NBER WORKING PAPER SERIES

EFFICIENT RECAPITALIZATION

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Working Paper 14929
http://www.nber.org/papers/w14929

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2009

We thank seminar participants at Harvard University, the NBER Monetary Economics Program Meeting, New York University, Princeton University, and our discussant Luigi Zingales for helpful comments and suggestions. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 14929
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JEL No. G01,G2,G28,G33,G38,H0,H2,H81

ABSTRACT

We analyze public interventions to alleviate debt overhang among private firms when the government has limited information and limited resources. We first compare the efficiency of buying equity, buying risky assets, and providing debt guarantees. With compulsory participation, all the interventions are equivalent. With endogenous participation, buying equity dominates the two other interventions. We extend our results to deposit insurance, debt covenants, and heterogeneity across assets. Finally, we propose a constrained-efficient mechanism where the government makes a subordinated loan in exchange for call options on equity.

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In today's financial environment, debt overhang problems can be seen everywhere.
Allen, Bhattacharya, Rajan, and Schoar (2008)

It is well understood since the seminal work of Myers (1977) that debt overhang can lead to under-investment. Companies facing a significant probability of financial distress find it difficult to raise capital to finance promising new investments because these new investments would mostly serve to increase the value of the existing senior debt.

If debt overhang is the problem, the standard prescription is to reorganize the capital structure. Frictionless bargaining between shareholders and debt holders would restore efficiency. This Coasian benchmark is unlikely to prevail, however, because of free-riding among multiple creditors (Asquith, Gertner, and Scharfstein (1994)), asymmetric information, or contract incompleteness (Bhattacharya and Faure-Grimaud (2001)). From a practical perspective, it is clear that reorganizing the capital structure of a large company is difficult, time consuming, and costly. From a theoretical perspective, these renegotiation costs should be expected. Indeed, it is the fact that debt is difficult to renegotiate that makes it useful for disciplining managers, as in Hart and Moore (1995). And if the problem is one of risk shifting instead of debt overhang, as in Jensen and Meckling (1976), then strong debt covenants are needed.

The financial crisis of 2007-2009 has underlined the importance of the debt overhang phenomenon. Banks have significantly reduced their lending since the start of the crisis. Ivashina and Scharfstein (2008) show that new lending was 68% lower in the three-month period around the Lehman bankruptcy relative to the three-month period before the Lehman bankruptcy. Using cross-sectional variation in bank access to deposit financing, the authors show that the reduction in lending reflects a reduction in credit supply by banks rather than a reduction in credit demand by borrowers. There is agreement among many observers that debt overhang is a major reason for the decline in lending (Zingales (2008b) and Fama (2009), among others).

The crisis has also shown the difficulty of finding effective solutions to the debt overhang problem. Several experts have expressed concerns that existing bankruptcy procedures are insufficient for dealing with complex bank failures. As an alternative, Zingales (2008a) argues for a law change that allows for forced debt-for-equity swaps. Coates and Scharfstein

(2009) suggest to restructure bank holding companies instead of bank subsidiaries. Ayotte and Skeel (2009) argue that Chapter 11 proceedings are adequate if managed properly by the government. Assuming that restructuring generates little deadweight loss, these approaches can reduce debt overhang at low cost to the government. However, Swagel (2009) argues that the government lacks the legal authority to force restructuring and that changing bankruptcy procedures in the midst of a financial crisis is politically infeasible.

Moreover, concerns for systemic risk and contagion make it difficult to restructure financial balance sheets in the midst of a crisis. Aside from the costs of its own failure, the bankruptcy of a large financial institution may trigger further bankruptcies because of counterparty risks and runs by creditors. A risk-averse government may decide to avoid restructuring because there is a positive probability of a breakdown of the entire financial system. Even if the government decides to let some institutions fail, there remains the question of how to deal with the debt overhang problem among the remaining financial institutions. This question becomes even more pressing because the remaining financial institutions may experience an outflow of capital due to an increased threat of future restructuring.

In this paper, we study how to alleviate debt overhang if the government chooses to avoid restructuring outstanding liabilities for a subset of financial institutions. We believe an analytical answer to this question is important because it allows the government to implement a principled approach in which all financial institutions are treated equally. This approach is preferable to an ad-hoc approach for several reasons. First, ad-hoc interventions create uncertainty for private investors, which makes them less willing to invest. Uncertainty also generates an option to wait for future interventions, further undermining private recapitalizations. Finally, ad-hoc interventions are more likely to be influenced and distorted by powerful incumbents (see Hart and Zingales (2008), Johnson (2009)). In other words, even in a third best world – with first best being frictionless bargaining and second best being bankruptcy without systemic risk – government interventions should still follow principles.

Surprisingly however, despite the wide agreement regarding the diagnostic (debt overhang), there is considerable disagreement about the optimal form of government intervention. The original bailout plan proposed by former Treasury Secretary Paulson favors asset buy backs over other forms of interventions. Stiglitz (2008) argues that equity injections are preferable to asset buy backs because the government can participate in the upside if financial institutions recover. Soros (2009a) also favors equity injections over asset buy backs because otherwise banks sell their least valuable assets to the government. Diamond, Kaplan, Kashyap, Rajan, and Thaler (2008) argue that the optimal government policy should be a combination of both asset buy backs and equity injections because asset buy backs establish prices in illiquid markets and equity injections encourage new lending. Bernanke (2009) suggests that in addition to equity injections and debt guarantees the government should purchase hard-to-value assets to alleviate uncertainty about bank solvency. Geithner (2009) argues that asset buy backs are necessary because they support price discovery of risky assets.

Other observers have pointed out common elements among the different interventions without necessarily endorsing a specific one. Ausubel and Cramton (2009) argue that both asset buy backs and equity injections require to put a price on hard-to-value assets. Bebchuk (2008) argues that both asset buy backs and equity injections have to be conducted at market values to avoid overpaying for bad assets. Soros (2009b) argues that bank recapitalization has to be compulsory rather than voluntary. Kashyap and Hoshi (2008) compare the financial crisis of 2007-2009 with the Japanese banking crisis and argue that in Japan both asset buy backs and capital injections failed because the programs were too small. Scharfstein and Stein (2008) argue that government interventions should restrict banks from paying dividends because, if there is debt overhang, equity holders favor immediate payouts over new investment. Acharya and Backus (2009) suggests that public lender of last resort interventions would be less costly if they borrowed some of the standard tools used in private contracts for lines of credit.

The goal of our paper is to analyze the efficient form of recapitalization in a standard model of debt overhang. We rule out restructuring by imposing that the government cannot alter the priority structure of financial contracts and that the government optimizes under the participation constraint of equity holders. We consider the three main interventions undertaken during the financial crisis of 2007-2009: asset buy backs, equity-against-cash, and debt guarantees. We also construct the optimal mechanism for government intervention.

In our benchmark model, banks differ across two dimensions. The first dimension is the quality of their investment opportunities. If the quality of investment opportunities is high, there is a welfare loss from not investing. The second dimension is the quality of assets in place. If asset quality is low, debt overhang is severe and banks under-invest. We allow for an arbitrary correlation between the two variables. The information structure is such that, under symmetric information, the government and the banks only know the distribution of future investment opportunities and asset values. Under asymmetric information, the private sectors knows the asset values and investment opportunities of each bank but the government does not.

We start by comparing the relative efficiency of three government interventions. The first intervention is the buy back of risky assets. The second intervention is capital injections in exchange for equity. The third intervention are government guarantees for new debt issues. All three interventions have been proposed and implemented during the financial crisis of 2007-2009.

The comparison of government interventions delivers two main results. First, if banks and the government have the same information, all interventions are equivalent. By equivalent, we mean that two interventions implement the same level of bank lending at the same expected cost for tax payers. Second, if banks have an informational advantage over the government, asset buy backs and debt guarantees are equivalent and equity injections dominate both asset buy backs and debt guarantees. By dominance, we mean that one intervention dominates another intervention if the former intervention has a lower expected cost than the latter intervention and both interventions implement the same level of investment.

The intuition for the symmetric information case comes in three steps. First, if banks and the government have the same information, the participation constraint for banks is the same under all programs. Hence, the government can extract the expected payoff from future investment projects by keeping equity holders to their reservation utility of not participating in the program. This result is independent of the particular form of the government program. Second, the cost to the government is the implicit transfer to debt holders minus the expected gain from future investment opportunities. We show that these transfers are similar (in expectation) across programs. Finally, we show that the incentives to invest – and therefore the expected gain from future investment opportunities – are the same under the three programs. In the most general case, we provide conditions on debt covenants and asset buybacks that are sufficient for the equivalence result to hold.

The result under information asymmetry is more surprising and more difficult to prove. When banks can opt in government programs based on private information about asset quality, the government faces the problem of endogenous selection. It is far from obvious whether asset buy backs, equity injections, and debt guarantees should face the same trade-off between welfare gains from increasing lending on the one hand, and adverse selection on the other hand.

It turns out that asset buy backs and debt guarantees are equivalent because both interventions charge a fixed price independent of the future bank equity value. For asset buy backs, the fixed price attracts banks with low quality assets because their assets would yield a lower price on private markets. Similarly, debt guarantees attracts banks with low quality assets because those banks are charged high interest rates to raise debt elsewhere. Since both interventions provide the same net benefit to banks, asset buy backs and debt guarantees attract the same set of participating banks, and thus yield the same investment at the same expected cost to the government.

Equity injections are different because the price of participation depends on the future bank equity value. The government takes an equity stake in the bank and participates in the upside of future investment opportunities. For a fixed size of the government program, the same low quality banks participate in equity injections as in the case of asset buy backs or debt guarantees. However, some firms with good investment opportunities but low-quality assets do not participate because they do not want to share the upside with the government and rather invest alone. As a result, there is less inefficient participation in equity injections than other interventions and the expected cost of equity injections is lower.

Our results on symmetric versus asymmetric information also shed light on the comparison between compulsory and voluntary programs. The symmetric information case is equivalent to compulsory participation under the constraint that the program is *acceptable for the average bank*. With voluntary participation, banks select to participate based on the value of assets in place and their investment opportunities. The endogenous selection can be costly because banks with assets of lower quality are more likely to participate, but it can also be beneficial because banks with good investment opportunities are also more likely to participate. We can show that compulsory programs dominate voluntary ones when the government intervention is large.

We then study two extensions of the model. The first extension is to allow for heterogeneity of bank assets. This extension has no effect on equity injections or debt guarantees but raises the costs of asset buy backs. The reason is that banks choose to sell their lowest quality assets to the government. This result indicates that equity injections dominate asset buy backs for two separate reasons: adverse selection *across* banks due to inefficient participation and adverse selection *within* banks due to inefficient asset sales.

The second extension is the introduction of deposit insurance. Deposit insurance decreases the net costs of intervention, because the government is partially bailing out the FDIC, not only debt holders. In the case where deposits are never strictly safe, the costs of intervention are actually negative and the government can and should implement the first best. In any case, deposit insurance does not alter our results on the relative efficiency of the different interventions.

Finally, we solve for the constrained optimum intervention. We construct an efficient mechanism where the government asks for call options on equity in exchange for providing a subordinated loan at below market rates. The optimal intervention implements the investment region at the same cost as an intervention in which investment opportunities and asset values are observable to the government. The intuition is that the government makes the loan subordinate to junior creditors to ensure that the government does not generate debt overhang itself. At the same time, the government uses call options to extract the surplus from new investment and thus keeps equity holders to their participation constraint. Swagel (2009) notes that the terms of the Capital Purchase Program, the first round of equity injections, consisted of a cash transfer in exchange for preferred shares combined with warrants. This structure is broadly consistent with the constrained optimal intervention.

We note that we assume throughout the analysis that debt covenants prevent banks from selling safe assets. If we weaken this assumption ad allow for the sale of safe assets, the cost of asset buy backs decreases because purchasing safe assets effectively gives priority to the government over senior debt holders. Conceptually, selling safe assets is identical to restructuring. We do not want to dismiss this solution but we think that debt covenants for safe assets are a good assumption because debt holders have a strong ex-ante incentive to impose such covenants and we observe such covenants in practice.

We also note that we assume that investment opportunities are riskless. We make this assumption because it allows us to separate the issue of debt overhang from incentives for risk shifting. However, risk shifting still plays an important role in our model: it explains why equity holders are reluctant to sell risky assets. Hence, an alternative interpretation of the time 1 dominance of equity-injections over asset buy backs is that making government intervention contingent on investment opportunities alleviates risk shifting incentives.

We view our work as following the tradition of Modigliani and Miller (1958) on the irrelevance of capital structure. We seek to distinguish the economic forces that matter from the ones that do not, by providing a benchmark in which the form of government interventions is irrelevant. In particular, we show that under symmetric information all interventions implement the same allocation at the same expected costs. However, under asymmetric information equity injections dominate over other forms of interventions because equity injection provide better incentives for program participation and thus alleviate adverse selection.

Veronesi and Zingales (2008) conduct the only empirical analysis of the different interventions to recapitalize the banking system. They perform an event study using data on stock returns and credit default swaps around the announcement of the revised Paulson Plan. They find that the revised Paulson plan, which combines capital injections with debt guarantees, increased the value of bank financial claims by $109 billion at a taxpayer's cost of $112-135 billions. They argue that there is no evidence that the revised Paulson Plan alleviated the debt overhang problem because improved investment opportunities should have created a net benefit of the program. They evaluate alternative interventions assuming the government was to achieve the same reduction in credit default swap prices as the revised Paulson plan and find that pure equity injections yield a higher net benefit than the revised Paulson plan or asset buy backs. Their preferred solution is a debt for equity swap as proposed by Zingales (2008a) and Zingales (2009).

This paper relates to the existing literature on bank bailouts. Kocherlakota (2009) analyzes resolutions to a banking crisis in a setup where insurance provided by the government generates debt overhang. Similar to our paper, he analyzes the optimal form of government intervention and finds an equivalence result similar to our symmetric information equivalence theorem. Our papers differ because we focus on debt overhang generated by debt holders instead of the government and we allow for heterogeneity in investment opportunities, heterogeneity in asset classes, and asymmetric information between the government and the private sector.

Gorton and Huang (2004) argue that there is a potential role for the government to bail out banks in distress because the government can provide liquidity more effectively than the private market. Diamond and Rajan (2005) show that bank bailouts can increase excess demand for liquidity, which can cause further insolvency and lead to a meltdown of the financial system. Diamond (2001) emphasizes that governments should only bail out banks that have specialized knowledge about their borrowers. Aghion, Bolton, and Fries (1999) show that bank bailout policies can be designed such that they do not distort ex-ante lending incentives relative to strict bank closure policies. Heider, Hoerova, and Holthausen (2008) emphasize the role of counterparty risk in the interbank market. In our model, we take the initial debt overhang problem as given. Recent research explains the rise of risky assets linked to mortgages (Mian and Sufi (2008), Keys, Mukherjee, Seru, and Vig (2010)) and the tendency of banks to become highly levered (Adrian and Shin (2008), Acharya and Schnabl (2009)).

With the exception of Kocherlakota (2009), our paper is different from the literature because we focus on the optimal form of the bank bailout instead of the effect of bank bailouts on lending incentives. The main question of our paper - how to recapitalize banks in the presence of debt overhang and asymmetric information - is not adequately addressed in the literature.

The paper proceeds as follows. Section 1 sets up the model. Section 2 solves for the decentralized equilibrium with and without debt overhang. Section 3 describes the government interventions. Section 4 compares the interventions. Section 5 extends the model to heterogenous assets and deposit insurance. Section 6 discusses optimal mechanisms. Section 7 concludes.

1 Model

The model has a continuum of banks of measure 1. Our abstract model is applicable to financial and industrial firms alike, but, for concreteness, we will to all of them as banks.

Figure 1 summarizes the timing, technology, and information structure of the model. The model has three dates $t = 0, 1, 2$. There is no discounting. Banks start time 0 with given initial assets and liabilities. At time 1 banks receive new investment opportunities, and they lend to and borrow from each other and from outside investors. To avoid confusion with inter-bank lending, we use the word "investments" to refer to the new loans that banks make to the non-financial sector at time 1. All returns are realized at time 2, and profits are paid out to investors.

The government announces its interventions at time 0, but the implementation can happen either at date 0 or at date 1. The difference matters because banks learn about the value of their existing assets and about their new investment opportunities at date 1. Interventions at date 1 are therefore subject to adverse selection, while interventions at date 0 are not. The two cases are empirically relevant, and we therefore analyze both.

1.1 Initial assets and liabilities

At time 0 banks have both assets and liabilities in place. All banks are ex-ante identical. On the liabilities side, banks have long term debt. Long term debt is due at time 2. Let D be the face value of long-term debt outstanding.

On the asset side, banks have three types of assets: cash, risk free long term assets, and risky long term assets assets. Cash is liquid and can be used for investments or for lending at date 1. Let c_t be cash holdings at the beginning of time t. All banks start time 0 with c_0 in cash. Cash holdings cannot be negative:

$$c_t \geq 0 \text{ for all } t.$$

Long-term risk free assets deliver payoff $\underline{A}$. Long-term risky assets deliver random payoff $a = A$ or $a = 0$ at time 2. We define the probability of a good outcome as

$$p \equiv \Pr(a = A).$$

At time 1 private investors learn the value of p for each bank.

We focus on the binary outcome model because it delivers the main insights while simplifying the algebra. We will later extend our equivalence theorem to a general distribution for a. Note that any binary asset payoff can be modeled using the risky/safe asset model.

For example, suppose that the payoffs are A^H in the good state and A^L in the bad state. To get back to the risky/safe model, we simply define $\underline{A} = A^L$ and $A = A^H - A^L$.

