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ABSTRACT

Since the fall of 2008, option smiles have been clearly asymmetric: out-of-the-money currency options point to large expected exchange rate depreciations (appreciations) for high (low) interest rate currencies, suggesting that disaster risk is priced in currency markets. To study the price of disaster risk, we propose a simple structural model that includes both Gaussian and disaster risk and can be estimated even in samples that do not contain disasters. Estimating the model over the 1996 to 2011 period using exchange rate spot, forward, and option data, we obtain a real-time index of world disaster risk premia. We find that disaster risk accounts for more than a third of currency risk premia in advanced countries over the period.

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Currency carry trades are investment strategies where one borrows in low–interest rate currencies and invests in high–interest rate currencies. Such simple strategies offer large expected excess returns, challenging the benchmark models in international macroeconomics. In this paper, we explore whether a class of disaster-based models that postulate the existence of rare but large adverse aggregate shocks to stochastic discount factors can explain these excess returns. This class of models, pioneered by Rietz (1988) and Barro (2006), has received much attention recently in the macroeconomics and finance literature. Those models, however, are difficult to estimate because of the small number of disasters in the samples. To address this difficulty, we turn to currency option markets.

Currency options reveal a stark contrast between the pre- and post-2008 crisis periods. As we shall see, before the fall of 2008, option prices were only mildly asymmetric across strikes, with small differences between the price of an out-of-the-money put and the price of an out-of-the-money call. During the fall of 2008, however, high interest rate currencies sharply depreciated and low interest rate currencies appreciated. Carry traders borrowing in Japanese yen and lending in New Zealand dollars lost close to 30% of their investment in October 2008. Since the fall of 2008, there have been significant differences between high and low interest rate currencies in the currency option markets. One the one hand, out-of-the money puts on high interest rate currencies have become much more expensive than out-of-the-money calls, indicating a high risk of large depreciations in those currencies. On the other hand, options on low-interest rate currencies show the opposite pattern, indicating a high risk for large appreciations. The fall of 2008 thus appears as a defining moment for the currency market, recalling the 1987 crisis for equity markets: before 1987, equity option smiles are non-existent, after 1987, they became central to equity option markets, pointing towards deviations from the lognormality assumption of the Black and Scholes (1973) option pricing formula. Before 2008, currency option smiles are mostly symmetric, after 2008 they are not.

Against this empirical background, we propose a parsimonious exchange rate model and a simple methodology using currency option prices to estimate world disaster risk premia even in samples that do not contain disasters. We find that, in our sample, disaster risk premia are statistically significant and account for more than a third of carry trade excess returns in the developed countries examined.

In our model, financial markets are complete and thus the log change in the exchange rate is the log difference between the domestic and foreign stochastic discount factors (SDFs). Following Backus, Foresi, and Telmer (2001), we write the law of motion of the SDF in each country. These SDFs incorporate both a traditional log-normal component, as in Lustig, Roussanov, and Verdelhan (2011), and a disaster component, as in Farhi and Gabaix (2011). The former responds to random shocks observed every period, while the latter responds to rare global disaster shocks that affect countries differently. For carry trade investors, the change in the exchange rate over the investment period is the sole source of risk. If investment currencies depreciate or funding currencies appreciate, then investors' returns decrease because they lose on their investment or must reimburse larger amounts. Such exchange rate movements can be due to the usual Gaussian shocks, or to more extreme disaster shocks. In the spirit of the macro-finance literature on disaster risk (Brunnermeier, Nagel, and Pedersen, 2008, Burnside et al., 2011, Seo and Wachter, 2013, and Wachter, forth.), we abstract from daily variation in exchange rate volatility and volatility risk premia, but allow volatility to change every month. Our model delivers closed-form solutions for call and put option prices in- and out-of-the-money, as well as expected currency excess returns when the investment horizon tends to zero. Conditional on no disaster in the sample, the expected currency excess returns are simply the sum of Gaussian and disaster risk premia.

We turn to currency data to estimate the compensation of disaster risk at each point in time and to test the model's implications. The data set comprises currency spot, forward, and option contracts collected by J.P. Morgan for the 10 most developed currency markets. The data set starts in January 1996 and ends in December 2011. Fall 2008 can be interpreted either as a financial disaster, or a moderate consumption disaster. Alternatively, it could be interpreted as a sharp increase in the probability of a macroeconomic disaster, but not a full-blown disaster. Our estimates of disaster risk premia do not depend on such interpretation, and we report them both for samples that include or exclude this period. We assume that the model parameters are constant over one month, but can vary non-parametrically from one month to the next. The model thus allows for monthly time variation in the expected exchange rate volatility, as well as changes in the disasters' probabilities and sizes.

In order to focus on carry trade risk, we sort currencies by their interest rates into three portfolios, as in Lustig and Verdelhan (2007). The average excess return on the highest interest rate currencies is large and significant at 5.9% and thus is our benchmark currency risk premium. The model parameters are estimated from the option prices of the five most liquid strikes. Currency option markets offer the perfect setting to measure the price of global disaster risk for three reasons: they are among the most liquid and developed option markets; exchange rates offer a direct measure of the pricing kernels, without any assumption on aggregate cash flows; and carry trade risk is a compensation for global, not local, shocks. The estimation proceeds in two steps: first, the minimization between the model implied option prices and the market prices is run on the portfolio of high interest rate currencies in order to determine the U.S. exposure to world disaster risk. Second, taking the U.S. exposure as given, the minimization is run for each currency and each date. The estimation procedure then delivers a time-series of the compensation for world disaster risk. To the best of our knowledge, this time-series is the first estimation of the compensation for global disaster risk.

On average over the whole sample, excluding the fall of 2008, investors who bear disaster risk on currency markets received a risk premium of 2.1%, which amounts to 36% of the total currency risk premium on carry trades. Consistent with the evidence on currency option smiles, the compensation for disaster risk increases a lot post-crisis. Although expected volatility is now back to its pre-crisis level, the price of disaster risk is still an order of magnitude higher than before. The large role of disaster risk is a robust finding: the inclusion of transaction costs leads to similar results, and the absence of counterparty risk in the analysis actually suggests that disaster risk might be even more important than estimated here.

In this paper, we propose a simple and successful structural estimation of global disaster risk. The model is parsimonious and flexible; despite its flexibility, it delivers closed-form expressions for the key object of interests. The closed-form expressions then lead to a simple and transparent estimation procedure. Such strengths come with a price. In the model, we assume that Gaussian shocks are jointly normal and independent of the disasters, an assumption that is not directly testable with changes in exchange rates, as they pertain to differences in shocks, not country-specific shocks. Yet, the model captures first-order economic links between interest rates, exchange rates, and disaster risk.

First, the model implies a strong link between interest rates and disaster premia. In the model, interest rates depend on the drift of the SDF and the exposure to disaster risk: interest rates are high in countries that tend to depreciate when disasters occur. In the data, we find a strong link between the average compensation for disaster risk implied in currency options and the average interest rates. Figure 1 reports the average estimated disaster risk premium, as well as the average interest rate differential for each country. Clearly, they align, suggesting that a large part of the cross-country differences in interest rates corresponds to different exposures to global disaster risk.

[Figure 1 about here.]

Second, the model implies that countries with small (large) exposures to global disaster risk should depreciate (appreciate) in times of disasters. This is the key risk that carry traders face and the core mechanism of the model. As Figure 2 shows, this is exactly what happened during the fall of 2008. Countries with estimated low exposure depreciated, while countries with estimated large exposure appreciated. The strong link between disaster's exposures and changes in exchange rates during that period appears whether disaster's exposure is measured during the fall of 2008 or in the three previous months (from May 2008 to August 2008).

[Figure 2 about here.]

Figures 1 and 2 provide strong support for the key mechanism and implications of the model. The model, however, could be easily rejected by additional data: the model ignores any potential market segmentation between currency markets and other asset markets; it does not attempt to model the full term structure of interest rates; it does not describe cash flows nor equity returns; it is written and used at a monthly frequency and ignores daily or intra-day exchange rate variation. The model could be extended in many dimensions, but we focus instead on its core and use it to reinterpret some recent results in the literature.

We derive closed form expressions for hedged currency excess returns when the investment horizon tends to zero. Hedged strategies protect investors against large exchange rate changes of two types: those due to jump-like disasters and those that might occasionally happen in a world of Gaussian shocks without any jump. We show that, in the limit of small time horizons, expected hedged currency excess returns are thus equal to a fraction of the Gaussian risk premium, which varies with the put option strike used to hedge the investment. The result is intuitive: if the option strike is far from the money, the investor bears a large amount of depreciation risk before the option contract pays off and delivers any insurance, and thus the investor expects a large return on the hedged carry trade as a compensation for this exchange rate risk. We show, however, that disaster risk cannot be fully hedged with a simple put option when the time horizon is not negligible. Therefore, average hedged currency excess returns offer only a biased estimation of disaster risk premia.

The paper is organized as follows. Section I rapidly reviews the literature. Section II compares the currency option smiles pre- and post-2008 for high versus low interest rate currencies. Section III presents our model and derives the estimation procedure. Section IV reports our estimation of time-varying disaster risk premia. Section V derives additional results on hedged currency excess returns that offer a structural interpretation of the previous literature. Section VI concludes. The online Appendix details all the mathematical proofs and reports additional simulation and estimation results.

I Literature Review

Our paper is related to three different literatures: the forward premium puzzle, disaster risk, and option pricing with jumps.

Since the pioneering work of Tryon (1979), Hansen and Hodrick (1980), and Fama (1984), many papers have reported deviations from the uncovered interest rate parity (UIP) condition. These deviations are also known as the forward premium puzzle. In a recent contribution, Lustig, Roussanov, and Verdelhan (2011) build a cross-section of currency excess returns and show that it can be explained by covariances between returns and return-based risk factors. In large baskets of currencies, foreign country-specific shocks average out. Currency carry trades, defined as the difference in baskets of currency returns, are thus dollar-neutral and depend only on world shocks. In order to replicate the dynamics of exchange rates, Lustig, Roussanov, and Verdelhan (2011) show that SDFs must have a common component across countries, as well as heterogenous loadings on this common component. While Lustig, Roussanov, and Verdelhan (2011) consider log-normal SDFs, Gavazzoni, Sambalaibat, and Telmer (2012) argue that SDFs should incorporate higher moments. Our paper builds on the disaster risk literature to satisfy these conditions.[1] World disaster risk is a common component of SDFs, but countries differ in their exposures to world disasters, which affect the higher moments of SDFs.

Our paper also relates to a recent literature using options to investigate the quantitative importance of disasters in currency markets.[2] Bhansali (2007) was the first to document the empirical properties of hedged carry trade strategies. Brunnermeier, Nagel, and Pedersen (2008) show that risk reversals increase with interest rates. In their view, the crash risk of the carry trade is due to a possible unwinding of hedge fund portfolios. This is consistent with one interpretation of disasters. Jurek (2008) provides a comprehensive empirical investigation of hedged carry trade strategies. He

[1] Other models replicate the forward premium puzzle. Using swap rates, exchange rate returns, and prices of at-the-money currency options, Graveline (2006) estimates a two-country term structure model that replicates the forward premium anomaly. Verdelhan (2010) uses habit preferences in the vein of Campbell and Cochrane (1999). Colacito (2008), Bansal and Shaliastovich (2012) and Colacito and Croce (2012) build on the long-run risk model pioneered by Bansal and Yaron (2004). Farhi and Gabaix (2011) propose a disaster risk explanation of the puzzle and the full term structure of interest rates, while Guo (2007) presents a disaster-based model with monetary frictions. Gourio, Siemer and Verdelhan (2013) study disaster risk in a two-country real-business cycle model.

[2] A large literature focuses instead on equity and bond markets: see Duffie, Pan and Singleton (2000), Ait-Sahalia, Wang and Yared (2001), Pan (2002), Liu, Pan and Wang (2005), Gourio (2008), Barro and Ursua (2009), Santa-Clara and Yan (2010), Backus, Chernov and Martin (2011), Bollerslev and Todorov (2011), Gabaix (2012), Julliard and Ghosh (2012), Bates (2012), Seo and Wachter (2013), Siriwardane (2013), Martin (forthcoming), and Wachter (forthcoming).

uses deep-out-of-the-money currency options to derive currency crash risk. Jurek's main result – that disaster risk explains 30% to 40% of carry trade returns – is consistent with the findings of this paper, but our approach differs in several dimensions. First, our model-based empirical strategy leads to a structural interpretation of the results. Second, the model allows us to use a variety of option strikes, including more-liquid at-the-money options, in order to disentangle Gaussian and disaster risk premia. Third, we take into account the time-varying volatilities in currency markets. Using at-the-money options, Burnside et al. (2011) also find that disaster risk can account for the carry trade premium, where disaster risk comes in the form of a high value of the SDF rather than large carry trade losses. In contrast to our approach, in their framework the only source of risk priced in carry trade returns is disaster risk and they only consider at-the-money options. Our model shows in closed form that average hedged excess returns at-the money are not zero in the presence of Gaussian risk. All those papers focus on the pre-crisis period, while our paper uncovers key differences in the post-crisis period. Finally, our paper is related to recent work by Chernov, Graveline, and Zviadadze (2012), who study daily changes in exchange rates and at-the-money implied volatilities. Unlike us, however, they specify the law of motion of stochastic volatility at high frequency, considering jumps in the volatility, as well as the level of exchange rates. They find that jump risk accounts for 25% of currency risk and show that many jumps in levels are related to macroeconomic news, while jumps in volatilities are not.

A related literature studies high-frequency data and option pricing with jumps, following pioneering work by Merton (1976) in the context of equity options. Borensztein and Dooley (1987) extend the use of models with jumps to currency options. Bates (1996a, 1996b) shows that exchange rate jumps are necessary to explain option smiles. More recently, Carr and Wu (2007) find great variations in the riskiness of two currencies (yen and British pound) against the U.S. dollar, and they relate it to stochastic risk premia. Campa, Chang, and Reider (1998) document similar results for some European cross-rates. Bakshi, Carr, and Wu (2008) find evidence that jump risk is priced in currency options. However, they consider jumps which occur at a high-frequency, whereas the disasters we have in mind are of very low frequency; in Barro and Ursua (2008), disasters happen every 30 years. As a result, the economic analysis and our econometric technique are very different from the traditional option pricing literature. Our focus is on the macro-finance explanations of currency risk. Our estimates of disaster risk premia and carry trade losses during fall 2008 are broadly consistent with the findings and calibration of Barro (2006) and Barro and Ursua (2008, 2009). While the option pricing literature tends to consider high frequency returns, the one-month frequency we consider is relevant for practitioners as well. Using actual returns from the hedge fund industry, we show that the exposure of global macro hedge funds to carry trade risk up to August 2008 is a good predictor of the extent of their losses in September 2008, a good example of very low returns on currency markets at the one-month frequency.

II Currency Option Smiles Pre- and Post-Crisis

We first describe our data, define some useful option-related terms, and then compare currency option smiles pre- and post-crisis.

A Spot and Forward Exchange Rates

Our data set comes from J.P. Morgan and focuses on the 10 largest and most liquid currency spot, forward, and option markets: Australia, Canada, Euro area, Japan, New Zealand, Norway, Sweden, Switzerland, United Kingdom, and United States. All exchange rates in our sample are expressed in U.S. dollars per foreign currency. As a result, an increase in the exchange rate corresponds to an appreciation of the foreign currency and a decline of the U.S. dollar. For each currency, the sample comprises spot and one-month forward exchange rates measured at the end of the month, as well as implied volatilities from currency options with one-month maturity for the same dates. Foreign interest rates are built using forward currency rates and the U.S. LIBOR, assuming that the covered interest rate parity condition holds.[3]

[3]In normal conditions, forward rates satisfy the covered interest rate parity condition (CIP): forward discounts (i.e., the log differences between forward and spot exchange rates) equal the interest rate differentials between two countries. Akram, Rime, and Sarno (2008) study high-frequency deviations from CIP and conclude that CIP holds at daily and

B Option Lexicon

Before turning to our option data, let us review some basic option terms. Figure 3 presents the payoffs of three option-based strategies we consider: (i) being long an out-of-the-money put option, (ii) being long an out-of-the-money call option, and (iii) being long a risk-reversal (i.e., being long an out-of-the-money put option and short an out-of-the-money call option with symmetric strikes).

[Figure 3 about here.]

A currency option is said to be at-the-money if its strike price is equal to the forward exchange rate. A put (call) option is said to be out-of-the-money if its strike price is below (above) the forward rate—that is, if it takes a large depreciation (appreciation) to make the option worth exercising. The value of an option changes with the value of its underlying asset: the delta of a currency option measures the sensitivity of the option price to changes in the exchange rate. Figure 4 presents the deltas of put options as a function of their strikes. The delta of a put varies between 0 for extremely out-of-the-money options to -1 for extremely in-the-money options. The exercise price of an option can thus be indirectly indirectly characterized by its corresponding delta. A 10 delta (25 delta) put is an option with a delta of -10% (-25%).

[Figure 4 about here.]

C Currency Options

In our data set, options are quoted using their Black and Scholes (1973) implied volatilities for five different deltas. The implied volatility of an option is a convenient normalization of the price of this option as a function of its strike. Our sample comprises monthly deep-out-of-the-money puts (denoted 10 delta puts), out-of-the-money puts (25 delta puts), at-the-money puts and calls, out-of-the money calls (25 delta calls), and deep-out-of-the money calls (10 delta calls) for the

lower frequencies. This relation, however, was violated during the extreme episodes of the financial crisis in the fall of 2008 [e.g., see Baba and Packer (2009)].

January 1996 to December 2011 period. Jorion (1995), Carr and Wu (2007), and Corte, Sarno, and Tsiakas (2011) study the features of currency implied volatilities pre-crisis.

D Smiles

If the underlying risk-neutral distributions of exchange rates were purely log-normal, then implied volatilities would not differ across strike prices. A graph of implied volatilities as a function of their strikes would be flat. Such a flat line is a good description of equity option markets until the crash of 1987. Since 1987, however, equity markets exhibit a different pattern: the price of out-of-the-money options is much higher than the price of at-the-money options. A graph of implied volatilities as a function of strikes thus looks like a "smile." Currency options exhibit a similar pattern. The dotted lines in Figure 5 are the average implied volatilities for different strikes during the first part of our sample, 1/1996 to 08/2008, for each country. Implied volatilities of out-of-the money options tend to be higher than those of at-the-money options, and out-of-the-money puts and calls roughly exhibit the same implied volatilities: in other words, implied volatilities "smile." The recent financial crisis introduces a clear change, breaking the symmetry between puts and calls.

[Figure 5 about here.]

E Risk-Reversals Pre- and Post-Crisis

Risk-reversals offer a simple summary statistic of the asymmetry of the smile: a high (low) price of an out-of-the-money put option (relative to the price of a call option with symmetric strike) implies a positive (negative) risk-reversal. Until the recent financial crisis, currency risk-reversals were small. Since the crisis, currency option smiles are no longer symmetric and risk-reversals are, in absolute value, an order of magnitude larger than before.

The risk-reversals of the Australian and New Zealand dollars, for example, have notably increased since the beginning of the crisis, while the risk-reversal of the Japanese yen has decreased. On the one hand, the Australian and New Zealand dollars are high interest rate currencies over the sample period, and market participants seem to price the risk of large depreciations of those currencies since the recent crisis. On the other hand, the Japanese yen is an example of a low interest rate currency with a negative risk-reversal: market participants seem thus to price the risk of a large appreciation. The Swiss franc is also a low interest rate currency; its risk-reversal, however, hovers around zero. In the recent period, the risk of a large appreciation of the Swiss franc is now mitigated by the intervention policy of the Swiss National Bank.[4]

In equity markets, a potential interpretation for the high price of out-of-the-money put options and the associated risk-reversal is that equity option prices reflects the possibility of large decreases in stock returns, a potential explanation for the large equity premium. Currency option markets tell a similar story, but only after 2008 and when comparing low versus high interest rate currencies.

The reason for conditioning on the level of interest rates is simple. Currency markets offer large average excess returns to carry trade investors who go long high interest rate currencies and short low interest rate currencies. In any risk-based view of currency markets, expected carry trade returns compensate investors for bearing the risk of a depreciation (appreciation) of the high (low) interest rate currencies in bad times. In other words, high interest rate currencies are risky, whereas low interest rate currencies are not. But currency markets do not offer significant returns for unconditional investments in any randomly chosen currency. Thus, research on currency returns focuses on conditional investment strategies. In order to study option prices conditional on interest rates, we sort risk-reversals by the level of foreign interest rates and allocate them into three portfolios, which are rebalanced every month. The first portfolio contains risk-reversals from the lowest–interest rate currencies while the last portfolio contains risk-reversals from the highest–interest rate currencies. Table 1 reports the portfolio average risk-reversals at 10 delta over different subsample periods.

[Table 1 about here.]

At the portfolio level, the contrast between currency option markets pre– and post–crisis is

[4]Since 2011, the Swiss National Bank is intervening massively in the currency market in order to resist currency appreciation and to maintain the exchange rate above 1.20 Swiss franc per euro.

striking. On average, risk-reversals of high interest rate currencies are equal to 0.5% over the 1996 to 2008 period, while those of low interest rate currencies are equal to -0.9%. During the crisis, the difference in risk-reversals escalates: the risk-reversal of high interest rate currencies reaches 6.4%, while the one for low interest rate currencies declines to -3.9%. After the crisis, on average over the 1/2009 to 12/2011 period, the average risk-reversal of high interest rate currencies is equal to 3.6%, while the one for low interest rate currencies is -0.1%. As Figure 6 shows, the difference between the risk-reversals of high and low interest rate currencies is more than twice as large after the recent crisis than before. For high interest rate currencies alone, risk-reversals are now more than six times larger than before the crisis.

[Figure 6 about here.]

The large risk-reversals suggest that market participants consider the risk of potential large depreciation of the high interest rate currencies, thus pointing to disaster concerns on currency markets. We now turn to a simple model that precisely establishes the link between currency options and disaster risk premia, and leads to a structural estimation of the historical compensation for disaster risk.

III Model

This section describes the pricing kernels, then turns to the implied interest rates, exchange rates, and expected currency returns, as well as the currency option prices in the model.

A Pricing Kernels

The model features two countries: home and foreign. The model is set up and estimated at the monthly frequency, assuming that the parameters that govern the SDF in each country are constant over one month. The model parameters, however, are allowed to change non-parametrically the next month. For the sake of clarity, we present the model in two periods. Section IV shows how to incorporate this building block in a multi-country, multi-period extension. There, a state variable Ω_t describes the state of the world. The parameters of the two-country, two-period model depend on Ω_t. All the results in this section should be understood as returns conditional on Ω_t, but for notational simplicity this dependence is implicit. In particular, all the expectations in this section are conditional on Ω_t.

