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THE PRICING OF DEFAULT-FREE MORTGAGES

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ABSTRACT

In this paper we examine the household's option to prepay or call a standard fixed-rate mortgage. Results based on simulation indicate that the value of this option is sensitive to the expected path of interest rates, the variation around that path, risk aversion and refinancing costs. Unfortunately, efforts to estimate the interest rate process (by us and by previous authors) have met with only limited success, and uncertainty exists regarding the degree of risk aversion and the magnitude of refinancing costs. Thus we conclude that the application of contingent-claims methodology to options on bonds is conceptually more difficult and operationally less reliable than is the analogous application to options on stocks.

Despite these reservations concerning the use of our model as a technique for absolute valuation, preliminary findings on the effects of changes in mortgage contract design on the value of the prepayment option are encouraging. For example, our estimate of the relative values of the call options on 30- and 15-year mortgages and on level-payment and graduated-payment mortgages appear to be reasonably robust with respect to specifications of the interest rate process and the other parameters. These findings suggest that our model may be of considerable use within the context of relative or comparative valuation.

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The Pricing of Default-Free Mortgages

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A number of recent studies have attempted to price the prepayment or call option in mortgage contracts (Dunn and McConnell, 1981a and b) or limitations on this option (Dietrich et. al., 1983). These studies: (1) assume that the spot rate of interest follows a mean-reverting process with a constant-elasticity standard deviation, (2) assign parameter values to the mean-reverting spot rate, the elasticity and scale of the standard deviation, the speed of adjustment and the "market price of risk", (3) assume zero tax rates and (4) calculate prices for the call option or limitations upon it.[1]

These studies suggest considerable confidence in the magnitudes of most, if not all, of the key parameters. Dunn and McConnell (1981b, p. 606) report that the parameter values used in their simulations were "similar to those estimated by Ingersoll" in an unpublished paper (1976) and did not report sensitivity results. The absence of such results could be taken as an indication that their model was sufficiently robust to permit reasonable inferences from the selected simulation results tabulated and reported. Dietrich, et. al., too, specify values with no discussion.

Unfortunately, there is substantial disagreement regarding many of these parameter values, as a comparison of the assumptions underlying Dunn-McConnell and Dietrich et. al. makes clear, and the results of these analyses are quite sensitive to a number of parameter values. Consideration of the speed at

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[1] If one were uncertain regarding the time path of one of these parameters (interest rate volatility or the market price of risk, for example), but fairly certain of the others, and were willing to posit market efficiency, then one could extract a time series on the unknown variable from observed mortgage price data. Bodie and Friedman (1978) did, in fact, extract an interest-rate volatility series from bond price and yield data.

which the interest rate adjusts to its mean-reverting value illustrates the point. Dietrich, et. al. posit a speed of 0.3, whereas Dunn and McConnell employ 0.8. With the lower 0.3 adjustment speed, Dunn and McConnell's estimates of the value of call would have doubled. Similarly, small changes in risk aversion (in the assumed rate on infinite-maturity default-free debt relative to the mean-reverting value of the spot rate) have significant impacts on the value of call.

We begin this paper with a description of arbitrage-free pricing. In Section II, we take a closer look at establishing "reasonable" parameter values. This includes estimation of the interest-rate process and consideration of the existing literature. Section III contains a detailed analysis of the value of the call option in GNMA pools of level-payment mortgages. Sensitivity analysis is conducted both with Dunn and McConnell parameter values and with the set we estimate in Section II. Both 8 and 13 percent GNMAs are analyzed. In Section IV, we turn to an analysis of GNMAs based upon pools of 15-year fixed-rate and 30-year graduated-payment mortgages. A summary concludes the paper.

I. The Theory of Mortgage Pricing and Numerical Solution Procedures

Price Paths for Default-Free Debt

Following others [Vasicek (1977), Richard (1978), Dothan (1978), and Cox, Ingersoll and Ross (1978)], we begin with two assumptions. First, the value of any default-free "bond" (B) is a deterministic function of time (t) and the spot rate of interest (r) with well-defined first and second partial derivatives

$$B = B(t, r). \tag{1}$$

Second, the time path of the taxable spot rate of interest follows a diffusion process[2]

$$dr = fdt + gdz, \tag{2}$$

where dz is a standardized Gauss-Weiner process and the mean or drift (f) and standard deviation (g) of the instantaneous rate are deterministic functions of time and the spot rate of interest. To simplify the notation, we suppress the functional form for these values as we do for the spot rate of interest, i.e., we use f for f(t, r), g for g(t, r) and r for r(t).

Given these technical assumptions, we can apply Ito's lemma [see McKean (1968)] which identifies an essential closure property that smooth functions (such as 1) of Ito processes (such as 2) are themselves Ito processes. Thus

[2] Merton (1973) shows that, in certain cases, poisson events can be incorporated via simple adjustments to the mean of instantaneous bond returns. In our simulation work (Section III), we employ shift parameters that serve to measure the sensitivity of the model to this extension.

$$dB = B_t dt + B_r dr + \tfrac{1}{2} B_{rr}(dr)^2, \tag{3}$$

where lower case subscripts denote partial derivatives. From (2) and the facts that $dtdz = (dt)^2 = 0$ and $(dz)^2 = dt$, this expression can be rewritten as

$$dB = f_B dt - g_B dz, \tag{3a}$$

where $$f_B = B_t + fB_r + \tfrac{1}{2}g^2 B_{rr} \tag{4}$$

and $$g_B = -gB_r. \tag{5}$$

Equilibrium Price Paths

Merton's (1973) equilibrium condition for asset returns, assuming none of the returns of the marginal investor are taxed, is:[3]

$$f_B + C - rB = \lambda g_B \tag{6}$$

where: C is the instantaneous cash payment (coupon plus amortization) on the mortgage; f_B and g_B are the drift and standard deviation of the price path for the mortgage; and λ, "the market price of risk," is a deterministic function of time and the spot rate of interest. Substituting (4) and (5) into (6) produces the following general expression for the equilibrium, or zero-arbitrage, price path of any default-free debt instrument

[3] Assuming an untaxed marginal investor conveniently allows us to abstract from tax-motivated trading.

$$B_t + (f+\lambda g)B_r + \tfrac{1}{2}g^2B_{rr} + C - rB = 0. \tag{6a}$$

The first term is the price drift including prepayment but abstracting from interest-rate changes, and the second term is due to the risk-adjusted interest-rate drift (as is the third which reflects the "Ito effect"). The remaining terms compare the cash flows of the mortgage with those of an alternative "pure cash" investment.

