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What Ties Return Volatilities to Price Valuations and Fundamentals?

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**ABSTRACT**

The relation between the volatility of stocks and bonds and their price valuations is strongly time-varying, both in magnitude and direction, defying traditional asset pricing models and conventional wisdom. We construct and estimate a model in which investors' learning about regular and unusual fundamental states leads to a non-monotonic V-shaped relation between volatilities and prices. Structural forecasts from our model predict future return volatility and covariances with  $R^2$  ranging between 40% and 60% at the 1-year horizon. The model's success stems largely from backing out the endogenous and time-varying pro (counter) cyclical weights that investors assign to earnings (inflation) news.

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# Introduction

While it is intuitive that the volatilities and comovements of stocks and bonds are strongly related to the state of economic fundamentals, it is surprising that the financial literature has been unable to empirically demonstrate such a strong link between them, as evidenced in the following quote from a recent paper by Nobel prize laureate Robert Engle.

*“After more than 25 years of research on volatility, the central unsolved problem is the relation between the state of the economy and aggregate financial volatility. The number of models that have been developed to predict volatility based on time series information is astronomical, but the models that incorporate economic variables are hard to find. Using various methodologies, links are found but they are generally much weaker than seems reasonable. For example, it is widely recognized that volatility is higher during recessions and following announcements but these effects turn out to be a small part of measured volatility.”* [Engle and Rangel (2008)]

For example, in a seminal paper, Schwert (1989) notes that stock market volatility is higher during recessions, which is now a widely accepted and influential stylized fact for volatility. While it is true that volatility is countercyclical, the regression of stock market volatility on the NBER recession indicator over 1960 – 2008, has a fairly low  $R^2$  of only about 13%. For bond volatilities the regression  $R^2$ s are only around 15% for alternative maturity bonds. In fact, the  $R^2$  of virtually all the macroeconomic variables that have been tested in the literature, which we will discuss in more detail in this paper, jointly explain a very small proportion of stock market volatility.

In this paper we argue that the relation between volatility and the macroeconomy is in fact very complex and goes beyond the simple boom-bust business cycle variation. This fact can be seen by looking at the top panel of Figure 1, which shows the massive variation, both in magnitude and in direction, of the correlation between stock return volatility and the price/earning (P/E) ratio. While conventional wisdom and most asset pricing models would predict a negative correlation between aggregate volatility and P/E ratio,<sup>1</sup> in line with the empirical finding that volatility tends to rise during recessions while the P/E ratio tends to drop, the top panel of Figure 1 shows that this is not really the case in the data. In fact, the correlation between volatility and P/E ratio is often strongly

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<sup>1</sup> For instance, both the habit formation model of Campbell and Cochrane (1999) [see Figures 3 and 5] and the long-run risk model of Bansal and Yaron (2004) [see Section II.C.3] imply a negative relation between valuation ratios and volatility.

positive.<sup>2</sup> What kind of economic mechanism generates this massive variation in the correlation between prices and volatility? Why is such mechanism unrelated to macroeconomic information? Why does this correlation switch sign?

Interestingly, the same economic mechanism that generates the time variation in the relation between stock prices and volatility is likely at play also for Treasury bonds, as the middle panel of Figure 1 shows. This panel plots the correlation between the yield of the 5-year Treasury bond and its return volatility. While the correlation has been mainly positive in the sample, it dipped to negative occasionally, and it has become strongly negative in the current decade spurring the question as to what has changed in this decade. Indeed, relatedly, the bottom panel of Figure 1 shows one additional interesting empirical fact, recently discussed in Baele, Bekaert, and Inghelbrecht (2009) and Campbell, Sunderam, and Viceira (2009), namely, that the covariance between stocks and bonds, normally positive, also changes substantially over time, as it became strongly negative around the two recessions of the new millennium. Once again, then, what economic model is consistent with this variation in the sign of the covariance between stock and bond prices? Is this economic mechanism consistent with the time variation in the relation between prices and volatilities, documented in the first two panels of Figure 1? The understanding of the economic mechanism driving asset prices, volatilities and correlations, is central to asset pricing and macro finance, and yet, as pointed out above is at odds with the most popular asset pricing models.

In this paper we argue that these stochastic changes in the relation between asset prices, volatilities, and cross-covariance, are due to market participants' variation in their beliefs about economic growth and future inflation. More specifically, we assume that the drift rates of earnings growth and inflation follow a joint  $n$ -state regime-switching model but that market participants cannot observe the current state of the economy and thus must learn about it by observing fundamentals and other signals. We obtain closed form formulas for stock prices, bond prices, their volatilities and cross-covariances. Because we, as econometricians, do not have full information about the signals used by market participants to form their beliefs, we exploit our analytical asset pricing formulas and estimate the time series of beliefs from fundamentals as well as prices, volatilities, and covariances.

The estimated model shows that Bayesian learning leads to a non-monotonic,  $V$ -shaped relation between valuation ratios and volatilities, both for stocks and bonds, an empirical regularity

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<sup>2</sup>Using data from Bob Shiller's web site, daily returns from 1926 to 2008 from CRSP, and daily returns before 1926 from Bill Schwert's data set, a similar time variation can be observed for the longer sample 1880 - 2008. The correlation between volatility and the price/earning ratio turned positive in the late 1920s, in the postwar period, and in the 1950s.

that we verify in the data. The intuition can be summarized as follows: Roughly speaking, there are “middle” economic states that are far more common than others. For instance, medium real growth is more common than very low growth and very high growth. Similarly, low/medium inflation is more common than very high inflation or deflation. Because of these characteristics, when the economy settles on the more common states, Bayesian learning induces a decline in uncertainty and thus of volatility for both stocks and bonds. However, when investors perceive a large deviation from these “regular” states, either positive or negative, they increase the subjective probability that the economy entered into a more unusual state. As a consequence, investors’ uncertainty about the true economic state increases and so does stock return volatility, as a natural consequence of the dynamics of Bayesian learning. Bond and stock valuation ratios, however, mainly react to the *direction* of the change, that is, to whether investors believe the transition is to a better or worse economic state. This decoupling of the direction of the change in beliefs and Bayesian uncertainty leads to a  $V$ -shaped relation between return volatility and price valuation ratios.

As a consequence of the endogenous non-monotonic relation between volatility and valuation ratios, our asset pricing model combined with the estimated time series of beliefs is able to largely explain the observed time variation in stock and bond prices, their volatilities, and cross-covariances. For instance, while the estimated model implies on average a negative correlation between the P/E ratios and return volatility (the conventional wisdom), it also explains why sometimes this relation may switch sign and become positive, as it did in the late 1990s. According to our estimates, this switch to a positive relation between volatility and prices during this time was due to a cautious increase in market participants’ beliefs about the US entering into a sustained high growth period. These beliefs increased both the P/E ratio, as they increased expected cash flows, and volatility, as they increased the uncertainty on whether this transition to a high growth state was true or not.

Similarly, the same economic mechanism explains the strong time variation in the relation between bond return volatility and yields. For instance, while in the late 1970s investors’ uncertainty about whether the economy would enter into a persistent stagflation (high inflation and low growth) state pushed up both bond return volatility and yields, the relation reversed in the new millennium because of an increase in uncertainty about whether the economy will enter into a persistent *deflationary* phase. The model then provides a consistent story of two periods that are characterized by a fundamentally different relation between bond yields and volatility.

Besides explaining the time-varying relation between valuation ratios and volatility of bonds and stocks individually, our model also explains the time variation in their comovement. Indeed, for

instance, while the estimated model produces on average a positive correlation between bonds' and stocks' yields (the conventional wisdom, also called the "Fed model"<sup>3</sup>), it also implies a negative correlation in the last decade, as investors' beliefs moved to give a somewhat higher probability that U.S. will enter a deflationary-low growth state. This belief lowered the P/E ratio and raised bond prices. Moreover, our estimated model suggests that the large uncertainty in the second half of 2008 about whether the transition to such a negative state in fact occurred caused a massive burst in the volatility of beliefs, which explains to a large extent the (negative) correlation between stocks and bonds, and record-setting stock volatility in this period.<sup>4</sup>

Our model not only provides a unified economic framework explaining several facts about the puzzling dynamic relation between prices, volatilities, and correlations, but it also improves substantially the predictability of future volatilities and covariances, even at relatively long horizons. For instance, the structural forecast of stock return volatility, whose construction we will discuss further below, explains over 42% of the 1-year ahead volatility, against only 5% of macro-variables and 15% of lagged volatility. The relatively limited success of macro variables is in line with the quote by Robert Engle at the beginning of the paper. In contrast to stocks, for bonds our model forecasts, macro variables, and lags, each forecast a similar amount, of nearly 60%, of the 1-year ahead realized return volatility. We return below on the explanation of the relative difference in performance of our model and macro variables for stocks and bonds. Finally, the structural model forecasts also perform well in predicting more than half the future variation of the covariance between stocks and bonds, slightly higher than lags and macro variables. The gap between the model and lags for each case improves substantially at longer horizons such as 2 years. The fact that the model *forecasts* future volatility and covariances shows that it aggregates relevant macroeconomic information, and provides additional evidence in support of the learning mechanism put forward in this paper.

It is important to emphasize why our model's forecast is more successful than forecasts based on past volatility or macroeconomic variables. First, our model and estimation strategy utilizes the information aggregation property of asset prices to capture the forward-looking information of investors in the economy. This information includes also expected future volatility, although the re-

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<sup>3</sup>Some authors have called the inverse relationship between P/E's and Treasury yields the "Fed Model," a term that has been attributed to Prudential Securities strategist Ed Yardeni. The relationship seems to hold in several countries across varying time samples [see, for example Aubert and Giot (2007) and Bekaert and Engstrom (2009)]. These papers however have failed to document the reversal of the relationship in the current decade.

<sup>4</sup>We thus provide a macroeconomic explanation for a large proportion of the high volatility in this period, which was in all probability boosted by the failures of Lehman Brothers and AIG.

lation is strongly non-linear due to the learning mechanism. Second, our structural learning model in a cash-flow discounting framework endogenously formulates stock and bond market volatilities as weighted averages of earnings and inflation uncertainty, so that macroeconomic information impacts volatility in a non-linear way. These endogenous weights crucially depend on both investors' beliefs about the state of the economy, and their subjective price valuations of stocks and bonds in those states, as determined by their preferences (summarized by their state price density in our model). It follows that while uncertainty about the state of the economy is a key determinant of volatility, there is also an important amplification mechanism generated by investors' marginal valuations of the various economic states, which may generate high volatility even when uncertainty is relatively low, and vice versa. Our calibrated model reveals that the weight investors place to earnings (inflation) news in stock market volatility is strongly pro (countercyclical), while the corresponding weights in bond market volatility are far more stable.<sup>5</sup> Therefore, even though the same fundamental information is useful in predicting both volatilities, due to the greater stability in the weights, bond volatility is better forecasted by simple linear regressions on lagged fundamentals than is stock market volatility.

Third, our model distinguishes among the fundamental conditions in the past several economic cycles on a finer filtration than the simple boom/bust states of a macroeconomic business cycle, which also affect market volatility. For instance, as mentioned, in the late 1990s and in the period between the two recessions in the current decade, investors' optimism that the U.S. economy had entered a new growth phase, which has also led to lofty stock price valuations, led to a burst of volatility due to uncertainty about the transition itself. This occasional positive correlation between volatility and stock prices defies the conventional wisdom, but it is naturally generated by our model. Similarly, variation in investors' expectations about inflation and deflation, and their uncertainty about it, changes both the sign of the relation between yield and bond return volatility, as well as the direction of stocks and bonds comovement.

This paper contributes to the asset pricing literature also from a methodological standpoint, as we put forward a new and different *equilibrium* structural form approach to volatility forecasting. In contrast to the GARCH literature, which effectively entails predicting future volatility by using lagged volatility and macroeconomic variables in a possibly non-linear regression, our approach starts from an economic specification of fundamentals and investors' pricing kernel – the main

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<sup>5</sup>These weights imply that generally discount rate news is relatively more important prior to recessions, which have generally been periods of rising inflation expectations, while cash flow news is relatively more important during boom periods.

ingredients of an asset pricing model – and explicitly solves for the equilibrium asset price levels and their volatilities in closed-form as functions of the state variables of the model. The model is then fit not only to fundamental data, but also to asset price levels and their volatilities with the use of overidentifying moments. Thus, in our framework the impact of the macroeconomic variables on volatility is explicit and completely specified.

It is important to note that our approach is different from the traditional GARCH literature in not only specifying a functional form for the volatilities of assets returns, but also in the way we fit our model parameters to financial data. In the former literature, the volatility of historical asset returns is not observed by the econometrician, who formulates alternative theoretical dynamic relationships for the volatility process to maximize the likelihood of observing the asset returns process. We instead follow an alternative approach in the volatility literature of treating realized volatility as an observed process.<sup>6</sup> This permits us to use volatility as an overidentifying moment in our estimation procedure described above. Forecasts of volatility can be optimally constructed using Monte Carlo simulations, a method that allow us to exploit the correct dynamics of beliefs as implied by Bayes rule, given the closed-form expression of the impact of beliefs on volatility and the estimated parameters of the structural form model. In addition, our estimation procedure allows for the econometrician's information set to be quite different from that of investors, by allowing for signals and state variables being observed only by the latter. Thus in our model, prices aggregate the information received by investors, and the econometrician extracts it and uses it to construct volatility forecasts.

Our paper is related to numerous strands of literature. First, it is related to the previous literature about Bayesian learning and asset prices, and especially the early purely theoretical contributions of David (1997), Veronesi (1999), and Veronesi (2000). In this paper we consider a much richer environment in which return volatility depends not only on investors' state-uncertainty, but crucially also on investors' marginal valuations of those economic states. This additional feature allows us to uncover a V-shaped relation between stock volatility and the P/E ratio against the *inverted U*–shape implied by Veronesi (1999). In addition, and differently from previous work, we also exploit the

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<sup>6</sup>In this sense we follow the approach of Schwert (1989), who approximates volatility at the monthly and quarterly frequencies as the squared daily average returns in these periods, but does not have a structural specification of volatility. More recently, there has been increasing interest in using higher frequency intraday data to approximate volatility in any given period [see, for example, Anderson, Bollerslev, Diebold, and Labys (2003)]. We continue to use daily returns to construct our volatility measures since we are interested in studying the relationship between volatility and the macroeconomy for a long sample during which intradaily data are not available.



model's implications to *estimate* the model's parameters from prices and volatilities, in addition to fundamentals, and form forecasts of future volatility. This estimation methodology is key to the success of the model. From this perspective, our methodology is closer to the recent paper of David (2008), who uses a similar model to ours, though he focuses on leverage and the level and variation of corporate credit spreads. The paper is also related to Pastor and Veronesi (2006) (henceforth PV), who also argue that uncertainty about long-term profitability pushed up both the NASDAQ index and its volatility in the late 1990s. It is crucial to observe that our model differs from PV in 3 key respects: First, our focus is on *aggregate* volatility, and not idiosyncratic volatility, which was the focus of PV. Second, we emphasize the time variation in the sign of the relation between prices and volatility, while PV do not have anything to say about these dynamics, as they mainly focus on the uncertainty and price level at one particular point in time, March 2000. Finally, unlike all these papers, our paper focuses not only on stocks, but also on Treasury bonds, uncovering a similar relation, and on the stock and bond comovement.

The paper is clearly related also to the large literature on time varying volatility, of which we do not attempt an exhaustive survey. See Bollerslev, Chou, and Kroner (1992) for a relatively early survey of its applications and Anderson, Bollerslev, and Diebold (2004) for more recent advances. As mentioned, we propose here a different and new methodology to use the asset prices implied by a structural model to improve upon the forecasting ability of macroeconomic information.

The paper develops as follows. In Section 1 we provide our economic model of fundamentals, asset prices, and asset price volatilities. In Section 2 we describe our empirical methodology to estimate the model and discuss its implications for the time series of fundamentals and asset prices. Section 3 shows that the estimated model fits well the data, and provides an intuitive explanation of the puzzling dynamics highlighted at the beginning of the introduction. Section 4 provides further evidence in favor of the model, as it performs the empirical tests on the forecastability of volatility and the covariation of asset returns. Section 5 concludes. An appendix contains the details of our empirical methodology to estimate the parameters of our continuous time model with discrete data using the Simulated Method of Moments (SMM).

# 1. Structure of the Model

Inflation is a key state variable in our model. We assume that the price of the single homogeneous good in the economy,  $Q_t$ , follows the process

$$\frac{dQ_t}{Q_t} = \beta_t dt + \sigma_Q dW_t, \quad (1)$$

where  $W_t = (W_{1t}, W_{2t}, W_{3t}, W_{4t})'$  is a four-dimensional vector of independent Weiner processes, the  $1 \times 4$  constant vector  $\sigma_Q = (\sigma_{Q,1}, 0, 0, 0)$  is assumed to be known by investors, and the process followed by  $\beta_t$  is described below.

The only real fundamental in the economy is the aggregate real earnings of all corporations,  $E_t$ , which follows the process

$$\frac{dE_t}{E_t} = \theta_t dt + \sigma_E dW_t, \quad (2)$$

where the  $1 \times 4$  constant vector  $\sigma_E = (0, \sigma_{E,2}, 0, 0)$  is assumed known by investors and the process for  $\theta_t$  is described below.<sup>7</sup> Investors learn about the hidden state of real earnings by observing its past history as well as an unbiased signal,  $S_t$  that they receive. The signal follows the process

$$\frac{dS_t}{S_t} = \theta_t dt + \sigma_S dW_t, \quad (3)$$

where the  $1 \times 4$  constant vector  $\sigma_S = (0, 0, 0, \sigma_{S,4})$  is assumed known by investors. The signals are not observed by the econometrician.

To simplify our analysis, we assume that market participants' preferences are summarized by a state price density (SPD)  $M_t$ , which evolves over time according to the process:<sup>8</sup>

$$\frac{dM_t}{M_t} = -k_t dt - \sigma_M dW_t, \quad (4)$$

where  $\sigma_M = (\sigma_{M,1}, \sigma_{M,2}, \sigma_{M,3}, \sigma_{M,4})$  is a  $1 \times 4$  constant vector of the market prices of risk, and  $k_t = \alpha_0 + \alpha_\theta \theta_t + \alpha_\beta \beta_t$  is the real short rate of interest *conditional* on the hidden state variables  $\theta_t$  and  $\beta_t$ . We restrict the conditional real rate of interest  $k_t$  to be a linear function of the two (hidden) state variables of the model, a restriction partly motivated by the drift in general equilibrium models with aggregate consumption. For example, in a Lucas (1978) economy where

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<sup>7</sup>By setting  $\sigma_{E,1} = 0$  we are assuming that the covariance between inflation and earnings is zero. We indeed find this to be the case for our inflation and earnings series described in Section 2.

<sup>8</sup>See e.g. Duffie and Kan (1997), Duffie and Singleton (1999) for seminal papers in the term structure and credit risk literature, respectively, Berk, Green, and Naik (1999) for an equilibrium theory determining the Fama-French factors, and Piazzesi (2005) for a theory of the Federal Reserve's monetary policy and bond yields, for prominent asset pricing contributions that use such a simplifying assumption.

investors have power utility  $U(C, t) = e^{-\phi t} \frac{C^{1-\gamma}}{1-\gamma}$ , we would have  $C_t = E_t$ ,  $M_t = U'(E_t, t)$ , and hence  $k_i = \phi + \gamma\theta_i + \frac{1}{2}\gamma(1-\gamma)\sigma_E\sigma'_E$  and  $\sigma_M = \gamma\sigma_E$ . In this case, the real rate is not affected by the inflation drift, that is,  $\alpha_\beta = 0$ . In general, however, the real rate may depend on the drift rate of inflation [see e.g. Feldstein (1980a), Feldstein (1980b), Auerbach (1979), Cohen and Hassett (1999) for micro-based motivations, and Cooley and Hansen (1989), Bakshi and Chen (1996) for macro and asset pricing models] and we leave the sign and magnitude of  $\alpha_\beta$  as an empirical issue. Note that while the state price density  $M_t$  (e.g.  $= U_c(E_t, t)$ ) is observed by investors, its drift rate  $k_t$  is not, as it depends on hidden state variables. In addition, we assume that  $M_t$  is not observed by the econometrician, who has to estimate it from data.

Given the state price density  $M_t$  and their information  $\mathcal{F}_t$  at time  $t$ , agents price a generic real payout flow of  $\{D_t\}$  as

$$M_t P_t = E \left[ \int_t^\infty M_s D_s ds | \mathcal{F}_t \right], \quad (5)$$

To streamline the notation in a paper we stack the fundamental processes and write  $X_t = (Q_t, E_t, M_t, S_t)'$ , which has the drift vector  $\nu_t = (\beta_t, \theta_t, -k_t, \theta_t)'$ , and volatility matrix  $\Sigma = (\sigma'_Q, \sigma'_E, -\sigma'_M, \sigma'_S)'$ . The drift vector  $\nu_t$  follows an  $n$ -state, continuous-time Markov chain with generator matrix  $\Lambda$ . We denote the states vectors as  $\nu^1, \dots, \nu^n$ . Thus, over the infinitesimal time interval of length  $dt$  the probability to move from state  $i$  to  $j$  is

$$\lambda_{ij} dt = \text{prob}(\nu_{t+dt} = \nu^j | \nu_t = \nu^i), \quad \text{for } i \neq j, \quad \lambda_{ii} = - \sum_{j \neq i} \lambda_{ij}.$$

The transition matrix over states in a finite interval of time,  $s$ , is  $\exp(\Lambda s)$  [see, for example, Karlin and Taylor (1982)]. It is important to note that we model joint shifts in inflation and real variables, which implies that as in the “proxy hypothesis” of Fama (1981), inflation signals future real economic activity.