The SDF for each country incorporates both a traditional log-normal component and a disaster component. SDFs are defined as nominal variables (i.e., expressed in units of local currency) because option data correspond to nominal exchange rates. In the home country, the log SDF evolves as:

$$\log M_{t,t+\tau} = -g\tau + \varepsilon\sqrt{\tau} - \frac{1}{2}\operatorname{var}(\varepsilon)\,\tau + \left\{ \begin{array}{ll} 0 & \text{if there is no disaster at time } t+\tau \\ \log(J) & \text{if there is a disaster at time } t+\tau \end{array} \right\}.$$

The log of SDF in the foreign country evolves as:

$$\log M^{\star}_{t,t+\tau} = -g^{\star}\tau + \varepsilon^{\star}\sqrt{\tau} - \frac{1}{2}\operatorname{var}(\varepsilon^{\star})\,\tau + \left\{ \begin{array}{ll} 0 & \text{if there is no disaster at time } t+\tau \\ \log(J^{\star}) & \text{if there is a disaster at time } t+\tau \end{array} \right\}.$$

Both SDFs have two components. The first one, $-g\tau + \varepsilon\sqrt{\tau} - \frac{1}{2}\operatorname{var}(\varepsilon)\,\tau$, is a country-specific Gaussian risk with an arbitrary degree of correlation across countries. Here, g and $g^{\star}$ are constants. The random variables $(\varepsilon, \varepsilon^{\star})$ are jointly normally distributed with mean 0 and are correlated across countries. The second component, $\log(J)$, captures the impact of a disaster on the country's SDF. Disasters are perfectly correlated across the two countries; they are world disasters. The probability of a disaster between t and $t+\tau$ is given by $p\tau$. The Gaussian shocks ε and $\varepsilon^{\star}$ are independent of the nonnegative random variables J and $J^{\star}$, which measure the magnitudes of the disaster event. All these variables are independent of the realization of the disaster event.

The term "disaster" can have several interpretations. One, championed by Rietz (1988) and

Barro (2006), is that of a macroeconomic drop in aggregate consumption, perhaps due to a war or a major economic crisis that affects many countries. Another interpretation is that of a financial stress or crisis affecting participants in world financial markets, perhaps via a drastic liquidity shortage or a violent drop in asset valuations. Both interpretations have merit, and we do not need to take a stand on the precise nature of a disaster. In our setting a disaster is a large increase in the SDFs. The laws of motion of the domestic and foreign SDFs are enough to compute all relevant asset prices, starting with interest rates, exchange rates, and expected currency returns.

B Interest Rates, Exchange Rates, and Expected Currency Excess Returns

Let us first define exchange rates. As in Bekaert (1996) and Bansal (1997), the change in the (nominal) exchange rate is given by the ratio of the SDFs:

$$\frac{S_{t+\tau}}{S_t} = \frac{M^{\star}_{t,t+\tau}}{M_{t,t+\tau}}, \tag{1}$$

where S is measured in home currency per foreign currency. An increase in S represents an appreciation of the foreign currency (we use the same sign convention as in the data analysis). Just like the exchange rate allows us to convert the home price of a good into foreign currency, it also allows us to convert the home currency SDF into the foreign currency SDF.

It might seem counterintuitive that when the foreign SDF increases more than the home SDF, the foreign currency appreciates. However, this robust implication of finance theory is a simple matter of accounting (and is not specific to disaster models) and can be thought as a version of the Law of One Price. The marginal investor can assess a given return either in home ($R_{t,t+\tau}$) or foreign currency ($R^*_{t,t+\tau} = R_{t,t+\tau}\frac{S_t}{S_{t+\tau}}$). The unit of account is simply a veil and has no impact on intrinsic valuation. The home currency SDF $M_{t,t+\tau}$ and foreign currency SDF $M^*_{t,t+\tau}$ encode the valuation of returns in home and foreign currency by the same marginal investor. This requires that $E[M_{t,t+\tau}R_{t,t+\tau}] = E[M^*_{t,t+\tau}R^*_{t,t+\tau}]$, for all equilibrium home currency returns, $R_{t,t+\tau}$. This immediately implies Equation (1).[5]

Let us turn now to interest rates; likewise, they are pinned down by the two SDFs. The home interest rate r is determined by the Euler equation $1 = E\left[M_{t,t+\tau} e^{r\tau}\right]$:

$$r = g - \log\left(1 + p\tau E\left[J - 1\right]\right)/\tau. \tag{2}$$

A similar expression determines the foreign interest rate. Currency carry trades then correspond to the following investment strategy: at date t, the investor borrows one unit of the home currency at rate r and invests the proceeds in the foreign currency at rate $r^\star$. At the end of the trade, at date $t + \tau$, the investor converts the proceeds back into the home currency. In units of the home currency, the payoff to the currency carry trade is:

$$X_{t,t+\tau} = e^{r^\star \tau} \frac{S_{t+\tau}}{S_t} - e^{r\tau}.$$

In the limit of small time intervals, interest rates and expected currency excess returns take a very simple form, presented in the Proposition below.

Proposition 1. *In the limit of small time intervals $\tau \to 0$, the interest rate r in the home country is:*

$$r = g - pE\left[J - 1\right].$$

Carry trade expected returns (conditional on no disasters) are given by:

$$X^e = \pi^D + \pi^G, \tag{3}$$

[5]An alternative derivation of Equation (1) starts from the Euler equations $E[M^\star_{t,t+\tau} R^\star_{t,t+\tau}] = 1$ and $E[M_{t,t+\tau} R^\star_{t,t+\tau} \frac{S_{t+\tau}}{S_t}] = 1$ of two different investors, home and foreign. If financial markets are complete, then the SDFs are unique, and the exchange rate is defined in terms of SDFs. Note that real exchange rates are time-varying even when financial markets are complete, as long as some frictions in the goods markets prevent perfect risk sharing across countries. An example of such a friction often used in the literature is the assumption that some goods are not traded.

where:

$$\pi^D = pE(J - J^*),$$

$$\pi^G = cov(\epsilon, \epsilon - \epsilon^*).$$

The interest rate has two components: the drift of the SDF and the disaster component. A foreign country whose currency tends to depreciate in times of disasters against the home currency (i.e., $E[J^*] < E[J]$) exhibits an interest rate that is above the home interest rate.[6] The currency risk premium has also two components. The first term in Equation (3) is the risk premium associated with disaster risk:

$$\pi^D \equiv pE[J - J^\star].$$

If $E[J - J^\star] > 0$, the expected return due to disaster risk is positive because the foreign currency tends to depreciate when disasters occur.

The second term in Equation (3) is the risk premium associated with "Gaussian risk" à la Backus, Foresi and Telmer (2001):[7]

$$\pi^G \equiv \text{cov}(\varepsilon, \varepsilon - \varepsilon^\star).$$

This is the covariance between the home SDF and the bilateral exchange rate, $S_{t+\tau}/S_t$. If a foreign currency tends to depreciate in bad times, investors expect to be compensated by a positive risk premium. In our model, the expected return of the carry trade compensates for the exposure to these two sources of risk.

[6]Farhi and Gabaix (2011) provide a detailed micro-foundation for the variables J and J^* that has two implications: (i) the more severe the world disasters (so that the world consumption of tradable goods falls more), the higher the values of J and J^*; (ii) if the foreign country fares worse than the home country in times of disasters (which implies that its currency depreciates when disasters occur), then J^* is less than J.

[7]Backus et al. (2001) show that, if markets are complete and SDFs are log normal, then expected log currency excess returns are equal to $E(\log R^e) = 1/2Var(\log M) - 1/2Var(\log M^\star)$. The focus here is instead on the log of expected currency excess returns, but the two expressions are naturally consistent.

C Option Prices

We turn now to option prices in the model. $P_{t,t+\tau}(K)$ is the home currency price of a put with strike K bought at date t and maturing at date $t+\tau$, thus yielding $(K - S_{t+\tau}/S_t)^+$ in the home currency, with the usual notation of $y^+ \equiv \max(0, y)$. The U.S. investor starts with one U.S. dollar, i.e., $1/S_t$ units of foreign currency. If the exchange rate at the end of the contract is lower than the strike ($KS_t > S_{t+\tau}$, where K is measured in units of foreign currency), then the put contract pays off the difference between the strike and the spot rate, $S_{t+\tau}$, for each unit of foreign currency invested; the payoff per U.S. dollar is thus $(K - S_{t+\tau}/S_t)^+$. Likewise, $C_{t,t+\tau}(K)$ is the home currency price of a call yielding $(S_{t+\tau}/S_t - K)^+$ in the home currency. Put and call prices in the models can be expressed using the Black and Scholes (1973) formula, even though the model features non Gaussian shocks. We first rapidly review the Black and Scholes (1973) formula for currency options and then turn to option prices in the model.

C.I Option prices in a Gaussian world

The Black and Scholes (1973) formula, developed originally in the context of stock markets, was adapted to a foreign exchange setting by Garman and Kohlhagen (1983). Let $V^P_{BS}(S, \kappa, \sigma, r, r^\star, \tau)$ and $V^C_{BS}(S, \kappa, \sigma, r, r^\star, \tau)$ denote the Black and Scholes (1973) prices for a put and a call, respectively, when the spot exchange rate is S, the strike is κ, the exchange rate volatility is σ, the home interest rate is r, the foreign interest rate is $r^\star$, and the time to maturity is τ. The prices of a call and a put are given by:

$$
\begin{aligned}
V^C_{BS}(S, \kappa, \sigma, r, r^\star, \tau) &= Se^{-r^\star\tau}\mathbb{N}(d_1) - \kappa e^{-r\tau}\mathbb{N}(d_2), \\
V^P_{BS}(S, \kappa, \sigma, r, r^\star, \tau) &= \kappa e^{-r\tau}\mathbb{N}(-d_2) - Se^{-r^\star\tau}\mathbb{N}(-d_1), \\
d_1 &= \frac{\log(S/\kappa) + (r - r^\star + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \\
d_2 &= d_1 - \sigma\sqrt{\tau},
\end{aligned}
$$

where $\mathbb{N}$ is the Gaussian cumulative distribution function. The Black and Scholes (1973) and Garman and Kohlhagen (1983) formula have a simple scaling property with respect to the time to maturity τ and the interest rates r and $r^\star$:

$$V_{BS}^{P}(S, \kappa, \sigma, r, r^\star, \tau) = V_{BS}^{P}(Se^{-r^\star \tau}, \kappa e^{-r\tau}, \sigma\sqrt{\tau}, 0, 0, 1).$$

For notational convenience, the arguments 0 and 1 are omitted and the value of a generic put is simply $V_{BS}^{P}(S, \kappa, \sigma) = V_{BS}^{P}(S, \kappa, \sigma, 0, 0, 1)$.

C.II Option prices in the model

Let us turn now to the price of a put in the model. The price of a call is derived similarly. We assume that J and $J^\star$ are constant over one month.[8] We define $\tilde{J} = \frac{pJ}{1-p\tau}$ and $\tilde{J}^* = \frac{pJ^*}{1-p\tau}$ and use them in the Proposition 2 below for mathematical convenience. Economically, however, $\tilde{J}$ and $\tilde{J}^*$ are empirically close to pJ and pJ^* at the one-month horizon for any reasonable disaster probability. In our estimation procedure, we cannot separately identify p, J and J^*; instead we are able to identify $\tilde{J}$ and $\tilde{J}^*$, and thus pJ and pJ^*. Proposition 2 decomposes the put price into a Gaussian and a disaster component, in the spirit of Merton (1976):

Proposition 2. *In a model with constant disaster sizes, the price of a put option can be decomposed into a non-disaster part $P^{ND}(K)$ and a disaster–related part $P^{D}(K)$ according to:*

$$P_{t,t+\tau}(K) = P_{t,t+\tau}^{ND}(K) + P_{t,t+\tau}^{D}(K),$$

[8] Recall that all parameters of the model, including J and $J^\star$, are allowed to move freely from one month to the next. Similar results can be obtained under the assumption that J and $J^\star$ are time-varying over one month and are log-normally distributed.

where:

$$P^{ND}_{t,t+\tau}(K) = V^{P}_{BS}\Big(\frac{e^{-r^*\tau}}{1+\tilde{J}^*\tau}, K\frac{e^{-r\tau}}{1+\tilde{J}\tau}, \sigma_h\sqrt{\tau}\Big),$$
$$P^{D}_{t,t+\tau}(K) = \tau V^{P}_{BS}\Big(\frac{e^{-r^*\tau}\tilde{J}^*}{1+\tilde{J}^*\tau}, K\frac{e^{-r\tau}\tilde{J}}{1+\tilde{J}\tau}, \sigma_h\sqrt{\tau}\Big),$$

where the strike is K, the time to maturity is τ, the home interest rate is r, the foreign interest rate is $r^\star$ and the volatility of the Gaussian part of exchange rates is $\sigma_h = \sqrt{var(\varepsilon - \varepsilon^*)}$.

C.III Estimation procedure

For each quoted strike K_i and at each date t, we consider the difference between the quoted put price, P_i, and its model counterpart, $P(\tilde{J}, \tilde{J}^*, \sigma_h, K_i)$. Put-call parity implies that call prices reflect the same information as put prices. The model parameters ($\tilde{J}$, $\tilde{J}^*$, σ_h) are obtained, at each date t and for each foreign country, by minimizing the sum of squared price differences for the five quotes:

$$\min_{\tilde{J},\ \tilde{J}^*,\ \sigma_h} \sum_{i=1}^{5} [P_i - P(\tilde{J}, \tilde{J}^*, \sigma_h, K_i)]^2,$$

where:

$$P(\tilde{J}, \tilde{J}^*, \sigma_h, K) \;\; = \;\; V^{P}_{BS}\Big(\frac{e^{-r^*\tau}}{1+\tilde{J}^*\tau}, K\frac{e^{-r\tau}}{1+\tilde{J}\tau}, \sigma_h\sqrt{\tau}\Big) + \tau V^{P}_{BS}\Big(\frac{e^{-r^*\tau}\tilde{J}^*}{1+\tilde{J}^*\tau}, K\frac{e^{-r\tau}\tilde{J}}{1+\tilde{J}\tau}, \sigma_h\sqrt{\tau}\Big).$$

Since the model parameters move freely across time periods, minimizations are independent across the time dimension. Each currency is characterized by its risk exposure, $\tilde{J}^*$, and its implied volatility, σ_h (at each date), but the estimations are not independent across our 9 currencies, since they all depend on the U.S. exposure to disaster risk, $\tilde{J}$. The estimation of the whole set of 19 parameters at once is feasible but highly compute-intensive.

Instead, at each date t, we proceed in two steps, preferring a transparent and easily replicable estimation technique. First, the minimization is run using the portfolio of high interest rate currencies in order to determine the U.S. exposure to world disaster risk $\tilde{J}$. The first step thus uses the information from a large set of countries, focusing on the economically-relevant case, without having to determine a large set of parameters at once. Second, taking the U.S. exposure as given, the minimization is run at the country level. Such country-level estimates allow for cross-sectional tests of the model. Since $\tilde{J}$ is given, the estimation is now independent across currencies, and the minimization problem, at each date and for each currency, is defined only over two parameters, $\tilde{J}^*$ and σ_h. A simple minimization on a grid ensures that the minimum obtained is the global minimum.

This simplification comes at a cost: instead of using a put on the portfolio of high interest rate currencies, we perform the first stage estimation using a portfolio of puts on these currencies. In section C.I, we show that this approximation does not affect our estimation results for the disaster premium.

D Key Assumptions

Before turning to the data to implement the estimation procedure above, let us pause to assess the validity of the experiment. The model is extremely tractable; indeed, it yields closed-form solutions for a number of key moments of interest. The model is also very flexible; it allows the realized and expected volatilities of exchange rates to be time-varying, in line with previous findings on currency markets [e.g., Diebold and Nerlove (1989)]. The volatilities are held constant over one month and then move non-parametrically from one month to the next.

The tractability and flexibility rely on two key assumptions: the shocks ϵ and $\epsilon^\star$ are (i) jointly normal, and (ii) independent from J, J^*, and the realization of the disaster. Excluding the fall of 2008, the difference $\epsilon^* - \epsilon$ appears conditionally normally distributed (as shown by a Jarque-Bera test), once one controls for the time-varying volatility of exchange rates. Yet, the model presumes not only that the difference $\epsilon^* - \epsilon$ is normal but also that the shocks ϵ and ϵ^* are both normal and independent of the realization of disasters. This log-normality and independence assumption on pricing kernels cannot be tested with exchange rates alone, but is common across macroeconomic models of exchange rates. The empirical experiment that follows is thus run under the assumption that SDF shocks at the monthly frequency are conditionally Gaussian when no disaster occurs.

IV Estimation of Disaster Risk Premia

This section reports estimates of currency excess returns and disaster risk premia using option prices.

A Currency Portfolios

We build portfolios of currency excess returns in order to focus on the sources of aggregate risk and to average out idiosyncratic variations. At the portfolio level, high interest rate currencies deliver average currency excess returns that are significantly different from zero; they capture expected excess returns from currency markets. We first describe the portfolio sorts and the sample period and then turn to the portfolio characteristics.

A.I Portfolios Sorts

For each individual currency, the corresponding excess return is built from the perspective of a U.S. investor. The first portfolio contains the lowest interest rate currencies while the last portfolio contains the highest interest rate currencies. Inside each portfolio, currencies are equally-weighted.

The connection with the theory developed in Section III is as follows. The different countries are indexed by $i \in I$. A state variable Ω_t describes the state of the world at date t. This state variable follows an arbitrary stationary stochastic process. All the parameters of the model are arbitrary functions of Ω_t. Correspondingly, all the computed variables r_i, X_i^e, π_i^D, and π_i^G depend on Ω_t. Underlying our three portfolios are three state-dependent sets: $I_1(\Omega_t)$, $I_2(\Omega_t)$, and $I_3(\Omega_t)$. Forming portfolios is a way to compute moments conditional on the three sets: I_1, I_2, and I_3. For instance, the expected return on portfolio k is simply the average return over the countries in the portfolio:

$$\overline{X}_k^e = E\left[\frac{\sum_{i \in I_k(\Omega_t)} X_i^e(\Omega_t)}{\# I_k(\Omega_t)}\right],$$

where I_k denotes the set of currencies in portfolio k and $\# I_k(\Omega_t)$ denotes their number.

A.II Sample Period

In the sample period, fall 2008 appears as the unique potential example of disasters and thus deserves special attention. Borrowing in Japanese yen and lending in New Zealand dollars would have incurred a loss of almost 30% in October 2008, and a total loss of close to 40% in the fall of 2008. In a diversified portfolio of high and low interest rate currencies, the average return of the carry trade strategy was −4.5% in the fall 2008, for a cumulative decline from September to December 2008 that amounts to 17.8%. This is a large drop, as the standard deviation of monthly returns over the whole sample is just 2%. Almost all of the 17.8% decline is due to losses on high interest rate currencies, which depreciated sharply. The large changes in exchange rates triggered the exercise of currency options. For example, in our sample, the share of 10-delta put options exercised reaches an all-time high in the fall of 2008.

These very low returns on currency markets occurred in bad times for U.S. and world investors [see Lustig and Verdelhan (2007, 2011)]. During fall 2008, the U.S. stock market declined by 33% in terms of the MSCI index. The closest event to this very strong decline in equity and currency returns is the 1987 stock market crash: from September to November 1987, the U.S. stock market lost 32.6%. Standard risk measures beyond those from equity markets point in the same direction. Very low currency excess returns (four standard deviations below their means) happened exactly when volatilities in equity and bond markets and credit spreads were high (four standard deviations above their means). These market-based indices offer real-time measures of risk that complement the approach based on marginal utilities and real consumption growth rates. U.S. national account statistics point toward an annualized decrease of 4.3% in real personal consumption expenditures in the fourth quarter of 2008, following an annualized decrease of 3.8% in the third quarter. These shocks represent declines of more than three standard deviations in the mean consumption growth rate.

Fall 2008 can be viewed as an example of disasters in our sample. This view is consistent with our model, which implies that, as long as a currency crash does not occur in the sample, *conditional* monthly changes in exchange rate are conditionally normally distributed. This is indeed the case if the fall of 2008 is excluded from the sample. To take into account exchange rate heteroscedasticity, a GARCH (1,1) model is estimated for each currency and then normality tests are run on exchange rate changes normalized by their volatility. After the GARCH (1,1) correction, all countries exhibit conditionally Gaussian exchange rates in the sample. Since our decomposition of expected currency excess returns is valid in samples without disasters, we report results on samples that excludes fall 2008 when that decomposition is used.

Fall 2008, however, could alternatively be viewed as an increase in the probability of disasters, not the realization of one particular disaster. For robustness checks, we also report average estimates of disaster risk premia on samples that include the fall of 2008. In that view, conditional changes in exchange rates are normally distributed in the fall of 2008 as in the rest of the sample. The results of conditional normality tests depend naturally on the information set and the conditioning variables used, and are thus subject to discussion. The main findings in this paper do not depend on such discussion.

A.III Portfolio Characteristics.

Let us turn now to the characteristics of the portfolios. Table 2 reports average changes in exchange rates, interest rates, risk-reversals at 10 and 25 delta, as well as average currency excess returns.

[Table 2 about here.]

Average currency excess returns increase monotonically from the first to the last portfolio. This is not a surprise: we know from the empirical literature on the uncovered interest rate parity that high interest rate currencies tend to appreciate on average. As a result, investors in these currencies gain both the interest rate differential and the foreign exchange rate appreciation. Excess returns on high interest rate currencies are 5.9% (5.0%) on average excluding (including) the fall of 2008 and more than two standard errors away from zero. The currency excess returns imply a 0.6 (0.5) Sharpe ratio, which is higher than the Sharpe ratio on the U.S. equity market over the same period.

If disaster risk is an important determinant of cross-country variations in interest rates, then a portfolio formed by selecting countries with high interest rates will, on average, select countries that feature a large risk of currency depreciation. We will come back to this point after estimating each country's disaster risk exposure, but risk-reversals give a preliminary hint. Intuitively, as already noted in Section II, higher probabilities of depreciation for the foreign currency should show up in higher levels of risk-reversals. Thus, if disaster risk matters for the cross-country differences in interest rates, high interest rate countries should exhibit high risk-reversals; Table 1 already shows that for risk-reversals at 10 delta. Table 2 reports similar evidence for risk-reversals at 25 delta. Risk-reversals at 10 and 25 delta increase monotonically across portfolios. Similar results are obtained when the fall of 2008 is included in the sample. The results confirm and extend the previous findings of Carr and Wu (2007), who report a high contemporaneous correlation between currency excess returns and risk reversals for the yen and the British pound against the U.S. dollar. Note that the risk reversals at 10 delta are more expensive than those at 25 delta. This is again consistent with a risk of depreciation for high interest rate currencies.

Currency markets thus exhibit large average excess returns that seem potentially linked to disaster risk. We now turn to the estimation of the market's compensation for bearing such risk.

B Disaster Risk

We first present the average compensation for disaster risk and then report its time-variation.

B.I Average Disaster Risk Premia

Estimates are obtained for each country and each date. For the sake of clarity, we then aggregate the results at the portfolio level and focus on the portfolio of high interest rate currencies, which exhibits significant average excess returns. Time-series of the country-level estimates are reported in the Online Appendix. Table 3 reports estimates of average disaster risk premia over different time-windows. Over the pre-crisis period, the role of disaster risk is statistically significant, but economically small: the compensation for disaster risk amounts to less than 0.7% and it accounts for less than 15% of total currency risk premia (Panel I). The 1996 to 2007 period thus offers only limited support to the disaster risk model. Over the post-crisis period, however, disaster risk appears as a major concern of market participants, as it accounts for more than half of the total currency risk compensation (Panel II). In the full sample (excluding the fall of 2008), the disaster risk premium is significantly different from zero: it amounts to 2.1% on average and it accounts for 36% of the 5.9% of total currency risk premia (Panel III). Including the fall of 2008, the disaster risk premium reaches 2.3% (Panel IV). Disaster risk is thus priced in currency markets and requires a sizable compensation, particularly over the recent period.

