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ABSTRACT

Wage setters take into account the future consequences of their current wage choices in the presence of downward nominal wage rigidities. Several interesting implications arise. First, nominal wages tend to be endogenously rigid also upward, at low inflation. Second, a closed-form solution for a long run Phillips curve relates average unemployment to average wage inflation; the curve is virtually vertical for high inflation rates but becomes flatter as inflation declines. Third, macroeconomic volatility shifts the Phillips curve outward, implying that stabilization policies can play an important role in shaping the trade-off. Fourth, when inflation decreases, volatility of unemployment increases whereas the volatility of inflation decreases: this implies a long-run trade-off also between the volatility of unemployment and that of wage inflation.

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This paper introduces downward wage rigidities in a dynamic stochastic general equilibrium model where forward-looking agents optimally set their wages taking into account the future implications of their choices. A closed-form solution for the long-run Phillips curve is derived. The inflation-unemployment trade-off is shown to depend on various factors, and particularly on the extent of macroeconomic volatility. The paper contributes to the argument that modern monetary models may underestimate the benefits of inflation and may hence suggest an optimal inflation rate that is too low (close to zero).

The conventional view argues against the presence of a long-run trade-off and in favor of price stability. Fifty years ago, Phillips (1958) showed evidence of a negative relationship between the unemployment rate and the changes in nominal wages for 97 years of British data, while Samuelson and Solow (1960) reported a similar fit for US data. The contributions of Friedman (1968), Phelps (1968) and Lucas (1973) as well as the oil shocks of the 1970s cast serious doubts on the validity of the Phillips curve. Although the empirical controversy has yet to settled down (see Ball et al., 1988; King and Watson, 1994; and Bullard and Keating, 1995), the textbook approach to monetary policy is based on the absence of a long-run trade-off between inflation and unemployment: the attempt to take advantage of the short-run trade-off would only generate costly inflation in the long run, so that price stability should be the objective of central banks (see for example Mishkin, 2008).

A wide range of recent monetary models exhibits a long-run relationship between inflation and real activity, due to (symmetric) nominal rigidities and asynchronized price-setting behavior in an intertemporal setup (see among others Goodfriend and King, 1997, and Woodford, 2003).[1] Nonetheless, this literature indicates that the optimal long-run inflation rate should be close to zero and unemployment at the natural rate:[2] even a moderate rate of inflation imposes high costs in terms of unemployment because firms that can adjust prices set a high markup in order to protect future profits from the erosion effect of inflation;[3] moreover, inflation creates costly price dispersion because of the asynchronized price setting. However, virtually no central

[1] State-dependent pricing would tend to weaken the long-run relationship between inflation and unemployment (see for example Golosov and Lucas, 2007).

[2] See Khan, King and Wolman (2003), Wolman (2001) and Schmitt-Grohe and Uribe (2004).

[3] It is a questionable assumption to impose price rigidity even at high inflation rates. But, this is a features of time-dependent price-setting models. A model with state-dependent pricing would instead imply a vertical Phillips curve at high inflation rates.

bank is adopting a policy of price stability, even though the number of countries adopting inflation targeting has been rapidly increasing over the past decade and a half.

This recent literature has mainly introduced symmetric price rigidities, while one of the most popular arguments against a zero-inflation policy relies on the existence of downward nominal rigidities.[4] A lower bound on wages and prices keeps them from falling and induces a drift: a negative demand shock would just reduce inflation if inflation remains positive, but would induce unemployment if prices would need to fall. A monetary policy committed to price stability can achieve its objective only by a very restrictive policy that increases the unemployment rate. It follows that at low inflation rates there is a high sacrifice-ratio of pursuing deflationary policies and the marginal benefit of inflation as "greasing" the labor market could be high. Akerlof et al. (1996) were the first to model labor market with downward wage rigidity and derive a trade-off between unemployment and inflation. But, at that time several researchers doubted the relevance of wage rigidities at low inflation and suggested the need for more international evidence.[5]

There is now a strong body of evidence indicating the presence of downward wage rigidities across a wide spectrum of countries, often even at low inflation (see for example Lebow, Saks, and Wilson, 2003, for the U.S., and the very long lists of citations in Akerlof, 2007, and in Holden, 2004).[6] Several explanations have been put forward for the existence of such rigidities, such as fairness and social norms (Bewley, 1999, and Akerlof, 2007) or labor market institutions (Holden, 2004). The combination of these factors is likely to imply that these rigidities could persist for a long time even in a low inflation environment, which would overturn one of the main arguments against the relevance of downward wage rigidities. Indeed, empirical studies about several European countries have found that downward wage rigidities

[4] Already in *The General Theory of Employment, Interest and Money,* Keynes leverages on the fact that workers usually resist a reduction of money-wages to question the conclusion of the classical analysis with regards the existence of a unique frictional rate of unemployment. Numerous authors, from Samuelson and Solow (1960) and Tobin (1972) to Akerlof (2007), stressed their importance for the existence of a long-run trade-off between inflation and unemployment.

[5] See the comments to Akerlof et al. (1996). Ball and Mankiw (1994) also claim that downward rigidities should disappear at low inflation.

[6] Evidence of downward rigidities on goods prices is not as conclusive (see for example Peltzman, 2000; Alvarez et al., 2006; and Chapter 18 in Blinder et al., 1998).

persist during low inflation periods.[7] Consistently, other works have found that the "grease" effect of inflation is more relevant in countries with highly regulated labor market (Loboguerrero and Panizza, 2006). It is thus not surprising that several studies on the U.S. labor market find that, despite a clear evidence of the presence of downward nominal rigidities, the evidence in favor of a "grease" effect of inflation is weaker (Groshen and Schweitzer, 1999, and Card and Hyslop, 1996). However, evidence for the U.S. should not be used to dismiss the broader implications of such rigidities in other countries.

In this paper, we introduce downward wage rigidity in an otherwise dynamic stochastic general equilibrium model, with forward looking optimizing agents that enjoy goods consumption and experience disutility from labor when working for profit-maximizing firms. Labor and goods markets are characterized by monopolistic competition, and goods prices are fully flexible. The economy is subject to an aggregate productivity shock and to stochastic perturbations to nominal spending.

The most important novelty with respect to the seminal contribution of Akerlof et al. (1996) is the focus on the dynamic implications of downward wage rigidities in a model otherwise similar to those that have been employed to argue against the existence or relevance of a long-run trade-off.[8] Moreover, we derive an analytical solution for the long-run distribution of inflation and unemployment and for the long-run Phillips curve. We find that the Phillips curve is almost vertical for medium-to-high inflation rates but can display a significant trade-off at low inflation rates, consistently with the literature on downward nominal rigidities.

An important determinant of the trade-off at low inflation rate is given by the volatility of nominal spending growth. Thus the unemployment-inflation trade-off should be different across countries experiencing different macroeconomic volatility (and not only across countries with different degrees of rigidity in the labor market as discussed in the literature). Hence, it is unlikely that a similar inflation target would

[7] See Agell and Lundburg (2003), Fehr and Gotte (2005), and Knoppik and Beissinger (2003).

[8] Andersen (2001) presents as well a static model which can be solved in a closed form, while Bhaskar (2003) offers a framework that endogeneize downward price rigidities. Our work is closely related to the literature on irreversible investment, since a dynamic problem in which wages cannot fall is similar to a problem in which capital cannot fall (see Bertola, 1998; Bertola and Caballero, 1994; Dixit, 1991; Dumas, 1991; Pindyck, 1988; and Stokey, 2006). In a recent work. Kim and Ruge-Murcia (2007) introduce asymmetric wage rigidities (stronger downward than upward) in a model similar to ours, but do not derive a closed form solution for the Phillips curve.

be ideal for all countries: countries experiencing higher macroeconomic volatility may want to target a higher inflation rate in order to reduce long-run unemployment. This result contrasts with the view that the gains from appropriate stabilization policies conducted by monetary and fiscal authorities are negligible, as found in Lucas (2003). In the framework we propose, the role of macroeconomic policies in stabilizing the shocks might have important first-order effects on unemployment at low inflation rates. Moreover, even for the same country the trade-off can change over time if macroeconomic volatility changes.

Downward wage rigidity in a dynamic model delivers several other interesting implications. When adjusting wages upward in the fact of a positive shock, wage setters have to take into account the future consequences of their wage choices. Indeed, they do not want to be constrained by too high wages in the future in case unfavorable shocks would require a wage cut. Hence, downward wage inflexibility in the presence of a forward-looking behavior implies an endogenous upward wage rigidity at low inflation rates. This effect mainly holds at low inflation, as at high inflation the downward rigidities are not effectively binding.