1.2 Investment opportunities

At time 1 banks receive investment opportunities. Investments cost the fixed amount x at time 1 and deliver safe income v at time 2. The value of v is between 0 and V and banks learn v at time 1. The joint distribution F of p and v is

$$F(p, v) \text{ for } p \in [0, 1] \text{ and } v \in [0, V]$$

And we use the notation

$$\bar{p} \equiv E[p]$$

To make the problem interesting, we assume that individual banks do not have enough cash to finance investment projects but the aggregate system has sufficient cash to finance all investments. To study debt overhang, we assume that debt is risky such that long term debt D is in default when $a = 0$, but not when $a = A$. We also assume that the payoff V from new investment is not sufficient to cover long term debt D.

Assumption A1: $c_0 < x < V < D - \underline{A} < A$

Assumptions A1 is maintained throughout the paper. Borrowing and lending at time 1 can be among banks, or between banks and outside investors. We assume risk neutral investors and we normalize the risk free rate to 0.

Assumption A2: Safe assets $\underline{A}$ are protected by debt covenants

Assumptions A2 protects debt holders from expropriation by equity holders. It is well known that equity holders \have incentives to engage in risk shifting at the expense of debt holders. For instance, shareholders might decide to sell the safe assets and invest the proceeds in risky projects. Debt covenants protect debt holders. Debt covenants play an important role when we discuss asset buyback programs.

2 Equilibrium without intervention

In this section, we study the equilibrium without government intervention. We characterize the first best outcome, and the debt overhang equilibrium.

2.1 Investor payoffs

Figure 2 summarizes the payoffs to equity holders. In order to finance investment, banks can lend to and borrow from each other. Let l be the face value of borrowing at time 1 and let r be the gross interest rate for interbank lending. At time 2 total bank income y is:

$$y = \underline{A} + a + c_2 + v \cdot i,$$

where i is dummy for the decision to invest at time 1. Let y^D, y^l and y^e be the payoffs at time 2 of long term debt, interbank lending and equity, respectively. Long term debt is senior to interbank lending l. Equity is junior to debt. There are no direct deadweight losses from bankruptcy. Under the usual seniority rules, the payoffs to investors are:

$$y^D = \min(y, D)\,; \quad y^l = \min(y - y^D, rl); \quad y^e = y - y^D - y^l.$$

Under assumption A1, the payoffs to investors depend on the realization of asset value in the following way. If $a = A$, all liabilities are fully repaid ($y^D = D$ and $y^l = rl$) and equity holders receive $y^e = y - D - rl$. If $a = 0$, then long term debt holders receive all income ($y^D = y$) and other investors receive nothing: $y^l = y^e = 0$.

2.2 First best

Figure 3 depicts the investment region in the first best equilibrium. Without intervention, the banks simply carry their cash holdings from period 0 to period 1, so $c_1 = c_0$. The interbank lending market opens at time 1. The first best assumption is that banks choose investments at time 1 to maximize total value $V_1 = \underline{A} + E_1[a] + c_2 + v \cdot i - E_1\left[y^l\right]$, subject to the time 1 budget constraint

$$c_2 = c_1 + l - x \cdot i. \tag{1}$$

The break even constraint for outside lenders is:

$$E_1\left[y^l\right] \geq l. \tag{2}$$

Using assumption A1, there is excess aggregate liquidity to finance the investment, and the break even constraint (2) binds: $E_1\left[y^l\right] = l$. Using (1), this implies that

$$V_1 = \underline{A} + E_1[a] + c_1 + (v - x) \cdot i.$$

Therefore, investment takes place in the domain:

$$I^* \equiv \{(p, v) \mid v > x\}.$$

Proposition 1 *The first best solution is for investment to take place at time 1 if and only if $v > x$, irrespective of the value of p.*

A few properties of the first best solution are worth mentioning. First, the interest rate is bank specific since equation (2) is simply $r = 1/p$. The other important property is the connection between shareholder value and total value. We can always write $V_1 = E_1\left[y - y^l\right] = E_1\left[y^e + y^D\right]$. The maximization program for total value is equivalent to the maximization of shareholder value $E_1\left[y^e\right]$ as long as we allow renegotiation and transfer payments between shareholders and debt holders.

2.3 Debt overhang

We assume that banks maximize shareholder value instead of total value. Under the risky debt assumption A1, shareholder value maximization leads to the classic debt overhang problem.

Figure 4 depicts the investment region in the debt overhang equilibrium. Consider the market at time 1. Shareholders get nothing if the bad state realizes at time 2, and if the good state is realized they get $c_2 + \underline{A} + A + v \cdot i - D - rl$. The bank maximizes shareholder value subject to budget constraint (1) and break even constraint for new investors (2). The condition for investment becomes

$$v - x > (r - 1)\, l \tag{3}$$

This is the investment condition under debt overhang.

Recall that the first best investment rule was simply $v - x > 0$. The difference with the First Best investment rule comes from two critical properties. First, the outside investors ask for a risk premium because they know that lending is risky. Hence $r > 1$. Second, shareholders perceive a high cost of funds because they do not get the returns of the investment project in the bad state. In the first best world, they would renegotiate with the debt holders. Debt overhang follows from the assumption that debt contracts cannot be renegotiated, or at least not quickly enough to seize the investment opportunity.

A constrained firm would always choose to invest its own cash first, so $c_2 = 0$, and $l = x - c_1$. Since $c_1 = c_0$, Equation (3) becomes $pv + (1-p)\, c_0 > x$ and we get the investment domain:

$$I^o \equiv \{(p,v) \mid L^o(p,v) > 0\}, \tag{4}$$

where we define

$$L^o(p,v) \equiv pv + (1-p)\, c_0 - x. \tag{5}$$

If $L^o(p,v) < 0$, no investment takes place. If $L^o(p,v) > 0$, investment takes place using the free cash c_0 and the additional borrowing $x - c_0$. The function $L^o(p,v)$ measures the value for shareholders of undertaking a new investment under debt overhang, given the quality of the existing assets p, the available liquidity c_0, and the fundamental value of new investment v. From the perspective of shareholders, the NPV of the investment is $pv - x$. Internal cash c_0 has a low opportunity cost since it would be given away to debt holders with probability $(1-p)$.

2.4 Shareholder value and welfare losses

We repeatedly use the time 0 and time 1 equity value to compute equity holder's optimal investment and participation decisions. The equity value at time 1 is

$$E_1[y^e|p,v] = p\,(N + c_0) + L^o(p,v)\, 1_{(p,v)\in I^o} \tag{6}$$

where

$$N \equiv \underline{A} + A - D.$$

Equity value at time 1 is the sum of two terms. The first term is the equity holder's expected value of long term assets and cash minus senior debt. The value is multiplied by probability p because equity holders only receive a payment in the high-payoff state. The second term is the equity holder's value of new investment opportunities $L^o(p,v)$ as defined above.

Taking expectations at time 0, the equity value is:

$$E_0[y^e] = \overline{p}\,(N + c_0) + \iint_{I^o} L^o(p,v)\, dF(p,v) \tag{7}$$

The first term is the expected equity value of long term assets and cash minus liabilities using the unconditional probability of solvency $\overline{p}$. The second term is the time 0 expected value of new investment opportunities. The domain I^o is defined in Equation (4). Since investment is chosen optimally, the value of new investment opportunities $L^o(p, v)$ is zero on the border of I^o.

Social welfare under debt overhang depends on the set of implemented investment projects I^o. We define $W(.)$ as the social welfare function, so that welfare under debt overhang is:

$$W(I^o). \tag{8}$$

As long as the second best investment set I^o is strictly smaller than the first best investment set I^*, there is a welfare loss. In the banking context, these deadweight losses capture missed trading and lending opportunities. We assume the social welfare function incorporates deadweight losses to both banks and borrowers. Hence, the welfare function is independent of how the benefits of investment projects are shared among banks and borrowers.

Note that equation (8) assumes that investment projects are bank specific. This formulation captures the idea that banks have proprietary information about their borrowers and it is costly for borrowers to switch to other banks. This assumption is based on a large literature in banking which argues that one of the main functions of financial intermediaries is to generate private information about their borrowers (see for instance Diamond (1984)).

3 Description of government interventions

We consider three government interventions: capital against equity, asset buy backs, and debt guarantees. We first discuss the government's objective function and then briefly describe each intervention.

3.1 Government objective function and constraints

The objective of the government is to minimize the welfare losses from missed investment opportunities and the costs of intervention. Let Ψ be the expected cost of a government intervention. Let χ be the marginal deadweight losses associated with raising taxes and administering government interventions. The objective function of the government is

$$\max_{\Gamma} W\left(I\left(\Gamma\right)\right) - \chi\Psi\left(\Gamma\right)$$

where Γ are the parameters chosen by a specific government intervention. For simplicity, we assume that the marginal cost χ is constant. This means that the government cares about expected costs, but not about the distribution of these costs.

The expected costs of the program depend on the time of participation. At time 0, all banks are identical and information is symmetric. At time 1, the banks learn the value of their investment opportunities and the expected value of their long term assets. The type of a bank is a two-dimensional random variable (p, v) realized at time 1.

We place constraints on the interventions of the government. First, we do not allow the government to change the priority rules of financial contracts and we assume that the government cannot make debt holders worse off. These restriction rules out government interventions such as forced bankruptcy, forced asses sales, and debt equity swaps, which would result in losses to debt holders. We also assume that the government cannot make payments directly contingent on the banks' new investments. This rules out directed lending. Finally, we assume that the government can restrict dividend payments to shareholders. Otherwise banks would simply pay out proceeds from government interventions as a dividend to shareholders.

3.2 Description of asset buy back program

The asset buy back program is parametrized by Z and p^z. The government announces at time 0 that it is willing to purchase risky assets up to an amount Z at a per unit price of p^z in exchange for cash. If a bank decides to participate and sell $z < Z$, long term assets become $A_1 = A - z$ and cash $c_1 = c_0 + zp^z$.

We note that the government can only buy risky but not safe long term assets. The reason is that under Assumption A2 debt covenants prevent equity holders from selling risky assets. This assumption is important because, as we show below, equity holders can extract rents from debt holders by selling safe assets. The intuition is that safe asset sales change the priority structure of financial claims and effectively give equity holders priority over debt holders.

The government can offer banks to participate in the asset buy back program at time 0, at time 1, or at both times. The time of participation is important because at time 1 banks learn about the value of investment opportunities and the expected value of long term assets. Due to the option value of new information, banks always choose to wait with their decision until time 1 if possible. Without loss of generality, we thus only consider government programs with participation at either time 0 or at time 1, not at both times.

At time 0, we can without loss of generality consider programs where all banks participate because all banks are identical and the government can always set $Z = 0$. The expected cost of the time 0 asset buy back program is

$$\Psi_0^a(Z, p^z) = z_0 \left(p^z - \bar{p}\right) \text{ with } z_0 < Z$$

where z_0 is the face value of assets purchased by the government. The government pays out $z_0 p^z$ at time 0 and receives z_0 in the high-payoff state with probability $\bar{p}$.

At time 1, the cost of the asset buy back program is different because banks learn the value of investment opportunity v and the value of long term assets p before deciding whether to participate. The expected cost is therefore

$$\Psi_1^a\left(Z, p^z\right) = \iint_{(v,p)} z_1(v, p; Z, p^z) \cdot (p^z - p) dF(v, p)$$

where z_1 is the face value of risky long term assets sold under the program. This formulation allows for adverse selection because banks may participate in the program depending on their type (v, p).

3.3 Description of capital injection program

Equity injection programs are parameterized by m and α. The government announces at time 0 that it is willing to offer cash m against a fraction α of equity returns. Similar to the asset buy back program, the government can offer banks to participate in this program at time 0 or time 1. If a bank decides to participate, its cash position becomes $c_1 = c_0 + m$. The expected cost of the program at time 0 is

$$\Psi_0^e\left(m, \alpha\right) = m - \alpha E_0\left[y^e\left(m\right)\right]$$

where $E_0\left[y^e\left(m\right)\right]$ is the expected equity return at time 0 conditional on cash injection m. In words, the government pays out m at time 0 and receives a share α of equity returns y^e

at time 2. There are no constraints on that program, except $m \geq 0$ and $\alpha \in [0,1]$. The expected cost of the date 1 program is

$$\Psi_1^e(m,\alpha) = \iint_{(v,p)} \delta^e(m,\alpha;v,p) \cdot (m - \alpha E_1[y^e(m)|v,p])dF(v,p)$$

where δ^e is an indicator variable whether a bank participates in the program, and $E_1[y^e(m)|v,p]$ is the expected equity return at time 1 conditional on cash injection m and and bank type (v,p). Similar to the asset buy back program, this formulation allows for adverse selection depending on bank type (v,p).

3.4 Description of debt guarantee program

Debt guarantee programs are parameterized by S and ϕ. The government announces at time 0 that it is willing to guarantee new bank debt up to a face value of S and charges banks a fee ϕ per unit of lending. There are several equivalent ways to define the parameters S and ϕ. In our notation, the fee is paid up-front and the upper bound applies to the face value of new bank debt. Let s be the face value of new bank debt issued under the program and let r_s be the interest rate on debt issued under the program. The amount of money raised at time is therefore $s/r_s - \phi s$ and the constraint is $s < S$ (we will see shortly that $r_s = 1$ in equilibrium). At time 0, the expected cost to the government is

$$\Psi_0^g(S,\phi) = s_0(1 - \phi - \bar{p}).$$

The expected cost to the government is the probability of the low-payoff state $(1-\bar{p})$ minus the guarantee fee ϕ.

At time 1, the expected cost of the government is

$$\Psi_1^g(S,\phi) = \iint_{(v,p)} s_1(v,p;S,\phi)(1 - p - \phi)\, dF(v,p). \tag{9}$$

Similar to the other programs, the time 1 debt guarantee allows for adverse selection depending on bank type (v,p).

4 Comparison of government interventions

Our main result is that all interventions are equivalent at time 0, but capital injections dominate both asset buy backs and debt guarantees at time 1. Equivalence of two interventions means that both interventions implement the same level of investment at the same expected cost to the government. Dominance of two interventions means that the dominant intervention implements the same level of investment as the dominated intervention but the dominant interventions has a lower costs than the dominated intervention. To build the intuition for our result we first present two useful lemmas, one for providing free cash to banks and one for debt guarantees.

The following investment domain $I(m)$ plays a key role in our discussions.

Definition 1 *Let the domain $I(m)$ be defined by*

$$I(m) \equiv \{(p, v) \mid L^o(p, v) + (1-p)m > 0\} \tag{10}$$

4.1 Equilibrium with free cash injections at time 0

We first discuss the case of providing free cash to banks at 0. That is, the government simply gives m to each bank, without asking for anything in return. This case is a useful because it illustrates how free cash injections affect the investment region. In terms of the government programs, free cash injections are equivalent to an equity injection m with equity share $\alpha = 0$, an asset buy back program with face value $Z \to 0$ and cash injection $p^z Z = m$, and a debt guarantee program with face value $S = m$ and guarantee fee $\phi = 0$. The following lemma characterizes free cash injections.

Lemma 1 *A free cash injection leads to the following welfare function for the government*

$$W(I(m)) - \chi m$$

Proof. Suppose the government injects m in each bank so that initial liquidity becomes $c_1 = c_0 + m$. From equation (4) and (5), we see that the investment domain becomes $I(m)$ and the total cost is $\Psi_0^e(m, 0) = m$ since the number of banks is normalized to one. ■

Figure 5 shows the effect of the free cash injection on the investment region $I(m)$. The cash injection m relaxes the investment constraint and therefore expands the set of implemented investment projects. If the cash injection is large enough to cover the entire financing need $x - c_0$, then the cash injection can eliminate the entire debt overhang. In other words, $I(x - c_0) = I^*$.

4.2 Equilibrium with debt guarantee at time 1

We now discuss the equilibrium with a debt guarantee at time 1. The comparison of the debt guarantee with free cash injection illustrates the main incentives effects of government interventions. To compute the banks' optimal investment and participation decision, we use the expected equity value. We obtain the equity value at time 0 and time 1 be replacing c_0 by $c_0 + m$ and I^o by $I(m)$ in equations (6) and (7).

Banks benefit from the debt guarantee by the government because it allows them to issue riskless debt. The equilibrium interest rate on riskless debt is $r_s = 1$ and the equilibrium interest rate on unsecured debt $r_u = 1/p$. The time 1 budget constraint (1) becomes

$$c_2 = c_0 + l_u + (1 - \phi) s - x, \tag{11}$$

and the investment condition (3) becomes

$$L^o(p, v) + s(1 - \phi - p) > 0. \tag{12}$$

It is clear from the budget constraint (11) that the government never wants to set S above $x - c_0$ since this could not possibly help the financing of new investment opportunities.

Also note that the government wants to design an intervention such that banks only participate in the program if they invest. Otherwise, the program would provide a subsidy to banks that make no investments. As discussed above, we assume that the government does not observe new lending and therefore cannot make participation contingent on new investments. It is therefore important to impose a 'no inefficient participation' constraint (NIP from now on). Payoffs to equity holders in the good state are $A - D + c_2 - s$, so from equation (11) it is clear that the NIP constraint is:

$$\phi > 0. \tag{13}$$

We summarize this brief discussion in the following lemma.