[Table 3 about here.]

B.II Time Series of Disaster Risk Premia

Figure 7 presents the time series estimates of the disaster premium (top panel) and of the volatility parameter (bottom panel) for the high interest rate currencies. Consistent with the averages presented in Table 3, the compensation for disaster risk is low over the 1996 to 2007 sample, but it increases markedly with the financial crisis of 2008 and has remained at high levels since then. This increase in disaster risk premia is intuitive; it mirrors the increase in risk-reversals noted in the previous section. At the country level, the correlations between risk-reversals and estimates of disaster risk premia vary between 0.70 and 0.93 depending on the country. The fall of 2008 is also characterized by a large increase in expected exchange rate volatility: yet, the volatility has decreased after the crisis while the compensation for disaster risk has not. The estimation also reveals that the Asian crisis of 1998 did not affect the price of disaster risk for the developed countries in our sample. In this perspective, the Asian crisis is not interpreted as a world disaster by currency option markets, but merely as a limited increase in expected exchange rate volatility.

[Figure 7 about here.]

The model and its associated estimation thus deliver the expected goal: a simple, time-varying, real-time estimate of the compensation for world disaster risk. This is a key contribution of the paper.

C Robustness

We assess the robustness of our results to four empirical issues: the initial step in the estimation procedure, the mis-measurement due to transaction costs, model mis-specifications, and the monthly frequency of the data.

C.I Two-step Estimation

The estimate of the disaster risk exposure for the home country at each date is obtained from the average interest rates and average option prices in the portfolio of high interest rate currencies. The choice of this portfolio is a natural starting point: it focuses on the countries that deliver large and significant currency excess returns on average and large and significant risk-reversals over the recent period. We check, however, that our average estimate of disaster risk premia does not depend on this initial step. We obtain similar results when the home country parameters are estimated on a given currency pair that exhibits large currency excess returns and significant risk-reversals over the recent period. For instance, using the Australian dollar in the first stage of the estimation (instead of the portfolio of high interest rate currencies) leads to a disaster risk premium that is slightly higher but clearly not statistically different from our benchmark estimate (2.4% vs 2.1%). The estimation results thus appear robust to variations in the first stage of the estimation.

C.II Transaction Costs

Our benchmark estimates of disaster risk premia do not take into account bid-ask spreads on currency markets. Transaction costs on forward and spot contracts reduce excess returns, while transaction costs on currency options increase insurance costs against disasters. As a result, transaction costs would most likely increase the share of disaster risk premia. We propose a preliminary estimation of their impact, constrained by data availability.

The dataset includes bid and ask quotes on the spot and the forward exchange rates for the entire sample. Unfortunately, bid and ask quotes on currency options are only available after 9/2004 and for a limited set of countries (Australia, Canada, Euro area, Japan, Switzerland, and U.K.) on Bloomberg. The bid-ask spreads are expressed in units of implied volatilities for each strike. On this limited sample, bid-ask spreads are clearly larger out-of-the-money than at-the-money. Bid-ask spreads appear stable pre-crisis, over the 9/2004 to 3/2007 period. To extend the bid and ask series to the earlier part of our sample (1/1996–8/2004), we thus use the cross-country average bid-ask spread measured on the pre-crisis period for each strike. To extend the series to Norway, New Zealand, and Sweden after 2004, the cross-country average bid-ask spread at each point in time and for each strike is used. As a result, bid-ask spreads widen when implied volatilities increase. The implied volatilities spreads are converted into bid-ask prices in order to re-estimate Gaussian and disaster risk premia.

The results are in line with the intuition. After bid-ask spreads, average currency excess returns on the high interest rate portfolio decrease from 5.9% to 5%. The average risk-reversals increase from 0.25% to 0.6%. The disaster risk premium over the full sample (excluding the fall of 2008) is relatively stable at 2% (vs. 2.1% without transaction costs). As a result, the share of currency risk premia explained by disaster risk increases from 36% to 41%. Overall, the results appear robust to the introduction of transaction costs.

Note, however, that the estimation above does not rule out more serious illiquidity issues. It is possible to imagine that the J.P. Morgan market maker simply gives indicative prices by using the

Black and Scholes (1973) formula (which generates a low option price), but there is little trading of out-of-the-money options. If someone wanted to aggressively buy these options, then she would end up moving prices against herself and paying higher prices. If this is the case, the potential trading prices are higher than the indicative prices in our data, and disaster risk is thus under-estimated.

C.III Model Misspecification

The model may be misspecified, not fully capturing the richness of exchange rate dynamics, ignoring any potential market segmentation between currency markets and other asset markets, and not modeling the full term structure of interest rates. One way to address those concerns would be to extend the model but at the cost of losing tractability and focus. A natural extension would be the introduction of small disasters. In such a specification, out-of-the-money options offer no protection against small disasters and would therefore be cheaper than at-the-money options. We choose instead to maintain the parsimony of the model and shows that its core mechanism offers two new insights on the average cross-country differences in interest rates over the sample and on the changes in exchange rates during the crisis.

First, as already noted in the introduction and shown in Figure 1, high interest rate countries are characterized by large disaster risk premia on average. The result is not mechanical because the model allows for a free drift parameter that could potentially account for the cross-country differences in interest rates. The finding is consistent with Brunnermeier, Nagel, and Pedersen (2008), who show that high interest rate countries tend to exhibit high risk-reversals in the pre-crisis sample. In the post-crisis sample, the link is much stronger, as Section II shows. Our estimation procedure extracts the disaster risk premium from option prices and highlights the link between interest rates and the risk of large currency movements.

Second, the core mechanism of the model is the risk of large currency changes in times of global disasters. If one interprets the fall of 2008 as an example of such global disaster, the model's implications are clearly borne out in the data. As Figure 2 shows, realized changes in exchange rates are consistent with estimates of disaster risk premia from currency options. This result is not mechanical either as the estimation of disaster risk does not use changes in exchange rates. The finding is consistent with the rest of the paper: in the model, high interest rate currencies bear the risk of large depreciations in times of disaster, and thus offer high expected excess returns due to large disaster risk premia. In the data, high interest rate currencies depreciated sharply in the fall of 2008, while low interest rate currencies appreciated. Again, the estimation procedure extracts the disaster risk premium from option prices, and it appears consistent with the behavior of exchange rates during the crisis.

C.IV Estimation Frequency

Our model is written and estimated at the monthly frequency and we focus on a simple carry trade strategy implemented through hypothetical portfolios. The model thus abstracts from higher frequency portfolio choices and more sophisticated investments. One could argue that sophisticated investors would not be sensitive to changes that take place over one month; however, data on hedge fund returns suggest otherwise.

The Morningstar CISDM database contains 158 hedge funds following a global macro strategy, including both currently active funds and defunct ones (135 funds were active in August 2008, and 131 in September 2008). The oldest hedge fund in the sample began operation in 1986, but the majority of the funds became active in the 2000s. Since actual hedge fund trades are not observable, we focus on funds whose returns load on the carry trade factor of Lustig, Roussanov and Verdelhan (2011) by estimating the following two-factor model:

$$R_{i,t} = \alpha_i + \beta_i HML_t^{FX} + \beta_i^w RW_t + \varepsilon_{i,t},$$

where $R_{i,t}$ is the return of hedge fund i at date t, HML_t^{FX} is the return of high interest rate currencies minus the return on low interest rate currencies, and RW_t is the world stock market return measured by the Dow Jones Global Index. The carry trade betas (β_i) and world market betas (β_i^w) are estimated on the 24-month period that ends in August 2008. Similar results are obtained with estimation windows of 36 and 48 months. The carry trade betas strongly predict currency returns in September 2008, even after controlling for world market betas:

$$R_i^{9/2008} = \gamma + \delta\beta_i + \delta^w \beta_i^w + \eta_i.$$

The R^2 of this regression is 47% (vs. 10% when only the world markets betas are included) and both slope coefficients are highly significant. All hedge funds versed in carry-trade strategies apparently did not get a chance to exit before the carry trade returns collapsed and some endured large related losses in September 2008. The mean return among the hedge funds with the largest carry trade betas (fifth quintile) is -5.1%. Subtracting the exposure to world stock markets ($\delta^W \beta_i^w$), the mean return is still -3.6%. It is low compared to the mean return over the previous year (1.0%) and compared to the standard deviation of around 0.8% of the portfolio return over the previous three years. The decrease of -3.6% on a portfolio of hedge funds thus represents a decrease of more than four standard deviations. Moreover, the averages per quintile hide very large losses for some hedge funds, some reaching a minimum of -24% in September 2008. The strong predictive power of the carry trade betas suggests that carry risk played a large role in the low returns experienced by hedged funds in the fall of 2008. Although our model ignores higher frequency variation, it captures a first-order economic effect of disasters.

Our estimation thus appears robust to several concerns. A final concern lies in the existence of counterpart risk, in the case of options without large enough margins. The counterparty risk issue relies on the possibility that the seller of a put might actually default during a disaster. Put premia take that risk into account and are lower than in the model. We expand on this question in the next section.

V Additional Model Implications and Link to Literature

In this section, we derive additional model implications on hedged returns and risk-reversals and use them to revisit the literature on disaster risk and on the forward premium puzzle. The section starts with a simple and novel approximation of hedged carry trade excess returns.

A Hedged Carry Trade Returns

We first define hedged carry trades and then propose a closed form expression for their expected returns.

A.I Definition of Hedged Payoffs

In what follows, we drop the time subscripts for notational simplicity. Let Δ^P be a Black-Scholes put delta, i.e., $\Delta^P < 0$ and let K_{Δ^P} be the corresponding strike; $\Delta^P \in (-1, 0)$ is decreasing in the option strike. The return $X(K_{\Delta^P})$ to the hedged carry trade is the payoff of the following zero-investment trade: borrow one unit of the home currency at interest rate r; use the proceeds to buy $\lambda^P(K_{\Delta^P})$ puts with strike K_{Δ^P}, protecting against a depreciation in the foreign currency below K_{Δ^P}; and invest the remainder $\left(1 - \lambda^P(K_{\Delta^P})P(K_{\Delta^P})\right)$ in the foreign currency at interest rate $r^\star$. So the hedged return is given by:

$$X(K_{\Delta^P}) = \Big(1 - \lambda^P(K_{\Delta^P})P(K_{\Delta^P})\Big)e^{r^\star\tau}\frac{S_{t+\tau}}{S_t} + \lambda^P\left(K_{\Delta^P}\right)\left(K_{\Delta^P} - \frac{S_{t+\tau}}{S_t}\right)^+ - e^{r\tau},$$

where the hedge ratio $\lambda^P(K_{\Delta^P})$ is given by:

$$\lambda^P(K_{\Delta^P}) = \frac{e^{r^*\tau}}{1 + e^{r^*\tau}P(K_{\Delta^P})}.$$

To summarize the notation: X is the carry trade return and X^e is its annualized expected value conditional on no disaster; $X(K_{\Delta^P})$ is the hedged carry trade return with strike K_{Δ^P}; $P\left(K_{\Delta^P}\right)$ is the home currency price of a put yielding $(K_{\Delta^P} - S_{t+\tau}/S_t)^+$ in the home currency; $X^e(K_{\Delta^P})$ is the annualized expected value of the hedged carry trade return conditional on no disaster; and E^{ND} denotes expectations under the assumption of no disaster:

$$X^e(K_{\Delta^P}) = \frac{E^{ND} X\left(K_{\Delta^P}\right)}{\tau}.$$

A.II A Simple and Intuitive Decomposition

Proposition 3 offers a closed form formula for the hedged returns.

Proposition 3. *We assume that the disaster sizes* (J, J^*) are constant *between* t *and* $t+\tau$ *with* $J > J^*$*. Let* Δ^P *be a Black-Scholes put delta i.e.,* $\Delta^P < 0$*, and let* K_{Δ^P} *be the corresponding strike. We define:*

$$\begin{aligned} \beta &= n\big(\mathbb{N}^{-1}(-\Delta^P)\big) - \mathbb{N}^{-1}(-\Delta^P)(1+\Delta^P), \\ \gamma &= \big(1+\Delta^P\big)\Delta^P \mathbb{N}^{-1}(-\Delta^P) - \big(2+\Delta^P\big)n\big(\mathbb{N}^{-1}(-\Delta^P)\big), \end{aligned}$$

where $\mathbb{N}(\ )$ *is the cumulative standard normal distribution and* $n(\ )$ *is the standard normal distribution. In the limit of small time intervals (*$\tau \to 0$*), the hedged carry trade expected return (conditional on no disasters) can be approximated by:*

$$X^e(K_{\Delta^P}) = \big(1+\Delta^P\big)\pi^G + \left(\beta\left(pJ + \frac{\pi^D\pi^G}{\sigma_h^2}\right) + \gamma\pi^G\right)\sigma_h\sqrt{\tau}, \tag{4}$$

where π^G *is the Gaussian premium,* σ_h *is the exchange rate volatility conditional on no disaster, and* π^D *is the disaster premium.*

Loosely speaking, in the limit of short time to maturity the Black–Scholes delta of the put option has a simple interpretation: it is the probability that the put will be exercised. The first term in Equation (4) is thus intuitive: the further away from the money, the more depreciation risk the investor bears and the higher the expected return of the hedged carry trade. For example, take the carry trade hedged with a put option at 10 delta. In the language of currency traders, this means that the strike is such that the Black-Scholes delta of the put is -0.10; thus the leading order of $X^e(K_{10^P})$ is equal to $0.9\pi^G$. Since the hedge uses a relatively deep-out-of-the-money put, investors bear 90% of the Gaussian risk.

The second term in Equation (4) depends on a mixture of Gaussian and disaster parameters. Our simulation of the model, which is discussed in the next section, shows that, for the one-month maturity, it accounts for 1/5 to 1/3 of the hedged returns (depending on Δ^P) and is positive for any reasonable values of the model parameters. Proposition 3 thus leads to a simple upper bound for the Gaussian risk premium and a lower bound for the disaster premium:

$$\pi^G < \frac{X^e(K_{\Delta^P})}{(1+\Delta^P)} \text{ and } \pi^D > X^e - \frac{X^e(K_{\Delta^P})}{(1+\Delta^P)}. \tag{5}$$

Table 4 reports portfolio average currency excess returns that are unhedged or hedged at 10 delta, at 25 delta, and at-the-money for three portfolios. In each case, the table reports the mean excess return and its standard error, along with the corresponding Sharpe ratio for excess returns. As expected, hedging downside risks decreases average returns. Unhedged excess returns in high interest rate currencies are, again, equal to 5.9% on average (Panel I). A hedge at 10 delta protects the investor against large drops in foreign currencies, whereas a hedge at-the-money protects the investor against any depreciation of the foreign currency: the latter insurance is obviously more expensive because it covers more states of nature and thus leads to lower excess returns. Average excess returns hedged at 10 delta are 5% (Panel II), whereas average excess returns hedged at 25 delta and at-the-money are 3.9% and 2.4% (Panels III and IV). Including the fall of 2008 in the sample leads to similar results: average excess returns hedged at 10 delta, 25 delta, and at-the-money are 4.4%, 3.4%, and 2.2% (not reported).

[Table 4 about here.]

Using, for example, currency excess returns hedged at 25 delta leads to an upper bound for the Gaussian risk premium of $3.9/0.75 = 5.2\%$ and to a lower bound bound for the disaster risk premium of 5.9 - 5.2% = 0.7%. Likewise, hedged excess returns at the money imply an upper bound for the Gaussian risk premium of 4.8% and a lower bound bound for the disaster risk premium of 1.1%. These bounds are consistent with the estimates reported in Table 3.

This methodology, however, suffers from three weaknesses when compared to our benchmark estimation: (i) it only delivers bounds instead of point estimates, (ii) it delivers an average disaster risk premium but not its time variation, and (iii) it relies on the estimation of two averages (hedged and unhedged excess returns), which are only known with large standard errors in small samples.

B Risk-Reversals

We now turn to our model's implications for risk-reversals. Given $\Delta > 0$, we can consider the corresponding Black-Scholes put delta, $\Delta^P = -\Delta$, as well as the Black-Scholes call delta $\Delta^C = \Delta$. Risk-reversals are defined as the difference between the implied volatility at the Black-Scholes put delta and the implied volatility at the Black-Scholes call delta:

$$RR_\Delta = \sigma_{-\Delta} - \sigma_\Delta. \tag{6}$$

Risk-reversals are an appealing metric that highlights the key role of disaster risk in the price of options as showed in Propositions 4 and 5:

Proposition 4. *If there is no disaster risk :* $RR_\Delta = 0$ *for all* Δ.

A similar result was derived by Bates (1991) for equity options. In the presence of disaster risk, Proposition 5 identifies conditions under which we can simplify the expression for risk-reversals.

Proposition 5. *We assume that the disaster sizes* (J, J^*) are constant *between* t *and* $t + \tau$. *Given a Black-Scholes delta* $\Delta > 0$, *risk-reversals can be approximated in the limit of small time intervals* $(\tau \to 0)$ *by:*

$$RR_\Delta = \frac{1 - 2\Delta}{n(\mathbb{N}^{-1}(\Delta))} \pi^D \sqrt{\tau}.$$

At short maturity, the risk–reversal is approximately proportional to the disaster premium and increases approximately linearly with the distance to the money measured by Δ. When the the foreign country is more exposed to disaster risk, the interest rate difference and the short-maturity risk-reversal increase. These characteristics appear in our data set. The simulations presented in the next session assess the accuracy of the risk-reversal approximation.

C Simulations

Propositions 1, 3, and 5 are derived in the limit of small time intervals. We check their validity for one-day and one-month horizons by simulating a calibrated version of the model. The model relies on eight parameters: the disaster probability (p), the domestic and foreign disaster sizes (J and $J^{\star}$), the domestic and foreign drifts (g and $g^{\star}$) of the pricing kernels, the domestic and foreign volatilities (σ and $\sigma^{\star}$) of the Gaussian shocks, as well as their correlation (ρ). The calibration thus relies on eight moments. The disaster probability is taken from Barro and Ursua (2008). The average domestic and foreign interest rates, the average domestic and foreign disaster sizes (scaled by p), the average currency excess returns, and the volatility of the bilateral exchange rate are all measured on the high interest rate currency portfolio during the 1996 to 2011 period (excluding fall 2008). The maximum Sharpe ratio is assumed to be 80%. The Online Appendix reports the parameters and simulation results.

The annualized, simulated unhedged returns are equal to 6.2% and 6% at the one-month and one-day horizons respectively, in line with the true value in the model (6%). Likewise, the simulated interest rates are equal to their calibrated targets. Proposition (1) thus delivers precise approximations of interest rates and average unhedged currency excess returns. These approximations are the only ones needed to derive and interpret our main empirical results.

At the one-month horizon, the simulated hedged returns are equal to 4.3% at 10 delta, 3.2% at 25 delta, and 2.0% at-the-money. The approximations in Proposition (3) deliver hedged returns equal to 4.1% at 10 delta, 3.1% at 25 delta, and 1.9% at-the-money, close to the true values in the model. The approximations are the sum of two terms. The first term in Proposition (3), i.e., the fraction of the Gaussian risk premium remaining, is equal to 2.70% at 10 delta, 2.25% at 25 delta, and 1.50% at-the-money. Thus, the second term, the unhedged component of the disaster premium, cannot be neglected.

At the one-day horizon, the risk-reversal in the model is equal to 0.6% at 10 delta and 0.2% at 25 delta. The simulation shows that the approximation derived in Proposition (5) is close to the actual value; the approximated risk-reversal is equal to 0.7% at 10 delta and 0.2% at 25 delta. At the one-month horizon, however, the distance between the true and approximated risk-reversal is larger. The risk-reversal in the model is equal to 2.4% at 10 delta and 0.9% at 25 delta. The approximated risk-reversal is equal to 4% at 10 delta and 1.4% at 25 delta. Overall, the limit values derived in Propositions 3 and 5 appear as precise approximations at the one-day horizon. At the one-month horizon, however, their precision declines, especially for risk-reversals. We thus do not use these approximations to estimate the compensation for disaster risk. Yet, Propositions 3 and 5 remain useful to understand intuitively hedged currency excess returns and risk-reversals.

D Comparison with the Literature

In the final part of the paper, we compare our results to the literature, using the closed-form expressions derived above. We start with the benchmark estimates of disaster risk in the macroeconomics literature and then turn to recent studies of currency markets.

D.I Comparison with Barro and Ursua (2008)

When a disaster occurs in our model, the SDF is multiplied by an amount J. The model of Farhi and Gabaix (2011) relates this amount to more primitive economic quantities. In that model, J equals $B^{-\gamma}F$, where $B^{-\gamma}$ is the growth of real marginal utility during a disaster and F is the growth of the value of one unit of the local currency in terms of international goods during the same disaster.

Hence, the disaster risk premium is:

$$\overline{\pi}^D = \overline{pE[J]}_L - \overline{pE[J]}_H = \overline{pE[B^{-\gamma}F]}_L - \overline{pE[B^{-\gamma}F]}_H,$$

where the subscripts L and H refer to low and high interest rate countries. Therefore, the disaster risk premium depends on the probability of disasters p, the relative value of the SDF $B^{-\gamma}$, and the payoff of the carry trade in disasters through the sufficient statistic $\overline{pE[B^{-\gamma}F]}_L - \overline{pE[B^{-\gamma}F]}_H$.

Using the episode of fall 2008 to calibrate the value of $\overline{F}_L - \overline{F}_H$ and assuming away a potential correlation between $B^{-\gamma}$ and $\overline{F}_L - \overline{F}_H$ sheds some light on the typical value of $pB^{-\gamma}$. This exercise should be viewed as a back-of-the-envelope calculation rather than a rigorous estimate, since the inference of $\overline{F}_L - \overline{F}_H$ relies on a single disaster. Moreover, it does not take into account the full path to recovery and, as Gourio (2008) shows, might overestimate the impact of disasters. With this caveat in mind, a value for $\overline{F}_L - \overline{F}_H$ of 20% implies a value of $\overline{pE[B^{-\gamma}]}$ equal to 10% in order to generate a disaster risk premium $\overline{\pi}^D$ of 2% as in the currency option data.

Barro and Ursua (2008) use long samples of consumption series for a large set of countries in order to estimate disaster sizes and probabilities.[9] They estimate a probability of disasters p equal to 3.63%. A coefficient of relative risk aversion $\gamma = 3.5$ then implies that $\overline{E[B^{-\gamma}]} = 3.88$, leading to a value of $\overline{pE[B^{-\gamma}]}$ equal to 14%, which rationalizes the equity premium. Barro and Ursua's (2008) value of 14% for $\overline{pE[B^{-\gamma}]}$ and a carry trade loss of 20% during disasters lead to a disaster risk premium of $0.14 \times 0.2 = 2.8\%$. Therefore, we view our estimates over the 1996 to 2011 period (2.3%) as consistent with Barro and Ursua's (2008) findings.