This mechanism also implies that there is a trade-off not only between mean wage inflation and unemployment, but also between their volatilities, as common also in the literature on monetary policy rules evaluation (see Clarida et al., 1999, Svensson, 1999, and Taylor, 1999). Also the trade-off between volatilities, and not just that between first moments, can be improved upon via stabilization policies aimed at reducing the volatility of nominal spending growth.

The paper is organized as follows. Section 1 describes the model. Section 2 and 3 present the solutions under flexible and downward-rigid wages, respectively. Section 4 solves for the long-run Phillips curve. Section 5 discusses the implication for volatilities. Section 6 draws conclusions.

1 The model

We describe a closed-economy model in which there are a continuum of infinitely lived households and firms (both in a [0,1] interval). Each household derives utility from the consumption of a continuum of goods aggregated using a Dixit-Stiglitz consumption index, and disutility from supplying one of the varieties of labor in a monopolistic-competitive market. The model assumes the presence of downward nominal rigidities: wages are chosen by optimizing households under the constraint that they cannot fall (this assumption will be relaxed in Section 4.3). Firms hire all varieties of labor to produce one of the continuum of consumption goods and operate in a monopolistic-competitive market where prices are set without any friction. The economy is subject to two aggregate shocks: a productivity and a nominal spending shock. The productivity shock is denoted by A_t, whose logarithmic a_t is distributed as a Brownian motion with drift g and variance σ_a^2

$$da_t = gdt + \sigma_a dB_{a,t} \tag{1}$$

where $B_{a,t}$ denotes a standard Brownian motion with zero drift and unit variance. The nominal spending shock is denoted by $\tilde{Y}_t$ whose logarithmic $\tilde{y}_t$ is also distributed as a Brownian motion, now with drift θ and variance σ_y

$$d\tilde{y}_t = \theta dt + \sigma_y dB_{y,t} \tag{2}$$

where $dB_{y,t}$ is a standard Brownian motion with zero drift and unit variance that might be correlated with $dB_{a,t}$.

Household j has preferences over time given by

$$E_{t_0}\left[\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\left(\ln C_t^j - \frac{l_t^{1+\eta}(j)}{1+\eta}\right) dt\right] \tag{3}$$

where the expectation operator $E_{t_0}(\cdot)$ is defined by the shock processes (1) and (2), and $\rho > 0$ is the rate of time preference. Current utility depends on the Dixit-Stiglitz consumption aggregate of the continuum of goods produced by the firms operating in the economy

$$C_t^j \equiv \left[\int_0^1 c_t^j(i)^{\frac{\theta_p}{\theta_p - 1}} di\right]^{\frac{\theta_p - 1}{\theta_p}}$$

where $\theta_p > 0$ is the elasticity of substitution among consumption goods and $c_t^j(i)$ is household j's consumption of the variety produced by firm i. An appropriate consumption-based price index is defined as

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta_p} di\right]^{\frac{1}{1-\theta_p}},$$

where $p_t(i)$ is the price of the single good i.

The utility flow is logarithmic in the consumption aggregate. In (3), labor disutility is assumed to be isoelastic with respect to the labor supplied $l_t(j)$, with $\eta \geq 0$ measuring the inverse of the Frisch elasticity of labor supply.[9]. Household j's intertemporal budget constraint is given by

$$E_{t_0}\left\{\int_{t_0}^{\infty} Q_t P_t C_t^j dt\right\} \leq E_{t_0}\left\{\int_{t_0}^{\infty} Q_t \left[w_t(j) l_t(j) + \Pi_t^j\right] dt\right\} \tag{4}$$

where Q_t is the stochastic nominal discount factor in capital markets where claims to monetary units are traded; $w_t(j)$ is the nominal wage for labor of variety j, and Π_t^j is the profit income of household j.

Starting with the consumption decisions, household j chooses goods demand, $\{c_t^j(i)\}$, to maximize (3) under the intertemporal budget constraint (4), taking prices as given. The first-order conditions for consumption choices imply

$$e^{-\rho(t-t_0)} C_t^{-1} = \chi Q_t P_t \tag{5}$$

$$\frac{c_t(i)}{C_t} = \left(\frac{p_t(i)}{P_t}\right)^{-\theta_p} \tag{6}$$

where the multiplier χ does not vary over time. The index j is omitted from the consumption's first-order conditions, because we are assuming complete markets through a set of state-contingent claims to monetary units.

Before we turn to the labor supply decision, we analyze the firms' problem. We assume that the labor used to produce each good i is a CES aggregate, $L(i)$, of the continuum of individual types of labor j defined by

$$L_t(i) \equiv \left[\int_0^1 l_t^d(j)^{\frac{\theta_w - 1}{\theta_w}} dj\right]^{\frac{\theta_w}{\theta_w - 1}}$$

with an elasticity of substitution $\theta_w > 1$. Here $l_t^d(j)$ is the demand for labor of type j. Given that each differentiated type of labor is supplied in a monopolistic-competitive market, the demand for labor of type j on the part of wage-taking firms is given by

$$l_t^d(j) = \left(\frac{w_t(j)}{W_t}\right)^{-\theta_w} L_t, \tag{7}$$

[9] These preferences are consistent with a balanced-growth path since we are assuming a drift in technology.

where W_t is the Dixit-Stiglitz aggregate wage index

$$W_t \equiv \left[\int_0^1 w_t(j)^{1-\theta_w} dj \right]^{\frac{1}{1-\theta_w}}; \tag{8}$$

and aggregate demand for labor L_t is defined as

$$L_t \equiv \int_0^1 L_t(i) di.$$

We assume a common linear technology for the production of all goods

$$y_t(i) = A_t L_t(i).$$

Profits of the generic firm i, $\Pi_t(i)$, are given by

$$\Pi_t(i) = p_t(i) y_t(i) - W_t L_t(i).$$

In a monopolistic-competitive market, given (6), each firm faces the demand

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta_p} Y_t$$

where total output is equal in equilibrium to aggregate consumption $Y_t = C_t$. Since firms can freely adjust their prices, standard optimality conditions under monopolistic competition imply that all firms set the same price

$$p_t(i) = P_t = \mu_p \frac{W_t}{A_t} \tag{9}$$

where $\mu_p \equiv \theta_p/(\theta_p - 1) > 1$ denotes the mark-up of prices over marginal costs. An implication of (9) is that labor income is a constant fraction of nominal income

$$\tilde{Y}_t = P_t Y_t = \mu_p W_t L_t. \tag{10}$$

Given firms' demand (7), a household of type j chooses labor supply in a monopolistic-competitive market to maximize (3) under the intertemporal budget constraint (4) taking as given prices $\{Q_t\}$, $\{P_t\}$ and the other relevant aggregate variables. An equivalent formulation of the labor choice is the maximization of the following objective

$$E_{t_0} \left[\int_{t_0}^{\infty} e^{-\rho(t-t_0)} \pi(w_t(j), W_t, \tilde{Y}_t) dt \right] \tag{11}$$

by choosing $\{w_t(j)\}_{t=t_0}^{\infty}$, where

$$\pi(w_t(j), W_t, \tilde{Y}_t) \equiv \frac{1}{\mu_p}\left(\frac{w_t(j)}{W_t}\right)^{1-\theta_w} - \frac{1}{\mu_p}\frac{1}{1+\eta}\left(\frac{w_t(j)}{W_t}\right)^{-(1+\eta)\theta_w}\left(\frac{\tilde{Y}_t}{W_t}\right)^{1+\eta}.$$

Households would then supply as much labor as demanded by firms in (7) at the chosen wages. In deriving $\pi(\cdot)$ we have used (5), (7) and (10). Note that the function $\pi(\cdot)$ is homogeneous of degree zero in $(w_t(j), W_t, \tilde{Y}_t)$.

2 Flexible wages

We first analyze the case in which wages are set without any friction, so that they can be moved freely and fall if necessary. With flexible wages, maximization of (11) corresponds to per-period maximization and implies the following optimality condition

$$\pi_w(w_t(j), W_t, \tilde{Y}_t) = 0$$

where $\pi_w(\cdot)$ is the derivative of $\pi(\cdot)$ with respect to the first argument. Since this holds for each j and there is a unique equilibrium, then $w_t(j) = W_t$. With our preference specification we thus obtain that nominal wages in the flexible case, W_t^f, are proportional to nominal spending

$$W_t^f = (\mu_w)^{\frac{1}{1+\eta}}\tilde{Y}_t \qquad (12)$$

where the factor of proportionality is given by the wage mark-up, defined by $\mu_w \equiv \theta_w/(\theta_w - 1)$, and by the elasticity of labor supply. We can also obtain the equilibrium level of aggregate labor in the flexible case, L^f, using (10) and (12)

$$L^f = \mu_p^{-1}\mu_w^{-\frac{1}{1+\eta}},$$

which is a constant and just a function of the price and wage mark-ups as well as of the labor elasticity. It follows that the unemployment rate, u_t^f, is given by

$$u^f = 1 - L^f,$$

where total labor force (equal to 1) is defined as the employment that would prevail if $\mu_w = \mu_p = 1$. Consumption and output follow from the production function. Prices, P_t^f, are given by

$$P_t^f = \mu_p\frac{W_t^f}{A_t}.$$

In this frictionless world, prices and wages move proportionally to nominal spending and unemployment is always constant. The Phillips curve is vertical.

