf{X}_t + \epsilon_t$, where “Covariance” is the theoretical formula (Panels A and B) or estimated from daily bond and stock returns (Panel C). Explanatory variables \mathbf{X}_t are identified on each row: ExpInf and ExpEarn are expected inflation and earnings, P_{HI} and P_{ZI} are the probabilities of high inflation or zero (or lower) inflation, UncInf and UncEarn are the inflation and earnings uncertainty. In Panels A and B, all the quantities are computed from the regime shift model. In Panel C, these quantities are proxied using either the Survey of Professional Forecasters (SPF) or the model’s fitted probabilities, as indicated on the heading of each column. The sample in Panel B is 1960-2010. The sample in Panel C is also 1960 - 2010, except when SPF data are used (columns 1a, 1b, 1c, 2a, 2b, and 3a), in which case the sample is 1968-2010. All t-statistics are Newey-West adjusted for heteroskedasticity and autocorrelation using 12 lags.

Table 4
Stock-Bond Covariance versus Asset Prices

Panel A. Model: Simulations					Panel B. Model: Fitted			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Const	-1.86 [-16.56, 4.29]	-9.73 [-336.13, 740.11]	-0.41 [-0.85, 0.46]	0.11 [-0.02, 0.25]	-4.51 (-3.69)	-56.94 (-2.83)	1.19 (3.35)	0.08 (1.78)
Y(5)	0.22 [0.13, 0.68]	-0.31 [-13.27, 3.37]			0.34 (6.19)	0.37 (1.03)		
pe	0.26 [-2.11, 5.09]	7.99 [-516.62, 235.85]			0.92 (2.70)	37.31 (2.80)		
Y(5) ²		-0.03 [-70.73, 63.62]				0.03 (1.15)		
pe ²		-1.62 [-41.35, 90.54]				-6.34 (-2.84)		
Y(5) ² ×pe ²		0.12 [-0.08, 0.25]				-0.00 (-0.49)		
Stock Vol			37.34 [-45.24, 75.07]				-7.65 (-2.80)	
Stock Vol×P _{HI}				107.43 [89.56, 899.65]				125.29 (12.95)
Stock Vol×P _{ZI}				-54.74 [-146.15, 86.99]				-35.69 (-4.35)
Adj R ²	0.41 [0.16, 0.80]	0.46 [0.34, 0.94]	0.35 [0.00, 0.92]	0.75 [0.63, 0.89]	0.67	0.70	0.25	0.77

Panel C. Data					
	(1)	(2)	(3)	(4a) Beliefs: SPF(t+1)	(4b) Beliefs: Model
Const	-1.04 (-2.38)	-6.97 (-2.48)	0.38 (3.24)	0.01 (1.60)	0.03 (0.48)
Y(5)	0.10 (4.20)	0.39 (3.22)			
pe	0.15 (1.42)	3.43 (2.20)			
Y(5) ²		-0.00 (-1.32)			
pe ²		-0.52 (-2.19)			
Y(5) ² ×pe ²		-0.00 (-1.80)			
Stock Vol			-2.42 (-2.13)		
Stock Vol×P _{HI}				8.44 (1.45)	48.59 (6.12)
Stock Vol×P _{ZI}				-45.39 (-4.04)	-7.43 (-9.33)
Adj R ²	0.40	0.47	0.20	0.38	0.27

Notes: Regression of $(\text{Covariance})_t = b_0 + \mathbf{b}_1 \mathbf{X}_t + \epsilon_t$, where “Covariance” is the theoretical formula (Panels A and B) or estimated from daily bond and stock returns (Panel C). Explanatory variables \mathbf{X}_t are identified on each row. Y(5) denotes the 5-year zero coupon bond yield, pe is the log P/E ratio, Stock Vol is the stock return volatility, and P_{HI} and P_{ZI} are the probabilities of high inflation or zero (or negative) inflation. In Panel A and B all quantities are computed from the model using the closed-form formulas. In Panel C, all quantities are observed from the data. Stock Vol refers to the quarterly volatility computed from daily returns, and the extreme inflation probabilities P_{HI} and P_{ZI} are computed from the Survey of Professional Forecasters. The sample in Panel B is 1960-2010. The sample in Panel C is also 1960 - 2010, except when SPF data are used (columns 4a), in which case the sample is 1968-2010. All t-statistics are Newey-West adjusted for heteroskedasticity and autocorrelation using 12 lags.

Table 5
Bond Return Volatility

	Panel A. Model: Simulations					Panel B. Model: Fitted				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Const	1.38 [-5.85, 7.19]	2.78 [1.13, 4.15]	0.88 [0.16, 1.60]	14.03 [-38.91, 135.78]	0.85 [-3913.9 3144.5]	2.81 (3.32)	3.49 (10.29)	1.51 (5.67)	20.83 (2.99)	149.55 (1.01)
ExpInf	.10 [-.011, 3.94]					0.92 (4.63)				
ExpEarn	-.58 [-1.82, -.22]					-0.81 (-6.56)				
P_{HI}		12.32 [7.54, 155.01]					16.86 (8.22)			
P_{ZI}		1.84 [-8.36, 23.75]					4.38 (1.97)			
UncInf			128.41 [101.31, 213.30]					85.82 (8.41)		
UncEarn			10.93 [-3.08, 23.55]					15.21 (3.73)		
Y(5)				0.62 [-1.39, 2.52]	-2.26 [-56.12 14.29]				0.38 (1.23)	-2.60 (-1.32)
pe				-5.23 [-46.48, 12.19]	-41.21 [-2047.3 2856.2]				-6.80 (-3.32)	-87.32 (-0.85)
Y(5) ²					-0.21 [-3.41, 4.58]					-0.06 (-0.35)
pe ²					4.59 [-519.00 344.77]					13.34 (0.76)
Y(5) ² × pe ²					0.07 [-0.54 1.28]					0.04 (1.37)
Adj R ²	0.51 [0.27, 0.84]	0.36 [0.24, 0.77]	0.89 [0.78, 0.97]	0.37 [0.09, 0.81]	0.43 [0.32, 0.95]	0.77	0.56	0.87	0.53	0.57

	Panel C. Data									
	(1a)	(1b)	(1c)	(1d)	(2a)	(2b)	(2c)	(3a)	(3b)	(4) (5)
	Exp	Exp	Exp	Model	Beliefs:	Beliefs:	Beliefs:	Beliefs:	Beliefs:	
	Earn: RCP	Earn: RGDP	Earn: Boom	Exp.	SPF(t+1)	SPF(t)	Model	SPF(t+1)	Model	
Const	3.93 (3.79)	5.20 (5.02)	9.14 (4.69)	3.37 (3.15)	4.50 (14.79)	4.64 (15.58)	3.92 (10.30)	2.28 (2.54)	2.59 (5.03)	0.84 (0.18) 21.17 (0.96)
ExpInf	0.45 (1.26)	0.39 (1.20)	0.32 (0.98)	0.87 (2.33)						
ExpEarn	-0.04 (-1.40)	-0.48 (-3.11)	-0.06 (-2.90)	-0.74 (-5.49)						
P_{HI}					4.85 (1.95)	4.72 (1.92)	19.02 (6.77)			
P_{ZI}					5.64 (2.54)	3.23 (2.32)	4.60 (6.29)			
UncInf								43.51 (1.39)	100.03 (4.02)	
UncEarn								19.17 (2.54)	5.41 (1.00)	
Y(5)										0.58 (2.13) -1.24 (-1.70)
pe										0.11 (0.09) -11.73 (-0.89)
Y(5) ²										0.12 (3.48)
pe ²										2.36 (1.15)
Y(5) ² × pe ²										-0.00 (-0.01)
Adj R ²	0.16	0.22	0.25	0.42	0.22	0.23	0.44	0.24	0.45	0.28 0.49

Notes: Regression of $(\text{Bond Volatility})_t = b_0 + \mathbf{b}_1 \mathbf{X}_t + \epsilon_t$, where “Bond Volatility” is the theoretical formula (Panels A and B) or estimated from daily bond returns (Panel C). Explanatory variables \mathbf{X}_t are identified on each row. ExpInf and ExpEarn are expected inflation and earnings, P_{HI} and P_{ZI} are the probabilities of high inflation or zero (or lower) inflation, UncInf and UncEarn are the inflation and earnings uncertainty, Y(5) denotes the 5-year zero coupon bond yield, and pe is the log P/E ratio. In Panels A and B, all the quantities are computed from the regime shift model. In Panel C, these quantities are empirical proxies either directly observable (Y(5) and pe), or computed from the Survey of Professional Forecasters (SPF) or from the model’s fitted probabilities, as indicated on the heading of each column. The sample in Panel B is 1960-2010. The sample in Panel C is also 1960 - 2010, except when SPF data are used (columns 1a, 1b, 1c, 2a, 2b, and 3a), in which case the sample is 1968-2010. All t-statistics are Newey-West adjusted for heteroskedasticity and autocorrelation using 12 lags.

