

NBER WORKING PAPER SERIES

A MODEL OF A SYSTEMIC BANK RUN

Harald Uhlig

Working Paper 15072

<http://www.nber.org/papers/w15072>

NATIONAL BUREAU OF ECONOMIC RESEARCH

1050 Massachusetts Avenue

Cambridge, MA 02138

June 2009

This paper has been written for and financially supported by the Carnegie-Rochester conference in April 2009. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2009 by Harald Uhlig. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

A Model of a Systemic Bank Run
Harald Uhlig
NBER Working Paper No. 15072
June 2009
JEL No. E44,G21,G28

ABSTRACT

The 2008 financial crisis is reminiscent of a bank run, but not quite. In particular, it is financial institutions withdrawing deposits from some core financial institutions, rather than depositors running on their local bank. These core financial institutions have invested the funds in asset-backed securities rather than committed to long-term projects. These securities can potentially be sold to a large pool of outside investors. The question arises, why these investors require steep discounts to do so. I therefore set out to provide a model of a systemic bank run delivering six stylized key features of this crisis. I consider two different motives for outside investors and their interaction with banks trading asset-backed securities: uncertainty aversion versus adverse selection. I shall argue that the version with uncertainty averse investors is more consistent with the stylized facts than the adverse selection perspective: in the former, the crisis deepens, the larger the market share of distressed core banks, while a run becomes less likely instead as a result in the adverse selection version.

I conclude from that that the variant with uncertainty averse investors is more suitable to analyze policy implications. This paper therefore provides a model, in which the outright purchase of troubled assets by the government at prices above current market prices may both alleviate the financial crises as well as provide tax payers with returns above those for safe securities.

Harald Uhlig
Dept. of Economics
University of Chicago
1126 E 59th Street
Chicago, IL 60637
and NBER
huhlig@uchicago.edu

1 Introduction

Bryant (1980) and Diamond and Dybvig (1983) have provided us with the classic benchmark model for a bank run. There, an individual bank engages in maturity transformation, using demand deposits to finance long-term loans, which can be liquidated in the short term only at a cost. If too many agents claim short-term liquidity needs and withdraw their demand deposits, the value of the bank assets are thus not sufficient to meet these liquidity demands, in turn justifying even patient depositors to get their money while they can: a bank run ensues. One policy conclusion then is for a central bank to follow the classic Bagehot principle of committing to inject liquidity to illiquid but otherwise solvent bank, in order to stop bank runs.

The financial crisis of 2007 and 2008 is reminiscent of a bank run, but not quite, see Brunnermeier (2008) and Gorton (2009). First, this was (with few exceptions) not a run of depositors on their local house bank, but a run of banks and money funds on some core financial institutions. Second, the health of some core financial institutions (I shall call them “core banks” for the purpose of this paper) was called into question not because of their commitment to costly-to-call long-term loans, but rather because of the questionable value of a variety of “exotic” securities, most notably their guarantees for particular tranches of mortgage-backed security derivatives and credit default swaps. These are assets which could be marked to market at least in principle. So, when a bank cannot repay its depositors because the market value of their assets is below the value of its liabilities, the traditional prescription is to declare the bank to be bankrupt and not to provide it with additional liquidity.

In the current situation, this would mean for the affected banks to sell their questionable assets at market prices to meet withdrawals. There is a widespread perception, however, that current market prices are below fundamental values, and that further sales of these assets are akin to fire sales, leading to further depression of the price of these assets, triggering additional bankruptcies. This conclusion appears unpalatable to many and therefore,

the Federal Reserve Bank and the Treasury have instead expanded interventions where these assets will be bought at above-laissez-faire prices. There is the perception that current events should be understood as some version of a systemic bank run, despite the inapplicability of the original Diamond-Dybvig framework. This creates a gap in our understanding. A new or at least a modified theory is needed.

This paper seeks to contribute to filling that gap, and provide a model (in two variants) of a systemic bank run. A systemic bank run is a situation, in which early liquidity withdrawals by long-term depositors at some bank are larger and a bank run more likely, if other banks are affected by liquidity withdrawals too, i.e. the market interaction of the distressed banks is crucial. This is different from a system-wide run, which may occur if all depositors view their banks as not viable, regardless of whether the depositors at other banks do to. The paper thereby seeks to provide a framework for analyzing or evaluating policy options in a financial crisis similar to the one experienced in 2007 and 2008 through the perspective of a bank run model, which allows for this market interaction.

It seeks to capture the following stylized view of the 2008 crisis.

1. The withdrawal of funds was done by financial institutions (in particular, money market funds and other banks) at some core financial institutions, rather than depositors at their local bank.
2. The troubled financial institutions held their portfolio in asset-backed securities rather than being invested directly in long-term projects.
3. These securities are traded on markets. In the crisis, the prices for these securities appears low compared to some benchmark fundamental value benchmark (“underpricing”).
4. There is a large pool of investors willing to purchase securities, as evidenced e.g. by market purchases of newly issued US government bonds or the volume on stock markets.

5. Nonetheless, these investors are only willing to buy these asset-backed securities at prices that are low compared to standard discounting of the entire pool of these securities.
6. The larger the market share of troubled financial institutions, the steeper the required discounts.

This view may be entirely incorrect: it is possible that the appropriate perspective is one of insolvency rather than illiquidity, and future research will hopefully eventually sort out which view is most appropriate. Absent that clarification, it is worthwhile to analyze the situation from a variety of perspectives: therefore, I shall proceed with the view as stated above.

It will turn out, that from this list, items 1 to 3 are straightforward to incorporate, merely requiring some additional notation. Item 4 is easy to incorporate in principle, but hard once one demands item 5 and 6 as well. In particular item 6 will turn out to be particularly thorny to achieve, and decisive in selecting one of two views of outside investors.

The key argument can be summarized as follows. Suppose that there are some unforeseen early withdrawals, e.g. due to a shaken confidence by, say, some local banks or money market funds with respect to the viability of their core bank. In order to provide resources to unforeseen withdrawals, the core financial institutions then need to sell part of their portfolio, thereby incurring opportunity costs in terms of giving up returns at some later date. Suppose that the remaining depositors (or depositing institutions) are the more inclined to withdraw early as well, the larger these opportunity costs are. If a larger market share of distressed banks and therefore larger additional liquidity needs drive these opportunity costs up, then a wide spread run on the core banks is more likely: this creates a systemic bank run. I therefore investigate, whether this increase in opportunity costs will happen.

After a literature review, I describe the model in section 3. I start from an environment inspired by Smith (1991), in which depositors interact with a local bank, which in turn refinances itself via an (uncontingent) deposit

account with one of a few core banks, who in turn invest in long-term securities backed by locally run projects (think: mortgage-backed securities). Clearly, the observable world of securities is considerably richer (and harder to describe), but this framework may capture the essence of the interactions. I assume that there are two aggregate states, a “boom” state and a (rare) “bust” state. In the “boom” state, everything follows from the well-known analysis in the benchmark bank run literature, see section 4: essentially, things are fine. More serious problems arise in the bust state. I assume that the long-term securities become heterogeneous in terms of their long-term returns, and that local banks (together with their local depositors) hold heterogeneous beliefs regarding the portfolio of their core bank. Therefore, some local banks may withdraw early, even in the local consumption demands are “late”.

I allow for outside investors, who in total have unbounded liquidity, to become active in the market for the long-term securities which the core banks seek to unload. I seek to understand why these investors demand steeper discounts for the long-term securities than one would expect to see under “normal” conditions, described in section 5.3. I investigate two variants in particular.

The first hypothesizes that a subset of outside investors with finite resources has the expertise to evaluate the asset which the core banks wish to sell, and that the remaining vast majority of investors is highly uncertainty averse: they fear getting “stuck” with the worst asset among a diverse portfolio, and are therefore not willing to bid more than the lowest price, see section 5. The second reason is assuming risk-neutral investors together with adverse selection, i.e. an Akerlof-style lemons problem: whatever the market price, liquid core banks have an incentive to sell assets that will be a good deal for them and a bad deal to the buyers, leading to a low market price, see section 6. Both models generate a downward sloping demand curve or, more accurately, an upward sloping period-2 opportunity cost for providing period-1 resources per selling long-term securities from the perspective of the

individual core bank, holding aggregate liquidity demands unchanged.

However, the two variants have sharply different implications regarding the last of the stylized features listed above. More precisely, with uncertainty averse investors beyond a small and fixed pool of expert investors, a larger market share of troubled institutions dilutes the set of expert investors faster, leading more quickly to steep period-2 opportunity costs for providing period-1 liquidity. As more local banks seek to withdraw early and steeper discounting sets in earlier, further local banks are encouraged to withdraw from this as well as other core banks. This creates a systemic bank run. By contrast and with adverse selection, a larger pool of troubled institutions forced into liquidating their long-term securities leads to less free-riding of unaffected core banks, thereby lowering the opportunity costs for providing liquidity, see section 7. Since the models also have sharply different policy conclusions, I shall therefore argue to rather trust the policy conclusions from the uncertainty averse model and to discard the policy conclusions emerging from the adverse selection framework. In extension, one may therefore seek a deeper analysis of the 2008 financial crisis, using the tools of uncertainty aversion.

2 Relation to the literature

There obviously is a large literature expanding the Diamond-Dybvig bank run paradigm, and it includes investigations into systemic risk and the occurrence of fire sales. Additionally and due to recent events, a plethora of papers have appeared, seeking to provide explanations and coherent frameworks. A number of these papers share questions and insights with the paper at hand, but differences remain. A complete discussion is beyond the scope of this paper and excellent surveys are available elsewhere. Allen and Gale (2007), for example, have succinctly summarized much of the bank run literature, including in particular their own contributions, in their Clarendon lectures. Rochet (2008) has collected a number of his contributions with

his co-authors which help to understand banking crises and the politics and policy of bank regulation. A number of papers regarding the recent financial crises and avenues towards a solution have been collected in Acharya and Richardson (2009), and that literature keeps evolving quickly. Nonetheless, it may be good to provide at least a sketch on some related ideas and to describe how this paper relates to them.

While the Diamond-Dybvig model is originally about multiple equilibria (“bank run” vs “no bank run”), Allen and Gale put considerable emphasis instead on fundamental equilibria, in which it is individually rational for a depositor to “run”, even if nobody else does. In this paper I lean towards this fundamental view, but take somewhat of a middle ground. For the “bust” state, I shall argue, that some investors may believe the situation to be sufficiently bad that they withdraw, even if few others or nobody else does, while others are more optimistic. This can generate a partial fundamental run (based on the underlying beliefs), which may tip into a full-fledged bank run, see section 5.3.

Allen and Gale (1994, 2004b) have investigated the scope and consequences of cash-in-the-market pricing to generate fire sale pricing and bank runs. In the context here, the idea is that the additional investors need to bring cash to period 1, in case the core banks need to sell securities in period 1 in the bust state. If the bust state is sufficiently unlikely, the incentives to do so and therefore the additional liquidity is small: asset prices in the bust state are then not determined by the usual asset pricing equations, but rather by the amount of liquidity available. This may suffice as an explanation for current events. However, there clearly are plenty of investors out there who have liquidity available, when, say, the US government seeks to sell additional Treasury bonds. Why, then, should one assume the same investors to forget to bring their wallet, when other securities are auctioned off at firesale prices? While technical and legal details and institutional frictions and barriers surely play a key role in preventing outside investors to enter this market quickly, see Duffie (2009), it still remains surprising that they have not done

so eventually. The cash-in-the-market pricing may be viewed as a stand-in assumption for an endogenous reluctance of an otherwise deep market to buy the securities which the core banks are desperate to sell. Thinking about this reluctance and its implications is one of the key goals of this paper.

Diamond and Rajan (2009) have argued that banks have become reluctant to sell their securities at present, if they foresee the possibility of insolvency due to firesale prices in the future: the option of waiting allows banks to redistribute losses of depositors from the insolvency state in the future into private gains in the case of continued solvency. Their paper helps to explain the reluctance of banks to resolve their predicament by trading, but additional reasons are needed to generate the firesale price in the first place: the latter is the focus of this paper.

While the popular press views financial crises and bank runs as undesirable disasters, e.g. Allen and Gale (1998, 2004a) have shown that they instead may be an integral part of business cycles and can serve a socially useful rule by partially substituting for a missing market due to the uncontractible nature of deposit contracts. A number of regulatory and policy issues arise as a result. It follows directly, that a policy avoiding bank runs or financial crises under all circumstances may be welfare decreasing. On a more subtle level, Ennis and Keister (2008) have shown that ex-post efficient policy responses to a bank run of allowing urgent depositors to withdraw may actually increase the incentives to participate in a bank run and the conditions for a self-fulfilling bank run in the first place. Given these and a number of related results, the focus of this paper is on the positive analysis rather than a normative “second-best” analysis, though this would be a desirable part of further research (or a future draft of this paper). Likewise (and regarding potentially welfare-improving private sector solutions), we assume a particular structure of the contracts, markets and asymmetries of beliefs and information, rather than requiring contracts to be optimal, as in Green and Lin (2003) or Ennis and Keister (2008).

