THE TERM STRUCTURES OF EQUITY AND INTEREST RATES

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ABSTRACT

This paper proposes a dynamic risk-based model capable of jointly explaining the term structure of interest rates, returns on the aggregate market and the risk and return characteristics of value and growth stocks. Both the term structure of interest rates and returns on value and growth stocks convey information about how the representative investor values cash flows of different maturities. We model how the representative investor perceives risks of these cash flows by specifying a parsimonious stochastic discount factor for the economy. Shocks to dividend growth, the real interest rate, and expected inflation are priced, but shocks to the price of risk are not. Given reasonable assumptions for dividends and inflation, we show that the model can simultaneously account for the behavior of aggregate stock returns, an upward-sloping yield curve, the failure of the expectations hypothesis and the poor performance of the capital asset pricing model.

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1   Introduction

Empirical studies of asset pricing have uncovered a rich set of properties of the time series of aggregate stock market returns, of the term structure of interest rates and of the cross-section of stock returns. Average returns on the aggregate stock market are high relative to short-term interest rates. Relative to dividends, aggregate stock returns are highly volatile. They are also predictable; the return on the aggregate market in excess of the short-term interest rate is predictably high when the price-dividend ratio is low and predictably low when the price-dividend ratio is high. The term structure of interest rates on U.S. government bonds is upward sloping, and excess bond returns are predictable by yield spreads and by linear combinations of forward rates. In the cross-section, stocks with low ratios of price to fundamentals (value stocks) have higher returns than stocks with high ratios of price to fundamentals (growth stocks), despite the fact that they have lower covariance with aggregate stock returns. These facts together are inconsistent with popular benchmark models and therefore represent an important challenge for theoretical modeling of asset prices.[1]

One approach to explaining these properties of asset prices is to propose a fully specified model of investor preferences, endowments and cash flows on the assets of interest. Under this approach, the returns investors demand for bearing risks (the prices of risk) are endogenously determined by the form of preferences and the process for aggregate consumption. These prices of risk in turn determine risk premia, volatility, and covariances on the assets in equilibrium. Models that follow this approach typically have a small number of free parameters and generate tight implications for asset prices. We refer to this as the *equilibrium approach*.

A second approach is to directly specify the stochastic discount factor (SDF) for the economy. Foundational work by Harrison and Kreps (1979) demonstrates that, in the absence of arbitrage, there exists a process (known as a stochastic discount factor) that determines current prices on the basis of future cash flows. Given that such a process exists, this second approach specifies the SDF process directly, without reference to preferences or endowments. The exogenously specified SDF implies processes for the prices of risk which determine asset pricing properties. Models based on the SDF typically have a large number of degrees of freedom and therefore allow for substantial

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[1]See Campbell (2003) and Cochrane (1999) for recent surveys of the empirical literature and discussion of these benchmark models.

flexibility in matching asset prices. Indeed, the parameters of the SDF and of cash flows are often backed out from asset prices. We refer to this as the *SDF approach*.

In this paper, we seek to explain the aggregate market, cross-sectional, and term structure facts within a single model. To do so, we combine elements of both approaches described above. We assume that only risk arising from aggregate cash flows is priced directly, thus maintaining the strict discipline about the number and nature of priced factors imposed by the equilibrium approach. We determine the parameters of the cash flow processes based on data from the cash flows themselves. This modeling approach maintains the parsimony that is typical of equilibrium models. However, rather than specifying underlying preferences, we directly specify the stochastic discount factor as in the SDF approach. Our goal is to introduce a small but crucial amount of flexibility in order to explain the facts listed in the first paragraph.

Our model's ability to match the data stems in part from properties of the time-varying price of risk, which results in time-varying risk premia on stocks and bonds. As in Brennan, Wang, and Xia (2004) and Lettau and Wachter (2007), we assume first-order autoregressive (AR(1)) processes for both the price of risk and the real interest rate. To model the nominal term structure of interest rates, we introduce an exogenous process for the price level (Cox, Ingersoll, and Ross (1985), Boudoukh (1993)) such that expected inflation follows an AR(1). Realized inflation can therefore be characterized as an ARMA(1,1). Following Bansal and Yaron (2004) and Campbell (2003), we assume an AR(1) process for the expected growth rate of aggregate cash flows.

We calibrate the dividend, inflation, and riskfree rate processes to their counterparts in U.S data. The price of risk is then calibrated to match aggregate asset pricing properties. Several properties of these processes key to the model's ability to fit the data. First, a volatile price of risk is necessary to capture the empirically demonstrated property that risk premia on stocks and bonds are time-varying. This time-varying price of risk also allows the model to match the volatility of stock and bond returns given low volatility of dividends, real interest rates, and inflation. Second, the real riskfree rate is negatively correlated with fundamentals. This implies a slightly upward-sloping real yield curve. Expected inflation is also negatively correlated with fundamentals, implying a yield curve for nominal bonds that is more upward-sloping than the real yield curve.

Our model illustrates a tension between the upward slope of the yield curve and the value

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premium. The value premium implies that value stocks, which are short-horizon equity, have greater returns than growth stocks, which are long-horizon equity. Therefore the "term structure of equities" slopes downward, not upward. However, the very mechanism that implies an upward-sloping term structure of interest rates, namely a negative correlation between shocks to fundamentals and shocks to the real interest rate, also implies a growth premium. We show that correlation properties of shocks to the price of risk are key to resolving this tension. Namely, when the price of risk is independent of fundamentals, the model can simultaneously account for the downward-sloping term structure of equities and the upward-sloping term structure of interest rates.

To summarize, our model generates quantitively accurate means and volatilities for the aggregate market and for Treasury bonds, while allowing for low volatilites in fundamentals. The model can replicate the predictability in excess returns on the aggregate market, the negative coefficients in Campbell and Shiller (1991) bond yield regressions and the tent-shaped coefficients on forward rates found by Cochrane and Piazzesi (2005). Finally, besides capturing the relative means of value and growth portfolios, our model also captures the striking fact that value stocks have relatively low risk according to conventional measures like standard deviation and covariance with the market. Therefore our model replicates the well-known outperformance of value, and underperformance of growth relative to the capital asset pricing model.

Our paper builds on studies that examining the implications of the term structure of interest rates for the stochastic discount factor. Dai and Singleton (2002, 2003) and Duffee (2002)) demonstrate the importance of a time-varying price of risk for explaining the predictability of excess bond returns. Like these papers, we also construct a latent factor model in which bond yields are linear. Ang and Piazzesi (2003), Bikbov and Chernov (2006) and Duffee (2006) introduce macroeconomic time series into the SDF as factors; in our work macroeconomic time series also are used to determine the SDF. Unlike our work, these papers focus exclusively on the term structure.

We also build on a literature that seeks to simultaneously explain prices in bond and the aggregate stock market (see Bakshi and Chen (1996), Bansal and Shaliastovich (2007), Bekaert, Engstrom, and Grenadier (2004), Buraschi and Jiltsov (2007), Gabaix (2007), Lustig, Van Nieuwerburgh and Verdelhan (2008) and Wachter (2006)). We extend these studies by exploring the con-

sequences of our pricing kernel for a cross-section of equities defined by cash flows. In particular, we show that the model can reproduce the high premium on value stocks relative to growth stocks and the fact that value stocks have a low variance and low covariance with the aggregate market.

Our paper also builds on work that seeks to simultaneously explain the aggregate market and returns on value and growth stocks. Several studies link observed returns on value and growth stocks to new sources of risk (Campbell and Vuolteenaho (2004), Campbell, Polk, and Vuolteenaho (2008), Lustig and Van Nieuwerburgh (2005), Piazzesi, Schneider, and Tuzel (2007), Santos and Veronesi (2006b) and Yogo (2006)). Others more closely relate to the present study in that they model value and growth stocks based on their underlying cash flows (Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), Gomes, Kogan, and Zhang (2003), Hansen, Heaton, and Li (2008), Kiku (2006), Lettau and Wachter (2007), Santos and Veronesi (2006a) and Zhang (2005)). Unlike these studies, our study also seeks to explain the upward slope of the nominal yield curve and time-variation in bond risk premia. As we show, jointly considering the term structure of interest rates and behavior of value and growth portfolios has strong implications for the stochastic discount factor.

2   The Model

2.1   Dividend growth, inflation, and the stochastic discount factor

Following Duffie and Kan (1996) we assume an affine structure for the underlying state of the economy. Let $\boldsymbol{H}_t$ be an $m \times 1$ vector of state variables at time t and let $\boldsymbol{\epsilon}_{t+1}$ be an $(m+2) \times 1$ vector of independent standard normal shocks. In what follows, we will use bold font to denote matrices and vectors. We assume that the state variables evolve according to the vector autoregression

$$\boldsymbol{H}_{t+1} = \boldsymbol{\Theta}_0 + \boldsymbol{\Theta} \boldsymbol{H}_t + \boldsymbol{\sigma}_H \boldsymbol{\epsilon}_{t+1}, \tag{1}$$

where $\boldsymbol{\Theta}_0$ is $m \times 1$, $\boldsymbol{\Theta}$ is $m \times m$, and $\boldsymbol{\sigma}_H$ is $m \times (m + 2)$. In Sections 2.3 and 2.4 we put more structure on $\boldsymbol{H}_t$. The fundamentals in the economy are represented by aggregate dividends. Let D_t denote the level of aggregate dividends at time t and $d_t = \log D_t$. We assume that the log of aggregate dividends are conditionally normally distributed with a time-varying mean z_t that is an

affine function of the state vector:

$$\Delta d_{t+1} = z_t + \boldsymbol{\sigma}_d \boldsymbol{\epsilon}_{t+1} \tag{2}$$

$$z_t = \delta_0 + \boldsymbol{\delta}' \boldsymbol{H}_t, \tag{3}$$

where $\boldsymbol{\sigma}_d$ is a $1 \times (m+2)$ vector of loadings on the shocks $\boldsymbol{\epsilon}$ and $\boldsymbol{\delta}$ is an $m \times 1$ vector of loadings on the variables $\boldsymbol{H}_t$. The conditional standard deviation of dividend growth is $\sigma_d = \sqrt{\boldsymbol{\sigma}_d \boldsymbol{\sigma}_d'}$. In what follows we will use the notation σ_i (without bold) to refer to the conditional standard deviation of i and $\sigma_{ij} = \boldsymbol{\sigma}_i \boldsymbol{\sigma}_j'$ to refer to the conditional covariance between i and j.

Because we are interested in pricing nominal bonds, we also specify a process for inflation. Let Π_t denote the price level and $\pi_t = \log \Pi_t$. Inflation follows the process

$$\Delta \pi_{t+1} = q_t + \boldsymbol{\sigma}_\pi \boldsymbol{\epsilon}_{t+1}, \tag{4}$$

$$q_t = \eta_0 + \boldsymbol{\eta}' \boldsymbol{H}_t, \tag{5}$$

where $\boldsymbol{\sigma}_\pi$ is $1 \times (m+2)$ and $\boldsymbol{\eta}$ is $m \times 1$. The conditional standard deviation of inflation is $\sigma_\pi = \sqrt{\boldsymbol{\sigma}_\pi \boldsymbol{\sigma}_\pi'}$.[2] All quantities are in real terms unless otherwise stated.

Discount rates are determined by the real riskfree rate and by the price of risk. Let $r_{f,t+1}$ denote the continuously-compounded riskfree return between times t and $t+1$. We assume that r_{t+1}^f is also an affine function of the state variables, and, because it is known at time t:

$$r_{t+1}^f = \alpha_0 + \boldsymbol{\alpha}' \boldsymbol{H}_t, \tag{6}$$

where $\boldsymbol{\alpha}$ is $m \times 1$. The variable that determines the price of risk, and therefore risk premia in this homoscedastic model, is denoted x_t. We assume

$$x_t = \xi_0 + \boldsymbol{\xi}' \boldsymbol{H}_t, \tag{7}$$

where $\boldsymbol{\xi}$ is $m \times 1$. To maintain a parsimonious model, we assume that only fundamental dividend risk is priced directly implying that the price of risk is proportional to the vector $\boldsymbol{\sigma}_d$. Other risks are priced insofar as they covary with aggregate cash flows. Besides reducing the degrees of freedom in

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[2]One could also include Δd_{t+1} and $\Delta \pi_{t+1}$ in the state variable vector $\boldsymbol{H}_t$. However, as will be shown, Δd_{t+1} and $\Delta \pi_{t+1}$ do not directly affect price-dividend ratios and nominal prices of nominal bonds. For this reason, it is more parsimonious to treat them separately.

the model, this specification allows for easier comparison to models based on preferences, such as those of Campbell and Cochrane (1999) and Menzly, Santos, and Veronesi (2004).[3] The stochastic discount factor (SDF) is given by

$$M_{t+1} = \exp\left\{-r^f_{t+1} - \frac{1}{2}\sigma_d^2 x_t^2 - x_t \boldsymbol{\sigma}_d \boldsymbol{\epsilon}_{t+1}\right\}.$$

Because the SDF is a quadratic function of x_t, the model is in the essentially affine class (Dai and Singleton (2002), Duffee (2002)). Asset prices are determined by the Euler equation

$$E_t\left[M_{t+1}R_{t+1}\right] = 1, \tag{8}$$

where R_{t+1} denotes the real return on a traded asset. Given the lognormal specification the maximal Sharpe ratio is given by (Campbell and Cochrane (1999), Lettau and Uhlig (2002), Lettau and Wachter (2007))

$$\max \frac{E_t R_{t+1} - R^f_{t+1}}{(\text{Var}_t[R_{t+1} - R^f_{t+1}])^{1/2}} = \frac{(\text{Var}_t[M_{t+1}])^{1/2}}{E_t[M_{t+1}]} = \sqrt{e^{x_t^2 \sigma_d^2} - 1} \approx |x_t|\sigma_d.$$

Note that the maximal Sharpe ratio is proportional to x_t and depends on the conditional standard deviation of dividend growth.[4]

2.2   Prices and Returns on Bonds and Equities

Real bonds

Let P^r_{nt} denote the real price of a zero-coupon bond with a fixed payoff of unity n periods from now. Because this asset has no intermediate payoffs, its real return between t and $t+1$ equals

$$R^r_{n,t+1} = \frac{P^r_{n-1,t+1}}{P^r_{nt}}. \tag{9}$$

The prices of real bonds at any maturity are determined recursively from the Euler equation (8), which implies that

$$E_t\left[M_{t+1}P^r_{n-1,t+1}\right] = P^r_{nt}, \tag{10}$$

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[3]The assumption that only dividends are priced does not affect the form of the solution. The formulas in the Appendix allow for a general price of risk.