CES Processes

Most of the applied work on arbitrage-free interest-dependent claims has focused on the special case of a spot rate of interest that follows a mean-reverting process with a constant-elasticity standard deviation, hereafter referred to as a CES process. For examples, see Ingersoll (1976), Ananthanarayanan and Schwartz (1980), Marsh (1980), Beckers (1980) and Dunn and McConnell (1981 a and b). We represent the general form of the CES model as:

$$dr = k(\mu - r)dt + \sigma r^{\alpha}dz, \tag{7}$$

where μ is the long-run mean of the process,

k measures the speed of adjustment,

σ is the "scale" of the standard deviation, and,

α is the elasticity of the standard deviation with respect to the spot rate of interest.

Comparison of (2) and (7) indicates that

$$f = k(\mu - r) \tag{7a}$$

and

$$g = \sigma r^{\alpha}. \tag{7b}$$

Part of the popularity of the CES model stems from its seemingly realistic properties: for μ, k, σ and α all positive, interest rates are cyclical yet always finite and never negative. Popularity also follows from the detailed theoretical work based on this model. Most important to us, Cox, Ingersoll and Ross (1978) show that the CES model is consistent with certain classes of general equilibrium economies. Moreover, for those economies the bond risk premium required in (6a) also exhibits constant elasticity with respect to the spot rate of interest:

$$\lambda g = \lambda' r^{(\alpha + \frac{1}{2})},$$

where λ' is a constant which we denote as the "transformed price of risk". Further, as Cox, et. al., note, for $\alpha = 0.5$ and $k > 0$,

$$\frac{R(\infty)}{\mu} = \frac{2k}{k - \lambda' + [2\sigma^2 + (k-\lambda')^2]^{\frac{1}{2}}}$$

or

$$\lambda' = k(1 - \frac{\mu}{R(\infty)}) + \frac{\sigma^2 R(\infty)}{2k\mu}, \tag{8}$$

where $R(\infty)$ is the limiting yield on default-free pure discount debt taken as term to maturity increases. Because one-period returns on long-term debt are uncertain owing to uncertain future interest rates, $R(\infty)/\mu$ is greater the higher is λ'.

Solutions

The equilibrium price path (6a) is a deterministic second-order differential equation. Given appropriate specifications of f, g, and g from (7a), (7b) and (8), it is possible in principle to solve the equation subject to appropriate boundary conditions. In the case of a noncallable debt instrument, these conditions are that (1) the mortgage becomes worthless at maturity full (amortization) or when r goes to infinity and (2) the mortgage equals the sum of the remaining payments when r goes to zero. To determine the price of a callable mortgage, we replace the second of these boundary conditions with the condition that refinancing will occur (the claim is paid off) when r takes on the value r_r such that

$$B(t, r_r) = (1 + RFW)\left[B(0, r) - \sum_{j=1}^{t} A_j\right]. \tag{9}$$

where the A's are the scheduled amortization payments and RFW, the refinancing wedge, equals the borrower's after-tax refinancing costs plus the value of the unused option.[4] In this event, the investor receives the prevailing book value (the initial par value less the cumulative amortization payments):

$$\hat{B}(t,r_r) = B(0,r) - \sum_{j=1}^{t} A_j. \tag{9a}$$

[4]For level-payment fully-amortizing mortgages, A rises over time and C (the cash flow) is constant. For graduated-payment mortgages, A is initially negative, and both A and C are positive functions of time.

The difference between the values of the noncallable and callable mortgages is the value of call protection to the investor (and the cost of the call option to the borrower). The coupon-equivalent cost of the call is the difference between the stated coupon and that needed on a noncallable mortgage to make it equal in value to the callable mortgage.

Analytic solutions to the fundamental differential equation (6a) subject to boundary conditions are known only in isolated cases of the CES model. As a rule, researchers rely on numerical alternatives such as the implicit difference method described by Brennan and Schwartz (1977). Thus our computer program solves a second-order difference equation subject to boundary conditions. As a check on our numerical and programming procedures, we have reproduced simulation results reported previously by Dunn and McConnell (1981a and 1981b) for what can be regarded as a special case of our model.

II. Estimation of the Parameters of the CES Process

The general form of the mean-reverting CES process governing the taxable spot rate of interest in continuous time was given in (7). Estimation requires a discrete-time analogue. If we view the spot rate as reverting to the beginning-of-period long-run mean value, then it is appropriate to estimate

$$\frac{r_t - r_{t-1}}{r_{t-1}^{\alpha}} = k \frac{(\mu_{t-1} - r_{t-1})}{r_{t-1}^{\alpha}} + \sigma\varepsilon_t. \tag{7'}$$

We estimate (7') for three values of α : 0.0, 0.5 and 1.0; with $\alpha = 0.0$, the dependent variable is the change in the spot rate; with $\alpha = 1.0$, it is the percentage change. The equation is estimated over two periods: January 1970 to October 1979 and November 1979 to December 1983.

The November 1, 1979 break point in our estimation corresponds to the change in the Federal Reserve's operating procedures, a change that resulted in substantially more volatile interest rates. To illustrate, the standard deviation of monthly changes in our one-month rate tripled from the first period to the second (0.0058 to 0.0183). Of course, some of the increase could be attributable to the higher average level of rates in the second period (0.1113 versus 0.0626). However, even after correcting for this (dividing the standard deviations by $r^{\frac{1}{2}}$), the "adjusted" standard deviation is nearly 2 1/2 times as large in the second period.

Two proxies for μ have been employed in the recent literature: Ingersoll (1976) treats μ as a constant in his preliminary estimates, and Bodie and Friedman (1978) use the long-term corporate rate. We test the hypotheses that μ is a linear function of the long-term Treasury rate minus a constant ("the" liquidity premium). First-day of month data for one-month (the spot rate) and 20-year Treasuries yields are from Salomon Brothers and are measured in decimals. The one-month discount rates have been converted to bond-equivalents.

