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# SPATIAL PRICE DISCRIMINATION WITH HETEROGENEOUS FIRMS

Jonathan Vogel

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**ABSTRACT**

In this paper we present and solve a three-stage game of entry, location, and pricing in a spatial price discrimination framework with arbitrarily many heterogeneous firms. We provide a unique characterization of all equilibria without imposing restrictions on the distribution of marginal costs.

Jonathan Vogel
Department of Economics
Columbia University
420 West 118th Street
New York, NY 10027
and NBER
jvogel@columbia.edu

# 1 Introduction

In this paper we present and solve a three-stage game of entry, location, and pricing in a spatial price discrimination framework with arbitrarily many heterogeneous firms. We provide a unique outcome of all pure undominated strategy SPNE without imposing restrictions on the distribution of marginal costs or the allocation of transportation costs between firms and consumers.

We answer the following question: What determines the pattern of firm entry, location, market share, and profit in an environment in which heterogeneous firms have the ability to spatially price discriminate? In the context of the present paper, spatial price discrimination represents the ability of a firm to charge different prices to consumers at different locations in space, but does not imply a restriction that consumers cannot arbitrage away price differences.

Spatial price discrimination is possible in markets in which firms are geographically differentiated, such as ready-mixed concrete. In such a market, a producer observes the location of each of its customers and can condition its customer-specific price on this location. While customers can arbitrage away price differences across space, they—like the producer itself—must incur a transportation cost to deliver the good. Spatial price discrimination is also possible in markets in which producers sell goods tailored to the desired specifications of their customers, such as differentiated intermediate input producers. In such a market, a producer customizes its output to match the requirements of each of its customers and can condition its price on these requirements. While customers can arbitrage away price differences, they—like the producer itself—must incur a customization cost to tailor the good. More generally, spatial price discrimination may occur in many markets in which buyers and sellers bargain over the joint surplus of trade, and in which surplus depends on a measure of distance between each potential buyer-seller pair.[1]

The present paper makes contributions along three dimensions. First, it reproduces recent results—in Vogel (2008)—that were obtained in a framework that applies to very different types of industries than the present paper. In particular, spatial price discrimination was not allowed in Vogel (2008). Second, it generalizes these predictions in several important respects: (*i*) It does not impose restrictions on the distribution of marginal costs across firms; (*ii*) It does not impose a restriction on the allocation of shipping costs between firms and their customers; and (*iii*) It includes an entry stage in which, in equilibrium, less productive

[1] In the present paper we assume that all bargaining power is allocated to buyers.

firms do not enter. Finally, by greatly simplifying the game—precisely by allowing for spatial price discrimination—the present paper highlights in a clearer way the economic intuition behind these recent results.

Section 2 introduces our theoretical framework. The market is represented by the unit circumference, which is populated with uniformly distributed consumers. There is a potentially large set of potential entrants with different constant marginal costs of production. These firms play a three-stage game of complete information. In the first stage, potential entrants simultaneously choose whether to enter and incur a fixed cost or to exit. In the second stage, the entrants simultaneously choose their locations in the market. In the final stage, firms simultaneously set their prices, where each firm can price discriminate, potentially choosing a different price for each location in the market.

In Sections 3-5 we solve for a subgame perfect Nash equilibrium using backward induction. The central testable implication of this paper, contained in Section 5, is similar to that in Vogel (2008): in equilibrium, more productive firms are more isolated—all else equal—, supply more consumers, and earn more profit. While the key result is similar, the interpretation of this result differs: in the present model, this result implies that more productive firms customize their products to a broader class of customers. Moreover, the specifics of the present model enable a cleaner link between theory and data along at least three dimensions. For example, $(i)$ four-digit SIC industries reviewed in Bartelsman and Doms (2000) have $85^{th} - 15^{th}$ total factor productivity ratios in the range of $2:1$ to $4:1$. Nevertheless, the new spatial competition models that incorporate firm heterogeneity all impose a restriction on the extent of permissible asymmetry between firms; see e.g. Aghion and Schankerman (2004), Syverson (2004), Alderighi and Piga (2008), and Vogel (2008). The present paper requires no such restriction. Moreover, $(ii)$ shipping costs are substantial in a wide range of industries. Whether a supplier or a consumer incurs the cost of transportation is typically an equilibrium outcome rather than an industry-wide restriction. Nevertheless, in most spatial competition frameworks, it is assumed that either suppliers, or more often consumers incur the full cost of transportation; see e.g. Hotelling (1929), Lancaster (1979), and Salop (1979). The present paper requires no such arbitrary assumption. Finally, $(iii)$ selection on productivity appears to be an important characteristic within a wide range of industries, see e.g. Syverson (2004). The present paper is the first to solve for both the set of heterogeneous firms that enter a given market and their locations within that market.

Finally, it is worth emphasizing that the assumption of spatial price discrimination is key to the present paper's ability to relax restrictions—on the distribution of marginal costs across firms and on the allocation of shipping costs between firms and their customers—that must be imposed in Vogel (2008). Spatial price discrimination greatly simplifies the game. As is well known, under the assumption of mill pricing—in which a firm charges a single price regardless of customer location—solving for an equilibrium to a location-and-pricing game is complicated because of issues that arise in the price stage; see e.g. d'Aspremont, Gabszewicz, and Thisse (1979). The model with spatial price discrimination is significantly more tractable: in the final stage, in which firms choose prices, firms engage in Bertrand competition with undifferentiated goods at each location.

This is not the first paper to consider price discrimination in a spatial competition model; see e.g. Hoover (1937), Lederer and Hurter (1986), Hamilton, Thisse, and Weskamp (1989), Hamilton, MacLeod, and Thisse (1991), and MacLeod, Norman, and Thisse (1992).[2] Building on these papers, the primary focus of which was existence of equilibria, we emphasize the determinants of isolation for arbitrarily many heterogeneous firms. This paper also contributes to a growing spatial competition literature concerned with heterogeneous firms; see e.g. Aghion and Schankerman (2004), Syverson (2004), Alderighi and Piga (2008), and Vogel (2008). Unlike Aghion and Schankerman (2004), Syverson (2004), and Alderighi and Piga (2008), the present paper considers not only endogenous prices, but also endogenous locations.

## 2 Setup

**Firms:** There is a set $N$ containing $|N| \geq 2$ potential entrants each of which is endowed with a unique marginal cost of production $c_i \in [0, v - t/2)$.[3] Firms play a three-stage game of complete information. In the first age, *the entry stage*, firms simultaneously choose whether or not to enter and incur a fixed cost $f > 0$. Those firms that do enter move to the second stage, *the location stage*, in which firms simultaneously choose their locations in the market, where the market is represented by the unit circumference the points of which are indexed in a clockwise direction by $z \in [0, 1]$. In the third stage, *the price stage*, firms simultaneously choose price schedules. Each firm $i$ can price discriminate, choosing a price $p_i(z)$ for each

[2] Spatial discrimination in the Cournot setting with homogeneous firms has also been studied; see e.g. Anderson and Neven (1991) and Chapter 9 of Combes, Mayer, and Thisse (2008). Bernard, Eaton, Jensen, and Kortum (2003) consider a spatial price discrimination model of international trade.

[3] The assumption of an upper bound on firm costs is to insure that at least one firm enters the market. The assumption that no two firms have the same marginal cost of production is for exposition only. Both assumptions could be dispensed with easily.

location $z$ on the circle. A firm $i$ that is located at point $\eta_i$ and sells to a consumer at point $z$ incurs a *delivered marginal cost* of $k_i(\eta_i, z) \equiv c_i + t\|\eta_i - z\|$, where $\|\eta_i - z\|$ is the shortest arc-length separating the firm from the consumer, and $t \in (0, 2v)$ is the cost of transportation.