There is a large literature on systemic risk and contagion, both for inter-

national financial crises (which I shall not even attempt to review here) as well as for banking crises. For example, Cifuentes et al (2005) have studied the interplay between uncontingent capital adequacy requirements and the endogenous collapse of prices and balance sheets, as banks need to unload assets in order to meet these requirements. They assume that demand for these assets is downward sloping: this paper seeks to investigate why. Allen and Gale (2000) have studied the possibility for contagion in a sparse network of banks interlinked by mutual demand deposits, where a collapse of one bank can lead to a domino effect per their large withdrawals on their direct neighbor. Here, a hierarchy is instead assumed, where local banks hold deposit contracts on core banks, who in turn use the market to obtain liquidity, rather than other core banks. Diamond and Rajan (2005) have investigated the contagious nature of bank failures, arguing that bank failures can shrink the common pool of liquidity, thereby possibly leading a meltdown of the entire system. They assume that the returns on long-term projects can only be obtained by banks, and that any securities written on these returns can only be traded by banks. While this paper shares the central idea of a shortage of a common pool of liquidity and the feature, that projects are run by “managing” (local) banks, I allow outside investors to buy the securities written on these projects and collect their returns. In essence, I assume that a mortgage-backed security will pay its return, irrespective of who actually holds that security. If that perspective is appropriate, then one needs to understand why outside deep-pocket investors do not buy these securities, if they are indeed severely undervalued.

Uncertainty aversion - or Knightian uncertainty - is a crucial ingredient in this paper. There obviously is a large literature investigating its implications for asset markets and equilibria. For some recent examples, one may want to consult Hansen and Sargent (2008) and Backus-Routledge-Zin (2009), and the references therein.

3 The model

There are three periods, $t = 0, 1, 2$. There are two fundamental aggregate states: “boom” and “bust”. The aggregate state will be learned by all participants in period 1. There are four types of agents or agencies:

1. Depositors in locations $s \in [0, 1]$.
2. Local banks in locations $s \in [0, 1]$.
3. Core banks, $n = 1, \dots, N$.
4. Outside investors $i \in [0, \infty)$.

There are two types of assets

1. A heterogeneous pool of long-term securities (“mortgage backed securities”), backed by long-term projects in locations $s \in [0, 1]$.
2. A short term security, providing a safe return of 1.

Figure 1 provides a graphical representation of the model: there, I have drawn the unit interval as a unit circle.

Let me describe each in turn. As in Allen and Gale (2007), I assume that depositors have one unit of resources in period 0, but that they care about consumption either in period 1 (“early consumer”) or in period 2 (“late consumer”). As in Smith (1983), I assume that all depositors at one location are of the same type. They learn their type in period 1. I assume that a fraction $0 < \varphi < 1$ of locations has early consumers and a fraction $1 - \varphi$ has late consumers. I assume that the realization of the early/late resolution is iid across locations and that depositors are evenly distributed across locations. I assume that depositors learn of their type in period 1. Ex-ante utility is therefore given by

$$U = \varphi E[u(c_1)] + (1 - \varphi) E[u(c_2)] \tag{1}$$

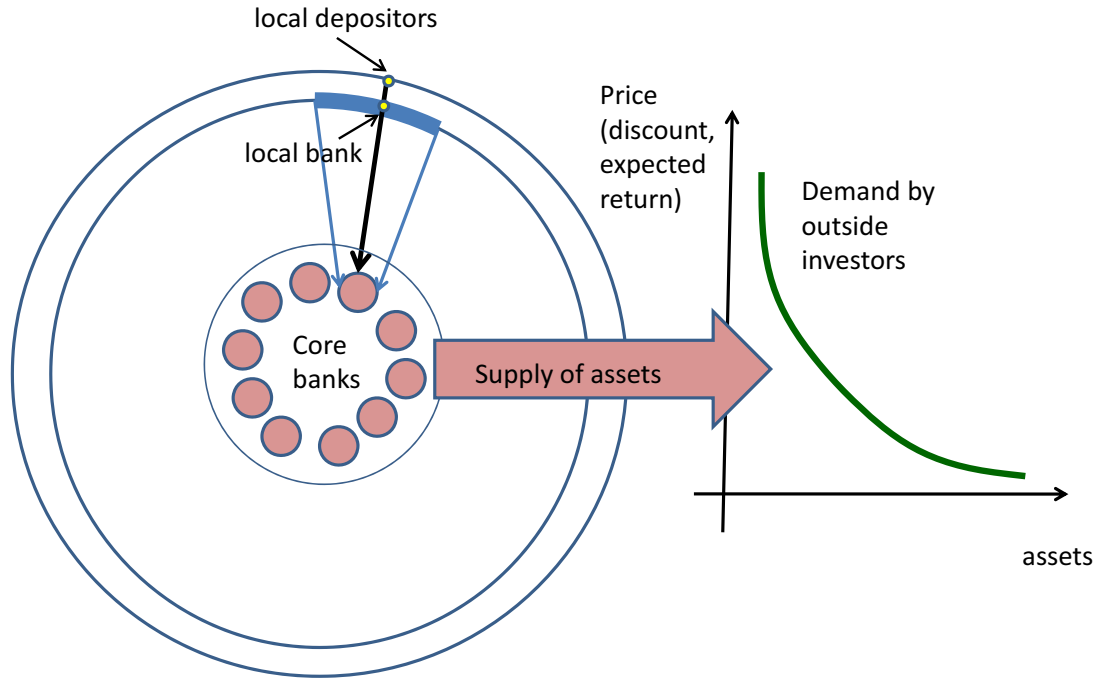


Figure 1: *A graphical representation of the model.*

where c_1 and c_2 denotes consumption at date 1, if the consumer is of the early type and c_2 denotes consumption at date 2, if the consumer is of the late type and where $u(\cdot)$ satisfies standard properties. This heterogeneity in consumption preference induces a role for liquidity provision and maturity transformation, as in Diamond and Dybvig (1983) and the related literature. I assume that depositors only bank with the local bank in the same location. This lack of diversification can be thought of as arising from some unspecified cost to diversification, e.g. the impossibility for banks or depositors to travel to other locations. An alternative way to think about this is that s actu-

ally enumerates the deposit banks in existence and each location denotes its customer base, noting that depositors are observed to typically spread their bank accounts across very few banks only.

At date zero, local banks can invest in long-term projects (“mortgages”) of location s or short-term securities, and they can invest in short-term securities in period 1, but they cannot invest in long-term securities. Long-term projects pay off only in period 2. I assume that long-term projects cannot be terminated (“liquidated”) prematurely and that they require nonnegative investments in period 0. I assume that local banks administer the local long-term projects, delivering their payment streams to whoever finances them originally.

I allow local banks to open accounts with the core banks, depositing resources in period 0 and taking withdrawals in period 1 and/or period 2. Again, for some unspecified cost reasons, we assume that local banks operate a deposit account only with one of the core banks.

Core banks invest the period-0 deposits received from local banks in local long-term projects, and turn their period-2 payments into long-term securities. In all periods, core banks can trade in short-term as well as long-term securities.

In the aggregate “boom” state, local long-term projects return $R_{\text{boom}} + \epsilon_s$, where ϵ_s is a random variable with mean zero, distributed independently and identically across locations $s \in [0, 1]$. Long-term securities pool these risks¹. Thus, in the aggregate “boom” state, the long-term securities all

¹To provide this with a bit of formal structure, suppose there are $m = 1, \dots, M$ long-term securities, suppose that $(A_m)_{m=1}^M$ is a partition of $[0, 1]$ with each A_m having equal Lebesgue measure, and suppose that the payoff for the long term security with index m is the integral of all long-term projects $s \in A_m$. The law of large numbers in Uhlig (1996) then implies the safe return here. Conversely, knowing the return of the long-term securities, one might directly assume that the long-term projects return this amount plus the idiosyncratic noise ϵ_s . This structure can also be used for the “bust” episode. I will not make further use of this formal structure, though.

return R_{boom} . I assume that

$$\varphi u(0) + (1 - \varphi)u(R_{\text{boom}}) < u(1) \quad (2)$$

$$1 < R_{\text{boom}} < \frac{u'(1)}{u'(R_{\text{boom}})} \quad (3)$$

If $u(c)$ is CRRA with an intertemporal elasticity of substitution below unity and if $R_{\text{boom}} > 1$, both equations are satisfied. Further, (2) is generally satisfied, if $(1 - \varphi)R_{\text{boom}} < 1$.

In the “bust” state, each long term securities offers a safe² return R , but these returns are heterogeneous and distributed according to $R \sim F$, drawn from some distribution F on some interval $[\underline{R}, \bar{R}]$, where $0 < \underline{R} \leq \bar{R} < \infty$, with unconditional expectation R_{bust} , satisfying

$$R_{\text{bust}} \leq R_{\text{boom}} \quad (4)$$

Once the aggregate state is revealed to be a “bust” in period 1, I assume that core banks all know the type of long-term securities in their portfolio, i.e. know the period-2 return of the securities in their portfolio, and by implication the return distribution of their securities. The entire portfolio of the long-term securities has the safe return R_{bust} , and I will assume the same for the portfolio for any core bank. Particular long-term securities within that portfolio have different returns, however. An outside investor who buys one particular security, and e.g. draws a random security from the entire pool therefore exposes himself to that return risk.

But even for the entire portfolio, the composition and its average (or total safe) return is assumed to be unknown to depositors and local banks. Instead, they form heterogeneous beliefs about that. I assume that local banks at location s and its depositors believe their core bank to hold a portfolio with return distribution $F(\cdot; s)$, where $F(\cdot; \cdot)$ is measurable and

²It is not hard to generalize this to risky returns, but the additional insights may be small. From the perspective of outside investors, who do not know the specific R , the returns will be uncertain, and this is what matters.

$F(\cdot; s)$ is a distribution function. One may wish to impose that

$$F(R) = \int F(R; s)ds \tag{5}$$

so that aggregate beliefs accurately reflect the aggregate distribution, but there is a potential disagreement at the local level. None of the results appear to depend on (5), however.

For simplicity, I shall assume that core banks actually all hold exactly the same portfolio, i.e. there is a mismatch between the beliefs of the local banks and the portfolio of their core bank. I assume that core banks do not know the belief $F(\cdot; s)$ of their local banks at date 0 and contracting time³ and cannot condition permitted withdrawals on these beliefs at time 1 or time 2. One possible interpretation of the heterogeneity in beliefs is that it arises from heterogeneous signals arriving at each location, otherwise starting from a common prior. With that interpretation, one needs to insist on local banks not updating their beliefs in light of the actions of other local banks in the analysis below, however. There may be a version of the model, where the local signal is sufficiently strong so as to overwhelm the market information contained in the withdrawal decisions of all other local banks: further research may be able to tell.

Finally, there is a large pool of outside investors $i \in [0, \infty)$. These investors can invest in the long-term securities or the short-term securities in period 1, though not in period 0. Each investor is endowed with one unit of resources. I do not allow them to engage in short-selling. They are assumed to be risk neutral, discounting the future at some rate β , with

$$\beta R_{\text{boom}} < 1 \tag{6}$$

It remains to specify the information and beliefs of these investors. I shall investigate three variants.

³E.g., suppose that the beliefs are $F(R; s) = F^*(R; s + X \text{ mod } 1)$, where X is a random variable uniformly distributed on $[0, 1]$ and drawn at date 1 and $F^*(\cdot; \cdot)$ is a commonly known function.

1. [**Benchmark:**] As a benchmark, I assume that outside investors are risk-neutral, discounting resources between period 1 and period 2 at rate β . Furthermore, I assume that core banks sell bundles of their long-term securities, which have the same return distribution as their total portfolio (or, equivalently, sell randomly selected long-term securities, but cannot “adversely select” the long-term security they wish to sell).
2. [**Uncertainty Aversion:**] I assume the investors to be uncertainty averse, following Schmeidler (1989) or Epstein (1999). Alternatively, one may interpret these investors as following robust control rules against downside risks, following Hansen and Sargent (2008). There may also be an interpretation as extreme loss aversion, following Tversky and Kahnemann (1991) and Barberis, Huang and Santos (2001). In either case, I presume the following starkly simplified structure: at the cost of more complexity, this is not hard to generalize. Given a security drawn from a pool of securities with some interval as the support of its returns, these investors are willing to pay β times the lower bound of this interval as the price per unit invested, i.e. the investor is risk neutral, but minimizes over all probability distributions with support on that interval.

Let $\omega \geq 0$. The group $i \in [0, \omega]$ of these investors is assumed to have the expertise of discerning the quality of the long-term securities, i.e. they know the return of a given long-term security, the support interval is a single number, and they are therefore willing to buy them when the return exceeds $1/\beta$. I call them the expert investors. All other investors $i > \omega$ only know the distribution F and the equilibrium, but not the specific return of some offered long-term security. They use the support interval $[\underline{R}, \bar{R}]$ and are therefore willing to pay

$$\beta \underline{R}$$

per unit invested.

