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ABSTRACT

The Bansal and Yaron (2004) model of long-run risks (LRR) in aggregate consumption and dividend growth and its cointegrated extension are tested on a cross-section of assets and rejected over 1930-2006. Reversal of earlier conclusions is due to the increased power of the tests resulting from two observations under the null: the latent state variables and, therefore, the pricing kernel are known affine functions of observables; and, the unconditional moments of the time series processes impose constraints in addition to the pricing constraints. The models perform better in postwar subperiods, consistent with evidence of structural-breaks.

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1 Introduction

Whereas the standard time-separable utility model parsimoniously links the returns of all assets to per capita consumption growth through the Euler equations of consumption, per capita consumption growth covaries too little with the returns of most classes of financial assets and this creates a host of asset pricing puzzles: the aggregate equity return and the returns of various subclasses of financial assets are too large, too variable, and too predictable. Several generalizations of essential features of the model have been proposed to mitigate its poor performance.[1]

In particular, Bansal and Yaron (2004) introduce a "long-run risks" (LRR) state variable that simultaneously drives aggregate consumption growth and aggregate dividend growth. In conjunction with Kreps and Porteus (1978) preferences, the LRR state variable has a rich set of pricing implications and shows promise in explaining the cross-section of expected returns of various classes of financial assets[2].

The first contribution of our paper is to propose a novel estimation approach for the LRR model. Unlike Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007) who treat the LRR variable and the conditional variance of its innovation as latent state variables, we argue that these state variables are observable because both the aggregate price-dividend ratio and interest rate are functions only of these two state variables under the model assumptions. In the particularly simple log-linearized version of the model, the aggregate log price-dividend ratio and log interest rate are affine functions of the two state variables, with coefficients that are known functions of the preference parameters and of the parameters of the time-series processes. This observation allows us to invert the affine system and express the two state variables as known affine functions of the observable aggregate log price-dividend ratio and log interest rate. Therefore, we are able to express the log pricing kernel as an affine function of the aggregate log price-dividend ratio, log interest rate, and their lags, in addition to consumption growth.

In GMM tests at the annual frequency over 1930-2006, we strongly reject the hypothesis that the above pricing kernel explains the equity premium when we impose the constraint that the parameters of the pricing kernel should be consistent with the preference parameters and the parameters that drive the time-series processes of consumption growth, aggregate dividend growth, the LRR variable, and the conditional

[1] This extensive literature is reviewed in a collection of essays in Mehra (2008); the textbooks by Campbell, Lo, and MacKinlay (1997) and Cochrane (2005); and the articles by Campbell (2000, 2003), Cochrane and Hansen (1992), Constantinides (2002), Kocherlakota (1996), and Mehra and Prescott (2003).

[2] See also, Alvarez and Jerman (2005), Bansal, Dittmar, and Lundblad (2005), Bansal, Gallant, and Tauchen (2007), Bansal, Kiku, and Yaron (2007), Bekaert, Engstrom, and Xing (2005), Hansen, Heaton, and Li (2008), Hansen and Scheinkman (2007), Kiku (2006), Lettau and Ludvigson (2008), and Malloy, Moskowitz, and Vissing-Jorgensen (2006).

variance of its innovation. We reject the hypothesis that it explains the value and size premia even when we do not impose the constraint; not surprisingly, we strongly reject the hypothesis that it explains the value and size premia when we impose the constraint.

The reversal of earlier conclusions by Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007) is due, in part, to the increased power of the tests brought about by the recognition that the state variables - LRR and the conditional variance of its innovation - are not latent after all but are known affine functions of the observable aggregate log price-dividend ratio and log interest rate and, therefore, the pricing kernel is an affine function of the aggregate price-dividend ratio and interest rate, in addition to consumption growth. Furthermore, the unconditional moments of consumption growth and aggregate dividend growth impose constraints in addition to the pricing constraints.

The second contribution of our paper is to re-examine the empirical evidence of an extended version of the model that introduces as a third state variable the co-integrating residual of the logarithms of consumption and aggregate dividend levels. Such a co-integrating relationship has been introduced in the LRR model by Bansal, Dittmar, and Kiku (2007) and Bansal, Gallant, and Tauchen (2007). A simple extension of our earlier observation is that the aggregate price-dividend ratio and the interest rate are affine functions of the three state variables, with coefficients that are known functions of the preference parameters and of the parameters of the time-series processes. As before, this allows us to express the pricing kernel as an affine function of the aggregate price-dividend ratio, the interest rate, the demeaned aggregate dividend-consumption ratio, and their lags, in addition to consumption growth. In GMM tests at the annual frequency over 1930-2006, we reject the hypothesis that the above pricing kernel explains the equity premium. Moreover, the value of the persistence parameter of the LRR variable that best fits the data is 0.8, implying that the half-life of the LRR variable is just 3 years. We also reject the hypothesis that the above pricing kernel explains the cross-section of returns over the period 1930-2006.

The third contribution of our paper is to explore the possibility that rejection of the LRR model and its cointegrated variant is due, in part, to failure to account for recent evidence on regime shifts. In particular, we repeat the tests over the post-war period 1947-2006, thereby allowing for a possible break at the end of the war; and over the post-war period 1947-1991, thereby recognizing possible additional breaks in the early nineties. Whereas the models are still formally rejected, we find that they perform considerably better in explaining the equity premium but not the cross-section of returns. The results suggest that regime shifts warrant further investigation.

We address the problem of temporal aggregation of consumption by repeating our estimation and tests using quarterly data over the post-war period. The results are very similar to those obtained using annual data over the post war subperiod. In particular, the models are statistically rejected but perform considerably better in explaining the equity premium than the cross-section of returns. This suggests that our findings are unlikely to be driven by the problems associated with temporal aggregation.

Finally, note that the methodology of expressing latent state variables as known functions of observables has been previously employed in testing affine models of the term structure of interest rates (see, Dai and Singleton (2000) and Duffee (2002)). In these models, bond yields are affine functions of the latent state variables. Hence, the system may be inverted to express the state variables as affine functions of the observable yields. To our knowledge, our paper is the first application of this methodology in testing models of the cross section of equity returns. The same approach may be applied to evaluate the empirical plausibility of other asset pricing models that rely on latent state variables.

The paper is organized as follows. In Section 2, we describe our estimation methodology of the LRR model. In Section 3, we discuss the data. Section 4 presents the estimation results and, hence, the empirical evidence on the LRR model for the market portfolio and the risk free rate. Section 5 examines the ability of the model to explain the cross-section of asset returns. In Section 6, we consider an extension of the LRR model that introduces, as a third state variable, the co-integrating residual of the logarithms of consumption and aggregate dividend levels. In Section 7, we address the possibility of structural breaks within the period 1930-2006 by repeating our tests in post-war subperiods. In Section 8 we address the issues related to temporal aggregation of consumption by repeating our estimation and tests with quarterly data. Section 9 concludes. The appendix contains details of the estimation methodology.

2 Model and Estimation Methodology

The Bansal and Yaron (2004) LRR model relies on Kreps and Porteus (1978) preferences that allow for separation between the intertemporal elasticity of substitution and risk aversion. They assume that the representative consumer has the version of Kreps and Porteus (1978) preferences adopted by Epstein and Zin (1989) and Weil (1989). The utility function is defined recursively as

$$V_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(E\left[V_{t+1}^{1-\gamma}|F(t)\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \qquad (1)$$

where δ denotes the subjective discount factor, $\gamma > 0$ is the coefficient of risk aversion, $\psi > 0$ is the elasticity of intertemporal substitution, and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. Note that the sign of θ depends on the relative magnitudes of γ and ψ. The standard time-separable power utility model is obtained as a special case when $\theta = 1$, i.e. $\gamma = \frac{1}{\psi}$.

The aggregate consumption and dividend growth rates, Δc_{t+1} and Δd_{t+1}, respectively, are modeled as containing a small persistent expected growth rate component (the LRR), x_t, and fluctuating volatility, σ_t, that captures time-varying economic uncertainty:

$$\begin{aligned} x_{t+1} &= \rho_x x_t + \psi_x \sigma_t z_{x,t+1}, \\ \sigma^2_{t+1} &= (1-\upsilon)\sigma^2 + \upsilon\sigma^2_t + \sigma_w z_{\sigma,t+1}, \\ \Delta c_{t+1} &= \mu_c + x_t + \sigma_t z_{c,t+1}, \\ \Delta d_{t+1} &= \mu_d + \phi x_t + \varphi\sigma_t z_{d,t+1}. \end{aligned} \tag{2}$$

The shocks $z_{x,t+1}$, $z_{\sigma,t+1}$, $z_{c,t+1}$, and $z_{d,t+1}$ are assumed to be $i.i.d.$ $N(0,1)$ and mutually independent.

The time-series specification, equation (2), imposes restrictions on the underlying time-series parameters: μ_c, μ_d, ϕ, φ, ρ_x, ψ_x, σ, υ, and σ_w. In particular, moments of the aggregate consumption and dividend growth processes are well-defined functions of these 9 parameters (see Appendix A.1 for expressions of the moments of consumption and dividend growth rates as functions of the time-series parameters).

For the specification of preferences in equation (1), Epstein and Zin (1989) and Weil (1989) show that, for any asset j, the first-order conditions of the consumer's utility maximization yield the following Euler equations,

$$E_t\left[\exp(m_{t+1} + r_{j,t+1})\right] = 1, \tag{3}$$

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{4}$$

where $E_t(.)$ denotes expectation conditional on time t information, m_{t+1} is the natural logarithm of the intertemporal marginal rate of substitution, $r_{j,t+1}$ is the log of the gross return on asset j, and $r_{c,t+1}$ is the unobservable log gross return on an asset that delivers aggregate consumption as its dividend each period.

Closed-form solutions for the model rely on log-linear approximations for the log return on the consumption claim, $r_{c,t+1}$, and that on the market portfolio (the return on the aggregate dividend claim), $r_{m,t+1}$, as in Campbell and Shiller (1988),

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1}, \tag{5}$$

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + \Delta d_{m,t+1}, \tag{6}$$

where z_t is the log price-consumption ratio and $z_{m,t}$ the log price-dividend ratio. In equation (5), $\kappa_1 = \frac{e^{\overline{z}}}{1+e^{\overline{z}}}$ and $\kappa_0 = log(1 + e^{\overline{z}}) - \kappa_1 \overline{z}$ are log-linearization constants, where $\overline{z}$ denotes the long-run mean of the log price-consumption ratio. Similarly, in equation (6), $\kappa_{1,m} = \frac{e^{\overline{z}_m}}{1+e^{\overline{z}_m}}$ and $\kappa_{0,m} = log(1 + e^{\overline{z}_m}) - \kappa_1 \overline{z}_m$, where $\overline{z}_m$ denotes the long-run mean of the log price-dividend ratio. Bansal and Yaron (2004) show that z_t and $z_{m,t}$, are affine functions of the state variables, x_t and σ^2_t,

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2, \tag{7}$$
$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2. \tag{8}$$

The coefficients A_0, A_1, A_2, $A_{0,m}$, $A_{1,m}$, and $A_{2,m}$ depend on the parameters of the utility function, equation (1), and those of the stochastic processes for consumption and dividend growth rates, equation (2) (see Appendix A.2.1 for expressions for A_0, A_1, A_2, $A_{0,m}$, $A_{1,m}$, and $A_{2,m}$).

For this model specification, the log risk free rate from period t to $t+1$ may also be expressed as an affine function of the state variables (see Appendix A.2.2 for expressions for $A_{0,f}$, $A_{1,f}$, and $A_{2,f}$),

$$\begin{aligned} r_{f,t} &= -\log E_t\left[\exp(m_{t+1})\right], \\ &= A_{0,f} + A_{1,f} x_t + A_{2,f} \sigma_t^2. \end{aligned} \tag{9}$$

Equations (8) and (9) express the observable variables, $z_{m,t}$ and $r_{f,t}$, as affine functions of the latent state variables, x_t and σ_t^2. These may be inverted to express the unobservable state variables, x_t and σ_t^2, in terms of the observables, $z_{m,t}$ and $r_{f,t}$, (see Appendix A.2.3 for details and expressions for α_0, α_1, α_2, β_0, β_1, and β_2),

$$x_t = \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 z_{m,t}, \tag{10}$$
$$\sigma_t^2 = \beta_0 + \beta_1 r_{f,t} + \beta_2 z_{m,t}. \tag{11}$$

Now, substituting the log-affine approximation for $r_{c,t+1}$ in equation (5) into the expression for the pricing kernel (equation (4)), and noting that z_t is given by equation (7), we have,

$$\begin{aligned} m_{t+1} = {} & \left(\theta \log \delta + (\theta - 1)\left[\kappa_0 + (\kappa_1 - 1) A_0\right]\right) + \left(-\frac{\theta}{\psi} + (\theta - 1)\right) \Delta c_{t+1} \\ & + (\theta - 1)\kappa_1 A_1 x_{t+1} + (\theta - 1)\kappa_1 A_2 \sigma_{t+1}^2 - (\theta - 1) A_1 x_t - (\theta - 1) A_2 \sigma_t^2. \end{aligned} \tag{12}$$

Equation (12) for the pricing kernel involves the unobservable (from the point of view of the econometrician) state variables, x_t and σ_t^2, and, hence, is not directly testable on a cross-section of asset returns. Substituting the expressions for x_t and σ_t^2 from equations (10) and (11) into the pricing kernel in equation (12), we have,

$$m_{t+1} = c_1 + c_2 \Delta c_{t+1} + c_3 \left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t}\right) + c_4 \left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t}\right), \tag{13}$$

The parameters $c = (c_1, c_2, c_3, c_4)'$ are functions of the parameters of the time-series processes and the preference parameters (see Appendix A.2.4 for details).

The above expression for the pricing kernel is entirely in terms of observables. We substitute this expression into the set of Euler equations (3) to obtain a set of moment restrictions that are expressed entirely in terms of observables.

