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ABSTRACT

This paper proposes an explanation of the international home bias in equity based on ambiguity aversion. We develop a simple dynamic model of consumption and portfolio decisions and derive the optimal portfolio allocation in terms of covariances between excess returns and the implied sources of risk. Under rational expectations and log utility, the only relevant risk underlying portfolio choices arises from fluctuations in non-tradeable labor income. We find that this hedging motif is empirically too weak to explain the observed lack of international diversification in equity portfolios. On the other hand, in an economy populated by ambiguity-averse agents, model uncertainty becomes an additional hedging reason which translates into long-run real exchange rate risk and is relevant even under log utility. We calibrate the degree of ambiguity aversion using detection error probabilities, and show that our framework is able to explain a large share of the observed U.S. home bias, as well as other stylized facts on U.S. cross-border asset holdings.

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1 Introduction

The lack of international diversification in equity portfolios is one the most persistent observations in international finance. Investors hold a large share of their wealth in domestic securities, more than what would be dictated by the share of these securities in the world market. This is known as the "the home-bias puzzle" (French and Poterba, 1991, Tesar and Werner, 1995).

This paper proposes an explanation of the home-bias puzzle based on ambiguity aversion. When investors are uncertain about the true data-generating model and fear misspecification, model uncertainty becomes an additional hedging reason underlying optimal portfolio choices. We show that this additional motif translates into hedging long-run real exchange rate risk and can help to explain a large share of the U.S. home bias in equity.

We develop a simple dynamic general equilibrium two-country model of consumption and portfolio choices under incomplete financial markets, where asset trading includes equities and bonds in two currencies. In particular, we derive the optimal portfolio allocation in terms of covariances between excess returns and the implied sources of risk. Along this dimension, we contrast the theoretical and empirical implications of ambiguity aversion with those of rational expectations.

Under rational expectations and log utility, the model implies that the cross-country variation in non-tradeable labor income is the only risk that investors should hedge in international financial markets. This channel has been emphasized, among others, by Baxter and Jermann (1997), Bottazzi et al. (1998), Heathcote and Perri (2004) and Coeurdacier and Gourinchas (2009). However, our empirical evaluation based on international data shows that labor-income risk alone is not able to explain the home-bias puzzle. Indeed, the empirical covariance between the model-implied labor-income risk and the excess return on foreign versus domestic equity is quantitatively very small, thereby implying quasi-full portfolio diversification. By relaxing the assumption of log utility, the model would imply an additional source of risk related to the inflation differential across countries which translates into fluctuations in the real exchange rate (as in Adler and Dumas, 1983, Cooper and Kaplanis, 1994, van Wincoop and Warnock, 2006, 2008 and Coeurdacier and Gourinchas, 2009, among others). However this channel is also weak empirically, as discussed among others by van Wincoop and Warnock (2006, 2008), because the covariance between the real exchange rate and the excess return on foreign versus domestic equity is small, once conditioned on the excess return on bonds. Hedging real exchange rate risk could become more important to explain the international home-bias puzzle by rising the risk-aversion coefficient. However, in this case, the model would imply a counterfactually high risk-free rate, a problem known as the risk-free rate puzzle (Weil, 1989).

The main result of this paper is to show that model misspecification implies hedging against fluctuations in the real exchange rate even if the elasticity of substitution is unitary. Therefore, this hedging motif can become empirically important without falling in the risk-free rate puzzle. There are, however, two important differences with respect to the existing literature. First, the real exchange rate risk emphasized by our model is not directly related to relative inflation risk but rather to the extent to which investors are averse to model uncertainty. This explains why this source of risk can arise also in the case of log utility. Second, the relevant horizon at which agents would like to evaluate real exchange rate risk is more the long run rather than the short run usually emphasized by the literature.

In our model economy, investors are endowed with a reference probability distribution, but they mistrust that such distribution is in fact the actual data-generating one. They suspect instead that the true one lies within a set of nearby distributions that are statistically difficult to distinguish in finite samples. Investors are averse to this ambiguity, and therefore seek decision rules that are robust to it. In particular, we use the sophisticated agents of the robust-control literature, developed by Hansen and Sargent (2005). These agents make their decisions considering the worst possible probability distribution, within the set of alternative ones that they consider. In this sense, the size of the set of alternative models captures the degree of aversion to model uncertainty: the larger the set, the more unfavorable the worst-case scenario.

The intuition for why we recover an additional hedging component related to real exchange rate risk works as follows. Agents fearing model misspecification make decisions considering the worst-case scenario. We show that such worst-case scenario takes the form of downward revisions in the expected cross-country consumption profile, over the entire planning horizon, and that these revisions are related to news on current and future appreciations of the real exchange rate. Ambiguity-averse investors want to hedge against this scenario and, therefore, overinvest in securities that pay relatively better when there are news on current or future appreciations. This additional hedging motif is the more relevant the more averse to model uncertainty the agents are.

Our empirical analysis shows that this channel is quantitatively more important than the hedging component related to non-tradeable income risk. Moreover, the hedging motif due to model uncertainty is able to explain a large share of the equity home bias. This result holds for reasonable degrees of ambiguity aversion, which we calibrate using detection error probabilities, i.e. imposing that alternative models must be difficult to tell apart in finite samples (Hansen and Sargent, 2005).

The result that real exchange rate risk is relevant to explain the home-bias puzzle may seem surprising given recent findings of van Wincoop and Warnock (2006, 2008) and Coeurdacier and Gourinchas (2009), who show that the real exchange rate does not co-vary much with the excess return on equities, once conditioning on bond returns. However, their result is specific to one-period ahead changes in the real exchange rate. On the contrary, in our dynamic model, long-run fluctuations in the real exchange rate are more relevant for investors. In this respect, we find that equities allow to hedge much better against this long-run risk and this is why we are able to explain the home-bias puzzle.

Finally, we show that ambiguity aversion is also able to reconcile the model with other stylized empirical facts on U.S. cross-border holdings that the rational-expectations benchmark has difficulties in replicating. In particular, the U.S. is a net creditor in equity instruments and a net debtor in bond instruments, its position in foreign-currency bonds is about balanced, whereas that in home-currency bonds is largely negative (Tille, 2005 and 2008).

While we work in discrete time, our paper is closely related to some recent continuous-time studies on international portfolio choices under ambiguity and information incompleteness. Uppal and Wang (2003) and Epstein and Miao (2005) use ambiguity aversion based on recursive multiple priors. Importantly, however, both contributions derive the result of under-diversification in international financial markets upon assuming that agents have more ambiguity in the foreign asset's return.[1] Van Nieuwerburgh and Veldkamp (2007) model an economy with imperfect information in which agents can learn and acquire better information on domestic and foreign stocks. However, to get home bias they need to assume that each home investor has more precise

[1]From a methodological perspective, our paper is also related to Maenhout (2004, 2006), who however does not deal with the home-bias puzzle. He develops a modification of the continuous-time robust-control literature to study portfolio and consumption choices in a closed-economy partial-equilibrium dynamic model. To get a closed-form solution he adopts a transformation of the objective function of the decision makers that changes the penalization of entropy from a constant Lagrange multiplier into a function of the value function. This modification deeply changes the nature of the approach proposed by Hansen and Sargent (2005) in a way that it is not comparable with the one proposed here. See the discussion in Pathak (2002).

prior information about home asset's payoff than foreigners have.

Unlike the contributions above, this paper derives a departure from full portfolio diversification that can go in either directions, and is not based upon an *a priori* asymmetry (that home agents have more ambiguity or less information with respect to foreign assets returns). Indeed, our world economy is completely symmetric *ex-ante*, and whether or not our results are consistent with the observed home bias in equity portfolios depends on the sign of the covariances in the data.

More broadly, this paper is also related to the very large literature addressing the puzzle of international under-diversification. Much effort has been made to develop general equilibrium models of portfolio choice, but no clear consensus has yet been reached. The proposed explanations range from the existence of information frictions to trade costs in goods and asset markets, home bias in consumption, sticky prices, terms of trade movements.[2]

Our paper departs from the existing literature mainly along three dimensions. First of all, most of the existing models derive the portfolio shares as a function of primitive parameters, like the risk-aversion coefficient, the share of traded goods, or the trade cost. This is clearly a desirable feature of general equilibrium models, but it has the drawback of hiding the hedging relationships based on observable variables that are at the root of the portfolio decisions. In fact, as shown by van Wincoop and Warnock (2006, 2008), the covariances between the asset returns and the sources of risk implied by these models are often counterfactual: once data restrictions on asset prices are considered, these models fail to solve the portfolio home-bias puzzle. On the other hand, the few contributions focusing on the hedging relationships that underlie portfolio choices (such as Coeurdacier and Gourinchas, 2009 and van Wincoop and Warnock, 2006, 2008) typically use static models, which, by construction, neglect any possible source of long-run risk. On the contrary, we focus on the risk-hedging motives implied by a dynamic model, which emphasizes the importance of risk related to long horizons.

Second, most of the existing literature adopts the expected-utility paradigm. However, it is well known that expected-utility preferences have counterfactual implications along several asset-price dimensions. Under complete markets, these preferences imply perfect correlation between the cross-country consumption growth and real exchange rate depreciations. In the data, instead, this correlation is extremely weak, as emphasized by Backus and Smith (1993). Moreover, these preferences are also unable to match other asset-price moments as the high and volatile returns on equities and the shape and volatility of the yield curve. Models that aim at explaining portfolio choices cannot fail in accounting for movements in asset prices, since both are the faces of the same coin. Our framework, instead, modifies the structure of preferences in a way that is desirable for at least two reasons. On the one hand, ambiguity aversion implies a risk-sensitive adjustment that has been shown to be successful in matching some properties of financial data, like the equity premium puzzle (Barillas et al., 2006) and the slope of the yield curve (Piazzesi and Schneider, 2006). On the other hand, model uncertainty acts like a preference shock in standard preferences and generates a multiplicative perturbation to the stochastic discount factor, relaxing the link between cross-country consumption and the real exchange rate, even with complete markets.[3]

Third, we depart from the complete-markets assumption, which is also very common in the

[2]An incomplete list of recent successful papers includes: Benigno and Kucuk-Tuger (2008), Bottazzi et al. (1998), Coeurdacier (2005), Coeurdacier and Gourinchas (2009), Coeurdacier et al. (2007), Cole and Obstfeld (1991), Engel and Matsumoto (2006), Heathcote and Perri (2004), Kollmann (2006), Obstfeld and Rogoff (2001), Pavlova and Rigobon (2007), Uppal (1993).

[3]Pavlova and Rigobon (2007) studies the role of preference shocks for the determination of asset prices and exchange rate.

current literature. This is a convenient device to obtain a closed-form solution, but it is unrealistic to describe the current stage of financial integration, as argued among others by Obstfeld (2006).

The structure of this paper is the following. Section 2 discusses the structure of model uncertainty. In Section 3 we present the model. We contrast the equilibrium portfolio allocation, implied by the standard framework with rational expectations in Section 4 with those implied by model uncertainty, in Section 5. Section 6 presents the empirical analysis and evaluate the empirical relevance of the model.

2 Model Uncertainty

We characterize model uncertainty as an environment in which agents are endowed with some probability distribution, but they are not sure that it is in fact the true data-generating one, and might instead act using a nearby distorted "subjective" probability distribution.

Consider a generic state of nature s_t at time t and define s^t as the history $s^t \equiv [s_t, s_{t-1}, ..., s_0]$. Let agents be endowed with $\pi(s^t)$ as the "approximating" or "reference" probability measure on histories s^t. Decision-makers may seek a different probability measure, a "subjective" one, denoted by $\widetilde{\pi}(s^t)$ which is absolutely continuous with respect to the "approximating" measure. Absolute continuity is obtained by using the Radon-Nykodym derivative.[4] First, the two probability measures agree on which events have zero probability. Second, there exists a non-negative martingale $G(s^t)$ with the property

$$E(G_t) \equiv \sum_{s^t} G(s^t)\pi(s^t) = 1 \tag{1}$$

such that, for a generic random variable $X(s^t)$,

$$\widetilde{E}(X_t) \equiv \sum_{s^t} \widetilde{\pi}(s^t)X(s^t) = \sum_{s^t} G(s^t)\pi(s^t)X(s^t) \equiv E(G_t X_t) \tag{2}$$

in which we have defined $E(\cdot)$ and $\widetilde{E}(\cdot)$ the expectation operators under the "approximating" and "subjective" probability measures, respectively. Specifically, $G(s^t)$ is a probability measure, equivalent to the ratio $\widetilde{\pi}(s^t)/\pi(s^t)$, that allows a change of measure from the "approximating" to the "subjective" measure.

Moreover, since G_t is a martingale, we can define its increment $g(s_{t+1}|s^t)$ as

$$g(s_{t+1}|s^t) \equiv \frac{G(s^{t+1})}{G(s^t)},$$

with the property $E_t g_{t+1} = 1$. It follows that $g(s_{t+1}|s^t)$ is equivalent to the likelihood ratio $\widetilde{\pi}(s_{t+1}|s^t)/\pi(s_{t+1}|s^t)$, and acts as a change of measure in conditional probabilities. High values of $g(s_{t+1}|s^t)$ imply that the decision-makers assign a higher subjective probability to state s_{t+1} conditional on history s^t.

For each random variable X_{t+1}, therefore, the martingale increment g_{t+1} defines a mapping between the conditional expectations under the two measures:

$$\widetilde{E}_t(X_{t+1}) = E_t(g_{t+1}X_{t+1}), \tag{3}$$

in which $E_t(\cdot)$ and $\widetilde{E}_t(\cdot)$ denote the conditional-expectation operators.

[4] This way of constructing subjective probability measures is borrowed from Hansen and Sargent (2005, 2007).

As in Hansen and Sargent (2005), we use conditional relative entropy as a measure of the divergence between the "approximating" and "subjective" probabilities,

$$E_t(g_{t+1} \ln g_{t+1}),$$

which approximately measures the variance of the distortions in the beliefs. When there are in fact no distortions this measure is zero: in this case, indeed, $g(s_{t+1}|s^t) = 1$ for each s_{t+1}. In particular, since we are going to work with a dynamic model, in what follows, it is more appropriate to exploit the discounted version of conditional relative entropy discussed in Hansen and Sargent (2005)

$$\eta_{t_0} \equiv E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t E_t(g_{t+1} \ln g_{t+1}) \right\}, \tag{4}$$

where $0 < \beta < 1$. A high value of entropy can be interpreted as a very large divergence between the "subjective" and the "approximating" beliefs. On the contrary a low value of entropy implies beliefs that are not too distorted or different from the reference model.

3 Model

We consider a model with two countries, denoted domestic (H) and foreign (F), each populated by a representative agent. Representative agents supply a fixed amount of labor.[5] In each country, there is a continuum of firms producing a continuum of goods in a market characterized by monopolistic competition. All goods are traded. Households enjoy consumption of both domestic and foreign goods and can trade in a set of financial assets. Specifically, there are four assets traded in the international markets: two risk-free nominal bonds, denominated in each of the currency, and shares in two equities that represent claims on the dividends of domestic and foreign firms, respectively.

The representative agent in the domestic economy maximizes utility given by

$$\widetilde{E}_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln c_t \right\} \tag{5}$$

where β, with $0 < \beta < 1$, is the intertemporal discount factor and $\widetilde{E}_{t_0}(\cdot)$ is the time$-t_0$ expectation operator taken with respect to the distorted probability measure. As discussed in the previous section, this distorted probability measure is absolutely continuous with respect to the "reference" measure and satisfies property (3). Therefore, the expected utility can be written also in terms of the "approximating" distribution as

$$\widetilde{E}_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \ln c_t \right\} = E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} G_t \ln c_t \right\}$$

where we have normalized $G_{t_0} = 1$. The representative agent in the other country has similar preferences but a possibly different subjective probability measure and therefore a different expectation operator $\widetilde{E}^*_{t_0}(\cdot)$.

