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ABSTRACT

Our paper examines whether intermittent portfolio re-balancing on the part of some investors can help to explain the counter-cyclical volatility of aggregate risk compensation in financial markets. To answer this question, we set up an incomplete markets model in which CRRA-utility investors are subject to aggregate and idiosyncratic shocks and have heterogeneous trading technologies. In our model, a large mass of investors do not re-balance their portfolio shares in response to aggregate shocks, but simply reinvest dividends, while a smaller mass of active investors adjust their portfolio each period to respond to changes in the investment opportunity set. We find that these intermittent re-balancers amplify the effect of aggregate shocks on the time variation in risk premia by a factor of three in a calibrated version of our model by forcing active traders to sell shares in good times and buy shares in bad times.

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1 Introduction

One of the largest challenges for standard dynamic asset pricing models is to explain the enormous countercylical variation in the risk-return trade-off in asset markets. Our paper establishes that the participation in equity markets of non-Mertonian investors who fail to continuously rebalance and who do not respond to changes in the investment opportunity set should be considered as an additional plausible explanation. The standard explanations rely on countercyclical risk aversion and heteroscedasticity in aggregate consumption growth, but these explanations either do not have a strong micro foundation or they fall short quantitatively. Our findings suggest that increased participation of non-Mertonian investors may lead to more volatility in risk premia.

There is a large group of households that invest in equities but only change their portfolio shares infrequently, even after large common shocks to asset returns. Ameriks and Zeldes (2004) find that over a period of 10 years 44% of households in a TIAA-CREF panel made no changes to either flow or asset allocations, while 17 % of households made only a single change. Calvet, Campbell, and Sodini (2009), in a comprehensive data set of Swedish households, found a weak response of portfolio shares to common variation in returns: between 1999 and 2002, the equal-weighted share of household financial wealth invested in risky assets drops from 57% to 45% in 2002, which is indicative of very weak re-balancing by the average Swedish household during the bear market. Finally, after examining two decades of panel data on US household asset allocation from the PSID and CEX surveys, Brunnermeier and Nagel (2008) conclude that inertia is the main driver of asset allocation in US household portfolios, while changes in wealth only play a minor role.

Without a specific model in mind, it is hard to know what effect, if any, intermittent re-balancing would have on equilibrium asset prices. In an equilibrium where all households are equally exposed to aggregate shocks, there is no need for any single household to re-balance his or her portfolio in response to an aggregate shock. This is clearest in a representative agent economy. However, in an environment in which households have heterogeneous exposures to aggregate shocks, the frequency of re-balancing may have important aggregate effects.

We conjecture that infrequent re-balancing on the part of non-Mertonian investors may contribute to countercylical volatility in risk prices because intermittent rebalancers mimic the portfolio behavior of households with countercyclical risk aversion. When the economy is affected by an adverse aggregate shock and the price of equity declines as a result, non-Mertonian investors who re-balance end up buying equities to keep their portfolio shares constant, while intermittent rebalancers do not. Hence, in the latter case, more aggregate risk is concentrated among the smaller pool of Mertonian investors whenever the economy is affected by a negative aggregate shock. These intermittent rebalancers act like households with countercyclical risk aversion.

To check the validity of this conjecture, we set up a standard incomplete markets model in which investors are subject to idiosyncratic and aggregate risk. The investors have heterogeneous trading technologies; a large mass of households are non-Mertonian investors who do not change their portfolio in response to changes in the investment opportunity set, but a smaller mass of active or Mertonian investors do. We consider two types of non-Mertonian investors: those that re-balance their portfolio each period to keep their portfolio shares constant, and those that rebalance intermittently. We assume that intermittent rebalancers reinvest the dividends in equities in non-rebalancing periods (see e.g. Duffie and Sun (1990)).

We find that the volatility of the price of aggregate risk is three times higher in the economy with intermittent rebalancers than in the economy with continuously re-balancing non-Mertonian investors. While the individual welfare loss associated with intermittent rebalancing is small relative to continuous rebalancing, and hence small costs would suffice to explain this behavior, the aggregate effects of non-rebalancing are large.

The automatic reinvestment assumption is critical for our results.[1] If intermittent rebalancers do not reinvest the dividends, the amplification of the volatility in risk prices delivered by intermittent rebalancing is much smaller. To see why, consider the case in which the dividend yield is constant. Then the average investor can simply consume the dividends, and there is no need for trade in equity shares between Mertonian and non-Mertonian investors. However, if the intermittent rebalancers reinvest dividends, then they need to buy shares from the active investors after good shocks and sell shares to these active investors after bad shocks. That is the crux of our

[1] We thank Fernando Alvarez for drawing our attention to this.

mechanism.

The heterogeneity in trading technologies allows us to match the volatility of risk premiums. We rely on two additional frictions to match the average risk-free rate and the average risk premium: (i) incomplete markets with respect to the idiosyncratic labor income risk and (ii) limited participation. The first friction produces reasonable risk-free rate implications in a growing economy. The second friction, limited participation, combined with the non-Mertonian trading technology of some market participants, produces a high average equity premium by concentrating aggregate risk, as in Chien, Cole, and Lustig (2010) but they only consider continuously rebalancing non-Mertonian investors.

We use our model as a laboratory for exploring the effects of changes in the composition of market participants. First, we show that increased participation by non-Mertonian investors decreases the average equity premium, but increases its volatility. Second, we find that financial innovation in the form of scope for hedging against idiosyncratic risk strengthens the amplification effect if we add a third friction: solvency constraints. When Mertonian investors can hedge against idiosyncratic risk, they have no precautionary motive to save and they run into frequently binding solvency constraints (Alvarez and Jermann (2001) and Chien and Lustig (2010)). This strengthens our amplification mechanism.

From the perspective of existing Dynamic Asset Pricing Models (DAPM's), there is a puzzling amount of variation in the risk-return trade-off in financial markets. In standard asset pricing models, the price of aggregate risk is constant (see, e.g., the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965)) or approximately constant (see, e.g., Mehra and Prescott (1985)'s calibration of the Consumption-CAPM). In the data, there is some variation in the conditional volatility of aggregate consumption growth that can deliver time-varying risk prices in a standard Consumption-CAPM, but probably not enough –and not of the right type– to explain the variation in the data.[2]

Habit formation preferences can help to match the counter-cyclicality of risk premia in the

[2]Recently, Campbell and Cochrane (1999) and Barberis, Huang, and Santos (2001), among others, have shown that standard representative agent models with different, non-standard preferences can rationalize counter-cyclical variation in Sharpe ratios.

data (Constantinides (1990),Campbell and Cochrane (1999)), as well as other features of the joint distribution of asset returns and macro-economic outcomes over the business cycle (see Jermann (1998),Boldrin, Christiano, and Fisher (2001)). However, it is not clear whether households actually have preferences defined over the difference between a habit and actual consumption. In fact, a key prediction of these preferences is that the household's risk aversion, and hence their allocation to risky assets, varies with wealth. According to Brunnermeier and Nagel (2008), there is little evidence of this in the data.

Moreover, these models do not seem to produce enough cyclical variation in Sharpe ratios to match the data. Lettau and Ludvigson (2001) measure the time-variation in the Sharpe ratio on equities in the data. This time variation is driven by variation in the conditional mean of returns (i.e. the predictability of returns) as well the variation in the conditional volatility of stock returns. In the data, these two objects are negatively correlated, according to Lettau and Ludvigson (2001), and this gives rise to a considerable amount of variation in the conditional Sharpe ratio: the annual standard deviation of the estimated Sharpe ratio is on the order of 50% per annum.

An annual calibration of the Campbell and Cochrane (1999) external habit model produces a volatility of 21%. The version of our model with the same i.i.d. aggregate consumption growth shocks and constant relative risk aversion investors delivers 15%. The volatility produced by our model increases to 25% if we use the Mehra and Prescott (1985) calibration of aggregate consumption growth, which builds in some negative autocorrelation in aggregate consumption growth.

Other channels for time-variation in risk premia that have been explored in the literature include differences in risk aversion (Chan and Kogan (2002),Gomes and Michaelides (2008)), differences in exposure to nontradeable risk (Garleanu and Panageas (2007)), participation constraints (Saito (1996), Basak and Cuoco (1998), Guvenen (2009)), differences in beliefs (Detemple and Murthy (1997)) and differences in information (Schneider, Hatchondo, and Krusell (2005)). Our paper imposes temporary participation constraints on the intermittent rebalancers instead of permanent ones, and it explores heterogeneity in trading technologies instead of heterogeneity in preferences.

Finally, there is a large literature on infrequent consumption adjustment starting with Grossman and Laroque (1990)'s analysis of durable consumption in a representative agent setting. Lynch (1996) specifically focuses on the aggregate effects of infrequent consumption adjustment by heterogeneous consumers to explain the equity premium puzzle. Gabaix and Laibson (2002) extend this analysis to a continuous-time setup that allows for closed-form solutions. In our approach, the intermittent rebalancers choose an intertemporal consumption path to satisfy the Euler equation in each period, including non-rebalancing periods, but, in between rebalancing times, their savings decisions can only affect their holdings of the risk-free assets. Only in rebalancing periods can they actually change their equity holdings.

2 Counter-cyclical and volatile Sharpe ratios

Lettau and Ludvigson (2001) measure the conditional Sharpe ratio on U.S. equities by forecasting stock market returns and realized volatility (of stock returns) using different predictors, and they obtain highly countercyclical and volatile Sharpe ratios. To get a clear sense of the link with business cycles, we consider a simple exercise. In expansions (recessions), the investor buys the stock market index in the n-th quarter after the NBER through (peak) and sells after 4 quarters. The NBER defines recessions as periods that stretch from the peak to the through. Strictly speaking, this is not an implementable investment strategy, because NBER peaks and troughs are only announced with a delay. [3] Nonetheless, the average returns on this investment strategy provide a clear indication of the cyclical behavior of the expected returns conditional on the aggregate state being expansion (recessions).

Figure 1 plots the Sharpe ratio on this investment strategy in the U.S. stock market, conditioning on the quarter of the NBER recession/expansion. We plot the (sample) Sharpe ratios obtained in both subsamples. This Sharpe ratio, which conditions only on the stage in the NBER business cycle, clearly increases in recessions (after the peak) and decreases in expansions (after

[3] However, there is recent evidence that agents realize a recession has started about 1 quarter after the peak (see Doms and Morin (2004)).

the through). The smoothed version of the conditional Sharpe ratio peaks three quarters into the recession at about 0.60, and it reaches its low three quarters after the trough at about 0.1. The details of the computation are in section C of the appendix.

[Figure 1 about here.]

3 Model

We consider an endowment economy in which households sequentially trade assets and consume. All households are ex ante identical, except for the restrictions they face on the menu of assets that they can trade. These restrictions are imposed exogenously. We refer to the set of restrictions that a household faces as a household trading technology. The goal of these restrictions is to capture the observed portfolio behavior of most households.