We first examine the empirical plausibility of the model when the asset menu consists of the market portfolio and the risk free rate. The lagged log price-dividend ratio of the market and the lagged log risk free rate are used as instruments. The Euler equations for the two assets along with the two chosen instruments give 6 moment restrictions. To this set of pricing restrictions, we add moment restrictions implied by the time-series specification of the model in equation (2). In particular, we include the following 9 moments of consumption and dividend growth rates: $E(\Delta c_{t+1})$, $Var(\Delta c_{t+1})$, $Cov(\Delta c_{t+1}, \Delta c_{t+2})$, $E(\Delta d_{t+1})$, $Var(\Delta d_{t+1})$, $Cov(\Delta d_{t+1}, \Delta d_{t+2})$, $Cov(\Delta c_{t+1}, \Delta d_{t+1})$, $Var\left[(\Delta c_{t+1})^2\right]$, and $Var\left[(\Delta d_{t+1})^2\right]$. Thus, we have a total of 15 moment conditions. The total number of parameters to be estimated is 12, including 9 time-series parameters and 3 preference parameters. We estimate the parameters with the GMM approach of Hansen (1982) and test the specification of the model using the overidentifying restrictions.

We next examine the ability of the model to explain the cross-section of returns. In this case, the asset menu consists of the market portfolio, the risk free rate, and portfolios of "Small" capitalization, "Large" capitalization, "Growth" and "Value" stocks. The Euler equations for the 6 assets give 6 moment restrictions. To this set of pricing restrictions, we add the 9 moment restrictions implied by the time-series specification of the model. This gives, once again, a total of 15 moment conditions in 12 parameters. We estimate the parameters and test the model specification with the GMM approach.

3 Data

We first estimate the model at the annual frequency, using annual data over the entire available sample period 1930 to 2006. We also repeat our analysis over the post-war periods 1947-2006 and 1947-1991, using both annual and quarterly data. The asset menu consists of the market, the risk free rate, and portfolios of "Value", "Growth", "Small" capitalization, and "Large" capitalization stocks. Our market proxy is the Centre for Research in Security Prices (CRSP) value-weighted index of all stocks on the NYSE, AMEX, and NASDAQ. The proxy for the risk free rate is the one-month Treasury Bill rate (from Ibbotson Associates). The construction of the size and book-to-market portfolios is as in Fama and French (1993). In particular, for the size sort, all NYSE, AMEX, and NASDAQ stocks are allocated across 10 portfolios according to their market capitalization at the end of June of each year. Value-weighted returns on these portfolios are then computed over the following twelve months. NYSE breakpoints are used in the sort. "Small" and "Large" denote the bottom and top market capitalization deciles, respectively. Similarly, value-weighted returns are computed for portfolios formed on the basis of BE/ME at the end of June of each year using NYSE breakpoints. The BE used in June of year t is the book equity for the last fiscal year end in $t-1$ and ME is the price times shares outstanding at the end of December of $t-1$. "Growth" and "Value" denote the bottom and top BE/ME deciles, respectively. Annual and quarterly returns for the above portfolios are computed by compounding monthly returns within each year and quarter, respectively. Also used in the empirical analysis are the price-dividend ratio and dividend growth rates of the above mentioned portfolios. Data on these are obtained from the CRSP files. All nominal quantities are converted to real, using the personal consumption deflator.

Table 1 provides descriptive statistics for the continuously compounded returns, the price-dividend ratios, and the dividend growth rates for the six assets mentioned above, for the annual sample over the period 1930-2006. The table illustrates the well documented equity premium and the size and value premia. Over the sample period, the annual equity premium over the 1-month Treasury bill rate has mean 5.8% and volatility of market returns is 19.3%. The annual risk free rate has mean 0.8% and standard deviation 5.0%. The annual mean premium of small over large stocks is 4.5% and of value over growth stocks is 4.1%. Value stocks are much more volatile than growth stocks and small stocks are much more volatile than large stocks.

The annual log price-dividend ratio on the market has a mean of 3.27 and standard error of 0.38 over the sample period. The price-dividend ratios of the "Small" and "Value" portfolios are much more volatile with annual volatilities at 0.71 and 1.14, respectively, compared to their counterparts, namely the "Large" and "Growth" portfolios that have volatilities 0.44 and 0.63, respectively.

The average annual log dividend growth rate on the market portfolio is 1.4% with volatility 10.8%. The mean and volatility of the "Small" (8.3% and 34.7%) and "Value" (7.0% and 56.8%) portfolios are much higher compared to their counterparts, namely the "Large" (1.2% and 13.6%) and "Growth" (0.7% and 20.6%) portfolios.

Finally, for consumption, we use real per capita consumption of non-durables and services from the National Income and Product Accounts (NIPA). We make the standard "end-of-period" timing assumption that consumption during period t takes place at the end of the period. Growth rates are constructed by taking the first difference of the corresponding log series. The annual log consumption growth has a mean of 1.5% and standard deviation of 2.6% over the sample period.

4 Empirical Evidence on the Equity Premium

We first examine the ability of the model to explain the returns of the market portfolio and the risk free rate, using annual data over the period 1930-2006. The LRR model was originally intended to explain the equity premium and the low risk free rate and it seems appropriate to start the empirical analysis by examining the ability of the model to explain the returns of these two assets. The lagged log price-dividend ratio of the market and the lagged log risk free rate are used as instruments giving 6 moment restrictions. To this set of pricing restrictions, we add moment restrictions implied by the time-series specification of the model in equation (2). In particular, as explained in Section 2, we include moments corresponding to the unconditional mean, variance, and first-order autocovariance of consumption and dividend growth rates, the covariance between consumption and dividend growth rates, and the variance of squared consumption and dividend growth rates (see Appendix A.1 for expressions of these moments in terms of the time-series parameters of the model). This gives 9 moment restrictions corresponding to the assumed time-series processes. Thus, we have a total of 15 moment conditions. The total number of parameters to be estimated is 12, including 9 time-series parameters and 3 preference parameters. We estimate the parameters with the GMM approach using the efficient weighting matrix and use the overidentifying restrictions to test the specification of the model.[3]

Note that the moment conditions, particularly the pricing restrictions, are highly nonlinear in the parameters making optimization difficult. In order to get accurate estimates, we first estimate the 9 time-series parameters using the 9 moment restrictions corresponding to the time-series specification of the model. This gives initial consistent estimates of these parameters along with their standard errors. Next, we update the initial estimates to obtain the final estimates of the time-series parameters and also estimate the preference parameters by performing a 12-dimensional grid search, over the 9 time series parameters and 3 preference parameters, using the full set of 15 moment restrictions. For the persistence parameter of the LRR variable, ρ_x, the grid covers the interval 0.10, 0.15, ..., 0.95, because its entire permissible range $(0, 1]$ is contained in the 95% confidence interval of the initial estimate. For each of the other time series parameters, the grid consists of evenly spaced points within two standard errors of the initial consistent point estimate. The grid for the risk aversion parameter is $2, 4, ..., 10$,

[3] Another common choice of the weighting matrix in the literature is the identity matrix. This puts equal weight on all the moment restrictions and arguments advanced in favour of it are its superior finite-sample properties and that it forces the estimates to minimize the sum of squared pricing errors. However, note that it ignores the covariance structure of the moment restrictions often leading to an identification problem. This issue is particularly serious in our setting where the moment restrictions include the pricing restrictions as well as time-series restrictions. Using the efficient weighting matrix avoids the identification issues by taking into account the covariance structure of the moment conditions and appropriately weighting them.

that for the IES is 0.3, 0.6, 0.9, 1.2, 1.5, and that for the rate of time-preference is 0.95, 0.97, 0.99. At each of the grid points, we compute the value of the GMM objective function and report the parameter vector at the grid point that minimizes the criterion function. We also report the standard errors of the parameter estimates that are Newey-West corrected using two lags, the average pricing errors of the assets and their associated standard errors[4], and the J-stat to test the null hypothesis that the model is correctly specified. Note that the J-stat has an asymptotic chisquared distribution with 3 degrees of freedom under the null.

The estimation results are reported in Table 2. Note that the persistence parameter, ρ_x, of the LRR variable is 0.90. It is very difficult, in finite samples, to statistically distinguish between a purely *i.i.d.* process and one with a very small, but persistent, predictable component (see Shephard and Harvey (1990)). Expressing the latent LRR variable as a known affine function of observables increases the power of our estimation approach making it easier to detect the presence of small, predictable components in consumption and dividend growth rates. The estimated values of the risk aversion and the IES parameters are 8 and 0.6, respectively, and both are quite imprecisely estimated.

However, note that the average pricing errors for the market portfolio and the risk free rate are substantial at 11.6% and 14.8%, respectively. Moreover, the J-stat is 10.41 and has an asymptotic p-value of 1.5%.

Note that the non-linearity of the moment restrictions with respect to the parameters, the large number of parameters to be estimated (12), and the relatively small sample size (76) calls into question the accuracy of the asymptotic inference in the preceding paragraph. We perform Monte Carlo simulations to examine the finite-sample performance of the estimators and obtain the finite-sample critical values of the J-stat for overidentifying restrictions. The details of the simulation design are in Appendix A.5. Using the point estimates of the time-series and preference parameters in Table 2, we simulate the time-series of the LRR variable, the stochastic variance process, the consumption and dividend growth rates, and returns on the market portfolio and the risk free rate, of the same length as the historical sample. We then perform the GMM estimation of the parameters using the pricing and time-series restrictions and obtain the J-stat, J^i. This procedure is repeated 100 times. The 90%, 95%, and 99% finite-

[4] The average pricing error for asset j is computed as

$$\left[\frac{1}{T}\sum_t exp\left(m_{t+1}\left(\widehat{\Theta}\right)+r_{j,t+1}\right)-1\right].$$

where $\widehat{\Theta}$ denotes the point estimates of the model parameters. The standard error of the average pricing error is computed as $se\left(exp\left(m_{t+1}\left(\widehat{\Theta}\right)+r_{j,t+1}\right)-1\right)/\sqrt{T}$, where se denotes standard error. Note that, under the model assumptions, $exp\left(m_{t+1}\left(\widehat{\Theta}\right)+r_{j,t+1}\right)-1$ is a martingale difference sequence and, hence, the above procedure gives valid standard errors for the average pricing error.

sample critical values of the J-stat are obtained from the percentiles of $\{J^i\}_{i=1}^{100}$. These critical values are 5.00, 5.01, and 5.07, respectively. Thus, the J-stat of 10.41 obtained using the historical sample in Table 2 has a finite-sample p-value much smaller than 1%. Furthermore, the means of the average pricing errors for the market portfolio and the risk free rate across the 100 simulations are 0.04% and -0.11%, respectively. These are, respectively, three and two orders of magnitude bigger than those obtained in the historical sample (13.0% and 16.7%). Thus, the simulation results reinforce the rejection of the model in Table 2 based on asymptotic inference.

To shed further light on the above results, we estimate the Euler equations (3) on annual data when the asset menu consists of the market portfolio and the risk free rate, treating the parameters of the pricing kernel, c, as free parameters. The lagged log price-dividend ratio of the market and the lagged log risk free rate are used as instruments giving six moment restrictions in four parameters. Note that this procedure does not impose the constraint that the estimated parameters c should be consistent with the preference parameters and the parameters that drive the time-series processes. Hence, non-rejection of the pricing equations does not necessarily imply support for the risk channels highlighted in the model - the low frequency movements and time-varying uncertainty in aggregate consumption and dividend growth rates. The above specification of the log of the stochastic discount factor as an affine function of the log consumption growth, the log price-dividend ratio of the market and the log risk free rate could arise from other asset pricing models which have two latent state variables quite unrelated to long run risks in consumption growth and fluctuating volatility.

The estimation results are reported in Table 3 for the efficient weighting matrix. The table reveals that the constant, the coefficient on the price-dividend ratio of the market, and the coefficient on the risk free rate rate are significantly negative, while the coefficient on consumption growth is significantly positive, at conventional levels of significance. The average pricing errors for the market and the risk free rate are very small at 0.2% and 0.4%, respectively, and insignificantly different from zero. Note that the average pricing errors in Table 3 are two orders of magnitude smaller than those in Table 2. The last row reports the J-stat for testing overidentifying restrictions. The J-stat has an asymptotic χ^2 distribution with two degrees of freedom. The computed statistic has p-value of 92.3%.

Thus, although the pricing kernel, equation (13), adequately explains the returns on the market portfolio and the risk free rate over the period 1930-2006 when its parameters are treated as free parameters, imposition of the restrictions imposed by the unconditional moments of consumption growth and aggregate dividend growth considerably worsens the ability of the model to explain the returns on the market portfolio and the risk free rate and leads to its rejection.

5 Empirical Evidence on the Cross-Section of Returns

We next explore the ability of the model to explain the cross-section of annual returns over the period 1930-2006. The asset menu consists of the market portfolio, the risk free rate, and portfolios of "Value", "Growth", "Small" capitalization, and "Large" capitalization stocks as detailed in Section 2. The Euler equations for these 6 assets along with the 9 time-series moment restrictions gives 15 moment restrictions in 12 parameters. The optimization algorithm used is similar to that in Section 4.

The estimation results are reported in Table 4. Note that the value of the persistence parameters of the LRR variable, ρ_x, that best fits the data is 0.95 and is statistically significant at conventional levels of significance. As in Section 5, this is indicative of the increased power of our estimation approach making it easier to detect the presence of small, but highly persistent, predictable components in consumption and dividend growth rates. The estimated values of the risk aversion and the elasticity of intertemporal substitution parameters are 10 and 0.9, respectively, and the former is statistically significant. However, note that the average pricing errors for the assets are substantial varying from 4.3% for the "Growth" portfolio to 12.3% for the portfolio of "Small" capitalization stocks. Also, the J-stat is 12.93 and has an asymptotic p-value smaller than 1%.

To examine the accuracy of the asymptotic inference, we perform Monte Carlo simulations to examine the finite-sample performance of the estimators and obtain the finite-sample critical values of the J-stat for overidentifying restrictions. The details of the simulation design are in Appendix A.5. Using the point estimates of the time-series and preference parameters in Table 4, we simulate time-series of the LRR variable, the stochastic variance process, the consumption and dividend growth rates, and returns on the market portfolio, the risk free rate, and the returns on the "Small", "Large", "Growth" and "Value" portfolios, of the same length as the historical sample. We then perform the GMM estimation of the parameters using the pricing and time-series restrictions and obtain the J-stat, J^i. This procedure is repeated 100 times. The 90%, 95%, and 99% finite-sample critical values of the J-stat are 5.21, 5.43, and 6.40, respectively. Thus, the J-stat of 12.93 obtained using the historical sample in Table 4 has a finite-sample p-value much smaller than 1%. Furthermore, the means of the average pricing errors for the 6 assets across the 100 simulations vary from -3.9% for the "Small" portfolio to 3.2% for the "Value" portfolio. These are considerably smaller in magnitude than the variation obtained in the historical sample (4.3% for the "Growth" portfolio to 12.3% for the portfolio of "Small" capitalization stocks). Thus, the simulation results reinforce the rejection of the model in Table 4 based on asymptotic inference.