The estimates are reported in Table 1. The speeds of adjustment are low, 4 percent in the 1970s and 20 percent in the more recent period of volatile interest rates.[5] Moreover, the k's are not measured with precision;

[5] It has come to our attention that Brennan and Schwartz (1982) have estimated equations similar to these (with $\alpha = 1.0$) for the December 1958-June 1969 and July 1969-December 1979 periods. For the latter, their estimate of k was 0.0377 with a standard error of 0.0369.

Table 1: Basic Estimates of the CES Process

		Constant $(1/r^{\alpha}_{t-1})$	Adjustment Speed $(\mu_{t-1}-r_{t-1})$	R^2	SEE	DW
January 1970–October 1979	0.0	−.0001 (.0007)	.039 (.040)	.008	.0058	1.85
	0.5	−.0001 (.0008)	.038 (.038)	.015	.0219	1.80
	1.0	−.0002 (.0008)	.043 (.039)	.022	.0867	1.74
November 1979–December 1983	0.0	−.0025 (.0028)	.205 (.115)	.063	.0179	1.85
	0.5	−.0016 (.0029)	.158 (.110)	.044	.0513	1.81
	1.0	−.0007 (.0030)	.122 (.107)	.040	.1502	1.74

they are only one (first period) or one-and-a-half (second period) times their standard errors, and the explanatory power of the equations is low. The latter should not be surprising. Shiller et al (1983) have reported results indicating that future expected spot rates have zero ability to explain changes in spot rates. In effect, changes in spot rates are predominantly unexpected, and the impact of unexpected events swamps the impact of expected changes. This does not mean, however, that expected future rate changes would not govern actual changes in the absence of unexpected events. In fact, Hendershott (1984) has shown that changes in the six-month bill rate are related to the expected change implied by forward rates when surprise variables (unexpected changes in anticipated inflation, industrial production and base money) are included in the estimation equation.

Say that these surprises show up in both μ and r, which would likely be the case when μ is proxied by the 20-year Treasury rate. The impact of surprises could be incorporated by estimation of

$$\frac{r_t - r_{t-1}}{r_{t-1}^{\alpha}} = k\,\frac{\mu_{t-1} - r_{t-1}}{r_{t-1}^{\alpha}} + k'\,\frac{\mu_t - \mu_{t-1}}{r_{t-1}^{\alpha}} + \sigma\varepsilon_t . \qquad (7'')$$

The results are reported in Table 2. As anticipated, the surprise proxy (the change in the 20-year Treasury rate) contributes enormously to the explanation of the change in the one-month rate. The coefficient of about 1 1/2 in the second period reflects the greater volatility of the short-term rate. The mean-reverting term is also significant. The standard errors of the speed-of-adjustment coefficients decline slightly and the coefficients themselves rise from 50 to 100 percent.

Table 2: Estimates of the CES Process When Surprises Are Included

	Elasticity (α)	Constant ($1/r^{\alpha}_{t-1}$)	Adj. Speed ($\mu_{t-1}-r_{t-1}$)	Surprise ($\mu_t-\mu_{t-1}$)	R^2	SEE	DW
January 1970–October 1979	0.0	–.0008 (.0007)	.070 (.036)	1.024 (.193)	.204	.0053	2.12
	0.5	–.0007 (.0007)	.065 (.036)	.879 (.186)	.176	.202	2.01
	1.0	–.0008 (.0008)	.066 (.037)	.755 (.178)	.154	.0810	1.92
November 1979–December 1983	0.0	–.0042 (.0024)	.293 (.098)	1.562 (.341)	.357	.0150	2.35
	0.5	–.0040 (.0025)	.269 (.097)	1.465 (.332)	.328	.0434	2.27
	1.0	–.0038 (.0027)	.256 (.097)	1.353 (.318)	.311	.1285	2.16

The sensitivity to the value of α of both the estimates and the ability of the equation to explain changes in the spot rate is low. The similarity of the coefficients is obvious. Comparability in explanatory power is attained by multiplying the equation standard errors of estimate (SEE) by the mean value of r_{t-1}^{α}. This multiplication provides standard errors on Δr in all cases. These estimates (from Table 1), for α's of 0.0, 0.5 and 1.0, for the 1970-79 span are 0.0058, 0.0054, and 0.0054. For the 1980-83 equation, the estimates are 0.0179, 0.0171, and 0.0167. That is, the explanatory power is greater the higher is α, but the differences are very small.

From the above results, our best an estimate of k for the first period is 0.05 to 0.075; for the second period, an estimate of 0.2 to 0.3 is reasonable. Because a solution for the risk coefficient is available when $\alpha = 0.5$ [equation (8)], we utilize this value. Estimates of σ for the different time periods are the standard errors of the equations in Table 1 for $\alpha = 0.5$. For 1970-79, when FHA mortgage rates averaged 8 1/2 to 9 percent, $\sigma = 0.0225$; for 1980-83, when the FHA rate averaged about 13 percent, $\sigma = 0.05$.

III. Values of the Call Option on Level-Payment GNMA Securities

In this section we provide estimates of the value of the call option for three base cases and show the sensitivity of the estimates to alternative parameter values. The three base cases are: the Dunn-McConnell (1981a and b) specification for an 8% GNMA, our specification for the 8% GNMA, and our specification for a 13% GNMA. The parameter values for these cases are listed in Table 3. The second base case is an alteration of the Dunn-McConnell values to obtain greater realism. The lower k and σ values were discussed earlier. The Dunn-McConnell spread between $R(\infty)$ and μ implies an enormous permanent spread between long and short rates and results in a large price of

Table 3: Assumed Parameter Values for Simulations

	DM 8's	BH 8's	BH 13's
Coupon	.08	.08	.13
$R(\infty)$	.08	.08	.125
μ	.056	.0775	.1175
k	.80	.10	.25
α	.5	.5	.5
σ	.09	.0225	.05
RFW	0	.03	.03
λ' (implied)	.247	.006	.020

risk based on equation (8). We have substantially narrowed the spread, and thereby lowered the price of risk, by raising μ. Dunn and McConnell assume a zero refinancing wedge: we assume a 3 percent refinancing cost which exceeds slightly the cost of originating a replacement mortgage. The third base case constitutes an attempt to analyze the early 1984 environment. The coupon rate is set at 13 percent, $R(\infty)$ at $12\frac{1}{2}$, μ at 11 3/4 percent and k at 0.25. The greater spread between $R(\infty)$ and μ and the larger k value, respectively, reflect the larger constant term and speed of adjustment estimated on data from the 1980-83 period. We also raise σ to 0.05, its value in the 1980-83 period.