Lemma 2 *It is enough to consider debt guarantees such that $S \in [0, x - c_0]$ and $\phi > 0$.*

Next we consider the choice between secured and unsecured borrowing. It is clear from (12) that banks take up the debt guarantee rather than the unsecured lending if and only if

$p < 1 - \phi$. Otherwise, banks prefer borrowing on the unsecured interbank lending market. This defines an upper-bound schedule for participation, $U_1^g(p, v; S, \phi) < 0$, where:

$$U_1^g(p, v; S, \phi) \equiv p + \phi - 1. \tag{14}$$

Because of the upper bound, if $p \in [1 - \phi, 1]$, banks do not participate in the program. However, if the bank type $(p, v) \in I^o$, the bank invests even without the debt guarantee from the program.

If $p < 1 - \phi$, banks prefer to participate in the debt guarantee program. Since the payoffs are linear in s, banks choose the maximum guarantee: $s = S$. This implies unsecured borrowing $l_u = x - (1 - \phi) S - c_0$ if the banks invest. Equation (12) leads to the lower bound schedule for investment, $L_1^g(p, v; S, \phi) > 0$, where:

$$L_1^g(p, v; S, \phi) \equiv L^o(p, v) + (1 - \phi - p) S \tag{15}$$

We now have a complete description of the participation and investment decisions. The structure comprises four elements and this structure is the same for all government interventions.

First, there is an NIP constraint (13) which means that the program cannot be too generous. The NIP constraint is like a haircut and defines an upper-schedule (14) above which banks do not participate in the government intervention. In the case of the debt guarantee program, the upper-schedule is vertical (it does not depend on v), but in general it is a function of p and v (as in the case of equity against capital, see below).

Second, there is a lower-schedule (15) under which banks are unwilling to invest even with the assistance of the government. These banks do not participate in the program and do not invest. In the case of the debt guarantee program, the lower schedule is a function of the bank type (p, v) and the guarantee fee ϕ. The NIP constraint and the lower-schedule bound define the participation set:

$$\Omega_1^g(S, \phi) = \{(p, v) \mid L_1^g(p, v; S, \phi) > 0 \wedge U_1^g(p, v; S, \phi) < 0\} \tag{16}$$

Third, the lower-schedule defines the investment domain. The investment domain is the combination of the initial debt overhang set I^o (banks that would invest even without the government's intervention) and the participation set Ω:

$$I_1^g(S, \phi) = I^o \cup \Omega_1^g(S, \phi) \tag{17}$$

Note that the overlap between the two sets, $I^o \cap \Omega_1^g(S,\phi)$, represents opportunistic participation. Opportunistic participation is inefficient, because the government provides a subsidy to banks that would have invested even in the absence of the government intervention.

Fourth, the participation set determines the expected cost of the government intervention. Using equation (9), the expected cost of the debt guarantee program is

$$\Lambda_1^g(S,\phi) \equiv S \int\int_{\Omega_1^g(S,\phi)} (1-p-\phi)\, dF(p,v)\,. \tag{18}$$

Figure 6 shows the investment set and participation set for time 1 debt guarantees. The figure distinguishes three regions of interest: efficient participation, opportunistic participation, and invest alone. The efficient participation region comprises the banks that participate in the intervention and that invest because of the intervention. The opportunistic region comprises the banks that participate in the intervention but would have invested even in the absence of the intervention. The invest alone region comprises the banks that do not participate in the program and invest without government intervention. As is clear from the figure, the government's trade-off is between expanding the efficient participation region and reducing the opportunistic participation region.

We summarize these results in the following lemma:

Lemma 3 *A time 1 debt guarantee program* (S,ϕ) *delivers welfare function* $W(I_1^g(S,\phi))$ *and has the expected cost* $\Lambda_1^g(S,\phi)$.

We can compare the debt guarantee with the free cash injections:

Proposition 2 *Debt guarantees at time 1 always dominate free cash injections at time 0.*

Proof. The proof is simple. Consider a debt guarantee with $\phi = 0$ and $S = m$. Both interventions achieve the same investment domain since $I_1^g(m,0) = I(m)$. However, the participation set for the time 1 debt guarantee is smaller than the participation set of the free cash injection. As a result, the expected cost of the debt guarantee $\Lambda_1^g(m,0)$ is smaller than the expected cost of the the free cash injection m. ■

In general, we see that debt guarantees are less costly than free cash injections because of three separate reasons. First, the NIP constraint ensures that only banks that invest participate in the debt guarantee program but all banks independent of investment participate in the free cash injection. Second, under the debt guarantee program the government only pays out insurance if the bank defaults and is compensated by fee ϕ otherwise but under free cash injection the government always pays out cash. Third, under the debt guarantee some healthy banks invest alone without participating in the program, but all banks participate in the free cash injection.

4.3 Comparison of time 0 programs

We now compare government programs at time 0. In these programs, the banks must opt in or out at time 0, when information is symmetric. We have the following proposition:

Theorem 1 ***Equivalence of time 0 programs - binary model.*** *A time 0 risky asset buy back program (Z, p^z) is equivalent to a time 0 debt guarantee program with $S = Z$ and $p^z = 1 - \phi$. It is also equivalent to a time 0 equity injection (m, α), where $m = Zp^z$ and p^z and α are chosen such that at time 0 all banks are indifferent between participating and not participating in the program. All programs deliver the same investment set $I(m)$ and have the same expected costs*

$$\Lambda_0(m) \equiv (1 - \bar{p})\, m - m \int\int_{I(m)} (1 - p)\, dF(p, v) - \int\int_{I(m) \setminus I^o} L^o(p, v)\, dF(p, v) \tag{19}$$

Proof. See Appendix. ■

The key to this equivalence result is that banks are forced to decide to participate in the programs before they receive information about investment opportunities and asset values. Banks are thus identical and the government optimally chooses the program parameters such that banks are indifferent between participating and not participating. For a fixed program amount, the government extracts all rents from the intervention. The cost to the government is thus independent of whether banks are charged through assets sales, guarantee fees, or equity shares.

It is important to emphasize that we are comparing pure date 0 interventions here, where no further interaction between the banks and the government occurs at date 1. We are not claiming that these pure date 0 interventions are optimal. In fact, they are not. It is always better for the government to sell at date 0 an option to participate in a date 1 program. We return to this idea later.

It is also important to understand the cost function $\Lambda_0(m)$ by looking at the three terms on the RHS of Equation (19). The first term reflects the fact that, in the bad state, the cash injection is received by long term debt holders. The second term is the gain in borrowing costs conditional on being in the investment set. The third is the subsidy to new investments. It contributes positively to the cost since $L^o(p,v) < 0$ for all $(p,v) \in I(m) \backslash I^o$. Note that $\Lambda_0(m) > 0$ since the first term dominates the second and the third is positive.

We can now also discuss the role of assumption A2.

Proposition 3 ***Safe assets sale.*** *If we relax Assumption A2, a program to sell $\underline{Z} < \underline{A}$ safe assets at time 0 in exchange for m in cash has an expected cost of $\Lambda_0(m) - (1-\bar{p})\underline{Z}$.*

The intuition is quite simple. An outright sale of *safe* assets changes the priority structure of financial claims. In the bad state, the government receives payoff $\underline{Z}$ which would otherwise have gone to the debt holders. This transfer from the debt holders lowers the cost for the government. This discussion shows that selling safe assets is yet another way to get around the renegotiation issue. But note that this is unlikely to be efficient in an ex-ante perspective, since covenants to protect debt holders are valuable only if they are credible.

In market value terms, debt holders do not lose, as long as $m \geq \underline{Z}$. If $\underline{Z}$ is high enough, then the government can implement $m = \underline{Z}$ and the cost becomes

$$-m \iint_{I^o} (1-p)\, dF(p,v) - \int \int_{I(m)\backslash I^o} \left(L^o(p,v) + (1-p)\, m\right) dF(p,v),$$

which is negative. In this case, the government would make money by capturing some of the rents. The debt holders break even if the firm does not invest since $m = \underline{Z}$, and are strictly better off if the firm invest since $v > m = \underline{Z}$. Of course, in practice it is difficult to separate assets just as it is difficult to do project financing. We do not argue that this is a realistic case, but we find it very helpful to understand the nature of the economic problem.

We can further extend the model to allow for a continuous asset distribution instead of the binary setup. Suppose at time 1, banks learn the parameter $p \in [0,1]$. and update the distribution to $G(.|p)$ over the support $[0, A]$ for all p. The ex-ante distribution of (p, v) is $F(p, v)$, so the ex-ante distribution of a is

$$f_0(a) = \int_{p=0}^{1} \int_{v=0}^{V} g(a|p) dF(p, v).$$

To compare the interventions, we need to define debt covenants for a continuous asset distribution. We assume covenants are efficient in the sense that for any distribution F debt holders receive at least the expected payoff they would receive without asset buy backs. This assumption ensures that debt holders have priority over asset buy backs.

We also need to define the priority structure of junior creditors and debt issued under the debt guarantee. We assume that junior creditors are senior to guaranteed debt. This assumption ensures that the government does not create its own debt overhang.

Theorem 2 ***Equivalence of time 0 programs - continuous distribution case.*** *A time 0 equity injection is equivalent to a time 0 asset buy back program with efficient covenants and equivalent to a time 0 debt guarantee program in which junior creditors have priority over guaranteed debt.*

Proof. See Appendix. ■

We think the generalization of the equivalence theorem to the continuous asset case is helpful for two reasons. First, and most importantly, we show that the equivalence theorem holds for the continuous asset case under reasonable specifications for debt covenants and debt guarantees.

Second, the continuous asset case clarifies the importance of the priority structure in designing government interventions. This is helpful because in the binary model all claims other than senior debt are either completely paid off or not paid at all.

Specifically, for debt guarantees the guaranteed debt has to be junior to borrowing at time 1. Under this assumption, the debt issued under the debt guarantee is effectively equivalent to buying preferred shares in the bank. The intuition is that the equivalence theorem holds as long as guaranteed debt does not generate its own debt overhang and therefore does not affect borrowing costs at time 1.

Similarly, efficient covenants ensure that senior debt holders have priority over asset buy backs. This assumption is equivalent to the covenant assumption A2 in the binary model. As discussed above, without covenants, the government can alter the priority structure by buying safe assets. We do not want to rule out such an intervention but we think ex-ante senior debt holders have strong incentives to demand efficient debt covenants. Hence, efficient debt covenants generate a priority structure that is equivalent to the two other interventions.

4.4 Comparison of date 1 programs

Let us now compare the date 1 programs. In these programs, the banks must opt in or out at time 1, when information is asymmetric. We have the following proposition:

Theorem 3 ***Equivalence of asset buy-backs and debt guarantees at time 1.*** *An asset buy back program (Z, p^z) with participation at time 1 is equivalent to a debt guarantee program with $S = Z$ and $p^z = 1 - \phi$.*

Proof. See Appendix. ■

Note that the allocation features adverse selection, such that banks only participate in the program if the expected value of their assets p is less than the price p^z offered by the government. This feature of the solution is a natural outcome of a setup in which banks know more about asset values than the government. The frequently made argument that asset buy backs or capital against equity have to occur at fair market value is not feasible because banks only participate in the program if the the program recapitalizes at rates above market value.

Theorem 4 ***Dominance of equity injection at time 1.*** *For any asset buy back program (Z, p^z) with participation at time 1, there is an equity program that achieves the same allocation at a lower cost for the government.*

Proof. See Appendix. ■

Figure 7 depicts the equilibrium with equity injection at date 1. The intuition is the following. First, we must understand the net effects of dilution. They are captured by the function:

$$X(p; m, \alpha) \equiv (1-\alpha)m - \alpha p(N + c_0)$$

This function is intuitive: $(1-\alpha)m$ is the net value of cash injected by the government, and $\alpha p(N + c_0)$ is the dilution of the claims on old assets. So X measures the cash value of government transfers under the program. The participation set in the equity program takes the generic form:

$$\Omega_1^e(m, \alpha) = \{(p, v) \mid L_1^e(p, v; m, \alpha) > 0 \wedge U_1^e(p, v; m, \alpha) < 0\}$$

This can be compared to the participation set in equation (16). The lower bound is defined by

$$L_1^e(p, v; m, \alpha) \equiv (1-\alpha)L^o(p, v) + X(p; m, \alpha)$$

The intuition is clear. X are the transfers, and $(1-\alpha)L^o(p, v)$ the diluted value of new investments. It is optimal to opt in and invest if $L_1^e(p, v; m, \alpha) > 0$. The upper bound for participation is

$$U_1^e(p, v; m, \alpha) \equiv \alpha L^o(p, v) - X(p; m, \alpha)$$

The intuition is also clear. By not participating, the firm foregoes the transfers $X(p; m, \alpha)$ but avoids the dilution of its new project. Hence, it is optimal to invest without the assistance of the government when $U_1^e(p, v; m, \alpha) > 0$. Finally, the NIP constraint is

$$X(1; m, \alpha) < 0$$

The intuition is once again clear. The government must avoid giving away money to banks that do not plan to invest. In this case, the comparison cash is transferred one for one between the good and the bad state, so the condition is $(1-\alpha)m < \alpha(N + c_0)$. This condition is the same as $X < 0$ for banks whose assets are risk free, i.e., the banks for which $p = 1$. It also means that $X(p; m, \alpha) \leq (1-\alpha)(1-p)m$ and therefore the investment domain is strictly smaller than in the pure cash injection: $I_1^e(m, \alpha) \subset I(m)$. The reason is that firms with high p and low v opt out to avoid dilution.

Now why is equity the better program? To understand, imagine a given asset buy back program. It has a lower schedule that determines the investment set, and thus the welfare function W. Now choose the equity program to have exactly the same lower schedule and thus the same investment set for an equity program.

The first point to understand is that equity induce less opportunistic participation. This is because it is costly for good banks to dilute their valuable equity. Hence the upper schedule is tighter. Of course, the two programs have different cost functions, so the fact that the participation set is smaller is not enough to show that equity is cheaper. However, the same reasons that make the upper schedule tighter also limit the rents earned as (p, v) move away from the lower frontier $L_1^e(p, v; m, \alpha) = 0$. Finally, it is easy to show that, once the lower schedule are the same, the NIP constraints are also equivalent. This shows that, for any asset buy back, or any debt guarantee program, there exists an equity injection program that delivers exactly the same investment set, but for a lower cost to the government. The lower cost comes from two sources: less opportunistic participation, and smaller rents conditional on participation.

4.5 Date 0 versus date 1

Let us now compare the programs at dates 0 and 1. From the perspective of the government, at date 0 there is adverse selection with respect to p since banks with bad assets are more likely to participate. There is also beneficial selection with respect to v since banks without investment projects are less likely to participate. We consider a change in the distribution of both p and v.

Proposition 4 ***Comparison of date 0 and date 1 programs.***

- *Consider two distribution functions F and $\tilde{F}$ for the parameters (p, v). If $\tilde{F}$ dominates F in the sense of first order stochastic dominance, then, for any investment domain I, the cost of the date 0 program is lower with $\tilde{F}$ than with F.*

- *Date 1 programs always dominate date 0 programs when few banks have positive NPV projects (i.e.,* $\Pr(v > x) \to 0$*).*

- *Date 0 programs always dominate date 1 programs when most banks have positive NPV projects (i.e.,* $\Pr(v > x) \to 1$*) and the government wants to implement a large program ($m \to x - c_0$, $I \to I^*$)*

Proof. See Appendix. ■

To understand this result, note that for every date 0 asset buy back program, we can construct a date 1 asset buy back program that generates the same investment region by setting the date 1 asset price p^z equal to one and choosing date 1 program size Z such that it generates the same cash injection as the date 0 asset buy back program.

If $\Pr(v > x) \to 0$ no bank receives an investment opportunity. Hence, there is no investment under any program. However, a date 0 asset buy back program yields a positive cost (because all banks participate) and a date 1 program yields zero cost (because nobody participates).

As more banks receive good investment opportunities, the cost of the date 0 program decreases because it extracts all rents from better investment opportunities. In contrast, the cost of date 1 asset buy back program increases because more banks participate.

A natural interpretation of data 0 versus date 1 is in terms of compulsory versus voluntary participation. Of course, compulsory participation without constraint does not make sense, so we impose the constraint that government offers be acceptable on average (for instance, a well diversified equity investor would accept the offer on behalf of all the banks). Our results can then be interpreted as follows: when interventions are large, and the government expect that most banks have positive NPV projects (positive franchise value), then it is better to do it early with compulsory participation. On the other hand, if the intervention is small, or if most banks do not have valuable new projects, then it is better to do it ex-post based on voluntary participation.

4.6 Discussion of the theorems

Let us briefly discuss our results. Some analysts advocate government interventions at market prices. In our model, this makes no sense since there would be no participation. The frequently made argument that asset buy backs or capital against equity have to occur at fair market value is not feasible because banks only participate in the program if the program recapitalizes at rates above market value. A subsidy is needed, the only question is how to do it.

We do not consider ex-ante incentives of banks, and we agree that bailouts create moral hazard. However, if bailouts are going to happen, they might as well be efficient. Our mechanisms are about minimizing the cost to tax payers, so they remain are relevant as soon as the government decides to intervene. In addition, our mechanisms minimize the rents to old shareholders and old debt holders (short of bankruptcy), so they also minimize moral hazard concerns conditional on any decision to bail out private investors.

We also need to discuss briefly the issue of risk shifting. Even though we assume that v is known at date 1, risk shifting is not absent from our model. Indeed, from the perspective of shareholders, selling risky assets is akin to anti-risk-shifting, and refusing to sell assets is like risk shifting. This is a very relevant issue. During his testimony to congress, Vikram Pandit, CEO of Citigroup, protested that he was not going to sell the assets at a dollar because it would not be right for shareholders: "When we look at some of the assets that we hold, we have a duty to our shareholders. The duty is that if it turns out they're marked so far below what our lifetime expected credit losses are, we can't sell them." Our model captures this issue and explains why it has been so hard to convince the banks to sell their risky assets.