As in Section 4, to further investigate the above results, we estimate the Euler equations (3) on annual data when the asset menu consists of the market portfolio, the risk free rate, and the "Small", "Large", "Growth", and "Value" portfolios, treating the parameters of the pricing kernel, c, as free parameters. This gives six moment restrictions in four parameters. In Table 5, we report results for the efficient weighting matrix. Note that the constant and the coefficient on the price-dividend ratio of the market are significantly negative while the coefficients on consumption growth and the risk free rate are statistically indistinguishable from zero at conventional levels of significance. The pricing errors for the assets, although substantially smaller than those in Table 4, are economically large varying from -2.7% for the risk free rate to 6.3% for the portfolio of "Small" capitalization stocks. Moreover, the J-stat reveals that the model is rejected at the 3% level.

As an additional robustness check, in Table 6, we report estimates using the same set of assets as in Table 5 but using the difference in the log price-dividend ratios of the "Value" and "Growth" portfolios, $z_{v-g,t}$, (instead of the risk free rate), and the log price-dividend ratio of the market, $z_{m,t}$, to express the unobservable state variables, x_t and σ_t^2, as affine functions of the observables, $z_{v-g,t}$ and $z_{m,t}$. This approach is valid under the assumption that the dividend growth processes of the "Growth" and "Value" portfolios are similar to that for the market. Under this assumption, similar calculations, as in Section 2, yield that the log price-dividend ratios of these portfolios are affine functions of the state variables and, hence, so is their difference. Table 6 reveals that this specification of the pricing kernel performs slightly better at explaining the cross-section than that in Table 5. Although the pricing errors for the assets are smaller varying from -1.1% for the risk free rate to 5.8% for the portfolio of "Small" capitalization stocks, they are economically large. Moreover, the J-stat reveals that the model is still rejected at the 5% significance level, confirming the findings in Table 5.

Thus, the specification of the pricing kernel in equation (13) fails to explain the cross-section of returns. As pointed out in Section 4, the above specification of the log of the stochastic discount factor as an affine function of the log consumption growth, the log price-dividend ratio of the market and the log risk free rate (or the difference in the log price-dividend ratios of the "Value" and "Growth" portfolios) could arise from other asset pricing models which have two latent state variables quite unrelated to long run risks in consumption growth and fluctuating volatility. Hence, failure of the above specification to explain the cross-section of returns suggests that a linear two-factor model is unlikely to succeed in explaining the cross-section of returns over the period 1930-2006.

6 A Cointegrated Long Run Risks Model

We consider a variant of the LRR model that imposes a cointegrating restriction between log aggregate stock market dividends, d_t, and log consumption, c_t. Bansal, Gallant, and Tauchen (2007) argue that this restriction is economically well motivated because aggregate consumption and aggregate stock market dividends cannot permanently deviate from each other and financial wealth cannot permanently deviate from aggregate wealth. Bansal, Dittmar, and Kiku (2007) highlight that this cointegrating relation measures long run covariance risks in dividends and is important in understanding sources of risk and explaining the equity risk premia across all investment horizons.[5]

Note that the Bansal and Yaron (2004) model implies that aggregate consumption and dividends are not cointegrated. Hence, the poor empirical performance of the model in explaining the equity premium and the cross-section of stock returns in Sections 5 and 6, respectively, may be due to its failure to account for the cointegrating relationship. Hence, we consider an extension of the model that imposes a cointegrating restriction between log aggregate stock market dividends and log consumption. In particular, we consider a variant of the model in Bansal, Gallant, and Tauchen (2007) that allows us to obtain closed-form expressions for asset prices. We estimate and test the model using an extension of our estimation methodology outlined in Section 2.

6.1 Model and Estimation Methodology

The aggregate consumption growth, Δc_{t+1}, the LRR variable, x_t, and the stochastic volatility, σ_t, processes are modeled as in Bansal and Yaron (2004),

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t z_{c,t+1}, \\
x_{t+1} &= \rho_x x_t + \psi_x \sigma_t z_{x,t+1}, \\
\sigma_{t+1}^2 &= \mu_\sigma + \rho_\sigma \sigma_t^2 + \sigma_w z_{\sigma,t+1}.
\end{aligned} \tag{14}$$

The point of departure is the imposition of a cointegrating restriction between log aggregate stock market dividends and log consumption

$$d_t - c_t = \mu_{dc} + s_t, \tag{15}$$

[5]In a different context, Lettau and Ludvigson (2001) and Menzly, Santos, and Veronesi (2004) apply the cointegrating residual between consumption, labour income, and aggregate stock market dividends to explain the cross-section of asset returns.

where the cointegrating residual, s_t, is an $I(0)$ process,

$$s_{t+1} = \lambda_{sx} x_t + \rho_s s_t + \psi_s \sigma_t z_{s,t+1}. \tag{16}$$

The shocks $z_{c,t+1}$, $z_{x,t+1}$, $z_{\sigma,t+1}$, and $z_{s,t+1}$ are assumed to be *i.i.d.* $N(0,1)$ and mutually independent. Note that the cointegrating coefficient is set at one in equation (15).[6]

From equation (15), we have,

$$\begin{aligned} \Delta d_{t+1} &= \Delta c_{t+1} + \Delta s_{t+1}, \\ &= \mu_c + (1+\lambda_{sx}) x_t + (\rho_s - 1) s_t + \sigma_t z_{c,t+1} + \psi_s \sigma_t z_{s,t+1}, \end{aligned} \tag{17}$$

where the second line follows from equations (14) and (16).

Thus, this extension of the LRR model involves three state variables - the LRR variable, x_t, the stochastic variance, σ_t^2, and the cointegrating residual between log aggregate dividends and log aggregate consumption, s_t. Note that the LRR model with two latent state variables obtains as a limiting special case when $\rho_s = 1$.

We solve the model using solution techniques similar to those in Bansal and Yaron (2004). We conjecture that the log price-consumption ratio and the log price-dividend ratio are affine functions of the three state variables

$$\begin{aligned} z_t &= A_0 + A_1 x_t + A_2 \sigma_t^2 + A_3 s_t, \\ z_{m,t} &= A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2 + A_{3,m} s_t. \end{aligned}$$

Appendix A.3 establishes that, for this extended specification, the log price-consumption ratio is a function only of x_t and σ_t^2. The log price-dividend ratio of the market, on the other hand, is an affine function of the three state variables, x_t, σ_t^2, and s_t (see Appendix A.3.1 for expressions for A_0, A_1, and A_2 and Appendix A.3.2 for expressions for $A_{0,m}$, $A_{1,m}$, $A_{2,m}$, and $A_{3,m}$)

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2, \tag{18}$$

$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2 + A_{3,m} s_t. \tag{19}$$

Also, the log risk free rate is an affine function only of x_t and σ_t^2 (see Appendix A.3.3 for expressions for $A_{0,f}$, $A_{1,f}$, and $A_{2,f}$)

$$r_{f,t} = A_{0,f} + A_{1,f} x_t + A_{2,f} \sigma_t^2. \tag{20}$$

[6] Bansal, Gallant, and Tauchen (2005) perform a heteroskedasticity-robust augmented Dickey-Fuller test for a unit root in $d_t - c_t$ and the results provide strong evidence for a cointegrating relationship between the variables with a coefficient equal to unity.

The estimation methodology outlined in Section 2 is readily adapted to the extended version of the model. Note that two of the three state variables, namely, the long run risk variable, x_t, and the stochastic variance, σ_t^2, are latent, while the cointegrating residual, s_t, is observable as the demeaned difference between log aggregate dividend and consumption levels (see equation (15)).

Equations (19) and (20) may be inverted to express the unobservable state variables, x_t and σ_t^2, in terms of the observables, $z_{m,t}$, $r_{f,t}$, and s_t, (see Appendix A.3.4 for details and expressions for α_0, α_1, α_2, α_3, β_0, β_1, β_2 and β_3),

$$\begin{aligned} x_t &= \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 z_{m,t} + \alpha_3 s_t, && (21) \\ \sigma_t^2 &= \beta_0 + \beta_1 r_{f,t} + \beta_2 z_{m,t} + \beta_3 s_t. && (22) \end{aligned}$$

Using equations (4), (5), and (18), we write the pricing kernel as,

$$\begin{aligned} m_{t+1} &= \left(\theta \log \delta + (\theta - 1)\left[\kappa_0 + (\kappa_1 - 1) A_0\right]\right) + \left(-\frac{\theta}{\psi} + (\theta - 1)\right) \Delta c_{t+1} \\ &\quad + (\theta - 1)\kappa_1 A_1 x_{t+1} + (\theta - 1)\kappa_1 A_2 \sigma_{t+1}^2 \\ &\quad - (\theta - 1) A_1 x_t - (\theta - 1) A_2 \sigma_t^2, \qquad (23) \end{aligned}$$

Substituting the expressions for x_t and σ_t^2 from equations (21) and (22) into the pricing kernel, equation (23), we have (see Appendix A.3.4 for details)

$$m_{t+1} = c_1 + c_2 \Delta c_{t+1} + c_3 \left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t}\right) + c_4 \left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t}\right) + c_5 \left(s_{t+1} - \frac{1}{\kappa_1} s_t\right). \qquad (24)$$

Thus, the pricing kernel is expressed entirely in terms of observables.

As in Section 2, we first examine the empirical plausibility of the model when the asset menu consists of the market portfolio and the risk free rate. The lagged log price-dividend ratio of the market and the lagged log risk free rate are used as instruments. The Euler equations for the two assets along with the two chosen instruments give 6 moment restrictions. To this set of pricing restrictions, we add moment restrictions implied by the time-series specification of the model in equations (14), (15), and (16). In particular, we include the following 7 moments of consumption and dividend growth rates: $E(\Delta c_{t+1})$, $Var(\Delta c_{t+1})$, $Cov(\Delta c_{t+1}, \Delta c_{t+2})$, $Cov(\Delta c_{t+1}, \Delta c_{t+2})$, $Var(\Delta d_{t+1})$, $Cov(\Delta d_{t+1}, \Delta d_{t+2})$, and $Cov(\Delta c_{t+1}, \Delta d_{t+1})$ (see Appendix A.4 for expressions for these moments in terms of the time-series parameters). Thus, we have a total of 13 moment conditions. The total number of parameters to be estimated is 12, including 9 time-series parameters and 3 preference parameters. We estimate the parameters with the GMM approach and test the specification of the model using the overidentifying restriction.

We next examine the ability of the model to explain the cross-section of returns. In this case, the asset menu consists of the market portfolio, the risk free rate, and portfolios of "Small" capitalization, "Large" capitalization, "Growth" and "Value" stocks. The Euler equations for the 6 assets give 6 moment restrictions. To this set of pricing restrictions, we add the 7 moment restrictions implied by the time-series specification of the model. This gives, once again, a total of 13 moment conditions in 12 parameters. We estimate the parameters and test the model specification with the GMM approach.

6.2 Empirical Evidence on the Cointegrated Model

We first explore the ability of the cointegrated model to explain the returns of the market portfolio and the risk free rate. The Euler equations for the two assets along with the two chosen instruments (namely, the lagged log price-dividend ratio of the market and the lagged log risk free rate) gives 6 moment restrictions. As mentioned in Section 6.1, to this set of pricing restrictions, we add moment restrictions implied by the time-series specification of the model in equations (14), (15), and (16). In particular, we include moments corresponding to the unconditional mean, variance, and first and second-order autocovariances of consumption growth, the variance and first-order autocovariance of the dividend growth rate, and the covariance between consumption and dividend growth rates. This gives 7 moment restrictions corresponding to the assumed time-series processes. Thus, we have a total of 13 moment conditions. The total number of parameters to be estimated is 12, including 9 time-series parameters and 3 preference parameters. We estimate the parameters with the GMM approach using the efficient weighting matrix and use the overidentifying restriction to test the specification of the model. The optimization algorithm is similar to that employed in Sections 4 and 5.

The estimation results are reported in Table 7. Note that, the value of the persistence parameter of the LRR variable, ρ_x, that best fits the data is 0.8, and is statistically significant at conventional levels of significance. As in Sections 5 and 6, this suggests the presence of a predictable component in consumption growth. However, the estimated value of the persistence parameter implies that the half-life of the LRR variable is three years and its implied frequency is about the same as or even slightly higher than that of the average business cycle, contrary to the notion of it being a very low-frequency component. The estimated values of the risk aversion and the elasticity of intertemporal substitution parameters are 8 and 1.5, respectively, and both are statistically significant. However, note that the average pricing errors for the market and the risk free rate are substantial at 16.5% and 20.6%, respectively. In fact, the average pricing errors are slightly larger than those obtained in Table 2 (13.0% and 16.7%) for the Bansal and Yaron (2004) two-factor model. Moreover, the J-statistic takes the value 12.76 and has a p-value smaller than 1% (note that the J-stat has an asymptotic chisquared distribution with one degree of freedom).

Next, we examine the empirical performance of the model in explaining the cross-section of returns. Table 8 reports the estimation results for the full sample 1930-2006 when the set of assets includes the market, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios. The Euler equations for the 6 assets along with the 7 moment restrictions implied by the time-series specification of the model gives a total of 13 moment conditions in 12 parameters. Table 8 reveals that the three-factor cointegrated model performs quite poorly at explaining the cross-section of asset returns. In fact, the average pricing errors for the cross-section of returns are higher for the cointegrated three-factor model than those obtained for the Bansal and Yaron (2004) two-factor model. For the cointegrated model, the average pricing errors for the assets vary from 25.7% for the "Growth" portfolio to 39.8% for the portfolio of "Small" capitalization stocks compared to the variation over 4.3% for the "Growth" portfolio to 12.3% for the portfolio of "Small" capitalization stocks obtained for the two-factor model. The J-statistic in Table 8 is 13.73 and has a p-value smaller than 1% showing that the model is statistically rejected over the period 1930-2006.