Each of the parts of Tables 4-6 contains three calculations for each of three values of the parameter for which sensitivity is being tested, and the results are reported for five values of the spot rate. The three calculated values are: the price of the \$100 par value mortgage, the value of the call option in dollars (price of a noncallable mortgage with the same coupon less the price of the callable mortgage), and the value of the call option in basis points (the stated coupon less the calculated coupon on a noncallable mortgage that would have the same price as the callable mortgage). To illustrate, when the spot rate is 0.059 in Table 4 and the parameters are set at the Dunn-McConnell base case values, the price of the mortgage is \$99.58, the call is worth \$3.69 (the price of a noncallable mortgage would be \$103.27) or a 29 basis point higher coupon (a noncallable mortgage priced at \$99.58 would carry a 7.71 percent coupon).

Before turning to the specific results, it is probably useful to indicate roughly how the variables should affect the call premium. They operate via the interest-rate process (k, μ and σ), the refinancing boundary condition (RFW), and the transformed price of risk [$R(\infty)$, k, μ and σ].

Table 4: The Sensitivity of Dunn-McConnell Call Values*

Infinite-Lived Default-Free Rate [R(∞)]

SPOT/	.075	.080	.085
.122	93.35	91.50	89.06
	4.53	2.15	0.74
	37.	18.	7.
.100	96.05	94.48	92.24
	5.06	2.42	0.83
	40.	20.	7.
.079	98.47	97.31	95.38
	5.86	2.84	0.96
	45.	23.	8.
.059	called	99.58	98.24
		3.69	1.23
		29.	10.
.041	called	called	called

Speed of Adjustment Coefficient (k)

SPOT/	.200	.500	.800
.122	77.72	87.75	91.50
	3.65	2.90	2.15
	36.	26.	18.
.100	94.40	91.88	94.48
	5.01	3.43	2.42
	45.	29.	20.
.079	91.06	95.82	97.31
	7.06	4.24	2.84
	58.	34.	23.
.059	96.82	99.04	99.58
	10.32	5.65	3.69
	77.	43.	29.
.041	called	called	called

Mean Reversion Value (μ)

SPOT/	.056	.066	.076
.122	91.50	92.59	93.43
	2.15	1.90	1.69
	18.	16.	14.
.100	94.48	95.22	95.79
	2.42	2.10	1.85
	20.	17.	15.
.079	97.31	97.71	98.02
	2.84	2.43	2.12
	23.	19.	17.
.059	99.58	99.70	99.79
	3.69	3.13	2.71
	29.	24.	21.
.041	called	called	called

Variance Scale Parameter (σ)

SPOT/	.050	.070	.090
.122	92.56	92.03	91.50
	1.13	1.64	2.15
	10.	14.	18.
.100	95.64	95.06	94.48
	1.31	1.87	2.42
	11.	15.	20.
.079	98.54	97.92	97.31
	1.66	2.26	2.84
	13.	18.	23.
.059	called	99.99	99.58
		3.30	3.69
		26.	29.
.041	called	called	called

Refinancing Wedge (RFW)

SPOT/	.000	2.000	4.000
.122	91.50	92.28	93.00
	2.15	1.37	0.65
	18.	12.	6.
.100	94.48	95.34	96.14
	2.42	1.56	0.75
	20.	13.	6.
.079	97.31	98.29	99.22
	2.84	1.86	0.92
	23.	15.	7.
.059	99.58	100.79	101.99
	3.69	2.47	1.27
	29.	19.	10.
.041	called	called	called

*The three numbers for each parameter value at each spot rate are: the price for a $100 par value mortgage, the difference between the price on the same coupon noncallable mortgage and the price on the callable mortgage, and the price difference translated into a coupon rate (basis points) equivalent.

Because the refinancing wedge and the infinite-maturity yield enter only one relation, their directional impacts on the call premium are necessarily unambiguous. The greater the refinancing wedge, the less the chance of call and thus the lower the premium. The impact of $R(\infty)$, and the role of the price of risk generally, requires more explanation. The call provision of a mortgage limits the extraordinarily large one-period gains received when interest rates decline (the mortgage is called) but also dampens losses when interest rates rise (the call premium is earned). Thus callable debt is less risky (has a lower standard deviation around the expected return) than noncallable debt. An increase in the price of risk increases yields on risky securities (noncallable) relative to less risky securities (callable). Hence call premia decline. Increasing $R(\infty)$ in the simulations is simply a means of analyzing a pure increase in risk aversion. The result will thus be a decrease in the call premium.

A higher mean-reverting value (μ) lowers the chance of call and thus tends to decrease the call premium. On the other hand, a higher μ, other things being equal, means the price of risk has declined, a decline which tends to raise the call premium. Thus the directional impact of an increase in μ is not readily apparent. A higher speed of adjustment (k) restricts declines in interest rates that tend to trigger call. Thus the higher is k, the lower is the call premium.[6] Finally, an increase in the variance (σ) of rates generally increases the likelihood of call and thus raises the call

[6] An exception can occur when the spot rate starts at such a high value relative to the mean-reverting value that much expected downward pull is necessary in order for an unexpected decline in rates to have any chance to trigger call. (The change in risk-aversion implied by a change in k is uncertain in direction and likely quite small in magnitude.)

premium.[7] (While an increase also implies an increase in the price of risk, the increase is slight and thus we would expect the direct effect to dominate.)

Dunn-McConnell 8% Coupon

Consider the base case, which is reported in the center section of all parts of Table 4 as a basis of comparison. At a spot rate equal to the mean-reverting value of 0.056, the call is worth about 30 basis points. At lower spot rates, instantaneous call occurs; at a roughly doubled spot rate, the call is worth about 20 basis points. These values seem low because the spread between GNMAs and a portfolio of Treasuries having an identical expected pattern of cash flows, taking into account differences in timing of payments, averaged about 65 basis points during the 1977-78 period when interest rates were in this range [Hendershott, Shilling and Villani, 1984].