**Consumers:** The market is populated by a unit mass of consumers who are uniformly distributed along the circumference of the circle. Each consumer is strategic and consumes one unit of a homogeneous good—buying from the lowest price source—if and only if the lowest price at which she can purchase the good, inclusive of transportation costs, is no greater than her reservation value, $v > 0$. With strategic consumers, there is a final, unmodeled stage, the *consumer choice stage*, in which consumers make their purchases.[4]

Although all results hold whether the firm, the consumer, or any combination thereof incurs the cost of shipping, for consistency we assume throughout that firms bear the cost of transportation. Nevertheless, consumers are not restricted from taking advantage of arbitrage opportunities. If one consumer ships a good a distance $d$ to another consumer, then the consumers incur a cost of transportation equal to $td$.

**Equilibrium concept and equilibrium outcome:** Throughout the paper we focus on pure strategy subgame perfect Nash Equilibria that are the limits of equilibria in weakly undominated strategies of approximation games with discrete price grids.[5] We refer to these simply as *equilibria*.

We define an *equilibrium outcome* as a vector $\{K, \mathbf{x}, \boldsymbol{\pi}\}$, where $K \subseteq N$ is the set of firms that enter the market, $\mathbf{x} \in \mathbb{R}^K$ is the vector of market shares of the entrants such that if $|K| \geq 1$ then $\sum_{i \in K} x_i = 1$ and $x_i \geq 0$ for all $i \in K$, and $\boldsymbol{\pi} \in \mathbb{R}^K$ is the vector of variable profits of the entrants. We say that there is a *unique equilibrium outcome* if the set of firms that enter and each firm's market share and profit are the same across all equilibria.

In the following sections we use backward induction to provide the unique equilibrium outcome in the limiting case in which the fixed cost of entry converges to zero, $f \to 0$. The assumption of an arbitrarily small fixed cost is important for obtaining a uniqueness result in the present setup.[6] However, we could obtain a uniqueness result without this assumption,

[4] We discuss the rationale for assuming strategic consumers below.

[5] Although standard in games of Bertrand competition with undifferentiated goods, we discuss the rationale for this equilibrium concept below.

[6] If entry costs are sufficiently large, then there may exist multiple equilibrium sets of entrants. This is a generic issue in oligopoly models with heterogeneous firms (e.g. Cournot). However, given the set of entrants, the equilibrium outcome is unique for an arbitrary fixed cost.

if we assumed instead sequential entry of firms (and a discretely positive fixed cost of entry) in the entry stage.

# 3 Price stage

Fix the integer number of firms in the market at $n \geq 1$, the vector of marginal costs of firms in the market at $\mathbf{c}$, and the location of all such firms $\boldsymbol{\eta}$. If $n = 1$, the monopoly charges each location the maximum price, $v$, at which consumers are willing to purchase the good. In what follows, suppose that $n \geq 2$.

With Bertrand competition, heterogeneous firms, and a continuum of prices there are two standard technical issues. For simplicity, suppose that there are two firms, 1 and 2 with $c_1 < c_2$, and one location. The first technicality is that there is no pure strategy equilibrium in a game in which consumers are not strategic and the tie-breaking rule places positive probability on both firms. To demonstrate this issues, fix any $p_2 \in (c_1, c_2]$. For any $p_1 < p_2$, firm 1 has an incentive to increase its price; and for any $p_1 \geq p_2$, firm 1 has an incentive to decrease its price. To avoid this technicality we assume that consumers are strategic. With strategic consumers, the only subgame perfect equilibria that exist satisfy: $(i)$ $p_1 = p_2 \in [c_1, c_2]$ and $(ii)$ consumers buy from firm 1. For this reason we assume that consumers are strategic.[7] The second technicality is that there exists a continuum of equilibria in which firm 2 sets any price $p_2 \in [c_1, c_2]$ and firm 1 charges $p_1 = p_2$, and because we assume that the price space is continuos, there is no equilibrium in weakly undominated strategies; that is, eliminating weakly dominated strategies in the game with a continuous price space would rule out the classical Bertrand outcome of $p_1 = p_2 = c_2$. We rule out equilibrium prices $p_2 < c_2$ because they are not limits of equilibria in weakly undominated strategies of approximation games with discrete price grids; see, e.g., Deneckere and Kovenock (1996) and page 260 of Mas-Colell, Whinston, and Green (1995).

With strategic consumers and Bertrand competition at each location $z$, the unique equilibrium price at almost every point $z$, denoted $p(z)$, is

$$\begin{aligned} p(z) &= \min_{i \neq \chi(z)} k_i(\eta_i, z) \\ \text{with } \chi(z) &\equiv \arg \min_{j=1,\dots,n} k_j(\eta_j, z). \end{aligned}$$

[7] Alternatively, we could assume an exogenous tie-breaking rule in which consumers always buy from the low cost firm when faced with equal prices.

The firm with the lowest delivered marginal cost sells the good to consumers at point $z$ at a price equal to the second lowest delivered marginal cost. At these prices consumers have no incentive to arbitrage: the upper bound on the price difference charged to two consumers separated by a distance of $d$ is $td$, and this upper bound equals the cost that the consumers would have to incur to transport the good from one to the other.

**Constructing market shares and profits**: In what follows in this section we construct market shares and profits in the special case in which all firms supply a positive mass of consumers. Denote by *boundary consumer* any consumer at the boundary between the sets of consumers supplied by two firms. Suppose that no two firms are located at the same point and that firm $i$ only sells to consumers located between its two closest neighbors, firms $i-1$ and $i+1$, where firm $i-1$ is the closest neighbor in the counterclockwise direction and firm $i+1$ is the closest neighbor in the clockwise direction. If all firms supply a positive mass of consumers, then the boundary consumer between $i$ and $i+1$ is a customer for whom the delivered costs of $i$ and $i+1$ are lower than the delivered costs of any other firms.

Let $d_{i,i+1}$ and $d_{i-1,i}$ denote the distance between firm $i$ and firm $i+1$ and between firm $i-1$ and firm $i$ in the clockwise direction, respectively. Let $x_{i,i+1}$ and $x_{i,i-1}$ denote the distance from firm $i$ in the clockwise direction of the boundary consumer between firm $i$ and firm $i+1$ and in the counterclockwise direction of the boundary consumer between firm $i$ and firm $i-1$, respectively. Firm $i+1$'s delivered marginal cost of supplying the boundary consumer between $i$ and $i+1$ is $c_{i+1}+t\left(d_{i,i+1}-x_{i,i+1}\right)$, which equals firm $i$'s delivered marginal cost of supplying the same consumer, $c_i+tx_{i,i+1}$. Hence,

$$x_{i,i+1} = \frac{1}{2t}\left[c_{i+1}-c_i+td_{i,i+1}\right], \tag{1}$$

$$x_{i,i-1} = \frac{1}{2t}\left[c_{i-1}-c_i+td_{i-1,i}\right]. \tag{2}$$

Letting $x_i \equiv x_{i,i-1}+x_{i,i+1}$ denote firm $i$'s market share, we have

$$x_i = \frac{1}{2t}\left[c_{i+1}+c_{i-1}-2c_i+tD_i\right]$$

where $D_i \equiv d_{i-1,i}+d_{i,i+1}$ denotes firm $i$'s isolation, the distance between its two neighbors.