3 Downward nominal wage rigidity

When nominal wages cannot fall below the level reached in the previous period, the constraint that $dw_t(j)$ should be non-negative needs to be added (Section 4.3 will explore alternative degrees of downward rigidities). The objective is then to maximize (11) under

$$dw_t(j) \geq 0 \tag{13}$$

with $w_{t_0} > 0$. In other words, agents choose a non-decreasing positive nominal wage path to maximize (11). Let us define the value function $V(\cdot)$ for this problem as

$$V(w_{t_0}(j), W_{t_0}, \tilde{Y}_{t_0}) = \max_{\{w_t(j)\} \in \mathcal{W}} E_{t_0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \pi(w_t(j), W_t, \tilde{Y}_t) dt \right\},$$

where $\mathcal{W}$ is the set of non-decreasing positive sequences $\{w_t(j)\}$. In the appendix we show that along the optimal path the following smooth-pasting condition holds (see Dixit, 1991)

$$V_w(w_t(j), W_t, \tilde{Y}_t) = 0 \quad \text{if } dw_t(j) > 0,$$
$$V_w(w_t(j), W_t, \tilde{Y}_t) \leq 0 \quad \text{if } dw_t(j) = 0,$$

where $V_w(\cdot)$ is the derivative of $V(\cdot)$ with respect to the first argument.

Moreover the maximization problem is concave and the above conditions are also sufficient to characterize a global optimum as shown in the appendix. It follows that all wage setters are going to set the same wage, $w_t(j) = W_t$ for all j. We define $v(W_t, \tilde{Y}_t) \equiv V_w(W_t, W_t, \tilde{Y}_t)$, and then $W(\tilde{Y}_t)$ as the function that solves

$$v(W(\tilde{Y}_t), \tilde{Y}_t) = 0.$$

In particular $W(\tilde{Y}_t)$ represents the current *desired* wage taking into account future downward-rigidity constraints, but not the current one (i.e. if agents were free to choose the current wage, even below the previous period wage, considering that future wages cannot fall). The agent will set $W_t = W(\tilde{Y}_t)$ whenever $dW_t \geq 0$, so that *actual* wages are the maximum of previous period wages and current *desired* wages. It follows that *actual* wages cannot fall below $W(\tilde{Y}_t)$, i.e. $W_t \geq W(\tilde{Y}_t)$. Either they are above the *desired* level, when the downward-rigidity constraint is binding, or they are equal, when an adjustment occurs. In particular, we show that

$$\begin{aligned} W(\tilde{Y}_t) &= c(\theta, \sigma_y^2, \eta, \rho) \cdot \mu_w^{\frac{1}{1+\eta}} \tilde{Y}_t \\ &= c(\theta, \sigma_y^2, \eta, \rho) \cdot W_t^f \end{aligned} \tag{14}$$

where $c(\cdot)$ is a non-negative function of the model's parameters as follows

$$c(\theta, \sigma_y^2, \eta, \rho) \equiv \left(\frac{\theta + \frac{1}{2}\gamma(\theta, \sigma_y^2, \rho) \cdot \sigma_y^2}{\theta + \frac{1}{2}(\gamma(\theta, \sigma_y^2, \rho) + \eta + 1) \cdot \sigma_y^2} \right)^{\frac{1}{1+\eta}} \leq 1$$

and $\gamma(\cdot)$ is a non-negative function of some parameters of the model

$$\gamma = \frac{-\theta + \sqrt{\theta^2 + 2\rho\sigma^2}}{\sigma^2}$$

as derived in the appendix.[10]

Agents' optimizing behavior in the presence of exogenous downward wage rigidities implies an endogenous tendency for *upward* wage rigidities. When wages adjust upward, they adjust to the *desired* level $W(\tilde{Y}_t)$, which is always below the *flexible*-case wage by a factor $c(\cdot)$. Indeed, optimizing wage setters choose an adjustment rule that tries to minimize the inefficiencies of downward wage inflexibility. Wage setters are worried to be stuck with an excessively high wage should future unfavorable shocks require a wage decline (as downward wage rigidities would imply a fall in employment). As a consequence, optimizing agents refrain from excessive wage increases when favorable shocks require upward adjustment, pushing current employment above the flexible-case level. Note that the fact that *desired* wages are always below the *flexible*-case wage does not imply that *actual* wages are always below the *flexible*-case wage: indeed, when the downward-rigidity constraint is binding, *actual* wages could be higher, and employment lower, than in the *flexible* case. As we will see in the next section, in the long run, unemployment would be higher on average then in the flexible-case wage.

The reaction of nominal wages to a nominal expenditure shock ($c(\cdot)$), when wages can adjust upward, depends on the properties of the nominal expenditure process

[10] It is possible that the desired wage, $W(\tilde{Y}_t)$, falls below the one associated with full employment. While temporary overemployment is not unrealistic, in the appendix we also solve the model with the additional constraint $l_t(j) \leq 1$ for each j.

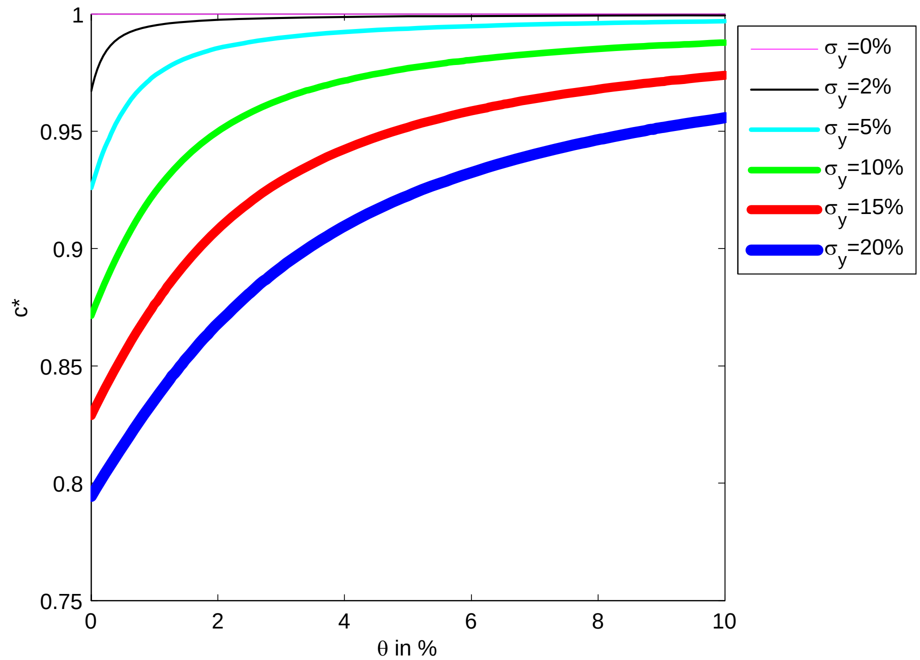

Figure 1: Plot of the function $c(\cdot)$ defined in (14) against the mean of nominal spending growth, θ, and for different standard deviations of nominal spending growth, σ_y. θ and σ_y are in percent and at annual rates. $\eta = 2.5$, $\rho = 0.01$ and $u^f = 6\%$.