Another way of reading these preferences is that investors are suspicious or perhaps even paranoid. If offered to trade a security from the described set, they will fear that they will always be offered the security with the lowest of these returns, even though this cannot happen to all investors in equilibrium. A third interpretation is that these are traders working on behalf of institutional investors drawn to the profit opportunities in the market, who face lopsided incentives for investing in a bust market: due to the complexity of these securities, they cannot afford to risk losing money ex post, as their managers may not be able to tell whether this was indeed just a case of bad luck or a case of poor research.

3. [**Adverse Selection:**] I assume that outside investors are risk-neutral, discounting period-2 payoffs at rate β , but cannot distinguish between the qualities of the long-term securities sold to them. I assume that core banks can “adversely select” the long-term security they wish to sell. I assume in this scenario, that all investors know that all core banks hold a portfolio of long-term securities with return distribution $R \sim F(R)$.

The timing of the events is now as follows. In period 0, core banks offer deposit contracts to local banks, offering state-uncontingent withdrawals of r in period 1 per unit deposited. Local banks offer state-uncontingent withdrawals of \tilde{r} in period 1 per unit deposited. In period 1 and depending on the aggregate state, local banks may withdraw r from their core bank. The core banks match these withdrawal demands from payoffs of their portfolio of short-term securities as well as sales of long-term securities. If they cannot meet all withdrawal demands, they declare bankruptcy. In that case, I assume that all local banks, who have decided to withdraw, obtain an equal pro-rata payment, splitting the entire resources of the bankrupt core bank across local banks in proportion to their withdrawal demands.

I assume that Bertrand competition in these contracts makes local banks pay out everything to their depositors⁴ and likewise makes core banks pay out everything to local banks. Therefore, any resources left in period 2 will be paid in proportion to the remaining deposits. Furthermore, local banks will be indifferent which particular core bank to choose. Let

$$\nu : [0, 1] \rightarrow \{1, \dots, N\}$$

be the core bank selection function, i.e. let $\nu(s)$ be the core bank selected by the local bank s . I assume $\nu(\cdot)$ to be measurable⁵ In the numerical examples, I will let $\nu(s) = \max\{n \mid n < Ns + 1\}$, i.e. assume that the local banks distribute themselves uniformly across core banks. To analyze what happens when the number of distressed banks increases, I consider in particular the case, where a fraction μ of the core banks (in terms of their market share) face the same heterogeneous beliefs of their local banks, whereas a fraction $1 - \mu$ of core banks has local banks, who all (accurately) believe the portfolio of their core bank to be given by securities with $R \sim F(R)$. It turns to the specifics for that assumption in the numerical example in subsection 5.4 for the uncertainty-aversion case and provide results as part of the general analysis in section 6 for the adverse selection case.

Finally and for simplicity, I assume that the “bust” state is sufficiently unlikely a priori, so that r and \tilde{r} are determined entirely from the “boom” state calculus⁶.

⁴For that, one may want to assume that there are at least two local banks in each location, though that assumption is immaterial for the rest of the analysis

⁵An alternative is to assume $\nu(\cdot)$ to be random and use Pettis integration, see Uhlig (1996).

⁶It would not make much difference for the analysis, if instead one were to calculate r and \tilde{r} from a full probabilistic analysis.

4 Analysis: Preliminaries

It is useful to first analyze some special cases in order to set the stage of the analysis of the bust state. The analysis of these special cases are the same, no matter which assumption has been made about the type of outside investors.

4.1 No core banks

Consider first the environment above without core banks. The investors then do not matter: they would love to short-sell the short-term securities, but they cannot do so (and that certainly seems reasonable, if one imagines the short-term securities to be Treasury bills). In that case, local banks offer contracts to their local depositors. Note that all their depositors wish to either only consume at date 1 or at date 2. Due to local Bertrand competition, the local banks will choose the deposit contract that maximizes expected utility (1).

Consider first the choice between investing everything in the long term project versus investing everything in short-term securities. In the first case, depositors only get to consume in case they turn out to be late consumers, and their ex ante utility is

$$U = \varphi u(0) + (1 - \varphi)E[u(R)] \leq \varphi u(0) + (1 - \varphi)u(R_{\text{boom}})$$

due to concavity of $u(\cdot)$ as well as (4). In the second case, depositors can consume in both periods, at ex ante utility equal to $u(1)$. If the choice is “either-or” and since the latter is larger than the former due to (2), local banks will only invest in short-term securities. One can view this as a version of 100% reserve banking. Note that there cannot be a bank run or financial crisis in this situation, but, as is well known and as we shall see, this solution is inefficient.

Generally,

$$U(y) = \varphi u(y) + (1 - \varphi)u((1 - y)R_{\text{boom}} + y)$$

is a concave function of the fraction y invested in the short-term security: the corner solution $y = 1$ obtains, if

$$(1 - \varphi)R_{\text{boom}} < 1 \tag{7}$$

and otherwise one obtains an interior solution. The inefficiency still remains, see the discussion in Allen and Gale (2007), chapter 3.

4.2 Only “boom” state

To set the stage of the “bust” state analysis as well as an important benchmark, consider the situation with only a boom state. Competition drives banks to maximize the ex-ante welfare of depositors. This amounts to choosing the amount x to be invested in the long-term securities, y to be invested in the short-term security and the amount z of the investment in the long-term security to be sold to outside investors at date 1 in order to solve

$$\begin{aligned} & \max_{x,y,z} \varphi u(c_1) + (1 - \varphi)u(c_2) \\ & s.t. \quad \varphi c_1 = y + \beta R_{\text{boom}} z \\ & \quad (1 - \varphi)c_2 = R_{\text{boom}}(x - z) \\ & \quad 0 \leq x, 0 \leq y, x + y = 1, 0 \leq z \leq x \\ & \quad c_2 \geq c_1 \geq 0 \end{aligned}$$

where the last constraint prevents local banks in locations with late consumers to withdraw their funds in period 1 and investing in the short security. Note that the optimal solution will have $z = 0$ due to (6): it is cheaper to deliver resources for period 1 per investing in the short-term security rather than investing it in the long-term security and selling it at a steep discount to the outside investors. With the interpretation of the sale to outside investors as the liquidation value of long-term projects, this problem is a baseline problem in the literature on banking and has been thoroughly analyzed in the literature, see e.g. Allen and Gale (2007), in particular chapter 3. A brief description of the solution is useful for the analysis below, however.

Due to (3) there will be an interior solution with $R_{\text{boom}} > c_2 > c_1 > 1$ with

$$\frac{u'(c_1)}{u'(c_2)} = R_{\text{boom}} \quad (8)$$

As a consequence of this as well as (6),

$$c_2 < \frac{r}{\beta} \quad (9)$$

holds: generally, this is rather far from being a sharp bound.

The period-1 withdrawals offered by the deposit contracts are

$$r = \tilde{r} = c_1 = \frac{y}{\varphi}$$

and the bank invests

$$x = 1 - \varphi r \quad (10)$$

in long term securities. As is well-understood, the solution is more efficient than the solution with 100% reserve banking of subsection 4.2, but potentially subject to bank runs. For example, if preferences are CRRA with an intertemporal elasticity of substitution below unity,

$$u(c) = \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma}, \quad \text{where } 0 < \sigma < 1 \quad (11)$$

and if $R_{\text{boom}} > 1$, then (3) is satisfied and

$$r = \left(\varphi + (1 - \varphi) R_{\text{boom}}^{\sigma-1} \right)^{-1}, \quad c_1 = r, \quad c_2 = R_{\text{boom}}^{\sigma} r = R_{\text{boom}} \frac{1 - \varphi r}{1 - \varphi} \quad (12)$$

There are perhaps two twists compared to the standard solution. First, core bank runs (i.e. local banks running on the core banks) can occur but they invoke the resale of long-term securities to outside investors at the market discount rate rather than the early termination of projects. This already could be viewed as a solution to the task set forth in the introduction of creating a bank-on-bank run in terms of marketable securities. It is obviously a rather trivial solution, as it simply amounts to one of many possible interpretations of the standard bank run model. That literature is typically silent

on what it means to “liquidate” the long-term projects, and selling them at a steep discount certainly is consistent with these models.

Second, aside from liquidity provision, the core banks also offer insurance against the idiosyncratic fluctuations in the returns of long-term projects. Consider a slightly different environment, in which local depositors split into fractions φ of early consumers and $(1 - \varphi)$ of late consumers at each location. The local bank may still solve a problem as above, but with the random return $R_{\text{boom}} + \epsilon_s$ in place of the safe return R_{boom} . It is obvious, that the solution involving securitization is welfare improving compared to this “local-only” solution, which exposes local depositors to additional local risks. Moreover, it is more likely to trigger “fundamental” bank runs, where long-term depositors run on the local bank, if $R_{\text{boom}} + \epsilon_s < c_1$. Indeed, absent intermediation by core banks, these fundamental bank runs are welfare-improving compared to regulating that deposit contracts need to avoid fundamental bank runs at the local level: these bank runs provide a partial substitute to the missing insurance market, see Allen and Gale (2007). Put differently, securitization improves welfare and makes the system less prone to local bank runs, but exposes it instead to the possibility of “systemic” runs on core banks and thereby to “contagion” across different locations. This interdependence has been analyzed in the literature previously, see e.g. the exposition in chapters 5 and 10 of Allen and Gale (2007), and the literature discussion there.

4.3 The “bust” state and the classic bank run case

To analyze the full model, we assume that the probability of the “bust” state is vanishingly small⁷. It therefore remains to analyze the “bust” state, fixing the first-period withdrawal r of the deposit contracts and the total investments r in the short-term securities and the long-term securities $1 - r$ as provided by the solution to the “boom”-only situation above.

⁷Alternatively, assume that the “bust” state was “irrationally” ignored at the time the deposit contracts were signed.

Note first, that in the absence of a run,

$$c_{2,\text{bust}}(0) = R_{\text{bust}} \frac{(1 - \varphi r)}{1 - \varphi}$$

where I use the argument “(0)” to denote that the fraction zero of local banks in locations with late consumers run. Therefore, if $c_{2,\text{bust}}(0) < r$, there will be a fundamental bank run, even if core banks hold the same “market” portfolio of long-term securities and local banks believe them to do so, as insurance against the “boom-bust” aggregate uncertainty is not available. For CRRA preferences (11) and therefore (12), this will be the case if

$$R_{\text{bust}} < R_{\text{boom}}^{1-\sigma} \quad (13)$$

Suppose even further, that all long-term securities offer the return R_{bust} and that a fraction θ of all local banks serving late consumers opt for early withdrawal. The following algebra is well understood, but will be useful for comparison to the more general case. The core banks meet the additional liquidity demands by selling a fraction ζ of its long-term portfolio or $z = x\zeta$ units of its long-term securities to obtain additional liquidity ℓ , where

$$r\theta(1 - \varphi) = \ell = \beta R_{\text{bust}}(1 - \varphi)\zeta \quad (14)$$

The securities are discounted by outside investors at $q = \beta$ and $1/\beta$ is the opportunity cost in terms of period-2 resources for providing one unit of resources of period-1 withdrawals. This leaves the remaining late-consumer local banks with

$$\begin{aligned} c_2(\theta) &= \frac{c_{2,\text{bust}}(0) - r\theta/\beta}{1 - \theta} \\ &= \frac{1}{1 - \theta} \left(\frac{1 - \varphi r}{1 - \varphi} R_{\text{bust}} - \frac{r}{\beta} \theta \right) \end{aligned} \quad (15)$$

in period 2. Let θ^* solve $c_2(\theta) = r$,

$$\theta^* = \frac{\beta}{1 - \beta} \left(\frac{1}{r} \frac{1 - \varphi r}{1 - \varphi} R_{\text{bust}} - 1 \right) \quad (16)$$

If $\theta^* < 0$, there is a fundamental bank run: all local banks will try to withdraw early, because even if no one else did, second-period consumption would be below the promised withdrawal at date 1, $c_2(0) < r$. Fundamental bank runs may actually be welfare-improving, as they partially complete missing markets, see Allen and Gale (2007). If $0 < \theta^* < 1$, there is scope for a Diamond-Dybvig “sunspot” bank run. If late-consumer local banks believe that the fraction of early withdrawals by late-consumer local banks exceeds θ^* , they will withdraw early too, so that $\theta = 1$ in equilibrium. If late-consumer local banks believe that the fraction of early withdrawals by late-consumer local banks is below θ^* , they will choose to wait until period 2, and $\theta = 0$ in equilibrium.

There are therefore three scenarios, namely a fundamental bank run, a Diamond-Dybvig “sunspot” bank run and no bank run. I call these the “classic bank run” scenarios, for comparison with the more general case to be analyzed below.