We assume that investors do not observe the realizations of  $\nu_t$  but know all the parameters of the model. They update their beliefs about the current state of  $\nu_t$  from the observations of past earnings, inflation, the signal, and the state price density  $M_t$ . Their learning process results in the time series of conditional probabilities of the hidden states,

$$\pi_{it} = \text{prob}(\nu_t = \nu^i | \mathcal{F}_t).$$

Given an initial condition  $\pi_0 = \hat{\pi}$  with  $\sum_{i=1}^n \hat{\pi}_i = 1$  and  $0 \leq \hat{\pi}_i \leq 1$  for all  $i$ , the probabilities  $\pi_{it}$  satisfy the  $n$ -dimensional system of stochastic differential equations:

$$d\pi_{it} = \mu_i(\pi_t) dt + \sigma_i(\pi_t) d\tilde{W}_t, \quad (6)$$

where,

$$\mu_i(\pi_t) = [\pi_t \Lambda]_i, \quad (7)$$

$$\sigma_i(\pi_t) = \pi_{it} [\nu^i - \bar{\nu}(\pi_t)]' \Sigma'^{-1}, \quad (8)$$

$$\bar{\nu}(\pi_t) = \sum_{i=1}^n \pi_{it} \nu^i = E_t \left( \frac{dX_t}{X_t} | \mathcal{F}_t \right). \quad (9)$$

In (6),  $d\tilde{W}$  are Brownian motions adapted to the investors' filtration, defined by the normalized expectation errors

$$d\tilde{W}_t = \Sigma^{-1} \left[ \frac{dX_t}{X_t} - E \left( \frac{dX_t}{X_t} | \mathcal{F}_t \right) \right] = \Sigma^{-1} (\nu_t - \bar{\nu}(\pi_t)) dt + dW_t. \quad (10)$$

The belief processes were first derived in Wonham (1964), and first introduced to the finance literature in David (1997). In particular, as was pointed out in that paper, the conditional variance of investors' expectations fluctuates indefinitely in this model, while in the more commonly used Kalman filter, it is deterministic and converges to a constant. These fluctuations in uncertainty are key to understanding the dynamics of volatility in this paper. An important generalization in this paper is that the econometrician estimates investors beliefs when her information set is smaller than the agents' since it does not contain the investors' state price density, or the additional signals about real growth received by agents. It is useful to note that investors' beliefs change with their inferred shocks,  $d\tilde{W}$ , in equation (10) as opposed to the true shocks,  $dW$ , that affect fundamentals.

## 1.1. Stock Prices and the Term Structure of Interest Rates

The following proposition provides expressions for the price-earnings (henceforth P/E) ratio and the nominal bond price:

### Proposition 1.

(a) The P/E ratio at time  $t$  is

$$\frac{P_t}{E_t}(\pi_t) = \sum_{j=1}^n C_j \pi_{jt} \equiv C \cdot \pi_t, \quad (11)$$

where for each  $j = 1, \dots, n$  the constant  $C_j$  is given by

$$C_j = E \left[ \int_t^\infty \frac{M_s}{M_t} \frac{E_s}{E_t} ds | \nu_t = \nu^j, \mathcal{F}_t \right] \quad (12)$$

In addition, the vector  $C = (C_1, \dots, C_n)$  satisfies  $C = A^{-1} \cdot \mathbf{1}_n$ ,

$$A = \text{Diag}(k^1 - \theta^1 + \sigma_M \sigma'_E, \dots, k^n - \theta^n + \sigma_M \sigma'_E) - \Lambda. \quad (13)$$

(b) The price of a nominal zero-coupon bond at time  $t$  with maturity  $\tau$  is

$$B_t(\pi_t, \tau) = \sum_{i=1}^n \pi_{it} B_i(\tau), \quad (14)$$

where the  $n \times 1$  vector valued function  $B(\tau)$  with element  $B_i(\tau) = E\left(\frac{M_{t+\tau}}{M_t} \cdot \frac{Q_t}{Q_{t+\tau}} | \nu_t = \nu^i\right)$  is given by

$$B(\tau) = \Omega e^{\omega \tau} \Omega^{-1} \mathbf{1}_n. \quad (15)$$

In (15),  $\Omega$  and  $\omega$  denote the matrix of eigenvectors and the vector of eigenvalues, respectively, of the matrix  $\hat{\Lambda} = \Lambda - \text{Diag}(r^1, r^2, \dots, r^n)$ , where each  $r^i = k^i + \beta^i - \sigma_M \sigma'_Q - \sigma_Q \sigma'_Q$ , is the nominal rate that would obtain in the  $i^{\text{th}}$  state, were the states observable. In addition,  $e^{\omega \tau}$  denotes the diagonal matrix with  $e^{\omega_i \tau}$  in its  $(i, i)$  position.

*Proof.* See the appendix.

In (a), we refer to the each constant  $C_i$  as the *conditional P/E ratio*, as it represents investors' P/E valuation of the stock conditional on the state being  $\nu^i$  today. As in the classic Gordon model, the valuation depends on the expected growth of earnings, the real rate, and the equity premium. Since investors do not observe the state  $\nu^i$ , they weight each conditional P/E ratio  $C_i$  by its conditional probability  $\pi_i$  thereby obtaining (11). Notice in particular that because  $k^i = \alpha_0 + \alpha_\theta \theta^i + \alpha_\beta \beta^i$ , the form of the constant vector  $C$  suggests that:<sup>9</sup> (i) if  $\alpha_\theta < 1$ , a higher growth rate of earnings implies a higher P/E, (ii) if  $\alpha_\beta > 0$ , a higher inflation state implies a lower P/E, which is the *real rate effect* of inflation, and (iii) in addition, the P/E ratio in a given state of growth depends on the future sustainability of the growth rate and is determined by the transition probabilities  $\lambda_{ij}$  as shown in the solution to the  $n$ -equations in (a).

Similarly, in (b) the bond price is a weighted average of the nominal bond prices that would prevail in each state  $\nu^i$ ,  $B_i(\tau)$ , which we refer to as the *conditional bond price*. Again, since investors do not actually observe the current state, they price the bond as a weighted average. Ceteris paribus, both higher inflation and higher growth rate of earnings lead to lower long term bond prices when  $\alpha_\beta > -1$  and  $\alpha_\theta > 0$ .<sup>10</sup> It is useful to notice that all asset prices follow continuous paths even though the drift rates for earnings and inflation jump between a discrete set of states. This results from the continuous updating process.

<sup>9</sup>Implications (i) and (ii) can be easily seen for the special case  $\Lambda = 0$ , which implies  $C_i = 1/[\alpha_0 + \alpha_\theta(\theta^i - 1) + \alpha_\beta \beta^i + \sigma_M \sigma'_E]$ .

<sup>10</sup>This can be easily seen in the special case  $\Lambda = 0$ , as in this case  $B_i(\tau) = \exp(-[\alpha_0 + \alpha_\theta \theta^i + (\alpha_\beta + 1) \beta^i - \sigma_M \sigma'_Q - \sigma_Q \sigma'_Q] \tau)$ .

Let  $P_t^N = P_t \cdot Q_t$  be the nominal value of stock, where  $P_t$  is the real value of stocks in Proposition 1. Using the dynamics of the inflation and earnings processes under the observed filtration, we now formulate the nominal return processes for stocks and bonds.

**Proposition 2.**

(a) *The nominal stock return process under the investor's filtration is given by*

$$\frac{dP_t^N}{P_t^N}(\pi_t) = (\mu^N(\pi_t) - \delta(\pi_t)) dt + \sigma^N(\pi_t) d\tilde{W}_t,$$

where  $\delta(\pi)$  is the dividend yield, and the nominal stock price volatility is

$$\sigma^N(\pi_t) = \sigma_E + \sigma_Q + \frac{\sum_{i=1}^n C_i \pi_{it} (\nu^i - \bar{\nu}(\pi_t))' (\Sigma')^{-1}}{\sum_{i=1}^n C_i \pi_{it}}. \quad (16)$$

(b) *The nominal zero-coupon bond return process is*

$$\frac{dB_t(\pi_t, \tau)}{B_t(\pi_t, \tau)} = \mu^B(\pi_t, \tau) dt + \sigma^B(\pi_t, \tau) d\tilde{W}_t,$$

where the nominal bond price volatility is

$$\sigma^B(\pi_t, \tau) = \frac{\sum_{i=1}^n B_i(\tau) \pi_{it} (\nu^i - \bar{\nu}(\pi_t))' (\Sigma')^{-1}}{\sum_{i=1}^n B_i(\tau) \pi_{it}}. \quad (17)$$

*Proof.* Follows from an application of Ito's Lemma to the pricing formulas in Proposition 1.

Stock price volatility has an exogenous component due to noise in the fundamental process and a learning-based endogenous component, which depends on the volatility of each state probability  $\pi_i$ . However, the volatility of stock prices depends additionally on the valuation of stocks in each state as measured by the conditional P/E vector,  $C$ . For a given news content, *states* in which valuations are higher contribute more to the total volatility. Since the valuations vary with both expected earnings growth and discount rates, stock price volatility in our model will endogenously load differently on news about these components in different stages of the business cycle as found in Boyd, Hu, and Jagannathan (2005). To provide an intuition, suppose that at some time  $t$  there are two states  $i$  and  $j$  for which  $\pi_{it} \approx (1 - \pi_{jt})$ . In this case, the volatility in (16) can be written as

$$\sigma^N(\pi_t) \approx \sigma_E + \sigma_Q + \frac{(C_i - C_j) \pi_{it} (1 - \pi_{it}) (\nu^i - \nu^j)' (\Sigma')^{-1}}{P/E(\pi_t)} \quad (18)$$

It follows that if the conditional P/E ratios in the two states are similar to each other,  $C_i \approx C_j$ , then even a large uncertainty on the states,  $\pi_{it} \approx \pi_{jt} \approx 0.5$ , would still generate a small learning-induced volatility. Vice versa, if  $C_i$  and  $C_j$  are very different from each other, even a milder uncertainty on the states may generate a high learning-induced volatility, especially if  $P/E(\pi_t)$  is small. These findings highlight that stock return volatility depends on a complex interaction of uncertainty across

states (e.g.  $\pi_{it}(1 - \pi_{it})$  in (18)) and market participants' price valuations of those states (e.g.  $(C_i - C_j)$  in (18)), which in turn depend on market participants' preferences through the state price density  $M_t$ , as shown in (12). The dynamics of volatility becomes much more complex when multiple states have positive posterior probability. In general, if in state  $i$  the earnings drift is far from its value in other states, so that the volatility of the probability of this state is most reactive to earnings news, then stock volatility will give a larger (smaller) weight to this earnings news if the conditional P/E ratio  $C_i$  is high (low) in this state. We will discuss this point further in Section 2 for our calibrated model.

The form of the bond volatility equation is very similar to the stock volatility equation, except that the conditional P/E ratio  $C_i$  in any state  $i$  is replaced by the conditional bond price  $B_i(\tau)$  in that state. In addition, there is no exogenous fundamental component to bond volatility. The two volatility equations (16) and (17) are the heart of this paper. From these, covariances between stocks and bonds, and bonds of different maturities, are derived straightforwardly. To simplify the exposition of the covariances as well as to develop sufficient conditions to determine their signs, we use the following “value-adjusted” probabilities:

$$\pi_{it}^C = \frac{\pi_{it} C_i}{\sum_{j=1}^n \pi_{jt} C_j} \quad \text{and} \quad \pi_{it}^{B(\tau)} = \frac{\pi_{it} B_i(\tau)}{\sum_{j=1}^n \pi_{jt} B_j(\tau)}. \quad (19)$$

Notice that the two sets of probabilities weight investors' true probabilities by the conditional valuations of stocks and bonds in each state, respectively. The normalizations ensure that each of the sets of probabilities sum to 1. Further, we define the expectation of the fundamental vector  $\nu$  as  $\bar{\nu}_t^C = \sum_{i=1}^n \pi_{it}^C \nu^i$ , using the value adjusted probability for stocks, and analogously for bonds. Using this notation we can write the stock volatility in (16) with the compact notation:  $\sigma^N(\pi_t) = \sigma_E + \sigma_Q + (\bar{\nu}_t^C - \bar{\nu}_t) \Sigma'^{-1}$ , and the bond volatility in (17) as  $\sigma^B(\pi_t) = (\bar{\nu}_t^{B(\tau)} - \bar{\nu}_t) \Sigma'^{-1}$ . In addition, we can write the covariance between stock and  $\tau$ -period bond returns as

$$\text{Cov} \left( \frac{dP_t^N(\pi_t)}{P_t^N(\pi_t)}, \frac{dB_t(\pi_t, \tau)}{B_t(\pi_t, \tau)} \right) = (\bar{\beta}_t^{B(\tau)} - \bar{\beta}_t) + (\bar{\theta}_t^{B(\tau)} - \bar{\theta}_t) + (\bar{\nu}_t^C - \bar{\nu}_t)(\Sigma \Sigma')^{-1}(\bar{\nu}_t^{B(\tau)} - \bar{\nu}_t)'. \quad (20)$$

While it is hard to place a sign on the stock-bond covariance in general, we can do so at points of time when only 2 of the  $n$  states have positive probability. In particular, we are interested in finding intuitive sufficient conditions for the covariance to be negative, which are provided in the following proposition.

**Proposition 3.** *Let there be 2 states  $i$  and  $j$  such that at date  $t$   $\pi_{it} + \pi_{jt} = 1$ . If the following conditions are met:*

$$(a) \beta_i > \beta_j; \quad (b) \theta_i > \theta_j; \quad (c) C_i > C_j; \quad (d) B_i(\tau) < B_j(\tau)., \quad (21)$$

and all elements of the volatility matrix  $\sigma_{Q,1}$ ,  $\sigma_{E,2}$ ,  $\sigma_{M,j}$ , for  $j = 1, \dots, 4$ , and  $\sigma_{S,4}$  are positive, then,  $\text{Cov}\left(\frac{dP_t^N(\pi_t)}{P_t^N(\pi_t)}, \frac{dB_t(\pi_t, \tau)}{B_t(\pi_t, \tau)}\right) < 0$ .

*Proof.* See the appendix.

This proposition shows that the covariance between stocks and bonds is negative if the earnings and inflation drift vectors  $\theta$  and  $\beta$  are positively correlated between the two states  $i$  and  $j$  (conditions (a) and (b)), while the conditional P/E ratios  $C$  and bond prices  $B(\tau)$  are negatively correlated (conditions (c) and (d)). The latter is intuitive, if stock valuations increase and bond valuations decrease, stock and bond returns will have a negative covariance. The interesting point of our model is that these conditions are tied to the fundamental drifts of earnings and inflation. The result can be theoretically extended to the case where all  $n$  state probabilities are non-zero, however, in our estimated model, we find that the fundamental vectors and price valuations do not have perfect correlations as required, so that in general it is hard to sign the covariance.

## 2. Estimation

In this section we provide a brief description of the estimation methodology and the parameter estimates of our model. Details are in the Appendix.

### 2.1 Estimation Methodology

We use both fundamentals as well as the first and second moments of financial variables to estimate the model's parameters as well as the time series of investors' beliefs over fundamental states. We employ a Simulated Method of Moments (SMM) method to carry out the estimation, as described next.

Let  $\Psi$  denote the set of structural parameters in the fundamental processes of inflation, earnings, earnings signals, and the state price density in equations (1), (2), (3), and (4), respectively. Let the *marginal* likelihood function for the fundamentals data observed at discrete points of time (quarterly) be  $\mathcal{L}$ , which we compute by simulating several sample paths of the state variables in smaller discrete subintervals using the Euler discretization scheme [see e.g. Kloeden and Platen (1992) and Brandt and Santa-Clara (2002)]. To extract information about investors' beliefs from the first and second moments of asset prices we use the pricing formulae for the P/E ratio and Treasury bond prices in Proposition 1, their volatilities in equations (16) and (17), and covariances in (20) to generate model-determined moments. Let  $\{e(t)\}$  denote the errors of the pricing and volatility



variables, and define  $\epsilon(t) = (e(t)', \frac{\partial \mathcal{L}}{\partial \Psi}(t)')'$ , where the second term is the score of the likelihood function of fundamentals with respect to  $\Psi$ . We minimize the usual SMM objective, given in (43) in the Appendix. The details of the procedure are in the Appendix.

It is worth emphasizing three key points about our choice of the SMM method of estimation. First, a simulation-based approach is necessary in our case since the likelihood function for the fundamental data observed at discrete points in time is not available in closed-form. In addition, series-based approximations of the likelihood function as in Ait-Sahalia (2002) are cumbersome given the high dimensionality of our state space (6 dimensions for the beliefs and 2 for fundamentals). Second, our SMM approach allows for the fact that the econometrician observes data only on fundamentals while investors in addition observe their marginal utility (state price density) and signals about earnings, and hence update their beliefs about fundamental drifts based on a finer information filtration. The simulation procedure ensures that we respect the dynamics of beliefs that are implied by the proper filter, explicitly given in (6). Finally, the procedure combines information in asset price and volatility moments with the information in fundamental data so that the extracted investors' beliefs are potentially quite different from estimation methods that rely only on fundamental information [see, for example, Hamilton (1989)].

## 2.2 Estimation Results for the Regime Switching Model

In this subsection, we briefly describe the results of the estimation of our model. Our data sample runs from 1958 to 2008. Our filtering process is started by using the stationary beliefs implied by the parameter values at the starting date. Since investors' beliefs at the initial date are likely influenced more strongly by most recent data received, as is standard in Bayesian econometric methods, we use a burn-in period of 8 quarters and thus report all results for the sub-sample from 1960-2008. We start with the description of the data series used.

Aggregate earnings for the economy are approximated as the operating earnings of S&P 500 firms, and these data are obtained from Standard and Poor's.<sup>11</sup> Similarly, the aggregate P/E ratio is estimated as the equity value of these firms divided by their operating earnings. Dividends for

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<sup>11</sup>Operating earnings typically exclude certain expense or income items that are nonrecurring or unusual in nature, such as restructuring charges and capital gains/losses on unusual asset sales, and are hence used in industry to assess the long term fundamentals of firms. For example, I/B/E/S uses this concept of earnings in analyst surveys. Sharpe (2002) uses operating earnings and analysts estimates of these earnings to study stock P/E valuations and returns. To adjust for seasonal factors we take the 4-quarter moving average of these earnings, which is also used in the construction of the P/E ratios.

these firms, also obtained from Standard and Poor's, are used with the prices to compute returns. We use the Consumer Price Index (CPI), obtained from the Federal Reserve Bank of St. Louis, as our inflation series, which is also used to discount nominal earnings. The time series of zero-coupon yields and returns on bonds of different maturities are obtained from the Fama-Bliss data set available at the University of Chicago. Finally, the realized volatilities of stocks and bonds are estimated as squared average of daily returns in any given quarter. Returns are not demeaned, although the demeaning of the series does not significantly affect our results. Use of such daily averages has a long tradition in finance [see, for example, Schwert (1989)]. More recently, authors such as Anderson, Bollerslev, Diebold, and Labys (2003) use higher frequency intradaily data to estimate realized volatilities; however, such data are not available for the long sample that we use.

We estimate a model with 4 regimes for inflation characterized by the states  $\beta^1 < 0 < \beta^2 < \beta^3 < \beta^4$  and 3 for earnings growth  $\theta^1 < 0 < \theta^2 < \theta^3$ , which lead to 12 composite states overall. In our estimation, we find that the unconstrained estimates of the transition matrix led to several zero elements, leading to a more parsimonious 6 state model with the following states:  $\{(\beta^1, \theta^1), (\beta^2, \theta^2), (\beta^3, \theta^1), (\beta^3, \theta^2), (\beta^4, \theta^1), (\beta^2, \theta^3)\}$ , which we also refer to as (D-LG), (LI-HG), (MI-LG), (MI-HG), (HI-LG), and (LG-NG), respectively. The letters I, and G, stand for inflation and growth, D for deflation, and L, M, H, and N stand for low, medium, high, and new economy, respectively. For example, the state (MI-LG) stands for medium inflation and low growth. We find that the remaining 6 states have close to zero probability of occurring in the sample. Overall, the 6-state and 12-state models lead to almost the same value for the SMM objective function. Gray (1996) and Bansal and Zhou (2002) use a similar criterion for the choice among alternative regime specifications.

The top 2 panels of Table 1 reports estimates of the drifts and volatilities of fundamentals and signals. The 3rd panel reports the transition probability (generator) matrix, while the 4th panel reports the estimates of the parameters of investors' SPD. We estimate that during deflationary periods the CPI on average declines 0.1 percentage points, and inflation averages 2%, 4%, and 8.1% in low, medium, and high states, respectively. Earnings growth averages -5%, 2.5%, and 6.1%, in the low, high, and "new economy" growth rate state, respectively.

The 2nd panel reports the volatilities of inflation and earnings and the earnings signal, which account for unexpected variation in the fundamentals. Notably, earnings are about 4 times as volatile as inflation. It is also worth noting that the instantaneous covariance between the two fundamentals is estimated to be close to zero, so that any nominal-real interaction in our model arises purely from

learning about the drift rates of earnings and inflation, which have a low frequency correlation.<sup>12</sup> The earnings signal, as suggested previously, accounts for information that investors obtain about earnings that is not reflected in historical earnings. Including the signal in the estimation enables us to better calibrate investors' belief response to earnings shocks, which affects all asset volatilities. We estimate volatility of the earnings signal is 9.6%, which is of similar magnitude to the earnings volatility itself.