[4]The definition of M_{t+1} is consistent with the riskfree rate r^f_{t+1} because, by conditional lognormality,

$$-\log E_t[M_{t+1}] = r^f_{t+1} + \frac{1}{2}\sigma_d^2 x_t^2 - \frac{1}{2}\sigma_d^2 x_t^2 = r^f_{t+1}$$

and therefore (8) is satisfied for the riskfree asset.

while the fact that the bond pays a single unit at maturity implies that $P_{0t}^r = 1$. Appendix A verifies that P_{nt}^r takes the exponential affine form

$$P_{nt}^r = \exp\{A_n^r + \boldsymbol{B}_n^r \boldsymbol{H}_t\}, \tag{11}$$

where A_n^r is a scalar and $\boldsymbol{B}_n^r$ is a $1 \times m$ vector. The sequences A_n^r and $\boldsymbol{B}_n^r$ satisfy difference equations, given in Appendix A, with boundary conditions $A_n^r = 0$ and $\boldsymbol{B}_n^r = \boldsymbol{0}_{1 \times m}$. We develop economic intuition about the properties of the A_n and $\boldsymbol{B}_n$ coefficients in the context of a simplified specification in section 2.3 below. Log yields are linear in the state variables:

$$y_{nt}^r = -\frac{1}{n} \log P_{nt}^r = -\frac{1}{n} \left(A_n^r + \boldsymbol{B}_n^r \boldsymbol{H}_t \right).$$

Equity

Let P_{nt}^d denote the price of the asset that pays the aggregate dividend n periods from now. It is convenient to scale P_{nt}^d by the aggregate dividend at time t to eliminate the need to consider D_t as a state variable. The return on this zero-coupon equity claim is equal to

$$R_{n,t+1}^d = \frac{P_{n-1,t+1}^d}{P_{nt}^d} = \frac{P_{n-1,t+1}^d/D_{t+1}}{P_{nt}^d/D_t} \frac{D_{t+1}}{D_t}. \tag{12}$$

The Euler equation implies that P_{nt}^d also satisfies a recursion

$$E_t \left[M_{t+1} \frac{D_{t+1}}{D_t} \frac{P_{n-1,t+1}^d}{D_{t+1}} \right] = \frac{P_{nt}^d}{D_t}, \tag{13}$$

with boundary condition $P_{0t}^d/D_t = 1$. The form of (13) is thus the same as that of (10). Appendix A verifies that

$$\frac{P_{nt}^d}{D_t} = \exp\{A_n^d + \boldsymbol{B}_n^d \boldsymbol{H}_t\}, \tag{14}$$

where A_n^d is a scalar and $\boldsymbol{B}_n^d$ is $1 \times m$. The difference equations that define A_n^d and $\boldsymbol{B}_n^d$ are given in Appendix A.

Nominal bonds

Let P_{nt}^π denote the real price of a zero-coupon nominal bond maturing in n periods. The real return on this bond equals

$$R_{n,t+1}^\pi = \frac{P_{n-1,t+1}^\pi}{P_{nt}^\pi} = \frac{P_{n-1,t+1}^\pi \Pi_{t+1}}{P_{nt}^\pi \Pi_t} \frac{\Pi_t}{\Pi_{t+1}}. \tag{15}$$

This asset is directly analogous to the dividend claim above: the "dividend" is the reciprocal of the price level, and the "price-dividend ratio" on this asset is its nominal price $P_{nt}^\pi \Pi_t$. The same argument can therefore be applied to show that

$$P_{nt}^\pi \Pi_t = \exp\{A_n^\pi + \boldsymbol{B}_n^\pi \boldsymbol{H}_t\}, \tag{16}$$

where the difference equations for A_n^π and $\boldsymbol{B}_n^\pi$ are reported in Appendix A. Following Campbell and Viceira (2001), we use the superscript \$ to denote nominal quantities for the nominal bond. The nominal (continuously-compounded) yield to maturity on this bond is equal to

$$y_{nt}^\$ = -\frac{1}{n}\log\left(P_{nt}^\pi \Pi_t\right) = -\frac{1}{n}\left(A_n^\pi + \boldsymbol{B}_n^\pi \boldsymbol{H}_t\right) \tag{17}$$

and, like the yield on the real bond, is linear in the state variables. Finally, we use the notation $R_{n,t+1}^\$$ to denote the nominal return on the nominal n-period bond:

$$R_{n,t+1}^\$ = \frac{P_{n-1,t+1}^\pi \Pi_{t+1}}{P_{nt}^\pi \Pi_t}.$$

Risk premia

Given the solutions for prices, it is straightforward to solve for risk premia. Since price are exponential affine functions of normally distributed random variables, returns on zero-coupon instruments are lognormally distributed. Let $r_{n,t}^r = \log R_{n,t}^r$ be the continuously compounded return on the real zero-coupon bond of maturity n. It follows from the Euler equation (8) that risk premia on real zero-coupon bonds satisfy

$$E_t[r_{n,t+1}^r - r_{t+1}^f] + \frac{1}{2}\sigma_{r,(n)}^2 = \boldsymbol{\sigma}_{(n)}^r \boldsymbol{\sigma}_d' x_t, \tag{18}$$

$$\boldsymbol{\sigma}_{(n)}^r = \boldsymbol{B}_{n-1}^r \boldsymbol{\sigma}_H, \tag{19}$$

where $\boldsymbol{\sigma}_{(n)}^r$ equals the vector of loadings on the shocks for the return on the n-period bond. In what follows, we let $\sigma_{r,(n)}$ denote the standard deviation of the bond return. Note that $\sigma_{r,(n)} = \boldsymbol{\sigma}_{(n)}^r (\boldsymbol{\sigma}_{(n)}^r)'$. The second term on the left hand side of (18), $\frac{1}{2}\sigma_{r,(n)}^2$, is therefore an adjustment for Jensen's inequality. Analogous notation is used for risk premia on equities and nominal bonds, as described below. The right hand side of (18) equals the covariance of the return on the bond with the stochastic discount factor, which is proportional to the covariance of the return with the

shock to dividends. Risk premia are time-varying and proportional to x_t. The average level of the risk premium for each bond is determined by $\boldsymbol{B}_{n-1}^r \boldsymbol{\sigma}_H \boldsymbol{\sigma}_d'$, where $\boldsymbol{B}_{n-1}^r$ represents the effect of each state variable on the bond price, and $\boldsymbol{\sigma}_H \boldsymbol{\sigma}_d'$ is the vector of covariances between fundamentals (dividends) and the state variables.

Let $r_{n,t}^d = \log R_{n,t}^d$ denote the continuously compounded return on zero-coupon equity of maturity n. Risk premia on zero-coupon equity claims are equal to

$$E_t[r_{n,t+1}^d - r_{t+1}^f] + \frac{1}{2}\sigma_{d,(n)}^2 = \boldsymbol{\sigma}_{(n)}^d \boldsymbol{\sigma}_d' x_t, \tag{20}$$

$$\boldsymbol{\sigma}_{(n)}^d = \boldsymbol{\sigma}_d + \boldsymbol{B}_{n-1}^d \boldsymbol{\sigma}_H, \tag{21}$$

where $\boldsymbol{\sigma}_{(n)}^d$ equals the vector of loadings on the shocks for the return on zero-coupon equity maturing in n periods. As is the case for zero-coupon bond premia, risk premia on equities depend on the loading on each state variable in $\boldsymbol{H}_t$ and the covariances of the elements of $\boldsymbol{H}_t$ with fundamentals as captured by the vector $\boldsymbol{\sigma}_H \boldsymbol{\sigma}_d'$. These covariances capture innovations to future risk premia (through x_t), innovations to the riskfree rate, and innovations to expected dividend growth. There is also an additional term, $\boldsymbol{\sigma}_d$, that represents cash flow risk due to unexpected dividend growth in the next period.

Risk premia for nominal bonds also satisfy an equation analogous to (18):

$$E_t[r_{n,t+1}^\pi - r_{t+1}^f] + \frac{1}{2}\sigma_{\pi,(n)}^2 = \boldsymbol{\sigma}_{(n)}^\pi \boldsymbol{\sigma}_d' x_t, \tag{22}$$

$$\boldsymbol{\sigma}_{(n)}^\pi = -\boldsymbol{\sigma}_\pi + \boldsymbol{B}_{n-1}^\pi \boldsymbol{\sigma}_H, \tag{23}$$

where $\boldsymbol{\sigma}_{(n)}^\pi$ equals the vector of loadings on the shocks for the real return on the n-period nominal bond. Real risk premia on nominal bonds are not only determined by the covariance of the state variable $\boldsymbol{H}_t$ with fundamentals, but also the covariance of unexpected inflation with fundamentals, $\boldsymbol{\sigma}_\pi \boldsymbol{\sigma}_d'$. It is also of interest to compute risk premia on nominal bonds relative to the one-period nominal bond (as opposed to the real bond, as in (22). It follows from the equation for nominal prices (16) that

$$E_t\left[r_{n,t+1}^\$ - y_{1t}^\$\right] + \frac{1}{2}\sigma_{\$,(n)}^2 = \boldsymbol{\sigma}_{(n)}^\$ \boldsymbol{\sigma}_d' x_t \tag{24}$$

$$\boldsymbol{\sigma}_{(n)}^\$ = \boldsymbol{B}_{n-1}^\pi \boldsymbol{\sigma}_H, \tag{25}$$

where $\boldsymbol{\sigma}_{(n)}^\$$ equals the vector of loadings on the shocks for the nominal return on the nominal bond and $\sigma_{\$,(n)}^2$ is the variance of the return.

This section shows that risk premia on all zero-coupon assets are proportional to x_t. While there is some conditional heteroscedasticity in the aggregate market that arises from time-varying weights on zero-coupon equity, this effect is small. A natural way to drive a wedge between time-variation in bond and stock premia is to allow for time-varying correlations as in Campbell, Sunderam, and Viceira (2007). For simplicity and to maintain our focus on the slope of the term structures of equity and interest rates, we do not pursue this route here.

2.3   A special case of four interpretable state variables

Consider a case where elements of $\boldsymbol{H}_t$ can be identified as expected dividend growth, expected inflation, the riskfree rate and the price-of-risk variable. This allows for interpretable closed-form solutions for the recursions in the previous section. We implement this special case by setting

$$
\boldsymbol{\delta} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \boldsymbol{\eta} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \ \boldsymbol{\alpha} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \ \boldsymbol{\xi} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{26}
$$

Further, we assume that each of these variables follows an AR(1) process, so that $\boldsymbol{\Theta}$ is a diagonal matrix:

$$
\boldsymbol{\Theta} = \begin{bmatrix} \phi_z & & & \\ & \phi_q & & \\ & & \phi_r & \\ & & & \phi_x \end{bmatrix}. \tag{27}
$$

Setting $\boldsymbol{\Theta}_0 = \boldsymbol{0}_{4\times 1}$ implies that δ_0 is mean dividend growth, η_0 is mean inflation, α_0 is the mean of the riskfree rate and ξ_0 is the mean of x_t. For notational simplicity, let $g = \delta_0$, $\bar{q} = \eta_0$, $\bar{r}^f = \alpha_0$ and $\bar{x} = \xi_0$. Partition $\boldsymbol{\sigma}_H$ as

$$
\boldsymbol{\sigma}_H = \begin{bmatrix} \boldsymbol{\sigma}_z \\ \boldsymbol{\sigma}_q \\ \boldsymbol{\sigma}_r \\ \boldsymbol{\sigma}_x \end{bmatrix}. \tag{28}
$$

The calibration is described in more detail in section 3.1. All results in this section are for parameter values given in Tables 1-3.

These assumptions imply that the vectors of coefficients $\boldsymbol{B}_n^r$, $\boldsymbol{B}_n^d$ and $\boldsymbol{B}_n^\pi$ can be written as

$$
\begin{aligned}
\boldsymbol{B}_n^r &= [B_{zn}^r, B_{qn}^r, B_{rn}^r, B_{xn}^r] \\
\boldsymbol{B}_n^d &= [B_{zn}^d, B_{qn}^d, B_{rn}^d, B_{xn}^d] \\
\boldsymbol{B}_n^\pi &= [B_{zn}^\pi, B_{qn}^\pi, B_{rn}^\pi, B_{xn}^\pi].
\end{aligned}
\tag{29}
$$

The scalar terms with z subscripts are loading on expected dividend growth z_t, the terms with q subscripts are loadings on expected inflation q_t, the terms with r subscripts are loadings on the real riskfree rate r_t^f, and terms with x subscripts are loadings on the price-of-risk variable x_t. Appendix B shows that the coefficients for expected dividend growth are

$$
\begin{aligned}
B_{zn}^r &= 0 \\
B_{zn}^d &= \frac{1 - \phi_z^n}{1 - \phi_z} \\
B_{zn}^\pi &= 0.
\end{aligned}
$$

Expected dividend growth does not directly influence the price of real or nominal bonds so that $B_{zn}^r = B_{zn}^\pi = 0$. For equities, the loadings on expected dividend growth B_{zn}^d are positive and increase in maturity. An increase in expected dividend growth increases the price of the asset that pays the aggregate dividend in the future. Because expected dividend growth is persistent, and because D_{t+n} cumulates shocks between t and $t+n$, the greater is the maturity n, the greater is the effect of changes in z_t on the price.

Expected inflation behaves like negative dividend growth for the nominal bond. Appendix B shows that the coefficients are therefore

$$
\begin{aligned}
B_{qn}^r &= 0 \\
B_{qn}^d &= 0 \\
B_{qn}^\pi &= -\frac{1 - \phi_q^n}{1 - \phi_q}.
\end{aligned}
$$

Expected inflation does not directly influence real bonds and equity claims because equity is a claim on the real quantity D_t. The coefficients on the real riskfree rate are identical for all scaled price ratios:

$$
B_{rn}^r = B_{rn}^d = B_{rn}^\pi = -\frac{1 - \phi_r^n}{1 - \phi_r}.
$$

All B_{rn} coefficients are negative because an increase in the interest rate increases discount rates of all assets leading to lower current prices. Moreover, B_r is decreasing in the maturity and increasing in the persistence of the interest rate process. This is the duration effect: the further out the cash flow, the larger the effect of a change in the one-period riskfree rate. This intuition holds for real and nominal bonds as well as for equity.

The coefficients on the price-of-risk variable x_t, B_{xn}, are more complicated and are given by

$$B_{xn}^r = (\phi_x - \sigma_{dx})B_{xn-1}^r - \left(-\frac{1-\phi_r^n}{1-\phi_r}\sigma_{dr} \right) \tag{30}$$

$$B_{xn}^\pi = (\phi_x - \sigma_{dx})B_{xn-1}^\pi - \left(-\sigma_{d\pi} - \frac{1-\phi_r^n}{1-\phi_r}\sigma_{dr} - \frac{1-\phi_q^n}{1-\phi_q}\sigma_{dq} \right) \tag{31}$$

$$B_{xn}^d = (\phi_x - \sigma_{dx})B_{xn-1}^d - \left(\sigma_d^2 - \frac{1-\phi_r^n}{1-\phi_r}\sigma_{dr} + \frac{1-\phi_z^n}{1-\phi_z}\sigma_{dz} \right) \tag{32}$$

Figure 1 shows these coefficients, and, for comparison, the coefficients on the other state variables. In our calibration, the covariance of unexpected dividend growth and interest rate shocks is negative ($\sigma_{dr} < 0$) implying from (30) that $B_{xn}^r < 0$. In other words an increase in the price of risk lowers prices of real zero-coupon bonds. Since $\sigma_{d\pi} = 0$ and $\sigma_{dq} < 0$, (31) nominal zero-coupon bond prices are more sensitive to changes in the price of risk than real zero-coupon bonds of the same maturity. Moreover, B_{xn}^r and B_{xn}^π increase in magnitude with the maturity. This is again a duration effect, which also is apparent in the coefficients B_{rn}. Compared with B_{rn} however, B_{xn} initially declines slowly in maturity. This is because short-horizon assets do not have large risk premia, and so their prices are not as sensitive to changes in the price of risk. Moreover, B_{xn} levels off more slowly in maturity than B_{rn}. As the horizon lengthens, risk premia increase, and therefore the sensitivity to x_t increases. The same effects are present for nominal bonds.