The value of the call is quite sensitive to some of the parameters. Most important are the yield on infinite-maturity default-free debt and the interest-rate speed of adjustment. A half percentage point reduction in the former, which constitutes a significant reduction in assumed risk aversion, roughly doubles the call premium (and triggers call at spot rates below 6 percent). So also does a reduction in the interest-rate speed of adjustment from 0.8 to 0.2 which sharply reduces the pull of the mean-reverting value on diverging spot rates. The high assumed values of either the infinite-maturity debt rate or the speed of adjustment could explain the low estimated call premium relative to that observed in the market place.

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[7] An exception can occur when the spot rate starts at such a low value relative to the mean-reverting value that call is likely unless there is a large unexpected increase in interest rates.

Changes in the scale parameter work in the opposite direction. Cutting the scale from 0.9 to 0.5 roughly halves the call premium. Changes in the other parameters have even smaller impacts. Addition of a 2 point wedge to deter refinancing cuts the premium by about a third, while raising the mean-reverting value by a full percentage point decreases the call premium by only 10 to 25 percent, depending on the level of the spot rate. The lesser sensitivity of the call premium to the variance scale parameter than to numerous other parameters, which could easily be changing over time, suggests great difficulty in extracting variance estimates from market data a' la' Bodie and Friedman (1978).

The Revised 8% Coupon

The base case result, reported in all parts of Table 5, indicates a call value of 100 basis points at a spot rate of 0.056. This is triple the value under the Dunn and McConnell assumptions. With a spot rate of 6 percent, the call is worth about 70 basis points, and a 50 percent higher spot lowers the call value to under 10 basis points.

In general, the call estimates are sensitive to the same parameters as was the case under the Dunn-McConnell assumptions, namely the yield on infinite-maturity default-free debt and the interest-rate speed of adjustment. A half percentage point decrease in the infinite-maturity yield increases the call premium by 50 to 100 percent, and a decline in the speed of adjustment from 0.15 to 0.05 doubles the premium. Also, a halving of the variance scale parameter cuts the call premia by 25 to 75 percent, with greater percentage reductions (but smaller absolute ones) occurring at higher spot rates.

Table 5: Sensitivity of Call Values on 8% GNMA's

Infinite-Lived Default-Free Rate [R(∞)]

SPOT/	.0775	.0800	.0825
.0975	91.32 0.66 8.	90.43 0.48 6.	89.58 0.33 4.
.0875	95.82 1.19 13.	95.02 0.90 10.	94.24 0.65 7.
.0775	99.94 2.31 24.	99.31 1.82 19.	98.70 1.39 15.
.0675	102.52 5.15 51.	102.17 4.37 43.	102.01 3.47 35.

Speed of Adjustment Coefficient (k)

SPOT/	.0500	.1000	.1500
.0975	87.38 0.46 5.	90.43 0.48 6.	92.25 0.39 4.
.0875	92.98 1.04 12.	95.02 0.90 10.	96.13 0.66 7.
.0775	98.30 2.40 25.	99.31 1.82 19.	99.78 1.25 13.
.0675	101.91 6.00 59.	102.17 4.37 43.	102.29 3.04 30.

Mean Reversion Value (μ)

SPOT/	.0750	.0775	.0800
.0975	90.29 0.48 6.	90.43 0.48 6.	90.58 0.47 5.
.0875	94.93 0.91 10.	95.02 0.90 10.	95.12 0.88 10.
.0775	99.28 1.85 19.	99.31 1.82 19.	99.36 1.77 18.
.0675	102.20 4.43 44.	102.17 4.37 43.	102.30 4.15 41.

Variance Scale Parameter (σ)

SPOT/	.0225	.0450
.0975	90.43 0.48 6.	88.44 2.23 26.
.0875	95.02 0.90 10.	92.61 3.22 35.
.0775	99.31 1.82 19.	96.57 4.73 49.
.0675	102.17 4.37 43.	99.99 7.18 71.

Refinancing WeQget (RFW)

SPOT/	.0200	.0300	.0400
.0975	90.34 0.56 7.	90.43 0.48 6.	90.53 0.38 4.
.0875	94.88 1.04 11.	95.02 0.90 10.	95.19 0.73 8.
.0775	99.07 2.06 21.	99.31 1.82 19.	99.62 1.51 16.
.0675	101.81 4.73 47.	102.17 4.37 43.	102.75 3.79 38.

13% Coupon

In general, the call premium in our third base case (see Table 6) is less sensitive than the premium was in the other two cases to equal absolute changes in parameter values, but the premium is equally sensitive to equivalent percentage changes. Consider the responses to changes in the yield on infinite-maturity default-free debt and the interest-rate speed of adjustment. A half point decline in the former increased the call premium by 50 percent in our analysis of 8% coupons; here a three-quarter point decline is required to cause the same impact. Similarly, the increase in the speed of adjustment from 0.05 to 0.15 halved the premium in the 8% coupon case; here an increase from 0.15 to 0.35 is necessary. Note, however, that in each of these examples the same percentage changes in values yield comparable premium impacts (e.g., a 0.005 increase on a base of 0.0775 is the same as a 0.0075 increase on a base of 0.1175). Changes in the refinancing wedge, the scale parameter and continue to have about the same impacts (negligible for the latter).

IV. Values of the Call Option in Alternative Maturity GNMA Securities

GNMAs based upon 15-year, level-payment mortgages and 30-year, graduated-payment mortgages with a 7½ percent annual graduation rate for 5 years are analyzed in this section.[8] The upper halves of Tables 7a - 8b have precisely the same format as Tables 4-6. That is, the value of the callable mortgage, the dollar value of the call option and the coupon rate equivalent are reported for different values of some parameter (the middle columns are identical, being the base case) for different levels of the spot rate. The lower halves are the differences between these values and those calculated for

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[8] Graduated-payment mortgages with 4.9 percent graduation for 10 years were also analyzed, but the results were so similar to the 7.5 percent instrument that they are not reported.