D.II Comparison with Jurek (2008) and Burnside et al. (2011)

Jurek (2008) studies one-month currency excess returns hedged at- and out-of-the money. Our model provides a structural interpretation to his empirical experiment. When the investment horizon

[9] Note, however, that interpreting our pricing kernel strictly as a simple function of consumption growth would open a large debate that is beyond the scope of this paper. Constant relative risk aversion and complete markets imply, for example, a very high correlation between consumption growth and exchange rates, a high correlation that is not evident in the data (Backus and Smith, 1993).

shrinks to zero, currency excess returns hedged out-of-the-money do not bear any disaster risk, but they offer biased estimates of the Gaussian risk premia, since they bear 90% of the Gaussian risk at 10 delta, and 75% of the Gaussian risk at 25 delta. At the one-month horizon, however, our simulations show that an additional bias is present. As a result, our work does not provide a simple adjustment that can be used to directly infer disaster risk premia from average realized hedged returns.

Burnside, Eichenbaum, Kleshchelski and Rebelo (2011) study currency excess returns hedged at-the-money in a sample that ends in July 2009. They find that average hedged carry trade excess returns are statistically different from zero and interpret this result as a rejection of a pure disaster risk explanation of the forward premium puzzle. Our decomposition of currency risk premia into a Gaussian and a disaster risk premium provides a simple interpretation of their findings. According to Equation (5), average currency excess returns hedged at-the-money are at least equal to half of the Gaussian risk premium.

D.III Counterparty Risk

All recent studies of disaster risk ignore counterparty risk. Yet, it is reasonable to think that the seller of a put might default with some probability ϕ if a disaster occurs, and that this risk is not fully-hedged by margin constraints. We are not able to measure default probabilities on option markets but obtain an order of magnitude of the potential impact on estimates of disaster risk premia.

In the presence of counterparty risk, an agent engaging in hedged carry trade still bears some disaster risk, even at short maturity. With probability ϕ, the agent is exposed to disasters and the compensation for the disaster risk is thus $\phi\pi^D$ and the expected excess return of the hedged carry trade is bounded below by $(1+\Delta)\pi^G + \phi\pi^D$. According Equation (5), in the limit of small time intervals, the disaster risk premium is bounded by:

$$\pi^D > \frac{X^e - X^e(K_{\Delta^P})/(1+\Delta^P)}{1-\phi/(1+\Delta^P)}.$$

For deep-out-of-the-money options ($\Delta = -0.1$), the lower bound for π^D that does not take into account counterparty risk must now be multiplied by approximately $1/(1-1.1\phi)$. When $\phi = 0.1$, it is multiplied by 1.12; when $\phi = 0.25$, it is multiplied by 1.38. For at-the-money options ($\Delta = -0.5$), the adjustment is even larger: when $\phi = 0.1$, it is multiplied by 1.25; when $\phi = 0.25$, it is multiplied by 2.

Counterparty risk can substantially increase estimates of disaster risk premia. Unfortunately, measuring expected default probabilities on option markets in disaster states is beyond the scope of this paper. The results above are only back-of-the-envelope estimates of the impact of counterparty risk.

D.IV Comparison with Lustig et al. (2011) and Menkhoff et al. (2012)

Lustig et al. (2011) show that time-varying volatility in global equity markets accounts for the cross-section of forward discount-based currency portfolio returns. This volatility measure does not use any exchange rate or interest rate data, but illustrates the systematic risk of currency markets. In times of high global volatility, high interest rate currencies tend to depreciate and low interest rate currencies tend to appreciate. Menkhoff et al. (2012) find that a measure of global volatility obtained from currency markets also explains the cross-section of interest rate-sorted currency portfolios.

How do these results relate to our paper? It turns out that large increases in global equity volatility corresponds to large increases in downside risk, and downside risk could as well account for the returns on the interest rate-sorted currency portfolios. Disentangling downside risk from volatility risk is not an easy task in a cross-sectional asset pricing experiment. To illustrate, the Online Appendix reports asset pricing tests on the six portfolios of Lustig et al. (2011) obtained with two risk factors: the average excess returns of a U.S. investor on currency markets (denoted

RX, as in the two papers above) and the risk-reversals at 25 delta on S&P500 index options (denoted RR). The U.S. S&P 500 index options are used to measure global disaster equity risk because of the lack of data on out-of-the-money equity options in other countries in the sample.

Risk-reversals are significantly priced in the cross-section of carry trade excess returns. Both factors explain jointly more than 90% of the cross-section of average excess returns. Loadings on the dollar risk factor are close to 1 and do not account for the cross-section of portfolio returns. Loadings on risk-reversals, however, differ markedly across portfolios: they range from 0.87 to -0.96. Unsurprisingly, the same pattern appears on our smaller set of countries and portfolios (for which betas vary from 0.81 to -0.76). High interest rate currencies tend to depreciate in bad times, when risk-reversals are high, while low interest rate currencies tend to appreciate in those times. Lettau, Maggiori and Weber (2013) report further evidence of downside risk in the cross-section of currency, equity, and commodity returns. Instead, this paper estimates a structural model on option prices in order to disentangle time-varying volatility from disaster risk premia.

VI Conclusion

The goal of this paper is to provide a simple, real-time, model-based estimation of the compensation for world disaster risk. We achieve this goal using currency options. The fall of 2008 appears as a turning point in currency option markets: option smiles are fairly symmetric before the financial crisis; post-crisis, they are clearly asymmetric, and those asymmetries depend on the level of interest rates. The model interprets the data in terms of disaster risk. High (low) interest rate currency options reflect the risk of large depreciations (appreciations) in bad times. The estimation of disaster risk premia shows that while the compensation for global disaster risk was low before the crisis, it remains an order of magnitude higher after the crisis. On average, the disaster premium explains more than a third of carry trade expected excess returns.

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Table 1: Average Risk-Reversals Before, During, and After the 2008 Crisis

Portfolios	1	2	3
		Panel I: 1/1996–8/2008	
Mean	−0.89	0.07	0.50
	[0.22]	[0.13]	[0.17]
		Panel II: 9/2008–12/2008	
Mean	−3.89	2.41	6.43
	[2.27]	[1.42]	[1.88]
		Panel III: 1/2009–12/2011	
Mean	−0.06	2.18	3.64
	[0.71]	[0.62]	[0.66]
		Panel IV: 1/1996–12/2011 (excl. fall 2008)	
Mean	−0.74	0.48	1.11
	[0.26]	[0.25]	[0.33]
		Panel V: 1/1996–12/2011 (incl. fall 2008)	
Mean	−0.79	0.51	1.21
	[0.27]	[0.23]	[0.33]

Notes: This table reports portfolio average risk-reversals at 10 delta over different subsamples (Panels I to V). Risk-reversals are sorted by the level of foreign interest rates and allocated into three portfolios, which are rebalanced every month. The first portfolio contains risk-reversals from the lowest interest rate currencies while the last portfolio contains risk-reversals from the highest interest rate currencies. Risk-reversals are reported in percentages. The standard errors, reported between brackets, are obtained by bootstrapping both the time-series using a block bootstrap of 10 months (1 month during the crisis) and the cross-section of countries. Data are monthly, from J.P. Morgan. The sample period is 1/1996 to 12/2011.

Table 2: Changes in Exchanges Rates, Risk-Reversals, and Currency Excess Returns

Portfolios	1	2	3
		Panel I: Exchange Rates	
Mean	0.20	1.47	3.04
	[2.16]	[2.39]	[2.50]
		Panel II: Interest Rates	
Mean	−2.19	0.03	2.30
	[0.42]	[0.34]	[0.35]
		Panel III: Risk-Reversals 10 Delta	
Mean	−0.74	0.48	1.11
	[0.26]	[0.25]	[0.33]
		Panel III: Risk-Reversals 25 Delta	
Mean	−0.39	0.27	0.60
	[0.14]	[0.14]	[0.18]
		Panel IV: Excess Returns	
Mean	−1.34	1.98	5.91
	[2.35]	[2.53]	[2.72]
Sharpe Ratio	−0.16	0.25	0.60

Notes: This table reports portfolio average changes in exchange rates, interest rates, risk-reversals, as well as average currency excess returns. Countries are sorted by the level of foreign interest rates and allocated into three portfolios, which are rebalanced every month. The first portfolio contains the lowest interest rate currencies while the last portfolio contains the highest interest rate currencies. The table reports the mean excess return and its standard error, along with the corresponding Sharpe ratio for excess returns. The mean and standard deviations for the exchange rates, the interest rates, and the excess returns are annualized (multiplied respectively by 12 and $\sqrt{12}$). The Sharpe ratio corresponds to the ratio of the annualized mean to the annualized standard deviation. The standard errors, reported between brackets, are obtained by bootstrapping both the time-series using a block bootstrap and the cross-section of countries. The block sizes are 10 months. Data are monthly, from J.P. Morgan. The sample period is 1996 to 2011, excluding the fall of 2008.

Table 3: Average Disaster Risk Premia

	π^D	$\pi^G + \pi^D$	Disaster Share
		Panel I: 1/1996–08/2008	
Mean	0.68	4.64	14.56
	[0.22]	[2.78]	[7.97]
		Panel II: 1/2009–12/2011	
Mean	8.24	13.72	60.09
	[1.78]	[7.95]	[22.44]
		Panel III: 1/1996–12/2011 (excl. fall 2008)	
Mean	2.14	5.91	36.22
	[0.76]	[2.68]	[28.28]
		Panel IV: 1/1996–12/2011	
Mean	2.34	5.05	46.45
	[0.75]	[2.49]	[30.21]

Notes: This table reports estimates of disaster risk premia over different time-windows. The estimation proceeds in two steps. First, the minimization is run at the portfolio level in order to determine the U.S. exposure to world disaster risk. Second, taking the U.S. exposure as given, the minimization is run at the country-level. The results are then aggregated into portfolios and the table reports the average estimates obtained for the portfolio of high interest rate currencies presented in Table 2. Standard errors are obtained by bootstrapping using a block bootstrap. Spot and forward exchange rates are from Datastream, while currency options are from J.P. Morgan. Data are monthly. The sample period is 1996 to 2011.

Table 4: Hedged Currency Excess Returns

	1	2	3
	Panel I: Excess Returns		
Mean	−1.34	1.98	5.91
	[2.43]	[2.58]	[2.83]
Sharpe Ratio	−0.16	0.25	0.60
	Panel II: Excess Returns Hedged at 10 Delta		
Mean	−2.21	0.99	5.05
	[2.32]	[2.47]	[2.71]
Sharpe Ratio	−0.28	0.13	0.55
	Panel III: Excess Returns Hedged at 25 Delta		
Mean	−1.58	0.56	3.91
	[1.92]	[2.14]	[2.41]
Sharpe Ratio	−0.23	0.08	0.49
	Panel IV: Excess Returns Hedged At-the-Money		
Mean	−0.66	0.24	2.39
	[1.34]	[1.55]	[1.90]
Sharpe Ratio	−0.14	0.05	0.39

Notes: This table reports portfolio average currency excess returns that are unhedged or hedged at 10 delta, at 25 delta, and at-the-money for three portfolios. Countries are sorted by the level of foreign interest rates and allocated into three portfolios, which are rebalanced every month. The first portfolio contains the lowest interest rate currencies while the last portfolio contains the highest interest rate currencies. The table reports the mean excess return and its corresponding Sharpe ratio. The mean and standard deviations are annualized (multiplied respectively by 12 and $\sqrt{12}$). The Sharpe ratio corresponds to the ratio of the annualized mean to the annualized standard deviation. Standard errors are obtained by bootstrapping. Data are monthly, from J.P. Morgan. The sample period is 1996 to 2011, excluding the fall of 2008.

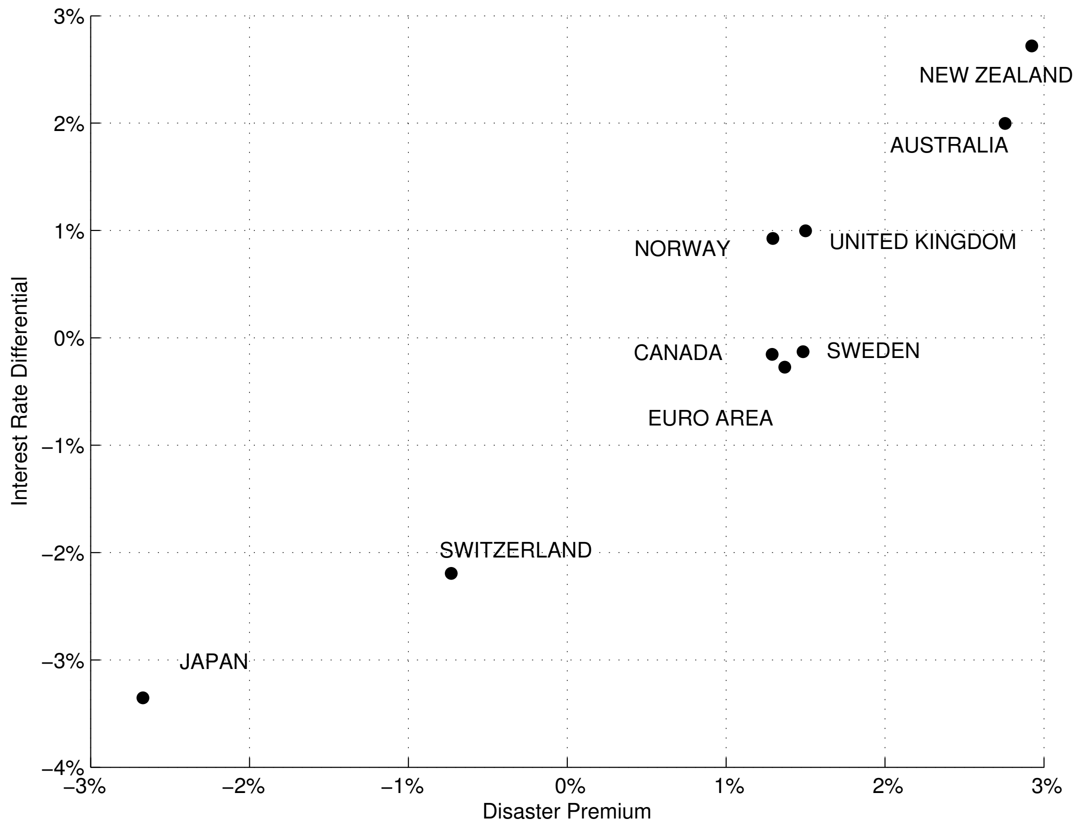

Figure 1: Average Disaster Risk Premia and Average Interest Rates

This figure reports the average disaster risk premium and the average interest rate differential for each country. Interest rates and risk premia are reported in percentage points per annum. Spot and forward exchange rates are from Datastream, while currency options are from J.P. Morgan. Data are monthly. The period is 1996 to 2011.

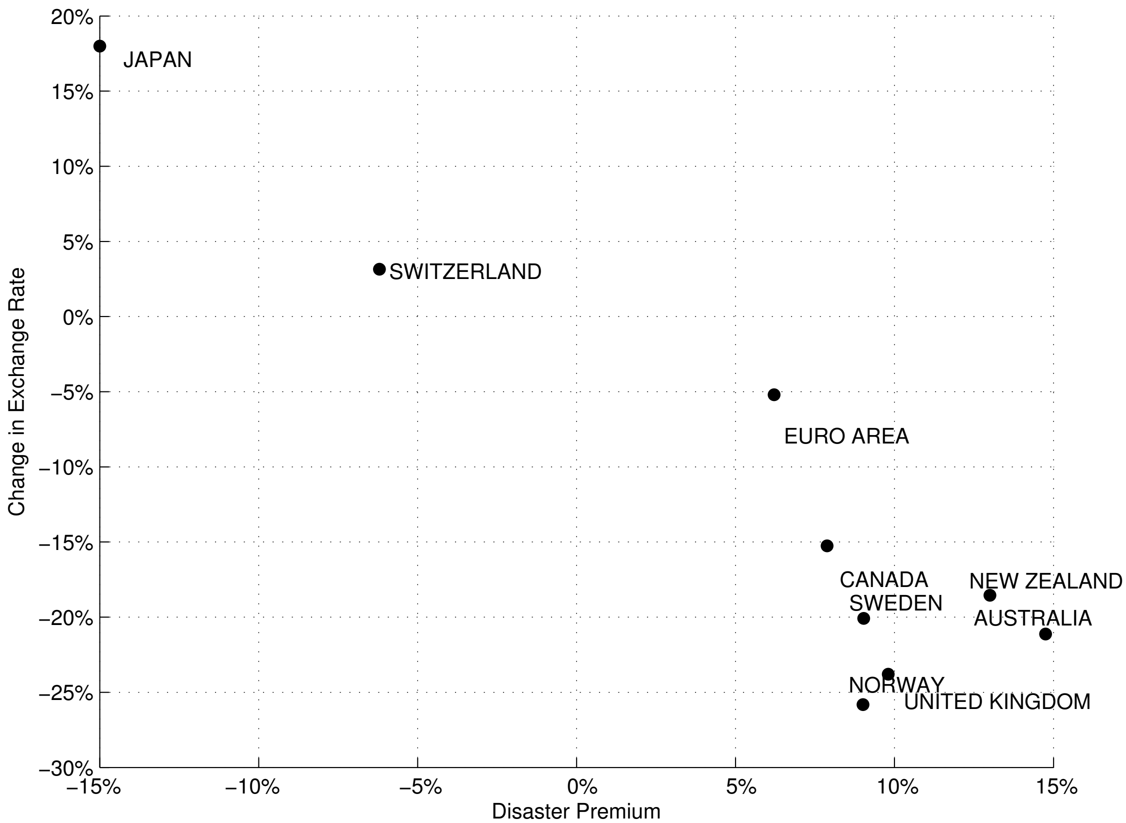

Figure 2: Disaster Risk Exposures and Changes in Exchange Rates During the Crisis

This figure reports the average estimated disaster risk premia and the cumulative percentage change in exchange rate for each country during the crisis (from September 2008 to December 2008). Spot and forward exchange rates are from Datastream, while currency options are from J.P. Morgan. Data are monthly.

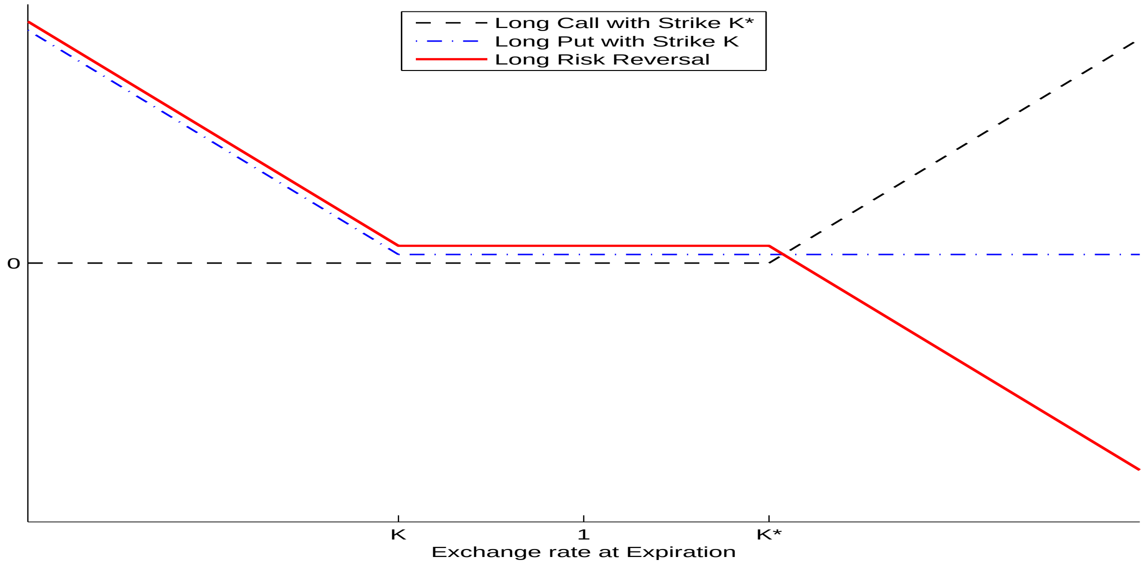

Figure 3: Option Payoffs

This figure presents the payoffs of three option investments as a function of the underlying asset at the expiration date. The underlying asset at the expiration date is normalized by the current forward price of the underlying asset. The three strategies consist of (i) buying an out-of-the money call (with strike $K^{\star}$); (ii) buying an out-of-the-money put option (with strike K); and (iii) a risk reversal that corresponds to selling an out-of-the-money call (with strike $K^{\star}$) and simultaneously buying an out-of-the-money put (with strike K).

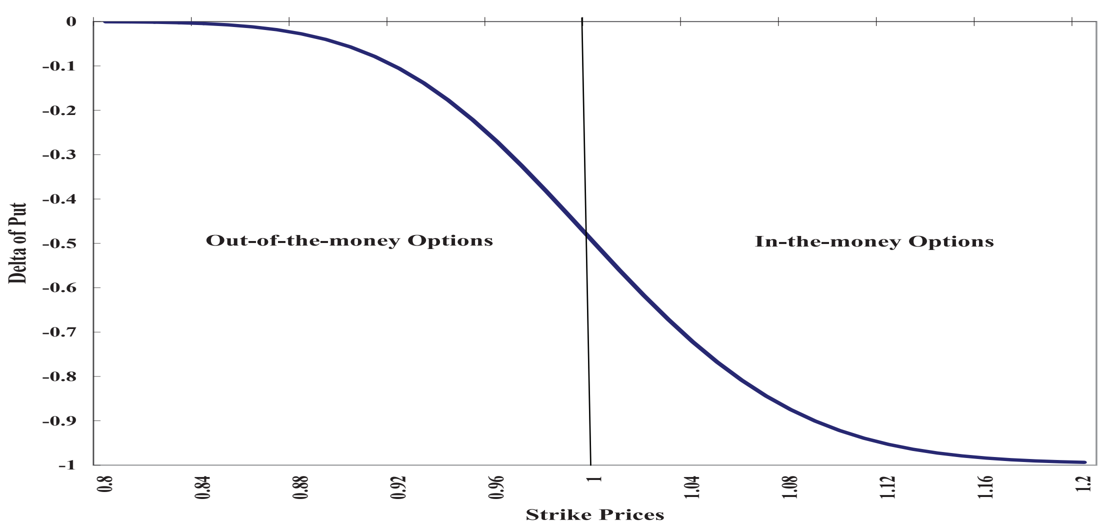

Figure 4: Option Deltas

This figure presents the deltas of put options as a function of their strikes. The strikes are normalized by the current forward price of the underlying asset. The delta of an option is defined as the rate of change of the option price with respect to the price of the underlying asset. The delta of a put varies between 0 for the most deep out-of-the-money options and -1 for the most deep in-the-money options. The figure is computed using the Black–Scholes formula for a currency put option with a one-month maturity, an annualized implied volatility of 10%, and foreign and domestic interest rates both set equal to 4% per annum.