Figure 1 reveals that, in this calibration, the loading on x_t for equities follows a very different pattern than that for nominal and real bonds. Over a range of zero to ten years, B_{xn}^d decreases in maturity. This is again the duration effect: the longer the maturity the more sensitive the price is to changes in the risk premium. After ten years B_{xn}^d increases, and then asymptotes to a level that is lower than B_{x0}^d. Given the duration effect, this decrease is surprising. It occurs because the covariance of interest rate shocks and dividend growth σ_{dr} is more negative than the covariance of expected dividend growth and dividend growth. Thus for short horizons, the last term in brackets in (32) is positive and the sequence B_{xn}^d is decreasing. However, since the persistence of expected dividend growth is higher than the persistence of interest rates, the term including the expected

dividend covariance dominates for longer horizons and the expression in brackets on the right hand side of (32) becomes positive. Intuitively, a negative correlation of expected dividend shocks and unexpected dividend growth lowers the premium on long-horizon equity. Because long-horizon equity has less of a premium, it is less sensitive to changes in x_t than medium-horizon equity.

Figure 2 displays ratios of zero-coupon equity prices to aggregate dividends as a function of maturity under the calibration described in Section 3.1. In all three panels, the solid line shows ratios when the state variables are at their long-run mean. This solid line is decreasing in maturity, as is necessary for convergence. The top panel varies expected dividend growth by plus and minus two unconditional standard deviations, the middle panel the riskfree rate, and the lower panel the price of risk. Prices are increasing functions of expected dividend growth z_t, decreasing functions of the real interest rate r_t^f and decreasing functions of x_t. This figure shows that, under our calibration, most of the variation of prices at all maturities comes from variations in risk premia as represented by x_t. Variation in expected dividend growth becomes important at the longest maturities, while variation in the real riskfree rate is relatively unimportant across all maturities.

Figure 3 plots the real yield to maturity on real zero coupon bonds as a function of maturity, assuming parameter values described in Section 3.1 and for various values of the state variables as described in the preceding paragraph. Real yields

$$y_{nt}^r = -\frac{1}{n} \log P_{nt}^r = -\frac{1}{n} \left(A_n^r + B_{rn}^r (r_{t+1}^f - \bar{r}^f) + B_{xn}^r (x_t - \bar{x}) \right) \tag{33}$$

are functions of the current values of the real riskfree rate and of the price of risk variable x_t. An increase in either type of discount rate increases yields at all maturities since both B^r coefficients are negative. The two lower panels of Figure 3 show that the real riskfree rate has a greater effect on short-term yields while x_t has more of an effect for long-term yields.

Finally, it follows from (17) that the nominal (continuously-compounded) yield to maturity on a nominal zero coupon bond, is equal to

$$y_{nt}^\$ = -\frac{1}{n} \log \left(P_{nt}^\pi \Pi_t \right) = -\frac{1}{n} \left(A_n^\pi + B_{rn}^\pi (r_{t+1}^f - \bar{r}^f) + B_{qn}^\pi (q_t - \bar{q}) + B_{xn}^\pi (x_t - \bar{x}) \right). \tag{34}$$

Figure 4 plots nominal yields for the parameter values described in Section 3.1 and for various values of the state variables as in the previous two figures. Nominal yields are functions of expected inflation, the real riskfree rate, and risk premia. Yields are decreasing in expected inflation, and,

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like real yields, increasing in the riskfree rate and in risk premia, with the riskfree rate having the greater impact for shorter maturities and risk premia having the greater impact for longer maturities. Note that since yields depend on the current value of the price of risk, the expectations hypothesis does not hold in this model. We will investigate the quantitative deviations from the expectations hypothesis in detail below in the context of Campbell-Shiller type yield regressions.

2.4 Qualitative implications for the term structures of equity and interest rates

Prior to describing the full calibration of the model and results from simulated data, we use the results developed above to describe the model's qualitative implications for risk premia on bonds and stocks. We first illustrate the issues by comparing bonds and equity maturing in two periods with those maturing in one period. It follows from (18) and (19) that the unconditional risk premium of the real bond maturing in two-periods equals

$$E[r_{2,t+1}^r - r_{t+1}^f] + \frac{1}{2}\sigma_{r,(2)}^2 = -\sigma_{dr}\bar{x}.$$

The risk premium on the one-period real bond is, by definition, equal to zero. The term σ_{dr} equals the conditional covariance between the real interest rate and dividend growth: a negative value of this covariance leads to a positive risk premium on the two-period bond. If unexpected changes in the riskfree interest rates are uncorrelated with unexpected dividend growth, the expected return of the two-period bond is the same as the current riskfree rate. The same term is reflected in the average yield spread between the one and the two-period bond:

$$E[y_2^r - y_1^r] = -\frac{1}{2}\sigma_{dr} - \frac{1}{2}\sigma_r^2, \tag{35}$$

where the second term represents an adjustment for Jensen's inequality and is relatively small.

The unconditional risk premia on one and two-period equity claims are equal to

$$E[r_{1,t+1}^d - r_{t+1}^f] + \frac{1}{2}\sigma_{d,(1)}^2 = \sigma_d^2\bar{x} \tag{36}$$

$$E[r_{2,t+1}^d - r_{t+1}^f] + \frac{1}{2}\sigma_{d,(2)}^2 = \left(\sigma_d^2 - \sigma_{dr} + \sigma_{dz} - \sigma_d^2\sigma_{dx}\right)\bar{x}. \tag{37}$$

The one-period equity premium depends only on the volatility of dividend shocks and $\bar{x}$ while the two-period equity claim is also exposed to unexpected changes in the interest rate, expected dividend and the price of risk process. If these processes are correlated with the priced fundamental

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dividend factor, the risk premium of two-period equity claim will be different from the one-period premium. These risk factors are represented by the covariance between shocks to expected dividends and unexpected dividends σ_{dz}, real interest rates and unexpected dividends σ_{dr}, and risk premia on unexpected dividends σ_{dx} in (37). If $\sigma_{dz} = \sigma_{dr} = \sigma_{dx} = 0$, the two-period equity claim has the same risk premium as the one-period claim; in other words the equity term structure is flat.

However, the positive premium of value (short horizon) stocks over growth (long horizon) stocks in the data suggests that the equity term structure is downward sloping. Thus the premium on two-period equity should be less that on one-period equity. Comparing (35) to (36) and (37) suggests that an upward sloping term structure of interest rates requires interest rate shocks to be negatively correlated with dividend shocks ($\sigma_{dr} < 0$). *Ceteris paribus*, this effect leads also to an upward sloping term structure of equity, which implies a growth premium rather than a value premium. As shown in Lettau and Wachter (2007), a key parameter for the slope of the equity term structure is the correlation of fundamental dividend risk and shocks to the price of risk process x_t. If σ_{dx} is negative, the equity term structure tends to be upward sloping, which is again inconsistent with the large value premium in the data.

Note that if $\sigma_{dx} = 1$ and interest rate and expected dividend shocks are uncorrelated with dividend shocks ($\sigma_{dr} = \sigma_{dz} = 0$), the two-period equity claim is riskless. Recall that returns of zero-coupon equity depend on dividend growth and the change in the price-dividend ratio (see (12)). If $\sigma_{dx} = 1$, positive dividend shocks are associated with positive price of risk shocks causing price-dividend ratios to decrease. In this special case, these two effects exactly offset each other creating a perfectly hedged one-period return. This example illustrates a general property of the model. If dividend shocks are associated with positive price of risk shocks ($\sigma_{dx} > 0$), long-term equity tends to be less risky than short-term equity.

Beyond two periods, the effect of changes in risk premia exacerbates this tension. The unconditional risk premium on the n-period real bond equals

$$E[r^r_{n,t+1} - r^f_{t+1}] + \frac{1}{2}\sigma^2_{r,(n)} = \left(-\frac{1 - \phi_r^{n-1}}{1 - \phi_r}\sigma_{dr} + B^r_{x,n-1}\sigma_{dx} \right)\bar{x} \qquad (38)$$

As Appendix B shows, $B^r_{x,n-1}$ has the same sign as $-\sigma_{dr}$ and is decreasing in maturity. A negative correlation between interest rates and fundamentals implies that long-term bonds have positive risk premia. Because bond prices are determined by risk premia, it follows that changes in risk premia

are another source of risk for these bonds. If changes in risk premia are negatively correlated with fundamentals, risk premia on bonds increase.

The risk premium on n-period equity is equal to

$$E[r_{n,t+1}^d - r_{t+1}^f] + \frac{1}{2}\sigma_{d,(n)}^2 = \left(\sigma_d^2 - \frac{1 - \phi_r^{n-1}}{1 - \phi_r}\sigma_{dr} + \frac{1 - \phi_z^{n-1}}{1 - \phi_z}\sigma_{dz} + B_{x,n-1}^d\sigma_{dx} \right) \bar{x}. \qquad (39)$$

As long as risk premia on zero-coupon equity are positive, $B_{x,n-1}^d$ is negative (see A.17). Just as in the two-period case, the same features of the model that deliver positive risk premia on long-term bonds causes risk premia of long-term equities to be higher than short-term equity premia. A negative correlation between discount rates and fundamentals leads to higher expected returns on long-term equities relative to short-term equities, the opposite of what cross-sectional asset pricing data suggest. This is because duration operates for both bonds and equities; when shocks to discount rates are priced risk premia on all long-term instruments are driven up relative to short-term instruments.

In the calibration that follows, we show that it is indeed possible to match both the upward slope of the term structure of interest rates and the downward slope of the term structure of equities in a model where the riskfree rate and the risk premium vary. Part of the answer lies in the role of expected dividend growth which appears in the equations for equities above and part of the answer lies in the role of expected inflation which influences risk premia on nominal bonds.

3 Implications for Returns on Stocks and Bonds

To study our model's implications for returns on the aggregate market, on real and nominal bonds, and for portfolios sorted on scaled-price ratios, we simulate 50,000 quarters from the model. Given simulated data on shocks ϵ_t, and on expected dividend growth z_t, expected inflation q_t, the real riskfree rate r_t^f, and the price-of-risk variable x_t, we compute real prices of real bonds given (11), ratios of prices to the aggregate dividend for zero-coupon equity (14), and nominal prices of nominal bonds (16). We also compute a series for realized dividend growth (2) and realized inflation (4). Real returns on zero-coupon instruments are then described by (9) for real bonds, (15) for nominal bonds, and (12) for zero-coupon equity. Computation of the price-dividend ratio for the market and for firms (which are then aggregated into portfolios) is described further below.

3.1 Calibration

We calibrate the special case of the model in Sections 2.3 and 2.4. This preserves intuition for the state variables. Means and autocorrelations are given in Table 1, conditional cross-correlations in Table 2 and conditional standard deviations of shocks in Table 3. As far as is practical, parameters for the dividend process and aggregate market are set as in Lettau and Wachter (2007). We compare these to the century-long annual data set of Campbell (2003) (the instability of many moments of the aggregate market in recent years makes a long data set especially desirable). Lettau and Wachter (2007) describes these data in further detail. Parameters for inflation are set as in Wachter (2006), and results for bond yields are compared to sample moments from the Fama-Bliss data on CRSP. These data begin in 1952 and end in 2004.[5] So that all data are at a comparable frequency, we use annual rather than quarterly observations on yields.

Table 1 shows the means and autocorrelation of expected inflation growth z_t, expected inflation q_t, the real riskfree rate r_t^f and the price of risk $\sigma_d x_t$.[6] The model is simulated at a quarterly frequency. Table 1 shows annualized parameter values, i.e. the means are multiplied by 4 and the autocorrelations are raised to the fourth power.

The mean of expected dividend growth is set at 2.28%, the mean of annual dividends on the S&P 500 over the 1881-2002 period. The mean of expected inflation is 3.68%, the sample mean over the 1952–2004 period. Given the mean of inflation, the mean of the riskfree rate implies reasonable values for the average yield on the short-term nominal bond. The mean of x_t determines Sharpe ratios in this economy. This variable is set to produce Sharpe ratios that are close to those in the data. The autocorrelations for expected dividend growth z_t and for expected inflation q_t are determined based on dividend and inflation data respectively (see Lettau and Wachter (2007) and Wachter (2006)). The autocorrelation for r_t^f is set to match the autocorrelation of the short-term nominal yield. The autocorrelation of x_t is determined to produce a realistic autocorrelation of the price-dividend ratio while still implying only a small amount of univariate mean reversion in returns as in the data.

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[5]As is customary in term structure and inflation studies, we calibrate inflation and yields to data after 1952 because of the Fed-Treasury accord (Campbell and Viceira (2001)).

[6]In Tables 1 and 3, we report statistics for $\sigma_d x_t$ rather than x_t. Because of its relation to the maximal Sharpe ratio, $\sigma_d x_t$ is the more intuitive quantity. Note that for Table 2, using $\sigma_d x_t$ of x_t would lead to the same answers.

Table 2 shows conditional cross-correlations of the shocks to the economy. The four state variables account for four shocks; there is also a shock to realized dividend growth and to realized inflation. To maintain a parsimonious model, we set many correlations to zero. Sharpe ratios in this model are determined by the correlation of the state variables with realized dividend growth. The correlation between expected and realized dividend growth is -0.83 is determined from dividend data (see Lettau and Wachter (2007)). The correlation between expected inflation q_t and realized dividend growth determines the premium for nominal over real bonds. A value of -0.20 implies that nominal bonds will carry a premium over real bonds, and moreover, that this premium increases in maturity. It is also consistent with the negative correlation of dividend growth and inflation in the data. The correlation between the real riskfree rate and expected dividend growth is -0.40. This implies an upward sloping real term structure. The correlations between dividend growth, q_t and r_t are consistent with empirical evidence that yields on indexed Treasury bonds are increasing in maturity, but that this slope is less than for nominal bonds (Roll (2004)). The implications of these parameters are discussed further in Section 3.3.

As in Lettau and Wachter (2007), the correlation between x and dividend growth is set to be zero. The implication of this parameter choice is discussed further in Section 3.4. The correlations between x and shocks to $\Delta \pi_t$, between x and z and between x and q are set to ensure that dividend growth and interest rates have reasonable time-series properties as explained below. The precise choice of the correlation between realized and expected inflation, set to be 0.50, has a very minor impact on asset prices. However, values that are too high or too low result in variance-covariance matrices for which the Cholesky decomposition does not exist.

Table 3 shows the conditional standard deviation of shocks. The shock to realized dividend growth is 10% per annum. This value falls between estimates in the long data ($\sim 14\%$), and in the post-war sample ($\sim 6\%$). The conditional standard deviation of expected dividend growth and its first-order autocorrelation are set at the same value as Lettau and Wachter (2007). The variance of expected inflation, as well as the first-order autocorrelation is determined by the ARMA estimation in Wachter (2006). While Wachter assumes perfect correlation between the shocks to expected and realized growth, the estimates can be adjusted so that the correlation is not perfect but that the likelihood function is the same as shown in the appendix to that paper. The standard deviation of the real riskfree rate is 0.15% per annum. This is set so that, given our process for

inflation, our model implies nominal bond yield volatility close to that in the data. The standard deviation of $\sigma_d x$ is set to 40; this high value is necessary to capture high volatility of stock returns and price-dividend ratios.

In this calibration we have set a number of interaction terms equal to zero. Richer models that are used to estimate the term structure allow nonzero off-diagonal elements in the matrix Θ_1 and arbitrary cross-correlations of shocks (see Dai and Singleton (2003) and Duffee (2002)). Results from these studies suggest that such interactions may be important for fully capturing the dynamics of the term structure of interest rates. We could introduce these effects without changing the analytic form of prices, as Section 2.2 shows. However, it is not clear how to cleanly identify these parameters with our macro-based approach. Moreover, our simpler model has the advantage that it is easier to connect the calibrated parameters with the results. While our model may miss some of the term structure properties captured by the more complex models, it nonetheless seems appropriate for our current purpose.

