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Douglas W. Diamond
Raghuram G. Rajan

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ABSTRACT

The cheapest way for banks to finance long term illiquid projects is typically to borrow short term from households. But when household needs for funds are high, interest rates will rise sharply, debtors will have to shut down illiquid projects, and in extremis, will face more damaging runs. Authorities may want to push down interest rates to maintain economic activity in the face of such illiquidity, but intervention may not always be feasible, and when feasible, could encourage banks to increase leverage or fund even more illiquid projects up front. This could make all parties worse off. Authorities may want to commit to a specific policy of interest rate intervention to restore appropriate incentives. For instance, to offset incentives for banks to make more illiquid loans, authorities may have to commit to raising rates when low, to counter the distortions created by lowering them when high. We draw implications for interest rate policy to combat illiquidity.

Douglas W. Diamond
Booth School of Business
University of Chicago
5807 S Woodlawn Avenue
Chicago, IL 60637
and NBER
d-diamond@uchicago.edu

Raghuram G. Rajan
Booth School of Business
University of Chicago
5807 South Woodlawn Avenue
Chicago, IL 60637
and NBER
rajan@chicagogsb.edu

There has been substantial recent debate on the role of central banks and interest rates, not so much in controlling inflation, but in dealing with episodes of illiquidity and financial fragility. For instance, Greenspan (2002) has argued that while the Federal Reserve cannot recognize or prevent asset price booms, it can "mitigate the fallout when it occurs and, hopefully, ease the transition to the next expansion." Others have responded that by following such an asymmetric interest rate policy – colloquially known as the Greenspan put -- a central bank can engender the kind of behavior that makes booms and busts more likely. In this paper, we ask whether a central bank can alleviate financial stress by altering the prevailing short term real interest rate. We then ask whether anticipation of such intervention distorts bank incentives, under what circumstances such distortions are most detrimental, and if so, what kinds of policies central banks may want to follow.

We start with a model where entrepreneurs borrow from banks to invest in long- term projects. Banks themselves borrow from risk-averse households, who receive endowments every period. Households deposit their initial endowment in banks in returns for demandable deposit claims (throughout the paper, we focus on demand deposits, though any form of overnight unsecured debt could be a close substitute). There is no uncertainty initially about the average quality of a bank's projects in our model, so the bank's asset side is not the source of the problem. However, there is uncertainty about household endowments (or equivalently, incomes) over time.

Once households have deposited their initial endowment, and projects have been started, households may have an unexpectedly high need to withdraw deposits. One possibility is that they suffer an adverse shock to current endowment that causes them to want to run down financial assets to consume. But another is that they anticipate significantly higher income or endowments in the future and want to smooth consumption. Thus anticipated prosperity, as well as current adversity, can increase current household demand for consumption goods substantially. We focus on the former (which has antecedents in the Austrian School of Von Mises and Hayek – see later), though the results in this paper apply for the most part to the latter also.

As households withdraw deposits to satisfy consumption needs, banks will have to call in loans to long gestation projects in order to generate the resources to pay them. The real interest rate will rise to equate the household demand for consumption goods and the supply of these goods from terminated projects.[2] Thus greater consumption demand will lead to higher real rates and more projects being terminated, as well as lower bank net worth. This last effect is because the bank's loans pay off only in the long run, and thus fall in value as real interest rates rise, while the bank's liabilities, that is demandable deposits, do not fall in value. Eventually, if rates rise enough, the bank may have negative net worth and experience runs, which are destructive of value because all manner of projects, including those viable at prevailing interest rates, are terminated. Anticipated future prosperity, as well as current adversity, can thus induce fragility in the banking system, and slow the real economy.

How can this tendency towards banking sector fragility be mitigated? One obvious possibility is to alter the structure of banks. If deposits were not demandable, a loss of net worth would not result in a destructive run. And if banks financed themselves with long term liabilities that fell in value as real interest rates rose, banks would be doubly stable. Even with demandable deposits, banks would be more stable if they financed with lower levels of deposits, or if deposits could be made state contingent, so they fluctuated appropriately in value with the aggregate consumption demands in each state. So can the structure of banks be altered easily while they continue to intermediate efficiently?

The answer from our previous work is no. Bank debt is demandable because it allows bankers to commit to pay out the value they collect (Diamond and Rajan (2001)). Put differently, given the skills needed to make loans to entrepreneurs and recover payments, financing with demandable debt is the

[2] While we focus on project liquidations (or equivalently, a halt in new projects) as affecting the aggregate supply of goods available for consumption, it is also plausible to think of them as affecting aggregate demand, as people lose jobs. We do not model this, though the qualitative effects would be similar.

cheapest form of financing for banks – financing with long term liabilities would reduce the efficiency of intermediation substantially. Given this, competitive conditions determine how much banks lever up. Competitive banks will be willing to accept some probability of financial distress and runs in order to commit to pay depositors more, and thus attract funds. Thus the commitment value of a hard liability structure, coupled with ex ante competition for funds, can lead to highly levered banks.

If altering bank structure is not easy, what about government intervention? It is not sufficient to just keep the volume of lending up if consumption is not altered. In our model, given the demand for consumption, the only way the market clears is if interest rates rise, bank lending falls (because projects are no longer so attractive at the higher interest rate), projects are terminated, and goods released for consumption. Put differently, the goods market has to clear and any intervention has to recognize this.

The government can always violate property rights and keep the banking system intact – for instance, by taxing households and gifting the proceeds to banks (or equivalently, lending to banks at rates the private market would not lend at). This sort of directed bailout would reduce household consumption while limiting project termination. It is well understood that anticipated government bailouts of particular failed institutions will generate poor ex-ante incentives. Poorly designed or heavily subsidized lender of last resort policy is criticized on similar grounds by Goodfriend and King (1998). While recognizing that such interventions may be necessary in extremis (and is taking place even as we write), we want to focus on more routine undirected central bank interventions – lending or borrowing from the market to alter interest rates – that seemingly do less violence to property rights.

Lending takes resources. Once we recognize the fact that these resources have to ultimately come from the households, we see there are a variety of circumstances in which attempts to alter the real interest rate could be ineffective. For instance, could authorities lower rates by taxing households' current endowments, lending them to banks at market rates, and then repaying the households in the future at the market interest rate so as to not violate their property rights? The problem is that households could respond to any tax on their current endowment by withdrawing an equivalent additional amount from banks so as to keep consumption constant. If so, interest rates would remain unchanged, a form of Ricardian Equivalence (Barro (1974)) that limits how much governments can do.[3]

However, some households may not participate in the banking system, either because they withdraw all their deposits (perhaps to compensate for current taxes) or because interest rates are too low, given their endowments, for them to deposit in the first place. A further tax on these households' current endowments will not be compensated for by equivalent withdrawals from the banking system, since these households already have no claims on the banking system. The consumption of these households will fall. The amount collected from this tax is then an incremental source of resources to the banking system, and by taxing and lending, the government can bring down the real interest rate, reducing project liquidation and thus bolstering the net worth of banks.

Interestingly, the reliance of banks on short-term funding allows current and anticipated future policy interventions on short-term interest rates to influence long-term investment and consumption choices. Such intervention, even if seemingly respectful of household property rights, has real effects only by forcibly changing household consumption opportunities and thus influencing real liquidation and investment decisions. Of course, if it prevents runs, this change could be Pareto improving. But if the objective is not to head off runs, some households could be made worse off, even if they are "fully" compensated for taxes at market interest rates.

An equally important concern is the effect of anticipation of such intervention on ex ante actions. A low ex post interest rate offers a very low reward for maintaining liquidity. If the authorities are expected to reduce interest rates when liquidity is at a premium, banks will take on more short-term

[3] It is well understood that government borrowing to finance lump sum tax and transfer programs have effect only when this Ricardian Equivalence does not hold. Government lending at market rates of interest (liquidity lending), using its taxation authority to raise resources, with the interest augmented principal transferred back to households in a lump sum manner is principally the same transaction in reverse, and should have no effect for similar reasons.

leverage or illiquid loans, thus bringing about the very states where intervention is needed. We show that the worst outcomes emerge when banks compete for funds thus focusing on household utility, while the central bank is particularly solicitous of the health of firms and banks.

Indeed, the current financial turmoil in the United States could be thought of as being partially caused by lenders, anticipating a continued environment of low interest rates following the implosion of the "tech bubble" in early 2000 and the subsequent collapse of corporate investment, choosing to take on more illiquid financial assets financed with short term debt. Anticipation of low interest rates may have been strengthened by the so-called Greenspan Put, whereby the financial sector believed that if it ever came under strain because of excessive expansion, the Federal Reserve would cut interest rates. Put differently, if policy drives short-term interest rates to a low level, and solvency and financial stability constraints on future policy rule out future high short-term rates, then financing illiquid assets with short-term debt will be profitable and safe, at least for a while.

Note that distortions are created in our framework when the private sector fully anticipates future short term interest rates will be held low. This differs from more well known "risk shifting" or "search for yield" explanations of the effects of low current long term interest rates; According to this, financial firms overestimate future interest rates, and consequently take on high interest rate long term liabilities. When the realized long term interest rate turns out to be unexpectedly low, they are "forced" to take on additional investment risk to provide returns and stave off bankruptcy.

In sum, interest rate interventions have the benefit of being untargeted, but are not benign. We discuss the kind of interest rate policy central banks may want to follow if they cannot commit not to bail out banks. Often, this requires raising interest rates in good times above the low market determined level, implying that stability-focused central banks may follow a smoother path for interest rates than suggested by underlying economic conditions. Finally, in this paper, we abstract from distortions in lending resulting from banker over-optimism or incentive problems which, while no doubt important, have been explored in other work. The rest of the paper is as follows. In section I, we lay out the basic model, in section II, we solve it, in section III we consider macroeconomic intervention on interest rates and its effect on ex ante choice of deposit level and asset liquidity. We then conclude.

I. The Framework

1.1. Agents, Endowments, Technology, Preferences.

Consider an economy with risk-neutral entrepreneurs and bankers, and risk-averse households. The economy has three dates, 0, 1, and 2. Each household is initially endowed with a unit of good at date 0. Households can invest their initial endowment in banks, which will lend the resources to entrepreneurs. At date 1, households will get endowment e_1. They also learn their date-2 endowment. Some households will get a high endowment, e_2^H at date 2 while the rest will get a low endowment e_2^L. Let there be two states of the world at date 1, the *exuberant* state where θ^E is the fraction of high endowment households and the *normal* state, where θ^N is the fraction, with $\theta^E > \theta^N$. Thus E is a state with greater anticipated prosperity, where more households expect a high endowment. The probability of state E is p^E. The state of nature and household types are not verifiable and in addition, household types are private information.

Each entrepreneur has a project, which requires the investment of a unit of good at date 0. The project produces $\tilde{Y}_2$ in goods at date 2 if it is not liquidated, and X_1 at (or after) date 1 if the project is liquidated. At date 0, $\tilde{Y}_2$ is uncertain for each entrepreneur, with outcomes becoming known at date 1 and distributed uniformly over the range $\left[\underline{Y}_2, \overline{Y}_2\right]$. Entrepreneurs have no goods to begin with, and the demand from entrepreneurs looking for funds at date 0 is greater than the supply with households.

Households maximize expected utility of consumption given by $E(\log C_1 + \log C_2)$. Risk neutral entrepreneurs and bankers maximize $E(C_1 + C_2)$.