In the interest of parameter parsimony, we estimate only 6 jump intensities describing the infinitesimal generator  $\Lambda$ , namely,  $\lambda_1 < \dots < \lambda_6$  for  $i = 1, \dots, 6$ , and restrict the infinitesimal generator to be characterized by these 6 intensities, or zero. Table 1 reports the resulting estimated infinitesimal generator. Table 2 reports the implied annual transition densities, which are easier to describe, and so we defer the discussion of our estimates to the next section, where we discuss the beliefs dynamics.

We finally turn to the SPD parameter estimates, the 4th set of parameters in Table 1. We notice immediately that  $\alpha_\theta$  is very close to zero, which implies that the real rate does not depend on the state of real fundamentals (earnings), and that  $\alpha_\beta = 0.685$ , which is significantly positive, which means that the real rate is higher in states of higher inflation. A positive relation between the real rate and expected inflation has been documented by, among others, Geske and Roll (1983), Evans (1998), and Brennan and Xia (2002). It is interesting to note that our estimate is close from that of Sharpe (2002), who estimates that a 1% increase in expected inflation of forecasters increases the required long term rate of return on stocks 0.75%. Ceteris paribus, these estimates imply lower P/E ratios and higher Treasury yields during periods with higher expected inflation. This real rate effect of expected inflation is further boosted by the signaling effect of inflation, which we discuss below, further strengthening the inverse relation between P/Es and Treasury yields.

Using the scores of the likelihood function and the errors of the price and volatility variables, we evaluate the SMM objective function, which serves as an omnibus test statistic [see for example Gray (1996) and Bansal and Zhou (2002)]. The overall SMM objective function value, which has a chi-squared distribution with 11 degrees of freedom, is 18.401, implying a  $p$ -value larger than 7%, so we fail to reject our model.

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<sup>12</sup> This distinguishes our approach from that of Campbell, Sunderam, and Viceira (2009), who assume that real and nominal shocks in the economy have a time-varying and non-trivial quarterly correlation.

## 2.3 The Dynamics of Investors' Beliefs

The estimated dynamics of investors' beliefs are key to understanding the puzzling behavior of stock and bond return volatility, their relation with valuation ratios, and macroeconomic variables.

We first provide some intuition for the dynamics of investors' beliefs by looking at the transition probability matrices and the stationary (long-run) beliefs of investors in Table 2, which are derived from the generator elements in Table 1. Notice first that the regular high growth rate of earnings,  $\theta^2$ , is far more persistent in the low inflation state: from the (LI-HG) state, there is only a 1.8% of chance of growth slowing in the following year. In contrast, from the (MI-HG) state, the chance of a slowdown in earnings increases more than 4-fold to 8.2%. This is the *signaling role of inflation* – it provides an early warning signal of an unsustainable high growth rate of fundamentals. In addition, the state of new economy growth can only be reached under the state of low inflation, so that either a pickup from the normal rate, or a substantial decline that triggers deflation fears will lower investors confidence of the new economy growth rate. Second, as can be seen from the middle panel of Table 2, even at the 5-year horizon there is only about a 2.3% chance of the economy entering the NG state from any other state. The bottom panel of Table 2 shows that the stationary (long run) probability of the NG state is only about 3.9%. However, as we will discuss in Section 4.1, the low probability of this event helps explain the spectacular contemporaneous rise of stock prices and stock volatility in the late 1990s, and their equally spectacular fall in the first half of the current decade. Finally, the dynamics of investors' worries about deflation accompanied by low growth (state 1) enables our model to explain the shift in the current decade to both a negative correlation between stocks and bonds, as well as the co-existence of low yields but high bond return volatility. Table 2 shows that the stationary probability of this state is only 3.8%, and once again the low probability of this state generated tremendous fundamental uncertainty and extremely high volatility at the end of 2008.

Moving to the time series, the 6 panels of Figure 2 report investors' beliefs over the 6 states, estimated from both fundamentals, asset prices, and their volatilities, as described in Section 2.1. As shown in the top-right panel, over the sample period investors have held fairly strong views that the U.S. economy has remained in the LI-HG states. For most of the decade of the 1960s this probability hovered around 85%, while in the 1990s it was lower and averaged around 70%. There were several switches within the medium inflation states in the 1970s, with two bouts of stagflation (high inflation and low growth) in the mid-1970s and early 1980s, as seen in the bottom-left panel. The mid-to-late 1980s were characterized by high (nearly 40%) probabilities of the MI-HG state, which steadily declined through the end of the 1990s, although the probabilities of this state rose

substantially to about 35% before the recession in 1990. This trend decline in expected inflation has also been noted by other authors [see, for example Sargent, Williams, and Zha (2006)]. In the current decade, the probability of the MI-HG state slowly trended up and averaged over 9% between 2004 and 2006, before falling again in the current recession.

One of the more striking aspects of the figure is that the large belief movements in the past 2 recessions have been very different from the previous 6 in our sample. In particular, the pre-recession pickup of the MI probability has been smaller than in the past 6 recessions in our sample. Instead, in both recessions there have been major corrections in investors' beliefs of the new economy growth rate (more dramatic in the 2001 recession), and rapid spikes of the deflation-low growth state probability (more dramatic in the current recession).<sup>13</sup> In fact, by our model estimates, investors' held close to a 50% chance that the economy was experiencing deflation at the end of 2008. This large movement of beliefs between the two states (LI, NG) and (D-LG) between 1995 and 2008 are largely responsible for the intriguing empirical regularities that have characterized the last 15 years: A positive relation between stock volatility and P/E ratio in the late 1990s, the negative relation between bond volatility and yields in the last decade, and the negative correlation between stock and bond returns around the last two recessions. The next section further illustrates these effects.

### 3 Volatilities, Valuations, and Fundamentals

The complex non-linear dynamics of beliefs and their relation to fundamentals explain the puzzling time varying relation existing between asset volatilities and their valuations. In addition, these dynamics also explain the time variation in the size and sign of the comovement between asset prices. This section discusses the performance of the model in accounting for this time-variation, and the intuition behind it. The next section shows that the model is in fact able to improve upon the forecast of future volatilities and covariances compared to lagged volatilities or macroeconomic variables lending additional evidence in favor of our economic mechanism.

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<sup>13</sup>This panel also shows an increase in beliefs about deflation around the 2001 recession, providing support for the Fed concerns about the (small) risk of deflation at that time, as evidenced by Chairman Greenspan's remarks on December 19, 2002, available at <http://www.federalreserve.gov/BOARDDOCS/SPEECHES/2002/20021219/default.htm>.

### 3.1 Stock Volatility and P/E Ratio

Figure 1 in the introduction documents that the relation between stock return volatility and the P/E ratio is strongly time varying, and apparently not explained by simple macroeconomic variables. Our model explains why.

First, we note that our model fits well both the time series of P/E ratio and return volatility. Table 3 shows that the  $R^2$  of a regression of historical P/E ratio on our fitted P/E ratio is about 62%, although notably, the model fails to fully fit the high valuations of the late 1990s, as investors were not fully convinced of the new economy growth state.<sup>14</sup> In particular, the model correctly fits a P/E in the high teens for most of the 1960s and low teens for much of the 1970s and early 1980s, as shown in the top-left panel of Figure 4.

Similarly, Table 3 shows the fitted stock return volatility explains 45% of the variation in historical volatility over the full sample and 59% if the fourth quarter of 1987 is excluded. Several authors have noted that the extreme volatility in this quarter was largely related to a breakdown in trading mechanisms rather than fundamental shocks [see, for example, Kyrillos and Tufano (1995)]. The top-right panel of Figure 4 shows that our model volatility is consistent with nearly all other episodes of historical volatility, and in particular, explains a large proportion of realized volatility in the second half of 2008. Realized volatility hits its record in our sample in the 4th quarter of 2008, rising to about 68% an annual rate, while our model fitted value was at 48%. This is consistent with our model measured earnings uncertainty being at its record level in our sample. Finally, consistent with the observation in Schwert (1989), stock market volatility is higher during recessions, however, the regression  $R^2$  of the historical volatilities on the NBER recession indicator is quite low at only 13%, which underscores the need for writing more sophisticated models that explain its dynamics.

The fitted model reveals a  $V$ -shaped relation between stock return volatility and the P/E ratio, as can be seen from the top-right panel of Figure 5. This panel shows the scatter plot of the model fitted stock return volatility ( $y$ -axis) and the fitted P/E ratio ( $x$ -axis). As can be seen, the volatility is higher both when the P/E ratio is low and when the P/E ratio is high, while the volatility is typically lower when the P/E ratio is in an intermediate range. This  $V$ -shaped pattern of return volatility and P/E ratios is also visible in the data as can be seen in the top-left panel of Figure 5 that plots the measured quarterly return volatility against the end-of-quarter P/E ratio. There are two exceptions, namely, the high measured volatility implied by the two price jumps in October

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<sup>14</sup>Recall that in our model the market price of risk is assumed constant, while there is some evidence of that lower expected return were partly responsible for the high valuations in the late 1990s (see e.g. Pastor and Veronesi (2006)).

1987 and in October 2008, which occurred when the P/E ratio was not (yet) low. Interestingly, and consistently with this last observation, also the fitted model contains occasions of high return volatility for intermediate P/E ratios, demonstrating that the dynamics of return volatility and its relation to prices is actually very complex.<sup>15</sup>

How can the model be consistent with both a positive and a negative relation between P/E ratios and aggregate volatility, as evidenced in the top-right panel of Figure 5? Isn't higher aggregate stock return volatility a symptom of higher "risk" in the economy and thus shouldn't it lead to lower P/E ratios, rather than higher P/E ratios? Indeed, this negative relation is predicted by leading asset pricing models, such as the habit formation model of Campbell and Cochrane (1999), and the long-run risk model of Bansal and Yaron (2004). What is then the intuition behind a *V*-shaped relation between volatility and P/E ratios in the context of our model?

In a nutshell, the answer is simply that uncertainty is higher at the extremes. More specifically, the high growth (HG) state of the economy is far more common than both a low growth (LG) state and a very high growth (NG) state. This fact can be seen from the stationary probabilities  $\pi^*$  in the bottom panel of Table 2. This fact implies that when the economy settles on any of the high growth states, volatility tends to decrease as learning reduces uncertainty. However, a sufficiently large piece of news, positive or negative, that move beliefs away from the current state will increase uncertainty, as investors now assign relatively more weight than before to a state of the economy, higher or lower, that is more unusual. In particular, bad news will move beliefs towards the LG regime while good news will move beliefs toward a NG regime. Either way, the uncertainty increases and with it also return volatility. However, the level of P/E ratios will follow the *direction* of the news: an increase in beliefs that the LG state is true decreases the P/E ratio, while an increase in beliefs that the NG state is true increases the P/E ratio. These dynamics imply that return volatility tends to be higher at the extremes, compared to the middle P/E.

Consistent with this explanation, the top panel of Figure 1 in the introduction provides evidence of the strong time variation in the correlation between P/E ratios and stock return volatility. The top panel of Figure 6 provides additional evidence of the Bayesian learning mechanism just discussed, as it plots the model-implied correlation between P/E and return volatility, along with the 95%

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<sup>15</sup>The empirical finding of a *V*-shape relation between volatility and P/E ratio, also distinguishes this paper from Veronesi (1999), which instead predicts an inverted U-shaped relation. In that paper, with only two states, uncertainty is the only driver of volatility, and thus stock volatility is low at the two extremes cases when investors learn that the state is either very good or very bad, which command a high P/E ratio or a low P/E ratio, respectively.

confidence bands already displayed in Figure 1. The figure suggests that the model is able to capture most of the variation in the correlation between P/E ratios and return volatility, with the main exception being the early 1960s. In this period, stock prices fluctuated from the political events surrounding the Cuban missile crisis and the assassination of JFK so that the data correlation between P/E and volatility was negative, as is the pattern in most periods of uncertainty from bad news.<sup>16</sup> In contrast, the model correlation was positive since the fundamentals entering this period were strong, and there were rising expectations of the very strong growth rate state, which increases uncertainty and thus volatility. Apart from this episode, the fitted model explains over 37% of the variation of the historical correlation.

It is useful to provide some additional intuition on this lack of monotonicity between P/E ratios and volatility by concentrating on the late 1990s. From the bottom right panel of Figure 2, the late 1990s saw a sharp rise in the posterior probability of the Low Inflation-New Economy growth state (LI, NG). This increase came at the expense of a lower posterior probability weight assigned to the Medium Inflation-High Growth state (MI, HG), as shown in the top-right panel of Figure 2. From the bottom panel in Table 2, the (LI, NG) state is characterized by a conditional P/E ratio  $C_6 = 34.1$ , against the much smaller  $C_4 = 12.3$  in the (MI, HG) state. That is, the contemporaneous decrease in inflation *and* increase in real earnings in the late 1990s increased market participants' assessed probability that the U.S. economy would enter into a sustained high growth state to almost 22%. This change in probabilities increase both the expected future earnings, and thus the stock P/E ratio, as well as the uncertainty about future earnings growth, as probability shifted from a "HG" state to the higher "NG" state. The spectacular increase in volatility, however, is compounded by the fact that the conditional P/E ratio in the (LI, NG) state is so large compared to the (MI HG) state, a fact that acts as a multiplier effect on uncertainty. This empirical finding is reminiscent of a similar point made in Pastor and Veronesi (2006) (PV), but it is key to notice 3 important differences: First, PV concentrate only on the price and volatility of NASDAQ, which in their model is considered as idiosyncratic volatility, and not the price and volatility of the aggregate market. Second, in our model the increase in P/E ratio is mainly due to an increase in expected cash flow, and not uncertainty per se. Third, PV's model cannot generate an *increase* in uncertainty and volatility, without relying to some external, out-of-the model mechanism, while our model endogenizes all of the variation.

The model also explains the spectacular decrease in both P/E ratios and volatilities that fol-

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<sup>16</sup>In a recent paper, Bloom (2009) lists these political events as some of the largest uncertainty shocks in post-war data.



lowed the late 1990s, as the probability of the New Economy state dropped, but the probability of *Deflation/Low Growth* state (D,LG) increased, as shown in the top-left panel of Figure 2. The conditional P/E ratio corresponding to this latter state is again low, of around  $C_1 = 12.9$ , explaining the drop in both the P/E ratio and volatility (again, a positive relation between the two variables). It is interesting to note from the bottom-right panel of Figure 2 that the bull market between the two recessions of the current decade was due to again an increase in the probability Low Inflation / New Economy growth, which again pushed up volatility, although to a much lower extent than in the late 1990s.

We conclude this section with an important caveat about uncertainty and return volatility, namely, the fact that while uncertainty is key in determining the variation in volatility, it is not sufficient, because return volatility not only depends on the level of uncertainty about the economic states, but also on the conditional P/E ratio attached to each state, as shows in expression (16) and discussed then. To clarify this important point, we compute model-based measures of fundamental uncertainty. Given investors' beliefs,  $\pi_t$ , at any point of time we can construct model-based measures of inflation and earnings uncertainty as

$$\text{RMSE}_I(t) = \sqrt{\sum_{i=1}^n \pi_{it} (\beta^i - \bar{\beta}(\pi_t))^2} \quad \text{and} \quad \text{RMSE}_E(t) = \sqrt{\sum_{i=1}^n \pi_{it} (\theta^i - \bar{\theta}(\pi_t))^2}, \quad (22)$$

respectively. For our estimated parameters, we find that the earnings and inflation uncertainty measures explain only 2% and 10% of the variation in historical volatility, respectively, while the model volatility explains 44%. This finding shows two facts: First, that the non-linearities built into the structural form model are extremely important for the understanding the volatility dynamics. Second, the size of conditional P/E ratios across the various states play a central role in determining the volatility level, an observation that directly links return volatility to asset valuations. As an example of this mechanism at work, consider again the example about the late 1990s. Uncertainty in that period was not extremely high, because investors only gave about 20% probability to be in the (LI, NG) state, while the remaining 80% was still assigned to the regular (LI,HG) state. Yet, volatility was extremely high. The reason is that the difference in conditional P/E ratios is large: From Table 2 the conditional P/E ratios are about 17.5 and 34.1 in the (LI,HG) and (LI, NG) states, respectively. From expression (18) the high state uncertainty as captured in the term  $\pi_{it}(1 - \pi_{it})$ , was further amplified by the difference in the valuation multiplies  $(C_j - C_i)$ , which was very large.

### 3.2 Bond Volatility and Long Term Yields

The model fits the variation in bond yields and volatilities well. Table 3 shows that the model explains between 64% to 67% of the variation of yields, and between 52% and 64% of the variation of bond return volatility. In what follows we concentrate only on the results for the long-maturity (5-year) bond. The results for the 1-year bond are similar, and in fact, stronger. The bottom-left panel of Figure 4 shows the fitted and actual 5-year yield, while the bottom-right panel shows the volatility of bond returns. We note that our model yield was at its record level in the early 1980s as in the data, however, it fails to fully explain the high yields in this period. It is relevant to note that the Federal Reserve experiment with targeting money growth as opposed to interest rates had a role to play in generating tight credit conditions that led to higher rates in this period than predicted by our model.<sup>17</sup> Outside this period, the model does capture the main time variation in the long term yield, be it in the 4% range until mid 70s, in the 8% range until the late 70s, again the 4% range in the 1990s, and below that thereafter.

Like stock market volatility, the bond return volatility is countercyclical, although the  $R^2$ s of the historical volatilities on the NBER recession indicator are again low at around 15%. More interestingly, while the volatility was at its highest levels during the periods of high interest rates in the early 1980s, it was remarkably high also in the two recessions of the current decade, in which interest rates and inflation expectations have been far lower. These trends highlight a non-linear relationship between yields and bond volatilities, which our model explains. Indeed, the bottom panels of Figure 5 plot the scatter plot of bond return volatility and yield in the data (left panel) and implied by the model (right panel). As was the case with P/E ratio and stock volatility, the relation is not monotonic, as the volatility was the smallest for intermediate long-term yields, and higher for high and low yields. Our model is able to replicate well this time varying conditional relation between these two variables.

The economic intuition of this finding is the same as for the case of stock return volatility and P/E ratios: uncertainty is larger at the extremes. In this case, the regular states are the medium/low inflation (MI and LI) states, while the unusual states are the high inflation (HI) and deflation (D) states. Uncertainty is relatively low when the economy is in the usual states, but increases when

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<sup>17</sup>It is worth pointing out that most term structure models that have an endogenous equilibrium short interest rate process have trouble matching the high rates in this period. For example, Campbell and Viceira (2001) report parameter estimates prior to and post the Fed experiment years. In this paper, we are mostly interested in understanding the dynamic comovement of yields and bond return volatility, making the exact fitting of asset prices in each time period less critical for our main results.

there is news pointing to the two unusual states. This increase in uncertainty pushes up bond return volatility. However, if news is about entering a HI inflation period, long-term yields reflect such an expectation, and thus yields are high. Vice versa, if news is about entering a deflation, then long-term yields are low. Therefore, high volatility occurs for both high and low bond yields.

Further evidence in favor of the learning mechanism is offered by the middle panel of Figure 6. This panel shows the correlation between bond return volatility and long-term yields from the model along with the confidence bands from the data, as shown already in Figure 1 in the introduction. As can be seen, the model successfully replicates the main time variation in this correlation. Indeed, the correlation from the data and from the model track each other well, as the model explains over 38% variation of the historical movement in the correlation. Moreover, the correlation was clearly positive in the mid 1970s and negative in the more recent history.

To provide some concrete intuition of the learning mechanism leading to a positive and negative relation between yields and bond return volatility, we contrast the dynamics of beliefs during the mid 1970s, when the correlation between yields and volatility was positive, with the dynamics of beliefs during the current decade, when the correlation was negative. From the middle panels and the bottom left panel of Figure 2, the mid 1970s were characterized by high probabilities assigned to medium and high inflation states. This expectation of high inflation generates a high long-term yield in our model, as can be seen from the bottom panel of Table 2. At the same time, the uncertainty to enter into a sustained high inflationary period increase the uncertainty of bond returns. The opposite effect occurred in the current decade, which is instead characterized by worries to enter into a deflationary state, as shown by the top-left panel in Figure 2. When the probability of the deflationary state increases, the long-term yield decreases but the uncertainty about future inflation (or, better, deflation) increases. This climb in deflation uncertainty then pushes up bond return volatility exactly when yields are lower, leading to a negative correlation between the two. The economic mechanism underlying our dynamic model then makes consistent with each other two historical periods that are characterized by a diametrically opposite behavior of yields and volatility.

### **3.3 The Stock-Bond Comovement and the Breakdown of the Fed Model**

The same learning mechanism highlighted in the previous two sections explains also the time variation in the relative valuation of stocks and bonds (the Fed Model) and the dramatic time variation in the comovement between stocks and bonds, as shown in Figure 1 in the introduction. To shed further light on the issue, we explicitly plot the historical (top panel) and model implied (bottom

panel) Fed model in Figure 7. As evident, the relation between the earnings yield and the 5-Year Treasury yield was strongly positive of around 0.75 both for the model and data in the sub-sample from 1960-1999. However, in the current decade the correlation is negative of around -0.6 both in the data and the model. This breakdown of the Fed model has important implications for the sign of the correlation between stocks and bonds, which also changed sign in the current decade and we discuss this further below.

Indeed, the bottom panel of Figure 6 reports the realized covariance between stocks and the long-term bonds, and the fitted value from the model's estimate. Realized covariances were at their highest levels in the early 1980s, at about the same periods as the bond volatilities. It is immediately evident from the plots that the stock-bond covariance changes sign, going from being mainly positive in the period from 1960-1999, to being mostly negative in the current decade, and our model captures this change in sign quite accurately. The fitted values explain around 50% of the variation of the covariance series (see Table 3).