Table 6: Sensitivity of Call Values on 13% GNMA's

Infinite-Lived Default-Free Rate [$R(\infty)$]

SPOT/	.1175	.1250	.1325
.1375	96.63	94.78	92.75
	2.92	1.48	0.65
	42.	22.	10.
.1275	99.02	97.43	95.59
	3.72	1.96	0.90
	51.	28.	13.
.1175	101.07	99.86	98.31
	4.88	2.69	1.29
	65.	37.	18.
.1075	102.48	101.83	100.74
	6.68	3.90	1.99
	87.	52.	27.
.0975	called	102.87	102.48
		6.04	3.39
		79.	46.

Speed of Adjustment Coefficient (k)

SPOT/	.1500	.2500	.3500
.1375	92.89	94.78	95.94
	1.86	1.48	1.08
	28.	22.	16.
.1275	96.14	97.43	98.19
	2.63	1.96	1.39
	38.	28.	20.
.1175	99.10	99.86	100.27
	3.80	2.69	1.86
	53.	37.	26.
.1075	101.43	101.83	101.98
	5.71	3.90	2.69
	76.	52.	36.
.0975	called	102.87	102.72
		6.04	4.47
		79.	59.

Mean Reversion Value (μ)

SPOT/	.1100	.1175	.1250
.1375	94.56	94.78	95.00
	1.53	1.48	1.40
	22.	22.	21.
.1275	97.30	97.43	97.59
	2.04	1.96	1.84
	29.	28.	26.
.1175	99.80	99.86	99.96
	2.82	2.69	2.52
	39.	37.	35.
.1075	101.84	101.83	101.89
	4.09	3.90	3.64
	55.	52.	49.
.0975	called	102.87	102.93
		6.04	5.66
		79.	74.

Variance Scale Parameter (σ)

SPOT/	.0250	.0500	.0750
.1375	95.75	94.78	93.65
	0.37	1.48	2.82
	5.	22.	42.
.1275	98.64	97.43	96.15
	0.62	1.96	3.45
	9.	28.	49.
.1175	101.22	99.86	98.48
	1.20	2.69	4.29
	17.	37.	59.
.1075	102.74	101.83	100.52
	2.87	3.90	5.43
	39.	52.	73.
.0975	called	102.87	102.07
		6.04	7.07
		79.	92.

Refinancing Wedge (RFW)

SPOT/	.0200	.0300	.0400
.1375	94.54	94.78	95.01
	1.71	1.48	1.25
	25.	22.	18.
.1275	97.12	97.43	97.72
	2.27	1.96	1.67
	32.	28.	24.
.1175	99.41	99.86	100.23
	3.14	2.69	2.32
	43.	37.	32.
.1075	101.13	101.83	102.31
	4.60	3.90	3.42
	62.	52.	46.
.0975	101.73	102.87	103.41
	7.18	6.04	5.51
	94.	79.	72.

the 30-year level payment mortgage. There are two tables (a and b) for each mortgage in order to compute sensitivities of the call to all six of the parameter values. Following analysis of these mortgage contracts, we briefly consider differences in call premia on 13 percent 30-year GNMAs with different remaining years to maturity.

15-Year Level Payment

At a 11.75 percent spot rate and the base case parameter values, the 15-year mortgage would be priced 44 cents above par and the call is valued at \$1.79 (see the middle column of any segment of Tables 7a or 7b except that for taxes). This is one-third (\$0.90) less than the call value on a 30-year mortgage; 15 years call protection is worth less than 30 years. Note, however, that on a coupon-equivalent basis the call value is only 10 percent less (33 versus 37 basis points). While 15 years of call is worth less than 30 years, the coupons on the 15-year mortgage will be received for a shorter period and on lower balances than the coupons on a 30-year mortgage.

The lower halves of all segments of Tables 7a and 7b suggest an insensitivity of the difference in call premia on 15- and 30-year level-payment mortgages to virtually all parameters: Except for low spot rates, the 15-year premium is 2 to 9 basis points less than the 30-year premium; at low spot rates a slight positive difference exists. The exceptions are for very low spot rates (instruments close to being called where the expected difference in lives is small) and for a low risk environment [$\mu = R(\infty)$] combined with high spot rates.

Table 7a: Call Values on 15-Year Fixed-Rate GNMAs; Both Absolute and Relative to Those on 30-Year Fixed-Rate GNMAs

Absolute

Infinite-Lived Default-Free Rate [R(∞)]

SPOT/	.1175	.1250	.1325
.1375	97.15	95.82	94.44
	1.37	0.73	0.37
	27.	14.	8.
.1275	99.36	98.26	97.01
	2.00	1.12	0.61
	38.	22.	12.
.1175	101.25	100.44	99.43
	2.96	1.79	1.04
	54.	33.	20.
.1075	102.55	102.05	101.47
	4.53	3.04	1.85
	81.	55.	34.
.0975	called	102.68	102.58
		5.28	3.61
		94.	65.

Speed of Adjustment Coefficient (k)

SPOT/	.1500	.2500	.3500
.1375	94.52	95.82	96.63
	0.93	0.73	0.54
	18.	14.	11.
.1275	97.41	98.26	98.75
	1.50	1.12	0.80
	29.	22.	15.
.1175	100.01	100.44	100.67
	2.46	1.79	1.25
	46.	33.	23.
.1075	101.88	102.05	102.10
	4.22	3.04	2.16
	76.	55.	40.
.0975	called	102.68	102.37
		5.28	4.23
		94.	76.

Relative to 30-Year

SPOT/	.1175	.1250	.1325
.1375	0.52	1.04	1.69
	-1.55	-0.75	-0.28
	-15.	-7.	-2.
.1275	0.34	0.83	1.43
	-1.72	-0.83	-0.29
	-14.	-6.	-1.
.1175	0.18	0.58	1.12
	-1.91	-0.90	-0.26
	-11.	-4.	1.
.1075	0.07	0.22	0.73
	-2.15	-0.65	-0.14
	-6.	3.	7.
.0975	called	-0.19	0.10
		-0.76	0.22
		15.	20.

SPOT/	.1500	.2500	.3500
.1375	1.63	1.04	0.70
	-0.93	-0.75	-0.54
	-9.	-7.	-5.
.1275	1.27	0.83	0.55
	-1.13	-0.83	-0.58
	-9.	-6.	-4.
.1175	0.91	0.58	0.40
	-1.35	-0.90	-0.62
	-7.	-4.	-3.
.1075	0.45	0.22	0.12
	-1.49	-0.65	-0.53
	0.	3.	3.
.0975	called	-0.19	-0.34
		-0.76	-0.24
		15.	17.