Table 6
Stock Return Volatility

	Panel A. Model: Simulations					Panel B. Model: Fitted				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Const	11.52 [6.48,20.10]	11.67 [9.43,13.77]	9.60 [8.52 12.27]	-0.75 [-60.78, 160.58]	-1.08 [-3599.2 3167.1]	20.78 (10.44)	12.92 (26.06)	11.90 (18.06)	29.41 (1.46)	587.16 (3.65)
ExpInf	.49 [-1.32,2.51]					-1.23 (-3.10)				
ExpEarn	-.50 [-2.41,-.14]					-1.25 (-5.00)				
P_{HI}		7.48 [2.08, 79.01]					4.71 (2.21)			
P_{ZI}		2.61 [-0.29,45.95]					18.91 (6.55)			
UncInf			57.56 [-71.23,110.88]					-66.77 (-1.36)		
UncEarn			29.28 [9.28, 46.56]					42.22 (2.35)		
Y(5)				0.76 [-2.61,2.43]	-7.11 [-70.31 12.19]				-0.81 (-0.98)	-13.03 (-4.96)
pe				3.11 [-50.93, 21.20]	33.13 [-2200.3 2625.4]				-3.76 (-0.68)	-351.36 (-3.35)
Y(5) ²					-31.30 [-4.10,5.85]					-0.40 (-1.42)
pe ²					-8.51 [-475.49 396.24]					57.00 (3.29)
Y(5) ² ×pe ²					0.15 [-0.75 1.18]					0.19 (3.02)
Adj R ²	0.22 [0.07, 0.85]	0.13 [0.06, 0.79]	0.59 [0.22, 0.97]	0.12 [0.04, 0.72]	0.34 [0.21, 0.94]	0.39	0.52	0.26	0.06	0.35

	Panel C. Data										
	(1a)	(1b)	(1c)	(1d)	(2a)	(2b)	(2c)	(3a)	(3b)	(4)	(5)
	Exp	Exp	Exp	Model	Beliefs:	Beliefs:	Beliefs:	Beliefs:	Beliefs:		
	Earn:	Earn:	Earn:	Exp.	SPF(t+1)	SPF(t)	Model	SPF(t+1)	Model		
Const	18.96 (7.04)	23.15 (6.17)	41.52 (5.46)	21.90 (4.84)	12.45 (10.70)	13.27 (11.16)	12.45 (11.42)	10.41 (5.14)	12.15 (7.97)	14.28 (0.99)	275.74 (3.52)
ExpInf	-0.85 (-1.83)	-0.95 (-2.01)	-1.37 (-3.31)	-1.51 (-1.70)							
ExpEarn	-0.22 (-2.84)	-1.83 (-2.82)	-0.26 (-3.56)	-1.44 (-2.15)							
P_{HI}					3.28 (1.34)	2.16 (0.92)	4.21 (1.29)				
P_{ZI}					46.87 (4.03)	29.48 (3.77)	24.06 (3.41)				
UncInf								-144.02 (-1.56)	-75.01 (-1.01)		
UncEarn								50.35 (3.66)	38.76 (1.50)		
Y(5)										-0.23 (-0.38)	-8.66 (-2.75)
pe										0.43 (0.11)	-154.97 (-3.53)
Y(5) ²											-0.03 (-0.34)
pe ²											24.92 (3.74)
Y(5) ² ×pe ²											0.09 (2.94)
Adj R ²	0.12	0.19	0.33	0.12	0.14	0.10	0.18	0.17	0.04	0.00	0.17

Notes: Regression of $(\text{Stock Volatility})_t = b_0 + \mathbf{b}_1 \mathbf{X}_t + \epsilon_t$, where “Stock Volatility” is the theoretical formula (Panels A and B) or estimated from daily stock returns (Panel C). Explanatory variables \mathbf{X}_t are identified on each row. ExpInf and ExpEarn are expected inflation and earnings, P_{HI} and P_{ZI} are the probabilities of high inflation or zero (or lower) inflation, UncInf and UncEarn are the inflation and earnings uncertainty, Y(5) denotes the 5-year zero coupon bond yield, and pe is the log P/E ratio. In Panels A and B, all the quantities are computed from the regime shift model. In Panel C, these quantities are empirical proxies either directly observable (Y(5) and pe), or computed from the Survey of Professional Forecasters (SPF) or from the model’s fitted probabilities, as indicated on the heading of each column. The sample in Panel B is 1960-2010. The sample in Panel C is also 1960 - 2010, except when SPF data are used (columns 1a, 1b, 1c, 2a, 2b, and 3a), in which case the sample is 1968-2010. All t-statistics are Newey-West adjusted for heteroskedasticity and autocorrelation using 12 lags.

Table 7

Out-of-Sample Forecasts

	2-Quarter Ahead Forecast							
	Model Forecast		Controls			Controls + Lag		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Adj. R^2	MAE	Adj. R^2	MAE	DM	Adj R^2	MAE	DM
Stock Vol.	26.46	3.54	15.66	3.74	-5.03	21.85	3.29	0.73
Stock Vol. (Ex-Crash)	32.54	3.22	3.45	3.45	-5.33	35.41	2.85	1.20
1Y Bond Vol.	6.39	0.33	0.33	0.26	5.83	5.00	0.19	2.74
5Y Bond Vol.	10.68	1.22	1.22	1.49	-3.77	2.75	1.02	1.27
Stock / 1Y Bond Cov.	32.38	0.04	0.04	0.02	4.24	17.34	0.02	2.74
Stock / 5Y Bond Cov.	47.14	0.12	0.12	0.13	-1.99	33.02	0.13	0.64
1Y / 5Y Bond Cov.	3.91	0.02	0.02	0.02	1.56	0.65	0.01	2.39

	4-Quarter Ahead Forecast							
	Model Forecast		Controls			Controls + Lag		
	Adj. R^2	MAE	Adj. R^2	MAE		Adj R^2	MAE	DM
Stock Vol.	35.35	4.85	10.20	5.46	-4.10	12.83	4.96	-0.25
Stock Vol. (Ex-Crash)	41.69	4.29	26.68	5.00	-4.28	26.43	4.47	-2.45
1Y Bond Vol.	2.01	0.48	-2.27	0.36	4.49	-2.66	0.33	1.59
5Y Bond Vol.	9.96	1.58	-2.25	2.25	-3.33	-2.52	1.58	-0.03
Stock / 1Y Bond Cov.	35.15	0.05	3.99	0.04	3.67	10.34	0.04	2.77
Stock / 5Y Bond Cov.	55.60	0.23	24.97	0.25	-2.68	30.65	0.25	-0.60
1Y / 5Y Bond Cov.	1.07	0.04	-2.28	0.04	0.90	-2.36	0.04	1.02