The utility flow is logarithmic in the consumption index c. The latter is a CES aggregator of domestic (c_H) and imported (c_F) goods:

$$c \equiv \left[n^{\frac{1}{\vartheta}} (c_H)^{\frac{\vartheta-1}{\vartheta}} + (1-n)^{\frac{1}{\vartheta}} (c_F)^{\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta-1}},$$

[5] The model can be also modified to include an elastic labor supply, without changing our results.

in which n, with $0 < n < 1$, is the weight given to the consumption of domestic goods and ϑ, with $\vartheta > 0$, is the intratemporal elasticity of substitution between domestic and foreign goods. The consumption sub-indexes c_H and c_F are Dixit-Stiglitz aggregators of the continuum of differentiated goods produced in country H and F, respectively:

$$c_H \equiv \left[\int_0^1 c(h)^{\frac{\sigma_t - 1}{\sigma_t}} dh\right]^{\frac{\sigma_t}{\sigma_t - 1}} \qquad c_F \equiv \left[\int_0^1 c(f)^{\frac{\sigma_t - 1}{\sigma_t}} df\right]^{\frac{\sigma_t}{\sigma_t - 1}},$$

in which σ_t is the time-varying elasticity of substitution across the continuum of measure one of goods produced in each country, with $\sigma_t > 1$, for all t. The appropriate consumption-based price indices expressed in units of the domestic currency are defined as

$$P \equiv \left[n(P_H)^{1-\vartheta} + (1-n)\,(P_F)^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}, \tag{6}$$

with

$$P_H \equiv \left[\int_0^1 p(h)^{1-\sigma_t} dh\right]^{\frac{1}{1-\sigma_t}} \qquad P_F \equiv \left[\int_0^1 p(f)^{1-\sigma_t} df\right]^{\frac{1}{1-\sigma_t}}.$$

A similar structure of preferences holds for the foreign agent marked with the appropriate asterisks. In particular the weight n^* in the consumption index might not be equal to n capturing, in the case in which $n > n^*$, home bias in consumption and therefore implying variation over time in the real exchange rate.

In each country, there is a continuum of firms of measure one producing the goods in a monopolistic-competitive market. A domestic firm of type h has a constant-return-to-scale production technology $y_t(h) = Z_t^{\phi} l_t^{1-\phi}$ where Z_t is a natural resource available in the country and l_t denotes labor which is employed at the wage rate W_t; ϕ is a parameter with $0 < \phi \leq 1$. When $\phi = 1$, the model collapses to an endowment economy.

Prices are set without frictions and the law-of-one price holds. Equilibrium implies that prices are equalized across all firms within a country and set as a time-varying markup $\mu_t \equiv \sigma_t/[(\sigma_t - 1)(1-\phi)] > 1$ over nominal marginal costs

$$P_{H,t} = \mu_t \frac{W_t l_t}{y_{H,t}},$$

which implies that the wage payments are inversely related to the mark-up

$$W_t l_t = \frac{P_{H,t} y_{H,t}}{\mu_t}.$$

Firms make profits and distribute them in the form of dividends. The aggregate dividends in the domestic economy are given by

$$D_{H,t} = P_{H,t} y_{H,t} - W_t l_t = \frac{(\mu_t - 1)}{\mu_t} P_{H,t} y_{H,t},$$

which displays a positive correlation between dividends and the mark-up. Hence, the model would be able to generate a negative correlation between dividends and labor income. Anticipating the discussion of the next section, the possibility that labor income correlates negatively with the equity return gives rise to an hedging motif for holding domestic equity and might rationalize the existence of home bias in equity. This channel has been emphasized and debated in the recent literature and, for the purpose of building comparisons, we allow for this possibility on theoretical grounds.[6] When $\phi = 1$ we are in a pure endowment economy, in which all income is diversifiable. In this case μ_t goes to infinity.

The market of foreign goods works in a similar way with the appropriate modifications.

There are two equity markets – one for each country – with shares that are traded internationally. The stock-market prices in local currency are $V_{H,t}$ and $V^*_{F,t}$ for the domestic and foreign country, respectively. Households can also trade in two risk-free nominal bonds, denominated in units of the two currencies. The flow-budget constraint of the domestic agent is

$$\begin{aligned} B_{H,t} + S_t B_{F,t} + x_{H,t}V_{H,t} + x_{F,t}S_t V^*_{F,t} \leq{} & R_{H,t}B_{H,t-1} + S_t R^*_{F,t} B^*_{F,t-1} \\ & + x_{H,t-1}(V_{H,t} + D_{H,t}) + x_{F,t-1}S_t(V^*_{F,t} + D^*_{F,t}) + W_t l_t - P_t c_t \end{aligned} \tag{7}$$

in which $B_{H,t}$ and $B_{F,t}$ are the amounts of one-period nominal bonds, in units of the two currencies, held at time t; $R_{H,t}$ and $R^*_{F,t}$ are the risk-free returns from period $t-1$ to period t, in the respective currencies; $x_{H,t}$ and $x_{F,t}$ are the shares of the domestic and foreign equity, respectively, held by the domestic agent. Finally S_t is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency. The flow-budget constraint (7) can be written in a more compact form as

$$A_t = R_{p,t}A_{t-1} + W_t l_t - P_t c_t \tag{8}$$

where we have defined

$$A_t \equiv B_{H,t} + S_t B_{F,t} + x_{H,t}V_{H,t} + x_{F,t}S_t V^*_{F,t}$$

and

$$R_{p,t} \equiv \alpha_{H,t-1}R_{H,t} + \alpha_{F,t-1}R^*_{F,t}\frac{S_t}{S_{t-1}} + \alpha^e_{H,t-1}R^e_{H,t} + \alpha^e_{F,t-1}R^{e*}_{F,t}\frac{S_t}{S_{t-1}}.$$

In the definition above, $\alpha_{H,t}$, $\alpha_{F,t}$, $\alpha^e_{H,t}$, $\alpha^e_{F,t}$ represent the shares of wealth that the domestic agent invests in the domestic bond, foreign bond, domestic equity and foreign equity, respectively, satisfying the following restriction:

$$\alpha_{H,t} + \alpha_{F,t} + \alpha^e_{H,t} + \alpha^e_{F,t} = 1. \tag{9}$$

Moreover $R^e_{H,t}$ and $R^{e*}_{F,t}$ are the returns in the two stock markets in their respective currencies.[7]

We can also express the flow-budget constraint in real terms – in units of the domestic consumption index – writing

$$a_t = r_{p,t}a_{t-1} + \xi_t - c_t, \tag{10}$$

where

$$r_{p,t} = \alpha_{H,t-1}r_{H,t} + \alpha_{F,t-1}r^*_{F,t}\frac{q_t}{q_{t-1}} + \alpha^e_{H,t-1}r^e_{H,t} + \alpha^e_{F,t-1}r^{e*}_{F,t}\frac{q_t}{q_{t-1}}$$

and in which lower-case variables denote the real counterpart of the respective upper-case variable. ξ_t denotes non-diversifiable real labor income, defined as $\xi_t \equiv W_t l_t/P_t$, while q_t is the real exchange rate defined as $q_t \equiv S_t P^*_t/P_t$.

The domestic agent's optimization problem is to choose consumption and the portfolio allocations to maximize (5) under the flow-budget constraint (10) and appropriate no-Ponzi game conditions.

[6] Mark-up shocks can fall in the category of redistributive shocks, discussed by Coeurdacier et al. (2007) and Coeurdacier and Gourinchas (2009).

[7] See the appendix for details on the derivations and definitions.

3.1 Optimality conditions

The optimality condition with respect to consumption implies an orthogonality condition, in expectation, between the real stochastic discount factor and the real portfolio return

$$\widetilde{E}_t(m_{t+1}r_{p,t+1}) = 1, \tag{11}$$

in which m_{t+1} is the real stochastic discount factor defined as

$$m_{t+1} \equiv \beta \frac{c_t}{c_{t+1}}. \tag{12}$$

A similar condition applies to the foreign economy:

$$\widetilde{E}_t^*(m_{t+1}^* r_{p,t+1}^*) = 1, \tag{13}$$

where the foreign stochastic discount factor is defined as

$$m_{t+1}^* \equiv \beta \frac{c_t^*}{c_{t+1}^*}. \tag{14}$$

The optimality conditions with respect to the portfolio allocation imply a set of four restrictions for each agent, one for each asset, given by:

$$\widetilde{E}_t\left(m_{t+1}r_{H,t+1}\right) = 1, \qquad \widetilde{E}_t^*\left(m_{t+1}^* r_{H,t+1}\frac{q_t}{q_{t+1}}\right) = 1, \tag{15}$$

$$\widetilde{E}_t\left(m_{t+1}r_{F,t+1}^*\frac{q_{t+1}}{q_t}\right) = 1, \qquad \widetilde{E}_t^*\left(m_{t+1}^* r_{F,t+1}^*\right) = 1, \tag{16}$$

$$\widetilde{E}_t\left(m_{t+1}r_{H,t+1}^e\right) = 1, \qquad \widetilde{E}_t^*\left(m_{t+1}^* r_{H,t+1}^e\frac{q_t}{q_{t+1}}\right) = 1, \tag{17}$$

$$\widetilde{E}_t\left(m_{t+1}r_{F,t+1}^{e*}\frac{q_{t+1}}{q_t}\right) = 1, \qquad \widetilde{E}_t^*\left(m_{t+1}^* r_{F,t+1}^{e*}\right) = 1. \tag{18}$$

Equilibrium in the goods market requires the production of each good to be equal to world consumption

$$y_{H,t} = c_{H,t} + c_{H,t}^*,$$

$$y_{F,t}^* = c_{F,t} + c_{F,t}^*.$$

The labor markets are in equilibrium at the exogenously supplied quantities of labor

$$l_t = \bar{l}_t,$$

$$l_t^* = \bar{l}_t^*.$$

Bonds are in zero-net supply worldwide

$$B_{H,t} + B_{H,t}^* = 0,$$

and

$$B_{F,t} + B_{F,t}^* = 0.$$

Equity shares sum to one

$$x_{H,t} + x_{H,t}^* = 1,$$

$$x_{F,t} + x^*_{F,t} = 1.$$

Given the path of the stochastic disturbances $\{\bar{l}_t,\ \bar{l}^*_t,\ Z_t,\ Z^*_t,\ \mu_t,\ \mu^*_t\}$, an equilibrium is an allocation of quantities $\{c_t, c_{H,t}, c_{F,t}, c^*_t, c^*_{H,t}, c^*_{F,t}, \alpha_{H,t}, \alpha_{H,t}, \alpha^e_{H,t}, \alpha^e_{F,t}, \alpha^*_{H,t}, \alpha^*_{H,t}, \alpha^{*e}_{H,t}, \alpha^{*e}_{F,t}, a_t, a^*_t\}$ and prices $\{r_{H,t}, r^*_{F,t}, r^e_{H,t}, r^{e*}_{F,t}, q_t, P_{H,t}/P_{F,t}, w_t, w^*_t\}$ such that each agent's consumption, portfolio shares and wealth are optimal given prices; and goods, labor, asset markets are in equilibrium.

Although we have written a general equilibrium model, in the next section we show that we do not really need to solve the entire model to understand the determinants of the portfolio allocation. Instead, we can determine the portfolio shares $\{\alpha_{H,t},\ \alpha_{H,t},\ \alpha^e_{H,t},\ \alpha^e_{F,t}, \alpha^*_{H,t},\ \alpha^*_{H,t},\ \alpha^{*e}_{H,t},\ \alpha^{*e}_{F,t}\}$ by taking as given the path of prices $\{r_{H,t},\ r^*_{F,t},\ r^e_{H,t},\ r^{e*}_{F,t},\ q_t\}$ and the processes of non-diversifiable labor incomes $\{\xi_t,\ \xi^*_t\}$. This is a convenient result because it forces our portfolio implications to be compatible with observable variables, which represent a harder test for the model. Since the stochastic structure of the model is rich enough, it should be eventually possible to build processes for the shocks or in any case to enrich the stochastic structure in a way to match the observed prices.

Recent papers in the literature on international portfolio choice assume a general equilibrium structure and explain portfolio choices in terms of primitive parameters or shocks. However, most of these models would be less successful if the portfolio implications were analyzed under data restrictions on prices and returns.[8]

4 A Simple Case: No Model Uncertainty

We start with the simple case in which there is no model uncertainty. This means that investors fully trust the "reference" probability distribution to be the true one, so that "approximating" and "subjective" measures coincide.

For a generic random variable X_{t+1}, it follows that $\widetilde{E}_t X_{t+1} = \widetilde{E}^*_t X_{t+1} = E_t X_{t+1}$. Accordingly, we can write each set of orthogonality conditions (15)–(18), by taking the difference between the two in each set:

$$E_t\left[\left(m_{t+1} - m^*_{t+1}\frac{q_t}{q_{t+1}}\right)\right] = 0, \tag{19}$$

$$E_t\left[\left(m_{t+1} - m^*_{t+1}\frac{q_t}{q_{t+1}}\right)\frac{q_{t+1}}{q_t}\right] = 0, \tag{20}$$

$$E_t\left[\left(m_{t+1} - m^*_{t+1}\frac{q_t}{q_{t+1}}\right)r^e_{H,t+1}\right] = 0, \tag{21}$$

$$E_t\left[\left(m_{t+1} - m^*_{t+1}\frac{q_t}{q_{t+1}}\right)r^{e*}_{F,t+1}\frac{q_{t+1}}{q_t}\right] = 0. \tag{22}$$

The above four conditions now require the cross-country difference in the real stochastic discount factors, evaluated in the units of the domestic discount factor, to be orthogonal to the assets returns.

First, we solve for the portfolio allocation under the assumption that all income is diversifiable. This is the case of pure endowment economies when $\phi = 1$ and $\xi_t = \xi^*_t = 0$. Under this restriction and log utility, consumption in each country is proportional to financial wealth

$$c_t = \frac{1-\beta}{\beta}a_t \qquad\qquad c^*_t = \frac{1-\beta}{\beta}a^*_t, \tag{23}$$

[8] See van Wincoop and Warnock (2006, 2008) and Coeurdacier and Gourinchas (2009) for a related argument and for models that are instead evaluated under data restrictions.

and financial wealth evolves according to the following laws of motion:

$$a_t = \beta r_{p,t} a_{t-1} \qquad\qquad a_t^* = \beta r_{p,t}^* a_{t-1}^*. \tag{24}$$

The portfolio allocation can be simply characterized by guessing that in equilibrium

$$m_{t+1} = m_{t+1}^* \frac{q_t}{q_{t+1}}, \tag{25}$$

through which (19)–(22) are automatically satisfied. Requiring equation (25) to hold means that risk is completely shared across the two agents. We can further write (25), by using (12), (14), (23) and (24), as

$$r_{p,t+1} = r_{p,t+1}^* \frac{q_{t+1}}{q_t}.$$

Our guess is verified when $\alpha_{H,t} = \alpha_{F,t} = \alpha_{H,t}^* = \alpha_{F,t}^* = 0$ and $\alpha_{H,t}^e = \alpha_{H,t}^{*e} = \alpha_{F,t}^e = \alpha_{F,t}^{*e} = 1/2$, which is indeed a feasible solution. In equilibrium, households do not hold any wealth in the bond markets and hold instead all their wealth in the equity market with an equal split between home and foreign stocks. In this case there is full risk sharing and full international portfolio diversification. Therefore, the model fails to account for the home bias in assets observed in the data. Moreover, this striking conclusion holds irrespectively of the degree of home bias in consumption and the elasticity of substitution between home and foreign goods.[9]

We now allow for non-diversifiable labor income. This small variation complicates the model solution in such a way that we are no longer able to get it in a non-linear closed form.[10] We can still derive many insights by using the approximation methods developed by Devereux and Sutherland (2006) and Tille and van Wincoop (2006). First, we solve for the paths of consumption and wealth, given returns and the steady-state portfolio shares, using a *first-order* approximation; then we use this result to solve for the steady-state portfolio shares as a function of prices, returns and non-diversifiable labor income, using a *second-order* approximation of the orthogonality conditions.

In what follows, a variable with an "upper-bar" denotes the symmetric steady state and a "hat" denotes the log-deviation with respect to such steady state. A first-order approximation of the Euler conditions (11) and (13) implies

$$E_t \Delta \hat{c}_{t+1} = E_t \hat{r}_{p,t+1}, \tag{26}$$

$$E_t \Delta \hat{c}_{t+1}^* = E_t \hat{r}_{p,t+1}^*. \tag{27}$$

In particular, the portfolio returns can be approximated to first order as

$$\hat{r}_{p,t+1} = \hat{r}_{H,t+1} + \bar{\boldsymbol{\alpha}}' \mathbf{exr}_{t+1},$$

$$\hat{r}_{p,t+1}^* = \hat{r}_{H,t+1} + \bar{\boldsymbol{\alpha}}^{*\prime} \mathbf{exr}_{t+1} - \Delta \hat{q}_{t+1},$$

where we have defined

$$\bar{\boldsymbol{\alpha}} \equiv \begin{bmatrix} \bar{\alpha}_F \\ \bar{\alpha}_H^e + \bar{\alpha}_F^e \\ \bar{\alpha}_F^e \end{bmatrix} \qquad\qquad \bar{\boldsymbol{\alpha}}^* \equiv \begin{bmatrix} \bar{\alpha}_F^* \\ \bar{\alpha}_H^{*e} + \bar{\alpha}_F^{*e} \\ \bar{\alpha}_F^{*e} \end{bmatrix}, \tag{28}$$

[9]See Coeurdacier and Gourinchas (2009) and Heathcote and Perri (2004) for a similar result obtained under the assumption that markets are indeed locally complete. Our model nests also the one-good model when ϑ goes to infinity.