Table 7B: Call Values on 15-Year Fixed-Rate GNMAs; Both Absolute and Relative to Those on 30-Year Fixed-Rate GNMAs

Absolute

	Mean Reversion Value (μ)				Variance Scale Parameter (σ)		
SPOT/	.1100	.1175	.1250	SPOT/	.0250	.0500	.0750
.1375	95.66	95.82	95.97	.1375	96.39	95.82	94.96
	0.76	0.73	0.70		0.09	0.73	1.70
	15.	14.	14.		2.	14.	34.
.1275	98.15	98.26	98.34	.1275	99.07	98.26	97.22
	1.19	1.12	1.08		0.24	1.12	2.26
	23.	22.	21.		5.	22.	43.
.1175	100.35	100.44	100.46	.1175	101.41	100.44	99.30
	1.93	1.79	1.72		0.76	1.79	3.03
	36.	33.	32.		14.	33.	57.
.1075	101.90	102.05	101.98	.1075	102.59	102.05	101.08
	3.35	3.04	2.97		2.44	3.04	4.11
	61.	55.	54.		44.	55.	75.
.0975	called	102.68	101.58	.0975	called	102.68	102.33
		5.28	6.14			5.28	5.73
		94.	109.			94.	102.

Relative to 30-Year

SPOT/	.1100	.1175	.1250	SPOT/	.0250	.0500	.0750
.1375	1.10	1.04	0.96	.1375	0.64	1.04	1.31
	-0.76	-0.75	-0.70		-0.28	-0.75	-1.12
	-7.	-7.	-7.		-4.	-7.	-8.
.1275	0.85	0.83	0.75	.1275	0.43	0.83	1.07
	-0.85	-0.83	-0.76		-0.38	-0.83	-1.19
	-6.	-6.	-6.		-4.	-6.	-6.
.1175	0.55	0.58	0.50	.1175	0.19	0.58	0.82
	-0.89	-0.90	-0.80		-0.44	-0.90	-1.26
	-3.	-4.	-3.		-3.	-4.	-3.
.1075	0.05	0.22	0.09	.1075	-0.14	0.22	0.56
	-0.74	-0.85	-0.68		-0.43	-0.85	-1.31
	6.	3.	5.		6.	3.	2.
.0975	called	-0.19	-1.35	.0975	called	-0.19	.16
		0.76	0.48			0.76	-1.34
		15.	35.			15.	10.

Graduated Payment Mortgages (GPMs)

Tables 8a - 8b indicate the results for the graduated-payment mortgage. Because these mortgages have a longer effective maturity than the 30-year level-payment mortgage, the call has a higher value. At a spot rate of 11.75 percent and the base case values, the difference is $0.33 or 10 percent. In this case, the premia in the coupon rates are nearly identical for virtually any configuration of parameter values and spot rates. While the value of call is greater with a GPM, the coupon will be earned on a larger valued mortgage over its life.

Differences in Remaining Years to Maturity

Table 9 contains prices, call values, and coupon-equivalent call premia for standard (30-year original term, level-payment) 13 percent GNMAs with differing remaining years to maturity for our base case parameter values. The first column reproduces the results from Table 6 for a new-issue (30 years remaining life) GNMA. The next two columns refer to the same underlying GNMA four and eight years after the date of issue (26 and 22 remaining years to maturity, respectively). At high spot rates, the call value (and coupon-equivalent premium) is about 25 percent less on the eight year old GNMA than on the newly issued one; at low spot rates, the difference is only about 5 percent. Because call is worth considerably less when spot rates are high, the absolute difference in call values is relatively constant. And this difference is a small 5 basis points in terms of coupon equivalents. Thus whether one purchases an old or new GNMA (with the same coupon) does not seem to be of great importance.

Table 8a: Call Values on 30-Year Graduated Payment Mortgage (graduation at 7½ percent per annum for 5 years), Both Absolute and Relative to the 30-Year Fixed-Rate Mortgage

Absolute

	Infinite-Lived Default-Free Rate [R(∞)]				Speed of Adjustment Coefficient (k)		
SPOT/	.1175	.1250	.1325	SPOT/	.1500	.2500	.3500
.1375	96.60 3.30 44.	94.62 1.68 23.	92.44 0.76 11.	.1375	92.56 2.11 29.	94.62 1.68 23.	95.87 1.26 17.
.1275	99.09 4.19 54.	97.42 2.21 29.	95.42 1.05 14.	.1275	95.99 2.97 40.	97.42 2.21 29.	98.24 1.59 21.
.1175	101.22 5.47 68.	99.97 3.02 39.	98.26 1.50 20.	.1175	99.11 4.26 55.	99.97 3.02 39.	100.41 2.12 27.
.1075	102.73 7.38 89.	102.03 4.33 54.	100.80 2.27 29.	.1075	101.58 6.32 78.	102.03 4.33 54.	102.09 3.12 39.
.0975	called	102.93 6.80 82.	102.38 4.03 50.	.0975	called	102.93 6.80 82.	102.79 5.07 62.

Relative to 30-Year

SPOT/	.1175	.1250	.1325	SPOT/	.1500	.2500	.3500
.1375	-0.03 0.38 2.	-0.15 0.21 1.	-0.31 0.11 1.	.1375	-0.33 0.25 2.	-0.15 0.21 1.	-0.06 0.18 1.
.1275	0.07 0.48 2.	-0.02 0.26 1.	-0.17 0.15 1.	.1275	-0.15 0.34 2.	-0.02 0.26 1.	0.05 0.21 1.
.1175	0.15 0.59 2.	0.10 0.33 1.	-0.05 0.21 1.	.1175	0.01 0.46 2.	0.10 0.33 1.	0.14 0.26 1.
.1075	0.25 0.69 2.	0.19 0.43 1.	0.06 0.28 2.	.1075	0.14 0.61 2.	0.19 0.43 1.	0.11 0.43 3.
.0975	called	0.06 0.76 3.	-0.10 0.64 5.	.0975	called	0.06 0.76 3.	0.07 0.60 3.