	8-Quarter Ahead Forecast							
	Model Forecast		Controls			Controls + Lag		
	Adj R^2	MAE	Adj. R^2	MAE	DM	Adj R^2	MAE	DM
Stock Vol.	34.78	6.87	3.31	8.18	-3.09	3.74	8.74	-3.70
Stock Vol. (Ex-Crash)	35.21	5.89	23.77	7.31	-3.14	19.91	7.70	-2.91
1Y Bond Vol.	0.16	0.65	1.60	0.39	2.39	3.37	0.57	0.67
5Y Bond Vol.	4.85	2.13	-2.29	2.81	-2.78	-0.60	2.81	-2.39
Stock / 1Y Bond Cov.	35.39	0.14	0.05	0.09	3.08	2.85	0.09	1.63
Stock / 5Y Bond Cov.	54.48	0.47	24.74	0.50	-2.26	25.08	0.49	-1.65
1Y / 5Y Bond Cov.	0.21	0.07	-2.34	0.08	-0.56	-1.35	0.07	0.08

Notes: Out-of-Sample forecast of second moments from model (columns 1 and 2), from controls (columns 3 and 4), and from controls, including lag of forecasted variable (columns 6 and 7). Model forecasts are computed by estimating the six-regime shift models every five years starting in 1984 (the middle of our overall sample), and feeding fundamentals to the estimated model to compute the model's beliefs over the following 5-year period. From beliefs, forecasts of future second moments are computed as the average across simulations of future model-implied second moments, the latter obtained by exploiting our analytical formulas, as described in equations (31) and (32). Forecasts from controls are obtained using a similar rolling regression estimate of realized second moments on variables used to forecast future volatility, given by NBER recession indicator, lagged stock return when negative, the P/E ratio, the 3-month yield, the 5-year yield, inflation volatility (from GARCH fit), and earnings volatility (from GARCH fit). Adj. R^2 is the R^2 of realized versus predicted variables, while MAE denotes the mean absolute errors. Columns 5 and 8 report the Diebold Moreno t-statistics on equality of absolute errors: A negative number indicates that the model has significant lower MAE than the controls.

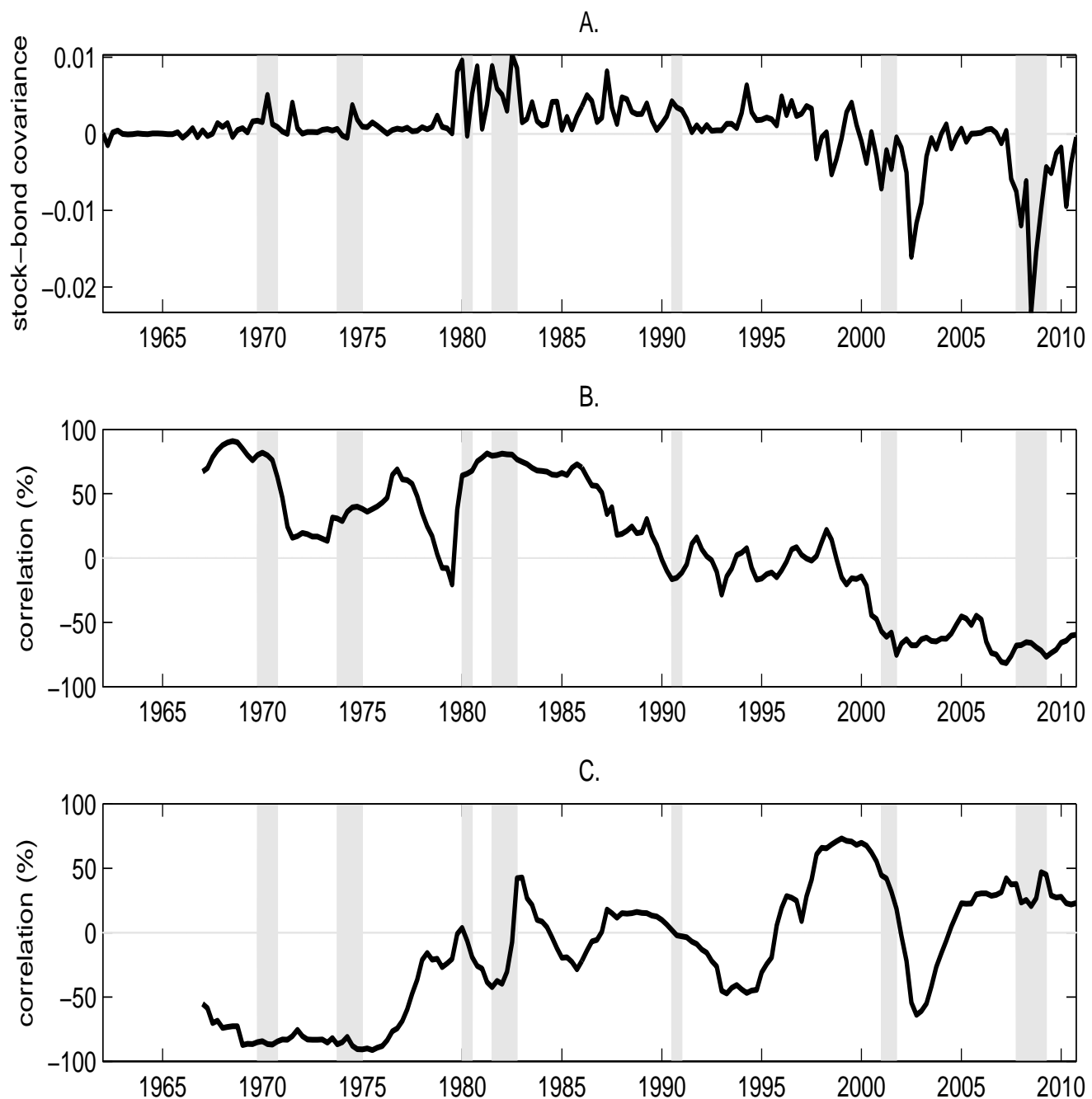


Figure 1. The Comovement of Bonds and Stocks, and their Volatilities. Panel A: Quarterly covariance between the S&P 500 index returns and the 5-year Treasury zero-coupon bond returns. Panel B: 5-year trailing correlation between the 5-year zero-coupon bond return volatility and its yield to maturity. Panel C: 5-year trailing correlation between the S&P 500 index return volatility and its price-earnings ratio. Shaded bands reflect NBER-dated recessions.