3.2 The Aggregate Market

Table 4 shows statistics for the aggregate market in simulated and in historical data. In our model, the aggregate market is the claim to all future dividends. Therefore its price-dividend ratio is given by

$$\frac{P_t^m}{D_t} = \sum_{n=1}^{\infty} \frac{P_{nt}}{D_t} = \sum_{n=1}^{\infty} \exp\left\{ A_n^d + \boldsymbol{B}_n^d \boldsymbol{H}_t \right\} \tag{40}$$

Appendix C describes necessary and sufficient conditions on the parameters such that (40) converges for all values of $\boldsymbol{H}_t$. The return on the aggregate market equals

$$R_{t+1}^m = \frac{P_{t+1}^m + D_t}{P_t^m} = \frac{(P_{t+1}^m / D_{t+1}) + 1}{P_t^m / D_t} \frac{D_{t+1}}{D_t} \tag{41}$$

In simulated, data, we calculate quarterly returns and compound to an annual frequency. We create an annual price-dividend ratio in simulated data by dividing price by dividends over the previous year.

As Table 4 shows, the volatility and the autocorrelation of the price-dividend ratio are close to those found in the data. This is not a surprise as model parameters were chosen in part to produce reasonable values for these moments. The model produces a mean price-dividend ratio of

18.6, while the mean in the data is 25.6. Matching this statistic is a common difficulty for models of this type: Campbell and Cochrane (1999), for example, find an average price-dividend ratio of 18.2. As they explain, this statistic is poorly measured due to the persistence of the price-dividend ratio. The present model fits the volatility of equity returns (19.1% in the model versus 19.4% in the data), though it produces an equity premium that is slightly higher than that in the data (8.45% in the model versus 6.33% in the data). Like the mean of the price-dividend ratio, this number is estimated with substantial noise. The annual autocorrelation of returns is near zero for both model and data.

Motivated by the substantial literature on return predictability, we report results of regressing excess aggregate market returns on the log of the price-dividend ratio and on the yield spread. Panel A of Table 5 reports results of long-horizon regressions of continuously compounded excess returns on the log price-dividend ratio. The model accounts for economically significant stock return predictability on the basis of the price-dividend ratio. The R^2 is equal to 11% at an annual horizon and rises to 39% at a ten-year horizon. These values are well within the range of empirical estimates (e.g. Campbell and Shiller (1988), Cochrane (1992), Fama and French (1989) and Keim and Stambaugh (1986)). Panel B reports results from regressing long-horizon excess returns on the difference between the five-year and the three-month nominal yields. R^2 values on this regression range from 7% for 1-year returns to 27% for ten-year returns. These values are again well within the range of estimates for stock return predictability by the yield spread (see Campbell (2003)).

Table 6 reports the results of long-horizon regressions of dividend growth on the price-dividend ratio and on yield spreads. Evidence indicates that there is little predictability in dividend growth by either variable (Ang and Bekaert (2007), Cochrane (2008), Lettau and Ludvigson (2005) and Lettau and Van Nieuwerburgh (2008)). Despite the fact that the mean of dividend growth is time-varying in our model, neither the price-dividend ratio, nor the yield spread, predicts dividend growth with R^2 values that exceed 2%, even at long horizons. This is both because the variation in expected dividend growth is relatively low, and because expected dividend growth is positively correlated with the price of risk x_t. Thus risk premia and expected dividend growth are negatively correlated, leading to less dividend growth predictability than what one would expect given the present-value nature of this model.

Our model does imply that dividend growth may be predictable by variables other than scaled

price ratios or yields. The bottom panel of Table 6 highlights this effect by regressing long horizon dividend growth on its expectation z_t. R^2 values range from 7% for a 1-year horizon to 35% for a ten-year horizon. Thus our model is consistent with findings of Lettau and Ludvigson (2005) that show that the cointegrating relation between consumption, dividends, and labor income predicts dividend growth.

3.3 The Term Structure of Interest Rates

Table 7 shows the implications of the model for means, standard deviations, and annual autocorrelations of nominal and real bond yields. Data moments for bond yields using the CRSP Fama-Bliss data set are provided for comparison. The mean of the 3-month nominal yield matches its value in the data. The mean for the 3-month real yield is lower, reflecting both a term for expected inflation, and a small, positive inflation premium. Both the real and nominal yield curves are upward sloping. This occurs because of the negative correlation between the real riskfree rate and fundamentals. Because bond prices fall when the real riskfree rate rises, bond prices fall when growth in fundamentals are low. This leads both real and nominal bonds to command a positive risk premium and implies an upward-sloping yield curve. In the case of nominal bonds, there is an additional effect arising from the negative correlation between fundamentals and expected inflation. This negative correlation, which is present in the data (Piazzesi and Schneider (2006), Wachter (2006)) implies that nominal bond prices fall when fundamentals are low, leading to a positive inflation risk premium.

The model implies volatilities for nominal bonds that are close to those in the data (the volatility on the short-term nominal bond is 2.9% in both data and model). Moreover, volatilities decrease in maturity, as in the data. This decrease follows from the stationary autoregressive nature of the underlying processes. Finally, the first-order autocorrelation of the three-month nominal yield is 0.82 per annum. This compares with 0.83 in the data.

Table 8 shows the outcome of regressions

$$y_{n-1,t+1}^r - y_{nt}^r = \alpha_n + \beta_n \frac{1}{n-1} \left(y_{nt}^r - y_{1t}^r \right) + e_t,$$

for real bonds and

$$y_{n-1,t+1}^\$ - y_{nt}^\$ = \alpha_n + \beta_n \frac{1}{n-1} \left(y_{nt}^\$ - y_{1t}^\$ \right) + e_t^\pi$$

for nominal bonds in simulated and historical data. These "long-rate" regressions are performed by Campbell and Shiller (1991) to test the hypothesis of constant risk premia on bonds.

The relation between risk premia and these regressions can be uncovered using the definition of yields and returns. For real bonds:

$$r^r_{n,t+1} = y^r_{nt} - (n-1)\left(y^r_{n-1,t+1} - y^r_{nt}\right)$$

Rearranging and taking expectations implies:

$$E_t\left[y^r_{n-1,t+1} - y^r_{nt}\right] = \frac{1}{n-1}\left(y^r_{nt} - y^r_{1t}\right) - \frac{1}{n-1}E_t\left[r^r_{n,t+1} - y^r_{1t}\right] \tag{42}$$

For nominal bonds, the analogous equation is

$$E_t\left[y^\$_{n-1,t+1} - y^\$_{nt}\right] = \frac{1}{n-1}\left(y^\$_{nt} - y^\$_{1t}\right) - \frac{1}{n-1}E_t\left[r^\$_{n,t+1} - y^\$_{1t}\right] \tag{43}$$

Thus the coefficient of a regression of changes in yields on the scaled yield spread produces a coefficient of one only if risk premia on bonds are constant. As found by Campbell and Shiller, the data coefficients are not only less than one, they are negative, indicating risk premia on bonds that strongly vary over time.

As Table 8 shows, the model captures the failure of the expectations hypothesis seen in the data. Coefficients β_n are negative for all maturities. Risk premia on bonds are time-varying in the model because of the time-varying price of risk x_t. Thus the model can parsimoniously explain both predictability in stock returns, and time-variation in risk premia on long-term bonds.[7]

Using the model, it is possible to write the coefficients β_n in terms of more fundamental quantities. This sheds light on the aspects of the model that lead to the failure of the expectations hypothesis, as well as tensions inherent in the model. For real bonds, by definition

$$\beta_n = \frac{\text{Cov}(y^r_{n-1,t+1} - y^r_n, y^r_{nt} - y^r_{1t})}{\text{Var}(y^r_{n-1,t+1} - y^r_{1t})}(n-1)$$

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[7]The failure of the expectations hypothesis is not as extreme as that seen in the data. This reflects the difficulty that models with a single homoscedastic factor have in matching these data. Indeed, Dai and Singleton (2002) find, within the affine class, only a model with three factors driving the price of risk is capable of fully matching the failure of the expectations hypothesis. In contrast, a single-factor model that allows for significant heteroscedasticity in the state variable can successfully match these data (Wachter (2006)). It is also possible that part of the deviation in the data is reflective of a peso problem (Bekaert, Hodrick, and Marshall (2001)).

Substituting in for changes in yields from (42), and noting that time-$(t + 1)$ shocks have zero correlation with time-t yields, we have

$$
\begin{aligned}
\beta_n &= \frac{\text{Cov}(y_{nt}^r - y_{1t}^r - E_t\left[r_{n,t+1}^r - y_{1t}^r\right], y_{nt}^r - y_{1t}^r)}{\text{Var}(y_{n-1,t+1}^r - y_{1t}^r)} \\
&= 1 - \boldsymbol{\sigma}_{(n)}^r \boldsymbol{\sigma}_d' \frac{\text{Cov}(x_t, y_{nt}^r - y_{1t}^r)}{\text{Var}(y_{n-1,t+1}^r - y_{1t}^r)},
\end{aligned}
\tag{44}
$$

where the second line follows from (18). If x_t were constant, then the covariance term in this expression would be zero and $\beta_n = 1$, its value implied by the expectations hypothesis. The deviation from the expectations hypothesis depends on two quantities. The first is $\boldsymbol{\sigma}_{(n)}^r \boldsymbol{\sigma}_d'$. This is the covariance between bond returns and fundamentals, and directly determines the risk premium on the bond as indicated by (18). The greater are risk premia on bonds, the greater the deviation from the expectations hypothesis. The second term is the coefficient from a regression of x_t on the yield spread. The more risk premia covary with yield spreads, then, the greater the deviation from the expectations hypothesis. Similar reasoning leads to the formula for nominal bonds:

$$
\beta_n = 1 - \boldsymbol{\sigma}_{(n)}^\$ \boldsymbol{\sigma}_d' \frac{\text{Cov}(x_t, y_{nt}^\$ - y_{1t}^\$)}{\text{Var}(y_{n-1,t+1}^\$ - y_{1t}^\$)}.
\tag{45}
$$

Figure 5 displays $\boldsymbol{\sigma}_{(n)} \boldsymbol{\sigma}_d'$, $\text{Cov}(x_t, y_{nt} - y_{1t})/\text{Var}(y_{n-1,t+1} - y_{1t})$, and β_n for real and nominal bonds.[8] As Panel A shows, the terms $\boldsymbol{\sigma}_{(n)} \boldsymbol{\sigma}_d'$ are increasing in maturity, reflecting the fact that risk premia increase in maturity and that the term spread is upward-sloping. Risk premia are greater for nominal bonds then for real bonds, and increase faster in the maturity. Despite this, as shown in Panel C, the model implies a greater deviation from the expectations hypothesis for real bonds than for nominal bonds. Moreover, the model predicts coefficients that are roughly constant in maturity over the range of zero to 5 years, while risk premia are upward sloping. The reason is that the upward slope for risk premia is canceled out by a downward slope in $\text{Cov}(x_t, y_{nt} - y_{1t})/\text{Var}(y_{n-1,t+1} - y_{1t})$, which results from the mean-reverting nature of x_t. Moreover, nominal bonds, whose yields are driven by expected inflation as well as by discount rates, have lower values of $\text{Cov}(x_t, y_{nt} - y_{1t})/\text{Var}(y_{n-1,t+1} - y_{1t})$. This explains why the model produces a less dramatic failure of the expectations hypothesis for nominal bonds, despite their higher risk premia.

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[8]While (42) and (43) can be interpreted at any frequency and are run at an annual frequency in the data and the model for Table 8, (44) and (45) require that the frequency be the same as the frequency at which the model is simulated, namely, quarterly. The implied differences for the coefficients β_n are very slight.

Finally, we ask whether the model can explain the findings of Cochrane and Piazzesi (2005). Cochrane and Piazzesi regress annual excess bond returns on a linear combination of forward rates, where the forward rate for loans between periods $t + n$ and $t + n + 1$ is defined as

$$f_{nt}^{\pi} = \log P_{n-1,t}^{\pi} - \log P_{nt}^{\pi}.$$

(in what follows, we refer to n as the forward rate maturity). Cochrane and Piazzesi show that the regression coefficients on the forward rates form a tent-shape pattern as a function of maturity (see also Stambaugh (1988)). Moreover, they show that a single linear combination of forward rates has substantial predictive power for bond returns across maturities.

These results offer support for our model's assumptions in that they imply that a single predictive factor drives much of the predictability in bond returns. In our model, that factor is represented by the latent variable x_t. Forward rates, like bond prices, are linear combinations of factors; therefore some linear combination of forward rates will uncover x_t. The model therefore predicts that some linear combination of forward rates will be the best predictor of bond returns, and that the regression coefficients for bonds of various maturities should be the same up to a constant of proportionality (as the true premia are all proportional to x_t). Given the 3-factor affine structure of the model, it is straightforward to solve for the linear combination of any three forward yields that is proportional to x_t. For example, the linear combination

$$-\phi_q^2 \phi_r^2 f_{1t} + (\phi_q^2 + \phi_r^2) f_{3t} - f_{5t} \tag{46}$$

of 1, 3 and 5 period forwards is perfectly proportional to x_t and is therefore the most powerful variable to predict excess bond holding returns. Consistent with Cochrane and Piazzesi (2005), the coefficients in (46) have a "tent" $(- + -)$ pattern. Moreover, the coefficients depend only on the persistence parameters of the state variables r_t^f and q_t. Intuitively, forward rates depend on all state variables but forwards of different maturities are affected differently by the state variables because of their different degrees of persistence. Thus the linear combination of forwards that is independent of r_t^f and q_t is a function of ϕ_r and ϕ_q.

Equation (46) holds on the same data frequency that is used for calibration but does not hold exactly in time-aggregated data. Since the model is simulated on a quarterly frequency and aggregated to annual data, we replicate the Cochrane and Piazzesi (2005) analysis in our simulated

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data. We report results for forward rates with $n = 1, 3$ and 5 years but the results are robust to alternative choices. Figure 6 shows the regression coefficients as a function of the forward rate maturity. As this figure shows, the model reproduces the tent shape in regression coefficients. [9]

3.4 The Cross-section of Equities

This section shows the implications of the model for portfolios formed by sorting on price ratios. As in Menzly, Santos, and Veronesi (2004) and we exogenously specify a share process for cash flows on long-lived assets. For each year of simulated data, we sort these assets into deciles formed on the ratio of price to fundamentals and form portfolios of the assets within each decile. We then calculate returns over the following year. This follows the procedure used in empirical studies of the cross section (e.g. Fama and French (1992)). We then perform statistical analysis on the portfolio returns.

We specify our share process so that assets pay a nonzero dividend at each time, so that the total dividends sum up to the aggregate dividend of the market, and so that the cross-sectional distribution of dividends, returns, and price ratios is stationary. The continuous-time framework of Menzly, Santos, and Veronesi (2004) allows the authors to specify the share process as stochastic, and yet keep shares between zero and one. This is more difficult in discrete time, and for this reason we adopt the simplifying assumption that the share process is deterministic. We assume the same process as in Lettau and Wachter (2007): shares grow at a constant rate of 5% per quarter for 100 quarters, and then shrink at the same rate for the next 100 quarters. Lettau and Wachter show that these parameters imply a cross-sectional distribution of dividend and earnings growth similar to that in the data.

At the start of each year in the simulation, we sort firms into deciles based on their price-dividend ratio. We then form equal-weighted portfolios of firms within each decile. As firms move through the life-cycle, they slowly shift (on average) from the growth category to the value category, and then revert back to the growth category. Having sorted firms into portfolios, we compute statistics on portfolio returns over the subsequent year.