Normalize firm $i-1$'s location as point zero and define all other points by their distance from $i-1$ in the clockwise direction. Firm $i$'s price at point $z$ is determined by firm $i-1$'s delivered cost if $c_{i-1}+tz \leq c_{i+1}+t\left(D_i-z\right)$. Firm $i$'s price at point $z$ is determined by firm

$i-1$'s delivered cost if and only if $z \leq z_i^*$, where

$$z_i^* \equiv \frac{1}{2t}\left(c_{i+1}-c_{i-1}\right)+\frac{1}{2}D_i.$$

Firm $i$'s price at a given $z$ is

$$p_i\left(z\right)=\begin{cases} c_{i-1}+tz & \text{if } z<z^* \\ c_{i+1}+t\left(D_i-z\right) & \text{if } z>z^* \end{cases}$$

Similarly, express the locations of the boundary consumers in terms of their distance from firm $i-1$ as $X_{i,i-1}$—which denotes the distance in the clockwise direction from firm $i-1$ of the boundary consumer between firm $i$ and firm $i-1$—and $X_{i,i+1}$—which denotes the distance in the clockwise direction from firm $i-1$ of the boundary consumer between firm $i$ and firm $i+1$. We then have

$$X_{i,i-1} = \frac{1}{2}\left[d_{i-1,i}-\frac{1}{t}\left(c_{i-1}-c_i\right)\right] \quad (3)$$

$$X_{i,i+1} = d_{i-1,i}+\frac{1}{2}d_{i,i+1}+\frac{1}{2t}\left(c_{i+1}-c_i\right) \quad (4)$$

Similarly, express firm $i$'s delivered marginal cost in terms of the distance, $z$, from firm $i-1$ as $k_i\left(z\right)$, where

$$k_i\left(z\right)=c_i+t\left\|d_{i-1,i}-z\right\|$$

Firm $i$'s variable profit, $\pi_i$, can be separated into two terms, the profit it earns over the range in which its price is determined by firm $i+1$, and the profit it earns over the range in which its price is determined by firm $i-1$. These ranges are $\left(X_{i,i-1}, z_i^*\right)$ and $\left(z_i^*, X_{i,i+1}\right)$ respectively, so that

$$\pi_i=\int_{X_{i,i-1}}^{z_i^*}\left[c_{i-1}+tz-k_i\left(z\right)\right]dz+\int_{z_i^*}^{X_{i,i+1}}\left[c_{i+1}+t\left(D_i-z\right)-k_i\left(z\right)\right]dz.$$

It can be shown that

$$\pi_i=t\left[\left(X_{i,i-1}\right)^2+\left(X_{i,i+1}\right)^2-\left(d_{i-1,i}\right)^2-\left(z_i^*\right)^2\right] \quad (5)$$

Figure 1 depicts the delivered marginal cost for firm $Y$, the price that firm $Y$ charges each of its customers, and firm $Y$'s market share. Firm $Y$'s profit is then the area under firm $Y$'s price and above its delivered marginal cost, integrated over firm $Y$'s market share.

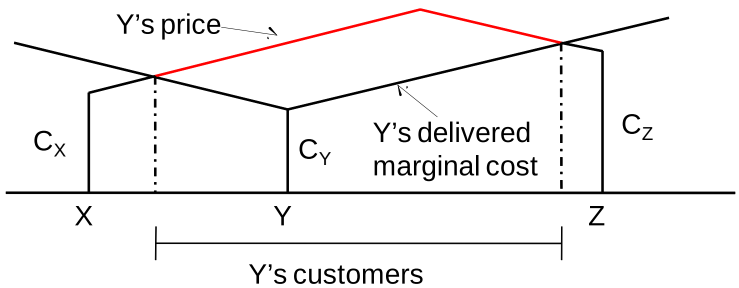


Figure 1: Firm $Y$'s market share and profit given locations

# 4 Location stage

In this Section, we obtain three results. First, we solve for the unique equilibrium outcome in the special case in which all firms that entered are "sufficiently productive," where this is defined below. In this case we show that every firm's market share and variable profit are strictly positive. Second, we show that if at least one firm that entered is not sufficiently productive, then there exists no equilibrium in which all entrants have strictly positive market shares or variable profits. The first and second results—together with the fact that each firm incurs a positive fixed cost of entry—suggest an important entry stage result: in the entry stage, only sufficiently productive firms choose to enter. Hence, the unique equilibrium outcome in the special case in which all firms are sufficiently productive is the relevant case in any equilibrium. Third, we show by construction that an equilibrium exists to an arbitrary location-stage subgame.

Let $K' \subseteq N$ denote the set of firms in the market. If $|K'| = 1$, then the monopoly firm is indifferent between all locations. In what follows, suppose that $|K'| \geq 2$. To crystallize ideas, we first consider a special case of the model in which each firm in the market is sufficiently productive such that it supplies a positive mass of customers in any equilibrium. Let

$$\lambda(K') \equiv \frac{1}{|K'|} + \frac{2}{t}\overline{c}(K'), \tag{6}$$

where $\overline{c}(K') \equiv \frac{1}{|K'|}\sum_{n \in K'} c_n$ is the average marginal cost of firms in the market. As shown below, in any equilibrium $\lambda(K')$ serves as an inverse measure of the toughness of competition in the market. Competition is tougher in a market with more (active) firms, holding fixed the average marginal cost (of active firms). Similarly, competition is tougher in a market with a lower average marginal cost (of active firms), holding fixed the number of (active) firms. The following Lemma provides a sufficient condition under which each firm in the market is productive enough to supply a positive mass of consumers. Under this condition, the following Lemma $(i)$ states that an equilibrium exists and $(ii)$ provides the unique equilibrium outcome.[8]

**Lemma 1** *Consider a location-stage subgame in which the set of the firms in the market is $K$, with $|K| \geq 2$, and each $n \in K$ satisfies*

$$c_n < \frac{t}{2}\lambda(K). \tag{7}$$

*There exists an equilibrium to the location-stage subgame. In any such equilibrium, the distance between any two neighbors $i$ and $i+1$ is given by*

$$d_{i,i+1}(K) = \lambda(K) - \frac{2}{t}\left(\frac{c_i + c_{i+1}}{2}\right), \tag{8}$$

*and, for all $i \in K$, firm $i$'s market share and variable profit are given by*

$$x_i(K) = \lambda(K) - \frac{2}{t}c_i \tag{9}$$

$$\pi_i(K) = \frac{t}{2}x_i(K)^2. \tag{10}$$

The key property of any equilibrium characterized in Lemma 1 is that each firm is centered in its market share; that is, each firm is equidistant from its boundary consumers in the clockwise and counter-clockwise directions. Because the cost of supplying consumers is increasing in distance (and the revenue from supplying consumers is decreasing in distance), each firm locates at the center of the mass of consumers it supplies. Figure 2 depicts firm $Y$'s market share, total cost, and total revenue given a non-equilibrium location for firm $Y$,

[8] All Proofs are relegated to the Appendix.

between firms $X$ and $Z$.

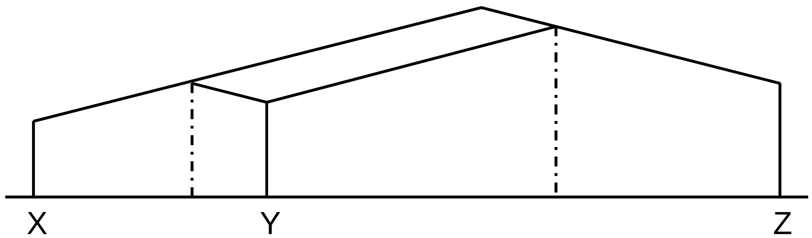


Figure 2: Out of equilibrium location

In Figure 2, firm $Y$ is closer to its boundary consumer on the side it shares with firm $X$ than it is to the boundary consumer on the side it shares with firm $Z$. If firm $Y$ were to move towards firm $Z$, firm $Y$'s market share would remain constant, its total revenue would increase, and its total costs would decrease. Hence, in equilibrium $Y$ must be centered in its market share, as depicted in Figure 3.