[10]Van Wincoop and Warnock (2006, 2008) obtained a closed-form solution, but in a partial-equilibrium two-period model. Coeurdacier and Gourinchas (2009), Coeurdacier et al. (2007), Heathcote and Perri (2004), Kollman (2006) obtain closed-form solutions by assuming that markets are locally complete.

and the vector of excess returns as

$$\mathbf{exr}_t \equiv \begin{bmatrix} \hat{r}^*_{F,t} + \Delta\hat{q}_t - \hat{r}_{H,t} \\ \hat{r}^e_{H,t} - \hat{r}_{H,t} \\ \hat{r}^{e*}_{F,t} + \Delta\hat{q}_t - \hat{r}^e_{H,t} \end{bmatrix}.$$

In a first-order approximation, the no-arbitrage conditions imply that excess returns have zero conditional means, $E_t\mathbf{exr}_{t+1} = 0$. It follows, using equations (26) and (27), that the cross-country differential in the expected consumption growth depends on the expected depreciation in the real exchange rate

$$E_t\Delta\hat{c}^R_{t+1} = E_t\Delta\hat{q}_{t+1}, \tag{29}$$

where an upper-script R denotes the difference between the domestic and foreign variables.

A first-order approximation of the flow budget constraint (10) together with the budget constraint of the foreign agent implies

$$\beta\hat{a}^R_t = \hat{a}^R_{t-1} + \bar{\boldsymbol{\lambda}}'\mathbf{exr}_t + \Delta\hat{q}_t + \beta s_\xi\hat{\xi}^R_t - \beta s_c\hat{c}^R_t, \tag{30}$$

where s_ξ is the steady-state ratio between non-traded income and financial wealth, given by $s_\xi \equiv \bar{\xi}/\bar{a}$, which is equal in the two countries; s_c is the steady-state ratio between consumption and financial wealth and such that $s_c = (1-\beta)/\beta + s_\xi$. Moreover, the vector $\bar{\boldsymbol{\lambda}}$ is defined as

$$\bar{\boldsymbol{\lambda}} \equiv \begin{bmatrix} 2\bar{\alpha}_F \\ 2(\bar{\alpha}^e_H + \bar{\alpha}^e_F) - 2 \\ 2\bar{\alpha}^e_F - 1 \end{bmatrix}. \tag{31}$$

The set of difference equations (29) and (30) can be solved forward to obtain relative consumption and relative wealth $(\hat{c}^R_t,\ \hat{a}^R_t)$ as a function of the states $(\hat{a}^R_{t-1},\ \hat{q}_{t-1})$ and the processes of excess returns, relative non-diversifiable income and the real exchange rate $\{\mathbf{exr}_t,\ \hat{\xi}^R_t,\ \hat{q}_t\}$. In particular, we obtain

$$(\hat{c}^R_t - \hat{q}_t) = \frac{(1-\beta)}{\beta s_c}(\hat{a}^R_{t-1} - \hat{q}_{t-1}) + \frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}'\mathbf{exr}_t + \frac{(1-\beta)s_\xi}{s_c}E_t\sum_{T=t}^{\infty}\beta^{T-t}(\hat{\xi}^R_T - \hat{q}_T), \tag{32}$$

$$(\hat{a}^R_t - \hat{q}_t) = (\hat{a}^R_{t-1} - \hat{q}_{t-1}) + \bar{\boldsymbol{\lambda}}'\mathbf{exr}_t + s_\xi(\hat{\xi}^R_t - \hat{q}_t) - (1-\beta)s_\xi E_t\sum_{T=t}^{\infty}\beta^{T-t}(\hat{\xi}^R_T - \hat{q}_T). \tag{33}$$

We determine the portfolio shares by using a second-order approximation of the moment conditions (19)–(22). In particular we just need three restrictions to determine the vector $\bar{\boldsymbol{\lambda}}$:

$$E_t\left[(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1})(\hat{r}^*_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1})\right] = 0,$$

$$E_t\left[(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1})(\hat{r}^e_{H,t+1} - \hat{r}_{H,t+1})\right] = 0,$$

$$E_t\left[(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1})(\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1})\right] = 0.$$

We can now use equations (32)–(33) in the conditions above and solve for the steady-state vector of portfolio shares:

$$\bar{\boldsymbol{\lambda}} = -s_\xi\frac{\beta}{1-\beta}\boldsymbol{\Sigma}_t^{-1}E_t(\mathbf{exr}_{t+1}\cdot\varepsilon_{l,t+1}), \tag{34}$$

in which we have defined $\varepsilon_{l,t+1}$ as the news at time $t+1$ in the growth path of the cross-country non-diversifiable labor incomes (in units of the domestic consumption index)

$$\varepsilon_{l,t+1} = \sum_{j=0}^{\infty} \beta^j [E_{t+1}(\Delta\hat{\xi}^R_{t+1+j} - \Delta\hat{q}_{t+1+j}) - E_t(\Delta\hat{\xi}^R_{t+1+j} - \Delta\hat{q}_{t+1+j})], \tag{35}$$

and $\mathbf{\Sigma}_t$ is the time$-t$ conditional variance-covariance matrix of the vector of excess returns $\mathbf{exr}_{t+1}$.

Equation (34) determines the portfolio allocation in the steady state. When $s_\xi = 0$, we confirm the result of the simple model in which all income risk is tradeable: indeed $\bar{\boldsymbol{\lambda}} = 0$ and accordingly $\bar{\alpha}^e_k = \bar{\alpha}^{*e}_k = 1/2$ and $\bar{\alpha}_k = \bar{\alpha}^*_k = 0$, for $k = H, F$.

When there is non-diversifiable income, instead, the model implies a departure from full diversification that depends on the covariances between labor-income risk and the excess returns.

The set of conditions in (34) can be written in a more simple form as

$$\bar{\alpha}_F = -\frac{s_\xi}{2}\frac{\beta}{1-\beta}\frac{cov_t(\varepsilon_{l,t+1}, \hat{r}^*_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}|exr^{de}_{t+1}, exr^{ie}_{t+1})}{var_t(\hat{r}^*_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}|exr^{de}_{t+1}, exr^{ie}_{t+1})}, \tag{36}$$

$$\bar{\alpha}^e_H + \bar{\alpha}^e_F = 1 - \frac{s_\xi}{2}\frac{\beta}{1-\beta}\frac{cov_t(\varepsilon_{l,t+1}, \hat{r}^e_{H,t+1} - \hat{r}_{H,t+1}|exr^{ib}_{t+1}, exr^{ie}_{t+1})}{var_t(\hat{r}^e_{H,t+1} - \hat{r}_{H,t+1}|exr^{ib}_{t+1}, exr^{ie}_{t+1})}, \tag{37}$$

$$\bar{\alpha}^e_F = \frac{1}{2} - \frac{s_\xi}{2}\frac{\beta}{1-\beta}\frac{cov_t(\varepsilon_{l,t+1}, \hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1}|exr^{ib}_{t+1}, exr^{de}_{t+1})}{var_t(\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1}|exr^{ib}_{t+1}, exr^{de}_{t+1})}, \tag{38}$$

in which variances and covariances are conditional on selected excess returns and previous-period information. We denote with exr^{ib}, exr^{ie} and exr^{de} the excess returns on international bonds, international equity and domestic equity, respectively:

$$exr^{ib}_{t+1} \equiv \hat{r}^*_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}, \tag{39}$$

$$exr^{ie}_{t+1} \equiv \hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1}, \tag{40}$$

$$exr^{de}_{t+1} \equiv \hat{r}^e_{H,t+1} - \hat{r}_{H,t+1}. \tag{41}$$

Using (36) to (38) together with (9), we are able to determine the split of wealth across the different assets.

In particular, equation (38) determines whether there will be home bias in equity holdings ($\bar{\alpha}^e_F < 1/2$). The home bias is optimal when, conditional on the other excess returns, the excess return on international equity co-varies positively with the surprises in the cross-country differential in the growth of non-diversifiable labor income. In this case, indeed, the return on domestic equity will increase, relative to that on foreign equity, when domestic agents receive a bad shock regarding their labor income. This makes domestic equity a better hedge against labor-income risk relative to foreign equity and rationalizes home bias in equity holdings.

Equation (37) instead determines the share of financial wealth invested overall in the equity market relative to the bond market. When $s_\xi = 0$, investors would like to invest all their wealth in equities, as we previously discussed. Instead when $s_\xi \neq 0$ and $\varepsilon_{l,t+1}$ co-varies positively with the excess return of domestic equity over domestic bonds, then domestic agents will also take an overall positive position in the bond markets ($\bar{\alpha}^e_H + \bar{\alpha}^e_F < 1$). In this case, indeed, in the face of a bad shock to labor income domestic bonds pay relative better than equities: bonds are, in relative terms, a good hedge with respect to labor-income risk.

Finally, equation (36) describes the position taken in the foreign bond market and as a consequence in the domestic bond market, given the overall position implied by (37). When the covariance between $\varepsilon_{l,t+1}$ and the excess return of the foreign bond with respect to the domestic bond is positive, then foreign bonds do not pay well when needed. In this case the domestic agent would like to take a short position in the foreign bond market ($\bar{\alpha}_F < 0$). Note that this does not necessarily imply a long position in the domestic bond market. Indeed, the overall position depends on equation (37), as previously discussed.

Although simpler versions of (36) and (38) have been treated in the literature, to our knowledge, this is the first complete analysis in a dynamic general equilibrium model with incomplete markets. Simple cases are nested in the above framework. When there is only trading in equities, the relevant condition for determining the portfolio allocation collapses to

$$\bar{\alpha}_F^e = \frac{1}{2} - \frac{s_\xi}{2}\frac{\beta}{1-\beta}\frac{cov_t(\varepsilon_{l,t+1}, \hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^e)}{var_t(\hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^e)}. \tag{42}$$

Domestic agents hold a smaller share of their wealth in the foreign equity market when the excess return of the foreign stock with respect to the domestic one co-varies positively with the surprises in the domestic-versus-foreign non-diversifiable labor incomes. Note that now these covariances are no longer conditional on the other excess returns, but they are only conditional on time–t information. There is home bias in equity holdings when home equity is a good hedge with respect to non-diversifiable income risk.

A popular argument for international diversification being worse is the neoclassical model of Baxter and Jermann (1997) in which labor income and dividends are correlated. In this case, the above covariance would be negative implying even larger holdings of foreign assets. Heathcote and Perri (2004) instead show a case in which the correlation can become positive when there is capital accumulation, or decumulation, and home bias in consumption preferences. Coeurdacier and Gourinchas (2009) discuss several theoretical cases that can rationalize a positive covariance and then imply home-bias in equity.[11]

Our theoretical model shows that the covariance can be positive or negative depending on the relative strength of the mark-up shocks. Conditional on a positive mark-up shock, profits and dividends increase, whereas labor income decreases. This might imply a negative correlation between labor income and the return on domestic equity. In this case, the domestic agent would hold more of its own asset to hedge against labor-income risk. However, at the end, whichever channel is relevant is a question of empirical evaluation of the covariances involved in (42).

Coeurdacier and Gourinchas (2009) consider a model in which agents can also trade in bonds, but in which shocks have a certain property of symmetry such that each country bond position is balanced to zero, so that a long position in one bond corresponds necessarily to a short position in the other. In our model, this is nested by requiring that $\bar{\alpha}_H + \bar{\alpha}_F = 0$. It follows that the relevant conditions for determining the portfolio allocations are

$$\bar{\alpha}_F = -\frac{s_\xi}{2}\frac{\beta}{1-\beta}\frac{cov_t(\varepsilon_{l,t+1}, \hat{r}_{F,t+1}^{*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1} | \hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^e)}{var_t(\hat{r}_{F,t+1}^{*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1} | \hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^e)} \tag{43}$$

$$\bar{\alpha}_F^e = \frac{1}{2} - \frac{s_\xi}{2}\frac{\beta}{1-\beta}\frac{cov_t(\varepsilon_{l,t+1}, \hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^e | \hat{r}_{F,t+1}^{*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1})}{var_t(\hat{r}_{F,t+1}^{e*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}^e | \hat{r}_{F,t+1}^{*} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1})}. \tag{44}$$

The above two conditions are similar to the ones discussed in Coeurdacieur and Gourinchas (2009) under their log-utility case. However, there are two important differences: 1) variances

[11] See also Coeurdacier et al. (2007) and Engel and Matsumoto (2006).

and covariances are conditional on the previous-period information while in their model (since it is static) they are unconditional;[12] 2) $\varepsilon_{l,t+1}$ is the model-implied measure of labor-income risk, defined as the surprises in the present discounted value of cross-country labor income growth over the entire planning horizon.[13]

We can easily generalize the model to recover another hedging motif – the one with respect to real-exchange-rate risk – if we were to assume non-log utility, as in the analysis of Adler and Dumas (1983), Coeurdacier and Gourinchas (2009) and van Wincoop and Warnock (2006, 2008). There are two important reasons for why we do not follow this strategy: on the one hand, estimates of the intertemporal elasticity of substitution are not far from the unitary value, as discussed in Vissing-Jœrgensen and Attanasio (2003); on the other hand, if we were to increase the risk-aversion coefficient to enhance the importance of hedging the real-exchange-rate risk we would lower the intertemporal elasticity of substitution, and raise in a counterfactual way the implied risk-free rate. We follow a different strategy.

5 Portfolio Choices under Model Uncertainty

Under rational expectations, agents form expectations using the "reference" probability measure because they fully trust it to be the true one. When agents face model uncertainty, on the contrary, they only regard the "reference" distribution as an approximation of the true one, which is believed to lie nearby but remains unknown. Therefore, they might want to use a distorted probability measure and form "subjective" conditional expectations. As shown in (3), the latter are linked to the "approximating" conditional expectations through the martingale increments g and g^*, for country H and F respectively.

Accordingly, we can write conditions (19)–(22) as

$$E_t\left[\left(m_{t+1}g_{t+1} - m^*_{t+1}g^*_{t+1}\frac{q_t}{q_{t+1}}\right)\right] = 0,$$

$$E_t\left[\left(m_{t+1}g_{t+1} - m^*_{t+1}g^*_{t+1}\frac{q_t}{q_{t+1}}\right)\frac{q_{t+1}}{q_t}\right] = 0,$$

$$E_t\left[\left(m_{t+1}g_{t+1} - m^*_{t+1}g^*_{t+1}\frac{q_t}{q_{t+1}}\right)r^e_{H,t+1}\right] = 0,$$

$$E_t\left[\left(m_{t+1}g_{t+1} - m^*_{t+1}g^*_{t+1}\frac{q_t}{q_{t+1}}\right)r^{e*}_{F,t+1}\frac{q_{t+1}}{q_t}\right] = 0.$$

This set of equations implies the three restrictions needed to determine the portfolio allocation. In a second-order approximation they read as

$$E_t\left[(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1} - \hat{g}^R_{t+1})(\hat{r}^*_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1})\right] = 0, \tag{45}$$

$$E_t\left[(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1} - \hat{g}^R_{t+1})(\hat{r}^e_{H,t+1} - \hat{r}_{H,t+1})\right] = 0, \tag{46}$$

$$E_t\left[(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1} - \hat{g}^R_{t+1})(\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1})\right] = 0. \tag{47}$$

The optimal portfolio allocation is going to be affected by the factor $\hat{g}^R_{t+1}$ which measures the cross-country difference between the subjective and approximating probability distributions.

[12] Conditional and unconditional moments in general coincides with white-noise processes.

[13] In the next section, we discuss how our empirical counterpart differs from Coeurdacier and Gourinchas (2009).

So far, we have put only a minimal structure on g_{t+1} and g^*_{t+1}. We now enrich our set of assumptions to endogenize the way beliefs are distorted.

Specifically, we will consider the sophisticated agents of the robust-control theory of Hansen and Sargent (2005, 2007). These agents are averse to model uncertainty, and seek decision rules that are robust to it. Following Hansen and Sargent (2005, 2007), we can regard such robust-decision-making process as a two-player game between the representative household and an "evil" agent. The household will surround the reference model with a set of alternative distributions, in which he/she believes the true one lies. The "evil" agent will, then, choose the most unfavorable distribution in this set, and the household will take expectations with respect to that.