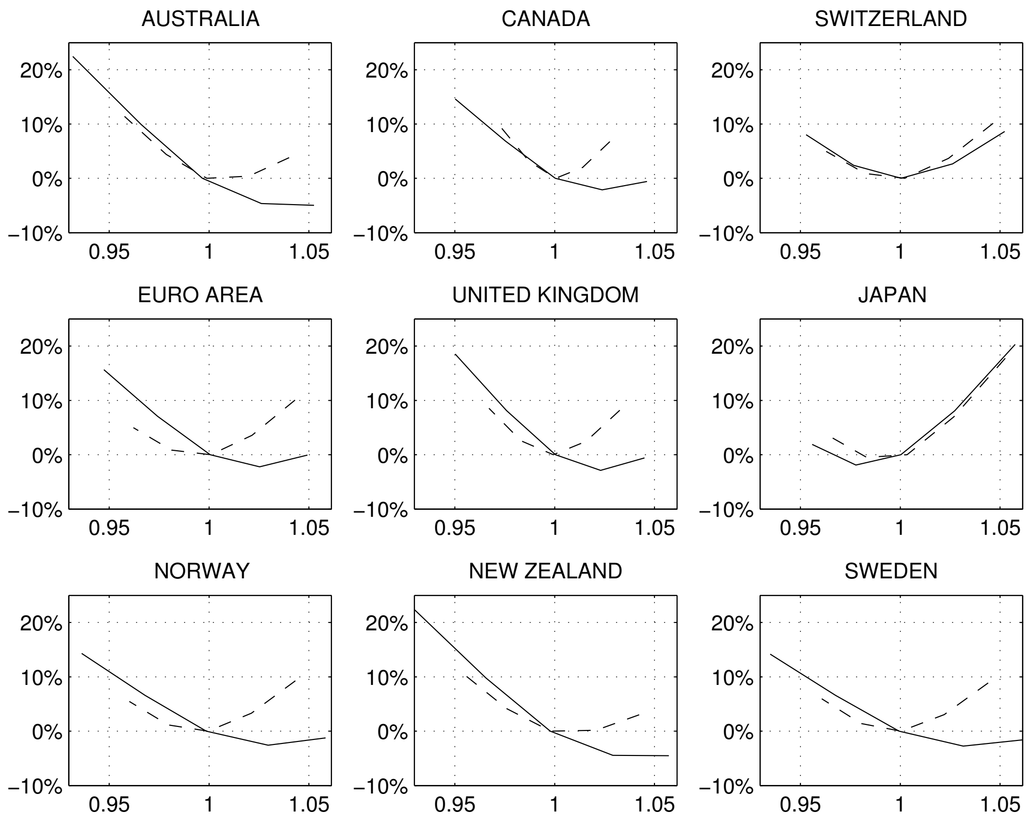

Figure 5: Average Currency Option Smiles Pre- and Post-Crisis

This figure presents the average quoted implied volatilities during the pre-crisis period (1/1996–08/2008, dotted line) and during the post-crisis period (1/2009–12/2011, full line) as a function of their strikes. To maintain comparability across currencies and periods, the implied volatilities at different strikes are scaled by the average implied volatility of at-the-money options during the corresponding period. The quoted strikes are normalized by the spot exchange rate. Spot and forward exchange rates are from Datastream, while currency options are from J.P. Morgan. Data are monthly.

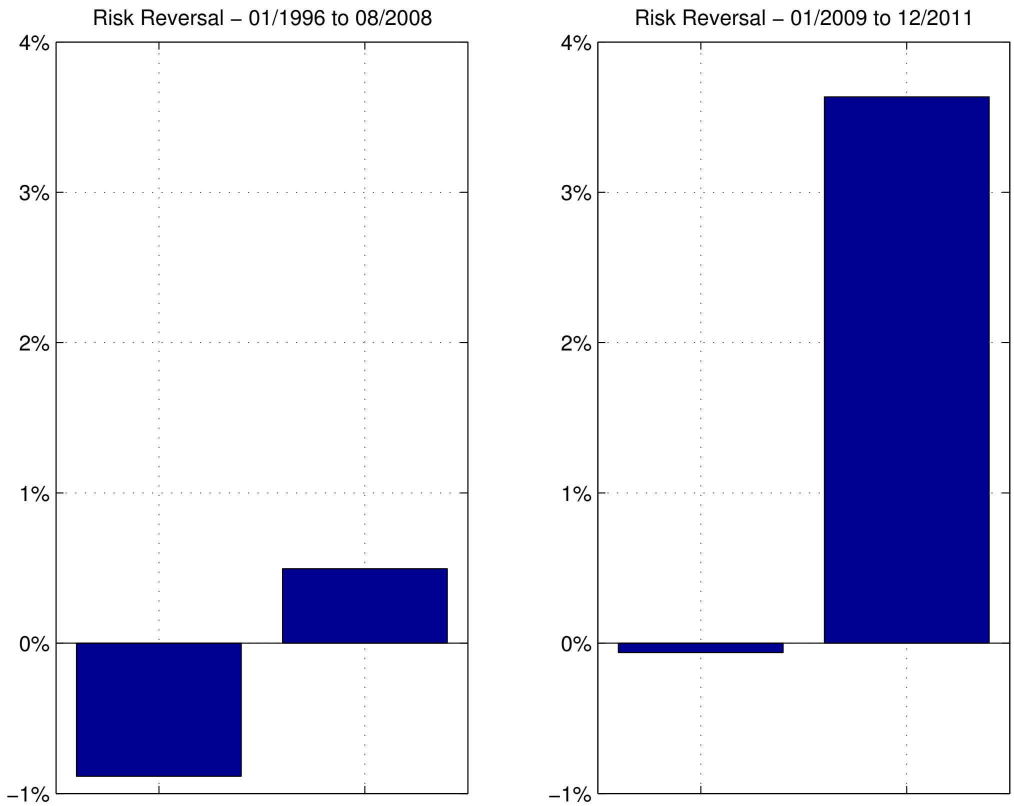

Figure 6: Average Risk Reversals Pre– and Post–Crisis

This figure presents the average risk-reversals of high and low interest rate currencies over two periods: 1/1996–8/2008 on the left panel, and 1/2009–12/2011 on the right panel. Risk-reversals are sorted by the level of foreign interest rates and allocated into three portfolios, which are rebalanced every month. The first portfolio contains risk-reversals from the lowest interest rate currencies while the last portfolio contains risk-reversals from the highest interest rate currencies. In each panel, the left bar corresponds to the portfolio of low interest rate currencies, while the right bar corresponds to the portfolio of high interest rate currencies. Data are monthly, from J.P. Morgan.

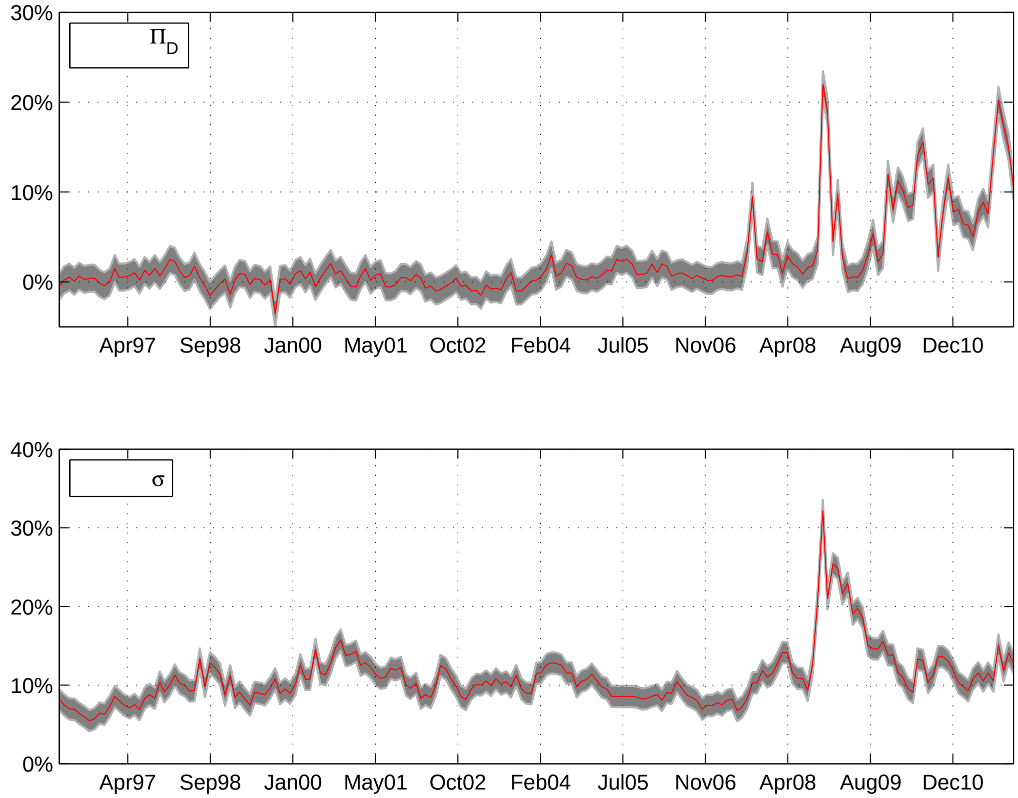

Figure 7: Time series of Disaster Risk Premia and Expected Currency Volatility

This figure presents the time series estimates of the average disaster risk premium (top panel) and of the average volatility parameter (bottom panel) among the currencies in the high interest rate portfolio. The shaded area corresponds to two standard errors above and below the mean estimates. The standard errors are obtained by bootstrapping both the time-series (using a block bootstrap of 10 months) and the cross-section of countries. Spot and forward exchange rates are from Datastream, while currency options are from J.P. Morgan. Data are monthly. The sample period is 1996 to 2011.

Crash Risk in Currency Markets

— Appendix —

Not For Publication

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The objective of this appendix is to present and derive the theoretical propositions of the paper regarding the model-implied prices of carry trade returns, interest rates, currency options, and risk-reversals. The appendix is self-contained, and thus reproduces some material presented in the paper. It is organized as follows. Section 1 recalls the laws of motion of the SDFs in both countries and the key model assumptions. Section 2 presents the Black and Scholes (1973) formula applied to currency options. Section 3 restates the discrete-time Girsanov's lemma in order to prove a key lemma that will be used for the derivations of all the proofs. Section 4 presents the propositions and their proofs. Section 5 reports simulation results. Finally, Section 6 presents our data set and reports country-level estimates.

1 Model Assumptions

Let us first summarize the notation used in the main text, starting with the two SDFs.

Pricing Kernels In the home country, the log SDF evolves as:

$$\log M_{t,t+\tau} = -g\tau + \varepsilon\sqrt{\tau} - \frac{1}{2}var(\varepsilon)\tau \\ + \left\{ \begin{array}{ll} 0 & \text{if there is no disaster at time } t+\tau \\ \log(J) & \text{if there is a disaster at time } t+\tau \end{array} \right\}.$$

Likewise, the log of SDF in the foreign country evolves as:

$$\log M^{\star}_{t,t+\tau} = -g^{\star}\tau + \varepsilon^{\star}\sqrt{\tau} - \frac{1}{2}var(\varepsilon^{*})\tau \\ + \left\{ \begin{array}{ll} 0 & \text{if there is no disaster at time } t+\tau \\ \log(J^{\star}) & \text{if there is a disaster at time } t+\tau \end{array} \right\}.$$

The parameters g and g^{*} are drift parameters which are constant between t and t+τ. The random variables ε and $\varepsilon^{\star}$ are jointly normally distributed with mean 0, variance σ and σ^{*} and correlation ρ. The probability of a disaster between t and $t+\tau$ is given by $p\tau$. J and J^{*}, which measure the magnitudes of the disaster, are independent of the process driving the realization of a disaster. The variables ε and $\varepsilon^{\star}$ are independent of random variables J and $J^{\star}$, and of the process driving the realization of a disaster.

Exchange Rates In a complete markets economy, the change in the nominal exchange rate is given by the ratio of the SDFs:

$$\frac{S_{t+\tau}}{S_t} = \frac{M^{\star}_{t,t+\tau}}{M_{t,t+\tau}}.$$

Risk Premia We define: $\tilde{J} = \frac{pJ}{1-p\tau}$ and $\tilde{J}^{*} = \frac{pJ^{*}}{1-p\tau}$. The disaster premium and the Gaussian premium as defined by:

$$\Pi_D = pE(J - J^{*}) = E(\tilde{J} - \tilde{J}^{*}) + O(\tau), \\ \Pi_G = cov(\epsilon, \epsilon - \epsilon^{*}) = \sigma^2 - \sigma\sigma^{*}\rho.$$

Conditional Moments Conditional on no disaster, the expected SDFs are:

$$\begin{aligned} E^{ND} M_{t,t+\tau} &= e^{-g\tau}, \\ E^{ND} M^{\star}_{t,t+\tau} &= e^{-g^{*}\tau}. \end{aligned}$$

where superscript ND indicates that expectations are taken conditional on no disaster. The expected ratio of the SDFs conditional on no disaster is:

$$\begin{aligned} E^{ND}\frac{M^{*}}{M} &= E^{ND} e^{\left(-g^{*}\tau+\epsilon^{*}\sqrt{\tau}-\frac{1}{2}\sigma^{*2}\tau\right)-\left(-g\tau+\epsilon\sqrt{\tau}-\frac{1}{2}\sigma^{2}\tau\right)} \\ &= e^{\left(g-g^{*}+\frac{1}{2}(\sigma^{2}-\sigma^{*2})\right)\tau} E^{ND} e^{(\epsilon^{*}-\epsilon)\sqrt{\tau}} \\ &= e^{\left(g-g^{*}+\frac{1}{2}(\sigma^{2}-\sigma^{*2})\right)\tau} e^{\frac{1}{2}\left(\sigma^{2}+\sigma^{*2}-2\rho\sigma\sigma^{*}\right)\tau} \\ &= e^{\left(g-g^{*}+\Pi_G\right)\tau} \end{aligned}$$

where we used the definition of the Gaussian risk premium ($\Pi_G = \sigma^2 - \sigma\sigma^{*}\rho$). We denote by σ_h^2 the volatility of the change in exchange rate conditional on no disaster:

$$var^{ND}(\log M_{t,t+\tau} - \log M^{\star}_{t,t+\tau}) = var(\epsilon - \epsilon^{*})\tau = \sigma_h^2\tau.$$

In all that follows, we drop the time subscripts for notational simplicity.

2 The Black and Scholes (1973) formula

$V^P_{BS}(S, K, \sigma, r, r^{\star}, \tau)$ and $V^C_{BS}(S, K, \sigma, r, r^{\star}, \tau)$ denote the Black and Scholes prices for a put and a call, respectively, when the spot exchange rate is S, the strike is K, the exchange rate volatility is σ, the time to maturity is τ, the home interest rate is r, and the foreign interest rate is $r^{\star}$. The Black and Scholes prices of a call and a put are given by:

$$\begin{aligned} V^C_{BS}(S, K, \sigma, r, r^{\star}, \tau) &= Se^{-r^{\star}\tau}\mathbb{N}(d_{+}) - Ke^{-r\tau}\mathbb{N}(d_{-}), && (1) \\ V^P_{BS}(S, K, \sigma, r, r^{\star}, \tau) &= Ke^{-r\tau}\mathbb{N}(-d_{-}) - Se^{-r^{\star}\tau}\mathbb{N}(-d_{+}), && (2) \\ d_{+} &= \frac{\log(S/K) + (r - r^{\star} + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, && (3) \\ d_{-} &= d_{+} - \sigma\sqrt{\tau}, && (4) \end{aligned}$$

where $\mathbb{N}$ is the cumulative standard normal distribution function. The Black–Scholes formula has a simple scaling property with respect to the time to maturity τ and the interest rates r and $r^\star$:

$$V_{BS}^P(S, K, \sigma, r, r^\star, \tau) = V_{BS}^P(Se^{-r^\star\tau}, Ke^{-r\tau}, \sigma\sqrt{\tau}, 0, 0, 1).$$

For notational convenience, we will omit the arguments 0 and 1 and simply write the value of a generic put as $V_{BS}^P(S, K, \sigma)$.

Options are quoted in terms of their implied volatility, the volatility parameter which has to be plugged into the Black–Scholes prices in order to retrieve the market option prices. In the case of currency options quotes, it is common to refer to the spot delta of a Black–Scholes option instead of its strike. The spot delta of an option is its first derivative with respect to the underlying asset S. We will refer to a spot delta as a delta for simplicity. In the case of Black–Scholes prices, the delta is given by:

$$\begin{aligned} \Delta_{BS}^C(S, K, \sigma, r, r^\star, \tau) &= +e^{-r^*\tau}\mathbb{N}(+d_+), \\ \Delta_{BS}^P(S, K, \sigma, r, r^\star, \tau) &= -e^{-r^*\tau}\mathbb{N}(-d_-) \end{aligned}$$

Let (Δ^C, Δ^P) be the values for the Black–Scholes call delta and a put delta respectively. Let $(\sigma_{\Delta^C}, \sigma_{\Delta^P})$ be the corresponding implied volatility. The strikes $(K_{\Delta^C}, K_{\Delta^P})$ can be obtained by inverting the formula for the Black–Scholes delta:

$$K_{\Delta^C} = Se^{-\mathbb{N}^{-1}\left(+e^{r^*\tau}\Delta^P\right)\sigma_{\Delta^C}\sqrt{\tau}+\left(r-r^*+1/2\sigma^2_{\Delta^C}\right)\tau} \tag{5}$$

$$K_{\Delta^P} = Se^{+\mathbb{N}^{-1}\left(-e^{r^*\tau}\Delta^P\right)\sigma_{\Delta^P}\sqrt{\tau}+\left(r-r^*+1/2\sigma^2_{\Delta^P}\right)\tau} \tag{6}$$

3 Some Useful Lemmas

We start with a well-known Lemma, whose proof we provide for completeness.

Lemma 1. *(Discrete-time Girsanov's lemma) Suppose that (x, y) are jointly Gaussian distributed random variables under probability measure P. Consider the measure Q such that $dQ/dP = \exp(x - E[x] - \mathrm{var}(x)/2)$. Then, under Q, y is Gaussian, with distribution*

$$y \sim^Q \mathcal{N}(E[y] + \mathrm{cov}(x, y), \mathrm{var}(y)), \tag{7}$$

where $E[y], \mathrm{cov}(x,y), \mathrm{var}(y)$ *are calculated under* P.

Proof. We calculate the characteristic function of y. For a purely imaginary number k, $E^Q\left[e^{ky}\right]$ is given by

$$E\left[e^{x-E[x]-\sigma_x^2/2}e^{ky}\right] = \exp\left(kE[y] + \frac{k^2\sigma_y^2}{2} + kcov(x,y)\right) = \exp\left(k\left(E[y] + \mathrm{cov}(x,y)\right) + \frac{k^2\sigma_y^2}{2}\right).$$

That is indeed the characteristic function of distribution (7). □

Lemma 2. *For* $\ln X$, $\ln Y$ *jointly Gaussian distributed,*

$$\begin{aligned} E\left[(X-Y)^+\right] &= V_{BS}^C\left(E[X], E[Y], var(\log X - \log Y)^{1/2}\right) \\ &= V_{BS}^P\left(E[Y], E[X], var(\log X - \log Y)^{1/2}\right), \end{aligned}$$

where $V_{BS}^C(S_0, K, \sigma)$ *and* $V_{BS}^P(S_0, K, \sigma)$ *are the Black and Scholes call and put prices with home and foreign interest rates equal to 0, and horizon equal to* 1.

Proof. Let us write $X = E[X]e^{x-var(x)/2}$ and $Y = E[Y]e^{y-var(y)/2}$, where (x,y) are jointly Gaussian distributed with mean 0 and respective variance $var(\log X)$ and $var(\log Y)$ under probability measure P. We define the probability measure Q by: $dQ/dP = exp(x - E[x] - var(x)/2) = \exp(x - var(x)/2)$ (as $E[x] = 0$). Using Lemma 1:

$$\begin{aligned} E\left[(X-Y)^+\right] &= E\left[\left(E[X]e^{x-var(x)/2} - E[Y]e^{y-var(y)/2}\right)^+\right] \\ &= E\left[e^{x-var(x)/2}\left(E[X] - E[Y]e^z\right)^+\right] \\ &= E^Q\left[\left(E[X] - E[Y]e^z\right)^+\right], \end{aligned}$$

where $z = y - var(y)/2 - x + var(x)/2$. Applying Lemma 1 implies that the variable z is distributed as: $z \sim^Q \mathcal{N}\left(E^Q[z], var(y-x)\right)$, where:

$$\begin{aligned} E^Q[z] &= -var(y)/2 + var(x)/2 + cov(x, y-x), \\ &= -var(y-x)/2, \end{aligned}$$

As a result, the variable z is distributed as:

$$z \sim^Q \mathcal{N}\left(-var(y-x)/2, var(y-x)\right).$$

Let us define the variable u as:

$$u = (z - E^Q(z))/var(y-x)^{1/2}.$$

Thus the variable u is gaussian, with mean zero and variance 1: $u \sim^{Q} \mathcal{N}(0,1)$. We can define z as a function of u:

$$\begin{aligned} u &= (z + var(y-x)/2)/var^{1/2}(y-x) \\ z &= u.var^{1/2}(y-x) - var(y-x)/2 \\ e^{z} &= e^{u.var^{1/2}(y-x)-var(y-x)/2} \end{aligned}$$

Therefore,

$$E\left[(X-Y)^{+}\right] = E^{Q}\left[\left(E\left[X\right] - E\left[Y\right] e^{u.var^{1/2}(y-x)-var(y-x)/2}\right)^{+}\right]$$

The Black and Scholes call and put prices are also given by:

$$\begin{aligned} V_{BS}^{P}(S,K,\sigma) &= E^{Q}\left[\left(K - Se^{\sigma u-\sigma^{2}/2}\right)^{+}\right], \\ V_{BS}^{C}(S,K,\sigma) &= E^{Q}\left[\left(Se^{\sigma u-\sigma^{2}/2} - K\right)^{+}\right], \end{aligned}$$

where u is a normal variable with mean 0 and variance 1 under probability measure Q. Using the previous result then implies:

$$E\left[(X-Y)^{+}\right] = V_{BS}^{P}\left(E\left[Y\right], E\left[X\right], var\left(\log X - \log Y\right)^{1/2}\right).$$

A similar reasoning implies that:

$$E\left[(X-Y)^{+}\right] = V_{BS}^{C}\left(E\left[X\right], E\left[Y\right], var\left(\log X - \log Y\right)^{1/2}\right).$$

□

Lemma 3. *For* $\log X$, $\log Y, \log Z$ *jointly Gaussian distributed,*

$$\begin{aligned} \operatorname{cov}\left(Z,(X-Y)^{+}\right) &= V_{BS}^{C}\left(E\left[ZX\right], E\left[ZY\right], \operatorname{var}\left(\log X - \log Y\right)^{1/2}\right) \\ &\quad - E\left[Z\right] V_{BS}^{C}\left(E\left[X\right], E\left[Y\right], \operatorname{var}\left(\log X - \log Y\right)^{1/2}\right) \\ &= V_{BS}^{P}\left(E\left[ZY\right], E\left[ZX\right], \operatorname{var}\left(\log X - \log Y\right)^{1/2}\right) \\ &\quad - E\left[Z\right] V_{BS}^{P}\left(E\left[Y\right], E\left[X\right], \operatorname{var}\left(\log X - \log Y\right)^{1/2}\right). \end{aligned}$$