5 The “bust” state with uncertainty averse investors.

Before proceeding to analyze the problem of a single core bank, consider the dependence of the market price for any security, in dependence of the aggregate liquidation L of long-term securities. If $L < \omega$, there is an “excess supply” of expert investors. They will bid more than non-expert uncertainty-averse investors for the securities sold: therefore, the market price will be the final payoff, discounted at β . If $L > \omega$ (and, by assumption, if $L = \omega$), however, the “marginal” investor is an uncertainty-averse investor, willing only to pay βR , regardless of the asset. This then must be the market price. Thus, given some specific security, its market price is a decreasing function of the aggregate liquidity needs L . This is the key feature needed in this section. The market price also happens to fall discontinuously, as L crosses ω : this is due to our particularly stark assumption regarding the uncertainty aversion of

the outside investors and assuming a discontinuity at ω . This is not essential to the results, and can be relaxed, at the price of higher complexity of the analysis. The required general construction of an equilibrium has therefore been postponed to appendix A, whereas the construction in subsection 5.2 relies on the two-step form of the demand function described above.

One way to read this section that it provides an alternative reason or interpretation for the cash-in-market pricing as in Allen and Gale (1994) or Allen and Gale (2007), chapter 4: there is unlimited amount of cash here, but the “expert cash” is limited indeed. The cash-in-the-market pricing scenario corresponds to an extreme version of the uncertainty averse investors, where the non-expert investors are bidding zero for all assets. In Allen and Gale (2007) therefore, the core banks cannot raise more liquidity than ω : should they reach that point, the sales price for their assets will be determined by the cash-in-the-market pricing, thereby determining the payoff for all depositors in period 1 by a now bankrupt system. In contrast, the analysis below allows for partial bankruns: as core banks need to provide more liquidity than is in the hands of the expert investors, they will suffer steep opportunity costs in terms of period 2 resources. Nonetheless, sufficient funds may be left over in period 2 to pay of the remaining late-withdrawing local banks.

5.1 The problem of a single core bank and its local banks.

Consider a core bank and suppose that a fraction θ of its local banks at late-consuming locations withdraw early. If $L < \omega$, so that only expert investors are present, the opportunity costs in terms of period-2 resources for providing one unit of resources for period-1 withdrawals is $1/\beta$. If $L \geq \omega$, however, the core bank obtains the market price $\beta \underline{R}$, regardless of the security sold. It will therefore sell its securities with the lowest period-2 payoff first. Suppose the core bank started initially with μ resources. It therefore purchased $(1 - \varphi r)\mu$ units of long-term securities. Given the early withdrawals, the core bank

needs to raise period-1 liquidity $\ell = r\theta(1 - \varphi)\mu$, and hence sell $\ell/(\beta\underline{R})$ units of its long-term securities, i.e. the fraction

$$\zeta(\theta) = \frac{1 - \varphi}{1 - \varphi r} \frac{r\theta}{\beta\underline{R}}$$

similar to equation (14).

Consider now one of its local banks and its beliefs $F(\dots; s)$ about the return distributions of the securities in the portfolio of its core bank (before selling any of its securities). For ease of notation, I shall write G in place of $F(\cdot; s)$. Let

$$G^{-1}(\tau) = \sup\{R \mid G(R) < \tau\}, \quad \tau \in [0, 1] \quad (17)$$

be the inverse function of G , see figure 2. Note that

$$E_G[R \mid R \leq G^{-1}(\zeta)] = \frac{\int_0^\zeta G^{-1}(\tau) d\tau}{\zeta} \quad (18)$$

is the expected return of all returns below the level given by $G^{-1}(\zeta)$, under the distribution G . Also note that $G^{-1}(\tau)$ is a continuous function of τ and $E_G[R \mid R \leq G^{-1}(\zeta)]$ is a continuous function of ζ .

From the perspective of this local bank, the period-2 opportunity costs for period-1 withdrawals are

$$\Gamma(\theta, L; G) = \frac{1}{\beta} \left(\mathbf{1}_{L < \omega} + \frac{E_G[R \mid R \leq G^{-1}(\zeta(\theta, L))]}{\underline{R}} \mathbf{1}_{L \geq \omega} \right) \quad (19)$$

Likewise,

$$q(\theta, L; G) = \frac{1}{\Gamma(\theta, L; G)}$$

is the effective liquidation discount rate of period-2 resources.

Proposition 1 1. $\Gamma(\theta, L; G)$ is increasing and continuous in θ .

2. $\Gamma(\theta, L; G)$ is increasing in L and satisfies $\beta\Gamma(\theta, L; G) \geq 1$. There is no dependence on L , if $\omega = 0$ or if $\omega = \infty$, i.e. in the absence of expert investors, or if all investors are experts.

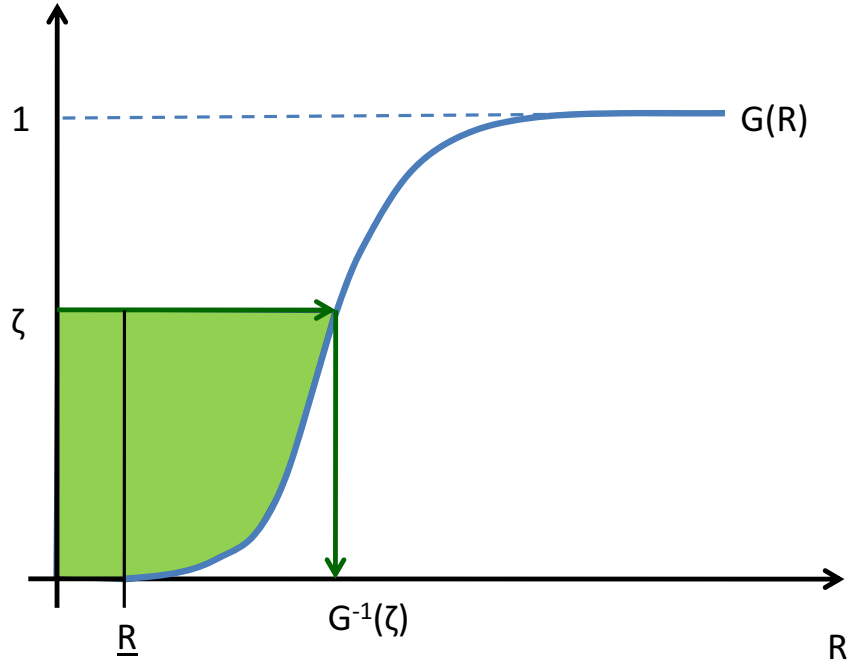


Figure 2: *The function G^{-1} and expected returns.*

3. *Suppose that H first-order stochastically dominates G . Then*

$$\Gamma(\theta, L; G) \leq \Gamma(\theta, L; H)$$

i.e. $\Gamma(\theta, L; G)$ is increasing in G , when ordering distributions by first-order stochastic dominance.

Proof:

1. *Note that $\zeta(\theta)$ and therefore $E_G[R \mid R \leq G^{-1}(\zeta(\theta))]$ is increasing in θ . Continuity is a consequence of the continuity of $\zeta(\theta, L)$ in θ .*

2. Note that $\underline{R}^{-1}E_G[R \mid R \leq G^{-1}(\zeta(\theta))] \geq 1$.
3. Define H^{-1} as the inverse of H as in 17. Since $H(R) \leq G(R)$ for all R , $H^{-1}(\tau) \geq G^{-1}(\tau)$ for all $\tau \in [0, 1]$. Equation (18) shows that $E_G[R \mid R \leq G^{-1}(\zeta)] \leq E_H[R \mid R \leq G^{-1}(\zeta)]$ and the claim follows.

•

As a result, a local bank with beliefs $G = F(\cdot; s)$ perceives the second-period payoff to be

$$c_2(0; G) = E_G[R] \frac{1 - \varphi r}{1 - \varphi} \quad (20)$$

if there are no withdrawals of late-consumer local banks in period 1, i.e. if $\theta = 0$. With withdrawals of a fraction θ of late-consumer local banks, the (perceived) remaining resources at period 2 per late consumer for this core bank is therefore

$$c_2(\theta, L; G) = \frac{c_2(0; G) - r\theta\Gamma(\theta, L; G)}{1 - \theta} \quad (21)$$

which generalizes (15). The local bank will therefore surely opt for period-1 withdrawal, if $c_2(\theta, L; G) < r$.

It may be useful to note that $c_2(\theta, L; G)$ is not monotone in G , when ordering G according to first-order stochastic dominance: while the first term is increasing in G , the second term is now decreasing, due to the negative sign. Indeed, it is easy to construct examples for both a decreasing or an increasing behaviour, by keeping one of the terms nearly unchanged while the other moves significantly.

However, $c_2(\theta, L; G)$ is monotonously decreasing in L and furthermore, it is decreasing in θ under the mild condition (22), which generalizes (9) and which essentially assures, that no late-withdrawal local bank will be happy about other late consumer local banks withdrawing early. For the following proposition, the properties of Γ in proposition (1) suffice: this is useful, if

generalizing the results in this paper to a smooth transition between expert investors and non-expert investors.

Proposition 2 *Assume that $\Gamma(\cdot, \cdot; \cdot)$ satisfies the properties listed in proposition (1). Then,*

1. $c_2(\theta, L; G)$ is monotonously decreasing in L .
2. $c_2(\theta, L; G)$ is continuous in θ .
3. Assume that

$$c_2(0; G) < \frac{r}{\beta} \quad (22)$$

Then $c_2(\theta, L; G)$ is strictly decreasing in θ .

Proof:

1. This follows directly from proposition 1.
2. Continuity follows from the continuity of $\Gamma(\theta, L; G)$ in θ .
3. Write $c_2(\theta, L; G)$ as

$$c_2(\theta, L; G) = c_2(0; G) - \frac{\theta}{1 - \theta} \chi(\theta, L; G) \quad (23)$$

where

$$\chi(\theta, L; G) = r\Gamma(\theta, L; G) - c_2(0; G) \quad (24)$$

is strictly positive and increasing in θ per (22) and proposition 1. Let $\theta_a < \theta_b$. Then

$$\begin{aligned} c_2(\theta_a, L; G) &= c_2(0; G) - \frac{\theta_a}{1 - \theta_a} \chi(\theta_a, L; G) \\ &> c_2(0; G) - \frac{\theta_b}{1 - \theta_b} \chi(\theta_a, L; G) \\ &\geq c_2(0; G) - \frac{\theta_b}{1 - \theta_b} \chi(\theta_b, L; G) \\ &= c_2(\theta_b, L; G) \end{aligned}$$

•

5.2 Equilibrium

To analyze the equilibrium, I shall now exploit that the dependence of Γ on L is the two-step threshold function given by (19). This particular form is rather special, however, and arises from the rather stark assumption regarding the difference between expert investors and other investors. Appendix A provides an analysis of the equilibrium also just under the assumption, that Γ satisfies the properties listed in proposition 1. It also contains a more formal definition of equilibrium.

With the two-step threshold function given by (19), each local bank needs to consider aggregate liquidity L only through the event that $L < \omega$ or that $L \geq \omega$.

Assume that (22) is true for all conjectured distributions $G = F(\cdot, s)$. Therefore, if local banks opt for early withdrawals at some level of market liquidity or some fraction of other early withdrawals, they will do also for higher levels of L and θ . As in appendix A, let

$$S_n(\theta, L) = \{s \mid \nu(s) = n, c_2(\theta, L; F(\cdot, s)) < r\} \quad (25)$$

be the set of local banks with deposits at core banks n , which will surely withdraw early, if a fraction θ of depositors at core bank n do, and if there is total liquidity demand L .

Given L and a core bank n , define the mappings

$$\eta_{n,L} : [0, 1] \rightarrow [0, 1]$$

per

$$\eta_{n,L}(\theta) = \lambda(S_n(\theta, L))$$

where $\lambda(\cdot)$ denotes the Lebesgue measure. Intuitively, if aggregate liquidity needs are given by L and if all local banks at core bank n conjecture the fractions θ of late consumer local banks to withdraw early at that core bank, then the fractions $\eta_{n,L}(\theta)$ surely will. Fixed points of η are bank runs, where withdrawers strictly prefer to do so.

Proposition 3 *Assume that (22) is true for all conjectured distributions $G = F(\cdot, s)$.*

1. $\eta_{n,L} : [0, 1] \rightarrow [0, 1]$ is increasing and continuous from the left, i.e. for $\theta_j \rightarrow \theta_\infty$, $\theta_j < \theta_\infty$, we have $\eta_{n,L}(\theta_j) \rightarrow \eta_{n,L}(\theta_\infty)$.
2. Given n, L , let $\theta_0 = 0$ and construct the sequence

$$\theta_{j;n,L} = \eta_{n,L}(\theta_{j-1;n,L})$$

Then $\theta_{j;n,L} \rightarrow \theta_{\infty;n,L}$, which satisfies $\theta_{\infty;n,L} = \eta_{n,L}(\theta_{\infty;n,L})$. Furthermore,

$$\theta_{\infty;n,L} = \min\{\theta \mid \theta \geq \eta_{n,L}(\theta)\} \quad (26)$$

Proof:

1. This follows from proposition 2.
2. The first part follows from the first part. For (26), consider any $\theta < \theta_{\infty;n,L}$. Therefore, for some j ,

$$\theta_{j-1;n,L} \leq \theta < \theta_{j;n,L} = \eta_{n,L}(\theta_{j-1;n,L}) \leq \eta_{n,L}(\theta)$$

or $\theta < \eta_{n,L}(\theta)$.