To choose the worst-case distribution, therefore, the "evil" agent seeks to minimize the utility of the decision-maker under an entropy constraint of the form similar to (4). The latter defines the size of the set of alternative models, and imposes a bound on the allowed divergence between the distorted and the approximating measures. In a more formal way, $\{g_t\}$ is chosen to minimize

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}G_t\ln C_t\right\},$$

under the entropy constraint

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}G_t\beta E_t(g_{t+1}\ln g_{t+1})\right\}\leq k,$$

and the restrictions given by the martingale assumption on G_t:

$$G_{t+1}=g_{t+1}G_t \tag{48}$$

$$E_t g_{t+1}=1. \tag{49}$$

The parameter k in the entropy constraint imposes an upper-bound on the divergence between the distorted and the approximating beliefs. The higher k, the more afraid of misspecification the agent is, because a higher k allows the "evil" agent to choose larger distortions.

Hansen and Sargent (2005) propose an alternative formulation of this problem in which the entropy constraint is added to the utility of the agent to form a modified objective function

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}G_t\ln c_t\right\}+\theta E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}G_t\beta E_t(g_{t+1}\ln g_{t+1})\right\}, \tag{50}$$

where $\theta>0$ is a penalty parameter on discounted entropy.

The problem of the "evil" agent, therefore, becomes that of choosing the path $\{g_t\}$ to minimize (50) under the constraints (48) and (49). Higher values of θ imply less fear of model misspecification, because the "evil" agent is penalized more by raising entropy when it minimizes the utility of the decision-maker. When θ goes to infinity, the optimal choice of the "evil" agent is to set $g_{t+1}=1$ at all times, meaning that the optimal distortion is zero: the rational expectations equilibrium is nested as a special case.

The problem of the decision-maker is instead that of choosing sequences for consumption and portfolio shares to maximize (50) taking into account the minimizing action of the evil agent. As discussed in the literature, among others by Barillas et al (2006), it can be shown that the solution to the inner minimization problem implies a transformation of the original utility function (50) into a non-expected recursive utility function of the form

$$v_t=c_t^{1-\beta}\left(\left[E_t(v_{t+1})^{1-\gamma}\right]^{\frac{1}{1-\gamma}}\right)^{\beta}. \tag{51}$$

This risk-adjusted utility function coincides with that of the preferences described in Kreps and Porteus (1978) and Epstein and Zin (1989), in which γ, the risk-aversion coefficient of the Kreps-Porteus-Epstein-Zin preferences, is related to θ through the following equation:

$$\theta = \frac{1}{(1-\beta)(\gamma-1)}. \tag{52}$$

The two frameworks imply the same equilibrium allocation, but the assumptions under the two models are different and in particular the parameters θ and γ have very different interpretations.[14] As discussed in Barillas et al. (2006), γ represents the risk-aversion coefficient whereas θ is a measure of the doubts that the decision-maker has with respect to the model probability distribution. While γ can indeed be calibrated as a parameter capturing the degree of risk aversion, θ can be related to detection error probabilities, which measure how difficult it is to tell apart competing models using a finite data set. The more the doubts (i.e. the lower θ), the more divergent the worst-case scenario and the lower the detection error probability (i.e. the easier is to distinguish the worst-case model from the reference model). A framework with model uncertainty can be more appealing than Kreps-Porteus-Epstein-Zin preferences because high values of γ can be considered implausible as a measure of risk aversion while, on the contrary, they can correspond to reasonable values for the detection error probabilities.

Moreover, the framework with model uncertainty is also observationally equivalent to a model in which there are preference shocks, which Pavlova and Rigobon (2007) have shown to be successful in explaining several features of asset prices in open economies. The Hansen-Sargent sophisticated agent seeking robust decisions, indeed, is just one of the possible classes of agents acting in a framework characterized by model uncertainty and distorted beliefs, such as the one defined in Section 2.

Using (51), the equilibrium real stochastic discount factor is

$$m_{t+1} = \beta \frac{c_t}{c_{t+1}} \left(\frac{v_{t+1}^{1-\gamma}}{[E_t(v_{t+1})^{1-\gamma}]} \right),$$

which implies that the optimal distortion is

$$g_{t+1} = \left(\frac{v_{t+1}^{1-\gamma}}{[E_t(v_{t+1})^{1-\gamma}]} \right).$$

Notice that in (51) we can scale continuation values by consumption to get

$$\frac{v_t}{c_t} = \left[E_t \left(\frac{v_{t+1}}{c_{t+1}} \frac{c_{t+1}}{c_t} \right)^{1-\gamma} \right]^{\frac{\beta}{1-\gamma}},$$

showing that g_{t+1} can be related to the current and future consumption path. Indeed, in a first-order approximation, which suffices to evaluate (45)–(47), we can write:

$$\hat{g}_{t+1} = -(\gamma-1) \sum_{j=0}^{\infty} \beta^j \left[E_{t+1} \Delta \hat{c}_{t+1+j} - E_t \Delta \hat{c}_{t+1+j} \right],$$

[14] The two models are observationally equivalent only with log utility. See Strzalecki (2009) for an analysis on how models of ambiguity aversion imply different preferences for the timing resolution of uncertainty.

in which $\hat{g}_{t+1}$ increases when the agent receives bad news with respect to the consumption-growth profile. Hence, the worst-case scenario takes the form of downward revisions in current and future consumption growth.[15]

Recall that $g(s_{t+1}|s^t)$ is equivalent to the ratio between the "subjective" and "approximating" probabilities, $\widetilde{\pi}(s_{t+1}|s^t)/\pi(s_{t+1}|s^t)$. Higher values of $g(s_{t+1}|s^t)$ therefore implies that the agent is assigning a higher probability on those states of nature where there are bad news on the consumption-growth profile. When $g(s_{t+1}|s^t)$ increases, the stochastic discount factor increases, measuring the appetite for receiving additional wealth. In this case, the agent would like to hold assets that pay well when there are indeed bad news on the consumption-growth profile.

The above derivations apply also to the foreign agent. Thereby, in the symmetric case in which $\gamma = \gamma^*$, we can show that the optimal relative distortion depends negatively on the surprises in the consumption-growth differential across countries:

$$\hat{g}^R_{t+1} = -(\gamma - 1)\sum_{j=0}^{\infty}\beta^j\left[E_{t+1}\Delta\hat{c}^R_{t+1+j} - E_t\Delta\hat{c}^R_{t+1+j}\right]. \tag{53}$$

It is important to notice, at this point, that the first-order approximation of the model equilibrium conditions is not affected by the assumption of distorted beliefs. Indeed, g_{t+1} and g^*_{t+1} do enter the Euler equations, but the martingale assumption implies that their expected value is zero, up to first order. However, g_{t+1} and g^*_{t+1} enter *indirectly* the first-order approximation because they affect the coefficients of the approximation, which depend on the steady-state portfolio allocation. As we have shown, indeed, the steady-state shares depend on the ratio of second-order moments.

It follows from the above that (29) and (30) still hold and can be used to write (53) as

$$\hat{g}^R_{t+1} = -(\gamma - 1)\frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}'\mathbf{exr}_{t+1} - (\gamma - 1)\varepsilon_{q,t+1} - (\gamma - 1)\frac{s_\xi}{s_c}\varepsilon_{l,t+1},$$

where we have defined the time-t surprises in the real exchange rate growth as

$$\varepsilon_{q,t+1} \equiv \sum_{j=0}^{\infty}\beta^j\left[E_{t+1}\Delta\hat{q}_{t+1+j} - E_t\Delta\hat{q}_{t+1+j}\right]. \tag{54}$$

Therefore, the left-hand side of the orthogonality conditions (45)–(47) can be written as

$$(\Delta\hat{c}^R_{t+1} - \Delta\hat{q}_{t+1} - \hat{g}^R_{t+1}) = \gamma\frac{(1-\beta)}{\beta s_c}\bar{\boldsymbol{\lambda}}'\mathbf{exr}_{t+1} + (\gamma - 1)\varepsilon_{q,t+1} + \gamma\frac{s_\xi}{s_c}\varepsilon_{l,t+1},$$

from which it follows that (45)–(47) imply

$$\bar{\boldsymbol{\lambda}} = -s_\xi\frac{\beta}{1-\beta}\boldsymbol{\Sigma}_t^{-1}E_t(\mathbf{exr}_{t+1}\cdot\varepsilon_{l,t+1}) - s_c\frac{(\gamma-1)}{\gamma}\frac{\beta}{1-\beta}\boldsymbol{\Sigma}_t^{-1}E_t(\mathbf{exr}_{t+1}\cdot\varepsilon_{q,t+1}). \tag{55}$$

Equation (55) determines the optimal steady-state portfolio shares under model uncertainty.

In particular, equation (55) displays an additional term with respect to equation (34), which depends on the covariances between the excess returns and the surprises in the real exchange rate. This term captures the hedging motif with respect to the real exchange rate risk, whose

[15] Hansen et al. (2008) show how to derive g_{t+1}in a closed-form solution including risk-premia terms, which, however, are not important in our approximation for computing the steady-state portfolio shares.

importance depends on the magnitude of parameter γ, capturing the degree of ambiguity aversion: the higher γ, the more averse to model uncertainty the investors, the more important this component. When investors are not concerned about model uncertainty (i.e. as $\theta \to \infty$) then γ is equal to 1, beliefs are not distorted and (55) coincides with (34).

Notice that, in contrast with rational expectations, model uncertainty and fear of misspecification can imply a departure from full diversification even when all income risk is tradeable, i.e. when $s_\xi = 0$.

In particular, equation (55) implies that, on top of the first component already discussed in Section 4, there should be a bias in holding domestic equity when, conditional on the other returns, domestic equity pays well relative to foreign equity when there are news about current or future appreciations of the real exchange rate. Bonds should be held when they pay better than equities when needed, and a higher share of foreign bonds when their return is negatively correlated with the surprises in the real exchange rate, conditioning on the other excess returns.

The intuition is the following. Under model uncertainty agents might use a distorted probability distribution. In particular, when they seek robust choices, this distortion comes from the fear of the worst-case scenario. Investors then tend to assign a higher probability to the states of nature in which they get bad news with respect to their consumption-growth profile. Bad news for domestic consumption growth relative to foreign one are captured by unexpected appreciations of the real exchange rate (a fall in q). Ambiguity-averse investors want to hedge against this scenario and will therefore overinvest in assets that pay relatively well when there are news of current or future appreciations.

There is an important distinction to underline at this point. The additional component in equation (55), capturing the hedge against real exchange rate risk, would be also present in a model with non-distorted beliefs and non-unitary risk aversion/or intertemporal elasticity of substitution. In that case, indeed, agents would want to hedge fluctuations in relative inflation rates and condition (55) would also apply. However, in a rational-expectation model, risk aversion is the reciprocal of the intertemporal elasticity of substitution. By rising risk aversion, to make the second component larger, the intertemporal elasticity of substitution is lowered and the implied risk-free rate increases in a counterfactual way. With our preference specifications, instead, the intertemporal elasticity of substitution is tied to one (a value close to recent empirical estimates) whereas the parameter γ can increase to give more weight to the second component without affecting the mean of the risk-free rate.[16]

Moreover, equation (54) shows that what matters is not only the risk of an immediate variation in the real exchange rate, but also the risk of future ones. As β gets close to one, only the *long-run risk* remains relevant. In this case, indeed, $\varepsilon_{q,t+1}$ becomes proportional to the revisions in the conditional expectations of the long-run real exchange rate

$$\varepsilon_{q,t+1} \cong E_{t+1}\hat{q}_\infty - E_t\hat{q}_\infty.$$

We can get further insights by looking at the simple case in which only equities are traded. It can be shown that in this case

$$\bar{\alpha}_F^e = \frac{1}{2} - \frac{1}{2}\frac{\beta}{1-\beta}s_\xi \frac{cov_t(\varepsilon_{l,t+1}, \hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1})}{var_t(\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1})}$$
$$- \frac{1}{2}\frac{(\gamma-1)}{\gamma}\frac{\beta}{1-\beta}s_c \frac{cov_t(\varepsilon_{q,t+1}, \hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1})}{var_t(\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^e_{H,t+1})}. \quad (56)$$

[16]See also Piazzesi and Schneider (2006) on how preferences of this kind are able to match moments on the US term structure.

On top of equation (42), agents would like to hold more domestic equities if their return is high when the real exchange rate is expected to appreciate. This requires that $\varepsilon_{q,t+1}$ co-varies positively with the excess returns of foreign-versus-domestic equity, $\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$. As the fear of model misspecification increases, then, this additional hedging motif matters more for determining home bias in international portfolio choice.

In the more general case in which also bonds are traded, the above condition still holds, although now variances and covariances are conditional on the residual excess returns:

$$\bar{\alpha}^e_F = \frac{1}{2} - \frac{1}{2}\frac{\beta}{1-\beta}s_\xi \frac{cov_t(\varepsilon_{l,t+1}, \hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}|exr^{ib}_{t+1}, exr^{de}_{t+1})}{var_t(\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}|exr^{ib}_{t+1}, exr^{de}_{t+1})}$$
$$- \frac{1}{2}\frac{(\gamma-1)}{\gamma}\frac{\beta}{1-\beta}s_c \frac{cov_t(\varepsilon_{q,t+1}, \hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}|exr^{ib}_{t+1}, exr^{de}_{t+1})}{var_t(\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}|exr^{ib}_{t+1}, exr^{de}_{t+1})}. \quad (57)$$

6 Empirical Evidence

One of the appealing features of the theoretical model presented in the previous section is that it derives clear implications about the second moments of variables that are directly observable. These implications can therefore be tested empirically without further assumptions on the empirical counterparts of our theoretical variables.

6.1 Data

To evaluate the implications of equations (34) and (55), we collect and use quarterly data for the G7 Countries, over the sample 1980q1-2007q4. We consider the US as the Home country and the aggregation of the rest of the G7 countries as the Foreign country.[17]

We define the CPI index for the foreign country, expressed in USD, as

$$S_t P^*_t = \sum_i \omega_{i,t} S_{i,t} P_{i,t},$$

in which $P_{i,t}$ is the CPI in local currency for country i, $S_{i,t}$ is the bilateral nominal exchange rate between the local currency in country i and the dollar (US dollars for one unit of local currency), and $\omega_{i,t}$ is the actual time-t GDP-weight of country i relative to the aggregation of the G6 countries:[18]

$$\omega_{i,t} = \frac{GDP_{i,t}}{\sum_i GDP_{i,t}}.$$

Accordingly, the real exchange rate between the US and the G6 countries is simply computed as:

$$\hat{q}_t = \log\left(\frac{S_t P^*_t}{P_t}\right) = \log\left(\frac{\sum_i \omega_{i,t} S_{i,t} P_{i,t}}{P_t}\right),$$

[17] In particular, we use data on aggregate nominal compensation of employees, from the OECD Quarterly National Accounts (**OCOS02B, where ** is the two-letter country code), the Consumer Price Indexes from the IFS database (**I64..F), nominal returns on short-term treasury bills from the IFS database (**I60C..), nominal National Price and Gross Return indexes on the domestic stock market, from MSCI Barra (MS****L), in local currency, and bilateral nominal exchange rate vis-à-vis the USD, constructed using the domestic stock-price indexes in USD, from the MSCI Barra (MS****$). Moving from the monthly National Price and Gross Return indexes from MSCI database, we construct series for the quarterly nominal returns on equity ($R^e_{i,t}$) following Campbell (1999).

[18] To check for robustness, we repeated the analysis using average GDP-weights as an alternative aggregation methodology, as in Coeurdacier and Gourinchas (2009), and using both aggregate and per-capita levels for the quantity variables. None of our results is significantly affected.

Table 1: Some Data Statistics (Annual rates)

	$\mu(\cdot)$	$\sigma(\cdot)$	$\rho(\cdot)$	$\rho(\cdot, \Delta\hat{\xi}^R - \Delta\hat{q})$	$\rho(\cdot, \Delta\hat{q})$
$\Delta\hat{\xi}^R - \Delta\hat{q}$	0.773	13.051	0.024	1.000	−0.438
$\Delta\hat{q}$	0.165	11.348	0.175	−0.438	1.000
$\hat{r}_F^{e*} + \Delta\hat{q} - \hat{r}_H^e$	0.699	13.535	0.108	−0.530	0.436
$\hat{r}_F^* + \Delta\hat{q} - \hat{r}_H$	0.984	10.718	0.030	−0.919	0.722
$\hat{r}_H^e - \hat{r}_H$	6.350	15.850	−0.004	−0.027	−0.139

Note: means and standard deviations are in percentage points

where P_t is the CPI index for the US.