(i.e. its mean and variance), the rate of time preference, and on the elasticity of labor supply. In particular the wage reaction is weaker ($c(\cdot)$ is low) when the variance of nominal expenditure growth is high (σ^2 is large), when the mean of nominal expenditure growth is small (θ is small), when agents discount less the future (ρ is low), and when the elasticity of labor is higher (η is low). First, when shocks are very volatile, future unfavorable shocks can be very large and hence very costly in terms of unemployment should the wage constraint be binding. As a limiting case, when $\sigma^2 = 0$, then $c(\cdot) = 1$ and $W(\tilde{Y}_t) = W_t^f$. Second, when the mean of nominal spending growth is low, it is better to have a muted reaction, since it is more likely that even small shocks would lead wages to hit the lower bound. When θ becomes very large, the drift in nominal spending growth is very sizable and the lower bound is not really effective, so that $c(\cdot)$ gets close to 1. In this case, it is unlikely that downward wage inflexibility is going to bind so that the flexible-wage level of employment will be achieved most of the time. Third, when wage setters discount less the future (high values of ρ) they are not going to anticipate future consequences of current wage decisions, and would set wages (when the downward rigidity is not binding) at a level close to the flexible-wage level implying higher unemployment in the long run. Indeed when ρ increases, $\gamma(\cdot)$ increases, and $c(\cdot)$ can get close to one. In this case, when shocks are unfavorable employment falls (due to the downward rigidities), but when shocks are favorable employment does not exceed much the flexible-wage level. Fourth, when labor supply is less elastic (η is high), wage setters want to avoid large fluctuations in hours worked so they set higher wages when adjusting ($c(\cdot)$ gets close to one), thus reducing the variability of employment fluctuation but also average employment.

In Figure 1 we plot $c(\cdot)$ as a function of the mean of the log of nominal spending growth, θ, with different assumptions on the standard deviation of nominal spending growth, σ_y, ranging from 0% to 20% at annual rates. The parameters' calibration is based on a discretized quarterly model. In particular, the rate of time preference ρ is equal to 0.01 as standard in the literature implying a 4% real interest rate at annual rates. The Frisch elasticity of labor supply is set equal to 0.4, as it is done in several studies, thus $\eta = 2.5$.[11] When $\sigma_y = 0\%$, $c(\cdot) = 1$. With positive standard deviations, $c(\cdot)$ decreases as θ decreases. The decline in c is larger the higher is the standard deviation of the nominal spending shock, as previously discussed.

4 The Phillips curve

4.1 Long-run Phillips curve

We can now solve for the equilibrium level of employment and characterize the inflation-unemployment trade-off in the presence of downward nominal wage rigidities. Equation (10) implies that

$$L_t = \frac{1}{\mu_p}\frac{\tilde{Y}_t}{W_t}.$$

Since we have shown that $W_t \geq c(\cdot)\left(\mu_w\right)^{\frac{1}{1+\eta}}\tilde{Y}_t$, it follows that $0 \leq L_t \leq L^f/c(\cdot)$. The existence of downward wage rigidities endogenously adds an upward barrier on the employment level. Moreover, since $\tilde{y}_t$ follows a Brownian motion with drift θ

[11] See Smets and Wouters (2003).

and standard deviation σ_y, also $l_t = \ln L_t$ is going to follow a Brownian motion but with a reflecting barrier at $\ln(L^f/c(\cdot))$. The probability distribution function for such process can be computed at each point in time.[12] We are here interested in studying whether this probability distribution converges to an equilibrium distribution when $t \to \infty$, in order to characterize the long-run probability distribution for employment, and thus unemployment. Standard results assure that this is the case when the drift of the Brownian motion of nominal-spending growth is positive, $\theta > 0$.[13] In this case, it can be shown that the long-run cumulative distribution of L_t, denoted with $P(\cdot)$, is given by

$$P(L_\infty \leq x) = \left(\frac{x}{L^f/c(\cdot)}\right)^{\frac{2\theta}{\sigma_y^2}}$$

for $0 \leq x \leq L^f/c(\cdot)$ where L_∞ denotes the long-run equilibrium level of employment. Since $u_t = 1 - L_t$, we can also characterize the long-run equilibrium distribution for the unemployment rate and evaluate its long-run mean

$$E[u_\infty] = 1 - \frac{1}{1 + \frac{\sigma_y^2}{2\theta}} \frac{(1-u^f)}{c(\theta, \sigma_y^2, \eta, \rho)}. \tag{15}$$

First note that when there is no uncertainty, $\sigma_y^2 = 0$ and $c(\cdot) = 1$, then the long-run unemployment rate coincides with the flexible-wage unemployment rate. In the stochastic case, two forces explain why the long-run equilibrium unemployment rate can differ from the flexible-wage level. On the one hand, a high variance-to-mean ratio of the nominal-expenditure shock (σ_y^2/θ) increases the equilibrium level of unemployment, because the downward wage constraint is more binding and downward rigidities are more costly in terms of lower employment. On the other hand, wage setters incorporate these costs by setting lower wages when adjusting ($c(\cdot)$ falls); this decreases the average unemployment rate, because, as discussed in the previous section, employment can increase above the flexible-wage level when there are favorable shocks. However, the first channel dominates the second one in the long run, and long-run average unemployment is never below the natural rate, i.e. $E[u_\infty] \geq u^f$

[12] See Cox and Miller (1990, pp. 223-225) for a detailed derivation.

[13] Otherwise, when the mean of nominal-spending growth is non-positive, the probability distribution collapses to zero everywhere, with a spike of one at zero employment and thus 100% unemployment rate in the long run. However, this is not a realistic case because nominal spending growth is rarely negative, and θ represents its mean.

since $(1+\sigma_y^2/2\theta)\cdot c(\cdot)\geq 1$.[14]

To construct the long-run Phillips curve, a relationship between average wage inflation and unemployment, we need to solve for the long-run equilibrium level of wage inflation. From the equilibrium condition (10), we note that

$$d\tilde{y}_t = \pi_t^w + dl_t$$

where π^w is the rate of wage inflation. Since $E(d\tilde{y}_t) = \theta$ and l_t converges to an equilibrium distribution implying $E(dl_\infty) = 0$, the long-run mean wage inflation rate is given by

$$E[\pi_\infty^w] = \theta. \tag{16}$$

Substituting (16) into (15), we obtain the long-run Phillips curve

$$E[u_\infty] = 1 - \frac{1}{1+\frac{\sigma_y^2}{2E[\pi_\infty^w]}}\frac{(1-u^f)}{c(E[\pi_\infty^w],\sigma_y^2,\eta,\rho)} \tag{17}$$

a relation between mean unemployment rate and mean wage inflation rate.

The long-run Phillips curve is no longer vertical. The “natural” rate of unemployment is not unique, but depends on the mean inflation rate. The shape of this long-run Phillips curve depends on the parameters of the model η, ρ, u^f and σ_y^2. It is important to note that σ_y^2 could in part be influenced by stabilization policies.[15] Indeed, in the real world, volatility of nominal spending growth is likely to result from real business cycle shocks, macroeconomic policies, and their interaction. It follows that the relation between average wage inflation and unemployment depends in a critical way on policy parameters and the business cycle fluctuations.[16]

When the mean wage inflation rate is high, $c(\cdot)$ is close to 1 and the average unemployment rate converges to u^f from above. The Phillips curve is virtually vertical for high inflation rates. In these cases, there is no long-run trade-off between inflation and unemployment. When instead the wage inflation is low, a trade-off emerges.

[14]Indeed $(1+\sigma_y^2/2\theta)\cdot c(\cdot)\geq 1$ when $\eta = 0$, and $c(\cdot)$ is non-decreasing function of η, as shown in the appendix.

[15]Structural policies affecting the degree of competition in the goods and labor markets could affect u^f.

[16]Lucas (1973) presents a model that displays a short-run trade-off between inflation and unemployment that depends on the macro volatility. Here a similar dependence is shown also for the long run.

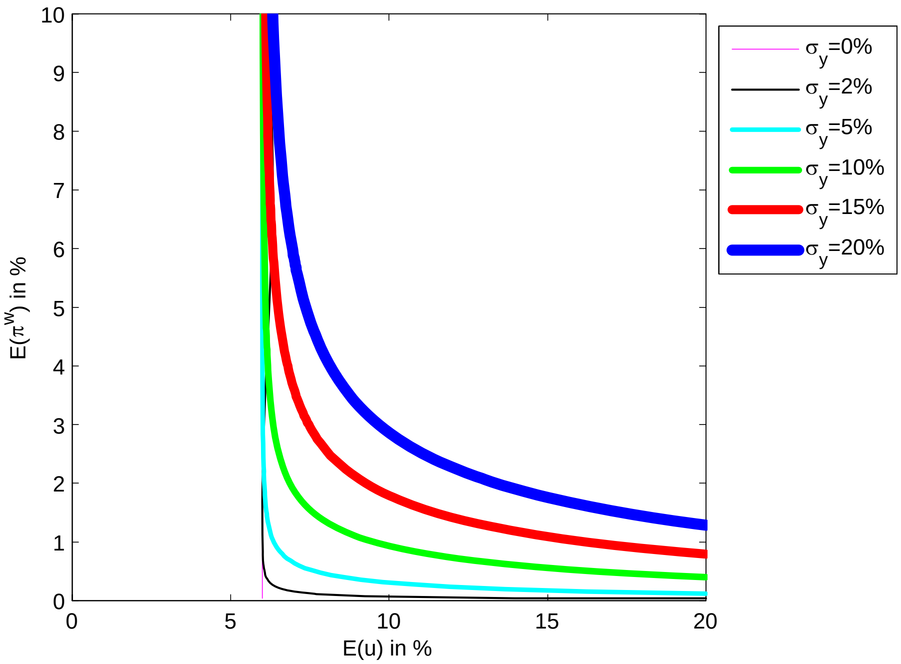

Figure 2: Long-run relationship between mean wage inflation rate, $E[\pi^w]$, and mean unemployment rate, $E[u]$, for different standard deviations of nominal spending growth, σ_y. All variables in % and at annual rates; $\eta = 2.5$, $\rho = 0.01$ and $u^f = 6\%$.