5 Extensions

5.1 Heterogenous assets within banks

Suppose that the face value of assets at time 0 is $A + A'$. All these assets are ex-ante identical. At time 1, the bank learns which assets are A' and which ones are A. The A assets are just like before, with probability p of A and $1 - p$ of 0. The A' assets are worth nothing. The ex-ante problems are unchanged, so all programs are still equivalent at date 0.

The equity and debt guarantee programs are unchanged at date 1. So equity still dominates debt guarantee. But the asset buy back program at date 1 is changed. For any price $p^z > 0$ the banks will always want to sell their A' assets. This will be true in particular of the banks outside the range of positive NPV investments

Proposition 5 *With heterogenous assets inside banks, there is a strict ranking of programs: equity is best, debt guarantee is intermediate, buy back program is worse.*

The key point here is that adverse selection across banks is very different from adverse selection across assets within each bank. This is exactly the point of the opponents of the asset buy back program.

Corollary 1 *For asset buy back to be optimal, the market failure must come from private information among private agents.*

Of course this is only necessary. It remains to be seen if and how an asset buy back program can be optimal in the case of adverse selection in the private sector (Philippon and Skreta (2009)).[1]

5.2 Deposit Insurance

Suppose long term debt consists of two types of debt: deposits Δ and unsecured long term debt B such that

$$D = \Delta + B.$$

Suppose that the government provides insurance for deposit holders and that deposit holders have priority over unsecured debt holders. Then the payoffs are are:

$$y^{\Delta} = \min(y, \Delta); y^{B} = \min\left(y - y^{\Delta}, B\right)$$

We consider two separate case. The first case is safe deposits if $\Delta < \underline{A} + c_0$. The second case is risky deposits if $\Delta \geq \underline{A} + c_0$

Proposition 6 *With safe deposits, the cost and benefits of both date 0 and date 1 programs remain unchanged.*

Proof. See Appendix. ■

If deposits are safe, banks always have sufficient date 2 income to repay deposit holders. Hence, the expected cost of deposit insurance is zero independent of whether there is a government intervention. Note that this results depends on the assumption that debt covenants

[1] It is worth pointing out that adverse selection can be mitigated by debt overhang. In our simple model, the maximization of shareholder value does not create adverse selection because a fixed rate would not attract the low type (low p). By contrast, total firm value maximization would lead to adverse selection. Hence it is clear that the two market failures are best studied in separate papers.

prevents equity holders from selling the safe assets. As a result, the costs and benefits of all programs remain unchanged.

Proposition 7 *With risky deposits, the costs of date 0 and date 1 programs decrease. The equivalence results and ranking of both date 0 and date 1 programs remain unchanged. If deposits are sufficiently large, date 0 programs dominate date 1 programs and the government can implement the first best at negative cost.*

Proof. See Appendix. ■

With risky deposits, the government has to pay out deposit insurance in the low-payoff state. Hence, every cash injection at date 0 lowers the expected cost of deposit insurance in the low-payoff state one-for-one. As a result, the government recoups its entire investment both in the high- and low-payoff state. Put differently, the date 0 cash injection represents a wealth transfer to depositors and, because of deposit insurance, a wealth transfer to the government. Also note that the government extracts all benefits of increased lending ex-ante by keeping equity holders to their reservation utility. As a result, the government receives the expected net benefit of increased lending and thus the expected cost is negative.

6 Optimal programs

6.1 Date 0

Proposition 8 *Any date 0 program can be improved by making participation at date 1 voluntary and selling at date 0 the option to participate at date 1.*

Proof. See Appendix. ■

A practical example is the debt guarantee program. It is inefficient to force banks to issue S at time 0. It is better to sell them at date 0 the right to issue secured debt at time 1. In this way, banks who end up without investment options do not participate, banks who can invest alone also do not participate, and everyone pays ex-ante the NPV of the option to participate.

Corollary 2 *An optimal date 0 program is to sell at date 0 the option to participate in an optimal date 1 program.*

6.2 Date 1

The following proposition extends the result of Theorem 4

Proposition 9 *Equity programs at date 1 cannot be improved by mixing them with a debt guarantee or asset buy back program. Pure equity programs always dominate.*

Proof. See Appendix. ■

Let us now consider optimal programs. The constraints we impose are that the debt holders cannot be worse off and that the government cannot alter the priority of claims. So all the government can do is to inject cash m at time 1 in exchange for state contingent payoffs at time 2. Date 1 junior creditor must be repaid, so as long as the government can commit, we can without loss of generality restrict our attention to the case where the government payoffs depend on the residual payoffs $y - y^D - y^l$.

In general, however, the government could offer a menu of contracts to the banks. Menus of contract can be used to obtain various investment sets. The optimal choice depends on the distribution of types $F(p, v)$ and the welfare function W so we cannot say in general which set is optimal. But we can say a lot about cost minimization for any given investment set. This is what we do now.

Lemma 4 *Any program with voluntary participation of shareholders over the set Ω and no renegotiation with debt holders has a minimum cost of*

$$\Psi^{\min}(\Omega) = -\iint_{\Omega} L^o(p, v)\, dF(p, v)$$

Proof. Voluntary participation means that shareholders must get at least $p(N + c_0)$. With no renegotiation with debt holders, the government and old equity must share the residual surplus whose value is

$$p(N + c_0) + L^o(p, v)$$

Hence the expected net payments to the government must be $\iint_{\Omega} L^o(p, v)\, dF(p, v)$. These are negative as long as Ω extends the debt overhang investment set, hence the positive minimum cost. ■

The intuition is simple. Suppose the government could make type contingent offers conditional on new investments. For type (p, v), the net value of new investment is $L^o(p, v)$ which is negative outside the I^o region. So the minimum the government would have to pay is $-L^o(p, v)$.

Proposition 10 *Consider the program $\Gamma = \{m, h, \varepsilon\}$ where the government offers a junior loan m at time 1 at the rate h in exchange for $(1-\varepsilon)/\varepsilon$ call options at the strike price $N + c_0$. This implements the investment set*

$$I(\Gamma) = I_1^g(S, \phi)$$

if we identify the cash injection $m = (1-\phi) S$ and the haircut $h = \phi/(1-\phi)$. In the limit $\varepsilon \to 0$, opportunistic participation disappears:

$$\lim_{\varepsilon \to 0} U(\Gamma) = L^o$$

Finally the program achieves the minimum cost in the limit:

$$\lim_{\varepsilon \to 0} \Psi(\Gamma) = \Psi^{\min}(I(\Gamma) \backslash I^o)$$

Proof. See Appendix. ■

Figure 8 depicts the equilibrium under the efficient mechanism. The intuition is that the payoff structure to old shareholders is now:

$$f(y^e) = \min(y^e, N + c_0) + \varepsilon \max(y^e - N - c_0, 0)$$

Old shareholders are full residual claimants up to the face value of old assets $N + c_0$ and ε residual claimants beyond. A few properties are worth mentioning. First, the loan must be junior to all creditors but senior to common stock holders. Hence it could also be implemented with preferred stock. It is crucial, however, that the government also takes a position that is junior to shareholders. Dilution should happen on the upside to induce participation of firms who need it, and to limit opportunistic participation.

The call option mechanism also has some advantages that are likely to be important for reasons outside the model. The first advantage is that it limits risk shifting incentives since the government owns the upside, not the old shareholders (see, for instance, Green (1984)).

Second, the government can credibly commit to protecting the shareholders since it owns call options. This is important because, conditional on not failing a bank, it makes no sense to try and punish the shareholders. This can only hurt the government, but political pressures to do so appear to be large nonetheless.

In practice, there might be lower bound on ε. It might be necessary to limit dilution to avoid fears of nationalization. An approximate optimal program would then be to determine first the minimum value of ε, and then to construct the program accordingly. Also the haircut h is chosen to rule out inefficient participation (the NIP constraint). In theory, any $h > 0$ would work, but in practice, parameter uncertainty would prevent h from being too close to zero.

7 Conclusion

In this paper we study the efficiency and welfare implications of different government interventions in a standard model with debt overhang. We consider asset buy backs, cash against equity injections, and debt guarantees. We find that under compulsory participation, all interventions are equivalent. Under voluntary participation, equity injections dominate both asset buy backs and debt guarantees, and buyback programs are strictly worse when there is adverse selection across asset classes, in addition to asymmetric information at the bank level.

Comparing voluntary and compulsory programs, we find that compulsory programs are more likely to be efficient if the intervention is large. We also show that deposit insurance reduces the expected cost of all government interventions. In the limit case in which deposits are always risky, the benefit of a bailout accrues to the government itself and the optimal solution is therefore to implement the first best at negative expected cost. Finally, we solve for the constrained optimum intervention. We find that the government should provide subsidized loans or debt guarantees in exchange for call options on equity.

It is important to keep in mind three caveats when interpreting our results. The first caveat is that we focus on one particular market failure, namely debt overhang. We allow for asymmetric information between the government and the private sector because the private sector is probably better informed about assets values than the government. However, we maintain the assumption that there is symmetric information about asset values within the private sector. We make this assumption because we believe that information asymmetry within the private sector is a separate market failure and we want to isolate the impact of debt overhang from other market distortions (see Philippon and Skreta (2009) for an analysis with adverse selection among private investors).

The second caveat is that we assume that the government faces the same constraints in reducing debt overhang as the private sector. Debt overhang arises because equity holders cannot renegotiate with debt holders and because new investment opportunities cannot be separated from existing assets. We thus assume that the government cannot renegotiate on behalf of shareholders or set up new banks to finance profitable investment opportunities. We do not want to dismiss such solutions but rather analyze a setting in which the government is bound by the same constraints as the private sector. In any case, even if the government decides to fail and restructure some banks, our results would still apply to the surviving ones.

The third caveat is that we do not address the issue of moral hazard regarding future interventions. Government interventions can be harmful if they create expectations of future bailouts. However, to the extent that some interventions are unavoidable, we would argue that the principled interventions that we analyze would create less moral hazard than ad-hoc interventions.

Proof of Theorem 1

We show the equivalence result in the binary case.

Cash against equity at time 0

Suppose the government dilutes existing equity holders. The government offers m in cash against a fraction α of the equity returns. The investment domain $I(m)$ is the same as in the case of pure cash injections. At time 0, we must impose the condition that shareholders are willing to sign in

$$(1-\alpha) E_0 [y^e (m)] \geq E_0 [y^e (0)] \tag{20}$$

The cost of the program is

$$\Psi_0^e (m, \alpha) = m - \alpha E_0 [y^e (m)]$$

Because the domain does not depend on α, it is clear that the government wants to satisfy the participation constraint (20) with equality.

$$\begin{aligned} E_0 [y^e] &= \bar{p} (N + c_0) + \iint_{I^o} L^o (p, v)\, dF (p, v) \\ E_0 [y^e (m)] &= \bar{p} (N + c_0 + m) + \int\int_{I(m)} (L^o (p, v) + (1-p) m)\, dF (p, v) \end{aligned}$$

Eliminating α from the cost function yields $\Psi_0^e (m, \alpha) = m - (E_0 [y^e | m] - E_0 [y^e])$ and

$$E_0 [y^e (m)] - E_0 [y^e] = \bar{p} m + m \int\int_{I(m)} (1-p)\, dF (p, v) + \int\int_{I(m) \setminus I^o} L^o (p, v)\, dF (p, v)$$

The expected cost of the optimally designed program is $\Lambda_0 (m)$ defined in equation (19).

Asset buy back at date 0

Under the buy back program $c_1 = c_0 + p^z Z$. Hence the investment domain becomes $I(p^z Z)$. The expected shareholder value at date 0 is

$$E_0 [y^e (z, p^z)] = \bar{p} (N + c_0 - (1-p^z) Z) + \int\int_{I(p^z Z)} (L^o (p, v) + (1-p) p^z Z)\, dF(p, v)$$

The bank's participation constraint is

$$E_0 [y^e (Z, p^z)] \geq E_0 [y^e (0, 0)] . \tag{21}$$

The government wants to satisfy participation constraint (21) with equality. We get

$$\bar{p} (1-p^z) Z = Z p^z \int\int_{I(p^z Z)} (1-p)\, dF(p, v) + \int\int_{I(p^z Z) \setminus I^o} L^o (p, v)\, dF (p, v)$$

Therefore the cost is

$$\begin{aligned}\Psi_0^a(Z,p^z) &= Zp^z - Z\bar{p} \\ &= (1-\bar{p})\,Zp^z - p^z Z \int\int_{I(p^z Z)} (1-p)\,dF(p,v) - \int\int_{I(p^z Z)\setminus I^o} L^o(p,v)\,dF(p,v) \\ &= \Lambda_0(Zp^z)\end{aligned}$$

This program is equivalent to the cash against equity program at date 0 when $m = Zp^z$. QED.

Debt guarantee at date 0

Under the program $c_1 = c_0 + (1-\phi)\,S$. Hence the investment domain becomes $I((1-\phi)\,S)$. The expected shareholder value at date 0 is

$$E_0\left[y^e(z,p^z)\right] = \bar{p}\,(N + c_0 - \phi S) + \int\int_{I((1-\phi)S)} \left(L^o(p,v) + (1-p)(1-\phi)\,S\right) dF(p,v)$$

This is equivalent to the asset buy-back program if we set $S = Z$ and $p^z = 1-\phi$.

A Proof of Theorem 2

We show the equivalence result in the case of a general distribution for asset value a.

A.1 Debt Overhang

As a benchmark, we first solve the model without government interventions. Banks invest if and only if

$$\begin{aligned}E[y^e|p,1] &> E[y^e|p,0] \\ \int_{D+rl-v}^{A} (a - D + v - rl)\,dG(a|p) &\geq \int_{D-c}^{A} (a - D + c)\,dG(a|p)\end{aligned}$$

which is equivalent to

$$v - rl > c \Longleftrightarrow v - c > r(x-c) \Longleftrightarrow v - x > (r-1)(x-c)$$

This is the same condition as in the model with binary payoffs. The break even constraint for junior creditors is

$$\begin{aligned}E[y^l|p] &\geq l \\ rl\int_{D+rl-v}^{A} dG(a|p) + \int_{D-v}^{D+rl-v} (a+v-D)\,dG(a|p) &\geq l\end{aligned}$$

Using assumption A1, this condition is binding and pins down the interest rate

$$r(p,v,D,c)\,.$$

The only difference to the binary payoff model is that the interest rate also depends on v.

Adding the two conditions yields the investment condition

$$\int_{D-v}^{A} (a+v-D)\, dG(a|p) \geq l + \int_{D-c}^{A} (a+c-D)\, dG(a|p).$$

Using $l = x - c$, we rearrange the terms to get the investment region

$$\begin{aligned} L^o(p,v) &= \int_{D-v}^{A} (a+v-D)\, dG(a|p) - \int_{D-c_0}^{A} (a+c_0-D)\, dG(a|p) - x + c_0 \\ &= \end{aligned}$$

$$L^o(p,v) = \underbrace{v\int_{D-c}^{A} dG(a|p)}_{\text{full NPV region}} + \underbrace{\int_{D-v}^{D-c} (a+v-D)\, dG(a|p)}_{\text{partial NPV region created by } v>c} + \underbrace{c\int_{0}^{D-c} dG(a|p)}_{\text{cash diverted from debt holders}} - x \tag{22}$$

We can also compute the transfer to senior debt holders in the investment region

$$\begin{aligned} T^o(p,v) &\equiv E[y^D|p,1] - E[y^D|p,0] \\ &= \int_0^{D-v} (a+v)\, dG(a|p) + D\int_{D-v}^{A} dG(a|p) - \int_0^{D-c_0} (a+c_0)\, dG(a|p) - D\int_{D-c_0}^{A} dG(a|p) \\ &= \underbrace{\int_0^{D-v} (v-c_0)\, dG(a|p)}_{\text{Debt Default Region w/ Inv}} + \underbrace{\int_{D-v}^{D-c_0} (D-a-c_0)\, dG(a|p)}_{\text{Debt Default Region w/o Inv}} \end{aligned}$$

Note that investment has two effects on senior debt holders. First, it reduces the probability of debt default by $P(D-v < a < D-c_0)$. Second, it increases the payoff in the case of default. Note that adding the investment payoff to equity holders $L^o(p,v)$ and debt holders $T^o(p,v)$ yields the investment payoff $(v-x)$.