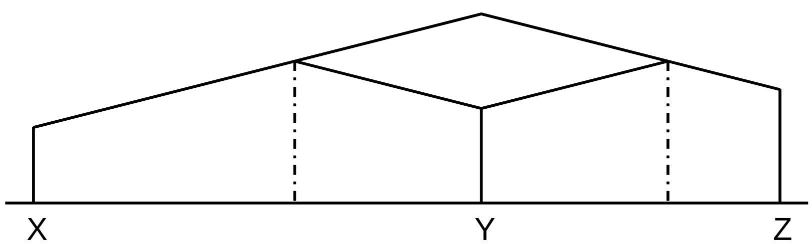


Figure 3: Equilibrium location

This property of any equilibrium, that each firm is centered in its market share, yields two related results. First, firm $i$'s equilibrium market share and profit depend on firm $j \neq i$'s marginal cost only through $j$'s impact on $\lambda$. Second, each firm's market share and profit is constant across all equilibria; see Equations (9) and (10).

**Unique equilibrium market shares and profits**: According to Lemma 1, if each $n \in K$ satisfies Condition (7) then a firm's market share and profit are identical in all equilibria and depend on another producer's marginal cost only through its impact on the average marginal cost, $\bar{c}(K)$.

Because each firm is centered in its market share, the delivered marginal cost of supplying all boundary consumers must be equal to $\frac{t}{2}\lambda(K)$, which directly implies that firm $i$'s market share and profit depend on its competitors' marginal costs only through their impact on $\lambda(K)$. This implies that there is a unique equilibrium outcome.

This result and its economic intuition are similar to those in Proposition 1 of Vogel (2008). Nevertheless, a unique equilibrium outcome arises in Vogel (2008) only after imposing a restriction that firms incur a positive shipping cost that they cannot pass along to consumers. Without this assumption in Vogel (2008), a firm's cost of supplying a set of consumers is independent of its location (with mill pricing, revenue per sale is always fixed). Here, no such assumption is needed. Intuitively, with price discrimination, the identity of the party that pays the cost of transportation is inconsequential. The firm can always pass along this cost to the consumer; but in equilibrium it will not, since its price at each location is pinned down by the costs of its competitors.

**Permissible asymmetries**: Lemma 1 provides an explicit bound—in Condition (7)—on the extent of marginal cost asymmetry under which there exists an equilibrium in which all firms supply a positive mass of consumers. According to Equation (9), if firm $i$ violates Condition (7) and all firms follow their equilibrium strategies, then firm $i$'s market share is zero. To understand the intuition behind Condition (7), suppose that all firms locate as prescribed by Lemma 1 and denote by $g(z) \equiv \min_{j\in K}\{k_j(z)\}$ the minimum delivered marginal cost, taken over all firms, to a consumer located at point $z$. As depicted in Figure 4, we have $g(z) \leq \frac{t}{2}\lambda(K)$ for all $z$, since $\frac{t}{2}\lambda(K)$ is the minimum delivered marginal cost at each boundary consumer and boundary consumers face the highest minimum delivered marginal costs in the market.

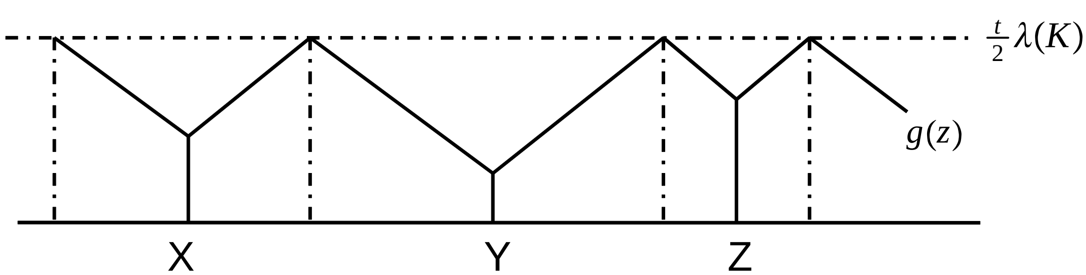


Figure 4: Boundary consumers face the same minimum delivered marginal cost

If firm $i$ violates Condition (7), then its revenue per sale is bounded above by its marginal cost and it is unable to earn positive variable profit. Of course, if all firms have symmetric marginal costs equal to $\overline{c}$, then all firms satisfy Condition (7) for any $|K|$.

Although this explicit bound on the extent of permissible marginal cost asymmetry represents an improvement with respect to Vogel (2008), in which no such explicit bound is derived, a goal of this paper is to provide a unique equilibrium outcome for an *arbitrary* distribution of marginal costs. It then remains to consider the case in which at least one firm violates Condition (7). The following Lemma takes a first step in this direction.

**Lemma 2** *Consider a location-stage subgame in which the set of the firms in the market is $K$, with $|K| \geq 2$. If at least one firm $n \in K$ violates Condition* (7)*, then there exists no equilibrium in which all firms have strictly positive market shares.*

The intuition for Lemma 2 is straightforward. Suppose that at least one firm $n \in K$ violates Condition (7) and—contrary to Lemma 2—that there exists an equilibrium in which all firms have strictly positive market shares. In such an equilibrium, each firm must be centered in its market share, as discussed above. If each firm is centered in its market share and all firms sell to a positive mass of consumers, then each firm's market share must be given by Equation (9). However, if any firm $n$ violates Condition (7), then Equation (9) predicts that firm $n$'s market share is not positive, a contradiction that proves the lemma.

According to Lemma 2, if there is sufficient heterogeneity in unit costs, then at least one firm earns no variable profit. Looking forward, Lemma 2 ensures that if the fixed cost of entry is positive and if an equilibrium exists, then all firms that enter in the first stage must satisfy Condition (7) relative to the set of entrants. If any firm enters that does not satisfy this condition, then at least one firm's total profit is negative, which cannot happen along the equilibrium path. Since, if an equilibrium exists, all entrants must satisfy Condition (7), then on the equilibrium path the location-stage equilibrium is as prescribed in Lemma 1.

It remains to provide an existence result for a location-stage subgame in which at least one firm violates Condition (7) relative to the set of entrants. This is carried out in the following lemma.

**Lemma 3** *There exists an equilibrium to an arbitrary location-stage subgame.*

$>$From Lemma 1 we know that an equilibrium exists in the special case in which all firms satisfy Condition (7) relative to the full set of entrants. To gain intuition in the general case, consider a location-stage subgame in which the non-empty set of firms in the market is $K' \subseteq N$ . The first step of the proof is to show that for any such set $K'$, there exists a unique subset $K^*(K')$, defined by $K^*(K') \equiv \left\{ i \in K' \middle| c_i < \frac{t}{2}\lambda \left[K^*(K')\right] \right\}$, that satisfies two properties. First, if only firms in $K^*(K')$ were in the market, then all firms in $K^*(K')$ would satisfy Condition (7) relative to $K^*(K')$. That is, in any equilibrium in which the set of entrants is $K^*(K')$, each firm would have a positive market share. Second, there exists no firm $i$ in $K'$ but not in $K^*(K')$ that would satisfy Condition (7) relative to $K^*(K') \cup i$. That is, in any equilibrium in which the set of entrants were $K^*(K') \cup i$, for any $i \in K' \backslash K^*(K')$, at least one firm would not have a strictly positive market share. Finally, note that this subset $K^*(K')$ contains all the firms in $K'$ with a marginal cost below a certain cutoff, where this cutoff depends on the set $K'$.