Panel A of Table 9 shows moments implied by the model. We compute the expected excess

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[9]The regression coefficients are larger in magnitude than those shown in Cochrane and Piazzesi (2005); this occurs because the correlation between bond returns in our model is greater than that in the data.

return, the volatility of the excess return and the Sharpe ratio. We also compute the abnormal return relative to the CAPM (α_i), and the coefficient on the market portfolio (β_i) from a time series regression of expected excess portfolio returns on expected excess market returns. Panel B shows counterparts from the data when portfolios are formed on the book-to-market ratio. Lettau and Wachter (2007) show that very similar results occur when portfolios are formed on earnings-to-price or cash-flow-to-price ratios.

Comparing the first line of Panel A with that of Panel B shows that the model is capable of matching the spread between expected returns on value and growth stocks. In both the model and the data, the expected excess return increases from about 6% per annum for the extreme growth portfolio to about 11% per annum for the extreme value portfolio. Comparing the second line of Panel A with that of Panel B shows that, in the model, the risk of value stocks is lower than that of growth stocks, just as in the data. Sharpe ratios increase from about 0.3 for the extreme growth portfolio to about 0.6 for the extreme value portfolio.

More importantly, the model is able to match the value *puzzle*. Even though the model predicts that value stocks have high expected returns, value stocks in the model have lower CAPM βs than growth stocks. The CAPM α in the model is -2.3% per annum for the extreme growth portfolio and rises to 3.33% per annum for the extreme value portfolio. The corresponding numbers in the data are -1.66% per annum and 3.97% per annum.

These results for value and growth stocks may at first seem counter-intuitive, especially given the implications of the model for the term structure of interest rates. The term structure results in the previous section show that long-run assets require higher expected returns than short-run assets. The results in this section show that the opposite is true for equities. For equities, it is the short-run assets that require high expected returns.

The model resolves this tension between the downward sloping term structure of equities and the upward-sloping term structure of interest rates by the dividend process, the inflation process, and the price-of-risk process x. As implied by the data, expected dividend growth is negatively correlated with realized dividend growth. This makes growth stocks a hedge and reduces their risk premium relative to what would be the case if, say, expected inflation were constant. Moreover, expected inflation is negatively correlated with realized dividend growth. This makes long-term nominal bonds riskier than short-term nominal bonds and riskier than real bonds.

The prices of inflation and dividend risks are important for accounting for the combined behavior of equities and bonds. However, they are not sufficient. As the discussion in Section 2.4 indicates, characteristics of the price-of-risk process x are also crucial. Because equities carry a higher risk premium than bonds, they are more sensitive to changes in x in the sense that a greater proportion of their variance comes from x than from r^f as compared to both real and nominal bonds. In our specification, variation in the price of risk is itself unpriced. This implies variability in returns on growth stocks (on account of duration), but, at the same time, low expected returns because this variability comes in the form of risk that the representative investor does not mind bearing.

4   Conclusion

This paper has shown that properties of the cross-section of returns, the aggregate market and the term structure of interest rates can all be understood within a single framework. We introduced a parsimonious model for the pricing kernel capable of accounting for the behavior of value and growth stocks, nominal bonds, and the aggregate market. At the root of the model are dividend, inflation, and interest rate processes calibrated to match their counterparts in the data. Time-varying preferences for risk, modeled using a first-order autoregressive process for the price of risk, capture the observed volatility in equity returns and bond yields, as well as time-varying risk premia in the equity and the bond market.

Our model highlights a challenge for any model that attempts to explain both bonds and the cross-section of equities. The upward-sloping yield curve for bonds indicates that investors require compensation in the form of a positive risk premium for holding high-duration assets. However, data on value and growth stocks imply the opposite: investors require compensation for holding value stocks, which are short-horizon equity. Our model addresses this tension by specifying a real riskfree rate that is negatively correlated with fundamentals and a price of risk shock that has zero correlation with fundamentals. We hope that future work will suggest microeconomic foundations for these specifications.

Appendix

A General Solution

In this section, we assume that the vector of state variables $\boldsymbol{H}_t$ follows the process (1), that the riskfree rate equals (6), that inflation follows the process (4) and that dividend growth follows (2). The variable x_t is given by (7). We assume a slightly more general form for the intertemporal marginal rate of substitution:

$$M_{t+1} = \exp\left\{-r^f_{t+1} - \frac{1}{2}\|\boldsymbol{\lambda}\|^2 x_t^2 - x_t \boldsymbol{\lambda}' \boldsymbol{\epsilon}_{t+1}\right\}. \tag{A.1}$$

The price of risk is therefore $x_t \boldsymbol{\lambda}$. In the main text, we impose the restriction $\boldsymbol{\lambda} = \boldsymbol{\sigma}'_d$. The more general formulas that follow are convenient in that they allow for a separation between the role of $\boldsymbol{\sigma}_d$ as the loadings on shocks for dividend growth and as the price of risk.

We describe the solution method for the case of zero-coupon equity. We conjecture a solution (14) to the recursion (13). Expanding out the exponential in the recursion implies

$$E_t\left[\exp\left\{-\alpha_0 - \boldsymbol{\alpha}'\boldsymbol{H}_t - \frac{1}{2}(\xi_0 + \boldsymbol{\xi}'\boldsymbol{H}_t)^2\|\boldsymbol{\lambda}\|^2 - (\xi_0 + \boldsymbol{\xi}'\boldsymbol{H}_t)\boldsymbol{\lambda}'\boldsymbol{\epsilon}_{t+1} + \delta_0 + \boldsymbol{\delta}'\boldsymbol{H}_t + \boldsymbol{\sigma}_d\boldsymbol{\epsilon}_{t+1} + \right.\right.$$
$$\left.\left. A^d_{n-1} + \boldsymbol{B}^d_{n-1}\left(\boldsymbol{\Theta}_0 + \boldsymbol{\Theta}\boldsymbol{H}_t + \boldsymbol{\sigma}_H\boldsymbol{\epsilon}_{t+1}\right)\right\}\right] = \exp\left\{A^d_n + \boldsymbol{B}^d_n\boldsymbol{H}_t\right\}.$$

It follows from properties of the lognormal distribution that

$$\exp\left\{-\alpha_0 - \boldsymbol{\alpha}'\boldsymbol{H}_t - \frac{1}{2}(\xi_0 + \boldsymbol{\xi}'\boldsymbol{H}_t)^2\|\boldsymbol{\lambda}\|^2 + \delta_0 + \boldsymbol{\delta}'\boldsymbol{H}_t + A^d_{n-1} + \boldsymbol{B}^d_{n-1}\left(\boldsymbol{\Theta}_0 + \boldsymbol{\Theta}\boldsymbol{H}_t\right) + \right.$$
$$\left. \frac{1}{2}\left(\boldsymbol{\sigma}_d - (\xi_0 + \boldsymbol{\xi}'\boldsymbol{H}_t)\boldsymbol{\lambda}' + \boldsymbol{B}^d_{n-1}\boldsymbol{\sigma}_H\right)\left(\boldsymbol{\sigma}_d - (\xi_0 + \boldsymbol{\xi}'\boldsymbol{H}_t)\boldsymbol{\lambda}' + \boldsymbol{B}^d_{n-1}\boldsymbol{\sigma}_H\right)'\right\} =$$
$$\exp\left\{A^d_n + \boldsymbol{B}^d_n\boldsymbol{H}_t\right\}.$$

Matching coefficients implies:[10]

$$\boldsymbol{B}^d_n = -\boldsymbol{\alpha}' + \boldsymbol{\delta}' + \boldsymbol{B}^d_{n-1}\boldsymbol{\Theta} - (\boldsymbol{\sigma}_d + \boldsymbol{B}^d_{n-1}\boldsymbol{\sigma}_H)\boldsymbol{\lambda}\boldsymbol{\xi}' \tag{A.2}$$
$$A^d_n = -\alpha_0 + \delta_0 + A^d_{n-1} + \boldsymbol{B}^d_{n-1}\boldsymbol{\Theta}_0 - (\boldsymbol{\sigma}_d + \boldsymbol{B}^d_{n-1}\boldsymbol{\sigma}_H)\boldsymbol{\lambda}\xi_0 +$$
$$\frac{1}{2}\boldsymbol{\sigma}_d\boldsymbol{\sigma}'_d + \boldsymbol{B}^d_{n-1}\boldsymbol{\sigma}_H\boldsymbol{\sigma}'_d + \frac{1}{2}\boldsymbol{B}^d_{n-1}\boldsymbol{\sigma}_H\boldsymbol{\sigma}'_H\boldsymbol{B}^{d'}_{n-1}, \tag{A.3}$$

with $\boldsymbol{B}^d_0 = \boldsymbol{0}_{1\times m}$ and $A^d_0 = 0$. Note that the terms that are quadratic in $\boldsymbol{H}_t$ cancel.

For real bonds, we note that (10) takes the same form as (13), except that there is no dividend growth term. We can therefore apply (A.2) and (A.3) directly, replacing δ_0 with 0, $\boldsymbol{\delta}_1$ with $\boldsymbol{0}_{m\times 1}$ and $\boldsymbol{\sigma}_d$ with

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[10]Because $\boldsymbol{\xi}'\boldsymbol{H}_t$ and $\boldsymbol{\lambda}'(\boldsymbol{\sigma}_d + \boldsymbol{B}^d_{n-1}\boldsymbol{\sigma}_H)'$ are each scalars,

$$\boldsymbol{\xi}'\boldsymbol{H}_t\boldsymbol{\lambda}'(\boldsymbol{\sigma}_d + \boldsymbol{B}^d_{n-1}\boldsymbol{\sigma}_H)' = \boldsymbol{\lambda}'(\boldsymbol{\sigma}_d + \boldsymbol{B}^d_{n-1}\boldsymbol{\sigma}_H)'\boldsymbol{\xi}'\boldsymbol{H}_t$$
$$= (\boldsymbol{\sigma}_d + \boldsymbol{B}^d_{n-1}\boldsymbol{\sigma}_H)\boldsymbol{\lambda}\boldsymbol{\xi}'\boldsymbol{H}_t.$$

$\mathbf{0}_{1\times(m+2)}$ to obtain

$$\boldsymbol{B}_n^r = -\boldsymbol{\alpha}' + \boldsymbol{B}_{n-1}^r\boldsymbol{\Theta} - \boldsymbol{B}_{n-1}^r\boldsymbol{\sigma}_H\boldsymbol{\lambda}\boldsymbol{\xi}' \tag{A.4}$$

$$A_n^r = -\alpha_0 + A_{n-1}^r + \boldsymbol{B}_{n-1}^r\boldsymbol{\Theta}_0 - \boldsymbol{B}_{n-1}^r\boldsymbol{\sigma}_H\boldsymbol{\lambda}\xi_0 + \frac{1}{2}\boldsymbol{B}_{n-1}^r\boldsymbol{\sigma}_H\boldsymbol{\sigma}_H'\boldsymbol{B}_{n-1}^{r\,\prime}, \tag{A.5}$$

with $\boldsymbol{B}_0^r = \mathbf{0}_{1\times m}$ and $A_0^r = 0$.

For nominal bonds, the Euler equation (8) implies that

$$E_t\left[M_{t+1}\frac{\Pi_t}{\Pi_{t+1}}P_{n-1,t+1}^\pi\Pi_{t+1}\right] = P_{nt}^\pi\Pi_t$$

This recursion has a form that is similar to (13), with $-\eta_0$ taking the place of δ_0, $-\boldsymbol{\eta}$ taking the places of $\boldsymbol{\delta}$ and $-\boldsymbol{\sigma}_\pi$ taking the place of $\boldsymbol{\sigma}_d$. We apply the same equation again to obtain

$$\boldsymbol{B}_n^\pi = -\boldsymbol{\alpha}' - \boldsymbol{\eta}' + \boldsymbol{B}_{n-1}^\pi\boldsymbol{\Theta} - (-\boldsymbol{\sigma}_\pi + \boldsymbol{B}_{n-1}^\pi\boldsymbol{\sigma}_H)\boldsymbol{\lambda}\boldsymbol{\xi}' \tag{A.6}$$

$$\begin{aligned}A_n^\pi = &-\alpha_0 - \eta_0 + A_{n-1}^\pi + \boldsymbol{B}_{n-1}^\pi\boldsymbol{\Theta}_0 - (-\boldsymbol{\sigma}_\pi + \boldsymbol{B}_{n-1}^\pi\boldsymbol{\sigma}_H)\boldsymbol{\lambda}\xi_0 + \\ &\frac{1}{2}\boldsymbol{\sigma}_\pi\boldsymbol{\sigma}_\pi' - \boldsymbol{B}_{n-1}^\pi\boldsymbol{\sigma}_H\boldsymbol{\sigma}_\pi' + \frac{1}{2}\boldsymbol{B}_{n-1}^\pi\boldsymbol{\sigma}_H\boldsymbol{\sigma}_H'\boldsymbol{B}_{n-1}^{\pi\,\prime},\end{aligned} \tag{A.7}$$

and $\boldsymbol{B}_0^\pi = \mathbf{0}_{1\times m}$ and $A_0^\pi = 0$.

B Special Case

We now specialize the above formulas under the assumptions of Section 2.3. We continue to assume that the intertemporal marginal rate of substitution is defined by (A.1). Setting $\boldsymbol{\lambda} = \boldsymbol{\sigma}_d'$ generates the results in Section 2.3.

For real bonds, (A.4) and (A.5) imply that

$$B_{zn}^r = B_{z,n-1}^r\phi_z \tag{A.8}$$

$$B_{qn}^r = B_{q,n-1}^r\phi_q \tag{A.9}$$

$$B_{rn}^r = -1 + B_{r,n-1}^r\phi_r \tag{A.10}$$

$$B_{xn}^r = B_{x,n-1}^r\phi_x - \boldsymbol{\sigma}_{(n)}^r\boldsymbol{\lambda} \tag{A.11}$$

$$A_n^r = -\bar{r}^f + A_{n-1}^r - \boldsymbol{\sigma}_{(n)}^r\boldsymbol{\lambda}\bar{x} + \frac{1}{2}\|\boldsymbol{\sigma}_{(n)}^r\|^2, \tag{A.12}$$

where $\boldsymbol{\sigma}_{(n)}^r$ is defined in Section 2.2 as the vector of loadings on the shocks for the return on the n-period real bond. Under the assumptions in Section 2.3,

$$\boldsymbol{\sigma}_{(n)}^r = B_{r,n-1}^r\boldsymbol{\sigma}_r + B_{q,n-1}^r\boldsymbol{\sigma}_q + B_{z,n-1}^r\boldsymbol{\sigma}_z + B_{x,n-1}^r\boldsymbol{\sigma}_x.$$