Now define the piecewise linear function

$$\zeta(a) \equiv c1_{[0,D-v]} + (v+a+c-D)\, 1_{[D-v,D-c]} + v1_{[D-c,A]}.$$

The investment condition can be written as $L^o(p,v) \geq 0$ where

$$L^o(p,v) = E[\zeta(a)\,|p] - x.$$

The function ζ is increasing over $[0,A]$, therefore FOSD of $G(a|p)$ implies that $L^o(p,v)$ is increasing in p. So there is a unique cutoff $\hat{p}$ such that

$$L^o(\hat{p},v) = 0.$$

It is also straightforward to see that $\zeta(a)$ is increasing in c, which implies that $L^o(p,v)$ is increasing in c. Therefore

$$\frac{d\hat{p}}{dc} = -\frac{L^o_c}{L^o_p} < 0.$$

Shareholder value in the investment region is

$$\begin{aligned}E_1\left[y^e|p,v\right] &= E_1\left[y|p,v\right]-E_1\left[y^D|p,v\right]-E_1\left[y^l|p,v\right]\\&= a+v-\int_0^{D-v}(a+v)\,dG(a|p)-D\int_{D-v}^{A}dG(a|p)-(x-c_0)\\&= \int_{D-v}^{A}(a+v-D)\,dG(a|p)-(x-c_0)\\&= \int_{D-c}^{A}(a+c-D)\,dG(a|p)+L^o(p,v)\end{aligned}$$

Expected shareholder value at time 1 is

$$E_1\left[y^e|p,v\right]=\int_{D-c}^{A}(a+c-D)\,dG(a|p)+L^o(p,v)\,1_{(p,v)\in I^o}$$

Expected shareholder value at time 0 is

$$E_0\left[y^e\right]=\int_{D-c}^{A}(a+c-D)\,dF_0(a)+\iint_{I^o}L^o(p,v)\,dF(p,v)$$

Senior debt holder value at time 1 is

$$E_1\left[y^D|p,v\right]=\int_0^{D-c}(a+c)\,dG(a|p)+D\int_{D-c}^{A}dG(a|p)+T^o(p,v)\,1_{(p,v)\in I^o}$$

Senior debt holder value at time 0 is

$$E_0\left[y^D\right]=\int_0^{D-c}(a+c)\,dF_0(a)+D\int_{D-c}^{A}dF_0(a)+\iint_{I^o}T^o(p,v)\,dF(p,v)$$

A.2 Equity injection

Suppose the government purchases equity share α in exchange for capital injection m. All equations to the debt overhang analysis still apply except $c=c_0+m$. Hence, expected shareholder value at time 0 is

$$E_0\left[y^e(m)\right]=\int_{D-c_0-m}^{A}(a+c_0+m-D)\,dF_0(a)+\iint_{I(m)}L^m(p,v)\,dF(p,v)$$

with

$$L^m(p,v)\equiv v\int_{D-c_0-m}^{A}dG(a|p)+\int_{D-v}^{D-c_0-m}(a+v-D)\,dG(a|p)+(c_0+m)\int_0^{D-c_0-m}dG(a|p)-x$$

The cost to the government is

$$\Psi(m)=m-\alpha E_0\left[y^e(m)\right].$$

Assuming the participation constraint for equity holders is binding, we have

$$\alpha E_0\left[y^e(m)\right] = E_0\left[y^e(m)\right] - E_0\left[y^e(0)\right]$$

and

$$\begin{aligned} E_0\left[y^e(m)\right] - E_0\left[y^e(0)\right] &= m\int_{D-c_0}^{A} dF_0(a) + \int_{D-c_0-m}^{D-c_0} (a+m+c_0-D)\, dF_0(a) \\ &+ \iint_{I(m)} L^m(p,v)\, dF(p,v) - \iint_{I^o} L^o(p,v)\, dF(p,v) \end{aligned}$$

Note that the transfer to senior debt holder is

$$\begin{aligned} E_0\left[y^D(m)\right] - E_0\left[y^D(0)\right] &= m\int_{0}^{D-c_0-m} dF_0(a) + \int_{D-c_0-m}^{D-c_0} (D-a-c_0)\, dF_0(a) \\ &+ \iint_{I(m)} T^m(p,v)\, dF(p,v) - \iint_{I^o} T^o(p,v)\, dF(p,v) \end{aligned}$$

where

$$T^m(p,v) = \int_0^{D-v} (v-c_0-m)\, dG(a|p) + \int_{D-v}^{D-c_0-m} (D-a-c_0-m)\, dG(a|p)$$

Also note that

$$T^m(p,v) + L^m(p,v) = v - x$$

and

$$E_0\left[y^e(m)\right] - E_0\left[y^e(0)\right] + E_0\left[y^D(m)\right] - E_0\left[y^D(0)\right] = m + (v-x)\int\int_{I(m)/I^0} dF(p,v)$$

Hence, we can interpret the cost of government intervention as

$$\Psi(m) = \underbrace{E_0\left[y^D(m)\right] - E_0\left[y^D(0)\right]}_{\text{Transfer to Senior Debt Holders}} - \underbrace{(v-x)\int\int_{I(m)/I^0} dF(p,v)}_{\text{Welfare Gain from Extra Investment}}$$

A.3 Asset buyback

To compute the cost of asset buy backs, it is important to specify the exact structure of debt covenants.

We define efficient debt covenants as follows. We assume banks starts with long-term assets with random payoffs a. The bank can sell assets to the government with bank payoff $a-\beta(a)$ and government payoff is $\beta(a)$. Debt covenants are restrictions on the function $\beta(a)$. We assume covenants are efficient if and only if for any distribution F, debt holders receive at least the expected payoff they would receive without asset buy backs:

$$\beta \text{ acceptable iff } \int_0^A \min(a+c-\beta(a), D)\, dF(a) \geq \int_0^A \min(a+c, D)\, dF(a) \geq \text{ for all } F$$

Now suppose there is an $\widehat{a} < D - c$ such that $\beta(\widehat{a}) > 0$. Then choose the distribution $f(\widehat{a}) = 1$. Note that $\widehat{a} + c - \beta(\widehat{a}) < \widehat{a} + c$ which violates the condition. Similarly, if there is an $\widehat{a} > D - c$ with $\widehat{a} + c - \beta(\widehat{a}) < D$, the condition is violated. So the solution is that β must satisfy:

$$\begin{aligned} \beta(a) &= 0 \text{ for all } a \leq D - c \\ \beta(a) &\leq a + c - D \text{ for all } a > D - c \end{aligned}$$

In words, efficient covenants ensure that senior debt holders have priority over asset purchasers for any distribution F. So bank payoff are $\tilde{a} = a - \beta(a)$ with $\beta(a) = 0$ for all $a \leq D - c$. The investment condition for asset buy backs is:

$$\begin{aligned} \int_{D+rl-v}^{D-c} (a + v - D - rl)\, dG(a|p) + \int_{D-c}^{A} (\tilde{a} + v - D - rl)\, dG(a|p) &> \int_{D-c}^{A} (\tilde{a} + c - D)\, dG(a|p) \\ \int_{D+rl-v}^{D-c} (a + v - D - rl)\, dG(a|p) + \int_{D-c}^{A} (v - rl - c)\, dG(a|p) &> 0 \Longleftrightarrow v - rl - c > 0 \end{aligned}$$

The participation constraint for junior creditors is:

$$rl \int_{D+rl-v}^{A} dG(a|p) + \int_{D-v}^{D+rl-v} (a + v - D)\, dG(a|p) \geq l$$

Note that the investment condition is the same as with equity injections and independent of $\tilde{a}$. Hence, the investment region is the same (with $c = c_0 + m$). The cost to the government is

$$\Psi = m - E_0[\beta(a)].$$

Assume that the participation constraint is binding. Note that $\beta(a)$ comes entirely from shareholder payoffs. So participation is simply

$$E_0[\beta(a)] = E_0[y^e(m)] - E_0[y^e]$$

Note that the expected cost to the government is the same as under equity injections.

A.4 Debt guarantee

Under the debt guarantee, net cash injected at time 1 is

$$c = c_0 + (1 - \phi) S$$

Banks invest if and only if

$$\begin{aligned} E[y^e|p, 1] &> E[y^e|p, 0] \\ \int_{D+S+rl-v}^{A} (a + v - D - rl - S)\, dG(a|p) &\geq \int_{D+S-c}^{A} (a + c - D - S)\, dG(a|p) \end{aligned}$$

So the investment condition from the perspective of shareholders is just $v - rl > c$ like before. Now it is important the the government is junior to time 1 creditors, otherwise the government creates its own debt overhang. In this case we get

$$\begin{aligned} E[y^l|p] &\geq l \\ rl \int_{D+rl-v}^{A} dG(a|p) + \int_{D-v}^{D+rl-v} (a + v - D)\, dG(a|p) &\geq x - c \end{aligned}$$

Note that this means that $r(p, v, D, c_0 + m)$ is the same as in the equity-injection case. In particular it does not depend on S. Hence, the investment region is the same. Regarding the costs of the program, there are two alternative interpretations. The first one is that the bank borrows directly from government. Then government recovers

$$\int_{(p,v)\in I(m)} \int_{D+rl-v}^{A} \min(a + v - D - rl, S)\, dG(a|p) + \int_{(p,v)\notin I(m)} \int_{D-c}^{A} \min(a + c - D, S)\, dG(a|p)$$

which is equivalent to $E_0[y^e(m, 0)] - E_0[y^e(m, S)]$ because the payoffs with shareholder add up to total shareholders without debt guarantee S. The binding participation constraint implies that

$$E_0[y^e(m, S)] = E_0[y^e(0)]$$

so

$$\Psi = (1 - \phi) S - (E_0[y^e(m, 0)] - E_0[y^e(0)]) = m - (E_0[y^e(m)] - E_0[y^e])$$

which is the same costs as for the other interventions.

An alternative interpretation is that the government covers losses ex-post. This yields the same calculation because the government gets ϕS up-front and then pays S minus the recovery in good states which is the same as above because $E_0[y^e(m, 0)] - E_0[y^e(m, S)]$.

Proof of Theorem 3

Let us analyze the asset buy-back program at date 1. To prove the theorem, we must show equivalence along 4 dimensions: (i) the NIP constraint, (ii) the upper schedule, (iii) the lower schedule, and (iv) the cost function.

Upon participation and investment, total equity value becomes

$$E_1[y^e(z, p^z)\,|p, v, i = 1] = p(N + c_0 - z) + L^o(p, v) + p^z z$$

Participation without investment yields

$$E_1[y^e(z, p^z)\,|p, v, i = 0] = p(N + c_0 - z + p^z z)$$

Now consider the three critical constraints:

- NIP: $E_1[y^e(z, p^z)\,|p, v, i = 0] < E_1[y^e(0, 0)\,|p, v, i = 0]$ or:

$$p^z < 1$$

- Upper schedule: $E_1[y^e(0, 0)\,|p, v, i = 1] > E_1[y^e(z, p^z)\,|p, v, i = 1]$ or:

$$p > p^z.$$

- Lower schedule: $E_1[y^e(z, p^z)\,|p, v, i = 1] > E_1[y^e(0, 0)\,|p, v, i = 0]$

$$L_1^a(p, v; z, p^z) = L^o(p, v) + (p^z - p)\, z.$$

It is therefore clear that z is either 0 or Z. It is also clear that, using the notations of the debt guarantee section, the participation set is simply

$$\Omega_1^g\left(Z, 1-p^z\right)$$

where Ω_1^g was defined above in equation (16). The expected cost of the program is therefore

$$\Psi_1^a\left(Z, p^z\right) = Z \int\int_{\Omega^g(Z,1-p^z)} \left(p^z - p\right) dF\left(p, v\right) = \Lambda_1^g\left(Z, 1-p^z\right)$$

and the investment domain is

$$I^o \cup \Omega_1^g\left(Z, 1-p^z\right)$$

Now if we set $S = Z$ and $p^z = 1 - \phi$, we see that the NIP constraint, the upper and lower schedules, and the cost function are the same for the asset buy back program as for the debt guarantee program. They are therefore equivalent.

Proof of Theorem 4

Let us analyze equity injections at date 1. Upon participation and investment, total equity value (including the share going to the government) becomes

$$E_1[y^e\left(m\right)|p, v, i = 1] = p\left(N + c_0\right) + L^o\left(p, v\right) + m$$

Participation without investment yields

$$E_1[y^e\left(m\right)|p, v, i = 0] = p(N + c_0 + m)$$

Now consider the three critical constraints

- NIP: $\left(1-\alpha\right) E_1[y^e\left(m\right)|p, v, i = 0] < E_1[y^e\left(0\right)|p, v, i = 0]$ or:

$$\left(1-\alpha\right) m < \alpha\left(N + c_0\right)$$

- Upper schedule: $E_1[y^e\left(0\right)|p, v, i = 1] > \left(1-\alpha\right) E_1[y^e\left(m\right)|p, v, i = 1]$ or:

$$\alpha\left(p\left(N + c_0\right) + L^o\left(p, v\right)\right) > \left(1-\alpha\right) m$$

- Lower schedule: $\left(1-\alpha\right) E_1[y^e\left(m\right)|p, v, i = 1] > E_1[y^e\left(0\right)|p, v, i = 0]$ or:

$$\left(1-\alpha\right)\left(L^o\left(p, v\right) + m\right) > \alpha p\left(N + c_0\right)$$

Now define the function

$$X\left(p; m, \alpha\right) \equiv \left(1-\alpha\right) m - \alpha p\left(N + c_0\right)$$

We can summarize the equity program by:

$$\begin{aligned} L_1^e\left(p, v; m, \alpha\right) &\equiv \left(1-\alpha\right) L^o\left(p, v\right) + X\left(p; m, \alpha\right) \\ U_1^e\left(p, v; m, \alpha\right) &\equiv \alpha L^o\left(p, v\right) - X\left(p; m, \alpha\right) \\ NIP &: X\left(1; m, \alpha\right) < 0 \end{aligned}$$

The participation becomes

$$\Omega_1^e(m,\alpha) = \{(p,v) \mid L_1^e(p,v;m,\alpha) > 0 \wedge U_1^e(p,v;m,\alpha) < 0\}$$

The cost function is therefore

$$\Psi_1^e(m,\alpha) = \int\int_{\Omega_1^e(m,\alpha)} (m - \alpha E_1[y^e(m,\alpha)\,|p,v,i=1])\,dF(p,v)$$

We can rewrite this in the convenient and intuitive form

$$\Psi_1^e(m,\alpha) = \int\int_{\Omega_1^e(m,\alpha)} X(p;m,\alpha)\,dF(p,v) - \alpha\int\int_{\Omega_1^e(m,\alpha)} L^o(p,v)\,dF(p,v)$$

The following table provides a comparison of the three programs:

	Debt guarantee	Asset buy back	Equity injection
Participation	$\Omega_1^g(S,\phi)$	$\Omega_1^g(Z,1-p^z)$	$\Omega_1^e(m,\alpha)$
Investment I_1	$I^o \cup \Omega_1^g(S,\phi)$	$I^o \cup \Omega_1^g(Z,1-p^z)$	$I^o \cup \Omega_1^e(m,\alpha)$
NIP constraint	$\phi > 0$	$p^z < 1$	$X(1,m,\alpha) < 0$
Cost function	$\Lambda_1^g(S,\phi)$	$\Lambda_1^g(Z,1-p^z)$	$\Psi_1^e(m,\alpha)$

Now let us prove that equity injections dominate the other two programs. Take a program S,ϕ. We are going to construct an equity program that has same welfare gains, and costs less. To get equity with same lower bound graph we need to ensure that:

$$L_1^e(p,v;m,\alpha) = L_1^g(p,v;S,\phi) \text{ for all } p,v$$

So we must have

$$X(p;m,\alpha) = (1-\alpha)(1-\phi-p)\,S \text{ for all } p \tag{23}$$

It is easy to see that this is indeed possible if we identify term by term: $\frac{\alpha}{1-\alpha} = \frac{S}{A+c_0-D}$ and $m = (1-\phi)\,S$. Therefore it is possible to implement exactly the same lower schedules. Formally, we have just shown that:

$$I_1^g(S,\phi) = I_1^e(m,\alpha).$$

Next notice that the NIP constraints are equivalent since:

$$X(1,m,\alpha) < 0 \Longleftrightarrow \phi > 0.$$

Now consider the upper bound. Consider the lowest point on the upper schedule of the guarantee program, i.e., the intersection of $U_1^g(p,v;S,\phi) = 0$ with $L^o(p,v) = 0$.

At that point, we have $\tilde{p} = 1 - \phi$ and $\tilde{v} = (x - \phi c_0) / (1 - \phi)$. But from (23), it is clear that $X(\tilde{p}; m, \alpha) = 0$, and therefore $U_1^e(\tilde{p}, \tilde{v}; m, \alpha) = \alpha L^o(\tilde{p}, \tilde{v}) - X(\tilde{p}; m, \alpha) = 0$. Therefore the upper schedule $U_1^e(p, v; m, \alpha) = 0$ also passes by this point. But the schedule $U_1^e(p, v; m, \alpha) = 0$ is downward slopping in (p, v), so the domain of inefficient participation is smaller (see Figure 7) than in the debt guarantee case. Formally, we have just shown that:

$$\Omega_1^e(m, \alpha) \subset \Omega_1^g(S, \phi).$$

As an aside, it is also easy to see that the schedule $U_1^e(p, v; m, \alpha) = 0$ is above the schedule $L^o(p, v) = 0$ so it does not get rid completely of opportunistic participation, but it helps.

The final step is to compare the cost functions.

$$\Lambda_1^g(S, \phi) \equiv S \int\int_{\Omega_1^g(S,\phi)} (1 - p - \phi)\, dF(p, v)$$

$$\Psi_1^e(m, \alpha) = \int\int_{\Omega_1^e(m,\alpha)} X(p; m, \alpha)\, dF(p, v) - \alpha \int\int_{\Omega_1^e(m,\alpha)} L^o(p, v)\, dF(p, v)$$

By definition of the participation domain, we know that $L_1^e(p, v; m, \alpha) > 0$. Therefore:

$$-\int\int_{\Omega_1^e(m,\alpha)} L^o(p, v)\, dF(p, v) < \frac{X(p; m, \alpha)}{1 - \alpha} \text{ for all } (p, v) \in \Omega_1^e(m, \alpha)$$

Therefore

$$\Psi_1^e(m, \alpha) < \frac{1}{1 - \alpha} \int\int_{\Omega_1^e(m,\alpha)} X(p; m, \alpha)\, dF(p, v) = S \int\int_{\Omega_1^e(m,\alpha)} (1 - \phi - p)\, dF(p, v)$$

Finally, since $1 - \phi - p > 0$ for all $(p, v) \in \Omega_1^e(m, \alpha)$, and since $\Omega_1^e(m, \alpha) \subset \Omega_1^g(S, \phi)$, we have

$$\Psi_1^e(m, \alpha) < \Lambda_1^g(S, \phi).$$

QED.