Moreover, the higher the variance of nominal-spending growth, the more a fall in the inflation rate would increase the average unemployment rate. An econometrician that observes realizations of inflation and unemployment at low inflation rates might have hard time uncovering a natural rate of unemployment as determined only by structural factors, unless macroeconomic volatility is properly accounted for. In Figure 2, for the same parameters' configuration as in Figure 1, we plot the Phillips curve for different values of the standard deviation σ_y ranging from 0% to 20% at annual rates. Wage inflation and unemployment are in percent and wage inflation is annualized. For high inflation rates the Phillips curve is virtually vertical at u^f, but for low inflation rates it becomes flatter.[17] When the standard deviation of the shocks

[17] If we were to take into account the constraint that employment should not exceed 1, there would be a kink in the Phillips curve at low inflation rates which would flatten the curve even more and reinforce our results.

is higher, the long-run average unemployment rate is higher for the same long-run average rate of wage inflation.

	σ_y					
$\Delta E[u_\infty]$	0%	2%	5%	10%	15%	20%
Reduction in $E[\pi^w_\infty]$ from:						
4% to 1%	0.0	0.0	0.4	3.4	9.5	16.9
5% to 2%	0.0	0.0	0.1	0.8	2.8	6.0
5% to 3%	0.0	0.0	0.0	0.2	1.0	2.3
6% to 3%	0.0	0.0	0.0	0.3	1.1	2.7

Table 1: Increase in long-run mean unemployment rate, $E[u_\infty]$, due to a reduction in long-run mean wage inflation, $E[\pi^w_\infty]$, for different standard deviations of nominal spending growth, σ_y. All variables are in percent and at annual rates. (Authors' calculations).

An illustrative example may be suggestive. On the basis of the parametrization underlying Figure 2, a country that is subject to low macroeconomic volatility (say a standard deviation of nominal GDP growth equal to 2%) may experience a negligible increase in unemployment when average wage inflation declines from 6 to 3 percent or even from 4 to 1 percent (see Table 1). However, a country with a significant macroeconomic volatility (say 10 percent) may face a cost in term of average unemployment of about 0.3% when inflation falls from 6 to 3 percent and of 3.4% when inflation falls from 4 to 1 percent. And for a country with very high volatility, the costs would be much higher. These calculations are purely illustrative: a more realistic assessment would need to be based on much more complex models. Nonetheless they are still indicative that significant unemployment costs are likely to be associated with achieving price stability for countries with moderate or high volatility in nominal spending growth.

Such range of volatilities have not been unusual over the past three decades. Several countries (mainly industrial ones, such as the U.S. and U.K.) exhibited low volatility, as witnessed by a standard deviation of both nominal and real quarterly GDP growth in the order of 2-3 percent. Other countries showed moderate levels at around 4-6 percent (Sweden and Korea) and it was not uncommon to find figures between 5 and 10 percent (Switzerland, Ireland, and Thailand). Some countries had volatility in excess of 10 percent (Israel) or even 20 percent (Brazil, Mexico, and Turkey).

Note that it is reasonable to expect that volatility of nominal GDP growth would decline as inflation declines. Endogenizing volatility to inflation would then steepen the Phillips curve. However, the decline in volatility is likely to be limited, and mainly due to a reduction in volatility of inflation rather than growth. Even at zero inflation, both inflation volatility and output volatility would persist.[18]

4.2 Short-run Phillips curve

The long-run Phillips curve is located to the right of the unique employment level under flexible wages, and it is tangent to such a level for high inflation rates. However, the short-run Phillips curve (defined as the relation between average unemployment and average inflation over a short period) would present a trade-off also in the region below the unemployment under flexible wages. The main reason lies in the endogenous upward rigidity described in Section 3: when agents can adjust their wage upward, they will set it at a level below the one that would prevail under flexible wages

[18] To gauge the potential decline we estimated the relation between the 3-year standard deviation of quarterly nominal GDP growth and the 3-year mean of quarterly GDP deflator inflation, in a panel regression with fixed effect and 9 periods over 1980-2006 for a sample of 24 industrial and 24 developing countries (from the IFS or WEO databases; for a subset of countries seasonal adjustment was not available in the original dataset and was implemented on the basis of the X12 method in EVIEWS). The relation was specified in either linear or logarithmic terms and with or without time effects. The effect of inflation on nominal GDP volatility was found to be positive and generally significant, although reasonably small. Additional regressions show that such an effect was mainly due to the effect of inflation on inflation volatility rather than on real growth volatility. Indeed, the effect on real volatility was invariably smaller than the one on nominal volatility and generally insignificant, while the one on inflation volatility was large and always significant. Results were quite similar when breaking the sample in industrial and developing countries. The largest effect of inflation on nominal volatility was found in the logarithmic specification without time dummies, with a coefficient of 0.23: a reduction in inflation by 10 percent (say from 10 to 9 percentage points) would be associated with a much less than proportional decline in volatility (by 2.5 percent of its initial level).

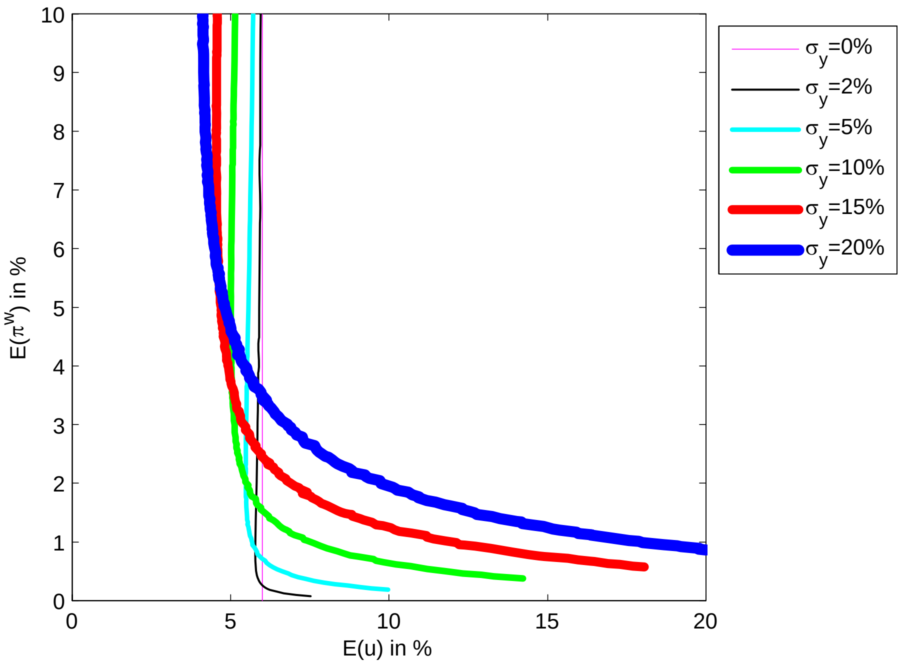

Figure 3: Short-run relationship between mean wage inflation rate, $E[\pi^w]$, and mean unemployment rate, $E[u]$, for different standard deviations of nominal spending growth, σ_y. All variables in % and at annual rates; $\eta = 2.5$, $\rho = 0.01$ and $u^f = 6\%$.

(and employment would be above the flexible-case one), as they anticipate the future binding effect of such a wage choice. When wages are low (not likely to be binding), the chance of a wage adjustment is high and on average unemployment will be below the flexible-case one. When wages are high, the chance of a wage adjustment is small and on average unemployment will be above the flexible-case one. Hence the shape of the short-run Phillips curve and the chance that it will span in areas when unemployment is below the flexible case depend on how likely wages are to be binding. The short-run Phillips curve would tend to shift to the right over time, as the extent to which wages are likely to be binding would tend to increase over time (until long-run convergence is achieved). Indeed, at the beginning of the agents' horizon, agents would set the wage to a low level, for the reasons discussed above. As time progresses, highly inflationary shocks would raise the wage and make it more likely to be binding in the future, especially in a low inflation environment.