The main intuition behind the success of the model to track the time variation in covariances between stocks and bonds relies on the time varying signaling role of inflation. More specifically, a period of rising but positive inflation expectations lead to higher Treasury yields, but due to the adverse impact of higher inflation on future earnings growth, they also lead to lower P/E ratios of stocks. Thus, bond and stock returns comove positively. Moreover, a large state-uncertainty implies a fast updating of beliefs, which drives up the covariance as beliefs act as a highly volatile common factor between stocks and bonds. However, when investors' expectations of deflation increase, bond yields decline, but since deflationary periods are also periods of low earnings growth, the signaling channel of inflation reverses, so that the P/E ratio also declines. Thus, the correlation between stock and bonds yields becomes negative. Similarly, deflationary expectations lead to higher bond returns and lower stock returns, reversing the normally positive relation. The higher the uncertainty, moreover, the larger is the impact of news on beliefs, and thus the more negative is the negative covariance between stocks and bonds. Because uncertainty was larger during the two most recent recessions, and especially in the 2007-2008 recession, the negative covariance between stocks and bonds became large during this period. More formally in the context of our model, Proposition 3 implies that the stock-bond covariance becomes negative when investors are concerned about the transitions between states in which stock and bond valuations are negatively related. As can be seen in Figure 2, the probabilities that investors assess of the economy being in the deflation and regular boom states (states 1 and 2) sum to about 95 percent in each of the recessions of the current decade. The bottom panel of Table 2 shows that the conditions in Proposition 3 are met during these periods

so that the model stock-bond covariance is negative in them.

As mentioned in the introduction, a number of recent papers have tried to explain the variation in the stock-bond covariance, and it is important to note the difference with our economic mechanism based on learning. Campbell, Sunderam, and Viceira (2009) build a quadratic term structure model whose distinctive feature is the introduction of a state variable which can change sign and has a direct impact on the covariance between inflation and real shocks. In this sense realized inflation signals real activity in their model as well, albeit in a Gaussian shock world rather than the regime switching framework of our model. A key difference between their model and ours is that in our model, the correlation between inflation and real growth is determined by investors learning about their hidden fundamental states, while in the Campbell, Sunderam, and Viceira (2009), the state variable is essentially unrelated to fundamentals. For example, we show that our estimated model directly implies that the covariance of stock and bond returns becomes negative in periods when the economy moves between regular expansion states and states with deflationary expectations, but is positive otherwise. Baele, Bekaert, and Inghelbrecht (2009) attribute the negative covariance between stocks and bonds in the current decade to liquidity factors in these markets, a channel that further contribute to the negative covariance related to fundamentals that is uncovered by our model. Both these papers use partial equilibrium models, in which the market prices of risk follow exogenous processes, which are in the spirit of time-varying risk aversion (e.g. Campbell and Cochrane (1999)). Our model instead, has constant prices of risk, which is consistent with a constant relative risk aversion framework with a homoskedastic consumption process. This assumption helps to identify the specific channel that we address in this paper — changing investors' uncertainty about fundamentals — in explaining the dynamics of volatilities and covariances in the absence of changing risk aversion. Finally, Hasseltoft (2009) builds a consumption-based model in which inflation has constant negative correlations with dividend and consumption growth. The model is able to fit the Fed model and the stock-bond correlation until 1999, but has trouble in replicating the breakdown of the Fed model and the negative stock-bond correlation in the current decade. Finally, while the main focus of all the aforementioned papers is chiefly the time variation in the correlation between stocks and bonds, our model also explains other facts of stocks and bonds, such as the variation in their volatility, and the changes in the relation between volatility and prices.

## 4 Volatility Forecasts

In this section we demonstrate that our approach is useful not only for understanding the fluctuations of these second moments of asset prices but also for forecasting their future values for medium-to-long term horizons ranging from 1 to 8 quarters. The success of our modeling approach relies on the predictability of business cycles and some subtle but important differences in the cycles of the past 5 decades. The success of the model in forecasting future volatility provides further support for the economic mechanism presented in this paper

We begin by providing a summary of our forecasting methodology. We use the dynamics of the fundamentals, signals, SPD and beliefs, and the derived closed-form expression for stock and bond volatilities to formulate optimal forecasts of volatilities and covariances over a finite horizon. Optimal forecasts are essentially the expected quadratic variations over the forecast interval of interest. In particular for a volatility forecast of asset  $A$  the optimal forecast of volatility between quarters  $T_1$  and  $T_2$  given the information that investors have at time  $t$  is

$$V^*(T_1, T_2; t) = \sqrt{E \left[ \int_{T_1}^{T_2} \sigma^A(\pi_s) \sigma^A(\pi_s)' ds | \mathcal{F}_t \right]}. \quad (23)$$

Similarly, the optimal forecast of covariance of returns of assets  $A$  and  $B$  is given by

$$C^*(T_1, T_2; t) = E \left[ \int_{T_1}^{T_2} \sigma^A(\pi_s) \sigma^B(\pi_s)' ds | \mathcal{F}_t \right]. \quad (24)$$

We approximate the expectations by Monte Carlo simulations sampling several paths of the state variables at small discrete intervals. Details are provided in the Appendix.

To compare our results to the existing volatility literature, and add to the intuition of the model, we provide results for regressions of the form:

$$\text{Vol}(t+1, t+k) = b_0 + b_1 \text{Vol}(t-k+1, t) + b_2 V^*(t+1, t+k; t) + \mathbf{b}_3 \mathbf{X}(t) + \varepsilon(t+1, t+k) \quad (25)$$

for  $k = 1, \dots, 8$ . The dependent variable,  $\text{Vol}(t+1, t+k)$ , is the realized volatility between quarters  $t+1$  and  $t+k$ ,  $\text{Vol}(t-k+1, t)$  is the realized volatility of the current and past  $k-1$  quarters, and  $V^*(t+1, t+k; t)$  is the optimal forecast of future volatility in the following  $k$  quarters, conditional on investors' beliefs at  $t$ . The vector  $\mathbf{X}(t)$  contains the following set of macroeconomic control variables:

1. A business cycle dummy variable taking value one during expansions, as defined by the NBER,  $\text{NBER}(t)$ .

2. Current returns on the asset in periods when that return is negative,  $R_A^{(-)}(t)$ , where  $A$  is the asset under consideration ( $S$  for stock and  $5y$  for the 5-year bond).
3. Term structure variables that include the short (3-month) Treasury Bill rate,  $r(t)$ , and the slope of the yield curve measure as the difference between the 5-year and 1-year Treasury yields,  $term(t)$ .
4. The current volatilities of inflation and earnings growth computed by fitting a GARCH(1,1) model to inflation or earnings growth,  $\sigma_I(t)$  and  $\sigma_E(t)$ , respectively.
5. The dispersion of inflation and earnings growth forecasts from the Survey of Professional Forecasters,  $\sigma_{PF}^I(t)$  and  $\sigma_{PF}^E(t)$ , respectively. These forecasts are obtained from the Federal Reserve Bank of Philadelphia. Details about the construction of dispersion measures are in the Appendix.

Including lag volatility improves the  $\bar{R}^2$  of volatility forecasts, but begs the question as to what causes volatility. The effects of persistent explanatory variables will result in the lag having a significant coefficient without increasing our understanding of the economic forces driving volatility. We therefore present results of regressions with and without volatility lags. In our regressions with controls we leave out lagged volatility to see which economic variables best explain the dynamics of volatilities.

## 4.1 Stock Market Volatility

We now present our results on forecasting stock market volatility. The forecast values with 4-quarter lagged data are shown in Figure 8. It is quite remarkable that besides the stock market crash, our model has successfully forecasted all the major increases and decreases in stock market volatility in our sample. The forecasting power prior the current decade came mainly from the increases in inflation uncertainty prior to recessions. Periods of low inflation were followed by low stock market volatility. The volatility in the two most recent recessions was also forecastable 4-quarters earlier from milder increases in inflation uncertainty, but the effect was even larger than in the past recessions due to the larger earnings uncertainty from investors' optimism of the NG state.

Formal results at the 1-year ahead forecast horizon are presented in Table 4. The forecasted value is the fitted value of the regression in (25) with  $b_1$  and all elements of  $b_3$  set equal to zero so that the forecast is based only on the optimal model-based forecast. As seen in the table, the model-based forecast is the only variable that is able to improve on pure lag-based volatility forecasts, and even makes lagged volatility insignificant in a joint regression. Lagged volatility explains about

15% of the variation in future volatility, while the model explains about 42%. Excluding the 4th quarter of 1987 increases the  $\bar{R}^2$  of the model forecast to 54%, but has little effect on the  $\bar{R}^2$  of the lag-only forecast. We have a single measure of cumulated four-quarter lagged volatility, but using several other combinations of lagged volatility fail to change the results significantly.

Line 5 of Table 4 shows that the six macroeconomic variables can jointly forecast less than 6% of the variation in future volatilities. The control list includes the popular term-structure variables, the short rate and the slope that are used in past studies. The former proxies for inflation risk that leads to macroeconomic instability and higher volatility in the future [see Glosten, Jagannathan, and Runkle (1993)], while the latter is known to proxy for risk premiums in the economy. In particular, note from Figures 4 and 8 that the episode of high stock volatility that started around 1996 was not immediately preceded by high rates. Line 6 of the table shows that using all the control variables and the model-based forecast increases the  $\bar{R}^2$  to 44%, an improvement of only about 3.5 percentage points. Lagged stock returns (in periods when negative) are significant in forecasting at the 10% level, but provide only a modest increase in the  $\bar{R}^2$  suggesting that any bad news that increases volatility is captured in the model-based forecast, which therefore subsumes the leverage effect in the literature. All other macroeconomic variables, including the NBER recession indicator and fundamental volatilities are also insignificant in the joint regression. Several authors now use survey-based dispersion measures of inflation and earnings to explain stock market volatility. Line 8 shows that survey-based dispersion measures of inflation and earnings can together explain only 1% of the variation in future volatility over the shorter sample in which they are available. In the presence of our model-based forecast, their forecasting power is insignificant (line 10).

Therefore, somewhat surprisingly, neither lags, nor the entire range of macroeconomic control variables, can forecast stock market volatility at the 4-quarter horizon while our model-based forecast has far greater success. We find similar results for horizons from 1-8 quarters. While we do not present analogous tables for each horizon for space considerations, in Figure 10, we report the  $R^2$  for the lag only and model forecast only regressions analogous to those in lines 1 and 2 of Table 4. For stock volatility, as seen in the top left panel of this figure, the  $R^2$  of the lag only regression declines monotonically with the horizon starting at 30% at the 1-quarter horizon, and declining to about 4% at the 8-quarter horizon. The model only  $R^2$  peaks at 4-quarters, but remains high at about 40% up to 8 quarters. The declining forecasting power of the lag has been noted by other authors, e.g., Diebold and Christoffersen (2000), who question the relevance of lag-based models for intermediate term decisions and risk management.

Our model volatility equation is built upon real and nominal macroeconomic variables, which



have clearly been used in the past literature, yet it performs better than simply including these variables in a linear way. As discussed below Proposition 2, the stock volatility equation puts time varying weights on the alternative shocks in the model. For our fitted model, we report the weights of each fundamental shock on stock variance (since the volatilities can change sign) in Figure 9.<sup>18</sup> The time varying weights arise from Bayesian updating and rational valuation effects as previously discussed. In particular, as seen in the plot, the weight given to earnings is the highest for stocks, averaging about 70% for the full sample, but it is pro-cyclical. In particular, prior to and during recessions, the weight to earnings shocks falls, while the weight to the SPD shocks increases. Therefore, the increases in inflation uncertainty prior to and during recessions has a larger impact on stock market volatility in these periods. The time variation in these weights in addition shows how investors' concerns about alternative macroeconomic shocks has varied through past cycles, and is critical to understanding volatility dynamics. The strength of our learning based model is that the changing relative importance to alternative economic shocks arises endogenously from Bayes' law alone. Our model thus provides an economic rationale for the different impact of real news on the stock market in different stages of the business cycle in Boyd, Hu, and Jagannathan (2005). Finally, simply using lags of volatility without accounting for the relative weight changes for alternative shocks will not substitute for the information content of our model.

## 4.2 Treasury Bond Market Volatility

Table 5 reports the performance of our model in forecasting volatility of the 5-year Treasury bonds at the 4-quarter horizon.<sup>19</sup> Figure 8 shows that our model forecasts all the major episodes of bond market volatility prior to the current recession, except for the immense surge in volatility in 2008, as it failed to forecast the massive decline in the CPI with information received 4-quarter earlier, which triggered off deflationary expectations. As seen in Figure 5, our model does fit this episode of volatility after investors observed the CPI declines in 2008. Still, the model-based forecasts explain 60% of the variation in future volatilities for the two maturities. In addition, the model-

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<sup>18</sup>The weight on inflation is the sum of the 1st and 3rd components of stock variance. The 1st component is the direct effect of inflation on stock returns, while the 3rd component is from shocks to the SPD, which by assumption are modeled as a linear combination inflation and earnings shocks. However, for our estimated model  $\alpha_\theta \simeq 0$ , so that the real rate in the model reacts mainly to inflation news. The weight on earnings news is the sum of the direct effect from observed earnings shocks and the inferred effect from the signals, which are unobserved by the econometrician.

<sup>19</sup>Again, we only report results for the long-term bond. Results for the 1-year bond are similar, and, in fact, stronger.

based forecast improves on the lag-based forecast, although the improvement is smaller than for stocks. The six macroeconomic control variables can together explain an additional 4 percentage points in the adjusted- $R^2$  relative to the model-only regression (lines 5). We also find that the lagged short rate as well as lagged returns help to forecast future volatility. Still, the model only forecasts is better than lagged volatility plus macro variables (lines 2 and 4). Finally, in lines 7-9 we look at the forecasting power of the survey based measures, and find that inflation based measures in particular are indeed useful in forecasting bond volatility, but the forecasting power is already present in either the lags or the model based forecasts.

We note that the macroeconomic control variables do a much better job at forecasting bond market volatility than stock market volatility. Our model again sheds light on why there is this difference. Looking at the weights given to the alternative shocks in the bottom panel of Figure 9, we notice that the weights given to earnings shocks is low (around 20%) and very stable. Most of the variation in the weights is from the inflation shocks that affect volatility directly and through their impact on the SPD. The stability in the weights implies that the non-linearities that are critical for forecasting stock market volatility are less important for the bond market.

Finally, we consider the forecasts for the bond volatilities at alternative horizons in Figure 10. The figures show that up to the 4-quarter horizon, the lagged volatilities are indeed as successful as the model in forecasting, but the model-based forecasts outperforms lagged volatility for longer horizons.

### 4.3 Covariance of Stocks and Bonds

As in the previous subsection, we run the following forecasting regressions:

$$\text{Cov}(t+1, t+k) = b_0 + b_1 \text{Cov}(t-k+1, t) + b_2 C^*(t+1, t+k; t) + \mathbf{b}_3 \mathbf{X}(t) + \varepsilon(t+1, t+4), \quad (26)$$

where  $\text{Cov}(t+1, t+k)$  is the realized covariance between the two return series between quarters  $t+1$  and  $t+k$  calculated from daily returns, and the other variables are as defined below equation (25). The historical series and their forecast values based on the optimal model-based forecast are shown in Figure 8. The regressions are reported in Table 6.<sup>20</sup>

The covariance between stocks and long term bonds is quite persistent and thus the lagged co-

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<sup>20</sup>We only report results for the covariance between stocks and the long-term bonds. The results related to the covariance between stocks and short term bonds, and between bonds of different maturity are similar, and omitted to save space. They are available in the on-line appendix to the paper.

variance in itself can forecast nearly 46% of the future variation (line 1). Our model forecasts are about 10 percentage points more accurate, though. Including both the lag and our model forecast makes the lag insignificant, implying that our model picks up the information that makes covariances persistent. Next, using the 7 macroeconomic control variables does not lead to much improvement in the forecast  $\bar{R}^2$ , and including controls, and our model lead to only modest improvements in the forecast. The survey based variables also have some forecasting power when our model or lag is not included in the regressions. As seen from Figure 8, the covariance forecasts of the model are fairly accurate for most of our 48 year sample, with the notable exception of the 2001 to 2003 period and 2008, when realized covariances were very negative, while our model-predicted covariances, though negative, are much closer to zero. Figure 10 shows our model forecasts covariances quite accurately for longer horizons up to 8 quarters as well, while lag based forecasts decline in accuracy.

We shed further light on the stock-bond covariance and comparing to existing research by studying the relation between the stock-bond covariance and stock market volatility. Using the historical data series, the regression of covariance of stock and 5-year bond on stock variance for data gives:

$$\text{Cov}^{data}(t) = 0.000 - 0.048 \text{Var}^{data}(t) \quad (27)$$

$$(3.120) \quad (-2.75) \quad R^2 = 0.215. \quad (28)$$

Regression of covariance of stock and 5-year bond on stock variance for model gives:

$$\text{Cov}^{model}(t) = 0.005 - 0.105 \text{Var}^{model}(t) \quad (29)$$

$$(1.240) \quad (-7.192) \quad R^2 = 0.347. \quad (30)$$

The regressions (t-stats in parenthesis) show that unconditionally, the stock-bond covariance becomes more negative in periods of higher stock market volatility, and our model has this feature as well. Connolly, Sun, and Stivers (2005) report a similar relationship, using implied volatility on S&P 500 options (the CBOE VIX index) and the stock-bond covariance and attribute it to a flight-to-quality phenomenon: in periods of high market turbulence, investors flee stocks for the safety of bonds, so that stocks have a negative return and bonds have a positive return. While this is certainly a plausible explanation, it fails to explain why most episodes of high stock market volatility prior to 1999, were not accompanied by a negative stock-bond covariance. Our model covariance, which arises from learning and valuation effects, and does not rely on portfolio reallocation effects, instead shows that the bad news signaled by high inflation in 1960-1999 hurt both stocks and bonds, but deflationary expectations in the current decade had opposite effects in the two markets.

## 5 Conclusion

What ties return volatilities to price valuations and fundamentals? Investors' learning about the current earnings growth/inflation state. Such learning dynamics about fundamentals allows us to provide a unified framework that contemporaneously explains the major variation in stocks and bonds volatilities in the last 50 years, their non-monotonic  $V$ –shaped relation with price valuations – an empirical finding that defies conventional wisdom and many extant asset pricing models – as well as the time variation in bonds and stocks comovement. Our economic mechanism is also supported by the fact that the model is successful in predicting future volatilities and covariances, above standard lagged values or macroeconomic variables.

The main intuitive mechanism at play in the model is that there are “middle” economic states that are much more common than others, such as moderately high earnings growth being more common than very low growth or very high growth, and low/medium inflation being more common than very high inflation or deflation. This fact implies that when the economy settles to one of these more common states, learning makes uncertainty decline and thus return volatility declines as well. However, when the state of the economy departs from these “normal” states in *either* direction (i.e. roughly speaking, to a worse or better state), then Bayesian learning implies an increase in uncertainty and thus an increase in return volatility. Because this increase occurs both when the transition is to a worse or better state, asset prices instead move with the direction of the change (as suggested by the learning dynamics as well) and not with the direction of the change in uncertainty. This decoupling of prices and volatility implied by the learning dynamics generates the  $V$ –shaped relation between prices and volatilities. Although this is the intuitive economic mechanism, the model does much more than this, as it shows the importance of market participants' price valuations of the various hidden states, which depend on their preferences through the state price density, and their key role in shaping the volatility of stock and bond returns, their relation to prices, and their time varying comovement. The learning dynamics, moreover, provides the correct frequency of the variation of state variables (beliefs) for long term forecasting.

From an empirical perspective, this paper introduces a new equilibrium structural form methodology for understanding the fluctuations and predictability of volatilities and covariances of asset returns. The methodology tests a particular economic mechanism – learning – by formulating exact functional forms of the second moment matrix on these asset returns and it thus explicitly quantifies the effects generated by the model. An important feature of our methodology is that the information set of the econometrician is smaller than that of investors in the economy, but the econometrician

uses the information aggregation property of asset prices to back out the information of investors. Our results help provide a new perspective to the huge literature on modeling volatility.

A significant empirical contribution of our paper is in providing an understanding of the major moves in stock market volatility over the past 50 years. Important as this statistic is, it appears that stock market volatility has a life of its own, and cannot be understood with simple regressions on macroeconomic variables, which is perhaps why the literature using lags has had such tremendous success. Our results indicate that once we use our model-based, optimally constructed volatility forecast, lagged volatility no longer matters. On its own, lagged volatility picks up some of the information that causes investors' uncertainty. The latter is in itself a persistent process since fundamental information is noisy and investors' rational learning of underlying trend shifts is gradual. The closed-form solution for stock volatility in our model reveals that investors assign pro (counter) cyclical weights to earnings (inflation) news through business cycles. The time-varying weights explain why linear regressions on fundamental news do not forecast stock market volatility well. Our model also forecasts the volatilities of Treasury bonds and the covariances of stocks and bonds of different maturities. In particular, it sheds light on the non-monotonic relation between interest rates and bond market volatility, and the success of macroeconomic fundamentals in forecasting bond market volatility with simple linear regressions.

Perhaps the most valuable insights of our model are the joint relationships between asset valuations and volatilities. In particular, the model explains that in the late 1990s the uncertainty of the new economy state of growth was strong enough to cause a conditional positive relationship between uncertainty, volatility, and price-earnings ratios, while for most of the remainder of our sample, these variables were negatively related. Similarly, the learning dynamics implies the reversal of the relation between volatility and yields, generally positive in the sample, but which turned strongly negative in the last decade. Learning and uncertainty about deflationary state explains the fact. Indeed, the *same* economic mechanism explain the simultaneous reversal of the sign of the covariance of stocks and bonds and the breakdown of the Fed model in the current decade.