1.2. Financing Entrepreneurs

Anyone will lend only to the extent that they can coerce borrowers into repaying, either by independently being able to generate value by seizing the borrower's assets, and thus being able to obtain some "collateral" protection in case of default, or by finding ways to commit to inflict serious damage to the borrower. Specifically, since entrepreneurs have no endowments, they need to borrow to invest. Each entrepreneur can borrow from a banker who has, or can acquire during the course of lending, knowledge about an alternative, but less effective, way to run the project. The banker's specific knowledge allows him to collect γY_2 from an entrepreneur whose project just matures, with $\gamma < 1$. Once a banker has lent, no one else (including other bankers) has the knowledge to collect from the entrepreneur.[4]

Because there are more entrepreneurs than households, not all projects are funded. Banks can ask entrepreneurs to repay the maximum possible for a loan – they will lend only if the entrepreneur promises to pay $\overline{Y}_2$ on demand. If the entrepreneur fails to pay on demand, he can make a counter-offer to the bank. If that offer is rejected, the bank takes over the project and either harvests date-2 cash flow γY_2 or liquidates it. [5] Banks can store any unused assets at a gross interest rate of 1.

1.3. Financing Banks

Since bankers have no resources initially, they have to raise them from households. But households have no collection skills (and bank loans are worthless in their hands), so how do banks commit to repaying households? By issuing demand deposits! In our previous work (Diamond and Rajan (2001)), we argued that the demandable nature of deposit contracts introduces a collective action problem for depositors that makes them run to demand repayment whenever they anticipate the banker cannot, or

[4] For a relaxation of these assumptions, see Diamond and Rajan (2001).

[5] While we assume here that households lend via the bank, we show in Diamond and Rajan (2001) that banks and their fragile liability structures arise endogenously to facilitate the flow of credit from investors with uncertain consumption needs to entrepreneurs who have hard-to-pledge cash flows.

will not, pay the promised amount. Because bankers will lose all rents when their bank is run, they will repay the promised amount on deposits whenever they can.

Deposit financing introduces rigidity into the bank's required repayments. Ex ante, this enables the banker to commit to repay if he can (that is, avoid strategic defaults by passing through whatever he collects to depositors), but it exposes the bank to destructive runs if he truly cannot pay (it makes non-strategic default more costly): when depositors demand repayment before projects have matured and the bank does not have the means of payment, it will be forced to liquidate projects to get X_1 immediately instead of allowing them to mature and generate γY_2. In addition to making banks fragile, short-term funding leaves households exposed to interest rate risk when reinvesting from date 1 to date 2. This is unavoidable because a bank cannot commit to pay a return above the market return from date 1 to date 2 (due to the bank's ability to renegotiate).

Banks are competitive, and we assume that if it offers a competitive rate, each bank attracts enough entrepreneurs so that the distribution of $\tilde{Y}_2$ among entrepreneurs it finances matches the aggregate distribution of entrepreneurs. Because the date-0 endowment is scarce relative to projects, banks will compete to offer the most attractive deposit face value D to households per unit of endowment deposited (henceforth, all values will be per unit of endowment). Since this is the face value repaid on a unit of good deposited, it is also the date-0 gross deposit interest rate.

Assumption 1: $\dfrac{e_2^H}{e_1} > \dfrac{\gamma \overline{Y}}{X_1}$

Assumption 1 implies that at the highest interest rate payable by firms, the H household wants to withdraw at least some of its deposits. Through much of the paper, we will focus on the situation where the date-1 interest rate exceeds 1 so that storage is not in use, but storage is easily handled.

Timeline

Date 0	Date 1	Date 2
Banks offer deposit terms and entrepreneurs offer loan repayment terms. Households get endowment and invest in banks. Banks lend to entrepreneurs.	Uncertainty over states, household date-2 endowments, and project outcomes revealed. Households decide how much to withdraw and how much to consume. Banks decide what projects to liquidate.	Projects mature, loans repaid, and deposits fully withdrawn from banks. All agents consume.

II. Solving the basic model

In what follows, we will start by solving the bank's decision vis a vis entrepreneurs at date 1, then the households' consumption and withdrawal decisions, and finally, the bank's date 0 decision on what level of deposit repayment D to offer to maximize household willingness to deposit.

2.1. Bank decisions vis a vis entrepreneurs

Let us start our analysis at date 1, once uncertainty is revealed. Let the interest rate households demand in state *s* for re-depositing between dates 1 and 2 be r_{12}^S. The bank can get X_1 at date 1 if the project is liquidated. The maximum it can collect from the entrepreneur is γY_2 at date 2, which is the maximum the entrepreneur can commit to pay at date 1. So the bank liquidates projects with

$$Y_2 < Y_2(r_{12}^S) = \frac{r_{12}^S X_1}{\gamma} \qquad (1.1)$$

and continues projects with $Y_2 \geq Y_2(r_{12}^S)$ in return for a promised payment of γY_2. Liquidated entrepreneurs get nothing, while entrepreneurs who are continued retain $(1-\gamma)Y_2$. Note that the present value of the bank's assets at date 1 (before withdrawals) is $\frac{1}{\overline{Y}_2 - \underline{Y}_2}\int_{\underline{Y}_2}^{Y_2(r_{12}^S)} X_1 dY_2 + \frac{1}{\overline{Y}_2 - \underline{Y}_2}\int_{Y_2(r_{12}^S)}^{\overline{Y}_2} \frac{\gamma Y_2}{r_{12}^S} dY_2$,

which is easily shown to fall in r_{12}^S.

2.2. Household decisions

Once uncertainty is revealed, and households know both the state and their endowments, they decide how much they want to withdraw and consume so as to maximize their expected utility of consumption. Of course, if they anticipate the bank will not be able to meet its obligations, they will collectively run on the bank, in which case all projects will be liquidated to pay households. We assume households can coordinate on a Pareto preferred Nash equilibrium. Thus we rule out panic based runs, which would only add to the inefficiencies we document. We start by considering situations where the bank will meet its obligations.

When a household does not withdraw all its deposit, its utility is maximized when the marginal rate of substitution between consumption at dates 1 and 2 is equal to the prevailing deposit rate, r_{12}^S, that is, when $\frac{U'(C_1)}{U'(C_2)} = \frac{C_2}{C_1} = r_{12}^S$.[6]

If household H (with high date-2 endowment) withdraws amount $w_1^{H,S}$ at date 1 (where $w_1^{H,S} \le D$), then $\frac{C_2^{\,H,S}}{C_1^{\,H,S}} = \frac{e_2^H + (D - w_1^{H,S})r_{12}^S}{e_1 + w_1^{H,S}}$. A similar expression can be derived for household L with low date-2 endowment. For both households to have an equal marginal rate of substitution (and deposit at a common rate r_{12}^S), it must be that H type households withdraw more so that $w_1^{H,S} > w_1^{L,S}$.

Lemma 1: *If* $r_{12}^S \ge \frac{e_2^H}{e_1 - D}$*, both households leave all their money in the bank at date 1. If* $\frac{e_2^H}{e_1 - D} > r_{12}^S > \frac{e_2^H}{e_1 + D}$*, neither household withdraws fully from the banking system, but the H household*

[6] The obvious intuition is that if the interest is lower, it could increase expected utility by withdrawing more at date 1 and consuming more, while if the interest rate is higher, it could increase expected utility by withdrawing less at date 1 and consuming more at date 2.

withdraws more than the L household. If $\frac{e_2^H}{e_1+D} \geq r_{12}^S > \frac{e_2^L}{e_1+D}$*, the H household withdraws fully, while the L household maintains some deposits. If* $\frac{e_2^L}{e_1+D} \geq r_{12}^S$*, both households withdraw fully.*

2.3. Equilibrium

Assume first the bank can repay depositors without failing. In equilibrium, markets for goods at date 1 and date 2 have to clear. At date 1, goods are produced when banks liquidate projects. Because all banks have the same distribution of projects and will liquidate projects with $Y_2 < Y_2(r_{12}^S) = \frac{r_{12}^S X_1}{\gamma}$, date-1 liquidation proceeds are $\frac{1}{\overline{Y}_2 - \underline{Y}_2} \int_{\underline{Y}_2}^{Y_2(r_{12}^S)} X_1 dY_2 = \left[\frac{\frac{r_{12}^S X_1}{\gamma} - \underline{Y}_2}{\overline{Y}_2 - \underline{Y}_2} \right] X_1$. Because this should equal the goods consumed by withdrawing households, it must be that (on simplifying)

$$\theta^S w_1^{H,S} + (1-\theta^S) w_1^{L,S} = \frac{r_{12}^S X_1{}^2 - \gamma X_1 \underline{Y}_2}{\gamma\left(\overline{Y}_2 - \underline{Y}_2\right)} \tag{1.2}$$

where θ^S is the fraction of H type households in state S. An equilibrium at date 1 in state S is an interest rate r_{12}^S and withdrawals by the H and L households, $w_1^{H,S}, w_1^{L,S}$ such that the date 1 supply of goods equals the date 1 consumption, banks liquidate enough projects to meet withdrawals, and households do not want to, or cannot, withdraw more.

Theorem 1:

(i) *If it exists, the equilibrium is unique.*

(ii) *If there is a set* $\{r_{12}^S, w_1^{H,S}, w_1^{L,S}\}$ *that solves* $\frac{e_2^H}{e_1 + 2w_1^{H,S} - D} = r_{12}^S$, $\frac{e_2^L}{e_1 + 2w_1^{L,S} - D} = r_{12}^S$, *and (1.2), with* $r_{12}^S > 0$, $w_1^{H,S} \in [0, D)$, $w_1^{L,S} < D$*, then that is the equilibrium, else* *if there are* r_{12}^S, $w_1^{H,S} = D$ *and* $w_1^{L,S}$ *that solve* $\dfrac{e_2^L}{e_1 + 2w_1^{L,S} - D} = r_{12}^S$, *and (1.2), with* $r_{12}^S > 0$

, and $w_1^{L,S} \in [0, D)$ *, then* $\{r_{12}^S, D, w_1^{L,S}\}$ *is the equilibrium, else*

$\{\left[\dfrac{\gamma D(\overline{Y}_2 - \underline{Y}_2) + \gamma \underline{Y}_2 X_1}{\left(X_1\right)^2}\right], D, D\}$ *is the equilibrium.*[7]

Proof: See Appendix.

Corollary 1:

(i) *H households with a higher date-2 endowment always withdraw (weakly) more than L households.*

(ii) *The interest rate* r_{12}^S*, total withdrawals,* $\theta^S w_1^{H,S} + (1-\theta^S) w_1^{L,S}$*, and the fraction of projects liquidated all (weakly) increase in the fraction* θ^S *of high endowment households and in the face value of deposits, D.*

(iii) *The net worth of banks (date 1 value of assets less deposits) decreases in the fraction* θ^S *of high endowment households and in the face value of deposits, D.*

Proof: See appendix.

Since H households have more date-2 endowment than L households, at any given interest rate they will consume (weakly) more at date 1, and hence will withdraw more. This means total withdrawals will (weakly) increase in the fraction θ^S of high endowment households, which means the interest rate and liquidation will have to increase to equilibrate the market. Given the present value of the bank's assets decrease with r_{12}^S, while the value of its deposit liabilities do not, its net worth decreases with θ^S. Turning to the effect of D, a higher face value of deposit claims increases the wealth of the depositing

[7] Note that we have assumed the household marginal rates of substitution are not so low that interest rates would fall below 1 in the absence of the bank's storage technology. If that were not the case, we would have to examine an interest rate of 1 as a candidate, with any excess deposits being stored by the bank. The three equations would now solve for the withdrawal by high households, the withdrawal by low households, and the amount invested by banks in the storage technology. This is straightforward.

household (provided the bank is not run), and increases its desire to consume immediately by withdrawing. The other implications follow.