The boundary conditions are $B_{zn}^r = B_{qn}^r = B_{rn}^r = B_{xn}^r = A_n^r = 0$. Equations (A.8) and (A.9) together with the boundary conditions imply that $B_{zn}^r = B_{qn}^r = 0$. The solution to (A.10) is given in the main text. The solution to (A.11) is

$$B_{xn}^r = \frac{\boldsymbol{\sigma}_r\boldsymbol{\lambda}}{1 - \phi_r}\frac{1 - \phi_\lambda^n}{1 - \phi_\lambda} - \frac{\boldsymbol{\sigma}_r\boldsymbol{\lambda}}{1 - \phi_r}\frac{\phi_r^n - \phi_\lambda^n}{\phi_r - \phi_\lambda}, \tag{A.13}$$

where

$$\phi_\lambda = \phi_x - \sigma_x\lambda.$$

In the case of equities, (A.2) and (A.3) imply that

$$
\begin{align}
B_{zn}^d &= 1 + B_{z,n-1}^d\phi_z \tag{A.14}\\
B_{qn}^d &= B_{q,n-1}^d\phi_q \tag{A.15}\\
B_{rn}^d &= -1 + B_{r,n-1}^d\phi_r \tag{A.16}\\
B_{xn}^d &= B_{x,n-1}^d\phi_x - \sigma_{(n)}^d\lambda \tag{A.17}\\
A_n^d &= -\bar{r}^f + g + A_{n-1}^d - \sigma_{(n)}^d\lambda\bar{x} + \frac{1}{2}\|\sigma_{(n)}^d\|^2, \tag{A.18}
\end{align}
$$

where

$$\sigma_{(n)}^d = \sigma_d + B_{r,n-1}^d\sigma_r + B_{q,n-1}^d\sigma_q + B_{z,n-1}^d\sigma_z + B_{x,n-1}^d\sigma_x$$

is the vector of loadings on the shocks for the return on n-period zero-coupon equity as defined in Section 2.2. The boundary conditions are $B_{zn}^d = B_{qn}^d = B_{rn}^d = B_{xn}^d = A_n^d = 0$. Equation (A.15) together with the boundary condition implies that $B_{qn}^d = 0$. The solutions to (A.14) and (A.16) are given in the main text. The solution to (A.17) is

$$B_{xn}^d = \left(-\sigma_d\lambda + \frac{\sigma_r\lambda}{1-\phi_r} - \frac{\sigma_z\lambda}{1-\phi_z}\right)\frac{1-\phi_\lambda^n}{1-\phi_\lambda} - \frac{\sigma_r\lambda}{1-\phi_r}\frac{\phi_r^n-\phi_\lambda^n}{\phi_r-\phi_\lambda} + \frac{\sigma_z\lambda}{1-\phi_z}\frac{\phi_z^n-\phi_\lambda^n}{\phi_z-\phi_\lambda}. \tag{A.19}$$

In the case of nominal bonds, (A.6) and (A.7) imply that

$$
\begin{align}
B_{zn}^\pi &= B_{z,n-1}^\pi\phi_z \tag{A.20}\\
B_{qn}^\pi &= -1 + B_{q,n-1}^\pi\phi_q \tag{A.21}\\
B_{rn}^\pi &= -1 + B_{r,n-1}^\pi\phi_r \tag{A.22}\\
B_{xn}^\pi &= B_{x,n-1}^\pi\phi_x - \sigma_{(n)}^\pi\lambda \tag{A.23}\\
A_n^\pi &= -\bar{r}^f - \bar{q} + A_{n-1}^\pi - \sigma_{(n)}^\pi\lambda\bar{x} + \frac{1}{2}\|\sigma_{(n)}^\pi\|^2, \tag{A.24}
\end{align}
$$

where

$$\sigma_{(n)}^\pi = -\sigma_\pi + B_{r,n-1}^\pi\sigma_r + B_{q,n-1}^\pi\sigma_q + B_{x,n-1}^\pi\sigma_x$$

is the vector of loadings on the shocks for the return on the n-period nominal bond as defined in Section 2.2. The boundary conditions are $B_{zn}^\pi = B_{qn}^\pi = B_{rn}^\pi = B_{xn}^\pi = A_n^\pi = 0$. Equation (A.20) together with the boundary condition implies that $B_{zn}^\pi = 0$. The solutions to (A.21) and (A.22) are given in the main text. The solution to (A.23) is

$$B_{xn}^\pi = \left(\sigma_\pi\lambda + \frac{\sigma_r\lambda}{1-\phi_r} + \frac{\sigma_q\lambda}{1-\phi_q}\right)\frac{1-\phi_\lambda^n}{1-\phi_\lambda} - \frac{\sigma_r\lambda}{1-\phi_r}\frac{\phi_r^n-\phi_\lambda^n}{\phi_r-\phi_\lambda} - \frac{\sigma_q\lambda}{1-\phi_q}\frac{\phi_q^n-\phi_\lambda^n}{\phi_q-\phi_\lambda}. \tag{A.25}$$

C Convergence

This Appendix derives conditions on the convergence of the price-dividend ratio. Let $\boldsymbol{K}_1 = \boldsymbol{\Theta} - \boldsymbol{\sigma}_H \boldsymbol{\lambda} \boldsymbol{\xi}'$ and $\boldsymbol{K}_2 = -\boldsymbol{\alpha}' + \boldsymbol{\delta}' - \boldsymbol{\sigma}_d \boldsymbol{\lambda} \boldsymbol{\xi}'$. Then (A.2) can be rewritten as

$$\boldsymbol{B}_n^d = \boldsymbol{B}_{n-1}^d \boldsymbol{K}_1 + \boldsymbol{K}_2.$$

The limit of $\boldsymbol{B}_n^d$ as n goes to infinity is the fixed point of this equation. A necessary and sufficient condition for a fixed point to exist is that the eigenvalues of $\boldsymbol{K}_1$ have absolute value less than 1. In this case $\boldsymbol{I}_m - \boldsymbol{K}_1$ is invertible, and the fixed point is

$$\overline{\boldsymbol{B}} = \boldsymbol{K}_2 (\boldsymbol{I}_m - \boldsymbol{K}_1)^{-1}.$$

It follows that for N sufficiently large,

$$A_n^d \approx \bar{A} n + \text{constant},$$

for $n \geq N$, where

$$\bar{A} = -\alpha_0 + \delta_0 + \overline{\boldsymbol{B}} \boldsymbol{\Theta}_0 - (\boldsymbol{\sigma}_d + \overline{\boldsymbol{B}} \boldsymbol{\sigma}_H) \boldsymbol{\lambda} \xi_0 + \frac{1}{2} \boldsymbol{\sigma}_d \boldsymbol{\sigma}_d' + \overline{\boldsymbol{B}} \boldsymbol{\sigma}_H \boldsymbol{\sigma}_d' + \frac{1}{2} \overline{\boldsymbol{B}} \boldsymbol{\sigma}_H \boldsymbol{\sigma}_H' \overline{\boldsymbol{B}}'.$$

Moreover,

$$\sum_{n=N}^{\infty} \exp\left\{ A_n^d + \boldsymbol{B}_n^d \boldsymbol{H}_t \right\} \approx \exp\left\{ \text{constant} + \overline{\boldsymbol{B}} \boldsymbol{H}_t \right\} \sum_{n=N}^{\infty} \exp\left\{ \bar{A} n \right\}.$$

Therefore necessary and sufficient conditions for convergence are that $\boldsymbol{K}_1$ has eigenvalues inside the unit circle, and that $\bar{A}$ is negative.

D Cochrane-Piazzesi Regressions

Up to an additive constant, the n-period forward rate can be written as $f_{nt} = -\phi_r^{n-1}(r_t^f - \bar{r}_t^f) - \phi_q^{n-1}(q_t - \bar{q}_t) - (B_{n-1,x}^\pi - B_{nx}^\pi)(x_t - \bar{x})$. Hence

$$\begin{pmatrix} f_{it} \\ f_{jt} \\ f_{kt} \end{pmatrix} = \begin{pmatrix} -\phi_r^{i-1} & -\phi_q^{i-1} & -(B_{i-1,x}^\pi - B_{ix}^\pi) \\ -\phi_r^{j-1} & -\phi_q^{j-1} & -(B_{j-1,x}^\pi - B_{jx}^\pi) \\ -\phi_r^{k-1} & -\phi_q^{k-1} & -(B_{k-1,x}^\pi - B_{kx}^\pi) \end{pmatrix} \begin{pmatrix} r_t^f \\ q_t \\ x_t \end{pmatrix},$$

or $\boldsymbol{f}_t = \boldsymbol{\Theta} \ (r_t^f, q_t, x_t)'$, where $\boldsymbol{f}_t$ is a vector of any three forwards rates with maturities i, j and k, and $\boldsymbol{\Theta}$ is a 3x3 matrix of known coefficients. Thus $x_t = \boldsymbol{e}_3' \boldsymbol{\Theta}^{-1} \boldsymbol{f}_t$ where $\boldsymbol{e}_3 = (0 \ 0 \ 1)'$. Since the first two columns of $\boldsymbol{\Theta}$ only depend on ϕ_r and ϕ_q, it follows that x_t is proportional to a linear combination of the three forward rates in $\boldsymbol{f}_t$ with coefficients that depend only on ϕ_r and ϕ_q. In other words, any 3x1 vector $\boldsymbol{\theta}$ with the property

$$\boldsymbol{\theta}' \begin{pmatrix} -\phi_r^{i-1} & -\phi_q^{i-1} \\ -\phi_r^{j-1} & -\phi_q^{j-1} \\ -\phi_r^{k-1} & -\phi_q^{k-1} \end{pmatrix} = (0 \ 0)$$

implies that $\boldsymbol{\theta}' \boldsymbol{f}_t$ is independent of r_t^f and q_t and thus proportional to x_t. Note that $\boldsymbol{\theta}$ is not identified up to a proportionality factor. (46) shows $\boldsymbol{\theta}$ for $\boldsymbol{f}_t = (f_{1t}, f_{3t}, f_{5t})'$ where the third element of θ is normalized to -1. Hence $e_3' \boldsymbol{\Theta}^{-1} = G\boldsymbol{\theta}'$ where G is a scalar. Since the expected excess bond return is linear in x_t (see (24-25)), we can solve for the Cochrane-Piazzesi expression that links risk premia to forward rates in closed form:

$$E_t[r_{n,t+1}^{\$} - r_{t+1}^f] + \frac{1}{2}\sigma_{\$,(n)}^2 = \boldsymbol{\sigma}_{(n)}^{\$}\boldsymbol{\sigma}_d' G(\boldsymbol{\theta}' \boldsymbol{f_t}). \tag{A.26}$$

$\boldsymbol{\theta}' \boldsymbol{f_t}$ corresponds to the common "tent" shaped factor of forwards (given a normalization) while $\boldsymbol{\sigma}_{(n)}^d \boldsymbol{\sigma}_d' G$ is the loading of the n-period bond. As noted above $\boldsymbol{\theta}$ depends only the persistence parameters ϕ_r and ϕ_q. The term $\boldsymbol{\sigma}_{(n)}^{\$}\boldsymbol{\sigma}_\pi' G$ is function of the other model parameters.

Let $J_n = \boldsymbol{\sigma}_{(n)}^{\$}\boldsymbol{\sigma}_d' G$ and write (A.26) as

$$E_t[r_{n,t+1}^{\$} - r_{t+1}^f] + \frac{1}{2}\sigma_{\$,(n)}^2 = J_n \boldsymbol{\theta}' \boldsymbol{f_t} \tag{A.27}$$

and normalize $\boldsymbol{\theta}$ setting the third element to -1 as in (46). J_n can be compared to the corresponding OLS coefficients $\hat{J}_n$ in Cochrane-Piazzesi where the tent coefficients $\hat{\theta}$ are normalized in the same way (i.e. by setting the third element of the tent to -1). The $\boldsymbol{\theta}$ coefficients in the model are quite close to $\hat{\boldsymbol{\theta}}$ in the data, however the elements in J_n are much larger that the elements in $\hat{J}_n$ (in absolute value) as shown in Figure 6. Yet, it turns out that the volatility of fitted values of the Cochrane-Piazzesi regressions are close to the volatility of expected returns in the model, i.e. $J_n \boldsymbol{\theta}' \boldsymbol{f_t}$ and $\hat{J}_n \hat{\boldsymbol{\theta}}' \boldsymbol{f_t}$ have similar volatilities. This implies that our model generates variations in expected returns that are reasonable, however, $\boldsymbol{\theta}' \boldsymbol{f_t}$ is "too smooth" compared to $\hat{\boldsymbol{\theta}}' \boldsymbol{f_t}$ in the data. The reason is that forward rates in the model are more highly correlated than in the data causing the linear combination $\boldsymbol{\theta}' \boldsymbol{f_t}$ to be too smooth compared to $\hat{\boldsymbol{\theta}}' \boldsymbol{f_t}$ in the data.

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Table 1: State Variable Means and Autocorrelations

State variable	Unconditional Mean	Autocorrelation
Expected dividend growth z_t	2.28%	0.89
Expected inflation q_t	3.68%	0.81
Real riskfree rate r_t^f	1.76%	0.89
Price of risk $\sigma_d x_t$	0.85	0.85

Notes: The second column reports means of state variables; means of expected dividend growth, expected inflation, and the real riskfree rate are in annual terms (i.e. multiplied by 4). Autocorrelations for all state variables are in annual terms (i.e. raised to the 4th power). The model is simulated at a quarterly frequency.

Table 2: Conditional Cross-correlations of Shocks

Variable	Δd_t	$\Delta \pi_t$	z_t	q_t	r_t^f	x_t
Δd_t	1	0	-0.83	-0.20	-0.40	0
$\Delta \pi_t$		1	0	0.50	0	0.40
z_t			1	0	0	0.30
q_t				1	0	0.20
r_t^f					1	0
x_t						1

Notes: Conditional cross-correlations of shocks to dividend growth Δd_t, inflation $\Delta \pi_t$, expected dividend growth z_t, expected inflation q_t, the real continuously compounded riskfree rate r_t^f, and the price-of-risk variable x_t. The model is simulated at a quarterly frequency.

Table 3: Conditional Standard Deviations of Shocks

Variable	Δd_t	$\Delta \pi_t$	z_t	q_t	r_t^f	$\sigma_d x_t$
Conditional standard deviation	10.00	1.11	0.32	0.41	0.15	40.00

Notes: Conditional standard deviations of shocks in annual percentage terms (i.e. multiplied by 200.) The model is simulated at a quarterly frequency.

Table 4: Simulated Moments for the Aggregate Market

	Data	Model
$E(P/D)$	25.55	18.60
$\sigma(p-d)$	0.38	0.32
AC of $p-d$	0.87	0.84
$E[R^m - R^f]$	6.33%	8.45%
$\sigma(R^m - R^f)$	19.41%	19.09%
AC of $R^m - R^f$	0.03	-0.07
Sharpe ratio of market	0.33	0.44

Notes: Data are annual from 1890 to 2002. The model is simulated at a quarterly frequency and returns, dividends, and price ratios are aggregated up to an annual frequency. AC refers to the annual autocorrelation, P refers to the value of the aggregate market, D the aggregate dividend, R^m the annual return on the aggregate market and R^f the annual return on the riskfree rate.

Table 5: Long-Horizon Regressions: Excess Stock Returns

	Horizon in Years					
	1	2	4	6	8	10
$\sum_{i=1}^{h} r_{t+i}^{m} - r_{t+i}^{f} = \beta_0 + \beta_1(p_t - d_t) + \epsilon_t$						
β_1	-0.18	-0.32	-0.54	-0.71	-0.84	-0.92
R^2	0.11	0.19	0.29	0.36	0.38	0.39
$\sum_{i=1}^{h} r_{t+i}^{m} - r_{t+i}^{f} = \beta_0 + \beta_1(y_{5,t}^{\$} - y_{0.25,t}^{\$}) + \epsilon_t$						
β_1	3.40	6.12	10.40	13.60	15.85	18.00
R^2	0.07	0.13	0.21	0.26	0.27	0.27

Notes: Continuously compounded excess returns on the market portfolio are regressed on the lagged price-dividend ratio on the market (top panel) and on the lagged spread between the yield on the 5-year nominal bond and the 1-quarter nominal bond (bottom panel) in simulated data. Returns are calculated over horizons ranging from 1 to 10 years. The price-dividend ratio and yields are expressed in annual terms.