In this paper we only considered one particular channel, learning about the drift rate of earnings growth and inflation, to explain the variation in prices and volatilities. In particular, risk prices are constant in our model. It seems a fruitful avenue of future research to investigate the case with time varying risk prices, either within a reduced form approach by modeling a stochastic variation in the market price of risk embedded in the state price density, or by explicitly introducing preferences, such as habit formation or long run risk. Such an extension would possibly bring about a better fit of both P/E ratios and the term structure of interest rates, and would allow a distinction between

volatility due to uncertainty versus volatility due to risk prices. However, this avenue presents serious challenges, not last the fact that there are additional state variables and parameters to estimate. While interesting for future research, we refrained from such an investigation in this paper, as our aim is to present the simplest possible explanation for the empirical facts outlined in the introduction. In particular, we find it quite appealing that a relatively simple learning mechanism with only 6 unobserved drift states is able to generate the large variation in bond and stock volatilities and covariances documented in the data, and offer a relatively simple and intuitive explanation of their variation and comovement in the last 50 years.

Still, we conclude by noting that our model is just one example of a broader structural form approach that explicitly incorporates the impact of a particular economic channel on asset volatilities and their covariances. Indeed, our results suggest that the line of research can be extended to incorporate the impact of additional economic channels, such as time varying risk prices, or models of flight-to-quality and flight-to-liquidity, which are natural and interesting extensions for future work.

## Appendix

### 1. Proofs of Propositions

*Proof of Proposition 1.* The proof of part (a) follows from an extension of the proof in Veronesi (2000). To prove part (b), consider the process of the nominal stochastic discount factor  $N_t = M_t/Q_t$

$$\frac{dN_t}{N_t} = -r_t dt - \sigma_N dW$$

where  $r_t = \kappa_t + \beta_t - \sigma_M \sigma'_Q - \sigma_Q \sigma'_Q$  and  $\sigma_N = \sigma_M + \sigma_Q$ . From the process for  $N_t$  we find

$$N_{t+\tau} = N_t \exp \left[ \int_t^{t+\tau} r_u - \frac{1}{2} \sigma_Q \sigma'_Q du + \sigma_Q (W_{t+\tau} - W_t) \right]$$

Hence, the bond at time  $t$  with maturity  $\tau$  is

$$B(\pi_t, \tau) = E \left[ \frac{N_{t+\tau}}{N_t} | \mathcal{F}_t \right] = \sum_{i=1}^n \left[ \frac{N_{t+\tau}}{N_t} | \nu_t = \nu^i \right] \pi_{it}$$

Consider now a small interval  $\delta$  and define  $B_i(\tau) = E \left[ \frac{N_{t+\tau}}{N_t} | \nu_t = \nu^i \right]$ . We have:

$$\begin{aligned} B_i(\tau) &= E \left[ \frac{N_{t+\tau}}{N_t} | \nu_t = \nu^i \right] = E \left[ \left( \frac{N_{t+\delta}}{N_t} \right) \left( \frac{N_{t+\tau}}{N_{t+\delta}} \right) | \nu_t = \nu^i \right] \\ &= E \left[ \left( e^{(-r^i - \frac{1}{2} \sigma_Q \sigma'_Q) \delta + \sigma_Q (W_{t+\delta} - W_t)} \right) \left( \frac{N_{t+\tau}}{N_{t+\delta}} \right) | \nu_t = \nu^i \right] \\ &= e^{-r^i \delta} E \left[ \frac{N_{t+\tau}}{N_{t+\delta}} | \nu_t = \nu^i \right] \end{aligned}$$

$$\begin{aligned}
&= e^{-r^i \delta} \left\{ (1 + \lambda_{ii} \delta) B_i(\tau - \delta) + \sum_{j \neq i} \lambda_{ij} \delta B_j(\tau - \delta) \right\} \\
&= e^{-r^i \delta} \left\{ B_i(\tau - \delta) + \sum_{j=1}^n \lambda_{ij} \delta B_j(\tau - \delta) \right\}
\end{aligned}$$

Rearranging, we obtain that

$$\frac{B_i(\tau) - B_i(\tau - \delta)}{\delta} = \frac{e^{-r^i \delta} - 1}{\delta} B_i(\tau - \delta) + e^{-r^i \delta} \left\{ \sum_{j=1}^n \lambda_{ij} B_j(\tau - \delta) \right\}$$

Taking the limit as  $\delta \rightarrow 0$ , and rearranging

$$B'_i(\tau) = (\lambda_{ii} - r^i) B_i(\tau) + \sum_{j \neq i} \lambda_{ij} B_j(\tau)$$

In vector form

$$B'(\tau) = \hat{\Lambda} B(\tau)$$

whose solution is (15). ■

*Proof of Proposition 3.* From (21) we have  $C_i > C_j$  so that  $C_i > \sum_j \pi_j C_j$ , which implies that  $\pi_i^C > \pi_i$ . Using these probabilities in computing expectations, (21) implies that  $\bar{\theta}^C > \bar{\theta}$  and (15) implies that  $\bar{\beta}^C > \bar{\beta}$ . Similar logic using the relative bond valuations in the two states in (21) imply that  $\pi_i^B(\tau) < \pi_i$ , so that  $\bar{\theta}^B < \bar{\theta}$  and  $\bar{\beta}^B < \bar{\beta}$ . The first element of the covariance,  $(\bar{\beta}^{B(\tau)} - \bar{\beta}) + (\bar{\theta}^{B(\tau)} - \bar{\theta})$  is therefore clearly negative. To establish the sign of the second term, notice that the matrix inverse of the fundamental variance-covariance matrix

$$(\Sigma \Sigma')^{-1} = \begin{pmatrix} \frac{\sigma_{M,1}^2 + \sigma_{M,3}^2}{\sigma_{M,3}^2 \sigma_{Q,1}^2} & \frac{\sigma_{M,1} \sigma_{M,2}}{\sigma_{M,3}^2 \sigma_{Q,1} \sigma_{E,2}} & \frac{\sigma_{M,1}}{\sigma_{M,3}^2 \sigma_{Q,1}} & \frac{\sigma_{M,1} \sigma_{M,4}}{\sigma_{S,4} \sigma_{M,3}^2 \sigma_{Q,1}} \\ \frac{\sigma_{M,1} \sigma_{M,2}}{\sigma_{M,3}^2 \sigma_{Q,1} \sigma_{E,2}} & \frac{\sigma_{M,2}^2 + \sigma_{M,3}^2}{\sigma_{M,3}^2 \sigma_{E,2}^2} & \frac{\sigma_{M,2}}{\sigma_{M,3}^2 \sigma_{E,2}} & \frac{\sigma_{M,2} \sigma_{M,4}}{\sigma_{S,4} \sigma_{M,3}^2 \sigma_{E,2}} \\ \frac{\sigma_{M,1}}{\sigma_{M,3}^2 \sigma_{Q,1}} & \frac{\sigma_{M,2}}{\sigma_{M,3}^2 \sigma_{E,2}} & \frac{1}{\sigma_{M,3}^2} & \frac{\sigma_{M,4}}{\sigma_{S,4} \sigma_{M,3}^2} \\ \frac{\sigma_{M,1} \sigma_{M,4}}{\sigma_{S,4} \sigma_{M,3}^2 \sigma_{Q,1}} & \frac{\sigma_{M,2} \sigma_{M,4}}{\sigma_{S,4} \sigma_{M,3}^2 \sigma_{E,2}} & \frac{\sigma_{M,4}}{\sigma_{S,4} \sigma_{M,3}^2} & \frac{\sigma_{M,3}^2 + \sigma_{M,4}^2}{\sigma_{S,4}^2 \sigma_{M,3}^2} \end{pmatrix},$$

has all positive elements so that the sign of the second element in (20) is the same as the sign of  $(\bar{\nu}^C - \bar{\nu})(\bar{\nu}^{B(\tau)} - \bar{\nu})'$ . This last term can be written as  $\alpha_\theta^2 (\bar{\theta}^C - \bar{\theta})(\bar{\theta}^B - \bar{\theta}) + \alpha_\theta \alpha_\beta (\bar{\theta}^C - \bar{\theta})(\bar{\beta}^B - \bar{\beta}) + \alpha_\theta \alpha_\beta (\bar{\beta}^C - \bar{\beta})(\bar{\theta}^B - \bar{\theta}) + \alpha_\beta^2 (\bar{\beta}^C - \bar{\beta})(\bar{\beta}^B - \bar{\beta})$ , but we have already established above that each of these products is negative, so that the entire stock-bond covariance is negative as claimed. ■

## 2. SMM Estimation of the Regime Switching Model

The SMM procedure is similar to that in David (2008) and we provide the details here only to make this paper self-contained for the reader. Despite the similar structure, there are 4 essential differences from the estimation procedure to the above paper:

1. The most important distinction is the inclusion of realized quarter volatility and covariance moments in the overidentification of the model. Thus the estimation procedure puts weights on fundamental processes, asset price valuations (stock P/E ratios and Treasury Bond Yields), and well as the second moments above. The current specification does not include credit spreads, nor does it model leverage or credit risk, which was the main focus David (2008).

2. The model has a 6-state specification as opposed to the 4-state specification in David (2008). The 4-state specification is rejected due to (a) the inclusion of the short term stock volatility, which was not important for fitting spreads on defaultable bonds with maturities of 10 years and (b) the extension of the sample period to the current decade in which the emergence of expectations of deflation in the US economy requires the modeling of such a state to capture important dynamics of all asset prices. We note that we use the same  $\chi^2$  criterion standard in the GMM/SMM methodology to select the number of regimes, but the inclusion of different moments and different sample period requires a richer regime specification.<sup>21</sup>
3. The current model specification includes the earnings signal observed by investors, but unobservable to the econometrician. This assumption opens up the gap between the information set of the econometrician and the investors in the economy.
4. To reduce the number of estimated parameters, we use only 6 parameters for the entire generator matrix as shown in the note to Table 1. We follow a 2-step estimation methodology. In the first step we estimate using the full set of generator elements, and in the second step we group the elements into 6 parameters according to their estimated values and reestimate the model with equality constraints estimated for generator elements within each group.

Since not all fundamentals are observed by the econometrician, we denote  $Y_t = (Q_t, E_t)'$ , and write

$$\frac{dY_t}{Y_t} = \varrho_t dt + \Sigma_2 dW_t, \quad (31)$$

where  $\frac{dY_t}{Y_t}$  is to be interpreted as “element-by-element” division,  $\varrho_t = (\beta_t, \theta_t)'$ , and  $\Sigma_2 = (\sigma'_E, \sigma'_Q)'$ .

Since fundamentals are stationary in growth rates, we start by defining logs of variables:  $y_t = \log(Y_t)$ ,  $s_t = \log(S_t)$ , and  $m_t = \log(M_t)$ , we can write

$$dy_t = (\bar{\varrho}(\pi_t) - \frac{1}{2}(\sigma_Q \sigma'_Q, \sigma_E \sigma'_E)')dt + \Sigma_2 d\tilde{W}_t, \quad (32)$$

$$ds_t = (\bar{\theta}(\pi_t) - \frac{1}{2}\sigma_E \sigma'_E)dt + \sigma_S d\tilde{W}, \quad (33)$$

$$dm_t = (-\bar{k}(\pi_t) - \frac{1}{2}\sigma_M \sigma'_M)dt - \sigma_M d\tilde{W}_t. \quad (34)$$

It is immediate that investors' beliefs  $\pi_t$  completely capture the state of the system  $(y_t, s_t, m_t)$  for forecasting future growth rates. The specification of the system is completed with the belief dynamics in (6).

The econometrician has data series  $\{y_{t_1}, y_{t_2}, \dots, y_{t_K}\}$ . Let  $\Psi$  be the set of parameters of the model. We start by specifying the likelihood function over data on fundamentals observed discretely using the procedure in the SML methodology of Brandt and Santa-Clara (2002). See also Duffie and Singleton (1993). Adapting their notation, let

$$\mathcal{L}(\Psi) \equiv p(y_{t_1}, \dots, y_{t_K}; \Psi) = p(\pi_{t_0}; \Psi) \prod_{k=1}^K p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi),$$

where  $p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi)$  is the marginal density of fundamentals at time  $t_{k+1}$  conditional on investors' beliefs at time  $t_k$ . Since  $\{\pi_{t_k}\}$  for  $k = 1, \dots, K$  is not observed by the

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<sup>21</sup>Complete tests of the number of regimes are computationally extremely demanding and beyond the scope of this paper (see Garcia (1998)). Instead, we follow the simpler and more practical methodology of using the overidentified SMM objective to determine a stopping rule on the number of regimes used in a number of papers modeling regimes shifts (e.g. Gray (1996), Bansal and Zhou (2002)).



econometrician, we maximize

$$E[\mathcal{L}(\Psi)] = \int \cdots \int \mathcal{L}(\Psi) f(\pi_{t_1}, \pi_{t_2}, \dots, \pi_{t_K}) d\pi_{t_1}, d\pi_{t_2}, \dots, d\pi_{t_K}, \quad (35)$$

where the expectation is over all continuous sample paths for the fundamentals,  $\tilde{y}_t$ , such that  $\tilde{y}_{t_k} = y_{t_k}$ ,  $k = 1, \dots, K$ . In general, along each path, the sequence of beliefs  $\{\pi_{t_k}\}$  will be different.

As a first step, we need to calculate  $p(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi)$ . Following Brandt and Santa-Clara (2002), we simulate paths of the state variables over smaller discrete units of time using the Euler discretization scheme (see also Kloeden and Platen (1992)):

$$\tilde{y}_{t+h} - \tilde{y}_t = (\bar{\varrho}(\pi_t) - \frac{1}{2}(\sigma_Q \sigma'_Q, \sigma_E \sigma'_E)') h + \Sigma_2 \sqrt{h} \tilde{\epsilon}_t; \quad (36)$$

$$s_{t+h} - s_t = (-\bar{\theta}(\pi_t) - \frac{1}{2} \sigma_S \sigma'_S) h + \sigma_S \sqrt{h} \tilde{\epsilon}_t; \quad (37)$$

$$m_{t+h} - m_t = (-\bar{k}(\pi_t) - \frac{1}{2} \sigma_M \sigma'_M) h + \sigma_M \sqrt{h} \tilde{\epsilon}_t; \quad (38)$$

$$\pi_{t+h} - \pi_t = \mu(\pi_t) h + \sigma(\pi_t) \sqrt{h} \tilde{\epsilon}_t \quad (39)$$

where  $\tilde{\epsilon}$  is a  $4 \times 1$  vector of standard normal variables, and  $h = 1/M$  is the discretization interval. The Euler scheme implies that the density of the  $2 \times 1$  fundamental growth vector  $y_t$  over  $h$  is bivariate (since  $\sigma_{Q,3} = \sigma_{Q,4} = \sigma_{E,3} = \sigma_{E,4} = 0$ ) normal.

We approximate  $p(\cdot|\cdot)$  with the density  $p_M(\cdot|\cdot)$ , which obtains when the state variables are discretized over  $M$  subintervals. Since the drift and volatility coefficients of the state variables in (6), and (32) to (34) are infinitely differentiable, and  $\Sigma \Sigma'$  is positive definite, Lemma 1 in Brandt and Santa-Clara (2002) implies that  $p_M(\cdot|\cdot) \rightarrow p(\cdot|\cdot)$  as  $M \rightarrow \infty$ .

The Chapman-Kolmogorov equation implies that the density over the interval  $(t_k, t_{k+1})$  with  $M$  subintervals satisfies  $p_M(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi) =$

$$\int \int \phi(y_{t_{k+1}} - y; \varrho(\pi) h, \Sigma_2 \Sigma'_2 h; \Psi) \times p_M(y - y_{t_k}, \pi, m, t_k + (M-1)h | \pi_{t_k}, t_k) d\pi dy, \quad (40)$$

where  $\phi(y; \text{mean}, \text{variance})$ , denotes a bivariate normal density. Now  $p_M(\cdot|\cdot)$  can be approximated by simulating  $L$  paths of the state variables in the interval  $(t_k, t_k + (M-1)h)$  and computing the average

$$\hat{p}_M(y_{t_{k+1}} - y_{t_k}, t_{k+1} | \pi_{t_k}, t_k; \Psi) = \frac{1}{L} \sum_{l=1}^L \phi(y_{t_{k+1}} - y^{(l)}; \varrho(\pi^{(l)}) h, \Sigma_2 \Sigma'_2 h; \Psi). \quad (41)$$

The Strong Law of Large Numbers (SLLN) implies that  $\hat{p}_M \rightarrow p_M$  as  $L \rightarrow \infty$ .

To compute the expectation in (35), we simulate  $S$  paths of the system (36) to (39) “through” the full time series of fundamentals. Each path is started with an initial belief,  $\pi_{t_0} = \pi^*$ , the stationary beliefs implied by the generator matrix  $\Lambda$ . In each time interval  $(t_k, t_{k+1})$  we simulate  $(M-1)$  successive values of  $\tilde{y}_t^{(s)}$  using the discrete scheme in (36), and set  $\tilde{y}_{t_k}^{(s)} = y_{t_k}$ . The results in the paper use  $M = 90$  for quarterly data, so that shocks are approximated at roughly a daily frequency. The SPD and beliefs along the entire path of the  $s^{th}$  simulation are obtained by iterating on (38) and (39). We approximate the expected likelihood as

$$\hat{\mathcal{L}}^{(S)}(\Psi) = \frac{1}{S} \sum_{s=1}^S \prod_{k=0}^{K-1} \hat{p}_M(y_{t_{k+1}}^{(s)} - y_{t_k}^{(s)}, t_{k+1} | \pi_{t_k}^{(s)}, t_k; \Psi), \quad (42)$$

where  $\hat{p}_M(\cdot|\cdot)$  is the density approximated in (41). The SLLN implies that  $\hat{\mathcal{L}}^{(S)}(\Psi) \rightarrow E[\mathcal{L}(\Psi)]$  as  $S \rightarrow \infty$ . We often report  $\bar{\pi}_{t_k} = 1/S \sum_{s=1}^S \pi_{t_k}^{(s)}$ , which is the econometrician's expectation of investors' belief at  $t_k$ .

To extract investors' beliefs from data on price levels and volatilities in addition to fundamentals we add overidentifying moments to the SML method above. From Proposition 1, we can compute the time series of model-implied price-earning ratios and bond yields at the discrete data points  $t_k$ ,  $k = 1, \dots, K$  as

$$\widehat{P/E}_{t_k} = C \cdot \bar{\pi}_{t_k}, \quad \widehat{i}_{t_k}(\tau) = -\frac{1}{\tau} \log(B(\tau) \cdot \bar{\pi}_{t_k}).$$

We note that the constants  $C$ s and the functions  $B(\tau)$  both depend on the parameters of the fundamental processes,  $\Psi$ . Hence, we let the pricing errors be denoted

$$e_{t_k}^P = \left( \widehat{P/E}_{t_k} - P/E_{t_k}, \widehat{i}_{t_k}(0.25) - i_{t_k}(0.25), \widehat{i}_{t_k}(1) - i_{t_k}(1), \widehat{i}_{t_k}(5) - i_{t_k}(5) \right).$$

Also note that since the pricing formulas are linear in beliefs,  $1/S \sum_{s=1}^S C \cdot \pi_{t_k}^{(s)} = C \cdot \bar{\pi}_{t_k}$  (and similarly for the bond yields) and no information is lost by simply evaluating the errors at the econometrician's conditional mean of beliefs. We similarly formulate the volatility errors as

$$e_{t_k}^V = \left( \widehat{\sigma}_{t_k}^N - \sigma_{t_k}^N, \widehat{\sigma}_{t_k}^B(1) - \sigma_{t_k}^B(1), \widehat{\sigma}_{t_k}^B(5) - \sigma_{t_k}^B(5) \right),$$

where the model-implied nominal stock volatility is obtained from the derived expression  $\sigma^N(\pi)$  in (16) and averaged over the simulations as  $\widehat{\sigma}_{t_k}^N = \frac{1}{S} \sum_{s=1}^S \sigma^N(\pi_{t_k}^{(s)})$ . Similarly, the model-implied nominal bond volatility is obtained from the derived expression  $\sigma^B(\pi, \tau)$  in (17) and averaged over the simulations as  $\widehat{\sigma}_{t_k}^B(\tau) = \frac{1}{S} \sum_{s=1}^S \sigma^B(\pi_{t_k}^{(s)}, \tau)$ , for  $\tau = 0.25, 1, 5$ . Using the model's market price of risk and estimated stock volatility at each date, we construct the Sharpe ratio at each date as  $(\sigma_M \widehat{\sigma}_{t_k}^N) / (||\widehat{\sigma}_{t_k}^N||)$ . We construct its time-series average and take its difference from an empirical unconditional estimate of 0.3 [see, e.g. Cochrane (2001)], although our results are not affected much by varying its exact value in a wide range from 0.2 to 0.5.

To estimate  $\Psi$  from data on fundamentals as well as financial variables, we form the overidentified SMM objective function

$$c = \left( \frac{1}{T} \sum_{t=1}^T \epsilon_t \right)' \cdot \Omega^{-1} \cdot \left( \frac{1}{T} \sum_{t=1}^T \epsilon_t \right). \quad (43)$$

The moments used are the scores of the log likelihood function from fundamentals, the pricing errors from financial variables, and their volatilities. Since the number of scores in  $\frac{\partial \log(\hat{\mathcal{L}})}{\partial \Psi}(t_k)$  equals the number of parameters driving the fundamental processes in  $\Psi$ , the number of pricing errors is 4, the number of volatility and covariance errors is 6, and the mean unconditional Sharpe Ratio imposed imply that the statistic  $c$  in (43) has a chi-squared distribution with 11 degrees of freedom. We correct the variance covariance matrix for autocorrelation and heteroskedasticity using the Newey-West method [see, for example, Hamilton (1994) equation 14.1.19] using a lag length of  $q = 12$ . A long lag length is chosen since interest rates and P/E ratios used in the error terms are highly persistent processes with autocorrelations at the 12-quarter horizon in excess of 0.5.