Note that all these implications would also hold if households differed, not in their date-2 endowments, but in their date-1 endowments, with H households receiving a *lower* date 1 endowment (poor current conditions) and, again, higher growth in marginal utility. Much of past analysis has followed Diamond-Dybvig (1983) and focused on liquidity shocks that are equivalent to poor current conditions, but it is useful to remember that exuberant views of the future could be equally problematic from the perspective of demands for liquidity. We will focus on this latter aspect through the paper, but because interest rates depend on anticipated consumption growth, pressures can stem both from anticipated future prosperity or current adversity.

2.4. Bank Fragility and the Ex Ante Choice of Deposit Rate

Let us now examine each bank's choice at date 0 of the face value D to offer for a deposit of a unit of good. Since the market is competitive and household endowment is scarce relative to entrepreneurs' projects, price-taking banks will have to offer a D that maximizes household utility, given the properly anticipated future interest rates and actions of households, banks and entrepreneurs.

Clearly, so long as the bank is not run in any state, a higher D makes households wealthier and better off. But the bank's net worth also falls by Corollary 1, and for a given D, is lower in state E because the fraction of H households is higher there than in state N (that is, $\theta^E > \theta^N$). When D is high enough that the bank's net worth is completely eroded in a state, the bank is run – which means all projects are liquidated to generate funds to pay depositors, regardless of whether liquidation is value maximizing at the prevailing interest rate.[8] Each running household gets X_1 immediately after a run.[9]

[8] This captures the idea that depositors simply want their money back before each bank runs out of resources. The results depend only on bank runs being inefficient, which could also stem from a fraction of projects being liquidated at fire-sale values, that is, at less than X_1.

[9] We assume that each household's deposits are evenly spread across banks and it joins enough lines to get its share of the proceeds with certainty. A run would be more problematic if it contributed to uncertainty about who gets

Let $D^{E,\max}$ be the deposit face value beyond which a run will be precipitated in the E state, while $D^{N,\max}$ be the corresponding face value in the N state. From our discussion above, $D^{E,\max} < D^{N,\max}$, so $D^{E,\max}$ is the highest safe level of deposits, where no bank runs are experienced in either state, while with deposits set at $D^{N,\max}$, runs are experienced in the E state. If deposit repayments could be state contingent, then at date 0, the bank would offer to pay $D^{E,\max}$ in state E and $D^{N,\max}$ in state N. But state-contingent deposit contracts may not be possible (as we discuss later).

What would be the D that competitive banks would set if they could not offer state contingent deposits? It is obvious that the only two candidates for D are $D^{E,\max}$ and $D^{N,\max}$.[10] $D^{E,\max}$ would mean the bank would not pay out the maximum it could in the N state. In contrast, setting the deposit face value $D^{N,\max}$ would ensure the bank would be run in the E state, which would reduce its value significantly below what would obtain if the deposit face value were $D^{E,\max}$ (since all projects are liquidated in a run even though at the prevailing interest rate some deserve to be continued – the desire to withdraw money independent of consumption needs is what distinguishes a run from a normal withdrawal). This then is the basic trade-off banks face is setting deposits at $D^{N,\max}$ rather than $D^{E,\max}$ – commit to paying more in the N state but have a run in the E state. The trade-off turns on the cost of a run relative to its probability.

Lemma 2: Ceteris paribus, the lower the probability p^E of the exuberant state, the lower the expected cost of bank runs relative to the benefit of paying out more in state N, and the more attractive is the higher face value $D^{N,\max}$ than $D^{E,\max}$ to households.

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what, with some households lucky enough to be at the front of the line getting more than households at the back. This would add to the cost of the run and complicate the algebra, but would not change the results qualitatively.

[10] We know a higher D, provided it does not precipitate a run, makes depositors wealthier at date-1, and thus is preferred. This means one candidate for the equilibrium D offered is the maximum no-run D, $D^{E,\max}$. Once a run is precipitated in the E state, a higher value of D makes no difference to outcomes or payouts in that state. Which means a second candidate equilibrium D is $D^{N,\max}$, which maximizes payout in the N state. Of course, going beyond $D^{N,\max}$ will reduce value because the bank will be run in all states.

Intuitively, since $D^{N,\max}$ and $D^{E,\max}$ maximize payouts for their specific states, the lower the probability of state E, the more attractive does $D^{N,\max}$ become as the ex ante choice.

2.5. Example With Comparative Statics

Let p^E =0.5, $e_1 = 1.2$, $e_2^L = 0.6$, e_2^H =3.6, X_1=0.95, $\underline{Y}_2$=0.5 , $\overline{Y}_2$=2.5, γ=0.9, $\theta^E = 0.6$, θ^N =0.3. First, assume that the deposit face value is set at $D^{E,Max}$, the maximum deposit face value with no run in either state, which is 1.016. In the E state, the interest rate r_{12}^E is 1.70, H households withdraw 0.97, while L households withdraw 0.08. In the N state, the interest rate r_{12}^N is 1.28, the H households withdraw their entire deposit, while L households withdraw 0.14. Now let the deposit face value be set at $D^{N,\max}$ =1.12. Now the bank is run in the E state, while in the N state, the interest rate is 1.39, the H households withdraws all their deposit while the L households withdraw only 0.18.

In Figure 1, we plot the optimal ex ante choice of D for different ex ante probabilities (blue line). The bank will set D high at 1.12 until the probability of the exuberant state exceeds 0.24, at which point households are better off having safe banks with D set at 1.016. Note from Figure 2 that the utility of the banks and entrepreneurs shifts up substantially when the bank moves to setting a lower safe face value of deposits – more rents are preserved for the banker/entrepreneur when deposit claims are lower.

2.6. Discussion

The point, thus far, is straightforward. In a competitive environment, the banking system can lever up to the point where it will fail with some probability when a significant fraction of households become exuberant about the future (or pessimistic about the present). Exuberance creates more pressure for current consumption, which the economy may be too illiquid to provide. Consequently, real interest rates rise to restore equilibrium, and projects are curbed. The more levered the banking system is to begin with, the more projects are liquidated, and higher the likelihood of systemic bank failures.

Ex post, in the exuberant state, the banking system looks over-levered. But ex ante, the extent of short-term leverage could be a competitive outcome. If banks could write state contingent deposit contracts, they would not have to resort to face values of deposits that risked runs. The problem really is that the state is hard to observe or verify in real time, and its correlation with the value of illiquid bank assets hard to determine precisely.[11] Moreover, making demandable deposits contingent on the state could lead to more instability, as depositors attempt to guess the state and attempt to front run any diminution of the value of their deposits.[12] Hellwig (1994) shows how to optimally set the value of state-contingent deposits to avoid runs when full state-contingency is possible. We assume that such state-contingency is not possible and examine the consequences.

Of course, if banks could finance substantially with long term renegotiable debt or equity, or write complete state-contingent contracts, they would be able to withstand significant variations in household consumption needs without failing. However, long term debt or equity would not be a competitive mode of finance for banks, given how central short term demandable debt is to reducing banker rents (see Diamond and Rajan (2001)). The rigidity of bank financing may therefore be an essential feature of the environment.

We are certainly not the first to place the emphasis for contraction and crises on the mismatch between the long duration before investment produces consumption goods, and the temporal pattern of consumption in an expansion. This dates back at least to Von Mises (1949) and the Austrian School. Von Mises placed the emphasis, though, on an artificially low initial rate of interest, induced by bank credit expansion, which makes the process of creating new goods excessively long compared with the tolerance of consumers to postpone consumption. While it is difficult to map his theory precisely to a rational expectations general equilibrium model, it would appear that Von Mises (1949) places the blame for

[11] Banks are thus unlike mutual funds holding traded assets, where there is a precise market value of assets.

[12] The problems caused by the need for short-term finance are not limited only to banks, although we do not separately analyze this here. When bank monitoring is not needed but the legal enforcement of debt contracts is costly, borrowers must issue short-term debt to fully exploit their debt capacity (Diamond (2004, 2008)), and leave their future fate subject to refinancing risk. Consequently, results similar to ours hold in an economy where debt is short-term, regardless of whether it is bank debt or not. Corporate debt maturity cannot be adjusted to facilitate stability without reducing access to capital.

crises squarely on the heads of overly optimistic, excessively aggressive, bankers (and on central bankers who encourage aggressive credit expansion). We will examine banker reactions to likely central bank interest rate interventions shortly. Thus far, though, our emphasis has been on the frictions stemming from the lack of perfect foresight about changes in the consumption patterns of depositors, and the intrinsic illiquidity of banking.

III. Macroeconomic Intervention

3.1. The Purpose of Intervention

As we have just argued, the inability of banks to offer state-contingent deposit contracts means households have to either accept too little value in one state or a destructive run in another. Moreover, the inability of banks to distinguish between H type households and L type households, and the inability to offer either type an above market return in the future, makes it difficult for banks to achieve better risk sharing between the types. One goal of intervention, regardless of whose interests the government serves, may simply be to avoid Pareto inefficient outcomes such as runs. A more controversial goal is to ensure that, not only are runs prevented, but also some agents benefit at the expense of others – for instance , when interest rates paid to households are repressed in order to favor banks and firms. Let us now examine intervention and the reactions it engenders in banks.

3.2. Limitations on Intervention

The interventions that we consider are tax financed lending and borrowing to alter the level of interest rates. Clearly, if the government could identify states, types, and implement any kind of tax and transfer scheme, it could achieve first-best outcomes. Such an omniscient, omnipotent, and error-free government authority is implausible. Instead, we will assume

(i) The government can identify the date-1 state of the world and modulate its actions accordingly.

(ii) The government cannot distinguish between households based on their future endowments.

Also, if the government can alter property rights, there is a lot they can do to enhance the welfare of a favored group – for instance, to rescue banks they can simply write down deposits, or tax households and transfer to banks. In extremis, this is what the authorities will do. But in more normal times, if the authorities have to return what they take from an agent, and they want to attempt an undirected "liquidity" infusion simply through borrowing and lending at market rates (albeit forced), the scope for intervention is more limited. This is what we will focus on.

(iii) The government can tax current household endowments and lend the proceeds to the banks (by depositing in them), or borrow from the banks (issue government bonds) and transfer the proceeds to households, but it cannot otherwise alter property rights. So what it taxes (or transfers) today has to be transferred (or taxed) back in the future as a lump sum augmented at the market interest rate. Similarly, what it borrows or lends has to be repaid at the market interest rate. It cannot directly write-down the face value of deposits.

(iv) The government cannot store goods between dates. Its net intake of goods at any date has to be zero.

The interventions we have described are typically thought of as being undertaken by different organs of the government – e.g., the central bank and the Treasury. In reality, they may be connected. For instance, we will describe an intervention which involves taxing and lending. In the real world, expansionary monetary policy by the central bank could be thought of as a combination of a seigniorage tax and lending to banks (offset eventually by contractionary monetary policy). At any rate, in what follows, we will refer to a single government entity, the central bank, performing both fiscal and monetary interventions.

3.3 When Interventions Do Not Work

One candidate intervention intended to lower interest rates could be to tax households at date 1 and lend the proceeds to the bank, transferring the repayment on the loan back to households at date 2. It turns out that a small intervention, when neither household withdraws entirely from the bank at date 1, will have no effect on interest rates or bank solvency. To see why, let Δt be the small tax on all households, which is lent to the bank at the post intervention rate $r_{12}^{S,i}$. The bank repayment at date 2, $\Delta t \; r_{12}^{S,i}$, is transferred back to households then.