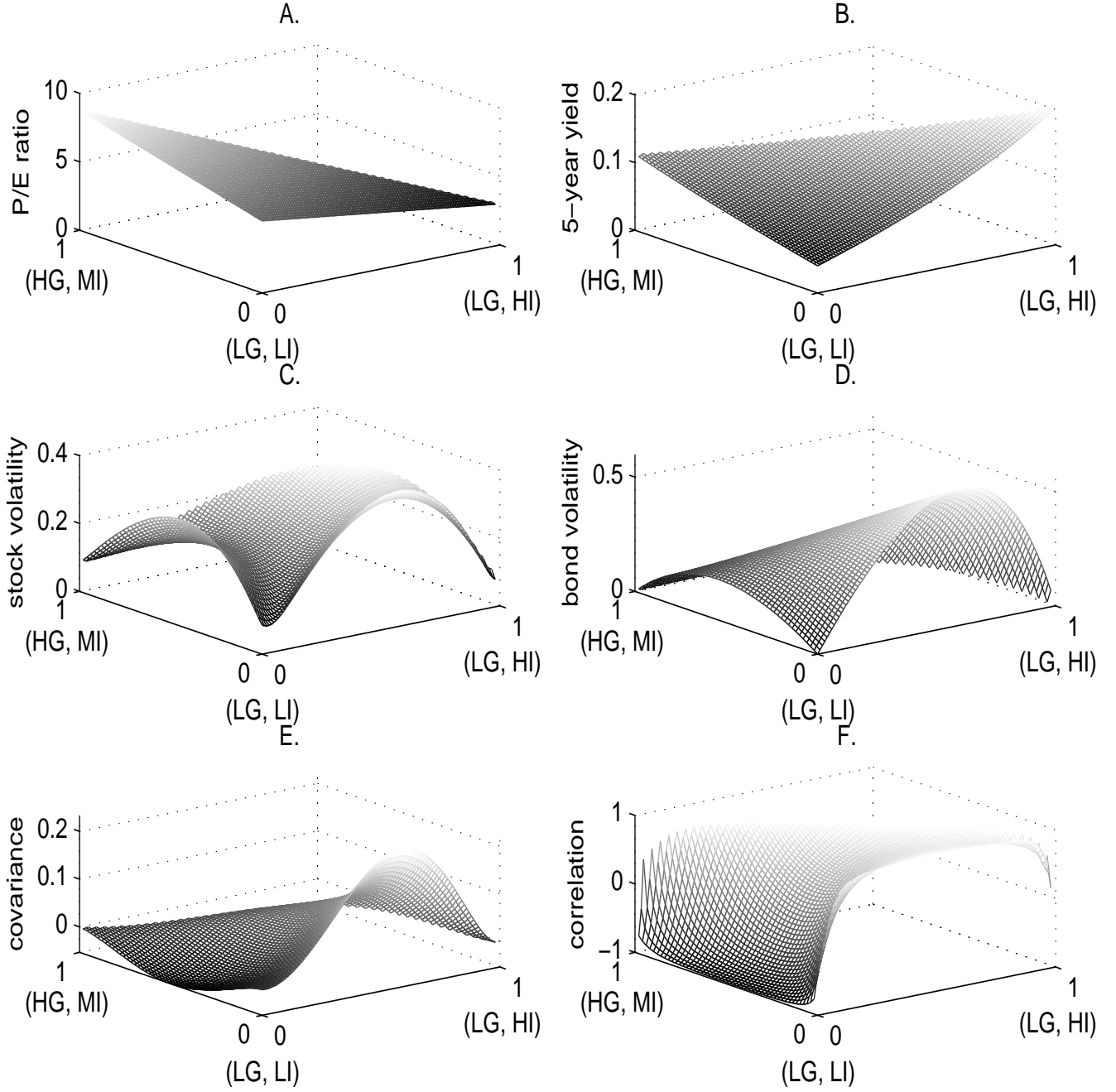


Figure 2. Prices, Volatilities and Comovement with Three Composite Regimes. Panel A: Price-earnings ratio. Panel B: Yield of a 5-year zero-coupon bond. Panel C: Stock return volatility. Panel D: 5-year bond return volatility. Panel E: Covariance between stock return and 5-year bond return. Panel F: Correlation between stock return and the 5-year bond return. All quantities are plotted on the probability simplex. Corners (HG,MI), (LG, LI) and (LG, HI) represents regimes with (High Growth, Medium Inflation), (Low Growth, Low Inflation), and (Low Growth, High Inflation), respectively. Parameters are as follows: risk aversion $\gamma = 10$, time preference $\rho = 0.02$, money illusion $\delta = 0.8$, dividend/earnings payout ratio $c = 0.5$, low growth $\theta^{LG} = -2\%$, high growth $\theta^{HG} = 5\%$, low inflation $\beta^{LI} = 0\%$, medium inflation $\beta^{MI} = 4\%$, high inflation $\beta^{HI} = 9\%$, consumption drift $\kappa^{LG} = \kappa^{HG} = 2\%$, earnings diffusion $\sigma_E = [0.1, 0, 0]$, inflation diffusion $\sigma_Q = [-0.02, 0.03, 0]$, consumption $\sigma_C = [0.03, 0, 0.05]$.

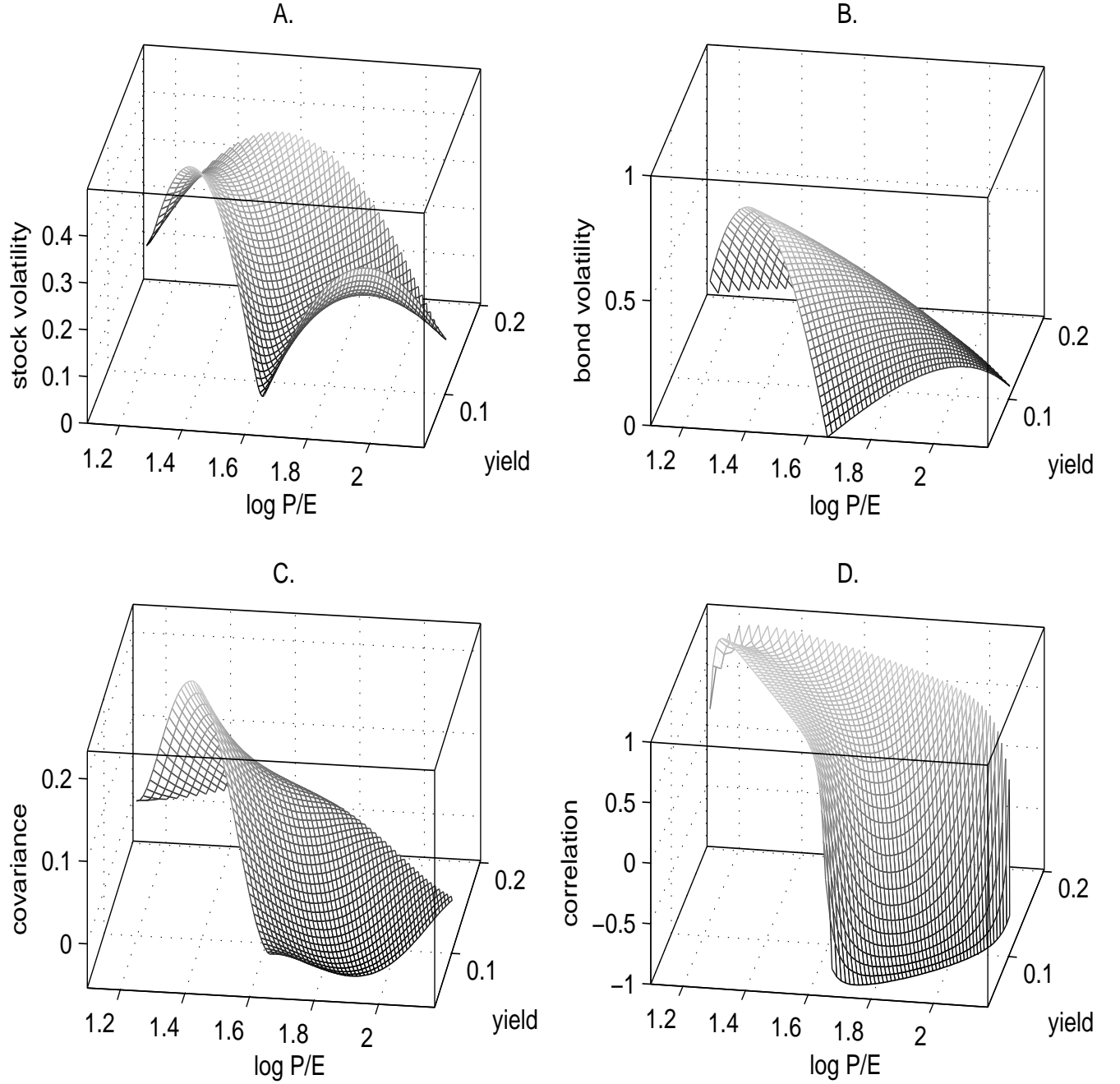


Figure 3. Volatilities and Comovement versus Bond Yield and log P/E Ratio with Three Composite Regimes. Panel A: Stock return volatility. Panel B: 5-year bond return volatility. Panel C: Covariance between stock return and 5-year bond return. Panel D: Correlation between stock return and 5-year bond return. All quantities are plotted against the log P/E ratio and the 5-year bond yield. Parameters are as follows: risk aversion $\gamma = 10$, time preference $\rho = 0.02$, money illusion $\delta = 0.8$, dividend/earnings payout ratio $c = 0.5$, low growth $\theta^{LG} = -2\%$, high growth $\theta^{HG} = 5\%$, low inflation $\beta^{LI} = 0\%$, medium inflation $\beta^{MI} = 4\%$, high inflation $\beta^{HI} = 9\%$, consumption drift $\kappa^{LG} = \kappa^{HG} = 2\%$, earnings diffusion $\sigma_E = [.1, 0, 0]$, inflation diffusion $\sigma_Q = [-.02, .03, 0]$, consumption $\sigma_C = [.03, 0, .05]$.