•

Given L , define

$$\vec{\theta}_{\infty;L} = (\theta_{\infty;1,L}, \dots, \theta_{\infty;N,L})$$

If $L(\vec{\theta}_{\infty;0}) < \omega$, then this is a **partial fundamental bank run**, but it does not have any systemic feature. If $L(\vec{\theta}_{\infty;0}) > \omega$, however, then pick any⁸ $\tilde{\omega} > \omega$. The **partial fundamental bank run** is now given by $\vec{\theta}_{\infty;\tilde{\omega}}$, and it involves a systemic spillover. Intuitively (and along the sequence constructed above), as more local banks become skeptical about the remaining resources at their core banks, more core banks need to obtain liquidity in period 1, eventually exceeding the resources supplied by expert investors. This leads to a decline (here, a collapse) in period 1 prices, exacerbating the problem.

Note that at $L(\vec{\theta}_{\infty;\tilde{\omega}}) > \omega$ and therefore, $c_2(\theta, L; F(\cdot, s))$ is continuous in L around $L = L(\vec{\theta}_{\infty;\tilde{\omega}})$, thereby satisfying the assumption in the last part of proposition (6).

5.3 The “bust” state with risk-neutral investors and no adverse selection.

Suppose instead (and as a benchmark for comparison), that investors are risk-neutral and that there is no adverse selection in selling the long-term securities. This may be a sensible assumption if all long-term securities return the same amount R_{bust} , despite the heterogenous beliefs of the local banks to the contrary. Or this may be sensible, if one were to assume that core banks can only sell well-defined (or well-audited) portfolios of long-term securities, whose risk-characteristics are known to the market. Finally, this may be sensible if one is to assume that $\omega = \infty$ in the analysis above. In all these cases, the outside investors discount future payments at rate β .

The analysis of the “bust” state is now a corollary to the analysis above by setting $\omega = \infty$ and using $\Gamma(\theta, L; G) = 1/\beta$ throughout. The details can be skipped, except perhaps for some useful formulas. With (21), second-period

⁸Technically, given my assumptions, it suffices to check $L(\vec{\theta}_{\infty;0}) = \omega$ and to pick $\tilde{\omega} = \omega$. But this is a knife edge case, which I have resolved somewhat arbitrarily per assumption.

consumption will assumed to be

$$\tilde{c}_2(\theta, L; G) = \frac{c_2(0; G) - r\theta/\beta}{1 - \theta} \quad (27)$$

which is monotone in G , when ordering distributions according to first-order stochastic dominance, and which does not depend on L (and where I use the $\tilde{\cdot}$ to distinguish it from the scenario above). As in (16), a late consumer local bank will withdraw early, if $\theta \geq \tilde{\theta}^*(G)$, where

$$\tilde{\theta}^*(G) = \frac{\beta}{1 - \beta} \left(\frac{1 - \varphi r}{r} \frac{1 - \varphi}{1 - \varphi} E_G[R] - 1 \right) \quad (28)$$

This scenario will serve as a benchmark. While there can also be a fundamental bank run in this case, there is no spillover to other core banks. A fundamental bank run in this scenario and the scenario with uncertainty averse investors start the same and affect the same core banks. However, a fundamental bank run with uncertainty averse investors can run considerably deeper.

5.4 A numerical example

To provide a specific, illustrative example, suppose that $\sigma = 1/2$, $R_{\text{boom}} = 1.44$ and $\varphi = 1/7$. Equation (12) then implies

$$c_1 = r = \frac{7}{6} = 1.1666, \quad x = \frac{5}{6}, \quad c_2 = \frac{7}{5} = 1.44 \frac{35}{36}$$

Assume that $\beta = 2/3$, therefore satisfying (6). Assume that 10% of the returns are uniformly distributed on $[0.6, 1.4]$, whereas 90% are equal to 1.4 in the bust state: this is the aggregate distribution F , see figure 3. Therefore, $R_{\text{bust}} = 1.36$. Note that (13) is violated, and that therefore there is no fundamental bank run with complete information in the bust state or if the beliefs $F(\cdot, s)$ of all local banks coincide with the asset distribution.

Assume that for a fraction $(1 - \mu)$ of core banks, local banks assume the correct aggregate distribution, and will therefore not run in a fundamental

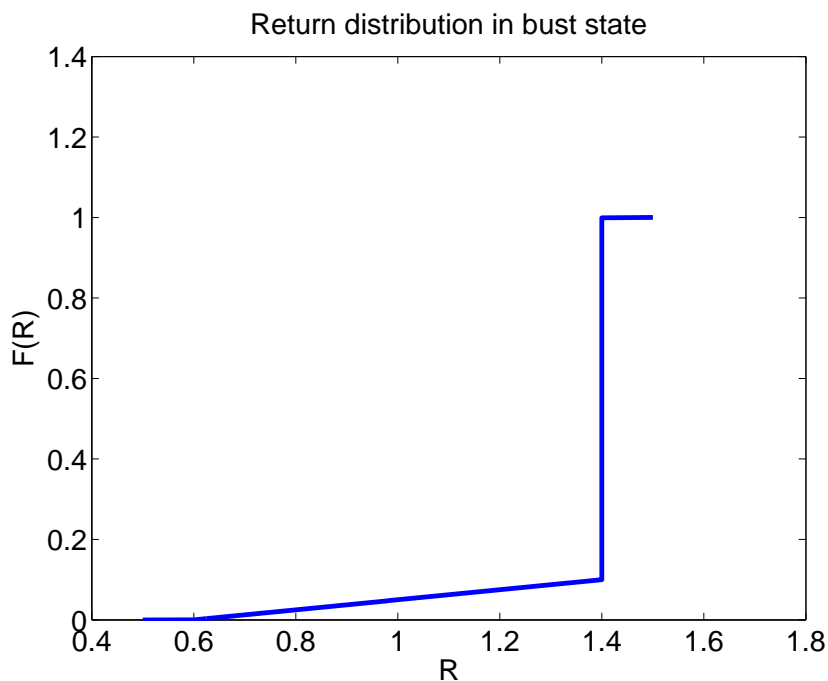


Figure 3: *Return distribution in the bust state.*

bank run equilibrium. However, for the remaining fraction μ of the core banks, the local banks believe with certainty that the return is some return R , where R is randomly drawn from F . I.e., if the local banks of these core banks are enumerated $\tau \in [0; 1]$, then $\Gamma(\tau) = 0.6 + 8\tau$ for $0 \leq \tau \leq 0.1$ and $\Gamma(\tau) = 1.4$ for $\tau \geq 0.1$. As a result, the local banks are correct in aggregate, but wrong individually, see figure 4

Absent a bank run, each late consumer local bank expects a pay out of

$$c_2(0; F(\cdot; \tau)) = \Gamma(\tau) \frac{35}{36}$$

Even for the most optimistic bank, I have

$$c_2(0; F(\cdot; 1)) = 1.4 \frac{35}{36} < \frac{r}{\beta} = \frac{7}{4} = 1.75$$

Therefore, the condition (22) is satisfied for all $G = F(\cdot; s)$.

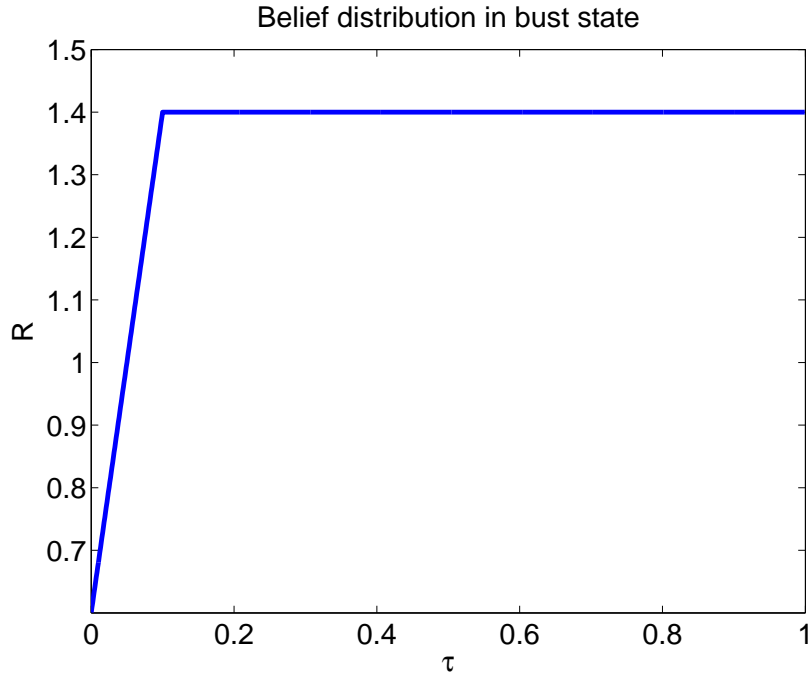


Figure 4: *Beliefs*.

Suppose first, that there are only risk neutral investors (or only expert investors), as in subsection 5.3. In that case, (28) can be used to calculate the fundamental bank run, if it exists, by calculating the smallest τ so that

$$\theta_{\infty;n,0} = \tilde{\theta}^*(F(\cdot; \tau)) = \tau$$

where I have also used the notation $\theta_{\infty;n,0}$ to denote the fraction of local banks at one of the affected core banks, say with index n , if aggregate liquidity demands L are believed to be below ω (or $L = 0$, for simplicity). The solution is approximately $\theta_{\infty;n,0} = 0.0811$, i.e. 8 percent of late consumer local banks will decide to run, see figure 5.

In the scenario with uncertainty averse investors, note that

$$L = L(\theta) = r\theta(1 - \varphi)\mu \tag{29}$$

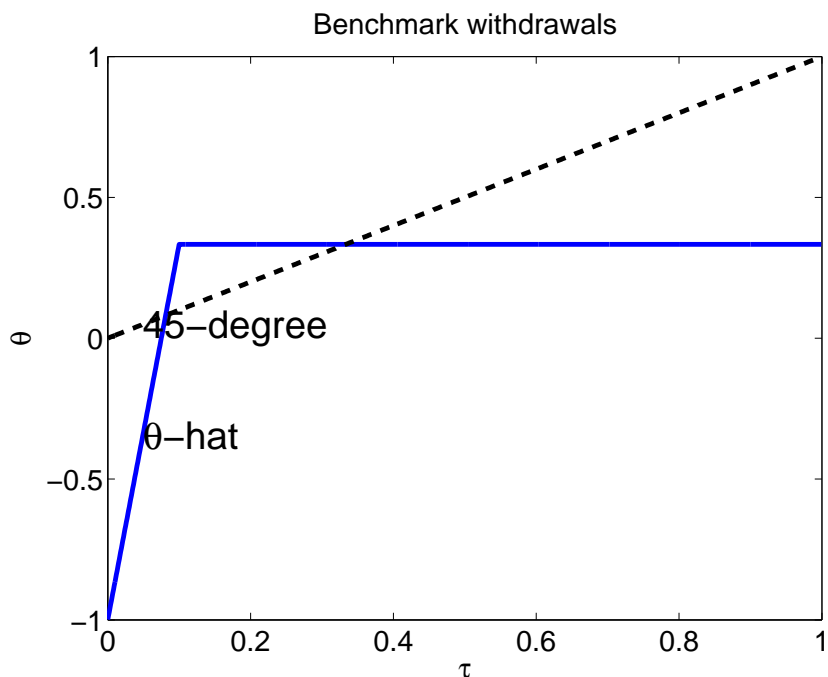


Figure 5: *Bankrun calculus when only expert investors are present.*

so that the market price drops to the uncertainty-averse investor price βR as a function of θ , when θ exceeds the threshold value θ_{crit} given by

$$\theta_{\text{crit}} = \frac{\omega}{\mu r(1 - \varphi)} = \frac{\omega}{\mu}$$

since my numerical values happen to imply $r(1 - \varphi) = 1$. Put differently, the given expertise of outside investors will be diluted, the more core banks are affected by withdrawals, “accelerating” the bank run compared to the experts-only scenario. It is in this sense, that the bank run is systemic.

Conversely, the experts-only partial bank run described above is not an equilibrium, if $\theta_{\infty;n,0} > \theta_{\text{crit}}$ or

$$\frac{\omega}{\mu} < \theta_{\infty;n,0} r(1 - \varphi) \approx 0.0811, \quad (30)$$

i.e. if the fraction of affected core banks is somewhat above 12 times the resources of the expert investors relative to the entire amount initially invested in all securities.