7 Further Results in Post-War Subperiods

We explore the possibility that rejection of the LRR model and its cointegrated extension is due, in part, to failure to account for regime shifts over the period 1930-2006. Pastor and Stambaugh (2001) document evidence of breaks in the equity premium in the early thirties and forties, and in the early and mid-nineties. Lettau, Ludvigson, and Wachter (2008) find evidence of a break in the consumption volatility around 1992 followed by a break in the log price-dividend ratio of the market around 1995. Lettau and Van Nieuwerburgh (2008) report evidence of two breaks in the mean of the aggregate price-dividend ratio around 1954 and 1994.

Since the period prior to 1947 was one of great economic uncertainty, including the Great Depression, World War II, and structural breaks in the equity premium, rejection of the LRR models in the full sample may be due to their poor performance in the pre-war period. To examine this, we first present empirical results for the Bansal and Yaron (2004) LRR model on the 2-asset system over the post-war subperiod 1947-2006. Recall that the 2-asset system consists of the market portfolio and the risk free rate, with the lagged log price-dividend ratio of the market and the lagged log risk free rate used as instruments. To this set of 6 pricing restrictions, we add 9 moment restrictions implied by the time-series specification of the model. Table 9 reports results for the efficient weighting matrix. The table reveals that the performance of the model improves dramatically over the postwar subsample. Note that the pricing errors for the market portfolio and the risk free rate in Table 9 are much smaller than those obtained for the full sample in Table 2. The average pricing error for the market is -0.6%, two orders of magnitude smaller than that in the full sample (13.0%), and that for the risk free rate is -1.3% which is also an order of magnitude smaller than that for the full period (16.7%). Although the J-stat is 15.06 leading to the model still being rejected at the 1% level of significance, this may well be due to the well known tendency of the efficient GMM estimation procedure to over-reject in finite samples.

As in Sections 5 and 6, the simulation results are largely in line with the asymptotic results. The 90%, 95%, and 99% finite-sample critical values of the J-stat are 5.05, 5.13, and 7.98, respectively. Thus, the J-stat of 15.16 obtained using the historical sample in Table 9 has a finite-sample p-value much smaller than 1%. However, the means of the average pricing errors for the market portfolio and the risk free rate across the 100 simulations are -0.05% and -0.08%, respectively, and are more in line with the small values obtained in the historical sample (-0.6% and -1.3%).

Similar results obtain when estimation is further restricted to the subperiod 1947-1991 which ends before the structural breaks in the nineties. The estimation results for the 2-asset system are reported in Table 10. The average pricing errors for the market portfolio and the risk free rate are very small at -0.9% and -1.1%, respectively, and are two and one orders of magnitude smaller than the errors in the full sample in Table 2. However, as in Table 9, the J-stat is 15.48 and has a p-value smaller than 1%. The simulation results yield very similar conclusions and are omitted for brevity.

Overall, the performance of the model in explaining the returns on the market portfolio and the risk free rate improves significantly in the postwar subperiods. A point worth noting is that the estimate of the mean of the stochastic volatility process, σ, is 1% in Table 9 over the period 1947-2006, half of the corresponding estimate in Table 10 over the period 1947-1991 (2%). Both estimates are statistically significant at conventional levels of significance. This is consistent with the findings in Lettau, Ludvigson, and Wachter (2008) who find evidence of a substantial decline in consumption volatility from 2.2% to 0.7% around 1992.

However, we find that the LRR model performs as poorly at explaining the cross-section of returns over the post-war subperiod 1947-2006 as over the full period. Table 11 reports results for the cross-section over the subperiod 1947-2006. The average pricing errors for the assets are economically large, varying from 11.8% for the "Growth" portfolio to 18.2% for the "Value" portfolio. The J-stat is 18.04 and the model is rejected at the 1% level of significance. Also note that the value of the persistence parameter of the LRR variable, ρ_x, that best fits the data is 0.7, implying that its half-life is two years and its implied frequency is much higher than that of the business cycle. Similar results are obtained for the subperiod 1947-1991 that are omitted for brevity.

Next, we examine the empirical performance of the cointegrated LRR model on the 2-asset and 6-asset systems over the postwar subperiods. Table 12 reports estimation results for the 2-asset system over the subperiod 1947-2006. The table reveals that the cointegrated model, like the two-factor model, performs significantly better at pricing the market portfolio and the risk free asset over the postwar subperiod. The average pricing errors for the market and the risk free rate are 1.7% and 2.0%, respectively, and both are an order of magnitude smaller than those for the full period in Table 7 (16.5% and 20.6%). However, note that the value of the persistence parameter of the LRR variable, ρ_x, that best fits the data is 0.7, implying that its half-life is two years and its implied frequency is much higher than that of the business cycle. This is similar to the findings in Table 7 which examines the performance of the cointegrated model in explaining the returns on the market portfolio and the risk free rate over the full period 1930-2006, where the value of the persistence parameter of the LRR variable, ρ_x, that best fits the data was found to be only 0.8.

The performance of the model improves further when we focus on the subperiod 1947-1991 which ends before the structural breaks in the nineties. Table 13 reports the estimation results for the 2-asset system. The average pricing errors for the market and the risk free rate are very small at 0.4% and -1.0%, respectively, and are considerably smaller than those for the subperiod 1947-2006 in Table 12. Note that the value of the persistence parameter of the LRR variable, ρ_x, that best fits the data is 0.6, implying that its half-life is about one-and-a-half years and its implied frequency is much higher than that of the business cycle, a conclusion similar to that arrived at in Table 12 for the period 1947-2006.

We find that the cointegrated model, too, performs quite poorly at explaining the cross-section of returns over the post-war subperiods. Table 14 reports results for the cross-section over the subperiod 1947-2006. The average pricing errors for the assets are economically large, varying from 13.6% for the "Growth" portfolio to 20.8% for the "Value" portfolio. The J-stat is 11.08 and the model is rejected at the 1% level of significance. Similar results are obtained for the subperiod 1947-1991 that are omitted for brevity.

8 Temporal Aggregation

Temopral aggregation of consumption is a relevant and important issue in the econometric analysis of long-run risks models. If the decision interval of the agent is of higher frequency than the frequency at which reliable data on aggregate consumption, dividend, and price-dividend ratio are available, specification of the low frequency stochastic discount factor is troublesome. Temporal aggregation of consumption growth makes consumption a more involved function of the low frequency predictive component (the LRR), x_t, and the high frequency risks, $z_{c,t+1}$, in equation (2). It also introduces autocorrelation in the dynamics of the innovations to (lower frequency) consumption and cross-correlations between the innovations to (lower frequency) consumption and its predictive component. Thus, although the Euler equations hold at a higher frequency corresponding to the decision interval of the agent, the need to use lower frequency consumption data breaks the linear relationship with the state variables, via temporal aggregation.

To examine whether our results are driven by problems of temporal aggregation, we repeat our estimation and tests using quarterly data. Since reliable quarterly data is only available over the post-war subperiod, we perform our analysis over 1947:2-2006:3. Table 15 reports the estimation results when the asset menu consists of the market portfolio and the risk free rate. As in Section 4, the lagged log price-dividend ratio of the market and the lagged log risk free rate are used as instruments giving 6 moment restrictions. To this set of pricing restrictions, we add moments corresponding to the unconditional mean, variance, and first-order autocovariance of consumption and dividend growth rates, the covariance between consumption and dividend growth rates, and the variance of squared consumption and dividend growth rates. Thus, we have a total of 15 moment conditions. The total number of parameters to be estimated is 12, including 9 time-series parameters and 3 preference parameters.

Note that the persistence parameter, ρ_x, of the LRR variable is 0.75. This implies that the variable has a half-life of just over 2 quarters and its implied frequency is much higher than that of the business cycle. This is similar to the findings in Table 12 which examines the performance of the cointegrated LRR model in explaining the returns on the market portfolio and the risk free rate using annual data over the post-war subperiod 1947-2006, where the value of the persistence parameter of the LRR variable, ρ_x, that best fits the data was found to be only 0.7, implying a half-life of only 2 years.

The average annualized pricing errors for the market portfolio and the risk free rate are 2.4% and -2.4%, respectively. Note that the average pricing errors obtained using annual data over the postwar subperiod 1947:2006 are much smaller at -0.6% and -1.3%, respectively (see Table 9). Thus, although the model prices the assets better at the annual frequency compared to the quarterly time horizon, the pricing errors obtained in the latter case are also economically small.

Table 16 reports results for the cross-section over the subperiod 1947:2-2006:3. In this case, the set of assets includes the market, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios. The average annualized pricing errors for the assets are economically large, including 15.2% for the "Value" portfolio, 14.4% for the "Small" portfolio, 9.6% for the "Large" portfolio, and 8.8% for the "Growth" portfolio. The J-stat is 53.13 and the model is rejected at the 1% level of significance. Also note that the value of the persistence parameter of the LRR variable, ρ_x, that best fits the data is 0.9, implying that its half-life is only about 7 quarters. The results are very similar to those in Table 11 that reports results for the cross-section using annual data over the subperiod 1947-2006. The average pricing errors for the assets in Table 11 are economically large, varying from 11.8% for the "Growth" portfolio to 18.2% for the "Value" portfolio. The J-stat is 18.04 and the model is rejected at the 1% level of significance. Also note that the value of the persistence parameter of the LRR variable, ρ_x, that best fits the data is 0.7, implying that its half-life is two years.

The results in this section suggest that our findings are unlikely to be driven by the problems associated with temporal aggregation.

9 Conclusion

In this paper we test and reject the Bansal and Yaron (2004) model of long-run risks in aggregate consumption and dividend growth and its extension that captures potential cointegration of the consumption and dividend levels over the sample period 1930-2006. The reversal of earlier empirical conclusions is partly due to the increase in the power of the tests resulting from two observations under the null hypothesis. First, the latent state variables, and, therefore, the pricing kernel are known affine functions of observables such as the interest rate and the market-wide price-dividend ratio. Second, the unconditional moments of consumption growth and aggregate dividend growth impose constraints in addition to the pricing constraints.

Recognizing that structural breaks have occurred in the prewar period and in the nineties, we repeat our tests for the subperiods 1947-2006 and 1947-1991. Whereas the models are still formally rejected, we find that they perform considerably better in explaining the equity premium but not the cross-section of returns. The results suggest that regime shifts warrant further investigation.

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A Appendix

A.1 Estimation of Time-Series Parameters

The decision interval of the agent is assumed to be annual. We estimate the model at the annual frequency, such that its annual growth rates of consumption and dividends match salient features of observed annual consumption and dividend data. There are 9 parameters to be estimated - μ_c, μ_d, ϕ, φ, ρ_x, ψ_x, σ, υ, and σ_w.

From the specification of the consumption growth process, we have

$$E\left(\Delta c_{t+1}\right) = \mu_c \tag{25}$$

We also, have

$$\begin{aligned} Var\left(\Delta c_{t+1}\right) &= Var\left(x_t\right) + Var\left(\sigma_t \eta_{t+1}\right) + 2Cov(x_t, \sigma_t \eta_{t+1}) \\ &= Var\left(x_t\right) + \sigma^2 + 0 \\ &= \frac{\psi_x^2 \sigma^2}{1-\rho_x^2} + \sigma^2 \end{aligned} \tag{26}$$

and,

$$Cov(\Delta c_{t+1}, \Delta c_{t+2}) = \rho_x \frac{\psi_x^2 \sigma^2}{1-\rho_x^2} \tag{27}$$

From the specification of the dividend process, we have

$$E\left(\Delta d_{t+1}\right) = \mu_d \tag{28}$$

$$Var\left(\Delta d_{t+1}\right) = \phi^2 \frac{\psi_x^2 \sigma^2}{1-\rho_x^2} + \sigma^2 \varphi^2 \tag{29}$$

$$Cov(\Delta d_{t+1}, \Delta d_{t+2}) = \phi^2 \rho_x \frac{\psi_x^2 \sigma^2}{1-\rho_x^2} \tag{30}$$

Also, from the consumption and dividend growth processes,

$$Cov(\Delta c_{t+1}, \Delta d_{t+1}) = \phi \frac{\psi_x^2 \sigma^2}{1-\rho_x^2} \tag{31}$$

Finally, we have

$$Var\left((\Delta c_{t+1})^2\right) = E\left[Var_t\left((\Delta c_{t+1})^2\right)\right] + Var\left[E_t\left((\Delta c_{t+1})^2\right)\right] \tag{32}$$

Now,

$$(\Delta c_{t+1})^2 = \mu_c^2 + x_t^2 + \sigma_t^2 z_{c,t+1}^2 + 2\mu_c x_t + 2x_t\sigma_t z_{c,t+1} + 2\mu_c\sigma_t z_{c,t+1} \tag{33}$$

Hence,

$$E_t\left((\Delta c_{t+1})^2\right) = \mu_c^2 + x_t^2 + \sigma_t^2 + 2\mu_c x_t$$

$$\begin{aligned} Var\left[E_t\left((\Delta c_{t+1})^2\right)\right] &= Var(x_t^2) + Var(\sigma_t^2) + 4\mu_c^2 Var(x_t) + 4\mu_c Cov(x_t, x_t^2) \\ &\quad +2Cov(x_t^2, \sigma_t^2) + 4\mu_c Cov(x_t, \sigma_t^2) \end{aligned} \tag{34}$$

Now, $Var(\sigma_t^2) = \frac{\sigma_w^2}{1-\upsilon^2}$, $Cov(x_t, \sigma_t^2) = 0$, $Cov(x_t^2, \sigma_t^2) = \frac{\psi_x^2\sigma_w^2\upsilon}{(1-\upsilon^2)(1-\upsilon\rho_x^2)}$, $Cov(x_t, x_t^2) = 0$, and

$$Var(x_t^2) = \frac{3\psi_x^4\sigma_w^2(1+\upsilon\rho_x^2)}{(1-\rho_x^4)(1-\upsilon^2)(1-\upsilon\rho_x^2)} + \frac{1}{1-\rho_x^4}\left[2\sigma^4 + \frac{4\rho_x^2\psi_x^4\sigma^4}{(1-\rho_x^2)}\right]$$