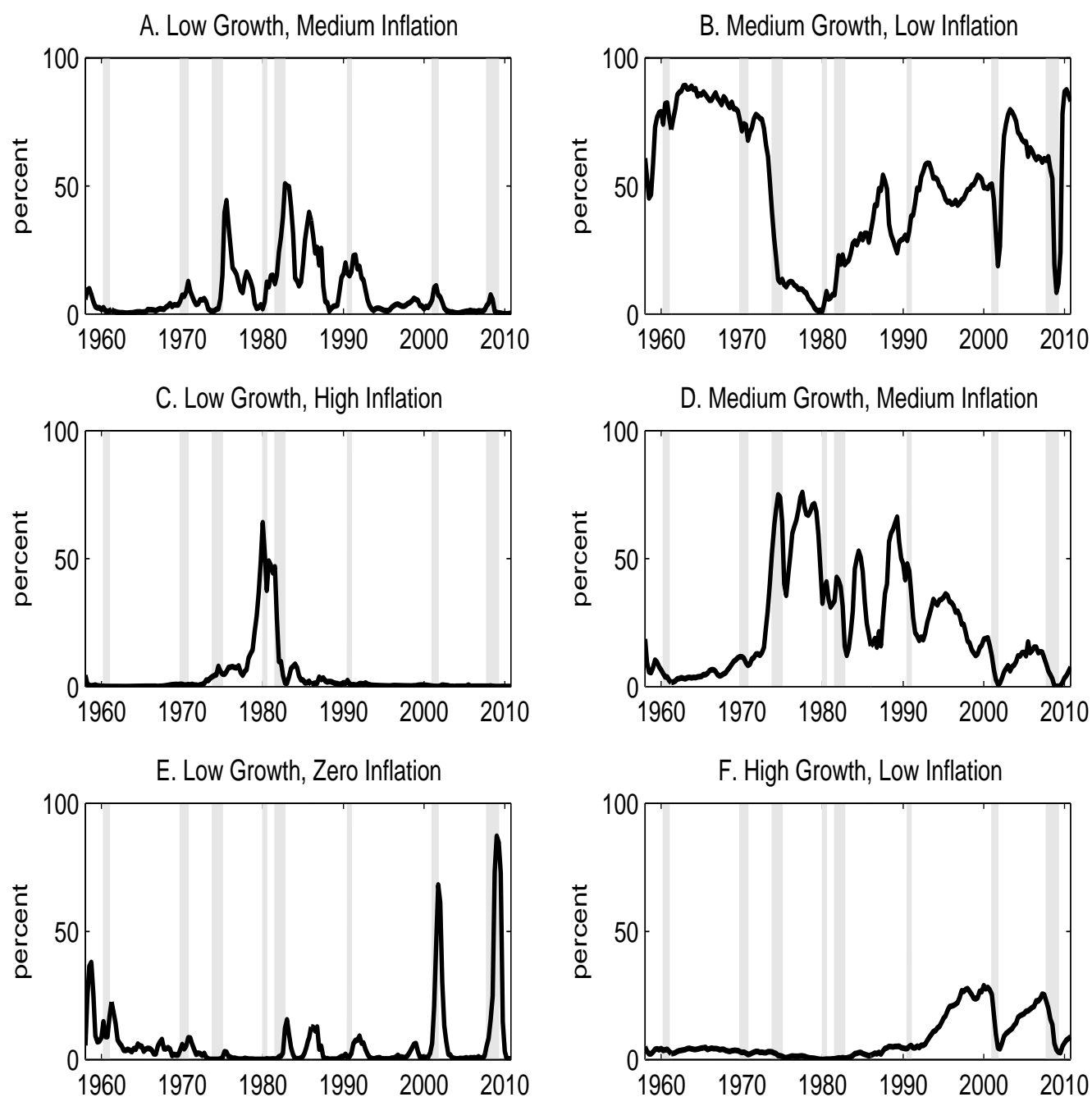


Figure 4. Composite Regime Probabilities. Model's fitted beliefs about each of six composite regimes from 1960 to 2010. Shaded areas correspond to NBER-dated recessions. The estimates of the six composite regimes are in Table 2.