To calculate the equilibrium in that case, consider a value $L < \omega$ and a value $L > \omega$. For each value, calculate $c_2(\theta, L; F(\cdot, \theta))$. Calculate the lowest $\theta = \tau$ so that

$$c_2(\theta, L; F(\cdot, \theta)) = r \tag{31}$$

or, absent that and depending on the boundary conditions, either $\theta = \tau = 0$, if $c_2 > r$ always, or $\theta = \tau = 1$, if $c_2 < r$ always.

The resulting second-period consumption is shown in figure 6. If $L > \omega$, the graph shows that $\theta = 1$, i.e. a run on all core banks affected by doubtful local banks, as the only solution. By contrast, there are multiple solutions to (31), if $L < \omega$. Therefore, if (30) holds, a system-wide bank run on the fraction μ of the core banks, which are subject to heterogeneous beliefs by their local banks, results, while the other $1 - \mu$ core banks remain unaffected (unless there is a Diamond-Dybvig sunspot-type bank run). Variations of this example can produce partial fundamental bank runs as well. Furthermore and in a generalized version of this model, if Γ varies smoothly with L , figure 6 suggests a critical value as the c_2 -curve is shifted downwards with increasing L , when the equilibrium close to the small expert-only partial bank run disappears and only the system-wide bank run on the affected core banks remains.

6 The “bust” state with adverse selection.

Consider now the variation of the model with adverse selection. More precisely, assume the outside investors to be risk-neutral, discounting the future at rate β . I assume that all outside investors are non-experts⁹, and can therefore not distinguish between long-term securities offered to them, while

⁹It would not be hard but a bit tedious, to generalize this and to include expert investors as well.

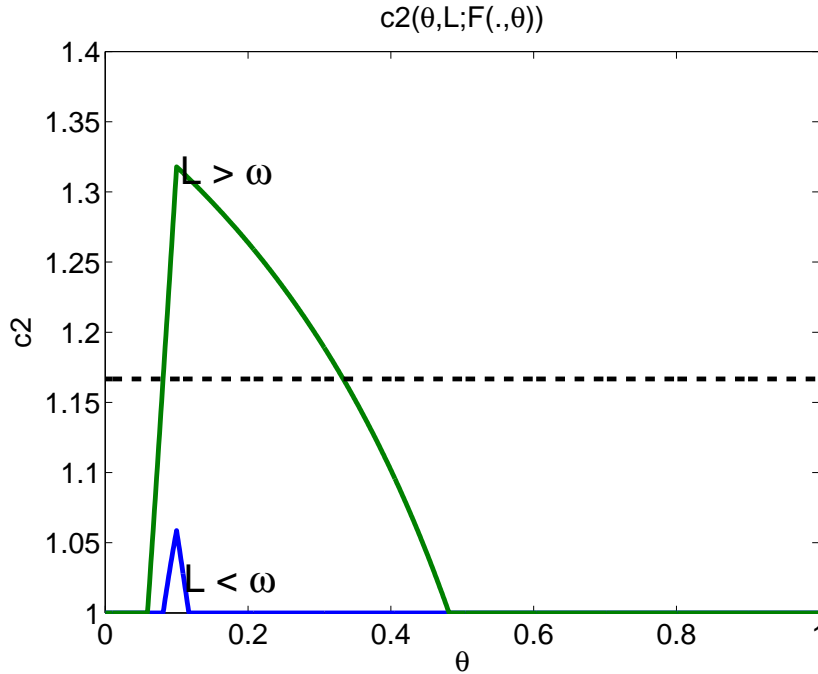


Figure 6: *Consumption of local banks that wait until the second period, assuming that all banks with $\tau < \theta$ run, and banks with $\tau > \theta$ do not. Comparison to $c_1 = r$.*

core banks selling them know the returns exactly, and can choose which security to sell. I assume that outside investors know the return distribution F . All the long-term securities are therefore sold at the same market price p . This creates adverse selection: not only will core banks sell the securities with their worst quality first (and this happens in the analysis above as well, when selling to non-expert investors), but furthermore, some core banks without liquidity needs due to withdrawals may sell long-term securities of low quality, if the price is right. The latter is a key difference between the adverse selection variant and the uncertainty aversion variant presented here: with uncertainty averse investors and sufficiently high discounting, there never is

a reason for “opportunistic” selling by liquid core banks¹⁰

To keep the analysis a bit more tractable, assume that the true portfolio F is atomless. Suppose that core banks with a market share μ face early withdrawals of the same¹¹ fraction θ of their late-consumer serving local banks, due to heterogeneous beliefs of their local banks. They need to sell a share ζ of their portfolio or $z = \mu x \zeta$ of their long-term securities, where

$$r\theta(1 - \varphi)\mu = p\mu x \zeta \quad (32)$$

On average, these securities pay $E_G[R \mid R \leq G^{-1}(\zeta)]$ per unit, see equation (18).

Assume that the other core banks have local banks who all correctly believe the core-bank portfolio to have securities with returns distributed according to $R \sim F(R)$. These core banks will sell long-term securities for purely opportunistic reasons, in case their price exceeds the expected return. Given a market price p for long-term securities, core banks without early withdrawals will sell all¹² securities with $R \leq p$, i.e. sell the fraction $F(p)$.

The outside investors are risk neutral, discounting the future at β , but understand this adverse selection problem. By assumption, they correctly assume the securities in the portfolios of the core banks to have the return distributions $R \sim F(R)$. Therefore, the market clearing price¹³ $p = p(\theta, \mu)$

¹⁰Clearly, the distinction here has been sharply drawn, for analytic purposes. It may well be that some mixture of the two variants is a better description than one of these two extreme variants.

¹¹It is straightforward, but tedious to extend this to the case, where θ differs from core bank to core bank.

¹²For equality, there is indifference, and therefore core banks may only sell a fraction of the securities for which there is a equality. The issue does not arise, if $F(\cdot)$ is atomless, as I have assumed in this section. In the more general case, it will be easy to patch that up at the final step, when calculating market clearing. For reasons of tractability, I shall not pursue this issue further.

¹³Note that the local banks considering withdrawals should be able to learn from the market price, that their beliefs G for their core bank and the market price together are inconsistent with the aggregate return distribution F , and should therefore somehow up-

and the fraction of the portfolio $\zeta = \zeta(\theta, \mu)$ sold by the distressed core banks solve the two equations

$$p = \beta \frac{\mu \int_{\underline{R}}^{F^{-1}(\zeta)} R dF + (1 - \mu) \int_{\underline{R}}^p R dF}{\mu \zeta + (1 - \mu) F(p)} \quad (33)$$

$$\zeta = \frac{r\theta}{p} \left(\frac{1 - \varphi}{1 - r\varphi} \right) \quad (34)$$

where (per notational convention or per calculation of the integral)

$$0 = \int_{\underline{R}}^{p(\theta, \mu)} R dF, \text{ if } p(\theta, \mu) < \underline{R}$$

and where $F^{-1}(\cdot)$ is defined as in (17). Note that the right hand side of (33) is simply the average return of the securities sold, discounted at β . Define

$$\bar{\theta} = \left(\frac{1 - r\varphi}{1 - \varphi} \right) \frac{\beta \underline{R}}{r} \quad (35)$$

as the maximal θ compatible with $\zeta \leq 1$, if $p = \beta \underline{R}$. Note that $\bar{\theta} < 1$.

Proposition 4 1. For every $\theta \in [0, \bar{\theta}]$ and $\mu \in (0, 1]$, there is a unique solution (p, ζ) to (33,34) with $\beta \underline{R} \leq p \leq \beta R_{bust}$ and $0 \leq \zeta \leq 1$, so that $p < F^{-1}(\zeta)$.

2. Given θ , $p(\theta, \mu)$ is a strictly increasing function in $\mu \in (0, 1]$.

Proof:

1. Recall that the support of F is $[\underline{R}, \bar{R}]$. Define the function $\rho(p)$ per the right hand side of (33), with ζ replaced with (34). Note that $\rho(p)$ is continuous on $p \in [\beta \underline{R}, \bar{R}]$ with

$$\rho(\beta \underline{R}) \geq \beta \underline{R}, \rho(\bar{R}) \leq \beta E_F[R] = \beta R_{bust} < \bar{R}$$

date their belief, learning from the information revealed in market prices. This may be a tough thing to do in practice, and I shall ignore this issue for the purpose of the analysis here.

By the mean value theorem, there is therefore a value p with $p = \rho(p)$. Suppose that $F^{-1}(\zeta) \leq p$ at this value. Then the right hand side of (33) is not larger than βp , a contradiction. To show uniqueness, suppose to the contrary that there are two solutions, say $p_a < p_b$, together with $1 \geq \zeta_a > \zeta_b$. Note generally that

$$\begin{aligned} \int_{\underline{R}}^{p_b} RdF - \int_{\underline{R}}^{p_a} RdF &\leq (F(p_b) - F(p_a))p_b \\ &\leq F(p_b)p_b - F(p_a)p_a \end{aligned}$$

Define the function

$$\psi(p, \zeta; \theta, \mu) = \frac{\beta\mu \int_{\underline{R}}^{F^{-1}(\zeta)} RdF + \beta(1-\mu) \int_{\underline{R}}^p RdF}{\mu r\theta \left(\frac{1-\varphi}{1-r\varphi}\right) + (1-\mu)F(p)p} \quad (36)$$

Note that $p_j = \rho(p_j)$ can be rewritten as

$$1 = \psi(p_j, \zeta_j; \theta, \mu) \quad (37)$$

for $j = a, b$. Therefore,

$$\begin{aligned} 1 &= \psi(p_a, \zeta_a; \theta, \mu) = \frac{\beta\mu \int_{\underline{R}}^{F^{-1}(\zeta_a)} RdF + \beta(1-\mu) \int_{\underline{R}}^{p_a} RdF}{\mu r\theta \left(\frac{1-\varphi}{1-r\varphi}\right) + (1-\mu)F(p_a)p_a} \\ &> \frac{\beta\mu \int_{\underline{R}}^{F^{-1}(\zeta_a)} RdF + \beta(1-\mu) \int_{\underline{R}}^{p_a} RdF + (1-\mu)(F(p_b)p_b - F(p_a)p_a)}{\mu r\theta \left(\frac{1-\varphi}{1-r\varphi}\right) + (1-\mu)F(p_a)p_a + (1-\mu)(F(p_b)p_b - F(p_a)p_a)} \\ &\geq \frac{\beta\mu \int_{\underline{R}}^{F^{-1}(\zeta_a)} RdF + \beta(1-\mu) \int_{\underline{R}}^{p_b} RdF}{\mu r\theta \left(\frac{1-\varphi}{1-r\varphi}\right) + (1-\mu)F(p_b)p_b} \\ &\geq \frac{\beta\mu \int_{\underline{R}}^{F^{-1}(\zeta_b)} RdF + \beta(1-\mu) \int_{\underline{R}}^{p_b} RdF}{\mu r\theta \left(\frac{1-\varphi}{1-r\varphi}\right) + (1-\mu)F(p_b)p_b} \\ &= \psi(p_b, \zeta_b; \theta, \mu) \end{aligned}$$

and therefore, (37) cannot hold for p_b , a contradiction.

2. Given θ, μ , denote the unique equilibrium with $p(\theta, \mu)$ and $\zeta(\theta, \mu)$. Let $\zeta(p)$ denote the expression on the right hand side of (34). The previous

calculation shows more generally that

$$\psi(p, \zeta(p); \theta, \mu) > 1 \quad \text{for } p < p(\theta, \mu) \quad (38)$$

$$\psi(p, \zeta(p); \theta, \mu) < 1 \quad \text{for } p > p(\theta, \mu) \quad (39)$$

Consider some $\bar{\mu}$ and write $\bar{p} = p(\theta, \bar{\mu})$ and $\bar{\zeta} = \zeta(\theta, \bar{\mu})$. Since $\psi(\bar{p}, \bar{\zeta}; \theta, \bar{\mu}) = 1$ and since

$$\beta \int_{\underline{R}}^{\bar{p}} R dF < F(\bar{p})\bar{p}$$

it follows that

$$\beta \int_{\underline{R}}^{F^{-1}(\bar{\zeta})} R dF > r\theta \frac{1 - \varphi}{1 - r\varphi}$$

Therefore, $\psi(\bar{p}, \bar{\zeta}; \theta, \mu)$ is increasing in μ . For $\mu' > \bar{\mu}$, one therefore has $\psi(\bar{p}, \bar{\zeta}; \theta, \mu') > 1$. It follows from (38) that $p(\theta, \mu_b) > p(\theta, \mu_a)$, as claimed.