### 3. Optimal Volatility and Covariance Forecasts

As for the likelihood function we formulate the expected quadratic variations in equation (23) and (24) across sample paths by Monte Carlo simulation while discretizing the dynamics of the state

variables of our system as in (36) to (39). We divide each quarter into  $M$  equally sized intervals of length  $h = 1/M$ . Then the volatility forecast for asset  $A$  is approximated as

$$V^{M*}(T_1, T_2, t) = \sqrt{\frac{1}{S} \sum_{j=1}^{(T_2-T_1)M} \sigma^A(\pi_{T_1+jh}^{(s)}) \sigma^A(\pi_{T_1+jh}^{(s)})' h},$$

where on each sample the process for the state variables is simulated starting with  $\pi_t^{(s)} = \pi_t$ , the assumed beliefs of investors at time  $t$ . The covariance forecasts in (24) are analogously formulated. We report forecasts for  $M = 90$ .

#### 4. Survey-Based Measures of Uncertainty

We obtain survey data from the Survey of Professional Forecasters, available at the Federal Reserve Bank of Philadelphia. The data are available since 1968, but we drop the first three years in our analysis since we find several missing entries, therefore restricting attention to the sample from 1971 to 2006. Forecasts are for horizons of  $\tau = 0, 1, \dots, 4$  quarters ahead, where  $\tau = 0$  indicates the forecast for the current quarter, which typically ends 1.5 months after the deadline to submit the questionnaire. The number of forecasters varies between 75 and 9 (for one quarter), and the mean number of forecasters is about 34.

We use the cross-sectional dispersion of the percentage inflation and earnings growth of individual forecasts as a measure of forecasters' uncertainty. Specifically, for each quarter  $t$ , let  $FI_i(t, \tau)$  be the forecast of individual  $i$  of the price index level at time  $t + \tau$ , where  $\tau$  is the horizon, and let  $I(t)$  be its current level (made available to the forecaster). If  $n_t$  is the number of individuals at time  $t$ , we then define the time  $t$  "uncertainty" on the inflation at time  $t + \tau$  as

$$\sigma_{PF}^I(t, \tau) = \sqrt{\frac{1}{n_t - 1} \sum_{i=1}^{n_t} \left( \left( \frac{FI_i(t, \tau)}{I(t)} \right) - \frac{1}{n_t} \sum_{i=1}^{n_t} \left( \frac{FI_i(t, \tau)}{I(t)} \right) \right)^2}. \quad (44)$$

To safeguard against typos and mistakes, we delete observations for  $FI_i(t, \tau) / I(t)$  that are four standard deviations away from the mean forecast.

A similar procedure is used for the forecast of real future corporate profits. Let  $FD_i(t, \tau)$  be the forecast of individual  $i$  about the level of corporate profits at time  $t + \tau$  and let  $D(t)$  be its current level (again, provided to the forecaster). We then define  $FRD_i(t, \tau) = FD_i(t, \tau) / FI_i(t, \tau)$  as a measure of the forecasted real future earnings by individual  $i$  and  $RD(t) = D(t) / I(t)$  as the current real earnings.<sup>22</sup> Then the empirical measure of uncertainty for profits (earnings) growth is

$$\sigma_{PF}^E(t, \tau) = \sqrt{\frac{1}{n_t - 1} \sum_{i=1}^{n_t} \left( \left( \frac{FRD_i(t, \tau)}{RD(t)} \right) - \frac{1}{n_t} \sum_{i=1}^{n_t} \left( \frac{FRD_i(t, \tau)}{RD(t)} \right) \right)^2}, \quad (45)$$

where again, we eliminate the observations of individuals that are more than four standard deviations away from the mean.

The two measures  $\sigma_{PF}^I(t, \tau)$  and  $\sigma_{PF}^E(t, \tau)$  introduced above are really measures of *dispersion* rather than *uncertainty*. However, there is an a priori reason to believe the two measures should

<sup>22</sup>It should be noted that the measure of forecasted real earnings defined as  $FRD_i(t, \tau) = FD_i(t, \tau) / FI_i(t, \tau)$  effectively uses the formula  $E_t[D(t + \tau)] / E_t[I(t + \tau)]$ , which is biased compared to the correct measure  $E_t[D(t + \tau) / I(t + \tau)]$  due to Jensen's inequality. However, since we are interested in the cross-sectional standard deviation of the forecasts, if the bias is reasonably constant across individuals, it is likely it will affect the measured "uncertainty" only very marginally.

be highly correlated: if all forecasters are uncertain about the future value of a macroeconomic variable, then it is also likely that their point forecasts have greater relative dispersion. Zarnowitz and Lambros (1987) find a positive relationship between similar disagreement and uncertainty measures.

We compare the 4-quarters-ahead survey-based measures of uncertainty with the asset-pricing-based measures of uncertainty obtained in Section 2. Figure 11 in the on-line appendix plots the survey-based measures alongside the model-based measures. In simple regressions the model-based uncertainty explains 58% and 24% of the variation in the survey-based measures. It must be noted that earnings growth is about four times more volatile as inflation, which leads to a lower  $R^2$  for the earnings series. The results are remarkable since we make no attempt to fit our model parameters to the survey measures, and hence the results are out-of-sample implications of our model.

Table 1: 6-State Model Estimated Parameters

Fundamental Drifts	$\beta^1$ -0.001 (0.001)	$\beta^2$ 0.020 (0.002)	$\beta^3$ 0.040 (0.002)	$\beta^4$ 0.081 (0.008)	$\theta^1$ -0.050 (0.004)	$\theta^2$ 0.025 (0.001)	$\theta^3$ 0.061 (0.018)
Volatilities	$\sigma_{Q,1}$ 0.021 (0.007)	$\sigma_{E,2}$ 0.085 (0.003)	$\sigma_{S,4}$ 0.096 (0.003)				
Generator Elements	$\lambda_1$ 0.004 (0.000)	$\lambda_2$ 0.014 (0.009)	$\lambda_3$ 0.033 (0.002)	$\lambda_4$ 0.105 (0.016)	$\lambda_5$ 0.157 (0.027)	$\lambda_6$ 0.263 (0.073)	
State Price Density (Real Rate)	$\alpha_0$ 0.013 (0.005)	$\alpha_\beta$ 0.685 (0.068)	$\alpha_\theta$ 0.000 (0.000)				
State Price Density (Prices of Risk)	$\sigma_{M,1}$ 0.324 (0.010)	$\sigma_{M,2}$ 0.535 (0.003)	$\sigma_{M,3}$ 0.324 (0.011)	$\sigma_{M,4}$ 0.000 (0.004)			
SMM Error Value ( $\chi^2(11)$ ):	18.401	P-Value: 0.073					

The table reports SMM estimates of the following model for CPI,  $Q_t$ , real earnings,  $E_t$ , earnings signals,  $S_t$ , and the SPD,  $M_t$  :

$$\begin{aligned}
\frac{dQ_t}{Q_t} &= \beta_t dt + \sigma_Q dW_t, \\
\frac{dE_t}{E_t} &= \theta_t dt + \sigma_E dW_t, \\
\frac{dM_t}{M_t} &= -k_t dt - \sigma_M dW_t, \\
\frac{dS_t}{S_t} &= \theta_t dt + \sigma_S dW_t,
\end{aligned}$$

where  $\sigma_Q = (\sigma_{Q,1}, \sigma_{Q,2}, 0)$ ,  $\sigma_E = (0, \sigma_{E,2}, 0)$ ,  $\sigma_M = (\sigma_{M,1}, \sigma_{M,2}, \sigma_{M,3})$ ,  $\sigma_S = (0, 0, 0, \sigma_{S,4})$ ,  $k_t = \alpha_0 + \alpha_\theta \theta_t + \alpha_\beta \beta_t$ , and the vector  $\nu_t = (\beta_t, \theta_t, -k_t, \theta_t)'$ , follows a 6-state regime switching model whose composite states are:  $\{(\beta^1, \theta^1), (\beta^2, \theta^2), (\beta^3, \theta^1), (\beta^3, \theta^2), (\beta^4, \theta^1), (\beta^2, \theta^3)\}$ , which we also refer to as (D-LG), (LI-HG), (MI-LG), (MI-HG), (HI-LG), and (LG-NG), respectively. The generator matrix  $\Lambda$  determining transitions between states uses the estimated parameters  $\lambda_i, i = 1, \dots, 6$  above to determine the elements  $\Lambda_{ij}$  as

	(D-LG)	(LI-HG)	(MI-LG)	(MI-HG)	(HI-LG)	(LI-NG)
(D-LG)	$-\sum_{j \neq 1} \Lambda_{1j}$	$\lambda_5$	$\lambda_3$	0	$\lambda_3$	0
(LI-HG)	$\lambda_2$	$-\sum_{j \neq 2} \Lambda_{2j}$	0	$\lambda_3$	0	$\lambda_1$
(MI-LG)	$\lambda_2$	$\lambda_3$	$-\sum_{j \neq 3} \Lambda_{3j}$	$\lambda_5$	$\lambda_6$	0
(MI-HG)	0	0	$\lambda_4$	$-\sum_{j \neq 4} \Lambda_{4j}$	0	0
(HI-LG)	$\lambda_2$	0	$\lambda_6$	0	$-\sum_{j \neq 5} \Lambda_{5j}$	0
(LI-NG)	$\lambda_2$	0	0	$\lambda_2$	0	$-\sum_{j \neq 6} \Lambda_{6j}$

The SPD,  $M_t$ , and the signal  $S_t$ , is observed by investors but not by the econometrician. Assets are priced using the formulas in Proposition 1. The moments used in the SMM procedure are the scores of the fundamental processes, 4 asset price valuations (stock P/E ratios and 3 Treasury yields), 3 price volatilities (stocks, and 1- and 5-year Treasury bonds) and the 3 covariances between these variables. We also include a moment of the model's Sharpe ratio to match its historical average. Details of the SMM methodology are in the Appendix. Standard errors are in parentheses.

Table 2: Model Implied Transition Probabilities, Stationary Probabilities, and Stock and Bond Price Valuations

Implied 1-Year Transition Probability Matrix						
	(D-LG)	(LI-HG)	(MI-LG)	(MI-HG)	(HI-LG)	(LI-NG)
(D-LG)	0.801	0.138	0.027	0.004	0.029	0.000
(LI-HG)	0.012	0.951	0.002	0.031	0.000	0.004
(MI-LG)	0.012	0.027	0.656	0.121	0.184	0.000
(MI-HG)	0.000	0.001	0.080	0.906	0.010	0.000
(HI-LG)	0.013	0.004	0.184	0.0158	0.783	0.000
(LI-NG)	0.012	0.001	0.001	0.0131	0.000	0.972
Implied 5-Year Transition Probability Matrix						
	(D-LG)	(LI-HG)	(MI-LG)	(MI-HG)	(HI-LG)	(LI-NG)
(D-LG)	0.345	0.420	0.075	0.066	0.087	0.005
(LI-HG)	0.037	0.793	0.022	0.120	0.011	0.017
(MI-LG)	0.032	0.084	0.291	0.289	0.304	0.001
(MI-HG)	0.009	0.022	0.188	0.675	0.105	0.000
(HI-LG)	0.037	0.053	0.304	0.160	0.445	0.000
(LI-NG)	0.039	0.017	0.011	0.054	0.008	0.869

Implied Stationary Probabilities, P/E Ratio, and Treasury Yields						
No.	State	$\pi^*$	$C$	$i_{0.25}$	$i_1$	$i_5$
1.	(D-LG)	0.038	13.209	0.008	0.012	0.026
2.	(LI-HG)	0.232	17.537	0.042	0.042	0.044
3.	(MI-LG)	0.174	9.832	0.078	0.082	0.088
4.	(MI-HG)	0.352	12.256	0.076	0.076	0.078
5.	(HI-LG)	0.170	8.545	0.142	0.136	0.115
6.	(LI-NG)	0.034	34.114	0.042	0.042	0.042

The **top** and **middle panels** report the 1-year and 5-year *implied* transition probability matrices between the 6 states implied from the generator matrix elements displayed in Table 1. Rows may not sum to one due to rounding. The **bottom panel** report the implied stationary probabilities and implied prices of the variables used in the SMM estimation procedure in the 6 states.  $C$  is the  $P/E$  ratio and  $i_T$  is the Treasury yield with maturity  $T$ . The  $P/E$  ratio and bond yields are computed as shown in Proposition 1.

Table 3: Model Fits for Expected Fundamental Growth, Asset Prices, and Asset Volatilities from SMM Procedure

Variable	$a$	$b$	$R^2$
Inflation	-0.002 [-2.528]	2.076 [7.880]*	0.504
Earnings	-0.03 [-1.41]	2.710 [4.277]*	0.125
P/E Ratio	-0.007 [-2.226]	1.424 [6.998]*	0.623
3-Month Yield	-0.002 [-2.071]	1.209 [7.956]*	0.642
1-Year Yield	-0.002 [-1.946]	1.306 [7.581]*	0.656
5-Year Yield	-0.002 [-1.579]	1.313 [5.989]*	0.674
Stock Volatility	$-1 \cdot 10^{-4}$ [0.805]	0.848 [9.324]*	0.445
Stock Volatility (EC)	$-1 \cdot 10^{-6}$ [-0.083]	1.201 [4.447]*	0.589
1-Year T. Bond Volatility	$2 \cdot 10^{-4}$ [1.256]	0.759 [4.960]*	0.646
5-Year T. Bond Volatility	$2 \cdot 10^{-4}$ [1.546]	0.812 [5.545]*	0.520
Covariance of Stocks and 1-Y T. Bond	$3 \cdot 10^{-6}$ [2.549]	0.532 [6.525]*	0.483
Covariance of Stocks and 5-Y T. Bond	$9 \cdot 10^{-6}$ [1.038]	0.979 [4.928]*	0.495
Covariance of 1-Y and 5-Y T. Bonds	$3 \cdot 10^{-5}$ [0.384]	0.820 [3.959]*	0.520

We display the fits of the variables used in our SMM procedure: the fundamentals, and the 10 of its overidentifying financial variables. The 11th overidentifying moment is the unconditional Sharpe Ratio for stocks, which is 0.32 for the model. For the two fundamentals we provide the regression results for the equation  $x(t) = a + b E[x|\mathcal{F}_t] + \epsilon(t)$ , where  $x(t)$  is the realized growth and  $E[x|\mathcal{F}_t]$  is investors' conditional expected growth of the fundamental under consideration. The conditional expected growth is obtained from the filtered probabilities  $\pi(t)$  displayed in Figure 2, and for earnings, for example, is given by  $\sum_i^N \theta_i \pi_i(t)$ . For the price (volatility) series, we present the regression results for the equation  $p(t) = \alpha + \beta p(\pi(t)) + \epsilon(t)$ , where  $p(t)$  and  $p(\pi(t))$  are the realized and model price (volatilities) conditional on investors' beliefs at  $t$  respectively. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols \* denotes significance at the 1% level. The term EC stands for ex-crash to denote that the fourth quarter of 1987 is removed from the sample.

Table 4: Forecasts of 4-Quarter Cumulative Volatility of Stocks and Macroeconomic Controls

No.	Sample	Const.	Vol( $t - 4, t$ )	$V^*(t + 1, t + 4)$	NBER( $t$ )	$R_S^{(-)}(t)$	$r(t)$	Term( $t$ )	$\sigma_I(t)$	$\sigma_E(t)$	$\sigma_{PF}^I(t)$	$\sigma_{PF}^E(t)$	$\bar{R}^2$
1	1962-2008	0.082 [4.072]*	0.429 [2.873]*										0.153
2	1962-2008	-0.008 [-3.373]*		1.393 [8.877]*									0.416
3	1962-2008 (EC)	-0.008 [-3.319]*		1.241 [8.821]*									0.542
4	1962-2008	-0.007 [-3.087]*	0.123 [1.986]	1.259 [7.099]*									0.419
5	1962-2008	0.171 [4.511]*			-0.037 [-2.060]	-0.459 [-1.621]	0.195 [0.609]	-0.447 [-0.442]	-0.989 [-0.639]	-0.16 [-0.105]			0.057
6	1962-2008	-0.004 [-1.477]		1.435 [9.161]*	-0.016 [-1.170]	-0.359 [-1.735]	-0.183 [-0.736]	-1.235 [-1.578]	-0.428 [-0.364]	-0.274 [-1.185]			0.45
7	1962-2008	-0.039 [-1.408]	0.063 [0.782]	1.407 [8.626]*	-0.017 [-1.232]	-0.339 [-1.606]	-0.175 [-0.688]	-1.281 [-1.670]	-0.525 [-0.437]	-0.293 [-1.230]			0.447
8	1971-2008	0.166 [7.816]*									0.069 [0.339]	-0.236 [-0.882]	0.009
9	1971-2008	-0.12 [-3.206]*		1.461 [7.760]*							2.499 [1.482]	-0.075 [-0.485]	0.352
10	1971-2008	-0.114 [-3.147]*	0.139 [2.483]	1.417 [6.753]*							2.425 [1.529]	-0.139 [-0.913]	0.362

This table reports the time series regressions  $\text{Vol}(t + 1, t + 4) = b_0 + b_1 \text{Vol}(t) + b_2 V^*(t + 1, t + 4; t) + \mathbf{b}_3 \mathbf{X}(t) + \varepsilon(t + 1, t + 4)$ , where  $\text{Vol}(t + 1, t + 4)$  is the realized volatility between quarters  $t + 1$  and  $t + 4$ ,  $V^*(t + 1, t + 4; t)$  is the optimal forecast of future volatility in the following four quarters in (23), and  $\mathbf{X}(t)$  contains a vector of the following controls: NBER( $t$ ) is a business cycle dummy variable taking value = 1 during expansions as defined by the NBER;  $R_S^{(-)}(t)$  is the return on the S&P 500 index in quarter  $t$  if it is negative and zero otherwise;  $r(t)$  is the 3-month Treasury Bill rate; Term( $t$ ) is the slope of the term structure defined as the five-year Treasury yield less the one-year Treasury yield;  $\sigma_I(t)$  and  $\sigma_E(t)$  are the current volatilities of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth; and  $\sigma_{PF}^I(t)$  and  $\sigma_{PF}^E(t)$  are the dispersion of forecasts from the Survey of Professional Forecasters. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols \* denotes significance at the 1% level. The term EC on line 3 stands for ex-crash to denote that the fourth quarter of 1987 is removed from the sample.



Table 5: Forecasts of 4-Quarter Cumulative Volatility of 5-Year Treasury Bonds and Macroeconomic Controls

No.	Sample	Const.	Vol ( $t - 4, t$ )	$V^*(t + 1, t + 4)$	NBER( $t$ )	$R_{5Y}^{(-)}(t)$	$r(t)$	Term( $t$ )	$\sigma_I(t)$	$\sigma_E(t)$	$\sigma_{PF}^I(t)$	$\sigma_{PF}^E(t)$	$\bar{R}^2$
1	1962-2008	0.014 [2.864]*	0.746 [7.126]*										0.572
2	1962-2008	0.099 [1.113]		0.941 [5.208]*									0.594
3	1962-2008	0.009 [1.762]	0.632 [5.110]*	0.707 [3.045]*									0.610
4	1962-2008	-0.005 [-0.401]			0.004 [0.424]	-0.492 [-2.977]*	0.535 [5.111]*	0.654 [2.198]	5.591 [2.201]	0.828 [0.621]			0.584
5	1962-2008	-0.008 [-0.571]		0.813 [3.764]*	0.008 [0.898]	-0.441 [-2.622]*	0.201 [1.211]	-0.172 [-0.371]	2.074 [2.293]	0.026 [0.262]*			0.653
6	1962-2008	0.003 [0.234]	0.475 [3.414]*	0.608 [2.552]	0.005 [0.565]	-0.363 [-2.345]	-0.008 [-0.052]	-0.684 [-1.697]	1.375 [1.436]	0.011 [0.146]			0.704
7	1971-2008	0.002 [2.090]									2.697 [1.790]	0.126 [1.271]	0.171
8	1971-2008			0.665 [2.983]*							1.734 [1.433]	0.007 0.081	0.523
9	1971-2008	0.001 [1.156]	0.619 [5.279]*	0.604 [2.665]*							1.514 [1.612]	-0.101 [-1.350]	0.595

This table reports the time series regressions

$$\text{Vol}(t + 1, t + 4) = b_0 + b_1 \text{Vol}(t - 4, t) + b_2 V^*(t + 1, t + 4; t) + \mathbf{b}_3 \mathbf{X}(t) + \varepsilon(t + 1, t + 4),$$

where  $\text{Vol}(t + 1, t + 4)$  is the realized volatility between quarters  $t + 1$  and  $t + 4$ ,  $V^*(t + 1, t + 4; t)$  is the optimal forecast of future volatility in the following four quarters in (23), and  $\mathbf{X}(t)$  contains a vector of the following controls: NBER( $t$ ) is a business cycle dummy variable taking value = 1 during expansions as defined by the NBER;  $R_{5Y}^{(-)}(t)$  is the return on the five-year Treasury Bond in quarter  $t$  if it is negative and zero otherwise;  $r(t)$  is the 3-month Treasury Bill rate; Term( $t$ ) is the slope of the term structure defined as the five-year Treasury yield less the one-year Treasury yield;  $\sigma_I(t)$  and  $\sigma_E(t)$  are the current volatilities of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth; and  $\sigma_{PF}^I(t)$  and  $\sigma_{PF}^E(t)$  are the dispersion of forecasts from the Survey of Professional Forecasters. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols \* denotes significance at the 1% level.