We will refer to households as being *non-Mertonian traders* if they take their portfolio composition as given and simply choose how much to save or dissave in each period. Other households optimally change their portfolio in response to changes in the investment opportunity set. We refer to these traders as *Mertonian traders* since they actively manage the composition of their portfolio each period. To solve for the equilibrium allocations and prices, we extend the method developed by Chien, Cole, and Lustig (2010) (hereafter CCL) to allow for non-Mertonian traders who only intermittently adjust their portfolio. In this section we describe the environment, and we describe the household problem for each of different asset trading technologies. We also define an equilibrium for this economy.

3.1 Environment

There is a unit measure of households who are subject to both aggregate and idiosyncratic income shocks. Households are ex ante identical, except for the trading technology they are endowed with. Ex post, these households differ in terms of their idiosyncratic income shock realizations. All of the households face the same stochastic process for idiosyncratic income shocks, and all households start with the same present value of tradeable wealth.

In the model time is discrete, infinite, and indexed by $t = 0, 1, 2, ...$ The first period, $t = 0$, is a planning period in which financial contracting takes place. We use $z_t \in Z$ to denote the aggregate shock in period t and $\eta_t \in N$ to denote the idiosyncratic shock in period t. z^t denotes the history of aggregate shocks, and, similarly, η^t, denotes the history of idiosyncratic shocks for a household. The idiosyncratic events η are i.i.d. across households. We use $\pi(z^t, \eta^t)$ to denote the unconditional probability of state (z^t, η^t) being realized. The events are first-order Markov, and we assume that

$$\pi(z^{t+1}, \eta^{t+1}|z^t, \eta^t) = \pi(z_{t+1}|z_t)\pi(\eta_{t+1}|z_{t+1}, \eta_t).$$

Since we can appeal to a law of large number, $\pi(z^t, \eta^t)/\pi(z^t)$ also denotes the fraction of agents in state z^t that have drawn a history η^t. We use $\pi(\eta^t|z^t)$ to denote that fraction. We introduce some additional notation: $z^{t+1} \succ z^t$ or $y^{t+1} \succ y^t$ means that the left hand side node is a successor node to the right hand side node. We denote by $\{z^\tau \succ z^t\}$ the set of successor aggregate histories for z^t including those many periods in the future; ditto for $\{\eta^\tau \succ \eta^t\}$. When we use $\succeq$, we include the current nodes z^t or η^t in the summation.

There is a single non-durable goods available for consumption in each period, and its aggregate supply is given by $Y_t(z^t)$, which evolves according to

$$Y_t(z^t) = \exp\{z_t\}Y(z^{t-1}), \tag{1}$$

with $Y(z^1) = \exp\{z_1\}$. This endowment goods comes in two forms. The first part is non-diversified income which is subject to idiosyncratic risk and is given by $\gamma Y(z^t)\eta_t$; hence γ is the share of income that is non-diversifiable. The second part is diversifiable income, which is not subject to the idiosyncratic shock, and is given by $(1-\gamma)Y_t(z^t)$.

All households are infinitely lived and rank stochastic consumption streams $\{c(z^t, \eta^t)\}$ according to the following criterion

$$U(\{c\}) = \sum_{t \geq 1, (z^t, \eta^t)}^{\infty} \beta^t \pi(z^t, \eta^t) \frac{c_t(z^t, \eta^t)^{1-\alpha}}{1-\alpha}, \tag{2}$$

where $\alpha > 0$ denotes the coefficient of relative risk aversion, and $c_t(z^t, \eta^t)$ denotes the household's consumption in state (z^t, η^t).

3.2 Assets Traded

Households trade assets in securities markets that re-open in every period. These assets are claims on diversifiable income, and the set of traded assets, depending on the trading technology, can include one-period Arrow securities as well as debt and equity claims. Households cannot directly trade claims to aggregate non-diversifiable income (labor income).

Debt and Equity We follow Abel (1999) in defining equity as a leveraged claim to aggregate diversifiable income ($(1-\gamma)Y_t(z^t)$). We use $V_t[\{X\}](z^t)$ to denote the no-arbitrage price of a claim to a payoff stream $\{X\}$ in period t with history z^t, and we use $R_{t+k,t}[\{X\}](z^{t+k})$ to denote the gross return between t and $t+k$. $R_{t+1,t}[\{1\}](z^t)$ denotes the one-period risk-free rate. To construct the debt and the equity claim, we will assume that aggregate diversifiable income in each period is split into a debt component (aggregate interest payments net of new issuance) and an equity component (aggregate dividend payments net of new equity issuance denoted $D_t(z^t)$). For simplicity, the bonds are taken to be one-period risk-free bonds. Since we assume a constant leverage ratio ψ, the supply of one-period non-contingent bonds $B_t^s(z^t)$ in each period needs to adjust such that:

$$B_t^s(z^t) = \psi\left[(1-\gamma)V_t[\{Y\}](z^t) - B_t^s(z^t)\right],$$

where $V[\{Y\}](z^t)$ denotes the value of a claim to aggregate income in node z^t. The payout to bond holders is given by $R_{t,t-1}[1](z^{t-1})B_{t-1}^s(z^{t-1}) - B_t^s(z^t)$, and the payments to shareholders, $D_t(z^t)$, are then determined residually as:

$$D_t(z^t) = (1-\gamma)Y_t(z^t) - R_{t,t-1}(z^{t-1})[1]B^s_{t-1}(z^{t-1}) + B^s_t(z^t).$$

A trader who invests a fraction $\psi/(1+\psi)$ in bonds and the rest in debt is holding the market portfolio. We can denote the value of the dividend claim as $V_t[\{D\}](z^t)$. $R_{t,t-1}[\{D\}](z^t)$ denotes the gross return on the dividend claim between $t-1$ and t.

We denote the price of a unit claim to the final good in aggregate state z^{t+1} acquired in aggregate state z^t by $Q_t(z_{t+1}, z^t)$. If there is a group of agents who trade claims with payoffs that are contingent on their idiosyncratic shocks, the absence of arbitrage would imply that the price $Q_t(\eta_{t+1}, z_{t+1}; \eta^t, z^t)$ of a claim to output in state (z^{t+1}, η^{t+1}) acquired in state (z^t, η^t) would be equal to $\pi(\eta^{t+1}|z^{t+1}, \eta^t)Q_t(z_{t+1}, z^t)$.

We consider a household entering the period with net financial wealth $\hat{a}_t(z^t, \eta^t)$. This household buys securities in financial markets (state contingent bonds $a_t(z^{t+1}, \eta^{t+1})$, non-contingent bonds $b_t(z^t, \eta^t)$, and equity shares $s^D_t(z^t, \eta^t)$) and consumption $c_t(z^t, \eta^t)$ in the good markets subject to this one-period budget constraint:

$$\begin{aligned}&\sum_{z^{t+1}\succ z^t, \eta^{t+1}\succ \eta^t} Q_t(\eta_{t+1}, z_{t+1}; \eta^t, z^t) a_t(z^{t+1}, \eta^{t+1}) + s^D_t(z^t, \eta^t) V_t[\{D\}](z^t) \\ &+ b_t(z^t, \eta^t) + c_t(z^t, \eta^t) \leq \hat{a}_t(z^t, \eta^t) + \gamma Y_t(z^t)\eta_t, \text{ for all } z^t, \eta^t, \end{aligned} \tag{3}$$

where $\hat{a}_t(z^t, \eta^t)$ is the agent's net financial wealth in state (z^t, η^t), and is given by his state-contingent bond payoffs from bonds acquired last period, the payoffs from his equity position and the non-contingent bond payoffs:

$$\hat{a}_t(z^t, \eta^t) = a_{t-1}(z^t, \eta^t) + s^D_t(z^{t-1}, \eta^{t-1}) \left[D_t(z^t) + V_t[\{D\}](z^t)\right] + R_{t,t-1}[1](z^{t-1}) b_{t-1}(z^{t-1}). \tag{4}$$

3.3 Trading Technology

A trading technology is a restriction on the menu of assets that the agent can trade in any given period. This includes restrictions on the frequency of trading as well; some trades are not allowed in each period for all households. The set of asset trading technologies that we consider can be divided into two main classes: Mertonian trading technologies and non-Mertonian trading technologies.

Agents with a Mertonian or active trading technology optimally choose their portfolio composition given the set of assets that they are allowed to trade in each period and given the state of the investment opportunity set. The set of assets that these traders are allowed to trade consist of a complete menu of state-contingent securities with payoffs contingent on aggregate but not idiosyncratic shocks, in addition to non-contingent debt and equity. Hence we refer to these as z-complete traders.

For all non-Mertonian trading technologies, the menu of traded assets only consists of debt and equity claims. A non-Mertonian trading technology also specifies an exogenously assigned and fixed target $\varpi^\star$ for the equity share. We refer to these traders as non-Mertonian precisely because the target does not respond to changes in the investment opportunity set.

There are two types of these traders. A *continuous-rebalancer* adjusts his equity position to the target $\varpi^\star$ in each period.[4] An *intermittent-rebalancer* adjusts his equity position to the target only every n periods; in non-rebalancing periods, all (dis-)savings occur through adjusting the holdings of the investor's risk-free asset.[5]

All households are initially endowed with a claim to their per capita share of both diversifiable and non-diversifiable income. Each agent's period 1 financial wealth is constrained by the value of

[4]One could think of this household delegating the management of its portfolio to a fund manager (see Abel, Eberly, and Panageas (2006))

[5]Abel, Eberly, and Panageas (2006) consider a portfolio problem in which the investor pays a cost to observe her portfolio, and they show that even small costs can rationalize fairly large intervals in which the household does not check its portfolio, and finances its consumption out of the riskless account. We do not endogenize the decision to observe the value of the portfolio, but, instead, we focus on the aggregate equilibrium implications of what Abel, Eberly, and Panageas (2006) call 'stock market inattention'. However, we assume that our investor knows the value of his holdings when making consumption decisions, even in non-rebalancing periods. Hence, we are implicitly assuming that it is the cost of reallocating his portfolio that is prevent continuous adjustment rather than the cost of finding out about the value of his portfolio.

their claim to tradeable wealth in the period 0 planning period, which is given by:

$$(1-\gamma)V_0[\{Y\}](z^0) \geq \sum_{z_1} Q(z_1, z^0)\hat{a}_0(z^1, \eta^0), \tag{5}$$

where both z^0 and η^0 simply indicate the degenerate starting values for the stochastic income process.[6]

Finally, the households face exogenous limits on their net asset positions, or solvency constraints,

$$\hat{a}_t(z^t, \eta^t) \geq 0. \tag{6}$$

Traders cannot borrow against their future labor income.

3.4 Measurability Restrictions

To capture these portfolio restrictions implied by the different trading technologies, we use measurability constraints (see Chien, Cole, and Lustig (2010) for a detailed discussion).