•

At the distressed core banks, local banks with beliefs G regarding their portfolio will therefore believe the opportunity costs for providing period-1 resources in terms of period-2 resources to be

$$\Gamma(\theta, \mu; G) = \frac{E_G[R \mid R \leq G^{-1}(\zeta(\theta, \mu))]}{p(\theta, \mu)} \quad (40)$$

It is instructive to compare this to (19) for the case $\omega = 0$: the two expressions coincide iff $p(\theta, \mu) = \beta \underline{R}$. Generally, the returns are quite different. In fact,

$$\Gamma(\theta, 1; G) = \frac{E_G[R \mid R \leq G^{-1}(\zeta(\theta, \mu))]}{\beta E_F[R \mid R \leq G^{-1}(\zeta)]} \quad (41)$$

as can be seen by direct calculation. In particular, for $G = F$, I obtain

$$\Gamma(\theta, 1; F) = \frac{1}{\beta} \quad (42)$$

More generally,

Proposition 5 $\Gamma(\theta, \mu; G)$ is decreasing in μ .

Proof: This is a direct consequence of (40) together with the fact that $p(\theta, \mu)$ is increasing in μ , implying that $\zeta(\theta, \mu)$ and thus $E_G[R \mid R \leq G^{-1}(\zeta)]$ are decreasing in μ . •

I obtain the key insight that an increasing market share of distressed banks lessens rather than deepens the crisis. Furthermore, with homogeneous beliefs, $F(\cdot, s) \equiv F$, and with the market share of distressed banks approaching unity, the moral-hazard scenario turns into the standard bank run scenario considered in section 4.3.

The remaining late-consumer local banks will obtain

$$c_2(\theta, \mu; G) = \frac{1}{1 - \theta} \left(\frac{x}{1 - \varphi} E_G[R] - r\theta \frac{E_G[R \mid R \leq G^{-1}(\zeta(\theta, \mu))]}{p(\theta, \mu)} \right) \quad (43)$$

Therefore, a late-consumer-serving local bank in location s , banking with a distressed core bank and believing that a fraction θ of local late-consumer banks will withdraw in period 1 will choose to do so itself, if

$$c_2(\theta, \mu; G) \leq r \quad (44)$$

The analysis of the resulting equilibrium appears to be similar to the analysis in section 5 and shall be omitted in the interest of space.

6.1 A numerical example

I use the same parameterization as in subsection 5.4. For low values of $\theta \leq \underline{\theta}$, the market price will be below $\underline{R} = 0.6$ and the required market discount $\Gamma(\theta, \mu; F)$ at the true distribution will equal $1/\beta$. For these low values of θ and due to the uniform distribution, the market price equals

$$p(\theta; \mu) = \beta \frac{F^{-1}(\zeta) + 0.6}{2} \quad (45)$$

Therefore, θ is low enough, iff $p(\theta; \mu) \leq 0.6$ or, equivalently, $F^{-1}(\zeta) \leq 1.2$. By the parameterization in (5.4),

$$F^{-1}(\zeta) = \min\{0.6 + 8\zeta, 1.4\}$$

Therefore, $F^{-1}(\zeta) \leq 1.2$ corresponds to $\zeta \leq 0.075$. To find p and ζ when $F^{-1}(\zeta) \leq 1.2$, I therefore need to solve

$$\zeta = \frac{r\theta}{\beta(0.6 + 4\zeta)} \left(\frac{1 - \varphi}{1 - r\varphi} \right) \quad (46)$$

Let

$$\kappa = \frac{r}{4\beta} \left(\frac{1 - \varphi}{1 - r\varphi} \right) = \frac{9}{20}$$

The solutions to (46) are therefore given by

$$\zeta = -0.075 + \sqrt{0.075^2 + \kappa\theta}$$

(where the negative root has been excluded as not sensible). Therefore, $\zeta \leq 0.075$, if

$$\theta \leq \underline{\theta} = 3 * 0.075^2 / \kappa = 0.0375.$$

For $\theta > \underline{\theta}$, the behavior of the price depends on market share of the distressed core banks. Two extreme scenarios can provide some general insights. If $\mu \rightarrow 0$, then the price will remain “stuck” at $\underline{p} = \underline{R} = 0.6$, as all remaining banks would sell arbitrarily large chunks of their worst assets otherwise. If $\mu = 1$, then discounting of future returns will remain to be done at the discount rate β .

For these as well as the in-between range of values of μ , equation (44) can then be used to determine the threshold value for τ , up to which local banks will decide to withdraw. Proposition (5) generally shows, that a bank run is the less likely, the larger the market share of distressed core banks.

7 Some policy implications

Given the length of this paper, a full discussion of the policy implications is beyond its scope. I shall also shy away from a welfare analysis. Instead, I

investigate the more modest question of the impact of certain policies for a policy maker who may be interested in learning the consequences for avoiding (or stopping) a crisis and for the government budget. This section is written under the assumption of the view described in the six-point list in the introduction: obviously, if that view is incorrect, then the following conclusions may no longer be applicable.

A key difference between the model with uncertainty averse investors and the adverse selection are the implications, as the “suspicion” of bad portfolios affect not only a fraction of the core bank but all banks. In the case of uncertainty averse investors, a given core bank now has even less access to expert investors, worsening the situation. In the case of adverse selection, and since all core banks need to obtain equal amounts of liquidity, they will all receive “fair value” for their assets, i.e., the situation essentially turns into a classic bank run. Therefore, the adverse selection scenario violates item six of the stylized description list in the introduction, while the scenario with uncertainty averse investors does not. For these reasons, I argue that it is more plausible to look at the 2008 financial crisis through the lense of the uncertainty averse investor scenario rather than the adverse selection scenario.

Consider, for example, a government guarantee of payoffs of the securities sold by the core banks, e.g. guaranteeing a return of at least R_{gov} . In that case, the uncertainty averse investors will pay βR_{gov} instead of $\beta \underline{R}$. In particular, if $R_{\text{gov}} = 1$, i.e. if the government guarantees that investments will not make losses, the “deep” bank run results in discounting with β throughout and turns the scenario with uncertainty averse investors into the “classic” bankrun situation of subsection 5.3 for the distressed core banks. The government will loose money on all securities with returns $R < R_{\text{gov}}$. Additionally, if $\beta R_{\text{gov}} > \underline{R}$, the government now creates an *additional* adverse selection problem at the core banks which are not distressed, and which now find it at their advantage to sell all assets with $\underline{R} \leq R \leq \beta R_{\text{gov}}$.

From the tax payers perspective, a more advantageous procedure in the

case of uncertainty averse investors seems to be the purchase of the troubled assets outright: if large parts of the portfolios of banks are bought, the tax payer will receive the average payoff and not the bottom payoff as feared by the outside investors. Consider a fixed government purchase price p at which the government stands ready to purchase assets from the core banks. The incentives of the participating core banks then become similar to the analysis in the adverse selection framework: while the distressed core banks will sell for sure, the core banks without distress will only do so if it is in their interest to sell the worst assets. Rather than imposing the equilibrium condition (33), one can then calculate the losses or gains to the tax payers at the mandated government purchase price. If that price is below the adverse selection scenario equilibrium price, the government will earn a return above $1/\beta$, and this scenario is possible and plausible, if investors are uncertainty averse. If the situation is as described in the adverse selection scenario, then the government would only find takers for its offers, if the government price is above the current market clearing price, in which case the government will make losses compared to the benchmark return of $1/\beta$.

Since I have argued that the uncertainty averse scenario is more plausible than the adverse selection scenario, the analysis here provides some support for the argument that an outright purchase of troubled assets by the government at prices above current market prices can both alleviate the financial crises as well as provide tax payers with returns above those for safe securities.

A number of private sector solutions may likewise provide reasonable avenues for resolving the crisis situation, e.g. the complete purchase of portfolios of a distressed core bank or the sale of a distressed core bank and a guarantee of its deposits through the buyer. It may be, however, that the same caution that drives uncertainty averse investors to demand steep discounts on asset backed securities might also prevent the sale of distressed financial institutions to the same investors at a price that can resolve the situation sufficiently well. Solutions that mix private sector involvement with

government intervention - an idea at the core of the Geithner plan - may likewise offer specific advantages or fallacies, that can be analyzed in this context.

Follow-up work, providing a deeper analysis of the various options and policy scenarios, is surely called for.

8 Conclusions

I have set out to provide a model of a systemic bank run delivering the following features

1. The withdrawal of funds was done by financial institutions at other financial institutions, rather than depositors at their bank.
2. The troubled financial institutions held their portfolio in asset-backed securities rather than being invested directly in long-term projects.
3. These securities are traded on markets. In the crisis, the prices for these securities appears low compared to some benchmark fundamental value benchmark (“underpricing”).
4. There is a large pool of investors willing to purchase securities, as evidenced e.g. by market purchases of newly issued US government bonds or the volume on stock markets.
5. Nonetheless, these investors are only willing to buy these asset-backed securities at prices that are low compared to standard discounting of the entire pool of these securities.
6. The larger the market share of troubled financial institutions, the steeper the required discounts.

To that end, I have hypothesized two different motives for outside investors and their interaction with banks trading asset-backed securities: uncertainty

aversion versus adverse selection. Both variants of the model are capable of delivering on the first five points of the list above. While the variant with uncertainty averse investors also delivers on the sixth point, this is not the case for the adverse selection scenario. Indeed there, as a larger share of financial institutions are distressed, the discounts lessen rather than rise.

I conclude from that that the variant with uncertainty averse investors rather than the adverse selection scenario is more suitable to analyze policy implications. This paper therefore provides a model, in which the outright purchase of troubled assets by the government at prices above current market prices may both alleviate the financial crises as well as provide tax payers with returns above those for safe securities.

A number of private sector solutions may likewise provide reasonable avenues for resolving the crisis situation. Solutions that mix private sector involvement with government intervention - an idea at the core of the Geithner plan - may likewise offer specific advantages or fallacies, that can be analyzed in this context. Follow-up work, providing a deeper analysis of the various options and policy scenarios, is surely called for.

Appendix

A Equilibrium with uncertainty aversion: a general case

I now seek to analyze the interplay between all $n = 1, \dots, N$ core banks, some of which may be subject to early withdrawals by their local banks. In terms of the period-2 opportunity costs $\Gamma(\theta, L; G)$ for providing period-1 liquidity, I shall only use the properties stated in proposition 1. Therefore, the analysis here generalizes to a situation, where the dependence on L is smooth, as e.g. in Allen and Gale (2007), chapter 4, rather than a step function as implied by our stark assumptions about uncertainty aversion. While this is more generality than is strictly needed for completing the analysis here, it is useful for applying the analysis in this paper more generally. I return to the special case of a liquidity threshold in subsection 5.2.

Let me start per defining the equilibrium.

Definition 1 *An equilibrium is collections $(S_n)_{n=1}^N$ of subsets of $[0, 1]$, withdrawal fractions $(\theta_n)_{n=1}^N$ and total additional liquidity L , so that*

1. $\nu(s) = n$ for all $s \in S_n$, i.e. S_n are locations of banks banking with core bank n .
2. $\theta_n = \lambda(S_n)$ is the Lebesgue measure of S_n .
3. For all $s \in S_n$: $c_2(\theta_n, L; F(\cdot, s)) \leq r$. For all $s \notin S_n$, $\nu(s) = n$: $c_2(\theta_n, L; F(\cdot, s)) \geq r$.
4. $L = L(\theta_1, \dots, \theta_n)$ where

$$L(\theta_1, \dots, \theta_n) = r(1 - \varphi) \sum_n \theta_n \lambda(\nu^{-1}(n)) \quad (47)$$

where $\lambda(\nu^{-1}(n))$ is the Lebesgue measure of $\nu^{-1}(n)$.

Note that the actual portfolio of the core banks does not matter for calculating the withdrawal fractions: only the perception of their portfolio matters. Obviously, the actual portfolio does matter for the realized date-2 payoff of the remaining late consumers.

Assume that (22) is true for all conjectured distributions $G = F(\cdot, s)$. Therefore, if local banks opt for early withdrawals at some level of market liquidity or some fraction of other early withdrawals, they will do also for higher levels of L and θ . Let

$$S_n(\theta, L) = \{s \mid \nu(s) = n, c_2(\theta, L; F(\cdot, s)) < r\} \quad (48)$$

be the set of local banks with deposits at core banks n , which will surely withdraw early, if a fraction θ of depositors at core bank n do, and if there is total liquidity demand L .

Define the mapping

$$h : [0, 1]^N \rightarrow [0, 1]^N$$

per

$$(\theta'_1, \dots, \theta'_N) = h(\theta_1, \dots, \theta_N)$$

where

$$\theta'_n = \lambda(S_n(\theta_n, L(\theta_1, \dots, \theta_N))) \quad (49)$$

and where $\lambda(\cdot)$ denotes the Lebesgue measure. Intuitively, if everyone conjectures the fractions $(\theta_1, \dots, \theta_N)$ of late consumer local banks to withdraw early, then the fractions $\tilde{\theta}_j$ surely will. Obviously h is an increasing function. Let

$$\Theta = \{\vec{\theta} = (\theta_1, \dots, \theta_N) \mid \vec{\theta} \leq h(\vec{\theta})\} \quad (50)$$

be the set of conservative withdrawal conjecture vectors, i.e. actual withdrawals will at least be as high, as the conjecture at each core bank, see figure 7.