Given the existence of this unique and non-empty set, the intuition for the proof is constructive. If $|K^*(K')| \geq 2$, we construct an equilibrium in which equilibrium locations of all firms in $K^*(K')$ are those that are prescribed by Lemma 1 if only firms in $K^*(K')$ were in the market. In constructing the locations of the firms $K^*(K')$, it is as if the firms in $K' \backslash K^*(K')$ had not entered the market. The remaining firms in $K'$ that are not in $K^*(K')$, if any, are all located at the same point as a boundary consumer. At these locations, the delivered marginal cost at any location $z$ of any firm $i \in K' \backslash K^*(K')$ is weakly greater than is the delivered marginal cost at that location of at least two firms in $K^*(K')$. Hence, no firm $i \in K' \backslash K^*(K')$ affects the price charged at any point in the market. That is, given these locations, it is as if the firms in $K' \backslash K^*(K')$ had not entered the market. Hence, no firm in $K^*(K')$ has an incentive to deviate. Moreover, given the locations of the firms in $K^*(K')$, no firm in $K' \backslash K^*(K')$ is sufficiently productive to earn positive variable profits from any location. Hence, no firm in $K' \backslash K^*(K')$ has an incentive to deviate. Thus, we have constructed an equilibrium if $|K^*(K')| \geq 2$. Using similar logic we show that if $|K^*(K')| = 1$, then there exists an equilibrium in which all firms locate together and firm $i \in K^*(K')$ supplies the entire market.

According to Lemma 3, an equilibrium exists in any location-stage subgame. Nevertheless, Lemma 3 does not state that there is a unique equilibrium outcome in an arbitrary subgame. To obtain a unique equilibrium outcome, we will include an entry stage with a positive, but vanishingly small fixed cost in the following section.

# 5 Entry stage

In the entry stage, each firm $i \in N$ chooses whether or not to enter. If a firm chooses to enter, it incurs a fixed cost $f > 0$ and proceeds to the location stage. Clearly, a firm chooses to enter if and only if it anticipates earning non-negative profit. Recall the discussion following Lemma 2: if the fixed cost of entry is positive and if an equilibrium exists, then all firms that enter in the first stage must satisfy Condition (7) relative to the set of entrants. The role of the fixed cost is to ensure that if the set of entrants is $K$, then all firms in $K$ satisfy Condition (7) relative to $K$. Hence, if an equilibrium exists in which at least two firms enter, then *given the set of entrants*, the unique equilibrium outcome is as prescribed by Lemma 1.

However, we still do not know which firms enter and whether the set of entrants is unique. In order to ensure that there is a unique set of entrants in equilibrium, in the following proposition we assume that the fixed cost of entry is sufficiently small. With a sufficiently small fixed cost, and using Lemmas 1-3, we obtain the central result of the paper in the following proposition.

**Proposition 1** *There exists a fixed cost* $f^* > 0$ *such that for all* $f < f^*$*:*
***1.*** *an equilibrium exists;*
***2.*** *the equilibrium outcome is unique; and*
***3.*** *if* $|K^*(N)| > 1$*, then firm i's market share and variable profit are given by Equations* (9) *and* (10) *in which* $K \equiv K^*(N)$*; and if* $|K^*(N)| = 1$ *then firm* $i \in K^*(N)$ *sets price* $v$ *at all locations and* $x_i = 1$.

**Intuition**: According to Proposition 1, if the fixed cost of entry is sufficiently low, then the unique set of entrants is $K^*(N)$ and there is a unique equilibrium outcome. Given the set of entrants, the unique equilibrium outcome is not surprising. But why is there a unique set of entrants? To understand this result, it is instructive to consider two potential equilibria ($A$ and $B$) in an environment in which there are three potential entrants, $N = \{1, 2, 3\}$, with $c_1 < c_2 < c_3$ and with $K^*(N) = \{1, 2\}$. Potential equilibria $A$ and $B$ differ in terms of the firms that enter along the proposed equilibria paths: along the equilibrium path in both potential equilibria, given the entrants the equilibrium in the location-stage subgame is as prescribed by Lemma 1.

In potential equilibrium $A$, firms 1 and 2 enter while firm 3 does not enter. In this potential equilibrium, suppose that firm 2's profits are strictly positive while firm 3's profits would be strictly negative if it entered. In potential equilibrium $B$, firms 1 and 3 enter while firm 2 does not enter. In this potential equilibrium, can it be the case that firm 3's profits are strictly positive while firm 2's profits would be strictly negative if it entered? For a sufficiently large fixed cost, the answer to the previous question can be "yes" (not only in the present model, but also in a basic model of Cournot competition). If the costs of firms 2 and 3 are sufficiently similar, then this is intuitive. Hence, for a sufficiently large fixed cost, the set of entrants is not necessarily unique.[9]

However, now consider the case in which the fixed cost of entry is sufficiently close to zero. In this case, potential equilibrium $A$ is an equilibrium. For a sufficiently small fixed cost of entry, firm 2's profits are positive (and therefore so are firm 1's) if only firms 1 and 2 enter. To see that firm 3 would have no incentive to deviate and enter, suppose that firms 1, 2, and 3 all enter. Because firm 3 violates Condition (7) relative to the set of entrants $N$, there is no equilibrium in which all firms have a strictly positive market share. Moreover, firm 3's market share must equal zero if it enters: if its market share were positive, then one of the more productive firms (e.g. firm 2) must have a market share of zero. But if firm 2's market share were zero, then it could deviate in the location stage by locating at the same point as firm 3; by doing so, firm 2 would obtain a positive market share and positive variable profits while firm 3 would have zero market share and zero variable profit. Hence, firm 3 would have no incentive to enter, so that potential equilibrium $A$ is an equilibrium.

On the other hand, for a sufficiently small fixed cost of entry, potential equilibrium $B$ (in which only firms 1 and 3 enter) is not an equilibrium because firm 2 would have an incentive to enter. If firm 2 does not enter, its profit is zero. However, in any equilibrium in which firm 2 does not have a positive market share (whether or not it enters), at least one boundary consumer faces a price $p \geq \frac{t}{2}\lambda\left[K^*\left(N\right)\right]$, which is strictly above firm 2's marginal cost. Hence, if firm 2 were to enter, it could locate at the same point as one such boundary consumer and obtain a strictly positive market share (and variable profit). For a sufficiently small fixed cost of entry, firm 2 would earn strictly positive profits by entering, so that potential equilibrium $B$ is not an equilibrium.

According to Proposition 1, an equilibrium exists in which at least one firm enters, the same set of firms enter in all equilibria, and each firm's market share and profit are the same across all equilibria. Moreover, this result holds for any distribution of marginal costs satisfying $c_i \in [0, v - t/2)$, where the restriction that $c_i < v - t/2$ ensures that almost all consumers are supplied in all equilibria.

[9]Even in this case, market shares and profits are uniquely determined by Lemma 1, given the entrants.