Z-complete Mertonian Trader Since idiosyncratic shocks are not spanned for the z-complete trader, his net wealth needs to satisfy:

$$\hat{a}_t\left(z^t, \left[\eta_t, \eta^{t-1}\right]\right) = \hat{a}_t\left(z^t, \left[\tilde{\eta}_t, \eta^{t-1}\right]\right), \tag{7}$$

for all t and η_t, $\tilde{\eta}_t \in N$.

Continuous-Rebalancing Non-Mertonian (*crb*) Trader Non-Mertonian traders who rebalance their portfolio in each period to a fixed fraction $\varpi^{\star}$ in levered equity and $1-\varpi^{\star}$ in

[6] In the quantitative analysis we only look at the ergodic equilibrium of the economy; hence, the assumptions about initial wealth are largely irrelevant. We assume that, during the initial trading period, households with portfolio restriction sell their claim to diversifiable income in exchange for their type appropriate fixed weighted portfolio of bonds and equities.

non-contingent bonds earn a return:

$$R_t^{crb}(\varpi^\star, z^t) = \varpi^\star R_{t,t-1}[\{D\}](z^t) + (1-\varpi^\star)R_{t,t-1}[1](z^{t-1})$$

Hence, their net financial wealth will satisfy the measurability restriction:

$$\frac{\hat{a}_t\left([z^{t-1}, z_t], [\eta_t, \eta^{t-1}]\right)}{R_t^{crb}(\varpi^\star, [z^{t-1}, z_t])} = \frac{\hat{a}_t\left([z^{t-1}, \tilde{z}_t], [\tilde{\eta}_t, \eta^{t-1}]\right)}{R_t^{crb}(\varpi^\star, [z^{t-1}, \tilde{z}_t])}, \tag{8}$$

for all t, z_t, $\tilde{z}_t \in Z$, and η_t, $\tilde{\eta}_t \in N$. If $\varpi^\star = \psi/(1+\psi)$, then this trader holds the market in each period and earns the return on a claim to all tradeable income: $R_{t,t-1}[\{(1-\gamma)Y\}](z^t)$. Without loss of generality, we can think of non-participants as *crb* traders with $\varpi^\star = 0$.

Intermittent-Rebalancing Non-Mertonian (*irb*) Trader An *irb* trader's technology is defined by his portfolio target (denoted $\varpi^\star$) and the periods in which he rebalances (denoted $\mathcal{T}$). We assume that rebalancing takes place at fixed intervals. For example, if he rebalances every other period, then $\mathcal{T} = \{1, 3, 5, ...\}$ or $\mathcal{T} = \{2, 4, 6, ...\}$.

After non-rebalancing periods, the *irb* trader with an equity share ϖ_{t-1} earns a rate of return:

$$R_t^{irb}(\varpi_{t-1}, z^t) = \varpi_{t-1}(z^{t-1})R_{t,t-1}[\{D\}](z^t) + (1-\varpi_{t-1}(z^{t-1}))R_{t,t-1}[1](z^{t-1})$$

and he faces the following measurability restriction on their net wealth:

$$\frac{\hat{a}_t\left([z^{t-1}, z_t], [\eta_t, \eta^{t-1}]\right)}{R_t^{irb}(\varpi_{t-1}, [z^{t-1}, z_t])} = \frac{\hat{a}_t\left([z^{t-1}, \tilde{z}_t], [\tilde{\eta}_t, \eta^{t-1}]\right)}{R_t^{irb}(\varpi_{t-1}, [z^{t-1}, \tilde{z}_t])}, \tag{9}$$

for all t, z_t, $\tilde{z}_t \in Z$, and η_t,$\tilde{\eta}_t \in N$, with $\varpi_t = \varpi^\star$ in rebalancing periods.

We define the trader's equity holdings as $e_t(z^t, \eta^t) = s_t(z^t, \eta^t)V_t[\{D\}](z^t)$. In re-balancing periods, this trader's equity holdings satisfy:

$$\frac{e_t(z^t, \eta^t)}{e_t(z^t, \eta^t) + b_t(z^t, \eta^t)} = \varpi^\star.$$

However, in non-rebalancing periods, the implied equity share is given by $\varpi_t = e_t/(e_t + b_t)$ where e_t evolves according to the following law of motion:

$$e_t(z^t, \eta^t) = e_{t-1}(z^{t-1}, \eta^{t-1}) R_{t,t-1}[\{D\}](z^t)$$

for each $t \notin \mathcal{T}$. This assumes that the *irb* trader automatically re-invests the dividends in equity in non-rebalancing periods.

Since this agent cannot hold any type of state-contingent bond, his flow budget constraint in non-rebalancing periods reduces to:

$$\gamma Y_t(z^t)\eta_t + b_{t-1}(z^{t-1}, \eta^{t-1}) R_{t,t-1}[1](z^{t-1}) \geq c_t(z^t, \eta^t) + b_t(z^t, \eta^t) \ \forall (z^t, \eta^t).$$

for each $t \notin \mathcal{T}$. Since setting $\mathcal{T} = \{1, 2, 3, ...\}$ generates the continuous-rebalancer's measurability constraint, the continuous-rebalancer can simply be thought of as a degenerate case of the intermittent-rebalancer. Hence, we can state without loss of generality that a non-Mertonian trading technology is completely characterized by $(\varpi^\star, \mathcal{T})$.

3.5 Equilibrium

We assume there is always a non-zero measure of z-complete traders to guarantee the uniqueness of the stochastic discount factor. For our Mertonian traders, we let μ_z denote the measure of z-complete traders. For our non-Mertonian traders, we will assume for simplicity that there are only two types participating non-Mertonian traders: *irb* (*crb*) traders with measure μ_{irb} (μ_{crb}) and portfolio target $\varpi^\star$, and nonparticipants with measure μ_{np} and portfolio target equal to zero. The non-state-contingent bond market clearing condition is given by

$$\sum_{\eta^t} \begin{bmatrix} \mu_z b_t^z(z^t, \eta^t) + \mu_{crb} b_t^{crb}(z^t, \eta^t) \\ +\mu_{irb} b_t^{irb}(z^t, \eta^t) + \mu_{np} b_t^{np}(z^t, \eta^t) \end{bmatrix} \pi(\eta^t | z^t) = V[\{(1-\gamma)Y - D\}](z^t)), \tag{10}$$

and the equity market clearing condition is given by

$$\sum_{\eta^t} \begin{bmatrix} \mu_z e_t^z(z^t,\eta^t) + \mu_{irb} e_t^{irb}(z^t,\eta^t) \\ +\mu_{crb} e_t^{crb}(z^t,\eta^t) \end{bmatrix} \pi(\eta^t|z^t) = V[\{D\}](z^t), \tag{11}$$

where we index the holdings by $\{z, crb, irb, np\}$ of the z-complete traders, continuous rebalancers, intermittent rebalancers and non-participants respectively. For the sake of clarity, we use (e.g.) $\eta^{t-1}(\eta^t)$ to denote the history from zero to $t-1$ contained in η^t. We use the same convention for the aggregate histories. Using this notation, the market clearing condition in the state-contingent bond market is given by:

$$\sum_{\eta^t} \left[\mu_z a_{t-1}^z(z^t, \eta^{t-1}(\eta^t))\right] \pi(\eta^t|z^t) = 0. \tag{12}$$

An equilibrium for this economy is defined in the standard way. It consists of a list of bond and dividend claim holdings, a consumption allocation and a list of bond and tradeable output claim prices such that: (i) given these prices, a trader's asset and consumption choices maximize her expected utility subject to the budget constraints, the solvency constraints and the measurability constraints, and (ii) the asset markets clear (eqs. (10), (11),(12)).

3.6 Solving for Equilibrium Allocations and Prices

To solve for the equilibrium allocations and prices, we develop an extension of the multiplier method developed by Chien, Cole, and Lustig (2010) to handle intermittent rebalancers. In continuous-time finance, Cuoco and He (2001) and Basak and Cuoco (1998) used stochastic weighting schemes to characterize allocations and prices. Our approach differs because it provides a tractable and computationally efficient algorithm for computing equilibria in environments with a large number of agents subject to idiosyncratic risk as well as aggregate risk, and heterogeneity in trading technologies. The use of cumulative multipliers in solving macro-economic equilibrium models was pioneered by Kehoe and Perri (2002), building on earlier work by Marcet and Marimon (1999). Our use of measurability constraints to capture portfolio restrictions is similar to that in

Aiyagari, Marcet, Sargent, and Seppala (2002) and Lustig, Sleet, and Yeltekin (2007), who consider an optimal taxation problem, while the aggregation result extends that in Chien and Lustig (2010) to an incomplete markets environment. Section A in the appendix contains a detailed description of the solution algorithm.

3.7 The Importance of Rebalancing

In a version of our economy with identically and independently distributed (i.i.d.) aggregate consumption growth, there is an equilibrium with non-Mertonian traders in the market in which all agents only trade the market, i.e. a claim to aggregate consumption, as shown by Krueger and Lustig (2009). In this equilibrium, the equity premium is the Breeden-Lucas-Rubenstein representative agent equity premium.

To explain the importance of rebalancing for aggregate risk sharing, we look at this stylized example in which the aggregate consumption growth is not predictable:

$$\phi(z'|z) = \phi(z'), \tag{13}$$

and the distribution of idiosyncratic shocks is independent of aggregate shocks:

$$\pi(\eta', z'|\eta, z)/\phi(z') = \varphi(\eta'|\eta). \tag{14}$$

Suppose that the non-Mertonian trader belongs to the class of continuous-rebalancers (*crb*), and holds the market portfolios: $\varpi^{\star} = \psi$. Also, suppose that there are no non-participants. One household consumption path that is feasible for the *crb* trader is proportional to aggregate output:

$$c_t(z^t, \eta^t) = \widehat{c}_t(\eta^t) Y_t(z^t). \tag{15}$$

Krueger and Lustig (2009) show that we can analyze an equivalent stationary economy without aggregate consumption growth (with a unit aggregate endowment), and with an adjusted transition probability matrix to solve for the equilibrium allocations and prices. To do so, we transform our growing model into a stationary model with a stochastic time discount rate and a growth-adjusted probability matrix, following Alvarez and Jermann (2001). First, we define growth deflated consumption allocations (or consumption shares) as in eq. (15). Next, we define *growth-adjusted* probabilities and the growth-adjusted discount factor as:

$$\hat{\phi}(z_{t+1}|z_t) = \frac{\phi(z_{t+1}|z_t)\exp(z_{t+1})^{1-\gamma}}{\sum_{z_{t+1}}\phi(z_{t+1}|z_t)\exp(z_{t+1})^{1-\gamma}} \text{ and } \hat{\beta} = \beta\sum_{z_{t+1}}\phi(z_{t+1}|z_t)\exp(z_{t+1})^{1-\gamma}.$$