Post-intervention, it must be that the marginal rates of substitution for household H equals the common interest rate, so $\frac{e_2^{H,S} + \Delta t \; r_{12}^{S,i} + (D - w_1^{H,S,i}) r_{12}^{S,i}}{e_1 - \Delta t + w_1^{H,S,i}} = r_{12}^{S,i}$ where $w_1^{H,S,i}$ is the post-intervention withdrawal. Simplifying, we get $\frac{e_2^H}{e_1 + 2(w_1^{H,S,i} - \Delta t) - D} = r_{12}^{S,i}$. We get a similar expression for household L. Finally, since the bank gets a loan of Δt , its date-1 resource constraint now is $\theta^S w_1^{H,S,i} + (1-\theta^S) w_1^{L,S,i} = \frac{r_{12}^{S,i} \left(X_1\right)^2 - \gamma X_1 \underline{Y}_2}{\gamma\left(\overline{Y}_2 - \underline{Y}_2\right)} + \Delta t$. Comparing to the equations in Lemma 2 (i), we see that if $\{r_{12}^S, w_1^{H,S}, w_1^{L,S}\}$ is the equilibrium before intervention, the equilibrium post-intervention is $\{r_{12}^{S,i} = r_{12}^S, w_1^{H,S,i} = w_1^{H,S} + \Delta t, w_1^{L,S,i} = w_1^{L,S} + \Delta t\}$, that is, the equilibrium interest rate and household consumption patterns remain unchanged.

Intuitively, households have no reason to change their consumption if they can withdraw an additional amount equivalent to the amount they are taxed today. With unchanged consumption, the interest rate does not change, nor does the amount the bank has to liquidate since the additional loan it receives from the government is completely exhausted by additional withdrawals. In short, because households have access to the capital market, government intervention has no effect on the household budget set, consumption, or the interest rate -- household choices perfectly offset government actions. Furthermore, because the interest rate does not change, the liquidity infusion has no effect on the net worth of the banking sector. To restate this, because households can undo government taxation and lending through their deposit withdrawals, a form of Ricardian equivalence (see Barro (1974)) holds.

3.4. Interventions that work.

There is, however, a way of breaking out of this zone of neutrality. Let us now consider a larger tax (and accompanying loan to banks) so that H households withdraw their deposits fully without being able to compensate for the tax, that is $\Delta t > D - w_1^{H,S}$. This means H households' date 1 consumption will fall by $\Delta t - (D - w_1^{H,S})$ relative to the no-intervention case, and their marginal rate of substitution will go up. Prima facie, this seems to go in the wrong direction. But these households no longer participate in the banking system, cannot borrow against their future endowment, and do not determine interest rates. The L households do participate, and their date-1 consumption will have to go up (to absorb some of the consumption given up by the H households). In the new equilibrium, their marginal rate of substitution falls, the interest rate falls, and L households do not make up entirely for the fall in consumption of H households. Overall date-1 consumption falls, and the required liquidation to meet consumption needs falls, and bank net worth rises.

Lemma 3: (i) If the prevailing no-intervention equilibrium has $w_1^{H,S} < D$, then government lending to the market financed by a tax of $\Delta t \le D - w_1^{H,S}$ has no effect on the interest rate, on consumption, or bank net worth. (ii) Government lending financed by a tax $\Delta t > D - w_1^{H,S}$ will reduce the interest rate the bank faces, increasing bank net worth and reducing project liquidation.

A very small intervention may have no effect, while a larger intervention may influence rates in the desired direction. However, there are limits to how much effect the government can have. First, it cannot tax more than the endowment, e_1 . Second, once it pushes gross interest rates down to 1, it cannot push them down any further since the banks will simply store any funds they obtain, and not pass on lower rates to borrowers.

What if the authorities wanted to raise rates? Here again, the authorities would have an effect only if at least the H type household withdrew its deposits entirely from the banking system in the absence of intervention. If the interest rate is so low to begin with that this is the case, authorities can reverse the above process to raise rates. They would borrow from the open market – that is from banks – by issuing bonds at date 1, and give the collected money back as a date-1 subsidy Δs to households. Date-2 taxes would be increased by $\Delta s\ r_{12}^{S,i}$.

Here is why this would raise the market interest rate. The marginal rate of substitution of the H household, prior to intervention, is higher than the pre-intervention market interest rate (which is why H households withdraw all deposits). H households would consume a small subsidy entirely, increasing their consumption. By contrast, L households would not change consumption if the interest rate did not change. But this would mean overall consumption would be higher than pre-intervention, which would require a higher market interest rate to clear the date-1 goods market and draw forth the necessary amount of liquidation. So it must be that in the new post-intervention equilibrium, the interest rate is higher, type L households consume less at date 1, while type H households consume more.

Lemma 4: If the prevailing no-intervention equilibrium has $w_1^{H,S} = D$ *, then government borrowing from the market and paying the collected resources as a subsidy* Δs *to households (with date-2 repayment of government debt financed by date-2 taxes on households) will increase the interest rate the bank faces and depositors get, decreasing bank net worth and increasing project liquidation*

3.5. Discussion

Government intervention has effect because one set of participants no longer uses the capital market at market interest rates. For example, when we tax type H households after they have withdrawn fully from the banking system, they would like to withdraw more to offset the tax at prevailing interest rates, but they cannot. Compensation at market interest rates at date-2 does not fully make up their loss (because their marginal rate of substitution is higher). More generally, a number of households do not participate in the financial system. Interventions "work" by effectively taxing them more heavily, and offering the proceeds to participants in the financial system, whose preferences set interest rates.

Note that the interventions that we are referring to could well be thought of as monetary policy interventions (with either seigniorage or a fiscal component attached), that are not targeted at specific banks, and are meant to bring down the real interest rate. To have effect, they must "penalize" one set of households – those who do not participate as strongly in financial markets -- in order to benefit the system. This differs from direct lender-of-last-resort lending, which is targeted, and undirected lending against illiquid collateral, which is not. In the former, the central bank lends to banks the market would not lend to, while in the latter, the central bank lends against collateral the market will not touch. While the market's aversion to lending may be due to a coordination problem or other friction, viewed at prevailing market prices the central bank's actions in both cases have an element of subsidy. By contrast, there is no explicit subsidized lending in our framework – the central bank picks a rate it will lend at and taxes enough to make sure that it has the resources to meet the market demand at that rate.

Finally, the government plays a similar role in our model as in Holmstrom and Tirole (1998) by making available for contracting household endowments that are inaccessible to the banking sector. However, the focus of the two models is very different, with the government providing new sources of collateral in their model, while it alters household consumption decisions in ours.

3.6. Objectives of Intervention

We begin by assuming that government will intervene at date 1 to maximize ex-post social welfare (with some weight on each type of agent). A government which maximizes ex-post social welfare with arbitrary weights will (if possible) reduce interest rates sufficiently to make banks solvent and prevent a run because runs are assumed to be so bad that they make all agents worse off. At date 1, we examine the effects of a central bank that will always choose to intervene in state s so as to avert a Pareto dominated run given the level banks have chosen a D at date 0.[13] Apart from this, we consider interventions that provide more extreme redistribution across the agents in the economy. A *household-friendly* (or equivalently, saver-friendly) central bank will want to undertake actions at date 1 that exclusively benefits the weighted sum of household utilities. An *institution-friendly* (or equivalently, borrower-friendly) central bank will undertake actions that exclusively benefit the sum of utilities of the entrepreneurs and bankers. We examine such interventions and their effect on the ex ante behavior of the banks in what follows. More general interventions with less extreme differences in the welfare weights across agents are qualitatively similar.

3.7. Intervention at Date 1.

Given a level of deposits and an equilibrium interest rate, central banks will reduce interest rates to avoid a run. Beyond that, the interests of households and institutions diverge.

Lemma 5: An institution-friendly central bank will reduce interest rates as far as possible. A household friendly central bank will raise interest rates to the maximum possible consistent with the solvency of the banks.

Proof: See Appendix

[13] We start by examining interest rate interventions sufficient to stop runs for a given level of deposits D. Later, we show that banks will all choose the same D and describe what level banks will choose

Clearly, lower interest rates benefit entrepreneurs as fewer of them will be liquidated. Bankers too prefer lower interest rates as it increases their net worth (and thus their consumption). So a central bank that cares about institutions wants to lower interest rates.

By contrast, high type households (who have withdrawn all deposits in the range where the central bank can affect interest rates) obtain a lower interest rate only through a further reduction in their date-1 consumption through the tax on endowment. Even though they are compensated at date 2, it is at an interest rate that is even lower than their already high marginal rate of substitution, so they are worse off on net. Conversely, higher interest rates make them better off. Turning to low type households, they use the financial system to save. For them, a reduction in interest rates reduces their investment opportunity set, making them worse off.

Governments with a sufficiently large weight on the welfare of institutions will behave as if they are institution-friendly, because the marginal utility of households remains finite for all possible interventions. Similarly, governments with a sufficiently large weight on the welfare of households will behave as if they are house-hold friendly, ensuring that interest rates are the maximum possible consistent with no run. For some interior weights on welfare, the government will always reduce rates to prevent runs, but will not otherwise alter interest rates.

In sum, borrowing institutions (firms and banks) are made better off through lower interest rates, while the households are made worse off. Now how will banks set deposits knowing the type of central bank they are dealing with?

3.8. Deposit Levels Chosen at Date 0 with Household-friendly Central Bank

Banks are forced by competition to choose deposit face values at date 0 that maximize the expected utility of households. From Lemma 5, the household-friendly central bank will raise interest rates at date 1 if it can, provided this does not cause banks to fail. It will lower interest rates only if banks would fail otherwise, and will not reduce interest rates below the rate at which banks are just solvent. Let us start with the deposit face values the banks would choose if the central bank were household-friendly. We have

Lemma 6: (i) In any state s where the central bank does not intervene, depositors will always be better off with a higher deposit face value so long as the bank stays solvent (ii) If $D^{S,Max}$ *is the highest deposit face value in state s at which the bank is solvent without central bank intervention, then the low type household would not be better off if the deposit face value were lower than* $D^{S,Max}$ *and the central bank intervened to raise interest rates to make the bank just solvent, or if the deposit face value were higher than* $D^{S,Max}$ *and the central bank lowered interest rates to make the bank solvent. (iii) If the central bank is household friendly there is a pair of numbers* $\{\theta_1, \theta_2\}$ *where* $\theta_1 < 1$ *and* $\theta_2 < 1$ *such that if* θ^s*, the proportion of high type household in state s, satisfy* $\theta^E < \theta_1$ *and* $\theta^N < \theta_2$*, the bank always chooses to set the deposit face value at date 0 such that* $D \in \{D^{E,\max}, D^{N,\max}\}$.

Proof: See Appendix.

Lemma 6 (i) simply states that, ceteris paribus, depositors are better off with a higher face value, provided the bank is solvent at that higher face value. However, we have to ask whether a bank may make its depositors better off by setting deposit face values low initially (e.g., even below $D^{E,\max}$) and have the household-friendly central bank raise interest rates by taxing future endowments of both types of households. Alternatively, the bank could set deposit face values high (e.g., above $D^{N,\max}$) and have the household-friendly central bank reduce interest rates to avoid a run. Lemma 6 (ii) shows that the low type household is better off at date 1 if the deposit face values were set at $D^{E,\max}$ ($D^{N,\max}$) than if it were set lower (higher) and the central bank intervened to raise (lower) rates – put differently, the household prefers the highest face value obtainable in that state without a run to any other combination of face value and intervention. Finally, since the commercial bank sets its deposit face values so as to maximize the expected utility of depositing households (who do not know their types at date 0), a sufficiently high probability of being a low-type household will ensure that expected utility is reduced if deposit face values are set below $D^{E,\max}$ or above $D^{N,\max}$.