# 6 Discussion

The result in the literature most closely related to Proposition 1 is Proposition 1 in Vogel (2008). As in Vogel (2008), in this paper there is a unique equilibrium outcome according to which more productive firms are more isolated—all else equal—, supply more consumers, and earn a higher profit. Unlike Vogel (2008), we obtain these results (*i*) for an arbitrary distribution of marginal costs, (*ii*) without imposing a restriction that firms incur a positive shipping cost that they cannot pass along to consumers, and (*iii*) while including an entry stage in which less productive firms do not enter. As discussed in the Introduction, each of these generalizations is potentially important for linking the theory to the data. In this section we focus on the impact of the cost of transportation, the fixed cost, market toughness, and a firm's marginal cost on its isolation, market share, and profit.

**Isolation**: The distance between two neighbors, firm $i$ and firm $i+1$, is greater than the average distance between firms, $1/|K|$, if and only if their average marginal cost is less than the average marginal cost of all active firms in the market. Moreover, holding the average marginal cost of active firms constant, the distance between neighbors is a strictly decreasing function of their average marginal cost. Intuitively, high-cost active firms shy away from the harsh competition of low-cost firms.

Neighbors are more isolated if there are fewer active firms in the market or if the average marginal cost of active firms in the market is greater. The impact of the number of active firms on isolation is straightforward and deserves no special mention since it is obtained in models with symmetric firms; see e.g. Salop (1979), Economides (1989), and Lancaster (1979). The impact of the average marginal cost of active firms on isolation is only obtained elsewhere, to the best of our knowledge, in Vogel (2008). If firm $j$'s marginal cost increases and the number of firms in the market remains constant, then firm $j$ must become less isolated. This requires that the distance between firms $i$ and $i+1$ increases, for any $j \neq i, i+1$.

The impact of the transportation cost, $t$, on isolation is more complex in the current model than in Vogel (2008). As in Vogel (2008), for a fixed set of active firms, a decrease in $t$ increases the benefit of a lower marginal cost in terms of isolation because consumers are relatively more sensitive to differences in marginal costs than differences in distances. In addition, in the current model a reduction in $t$ also restricts entry, because profits are increasing in $t$ for fixed locations. This provides an additional benefit of a lower marginal cost in terms of isolation. The fixed cost only affects isolation by restricting entry because it is sunk at the point at which firms choose their locations and prices.

**Market share and profit**: More productive firms have larger market shares and earn higher profits. In particular, a firm's market share and profit are greater than average if and only if its marginal cost is less than average, and a firm's market share and profit are decreasing in its marginal cost. The reason more productive firms supply a larger mass of consumers derives entirely from the fact that these firms are more isolated, as all firms charge the same average FOB price of $\lambda t/2$. In particular, firm $i$ charges a minimum FOB price of $\frac{t}{2}\left[\lambda\left(K\right)-x_i\left(K\right)\right]$, which equals $c_i$, to its boundary consumers and charges a maximum FOB price of $\frac{t}{2}\left[\lambda\left(K\right)+x_i\left(K\right)\right]$ to those consumers jointly located with the firm.

In the present paper, more productive firms earn higher profits both because ($i$) they supply a larger mass of consumers and ($ii$) they charge higher absolute markups, on average. In the model, firm $i$'s average absolute (FOB) markup is $\lambda t/2 - c_i$, which is decreasing in its marginal cost. Intuitively, productive firms set higher average absolute markups because their greater isolation provides increased monopoly power. In fact, all firms charge identical absolute (FOB) markups to consumers located $z$ units away from their boundary customers. However, a more productive firm supplies customers located farther from its boundary customer.

The number of active firms and their average productivity affect a firm's market share and profit in the expected directions. A reduction in the cost of transportation reduces a low productivity firm's profit and market share and has an ambiguous effect on a high productivity firm's profit and market share. The direct effect of a reduction in $t$ reduces each firm's profit. However, as noted above, reducing $t$ both restricts entry—which increases isolation for all remaining active firms—and increases the relative return to higher productivity. An increase in the fixed cost $f$ reduces the profit of a low productivity firm and has an ambiguous effect on a high productivity firm. The direct effect of an increase in $f$ is to lower each active firm's profit. However, increasing $f$ restricts entry, which increases variable profit for all firms that remain active. Clearly the direct effect wins out for the high cost firms that exit.

# 7 Conclusion

In this paper we presented and solved a three-stage game of entry, location, and pricing in a spatial price discrimination framework with arbitrarily many heterogeneous firms. In contrast to the spatial competition literature of which we are aware, we did not impose restrictions on the distribution of marginal costs across firms or the allocation of transportation costs between firms and consumers. Our main empirical prediction is that more productive firms are more isolated, all else equal.

Our analysis is limited in (at least) three important respects. We have assumed that consumers are uniformly distributed through space, that space is one dimensional, and that the game is static. These are strong and unrealistic assumptions that we made for tractability. Nonetheless, we hope that the paper provides useful insight into the determinants of firm isolation.

# A Proofs

**Proof of Lemma 1.** The proof proceeds in 3 Steps.

**Step 1:** *Consider a location-stage subgame in which the set of firms in the market is $K$, with $|K| \geq 2$. If there exists an equilibrium to this subgame in which all firms supply a positive mass of consumers, then the distance between any two neighbors $i$ and $i+1$ is given by Equation* (8) *and firm $i$'s market share and variable profit are given by Equations* (9) *and* (10)*.*

**Proof:** Suppose there exists an equilibrium to the subgame beginning in the location stage in which all firms supply a positive mass of consumers. Fix the location of all firms and consider the effect of firm $i$'s unilateral $\varepsilon$-deviation towards firm $i+1$ (if $\varepsilon > 0$) or towards firm $i-1$ (if $\varepsilon < 0$). From Equations (3), (4), and (5), firm $i$'s first-order condition for a maximum—conditional on all firms supplying a positive mass of consumers—is given by

$$d_{i,i+1}(K) = d_{i-1,i}(K) + \frac{1}{t}(c_{i-1} - c_{i+1}). \tag{11}$$

Such a location locally maximizes firm $i$'s profits as the second-order condition is satisfied. If an equilibrium exists in which all firms supply a positive mass of consumers, then given an order of firms around the circle ($i$) each firm's location must satisfy Equation (11) and ($ii$) the sum of distances between all pairs of firms must sum to 1, i.e.

$$d_{n,1}(K) + \Sigma_{i=1}^{n-1} d_{i,i+1}(K) = 1. \tag{12}$$

Solving Equation (11) recursively yields

$$d_{i+j,i+j+1}(K) = d_{i-1,i}(K) + \frac{1}{t}(c_{i-1} + c_i - c_{i+j} - c_{i+j+1}).$$

The distance between two arbitrary neighbors as a function of the distance between firms 1 and $n$, where firm 1 is firm $n$'s clockwise neighbor, is

$$d_{j,j+1}(K) = d_{n,1}(K) + \frac{1}{t}(c_n + c_1 - c_j - c_{j+1}). \tag{13}$$

Substituting Equation (13) into Equation (12) provides the solution for the distance between firm 1 and firm $n$

$$d_{n,1}(K) = \lambda(K) - \frac{2}{t}\left(\frac{c_n + c_1}{2}\right),$$

Substituting the solution for $d_{n,1}(K)$ into Equation (13) yields Equation (8). Given Equation (8), it is straightforward to show that market shares and variable profits are given by Equations (9) and (10).