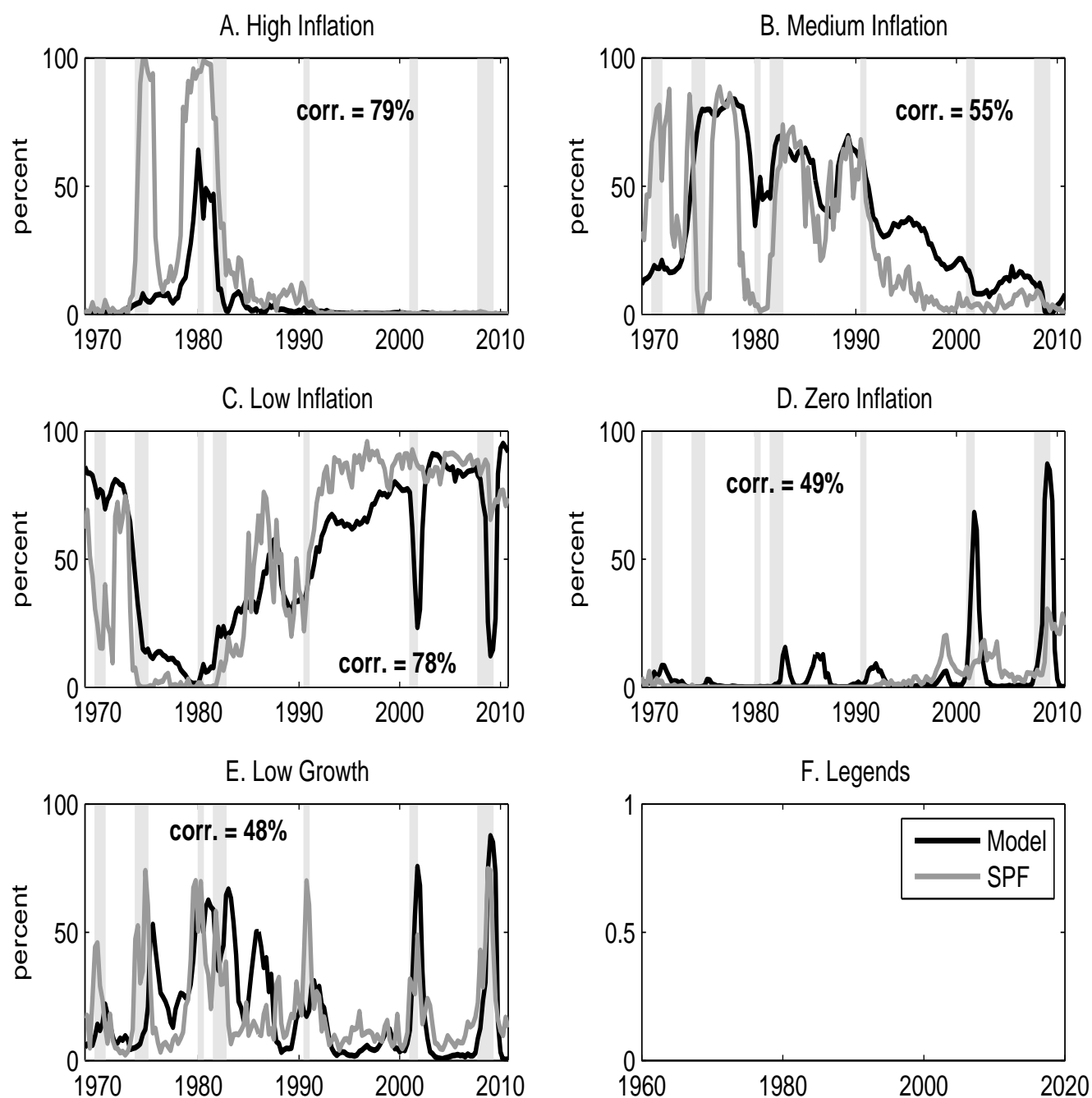


Figure 5. Marginal Probability of Inflation Regimes and Low Growth. Panels A to D: Model's fitted marginal posterior probabilities about the four possible inflation regimes (black lines) and professional forecasters' probability assessments of similar levels of next-year inflation (grey lines). Panel E: model's fitted marginal probability of low growth (black line) and professional forecasts' beliefs of a recession in the following quarter (grey line). Shaded areas correspond to NBER-dated recessions.

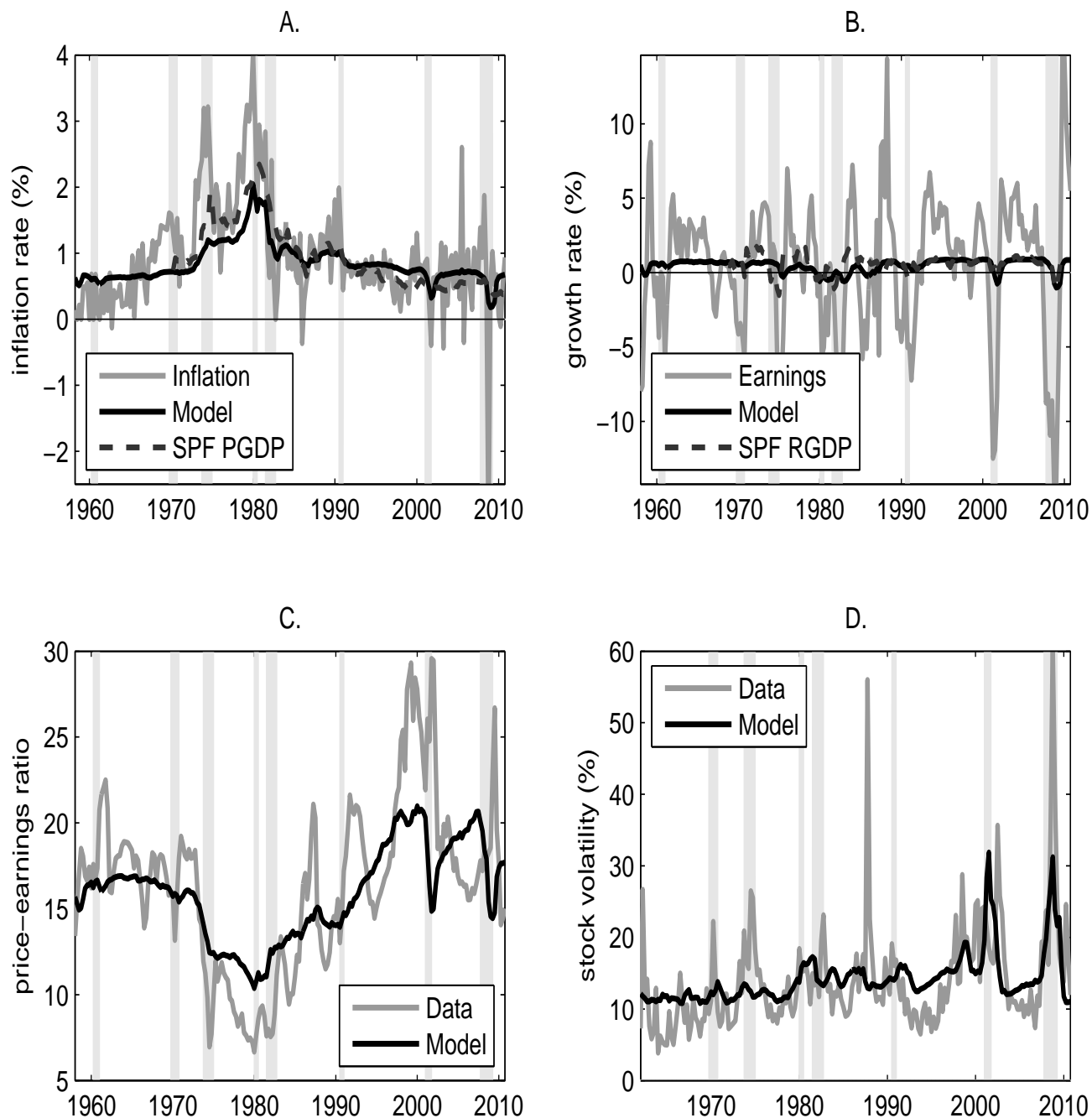


Figure 6. Fitted and Data Series. Panel A: Realized CPI inflation (grey line), model fitted expected inflation (black line), and consensus forecast of the following year GDP deflator (dashed line). Panel B: Realized real operating earnings growth (grey line), model's fitted expected earnings (black line), and the consensus forecasts of real GDP growth. Realized real earnings growth data are winsorized to 1% level. Panel C: Realized price-earnings ratio of S&P 500 index (grey line) and model-fitted price-earnings ratio (black line). Panel D: Realized quarterly volatility of the S&P 500 index return (grey line) and model-fitted stock return volatility (black line). Shaded areas correspond to NBER-dated recessions.