The above discussion is couched as if we had a single bank and it could rely on government intervention. More generally, there are three alternative foundations for this deposit choice by banks when there are identical competitive banks. The first is based on banks' choice of their preferred Nash equilibrium, where a bank takes as given the future state-contingent interest rates and the level of D selected by other banks when it chooses to set the level of its own D. For a bank to attract deposits in the absence of interventions, it must choose a D that maximizes the ex-ante welfare of depositors given the equilibrium rates. Given household friendly interventions, a bank's beliefs about the interest rates depend on its beliefs about the D chosen by other banks. For levels of D that lead to insolvency absent intervention in a state, s, the higher the D that is anticipated, the lower the interest rate at date 1 in state s. Lower anticipated future interest rates induce a bank to set a higher D at date 0. There are then possibly multiple Nash equilibria, and multiple future equilibrium interest rates. We assume that banks can coordinate to play the Nash strategy that is best for them collectively, which is level of D identified in Lemma 6 and which maximizes the ex-ante expected utility of households. No individual bank has an incentive to offer values of D greater than these because for $D \in \{D^{E,\max}, D^{N,\max}\}$, bank profits are zero given household friendly interventions. A second alternative foundation is that the level of D is set by regulation (an upper bound on D) to maximize the ex-ante welfare of households given the ex-post interventions on interest rates.

A third foundation is a "too important to fail" equilibrium which would prevail if any given bank was assured that interest rates would be reduced to keep it solvent (and the choice of D then directly influenced interest rates). Banks are forced by competition to set sufficiently high *D* such that each earns zero profits with household friendly interventions. Again there are multiple Nash equilibria and if banks can coordinate to play the Nash strategy that is best for them collectively, D is the one identified in Lemma 6. Absent this ex-ante coordination, there can be Nash equilibria with excessively high levels of D, producing unnecessary tax-financed interventions that reduce everyone's welfare. We rule out these Pareto-dominated equilibria.

Let us return to our example. Recall that without anticipated intervention, banks set deposit face values high at $D^{N,\max}$ =1.12 only if the probability of the exuberant state is low, for that face value of deposits would set off costly runs in the exuberant state. But if the probability of the exuberant exceeds 0.24, the bank switches to setting the face value of deposits low at $D^{E,\max}$.

We can now examine the effects on depositor welfare of ex-ante promised payments to depositors knowing that if the bank turns out to be insolvent at date 1, the central bank will reduce date-1 interest rates just enough to make it solvent, while if it is solvent, the central bank will increase rates as high as possible. For the parameters in our earlier example, we plot the ex ante date-0 deposit face values set in the expectation of household-friendly central bank intervention in Figure 1. The green line indicates that deposit face values are set at the higher $D^{N,\max}$ for a broader range of probabilities of the exuberant state, and deposit face values come down to $D^{E,\max}$ only when the probability of the exuberant state is very high. If the central bank will intervene to reduce interest rates ex post if banks are insolvent, the cost to banks (and consequently depositors) of having too high leverage is greatly diminished. Competition forces banks to offer higher deposit face values that will necessitate central bank intervention if the state turns out to be exuberant. As Figure 2 suggests, the expected utility of the households is uniformly higher with intervention by a household-friendly central bank than with no intervention.[14]

[14] In a related vein, Allen, Carletti, and Gale (forthcoming) examine how central banks can intervene to determine short term interest rates in the interbank market and improve outcomes.

The interesting question is why depositors do not prefer that the bank set the deposit face value at $D^{N,\max}$ for all possible ex-ante probabilities of the exuberant state, including probability one, and allow the central bank to make the payouts to depositors effectively state-contingent through intervention. The reason is that, as we have seen in Lemma 6, intervention that pushes down interest rates when D is unnecessarily high is distortionary and hurts depositing households. As the probability of the exuberant state nears certainty, it makes everyone better off if each banks sets the deposit face value at the maximum consistent with that state, $D^{E,\max}$, instead of setting it higher and wait for intervention to bring rates down.

3.9. Deposit Face values Chosen at Date 0 with Institution-friendly Central Bank

Now let us turn to an institution-friendly central bank, which wants to bring rates down as low as possible. In Figure 3, we plot the utility of entrepreneurs and bankers without and with the possibility of intervention by the central bank. Without anticipated intervention, deposit face values are set at the lower $D^{E,\max}$ once the probability of the exuberant state exceeds 0.24. This low face value of deposits generates low date-1 interest rates, low liquidation, and thus high utility for bankers and entrepreneurs in the normal state, as well as moderate interest rates and liquidation in the exuberant state. Hence entrepreneur/banker utility shoots up (the blue dotted line in Figure 3) once deposit face values come down, and falls slowly as the probability of the exuberant state increases.

By contrast, if banks anticipate an interventionist central bank that will reduce interest rates sufficiently to always leave ex-post profit to the bank at date 1, the banks will be forced to offer as high a deposit face value as they can up front. For any level of D chosen by banks, the central bank will reduce rates as much as possible and if this will leave the bank with a profit and solvent, the bank has an incentive to offer a higher D to attract a larger share of the deposit market away from competitors. Beyond a certain face value of deposit, though, the central bank will not be able to bring rates down to a level that always keeps the bank solvent, either because the bank is insolvent even at interest rate 1, or because even by taxing all date-1 endowments and lending them to the banks, the rate cannot be brought down low enough. This then determines the face value of deposits set up front, which turns out to be 1.41 in the example. The utility of the entrepreneur/banker is moderate (the red solid line in Figure 3) regardless of the probability of the exuberant state.

Note that with multiple identical banks, the excessively high D is the unique Nash equilibrium if the authorities are anticipated to intervene to reduce rates excessively even if they do not see each bank as "too important to fail" – each bank wants to attract more deposits away from others by promising a slightly higher rate until they exhaust the capacity of the authorities to intervene.

Interestingly, therefore, the presence of an institution-friendly interventionist central bank can make entrepreneurs and bankers worse off for a range of intermediate probabilities. Indeed, all agents are worse off over a range of parameters relative to a policy of no intervention. The intuition is interesting. When the central bank is entrepreneur/bank friendly – that is, it has a bias towards lowering interest rates, there is a divergence between the ex ante competitive focus of banks (to raise promised deposit rates to levels that maximize household expected utility) and the ex post interest of central banks (to protect entrepreneurs and banks by reducing interest rates). These can work at cross-purposes to make all participants worse off.

Contrast this with the situation when the central bank is household friendly. In this case, the central bank has a bias towards raising rates except when that would trigger default, and the households are invariably better off because we get more state-contingency into payouts.

Catastrophic Failure

In our previous example, at the lowest achievable interest rate, both types of households withdraw fully. Therefore, the state (Exuberant or Normal) does not affect the (low) interest rate that can be achieved. If, however, the date-1 endowment is low relative to deposit levels and the date-2 endowment, it is possible that the Low households do not withdraw their deposit fully in at least one of the states at the lowest achievable interest rate. In this case, the lowest achievable interest rate will be influenced by the relative fraction of High and Low households, that is, it will depend on the state. In turn, this means that the maximum possible D for which the central bank can intervene to save banks varies with the state. It is easy to show that if the probability of the exuberant state is low enough, banks will compete to choose a D that ensure that even the authorities cannot save the banking system because their taxation power is too low. For instance, when $e_1 = 0.5$, $e_2^L = 0.5$, $e_2^H = 2.5$, $X_1 = 0.95$, $\underline{Y}_2 = 0.5$, $\overline{Y}_2$ =4.5, γ =0.9, $\theta^E = 0.6$, θ^N =0.3, the highest possible D for which the banking system can be saved through intervention is 1.097 in the exuberant state and 1.131 in the normal state. Banks will set D at 1.131 if the probability of the exuberant state is less than or equal to 0.1, thus ensuring the system fails catastrophically if the exuberant state occurs, with not even the authorities having the capacity to rescue it.

3.10 Discussion.

The more general point we are making is not just the straightforward one that government intervention will prompt reaction from the regulated, but that reaction will take specific forms. What we have shown above is a reaction that takes the form of higher up-front promised payments and thus effective leverage of the banking system. The central bank may be better off committing not to intervene, or to intervene only to avoid runs, than to be perceived as institution-friendly, which can make everyone worse off under a variety of circumstances, or lead to catastrophic failures. Alternatively, knowing that it cannot refrain from institution-friendly intervention ex post, the central bank may want to intervene ex ante by limiting the face value of deposits through interest rate ceilings and maximum leverage ratios.

IV. Distorted Choice of Liquidity

Before concluding, let us examine another way ex post intervention can affect ex ante incentives – through the choice of the liquidity of projects banks undertake. To focus the analysis, let us abstract from the choice of deposit face values, taking them as given at say *D*. Our goal is to briefly illustrate how the interaction anticipated interest rate policy and potential insolvency influence bank's choice of asset liquidity. A full analysis of liquidity choice, however, is beyond the scope of this paper (see Diamond-Rajan (2009a)).

This effect of the price of liquidity on the incentive of banks to hold liquid assets was first studied by Bhattacharya and Gale (1987), for banks with access to interbank markets for liquidity. Work in this area includes Allen, Carletti and Gale (forthcoming), Diamond (1997), Freixas, Martin, and Skeie (2009), Jacklin (1987) and Lorenzoni (2001). None of these models consider insolvency or feedback from insolvency to interest rate interventions.

4.1. Choice of portfolio liquidity

Let the liquidity of a project portfolio be given by a factor $Z > 1$, whereby more liquid portfolios pay $\frac{\tilde{Y}_2}{Z}$ in goods at date 2 if the project is not liquidated, and ZX_1 at (or after) date 1 if the project is liquidated. Thus more liquid portfolios have a greater liquidation value, while sacrificing long run returns.

A banker will choose the liquidity of his portfolio, taking future interest rates as given, to maximize the discounted value of his consumption; which is his residual claim, net of costs of financing and refinancing at date 1. The total value (at date 1) of the bank's assets for a given date 1 interest rate in state S is $\frac{Z}{\overline{Y}_2 - \underline{Y}_2}\left[\int_{\underline{Y}_2}^{Y_2(r_{12}^s)} X_1 dY_2 + \frac{\gamma}{Z r_{12}^s}\int_{Y_2(r_{12}^s)}^{\overline{Y}_2} Y_2 dY_2\right] \equiv V_1\ (r_{12}^s, Z)$. It is easily shown that $\frac{\partial V_1}{\partial r_{12}^S \partial Z} > 0$, that is, a higher prospective date-1 interest rate makes it more valuable to have a more liquid project portfolio.

After refinancing deposits and meeting withdrawals at date 1, the date-1 present value of a solvent banker's date-2 payoff is $V_1\ (r_{12}^S, Z) - D$, and the date 2 value of this payoff is $r_{12}^S(V_1\ (r_{12}^S, Z) - D)$. A banker's payoff is zero if the bank is insolvent and both terms are negative.

Suppose that the bank has a choice between two portfolios with liquidity Z_1 and Z_2 respectively, with $Z_1 < Z_2$. It is easily shown that there is an interest rate r_{12}^{IC} whereby the bank prefers the portfolio with asset liquidity Z_1 in state S iff $r_{12}^S < r_{12}^{IC}$. As a result, anticipated institution-friendly interventions by the central bank which reduce interest rates below r_{12}^{IC} at all times (or which reduce rates below r_{12}^{IC} in the normal state and set interest rates in the exuberant state such that $V_1\ (r_{12}^E, Z) - D$ is zero or close to zero), will cause banks to choose the more illiquid portfolio with $Z = Z_1$. Even interventions that merely reduce interest rates to prevent insolvency can cause banks to prefer to be illiquid. If rates are reduced to keep banks just solvent in the exuberant state, then they must not be kept too low (and may need to be increased above the no-intervention level) in normal times. If not, in anticipation of interest rate intervention, banks will choose portfolios that make future liquidity crises extremely severe.