Analogously, we compute nominal labor income in US dollars for the Foreign country as:

$$S_t W_t^* \bar{l}_t^* = \sum_i \omega_{i,t} S_{i,t} W_{i,t} \bar{l}_{i,t},$$

in which we measure $W_{i,t}\bar{l}_{i,t}$ using data on aggregate nominal compensation of employees in country i. Accordingly, relative labor income in units of US dollars is the log difference between the aggregate nominal compensation in the US and that in the rest of the world:

$$\log\left(\frac{W_t\bar{l}_t}{S_t W_t^* \bar{l}_t^*}\right) = \log\left(\frac{W_t\bar{l}_t}{P_t}\frac{P_t}{S_t P_t^*}\frac{P_t^*}{W_t^*\bar{l}_t^*}\right) = \log\left(\frac{\xi_t}{q_t\xi_t^*}\right) = \hat{\xi}_t^R - \hat{q}_t.$$

Given nominal quarterly returns on the stock market, defined by $R_{i,t}^e$ for each country i and R_t^e for the US, and nominal quarterly returns on bonds, defined by $R_{i,t}$ for each country i and R_t for the US, we can obtain the real returns as $r_{i,t} \equiv R_{i,t}P_{i,t-1}/P_{i,t}$ and $r_{i,t}^e \equiv R_{i,t}^e P_{i,t-1}/P_{i,t}$ for each country i and for the US, respectively. Using those, we construct the three excess returns of interest as

$$exr_t^{ie} \equiv \hat{r}_{F,t}^{e*} + \Delta\hat{q}_t - \hat{r}_{H,t}^e = \log\left(\frac{\sum_i \omega_{i,t} r_{i,t}^e \frac{q_{i,t}}{q_{i,t-1}}}{r_t^e}\right),$$

$$exr_t^{ib} \equiv \hat{r}_{F,t}^* + \Delta\hat{q}_t - \hat{r}_{H,t} = \log\left(\frac{\sum_i \omega_{i,t} r_{i,t} \frac{q_{i,t}}{q_{i,t-1}}}{r_t}\right),$$

$$exr_t^{de} \equiv \hat{r}_{H,t}^e - \hat{r}_{H,t} = \log\left(\frac{r_t^e}{r_t}\right).$$

Table 1 reports some summary statistics for the variables of interest. We report the average level $\mu(\cdot)$ and the standard deviation $\sigma(\cdot)$, both annualized and in percentage points, the serial correlation coefficient $\rho(\cdot)$ and the correlation with the growth rate in relative labor income $\rho(\cdot, \Delta\hat{\xi}^R - \Delta\hat{q})$ and with the real exchange rate $\rho(\cdot, \Delta\hat{q})$. These simple correlations already suggest that domestic equity seems a poor hedge against labor income risk, relative to foreign stocks, while both domestic equity and domestic bonds seem somewhat useful in providing the right co-movement to hedge against real exchange rate fluctuations. In the next sections we will refine and articulate these results.

In order to evaluate the optimal portfolio allocation implied by our model, we need to calibrate the steady-state ratio of consumption-to-financial wealth, s_c. To this end, we use the average financial wealth-to-disposable income ratio for the US computed by Bertaut (2002), and the average consumption-to-disposable income ratio for the US, computed using data on personal consumption of non-durable goods and personal disposable income. The former, on a quarterly frequency, amounts to about 20, while the latter to around .3: by using these numbers we get a calibrated consumption-to-wealth ratio $s_c = .3/20 = .015$. We calibrate the quarterly time discount factor following Tallarini (2000) and Barillas et al (2006): $\beta = .995$. Using the value of s_c obtained above, we derive the model-consistent steady-state value of the labor income-to-financial wealth ratio, by using $s_\xi = s_c - (1-\beta)/\beta = .01$.

6.2 The statistical model

We define the following vector

$$\mathbf{y}_t \equiv \begin{bmatrix} \Delta\hat{\xi}_t \\ \Delta\hat{\xi}_t^* \\ \Delta\hat{q}_t \\ \hat{r}_{F,t}^* + \Delta\hat{q}_t - \hat{r}_{H,t} \\ \hat{r}_{H,t}^e - \hat{r}_{H,t} \\ \hat{r}_{F,t}^{e*} + \Delta\hat{q}_t - \hat{r}_{H,t}^e \\ \hat{r}_{H,t} \\ \mathbf{x}_t \end{bmatrix}, \tag{58}$$

and estimate the following VAR(1) model

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t, \tag{59}$$

in which $\mathbf{e}_t$ is distributed as a multivariate normal with zero mean and variance-covariance matrix $\boldsymbol{\Omega}$.[19] In the data vector $\mathbf{y}$ we also include a series of additional controls, collected into the vector $\mathbf{x}$, which might be useful in describing the dynamic path of the variables of interest. In practice, $\mathbf{x}$ includes the growth rate of relative GDP, the slope of the US yield curve, the international excess return on ten-year government bonds and the growth rate in the US trade balance.[20]

We define $\boldsymbol{\iota}_z$ as the vector that selects the element z from vector $\mathbf{y}$. In particular, the vector of excess returns, in deviation from its conditional mean, can be written as

$$\mathbf{exr}_{t+1} - E_t\mathbf{exr}_{t+1} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \dots 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \dots 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \dots 0 \end{bmatrix} (\mathbf{y}_{t+1} - E_t\mathbf{y}_{t+1}) = \begin{bmatrix} \boldsymbol{\iota}_{ib}' \\ \boldsymbol{\iota}_{de}' \\ \boldsymbol{\iota}_{ie}' \end{bmatrix} \mathbf{e}_{t+1} = \boldsymbol{\iota}_{exr}'\mathbf{e}_{t+1}.$$

We can then use the estimated statistical model to evaluate $\varepsilon_{l,t+1}$ and $\varepsilon_{q,t+1}$, as

$$\varepsilon_{l,t+1} \equiv \sum_{j=0}^{\infty} \beta^j [E_{t+1}(\Delta\hat{\xi}_{t+1+j}^R - \Delta\hat{q}_{t+1+j}) - E_t(\Delta\hat{\xi}_{t+1+j}^R - \Delta\hat{q}_{t+1+j})] = \boldsymbol{\iota}_l'(I - \beta\mathbf{A})^{-1}\mathbf{e}_{t+1},$$

in which $\boldsymbol{\iota}_l \equiv \boldsymbol{\iota}_\xi - \boldsymbol{\iota}_{\xi^*} - \boldsymbol{\iota}_q$, and

$$\varepsilon_{q,t+1} \equiv \sum_{j=0}^{\infty} \beta^j \left(E_{t+1}\Delta\hat{q}_{t+1+j} - E_t\Delta\hat{q}_{t+1+j}\right) = \boldsymbol{\iota}_q'(I - \beta\mathbf{A})^{-1}\mathbf{e}_{t+1}.$$

[19] The length of the VAR is chosen optimally using the Schwarz's Bayesian Criterion for each estimation, and turns out to be always 1.

[20] Gourinchas and Rey (2007) show that the net export growth rate is a useful predictor for portfolio returns at long horizons, while the other variables are among the forecasting variables commonly used for predicting asset returns and labor income. See also Campbell (1996).

Finally, by setting $\mathbf{H} \equiv (\mathbf{I} - \beta\mathbf{A})^{-1}$, we can evaluate the relevant time–t conditional covariance vectors and matrices as implied by our estimated statistical model:

$$E_t\left\{(\mathbf{exr}_{t+1} - E_t\mathbf{exr}_{t+1}) \cdot \varepsilon_{l,t+1}\right\} = \boldsymbol{\iota}_{exr}'\boldsymbol{\Omega}\mathbf{H}'\boldsymbol{\iota}_l$$
$$E_t\left\{(\mathbf{exr}_{t+1} - E_t\mathbf{exr}_{t+1}) \cdot \varepsilon_{q,t+1}\right\} = \boldsymbol{\iota}_{exr}'\boldsymbol{\Omega}\mathbf{H}'\boldsymbol{\iota}_q$$
$$\boldsymbol{\Sigma}_t \equiv E_t\left\{(\mathbf{exr}_{t+1} - E_t\mathbf{exr}_{t+1})(\mathbf{exr}_{t+1} - E_t\mathbf{exr}_{t+1})'\right\} = \boldsymbol{\iota}_{exr}'\boldsymbol{\Omega}\boldsymbol{\iota}_{exr}.$$

Using the above, we can evaluate the theoretical implications of our framework, and relate the results to existing literature. At this point, it is important to underline that a common procedure in the literature is to rely on static models, and to evaluate covariances and variances using *unconditional* distributions (see van Wincoop and Warnock, 2006, 2008, and Coeurdacier and Gourinchas, 2009) or equivalently by running regressions of the form

$$\varepsilon_{l,t+1} = \kappa_l + \boldsymbol{\psi}_l'\mathbf{exr}_{t+1} + u_{l,t+1}, \tag{60}$$
$$\varepsilon_{q,t+1} = \kappa_q + \boldsymbol{\psi}_q'\mathbf{exr}_{t+1} + u_{q,t+1}, \tag{61}$$

for given parameters κ_k, $\boldsymbol{\psi}_k$, for $k = l, q$ and well-behaved residuals u, where indeed OLS regression coefficients imply

$$\boldsymbol{\psi}_k = \boldsymbol{\Sigma}^{-1}E(\mathbf{exr}_{t+1} \cdot \varepsilon_{k,t+1}). \tag{62}$$

It is important to notice, however, that in the context of a general dynamic model this procedure is appropriate only as long as $\mathbf{y}_t$ is a multivariate white-noise process.

6.3 The case of no model uncertainty

In the absence of model uncertainty, with log utility, the only possible reason for home bias in equity is hedging against non-diversifiable labor income risk. In particular, this depends on the positive covariance between the present discounted value of domestic-versus-foreign labor income and the excess return of foreign-versus-domestic equity.

This hedging motif has been emphasized by several studies without reaching a clear consensus. Baxter and Jermann (1997) show that when equity is the only asset that can be traded internationally, the presence of non-diversifiable income risk actually implies a foreign-equity bias. On the other hand, Bottazzi et al. (1996) and more recently Julliard (2003) and Coeurdacier and Gourinchas (2009) bring evidence supporting the view that hedging against labor-income risk can explain some degree of home-bias in equity holdings. Heathcote and Perri (2004) and Coeurdacier and Gourinchas (2009), moreover, discuss some theoretical examples that can produce the required co-movements to explain home-bias.

We analyze this interaction in the context of our dynamic model, starting with a simple case in which the asset menu available for international trade includes only equities (henceforth *Asset Menu I*). In this case the relevant equilibrium condition, described by equation (42), involves a covariance-to-variance ratio which is conditional on time-t information, but *un*conditional on the residual asset space, the latter being empty.

To evaluate the relevant covariance and derive the portfolio allocation, we first estimate our statistical model.[21] Using the output of the VAR we construct the surprises in the path of rela-

[21] For what concerns the statistical model, as a robustness check, we estimated three alternative specifications. The first specification is the minimal requirement to describe the model economy and include only data on labor income and the excess return on foreign equity. The second and third specifications augment the first one by introducing data on the residual excess returns. Moreover, for each of the specifications above, we also varied the informational content of the data-vector by adding the real exchange rate, in changes, and the auxiliary regressors included in vector $\mathbf{x}$. In the text we report results for the extensive specification only, corresponding to equation (58), since results are robust to the other alternatives. The full set of results is available upon request.

tive labor income across countries. In our model, we need to evaluate covariances and variances conditional on previous-period information, but for comparisons with Coeurdacier and Gourinchas (2009) we also compute the unconditional moments. The unconditional ratios are obtained through straightforward OLS projection of the surprises in relative labor income, obtained from the VAR, on the excess returns of the assets available to trade (in this case just the excess return of foreign-versus-domestic equity). Following equation (60), we obtain:

$$\varepsilon_{l,t+1} = \underset{(0.081)}{-\mathbf{0.479}} \cdot (\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}) + u_{l,t+1},$$

where the standard error is reported in parenthesis. As it is clear from the equation above, it turns out that the relevant covariance-to-variance ratio is negative, statistically significant and economically rather large. The result is therefore that hedging labor-income risk in this setup is unable to produce home-bias in equity, but it rather implies a *foreign*-equity bias. This result on the one hand supports Baxter and Jermann (1997), and on the other hand weakens the argument of Heathcote and Perri (2007).

Notice, however, that this procedure is consistent with our theoretical model only as long as the statistical evidence suggests that the process $\mathbf{y}_t$ is in fact a multivariate white noise. This representation is not supported by our data, especially when we include additional regressors to help predicting the future path of labor income and excess returns. In this case, it is more appropriate to compute the covariance-to-variance ratio conditional on time t following equation (42). To this end, we use the output of the estimated VAR model and compute the relevant ratios directly, using

$$\begin{aligned} cov_t(\varepsilon_{l,t+1}, \hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}) &= \boldsymbol{\iota}'_{ie}\boldsymbol{\Omega}\mathbf{H}'\boldsymbol{\iota}_l \\ var_t(\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}) &= \boldsymbol{\iota}'_{ie}\boldsymbol{\Omega}\boldsymbol{\iota}_{ie}. \end{aligned}$$

Also the computation of the *conditional* covariance-to-variance ratio supports the results of Baxter and Jermann (1997), that the portfolio diversification puzzle is even worse than expected. When the only asset that can be traded internationally is equity, the relevant covariance-to-variance ratio is negative, and so large that the implied position on the international equity market is consistent with allocating basically the whole financial wealth in foreign equity, as shown in Table 2.

Table 2: Model with equities only – Rational Expectations

Conditional Covariance-variance Ratios	
$\frac{cov_t(\varepsilon_{l,t+1}, \hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1})}{var_t(\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1})}$	$-\mathbf{0.524}$
Optimal Portfolio Allocation	
$\bar{\alpha}^e_F$	**1.020**

Note: $\bar{\alpha}^e_F$ denotes the share of wealth invested in foreign equity

This result has been recently challenged by Coeurdacier and Gourinchas (2009), who point out that, once also riskless bonds are traded, variances and covariances should be computed conditional on the other asset returns, as also shown by equations (43)–(44). Their claim is that, with the appropriate conditioning, the previous result would be overturned, and their empirical findings indeed support this claim.

We repeat the analysis of Coeurdacier and Gourinchas (2009) within our dynamic framework. Accordingly, the asset menu, in this case, includes both equities and bonds, and the latter are balanced to an overall zero-position (for short, *Asset Menu II*).

Table 3 contrasts our findings with theirs.[22] In the second column we report the findings of Coeurdacier and Gourinchas (2009) which show that, conditioning on the residual excess returns, there is a positive covariance between the excess return on foreign-versus-domestic equity and non-diversifiable labor-income risk, whereas the unconditional covariance is instead negative. Thereby, they conclude that the results in Baxter and Jermann (1997) are driven by their particular asset structure, and do not hold when bonds are included. In the third column, we report our estimation's results, which show instead a negative (although insignificant) conditional covariance-to-variance ratio.

Table 3: Unconditional covariance-variance ratios

Loadings of:	Coeurdacier–Gourinchas (2009)*	Benigno–Nisticò (2009)**
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	**0.260** (0.070)	**−0.027** (0.067)
$\hat{r}^{*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}$	**−1.170** (0.110)	**−0.982** (0.084)

Note: standard errors in parentheses. * Dependent variable is $\hat{r}^{w}_{t+1} - E_t\hat{r}^{w}_{t+1}$. ** Dependent variable is $\varepsilon_{l,t+1}$.

The difference between the two results can be explained by the different approach to measuring labor-income risk. Coeurdacier and Gourinchas (2009) use the unexpected component of the (home relative to foreign) return-to-labor, which they construct as

$$\hat{r}^{w}_{t+1} - E_t\hat{r}^{w}_{t+1} = \sum_{j=0}^{\infty}\rho^j(E_{t+1} - E_t)(\Delta\hat{\xi}^{R}_{t+1+j} - \Delta\hat{q}_{t+1+j})$$
$$-\sum_{j=1}^{\infty}\rho^j(E_{t+1} - E_t)(\hat{r}^{e}_{H,t+1+j} - \Delta\hat{q}_{t+1+j} - \hat{r}^{e*}_{F,t+1+j}) = (\boldsymbol{\iota}'_l - \rho\boldsymbol{\iota}'_{ie}\mathbf{A})(\mathbf{I} - \rho\mathbf{A})^{-1}\mathbf{e}_{t+1}, \quad (63)$$

where $\rho \equiv 1 - s_c$ is a constant of linearization that depends on the average consumption-to-wealth ratio. It is worth noticing that this measure is not directly implied by their model, which is static, but rather it is borrowed from Campbell (1996). Two important assumptions underlie this formulation, which are critical to distinguish their approach from ours. First, it is assumed that there exists a market for tradeable claims on the stream of future labor-income flows, which implies that the return on labor is computed in analogy to the return on the financial assets. Second, the expected relative return on domestic non-financial wealth is equated to the expected excess return on domestic-versus-foreign equities. This is a strong assumption, as also discussed by Campbell (1996), and explains why the first term on the second line of equation (63) arises.

With this definition, it follows that the return-to-labor is likely to be positively related, by construction, with the excess return on foreign-versus-domestic equity.