Table 6: Forecasts of 4-Quarter Cumulative Covariance of Stocks and 5-Year Treasury Bonds and Macroeconomic Controls

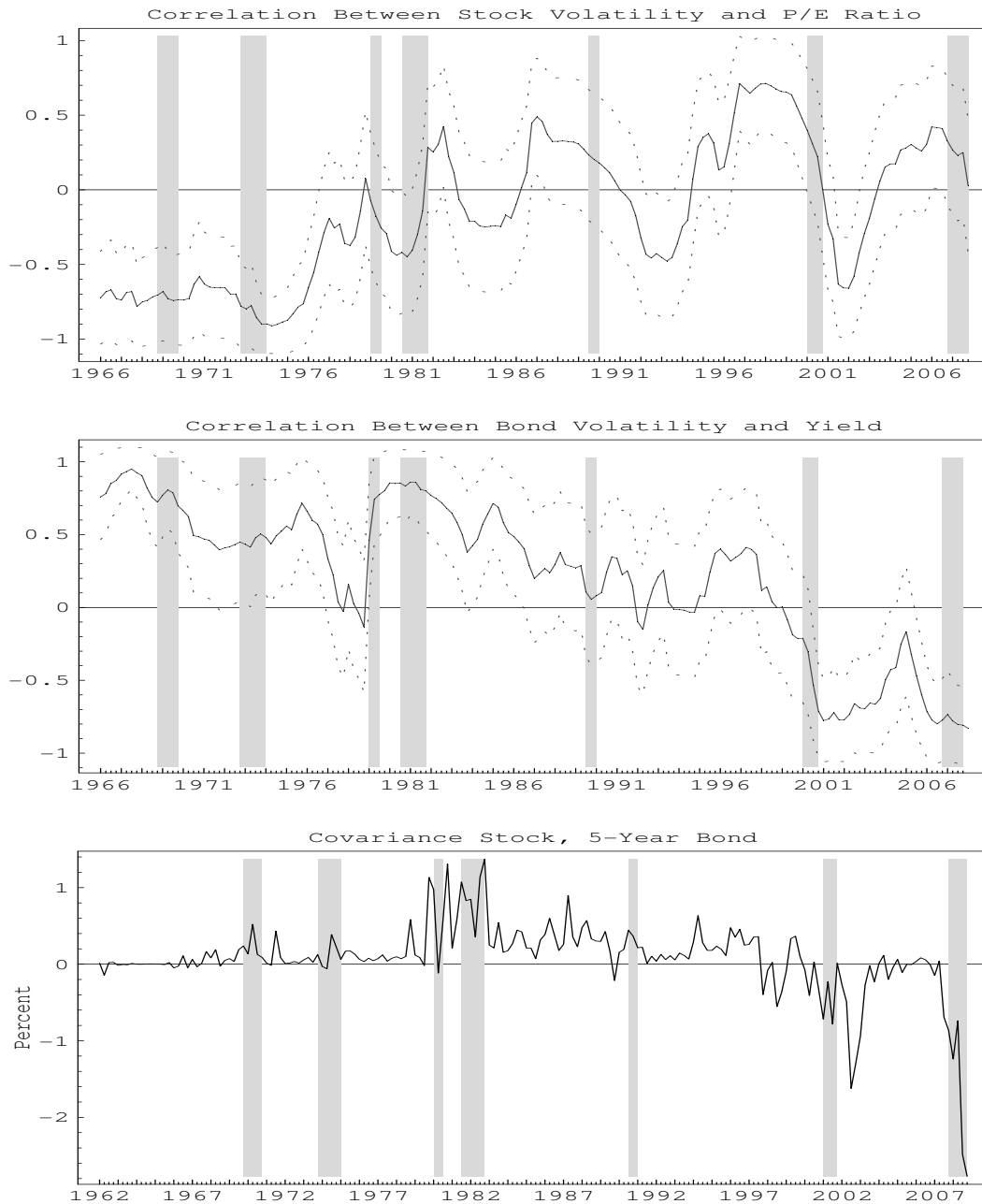
No.	Sample	Const.	$\text{Cov}(t-4, t)$	$C^*(t+1, t+4)$	NBER( $t$ )	$R_{5Y}^{(-)}(t)$	$R_{SY}(t)$	$r(t)$	Term( $t$ )	$\sigma_I(t)$	$\sigma_E(t)$	$\sigma_{PF}^I(t)$	$\sigma_{PF}^E(t)$	$\bar{R}^2$
1	1962-2008	-0.001 [-0.004]	0.795 [4.265]*											0.428
2	1962-2008	-0.119 [-2.072]		0.752 [6.183]*										0.529
3	1962-2008	-0.099 [-1.841]	0.339 [2.032]	0.538 [3.997]*										0.551
4	1962-2008	-0.6 [-2.807]*			0.167 [1.581]	-2.434 [1.127]	1.843 [1.022]	0.083 [4.961]*	0.119 [2.289]	0.102 [1.193]	-0.026 [-2.229]			0.474
5	1962-2008	-0.324 [-1.707]		0.694 [4.029]*	0.237 [2.329]	-2.257 [-1.137]	1.097 [0.872]	0.008 [0.426]	-0.004 [-0.699]	0.004 [0.661]	-0.002 [-1.297]			0.58
6	1962-2008	-0.246 [-0.883]	0.247 [1.539]	0.607 [3.636]*	0.219 [2.256]	-2.452 [-1.327]	0.575 [0.449]	-0.675 [-0.417]	-0.007 [-.417]	-0.014 [-0.883]	0.067 [0.977]	0.059 [1.029]		0.593
7	1971-2008	-0.419 [-2.23]										0.357 [2.877]*	0.276 [1.067]	0.185
8	1971-2008	-0.258 [1.939]		0.833 [5.154]*								-0.038 [-0.345]	1.526 [1.002]	0.576
9	1971-2008	-0.246 [-1.967]	0.282 [1.662]	0.632 [3.456]*								-0.021 [-0.209]	0.015 [1.051]	0.59

This table reports the time series regressions

$$\text{Cov}(t+1, t+k) = b_0 + b_1 \text{Cov}(t-4, t) + b_2 C^*(t+1, t+4; t) + \mathbf{b}_3 \mathbf{X}(t) + \varepsilon(t+1, t+4),$$

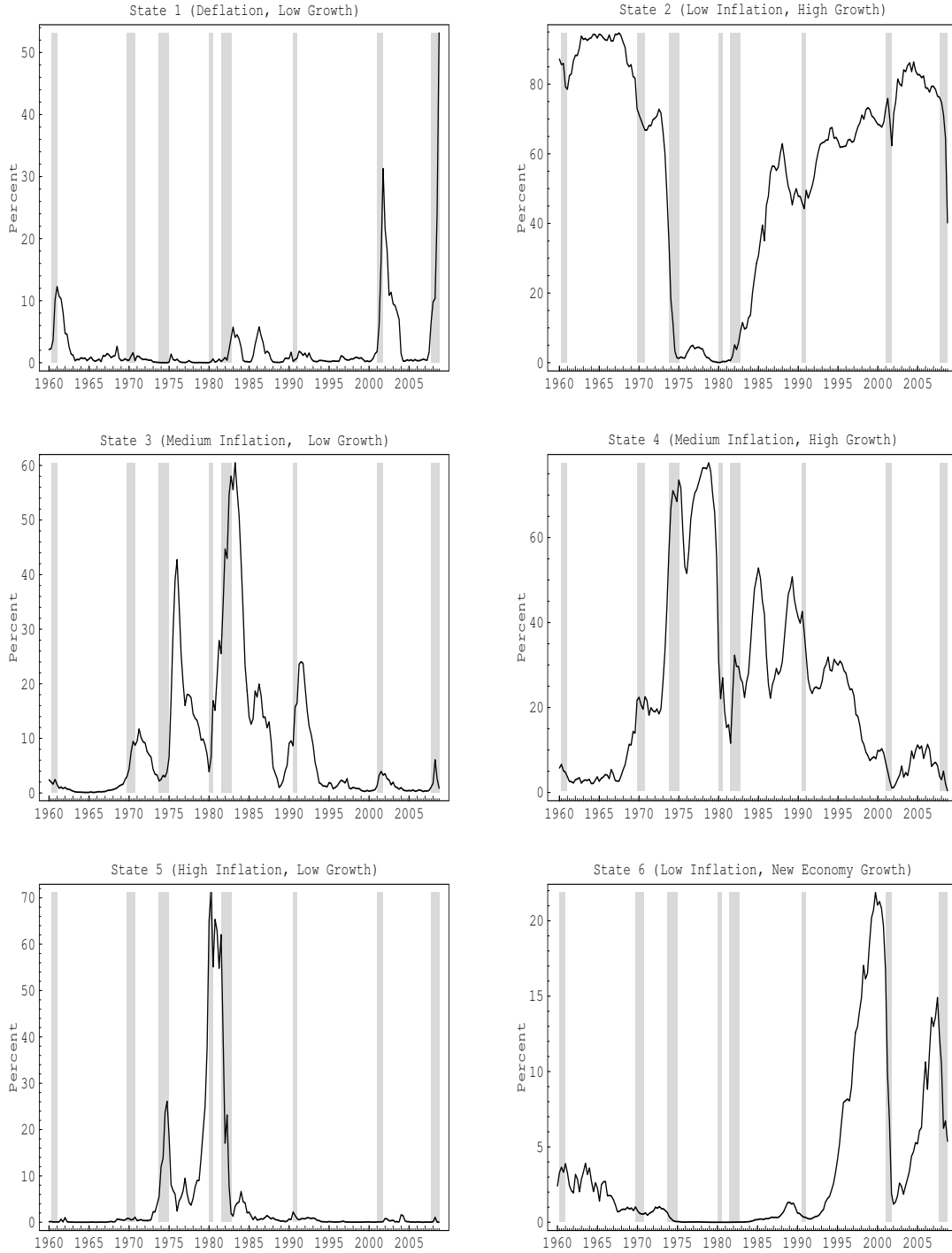
where  $\text{Cov}(t+1, t+k)$  is 100 times the realized covariance in between quarters  $t+1$  and  $t+k$ ,  $C^*(t+1, t+4; t)$  is 100 times the optimal forecast of future covariance in the following four quarters in (24), and  $\mathbf{X}(t)$  contains a vector of the following controls: NBER( $t$ ) is a business cycle dummy variable taking value = 1 during expansions as defined by the NBER;  $R_{5Y}(t)$  is the return on the five-year Treasury Bond in quarter  $t$ ;  $R_S^{(-)}(t)$  is the return on the S&P 500 in quarter  $t$  if it is negative and zero otherwise;  $R_{5y}^{(-)}(t)$  is the return on the five-year Treasury Bonds in quarter  $t$  if it is negative and zero otherwise;  $r(t)$  is the 3-month Treasury Bill rate; Term( $t$ ) is the slope of the term structure defined as the five-year Treasury yield less the one-year Treasury yield;  $\sigma_I(t)$  and  $\sigma_E(t)$  are the current volatilities of inflation and earnings growth, respectively, computed by fitting a GARCH(1,1) model to inflation or earnings growth; and  $\sigma_{PF}^I(t)$  and  $\sigma_{PF}^E(t)$  are the dispersion of forecasts from the Survey of Professional Forecasters. T-statistics are in parenthesis and are adjusted for heteroskedasticity and autocorrelation. The symbols \* denotes significance at the 1% level.

Figure 1: Asset Price Valuations and Volatilities and the Stock-Bond Correlation (1960-2008)



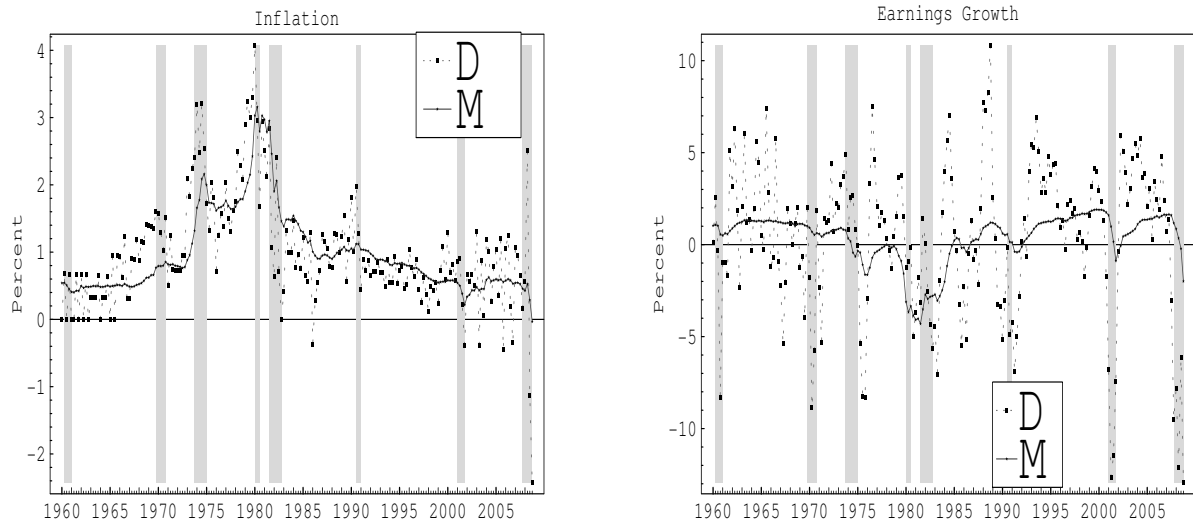
The **top panel** shows the quarterly time series of the correlation between the S&P 500 P/E ratio measured at the end of the quarter, and realized stock market volatility in that quarter constructed from daily returns. The correlation is based on a rolling sample of 5 years. The **middle panel** shows the analogous correlation between the 5-year Treasury bond yield and its realized volatility in the quarter. The **bottom panel** shows the quarterly realized correlation between stock and bond returns obtained from daily stock and bond returns. All data definitions are in Section 2.

Figure 2: Conditional Probabilities of Six States (1960-2008)



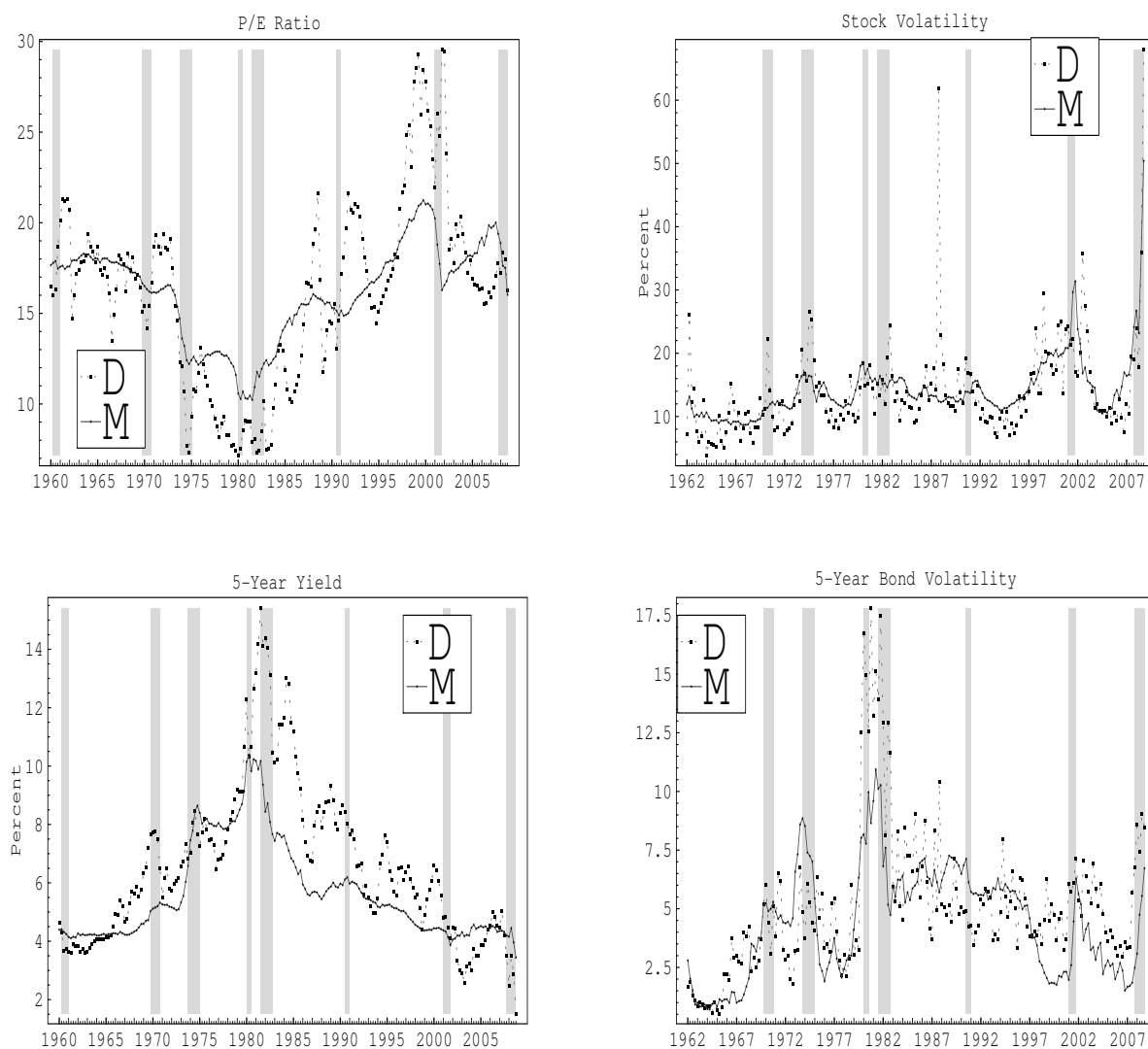
The states are numbered as  $(\beta^1, \theta^1)$ ,  $(\beta^2, \theta^2)$ ,  $(\beta^3, \theta^1)$ ,  $(\beta^3, \theta^2)$ ,  $(\beta^4, \theta^1)$ ,  $(\beta^2, \theta^3)$ , and  $(\beta^1, \theta^1)$ , where  $\beta^i$ ,  $i = 1, 2, 3, 4$  are the deflation, and low, medium, and high states of inflation, respectively, and  $\theta^1$ ,  $\theta^2$ , and  $\theta^3$ , are the regular low and high states, and the “new economy” rates of earnings growth. The filtered beliefs are obtained from the SMM procedure in the Appendix. The estimated values of the parameters are shown in Table 1. Shaded areas represent NBER-dated recessions.

Figure 3: Fundamental Variables: Empirical and Model Fitted (1960-2008)



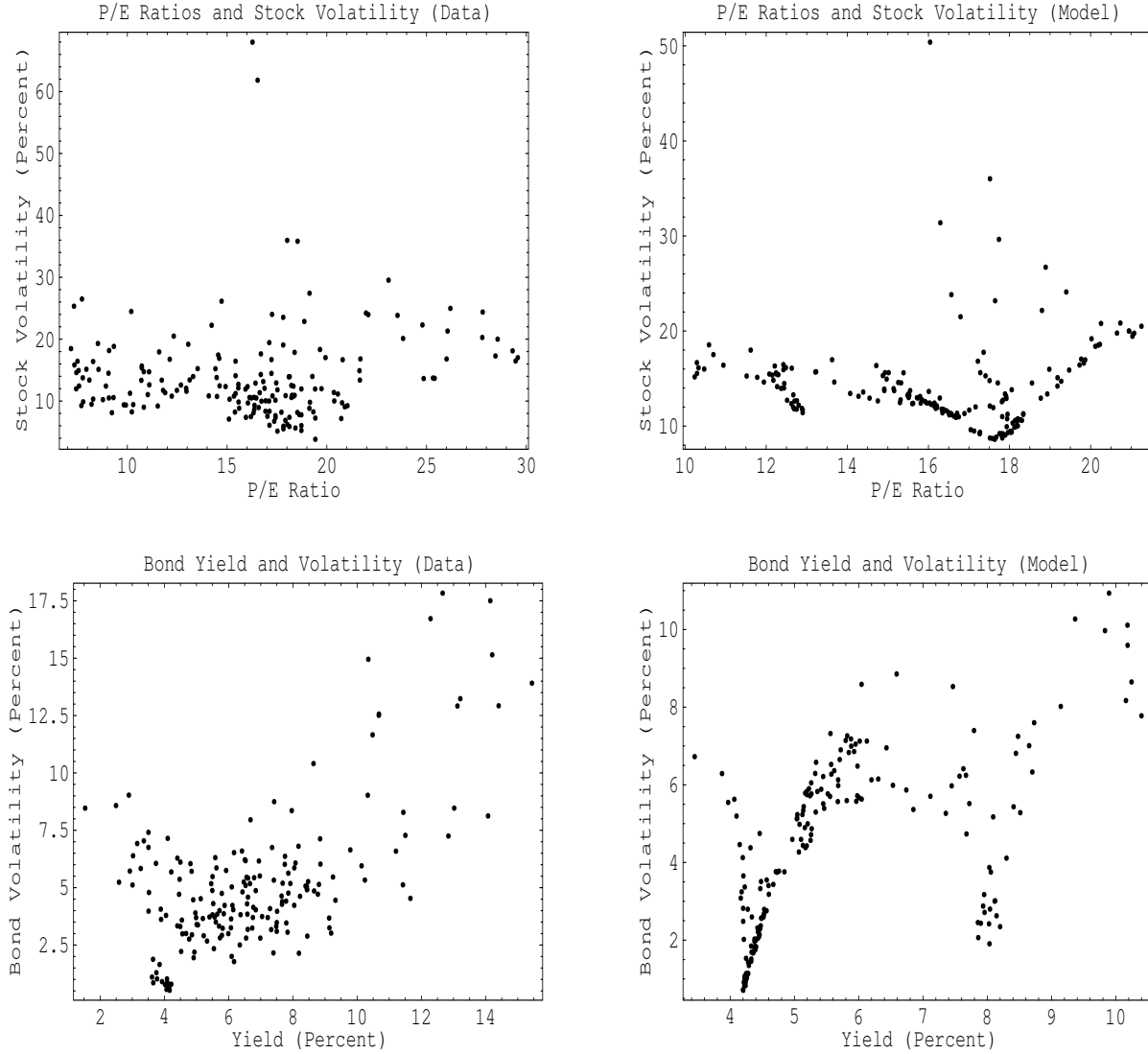
Historical values of financial and fundamental variables series (D) are in solid lines and their fitted values (M) from the SMM estimation procedure in the Appendix are in dashed lines. The estimated values of the parameters are shown in Table 1 and the implied asset price valuations are in Table 2. The filtered beliefs series of investors used to generate the fitted values are shown in Figure 2. Shaded areas represent NBER-dated recessions.