It is important to note that also the short-run Phillips curve implies a significant trade-off between unemployment and wage inflation in a low inflation environment, and that such a trade-off is again largely dependent on the degree of volatility present in the economy. This is shown in Figure 3 for the same calibration as in Figure 2.[19] Volatility would have two effects on the short-run Phillips curve. First, it would increase the chance of a binding downward rigidities, thus increasing unemployment. Second it would make agents more cautious in setting their wage claims. The first effect would dominate at low inflation levels (and is the one that would dominate also in the long run), while the second one would dominate at moderate inflation rates. Hence the relative positions of the short-run Phillips curve for countries with different degrees of volatility would depend on the level of inflation: the country with higher volatility would face a short-run trade-off that is placed more to the right for low inflation and to the left for moderate inflation. As inflation increases however, also the short-run Phillips curve converges to the flexible-wage employment level, so that the curve becomes concave. As time progresses, the Phillips curve (for any degree of volatility) shifts to the right and converges to the long run depicted in Figure 2. [20]

4.3 Varying the degree of downward rigidities

The main criticism of an approach that includes downward wage rigidities is that this inflexibility should disappear as the inflation rate declines toward zero (see the comments to Akerlof et al., 1996, and Ball and Mankiw, 1994) As we discussed in the introduction, there is now more evidence that downward wage rigidities persist even during low inflation periods. Nonetheless, we explore the implications of a link between the degree of downward rigidities and inflation, by replacing the assumption

[19]Figure 3 is obtained through simulations of the model in which the first 400 observations are repeated 10000 times. In the short run, average wage inflation is slightly above θ for very low θ, so that the curves do not reach the x-axis even when θ is close to zero. This is because agents are very cautious and set very low wages at the beginning of the horizon when θ is very low, implying that upward adjustment would occur quite frequently at the beginning of the horizon.

[20]In the short run, it is not true that average wage inflation is equal to θ. Actually, it is the case that the average wage inflation is above θ for very low θ. This is because agents are very cautious and set very low wages at the beginning of the horizon when θ is very low. So it is likely that shocks that require upward adjustment occur quite frequently at the beginning of the horizon. This appears in Figure 3 since the curves do not reach the x-axis even when θ is close to zero. In Figure 3, θ varies in the range $(0, 10]$ in percent and at annual rates.

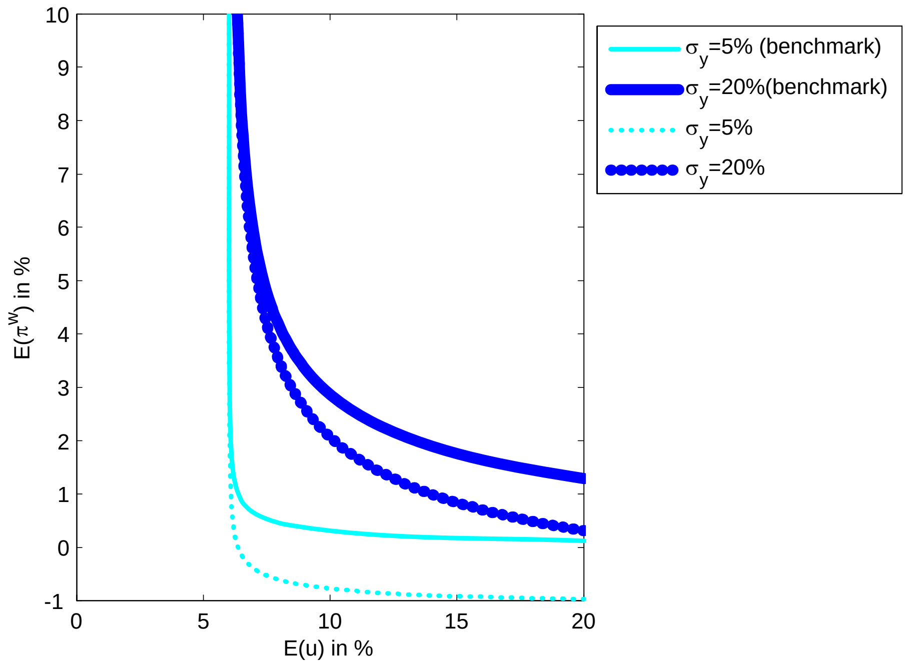

Figure 4: Long-run relationship between mean wage inflation rate, $E[\pi^w]$, and mean unemployment rate, $E[u]$, for different standard deviations of nominal spending growth, σ_y under both the benchmark case (wages cannot fall) and the alternative hypothesis in which wages can fall according to rules (18) and (19). All variables (including $\kappa_1$below) in % and at annual rates; $\eta = 2.5$, $\rho = 0.01$, $u^f = 6\%$, $\kappa_1 = 1\%$ and $\kappa_2 = 0.1$.

$dw_t^j \geq 0$ with

$$dw_t^j \geq -\kappa(\theta)w_t^j dt \tag{18}$$

which nests the previous model. Nominal wages are now allowed to fall, but the percentage decline cannot exceed $\kappa(\theta)$, where $\kappa(\theta)$ is a non-increasing function of the mean of nominal-spending growth, θ. It is easy to see that the solution of the model is similar to the previous case except that θ should now be replaced by $\lambda(\theta)$ with

$\lambda(\theta) \equiv \theta + \kappa(\theta)$.[21] In particular, the long-run Phillips curve becomes

$$E[u_\infty] = 1 - \frac{1}{1 + \frac{\sigma_y^2}{2\lambda(E[\pi_\infty^w])}} \frac{(1-u^f)}{c(\lambda(E[\pi_\infty^w]), \sigma_y^2, \eta, \rho)},$$

since it is still true that $E[\pi_\infty^w] = \theta$. Obviously the way in which the rigidities endogenously decline (i.e. the functional form of $\kappa(\theta)$) is crucial in shaping the Phillips curve. For example if the percentage decline could not exceed a fixed amount κ_1(hence $\kappa(\theta)$=κ_1), then the Phillips curve would simply shift down by κ_1 (when compared to the one presented in Figure 2). If $\kappa(\theta)$ would increase as θ declines, as suggested by the main argument against downward wage rigidities, the Phillips curve would tilt clockwise.[22] For illustrative purposes, Figure 4 shows a Phillips curve resulting from the following linear function

$$\kappa(\theta) = \kappa_1 - \kappa_2\theta \tag{19}$$

where $\kappa_1 = 1\%$ at annual rates and $\kappa_2 = 0.1$. The unemployment costs of low inflation would clearly decline, but would by no means disappear if macroeconomic volatility is large.

5 Implications for long-run inflation and unemployment volatilities

We discuss now other interesting implications of our model: i) volatility of wage inflation increases as the mean inflation rate increases; ii) volatility of unemployment increases as the mean wage inflation rate decreases; iii) as a consequence, there is a long-run trade-off between the volatility of inflation and that of unemployment.

As discussed in Section 3, exogenous downward nominal wage rigidities imply endogenous upward nominal wage rigidities, as a consequence of the optimizing behavior of wage setters. In the long run, the degree of overall rigidity is high when

[21] In this case, the condition ensuring that the probability distributions converge to their equilibrium ones in the long run becomes $\lambda(\theta) > 0$. A supplementary appendix that presents the model solution under this general case is available upon request.

[22] Obviously, if $\kappa(\theta)$ were to be very large for any theta, then the Phillips curve would become virtually vertical, similarly to the flexible wage case. However, as discussed extensively in the introduction, there is substantial evidence that, at least in some countries, downward wage rigidities persist even at low inflation.

wage inflation rate is low and when the variance of nominal spending shocks is high, implying that nominal disturbances have strong effects on real variables. At high inflation rate or with very small variance of nominal spending, however, wages are much more flexible, and monetary policy is virtually neutral.