Note that $\hat{\pi}$ is a well-defined Markov matrix in that $\sum_{z_{t+1}}\hat{\phi}(z_{t+1}|z_t) = 1$ for all z_t. Finally, we let $\hat{U}(\hat{c})(s^t)$ denote the lifetime expected continuation utility in node s^t, under the new transition probabilities and discount factor, defined over consumption shares $\{\hat{c}_t(s^t)\}$[7]

$$\hat{U}(\hat{c})(s^t) = u(\hat{c}_t(s^t)) + \hat{\beta}\sum_{s_{t+1}}\hat{\pi}(s_{t+1}|s_t)\hat{U}(\hat{c})(s^t, s_{t+1}). \tag{16}$$

Finally, we use $\widehat{V}_t[\{\widehat{X}\}$ to denote the no-arbitrage price of a claim to $\widehat{X}_t$ in the stationary economy, where the payoffs $\widehat{X}_t$ only depend on η^t. In this stationary economy, the measurability constraints of the non-Mertonian continuous-rebalancers can be stated as:

$$\widehat{V}_t\left[\{\gamma\eta - \widehat{c}\}\right](\eta^t) \textit{ is measurable w.r.t. } \eta^{t-1}. \tag{17}$$

These measurability constraints in the stationary economy do not depend on the aggregate history. As a result, the Mertonian z-complete traders face the exact same measurability constraint in the stationary economy as the non-Mertonian *crb* traders. Hence, given the assumptions we have imposed on the nature of aggregate and idiosyncratic shocks, the distinction between Mertonian and non-Mertonian trader becomes moot. We can solve for $\{\widehat{c}_t\}$ in the stationary economy, scale it by Y_t, to obtain equilibrium household consumption $\{\widehat{c}_t Y_t\}$. In this equilibrium, the relative wealth

[7]It is easy to show that this transformation does not alter the agents' ranking of different consumption streams. Households rank consumption share allocations in the de-trended model in exactly the same way as they rank the corresponding consumption allocations in the original model with growth.

of the non-Mertonian crb traders $\widehat{A}_t^{crb}(z^t)/\sum_{j\in\{z,crb\}}\widehat{A}_t^j(z^t)$ is invariant to aggregate shocks. The equilibrium equity premium in this economy is identical to that in a representative agent economy.

Now, this particular consumption path in eq. (15) is feasible for the non-Mertonian trader simply by trading a claim to aggregate consumption (the market), .i.e., maintaining a portfolio with $\varpi^\star = 1/(1+\psi)$ invested in equity. However, for the irb trader, this consumption path is not feasible, because holding the market requires re-balancing every period.

Instead, consider what happens to an irb trader who starts out by holding the aggregate consumption claim in his portfolio. After a negative aggregate consumption growth shock z_t, the equity share of his portfolio drops below $1/(1+\psi)$, and the non-Mertonian trader holds too little equity. After a positive aggregate consumption growth shock z_t, the equity share of his portfolio increases above ψ, and the non-Mertonian trader holds too much equity.

To see why, note that the p/d ratio is constant in the original equilibrium. If the p/d ratio is constant, then the average agent should simply consume the dividends to hold the market portfolio. The equity share in his portfolio remains constant at $1/(1+\psi)$ if he does so. Instead, the average irb trader, in non-rebalancing periods, buys more shares after high aggregate consumption and hence dividend growth realizations, because he re-invests the dividends automatically. After low aggregate consumption growth realizations, the opposite happens. As a result, the active traders have to buy shares from the irb traders after low aggregate consumption growth realizations and sell shares after high aggregate consumption growth realizations.

Thus, after a series of negative aggregate consumption growth shocks, the irb's equity share ϖ_{t-1} would be much lower than what is prescribed to hold the market, and $R_t^{irb}(\varpi_{t-1},\tilde{z}_t)$ is increasingly less exposed to aggregate consumption risk. In this new equilibrium, the relative wealth of the non-Mertonian irb , traders $\widehat{A}_t^{irb}(z^t)/\sum_{j\in\{z,irb\}}\widehat{A}_t^j(z^t)$ cannot be invariant w.r.t aggregate shocks.

Hence, these intermittent rebalancers act like households with counter-cyclical risk aversion, because of the nature of the trading technology: adverse aggregate shocks endogenously concentrate aggregate risk among the Mertonian traders. Even in the case of i.i.d. aggregate shocks without non-participants, the irrelevance result in Krueger and Lustig (2009) no longer holds if some of the non-Mertonian traders do not continuously rebalance.

In the calibrated version of the model, we introduce non-participants as well. These non-participants create residual aggregate risk that needs to be transferred to the other market participants. This concentration of aggregate risk allows us to match the average equity premium.

4 Calibration

Section 4 and 5 evaluate a calibrated version of the model to examine the extent to which our model can account for the empirical moments of asset prices, and in particular the counter-cyclical volatility at market price of risk. This section discusses the calibration of the parameters and the endowment processes, and the composition of trader pools.

To compute the equilibrium of this economy, we follow the algorithm described by Chien, Cole, and Lustig (2010), which uses truncated aggregate histories as state variables. We keep track of lagged aggregate histories up to 7 periods. The details are in section D of the appendix.

Our objective is to examine the response of the moments of equilibrium asset prices, consumption growth, portfolio returns and the welfare to changes in the frequency of rebalancing by non-Mertonian equity holders and the level of their equity target.

4.1 Preferences and Endowments

The model is calibrated to annual data. We choose a coefficient of relative risk aversion α of five and a time discount factor β of .95. These preference parameters allow us to match the collaterizable wealth to income ratio in the data when the tradeable or collateralizable income share $1-\gamma$ is 10%, as discussed below. The average ratio of household wealth to aggregate income in the US is 4.30 between 1950 and 2005. The wealth measure is total net wealth of households and non-profit organizations (Flow of Funds Tables). With a 10% collateralizable income share, the implied ratio of wealth to consumption is 5.28 in the model's benchmark calibration.[8]

[8] As is standard in this literature, we compare the ratio of total outside wealth to aggregate non-durable consumption in our endowment economy to the ratio of total tradeable wealth to aggregate income in the data. Aggregate income exceeds aggregate non-durable consumption because of durable consumption and investment.

Our benchmark model is calibrated to match the aggregate consumption growth moments from Alvarez and Jermann (2001) and Mehra and Prescott (1985). The average consumption growth rate is 1.8% and the standard deviation is 3.15%. Recessions are less frequent than expansions: 27% of realizations are low aggregate consumption growth states. The first-order autocorrelation coefficient of aggregate consumption growth (ρ_z) is -.14. We will refer to this as the Mehra-Prescott (MP) Economy henceforth.

We also consider a second calibration without any predictability of aggregate consumption growth. This i.i.d consumption process is the same one as described in subsection 3.7. In this calibration, we simply set the autocorrelation of aggregate consumption growth (ρ_z) to zero, but we keep all others moments are unchanged. We refer to this calibration as IID Economy.

We calibrate the labor income process as in Storesletten, Telmer, and Yaron (2004) and Storesletten, Telmer, and Yaron (2007), except that we eliminate the counter-cyclical variation of labor income risk. The variance of labor income risk is constant in our model. This allows us to focus on the effects of changes in composition of non-Mertonian traders pool and their target equity share. The Markov process for $\log \eta(y, z)$ has a standard deviation of .71, and the autocorrelation is 0.89. We use a 4-state discretization for both aggregate and idiosyncratic risk. The elements of the process for $\log \eta$ are $\{0.38, 1.61\}$.

Equity in our model is simply a leveraged claim to diversifiable income. In the Flow of Funds, the ratio of corporate debt-to-net worth is around 0.65, suggesting a leverage parameter ψ of 2. However, Cecchetti, Lam, and Mark (1990) report that standard deviation of the growth rate of dividends is at least 3.6 times that of aggregate consumption, suggesting that the appropriate leverage level is over 3. Following Abel (1999) and Bansal and Yaron (2004), we choose to set the leverage parameter ψ to 3.

4.2 Composition of Trader Pools and Equity Share

In the most recent Survey of Consumer Finances, 51.1 % reported owning stocks directly or indirectly. Therefore, the fraction of non-Mertonian trader with zero equity holding (Non-participants) is calibrated to 50%. In order to match the large equity premium (7.53 %) measured in post-war US data, a small fraction of Mertonian trader needs to bear the residual aggregate risk created by non-participant. Hence, we set the share of the share of Mertonian traders equal to 5%, and non-Mertonian traders who hold equities to 45%.

We consider two types of non-Mertonian equity holders: (1) those who rebalance every period (crb) and (2) those who rebalance every 3 years (irb). This level of inertia is modest compared to what researchers have documented in the data (see, e.g., the evidence reported by Ameriks and Zeldes (2004), Calvet, Campbell, and Sodini (2009) and Brunnermeier and Nagel (2008)).

In the case with irb trader, the optimal target equity share of irb traders turns out to be 37 % in MP economy and 42 % in IID economy. Hence, these cases are natural benchmark cases to look at. To evaluate the effects of changes in equity share, we consider additional three different equity share targets for our non-Mertonian equity holders: 30%, 35%, and 40%.

5 Quantitative Results in Benchmark Economy

Table I reports moments of asset prices generated by simulating data from a model with 3,000 agents for 10,000 periods. Panel I and II report results for the case of MP economy and IID economy respectively. Each panel consists of two cases: one with 45% crb trader and the other with 45% irb trader.

5.1 Benchmark Mehra-Prescott Economy

In the upper part of Table I, we report the maximum unconditional Sharpe ratio ($\frac{\sigma(m)}{E(m)}$), the standard deviation of the maximum SR ($Std(\frac{\sigma_t(m)}{E_t(m)})$), the equity risk premium $E\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$, the standard deviation of excess returns $\sigma\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$, the Sharpe ratio on equity, the mean risk-free rate and the standard deviation of the risk-free rate. In the lower part of Table I, we report the standard deviation of the conditional risk premium on equity, the standard deviation of the conditional volatility of risk premium on equity and the standard deviation of the conditional SR on equity.

CRB In the case with *crb* traders, the maximum SR is 0.39 and the standard deviation of the maximum SR is 8.22%. The equity premium is 8.48% and the Sharpe ratio on equity is .38. The average risk-free rate is 1.53% and its volatility is 2.80%. Finally, we also decompose the variation in the SR on equity; the standard deviation of the conditional risk premium on equity is 1.63%, the standard deviation of the conditional volatility is 1.68% and this produces a standard deviation of the conditional SR is 8.22%.

IRB In the case with *irb* traders, the maximum SR is 0.41 and the standard deviation of the maximum SR is 25.25%, which represents a threefold increase in the volatility. The equity premium drops to 7.85% while the standard deviation of stock returns increases to 27.67%. The Sharpe ratio on equity drops to .28. The moments of the risk-free rate are virtually unchanged. So, while the unconditional risk premia are lower in the economy with intermittent rebalancing, the volatility of conditional risk premia triples. The behavior of interest rates is largely unaffected.