Table 6: Long-Horizon Regressions: Aggregate Dividend Growth

	Horizon in Years					
	1	2	4	6	8	10
$\sum_{i=1}^{h} \Delta d_{t+i} = \beta_0 + \beta_1 (p_t - d_t) + \epsilon_t$						
β_1	0.02	0.04	0.07	0.09	0.09	0.10
R^2	0.00	0.01	0.01	0.02	0.01	0.01
$\sum_{i=1}^{h} \Delta d_{t+i} = \beta_0 + \beta_1 (y_{5,t}^{\$} - y_{0.25,t}^{\$}) + \epsilon_t$						
β_1	0.47	0.89	1.64	1.64	2.75	2.50
R^2	0.00	0.01	0.01	0.01	0.02	0.02
$\sum_{i=1}^{h} \Delta d_{t+i} = \beta_0 + \beta_1 z_t + \epsilon_t$						
β_1	3.74	7.20	12.82	16.77	20.58	24.26
R^2	0.07	0.12	0.21	0.26	0.31	0.35

Notes: Aggregate dividend growth on the market portfolio is regressed on the lagged price-dividend ratio (top panel), on the lagged spread between the yield on the 5-year nominal bond and the 1-quarter nominal bond (middle panel), and on expected dividend growth (bottom panel) in simulated data. Dividend growth is calculated over horizons ranging from 1 to 10 years and is in real terms.

Table 7: Simulated Moments for Zero-Coupon Bond Yields

Maturity (yrs)	0.25	1	2	3	4	5
			Real bonds			
$E(y_{nt})$	1.74	1.89	2.07	2.23	2.38	2.52
$\sigma(y_{nt})$	1.22	1.18	1.13	1.10	1.09	1.07
$\mathrm{Corr}(y_{nt}, y_{n,t+1})$	0.88	0.88	0.88	0.88	0.88	0.87
			Nominal bonds			
$Ey_{nt}^{\$}$	5.32	5.66	6.06	6.41	6.73	7.00
$\sigma(y_{nt}^{\$})$	2.85	2.73	2.61	2.53	2.47	2.41
$\mathrm{Corr}(y_{nt}^{\$}, y_{n,t+1}^{\$})$	0.82	0.82	0.83	0.83	0.83	0.83
			Data			
$Ey_{nt}^{\$}$	5.30	5.48	5.70	5.87	5.99	6.07
$\sigma(y_{nt}^{\$})$	2.89	2.82	2.76	2.69	2.67	2.64
$\mathrm{Corr}(y_{nt}^{\$}, y_{n,t+1}^{\$})$	0.83	0.84	0.86	0.87	0.88	0.89

Notes: Each panel displays means, standard deviations, and annual autocorrelations of bond yields. The top two panels describe yields on zero-coupon bonds in simulated data. Yields on real bonds (expressed in real annual percentage terms) are denoted y_{nt}, where n denotes the maturity and t denotes time. Yields on nominal bonds (expressed in nominal annual percentage terms) are denoted $y_{nt}^{\$}$. Data on nominal zero-coupon bond yields, expressed in annual percentage terms, are annual and from 1952–2004.

Table 8: Long-Rate Regressions on Bond Yields

Maturity (yrs)	2	3	4	5
	Real bonds			
β_n	-0.76	-0.78	-0.80	-0.82
	Nominal bonds			
β_n	-0.44	-0.43	-0.41	-0.39
	Data			
β_n	-0.91	-1.49	-1.65	-1.74

Notes: Coefficients β_n from the regression

$$y^r_{n-1,t+1} - y^r_{nt} = \alpha_n + \beta_n \frac{1}{n-1} \left(y^r_{nt} - y^r_{1t}\right) + \text{error},$$

for real bonds and

$$y^\$_{n-1,t+1} - y^\$_{nt} = \alpha_n + \beta_n \frac{1}{n-1} \left(y^\$_{nt} - y^\$_{1t}\right) + \text{error}$$

for nominal bonds in simulated and historical data, where y^r_{nt} denotes the annual real yield on the n-year real bond and $y^\$_{nt}$ denotes in the annual nominal yield on the n-year nominal bond. Historical data are annual and from 1952–2004. The model is simulated at a quarterly frequency and aggregated to an annual frequency. The expectations hypothesis implies $\beta_n = 1$ for all n.

Table 9: Simulated Moments for Equity Portfolios

|	G				Growth to Value					V	V-G
Portfolio	1	2	3	4	5	6	7	8	9	10	10-1
					Model						
$ER^i - R^f$	6.05	6.23	6.51	6.92	7.44	8.05	8.70	9.31	9.85	10.74	4.69
$\sigma(R^i - R^f)$	19.17	19.44	19.67	19.79	19.69	19.27	18.50	17.60	17.00	17.23	7.42
Sharpe Ratio	0.32	0.32	0.33	0.35	0.38	0.42	0.47	0.53	0.58	0.62	0.63
α_i	-2.32	-2.27	-2.11	-1.79	-1.25	-0.48	0.51	1.57	2.45	3.33	5.65
β_i	0.99	1.01	1.02	1.03	1.03	1.01	0.97	0.92	0.88	0.88	-0.11
					B/M sorted portfolios						
$ER^i - R^f$	5.67	6.55	6.98	6.51	8.00	8.33	8.27	10.08	9.98	10.55	4.88
$\sigma(R^i - R^f)$	17.77	15.89	15.82	15.42	14.65	14.73	14.74	15.11	15.71	18.46	15.15
Sharpe Ratio	0.32	0.41	0.44	0.42	0.55	0.57	0.56	0.67	0.64	0.57	0.32
α_i	-1.66	-0.17	0.33	0.22	2.12	2.37	2.59	4.30	4.05	3.97	5.63
β_i	1.11	1.02	1.01	0.95	0.89	0.90	0.86	0.87	0.90	1.00	-0.11

Notes: For the top panel, firms in simulated data are sorted into deciles on their price-dividend ratios in each simulation year. Returns are calculated over the subsequent year. Intercepts and slope coefficients are from OLS time-series regressions of excess portfolio returns on the excess market return. Data are on monthly returns on book-to-market portfolios from 1952–2002. Data moments are annualized (multiplied by 12 in the case of means and intercepts and $\sqrt{12}$ in the case of standard deviations). Means, intercepts, and standard deviations are reported in percentage terms.

Figure 1: Model Solution
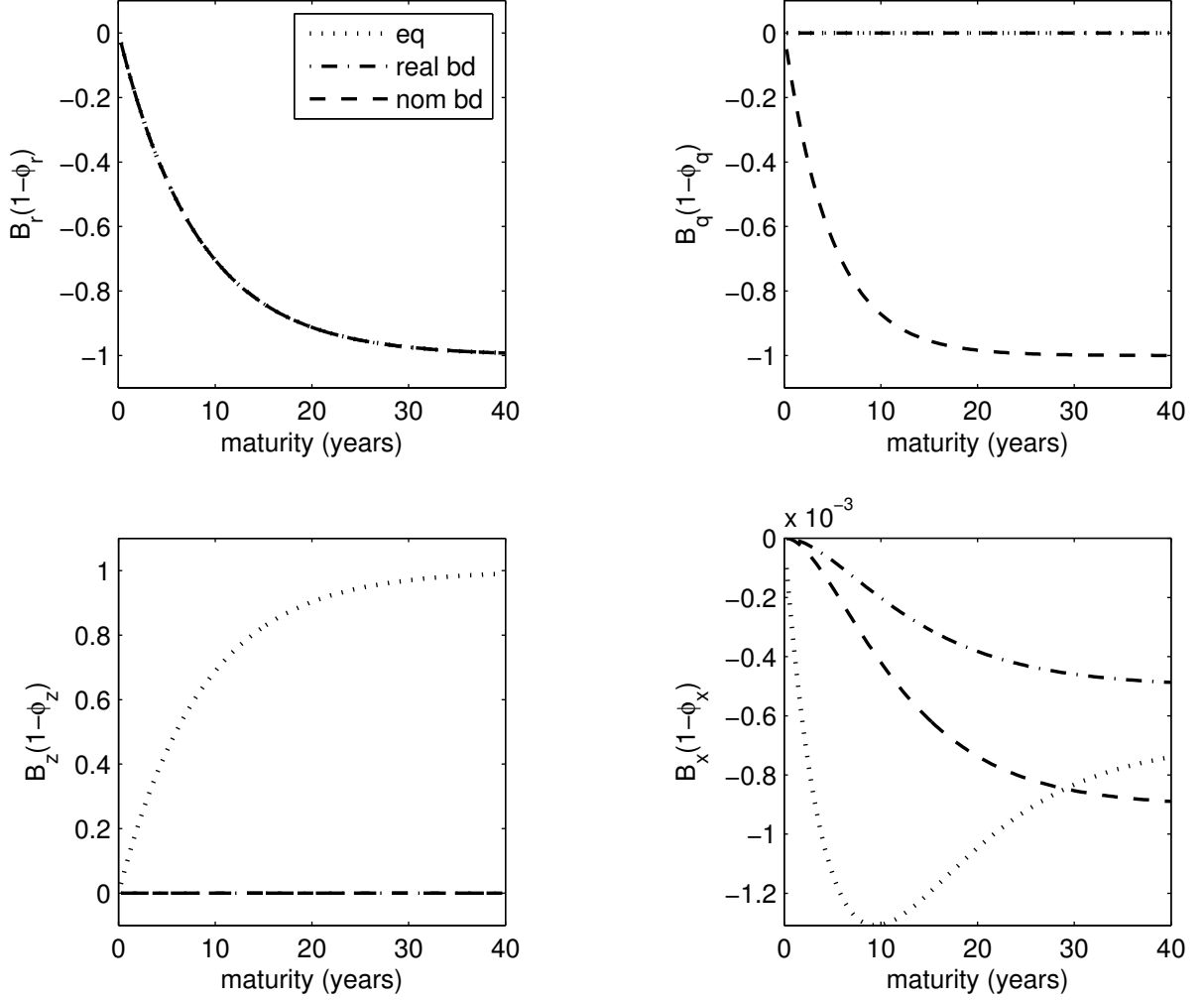
Notes: Solutions to B_{rn}, the sensitivity of prices to the real riskfree rate (top left); to B_{qn}, the sensitivity of prices to expected inflation (top right); to B_{zn}, the sensitivity of prices to expected dividend growth (bottom left); and to B_{xn}, the sensitivity of prices to the price of risk variable, where n is the maturity in years. Dotted lines denote the solutions for zero-coupon equity prices expressed in real terms, dashed-dotted lines denote the solutions for real bond prices expressed in real terms, dashed lines denote the solutions for nominal bond prices expressed in nominal terms. The solutions are scaled by the persistence ϕ of the variables. The solution for B_r is identical for all three asset classes. The solution for B_q is identical for equities and real bonds and equal to zero. The solution for B_z is identical for real and nominal bonds and equal to zero.