Proof of proposition 4

Date 0 Cost Function

Define $v_0^m(p)$ by

$$L^o(p, v_0^m(p)) + (1 - p)\, m = 0$$

Then define the ex-post cost function:

$$\zeta(v, p) \equiv pm_{[0, v_0^m(p)]} + (L^o(p, v) + (1 - p)m))\, 1_{[v_0^m(p), v_0(p)]} + m 1_{[v_0(p), 1]}$$

Note that we can rewrite the cash-against-equity cost function as

$$\begin{aligned} \Lambda_0(m; F) &= m - \iint_{p,v} \zeta(v, p)\, dF(p, v) \\ &= m - E[\zeta(v, p) | F] \end{aligned}$$

The function ζ is increasing in v and in p, therefore FOSD of $\tilde{F}$ on F, implies that $E\left[\zeta(v,p)\,|\tilde{F}\right] > E\left[\zeta(v,p)\,|F\right]$. Hence, $\Lambda_0\left(m;\tilde{F}\right) < \Lambda_0(m;F)$.

Date 1 can dominate

Choose any date 0 program, with cash m and optimal cost $\Lambda_0(m)$. Choose $\phi = 0$ and $S = m$ to get same investment set. Then,

$$\begin{aligned}
\Lambda_1^g(m,0) &= m\iint_{I(m)} (1-p)\,dF(p,v) \\
\Lambda_0(m) &= m\iint_{T\setminus I(m)} (1-p)\,dF(p,v) - \iint_{I(m)\setminus I^o} L^o(p,v)\,dF(p,v)
\end{aligned}$$

Where $T = [0,V]\times[0,1]$. Clearly, we have $\Lambda_1^g(m,0) < \Lambda_0(m)$ when $\iint_{I(m)} dF(p,v)$ is small enough. Just because of the fact that date 1 program do not give away money to banks that do not need it. This is *a fortiori* true for equity injection since they dominate at date 1.

Date 0 can dominate

Consider a very large government program such that all firms invest. Then it must be that $Zp^z = x - c_0$ and $p^z = 1$ (see Figure). Then choose $m = x - c_0$. Then $I(m) = I^* = \Omega(Z, p^z)$. Now

$$\begin{aligned}
\Lambda_1^g(Z,0) &> \Lambda_0(m) \\
&\Longleftrightarrow \\
(x-c_0)\iint_{I^*} (1-p)\,dF(p,v) &> (x-c_0)\iint_{T\setminus I^*} (1-p)\,dF(p,v) - \iint_{I^*\setminus I^o} L^o(p,v)\,dF(p,v)
\end{aligned}$$

Now clearly when $\Pr(I^*) \to 1$, then $\Pr(T\setminus I^*) \to 0$, so $(x-c_0)\iint_{T\setminus I^*} (1-p)\,dF(p,v) \to 0$.

Over I^*, we know that $v > x$, hence $-L^o(p,v) < (1-p)(x-c_0)$, therefore

$$-\iint_{I^*\setminus I^o} L^o(p,v)\,dF(p,v) < (x-c_0)\iint_{I^*\setminus I^o} (1-p)\,dF(p,v) \le (x-c_0)\iint_{I^*} (1-p)\,dF(p,v)$$

QED.

B Proof of proposition 6

First note that the optimization problem from the equity holders perspective remains unchanged because the investment and participation decision only depend on total debt D.Now consider the expected cost of deposit insurance. Note that the date 0 expected value of deposits is Δ because $\Delta \le \underline{A} + c_0$ Hence, the cost of government intervention is unchanged and therefore the cost and benefits of both date 0 and date 1 programs remain unchanged.

C Proof of proposition 7

Date 0 Programs

Cash against equity injection

- Full Transfer: $\underline{A} + v < \Delta$

  The date 1 expected value of deposits is

$$\begin{aligned} E_1\left[y^{\Delta}(m)\,|p,v\right] &= p\Delta + (1-p)\left(\underline{A} + c_0 + m\right) \text{ if } (p,v) \in T\backslash I_0(m) \\ &= p\Delta + (1-p)\left(\underline{A} + v\right) \text{ if } (p,v) \in I_0(m) \end{aligned}$$

The date 0 expected value of deposits is

$$\begin{aligned} E_0\left[y^{\Delta}(m)\right] &= \overline{p}\Delta + (1-\overline{p})\,\underline{A} + \iint\limits_{T\backslash I_0(m)} (1-p)(c_0+m)\,dF(p,v) + \iint\limits_{I_0(m)} (1-p)\,v dF(p,v) \\ &= \overline{p}\Delta + (1-\overline{p})\left(\underline{A} + c_0 + m\right) + \iint\limits_{I_0(m)} (1-p)(v - c_0 - m)\,dF(p,v) \end{aligned}$$

Assume the FDIC provides deposit insurance, i.e. the FDIC covers the face value of deposits. The date 0 expected cost of deposit insurance is

$$\begin{aligned} \Psi_0^F(m) &= \Delta - E_0\left[y^{\Delta}(m)\right] \\ &= (1-\overline{p})\left(\Delta - \underline{A} - c_0 - m\right) - \iint\limits_{I_0(m)} (1-p)(v - c_0 - m)\,dF(p,v) \end{aligned}$$

The date 0 cost of government intervention without accounting for FDIC is $\Lambda_0(m)$ defined earlier. Now consider the change in the expected cost of deposit insurance

$$\begin{aligned} \Lambda_0^F(m) &= \Psi_0^F(m) - \Psi_0^F(0) \\ &= -(1-\overline{p})\,m + m\iint\limits_{I_0(m)} (1-p)\,dF(p,v) - \iint\limits_{I_0(m)\backslash I^o} (1-p)(v-c_0)\,dF(p,v) \end{aligned}$$

For simplicity, assume that the FDIC and the government have the same marginal deadweight loss of raising taxes. The net cost of government intervention is

$$\Lambda_0(m) + \Lambda_0^F(m) = -\iint\limits_{I_0(m)\backslash I^o} (v-c_0)\,dF(p,v)$$

Note that this term is the expected benefit from investments taken because of the government intervention.

- Partial Transfer: $\underline{A} + c_0 < \Delta < \underline{A} + v$

The date 1 expected value of deposits is

$$
\begin{aligned}
E_1\left[y^{\Delta}(m)\,|p,v\right] &= p\Delta + (1-p)\max\left(\Delta, \underline{A} + c_0 + m\right) \text{ if } (p,v) \in T \backslash I_0(m) \\
&= \Delta \text{ if } (p,v) \in I_0(m)
\end{aligned}
$$

The date 0 expected value of deposits is

$$
\begin{aligned}
E_0\left[y^{\Delta}(m)\right] &= \Delta \Pr\left(I_0(m)\right) + \Pr\left(T \backslash I_0(m)\right)\overline{p}\Delta + \int\int_{T \backslash I_0(m)} (1-p)\max\left(\Delta, \underline{A} + c_0 + m\right) dF(p,v) \\
&= \Delta - \int\int_{T \backslash I_0(m)} (1-p)\left(\Delta - \max\left(\Delta, \underline{A} + c_0 + m\right)\right) dF(p,v)
\end{aligned}
$$

The expected cost of deposit insurance is

$$
\Psi_0^F(m) = \int\int_{T \backslash I_0(m)} (1-p)\left(\Delta - \max\left(\Delta, \underline{A} + c_0 + m\right)\right) dF(p,v)
$$

Now consider the change in the expected cost of deposit insurance

$$
\Lambda_0^F(m) = \int\int_{T \backslash I_0(m)} (1-p)\left(\Delta - \max\left(\Delta, \underline{A} + c_0 + m\right)\right) dF(p,v) - \int\int_{T \backslash I^o} (1-p)\left(\Delta - \underline{A} - c_0\right) dF(p,v)
$$

Note that when $\Delta \to (\underline{A} + c_0)$, then $\Lambda_0^F(m) \to 0$. This means the expected change in the cost of deposit insurance goes to zero as deposits become safe. Also note that when $\Delta \to (\underline{A} + v)$, then

$$
\Lambda_0^F(m) \to -(1-\overline{p})\, m + m \int\int_{I_0(m)} (1-p)\, dF(p,v) - \int\int_{I_0(m) \backslash I^o} (1-p)(v - c_0)\, dF(p,v)
$$

which is the change in expected cost of deposit insurance in the full transfer case. The government cost is $\Lambda_0^F(m) + \Lambda_0(m)$.

The net cost of date 0 debt guarantees and date 0 asset buy back programs is equivalent because all date 0 programs have the same cost function. Note that the equivalence result depends on the assumption of debt covenants that prevent the sale of safe assets. Without debt covenants, equity holders can sell safe assets and the cost of date 0 asset buy backs changes relative to the other programs because deposit holders may not receive the full value of safe assets $\underline{A}$ in the low-payoff state.

Date 1 Programs

Asset Buy Back Program

- Full Transfer: $\underline{A} + v < \Delta$

Date 1 expected value of deposits is

$$\begin{aligned} E_1\left[y^{\Delta}(Z,p^z)\,|p,v\right] &= p\Delta + (1-p)\left(\underline{A}+c_0\right) \text{ if } (p,v)\in T\backslash\left(I^o\cup\Omega_1^g(Z,1-p^z)\right) \\ &= p\Delta + (1-p)\left(\underline{A}+v\right) \text{ if } (p,v)\in I^o\cup\Omega_1^g(Z,1-p^z) \end{aligned}$$

Date 0 expected value of deposits is

$$E_0\left[y^{\Delta}(Z,p^z)\right] = \overline{p}\Delta + (1-\overline{p})\left(\underline{A}+c_0\right) + \int\int_{I^o\cup\Omega_1^g(Z,1-p^z)} (1-p)(v-c_0)\,dF(p,v)$$

The expected cost of deposit insurance is

$$\Psi_0^F(Z,p^z) = (1-\overline{p})\left(\Delta-\underline{A}-c_0\right) - \int\int_{I^o\cup\Omega_1^g(Z,1-p^z)} (1-p)(v-c_0)\,dF(p,v)$$

The change in the cost of deposit insurance is

$$\Lambda_0^F(Z,p^z) = -\int\int_{\Omega_1^g(Z,1-p^z)/I^o} (1-p)(v-c_0)\,dF(p,v)$$

Expected government cost is

$$\Psi_1^a(Z,p^z) = Z\int\int_{\Omega_1^g(Z,1-p^z)} (p^z-p)\,dF(p,v) = \Lambda(Z,1-p^z) - \int\int_{\Omega_1^g(Z,1-p^z)/I^o} (1-p)(v-c_0)\,dF(p,v)$$

- Partial Transfer: $\underline{A}+c_0<\Delta<\underline{A}+v$:

  The date 1 expected value of deposits is

$$\begin{aligned} E_1\left[y^{\Delta}(Z,p^z)\,|p,v\right] &= p\Delta + (1-p)\left(\underline{A}+c_0\right) \text{ if } (p,v)\in T\backslash\left(I^o\cup\Omega_1^g(Z,1-p^z)\right) \\ &= \Delta \text{ if } (p,v)\in I^o\cup\Omega_1^g(Z,1-p^z) \end{aligned}$$

The date 0 expected value of deposits is

$$E_0\left[y^{\Delta}(Z,p^z)\right] = \Delta - \int\int_{T\backslash\left(I^o\cup\Omega_1^g(Z,1-p^z)\right)} (1-p)\left(\Delta-\underline{A}-c_0\right)dF(p,v)$$

The expected cost of government insurance is

$$\Psi_0^F(Z,p^z) = \int\int_{T\backslash\left(I^o\cup\Omega_1^g(Z,1-p^z)\right)} (1-p)\left(\Delta-\underline{A}-c_0\right)dF(p,v)$$

The change in expected cost of deposit insurance is

$$\Lambda_0^F(Z,p^z) = -\int\int_{\Omega_1^g(Z,1-p^z)/I^o} (1-p)\left(\Delta-\underline{A}-c_0\right)dF(p,v)$$

Note that when $\Delta \to (\underline{A} + c_0)$, then $\Lambda_0^F(Z, p^z) \to 0$. Also note that when $\Delta \to (\underline{A} + v)$, then

$$\Lambda_0^F(Z, p^z) \to -\int\int_{\Omega_1^g(Z,1-p^z)/I^o} (1-p)(v - c_0)\, dF(p, v)$$

Total government cost is

$$\Psi_1^a(Z, p^z) = Z \int\int_{\Omega_1^g(Z,1-p^z)} (p^z - p)\, dF(p, v) = \Lambda(Z, 1-p^z) - \int\int_{\Omega_1^g(Z,1-p^z)/I^o} (1-p)(\Delta - \underline{A} - c_0)\, dF(p, v)$$

The results also apply to date 1 debt guarantees because date 1 asset buy backs and date 1 debt guarantees have the same cost function.

Date 1 Equity Injection

Note that we can compute the expected cost of date 1 equity injections similarly to the date 1 asset buy back program. The only difference is the participation region is capital 1 cash injection $\Omega^e(m, \alpha)$ and the participation region in date 1 asset buy back $\Omega_1^g(Z, 1-p^z)$. It turns out that the change in the expected cost of deposit insurance $\Lambda_0^F(m)$ is equivalent under both programs because both in the full and partial transfer case the difference in the participation region cancels out when computing the difference in expected cost of deposit insurance. It follows that the relative ranking of programs is unchanged because all programs have the same reduction in costs due to deposit insurance.

Optimal programs

Proof of proposition 8

Just to illustrate the logic, let us compare the date 0 debt guarantee with the optional date 0 debt guarantee. Participation is decided at date 0. Banks give an equity share α in exchange for the right (not the obligation) to use the debt guarantee program (S, ϕ) at date 1. The increase in shareholder value is

$$\begin{aligned} E_0[y^e(\Gamma)] - E_0[y^e] &= \int\int_{I_1^g(S,\phi)} L_1^g(p, v; S, \phi)\, dF(p, v) - \iint_{I^o} L^o(p, v)\, dF(p, v) \\ &= S \int\int_{I_1^g(S,\phi)} (1 - \phi - p)\, dF(p, v) + \int\int_{I_1^g(S,\phi)\setminus I^o} L^o(p, v)\, dF(p, v) \\ &= \Lambda_1^g(S, \phi) + \int\int_{I_1^g(S,\phi)\setminus I^o} L^o(p, v)\, dF(p, v) \end{aligned}$$

If the government asks for equity ex-ante, then the net cost to the government is

$$\tilde{\Lambda}_0(S, \phi) = -\int\int_{I_1^g(S,\phi)\setminus I^o} L^o(p, v)\, dF(p, v)$$

To compare with:

$$\Lambda_0(m) = m\int\int_{T\backslash I(m)} (1-p)\,dF(p,v) - \int\int_{I(m)\backslash I^o} L^o(p,v)\,dF(p,v)$$

Where $T = [0, V] \times [0,1]$. So it is clear that with $s = m$, the investment domains are the same, and the cost saving is

$$\Lambda_0(s) - \tilde{\Lambda}_0(S,\phi) = m\int\int_{T\backslash I(m)} (1-p)\,dF(p,v) - \int\int_{I(m)\backslash I^o} L^o(p,v)\,dF(p,v) + \int\int_{I_1^g(S,\phi)\backslash I^o} L^o(p,v)\,dF(p,v)$$

So it is clear that the ex-ante optional program strictly dominates in all cases. First, one can always set $\phi = 0^+$ and $S = m$ in which case $I_1^g(S,\phi) = I(m)$ and the cost reduction is

$$m\int\int_{T\backslash I(m)} (1-p)\,dF(p,v)$$

which corresponds to idle cash wasted on banks that do not make new investment. In addition, the optional program allows for greater flexibility in the design on the investment set. In particular, Figure 6 shows that the optional program is better at getting the high v in the low p region without admitting a lot of low v in the high p region.

Proof of proposition 9

Let us show that pure equity dominates. Let m be total money injection, sum of $m\prime$ from equity and $p^z Z$ from asset buy-back. Now define the function

$$X(p) \equiv (1-\alpha)(m - pZ) - \alpha p (N + c_0)$$

The usual calculations lead to

$$\begin{aligned} L &\equiv (1-\alpha) L^o(p,v) + X(p) \\ U &\equiv \alpha L^o(p,v) - X(p) \\ NIP &: X(1) < 0 \end{aligned}$$

The participation becomes

$$\Omega = \{(p,v) \mid L > 0 \wedge U < 0\}$$

The cost function is therefore

$$\Psi = \iint_{\Omega} X(p)\,dF(p,v) - \alpha\iint_{\Omega} L^o(p,v)\,dF(p,v) \tag{24}$$

Now take any program. To get the same investment curve, we need the same lower bound, and therefore the same function $X(p)$. But then we now from (24) that the cost function is the same. Also we know that the NIP constraint is $X(1) < 0$, so it is also the same. Thus, all that matters is the participation domain Ω. So we need only to look at the upper bound U. We want to exclude as many banks as possible, so we want U to be as high as possible. The way to do so is obviously to have α as high as possible. But of course we must keep the function $X/(1-\alpha)$ constant. Therefore we must keep $Z + \frac{\alpha}{1-\alpha}(N + c_0)$ constant. As α goes up, Z must go down. Therefore we want to set $Z = 0$. Therefore asset buy back cannot improve the equity program.

The same exact proof works for debt guarantees. QED.