**Step 2:** *In any location-stage subgame in which all $n \in K$ satisfy Condition* (7)*, each $i \in K$ supplies a positive mass of consumers in any equilibrium.*

**Proof:** To obtain a contradiction, suppose that there is an equilibrium to this subgame in which a firm $i \in K$ does not supply a positive mass of consumers. The entire market must be supplied by at most the remaining $|K| - 1$ firms. Moreover, $c_j < \frac{t}{2}\lambda(K)$ is equivalent to

$$c_j < \frac{t}{2}\frac{1}{|K|} + \frac{|K|-1}{|K|}\bar{c}(K\backslash j) + \frac{1}{|K|}c_j$$

so that $c_j < \frac{t}{2}\lambda(K) \Leftrightarrow c_j < \frac{t}{2}\lambda(K\backslash j)$. Hence, the distance between at least one firm $j \in K$, $j \neq i$ and at least one of its boundary consumers is strictly greater than $\frac{1}{2}x_j(K)$, given in Equation (9). Thus, the delivered marginal cost to this boundary consumer is strictly greater than $\frac{t}{2}\lambda(K)$. Because $c_i$ satisfies Condition (7), firm $i$ could locate at the point at which this boundary consumer is located and supply a positive mass of consumers while earning a strictly positive variable profit, a contradiction.

**Step 3:** *There exists an equilibrium to any location-stage subgame in which each $n \in K$ satisfies Condition* (7)*.*

**Proof:** Suppose that all firms $n \in K\backslash i$ locate as prescribed by Lemma 1. Let $g_i(z) \equiv \min_{j\neq i} k_j(z)$ denote the minimum delivered marginal cost, taken over all firms but firm $i$, to a consumer located at point $z$. Then $g_i(z)$ is continuous and $\int_{z\in\vartheta} g(z)\,dz$ denotes firm $i$'s revenue from selling to a set $\vartheta$ of consumers. Let $\vartheta_i^*$ denote the set of consumers to whom firm $i$ sells if firm $i$ does not deviate from the location prescribed by Lemma 1. The lowest cost location from which to supply all $z \in \vartheta_i^*$ is the location prescribed by Lemma 1. Step 3 then follows directly from the fact that $g(z) > g(z')$ for almost all $z \in \vartheta_i^*$ and $z' \notin \vartheta_i^*$.

Lemma 1 follows directly from Steps 1-3. **QED.** ■

**Proof of Lemma 2.** To obtain a contradiction, suppose ($i$) that the set of the firms in the market is $K$, with $|K| \geq 2$; ($ii$) that at least one firm $n \in K$ violates Condition (7);and ($iii$) that there exists an equilibrium in which all firms have strictly positive market shares. According to Step 1 in the proof of Lemma 1, in any such equilibrium each firm $i$'s market share must be given by $\lambda(K) - \frac{2}{t}c_i$. Because firm $n$ violates Condition (7), we have $\frac{2}{t}c_n \geq \lambda(K)$. Hence, firm $n$'s market share is bounded above by zero, a contradiction. **QED.** ■

**Proof of Lemma 3.** Suppose that the non-empty set of entrants is $K'$. The proof requires four steps.

**Step 1:** *If $i \in K^*(K')$ and $c_j < c_i$, then $j \in K^*(K')$.*

**Proof:** To obtain a contradiction, suppose that $i \in K^*(K')$, $c_j < c_i$, and $j \notin K^*(K')$. If

$j \notin K^*(K')$, then $c_j \geq \frac{t}{2}\lambda[K^*(K') \cup j]$, which is equivalent to $c_j \geq \frac{t}{2}\lambda[K^*(K')]$. However, $i \in K^*(K')$ implies $c_i < \frac{t}{2}\lambda[K^*(K')]$. Hence, $c_j > c_i$, a contradiction.

**Step 2:** *There exists a unique non-empty* $K^*(K') \equiv \left\{i \in K' | \, c_i < \frac{t}{2}\lambda[K^*(K')]\right\}$.

**Proof:** To obtain a contradiction, suppose that there exists a $K_1 \equiv \left\{i \in K' | \, c_i < \frac{t}{2}\lambda(K_1)\right\}$ and a $K_2 \equiv \left\{i \in K' | \, c_i < \frac{t}{2}\lambda(K_2)\right\}$ with $K_1 \neq K_2$. According to Step 2, we have either $K_1 \subset K_2$ or $K_2 \subset K_1$. Suppose that $K_1 \subset K_2$. We have

$$\lambda(K_2 \backslash V) \geq \lambda(K_2) > \lambda(K_1) \text{ for any } V \subseteq K_2 \backslash K_1 \tag{14}$$

The first inequality in Equation (14) follows from $c_j < \frac{t}{2}\lambda(K) \Leftrightarrow c_j < \frac{t}{2}\lambda(K \backslash j)$. The second inequality in Equation (14) follows from the fact that $K_1 \subset K_2$ requires the existence of at least one firm $j$ such that $c_j \in [\lambda(K_1), \lambda(K_2))$, which implies $\lambda(K_1) < \lambda(K_2)$. Because Equation (14) holds for any $V \subseteq K_2 \backslash K_1$, it must hold for $V = K_2 \backslash K_1$. However, when $V = K_2 \backslash K_1$ Equation (14) requires $\lambda(K_1) > \lambda(K_1)$, a contradiction. Hence, there exists a unique $K^*(K') \equiv \left\{i \in K' | \, c_i < \frac{t}{2}\lambda[K^*(K')]\right\}$. Moreover, $K^*(K')$ is non-empty as $\frac{t}{2}\lambda(i) \equiv \frac{t}{2} + c_i > c_i$ for any firm $i$.

**Step 3:** Consider the case in which $|K^*(K')| \geq 2$. First consider an arbitrary $j \in K' \backslash K^*(K')$. Given the locations of all firms $i \in K^*(K')$, the maximum delivered marginal cost, taken over all $i \in K^*(K')$, at any point in the market is $\frac{t}{2}\lambda[K^*(K')] \leq c_j$. Hence, firm $j$ has no incentive to deviate as it cannot earn positive variable profits from any location. Second, consider an arbitrary firm $i \in K^*(K')$. Given the locations of all $j \in K' \backslash K^*(K')$ and the fact that $c_j \geq \frac{t}{2}\lambda[K^*(K')]$, each firm $j \in K' \backslash K^*(K')$ does not impact the potential variable profits of firm $i$ at any location that firm $i$ chooses. Then according to Lemma 1, firm $i$ has no incentive to deviate. This concludes the proof of Lemma 3 in the case in which $|K^*(K')| \geq 2$.

**Step 4:** Consider the case in which $|K^*(K')| = 1$. No firm $j \in K' \backslash K^*(K')$ can make positive variable profits locating anywhere in the market, so these firms have no incentive to deviate. Given that all firms $j \in K' \backslash K^*(K')$ locate together, firm $i \in K^*(K')$ earns the same variable profit no matter where it chooses to locate, so it too has no incentive to unilaterally deviate. This concludes the proof of Lemma 3 in the case in which $|K^*(K')| = 1$.