Substituting the above expressions into equation (34), we have

$$\begin{aligned} Var\left[E_t\left((\Delta c_{t+1})^2\right)\right] &= \frac{3\psi_x^4\sigma_w^2(1+\upsilon\rho_x^2)}{(1-\rho_x^4)(1-\upsilon^2)(1-\upsilon\rho_x^2)} + \frac{1}{1-\rho_x^4}\left[2\sigma^4 + \frac{4\rho_x^2\psi_x^4\sigma^4}{(1-\rho_x^2)}\right] \\ &\quad + \frac{\sigma_w^2}{1-\upsilon^2} + 4\mu_c^2\frac{\psi_x^2\sigma^2}{1-\rho_x^2} + \frac{2\psi_x^2\sigma_w^2\upsilon}{(1-\upsilon^2)(1-\upsilon\rho_x^2)} \end{aligned} \tag{35}$$

Also, from equation (33),

$$Var_t\left((\Delta c_{t+1})^2\right) = 2\sigma_t^4 + 4x_t^2\sigma_t^2 + 4\mu_c^2\sigma_t^2 + 8\mu_c x_t\sigma_t^2$$

Hence,

$$E\left[Var_t\left((\Delta c_{t+1})^2\right)\right] = 2\frac{\sigma_w^2}{1-\upsilon^2} + 2\sigma^4 + \frac{4\psi_x^2\sigma_w^2\upsilon}{(1-\upsilon^2)(1-\upsilon\rho_x^2)} + \frac{4\psi_x^2\sigma^4}{1-\rho_x^2} + 4\mu_c^2\sigma^2 \tag{36}$$

Substituting equations (35) and (36) into equation (32), we have

$$\begin{aligned} Var\left((\Delta c_{t+1})^2\right) &= \frac{3\psi_x^4\sigma_w^2(1+\upsilon\rho_x^2)}{(1-\rho_x^4)(1-\upsilon^2)(1-\upsilon\rho_x^2)} + \frac{1}{1-\rho_x^4}\left[2\sigma^4 + \frac{4\rho_x^2\psi_x^4\sigma^4}{(1-\rho_x^2)}\right] + \frac{3\sigma_w^2}{1-\upsilon^2} \\ &\quad +4\mu_c^2\frac{\psi_x^2\sigma^2}{1-\rho_x^2} + \frac{6\psi_x^2\sigma_w^2\upsilon}{(1-\upsilon^2)(1-\upsilon\rho_x^2)} + \frac{4\psi_x^2\sigma^4}{1-\rho_x^2} + 2\sigma^4 + 4\mu_c^2\sigma^2 \end{aligned} \tag{37}$$

Similar calculations yield,

$$
\begin{aligned}
Var\left[E_t\left((\Delta d_{t+1})^2\right)\right] &= \phi^4\left[\frac{3\psi_x^4\sigma_w^2(1+\upsilon\rho_x^2)}{(1-\rho_x^4)(1-\upsilon^2)(1-\upsilon\rho_x^2)}+\frac{1}{1-\rho_x^4}\left(2\sigma^4+\frac{4\rho_x^2\psi_x^4\sigma^4}{(1-\rho_x^2)}\right)\right] \\
&\quad +\frac{\sigma_w^2}{1-\upsilon^2}\varphi^4+4\mu_c^2\frac{\psi_x^2\sigma^2}{1-\rho_x^2}\phi^2+\frac{2\psi_x^2\sigma_w^2\upsilon}{(1-\upsilon^2)(1-\upsilon\rho_x^2)}\phi^2\varphi^2
\end{aligned}
$$

$$
\begin{aligned}
E\left[Var_t\left((\Delta d_{t+1})^2\right)\right] &= \left[2\frac{\sigma_w^2}{1-\upsilon^2}+2\sigma^4\right]\varphi^4+\left[\frac{4\psi_x^2\sigma_w^2\upsilon}{(1-\upsilon^2)(1-\upsilon\rho_x^2)}+\frac{4\psi_x^2\sigma^4}{1-\rho_x^2}\right]\phi^2\varphi^2 \\
&\quad +4\mu_d^2\varphi^2\sigma^2
\end{aligned}
$$

Hence, we have

$$
\begin{aligned}
Var\left((\Delta d_{t+1})^2\right) &= \phi^4\left[\frac{3\psi_x^4\sigma_w^2(1+\upsilon\rho_x^2)}{(1-\rho_x^4)(1-\upsilon^2)(1-\upsilon\rho_x^2)}+\frac{1}{1-\rho_x^4}\left(2\sigma^4+\frac{4\rho_x^2\psi_x^4\sigma^4}{(1-\rho_x^2)}\right)\right]+\frac{3\sigma_w^2}{1-\upsilon^2}\varphi^4 \\
&\quad +4\mu_c^2\frac{\psi_x^2\sigma^2}{1-\rho_x^2}\phi^2+\frac{6\psi_x^2\sigma_w^2\upsilon}{(1-\upsilon^2)(1-\upsilon\rho_x^2)}\phi^2\varphi^2+\frac{4\psi_x^2\sigma^4}{1-\rho_x^2}\phi^2\varphi^2 \\
&\quad +2\sigma^4\varphi^4+4\mu_d^2\varphi^2\sigma^2 \qquad (38)
\end{aligned}
$$

Equations (25)-(31), (37), and (38) give 9 moments restrictions in the 9 time-series parameters.

A.2 Details of Estimation Methodology

The model is given by the equations

$$
\begin{aligned}
x_{t+1} &= \rho_x x_t+\psi_x\sigma_t z_{x,t+1}, \\
\sigma_{t+1}^2 &= (1-\upsilon)\sigma^2+\upsilon\sigma_t^2+\sigma_w z_{\sigma,t+1}, \\
\Delta c_{t+1} &= \mu_c+x_t+\sigma_t z_{c,t+1}, \\
\Delta d_{t+1} &= \mu_d+\phi x_t+\varphi\sigma_t z_{d,t+1}.
\end{aligned}
$$

The shocks $z_{x,t+1}$, $z_{\sigma,t+1}$, $z_{c,t+1}$, $z_{d,t+1}$ are assumed to be *i.i.d.* $N(0,1)$ and mutually independent.

A.2.1 Expressions for $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_{0,m}$, $\mathbf{A}_{1,m}$, and $\mathbf{A}_{2,m}$

Bansal and Yaron (2004) show that z_t and $z_{m,t}$, are affine functions of the state variables, x_t and σ_t^2,

$$
\begin{aligned}
z_t &= A_0 + A_1 x_t + A_2 \sigma_t^2, \\
z_{m,t} &= A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2,
\end{aligned}
$$

where

$$
\begin{aligned}
A_1 &= \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho_x} \\
A_2 &= \frac{0.5 \left[\left(-\frac{\theta}{\psi} + \theta \right)^2 + (\theta \kappa_1 A_1 \psi_x)^2 \right]}{\theta \left(1 - \kappa_1 \upsilon\right)} \\
A_0 &= \frac{log(\delta) + \left(1 - \frac{1}{\psi}\right) \mu_c + \kappa_0 + \kappa_1 A_2 \sigma^2 (1 - \upsilon) + 0.5 \theta \kappa_1 A_2 \sigma_w^2}{1 - \kappa_1}
\end{aligned}
$$

$$
\begin{aligned}
A_{1,m} &= \frac{\phi - \frac{1}{\psi}}{1 - \kappa_{1,m} \rho_x} \\
A_{2,m} &= \frac{(1-\theta) A_2 \left(1 - \kappa_1 \upsilon\right) + 0.5 \left[\gamma^2 + \varphi^2 + \left(\left(\theta - 1\right) \kappa_1 A_1 + \kappa_{1,m} A_{1,m} \right)^2 \psi_x^2 \right]}{1 - \kappa_{1,m} \upsilon} \\
A_{0,m} &= \frac{\theta \log(\delta) + \left(-\frac{\theta}{\psi} + \theta - 1 \right) \mu_c + \left(\theta - 1\right) \kappa_0 + \left(\theta - 1\right) \left(\kappa_1 - 1\right) A_0 + \left(\theta - 1\right) \kappa_1 A_2 \sigma^2 (1 - \upsilon)}{1 - \kappa_{1,m}} \\
&\quad + \frac{\kappa_{0,m} + \mu_d + \kappa_{1,m} A_{2,m} \sigma^2 (1 - \upsilon) + 0.5 \left[\left(\theta - 1\right) \kappa_1 A_2 + \kappa_{1,m} A_{2,m} \right]^2 \sigma_w^2}{1 - \kappa_{1,m}}
\end{aligned}
$$

A.2.2 Risk Free Rate

To derive the expression for the risk free rate, note that

$$
E_t \left[\exp \left(\theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1} + r_{f,t} \right) \right] = 1
$$

Hence,

$$
\begin{aligned}
\exp\left(-r_{f,t}\right) &= E_t\left[\exp\left(\theta\log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{c,t+1}\right)\right] \\
&= \exp(\theta\log\delta - \frac{\theta}{\psi}\mu_c - \frac{\theta}{\psi}x_t + (\theta-1)\kappa_0 + (\theta-1)\kappa_1 A_0 \\
&\quad +(\theta-1)\kappa_1 A_1\rho_x x_t + (\theta-1)\kappa_1 A_2(1-\upsilon)\sigma^2 + (\theta-1)\kappa_1 A_2\upsilon\sigma_t^2 \\
&\quad -(\theta-1)A_0 - (\theta-1)A_1 x_t - (\theta-1)A_2\sigma_t^2 + (\theta-1)\mu_c + (\theta-1)x_t \\
&\quad +0.5\left[\left(-\frac{\theta}{\psi}+\theta-1\right)^2\sigma_t^2 + (\theta-1)^2\kappa_1^2A_1^2\psi_x^2\sigma_t^2 + (\theta-1)^2\kappa_1^2A_2^2\sigma_w^2\right])
\end{aligned}
$$

Therefore, the risk free rate is

$$
\begin{aligned}
r_{f,t} &= -\theta\log\delta - \left(-\frac{\theta}{\psi}+\theta-1\right)\mu_c - (\theta-1)\kappa_0 - (\theta-1)(\kappa_1-1)A_0 - (\theta-1)\kappa_1 A_2(1-\upsilon)\sigma^2 \\
&\quad -0.5(\theta-1)^2\kappa_1^2A_2^2\sigma_w^2 - \left[(-\frac{\theta}{\psi}+\theta-1) + (\theta-1)(\kappa_1\rho_x-1)A_1\right]x_t \\
&\quad -\left[(\theta-1)(\kappa_1\upsilon-1)A_2 + 0.5\left(\left(-\frac{\theta}{\psi}+\theta-1\right)^2 + (\theta-1)^2\kappa_1^2A_1^2\psi_x^2\right)\right]\sigma_t^2 \\
&= A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2
\end{aligned}
$$

where

$$
\begin{aligned}
A_{0,f} &= -\theta\log\delta - \left(-\frac{\theta}{\psi}+\theta-1\right)\mu_c - (\theta-1)\kappa_0 - (\theta-1)(\kappa_1-1)A_0 - (\theta-1)\kappa_1 A_2(1-\upsilon)\sigma^2 \\
&\quad -0.5(\theta-1)^2\kappa_1^2A_2^2\sigma_w^2 \\
A_{1,f} &= -\left[(-\frac{\theta}{\psi}+\theta-1) + (\theta-1)(\kappa_1\rho_x-1)A_1\right] \\
A_{2,f} &= -\left[(\theta-1)(\kappa_1\upsilon-1)A_2 + 0.5\left(\left(-\frac{\theta}{\psi}+\theta-1\right)^2 + (\theta-1)^2\kappa_1^2A_1^2\psi_x^2\right)\right]
\end{aligned}
$$

A.2.3 Latent state variables in terms of observable variables

The model implies

$$
\begin{aligned}
z_{m,t} &= A_{0,m} + A_{1,m}x_t + A_{2,m}\sigma_t^2, \\
r_{f,t} &= A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2.
\end{aligned}
$$

These equations may be inverted to express the state variables in terms of the observables,

$$x_t = \alpha_0 + \alpha_1 r_{f,t+1} + \alpha_2 z_{m,t},$$

where

$$\begin{aligned}
\alpha_0 &= \frac{A_{2,m}A_{0,f} - A_{0,m}A_{2,f}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}}, \\
\alpha_1 &= \frac{-A_{2,m}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}}, \\
\alpha_2 &= \frac{A_{2,f}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}},
\end{aligned}$$

and

$$\sigma_t^2 = \beta_0 + \beta_1 r_{f,t+1} + \beta_2 z_{m,t},$$

where

$$\begin{aligned}
\beta_0 &= \frac{A_{0,m}A_{1,f} - A_{1,m}A_{0,f}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}}, \\
\beta_1 &= \frac{A_{1,m}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}}, \\
\beta_2 &= \frac{-A_{1,f}}{A_{1,m}A_{2,f} - A_{2,m}A_{1,f}}.
\end{aligned}$$

A.2.4 The pricing kernel in terms of observables

The pricing kernel is given by (12),

$$\begin{aligned}
m_{t+1} &= (\theta \log \delta + (\theta - 1)\left[\kappa_0 + (\kappa_1 - 1)\, A_0\right]) + \left(-\frac{\theta}{\psi} + (\theta - 1)\right)\Delta c_{t+1} \\
&\quad + (\theta - 1)\kappa_1 A_1 x_{t+1} + (\theta - 1)\kappa_1 A_2 \sigma_{t+1}^2 - (\theta - 1) A_1 x_t - (\theta - 1) A_2 \sigma_t^2
\end{aligned}$$