Proof. The proof is a straightforward application of the previous Lemma. □

4 Propositions and Proofs.

4.1 Interest rates and currency excess returns

Let us now turn to carry trade returns. Consider the case of an investment in foreign currency funded by borrowing in domestic currency. The symmetric case — borrowing in foreign currency to invest in domestic currency — would yield the opposite of the result. The trade has return X in domestic currency, and does not require any investment:

$$X = e^{r^* \tau} \frac{S_{t+\tau}}{S_t} - e^{r\tau}$$

Let X^e be the annualized expected value of the carry trade return conditional on no disaster:

$$X^e = \frac{E^{ND} X}{\tau}$$

Proposition 1. *Recall that* $\tilde{J} = \frac{pJ}{1-p\tau}$ *and* $\tilde{J}^* = \frac{pJ^*}{1-p\tau}$. *In the limit of small time intervals* ($\tau \to 0$)*, the interest rate* r *in the home country is approximately equal to:*

$$r = g - pE\big(J-1\big) + O(\tau)$$

In the limit of small time intervals ($\tau \to 0$)*, the carry trade expected returns (conditional on no disasters) are approximately equal to:*

$$X^e = \Pi_D + \Pi_G + O(\tau)$$

where:

$$\Pi_D = pE(J - J^*) = E(\tilde{J} - \tilde{J}^*) + O(\tau)$$
$$\Pi_G = cov(\epsilon, \epsilon - \epsilon^*)$$

Proof. Let us first focus on interest rates. The home interest rate r is determined by the Euler equation:

$$E\left[Me^{r\tau}\right] = 1$$

The Euler equation implies:

$$e^{-r\tau} = E\left[M\right] = e^{-g\tau}\Big(1 + p\tau E\big(J-1\big)\Big) \tag{8}$$

Taking logs gives:

$$r\tau = g\tau - \log\Big(1 + p\tau E\big(J-1\big)\Big)$$

A Taylor expansion yields:

$$\log\Big(1 + p\tau E\big(J-1\big)\Big) = p\tau E\big(J-1\big) + O(\tau^2)$$

The interest rate is thus:

$$r = g - pE\big(J-1\big) + O(\tau)$$

Now let's turn to the carry trade excess return. The Euler equation for the unhedged carry trade excess return X is:

$$E\left[MX\right] = 0,$$

Recall that the change in the nominal exchange rate is given by the ratio of the SDFs:

$$\frac{S_{t+\tau}}{S_t} = \frac{M^\star}{M}$$

Therefore, the currency excess return is equal to:

$$X = e^{r^*\tau}\frac{M^*}{M} - e^{r\tau}$$

The Euler equation can be decomposed in two terms, corresponding to the expected gaussian and disaster states:

$$\begin{aligned} E\left[MX\right] &= (1-p\tau)E^{ND}\left[MX\right] + p\tau E^{D}\left[MX\right], \\ &= (1-p\tau)E^{ND}[M]E^{ND}[X] + (1-p\tau)cov^{ND}(M,X) \\ &+ p\tau E^{D}\left[MX\right], \\ &= 0, \end{aligned}$$

where superscripts ND and D denote moments conditional on no disasters and disasters respectively. The expected currency excess returns conditional on no disasters is thus:

$$\begin{aligned} E^{ND}[X] &= \frac{-p\tau E^{D}[MX] - (1-p\tau)cov^{ND}(M,X)}{(1-p\tau)E^{ND}[M]}, \\ &= -\frac{p}{(1-p\tau)}e^{g\tau}\tau E^{D}[MX] - e^{g\tau}cov^{ND}(M,X). \end{aligned} \tag{9}$$

The first term in Equation (9) yields:

$$-\frac{p}{(1-p\tau)}\tau e^{g\tau}E^{D}[MX] = \frac{p}{(1-p\tau)}\tau e^{g\tau}\Big(E[J]e^{(r-g)\tau} - E[J^*]e^{(r^*-g^*)\tau}\Big)$$

which leads to:

$$-\frac{p}{(1-p\tau)}\tau e^{g\tau}E^{D}[MX] = \Pi_D\tau + O(\tau^2)$$

The second term in Equation (9) is given by:

$$-e^{g\tau}cov^{ND}(M,X) = -e^{g\tau}e^{r^*\tau}cov^{ND}(M,\frac{M^*}{M}),$$

The covariance between the home SDF and the ratio of SDFs is:

$$\begin{aligned} cov^{ND}\Big(M,\frac{M^*}{M}\Big) &= E^{ND}M^* - E^{ND}\frac{M^*}{M}E^{ND}M \\ &= e^{-g^*\tau} - e^{-g^*\tau}e^{\Pi_G\tau} \\ &= e^{-g^*\tau}\Big(-\Pi_G\tau + O(\tau^2)\Big) \end{aligned}$$

The second term in Equation (9) is finally:

$$-e^{g\tau} cov^{ND}(M,X) = e^{(g-g^*+r^*)\tau}\Big(\Pi_G\tau + O(\tau^2)\Big) = \Pi_G\tau + O(\tau^2)$$

Collecting terms of Equation (9), the approximation at the order τ of the expected return, conditional on no-disaster is given by:

$$X^e = \frac{E^{ND}[X]}{\tau} = \Pi_D + \Pi_G + O(\tau)$$

Substituting in the definition of the Gaussian and the disaster risk premia gives therefore:

$$\begin{aligned} X^e &= pE(J - J^*) + cov(\epsilon, \epsilon - \epsilon^*) + O(\tau) \\ &= E(\tilde{J} - \tilde{J}^*)(1 - p\tau) + cov(\epsilon, \epsilon - \epsilon^*) + O(\tau) \\ &= E(\tilde{J} - \tilde{J}^*) + cov(\epsilon, \epsilon - \epsilon^*) + O(\tau) \end{aligned}$$

□

4.2 Option prices

In all that follows, options prices and strikes are normalized by the spot. The main results are derived in the case of a put option but can easily be generalized to a call option.

Proposition 2. *We assume that the disaster sizes (J, J^*) are constant between t and $t+\tau$. Recall that $\tilde{J} = \frac{pJ}{1-p\tau}$ and $\tilde{J}^* = \frac{pJ^*}{1-p\tau}$.*

The price of a put option can be decomposed into a non-disaster part $P^{ND}(K)$ and a disaster–related part $P^D(K)$ according to:

$$\begin{aligned} P(K) &= P^{ND}(K) + P^D(K) \\ P^{ND}(K) &= V^P_{BS}\Big(\frac{e^{-r^*\tau}}{1+\tilde{J}^*\tau}, K\frac{e^{-r\tau}}{1+\tilde{J}\tau}, \sigma_h\sqrt{\tau}\Big) \\ P^D(K) &= \tau V^P_{BS}\Big(\frac{e^{-r^*\tau}\tilde{J}^*}{1+\tilde{J}^*\tau}, K\frac{e^{-r\tau}\tilde{J}}{1+\tilde{J}\tau}, \sigma_h\sqrt{\tau}\Big) \end{aligned}$$

where the strike is K, the exchange rate volatility conditional on no disaster is σ_h, the time to maturity is τ, the home interest rate is r, and the foreign interest rate is $r^\star$.

Proof. Using the model expression for the exchange rate movement, we obtain the following put option price:

$$P(K) = E\Big(KM - M^*\Big)^+$$

Decomposing the expectation into its non-disaster and disaster components gives:

$$P(K) = (1 - p\tau)E^{ND}\Big(KM - M^*\Big)^+ + p\tau E^D\Big(KM - M^*\Big)^+$$

The non-disaster part can be expressed in terms of Black–Scholes formula using Lemma (2):

$$\begin{aligned} P^{ND}(K) &= (1 - p\tau)E^{ND}\Big(KM - M^*\Big)^+ \\ &= (1 - p\tau)V^P_{BS}\Big(e^{-g^*\tau}, Ke^{-g\tau}, \sigma_h\sqrt{\tau}\Big) \end{aligned}$$

Plugging the equation for the interest rate given in equation (8) gives:

$$P^{ND}(K) = V^P_{BS}\Big(e^{-r^*\tau}\frac{1 - p\tau}{1 + p\tau(J^* - 1)}, Ke^{-r\tau}\frac{1 - p\tau}{1 + p\tau(J - 1)}, \sigma_h\sqrt{\tau}\Big)$$

Using the definition of $\tilde{J}$ and $\tilde{J}^*$ implies:

$$P^{ND}(K) = V^P_{BS}\Big(\frac{e^{-r^*\tau}}{1 + \tilde{J}^*\tau}, K\frac{e^{-r\tau}}{1 + \tilde{J}\tau}, \sigma_h\sqrt{\tau}\Big)$$

When J, J^* are constant between t and $t + \tau$, the SDFs conditional on a disaster are also log-normally distributed and so we can apply Lemma 2 to compute $P^D(K)$:

$$
\begin{aligned}
P^D(K) &= p\tau E^D\Big(KM - M^*\Big)^+ \\
&= \tau V^P_{BS}\Big(e^{-g^*\tau} pJ^*, Ke^{-g\tau} pJ, \sigma_h\sqrt{\tau}\Big) \\
&= \tau V^P_{BS}\Big(e^{-r^*\tau}\frac{pJ^*}{1+p\tau(J^*-1)}, Ke^{-r\tau}\frac{pJ}{1+p\tau(J-1)}, \sigma_h\sqrt{\tau}\Big) \\
&= \tau V^P_{BS}\Big(\frac{e^{-r^*\tau}\tilde{J}^*}{1+\tilde{J}^*\tau}, K\frac{e^{-r\tau}\tilde{J}}{1+\tilde{J}\tau}, \sigma_h\sqrt{\tau}\Big)
\end{aligned}
$$

□

The following lemma identifies some conditions under which we can simplify the non-disaster component of a put price P^{ND}. Similar results can be obtained for a call price.

Lemma 4. *We assume that the disaster sizes* (J, J^*) are constant *between* t *and* $t+\tau$*. Given a Black–Scholes delta* Δ*, let* σ_Δ *be the option implied volatility and* K_Δ *be the corresponding strike.*

Let $\mathbb{N}()$ *be the cumulative standard normal distribution and let n() be the standard normal distribution. We define* $\alpha = -\phi\mathbb{N}^{-1}\big(\phi\Delta e^{r^*\tau}\big)$ *where* $\phi = 1$ *for a call delta and* $\phi = -1$ *for a put delta.*

In the limit of small time intervals ($\tau \to 0$*), the non–disaster component of a put price with a strike* K_Δ *can be approximated by:*

$$
\begin{aligned}
P^{ND}(K_\Delta) &= \Big(n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_h + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\Big)\sqrt{\tau} \\
&+ \Big(\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(1/2\big(1+\alpha^2\big)\sigma_\Delta^2 - \Pi_D\Big) + 1/2\alpha n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\sigma_h\Big)\tau + O(\tau\sqrt{\tau})
\end{aligned}
$$

where the exchange rate volatility conditional on no disaster is σ_h *and* Π_D *is the disaster premium.*

Proof. Recall that the strike price given a Black-Scholes delta Δ is given by equation (5) and (6):

$$
K_\Delta = e^{-\phi\mathbb{N}^{-1}\left(\phi e^{r^*\tau}\Delta\right)\sigma_\Delta\sqrt{\tau} + \left(r - r^* + 1/2\sigma_\Delta^2\right)\tau}
$$

where $\phi = 1$ for a call delta and $\phi = -1$ for a put delta. Let's define:

$$
\begin{aligned}
\alpha &= -\phi \mathbb{N}^{-1}(\phi e^{r^* \tau} \Delta) \\
\beta &= \frac{1/2\sigma_\Delta^2 - \Pi_D}{\sigma_h^2} \\
\gamma &= \Big(1/2\big(1+\alpha^2\big)\sigma_\Delta^2 - \Pi_D\Big)
\end{aligned}
$$

Using these notations, the non-disaster component of the option price can be written :

$$
\begin{aligned}
P^{ND}(K_\Delta) &= (1-p\tau)V_{BS}^{P}(e^{-g^*\tau}, K_\Delta e^{-g\tau}, \sigma_h\sqrt{\tau}) \\
&= e^{-g^*\tau}(1-p\tau)\Big(K_\Delta e^{-(g-g^*)\tau}\mathbb{N}(-d_-) - \mathbb{N}(-d_+)\Big)
\end{aligned}
$$

where:

$$
\begin{aligned}
-d_\pm &= \frac{\log(K_\Delta) + (g^* - g \mp 1/2\sigma_h^2)\tau}{\sigma_h\sqrt{\tau}} \\
&= \frac{-\phi\mathbb{N}^{-1}\big(\phi e^{r^*\tau}\Delta\big)\sigma_\Delta\sqrt{\tau} + \big(r - r^* + 1/2\sigma_\Delta^2 + g^* - g \mp 1/2\sigma_h^2\big)\tau}{\sigma_h\sqrt{\tau}} \\
&= \alpha\frac{\sigma_\Delta}{\sigma_h} + \Big(\beta \mp 1/2\Big)\sigma_h\sqrt{\tau} + O(\tau\sqrt{\tau})
\end{aligned}
$$

When τ is small, the Taylor expansion of $\mathbb{N}(-d_\pm)$ gives :

$$
\mathbb{N}(-d_\pm) = \mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big) + n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(\beta \mp 1/2\Big)\sigma_h\sqrt{\tau} + 1/2n'\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(\beta \mp 1/2\Big)^2\sigma_h^2\tau + O(\tau\sqrt{\tau})
$$

Similarly, the Taylor expansion of $K_\Delta e^{-(g-g^*)\tau}$ gives :

$$
\begin{aligned}
K_\Delta e^{-(g-g^*)\tau} &= e^{-\phi\mathbb{N}^{-1}\big(\phi e^{r^*\tau}\Delta\big)\sigma_\Delta\sqrt{\tau} + \big(r - r^* - (g-g^*) + 1/2\sigma_\Delta^2\big)\tau} \\
&= 1 + \alpha\sigma_\Delta\sqrt{\tau} + \gamma\tau + O(\tau^2)
\end{aligned}
$$

Combining the terms, we get:

$$
\begin{aligned}
P^{ND}(K_\Delta) = (1 - g^*\tau - p\tau)\Bigg[&\Big(1 + \alpha\sigma_\Delta\sqrt{\tau} + \gamma\tau\Big).\\
&\Bigg(\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big) + n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)(\beta + 1/2)\sigma_h\sqrt{\tau} + 1/2n'\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(\beta + 1/2\Big)^2\sigma_h^2\tau\Bigg)\\
&-\Bigg(\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big) + n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)(\beta - 1/2)\sigma_h\sqrt{\tau} + 1/2n'\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(\beta - 1/2\Big)^2\sigma_h^2\tau\Bigg)\Bigg] + O(\tau\sqrt{\tau})\\
&= n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_h\sqrt{\tau} + n'\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\beta\sigma_h^2\tau + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\sqrt{\tau}\\
&+\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\gamma\tau + \alpha n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)(\beta + 1/2)\sigma_\Delta\sigma_h\tau + O(\tau\sqrt{\tau})
\end{aligned}
$$

By plugging the expression for γ, and using $n'(x) = -xn(x)$ we get:

$$
\begin{aligned}
P^{ND}(K_\Delta) = &\Bigg(n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_h + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\Bigg)\sqrt{\tau}\\
&+\Bigg(\mathbb{N}\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\Big(1/2(1+\alpha^2)\sigma_\Delta^2 - \Pi_D\Big) + 1/2\alpha n\Big(\alpha\frac{\sigma_\Delta}{\sigma_h}\Big)\sigma_\Delta\sigma_h\Bigg)\tau + O(\tau\sqrt{\tau})
\end{aligned}
$$

□

The following lemma identifies some conditions under which we can simplify the disaster component of a put price P^D. Similar results can be obtained for a call price.

Lemma 5. *We assume that the disaster sizes* (J, J^*) are constant *between* t *and* $t+\tau$*. Given a Black–Scholes delta* Δ*, let* σ_Δ *be the option implied volatility and* K_Δ *be the corresponding strike.*

In the limit of small time intervals ($\tau \to 0$)*, the disaster component of a put price with a strike* K_Δ *can be approximated by:*

$$
P^D(K_\Delta) = \begin{cases} (K_\Delta pJ - pJ^*)\tau + O(\tau^2) & \text{if } J > J^*;\\ O(\tau^2) & \text{if } J < J^*. \end{cases}
$$

Proof. When J, J^* are constant between t and $t+\tau$, we get from proposition (2):

$$P^D(K_\Delta) = K_\Delta pJe^{-g\tau}\mathbb{N}(-d_-)\tau - pJ^* e^{-g^*\tau}\mathbb{N}(-d_+)\tau$$

where:

$$\begin{aligned} -d_\pm &= \frac{\log\left((JK_\Delta)/J^*\right) - (g - g^* \pm \frac{1}{2}\sigma_h^2)\tau}{\sigma_h\sqrt{\tau}} \\ &= \frac{\log\left(J/J^*\right)}{\sigma_h\sqrt{\tau}} + O(1) \end{aligned}$$

We want to approximate the cumulative normal terms $\mathbb{N}(-d_\pm)$ by zero or one with sufficient precision. We apply Chebychev's inequality and derive the following bounds:

$$\begin{cases} 1 - \frac{1}{2d_\pm^2} < \mathbb{N}(-d_\pm) < 1 & \text{if } d_\pm < 0; \\ 0 < \mathbb{N}(-d_\pm) < \frac{1}{2d_\pm^2} & \text{if } d_\pm > 0. \end{cases}$$

The Taylor expansion of $\frac{1}{2d_\pm^2}$ gives :

$$\frac{1}{2d_\pm^2} = O(\tau)$$

Notice from the Taylor expansion of $-d_\pm$ that when τ is small enough we have:

$$\text{sign}(d_\pm) = \text{sign}(J^* - J)$$

By plugging the above approximation for $\mathbb{N}(-d_\pm)$ into the expression for $P^D(K_\Delta)$, we obtain:

$$P^D(K_\Delta) = \begin{cases} (K_\Delta pJ - pJ^*)\tau + O(\tau^2) & \text{if } J > J^*; \\ O(\tau^2) & \text{if } J < J^*. \end{cases}$$

□

The following lemma identifies some conditions under which we can simplify the implied volatility σ_Δ for a given Black–Scholes delta Δ.

Lemma 6. *We assume that the disaster sizes* (J, J^*) are constant *between* t *and* $t+\tau$*. For a given Black–Scholes delta* Δ*, let* σ_Δ *be the corresponding implied implied volatility.*

In the limit of small time intervals ($\tau \to 0$)*, the implied volatility can be approximated by:*

$$\sigma_\Delta = \begin{cases} \sigma_h + \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ^* - pJ\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)}\sqrt{\tau} + O(\tau) & \textit{if } \Delta > 0; \\ \\ \sigma_h + \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ - pJ^*\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)}\sqrt{\tau} + O(\tau) & \textit{if } \Delta < 0. \end{cases}$$

where $\mathbb{N}()$ *is the cumulative standard normal distribution and n() the standard normal distribution.*

Proof. By definition, the implied volatility σ_Δ verifies:

$$P^{ND}(K_\Delta) + P^D(K_\Delta) = V^P_{BS}\left(e^{-r^*\tau}, K_\Delta e^{-r\tau}, \sigma_\Delta\sqrt{\tau}\right) \tag{10}$$

We guess that the Taylor expansion of the implied volatility with respect to the maturity has the following form :

$$\sigma_\Delta = \sigma_h + A\sqrt{\tau} + O(\tau) \tag{11}$$

and we want to find the value for A. We proceed in 3 steps:

1. Recall that when $\alpha = -\phi\mathbb{N}^{-1}(\phi e^{r^*\tau}\Delta)$, with $\phi = 1$ for a call delta and $\phi = -1$ for a put delta, the approximation for the non-disaster component of the put price is given by lemma (4):

$$\begin{aligned} P^{ND}(K_\Delta) = &\left(n\left(\alpha\frac{\sigma_\Delta}{\sigma_h}\right)\sigma_h + \alpha\mathbb{N}\left(\alpha\frac{\sigma_\Delta}{\sigma_h}\right)\sigma_\Delta\right)\sqrt{\tau} \\ &+ \left(\mathbb{N}\left(\alpha\frac{\sigma_\Delta}{\sigma_h}\right)\left(1/2\left(1+\alpha^2\right)\sigma_\Delta^2 - \Pi_D\right) + 1/2\alpha n\left(\alpha\frac{\sigma_\Delta}{\sigma_h}\right)\sigma_\Delta\sigma_h\right)\tau + O(\tau\sqrt{\tau}) \end{aligned}$$

Plugging the guess from equation (11) into this expression for P^{ND} gives:

$$
\begin{aligned}
P^{ND}(K_\Delta) = & \Bigg(n\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big)\sigma_h + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big)\big(\sigma_h + A\sqrt{\tau}\big)\Bigg)\sqrt{\tau} \\
& + \Bigg(\mathbb{N}\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big)\Big(1/2\big(1+\alpha^2\big)\big(\sigma_h + A\sqrt{\tau}\big)^2 - \Pi_D\Big) \\
& + 1/2\alpha n\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big)\big(\sigma_h + A\sqrt{\tau}\big)\sigma_h\Bigg)\tau + O(\tau\sqrt{\tau}) \\
\\
= & \Bigg(n\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big) + \alpha\mathbb{N}\Big(\alpha\frac{\sigma_h + A\sqrt{\tau}}{\sigma_h}\Big)\Bigg)\sigma_h\sqrt{\tau} \\
& + \Bigg(\mathbb{N}\Big(\alpha\Big)\Big(1/2(1+\alpha^2)\sigma_h^2 - \Pi_D\Big) + 1/2\alpha n\Big(\alpha\Big)\sigma_h^2 + \alpha\mathbb{N}\Big(\alpha\Big)A\Bigg)\tau + O(\tau\sqrt{\tau})
\end{aligned}
$$

A Taylor expansion of the standard normal terms gives :

$$
\begin{aligned}
P^{ND}(K_\Delta) = & \Bigg(n\Big(\alpha\Big)\sigma_h + n'\Big(\alpha\Big)\alpha A\sqrt{\tau} + \alpha\mathbb{N}\Big(\alpha\Big)\sigma_h + \alpha^2 n\Big(\alpha\Big)A\sqrt{\tau}\Bigg)\sqrt{\tau} \\
& + \Bigg(\mathbb{N}\Big(\alpha\Big)\Big(1/2(1+\alpha^2)\sigma_h^2 - \Pi_D\Big) + 1/2\alpha n\Big(\alpha\Big)\sigma_h^2 + \alpha\mathbb{N}\Big(\alpha\Big)A\Bigg)\tau + O(\tau\sqrt{\tau}) \\
\\
= & \Bigg(n\Big(\alpha\Big) + \alpha\mathbb{N}\Big(\alpha\Big)\Bigg)\sigma_h\sqrt{\tau} \\
& + \Bigg(\mathbb{N}\Big(\alpha\Big)\Big(1/2(1+\alpha^2)\sigma_h^2 - \Pi_D\Big) + 1/2\alpha n\Big(\alpha\Big)\sigma_h^2 + \alpha\mathbb{N}\Big(\alpha\Big)A\Bigg)\tau + O(\tau\sqrt{\tau})
\end{aligned}
$$