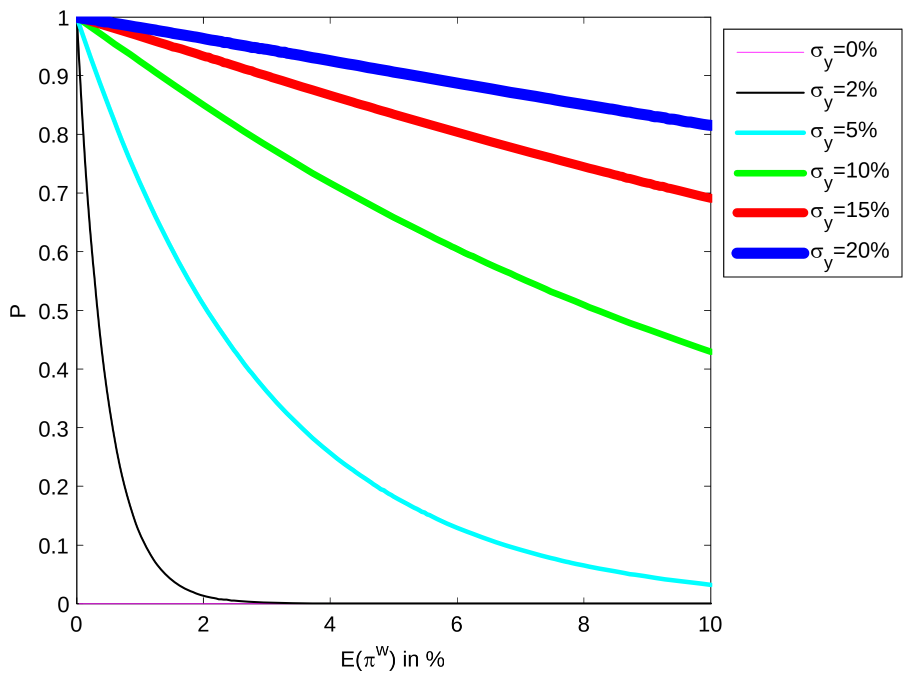

Figure 5: Plot of the long-run probability of wage rigidity, defined in (20), by varying the mean wage inflation rate, $E[\pi^w]$, for different standard deviations of nominal spending growth, σ_y. All variables in % and at annual rates; $\eta = 2.5$, $\rho = 0.01$, $u^f = 6\%$ and $\epsilon = 0.01$.

To illustrate this point, we recall that l_t follows a Brownian motion with a reflecting barrier at $\ln(L^f/c(\cdot))$ and that the barrier is reached when wages are adjusted upward. Hence, the probability that wages are rigid is given by $P(0 \leq L_t < L^f/c(\cdot))$. Since the probability distribution function of L_t is continuous, this can be approximated by $P(0 \leq L_t \leq L^f/c(\cdot) - \epsilon)$ for a small $\epsilon > 0$. Focusing on the long run, we obtain that

$$P(0 \leq L_\infty \leq L^f/c(\cdot) - \epsilon) = \left(1 - \frac{\epsilon c(\cdot)}{L^f}\right)^{\frac{2\theta}{\sigma_y^2}} \approx 1 - \frac{2E[\pi_\infty^w]}{\sigma_y^2}\frac{c(\cdot)}{L^f}\epsilon \tag{20}$$

which shows that when wage inflation is very low, the probability that wages remain fixed is close to one (Figure 5 plots the long-run probability that wages remain fixed against the long-run mean wage inflation rate, for different variances of nominal-spending growth.). Similarly when the variance of nominal spending is high, the probability gets also close to one. The probability declines when inflation increases, and it declines faster when macroeconomic volatility is lower.

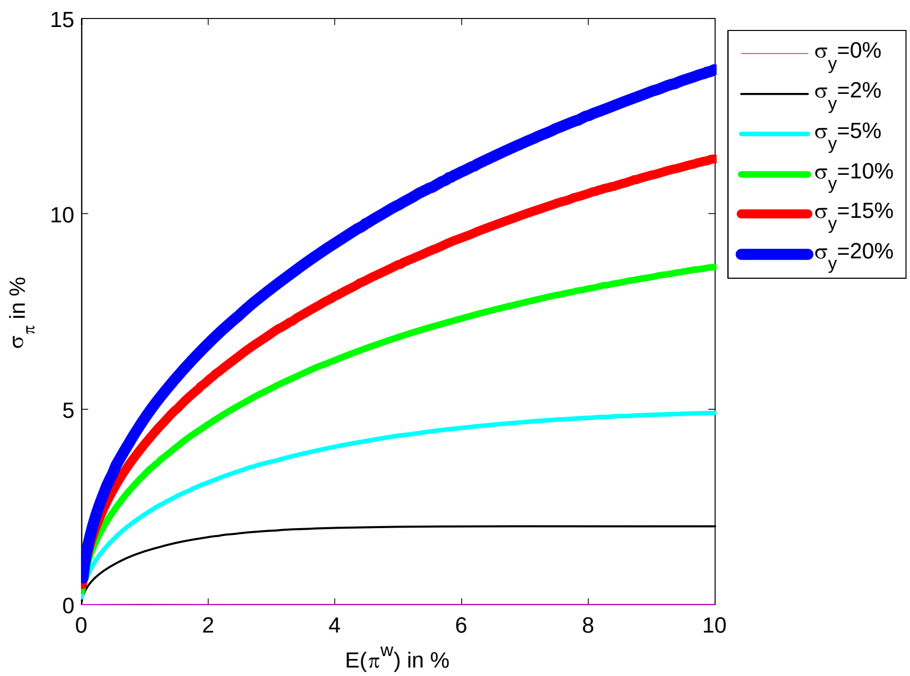

Figure 6: Long-run relationship between the standard deviation of the wage inflation, $\sigma(\pi^w)$, and the mean wage inflation rate, $E[\pi^w]$, for different standard deviations of nominal spending growth, σ_y. All variables in % and at annual rates; $\eta = 2.5$, $\rho = 0.01$ and $u^f = 6\%$.

This has clear implications for the long-run volatilities of inflation and unemployment. Indeed, as shown in Figure 6, the volatility of wage inflation is low when the mean inflation rate is low (for given volatility of nominal-spending growth), but increases when mean inflation increases.[23] By the same token, at low inflation rates,

[23] When the mean of nominal expenditure growth is high, long-run mean wage inflation is high and wages tend to adjust always and proportionally to nominal expenditure shocks, so that the

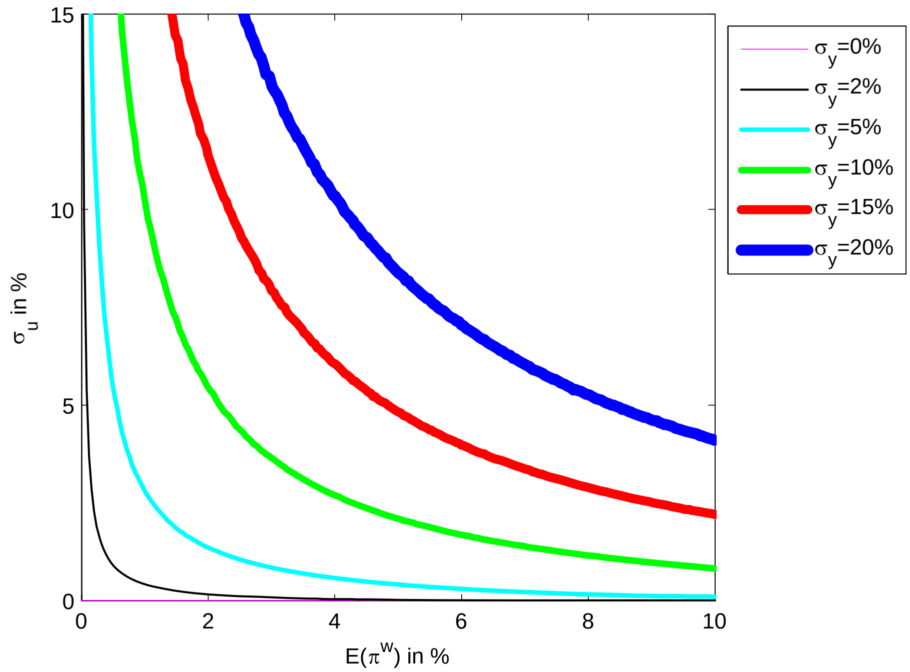

Figure 7: Long-run relationship between the standard deviation of the unemployment rate, $\sigma(u)$, and the mean wage inflation rate, $E[\pi^w]$, for different standard deviations of nominal spending growth, σ_y. All variables in % and at annual rates; $\eta = 2.5$, $\rho = 0.01$ and $u^f = 6\%$.

nominal expenditure affects the real allocation, causing large fluctuations of employment and output, since wages are sticky. Using the long-run probability distribution, it is possible to show that the variance of the long-run unemployment rate is given by

$$Var[u_\infty] = \frac{1}{2\left(1 + \frac{\sigma_y^2}{E[\pi_\infty^w]}\right)\left(1 + \frac{E[\pi_\infty^w]}{2\sigma_y^2}\right)^2} \left(\frac{L^f}{c(E[\pi_\infty^w], \sigma_y^2, \eta, \rho)}\right)^2$$

which is bounded above by

$$Var[u_\infty] \leq \frac{1}{2\left(1 + \frac{\sigma_y^2}{E[\pi_\infty^w]}\right)} (L^f)^2$$

volatility of nominal wages converges to the volatility of nominal expenditure growth, as shown in Figure 6.