Substituting the expressions for x_t and σ_t^2 from Section A.1.2 into the pricing kernel, we have

$$m_{t+1} = c_1 + c_2 \Delta c_{t+1} + c_3 \left(r_{f,t+1} - \frac{1}{\kappa_1} r_{f,t}\right) + c_4 \left(z_{m,t+1} - \frac{1}{\kappa_1} z_{m,t}\right)$$

where

$$\begin{aligned}
c_1 &= \theta \log \delta + (\theta - 1)[\kappa_0 + (\kappa_1 - 1)(A_0 + A_1\alpha_0 + A_2\beta_0)] \\
c_2 &= -\frac{\theta}{\psi} + (\theta - 1) \\
c_3 &= (\theta - 1)\kappa_1[A_1\alpha_1 + A_2\beta_1] \\
c_4 &= (\theta - 1)\kappa_1[A_1\alpha_2 + A_2\beta_2]
\end{aligned}$$

A.3 Estimation Methodology for Cointegrated Model

The model is given by the equations

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t z_{c,t+1} \\
x_{t+1} &= \rho_x x_t + \psi_x \sigma_t z_{x,t+1} \\
\sigma_{t+1}^2 &= \mu_\sigma + \rho_\sigma \sigma_t^2 + \sigma_w z_{\sigma,t+1} \\
d_t - c_t &= \mu_{dc} + s_t \\
s_{t+1} &= \lambda_{sx} x_t + \rho_s s_t + \psi_s \sigma_t z_{s,t+1} \\
\Delta d_{t+1} &= \mu_c + (1 + \lambda_{sx}) x_t + (\rho_s - 1) s_t + \sigma_t z_{c,t+1} + \psi_s \sigma_t z_{s,t+1}
\end{aligned} \tag{39}$$

With Epstein and Zin (1989) preferences, the asset pricing Euler condition for asset j is

$$E_t\left[\exp(m_{t+1} + r_{j,t+1})\right] = 1,$$

where

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}$$

Using the log-affine approximations for the continuous return on the consumption claim, $r_{c,t+1}$, and that on the market portfolio, $r_{m,t+1}$,

$$\begin{aligned}
r_{c,t+1} &= \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1} \\
r_{m,t+1} &= \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + \Delta d_{t+1}
\end{aligned}$$

where z_t and $z_{m,t}$ are the log price-dividend ratios of the consumption and the dividend claims, respectively, and conjecturing that these ratios are affine functions of the state variables, x_t, σ_t^2, and s_t,

$$
\begin{aligned}
z_t &= A_0 + A_1 x_t + A_2 \sigma_t^2 + A_3 s_t \\
z_{m,t} &= A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2 + A_{3,m} s_t
\end{aligned}
$$

the coefficients A_0, A_1, A_2, A_3, $A_{0,m}$, $A_{1,m}$, $A_{2,m}$, and $A_{3,m}$ may be computed using the method of undetermined coefficients.

A.3.1 The Consumption Claim

From equations (3) and (4), for the unobservable return on the consumption claim, $r_{c,t+1}$, the Euler equation is,

$$
E_t \left[\exp \left(\theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + \theta r_{c,t+1} \right) \right] = 1
$$

Substituting the expression for $r_{c,t+1}$ from equation (5) into the above Euler condition and noting that z_t is given by equation (18), we have

$$
\begin{aligned}
& E_t[\exp(\theta \log \delta - \frac{\theta}{\psi} \mu_c - \frac{\theta}{\psi} x_t - \frac{\theta}{\psi} \sigma_t z_{c,t+1} + \theta \kappa_0 + \theta \kappa_1 A_0 \\
& + \theta \kappa_1 A_1 \rho_x x_t + \theta \kappa_1 A_1 \psi_x \sigma_t z_{x,t+1} + \theta \kappa_1 A_2 \mu_\sigma + \theta \kappa_1 A_2 \rho_\sigma \sigma_t^2 + \theta \kappa_1 A_2 \sigma_w z_{\sigma,t+1} \\
& + \theta \kappa_1 A_3 \lambda_{sx} x_t + \theta \kappa_1 A_3 \rho_s s_t + \theta \kappa_1 A_3 \psi_s \sigma_t z_{s,t+1} - \theta A_0 - \theta A_1 x_t - \theta A_2 \sigma_t^2 - \theta A_3 s_t \\
& + \theta \mu_c + \theta x_t + \theta \sigma_t z_{c,t+1})] \\
= & \; 1
\end{aligned}
$$

Using the assumed conditional log-normality of the stochastic processes, the above expression may be simplified to

$$
\begin{aligned}
\exp(\quad & \theta \log \delta + \left(-\frac{\theta}{\psi} + \theta \right) \mu_c + \theta \kappa_0 + \theta \left(\kappa_1 - 1 \right) A_0 + \theta \kappa_1 A_2 \mu_\sigma \\
& + \left[-\frac{\theta}{\psi} + \theta + \theta \left(\kappa_1 \rho_x - 1 \right) A_1 + \theta \kappa_1 A_3 \lambda_{sx} \right] x_t \\
& + \theta \left(\kappa_1 \rho_s - 1 \right) A_3 s_t + \theta \left(\kappa_1 \rho_\sigma - 1 \right) A_2 \sigma_t^2 \\
& + 0.5 \left\{ \left(-\frac{\theta}{\psi} + \theta \right)^2 \sigma_t^2 + \left(\theta \kappa_1 A_1 \psi_x \right)^2 \sigma_t^2 + \left(\theta \kappa_1 A_2 \sigma_w \right)^2 \right\}) \\
= & \; 1 \qquad (40)
\end{aligned}
$$

Since the Euler equation (40) must hold for all values of the state variables, we have

$$\theta\left(\kappa_1\rho_s - 1\right)A_3 = 0$$

Hence,

$$A_3 = 0 \tag{41}$$

Similarly,

$$-\frac{\theta}{\psi} + \theta + \theta\left(\kappa_1\rho_x - 1\right)A_1 + \theta\kappa_1 A_3\lambda_{sx} = 0$$

implying

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1\rho_x} \tag{42}$$

Also

$$\theta\left(\kappa_1\rho_\sigma - 1\right)A_2 + 0.5\left\{\left(-\frac{\theta}{\psi} + \theta\right)^2 + \left(\theta\kappa_1 A_1\psi_x\right)^2\right\} = 0$$

yielding

$$A_2 = \frac{0.5\left[\left(-\frac{\theta}{\psi} + \theta\right)^2 + \left(\theta\kappa_1 A_1\psi_x\right)^2\right]}{\theta\left(1 - \kappa_1\rho_\sigma\right)} \tag{43}$$

and

$$\theta\log\delta + \left(-\frac{\theta}{\psi} + \theta\right)\mu_c + \theta\kappa_0 + \theta\left(\kappa_1 - 1\right)A_0 + \theta\kappa_1 A_2\mu_\sigma + 0.5\left(\theta\kappa_1 A_2\sigma_w\right)^2 = 0$$

implying

$$A_0 = \frac{\log\delta + \left(-\frac{1}{\psi} + 1\right)\mu_c + \kappa_0 + \kappa_1 A_2\mu_\sigma + 0.5\theta\left(\kappa_1 A_2\sigma_w\right)^2}{1 - \kappa_1} \tag{44}$$

A.3.2 The Dividend Claim

The Euler equation for the observable return on the aggregate dividend claim, $r_{m,t+1}$, is,

$$E_t\left[\exp\left(\theta\log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)r_{c,t+1} + r_{m,t+1}\right)\right] = 1 \tag{45}$$

Substituting the expression for $r_{m,t+1}$ from equation (6) into the above Euler condition and noting that $z_{m,t}$ is given by equation (19), we have

$$\begin{aligned}
& E_t[\exp(\theta \log \delta - \frac{\theta}{\psi}\mu_c - \frac{\theta}{\psi}x_t - \frac{\theta}{\psi}\sigma_t z_{c,t+1} + (\theta-1)\kappa_0 + (\theta-1)\kappa_1 A_0 \\
& +(\theta-1)\kappa_1 A_1 \rho_x x_t + (\theta-1)\kappa_1 A_1 \psi_x \sigma_t z_{x,t+1} \\
& +(\theta-1)\kappa_1 A_2 \mu_\sigma + (\theta-1)\kappa_1 A_2 \rho_\sigma \sigma_t^2 + (\theta-1)\kappa_1 A_2 \sigma_w z_{\sigma,t+1} \\
& +(\theta-1)\kappa_1 A_3 \lambda_{sx} x_t + (\theta-1)\kappa_1 A_3 \rho_s s_t + (\theta-1)\kappa_1 A_3 \psi_s \sigma_t z_{s,t+1} \\
& -(\theta-1)A_0 - (\theta-1)A_1 x_t - (\theta-1)A_2 \sigma_t^2 - (\theta-1)A_3 s_t \\
& +(\theta-1)\mu_c + (\theta-1)x_t + (\theta-1)\sigma_t z_{c,t+1} \\
& +\kappa_{0,m} + \kappa_{1,m}A_{0,m} + \kappa_{1,m}A_{1,m}\rho_x x_t + \kappa_{1,m}A_{1,m}\psi_x \sigma_t z_{x,t+1} + \kappa_{1,m}A_{2,m}\mu_\sigma \\
& +\kappa_{1,m}A_{2,m}\rho_\sigma \sigma_t^2 + \kappa_{1,m}A_{2,m}\sigma_w z_{\sigma,t+1} + \kappa_{1,m}A_{3,m}\lambda_{sx} x_t + \kappa_{1,m}A_{3,m}\rho_s s_t \\
& +\kappa_{1,m}A_{3,m}\psi_s \sigma_t z_{s,t+1} - A_{0,m} - A_{1,m}x_t - A_{2,m}\sigma_t^2 - A_{3,m}s_t \\
& +\mu_c + (1+\lambda_{sx})x_t + (\rho_s - 1)s_t + \sigma_t z_{c,t+1} + \psi_s \sigma_t z_{s,t+1})] \\
= \; & 1
\end{aligned}$$

Using the assumed conditional log-normality of the stochastic processes, the left-hand-side of the above expression simplifies to

$$\begin{aligned}
\exp(\quad & \theta \log \delta + \left(-\frac{\theta}{\psi} + \theta\right)\mu_c + (\theta-1)\kappa_0 + (\theta-1)\left(\kappa_1 - 1\right)A_0 + (\theta-1)\kappa_1 A_2 \mu_\sigma \\
& +\kappa_{0,m} + (\kappa_{1,m} - 1)A_{0,m} + \kappa_{1,m}A_{2,m}\mu_\sigma \\
& + \left[\left(-\frac{\theta}{\psi} + \theta - 1\right) + (\theta-1)\left(\kappa_1\rho_x - 1\right)A_1 + (\theta-1)\kappa_1 A_3 \lambda_{sx} + \left(\kappa_{1,m}\rho_x - 1\right)A_{1,m} + (1+\lambda_{sx})\right] x_t \\
& + \left[\kappa_{1,m}A_{3,m}\lambda_{sx}\right] x_t + \left[(\theta-1)\left(\kappa_1\rho_s - 1\right)A_3 + \left(\kappa_{1,m}\rho_s - 1\right)A_{3,m} + \rho_s - 1\right] s_t \\
& + \left[(\theta-1)\left(\kappa_1\rho_\sigma - 1\right)A_2 + \left(\kappa_{1,m}\rho_\sigma - 1\right)A_{2,m}\right]\sigma_t^2 \\
& +0.5\{\left(-\frac{\theta}{\psi} + \theta\right)^2 \sigma_t^2 + \left[(\theta-1)\kappa_1 A_3 + \kappa_{1,m}A_{3,m} + 1\right]^2 \psi_s^2 \sigma_t^2 \\
& + \left[(\theta-1)\kappa_1 A_1 + \kappa_{1,m}A_{1,m}\right]^2 \psi_x^2 \sigma_t^2 + \left[(\theta-1)\kappa_1 A_2 + \kappa_{1,m}A_{2,m}\right]^2 \sigma_w^2\}) \\
= \quad & 1
\end{aligned} \tag{46}$$

Since the Euler equation (46) must hold for all values of the state variables, we have

$$\left[(\theta-1)\left(\kappa_1\rho_s - 1\right)A_3 + \left(\kappa_{1,m}\rho_s - 1\right)A_{3,m} + \rho_s - 1\right] = 0$$

$$A_{3,m} = \frac{\rho_s - 1}{1 - \kappa_{1,m}\rho_s} \tag{47}$$

$$\left(-\frac{\theta}{\psi}+\theta-1\right)+(\theta-1)\left(\kappa_1\rho_x-1\right)A_1+(\theta-1)\kappa_1A_3\lambda_{sx}+\left(\kappa_{1,m}\rho_x-1\right)A_{1,m}+\kappa_{1,m}A_{3,m}\lambda_{sx}+1+\lambda_{sx}=0$$

$$A_{1,m}=\frac{1-\frac{1}{\psi}+\lambda_{sx}(1+\kappa_{1,m}A_{3,m})}{1-\kappa_{1,m}\rho_x} \tag{48}$$

$$\begin{aligned}&(\theta-1)\left(\kappa_1\rho_\sigma-1\right)A_2+\left(\kappa_{1,m}\rho_\sigma-1\right)A_{2,m}+0.5\{\left(-\frac{\theta}{\psi}+\theta\right)^2\\&+\left[(\theta-1)\kappa_1A_3+\kappa_{1,m}A_{3,m}+1\right]^2\psi_s^2+\left[(\theta-1)\kappa_1A_1+\kappa_{1,m}A_{1,m}\right]^2\psi_x^2\}\\=\;&0\end{aligned}$$

$$\begin{aligned}A_{2,m} &= \frac{(\theta-1)\left(\kappa_1\rho_\sigma-1\right)A_2+C}{1-\kappa_{1,m}\rho_\sigma} \\ C &= 0.5\{\left(-\frac{\theta}{\psi}+\theta\right)^2+\left[\kappa_{1,m}A_{3,m}+1\right]^2\psi_s^2 \\ &\quad+\left[(\theta-1)\kappa_1A_1+\kappa_{1,m}A_{1,m}\right]^2\psi_x^2\}\end{aligned} \tag{49}$$

$$\begin{aligned}&\theta\log\delta+\left(-\frac{\theta}{\psi}+\theta\right)\mu_c+(\theta-1)\kappa_0+(\theta-1)\left(\kappa_1-1\right)A_0+(\theta-1)\kappa_1A_2\mu_\sigma\\&+\kappa_{0,m}+(\kappa_{1,m}-1)A_{0,m}+\kappa_{1,m}A_{2,m}\mu_\sigma+0.5\left[(\theta-1)\kappa_1A_2+\kappa_{1,m}A_{2,m}\right]^2\sigma_w^2\\=\;&0\end{aligned}$$

$$\begin{aligned}A_{0,m} &= \frac{\theta\log\delta+\left(-\frac{\theta}{\psi}+\theta\right)\mu_c+(\theta-1)\kappa_0+(\theta-1)\left(\kappa_1-1\right)A_0}{1-\kappa_{1,m}} \\ &\quad+\frac{(\theta-1)\kappa_1A_2\mu_\sigma+\kappa_{0,m}+\kappa_{1,m}A_{2,m}\mu_\sigma+0.5\left[(\theta-1)\kappa_1A_2+\kappa_{1,m}A_{2,m}\right]^2\sigma_w^2}{1-\kappa_{1,m}}\end{aligned} \tag{50}$$