Figure 2: Ratios of Prices to Aggregate Dividends for Zero-Coupon Equity
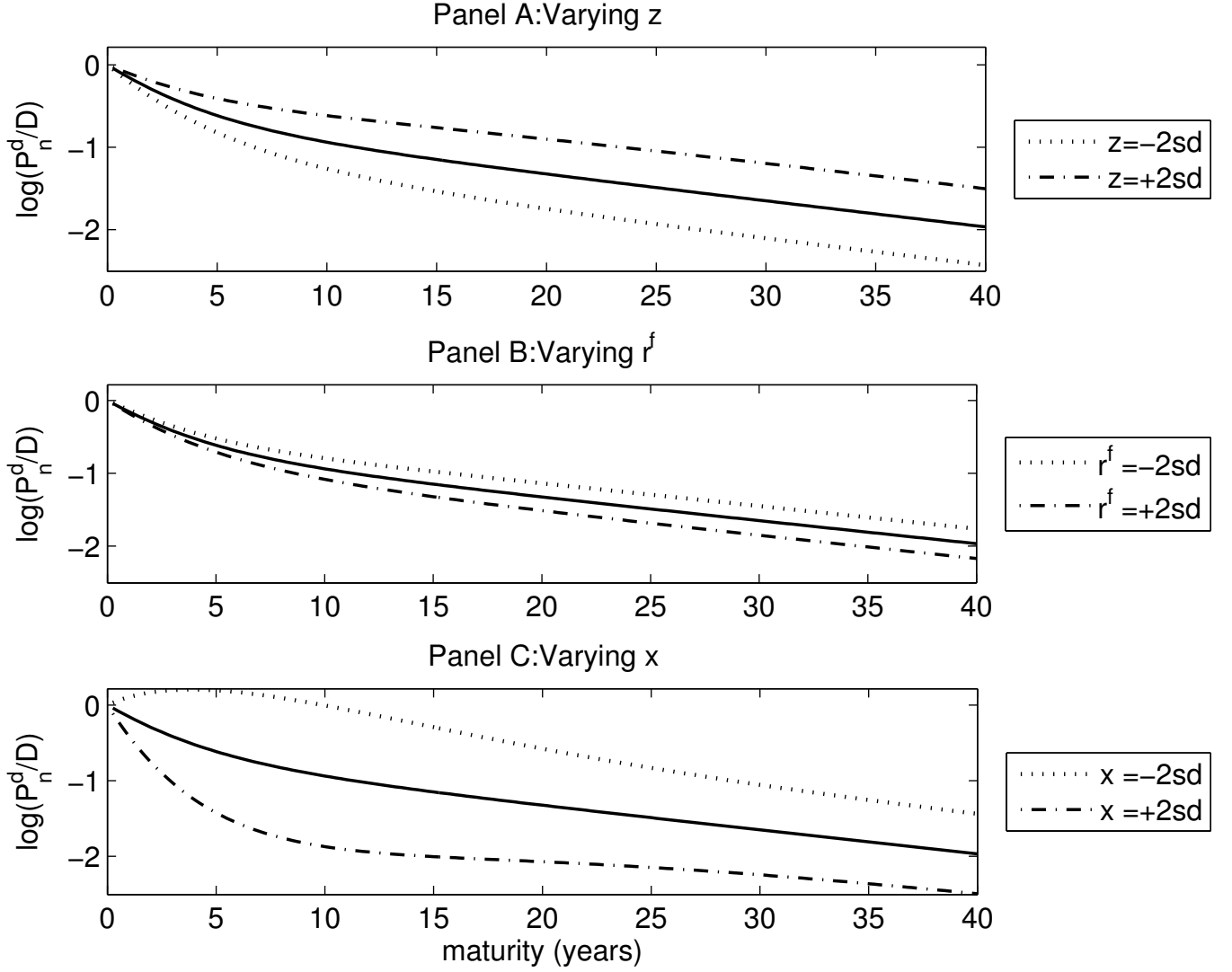
Notes: Panel A shows the log of ratios of zero-coupon equity prices to the aggregate dividend as a function of maturity when the state variables are equal to their long-run mean (solid line), and when expected dividend growth z_t is equal to the long-run mean plus (dashed-dotted line) or minus (dotted line) two unconditional quarterly standard deviations. All other state variables are kept at their long-run mean. Panel B shows analogous results when the real riskfree rate r_t^f is varied by plus or minus two unconditional quarterly standard deviations. Panel C shows analogous results when the price-of-risk variable x_t is varied by plus or minus two unconditional quarterly standard deviations.
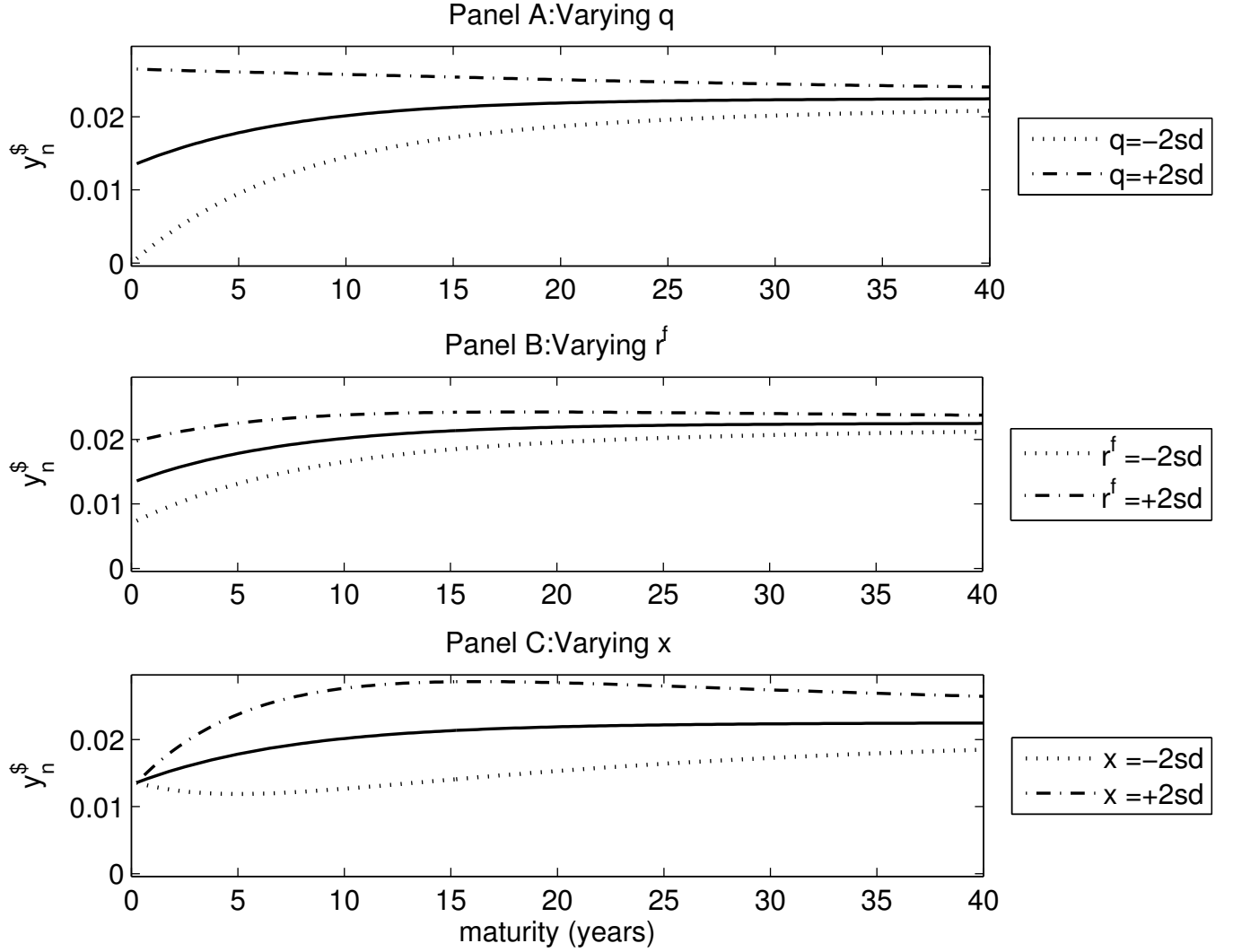
Figure 3: Yields on Zero-Coupon Real Bonds
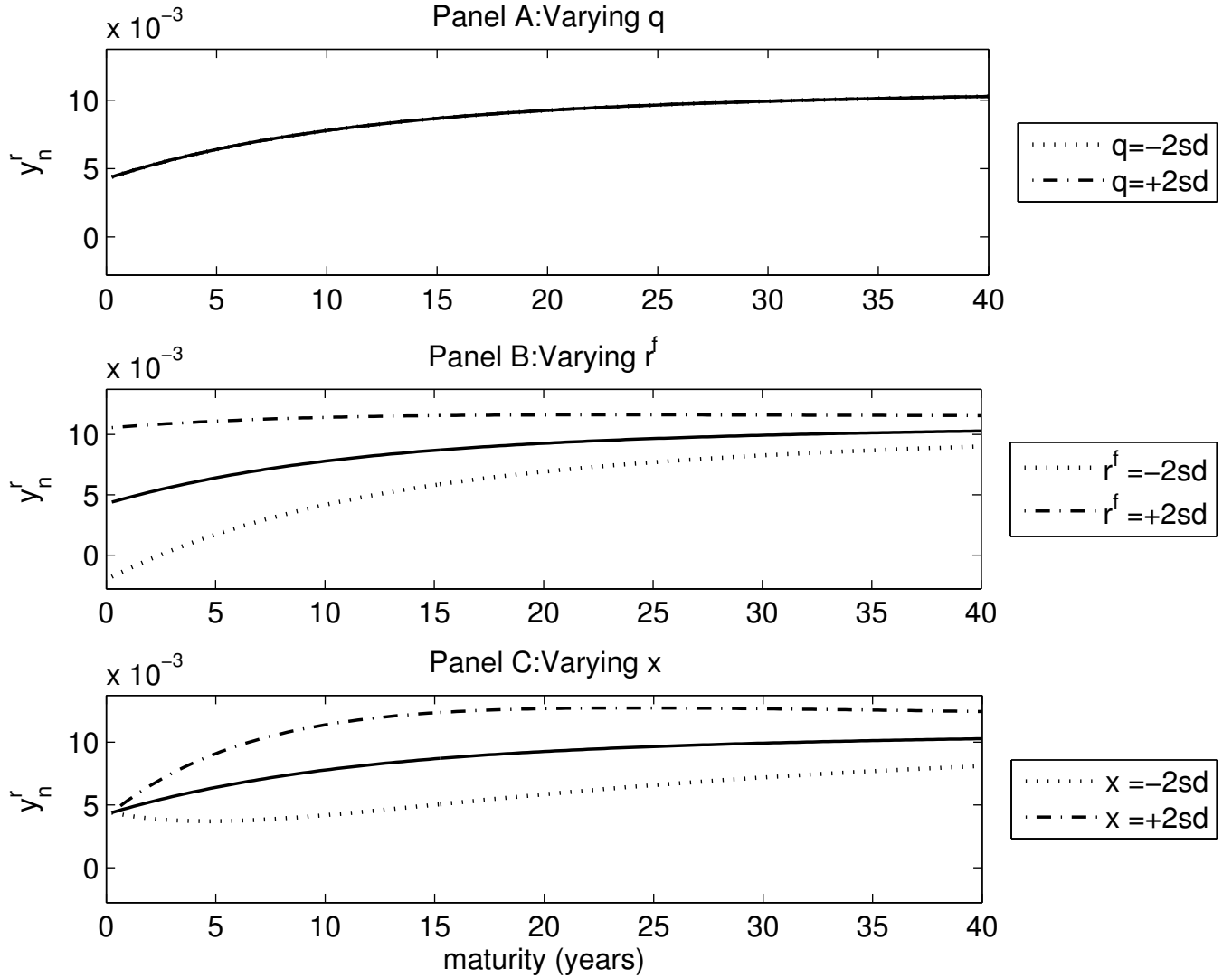
Notes: Panel A shows quarterly yields on real bonds as a function of maturity when the state variables are equal to their long-run mean (solid line), and when expected inflation q_t is equal to the long-run mean plus (dashed-dotted line) or minus (dotted line) two unconditional quarterly standard deviations. All other state variables are kept at their long-run mean. Panel B shows analogous results when the real riskfree rate r_t^f is varied by plus or minus two unconditional quarterly standard deviations. Panel C shows analogous results when the price-of-risk variable x_t is varied by plus or minus two unconditional quarterly standard deviations.

Figure 4: Yields on Zero-Coupon Nominal Bonds

Notes: Panel A shows quarterly nominal yields on nominal bonds as a function of maturity when the state variables are equal to their long-run mean (solid line), and when expected inflation q_t is equal to the long-run mean plus (dashed-dotted line) or minus (dotted line) two unconditional quarterly standard deviations. All other state variables are kept at their long-run mean. Panel B shows analogous results when the real riskfree rate r_t^f is varied by plus or minus two unconditional quarterly standard deviations. Panel C shows analogous results when the price-of-risk variable x_t is varied by plus or minus two unconditional quarterly standard deviations.

Figure 5: Decomposition of Coefficients from Long-Rate Regressions
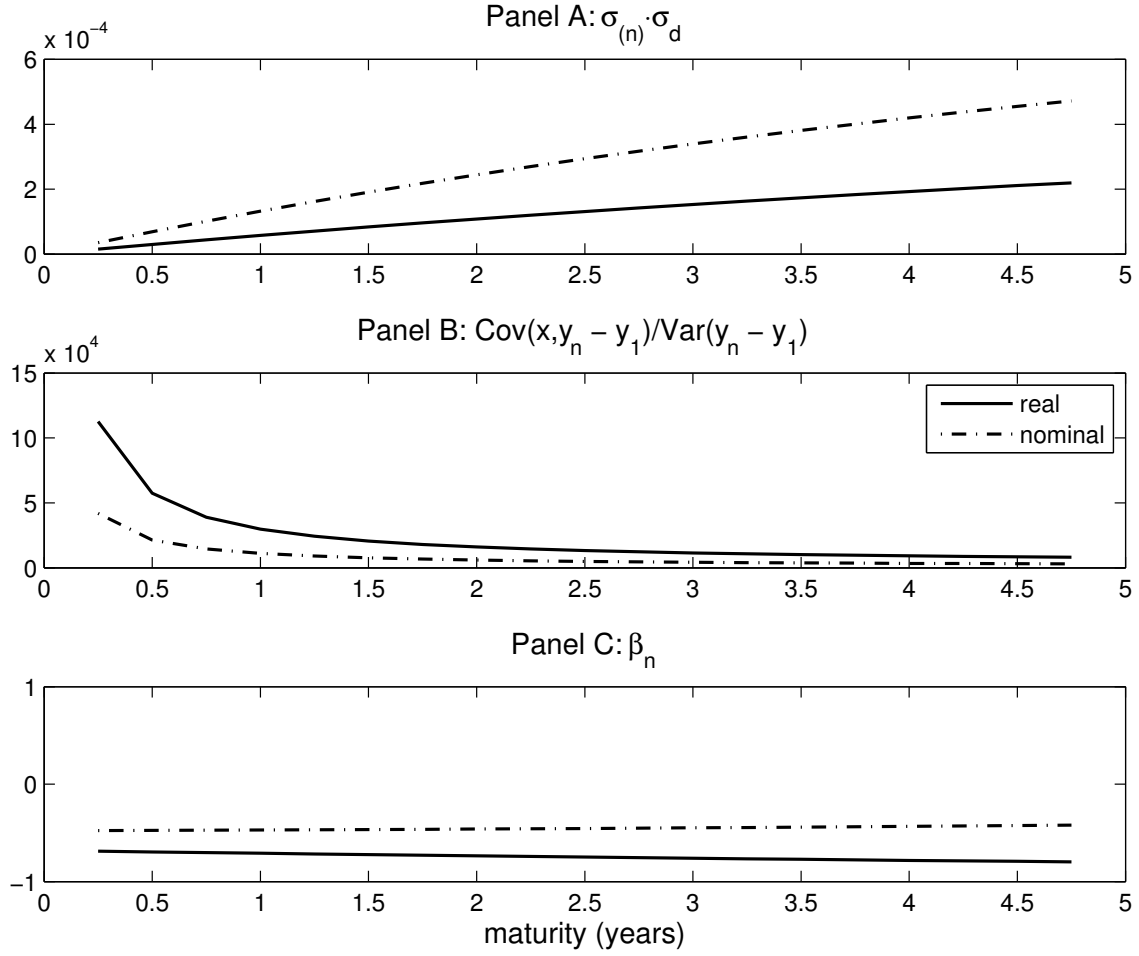
Notes: Panel A shows the covariance between returns on n-period real and nominal bonds and fundamentals, Panel B shows the coefficient from a regression of the price-of-risk variable x_t on the yield spread, and Panel C shows the coefficients β_n from the regression
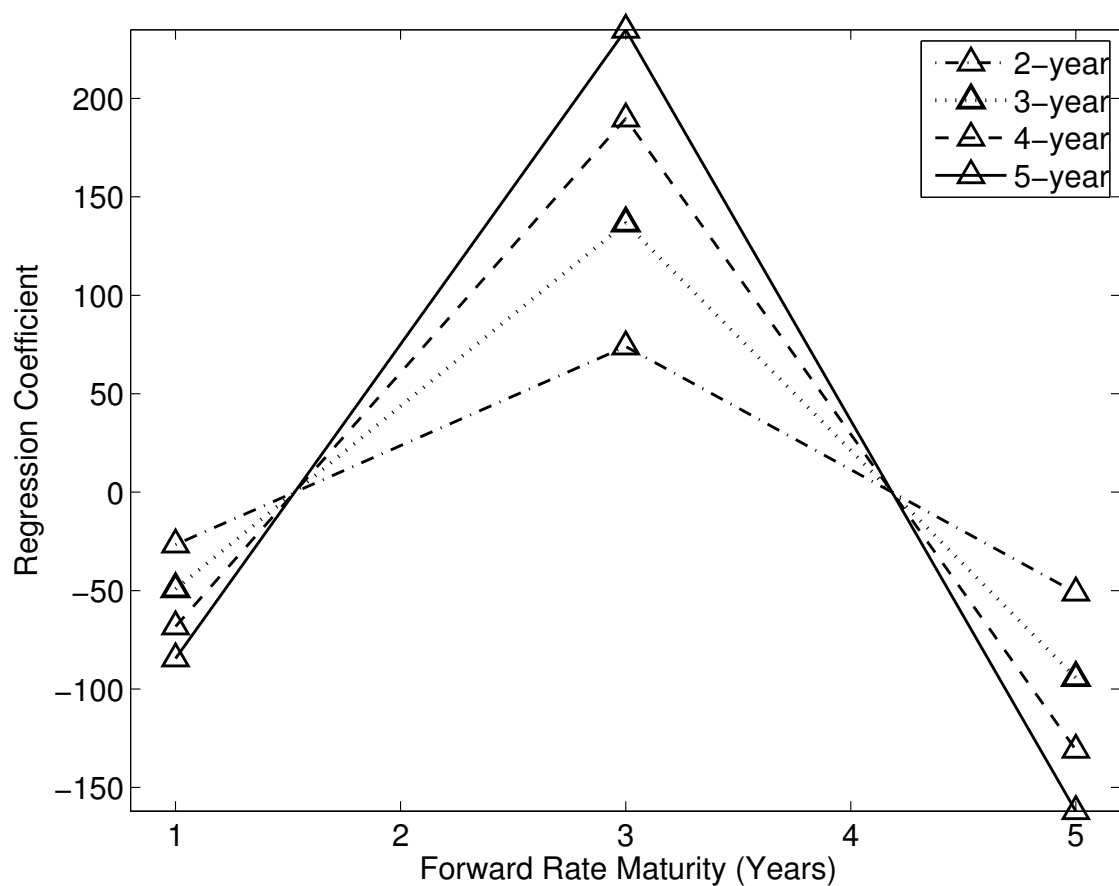
$$y_{n-1,t+1} - y_{nt} = \alpha + \beta \frac{1}{n-1} (y_{nt} - y_{1t}) + \text{error},$$

These quantities are related by the equation

$$\beta_n = 1 - (\boldsymbol{\sigma}_{(n)} \cdot \boldsymbol{\sigma}_d) \frac{\text{Cov}(x_t, y_{nt} - y_{1t})}{\text{Var}(y_{n-1,t+1} - y_{1t})}.$$

Covariances, variances, and regressions are measured using data simulated at a quarterly frequency, n is in quarters, and y_{1t} refers to the 1-quarter yield.

Figure 6: Regressions of Excess Bond Returns on Forward Rates

Notes: Annual returns on 2, 3, 4 and 5-year nominal bonds (in excess of the return on the 1-year bond) are regressed on the 1, 3 and 5-year forward rates. The figure plots the resulting regression coefficients.