Table 8b: Call Values on 30-Year Graduated Payment Mortgage (graduation at 7½ percent per annum for 5 years), Both Absolute and Relative to the 30-Year Fixed-Rate Mortgage

Absolute

Mean Reversion Value (μ)

SPOT/	.1100	.1175	.1250
.1375	94.36	94.62	94.84
	1.76	1.68	1.63
	24.	23.	22.
.1275	97.23	97.42	97.54
	2.35	2.21	2.14
	31.	29.	28.
.1175	99.83	99.97	100.00
	3.24	3.02	2.90
	41.	39.	37.
.1075	101.89	102.03	101.97
	4.69	4.33	4.17
	58.	54.	52.
.0975	called	102.93	102.98
		6.80	6.39
		82.	78.

Variance Scale Parameter (σ)

SPOT/	.0250	.0500	.0750
.1375	95.72	94.62	93.37
	0.43	1.68	3.17
	6.	23.	43.
.1275	98.78	97.42	96.00
	0.70	2.21	3.87
	9.	29.	51.
.1175	101.55	99.97	98.44
	1.30	3.02	4.78
	17.	39.	61.
.1075	called	102.03	100.58
		4.33	6.01
		54.	75.
.0975	called	102.93	102.23
		6.80	7.74
		82.	94.

Refinancing Wedge (RFW)

SPOT/	.0200	.0300	.0400
.1375	94.39	94.62	94.88
	1.92	1.68	1.43
	26.	23.	20.
.1275	97.10	97.42	97.73
	2.53	2.21	1.90
	34.	29.	25.
.1175	99.51	99.97	100.36
	3.47	3.02	2.63
	45.	39.	34.
.1075	101.28	102.03	102.52
	5.08	4.33	3.84
	63.	54.	48.
.0975	101.64	102.93	103.83
	8.08	6.80	5.90
	98.	82.	71.

Relative to 30-Year

SPOT/	.1100	.1175	.1250
.1375	-0.20	-0.15	-0.16
	0.24	0.21	0.23
	2.	1.	2.
.1275	-0.07	-0.02	-0.05
	0.31	0.26	0.29
	2.	1.	2.
.1175	0.03	0.10	0.04
	0.41	0.33	0.38
	2.	1.	2.
.1075	0.04	0.19	0.09
	0.60	0.43	0.52
	3.	1.	3.
.0975	called	0.06	0.05
		0.76	0.73
		3.	3.

SPOT/	.0250	.0500	.0750
.1375	-0.02	-0.15	-0.28
	0.06	0.21	0.35
	0.	1.	2.
.1275	0.14	-0.02	-0.15
	0.09	0.26	0.42
	1.	1.	2.
.1175	0.32	0.10	-0.04
	0.10	0.33	0.49
	0.	1.	2.
.1075	called	0.19	0.06
		0.43	0.58
		1.	2.
.0975	called	0.06	0.16
		0.75	0.68
		3.	1.

SPOT/	.0200	.0300	.0400
.1375	-0.15	-0.15	-0.13
	0.21	0.21	0.18
	1.	1.	1.
.1275	-0.02	-0.02	0.01
	0.26	0.26	0.23
	1.	1.	1.
.1175	0.10	0.10	0.12
	0.33	0.33	0.31
	1.	1.	2.
.1075	0.15	0.19	0.21
	0.48	0.43	0.42
	1.	1.	2.
.0975	-0.09	0.06	0.42
	0.90	0.76	0.40
	4.	3.	-1.

Table 9: The Sensitivity of Call Values to Differences in Remaining Years to Maturity on 30-Year 13% GNMA (base case parameter values)

Remaining Years to Maturity

SPOT/	30 Yrs.	26 Yrs.	22 Yrs.
.1375	94.78	94.94	95.17
	1.48	1.34	1.16
	22.	20.	17.
.1275	97.43	97.56	97.76
	1.96	1.82	1.62
	28.	26.	23.
.1175	99.86	99.93	100.08
	2.69	2.58	2.37
	37.	36.	33.
.1075	101.83	101.79	101.87
	3.90	3.87	3.66
	52.	52.	49.
.0975	102.87	102.62	102.90
	6.04	6.19	5.72
	79.	81.	75.

V. Summary

Substantial effort has been expended in estimating the magnitudes of the parameters governing the interest-rate process and measuring the sensitivity of call values to these parameters and refinancing costs. Unfortunately, the results are not encouraging for application of the option pricing model to real world data. The interest-rate process is not estimated with great precision; substantial uncertainty remains regarding the speed of adjustment (k), and the data do not discriminate among different values of the elasticity of the standard deviation (α) with respect to the spot rate of interest. All and all, there is not overwhelming evidence for the existence of a stable, single state variable mean-reverting CES process. Substantial uncertainty also exists regarding the magnitude of the refinancing wedge. These difficulties are compounded by the sensitivity of the estimated call values to virtually all the parameters about which we are uncertain.

Nonetheless, some interesting results have been obtained. The differences in call premia in coupon rates among alternative long-term fixed-rate mortgage contracts (15- and 30-year level-payment and 30-year graduated-payment mortgages) are relatively small and insensitive to alternative parameter values. While the dollar value of call is less on the 15-year mortgage than on the 30-year level-payment mortgage, the premium built into the coupon to earn a given call value has to be relatively greater on the former because the coupon will be received for a shorter period. Thus the coupon-equivalents are roughly equal and the differences between them are not sensitive to differences in parameter values. The reverse is true for graduated-payment mortgages (GPMs); the dollar value of call is more than that on the 30-year level-payment mortgage, but a somewhat smaller premium needs to be changed to earn a given call value because the mortgage principal on the GPM will continually exceed that on the 30-year level-payment mortgage. Here the coupon equivalents are nearly identical and are quite insensitive to differences in parameter values. Finally, differences in call premia on near-par GNMAs with the same coupons but with different remaining lives are likewise quite small. On near-par GNMAs the difference between 8-year old GNMAs (22 remaining years to maturity) and new GNMAs with the same coupon is only 5 basis points in terms of coupon equivalents. Thus whether one purchases an old or new GNMA (with same coupon) does not appear to be of great importance.

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