In sum, if liquid assets offer lower long-run returns, the prospect of lower future interest rates will make banks prefer less liquid assets. Specifically, if the central bank intervenes to make banks only just solvent when the demand for liquidity is high (that is, in the exuberant state), banks will choose portfolios based primarily on their relative attractiveness at interest rates that will prevail when the demand for liquidity is low. This will typically mean they will choose illiquid portfolios. In turn, this will require larger interventions and more tax-financed lending to achieve a given interest rate reduction in times of stress.

4.2. Discussion.

Summarizing our analysis, a central bank that promises to cut interest rates conditional on stress, or that is biased towards low interest rates favoring entrepreneurs, will induce banks to promise higher payouts or take more illiquid projects. This in turn can make the illiquidity crisis more severe and require a greater degree of intervention, a view reminiscent of the Austrian theory of cycles. According to this theory (Rothbard (2008, p11)) the conditions for a crisis are put in place because

"businessmen were misled by bank credit inflation to invest too much in higher-order capital goods [i.e., investment projects that take a long time to mature] …Businessmen were led to this error by the credit expansion and its tampering with the free-market rate of interest."

Bank credit inflation is, in turn, caused by an extremely accommodative central bank. The immediate trigger for the crisis is that (Rothbard (2008, p11))

"new money percolates downwards from the business borrowers to the factors of production in wages, rents, and interest…people will rush to spend the higher incomes …and demand will shift from the higher to the lower orders [i.e., from long dated capital projects to consumption goods]. Capital goods industries will find that their investments have been in error…Higher orders of production have turned out to be wasteful, and the malinvestment must be liquidated."

While the Austrian view seems to rely on banks that are excessively optimistic, our model can produce crises with rational optimizing banks. Crises stem from the mismatch between the household desire for immediate consumption goods as a result of anticipated higher incomes (coupled with their ability to withdraw cash needed to pay for these goods) and the illiquidity of projects the bank has invested in, and can occur in states where households anticipate high future incomes. An interventionist central bank, especially one biased towards cutting interest rates excessively, can make crises worse.

Our model suggests that the crisis of 2007-2009 may not be unrelated to the actions of the Federal Reserve earlier in the decade, not only in convincing the market that interest rates would remain low for a sustained period following the dot-com bust because of its fears of deflation, but also in promising to intervene to pick up the pieces in case of an asset price collapse – the so-called Greenspan put. The behavior of the central bank may have contributed to bank investment in illiquid assets (the now infamous mortgage-backed securities as well as vehicles such as SIVs and conduits) as well as bank leverage (see Adrian and Shin (2008) for compelling evidence).

From a policy perspective, it is hard to imagine that central banks will stand by and let large parts of the banking system fail – a form of time inconsistency identified by Bagehot (1873). Given their inability to commit on a particular form of future intervention (or commit not to intervene), central banks should be especially wary of keeping rates low other than in times of market stress. Indeed, in the interest of stability, it could be useful for a central bank to convince the system it will have rates higher than the otherwise "natural" rate in normal times to offset the distortions created by abnormally low rates in times of financial market stress.[15] Of course, the precise extent of the bias towards higher rates has to be weighed against the effects on activity, which is beyond the scope of this paper.

4.3. Possible Extensions

We have assumed throughout the paper that household endowments are independent of lending activity. An important extension would be to relate future endowments to the fraction of long dated projects that are allowed to continue, as in Diamond-Rajan (2005). On the one hand, project liquidation would reduce future household endowments and thus moderate the immediate demand for consumption. This reflects the Austrian School view that allowing unfettered liquidation would itself allow the system to equilibrate more quickly. On the other hand, to the extent that liquidation set off financial panics a la Diamond and Dybvig (1983), it could create its own momentum for liquidity. A more careful examination is certainly warranted.

We have also assumed a homogenous set of banks. Heterogeneity amongst banks could suggest interesting additional effects. For instance, central bank intervention to keep one set of banks alive could alter incentives for other banks. Acharya and Yorulmazer (2007) examine the incentives for two banks to have correlated asset portfolios – which would increase as the extent of bailout in case of failure

[15] There is some evidence that periods of lower interest rates are followed by episodes of financial stress.

increases. In a closely related paper to ours, Farhi and Tirole (2009) examine the strategic complementarities in bank leverage – the possibility of central bank intervention is increased if more banks lever up, which gives yet more banks the incentive to lever up.

Finally, we have assumed the policy bias of the central bank (household friendly or institution-friendly) is independent of the underlying economy. Yet there may indeed be a relationship. For instance, authorities in developing countries have historically favored producers at the expense of consumers, and may thus have had a bias towards low rates. If so, our model suggests there would be a tendency for developing country banks to have greater fragility (higher effective leverage, more illiquid projects). Indeed, the move in these countries towards monetary policies more focused on controlling inflation than on maintaining low real rates may have contributed towards greater stability.

V. Conclusion

Our work has been done in very interesting times. Following a period of extremely benign financial conditions, we are in the midst of a financial panic caused by financial firms overloading on illiquid assets and taking on too much leverage. Why might we have arrived in this state and what can we do about it? While agency problems in banks and the breakdown in risk controls and compensation structures (see Kashyap, Rajan, and Stein (2008) and Diamond-Rajan (2009b)) must be part of the explanation, our model offers another one; the anticipated benign environment, perhaps accentuated by hopes of central bank intervention if interest rates rose too high and asset prices fell substantially, must have also been an important causal explanation for both the illiquidity of assets and the excessive leverage.

Our analysis points to the dangers of one-sided interventions, for they distort the price of liquidity and increase the attractiveness for banks of relying on publicly provided liquidity to fund mismatches. For instance, reducing interest rates drastically when the financial sector is in trouble, but not raising them quickly as the sector recovers could create incentives for banks to seek out more illiquidity than good for the system. Such incentives may have to be offset by raising rates in normal times more than strictly warranted by macroeconomic conditions. While our entire model is couched in terms of real goods, we hope in future work to describe monetary implications, as well as explore the dynamic implications.

References

Acharya, Viral and Tanjul Yorulmazer (2007), “Too many to fail: An analysis of time-inconsistency in bank closure policies”, Working Paper, Bank of England.

Adrian, Tobias and Hyun Shin (2008), “Financial Intermediaries, Financial Stability, and Monetary Policy”, forthcoming, Proceedings of the Kansas City Federal Reserve Symposium at Jackson Hole.

Allen, Franklin, Elena Carletti and Douglas Gale (forthcoming), "Interbank Market Liquidity and Central Bank Intervention”, Journal of Monetary Economics

Barro, Robert (1974) “Are Government Bonds Net Wealth?”, Journal of Political Economy, vol. LXXXII (1974), 109-175.

Bhattacharya, Sudipto, and Douglas Gale (1987),"Preference Shocks, Liquidity and Central Bank Policy," in New Approaches to Monetary Economics, edited by W. Barnett and K. Singleton, (Cambridge: Cambridge University Press).

Diamond, Douglas, (2004) “Presidential Address, Committing to Commit: Short-Term Debt When Enforcement Is Costly,” The Journal of Finance, Vol. 59, No. 4 (Aug., 2004), pp. 1447-1479

Diamond, D. W. and P. H. Dybvig [1983], “Bank Runs, Deposit Insurance, and Liquidity,” *Journal of Political Economy* 91, 401-419.

Diamond, Douglas W. and Raghuram Rajan (2001)“Liquidity risk, liquidity creation and financial fragility: A theory of banking,” *Journal of Political Economy* 109 (April 2001).

Diamond, Douglas W. and Raghuram Rajan “Liquidity Shortages and Banking Crises,” *Journal of Finance* (April 2005).

Diamond, Douglas W. and Raghuram Rajan “Notes on Liquidity Choice,” mimeo (2009a).

Diamond, Douglas W. and Raghuram Rajan, 2009b. "The Credit Crisis: Conjectures about Causes and Remedies," *American Economic Review*, Papers and Proceedings: 99: 606-10.

Farhi, Emmanuel and Jean Tirole (2009), “Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts”, working paper, June 29, 2009,Toulouse.

Hayek, Friederich, (1935), *Prices and Production*, Routledge and Kegan Paul, London.

Freixas, Xavier, Antoine Martin and David Skeie (2009), “Bank liquidity, interbank markets and monetary policy,” working paper, Federal Reserve Bank of New York.

Goodfriend M. and R. King (1988). ”Financial Deregulation, Monetary Policy and Central Banking”, *Federal Reserve Bank of Richmond Economic Review*, 74, 3-22.

Greenspan, Alan (2002), Opening Remarks, Jackson Hole Symposium organized by the Kansas City Federal Reserve Bank.

Hellwig, Martin (1994): "Liquidity Provision, banking and the allocation of interest rate risk," European Economic Review, 38(7), 1363-89.

Holmstrom, Bengt and Jean Tirole (1998), "Private and Public Supply of Liquidity", *Journal of Political Economy*, 106 (1), pp 1-40.

Jacklin, Charles. "Demand Deposits, Trading Restrictions, and Risk Sharing." In Contractual Arrangements for Intertemporal Trade, edited by Edward Prescott and Neil Wallace. University of Minnesota Press, 1987.

Lorenzoni, Guido (2001) "Excess Interest Rate Volatility in an Intermediated Economy," MIT economics working paper.

Kashyap, A., R. Rajan, and Jeremy Stein, (2008), Rethinking Capital Regulation, forthcoming, Proceedings of the Kansas City Federal Reserve Symposium at Jackson Hole.

Rothbard, Murray, 2008, *America's Great Depression,* Ludwig von Mises Institute, Auburn, Alabama.

Von Mises, Ludwig (1949), *Human Action*, Yale University Press, New Haven Connecticut.

Appendix

Proof of Theorem 1:

(i) The right hand side of (1.2) is strictly increasing in r_{12}^S. The left hand side is weakly decreasing -- withdrawals decrease in the interest rate, but when D is withdrawn, the household cannot withdraw more. Thus there is a unique interest rate and withdrawals at which the supply of goods at date 1 equals demand, and the depositors do not want to, or cannot, withdraw more.

Suppose not. Can there, for instance, be another equilibrium with the same interest rate and different withdrawal levels? If the households are on their first order conditions at the equilibrium interest rate, there is a unique withdrawal level consistent with the interest rate. If they are off their first order condition, they have to withdraw D. So there cannot be another equilibrium with the same interest rate and different withdrawal levels. Can there be another equilibrium with a higher interest rate? No, because withdrawals would be (weakly) lower while liquidation would be strictly higher, so (1.2) would not hold. Similarly for a lower interest rate.

(ii) Follows by construction.

Proof of Corollary 1:

(i) When both households are on their first order condition, the H household withdraws more than the L Household. The H household withdraws D fully before the L household withdraws. Hence it (weakly) withdraws more and $w_1^{H,S} \geq w_1^{L,S}$.

(ii) Totally differentiating (1.2) with respect to θ^S, we get

$$\left(w_1^{H,S} - w_1^{L,S}\right) + \theta^S \frac{dw_1^{H,S}}{d\theta^S} + (1-\theta^S)\frac{dw_1^{L,S}}{d\theta^S} - \frac{X_1^{\ 2}}{\gamma\left(\overline{Y}_2 - \underline{Y}_2\right)}\frac{dr_{12}^S}{d\theta^S} = 0$$

Now $\frac{dw_1^{H,S}}{d\theta^S} = \frac{\partial w_1^{H,S}}{\partial r_{12}^S}\frac{dr_{12}^S}{d\theta^S}$ where $\frac{\partial w_1^{H,S}}{\partial r_{12}^S} \leq 0$ from the expressions for $w_1^{H,S}$. Similarly for $\frac{dw_1^{L,S}}{d\theta^S}$. Thus $\frac{dr_{12}^S}{d\theta^S} = \frac{\left(w_1^{H,S} - w_1^{L,S}\right)}{\frac{X_1^{\ 2}}{\gamma\left(\overline{Y}_2 - \underline{Y}_2\right)} - \theta^S \frac{\partial w_1^{H,S}}{\partial r_{12}^S} - (1-\theta^S)\frac{\partial w_1^{L,S}}{\partial r_{12}^S}} \geq 0$ because $w_1^{H,S} \geq w_1^{L,S}$.