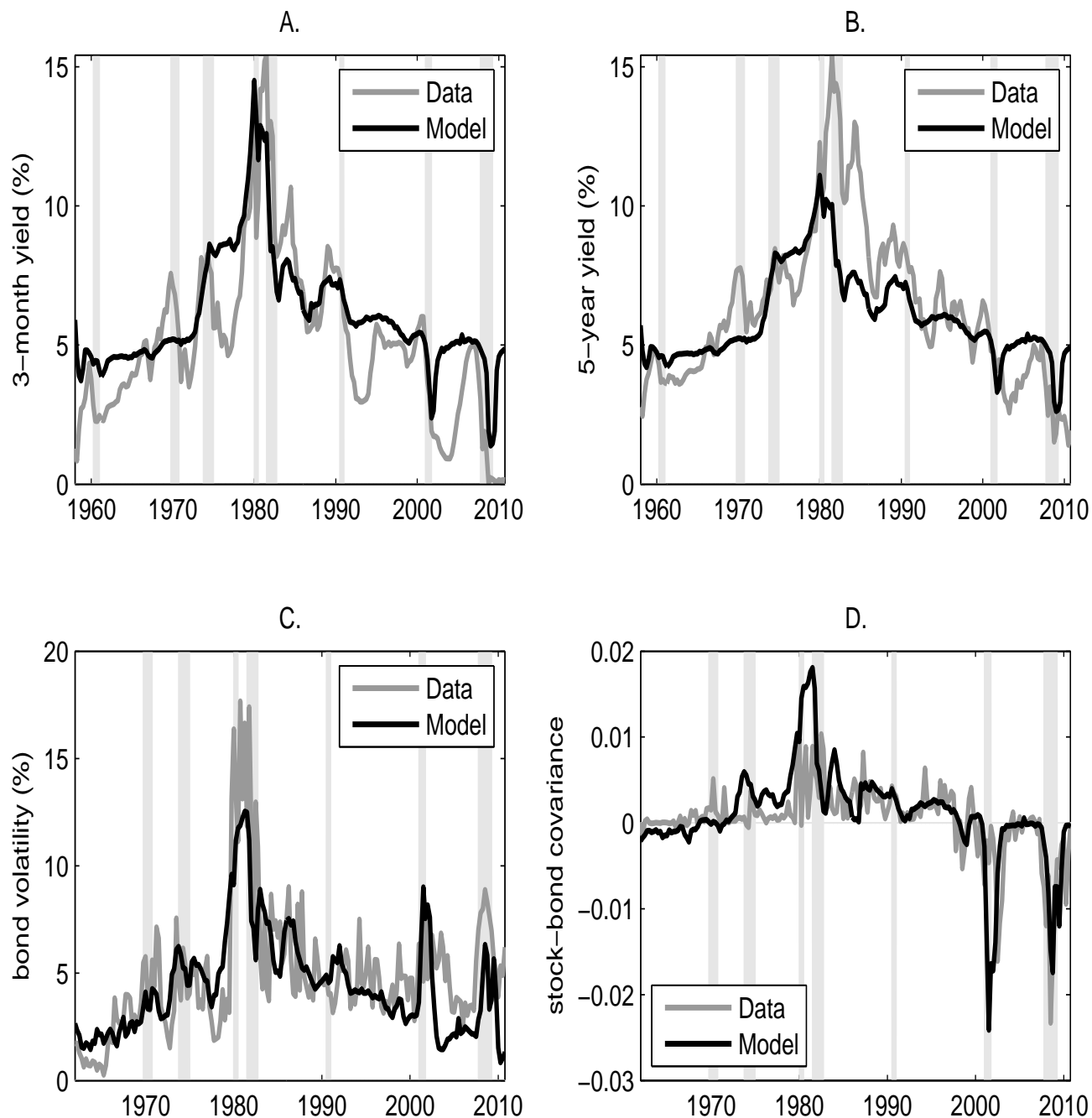


Figure 7. Fitted and Data Series. Panel A. Realized 3-month U.S. Treasury bill rate (grey line) and model-fitted 3-month rate (black line). Panel B. Realized 5-year U.S. Treasury zero-coupon yield (grey line) and model fitted 5-year yield (black line). Panel C. Realized annualized volatility of the 5-year zero coupon bond (grey line) and model fitted volatility (black line). Panel D. Realized covariance between the S&P 500 index return and the 5-year zero-coupon bond return (grey line) and model- fitted covariance (black line). Shaded areas correspond to NBER-dated recessions.

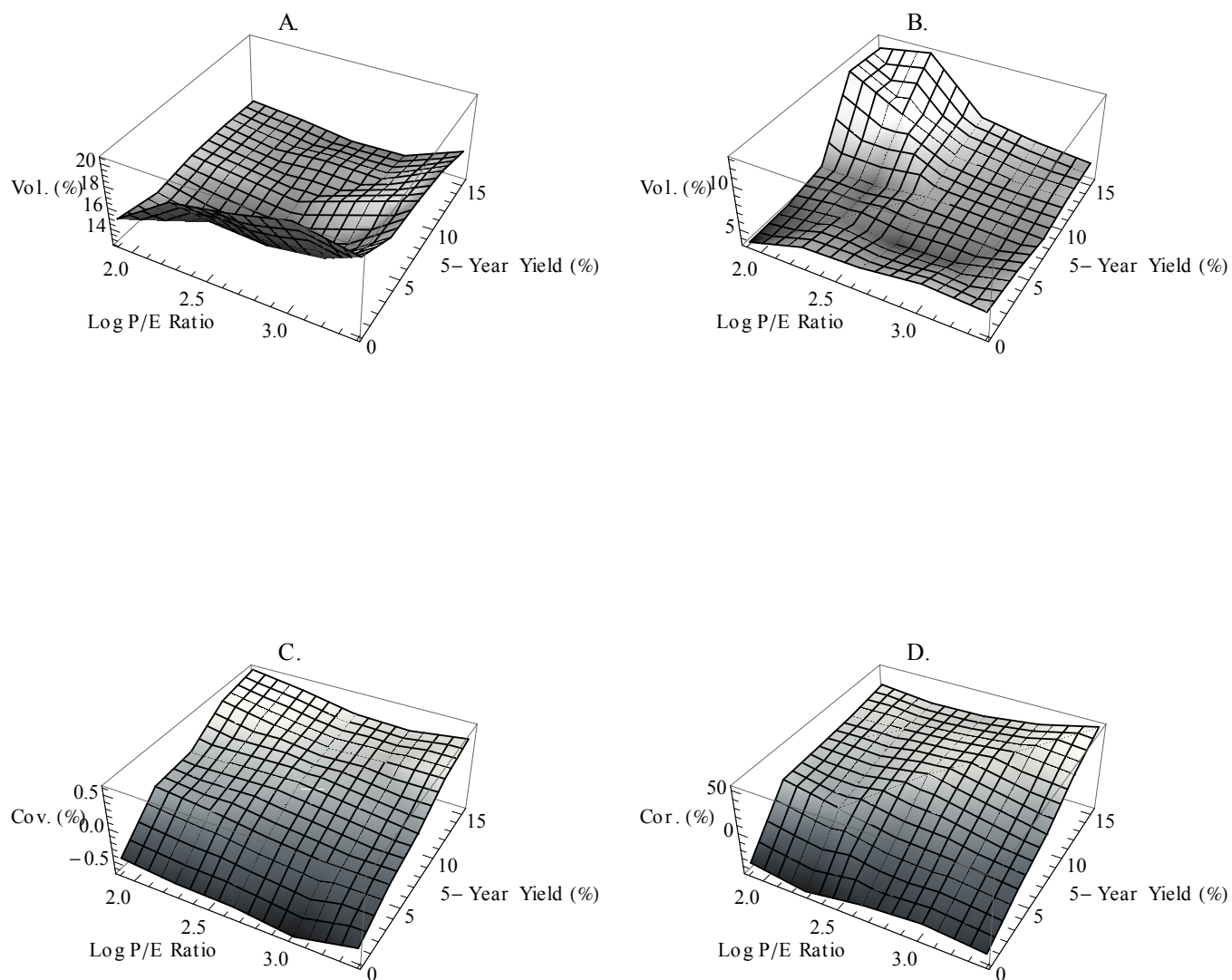


Figure 8. The Non-linear Relation between Second Moments, 5-year Yield and log P/E. Panel A. Stock return volatility. Panel B. 5-year bond return volatility. Panel C. Covariance between stock returns and 5-year bond returns. Panel D. Correlation between stock returns and 5-year bond returns. Each panel plots the fitted value of a bivariate kernel regression of each quantity on contemporaneous 5-year yield and log price-earnings ratio.