Figure 4: Fundamental and Financial Variables: Empirical and Model Fitted (1960-2008)



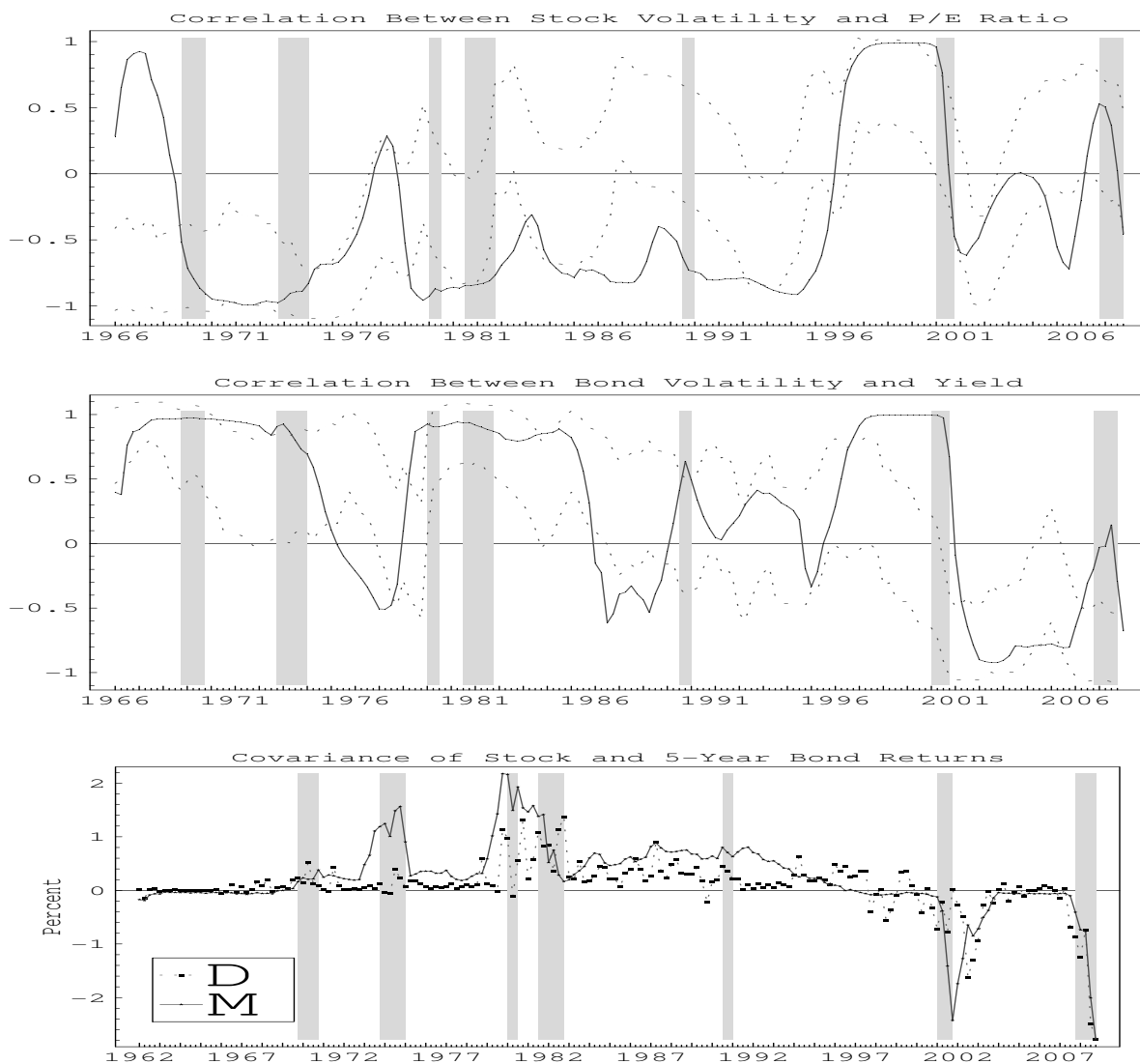
Historical values of financial and fundamental variables series (D) are in solid lines and their fitted values (M) from the SMM estimation procedure in the Appendix are in dashed lines. The estimated values of the parameters are shown in Table 1 and the implied asset price valuations are in Table 2. The filtered beliefs series of investors used to generate the fitted values are shown in Figure 2. Shaded areas represent NBER-dated recessions.

Figure 5: Relation between Stock and Bond Valuations and Volatilities (1962 - 2008)



Historical realized values of volatilities and covariances (D) are in dotted lines and their fitted values (M) from the SMM estimation procedure in the Appendix are in dashed lines. Stock and bond volatilities in each quarter are calculated using the closed-form expressions in (16) and (17), respectively, and covariances are calculated as in (20) using estimated values of the parameters are shown in Table 1 and the filtered beliefs shown in Figure 2. Shaded areas represent NBER-dated recessions.

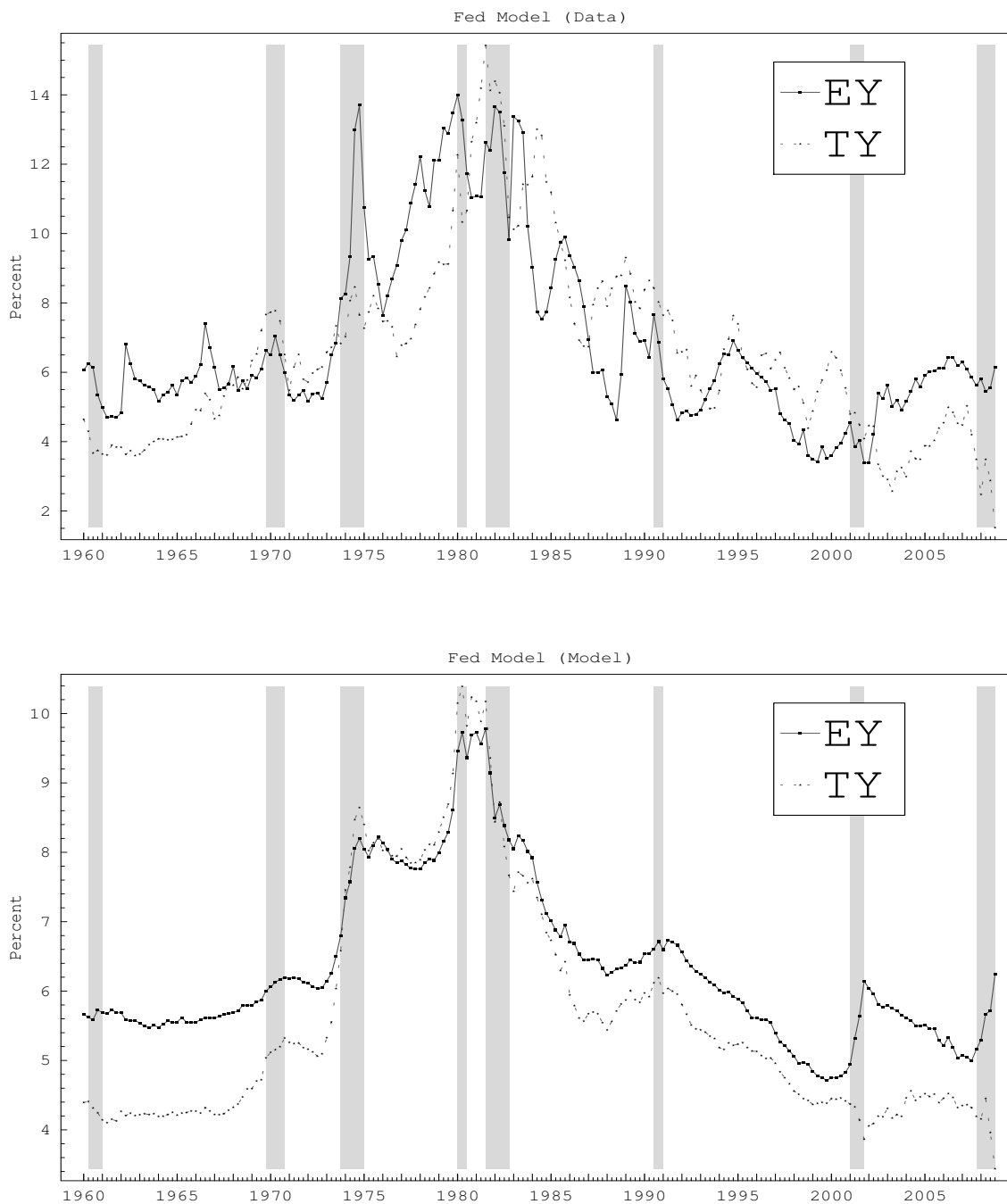
Figure 6: Valuation, Volatilities, and Comovement: Empirical and Model Fitted (1960-2008)



Historical values of financial and fundamental variables series (D) are in solid lines and their fitted values (M) from the SMM estimation procedure in the Appendix are in dashed lines. The estimated values of the parameters are shown in Table 1 and the implied asset price valuations are in Table 2. The filtered beliefs series of investors used to generate the fitted values are shown in Figure 2. Shaded areas represent NBER-dated recessions.

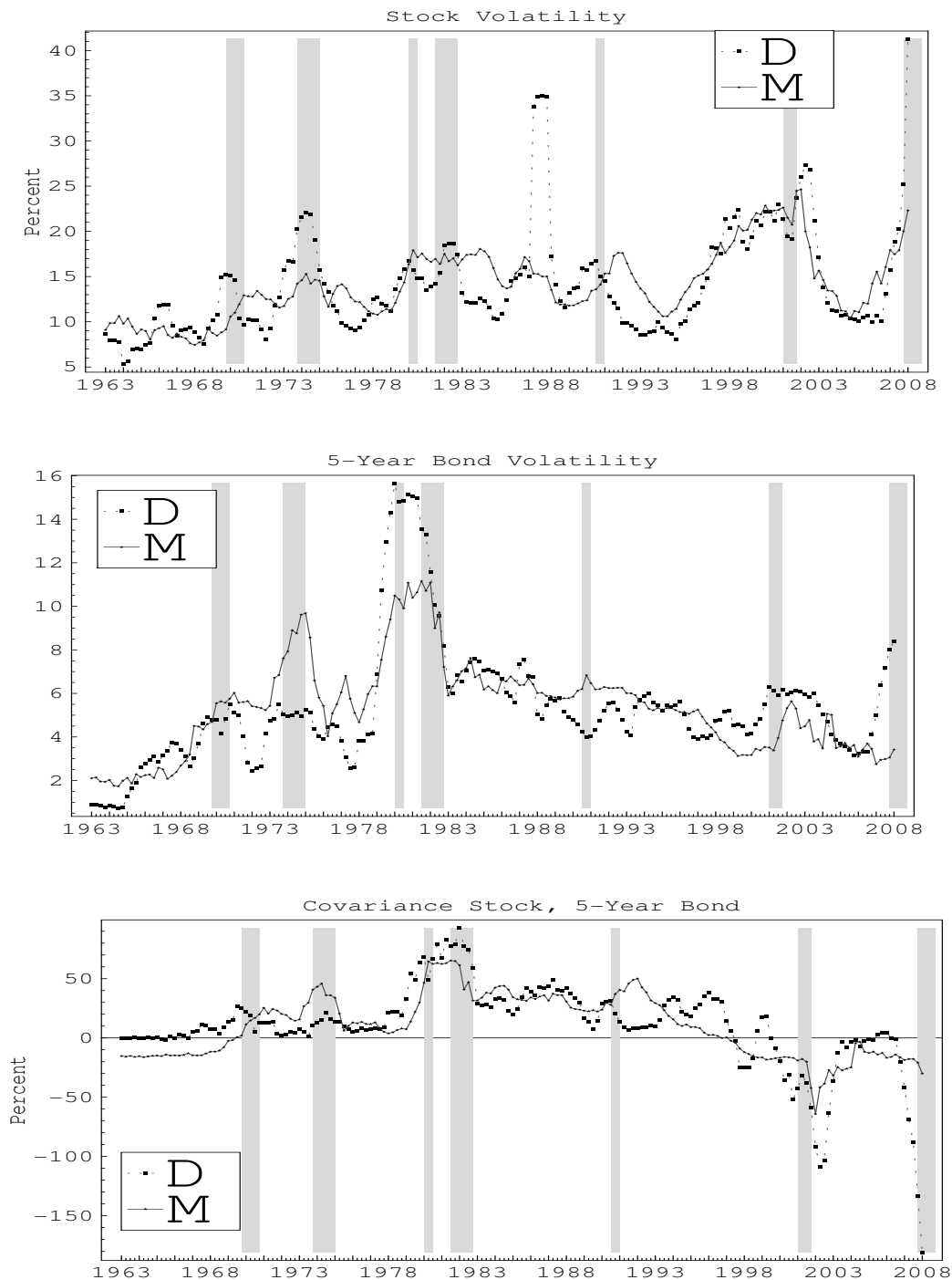


Figure 7: 'Fed Model': Data and Model Fitted (1960-2008)



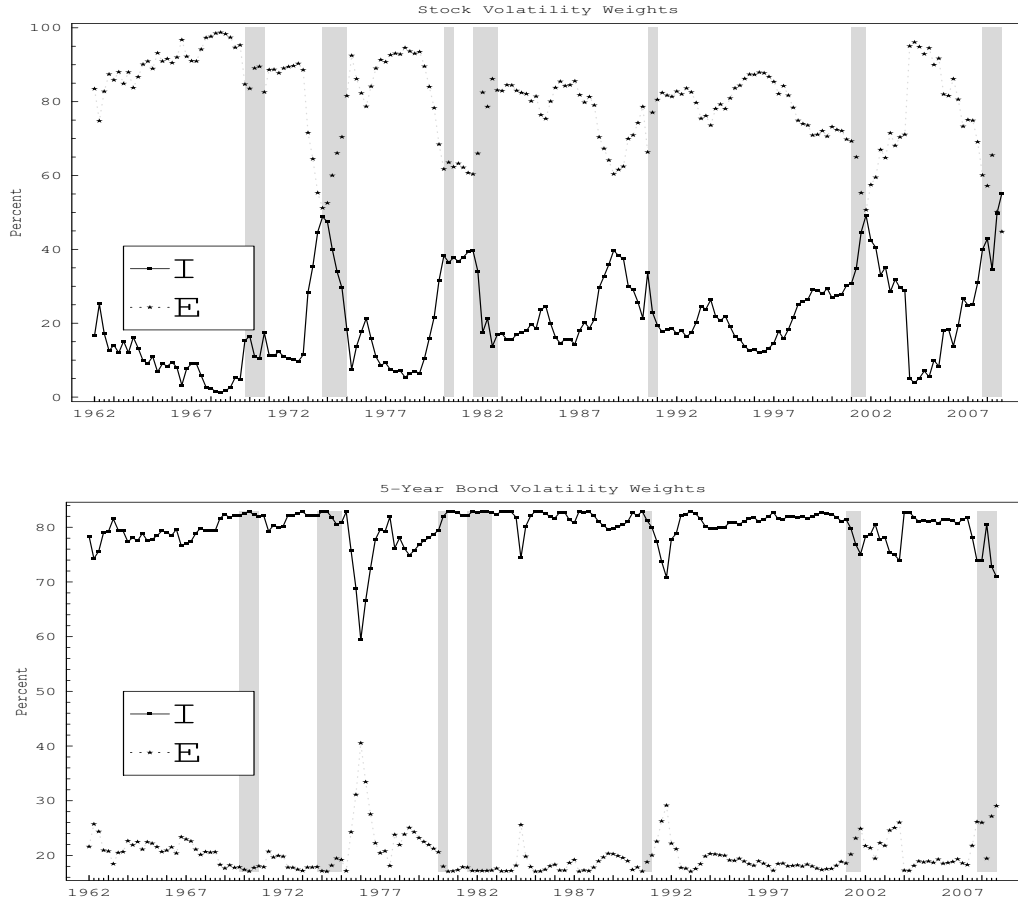
Historical values of the S&P 500 earnings yield (EY, solid line) and the 5-year Treasury Bond yield (TY, dashed lines) are in the top panel, while their model fitted values are in the bottom panel. The model values are obtained from the SMM estimation procedure in the Appendix. The estimated values of the parameters are shown in Table 1 and the implied asset price valuations are in Table 2. The filtered beliefs series of investors used to generate the fitted values are shown in Figure 2. Shaded areas represent NBER-dated recessions.

Figure 8: Optimal Data and and 4-Quarter Ahead Model Forecasts of Volatilities and Covariance (1962 - 2008)



The volatility forecast for each variable is the fitted value from the regression:  $\text{Vol}(t+1, t+4) = b_0 + b_2 V^*(t+1, t+4; t) + \varepsilon(t+1, t+4)$ , where the optimal model-based forecast  $V^*(t+1, t+4; t)$  is formulated as in (23). Similarly, the covariances are forecasted as the fitted values of the regression  $\text{Cov}(t+1, t+k) = b_0 + b_2 C^*(t+1, t+4; t) + \varepsilon(t+1, t+4)$ , where the optimal covariance forecast is formulated as in (24). The estimated values of the parameters are shown in Table 1. The filtered beliefs series of investors used to generate the fitted values are shown in Figure 2. Shaded areas represent NBER-dated recessions.

Figure 9: Weights Given to Earnings and Inflation Shocks in Stock Variance

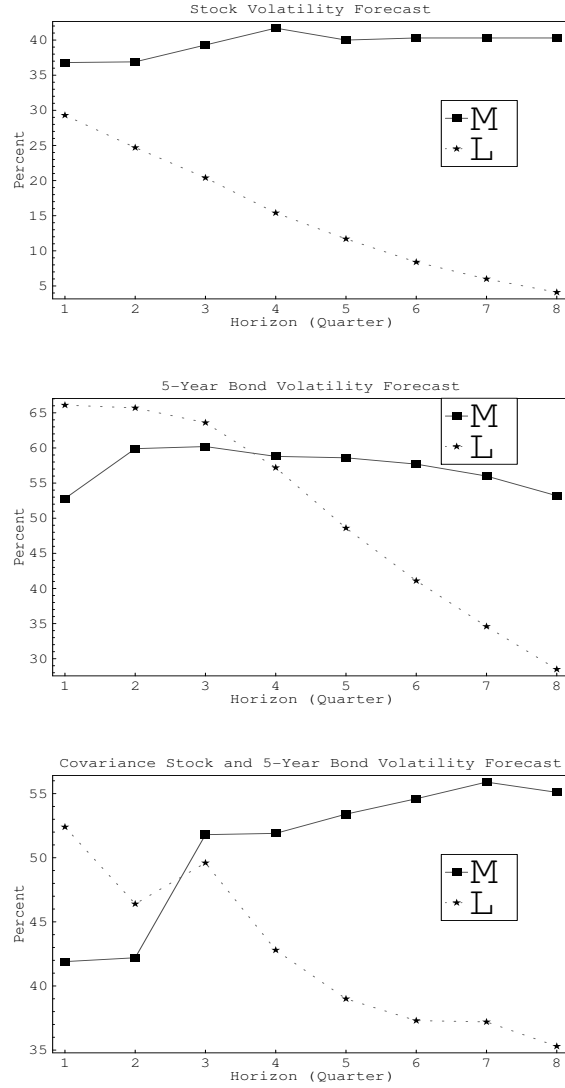


The weights given to inflation and earnings shocks at time  $t$  for asset A

$$w_I(t) = \frac{(\sigma_1^A(\pi_t))^2 + (\sigma_3^A(\pi_t))^2}{\|\sigma^A(\pi_t)\|}, \quad \text{and} \quad w_E(t) = \frac{(\sigma_2^A(\pi_t))^2 + (\sigma_4^A(\pi_t))^2}{\|\sigma^A(\pi_t)\|},$$

respectively, where the volatilities for stocks and bonds are given in Proposition 2. The volatilities are evaluated using the estimated values of the parameters are shown in Table 1, and conditional on investors filtered beliefs, which are shown in Figure 2.

Figure 10:  $R^2$  of Full Sample Forecasts at 1 to 8 Quarters Ahead (1962 - 2008)



The volatility forecast for each variable is the fitted value from the regression:  $\text{Vol}(t+1, t+k) = b_0 + b_2 V^*(t+1, t+k; t) + \varepsilon(t+1, t+4)$ , where the optimal model-based forecast  $V^*(t+1, t+k; t)$  is formulated as in (23). Similarly, the covariances are forecasted as the fitted values of the regression  $\text{Cov}(t+1, t+k) = b_0 + b_2 C^*(t+1, t+k; t) + \varepsilon(t+1, t+k)$ , where the optimal covariance forecast is formulated as in (24). The estimated values of the parameters are shown in Table 1. The filtered beliefs series of investors used to generate the fitted values are shown in Figure 2.

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