Because $\frac{dr_{12}^S}{d\theta^S} \geq 0$, it must be that the right hand side of (1.2) increases in θ^S, so the left hand side must also increase. Hence $\theta^S w_1^{H,S} + (1-\theta^S) w_1^{L,S}$ increases in θ^S. Finally, the fraction liquidated increases in r_{12}^S, hence it must increase in θ^S.

Turning to the effect of D, we have on totally differentiating (1.2)

$\theta^S \frac{dw_1^{H,S}}{dD} + (1-\theta^S)\frac{dw_1^{L,S}}{dD} - \frac{X_1^{\ 2}}{\gamma\left(\overline{Y}_2 - \underline{Y}_2\right)}\frac{dr_{12}^S}{dD} = 0$. Now either $\frac{dw_1^{\bullet,S}}{dD}$ =1 if the household is off its first order condition and has withdrawn everything, or it is given by $\frac{1}{2}\left[1-e_2^{\bullet}\left(\frac{1}{r_{12}^S}\right)^2\frac{dr_{12}^S}{dD}\right]$.

Substituting and simplifying, we get $\frac{dr_{12}^S}{dD}\geq 0$. Given r_{12}^S increases in D, the other comparative statics follow for the same reasons as above.

(iii) The date-1 asset value of banks is $\frac{1}{\overline{Y}_2-\underline{Y}_2}\int_{\underline{Y}_2}^{Y_2(r_{12}^S)} X_1 dY_2+\frac{1}{\overline{Y}_2-\underline{Y}_2}\int_{Y_2(r_{12}^S)}^{\overline{Y}_2}\frac{\gamma Y_2}{r_1^S}dY_2$ while the liabilities are D. The asset value falls in r_{12}^S, so it falls in θ^S and D. The liabilities increase in D. Hence the net worth (=assets-liabilities) falls in θ^S and D.

Proof of Lemma 5:

Since project liquidation and the banks' net worth increase as r_{12}^S decreases, both entrepreneurs and banks are better off with an institution-friendly central bank. Turning to the households, their utility is

$U^L=\log(e_1+w_1^{LS}+s)+\log(e_2^L+(D-w_1^{LS}).r_{12}^S-sr_{12}^S)$

$U^H=\log(e_1+D+s)+\log(e_2^H-sr_{12}^S)$

where s is the subsidy transferred to households by the government at date 1.

$\frac{dU^H}{ds}=\frac{1}{e_1+D+s}-\frac{r_{12}^S}{e_2^H-sr_{12}^S}$. But $r_{12}^S<\frac{e_2^H-sr_{12}^S}{e_1+D+s}$ given that H households have withdrawn completely and do not want to deposit in the system. Therefore $\frac{dU^H}{ds}$ >0.

$\frac{dU^L}{ds}=\frac{1}{e_1+w_1^{LS}+s}\left(1+\frac{dw_1^{LS}}{ds}\right)-\frac{1}{e_2^L+(D-w_1^{LS}).r_{12}^S-sr_{12}^S}\left(r_{12}^S\left(1+\frac{dw_1^{LS}}{ds}\right)-\left(D-w_1^{LS}-s\right)\frac{dr_{12}^S}{ds}\right)$. Now L type households are on their first order condition, so

$\frac{1}{e_1+w_1^{LS}+s}\left(1+\frac{dw_1^{LS}}{ds}\right)-\frac{1}{e_2^L+(D-w_1^{LS}).r_{12}^S-sr_{12}^S}\left(r_{12}^S\left(1+\frac{dw_1^{LS}}{ds}\right)\right)=0$. This means

$\frac{dU^L}{ds}=\frac{1}{e_2^L+(D-w_1^{LS}).r_{12}^S-sr_{12}^S}\left(\left(D-w_1^{LS}-s\right)\frac{dr_{12}^S}{ds}\right)\geq 0$ because $\left(D-w_1^{LS}-s\right)>0$ and $\frac{dr_{12}^S}{ds}>0$.

Thus both types of households prefer transfers followed by taxes that raise the interest rate.

Proof of Lemma 6:

(i) Suppose we increase D keeping the bank solvent.

$$\frac{dU^L}{dD} = \frac{1}{e_1 + w_1^{LS}}\left(\frac{dw_1^{LS}}{dD}\right) - \frac{r_{12}^S}{e_2^L + (D - w_1^{LS}).r_{12}^S}\left(\frac{dw_1^{LS}}{dD}\right) + \frac{r_{12}^S + \left(D - w_1^{LS}\right)\frac{dr_{12}^S}{dD}}{e_2^L + (D - w_1^{LS}).r_{12}^S}$$

Because $\frac{1}{e_1 + w_1^{LS}} - \frac{r_{12}^S}{e_2^L + (D - w_1^{LS}).r_{12}^S} = 0$, the first two terms are zero. The last term is positive. So $\frac{dU^L}{dD} > 0$. Similarly, if H households are on their first order condition, we can show $\frac{dU^H}{dD} > 0$. If they are not – either they withdraw everything or leave everything in the bank – their utility clearly increases in D.

(ii)Let $D^{S,Max}$ be the maximum possible deposit level without precipitating a run and $r_{12}^{S,\max}$ be the corresponding interest rate and $w_1^{L,S\max}$ be the withdrawal by the L household. Consider setting the deposit level at $D^{S,Max} - \Delta D$ and then offering a subsidy *s* (and taxing it back at date 2). Let the withdrawal be $w_1^{L,S}$ and r_{12}^S be the interest rate.

First consider $s > \Delta D$. Let us check if the bank is solvent at the old equilibrium interest rate $r_{12}^{S,\max}$.

Now $r_{12}^S = \frac{e_2^L}{e_1 + 2(w_1^{L,S} + s) - (D^{S,Max} - \Delta D)}$ and $r_{12}^{S,\max} = \frac{e_2^L}{e_1 + 2w_1^{L,S\max} - D^{S,Max}}$. For $r_{12}^S = r_{12}^{S,\max}$, it must be that the denominator $2(w_1^{L,S\max} - (w_1^{L,S} + s)) + \Delta D = 0$ so that $w_1^{L,S\max} - w_1^{L,S} = s + \frac{\Delta D}{2}$. The net liabilities of the bank are

$D^{S,Max} - \Delta D - w_1^{L,S} = D^{S,Max} - \Delta D - (w_1^{L,S\max} - s - \frac{\Delta D}{2}) > D^{S,Max} - w_1^{L,S\max}$ if $s > \Delta D$. But the residual assets of the bank at the interest rate $r_{12}^{S,\max}$ were just enough to meet liabilities $D^{S,Max} - w_1^{L,S\max}$. They cannot be enough to meet a greater level of liabilities. So we cannot have $s > \Delta D$ and $r_{12}^S = r_{12}^{S,\max}$, or indeed $r_{12}^S \geq r_{12}^{S,\max}$.

Can we have $s \leq \Delta D$ and $r_{12}^S \geq r_{12}^{S,\max}$? For the interest rate to be higher, given the H household consumes less, it must be that the L household consumes (weakly) more. This implies $w_1^{L,S} + s \geq w_1^{L,S\max}$.

But we also need $\frac{e_2^L}{e_1 + 2(w_1^{L,S} + s) - (D^{S,Max} - \Delta D)} \geq \frac{e_2^L}{e_1 + 2w_1^{L,S\max} - D^{S,Max}}$, which means $w_1^{L,S} + s + \frac{\Delta D}{2} \leq w_1^{L,S\max}$, a contradiction. Therefore, we cannot have $r_{12}^S \geq r_{12}^{S,\max}$. But if $r_{12}^S < r_{12}^{S,\max}$, the

L household is always worse off than in the equilibrium with $D^{S,Max}$ because its financial holdings are lower at $D^{S,Max} - \Delta D$ and the investment opportunity set determined by r_{12}^{S} is lower.

Now consider the possibility of raising D above $D^{S,Max}$ while lowering interest rates through intervention to keep the bank solvent. Post intervention, $U^L = \log(e_1 + w_1^{LS} - t) + \log(e_2^L + (D - w_1^{LS} + t).r_{12}^S)$ such that $V(r_{12}^S) = D$ where V is the value of the bank at date 1 (before withdrawals). We then have

$$\frac{dU^L}{dD} = \frac{1}{e_1 + w_1^{LS} - t}\left(\frac{dw_1^{LS}}{dD} - \frac{dt}{dD}\right) - \frac{r_{12}^S}{e_2^L + (D - w_1^{LS} + t).r_{12}^S}\left(\frac{dw_1^{LS}}{dD} - \frac{dt}{dD}\right) + \frac{r_{12}^S + \left(D - w_1^{LS} + t\right)\frac{dr_{12}^S}{dD}}{e_2^L + (D - w_1^{LS} + t).r_{12}^S}.$$

Because the L households are on their FOC, the first two terms sum to zero again. We also have from the requirement of solvency that $\frac{dr_{12}^S}{dD} = \frac{-1}{\frac{dV}{dr_{12}^S}}$. Simple algebra suggests that

$$\frac{dV}{dr_{12}^S} = \frac{1}{\left(\overline{Y}_2 - \underline{Y}_2\right)}\left(\frac{\left(X_1\right)^2}{2\gamma} - \frac{\overline{Y}^2\gamma}{2\left(r_{12}^S\right)^2}\right) = \frac{-1}{r_{12}^S}\left(\text{Present Value of Bank (after withdrawals)}\right).$$ This then

means that $\frac{dU^L}{dD} = \frac{r_{12}^S\left(1 - \frac{\left(D - w_1^{LS} + t\right)}{\text{Present value of bank}}\right)}{e_2^L + (D - w_1^{LS} + t).r_{12}^S} < 0$ because if the bank is just solvent after withdrawals, the present value of the bank = $(1-\theta)(D - w_1^{L,S}) + t < D - w_1^{L,S} + t$.

Given that the L type household is always worse off when the deposit level is set either above or below $D^{S,Max}$ keeping in mind the prospect of intervention, it must be that if their weight is sufficient, the bank will set $D \in [D^{E,\max}, D^{N,\max}]$.