The intermittent rebalancing behavior also increases the volatility of conditional moments on equity returns significantly. The standard deviation of the conditional risk premium increases from 1.63% to 5.99%, the standard deviation of the conditional volatility increases from 1.68 to 3.20%, and the standard deviation of the conditional SR on equity increases from 8.22% to 26.23%.

Approximation The last line in Table I reports the standard deviation of the allocation error that results from our approximation in percentage points. The standard deviation of the percentage forecast error are between 0.14 and 0.07 % in the benchmark case of both MP and IID economy. This means our approximation is highly accurate compared to other results reported in the literature for models with heterogeneous agents and incomplete markets. The implied R^2 in a linear regression of the actual, realized SDF's rates on the SDF that we predicted based on the truncated aggregate histories exceed 0.998 in all cases.

[Table 1 about here.]

Economy with IID Aggregate Consumption Growth Dynamics Alvarez and Jermann (2001) match the first-order autocorrelation of aggregate consumption growth shocks reported by Mehra and Prescott (1985) ($\rho_z = -.14$). We check the sensitivity of our results to the negative autocorrelation of aggregate consumption growth shocks by choosing an IID calibration of aggregate consumption growth shocks ($\rho_z = 0$), and we compare our model quantitatively to Campbell and Cochrane (1999) with the same aggregate consumption growth process. This IID economy satisfies the assumptions we imposed in the IID example (see subsection 3.7). In this version of model, without non-participants, the risk premium of representative agent model is obtained if all non-Mertonian traders are of *crb* type.

The key moments of the stochastic discount factor are reported in Panel II of Table I. If the *irb* traders hold the optimal equity share (42%), the volatility increases from 6.35% to 15.11%. This represents a 238% increase, smaller than the increase reported in the non-IID economy. The volatility is smaller in IID economy but the *irb* traders still amplify the volatility of the market prices of risk significantly.

As a benchmark, we can use the implied standard deviation of the market price of risk in an annual calibration of the Campbell and Cochrane (1999) external habit model. We used the same i.i.d aggregate consumption growth process as in our IID economy. All of the other parameters were taken directly from Campbell and Cochrane (1999). In this annual calibration of their model, the standard deviation of the market price of risk is 21%. In our benchmark IID economy with CRRA (constant relative risk aversion) agents, our model generates 15.11%, 1/4-th less than the external habits model. However, as we shall show in section 6, the volatility generated by our model increases to 23.20% if Mertonian traders can hedge against their idiosyncratic risk, slightly higher than the volatility produced by the external habits model.

Reinvestment of Dividends Our results critically rely on our assumption that the *irb* traders reinvest the dividends in non-rebalancing periods. In the benchmark case of MP economy, the standard deviation of the market price of risk actually drops from 8.22% to 6.63% in the *irb* case *without* reinvestment of dividends (not shown in Table). Recall that with reinvestment the volatility increases to 25.25%.

In the benchmark case, without intermittent rebalancers, our model does not generate much variation in the price/dividend ratio, and hence, most of the effects come from the reinvestment of dividends. Hence, if the dividends are not reinvested and the price/dividend ratio is constant, there is really no need for the *irb* traders to rebalance. There is no net new issuance of shares at the aggregate level. In equilibrium, the average investor simply consumes his or her dividends. There is no need for trade in shares between the average non-Mertonian and the average Mertonian trader. Once the *irb* trader reinvests dividends, then the Mertonian traders have to sell shares after good aggregate shocks and buy shares after bad aggregate shocks.

Figure 2 plots a 100-year simulation of the equity share (full line) of the Mertonian trader in the case with *irb* traders in the top panel; the bottom panel shows the case with *crb* traders. The shaded areas are low aggregate consumption growth states and the dashed line is a 4-period moving average of aggregate consumption growth. Clearly, there is much more counter-cyclical variation in the equity share of the Mertonian traders in the *crb* case, especially on the downside. This variation in the equity share of the Mertonian traders is the driving force behind our amplification mechanism.

[Figure 2 about here.]

Countercyclical Variation The variation in market price of risk created by the *irb* traders is counter-cyclical; it mirrors the variation in the active trader's equity share. Figure 3 plots the conditional Sharpe ratio on equity against the history of aggregate consumption growth shocks for the benchmark case of MP economy. The shaded areas denote the low aggregate consumption growth realizations. The dotted line shows 4-period moving average of aggregate consumption growth; the full line shows the conditional Sharpe ratio.

[Figure 3 about here.]

In the *irb* case, the conditional risk premium on equity increases with each low aggregate consumption growth realization, and decreases with each high aggregate consumption growth realization. The conditional Sharpe ratio on equity is even more counter-cyclical, because the conditional volatility decreases with each negative aggregate consumption growth realization (not shown in the picture). Figure 4 shows this in a scatter plot representation of the same 100 simulations, with the weighted average of aggregate consumption growth shocks on the x-axis and the conditional Sharpe ratio. On the other hand, in the *crb* case, shown in figure 5, the conditional Sharpe ratio is only weakly counter-cyclical.

[Figure 4 about here.]

[Figure 5 about here.]

5.2 Portfolio, Wealth, Consumption and Welfare Costs

The first panel in Table II reports the moments of household portfolio returns in benchmark case of MP economy. The Mertonian z-complete traders realize an excess return of 6.02% and a SR of .51, compared to only 2.72% and .26 respectively for the *irb* trader. The optimal average portfolio share for a non-Mertonian *crb* trader is only 51% (compared to 72% in the crb case), because the equity premium is lower.

We also evaluate the welfare cost of being a non-Mertonian *crb* or *irb* trader. This cost is measured by the percentage of consumption compensation to Mertonian traders so that they are indifference to become non-Mertonian traders. Given the optimal equity share target in each case, the welfare cost of being a non-Mertonian *irb* trader in *irb* case is 4 times higher than that in the *irb* case (11.94% v.s. 2.66%), simply because the risk premium is much more volatile and hence the cost of not responding to variation in the investment opportunity set is much larger.

We also report the welfare cost of being an *irb* trader compared to a *crb* trader holding fixed the equity share target at 37%. The cost is small (-1.23%) and negative: a *crb* trader would be willing to yield 1.23% of his consumption to become an *irb* trader. This is surprising, but it has a simple reason. The optimal equity target of *crb* trader is 51%, which is higher than 37%, the optimally chosen target for the *irb* trader. Hence, increasing the equity share actually benefits the *crb* trader because of high average equity premium. The actual average equity share holding of *irb* traders is actually higher than his target equity share, because expansions are more frequent than recessions, and because their equity share drifts up in expansions. As it turns out, this benefit outweighs the cost of intermittent rebelancing.[9]

On the other hand, an *irb* trader –setting his target share optimally– would be willing to pay 2.7% of consumption to become a *crb* trader who can optimally choose his target. This number is the difference between 11.9% (reported as the welfare cost(%) of *irb* to z at the optimal equity share for *irb*) and 9.2% in the table (reported as the welfare cost(%) of *crb* to z at the optimal equity share for *crb*). This 2.6% is the true cost of not rebalancing. It is small relative to the cost of not responding to changes in the investment opportunity set. The costs of not rebalancing are small; the costs of having a fixed equity target are large.

The second panel in Table II reports the moments of household consumption growth, and the moments of aggregate consumption growth for each group of traders. In the case of *crb* traders, the volatility of household consumption growth is inversely related to the degree of sophistication of the trader: 3.12% for Mertonian traders, 3.33% for the *crb* traders, for 3.76% for non-participants. However, the relation between consumption volatility and trader sophistication reverses itself at the group level. The volatility for the Mertonian trader segment is 1.85%, compared to 1.19% for the non-Mertonian equity holders, and 0.60% for the non-participants. These results highlight the fact that these traders are exposed to different types of risk. Mertonian traders are more exposed to aggregate risk and non-Mertonian traders are more subject to idiosyncratic risk.

Now, in the case of the *irb* traders, the volatility of the Mertonian trader's consumption growth (at the group level) decreases to 1.80%, while, at the household level, the volatility of household consumption growth for non-Mertonian equity holders increases from 3.33% to 3.88%. Other than that, the second moment of consumption is very similar to *crb* case at both individual and group level.

Overall, what is striking is how similar the unconditional moments are in the case of *crb* and *irb* traders, both in terms of portfolio returns and household consumption. The main quantitative

[9]However, if we would force the average equity shares to be the same for these traders, the cost would obviously be positive. In any case, this shows that the direct cost of intermittent rebalancing has to be small, much smaller than the cost of being a non-Mertonian.

difference is the increase in the volatility of household consumption growth for the non-Mertonian equity holders.

Finally, the third panel in Table II reports the household wealth statistics. The Mertonian (z-complete) trader accumulates 1.88 times as much wealth as the average household in the baseline *crb* case, while the non-Mertonian trader accumulates 1.19 times as much and the non-participant .74. These fractions are virtually unchanged in the *irb* case. However, the wealth of the non-Mertonian trader (expressed as a fraction of average wealth) becomes more volatile –it increases from 9.4% to 15.2%.

Our model has reasonable cross-sectional consumption implications. In our model, Mertonian investors load up on aggregate consumption risk, earn higher portfolio return and end up richer. This is consistent with the data. The consumption of the 10% wealthiest households is five times more exposed to aggregate consumption growth than that of the average US household (Parker and Vissing-Jorgensen (2009)). In contrast, less sophisticated investors (Non-participants in our model) are more exposed to idiosyncratic risk. This is broadly in line with the data. Malloy, Moskowitz, and Vissing-Jorgensen (2009) find that the average consumption growth rate for stockholders is between 1.4 and two times as volatile as that of non-stock holders. They also find that aggregate stockholder consumption growth for the wealthiest segment (upper third) is up to 3 times as sensitive to aggregate consumption growth shocks as that of non-stock holders.

[Table 2 about here.]

5.3 Composition of the non-Mertonian Trader Pool and Equity Share

The participation of non-Mertonian traders tends to increase the volatility in risk premia. In our model, this force operates in two ways: (i) as we shift non-Mertonian traders from the *crb* type to the *irb* type and (ii) as we increase the target share of equity in the non-Mertonian trader's portfolio. We discuss both of these effects below.

Table III varies the target equity share from 30% to 40%. The first panel reports result for the case when $\varpi^{\star}$, the target equity share of the non-Mertonian trader is 30%, the second panel consider 35%,and finally, the last panel looks at the case of 40 %.

Increasing the target share of equity for Non-Mertonian equity traders also increases the volatility substantially, from 6.8% in the *crb* case (17.3 % in the *irb* case), with 30% target equity share (see left panel of Table III), to 10.02% (24.80%) with 40% target share (see right panel of Table III). However, as we increase the equity holdings of the non-Mertonian equity traders from 30% to 40%, the unconditional market price of risk, the equity premium and Sharpe ratio all decrease. The more equity non-Mertonian traders hold, the higher the volatility of risk prices. A 10 percentage point increase in the target share for equities delivers a 50% increase in the volatility of risk prices. As we increase the target equity share to 40 %, the equity risk premium actually turns negative after a series of high aggregate consumption growth shocks. This explains why the volatility of the Sharpe ratio surpasses that of the market price of risk.