Proof of Proposition 10

In the good state, the residual payoffs conditional on investment are

$$N + c_0 + m + \frac{L^o(p,v) + (1-p)m}{p}$$

The loan gets repaid first, then shareholders get

$$\begin{aligned} y^e &= \max\left(N + c_0 + \frac{L^o(p,v) + (1-p)m}{p} - hm, 0\right) \text{ if } i = 1 \text{ and } a = A \\ y^e &= \max(N + c_0 - hm, 0) \text{ if } i = 0 \text{ and } a = A \end{aligned}$$

As soon as $y^e > N + c_0$, the options are in the money and the number of shares jumps to $1 + \frac{1-\varepsilon}{\varepsilon} = \frac{1}{\varepsilon}$. So the old shareholders get only a fraction ε of the value beyond $N + c_0$. Their payoff function is therefore:

$$f(y^e) = \min(y^e, N + c_0) + \varepsilon \max(y^e - N - c_0, 0)$$

So old shareholders are full residual claimants up to the face value of old assets $N + c_0$ and ε residual claimants beyond. Now let us think about their decisions at time 1. As usual only the payoffs in the non default state matter. If they do not invest they get $N + c_0$. If they do investment, they can get more if and only if $L^o(p,v) + (1-p)m > phm$. The lower participation constraint is therefore

$$L^o(p,v) + (1 - (1+h)p)m > 0$$

It converges to $L^o(p,v) + (1-p)m$ if $h \to 0$. We can compare this to the equity injection schedule $L_1^e(p,v;m,\alpha)$, we can identify the same cash injection m, and the dilution factor

$$\alpha = \frac{m(1+h)}{N + c_0 + m(1+h)}$$

If we compare to debt guarantee $L_1^g(p,v;S,\phi) = L^o(p,v) + (1-\phi-p)S$. Then

$$m = (1-\phi)S \text{ and } h = \frac{\phi}{1-\phi}$$

Next consider the upper schedule. Investing alone gets $N + c_0 + L^o(p,v)/p$ so they opt in if and only if $L^o(p,v) > \varepsilon(L^o(p,v) + (1-(1+h)p)m)$

$$U = L^o(p,v) - m\varepsilon\frac{1-(1+h)p}{1-\varepsilon}$$

It converges to $L^o(p,v)$ when $\varepsilon \to 0$. The NIP constraint is simply

$$h > 0.$$

Finally, the cost of the program is small because the government gets all the upside value of the new projects. The expected payments to the old shareholders converge to $p(N + c_0)$. So the government gets expected value $L^o(p,v) + m$ by paying m at time 1. The total cost is therefore:

$$-\int\int_{I(\Gamma)\backslash I^o} L^o(p,v)\,dF(p,v)$$

It is positive since $L^o(p,v) < 0$ for all $(p,v) \in I(m)\backslash I^o$.

References

ACHARYA, V., AND D. BACKUS (2009): "Private Lessons for Public Banking: The Case for Conditionality in LOLR Facilities," in *Restoring Financial Stability: How to Repair a Failed System*, ed. by Acharya, and Richardson. Wiley.

ACHARYA, V., AND P. SCHNABL (2009): "Securitization without risk transfer," mimeo NYU.

ADRIAN, T., AND H. S. SHIN (2008): "Financial Intermediary Leverage and Value at Risk," Federal Reserve Bank of New York Staff Reports, 338.

AGHION, P., P. BOLTON, AND S. FRIES (1999): "Optimal Design of Bank Bailouts: The Case of Transition Economies," *Journal of Institutional and Theoretical Economics*, 155, 51–70.

ALLEN, F., S. BHATTACHARYA, R. RAJAN, AND A. SCHOAR (2008): "The Contributions of Stewart Myers to the Theory and Practice of Corporate Finance," *Journal of Applied Corporate Finance*, 20, 8–19.

ASQUITH, P., R. GERTNER, AND D. SCHARFSTEIN (1994): "Anatomy of Financial Distress: An Examination of Junk-Bond Issuers," *The Quarterly Journal of Economics*, 109, 625–658.

AUSUBEL, L. M., AND P. CRAMTON (2009): "No Substitute for the "P" Word in Financial Rescue," *Economist's Voice*, 6(2).

AYOTTE, K., AND D. SKEEL (2009): "Bankruptcy or Bailouts?," Working Paper.

BEBCHUK, L. (2008): "A Plan for Addressing the Financial Crisis," Harvard Law and Economics Discussion Paper No. 620.

BERNANKE, B. (2009): "The Crisis and the Policy Response," Stamp Lecture, London School of Economics.

BHATTACHARYA, S., AND A. FAURE-GRIMAUD (2001): "The Debt Hangover: Renegotiation with Non-contractible Investment," *Economics Letters*, 70, 413–419.

COATES, J., AND D. SCHARFSTEIN (2009): "Lowering the Cost of Bank Recapitalization," Working Paper, Harvard University.

DIAMOND, D. (2001): "Should Japanese Banks Be Recapitalized?," *Monetary and Economic Studies*, pp. 1–20.

DIAMOND, D., S. KAPLAN, A. KASHYAP, R. RAJAN, AND R. THALER (2008): "Fixing the Paulson Plan," *Wall Street Journal*.

DIAMOND, D., AND R. RAJAN (2005): "Liquidity Shortages and Banking Crises," *Journal of Finance*, 60(2), 615–647.

DIAMOND, D. W. (1984): "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, 51, 393–414.

FAMA, E. (2009): "Government Equity Capital for Financial Firms," www.dimensional.com/famafrench.

Geithner, T. (2009): "My Plan for Bad Bank Assets," *Wall Street Journal*.

Gorton, G., and L. Huang (2004): "Liquidity, Efficiency and Bank Bailouts," *American Economic Review*, 94, 455–483, NBER WP9158.

Green, R. C. (1984): "Investment Incentives, Debt, and Warrants," *Journal of Financial Economics*, 13, 115–136.

Hart, O., and J. Moore (1995): "Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management," *American Economic Review*, 85(3), 567–585.

Hart, O., and L. Zingales (2008): "Economists Have Abandoned Principle," *Wall Street Journal*.

Heider, F., M. Hoerova, and C. Holthausen (2008): "Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk," Working Paper, European Central Bank.

Ivashina, V., and D. Scharfstein (2008): "Bank Lending During the Financial Crisis of 2008," Harvard Business School Working Paper.

Jensen, M., and W. Meckling (1976): "Theory of the firm: managerial behavior, agency costs and ownership structure," *Journal of Financial Economics*, 3, 305Ű360.

Johnson, S. (2009): "The quiet coup," *The Atlantic*.

Kashyap, A., and T. Hoshi (2008): "Will the U.S. Bank Recapitalization Succeed? Lessons from Japan," NBER Working Paper No. 14401.

Keys, B., T. Mukherjee, A. Seru, and V. Vig (2010): "Did Securitization Lead to Lax Screening? Evidence From Subprime Loans," *Quarterly Journal of Economics*, 125.

Kocherlakota, N. (2009): "Assessing Resolutions of the Banking Crisis," Working Paper.

Mian, A., and A. Sufi (2008): "The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis," forthcoming, Quarterly Journal of Economics.

Modigliani, F., and M. H. Miller (1958): "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review*, 48, 261–297.

Myers, S. C. (1977): "Determinants of Corporate Borrowing," *Jounral of Financial Economics*, 5, 147–175.

Philippon, T., and V. Skreta (2009): "Optimal Bailouts under Adverse Selection," work in progress, NYU Stern.

Scharfstein, D., and J. Stein (2008): "This Bailout Doesn't Pay Dividends," *The New York Times*, October 21.

Soros, G. (2009a): "The Game Changer," *Financial Times*, January 28.

——— (2009b): "The right and wrong way to bail out the banks," *Financial Times*, January 22.

STIGLITZ, J. (2008): "We Aren't Done Yet: Comments on the Financial Crisis and Bailout," *The Economists' Voice*, 5 (5).

SWAGEL, P. (2009): "The Financial Crisis: An Inside View," Brookings Panel on Economic Activity.

VERONESI, P., AND L. ZINGALES (2008): "Paulson's Gift," Mimeo University of Chicago.

ZINGALES, L. (2008a): "Plan B," *Economist's Voice*, 5 (6).

——— (2008b): "Why Paulson is Wrong," *Economist's Voice*, 5 (5).

——— (2009): "Yes We Can, Secretary Geithner," *Economist's Voice*, 6 (2).