A.3.3 The Risk Free Rate

To derive the expression for the risk free rate, note that

$$E_t\left[\exp\left(\theta\log\delta-\frac{\theta}{\psi}\Delta c_{t+1}+(\theta-1)r_{c,t+1}+r_{f,t}\right)\right]=1$$

Hence,

$$\begin{aligned}
\exp\left(-r_{f,t}\right) &= E_t\left[\exp\left(\theta\log\delta-\frac{\theta}{\psi}\Delta c_{t+1}+(\theta-1)r_{c,t+1}\right)\right] \\
&= \exp(\theta\log\delta-\frac{\theta}{\psi}\mu_c-\frac{\theta}{\psi}x_t \\
&\quad+(\theta-1)\kappa_0+(\theta-1)\kappa_1A_0+(\theta-1)\kappa_1A_1\rho_x x_t+(\theta-1)\kappa_1A_2\mu_\sigma \\
&\quad+(\theta-1)\kappa_1A_2\rho_\sigma\sigma_t^2+(\theta-1)\kappa_1A_3\lambda_{sx}x_t+(\theta-1)\kappa_1A_3\rho_s s_t \\
&\quad-(\theta-1)A_0-(\theta-1)A_1x_t-(\theta-1)A_2\sigma_t^2-(\theta-1)A_3s_t \\
&\quad+(\theta-1)\mu_c+(\theta-1)x_t \\
&\quad+0.5\left[\left(-\frac{\theta}{\psi}+\theta-1\right)^2\sigma_t^2++(\theta-1)^2\kappa_1^2A_1^2\psi_x^2\sigma_t^2+(\theta-1)^2\kappa_1^2A_2^2\sigma_w^2\right])
\end{aligned}$$

Therefore, the risk free rate is

$$\begin{aligned}
r_{f,t} &= -\theta\log\delta-\left(-\frac{\theta}{\psi}+\theta-1\right)\mu_c-(\theta-1)\kappa_0-(\theta-1)(\kappa_1-1)A_0-(\theta-1)\kappa_1A_2\mu_\sigma \\
&\quad-0.5(\theta-1)^2\kappa_1^2A_2^2\sigma_w^2-\left[(-\frac{\theta}{\psi}+\theta-1)+(\theta-1)(\kappa_1\rho_x-1)A_1\right]x_t \\
&\quad-\left[(\theta-1)(\kappa_1\rho_s-1)A_3\right]s_t+ \\
&\quad-\left[(\theta-1)(\kappa_1\rho_\sigma-1)A_2+0.5\left\{\left(-\frac{\theta}{\psi}+\theta-1\right)^2+(\theta-1)^2\kappa_1^2A_1^2\psi_x^2\right\}\right]\sigma_t^2 \\
&= A_{0,f}+A_{1,f}x_t+A_{2,f}\sigma_t^2
\end{aligned}$$

where

$$\begin{aligned}
A_{0,f} &= -\theta\log\delta-\left(-\frac{\theta}{\psi}+\theta-1\right)\mu_c-(\theta-1)\kappa_0-(\theta-1)(\kappa_1-1)A_0-(\theta-1)\kappa_1A_2\mu_\sigma \\
&\quad-0.5(\theta-1)^2\kappa_1^2A_2^2\sigma_w^2 \\
A_{1,f} &= -\left[(-\frac{\theta}{\psi}+\theta-1)+(\theta-1)(\kappa_1\rho_x-1)A_1\right] \\
A_{2,f} &= -\left[(\theta-1)(\kappa_1\rho_\sigma-1)A_2+0.5\left\{\left(-\frac{\theta}{\psi}+\theta-1\right)^2+(\theta-1)^2\kappa_1^2A_1^2\psi_x^2\right\}\right]
\end{aligned}$$

A.3.4 Latent State Variables in terms of Observable Variables

We have

$$\begin{aligned}
z_{m,t} &= A_{0,m} + A_{1,m}x_t + A_{2,m}\sigma_t^2 + A_{3,m}s_t \\
r_{f,t} &= A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2
\end{aligned}$$

The above equations may be inverted to express the unobservable state variables, x_t and σ_t^2, in terms of the observables, $z_{m,t}$, $r_{f,t}$, and s_t.

Define,

$$D \equiv A_{1,m}A_{2,f} - A_{1,f}A_{2,m}$$

We have,

$$\begin{aligned}
x_t &= \alpha_0 + \alpha_1 r_{f,t} + \alpha_2 z_{m,t} + \alpha_3 s_t \\
\alpha_0 &= \frac{A_{0,f}A_{2,m} - A_{0,m}A_{2,f}}{D} \\
\alpha_1 &= \frac{-A_{2,m}}{D} \\
\alpha_2 &= \frac{A_{2,f}}{D} \\
\alpha_3 &= \frac{-A_{3,m}A_{2,f}}{D}
\end{aligned}$$

$$\begin{aligned}
\sigma_t^2 &= \beta_0 + \beta_1 r_{f,t} + \beta_2 z_{m,t} + \beta_3 z_{v-g,t} \\
\beta_0 &= \frac{A_{0,m}A_{1,f} - A_{1,m}A_{0,f}}{D} \\
\beta_1 &= \frac{A_{1,m}}{D} \\
\beta_2 &= \frac{-A_{1,f}}{D} \\
\beta_3 &= \frac{A_{1,f}A_{3,m}}{D}
\end{aligned}$$

Now, from equations (4), (5), and (18), the pricing kernel is given by the expression

$$\begin{aligned}
m_{t+1} &= \left(\theta \log \delta + (\theta - 1)\left[\kappa_0 + (\kappa_1 - 1) A_0\right]\right) + \left(-\frac{\theta}{\psi} + (\theta - 1)\right) \Delta c_{t+1} \\
&\quad + (\theta - 1)\kappa_1 A_1 x_{t+1} + (\theta - 1)\kappa_1 A_2 \sigma_{t+1}^2 \\
&\quad - (\theta - 1)A_1 x_t - (\theta - 1)A_2 \sigma_t^2
\end{aligned}$$

Substituting the expressions for x_t and σ_t^2 from equations (21) and (22) into the above expression for the pricing kernel, we have

$$m_{t+1} = c_1 + c_2\Delta c_{t+1} + c_3\left(r_{f,t+1} - \frac{1}{\kappa_1}r_{f,t}\right) + c_4\left(z_{m,t+1} - \frac{1}{\kappa_1}z_{m,t}\right) + c_5\left(s_{t+1} - \frac{1}{\kappa_1}s_t\right)$$

where

$$\begin{aligned}
c_1 &= \theta\log\delta + (\theta-1)[\kappa_0 + (\kappa_1 - 1)(A_0 + A_1\alpha_0 + A_2\beta_0)] \\
c_2 &= -\frac{\theta}{\psi} + (\theta - 1) \\
c_3 &= (\theta-1)\kappa_1[A_1\alpha_1 + A_2\beta_1] \\
c_4 &= (\theta-1)\kappa_1[A_1\alpha_2 + A_2\beta_2] \\
c_5 &= (\theta-1)\kappa_1[A_1\alpha_3 + A_2\beta_3]
\end{aligned}$$

A.4 Estimation of Time-Series Parameters of the Extended Model

In this specification, there are 10 parameters to be estimated - μ_c, μ_{dc}, ρ_x, ψ_x, μ_σ, ρ_σ, σ_w, λ_{sx}, ρ_s, and ψ_s.

We have

$$E(\Delta c_{t+1}) = \mu_c \tag{51}$$

Define $\sigma^2 = \frac{\mu_\sigma}{1-\rho_\sigma}$. We then have

$$\begin{aligned}
Var(\Delta c_{t+1}) &= Var(x_t) + Var\left(\sigma_t\eta_{t+1}\right) + 2Cov(x_t, \sigma_t\eta_{t+1}) \\
&= Var(x_t) + \sigma^2 + 0 \\
&= \frac{\psi_x^2\sigma^2}{1-\rho_x^2} + \sigma^2 \qquad (52)
\end{aligned}$$

and,

$$Cov(\Delta c_{t+1}, \Delta c_{t+2}) = \rho_x\frac{\psi_x^2\sigma^2}{1-\rho_x^2} \tag{53}$$

$$Cov(\Delta c_{t+1}, \Delta c_{t+3}) = \rho_x^2\frac{\psi_x^2\sigma^2}{1-\rho_x^2} \tag{54}$$

From the specification of the dividend growth process, we have

$$\begin{aligned} Var\left(\Delta d_{t+1}\right) &= \left(1+\lambda_{sx}\right)^2 Var\left(x_t\right) + \left(\rho_s - 1\right)^2 Var\left(s_t\right) + \\ &\quad \left(1+\psi_s^2\right)\sigma^2 + 2\left(1+\lambda_{sx}\right)\left(\rho_s - 1\right) Cov(x_t, s_t) \end{aligned} \tag{55}$$

where $Var\left(x_t\right) = \frac{\psi_x^2\sigma^2}{1-\rho_x^2}$, $Cov(x_t, s_t) = \frac{\lambda_{sx}\rho_x}{1-\rho_x\rho_s} Var\left(x_t\right)$, and

$$Var\left(s_t\right) = \frac{\lambda_{sx}^2 Var\left(x_t\right) + \psi_s^2\sigma^2 + \frac{2\lambda_{sx}^2\rho_x\rho_s Var(x_t)}{1-\rho_x\rho_s}}{1-\rho_s^2}$$

Also,

$$\begin{aligned} Cov(\Delta d_{t+1}, \Delta d_{t+2}) &= \left(1+\lambda_{sx}\right)^2 Cov(x_{t+1}, x_t) + \left(\rho_s - 1\right)^2 Cov(s_{t+1}, s_t) \\ &\quad + \left(1+\lambda_{sx}\right)\left(\rho_s - 1\right)\left[Cov(x_{t+1}, s_t) + Cov(x_t, s_{t+1})\right] \\ &\quad + \left(\rho_s - 1\right)\psi_s Cov(s_{t+1}, \sigma_t z_{s,t+1}) \end{aligned} \tag{56}$$

where $Cov(x_{t+1}, x_t) = \rho_x Var\left(x_t\right)$, $Cov(s_{t+1}, s_t) = \lambda_{sx} Cov(x_t, s_t) + \rho_s Var\left(s_t\right)$, $Cov(x_t, s_{t+1}) = \lambda_{sx} Var\left(x_t\right) + \rho_s Cov(x_t, s_t)$, $Cov(x_{t+1}, s_t) = \rho_x Cov(x_t, s_t)$, and $Cov(s_{t+1}, \sigma_t z_{s,t+1}) = \psi_s \sigma^2$.

Finally,

$$Cov(\Delta c_{t+1}, \Delta d_{t+1}) = \left(1+\lambda_{sx}\right) Var\left(x_t\right) + \left(\rho_s - 1\right) Cov(x_t, s_t) + \sigma^2 \tag{57}$$

and

$$E(d_t - c_t) = \mu_{dc} \tag{58}$$

Equations (51)-(58) give 8 moment restrictions in the 8 parameters μ_c, μ_{dc}, ρ_x, ψ_x, μ_σ, λ_{sx}, ρ_s, ψ_s

A.5 Simulation Design

We obtain the finite-sample distribution of the J-stat for the overidentifying restrictions with Monte Carlo simulation. We calibrate the parameters of the time series to their GMM point estimates and set the initial conditions of the state variables to their unconditional means, $x_0 = 0$ and $\sigma_0^2 = \sigma^2$. We simulate the time-series of the LRR variable, the stochastic volatility process, and the aggregate consumption and dividend growth rates to obtain a simulated sample of the same size as the historical sample. For the 2-asset system, we simulate the time-series of log returns on the market portfolio and the log risk free rate, using the log-linearization in equation (6) and the model solution in equation (9), respectively. For the 6-asset system, we simulate the series for the log returns on the Small, Large, Growth, and Value portfolios, using similar log-linearizations as for the market portfolio. We then perform the GMM estimation of the time-series and preference parameters using jointly the pricing and the time-series restrictions for the two-asset and six-asset systems, as in the empirical Sections 5 and 6, respectively. We also compute the J-stat for the overidentifying restrictions. We repeat the simulation 100 times and obtain the 90%, 95%, and 99% critical values of the J-stat from its finite-sample distribution. We perform the simulation for the 2-asset and the 6-asset systems for the full-sample period 1930-2006, as well as the two postwar subperiods, 1947-2006 and 1947-1991.

Table 1: Descriptive Statistics

	$log(returns)$		$log(P/D)$		$log(D_{t+1}/D_t)$	
	$Mean$	$Std.Dev.$	$Mean$	$Std.Dev$	$Mean$	$Std.Dev$
$SizePortfolios$						
$Small$	0.105	0.333	4.147	0.711	0.083	0.347
$L\arg e$	0.060	0.184	3.289	0.440	0.012	0.136
$B/M\ Portfolios$						
$Growth$	0.052	0.206	3.725	0.630	0.007	0.206
$Value$	0.093	0.302	3.588	1.135	0.070	0.568
$Market$	0.066	0.193	3.267	0.384	0.014	0.108
$Risk\ free\ rate$	0.008	0.050				

This table reports the descriptive statistics for the annual log returns, the log price-dividend ratios, and the log dividend growth rates of the market, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios. The sample period is 1930-2006.