We do not make either of the assumptions above. Instead, in our framework, the relevant measure of non-diversifiable labor risk is directly implied by the theoretical model, and corresponds to the revision in the present-discounted value of cross-country labor income $\varepsilon_{l,t+1}$, as

[22] Note that we have defined the excess returns as foreign-versus-domestic returns, the opposite of Coeurdacier and Gourinchas (2009). Accordingly, for comparison, in Table 3 we report their results multiplied by -1.

Table 4: Model with equities and balanced bonds – Rational Expectations

Conditional Covariance-variance Ratios	
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{ie}_{t+1} \mid exr^{ib}_{t+1})}{var_t(exr^{ie}_{t+1} \mid exr^{ib}_{t+1})}$	**0.016**
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{ib}_{t+1} \mid exr^{ie}_{t+1})}{var_t(exr^{ib}_{t+1} \mid exr^{ie}_{t+1})}$	**−1.116**
Optimal Portfolio Allocation	
$\bar{\alpha}^e_F$	**0.484**
$\bar{\alpha}_F$	**1.108**
$\bar{\alpha}^e_F + \bar{\alpha}_F$	**1.592**

Note: $\bar{\alpha}^e_F$ denotes the share of wealth invested in foreign equity; $\bar{\alpha}_F$ denotes the share of wealth invested in foreign bonds; $\bar{\alpha}^e_F + \bar{\alpha}_F$ measures the overall share of wealth invested in foreign assets

shown by equation (35).[23] It is worth noticing that our measure of labor-income risk is instead similar to those used by Shiller (1995) and Baxter and Jermann (1997), which coincide with the first summation on the right-hand-side of (63).[24] Using this definition, we find that domestic equity is not a good hedge, even if we condition on bond returns. Therefore, we reinforce Baxter and Jermann's (1997) results even when we condition on other excess returns.

To derive the equilibrium portfolio allocations implied by our theoretical model, we compute the relevant covariance-to-variance ratios conditioning them also on the information set available at time$-t$, and report the results in Table 4. The covariance between labor-income risk and the excess return on equities becomes of the right sign, but it is quantitatively negligible, and it does not imply a substantial degree of home-bias. On this respect, therefore, our results again contrast with Coeurdacier and Gourinchas (2009). However, we share the finding that agents should go long in foreign bonds and short in domestic ones, with the counterfactual implication that almost 160% of domestic wealth is allocated to foreign assets.

We now turn to the more general specification of our model, by relaxing the assumption that the bond position is balanced, thus allowing for leveraged positions between different types of securities (*Asset Menu III*). The relevant conditions are now (36), (37), (38). In this more general case, we can evaluate the ability of the model to replicate other stylized facts that are receiving increasing attention by the empirical literature. Tille (2005, 2008), for example, reports a detailed breakdown of the composition of US foreign assets and liabilities, and documents four basic features: 1) the U.S. is a large net creditor in equity instruments and 2) a net debtor in bond instruments; 3) the net position on foreign-currency bonds is about balanced, while 4) the position in bonds denominated in US dollars is largely negative.

In our model we note that the steady-state net-foreign asset position (as a share of steady-state domestic wealth), defined by NFA, is given by

$$NFA \equiv \bar{\alpha}^e_F + \bar{\alpha}_F - \bar{\alpha}^{e*}_H - \bar{\alpha}^*_H.$$

[23] Note that in a first-order approximation (which is all is needed to evaluate the orthogonality conditions and derive the portfolio allocation) expected excess returns are always zero, so the last terms in (63) would drop even if we did make the two assumptions discussed above.

[24] Indeed, the only difference between (35) and the measure in Shiller (1995) and Baxter and Jermann (1997) is the discount parameter: while they use $\rho \equiv 1 - s_c$, we use the time discount factor β. Numerically, however, they are also very close to each other.

Table 5: General model with equities and bonds – Rational Expectations

Conditional Covariance-variance Ratios	
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{ie}_{t+1} \mid exr^{ib}_{t+1}, exr^{de}_{t+1})}{var_t(exr^{ie}_{t+1} \mid exr^{ib}_{t+1}, exr^{de}_{t+1})}$	**0.040**
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{ib}_{t+1} \mid exr^{ie}_{t+1}, exr^{de}_{t+1})}{var_t(exr^{ib}_{t+1} \mid exr^{ie}_{t+1}, exr^{de}_{t+1})}$	**−1.129**
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{de}_{t+1} \mid exr^{ie}_{t+1}, exr^{ib}_{t+1})}{var_t(exr^{de}_{t+1} \mid exr^{ie}_{t+1}, exr^{ib}_{t+1})}$	**0.062**
Optimal Portfolio Allocation	
$\bar{\alpha}^e_F$	**0.460**
$\bar{\alpha}_F$	**1.121**
$\bar{\alpha}^e_F + \bar{\alpha}_F$	**1.580**
$\bar{\alpha}^e_H + \bar{\alpha}^e_F$	**0.939**
$\bar{\alpha}^e_H$	**0.479**

Note: $\bar{\alpha}^e_F$ denotes the share of wealth invested in foreign equity; $\bar{\alpha}_F$ denotes the share of wealth invested in foreign bonds; $\bar{\alpha}^e_F + \bar{\alpha}_F$ measures the overall share of wealth invested in foreign assets; $\bar{\alpha}^e_H + \bar{\alpha}^e_F$ measures the overall share of wealth invested in equity; $\bar{\alpha}^e_H$ denotes the share of wealth invested in domestic equity

Moreover the net-foreign asset position in equities is given by $NFE \equiv \bar{\alpha}^e_F - \bar{\alpha}^{e*}_H$ and that in bonds by $NFB \equiv \bar{\alpha}_F - \bar{\alpha}^*_H$. We write them as[25]

$$NFE = \bar{\alpha}^e_F + \bar{\alpha}^e_H - 1 \quad (64)$$

$$NFB = \bar{\alpha}_F + \bar{\alpha}_H, \quad (65)$$

in which $\bar{\alpha}_F$ and $\bar{\alpha}_H$ capture country H's position on bonds denominated in foreign and domestic currency, respectively.

Accordingly, to replicate the U.S. features documented by Tille (2005, 2008), our model should imply that $\bar{\alpha}^e_F + \bar{\alpha}^e_H > 1$, $\bar{\alpha}_F + \bar{\alpha}_H < 0$, $\bar{\alpha}_F \approx 0$ and $\bar{\alpha}_H < 0$.

To evaluate the implications of our general framework along these dimensions, we compute the time-t conditional covariance-to-variance ratios that are needed to derive the optimal portfolio allocation, and report the results in Table 5. Allowing for a non-zero position in the international bond market does not change the result of *quasi*-full international portfolio diversification. Indeed, empirical co-movements imply that domestic investors would like to allocate more than 150% of their wealth in foreign assets ($\bar{\alpha}^e_F + \bar{\alpha}_F$=1.580). Moreover, the empirical co-movement of labor-income risk with the domestic equity premium would imply an overall long position in the international bond market ($\bar{\alpha}^e_H + \bar{\alpha}^e_F < 1$), contrary to what documented by Tille (2005, 2008). Note that this result further exacerbates the inability of labor-income risk alone to support home-bias in equity: indeed, even though less than half of the steady-state wealth is allocated to foreign equities, the share allocated to domestic ones is also smaller than 50%, as reported by the last line of Table 5. Finally, the largely long position on foreign bonds is also at odds with the empirical facts documented by Tille (2005, 2008).

[25]It can be shown that in a symmetric steady state in which $\bar{A} = \bar{S}\bar{A}^*$, $\bar{\alpha}^{e*}_H = 1 - \bar{\alpha}^e_H$ and $\bar{\alpha}^*_H = -\bar{\alpha}_H$.

We will show in the next section that our model with distorted beliefs is more successful in accounting for this empirical evidence.

6.4 Real exchange rate risk and model uncertainty

In the above section we showed that there is no support for the view that domestic equity is a good hedge against non-diversifiable labor-income risk to explain the home-bias in U.S. equity holdings. We now move to analyze the portfolio implications of model uncertainty; in this case, as we have shown, the fear of model misspecification translates into long-run real exchange rate risk, that needs to be hedged even with log-utility.

The role of hedging real exchange rate fluctuations as an explanation for the home-bias puzzle – which arises also in a rational-expectation model with non-log utility – has been recently questioned by van Wincoop and Warnock (2006, 2008) and Coeurdacier and Gourinchas (2009). Their main argument is based on the evidence that the covariance between real exchange rate changes and the excess return on foreign-versus-domestic equity becomes negligible once this covariance is taken conditional on other returns, like the excess return on riskless bonds.

The results of a simple OLS regression between real exchange rate changes and the vector of excess returns

$$\Delta\hat{q}_{t+1} = \kappa_q + \boldsymbol{\psi}_q'\mathbf{exr}_{t+1} + u_{q,t+1}, \tag{66}$$

are reported in Table 6. While the loading of the excess returns on foreign equity is significant and positive if equity is the only tradeable asset, once the vector of excess returns is augmented to include also the excess return on foreign-versus-domestic bonds, the covariance-to-variance ratio between the real exchange rate and the excess return on equity becomes negligible.

Table 6: Loadings of excess returns on real exchange rate depreciations

Loadings of:	Asset Menu I	Asset Menu II	Asset Menu III
$\hat{r}^{e*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}^{e}_{H,t+1}$	**0.365** (0.072)	**0.021** (0.068)	**−0.026** (0.071)
$\hat{r}^{*}_{F,t+1} + \Delta\hat{q}_{t+1} - \hat{r}_{H,t+1}$	– –	**0.747** (0.086)	**0.781** (0.086)
$\hat{r}^{e}_{H,t+1} - \hat{r}_{H,t+1}$	– –	– –	**−0.098** (0.048)

Note: standard errors in parentheses. Dependent variable is $\Delta\hat{q}_{t+1}$.

In a rational-expectation model such small covariances (provided they are of the right sign) would require an unreasonably large degree of risk aversion to justify the hedging role of domestic equities, which would then open room for other puzzles, like the already mentioned risk-free rate puzzle.

Instead, our dynamic model with distorted beliefs provides two additional features which give a new role to real exchange rate risk: on the one hand, what matters is not only the current real-exchange-rate risk but also the revisions in the entire future expected path of the real exchange rate. What is relevant, therefore, is not so much the role of equity to hedge against *short-run* exchange rate risk, but rather its hedging properties against *long-run* fluctuations. On the other hand, for a given positive covariance between $\varepsilon_{q,t+1}$ and the excess return on equity, a stronger fear of model misspecification translates into larger home biases, without requiring implausible coefficients of the intertemporal elasticity of substitution and therefore without falling in the risk-free rate puzzle. In what follows, we provide an empirical evaluation of these two additional features.

6.4.1 Short-run versus long-run risk

First, we study whether shifting from a short-run to a long-run perspective affects the hedging properties of equity with respect to real exchange rate risk. To this purpose, note that equation (54) can be written in terms of levels instead of growth rates:

$$\varepsilon_{q,t+1} \equiv \sum_{j=0}^{\infty} \beta^j \left[E_{t+1}\Delta\hat{q}_{t+1+j} - E_t\Delta\hat{q}_{t+1+j}\right] = (1-\beta)\sum_{j=0}^{\infty} \beta^j \left[E_{t+1}\hat{q}_{t+1+j} - E_t\hat{q}_{t+1+j}\right]. \tag{67}$$

By looking at different terms in the summation above, we can investigate the co-movement between asset returns and surprises in the real exchange rate path, at different time horizons. In particular, we can evaluate whether the hedging properties of equity and bonds change when the risk to be hedged is farther away in the future, as opposed to very soon.

Indeed, given our estimated model (59) and given the vector $\boldsymbol{\iota}_q$ which selects the depreciation rate from the vector $\mathbf{y}$, we can write the time$-t+1$ news about the real exchange rate k periods ahead as

$$\Delta E_{t+1}\hat{q}_{t+1+k} = \boldsymbol{\iota}_q' \sum_{j=0}^{k} \Delta E_{t+1}\mathbf{y}_{t+1+j} = \boldsymbol{\iota}_q' \sum_{j=0}^{k} \mathbf{A}^j \mathbf{e}_{t+1} = \boldsymbol{\iota}_q'(\mathbf{I}-\mathbf{A})^{-1}\left(\mathbf{I}-\mathbf{A}^{k+1}\right)\mathbf{e}_{t+1},$$

and the news about the long-run component as

$$\Delta E_{t+1}\hat{q}_{\infty} = \boldsymbol{\iota}_q'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{e}_{t+1},$$

in which $\Delta E_{t+1} \equiv (E_{t+1} - E_t)$ denotes the time-$t+1$ revisions in conditional expectations.

For each asset structure, we can use the above equation to evaluate the covariance-to-variance ratios with respect to all excess returns of interest, conditional on time-t information and on the residual asset space.

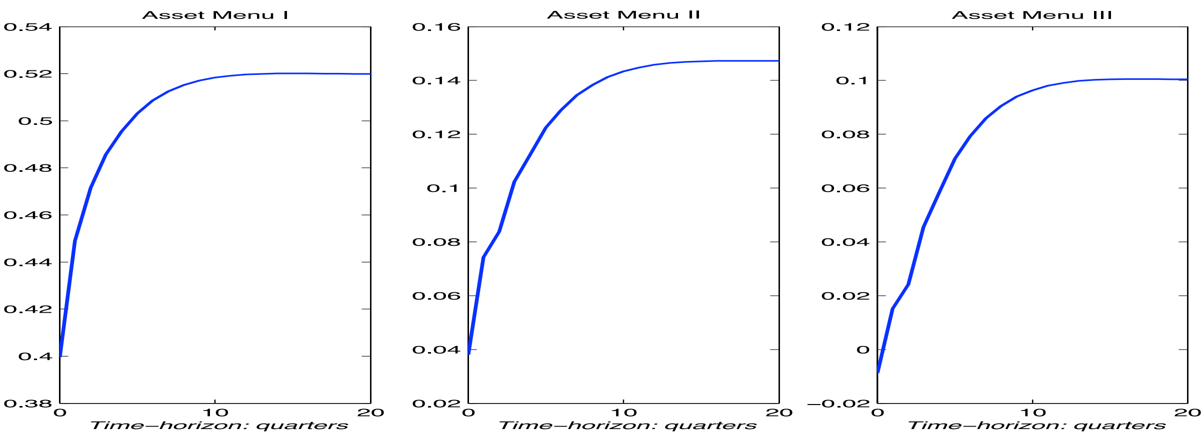

Figure 1: The covariance-to-variance ratio between $\Delta E_{t+1}\hat{q}_{t+1+k}$ and exr^{ie}_{t+1}, for increasing k (horizontal axis). *Asset Menu I*: equities only. *Asset Menu II*: equities and balanced bonds. *Asset Menu III*: general model with equities and bonds.

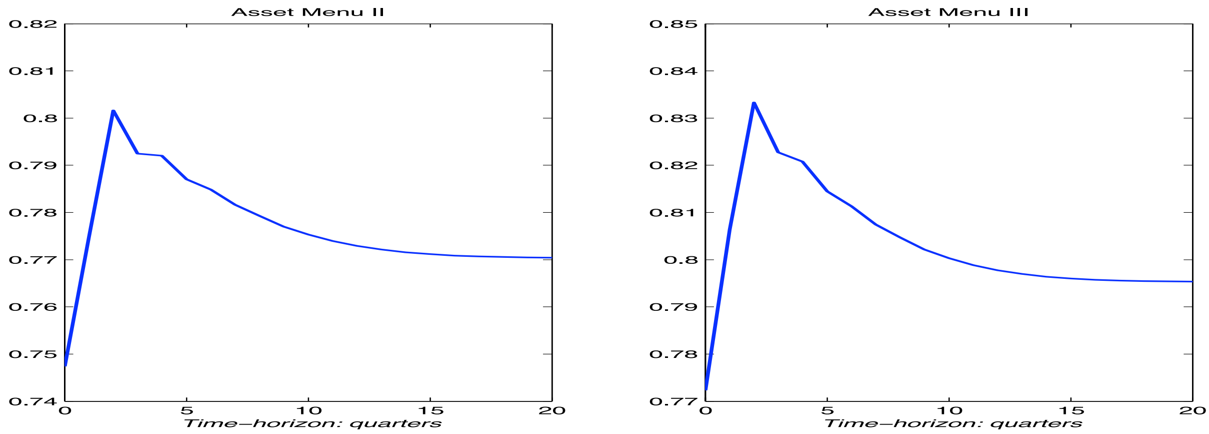

Figure 2: The covariance-to-variance ratio between $\Delta E_{t+1}\hat{q}_{t+1+k}$ and exr^{ib}_{t+1}, for increasing k (horizontal axis). Asset Menu II: equities and balanced bonds. Asset Menu III: general model with equities and bonds.