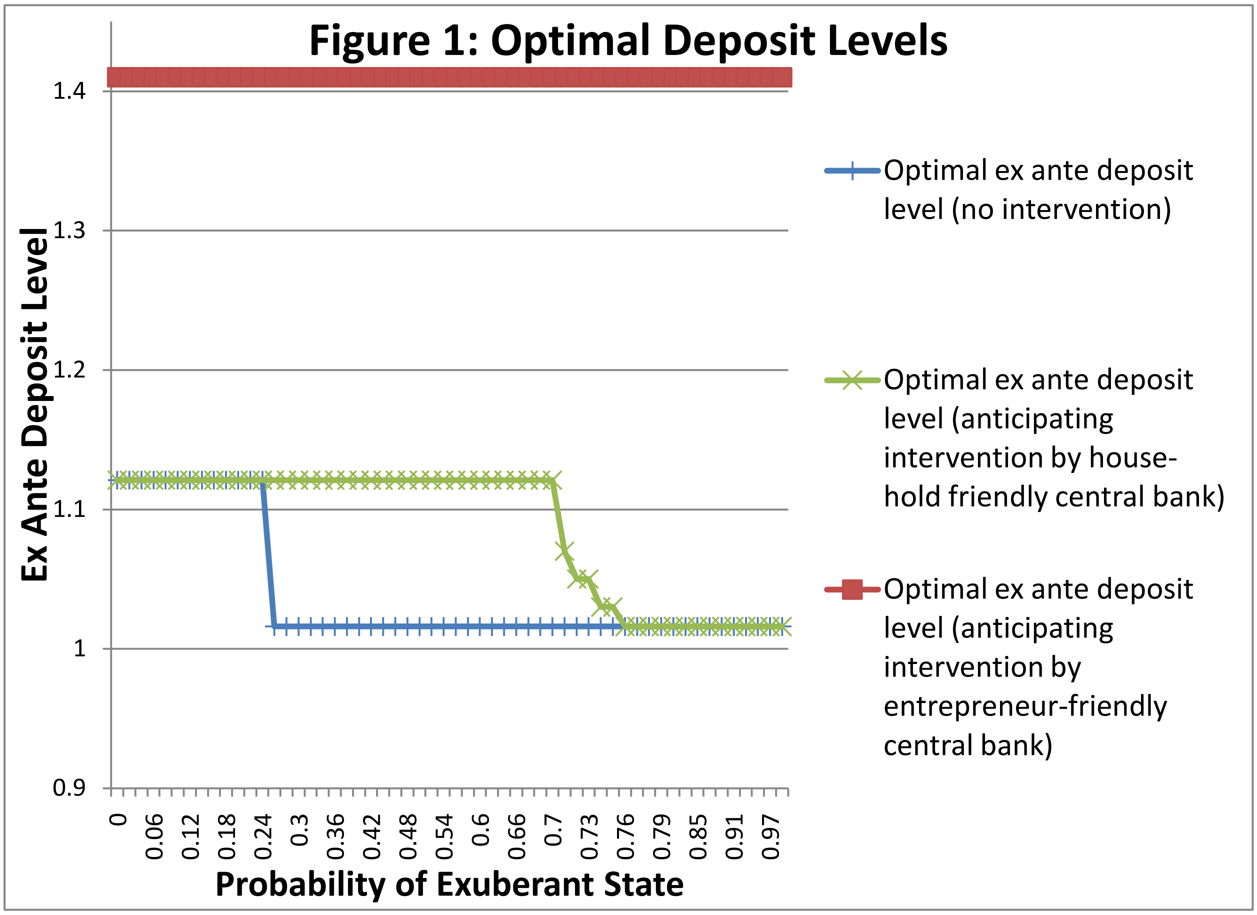
Figure 1: Optimal Deposit Levels
Ex Ante Deposit Level
1.4
1.3
1.2
1.1
1
0.9
0
0.06
0.12
0.18
0.24
0.3
0.36
0.42
0.48
0.54
0.6
0.66
0.7
0.73
0.76
0.79
0.85
0.91
0.97
Probability of Exuberant State
Optimal ex ante deposit level (no intervention)
Optimal ex ante deposit level (anticipating intervention by house-hold friendly central bank)
Optimal ex ante deposit level (anticipating intervention by entrepreneur-friendly central bank)

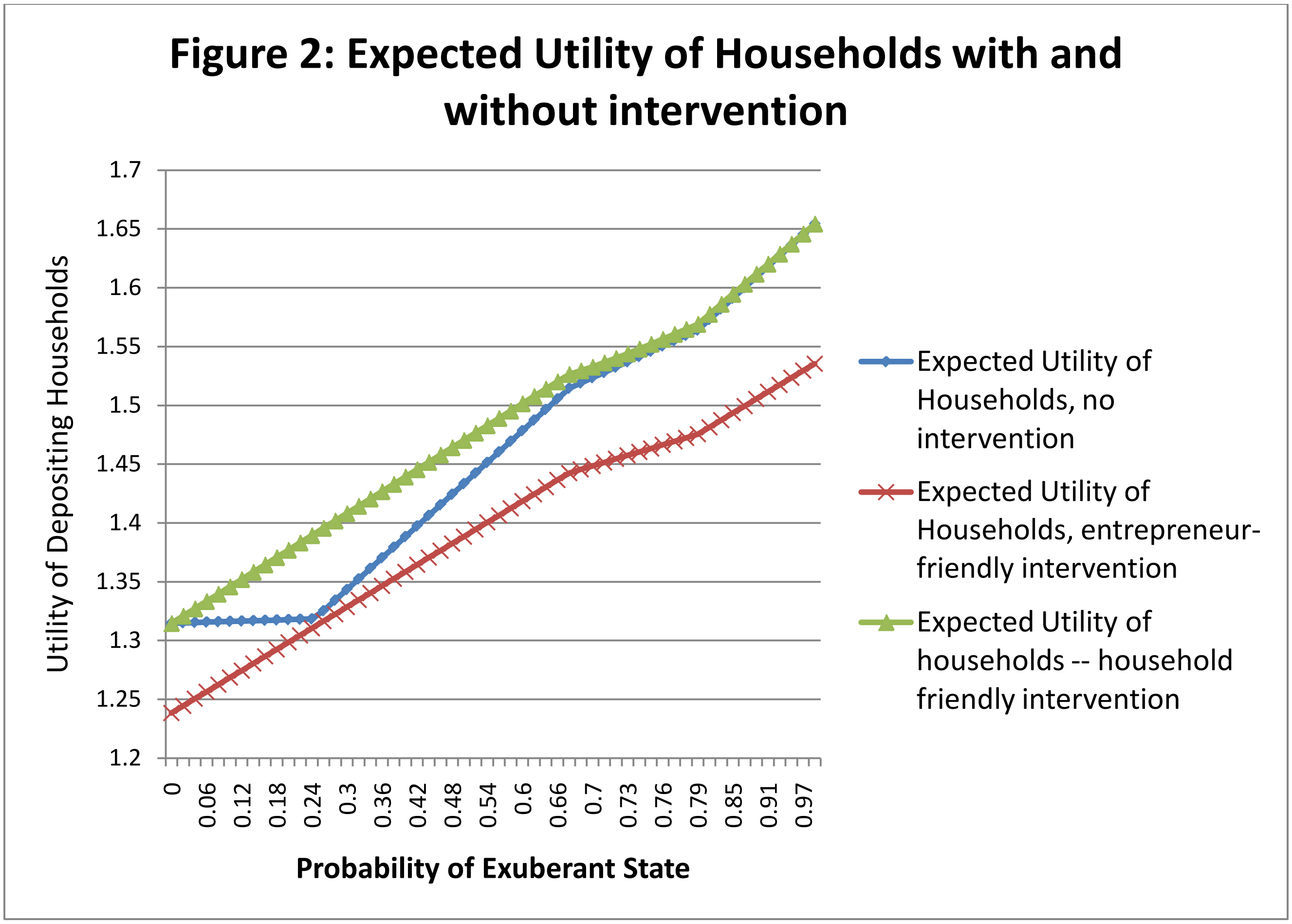
Figure 2: Expected Utility of Households with and without intervention
Utility of Depositing Households
1.7
1.65
1.6
1.55
1.5
1.45
1.4
1.35
1.3
1.25
1.2
0
0.06
0.12
0.18
0.24
0.3
0.36
0.42
0.48
0.54
0.6
0.66
0.7
0.73
0.76
0.79
0.85
0.91
0.97
Probability of Exuberant State
Expected Utility of Households, no intervention
Expected Utility of Households, entrepreneur-friendly intervention
Expected Utility of households -- household friendly intervention

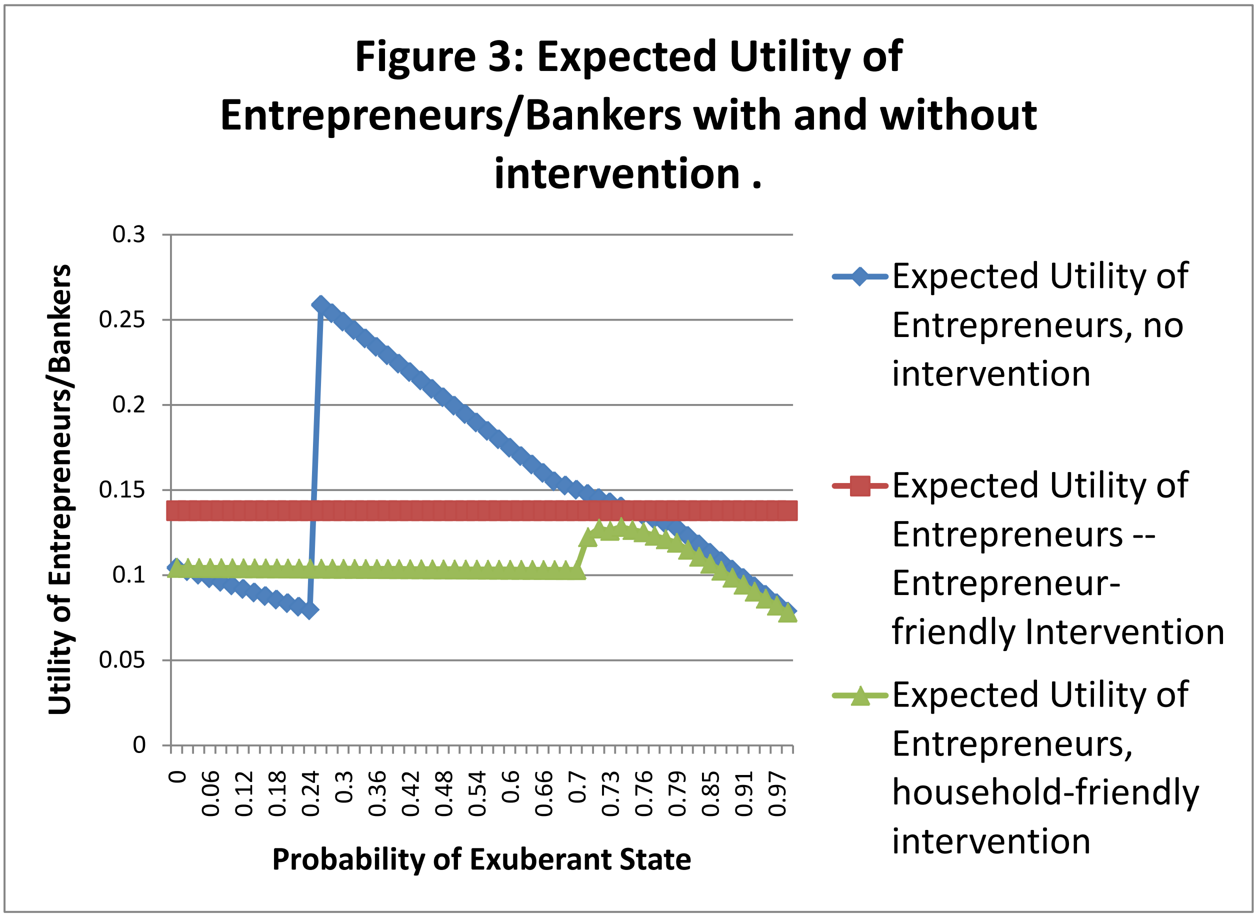
Figure 3: Expected Utility of Entrepreneurs/Bankers with and without intervention .
0.3
0.25
0.2
0.15
0.1
0.05
0
Utility of Entrepreneurs/Bankers
0
0.06
0.12
0.18
0.24
0.3
0.36
0.42
0.48
0.54
0.6
0.66
0.7
0.73
0.76
0.79
0.85
0.91
0.97
Probability of Exuberant State
Expected Utility of Entrepreneurs, no intervention
Expected Utility of Entrepreneurs -- Entrepreneur-friendly Intervention
Expected Utility of Entrepreneurs, household-friendly intervention