Fig 1: Information & Technology

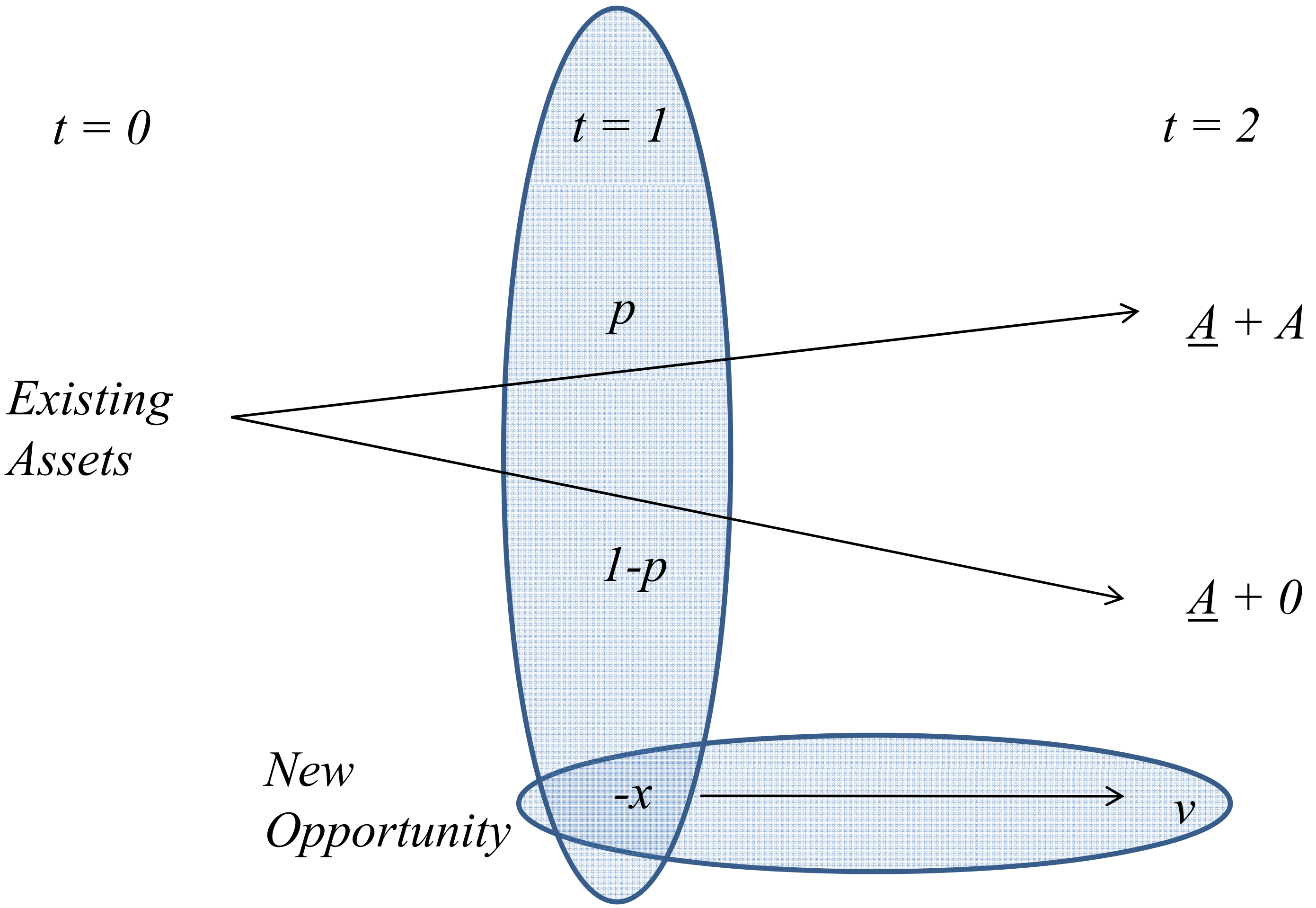

Fig 2: Payoffs

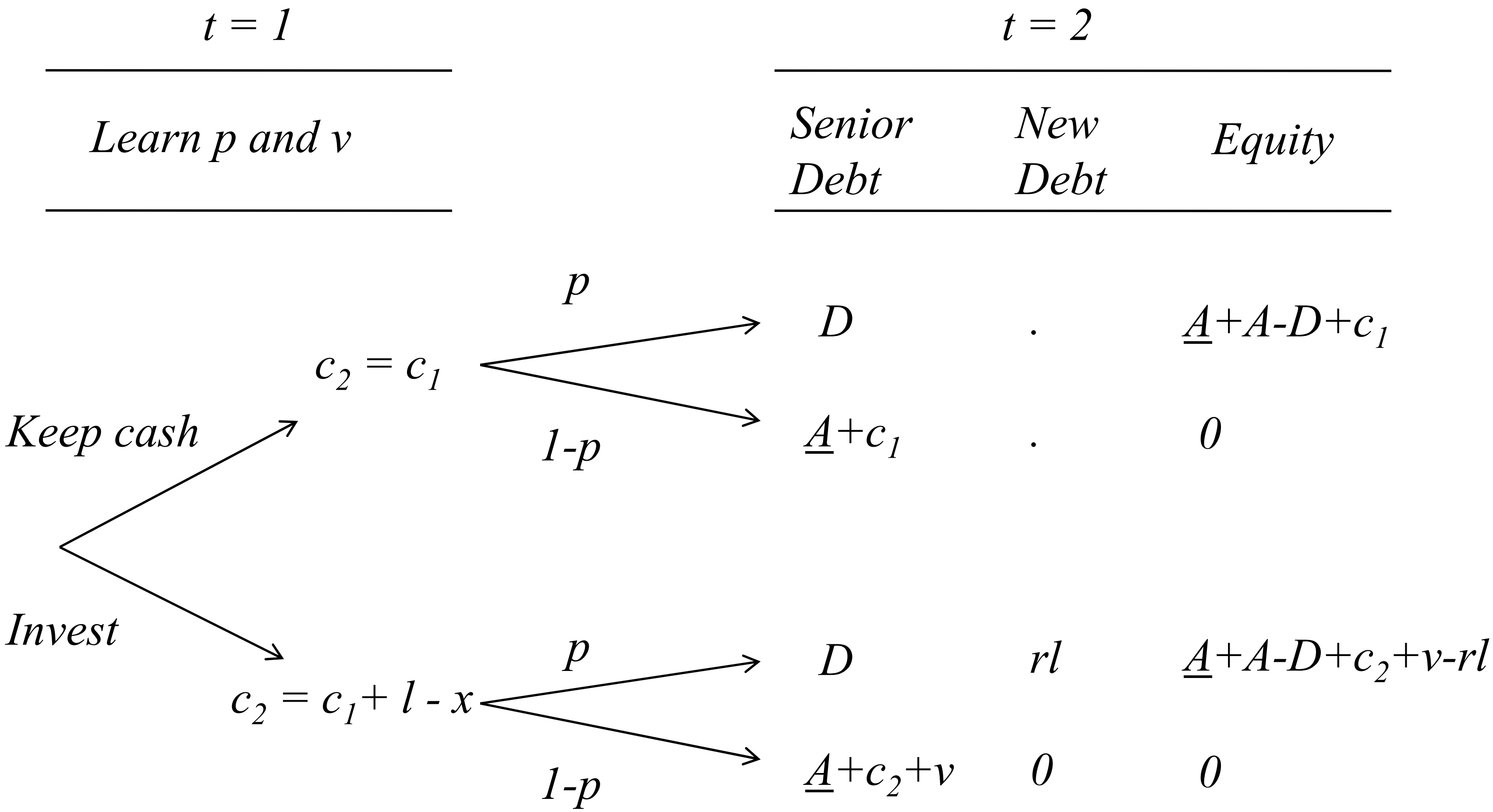

Fig 3: First Best

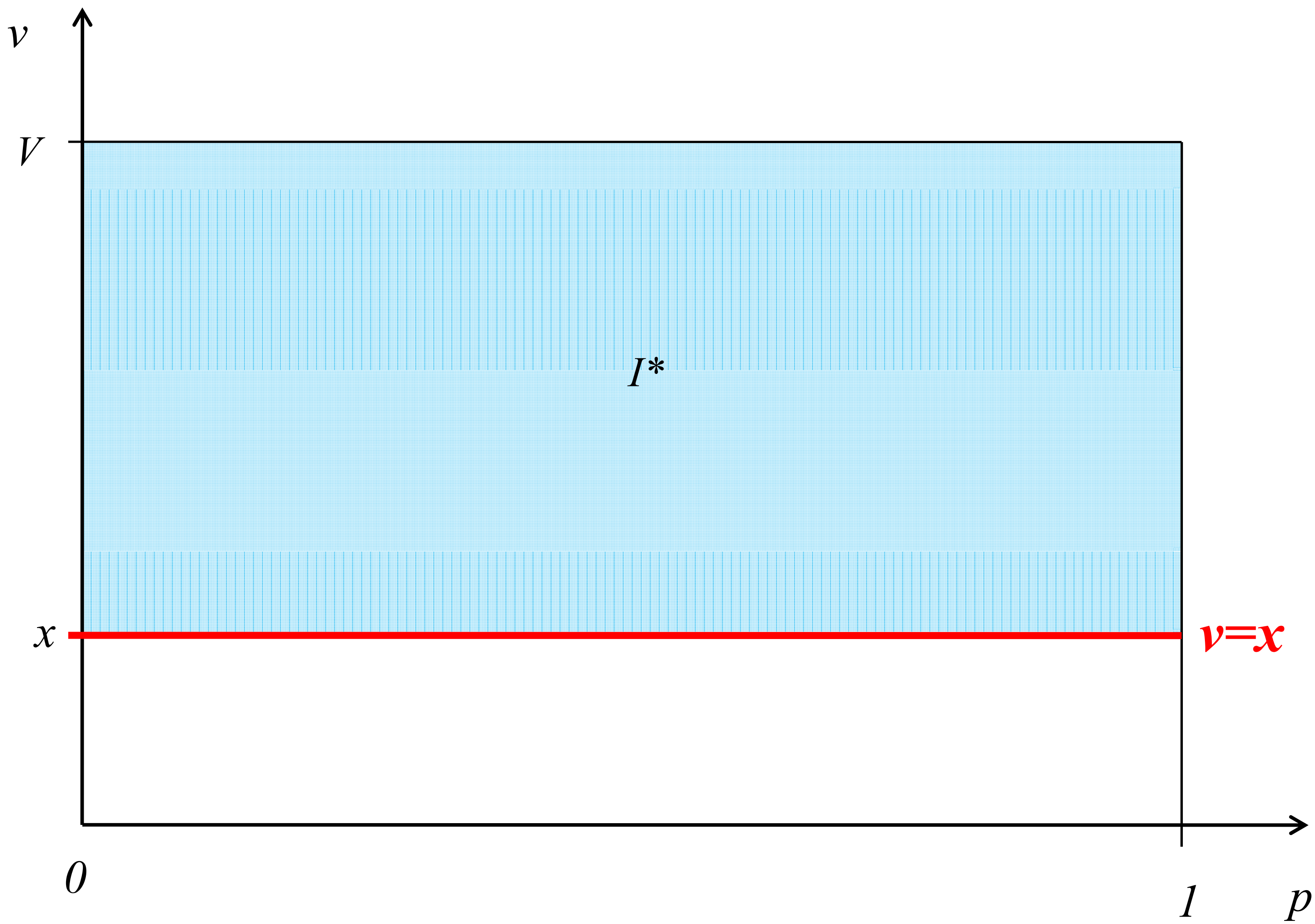

Fig 4: Debt Overhang

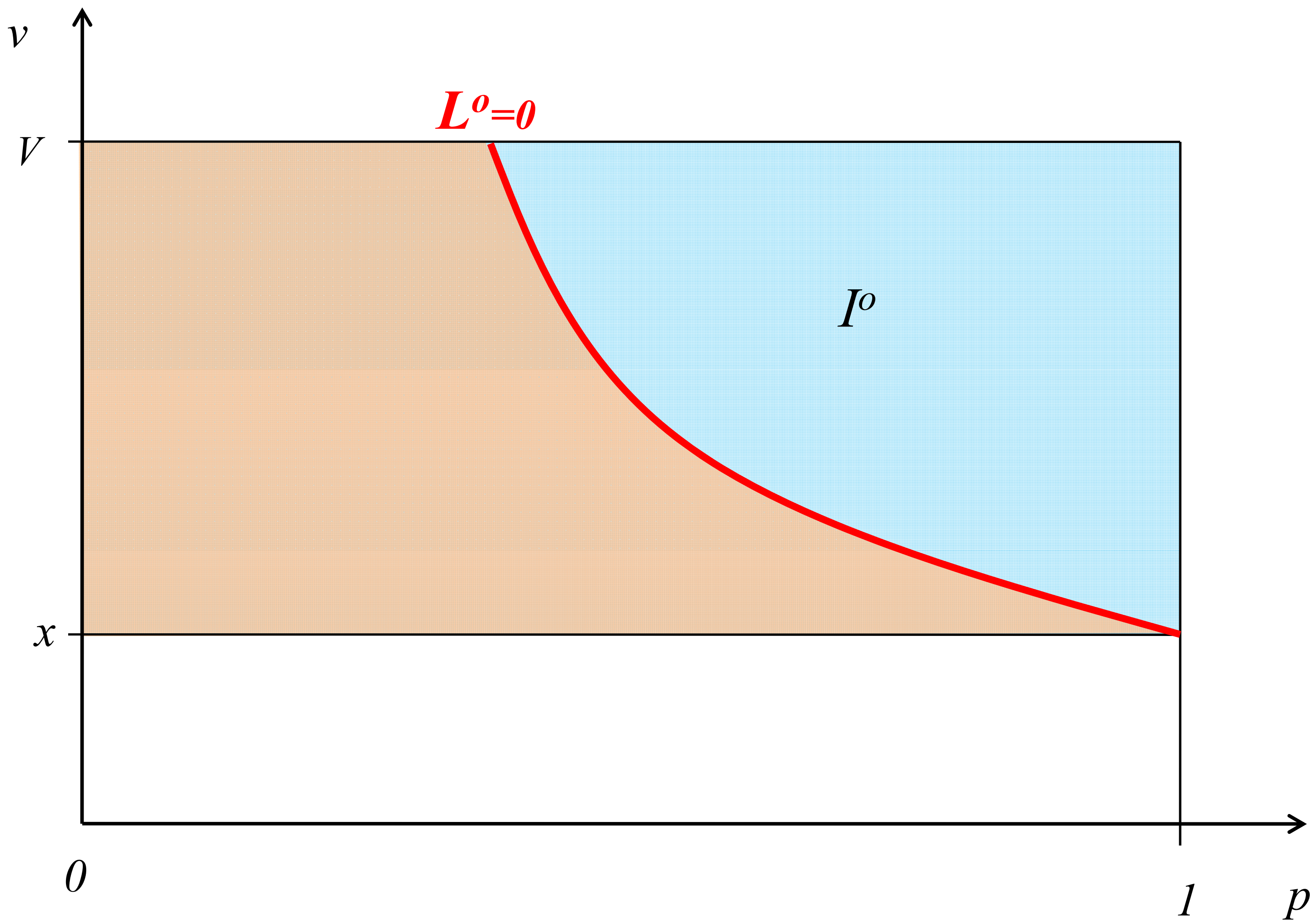

Fig 5: Cash at time 0

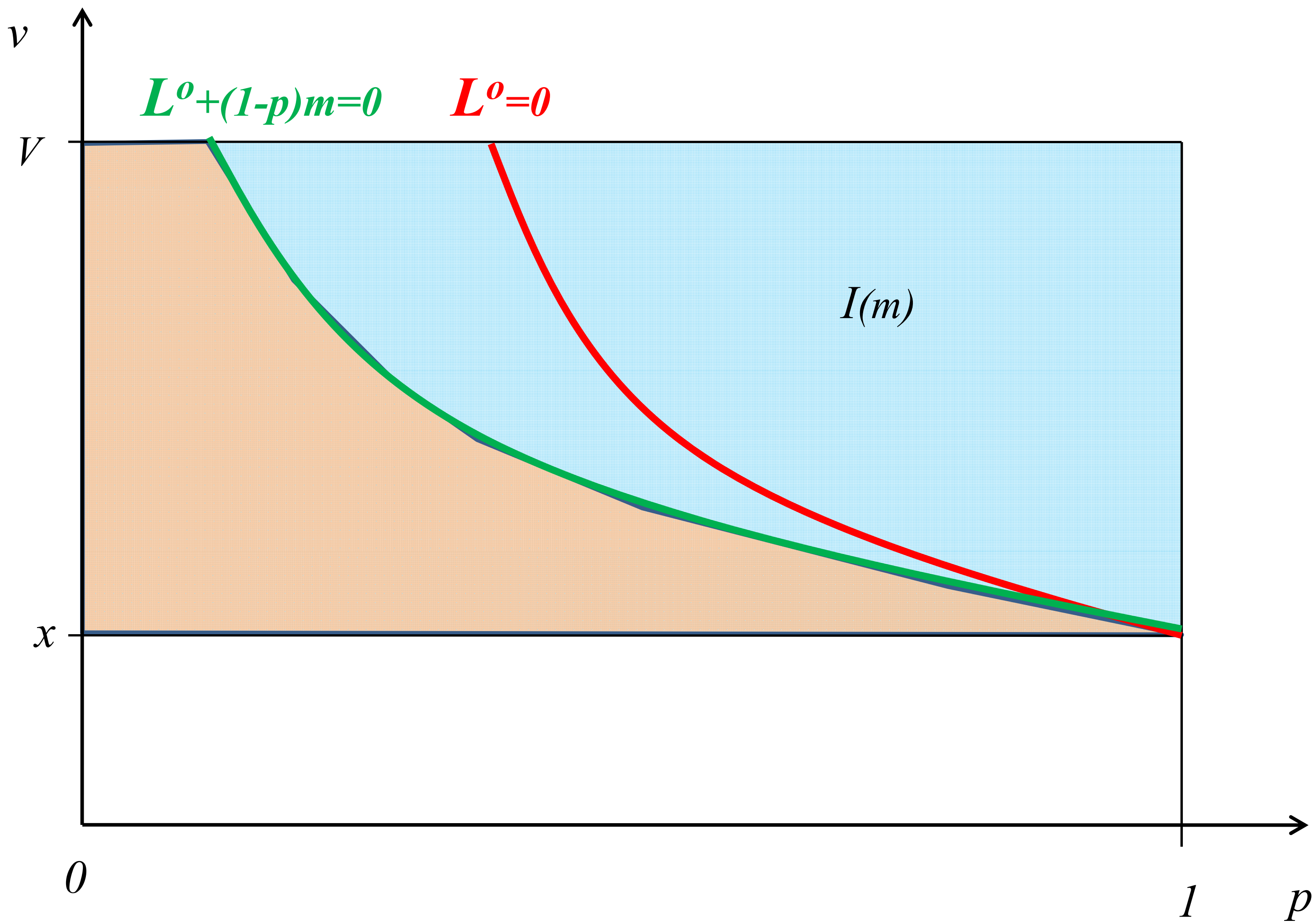

Fig 6: Debt Guarantee at time 1

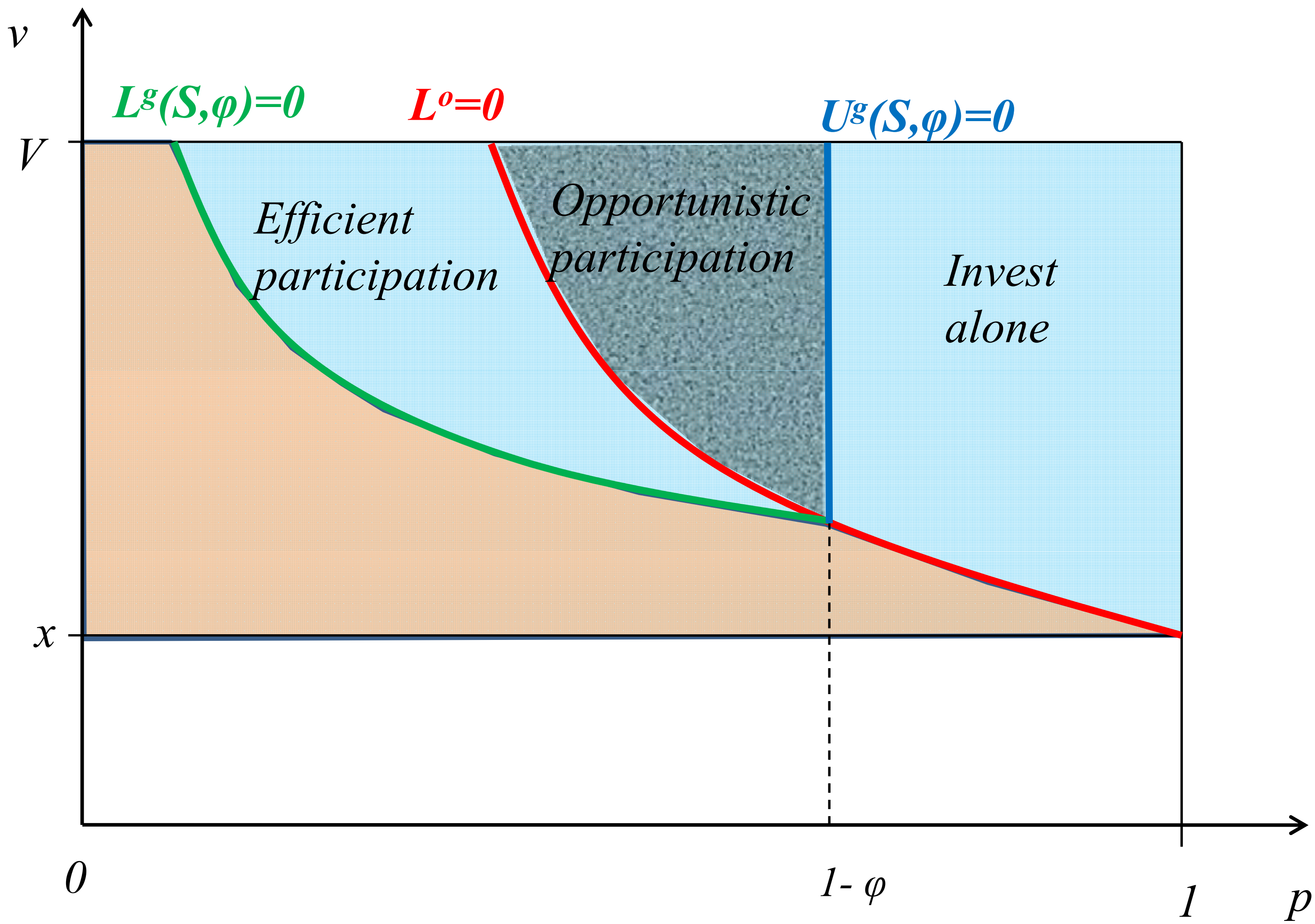

Figure 7: Equity injection at time 1

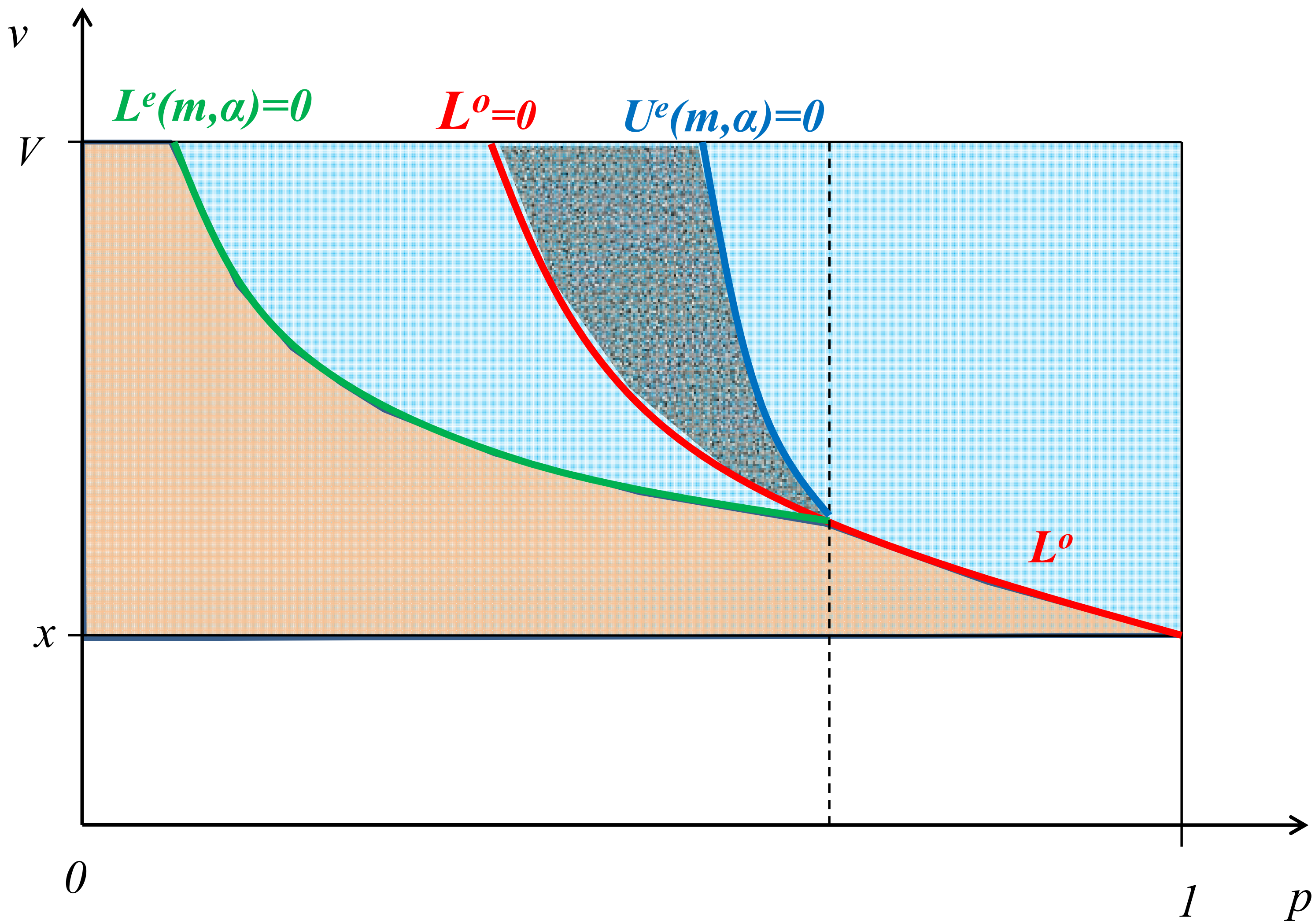

Figure 8: Efficient Mechanism