[Table 3 about here.]

6 Quantitative Results in Economy with Binding Solvency Constraints for Mertonian Traders

While the results reported sofar show that *irb* non-Mertonian traders amplify the volatility of risk prices, the numbers are still small compared to the 50 % standard deviation of the SR reported by Lettau and Ludvigson (2001). However, the composition of the Mertonian trader pool is equally important for the volatility of the market price of risk. The z-complete Mertonian traders are subject to idiosyncratic risk and hence have a precautionary motive to accumulate wealth. As a result, their solvency constraints rarely bind in equilibrium. We now look at what happens when we introduce another type of Mertonian traders who are not subject to idiosyncratic risk or can hedge against it. We refer to this type of Mertonian trader as complete trader. We can think of complete traders as a stand-in for highly levered, Mertonian market participants like hedge funds. These participants will tend to increase the volatility of risk premia if they are subject to occasionally binding solvency constraints (see Alvarez and Jermann (2001) and Chien and Lustig

(2010)).

6.1 Composition of the Mertonian Trader Pool

The first panel of Table IV reports the result of MP economy with complete traders. As we change the Mertonian trader from z-complete traders to complete traders, the volatility of the market price of risk increases from 25.25% to 34.41% in our MP economy as shown in Table I and IV. Moreover, the volatility of the conditional Sharpe ratio on equity increases from 26.23% to 39.83% and the maximum SR increases as well from .41 to .54. This means we do get much closer to the target in the data if we introduce these complete Mertonian traders. Clearly, the binding solvency constraints strengthen our mechanism considerably.

Economy with IID Aggregate Consumption Growth Dynamics The second panel of Table IV reports the result for the IID economy. The volatility of the market price of risk is 23.20%, which is comparable to that (21%) in the annual calibration version of Campbell and Cochrane (1999) model.

[Table 4 about here.]

In addition, these complete traders load up on more aggregate risk, as is apparent from the results in the right panel of Table V. The complete traders realize average excess returns of up to 14% per annum. At the household level, in the baseline case with *crb* traders, we get the same relation between trader sophistication and consumption growth volatility: the standard deviation of household consumption growth is 2.39% for the Mertonian traders, compared to 3.25% for the non-Mertonian equity holders and 3.78 % for the non-participants. However, the composition is very different: the group volatility is 2.10 for the Mertonian traders, compared to 1.22 for the non-Mertonian equity holders and .63 for the non-participants. Finally, the welfare cost of being a non-Mertonian trader increases significantly from 11.94% to 25.15% of consumption, simply because the volatility of risk premia is so much higher.

[Table 5 about here.]

For the same 100 simulations, we also plot the conditional Sharpe ratio against weighted average of consumption growth shocks in Figure 6. The conditional SR declines monotonically as the weighted average of aggregate consumption growth shocks increases. The pattern here is similar to that in the case of z-complete traders (4) but with a larger amplitude. The range of variation increases from $[0.15, 0.75]$ in the case with z-complete traders to $[0.1, 1.2]$ in the case with complete traders.

[Figure 6 about here.]

[Figure 7 about here.]

6.2 Size of Mertonian Trader Pool

The volatility of risk premia depends critically on the size of the Mertonian trader pool. We fix the target equity share at 37% in MP economy. As we grow the size of the Mertonian trader pool, the volatility of the market price of risk decreases at a fast rate. Table VI reports the conditional moments in the case of a 10% Mertonian trader pool (up from 5% in the benchmark case) in MP economy.

The first two columns report the case with z-complete traders. The amplification channel is still operative, but the effect is smaller. In the case with 10% z-complete traders, the volatility of the market price of risk is 5.2%, and this number increases to 13.3% when we replace the *crb* traders with *irb* traders. In the benchmark case with only 5% Mertonian traders, these numbers were 8.22% and 25.25% respectively, as reported in Table I. So, the amplification channel has weakened considerably. The standard deviation of the conditional Sharpe ratio on equity increases from 5.23% to 13.3%, compared to an increase from 8.22% to 26.23% in the benchmark case with 5% Mertonian traders.

As the mass of z-complete traders increases, the amplification channel weakens partly because aggregate risk is not concentrated enough. This reason is vanished if we replace these z-complete traders with complete traders because the latter have no precautionary motive to accumulate wealth, and as result, their solvency constraints will still bind in equilibrium. These results are reported in the last two columns of Table VI. In this case, the standard deviation of the market price of risk increases from 10.96% to 25.54%, compared to 16.32% and 34.4% respectively in the benchmark case with only 5% Mertonian traders. Therefore, the amplification channel is not mitigated as much as the case with z-complete trader by an increase in the supply of Mertonian trader.

[Table 6 about here.]

7 Conclusion

Our paper shows that intermittent re-balancing should be considered as one potential explanation for the puzzling volatility of Sharpe ratios in equity markets, but not the only explanation. This explanation does not rely on non-standard preferences, but instead it simply assumes that some traders fail to continuously re-balance their portfolios. However, the welfare cost calculations suggest that small costs might suffice to deter households from continuously re-balancing. Even though the individual welfare loss from not rebalancing may be small, the aggregate impact on pricing is large. This makes it an appealing friction.

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Table I: Moments of Asset Prices: Benchmark Case

	Panel I: MP Economy		Panel II: IID Economy	
target equity share ($\varpi^{\star}$)	37%		42%	
Non-Mertonian equity holder	**crb**	**irb**	**crb**	**irb**
Mertonian ***z-complete***	5%	5%	5%	5%
Non-Mertonian ***crb***	45%	0%	45%	0%
Non-Mertonian ***irb***	0%	45%	0%	45%
Non-Mertonian ***np***	50%	50%	50%	50%
$\frac{\sigma(M)}{E(M)}$	0.394	0.412	0.297	0.282
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	8.218	25.252	6.352	15.106
$E\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	8.487	7.849	4.255	3.830
$\sigma\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	22.294	27.674	14.794	16.647
Sharpe Ratio	0.381	0.284	0.288	0.230
$E\left(R_{t+1,t}[1]\right)$	1.526	1.776	2.381	2.462
$\sigma\left(R_{t+1,t}[1]\right)$	2.802	2.312	0.202	0.323
$Std\left[E_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.625	5.987	0.897	2.396
$Std\left[\sigma_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.684	3.203	0.166	0.412
$Std\left[SR_t\right]$	8.218	26.226	6.352	15.106
$Std[\log(e)](\%)$	0.110	0.141	0.068	0.086

Moments of annual returns. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without CCV; Panel I uses the Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Panel II uses an i.i.d. calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

Table II: Moments of Household Portfolio Returns and Consumption in MP Economy

Non-Mertonian equity holder	**crb**	**irb**
*Mertonian **z-complete***	5%	5%
*Non-Mertonian **crb***	45%	0%
*Non-Mertonian **irb***	0%	45%
*Non-Mertonian **np***	50%	50%
	Panel I: Household Portfolio	
	Excess Return	
Mertonian Trader	5.532	6.019
Non-Mertonian Equity Holder	3.128	2.719
	Sharpe Ratio	
Mertonian Trader	0.403	0.505
Non-Mertonian Equity Holder	0.381	0.262
	Additional Stats	
Optimal Equity Share for irb	0.580	0.370
Welfare cost(%) of irb to z at optimal equity share for irb	2.659	11.941
Optimal Equity Share for crb	0.720	0.510
Welfare cost(%) of crb to z at optimal equity share for crb	1.580	9.274
Welfare cost(%) of irb to crb at 37% equity share	0.434	−1.225
	Panel II Household Consumption	
	Std. Dev. at Household level	
Mertonian Trader	3.121	3.107
Non-Mertonian Equity Holder	3.331	3.879
Non-Mertonian non-participant	3.756	3.714
	Std. Dev. of Group Average	
Mertonian Trader	1.850	1.804
Non-Mertonian Equity Holder	1.266	1.332
Non-Mertonian non-participant	0.602	0.606
	Panel III: Household Wealth	
	Average Household Wealth Ratio	
Mertonian Trader	1.883	1.979
Non-Mertonian Equity Holder	1.191	1.162
Non-Mertonian non-participant	0.740	0.756
	Stdev. of Household Wealth Ratio	
Mertonian Trader	0.380	0.640
Non-Mertonian Equity Holder	0.094	0.152
Non-Mertonian non-participant	0.118	0.119
	Stdev. of Aggregate Equity Share	
Non-Mertonian Equity Holder	0.050	0.139
	Correlation of Aggregate Equity Share	
Non-Mertonian Equity Holder	0.771	0.706

Panel I reports moments of household portfolio returns, Panel II reports moments of household consumption, and Panel III reports moments of household wealth: we report the average excess returns on household portfolios and the Sharpe ratios, we report the standard deviation of household consumption growth (as a multiple of the standard deviation of aggregate consumption growth), and we report the standard deviation of group consumption growth (as a multiple of the standard deviation of aggregate consumption growth); the last panel reports the average household wealth as a share of total wealth, and the standard deviation of household wealth, as a share of total wealth. Results for 37% equity share non-Mertonian target ($\varpi^\star$).Moments of annual returns and consumption flows. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without CCV; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

Table III: Target Equity Share and Moments of Asset Prices in MP Economy

equity share target ($\varpi^\star$)	30%		35%		40 %	
Non-Mertonian equity holder	**crb**	**irb**	**crb**	**irb**	**crb**	**irb**
Mertonian ***z-complete***	5%	5%	5%	5%	5%	5%
Non-Mertonian ***crb***	45%	0%	45%	0%	45%	0%
Non-Mertonian ***irb***	0%	45%	0%	45%	0%	45%
Non-Mertonian ***np***	50%	50%	50%	50%	50%	50%
$\frac{\sigma(M)}{E(M)}$	0.479	0.466	0.421	0.422	0.330	0.395
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	6.798	17.322	7.749	23.740	10.018	24.794
$E\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	10.620	10.172	9.119	8.480	6.865	6.576
$\sigma\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	22.558	24.333	22.275	26.313	22.239	28.589
Sharpe Ratio	0.471	0.418	0.409	0.322	0.309	0.230
$E\left(R_{t+1,t}[1]\right)$	0.992	1.195	1.370	1.602	1.905	2.066
$\sigma\left(R_{t+1,t}[1]\right)$	2.848	2.474	2.813	2.348	2.754	2.273
$Std\left[E_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.341	3.373	1.533	5.114	2.013	6.713
$Std\left[\sigma_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.649	2.042	1.673	2.714	1.710	3.953
$Std\left[SR_t\right]$	6.798	17.322	7.749	23.740	10.018	28.365
$Std[\log(e)](\%)$	0.151	0.126	0.110	0.128	0.106	0.156

Moments of annual returns. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without CCV; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