Table 2: Tests of the LRR Model on the 2-Asset System over 1930-2006

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	μ_d	ϕ	φ	ρ_x	ψ_x	σ	ρ_σ	σ_w
Estimate	0.99 (0.064)	8 (7.485)	0.6 (1.533)	0.015 (0.003)	0.014 (0.012)	2.734 (1.695)	2.840 (1.364)	0.90 (0.733)	0.307 (1.231)	0.016 (0.004)	0.70 (144.3)	3.2×10^{-5} (0.004)
Pr*icing Errors*		*Mean*	*Std.Err.*									
Market		0.130	0.116									
Risk free rate		0.167	0.148									
$Dist - stat$	10.41 (0.015)											

The table reports GMM estimates of the model using annual data over the period 1930-2006. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom. The 90%, 95%, and 99% critical values of the finite-sample distribution are 5.00, 5.01, and 5.07, respectively.

Table 3: Tests of the LRR Model on the 2-Asset System

	Efficient weighting matrix			
	c_1	c_2	c_3	c_4
Estimated	-1.447 (0.638)	20.36 (11.79)	-15.81 (6.300)	-4.725 (1.616)
Pr*icing Errors*		*Mean*		*Std.Err.*
Market		0.002		0.163
Risk free rate		0.004		0.169
$J - stat$	0.160 (0.923)			

The table reports GMM estimates of the LRR Model using annual data over the period 1930-2006. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The statistic has an asymptotic χ^2-distribution with two degrees of freedom.

Table 4: Tests of the LRR Model on the 6-Asset System over 1930-2006

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	μ_d	ϕ	φ	ρ_x	ψ_x	σ	ρ_σ	σ_w
Estimate	0.99 (0.025)	10 (1.112)	0.9 (0.872)	0.015 (0.003)	0.014 (0.011)	3.280 (0.721)	11.93 (1.133)	0.95 (0.172)	0.307 (0.607)	0.009 (0.001)	0.90 (0.132)	3.2×10^{-5} (0.0002)
Pricing Errors		*Mean*	*Std.Err.*									
Small		0.123	0.100									
Large		0.059	0.091									
Growth		0.043	0.086									
Value		0.101	0.095									
Market		0.063	0.091									
Risk free rate		0.078	0.114									
$Dist - stat$	12.93 (0.005)											

The table reports GMM estimates of the model using annual data over the period 1930-2006. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios.The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom. The 90%, 95%, and 99% critical values of the finite-sample distribution are 5.21, 5.43, and 6.40, respectively.

Table 5: Tests of the LRR Model on the 6-Asset System

	Efficient weighting matrix			
	c_1	c_2	c_3	c_4
Estimated	-0.840 (0.498)	10.94 (18.22)	-13.00 (10.67)	-3.134 (1.256)
Pricing Errors		*Mean*		*Std.Err.*
Small		0.063		0.105
Large		-0.013		0.090
Growth		-0.016		0.095
Value		0.023		0.089
Market		-0.007		0.091
Risk free rate		-0.027		0.097
$J-stat$	7.151 (0.028)			

The table reports GMM estimates of the LRR Model using annual data over the period 1930-2006. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios. The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The statistic has an asymptotic χ^2-distribution with two degrees of freedom.

Table 6: Tests of the LRR Model with Alternative SDF

	Efficient weighting matrix			
	c_1	c_2	c_3	c_4
Estimated	-0.243 (1.834)	-7.318 (99.92)	-0.911 (2.402)	-1.174 (7.039)
Pricing Errors		*Mean*		*Std.Err.*
Small		0.058		0.073
Large		0.009		0.077
Growth		0.004		0.076
Value		0.027		0.066
Market		0.012		0.076
Risk free rate		-0.011		0.090
$J - stat$	6.437 (0.040)			

The table reports GMM estimates of the LRR Model using annual data over the period 1930-2006. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the difference in the log price-dividend ratios of "Value" and "Growth" portfolios and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios. The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The statistic has an asymptotic χ^2-distribution with two degrees of freedom.

Table 7: Tests of the Cointegrated LRR Model on the 2-Asset System over 1930-2006

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	ρ_x	ψ_x	μ_σ	ρ_σ	σ_w	λ_{sx}	ρ_s	ψ_s
Estimate	0.99 (0.263)	8 (0.365)	1.5 (0.359)	0.015 (0.002)	0.80 (0.250)	0.288 (0.258)	5.1×10^{-6} (1.9×10^{-5})	0.90 (0.351)	3.2×10^{-5} (0.254)	3.879 (0.364)	0.90 (0.046)	5.931 (0.365)
Pr*icing Errors*		*Mean*	*Std.Err.*									
Market		0.165	0.166									
Risk free rate		0.206	0.198									
$Dist - stat$	12.76 (0.0004)											

The table reports GMM estimates of the model using annual data over the period 1930-2006. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, the log risk free rate and its lag, and the demeaned log dividend-consumption ratio and its lag. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with one degree of freedom.

Table 8: Tests of the Cointegrated LRR Model on the 6-Asset System over 1930-2006

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	ρ_x	ψ_x	μ_σ	ρ_σ	σ_w	λ_{sx}	ρ_s	ψ_s
Estimate	0.99 (0.887)	4.0 (1.098)	0.9 (0.764)	0.015 (0.003)	0.95 (0.103)	0.288 (0.385)	5.1×10^{-6} (5.7×10^{-5})	0.9 (1.118)	3.2×10^{-5} (0.740)	3.879 (1.151)	0.7 (0.534)	5.931 (1.139)
Pricing Errors		*Mean*	*Std.Err.*									
Small		0.398	0.196									
Large		0.278	0.160									
Growth		0.257	0.153									
Value		0.368	0.188									
Market		0.287	0.161									
Risk free rate		0.325	0.203									
$Dist-stat$	13.73 (0.0003)											

The table reports GMM estimates of the model using annual data over the period 1930-2006. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, the log risk free rate and its lag, and the demeaned log dividend-consumption ratio and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios.The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with one degree of freedom.

Table 9: Tests of the LRR Model on the 2-Asset System over 1947-2006

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	μ_d	ϕ	φ	ρ_x	ψ_x	σ	ρ_σ	σ_w
Estimate	0.99 (0.068)	6 (7.321)	1.2 (8.424)	0.013 (0.002)	0.023 (0.008)	1.930 (2.242)	3.059 (2.712)	0.90 (0.867)	0.327 (1.612)	0.010 (0.005)	0.70 (7.074)	3.4×10^{-6} (0.008)
Pr*icing Errors*		*Mean*	*Std.Err.*									
Market		-0.006	0.091									
Risk free rate		-0.013	0.117									
$Dist - stat$	15.06 (0.002)											

The table reports GMM estimates of the model using annual data over the period 1947-2006. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. Results are reported for the efficient weighting matrix. The table presents the parameter estimates along with the associated standard errors in parentheses. Both the pricing restrictions and the time-series restrictions are used in the estimation. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom. The 90%, 95%, and 99% critical values of the finite-sample distribution are 5.05, 5.13, and 7.98, respectively.

Table 10: Tests of the LRR Model on the 2-Asset System over 1947-1991

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	μ_d	ϕ	φ	ρ_x	ψ_x	σ	ρ_σ	σ_w
Estimate	0.99 (0.022)	6 (8.716)	1.5 (7.473)	0.013 (0.003)	0.023 (0.009)	1.930 (0.539)	3.059 (0.563)	0.95 (0.029)	0.227 (0.051)	0.020 (0.003)	0.70 (6.818)	3.4×10^{-6} (0.009)
Pricing Errors		*Mean*	*Std.Err.*									
Market		-0.009	0.097									
Risk free rate		-0.011	0.123									
Dist − stat	15.48 (0.001)											

The table reports GMM estimates of the model using annual data over the period 1947-1991. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. Results are reported for the efficient weighting matrix. The table presents the parameter estimates along with the associated standard errors in parentheses. Both the pricing restrictions and the time-series restrictions are used in the estimation. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom.

Table 11: Tests of the LRR Model on the 6-Asset System over 1947-2006

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	μ_d	ϕ	φ	ρ_x	ψ_x	σ	ρ_σ	σ_w
Estimate	0.99 (8.289)	8 (35.03)	1.5 (11.86)	0.013 (0.002)	0.023 (0.008)	1.930 (4.097)	3.059 (1.369)	0.70 (1.282)	0.327 (0.949)	0.015 (0.004)	0.70 (1.590)	3.4×10^{-6} (0.008)
Pr*icing Errors*		*Mean*	*Std.Err.*									
Small		0.178	0.114									
Large		0.135	0.112									
Growth		0.118	0.108									
Value		0.182	0.115									
Market		0.139	0.111									
Risk free rate		0.103	0.130									
$Dist - stat$	18.04 (0.0004)											

The table reports GMM estimates of the model using annual data over the period 1947-2006. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios.The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom.

Table 12: Tests of the Cointegrated LRR Model on the 2-Asset System over 1947-2006

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	ρ_x	ψ_x	μ_σ	ρ_σ	σ_w	λ_{sx}	ρ_s	ψ_s
Estimate	0.99 (0.465)	6 (1.272)	1.5 (1.252)	0.015 (0.002)	0.7 (0.735)	0.388 (0.511)	5.1×10^{-6} (3.5×10^{-5})	0.9 (0.708)	3.2×10^{-5} (1.204)	4.879 (1.274)	0.9 (0.147)	5.931 (1.178)
Pr*icing Errors*		*Mean*	*Std.Err.*									
Market		0.017	0.081									
Risk free rate		0.020	0.099									
$Dist - stat$	13.57 (0.0002)											

The table reports GMM estimates of the model using annual data over the period 1947-2006. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. Results are reported for the efficient weighting matrix. The table presents the parameter estimates along with the associated standard errors in parentheses. Both the pricing restrictions and the time-series restrictions are used in the estimation. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with one degree of freedom.

Table 13: Tests of the Cointegrated LRR Model on the 2-Asset System over 1947-1991

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	ρ_x	ψ_x	μ_σ	ρ_σ	σ_w	λ_{sx}	ρ_s	ψ_s
Estimate	0.99 (0.096)	4.0 (1.465)	1.2 (1.396)	0.015 (0.002)	0.60 (0.869)	0.488 (0.685)	5.1×10^{-6} (4.0×10^{-5})	0.9 (0.828)	3.2×10^{-5} (1.488)	4.879 (1.454)	0.9 (0.193)	3.931 (1.319)
Pr*icing Errors*		*Mean*	*Std.Err.*									
Market		0.004	0.066									
Risk free rate		−0.010	0.083									
$Dist - stat$	12.32 (0.0004)											

The table reports GMM estimates of the model using annual data over the period 1947-1991. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. Results are reported for the efficient weighting matrix. The table presents the parameter estimates along with the associated standard errors in parentheses. Both the pricing restrictions and the time-series restrictions are used in the estimation. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with one degree of freedom.

Table 14: Tests of the Cointegrated LRR Model on the 6-Asset System over 1947-2006

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	ρ_x	ψ_x	μ_σ	ρ_σ	σ_w	λ_{sx}	ρ_s	ψ_s
Estimate	0.99 (0.903)	6.0 (1.204)	1.2 (1.261)	0.015 (0.002)	0.90 (0.290)	0.488 (0.577)	5.1×10^{-6} (2.14×10^{-5})	0.90 (0.433)	3.2×10^{-5} (0.924)	4.879 (1.280)	0.7 (0.382)	5.931 (1.213)
Pr*icing Errors*		*Mean*	*Std.Err.*									
Small		0.196	0.190									
Large		0.164	0.189									
Growth		0.136	0.176									
Value		0.208	0.197									
Market		0.166	0.188									
Risk free rate		0.229	0.273									
$Dist - stat$	11.08 (0.001)											

The table reports GMM estimates of the model using annual data over the period 1947-2006. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, the log risk free rate and its lag, and the demeaned log dividend-consumption ratio and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios.The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with one degree of freedom.

Table 15: Tests of the LRR Model on the 2-Asset System with Quarterly Data

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	μ_d	ϕ	φ	ρ_x	ψ_x	σ	ρ_σ	σ_w
Estimate	0.97 (0.048)	10 (0.665)	0.9 (0.624)	0.004 (0.001)	0.006 (0.005)	1.371 (0.609)	2.286 (0.153)	0.75 (0.186)	0.113 (0.050)	0.047 (0.002)	0.70 (0.146)	8.9×10^{-5} (0.006)
Pr*icing Errors*		*Mean*	*Std.Err.*									
Market		0.006	0.048									
Risk free rate		-0.006	0.046									
$Dist - stat$	75.35 (0.000)											

The table reports GMM estimates of the model using quarterly data over the period 1947:2-2006:3. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio and the risk free rate. The lagged price-dividend ratio of the market and the lagged risk free rate are used as instruments. The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom.

Table 16: Tests of the LRR Model on the 6-Asset System with Quarterly Data

	Efficient weighting matrix											
Parameter	δ	γ	ψ	μ_c	μ_d	ϕ	φ	ρ_x	ψ_x	σ	ρ_σ	σ_w
Estimate	0.97 (5.671)	8 (6.566)	0.3 (2.003)	0.004 (0.001)	0.006 (0.005)	1.917 (6.656)	6.174 (1.197)	0.90 (0.473)	0.013 (0.355)	0.011 (0.001)	0.80 (0.779)	8.9×10^{-5} (0.0004)
Pricing Errors		*Mean*	*Std.Err.*									
Small		0.036	0.019									
Large		0.024	0.017									
Growth		0.022	0.017									
Value		0.038	0.018									
Market		0.025	0.017									
Risk free rate		0.010	0.017									
$Dist-stat$	53.13 (0.000)											

The table reports GMM estimates of the model using annual data over the period 1947:2-2006:3. Both the pricing restrictions and the time-series restrictions are used in the estimation. Results are reported for the efficient weighting matrix. The pricing kernel is a function of the consumption growth, the log price-dividend ratio of the market and its lag, and the log risk free rate and its lag. The asset menu consists of the market portfolio, the risk free rate, the "Small", "Large", "Growth", and "Value" portfolios.The table presents the parameter estimates along with the associated standard errors in parentheses. Average pricing errors and their standard errors are presented for each asset. The bottom line reports the J-stat for the overidentifying restrictions along with the associated p-value in parentheses. The J-stat has an asymptotic χ^2-distribution with three degrees of freedom.