Let

$$\Theta_{\text{lim}} = \{\vec{\theta} \in [0, 1]^N \mid \text{There is a sequence } \vec{\theta}_j \in \Theta \text{ with } \vec{\theta}_j \leq \vec{\theta} \text{ and } \vec{\theta}_j \rightarrow \vec{\theta}\} \quad (51)$$

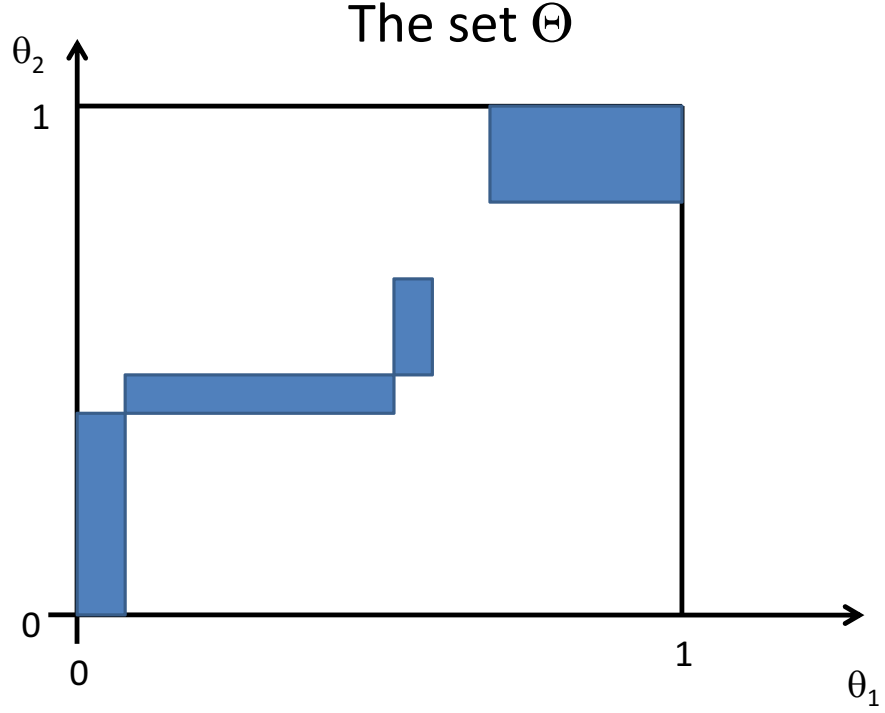


Figure 7: *The set Θ .*

be a set of upper bounds for Θ and let

$$\Theta_{\max} = \{ \vec{\theta} \mid \text{There is } \epsilon \in \mathbf{R}_{++}^N \text{ so that } [\vec{\theta}, \vec{\theta} + \epsilon] \cap \Theta_{\lim} = \{ \vec{\theta} \} \} \quad (52)$$

be the set of all local maxima in Θ_{\lim} . For notation, recall that $[\vec{\theta}, \vec{\theta} + \epsilon]$ is the set of all $\tilde{\theta}$ with $\vec{\theta} \leq \tilde{\theta} \leq \vec{\theta} + \epsilon$.

Proposition 6 *Assume that (22) is true for all conjectured distributions $G = F(\cdot, s)$.*

1. *If*

$$\vec{\theta} = h(\vec{\theta}) \quad (53)$$

then $\vec{\theta}$ together with $L = L(\vec{\theta})$ and $S_n = S_n(\theta_n, L)$, $n = 1, \dots, N$ is an equilibrium¹⁴.

2. If $\vec{\theta} \in \Theta$, then

$$[\vec{\theta}, h(\vec{\theta})] \subset \Theta \quad (54)$$

3. $\emptyset \neq \Theta_{\max} \subset \Theta_{\lim} \subset \Theta$.

4. Any $\vec{\theta} \in \Theta_{\max}$ satisfies (53) and therefore has an equilibrium associated with it.

5. Let $\vec{\theta}_0 = (0, 0, \dots, 0)$. Consider the sequence

$$\vec{\theta}_j = h(\vec{\theta}_{j-1}) \quad (55)$$

This sequence converges to some $\vec{\theta}_\infty \in [0, 1]^N$. Any $\vec{\theta} \in \Theta_{\max}$ satisfies $\vec{\theta} \geq \vec{\theta}_\infty$. Suppose that all $c_s(\theta, L; F(\cdot, s))$ are continuous in L at $L = L(\vec{\theta}_\infty)$. Then $h(\vec{\theta}_\infty) = \vec{\theta}_\infty$ and therefore has an equilibrium associated with it.

Proof:

1. Check the equilibrium definition.

2. Let $\tilde{\theta} \in [\vec{\theta}, h(\vec{\theta})]$. Then

$$h(\tilde{\theta}) \geq h(\vec{\theta}) \geq \vec{\theta}$$

3. Per (22) or better (23),

$$h(1, 1, \dots, 1) = (1, 1, \dots, 1) \quad (56)$$

for if everyone else withdraws early, so should you. Thus $(1, 1, \dots, 1) \in \Theta_{\max}$, which is therefore not empty. The first inclusion is trivial. For

¹⁴Since I have focussed here on finding equilibria per strict preference for withdrawal in period 1, the converse may generally not be true.

the second inclusion, let $\vec{\theta}_j \rightarrow \vec{\theta}$ with $\vec{\theta}_j \in \Theta$ and $\vec{\theta}_j \leq \vec{\theta}$. Note that

$$h(\vec{\theta}) \geq h(\vec{\theta}_j) \geq \vec{\theta}_j$$

for all j . Therefore,

$$h(\vec{\theta}) \geq \vec{\theta}$$

and hence $\vec{\theta} \in \Theta$.

4. Assume additionally that $\vec{\theta} \in \Theta_{\max}$. I need to show that

$$h(\vec{\theta}) = \vec{\theta}$$

But if instead $(h(\vec{\theta}))_n > (\vec{\theta})_n$ for some entry n , say, then this together with $[\vec{\theta}, h(\vec{\theta})] \subset \Theta$ would be a contradiction to local maximality.

5. Note that $\vec{\theta}_j$ is an increasing sequence in $[0, 1]^N$: it therefore must converge. Let $s \in S_n(\vec{\theta}_\infty, L)$. Therefore, $c_s(\vec{\theta}_\infty, L; F(\cdot, s)) < r$. By continuity in θ , $c_s(\vec{\theta}_j, L; F(\cdot, s)) < r$ for all j sufficiently large. Conversely, if $c_s(\vec{\theta}_j, L; F(\cdot, s)) < r$, then $c_s(\vec{\theta}_\infty, L; F(\cdot, s)) < r$, due to proposition 2. Therefore,

$$S_n(\vec{\theta}_\infty, L) = \bigcup_{j=1}^{\infty} S_n(\vec{\theta}_j, L)$$

and hence

$$h(\vec{\theta}_\infty) = \lim_j h(\vec{\theta}_j) = \lim_j \vec{\theta}_{j+1} = \vec{\theta}_\infty$$

Let $\vec{\theta} \in \Theta_{\max}$. Since trivially $\vec{\theta} \geq (0, 0, \dots, 0)$, the conclusion follows per repeated application of $h(\cdot)$.

•

If indeed $h(\vec{\theta}_\infty) = \vec{\theta}_\infty$, I call $\vec{\theta}_\infty$ the **partial fundamental bank run**. If $(h(0, \dots, 0))_n = 0$ for $n \in \mathcal{N} \subset \{1, \dots, N\}$, then the same is true for

$\vec{\theta}_\infty$. The fundamental bank run therefore only affects the core banks, which experience withdrawals “at the start” of the run, i.e. experience withdrawals of late consumer local banks, even if all local banks assume that nobody else withdraws. Nonetheless, withdrawals at one core bank can spill over to withdrawals at other core banks within this set, due to the dependence of $\Gamma(\theta, L; G)$ on aggregate liquidity, provided that ω is nonzero and sufficiently small.

References

- [1] Acharya, Viral V. and Matthew Richardson, editors (2009), *Restoring Financial Stability*, John Wiley & Sons, Hoboken, NJ.
- [2] Allen, Franklin and Douglas Gale (1994), “Limited Market Participation and Volatility of Asset Prices,” *American Economic Review* 84, 933-955.
- [3] Allen, Franklin and Douglas Gale (1998), “Optimal Financial Crises,” *Journal of Finance* 53, 1245-1284.
- [4] Allen, Franklin and Douglas Gale (2000), “Financial Contagion,” *Journal of Political Economy* 108, 1-33.
- [5] Allen, Franklin and Douglas Gale (2004a), “Financial Intermediaries and Markets,” *Econometrica* 72, 1023-1061.
- [6] Allen, Franklin and Douglas Gale (2004b), “Financial Fragility, Liquidity, and Asset Prices,” *Journal of the European Economic Association* 2, 1015-1048.
- [7] Allen, Franklin and E. Carletti (2006), “Credit Risk Transfer and Contagion,” *Journal of Monetary Economics* 53, 89-111.
- [8] Allen, Franklin and Douglas Gale (2007), *Understanding Financial Crises*, Clarendon Lectures in Finance, Oxford University Press, Oxford.

- [9] Backus, David K., Bryan R. Routledge and Stanley E. Zin (2009), “Uncertainty and Sudden Stops in Real Investment,” draft, Stern School of Business, New York University.
- [10] Barberis, Nicholas, Ming Huang and Tano Santos, “Prospect Theory and Asset Prices,” *Quarterly Journal of Economics* 116(1) (February), 1-53.
- [11] Brunnermeier, Markus (2008), “Deciphering the 2007-2008 liquidity and credit crunch,” draft, Princeton University.
- [12] Bryant, J., (1980), “A Model of Reserves, Bank Runs and Deposit Insurance,” *Journal of Banking and Finance* 4, 335-344.
- [13] Cifuentes, R., G. Ferrucci, and H. Shin (2005), “Liquidity Risk and Contagion,” *Journal of the European Economic Association* 3, 556-566.
- [14] Diamond, Douglas W., and Philip H. Dybvig. 1983. “Bank Runs, Deposit Insurance, and Liquidity.” *Journal of Political Economy* 91 (5): 401-19.
- [15] Diamond, Douglas W. and Raghuram Rajan (2005), “Liquidity Shortages and Banking Crises”, *The Journal of Finance*, vol. 60, no. 2 (April), 615-647.
- [16] Diamond, Douglas W. and Raghuram Rajan (2009), “Fear of fire sales and the credit freeze,” draft, University of Chicago, Booth Graduate School of Business.
- [17] Duffie, Darrell, “The Failure Mechanics of Dealer Banks,” draft, Stanford University.
- [18] Ennis, Huberto M. and Todd Keister (2008), “Bank runs and institutions: the perils of intervention”, draft, New York Federal Reserve Bank, *American Economic Review*, forthcoming.
- [19] Ennis, Huberto M. and Todd Keister (2008), “Run Equilibria in the

- Green-Lin Model of Financial Intermediation”, draft, New York Federal Reserve Bank.
- [20] Epstein, Larry G. (1999), “A definition of uncertainty aversion,” *The Review of Economic Studies*, vol. 66, pp. 579-608.
- [21] Gorton, Gary (2009), “Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007”, draft, Yale University.
- [22] Green, Edward J. and Ping Lin (2003), “Implementing efficient allocations in a model of financial intermediation,” *Journal of Economic Theory* 109, 1-23.
- [23] Hansen, Lars Peter and Thomas J. Sargent (2008), *Robustness*, Princeton University Press, 2008.
- [24] Rochet, Jean-Charles (1998), *Why are there so many banking crises?*, Princeton University Press, Princeton, NJ.
- [25] Schleifer, A. and R. Vishny (1992), “Liquidation Values and Debt Capacity – A Market Equilibrium Approach,” *Journal of Finance* 47, pp. 1343-1366.
- [26] Schmeidler, David (1989), “Subjective Probability and Expected Utility without Additivity”, *Econometrica*, vol 57 (3), pp. 571-587.
- [27] Smith, Bruce D. (1991), “Bank panics, suspensions, and geography: some notes on the ‘contagion of fear’ in banking”, *Economic Inquiry*, April 19991, 29(2), 230-248.
- [28] Tversky, A. and Kahneman, D. (1991), “Loss Aversion in Riskless Choice: A Reference Dependent Model,” *Quarterly Journal of Economics* 106, 1039-1061.
- [29] Uhlig, Harald (1996), ”A Law of Large Numbers for a Large Economy,” *Economic Theory*, vol. 8, pp. 41-50.
- [30] Zingales, Luigi (2008), “Plan B”, draft, University of Chicago.