2. Now, let's write the non-disaster component of the put price $P^{ND}(K_\Delta)$ as:

$$
P^{ND}(K_\Delta) = P^{ND}(K_\Delta, p, g, g^*, \sigma_h)
$$

So $P^{ND}(K_\Delta)$ is formally equivalent to the right–hand side of equation (10) when $p = 0$, $g = r$, $g^* = r^*$ and $\sigma_h = \sigma_\Delta$:

$$P^{ND}(K_\Delta, p=0, g=r, g^*=r^*, \sigma_h=\sigma_\Delta) = V^P_{BS}\Big(e^{-r^*\tau}, K_\Delta e^{-r\tau}, \sigma_\Delta\sqrt{\tau}\Big)$$

So we can apply lemma (4) to compute the Taylor expansion for the right–hand side of equation (10):

$$\begin{aligned} V^P_{BS}\Big(e^{-r^*\tau}, K_\Delta e^{-r\tau}, \sigma_\Delta\sqrt{\tau}\Big) &= \Big(n\Big(\alpha\Big) + \alpha\mathbb{N}\Big(\alpha\Big)\Big)\sigma_\Delta\sqrt{\tau} \\ &+ 1/2\Big(\mathbb{N}\Big(\alpha\Big)(1+\alpha^2) + \alpha n\Big(\alpha\Big)\Big)\sigma^2_\Delta\tau + O(\tau\sqrt{\tau}) \end{aligned}$$

Plugging the expression for σ_Δ in equation (11) gives :

$$\begin{aligned} V^P_{BS}\Big(e^{-r^*\tau}, K_\Delta e^{-r\tau}, \sigma_\Delta\sqrt{\tau}\Big) &= \Big(n\Big(\alpha\Big) + \alpha\mathbb{N}\Big(\alpha\Big)\Big)\sigma_h\sqrt{\tau} \\ &+ \Big(n\Big(\alpha\Big) + \alpha\mathbb{N}\Big(\alpha\Big)\Big)A\tau \\ &+ 1/2\Big(\mathbb{N}\Big(\alpha\Big)(1+\alpha^2) + \alpha n\Big(\alpha\Big)\Big)\sigma^2_h\tau + O(\tau\sqrt{\tau}) \end{aligned}$$

3. Finally, when τ is small enough we can simplify the disaster component of the put price using lemma (5):

$$P^D(K_\Delta) = \big(pJ - pJ^*\big)^+\tau + O(\tau\sqrt{\tau})$$

Plugging these results into equation (10) gives:

$$-\mathbb{N}\big(\alpha\big)\big(pJ - pJ^*\big) + \big(pJ - pJ^*\big)^+ = n\big(\alpha\big)A$$

So the unknown coefficient A is given by:

$$A = \frac{\left(pJ - pJ^{*}\right)^{+} - \mathbb{N}\left(\alpha\right)\left(pJ - pJ^{*}\right)}{n\left(\alpha\right)}$$

Recall the expression for α:

$$\begin{aligned}\alpha &= -\phi \mathbb{N}^{-1}(\phi e^{r^{*}\tau}\Delta) \\ &= -\phi \mathbb{N}^{-1}(\phi\Delta) + O(\tau)\end{aligned}$$

So we can simplify the expression for A depending on the type of option delta ($\phi = 1$ for a call delta and - 1 for a put delta):

$$A = \begin{cases} \frac{\Delta\left(pJ - pJ^{*}\right) + \left(pJ^{*} - pJ\right)^{+}}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)} + O(\tau) & \text{if } \phi = 1; \\ \\ \frac{\Delta\left(pJ - pJ^{*}\right) + \left(pJ - pJ^{*}\right)^{+}}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)} + O(\tau) & \text{if } \phi = -1. \end{cases}$$

Recall that by construction a call delta is positive and a put delta is negative. So by plugging the value of A into equation (11) gives the expression for the implied volatility:

$$\sigma_{\Delta} = \begin{cases} \sigma_h + \frac{\Delta\left(pJ - pJ^{*}\right) + \left(pJ^{*} - pJ\right)^{+}}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)}\sqrt{\tau} + O(\tau) & \text{if } \Delta > 0; \\ \\ \sigma_h + \frac{\Delta\left(pJ - pJ^{*}\right) + \left(pJ - pJ^{*}\right)^{+}}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)}\sqrt{\tau} + O(\tau) & \text{if } \Delta < 0. \end{cases}$$

□

4.3 Hedged currency excess returns

In all that follows, we assume that the foreign currency is the investment currency.

Let Δ^{P} be a Black–Scholes put delta, i.e. $\Delta^{P} < 0$ and let $K_{\Delta^{P}}$ be the corresponding strike. The return $X(K_{\Delta^{P}})$ to the hedged carry trade is the payoff of the following zero-investment trade:

borrow one unit of the home currency at interest rate r ; use the proceeds to buy $\lambda^P(K_{\Delta^P})$ puts with strike K_{Δ^P}, protecting against a depreciation in the foreign currency below K_{Δ^P}; and invest the remainder $\left(1-\lambda^P(K_{\Delta^P})P(K_{\Delta^P})\right)$ in the foreign currency at interest rate $r^\star$. So the hedged return is given by:

$$X(K_{\Delta^P}) = \left(1-\lambda^P(K_{\Delta^P})P(K_{\Delta^P})\right)e^{r^\star\tau}\frac{S_{t+\tau}}{S_t} + \lambda^P\left(K_{\Delta^P}\right)\left(K_{\Delta^P} - \frac{S_{t+\tau}}{S_t}\right)^+ - e^{r\tau},$$

where the hedge ratio $\lambda^P(K_{\Delta^P})$ is given by:

$$\lambda^P(K_{\Delta^P}) = \frac{e^{r^*\tau}}{1+e^{r^*\tau}P(K_{\Delta^P})}$$

To summarize the notation: X denotes the carry trade return and X^e is its annualized expected value conditional on no disaster; $X(K_{\Delta^P})$ denotes the hedged carry trade return with strike K_{Δ^P}; $P\left(K_{\Delta^P}\right)$ is the home currency price of a put yielding $\left(K_{\Delta^P} - S_{t+\tau}/S_t\right)^+$ in the home currency; $X^e(K_{\Delta^P})$ is the annualized expected value of the hedged carry trade return conditional on no disaster and E^{ND} denotes expectations under the assumption of no disaster:

$$X^e(K_{\Delta^P}) = \frac{E^{ND}X\left(K_{\Delta^P}\right)}{\tau}.$$

The following proposition offers a closed form formula for the hedged returns.

Proposition 3. *We assume that the disaster sizes* (J, J^*) are constant *between* t *and* $t+\tau$ *with* $J > J^*$. *Let* Δ^P *be a Black-Scholes put delta i.e.* $\Delta^P < 0$, *and let* K_{Δ^P} *be the corresponding strike. We define:*

$$\begin{aligned}\beta &= n\big(\mathbb{N}^{-1}(-\Delta^P)\big) - \mathbb{N}^{-1}(-\Delta^P)(1+\Delta^P)\\ \gamma &= \big(1+\Delta^P\big)\Delta^P\mathbb{N}^{-1}(-\Delta^P) - \big(2+\Delta^P\big)n\big(\mathbb{N}^{-1}(-\Delta^P)\big)\end{aligned}$$

where $\mathbb{N}()$ *is the cumulative standard normal distribution and n() the standard normal distribution.*

In the limit of small time intervals ($\tau \to 0$*), the hedged carry trade expected return (conditional on no disasters) can be approximated by:*

$$X^e(K_{\Delta^P}) = \big(1+\Delta^P\big)\Pi_G + \left(\beta\left(pJ + \frac{\Pi_D\Pi_G}{\sigma_h^2}\right) + \gamma\Pi_G\right)\sigma_h\sqrt{\tau} + O(\tau)$$

where Π_G is the Gaussian premium, σ_h is the exchange rate volatility conditional on no disaster and Π_D is the disaster premium.

Proof. Let's write the hedged return as a function of the unhedged return X:

$$X(K_{\Delta^P}) = X - \lambda^P(K_{\Delta^P})P(K_{\Delta^P})e^{r^*\tau}\frac{M^*}{M} + \lambda^P(K_{\Delta^P})\left(K_{\Delta^P} - \frac{M^*}{M}\right)^+, \tag{12}$$

Like in proposition (1), we can write the Euler equation for the hedged return as the sum of a disaster component Π_{HD} and a non–disaster component Π_{HG}:

$$\frac{E^{ND}X(K_{\Delta^P})}{\tau} = \Pi_{HD} + \Pi_{HG}$$

where:

$$\Pi_{HD} = -\frac{pe^{g\tau}}{1-p\tau}E^D MX(K_{\Delta^P})$$

and:

$$\Pi_{HG} = -\frac{e^{g\tau}}{\tau}cov^{ND}\Big(M, X(K_{\Delta^P})\Big)$$

We then proceed in 3 steps:

1. First let's replace the expression for the hedged return given by equation (12) in Π_{HD}.

   From proposition (1), the first term in Π_{HD} can be approximated by:

   $$-\frac{pe^{g\tau}}{1-p\tau}E^D MX = (pJ - pJ^*) + O(\tau)$$

   Recall from lemma (4) and (5) that $P(K_{\Delta^P})$ is of order $O(\sqrt{\tau})$. So the second term in Π_{HD} can be approximated by:

   $$\frac{pe^{g\tau}}{1-p\tau}\lambda^P(K_{\Delta^P})P(K_{\Delta^P})e^{r^*\tau}E^D M^* = \lambda^P(K_{\Delta^P})P(K_{\Delta^P})pJ^* + O(\tau)$$

   Recall that K_{Δ^P} is of order $O(\sqrt{\tau})$. So when $J > J^*$ and τ is small enough the third term in Π_{HD} can be approximated using lemma (5) by:

   $$-\frac{pe^{g\tau}}{1-p\tau}\lambda^P(K_{\Delta^P})E^D\big(K_{\Delta^P}M - M^*\big)^+ = -\lambda^P(K_{\Delta^P})\big(K_{\Delta^P}pJ - pJ^*\big) + O(\tau)$$

Summing up the components of Π_{HD}, we get:

$$\begin{aligned}\Pi_{HD} &= (pJ - pJ^*) + \lambda^P(K_{\Delta^P})P(K_{\Delta^P})pJ^* - \lambda^P(K_{\Delta^P})\big(K_{\Delta^P}pJ - pJ^*\big),\\ &= pJ\Big(1 - K_{\Delta^P}\lambda^P(K_{\Delta^P})\Big) - pJ^*\Big(1 - \lambda^P(K_{\Delta^P})\big(1 + P(K_{\Delta^P})\big)\Big) + O(\tau)\end{aligned}$$

The hedge ratio can be approximated by:

$$\lambda^P(K_{\Delta^P}) = \frac{1}{1 + P(K_{\Delta^P})} + O(\tau)$$

So the expression for Π_{HD} can be simplified to:

$$\Pi_{HD} = \Big(1 + P(K_{\Delta^P}) - K_{\Delta^P}\Big).pJ + O(\tau)$$

Let's use the notation: $\alpha = \mathbb{N}^{-1}(-\Delta^P)$. Lemma (4) and (5) give:

$$P(K_{\Delta^P}) = \Big(n\big(\alpha\big) + \alpha\mathbb{N}\big(\alpha\big)\Big)\sigma_h\sqrt{\tau} + O(\tau)$$

Recall that given a Black-Scholes put delta Δ^P the strike K_{Δ^P} is given by equation (6):

$$K_{\Delta^P} = e^{\mathbb{N}^{-1}\left(-e^{r^*\tau}\Delta^P\right)\sigma_{\Delta^P}\sqrt{\tau} + \left(r - r^* + 1/2\sigma^2_{\Delta^P}\right)\tau}$$

which can be simplified to:

$$K_{\Delta^P} = 1 + \alpha\sigma_h\sqrt{\tau} + O(\tau)$$

Let's define $\beta = n\big(\mathbb{N}^{-1}(-\Delta^P)\big) - \mathbb{N}^{-1}(-\Delta^P)\big(1 + \Delta^P\big)$. We can then Π_{HD}:

$$\Pi_{HD} = \Big(n(\alpha) + \alpha\mathbb{N}(\alpha) - \alpha\Big).pJ\sigma_h\sqrt{\tau} + O(\tau)$$
$$= \beta.pJ.\sigma_h.\sqrt{\tau} + O(\tau)$$

2. Now let's replace the expression for the hedged return given by equation (12) in Π_{HG}.

   From proposition (1) the first term in Π_{HG} can be approximated by:

$$-\frac{e^{g\tau}}{\tau} cov^{ND}(M, X) = \Pi_G + O(\tau)$$

   In proposition (1) we derived:

$$cov^{ND}\Big(M, \frac{M^*}{M}\Big) = -\Pi_G\tau + O(\tau^2)$$

   So the second term in Π_{HG} can be approximated by:

$$\frac{e^{g\tau}}{\tau}\lambda^P(K_{\Delta^P})P(K_{\Delta^P})e^{r^*\tau} cov^{ND}\Bigg(M, \frac{M^*}{M}\Bigg) = -\lambda^P(K_{\Delta^P})P(K_{\Delta^P})\Pi_G + O(\tau)$$

   Recall that the ratio of the SDFs is given by:

$$E^{ND}\frac{M^*}{M} = e^{\big(g-g^*+\Pi_G\big)\tau}$$

   So using lemma 3 we have:

$$cov^{ND}\Bigg(M, \Big(K_{\Delta^P} - \frac{M^*}{M}\Big)^+\Bigg) = V^P_{BS}\Big(e^{-g^*\tau}, K_{\Delta^P}e^{-g\tau}, \sigma_h\sqrt{\tau}\Big) - V^P_{BS}\Big(e^{(-g^*+\Pi_G)\tau}K_{\Delta^P}e^{-g\tau}, \sigma_h\sqrt{\tau}\Big)$$

   Recall that a put delta is the first derivative of the put price with respect to the spot. So the third term in Π_{HG} can be approximated by:

$$\frac{-e^{g\tau}\lambda^P(K_{\Delta^P})}{\tau} cov^{ND}\Bigg(M, \Big(K_{\Delta^P} - \frac{M^*}{M}\Big)^+\Bigg) = \lambda^P(K_{\Delta^P})\Delta^P_{BS}\Big(e^{(g-g^*)\tau}, K_{\Delta^P}, \sigma_h\sqrt{\tau}\Big)\Pi_G + O(\tau)$$

Summing up the components of Π_{HG} gives:

$$\Pi_{HG} = \left(1 + \lambda^P(K_{\Delta^P})\Big(\Delta^P_{BS}\big(e^{(g-g^*)\tau}, K_{\Delta^P}, \sigma_h\sqrt{\tau}\big) - P(K_{\Delta^P})\Big)\right)\Pi_G + O(\tau)$$

which can be simplified using the approximation for $\lambda^P(K_{\Delta^P})$:

$$\Pi_{HG} = \Big(1 + \Delta^P_{BS}\big(e^{(g-g^*)\tau}, K_{\Delta^P}, \sigma_h\sqrt{\tau}\big)\Big).\Big(1 - P(K_{\Delta^P})\Big)\Pi_G + O(\tau)$$

Recall the expression for a Black–Scholes put delta:

$$\Delta^P_{BS}\big(e^{(g-g^*)\tau}, K_{\Delta^P}, \sigma_h\sqrt{\tau}\big) = -\mathbb{N}\big(-d_-\big)$$

where:

$$-d_- = \frac{\log(K_{\Delta^P}) + (g^* - g + 1/2\sigma_h^2)\tau}{\sigma_h\sqrt{\tau}}$$

and the strike K_{Δ^P} is given by equation (6):

$$K_{\Delta^P} = e^{\mathbb{N}^{-1}\left(-e^{r^*\tau}\Delta^P\right)\sigma_{\Delta^P}\sqrt{\tau} + \left(r - r^* + 1/2\sigma^2_{\Delta^P}\right)\tau}$$

When $J > J^*$, the implied volatility σ_{Δ^P} given by lemma (6) is:

$$\sigma_{\Delta^P} = \sigma_h + \frac{\big(1 + \Delta^P\big)\Pi_D}{n\big(\mathbb{N}^{-1}(-\Delta^P)\big)}\sqrt{\tau} + O(\tau)$$

So when τ is small, the Taylor expansion of $-d_-$ is:

$$
\begin{aligned}
-d_- &= \frac{\mathbb{N}^{-1}\big(-e^{r^*\tau}\Delta\big)\sigma_\Delta\sqrt{\tau} + \big(r - r^* + 1/2\sigma_\Delta^2 + g^* - g + 1/2\sigma_h^2\big)\tau}{\sigma_h\sqrt{\tau}} \\
&= \alpha + \Big(\sigma_h - \frac{\Pi_D}{n(\alpha)\sigma_h}\beta\Big)\sqrt{\tau} + O(\tau)
\end{aligned}
$$

where $\alpha = \mathbb{N}^{-1}(-\Delta^P)$ and $\beta = n\big(\mathbb{N}^{-1}(-\Delta^P)\big) - \mathbb{N}^{-1}(-\Delta^P)(1+\Delta^P)$. So the Taylor expansion of $-\mathbb{N}()$ around α gives :

$$
\Delta_{BS}^P\big(e^{(g-g^*)\tau}, K_{\Delta^P}, \sigma_h\sqrt{\tau}\big) = \Delta^P - \Big(n\big(\alpha\big)\sigma_h - \beta\frac{\Pi_D}{\sigma_h}\Big)\sqrt{\tau} + O(\tau)
$$

Plugging this expression and the approximation for $P(K_\Delta)$ given by lemma (4) and (5) back into Π_{HG} gives :

$$
\begin{aligned}
\Pi_{HG} =& \bigg(\Big(1 + \Delta^P\Big) - \Big(n\big(\alpha\big)\sigma_h - \beta\frac{\Pi_D}{\sigma_h}\Big)\sqrt{\tau}\bigg).\bigg(1 - \Big(n\big(\alpha\big) + \alpha\mathbb{N}\big(\alpha\big)\Big)\sigma_h\sqrt{\tau}\bigg)\Pi_G + O(\tau) \\
=& \Big(1 + \Delta^P\Big)\Pi_G - \bigg(n\big(\alpha\big)\sigma_h - \beta\frac{\Pi_D}{\sigma_h} + \big(1 + \Delta^P\big)\Big(n\big(\alpha\big) - \Delta^P\alpha\Big)\sigma_h\bigg)\Pi_G\sqrt{\tau} + O(\tau) \\
=& \big(1 + \Delta^P\big)\Pi_G + \bigg(\beta\frac{\Pi_D}{\sigma_h} + \gamma\sigma_h\bigg)\Pi_G\sqrt{\tau} + O(\tau)
\end{aligned}
$$

where $\gamma = \big(1 + \Delta^P\big)\Delta^P\mathbb{N}^{-1}(-\Delta^P) - \big(2 + \Delta^P\big)n\big(\mathbb{N}^{-1}(-\Delta^P)\big)$.

3. Finally, we combine the terms to compute $X^e\big(K_{\Delta^P}\big)$:

$$
X^e(K_{\Delta^P}) = \big(1 + \Delta^P\big)\Pi_G + \bigg(\beta\Big(pJ + \frac{\Pi_D\Pi_G}{\sigma_h^2}\Big) + \gamma\Pi_G\bigg)\sigma_h\sqrt{\tau} + O(\tau)
$$

$\square$

4.4 Risk-reversals

Given a $\Delta > 0$ we can consider the corresponding Black–Scholes put delta : $\Delta^P = -\Delta$ and the Black–Scholes call delta $\Delta^C = \Delta$. A risk-reversal is defined as the difference between the implied volatility at the Black–Scholes put delta and the implied volatility at the Black–Scholes call delta:

$$RR_\Delta = \sigma_{-\Delta} - \sigma_\Delta$$

Proposition 4. *Given $\Delta > 0$, when there is no disaster risk:*

$$RR_\Delta = \sigma_{-\Delta} - \sigma_\Delta = 0$$

Proof. The result follows by taking $p = 0$ in proposition (2):

$$\sigma_\Delta = \sigma_{-\Delta} = \sigma_h$$

So:

$$RR_\Delta = 0$$

□

In the presence of disaster risk the following proposition identifies conditions under which we can simplify the expression for the risk–reversal.

Proposition 5. *We assume that the disaster sizes (J, J^*)* are constant *between t and $t + \tau$. Given a Black–Scholes delta $\Delta > 0$, the risk–reversal $\sigma_{-\Delta} - \sigma_\Delta$ can be approximated in the limit of small time intervals ($\tau \to 0$) by:*

$$RR_\Delta = \sigma_\Delta - \sigma_{-\Delta} = \frac{1 - 2\Delta}{n\big(\mathbb{N}^{-1}(\Delta)\big)} \Pi_D \sqrt{\tau} + O(\tau)$$

where $\mathbb{N}()$ is the cumulative standard normal distribution, $n()$ the standard normal distribution and Π_D is the disaster premium.

Proof. From lemma (6), we have:

$$\sigma_\Delta = \begin{cases} \sigma_h + \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ^* - pJ\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)}\sqrt{\tau} + O(\tau) & \text{if } \Delta > 0; \\ \\ \sigma_h + \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ - pJ^*\right)^+}{n\left(\mathbb{N}^{-1}(|\Delta|)\right)}\sqrt{\tau} + O(\tau) & \text{if } \Delta < 0. \end{cases}$$

Notice that $\left(pJ - pJ^*\right)^+ - \left(pJ^* - pJ\right)^+ = \left(pJ - pJ^*\right)$. So given $\Delta > 0$, we obtain:

$$\begin{aligned} RR_\Delta &= \sigma_{-\Delta} - \sigma_\Delta \\ &= \frac{-\Delta\left(pJ - pJ^*\right) + \left(pJ - pJ^*\right)^+}{n\left(\mathbb{N}^{-1}(\Delta)\right)}\sqrt{\tau} - \frac{\Delta\left(pJ - pJ^*\right) + \left(pJ^* - pJ\right)^+}{n\left(\mathbb{N}^{-1}(\Delta)\right)}\sqrt{\tau} + O(\tau) \\ &= \frac{1 - 2\Delta}{n\left(\mathbb{N}^{-1}(\Delta)\right)}\Pi_D\sqrt{\tau} + O(\tau) \end{aligned}$$

□

5 Simulation

Propositions (1), (3) and (5) are derived in the limit of small time intervals. We check their validity for one-day and one-month horizons by simulating a calibrated version of the model.

Table 1 reports the parameter values used in the calibration, while Table 2 reports simulation results.

[Table 1 about here.]

[Table 2 about here.]