Combining Steps 3 and 4 directly yields the desired result. **QED.** ■

**Proof of Proposition 1.** We first prove Part 1 of the proposition, that an equilibrium exists. Let $f_0 \equiv \min_{j \in K^*(N)} \pi_j[K^*(N)]$, where $f_0 > 0$ follows from Lemma 1 and the definition of $K^*(N)$. Suppose that $0 < f \leq f_0$ and consider a potential equilibrium in which the set of entrants is $K^*(N)$ and the equilibrium to the subgame beginning in the location stage is as prescribed by Lemma 1. If firms follow the strategies prescribed by this potential equilibrium, then no firm $i \in K^*(N)$ has an incentive to deviate in the location or price stages, according to Lemma 1. Moreover, $\pi_i[K^*(N)] \geq f_0$ for all $i \in K^*(N)$. Hence, no firm $i \in K^*(N)$ has an incentive to deviate in any stage for any $f < f_0$. Now suppose that the set of entrants is $K^*(N) \cup j$ for some $j \in N \backslash K^*(N)$. In this case, an equilibrium exists to the location-stage subgame, according to Lemma 3, and at least one firm has a market share of zero, according to Lemma 2. To obtain a contradiction, suppose that firm $j$'s market share is positive. If $j$'s market share were positive, then one of the more productive firms (e.g. firm $i$) must have a market share of zero. But if firm $i$'s market share were zero in equilibrium, then it could deviate in the location stage by locating at the same point as firm $j$; by doing so, firm $i$ would obtain a positive market share and positive variable profits, a contradiction. Thus, firm $j$'s market share must equal zero and its profit must be negative if deviates from the strategy prescribed by the potential equilibrium. Hence, the potential equilibrium is an actual equilibrium if $0 < f < f_0$, concluding the proof of Part 1.

We now prove Part 2 of the proposition. To obtain a contradiction, suppose that the set of entrants is $K' \neq K^*(N)$. There are three cases to consider: $(i)$ $K' \subset K^*(N)$, $(ii)$ $K^*(N) \subset K'$, and $(iii)$ there exist $i \in K^*(N) \cap (N \backslash K')$ and $j \in (N \backslash K^*(N)) \cap K'$.[10] First, consider case $(i)$: $K' \subset K^*(N)$. Note that firm $i$ satisfies Condition (7) relative to $K$ if and only if firm $i$ satisfies Condition (7) relative to $K \backslash i$ for any $|K| > 1$:

$$c_i < \frac{t}{2}\lambda(K) \Leftrightarrow c_i < \frac{t}{2}\frac{1}{|K|} + \frac{|K|-1}{|K|}\bar{c}(K \backslash i) + \frac{1}{|K|}c_i \Leftrightarrow c_i < \frac{t}{2}\lambda(K \backslash i). \tag{15}$$

Using Condition (15), we can show that if $K' \subset K^*(N)$, then there exists at least one firm $i \in K^*(N) \backslash K'$ that could enter and earn strictly positive variable profit. Therefore, if $f$ is sufficiently close to zero, then this firm $i$ would have an incentive to deviate and enter, a contradiction. Hence, for $f > 0$ sufficiently close to zero, case $(i)$ cannot be an equilibrium. Second, consider case $(ii)$: $K^*(N) \subset K'$. Using the same argument as in the proof of Part 1 of the proposition, there is at least one firm $i \in K' \backslash K^*(N)$ that has a market share of zero and a negative profit, a contradiction. Hence, case $(ii)$ cannot be an equilibrium. Finally, consider case $(iii)$: there exist $i \in K^*(N) \cap (N \backslash K')$ and $j \in \{N \backslash K^*(N)\} \cap K'$. To obtain a contradiction, suppose that firm $i \in K^*(N) \cap (N \backslash K')$ enters but does not obtain a strictly positive market share in the resulting subgame equilibrium. In this case, each boundary consumer must face a price $p < \frac{t}{2}\lambda[K^*(N)]$; otherwise firm $i$ could locate on a boundary consumer facing a price $p \geq \frac{t}{2}\lambda[K^*(N)]$ and would obtain a strictly positive market share. If each boundary consumer faces a price $p < \frac{t}{2}\lambda[K^*(N)]$, then no firm $j \in \{N \backslash K^*(N)\} \cap K'$ can obtain a strictly positive market share. This implies that the market is being served by the set of firms $K^*(N) \cap K'$. By the same argument in case $(i)$, firm $i$ can choose a location at which it serves a strictly positive market share, a contradiction. Hence, for $f > 0$ sufficiently close to zero, case $(iii)$ cannot be an equilibrium, concluding the proof of Part 2.

[10] Note that case $(iii)$—the most complex case—is the case discussed following Proposition 1 in the text.

Finally, Part 3 follows directly from Parts 1 and 2. According to Part 2, for an $f > 0$ sufficiently close to zero, there exists no equilibrium in which the set of entrants is $K' \neq K^*(N)$. According to Part 1, for an $f > 0$ sufficiently close to zero, there exists an equilibrium in which the set of entrants is $K^*(N)$. Given that the set of entrants is $K^*(N)$ in any equilibrium with $f > 0$ sufficiently close to zero, in any such equilibrium market shares and variable profits are as in Equations (9) and (10). ■

# References

Aghion, P., and M. Schankerman (2004): "On the Welfare Effects and Political Economy of Competition-Enhancing Policies," *Economic Journal*, 114, 804–834.

Alderighi, M., and C. A. Piga (2008): "The Circular City with Heterogeneous Firms," *Loughborough University: Department of Economics WP 2008-03*.

Anderson, S. P., and D. J. Neven (1991): "Cournot Competition Yields Spatial Agglomeration," *International Economic Review*, 32, 793–808.

Bartelsman, E. J., and M. Doms (2000): "Understanding Productivity: Lessons from Longitudinal Microdata," *Journal of Economic Literature*, 38, 569–95.

Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003): "Plants and Productivity in International Trade," *American Economic Review*, 93, 1268–1290.

Combes, P.-P., T. Mayer, and J.-F. Thisse (2008): *Economic Geography: The Integration of Regions and Nations*. Princeton University Press, Princeton.

d'Aspremont, C., J. J. Gabszewicz, and J.-F. Thisse (1979): "On Hotelling's "Stability in Competition"," *Econometrica*, 47(5), 1145–50.

Deneckere, R. J., and D. Kovenock (1996): "Bertrand-Edgeworth Duopoly with Unit Cost Asymmetry," *Economic Theory*, 8(1), 1–25.

Economides, N. (1989): "Symmetric Equilibrium Existence and Optimality in Differentiated Product Markets," *Journal of Economic Theory*, 47(1), 178–94.

Hamilton, J. H., W. B. MacLeod, and J.-F. Thisse (1991): "Spatial Competition and the Core," *The Quarterly Journal of Economics*, 106, 925–937.

Hamilton, J. H., J.-F. Thisse, and A. Weskamp (1989): "Spatial Discrimination: Bertrand vs. Cournot in a Model of Location Choice," *Regional Science and Urban Economics*, 19, 87–102.

Hoover, E. (1937): "Spatial price discrimination," *Review of Economic Studies*, 4, 182–191.

Hotelling, H. (1929): "Stability in Competition," *The Economic Journal*, 39(153), 41–57.

Lancaster, K. J. (1979): *Variety, Equity, and Efficiency: Product Variety in an Industrial Society*. Columbia University Press, New York.

Lederer, P. J., and A. P. Hurter (1986): "Competition of Firms: Discriminatory Pricing and Location," *Econometrica*, 54(3), 623–40.

MacLeod, W. B., G. Norman, and J.-F. Thisse (1992): "Price discrimination and equilibrium in monopolistic competition," *International Journal of Industrial Organization*, 6(4), 429–446.

Mas-Colell, A., M. D. Whinston, and J. R. Green (1995): *Microeconomic Theory*. Oxford University Press, New York, New York.

Salop, S. C. (1979): "Monopolistic Competition with Outside Goods," *Bell Journal of Economics*, 10(1), 141–56.

Syverson, C. (2004): "Market Structure and Productivity: A Concrete Example," *Journal of Political Economy*, 112(6), 1181–1222.

Vogel, J. (2008): "Spatial Competition with Heterogeneous Firms," *Journal of Political Economy*, 116(3), 423–466.