Indeed, for each time-horizon $k = 0, 1, 2, ...$ we get:

$$\boldsymbol{\Sigma}_t^{-1} E_t(\Delta E_{t+1}\hat{q}_{t+1+k} \cdot \mathbf{exr}_{t+1}) = (\boldsymbol{\iota}'_{exr}\boldsymbol{\Omega}\boldsymbol{\iota}_{exr})^{-1}\boldsymbol{\iota}'_q(\mathbf{I}-\mathbf{A})^{-1}\left(\mathbf{I}-\mathbf{A}^{k+1}\right)\boldsymbol{\Omega}\boldsymbol{\iota}_{exr}. \qquad (68)$$

Hence, Figure 1 plots the covariance-to-variance ratios of the news in the real exchange rate path with the excess return on foreign-versus-domestic equity, against the time-horizon k, under the three specifications: I) the model in which international trade is restricted to equities only, II) the model in which there is trading in equities and bonds, but the latter are balanced; III) the general model with unrestricted trading in equities and bonds. Figure 2 does the same for the excess return on foreign-versus-domestic bonds, for the two specifications including bonds (II and III). The first point (for $k = 0$) in each plot corresponds to the covariance-to-variance ratio of a static model, in which only the short-run risk matters. Moving from the left to the right panel of Figure 1, the first point drops from about .4 to virtually zero, implying that the hedging power of equity against real exchange rate risk fades away, when we condition on other excess returns and in particular on bonds. This is the core of the results in van Wincoop and Warnock (2006) and Coeurdacier and Gourinchas (2009).

However, we note that as we look at longer horizons the hedging properties of equity sharply improve, even when we condition on other excess returns. Figure 2, instead, shows that the hedging properties of bonds are only marginally affected.

We view this evidence as suggesting that domestic equity can have a relatively more important role in hedging the real exchange rate risk at longer horizons in a way to explain the international home-bias puzzle.[26]

6.4.2 The role of model uncertainty

Equation (55) shows that when agents have distorted beliefs and log utility, the equilibrium international portfolio allocation is made of two components. The first component is driven only by the desire to insure against labor-income risk, and is common to the case of undistorted beliefs, since it does not depend on γ. The second component is instead directly driven by ambiguity

[26] A recent literature documents the quantitatively substantial implications of long-run risk for asset valuation, in the context of non-expected utility frameworks. See, among others, Hansen et al. (2008), who also provide an interpretation related to model uncertainty.

Table 7: Model with equities only – Distorted Beliefs

Covariance-variance Ratios	
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{ie}_{t+1})}{var_t(exr^{ie}_{t+1})}$	**−0.524**
$\frac{cov_t(\varepsilon_{q,t+1}, exr^{ie}_{t+1})}{var_t(exr^{ie}_{t+1})}$	**0.518**

Optimal Portfolio Allocation

		γ=1	γ=2	γ=5	γ=10
$\bar{\alpha}^e_F$:	first component	1.020	1.020	1.020	1.020
	second component	0.500	0.113	−0.118	−0.196
	total	**1.020**	**0.634**	**0.402**	**0.324**

Note: $\bar{\alpha}^e_F$ denotes the share of wealth invested in foreign equity. "first component" refers to hedging labor-income risk only ($\gamma = 1$); "second component" refers to hedging real-exchange-rate risk only ($s_\xi = 0$); "total" refers to hedging both risks (total $\bar{\alpha}^e_F$= first component + second component − 1/2).

aversion, and is related to hedging real-exchange-rate risk; the relative weight of this additional component depends on the extent to which agents fear model misspecification (captured by γ).[27]

Table 7 studies the implications when the only asset traded internationally is equity (*Asset Menu I*). The first two rows show that the conditional covariance-to-variance ratio between the surprises in the real exchange rate and the international excess return on equity is positive and economically large, while the covariance with labor-income risk is negative and even larger. As a consequence, the first component is qualitatively consistent with a foreign-equity bias, as we found under undistorted beliefs, while the second component is qualitatively consistent with a home-bias.

For $\gamma = 1$ there is no model uncertainty and the first component is the only relevant: the model implies a foreign-equity bias. When we consider model uncertainty, instead, the picture changes substantially: the more the doubts about the true model specification (the higher γ), the more the second component becomes relevant and the more the portfolio allocation is biased towards domestic equity. Indeed, a moderate fear of model misspecification is able to overturn the effect of hedging labor-income risk and produce a good degree of home-bias: (around 60% of wealth allocated in domestic equity for $\gamma = 5$ and about 70% for γ between 10 and 20).

By enlarging the asset structure to include also trading in riskless bonds (*Asset Menu II*), we get further qualifications to previous results, as shown in Table 8. First, as in Coeurdacier and Gourinchas (2009), the presence of bonds provides a valuable hedge against real exchange rate risk and, as a consequence, the conditional covariance-to-variance ratio with equity becomes smaller, inducing a smaller hedging role for equities. However, unlike in Coeurdacier and Gourinchas (2009), this role is not ruled out altogether because of the importance of both *long-run* risk and model uncertainty. First, an increasing concern about model misspecification is still able to

[27]Specifically, we compute the "first component" as the optimal allocation when labor-income risk is the only risk to hedge (i.e. when γ is 1) and the "second component" as the optimal allocation when the only relevant risk is the one related to real exchange rate fluctuations (i.e. when s_ξ=0). Accordingly, given the definition (31) of vector $\bar{\boldsymbol{\lambda}}$, the total allocation in foreign equity ($\bar{\alpha}^e_F$) is given by: first component + second component − 1/2; the total allocation in foreign bonds ($\bar{\alpha}_F$) is given by: first component + second component; the total allocation in equity instruments ($\bar{\alpha}^e_F + \bar{\alpha}^e_H$) is given by: first component + second component − 1.

Table 8: Model with equities and balanced bonds – Distorted Beliefs

Covariance-variance Ratios	
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{ie}_{t+1} \| exr^{ib}_{t+1})}{var_t(exr^{ie}_{t+1} \| exr^{ib}_{t+1})}$	**0.016**
$\frac{cov_t(\varepsilon_{q,t+1}, exr^{ie}_{t+1} \| exr^{ib}_{t+1})}{var_t(exr^{ie}_{t+1} \| exr^{ib}_{t+1})}$	**0.145**
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{ib}_{t+1} \| exr^{ie}_{t+1})}{var_t(exr^{ib}_{t+1} \| exr^{ie}_{t+1})}$	**−1.116**
$\frac{cov_t(\varepsilon_{q,t+1}, exr^{ib}_{t+1} \| exr^{ie}_{t+1})}{var_t(exr^{ib}_{t+1} \| exr^{ie}_{t+1})}$	**0.771**

Optimal Portfolio Allocation

		γ=1	γ=2	γ=5	γ=10
$\bar{\alpha}^e_F$:	first component	0.484	0.484	0.484	0.484
	second component	0.500	0.392	0.327	0.305
	total	**0.484**	**0.376**	**0.311**	**0.290**
$\bar{\alpha}_F$:	first component	1.108	1.108	1.108	1.108
	second component	0.000	−0.575	−0.921	−1.036
	total	**1.108**	**0.532**	**0.187**	**0.072**
$\bar{\alpha}^e_F + \bar{\alpha}_F$:	**total**	**1.592**	**0.908**	**0.498**	**0.361**

Note: $\bar{\alpha}^e_F$ denotes the share of wealth invested in foreign equity; $\bar{\alpha}_F$ denotes the share of wealth invested in foreign bonds; $\bar{\alpha}^e_F + \bar{\alpha}_F$ measures the overall share of wealth invested in foreign assets. "first component" refers to hedging labor-income risk only ($\gamma = 1$); "second component" refers to hedging real-exchange-rate risk only ($s_\xi = 0$); "total" refers to hedging both risks (total $\bar{\alpha}^e_F$= first component + second component − 1/2; total $\bar{\alpha}_F$= first component + second component).

induce a substantial home-bias in equity holdings (up to over 70% allocated to domestic equity). Second, the hedging role of domestic bonds against fluctuations in the real exchange rate implies that the position on the foreign-bond market turns progressively from long to balanced, as γ increases.

Hence, the optimal portfolio allocation results biased towards domestic assets (around 65% of wealth invested in local assets), and the country has an almost balanced position in foreign-currency debt instruments, consistently with the evidence of Tille (2005, 2008).

Finally, we analyze the case in which we do not impose the zero-balanced position in the international bond market, to allow for leveraged positions between different kinds of assets (*Asset Menu III*). Table 9 shows that the role of domestic equity as a hedge against real exchange rate risk is still twice as important as its role to hedge against labor-income risk, and is able to imply a substantial degree of home bias. In this respect, a further important implication of model uncertainty and fear of model misspecification comes from the negative covariance between surprises in the real exchange rate path and the domestic equity premium. Indeed, a negative covariance implies that domestic equities pay relatively better than domestic bonds precisely when an unexpected appreciation of the real exchange rate occurs. In equilibrium, this implies that agents with distorted beliefs optimally take an overall short position in the international bond market in order to buy a higher share of domestic equity and hedge against the uncertainty

Table 9: General model with equities and bonds – Distorted Beliefs

Covariance-variance Ratios	
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{ie}_{t+1} \vert exr^{ib}_{t+1}, exr^{de}_{t+1})}{var_t(exr^{ie}_{t+1} \vert exr^{ib}_{t+1}, exr^{de}_{t+1})}$	**0.040**
$\frac{cov_t(\varepsilon_{q,t+1}, exr^{ie}_{t+1} \vert exr^{ib}_{t+1}, exr^{de}_{t+1})}{var_t(exr^{ie}_{t+1} \vert exr^{ib}_{t+1}, exr^{de}_{t+1})}$	**0.098**
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{ib}_{t+1} \vert exr^{ie}_{t+1}, exr^{de}_{t+1})}{var_t(exr^{ib}_{t+1} \vert exr^{ie}_{t+1}, exr^{de}_{t+1})}$	**−1.129**
$\frac{cov_t(\varepsilon_{q,t+1}, exr^{ib}_{t+1} \vert exr^{ie}_{t+1}, exr^{de}_{t+1})}{var_t(exr^{ib}_{t+1} \vert exr^{ie}_{t+1}, exr^{de}_{t+1})}$	**0.796**
$\frac{cov_t(\varepsilon_{l,t+1}, exr^{de}_{t+1} \vert exr^{ie}_{t+1}, exr^{ib}_{t+1})}{var_t(exr^{de}_{t+1} \vert exr^{ie}_{t+1}, exr^{ib}_{t+1})}$	**0.062**
$\frac{cov_t(\varepsilon_{q,t+1}, exr^{de}_{t+1} \vert exr^{ie}_{t+1}, exr^{ib}_{t+1})}{var_t(exr^{de}_{t+1} \vert exr^{ie}_{t+1}, exr^{ib}_{t+1})}$	**−0.117**

Optimal Portfolio Allocation

		γ=1	γ=2	γ=5	γ=10
$\bar{\alpha}^e_F$:	first component	0.460	0.460	0.460	0.460
	second component	0.500	0.427	0.383	0.369
	total	**0.460**	**0.387**	**0.343**	**0.328**
$\bar{\alpha}_F$:	first component	1.121	1.121	1.121	1.121
	second component	0.000	−0.594	−0.951	−1.069
	total	**1.121**	**0.526**	**0.170**	**0.051**
$\bar{\alpha}^e_F + \bar{\alpha}_F$:	**total**	**1.580**	**0.913**	**0.513**	**0.380**
$\bar{\alpha}^e_H + \bar{\alpha}^e_F$:	first component	0.939	0.939	0.939	0.939
	second component	1.000	1.087	1.140	1.157
	total	**0.939**	**1.026**	**1.079**	**1.096**
$\bar{\alpha}^e_H$:	**total**	**0.479**	**0.639**	**0.736**	**0.768**

Note: $\bar{\alpha}^e_F$ denotes the share of wealth invested in foreign equity; $\bar{\alpha}_F$ denotes the share of wealth invested in foreign bonds; $\bar{\alpha}^e_F + \bar{\alpha}_F$ measures the overall share of wealth invested in foreign assets; $\bar{\alpha}^e_H + \bar{\alpha}^e_F$ measures the overall share of wealth invested in equity; $\bar{\alpha}^e_H$ denotes the share of wealth invested in domestic equity. "first component" refers to hedging labor-income risk only ($\gamma = 1$); "second component" refers to hedging real-exchange-rate risk only ($s_\xi = 0$); "total" refers to hedging both risks (total $\bar{\alpha}^e_F$= first component + second component − 1/2; total $\bar{\alpha}_F$= first component + second component; total $\bar{\alpha}^e_F + \bar{\alpha}^e_H$= first component + second component − 1).

about the model specification. As γ increases, therefore, the overall share of wealth allocated to equity ($\bar{\alpha}^e_H + \bar{\alpha}^e_F$) also increases, well above the unitary value. This result has important implications for the home-bias in equity, as shown by the last row of Table 9. The empirical co-movement of the domestic equity premium with real exchange rate risk indeed reinforces the role of domestic equity as a hedge against such risk, with respect to the previous specification, bringing the overall share of wealth optimally allocated to domestic equity to about 75 percentage points, for γ between 5 and 10.

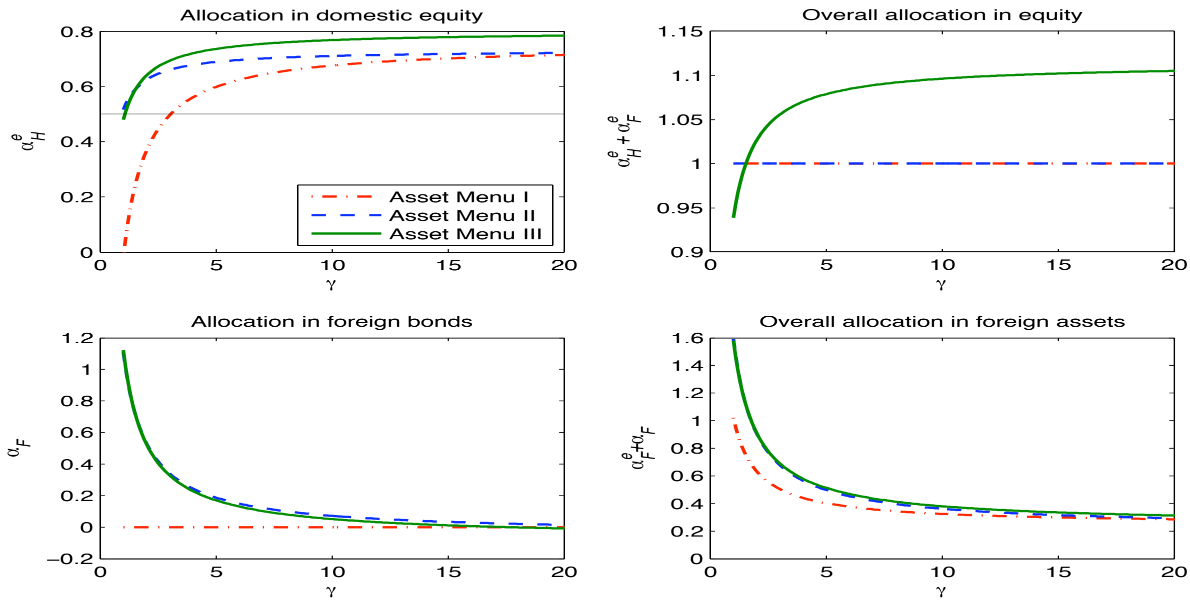

Figure 3: Optimal portfolio allocation: the effect of increasing degrees of concern about model misspecification. Asset Menu I: equities only. Asset Menu II: equities and balanced bonds. Asset Menu III: general model with equities and bonds.

Notice that the result of an overall short position in the international bond market reconciles the model with all the stylized facts on U.S. cross-border holdings documented by Tille (2005, 2008). Indeed, when accounting for fear of model misspecification, our framework is able to produce a positive net equity position ($\bar{\alpha}_H^e + \bar{\alpha}_F^e > 1$) coupled with a negative net position in debt instruments ($\bar{\alpha}_H + \bar{\alpha}_F < 0$). Moreover, the position on foreign-currency bonds is now about balanced ($\bar{\alpha}_F \approx 0$) while the one on home-currency bonds is negative ($\bar{\alpha}_H < 0$) and therefore the overall position in the international bond market is short.

Overall, agents with distorted beliefs tend to invest more in (domestic) equity rather than in bonds, and finance their leveraged position by using domestic bonds.

The role of model uncertainty in driving the equilibrium portfolio allocation is further explored in Figure 3, which plots the share of wealth invested in equity (domestic and overall) and foreign assets (bonds and overall) for increasing degrees of concern about model misspecifications and for each of the three asset menus analyzed.