Table IV: Moments of Asset Prices with Mertonian Complete Traders

	Panel I: MP Economy		Panel II: IID Economy	
target equity share ($\varpi^\star$)	37%		42%	
Non-Mertonian equity holder	**crb**	**irb**	**crb**	**irb**
Mertonian ***complete***	5%	5%	5%	5%
Non-Mertonian ***crb***	45%	0%	45%	0%
Non-Mertonian ***irb***	0%	45%	0%	45%
Non-Mertonian ***np***	50%	50%	50%	50%
$\frac{\sigma(M)}{E(M)}$	0.507	0.541	0.370	0.391
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	16.318	34.409	9.960	23.197
$E\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	9.696	8.328	4.889	3.587
$\sigma\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)$	20.888	30.712	13.940	18.767
Sharpe Ratio	0.464	0.271	0.351	0.191
$E\left(R_{t+1,t}[1]\right)$	1.394	1.946	2.296	2.645
$\sigma\left(R_{t+1,t}[1]\right)$	2.517	1.921	0.422	0.814
$Std\left[E_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	2.546	9.266	1.219	4.366
$Std\left[\sigma_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.839	5.536	0.512	3.552
$Std\left[SR_t\right]$	16.318	39.827	9.961	29.051
$Std[\log(e)](\%)$	0.076	0.172	0.059	0.082

Moments of annual returns. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without CCV; Panel I uses the Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Panel II uses an i.i.d. calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

Table V: Moments of Household Portfolio Returns and Consumption in MP economy with Complete Traders

Non-Mertonian equity holder	**crb**	**irb**
*Mertonian **complete***	5%	5%
*Non-Mertonian **crb***	45%	0%
*Non-Mertonian **irb***	0%	45%
*Non-Mertonian **np***	50%	50%
	Panel I: Household Portfolio	
	Excess Return	
Mertonian Trader	12.888	14.133
Non-Mertonian Equity Holder	3.578	2.734
	Sharpe Ratio	
Mertonian Trader	0.082	0.163
Non-Mertonian Equity Holder	0.468	0.243
	Additional Stats	
Optimal Equity Share for irb	0.750	0.290
Welfare cost(%) of irb to z at optimal	7.174	25.151
Optimal Equity Share for crb	1.020	0.510
Welfare cost(%) of crb to z at optimal	5.320	21.422
Welfare cost(%) of irb to crb at 37% equity	0.718	−2.481
	Panel II Household Consumption	
	Std. Dev. at Household level	
Mertonian Trader	2.390	2.580
Non-Mertonian Equity Holder	3.248	3.889
Non-Mertonian non-participant	3.775	3.687
	Std. Dev. of Group Average	
Mertonian Trader	2.100	2.213
Non-Mertonian Equity Holder	1.220	1.331
Non-Mertonian non-participant	0.633	0.611
	Panel III: Household Wealth	
	Average Household Wealth Ratio	
Mertonian Trader	0.535	0.911
Non-Mertonian Equity Holder	1.31	1.23
Non-Mertonian non-participant	0.764	0.802
	Stdev. of Household Wealth Ratio	
Mertonian Trader	0.108	0.534
Non-Mertonian Equity Holder	0.097	0.149
Non-Mertonian non-participant	0.095	0.121
	Stdev. of Aggregate Equity Share	
Non-Mertonian Equity Holder	0.023	0.135
	Correlation of Aggregate Equity Share	
Non-Mertonian Equity Holder	0.738	0.662

Panel I reports moments of household portfolio returns, Panel II reports moments of household consumption, and Panel III reports moments of household wealth: we report the average excess returns on household portfolios and the Sharpe ratios, we report the standard deviation of household consumption growth (as a multiple of the standard deviation of aggregate consumption growth), and we report the standard deviation of group consumption growth (as a multiple of the standard deviation of aggregate consumption growth); the last panel reports the average household wealth as a share of total wealth, and the standard deviation of household wealth, as a share of total wealth. Results for 37% equity share non-Mertonian target ($\varpi^\star$).Moments of annual returns and consumption flows. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without CCV; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

Table VI: Conditional Moments and size of Mertonian Trader Pool in MP economy

Mertonian trader	**z-complete**		**complete**	
Non-Mertonian equity holder	**crb**	**irb**	**crb**	**irb**
Mertonian ***z-complete***	10%	10%	0%	0%
Mertonian ***c***	0%	0%	10%	10%
Non-Mertonian ***crb***	40%	0%	40%	0%
Non-Mertonian ***irb***	0%	40%	0%	40%
Non-Mertonian ***np***	50%	50%	50%	50%
$\frac{\sigma(m)}{E(m)}$	0.347	0.333	0.457	0.462
$Std\left[\frac{\sigma_t(M)}{E_t(M)}\right]$	5.237	13.303	10.957	25.543
$Std\left[E_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.099	2.544	1.524	4.304
$Std\left[\sigma_t\left(R_{t+1,t}[D] - R_{t+1,t}[1]\right)\right]$	1.645	2.519	1.482	5.906
$Std\left[SR_t\right]$	5.237	13.303	10.957	26.653

Moments of annual returns conditional on history of aggregate shocks z^t. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) calibration of idiosyncratic shocks without CCV; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. Results for 37% equity share non-Mertonian target ($\varpi^{\star}$). The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

Table VII: The Conditional Sharpe Ratio in Equity Markets and the Business Cycle

In expansions (recessions), the investor buys the stock market index in the n-th quarter after the NBER hrough (peak) and sells after 4 quarters. Table reports moments of excess returns for thhis investment strategy implemented on the CRSP-VW index of NYSE-AMEX-NASDAQ realized. The riskfree rate is the 90-days T-bill rate (also from CRSP). The entire sample comprises 1925.IV-2009.II. The postwar sample comprises 1945.I-2009.II. We report the average excess return on this investment strategy (annualized) in the first panel, the standard deviation (not annualized) in the second panel and the Sharpe ratio (annualized) in the third panel.

			Expansions		
			Buy in $n-th$ quarter after through		
	1st	2nd	3rd	4th	5th
		Conditional Expected Excess Return			
whole	17.30%	4.29%	3.48%	8.22%	2.11%
postwar	9.85%	1.45%	0.86%	5.51%	5.51%
		Conditional Stdev. of Excess Return			
whole	14.75%	9.17%	8.70%	7.74%	8.57%
postwar	8.62%	8.95%	8.50%	7.77%	6.97%
		Conditional Sharpe Ratio			
whole	0.586	0.234	0.200	0.531	0.123
postwar	0.571	0.081	**0.051**	0.355	0.396
			Recessions		
			Buy in $n-th$ quarter after peak		
	1st	2nd	3rd	4th	5th
		Conditional Expected Excess Return			
whole	2.88%	8.43%	11.73%	10.76%	2.46%
postwar	6.22%	10.70%	12.63%	10.57%	3.82%
		Conditional Stdev. of Excess Return			
whole	12.54%	12.61%	12.07%	10.84%	10.79%
postwar	10.47%	11.20%	11.00%	9.78%	9.87%
		Conditional Sharpe Ratio			
whole	0.115	0.334	0.486	0.496	0.114
postwar	0.297	0.478	**0.574**	0.540	0.193

Figure 1: Counter-Cyclical Time Variation in the Sharpe Ratio on Equities

Conditional Sharpe Ratio on Market (VW-CRSP). Computed by buying equity N quarters after NBER peak/trough and holding for one year. N is on horizontal axis. We plot the results that we obtained on the 1925-2009 sample (dotted line) and the 1945-2009 sample (dashed line). The data is quarterly. The full line is a 4-th degree polynomial approximation.

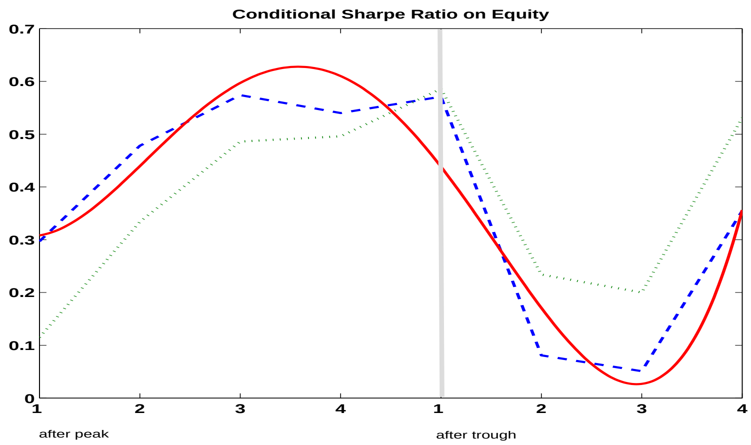

Figure 2: Equity Share of Mertonian Trader: MP Economy with z-Complete Mertonian Traders

The full line shows the equity share for the Mertonian trader (axis on the left hand side). The dashed line is a 4-period moving average of aggregate consumption growth with linearly decreasing weights. This calibration has 50 % non-participants, 5% complete and 45 % either in *crb* or *irb* traders. The target equity share is 33 %. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

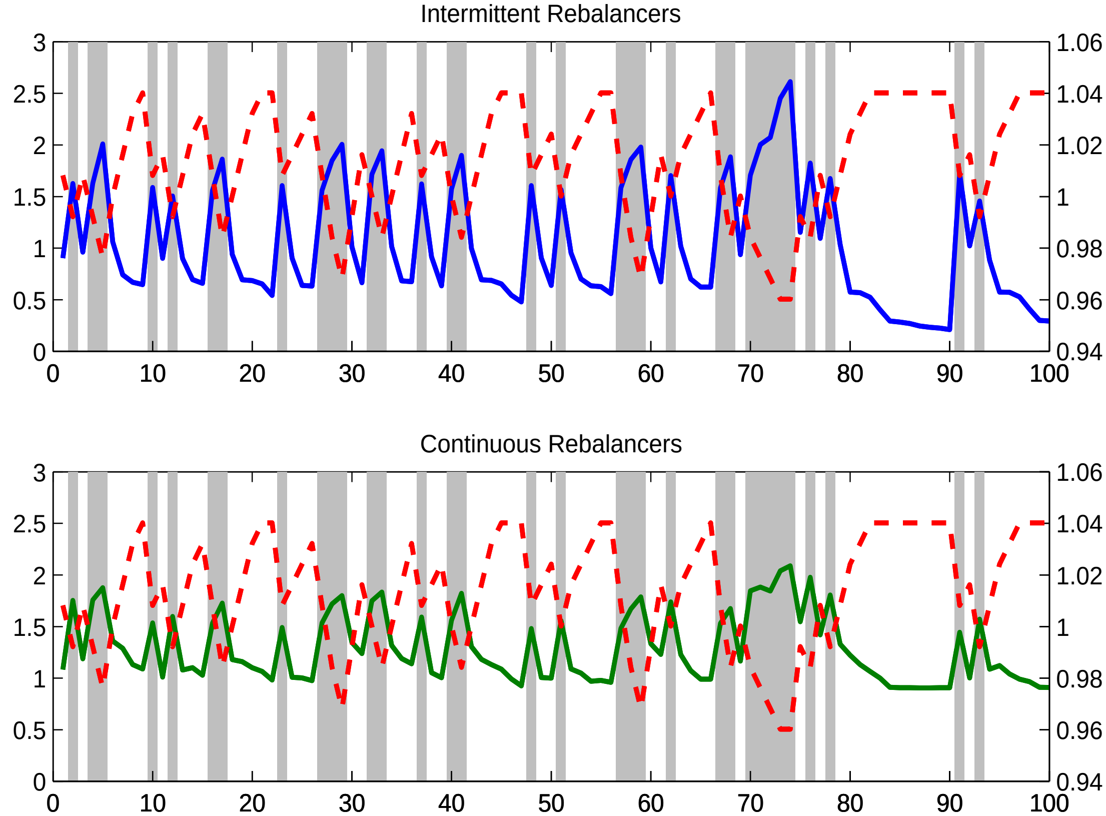

Figure 3: Conditional Sharpe Ratio: MP Economy with z-Complete Mertonian Traders

The full line is the conditional Sharpe ratio (on the left hand side axis). The dashed line is a 4-period moving average of aggregate consumption growth with linearly decreasing weights. This calibration has 50 % non-participants, 5% complete and 45 % either in *crb* or *irb* traders. The target equity share is 33 %. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