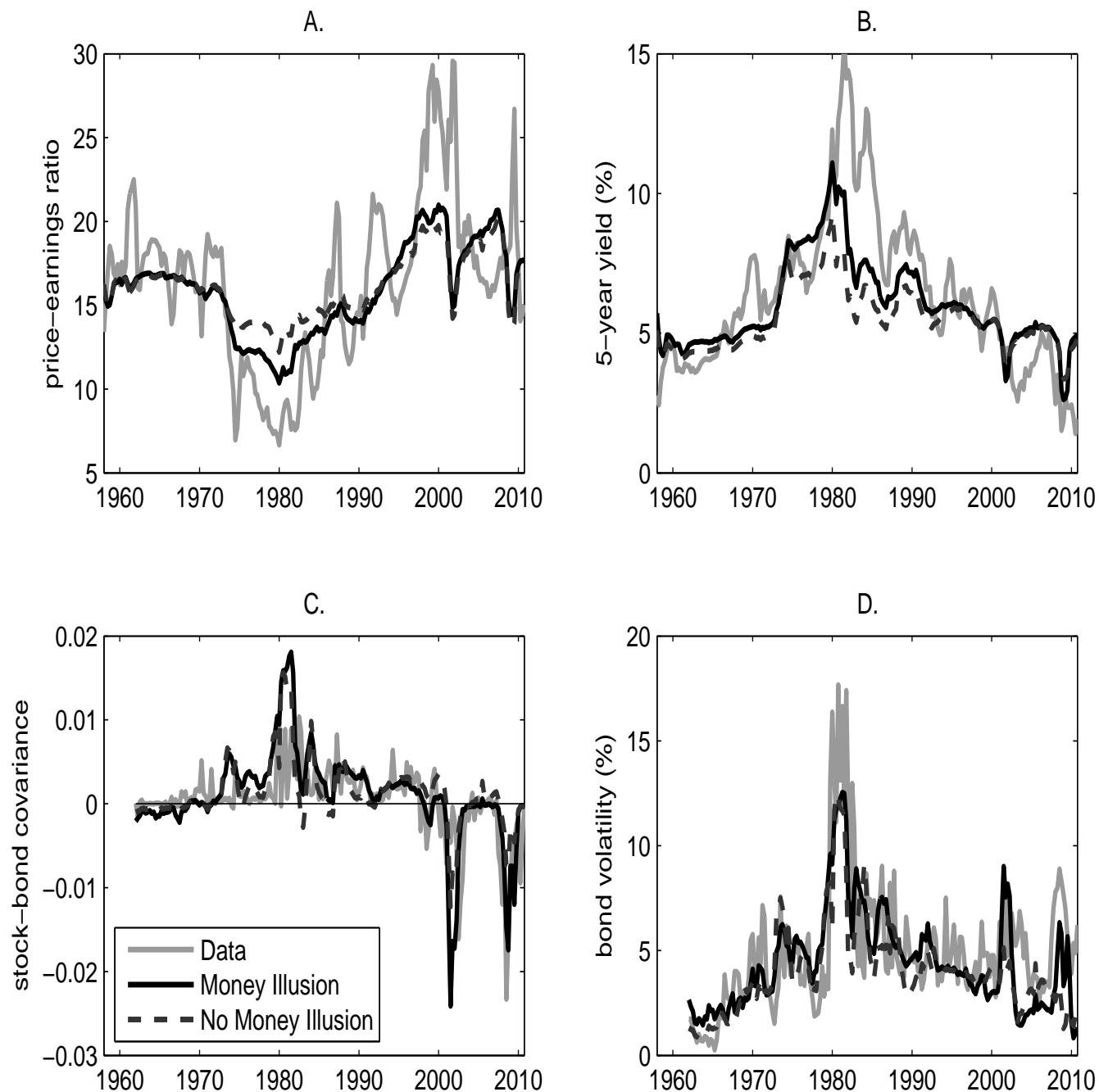


Figure 9. Money Illusion versus No Money Illusion. Panel A plots the fitted P/E ratio under the model with money illusion (black line) and the restricted model without money illusion (dashed line), along with the realized P/E ratio (grey line). The case without money illusion is re-estimated under the parameter constraint $\delta = 0$. Panel B, C, and D plot the time series of the 5-year bond yield, the stock/bond covariance, and the bond return volatility, respectively.

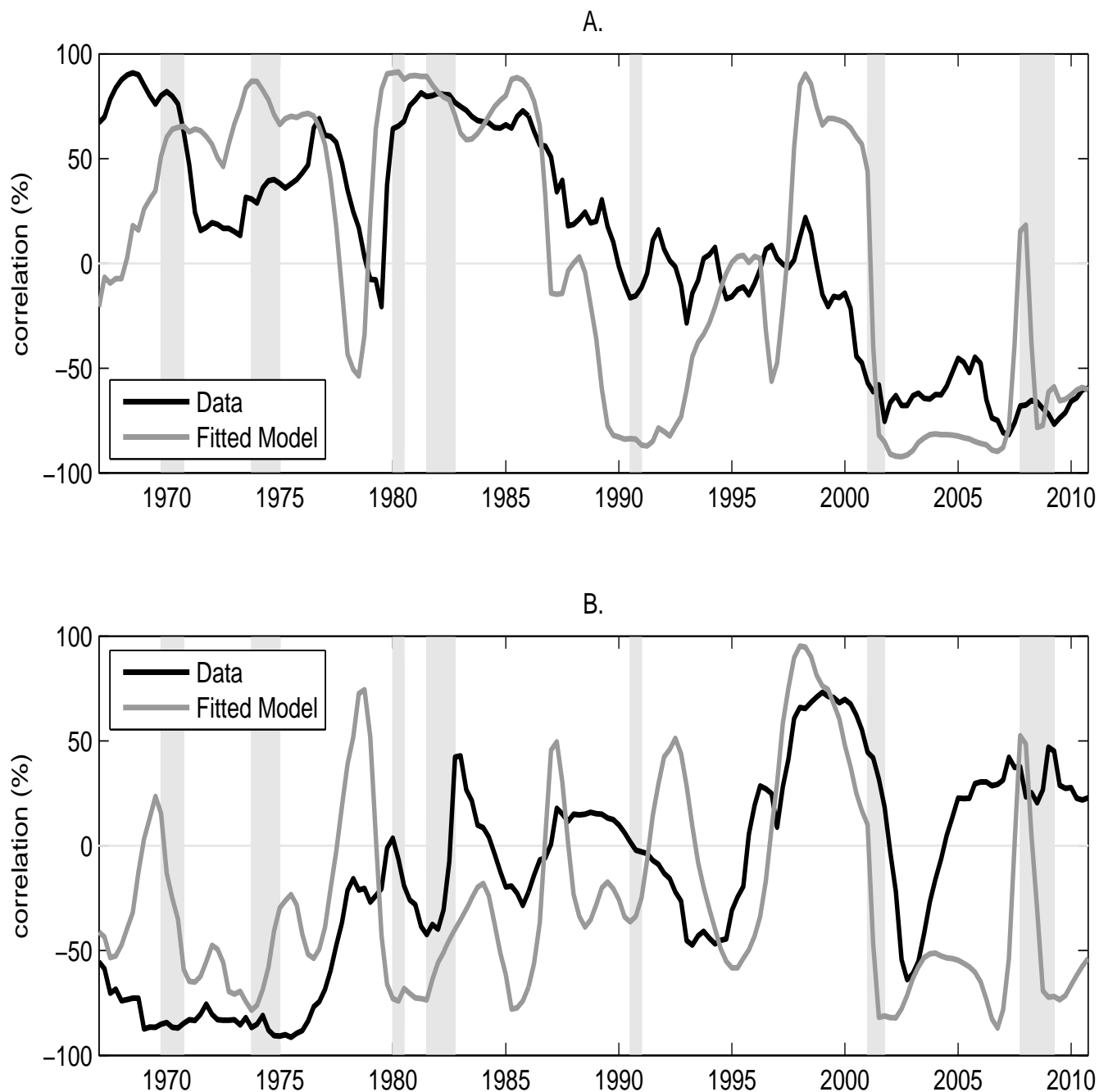


Figure 10. The Time-Varying Correlation between Volatility and Asset Prices. Panel A plots 5-year trailing correlation between the 5-year yield and the 5-year bond return volatility in the data (black line) and in the fitted model (grey line). The correlation between the two series is 67%. Panel B plots the 5-year trailing conditional correlation between the P/E ratio and stock return volatility in the data (black line) and in the fitted model (grey line). The correlation between the two series is 36%.