Figure 3 synthesizes the main findings of this section. First, without model uncertainty, hedging against non-diversifiable labor-income risk is not sufficient to imply home bias in equity, contrary to Coeurdacier and Gourinchas (2009), and instead implies a foreign-equity bias when international trade is restricted to equities only, as in Baxter and Jermann (1997). Moreover, the position in foreign-currency bonds is, in this case, long, as in Coeurdacier and Gourinchas (2009), and very large, and the economy is a net debtor in equities and a net creditor in bonds. All these implications contrast with the evidence of Tille (2005, 2008). Each of these findings is implied by the position of the first point in the various lines plotted in Figure 3 (corresponding to $\gamma = 1$).

Second, the model in which agents have distorted beliefs, instead, proves more successful along all the dimensions. By increasing ambiguity aversion, the allocation in domestic equities monotonically increases (top-left panel), implying a rising degree of home-bias. Moreover, increasing concerns for model misspecification make the model's implications for cross-border holdings consistent with the evidence in Tille (2005, 2008). Indeed, the position of the Home country in foreign-currency bonds shrinks towards a balanced position (bottom-left panel), and the overall share allocated to equities, both domestic and foreign, increases above unity, implying an overall short position in the international bond market (top-right panel). This latter result reinforces the role of domestic equity as a good hedge against real exchange rate risk and makes the home country a net creditor in equity instruments and a net debtor in debt instruments. Finally, given a balanced position in foreign-currency bonds, it also implies a short position in bonds denominated in the domestic currency, consistently with Tille (2005, 2008).

6.5 Calibrating γ using detection error probabilities

This section describes how to appropriately calibrate γ as a parameter capturing the concern about model misspecification of the consumers in the two economies. We follow Anderson et al. (2003) and Hansen and Sargent (2007) in using detection error probabilities. Let us call model A as the approximating model and model $B(\gamma)$ as the worst-case model associated with a particular γ. Agents start with the belief that the models are equally likely. That is, they assign 50% prior probability to each model. After having seen T observations, they can perform a likelihood ratio test for distinguishing the two models. Under the hypothesis that model A is correct, we denote with $p_A(\gamma)$ the probability that a likelihood ratio test would instead falsely say that model $B(\gamma)$ generated the data. Conversely, we denote with $p_B(\gamma)$ the probability that a likelihood ratio test would falsely say that model A generated the data, when in fact model $B(\gamma)$ is correct. The detection error probability, then, is the weighted average of $p_A(\gamma)$ and $p_B(\gamma)$ with the weights given by the prior probabilities:

$$p(\gamma) = \frac{1}{2}\Big(p_A(\gamma) + p_B(\gamma)\Big).$$

Notice that the detection error probability is a decreasing function of γ, since a larger γ (and therefore a smaller θ) implies a lower penalization upon relaxing the entropy constraint in equation (50). Indeed, the higher γ the wider is the entropy ball inside which the consumer allows the evil agent to choose the worst-case distortion, and therefore the more afraid of misspecifications the consumer is. Accordingly, higher values of γ imply a larger divergence between the worst-case model and the approximating one, and is therefore less probable that the likelihood-ratio test will favor the wrong model. When $\gamma = 1$, on the contrary, the two models are equivalent and $p(\gamma)$ is therefore equal to 1/2.

In our context the approximating model is given by the VAR in (59):

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t$$

where $\mathbf{e}_t$ is distributed as a multivariate normal with zero mean and variance-covariance matrix $\boldsymbol{\Omega}$. Accordingly, the probability density of $\mathbf{e}_t$, denoted by $f(\mathbf{e}_t)$, is proportional to

$$\exp\left(-\frac{1}{2}\mathbf{e}_t'\boldsymbol{\Omega}^{-1}\mathbf{e}_t\right).$$

The approximating and the worst-case models are linked through the martingale increments g and g^* for the agents of country H and F, respectively. We showed in Section 5 that in a first-order approximation g and g^* are related to the revisions in the expected future path of the respective consumption growth:

$$\hat{g}_t = -(\gamma - 1)\sum_{j=0}^{\infty}\beta^j\Big(E_t\Delta\hat{c}_{t+j} - E_{t-1}\Delta\hat{c}_{t+j}\Big)$$
$$\hat{g}_t^* = -(\gamma^* - 1)\sum_{j=0}^{\infty}\beta^j\Big(E_t\Delta\hat{c}_{t+j}^* - E_{t-1}\Delta\hat{c}_{t+j}^*\Big),$$

in which we are allowing for different γ and γ^*.

Using equations (26)–(27) and a first-order approximation of the flow-budget constraint (10), we can solve for the growth rate of domestic consumption, as a function of steady-state portfolio shares, asset returns and labor income. It follows that we can write g and g^* as linear combinations of the VAR innovations:

$$\hat{g}_t = -(\gamma - 1)\mathbf{z}(\gamma)'\mathbf{e}_t$$
$$\hat{g}_t^* = -(\gamma^* - 1)\mathbf{z}^*(\gamma^*)'\mathbf{e}_t,$$

in which vectors $\mathbf{z}$ and $\mathbf{z}^*$ depend on γ and γ^* through the steady-state portfolio shares. Indeed, simple algebra shows that

$$\mathbf{z}(\gamma) \equiv \frac{1-\beta}{\beta s_c}\boldsymbol{\iota}_{exr}\bar{\boldsymbol{\alpha}}(\gamma) + \mathbf{H}'\boldsymbol{\iota}_r + \frac{s_\xi}{s_c}\mathbf{H}'(\boldsymbol{\iota}_\xi - \boldsymbol{\iota}_r)$$

for country H, and

$$\mathbf{z}^*(\gamma^*) \equiv \frac{1-\beta}{\beta s_c}\boldsymbol{\iota}_{exr}\bar{\boldsymbol{\alpha}}^*(\gamma^*) + \mathbf{H}'(\boldsymbol{\iota}_r - \boldsymbol{\iota}_q) + \frac{s_\xi}{s_c}\mathbf{H}'(\boldsymbol{\iota}_\xi + \boldsymbol{\iota}_q - \boldsymbol{\iota}_r),$$

for country F, in which $\bar{\boldsymbol{\alpha}}$ and $\bar{\boldsymbol{\alpha}}^*$ are defined in (28).

It follows that the probability distribution of the distorted model for the agent in country H, denoted by $\tilde{f}(\mathbf{e}_t)$, is given by

$$\tilde{f}(\mathbf{e}_t) \equiv f(\mathbf{e}_t)\cdot g_t \propto \exp\left(-\frac{1}{2}\mathbf{e}_t'\boldsymbol{\Omega}^{-1}\mathbf{e}_t\right)\exp\Big(-(\gamma-1)\mathbf{z}(\gamma)'\mathbf{e}_t\Big).$$

Completing the square finally allows us to write $\tilde{f}(\mathbf{e}_t)$ as

$$\tilde{f}(\mathbf{e}_t) \propto \exp\left(-\frac{1}{2}\Big(\mathbf{e}_t - \mathbf{w}(\gamma)\Big)'\boldsymbol{\Omega}^{-1}\Big(\mathbf{e}_t - \mathbf{w}(\gamma)\Big)\right),$$

in which $\mathbf{w}(\gamma) \equiv -(\gamma-1)\boldsymbol{\Omega}\mathbf{z}(\gamma)$ is the mean distortion implied by the preference for robustness. Similarly, the distorted probability distribution function for the agent in country F, $\tilde{f}^*(\mathbf{e}_t)$, is given by

$$\tilde{f}^*(\mathbf{e}_t) \equiv f(\mathbf{e}_t)\cdot g_t^* \propto \exp\left(-\frac{1}{2}\Big(\mathbf{e}_t - \mathbf{w}^*(\gamma^*)\Big)'\boldsymbol{\Omega}^{-1}\Big(\mathbf{e}_t - \mathbf{w}^*(\gamma^*)\Big)\right),$$

in which $\mathbf{w}^*(\gamma^*) \equiv -(\gamma^*-1)\boldsymbol{\Omega}\mathbf{z}^*(\gamma^*)$.

Therefore, the worst-case models are distorted in the means, and will be given by

$$\mathbf{y}_t = \boldsymbol{\mu} - (\gamma-1)\boldsymbol{\Omega}\mathbf{z}(\gamma) + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t$$

for consumers in country H and

$$\mathbf{y}_t = \boldsymbol{\mu} - (\gamma^*-1)\boldsymbol{\Omega}\mathbf{z}^*(\gamma^*) + \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t$$

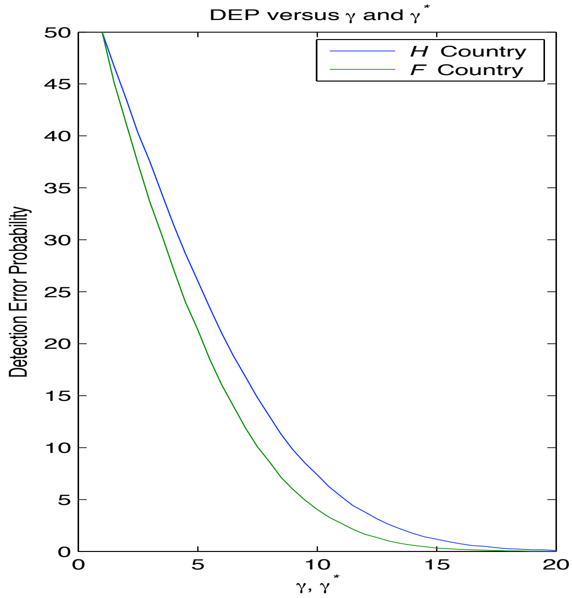

Figure 4: Detection Error Probabilities (DEP) versus fear of model misspecification (γ and γ^*, left panel) and versus discounted conditional relative entropy (η and η^*, right panel).

for those in country F.

We simulated 100,000 samples, each of size 112 observations (corresponding to the sample 1980q1-2007q4 that we use in the VAR estimation), and computed the detection error probabilities associated with the approximating and the worst-case models, by varying the parameters γ and γ^*. The results are displayed in Figure 4.

The left panel of Figure 4 shows the detection error probabilities, $p(\gamma)$ and $p(\gamma^*)$, plotted against γ and γ^*. It is important to notice that the mapping between γ and $p(\gamma)$, is model-specific and varies in different contexts. This is why the plausibility of a given value of γ, as a measure of the concern about model misspecification, should be appropriately determined in terms of the detection error probability that it implies, which can be instead regarded as a context-invariant measure. As to what "plausible" means in this respect, we follow Anderson et al. (2003), Maenhout (2006) and Barillas et al. (2006), and consider alternative models whose detection error probabilities are around 10 per cent as "difficult to detect". Figure 3 has shown that values of γ or γ^* between 5 and 10 are sufficient to get the most of the model fit in terms of home-bias in equity and other empirical evidence on cross-border holdings. Figure 4 then shows that values of γ and γ^* between 7 and 10 are still associated with detection error probabilities around 0.10.

The degree of ambiguity aversion needed to explain the empirical facts is therefore consistent with conservative values of the detection error probabilities, thus validating the empirical relevance of the model's implications. Given that the left panel shows that for similar detection error probability γ and γ^* are very close, we can also conclude that the assumption $\gamma = \gamma^*$ is generally innocuous.[28]

The right panel of Figure 4 plots the detection error probabilities against the discounted

[28]At the threshold value of 10%, the values for γ and γ^* are, respectively, 9 and 7.5.

conditional relative entropy defined in (4), which in our case is time-invariant and equals

$$\eta(\gamma) = .5\frac{\beta}{1-\beta}\mathbf{w}(\gamma)'\mathbf{\Omega}^{-1}\mathbf{w}(\gamma)$$

for agents in country H and

$$\eta^*(\gamma^*) = .5\frac{\beta}{1-\beta}\mathbf{w}^*(\gamma^*)'\mathbf{\Omega}^{-1}\mathbf{w}^*(\gamma^*)$$

for those in country F. This panel reveals that for each value of detection error probability, the discounted entropies are the same for the two agents, in further support of the view that a bound on detection error probabilities, rather than a given value for γ, appropriately defines a context-invariant measure of concern about model uncertainty.

7 Conclusions

The observation that international investors hold a disproportionate share of their wealth in domestic rather than foreign assets is one of the most persistent ones in international finance. This is named the international home-bias puzzle, that the literature has been dealing with for a couple of decades.

This paper develops a dynamic general equilibrium model of portfolio and consumption choices, with incomplete markets and distorted beliefs. Households might use a "subjective" probability distribution that is generally different from the "approximating" one (although the two distributions are close enough, in an absolute continuity sense, to be difficult to tell apart in finite samples) and make robust optimal choices against model uncertainty. This framework assigns a new role to real exchange rate risk for portfolio allocation even in a model with log utility. Importantly, moreover, what matters is not only the short-run risk but also and foremost the long-run risk of real exchange rate fluctuations.

Within this framework we characterize optimal portfolio allocations in terms of covariances between measurable sources of risk to be hedged (non-diversifiable labor income risk and real exchange rate risk) and a vector of cross-country excess returns, and evaluate their empirical relevance using financial and macro data on the G7 countries.

Our results suggest that, contrary to what claimed in recent related contributions, hedging non-diversifiable labor-income risk is not sufficient to account for the lack of international portfolio diversification. Indeed, in a setting in which equity is the only available asset, correlations in financial data support a large foreign-equity bias, as in Baxter and Jermann (1997). Adding further assets does not help in identifying a clear role for this risk in explaining the home-bias puzzle, once the former is measured in a model-consistent way. On the other hand, a "plausible" concern about model misspecification is able to generate a substantial equilibrium home bias in equity holdings, and allows to match other empirical facts regarding the U.S. cross-border holdings. We evaluate the "plausibility" of the concern for model uncertainty by resorting to detection error probabilities, which measure how easily the competing models can be told apart using a finite amount of data.

The methodological contribution of the paper goes beyond the analysis of the home-bias puzzle. The class of preferences that we suggest, in fact, produces a perturbation of the equilibrium stochastic discount factor which decouples the attitudes towards intertemporal substitution with those towards risk and ambiguity, and can prove useful in addressing other failures of standard preference specifications along the asset-price dimension.[29] Indeed, it has been shown in closed-economy settings, that disentangling the elasticity of intertemporal substitution from the degree of risk aversion helps in accounting for the equity premium puzzle. Once we open the economy to international trade in assets, there are additional puzzling features of financial data, among which the international equity- and bond-premia puzzles and the Backus-Smith anomaly are notable examples.[30] All these stylized facts imply restrictions on the stochastic discount factor that standard preferences cannot meet at the same time, and that might be all reconnected to some common misspecification.[31] The modification of the stochastic discount factor that our preference specification implies is a promising tool to correct this misspecification and build macro models whose predictions are closer to the empirical implications of financial data.

[29] See for a discussion Backus et al. (2004).

[30] Ilut (2008) studies how ambiguity aversion can help explain the uncovered-interest-rate puzzle.

[31] All excess-return puzzles, for example, imply "high" lower bounds on the volatility of the equilibrium stochastic discount factor, as discussed for the equity premium by Hansen and Jagannathan (1991).

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8 Appendix

To get equation (8), we have defined

$$A_t \equiv B_{H,t} + S_t B_{F,t} + x_{H,t} V_{H,t} + x_{F,t} S_t V^*_{F,t},$$

and

$$R_{p,t} = \alpha_{H,t-1} R_{H,t} + \alpha_{F,t-1} R^*_{F,t} \frac{S_t}{S_{t-1}} + \alpha^e_{H,t-1} R^e_{H,t} + \alpha^e_{F,t-1} R^{e*}_{F,t} \frac{S_t}{S_{t-1}},$$

with

$$R^e_{H,t} \equiv \frac{V_{H,t} + D_{H,t}}{V_{H,t-1}},$$

$$R^{e*}_{F,t} \equiv \frac{V^*_{F,t} + D^*_{F,t}}{V^*_{F,t-1}},$$

and

$$B_{H,t} \equiv \alpha_{H,t} A_t,$$

$$S_t B_{F,t} \equiv \alpha_{F,t} A_t,$$

$$x_{H,t} V_{H,t} \equiv \alpha^e_{H,t} A_t,$$

$$x_{F,t} S_t V^*_{F,t} = \alpha^e_{F,t} A_t,$$

and analogously for the foreign country:

$$B^*_{H,t} \equiv \alpha^*_{H,t} A^*_t S_t,$$

$$B^*_{F,t} \equiv \alpha^*_{F,t} A^*_t,$$

$$x^*_{H,t} V_{H,t} \equiv \alpha^{e*}_{H,t} A^*_t S_t,$$

$$x^*_{F,t} V^*_{F,t} = \alpha^{e*}_{F,t} A^*_t.$$