We verify that the higher order term in Proposition 3 remains positive given reasonable values for the parameter estimates. Notice that the higher order term is the sum of two positive terms $\left(pJ + \frac{\pi^D\pi^G}{\sigma_h^2}\right)$ and π^G which are multiplied by 2 coefficients β and γ respectively. In our simulation $\left(pJ + \frac{\pi^D\pi^G}{\sigma_h^2}\right)$ is equal to 34% and π^G is equal to 3% and in each of the subsample estimations we considered pJ is roughly one order of magnitude larger than π^G. At 10 delta, β is equal to 1.3 and

γ is equal to -0.2. At 25 delta, β is equal to 0.8 and γ is equal to -0.4. At 50 delta, β is equal to 0.4 and γ is equal to -0.6. These results supports the claim that the higher order term is positive given a large range of values for the parameters of the model.

6 Data and Additional Estimation Results

6.1 Data

Our dataset contains spot exchange rates, one-month forward exchange rates and the U.S. LIBOR interest rate obtained from Datastream, as well as one-month implied volatilities obtained from JP Morgan, for the period 1/1996 to 12/2011. For each country, the spot and forward exchange rates are expressed in U.S. dollars per unit of foreign currency. The foreign interest rates are computed by using covered interest rate parity :

$$F = Se^{(r_{us}-r^*)\tau}.$$

In the JP Morgan volatility datasets, all the implied volatilities refer to options on spot exchange rates for which the foreign currency is the base currency and the dollar is the term currency. The convention in the forex market is to quote these implied volatilities by using the value of a Black and Scholes (1973) delta. With one-month maturity options (or any option with maturity less than one year), if the country is a G7 country the convention is to use a Black–Scholes spot delta. For instance, we call σ_{10C} (σ_{10P}) is the implied volatility at 10 delta call (put). It is the implied volatility given that the option Black–Scholes spot delta is equal to 0.1 (-0.1). We can then retrieve the strikes K_{10C} and K_{10P} by using equation (5) and (6).

When the country is not a G7 country, then the spot delta is replaced by the forward delta in the calculation of K_{10C} and K_{10P}:

$$\Delta_{BS,F}(\phi) = \phi\mathbb{N}(\phi d_+),$$

where $\phi = 1$ for a delta call and $\phi = -1$ for a delta put.

The at-the-money strike is a special case. With one-month maturity options (or any option with maturity less than 10 years), if the country is a G7 country except Japan, the at-the-money strike is the strike which cancels the spot delta of a straddle (i.e., the sum of a put and a call):

$$\Delta_{BS}^{C} + \Delta_{BS}^{P} = 0$$

This implies:

$$K_{atm} = Se^{(r_{us}-r^*+\frac{1}{2}\sigma^2_{\text{atm}})\tau}.$$

Figure 1 presents the time-series of risk-reversals at 10-delta in percentage points for all the currencies in our sample.

[Figure 1 about here.]

6.2 Country-Level Disaster Risk Premia

Figures (2), (3), and (4) report the time-series estimates for the disaster risk premium for each country.

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

Finally, Figure 5 reports similar findings as in Figure **??** in the text. In Figure 5, however, the average estimated disaster risk premia is estimated over the period leading to the crisis (from May 2008 to August 2008) and compared to the cumulative percentage change in exchange rate for each country during the crisis (from September 2008 to December 2008), while in Figure **??**, both disaster risk and changes in exchange rates are estimated during the crisis.

[Figure 5 about here.]

6.3 Asset Pricing

Table 3 reports asset pricing results obtained with two risk factors: the average excess returns of a U.S. investor on currency markets (denoted RX) and the risk-reversals at 25-delta on S&P500 index options (denoted RR). The test assets are the six portfolios of Lustig, Roussanov and Verdelhan (2011).

[Table 3 about here.]

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Table 1: Simulation Parameters

Parameter	Symbol	Value
Disaster probability	p	3.60%
Disaster size (domestic)	J	7.50
Disaster size (foreign)	$J^{\star}$	6.67
SDF drift (domestic)	g	26.17%
SDF drift (foreign)	$g^{\star}$	26.23%
Volatility of gaussian shocks (domestic)	σ	82.94%
Volatility of gaussian shocks (foreign)	$\sigma^{\star}$	80.00%
Correlation of gaussian shocks	ρ	99.15%

Notes: This table shows the parameters used in the simulation. The disaster probability is taken from Barro (2006). The domestic and foreign disaster sizes (J and $J^{\star}$) as well as the domestic and foreign drifts (g and $g^{\star}$) of the pricing kernel come from the estimation results for the high interest currency portfolio during the period 1/1996–12/2011 excluding fall 2008. The domestic and foreign volatility (σ and $\sigma^{\star}$) of the Gaussian shocks, as well as their correlation (ρ), are calibrated to match a Gaussian premium of 3% and a volatility of the bilateral exchange rate equal to 10% to match their counterparts on the high interest currency portfolio, as well as a maximum Sharpe Ratio equal to 80%.

Table 2: Simulation Results

	One-Month Horizon		One-Day Horizon	
	Model	Approximation	Model	Approximation
		Excess Returns		
Unhedged Returns	6.20	6.00	6.02	6.00
Hedged Returns at 10 delta	4.27		3.06	
Order 0		2.70		2.70
Order $\sqrt{\tau}$		4.13		2.96
Hedged Returns at 25 delta	3.22		2.53	
Order 0		2.25		2.25
Order $\sqrt{\tau}$		3.11		2.41
Hedged Returns at-the-money	1.99		1.66	
Order 0		1.50		1.50
Order $\sqrt{\tau}$		1.88		1.57
		Risk Reversals		
Risk-Reversals at 10 delta	2.39	3.95	0.58	0.72
Risk-Reversals at 25 delta	0.88	1.36	0.23	0.25

Notes: This table compares the simulation results obtained by running a Monte-Carlo simulation on the model quantities of interest for a one-month and a one-day horizon to the closed form formula that derived in the paper. This simulation uses the parameters described in Table 1. The results are expressed in percentage points. The excess returns are annualized (multiplied by 12). The risk reversal is computed as the difference between the volatility of an out-of-the-money put and an out-of-the-money call.

Table 3: Asset Pricing with Risk Reversals

Panel I: Risk Prices							
	λ_{RX}	λ_{RR}	b_{RX}	b_{RR}	R^2	$RMSE$	χ^2
GMM_1	1.03	−4.66	−1.16	−60.09	90.80	0.75	
	[5.23]	[2.96]	[0.86]	[37.38]			85.85
GMM_2	2.20	−4.76	−0.95	−60.97	67.82	1.40	
	[3.85]	[2.48]	[0.77]	[31.62]			87.62
FMB	1.03	−4.66	−1.15	−59.77	90.83	0.75	
	[1.63]	[1.18]	[0.47]	[15.23]			46.83
	(1.64)	(2.27)	(0.73)	(29.28)			91.82
Panel II: Factor Betas							
$Portf.$	α_0^j	β_{RX}^j	β_{RR}^j	R^2			
1	−8.74	0.91	0.87	66.76			
	[2.24]	[0.07]	[0.38]				
2	−3.04	0.86	0.14	69.88			
	[2.24]	[0.06]	[0.34]				
3	−1.78	0.93	0.18	79.60			
	[1.90]	[0.05]	[0.31]				
4	0.99	0.94	−0.08	78.75			
	[1.94]	[0.05]	[0.30]				
5	2.48	1.12	−0.15	77.56			
	[2.95]	[0.07]	[0.47]				
6	10.09	1.24	−0.96	68.01			
	[3.45]	[0.08]	[0.59]				

Notes: Panel I reports results from GMM and Fama-McBeth asset pricing procedures. The market prices of risk λ, the adjusted R^2, the square-root of mean-squared errors $RMSE$ and the p-values of χ^2 tests on pricing errors are reported in percentage points. The log pricing kernel is here: $m_{t+1}^{US} = 1 - b_{RX}RX_{t+1} - b_{RR}RR_{t+1}$, where b denotes the vector of factor loadings. Excess returns used as test assets and risk factors do not take into account bid-ask spreads. All excess returns are multiplied by 12 (annualized). Shanken (1992)-corrected standard errors are reported in parentheses. We do not include a constant in the second step of the FMB procedure. Panel II reports OLS estimates of the factor betas. R^2s and p-values are reported in percentage points. The alphas are annualized and in percentage points. The standard errors in brackets are Newey and West (1987) standard errors computed with the optimal number of lags according to Andrews (1991). Note that risk reversals are not excess returns. As a result, constants in the time-series regressions reported in the second panel do not have to be zero. (Each constant α_0^j is equal to $\beta^j(\lambda - E(f)$, where λ denotes the vector of risk prices and $E(f)$ the mean of the risk factors. When the factor is an excess return, then the Euler equation implies that $\alpha_0^j = 0$). The test assets are the six currency portfolios of Lustig et al. (2011). Countries are sorted on the basis of their interest rates. The first portfolio corresponds to low interest rate countries while the last portfolio corresponds to high interest rate countries. Data are monthly, from the Datastream and CRSP databases. The sample period is 2/1996–12/2011.

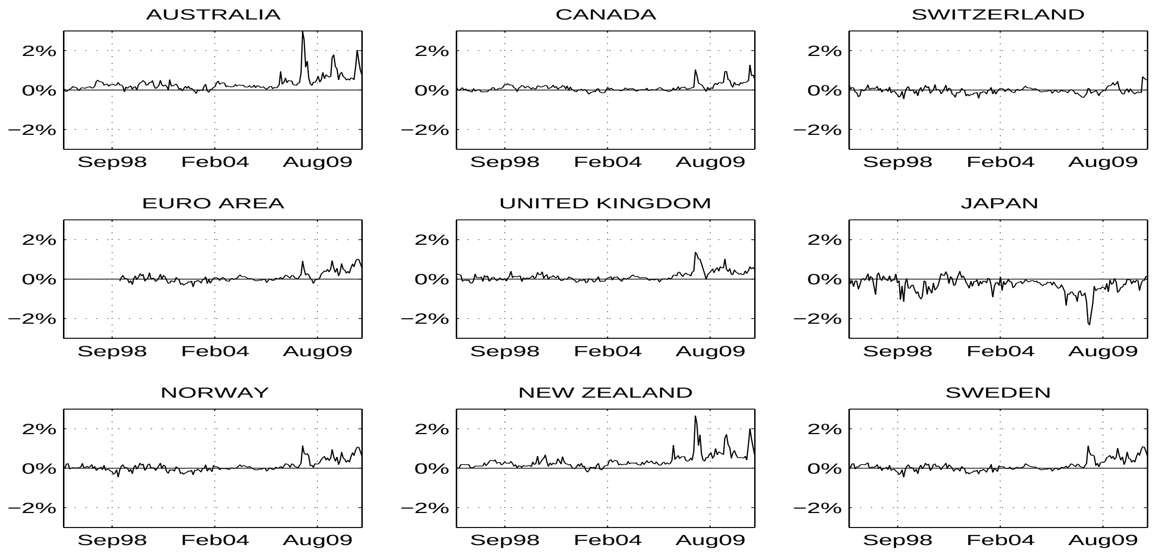

Figure 1: Risk Reversals

This figure presents the time-series of risk-reversals at 10-delta in percentage points for all the currencies in our sample: the Australian dollar, the Canadian dollar, the Euro, the Japanese yen, the New Zealand dollar, the Norwegian Krone, the Swedish Krona, the Swiss Franc, and the U.K. pound, all defined versus the U.S. dollar. Risk-reversals are computed as the difference between the price of a put at 10-delta and the price of a call at 10-delta. The put and call prices are normalized by the spot exchange rate and annualized (multiplied by 12). Spot and forward exchange rates are from Datastream, currency options are from JP Morgan. Data are monthly. The sample period is 1/1996 – 12/2011.

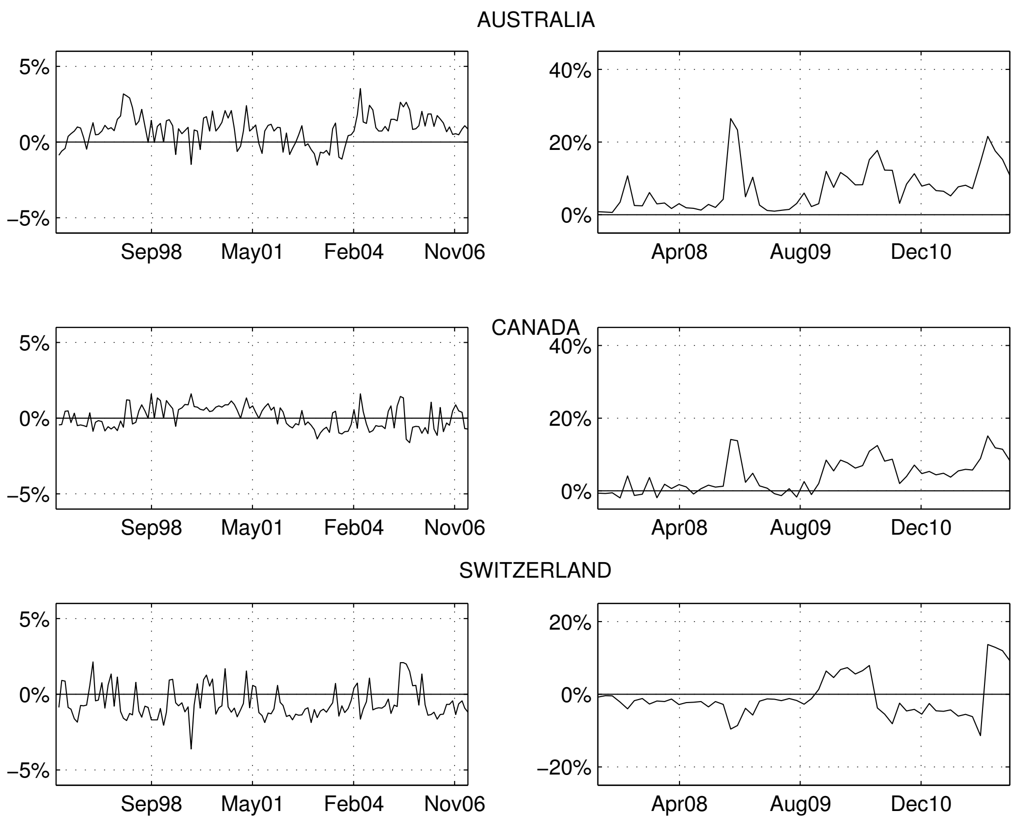

Figure 2: Country-level Estimates of Disaster Risk Premia (I/III)

This figure shows time-series estimates for the disaster risk premium for each country. The left panel focuses on the pre-crisis period, while the right panel focuses on the crisis and post-crisis periods. Spot and forward exchange rates are from Datastream, currency options are from JP Morgan. Data are monthly. The sample period is 1/1996–12/2011.

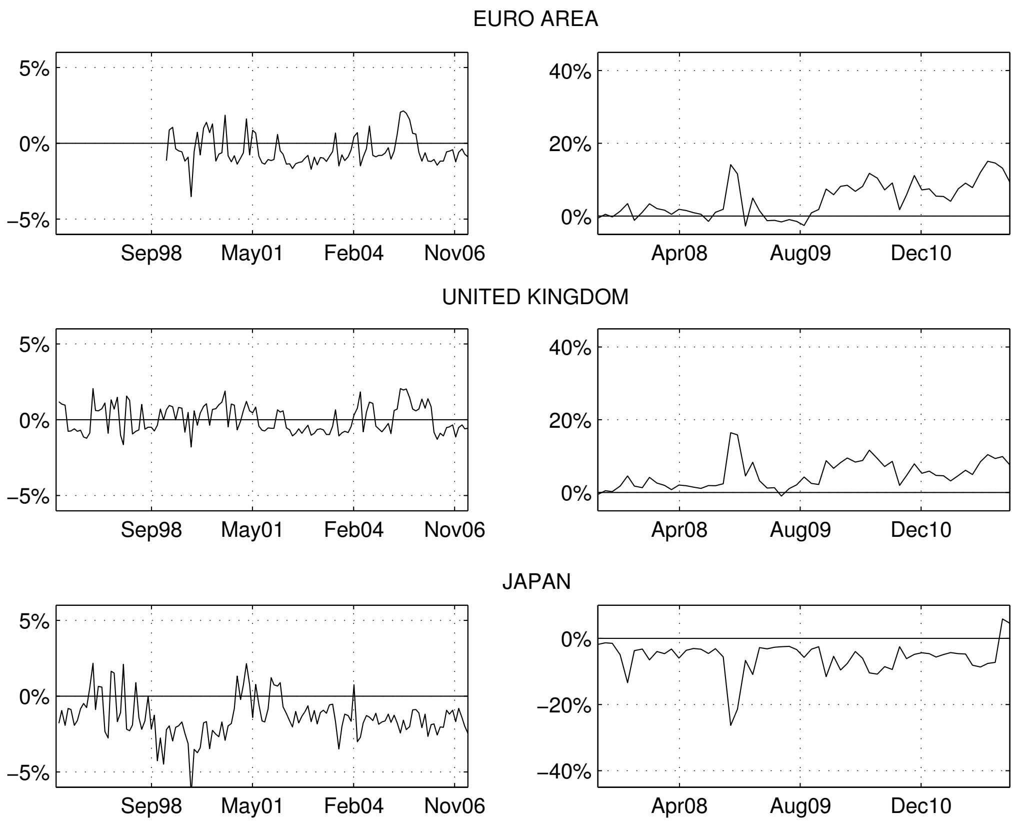

Figure 3: Country-level Estimates of Disaster Risk Premia (II/III)

This figure shows time-series estimates for the disaster risk premium for each country. The left panel focuses on the pre-crisis period, while the right panel focuses on the crisis and post-crisis periods. Spot and forward exchange rates are from Datastream, currency options are from JP Morgan. Data are monthly. The sample period is 1/1996–12/2011.

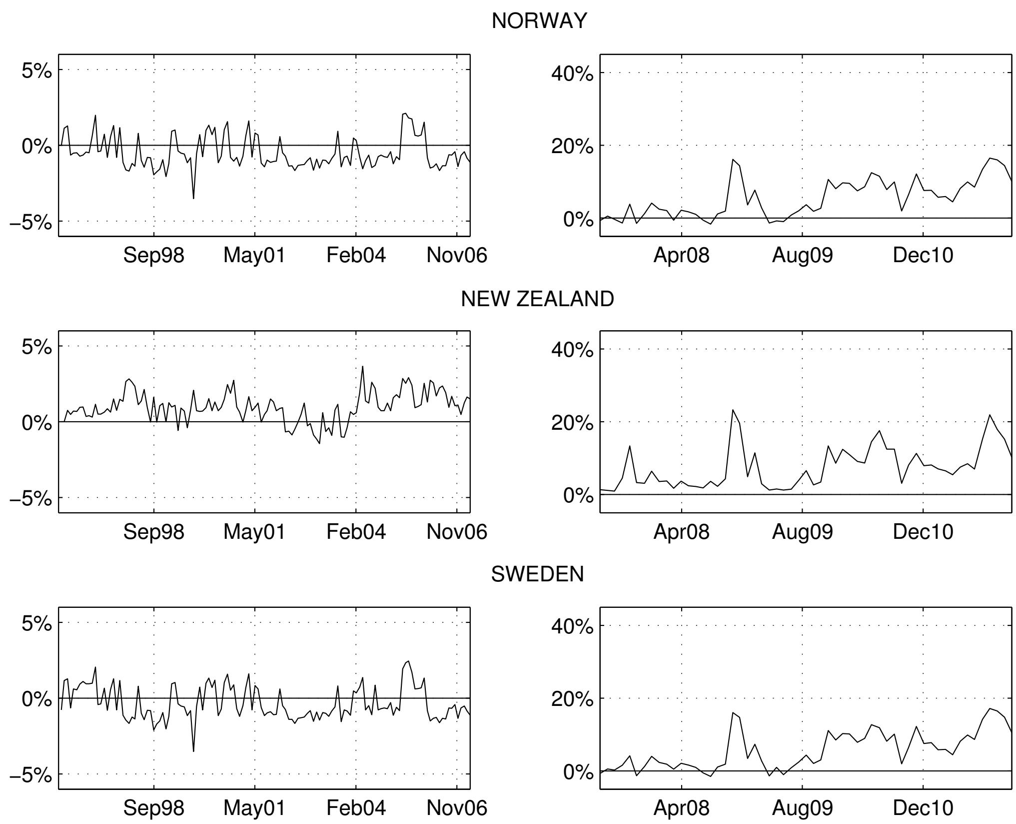

Figure 4: Country-level Estimates of Disaster Risk Premia (III/III)

This figure shows time-series estimates for the disaster risk premium for each country. The left panel focuses on the pre-crisis period, while the right panel focuses on the crisis and post-crisis periods. Spot and forward exchange rates are from Datastream, currency options are from JP Morgan. Data are monthly. The sample period is 1/1996–12/2011.

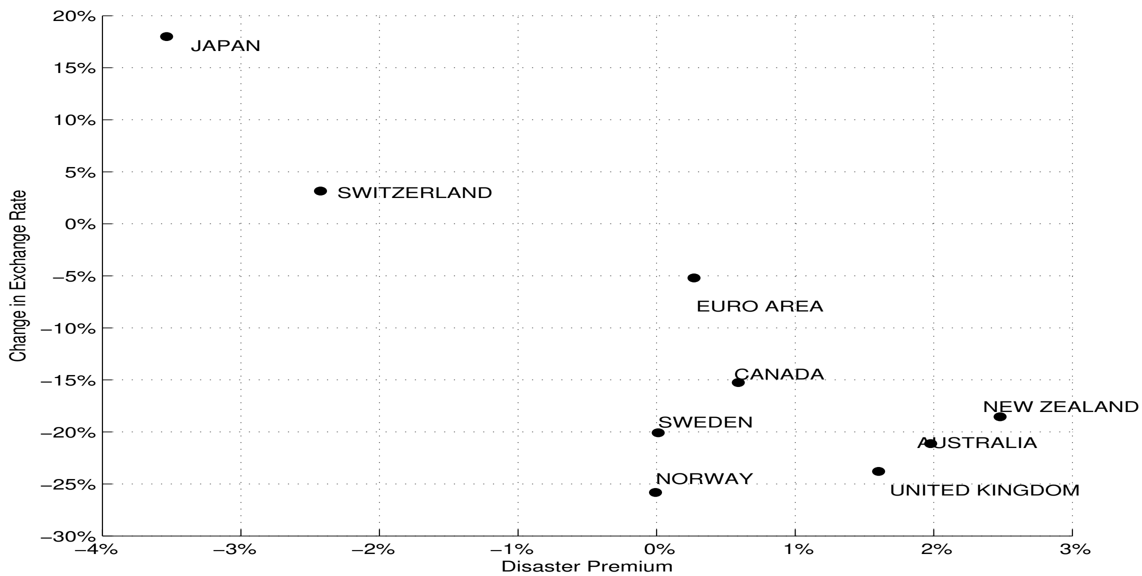

Figure 5: Disaster Risk Exposures and Changes in Exchange Rates During the Crisis

This figure reports the average estimated disaster risk premia leading to the crisis (from May 2008 to August 2008) and the cumulative percentage change in exchange rate for each country during the crisis (from September 2008 to December 2008). Spot and forward exchange rates are from Datastream, currency options are from JP Morgan. Data are monthly.