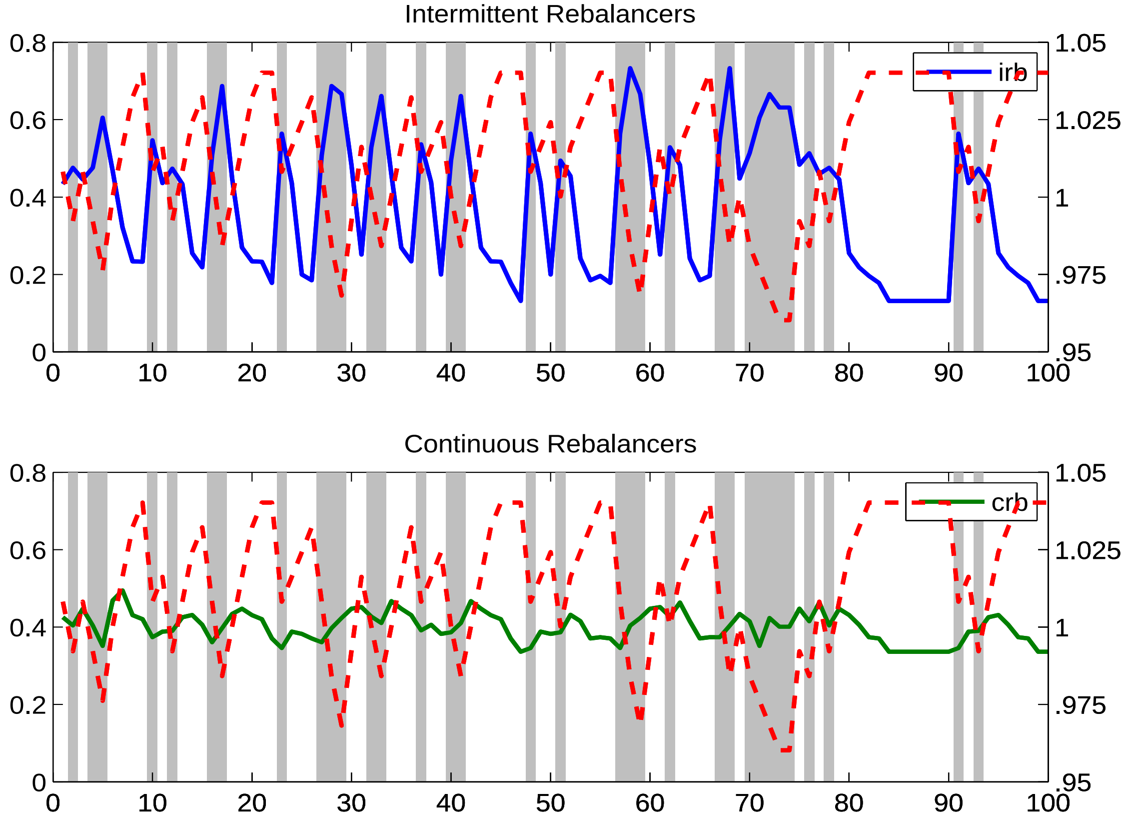

Figure 4: Conditional Sharpe Ratio -Benchmark MP Economy with z-Complete Mertonian Traders and *irb* Non-Mertonian Traders.

Scatter plot of the 100 data points in figure 3. On the x-axis is a 4-period moving average of aggregate consumption growth with linearly decreasing weights. On the y-axis is the conditional expected excess return on equity. This calibration has 50 % non-participants, 5% complete and 45 % *irb* traders. The target equity share is 33 %. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

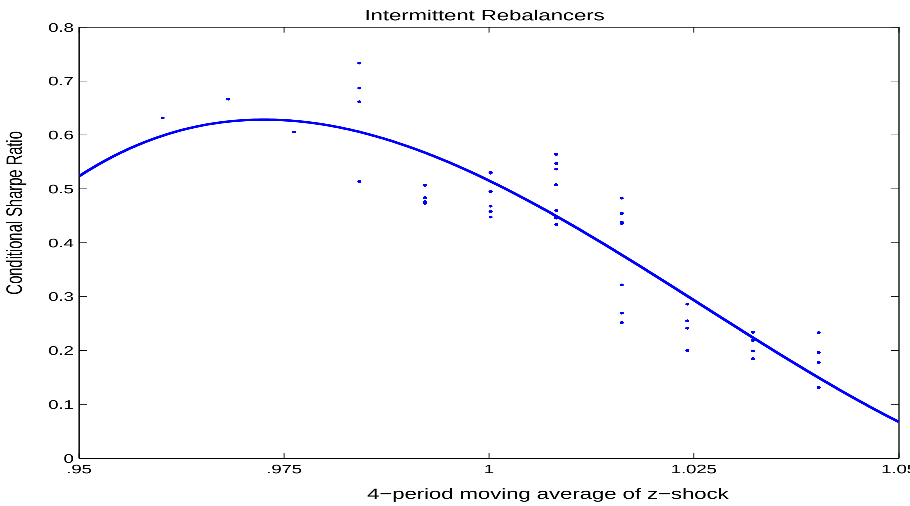

Figure 5: Conditional Sharpe Ratio-Benchmark MP Economy with z-Complete Mertonian Traders and *crb* Non-Mertonian Traders.

Scatter plot of the 100 data points in figure 3. On the x-axis is a 4-period moving average of aggregate consumption growth with linearly decreasing weights. On the y-axis is the conditional expected excess return on equity. This calibration has 50 % non-participants, 5% complete and 45 % *crb* traders. The target equity share is 33 %. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

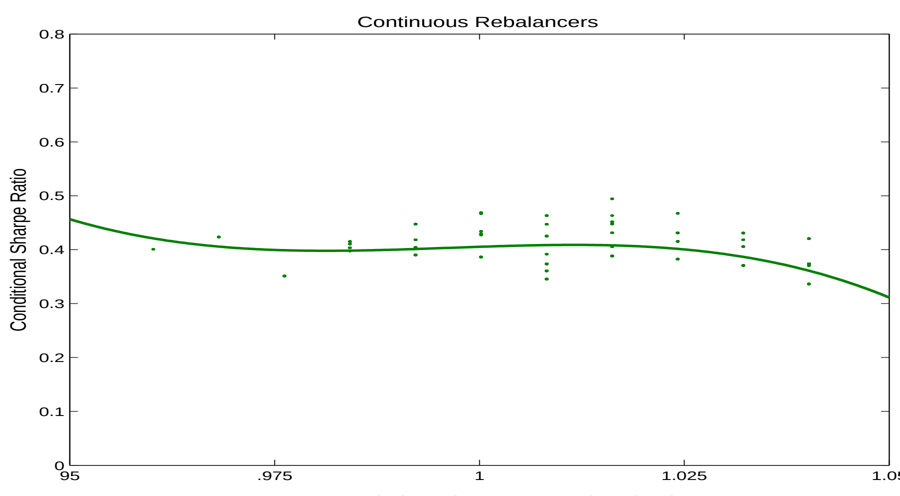

Figure 6: Conditional Sharpe Ratio-Benchmark MP Economy with Complete Mertonian Traders and *irb* Non-Mertonian Traders.

On the x-axis is a 4-period moving average of aggregate consumption growth with linearly decreasing weights. On the y-axis is the conditional expected excess return on equity. This calibration has 50 % non-participants, 5% complete and 45 % *irb* traders. The target equity share is 33 %. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.

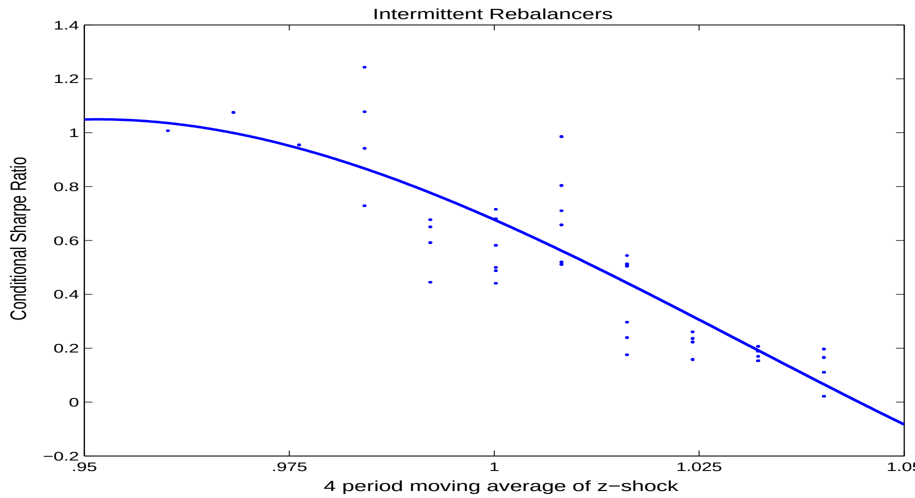

Figure 7: Conditional Sharpe Ratio-Benchmark MP Economy with Complete Mertonian Traders and *crb* Non-Mertonian Traders.

On the x-axis is a 4-period moving average of aggregate consumption growth with linearly decreasing weights. On the y-axis is the conditional expected excess return on equity. This calibration has 50 % non-participants, 5% complete and 45 % *crb* traders. The target equity share is 33 %. The *irb* traders re-balance every three periods in a staggered fashion (1/3 each year). Storesletten, Telmer, and Yaron (2007) of idiosyncratic shocks; Alvarez and Jermann (2001) calibration of aggregate consumption growth shocks. Parameters: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 10%. The results are generated by simulating an economy with $3,000$ agents and $10,000$ periods.