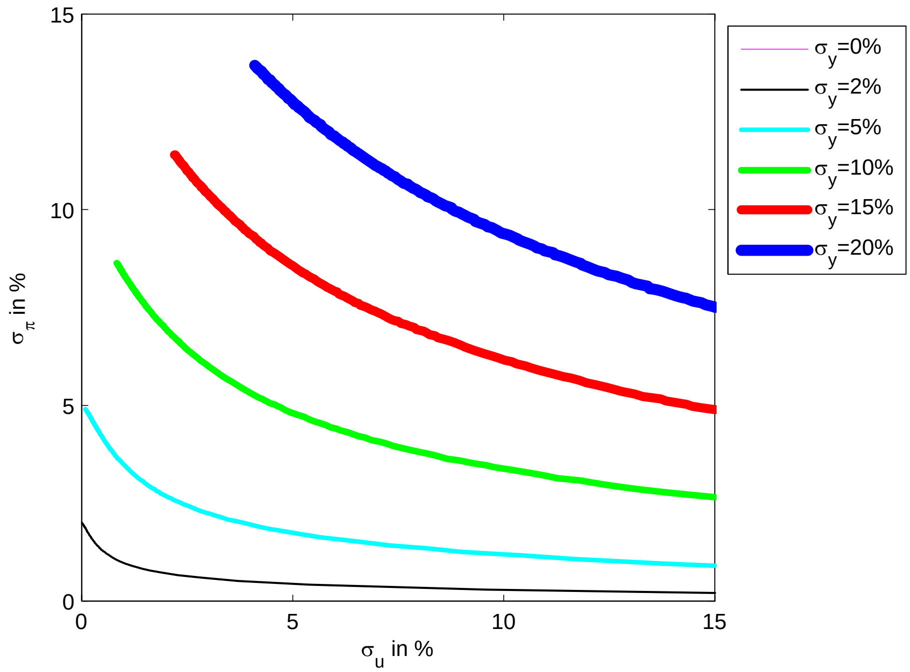

Figure 8: Long-run relationship between the standard deviation of the unemployment rate, $\sigma(u)$, and of the wage inflation rate, $\sigma(\pi^w)$, for different standard deviations of nominal spending growth, σ_y. All variables in % and at annual rates; $\eta = 2.5$, $\rho = 0.01$ and $u^f = 6\%$.

Figure 7 shows (for different choices of σ_y) that the volatility of unemployment is high when inflation is low and decreases as inflation increases, because unemployment will converge to the flexible-wage level. These two results imply the presence of a long-run trade-off between the variability of inflation and that of unemployment, for given volatility of nominal spending growth (as shown in Figure 8). [24]

Trade-offs of this nature have been generally assumed in monetary policy analysis over the past thirty years (see Kydland and Prescott, 1977; Barro and Gordon, 1983). Woodford (2003) has recently provided microfoundation for these trade-offs and for

[24] Note, however, that when inflation is really low (nominal spending growth close to zero) the unemployment distribution collapses to a mass at 100% unemployment rate. In this limiting case the volatility of unemployment collapses to zero and the trade-off between volatilities disappears. Note also that this reversal occurs only in the long run: in the short run we always find a clear trade-off.

their link to the monetary reaction functions that have been so widely employed in inflation targeting models. However, in our model this trade-off is a feature of the global equilibria and not just of the local approximation as in Woodford (2003).

6 Conclusions

This paper offers a theoretical foundation for the long-run Phillips curve in a modern framework. It introduces downward nominal wage rigidities in a dynamic stochastic general equilibrium model with forward-looking agents and flexible-goods prices. The main difference with respect to current monetary models is that nominal rigidities are assumed to be asymmetric rather than symmetric (and on wages rather than prices).[25] Downward nominal rigidities have been advocated for a long time as a justification for the Phillips curve, but with weak theoretical and empirical support. Over the past decade and a half, a substantial body of theoretical and empirical research across numerous countries (see for example the large list of references in Akerlof, 2007, and in Holden, 2004) has offered a conceptual justification for these rigidities and has confirmed not only their existence, but also their relevance in a low inflation environment.

A closed-form solution uncovers a highly non-linear relation for the long-run trade-off between average inflation and unemployment: the trade-off is virtually inexistent at high inflation rates, while it becomes relevant in a low inflation environment. The relation shifts with several factors, and in particular with the degree of macroeconomic volatility. In a country with significant macroeconomic stability, the Phillips curve is virtually vertical also at low inflation. However, a country with moderate to high volatility may face a substantial costs in terms of unemployment if attempting to reach price stability.[26]

It is interesting to note that the forward-looking behavior of optimizing agents in the presence of downward wage rigidities generates an endogenous tendency for

[25] See Blanchard (1997) and Erceg et al. (2000) for a discussion on the importance of assuming rigidities in wages rather than prices in modern macro models.

[26] With respect to the other parameters of the model, the Phillips curve would flatten when labor elasticity is lower and agents heavily discount the future; it would steepen if the degree of downward rigidities weakens at low inflation; and it would shift outward if labor and goods market competition weakens.

upward wage rigidities. Indeed, when choosing the wage increase in the presence of an inflationary shock, agents anticipate the negative effect of downward rigidities on their future employment opportunities, and thus moderate their wage adjustment. Hence, in our model the overall degree of wage rigidity is endogenously stronger at low inflation rates and disappears at high inflation rates, while in time-dependent models of price rigidities, prices remain sticky even in a high inflation environment. The endogenous wage rigidity introduces a trade-off also between the volatility of unemployment and the one of inflation.

The degree to which downward rigidities soften when inflation declines can reduce the extent of the trade-off (as argued by Mankiw and Ball, 1994). However, numerous recent empirical studies have confirmed the persistence of such rigidities at low inflation for various countries. More evidence is nonetheless necessary to assess the degree of such persistence and the corresponding implications for the trade-off.

Several policy implications arise. First, not every country should target the same inflation rate: differences in, among other things, the degree of macroeconomic volatility should matter for the choice of the inflation rate. Second, policymakers can influence the inflation unemployment trade-off: stabilization policies aimed at reducing macroeconomic volatility would improve the trade-off, thus reducing the unemployment costs of lowering long-run inflation.

It is useful to highlight that multiple sectors are not necessary for downward rigidities to generate a trade-off between inflation and unemployment, unlike commonly thought (see for example Akerlof et al. 1996). An intertemporal stochastic framework is sufficient. Obviously, multiple sectors would increase the relevance of the downward nominal rigidities, because they would generate additional need for relative price adjustment, which could be achieved via inflation.

The results suggest that the "great moderation" experienced by the U.S. over the past two decades may have significantly steepened the Phillips curve in the U.S., making it even more unlikely that empirical analyses would uncover such a curve, thus potentially strengthening the case for the conventional view of a vertical long-run curve in this country. However, this does not need to apply to other countries. Indeed, macroeconomic volatility is typically larger in emerging markets, as well as in some industrial such as Switzerland, pointing to a more costly trade-off at low inflation. It may then not be surprising that Groshen and Schweitzer (1997) and Card and Hyslop (1996) find that the grease effect of inflation are not particularly relevant for the U.S., while Fehr and Gotte (2005) find that downward wage rigidities are very relevant for Switzerland. Surely some emerging markets (such as Brazil, Mexico, and Turkey) that experienced highly volatility of nominal GDP over the past decades may enjoy lower volatility going forward if, other things equal, inflation stays low. However, their macroeconomic volatility is unlikely to reach the low to moderate levels of, say, U.S. and Sweden simply as a result of a decline in inflation.

A recent literature has shown that ignorance of the model economy can lead to very costly choices (Primiceri, 2006; Sargent, 2007). Primiceri (2006) argues that the explanation for the large increase in inflation and unemployment in the 1970s relates to the government's misperception about, among other things, the presence of a trade-off between unemployment and inflation. But this argument would work also in reverse. While our results would concur on the lack of such a trade-off at the high inflation levels of the 1970s, they would point at the risk of an opposite misperception (ignoring the presence of a trade-off) in low inflation periods, a risk that can result in significantly higher unemployment. More generally Cogley and Sargent (2005) offers a view in which policymakers have doubts about the true model of the economy and can assign a positive probability to a model in which there is a long-run trade-off, and Sargent (2008) concludes that a "reason for assigning an inflation target to the monetary authority is to prevent it from doing what it might want to do because it has a misspecified model". Our analysis would suggest that the probability that the true model should encompass a long-run trade-off should be made dependent on both the rate of wage inflation and the volatility of nominal spending growth.

Our model is also related to another important controversy in modern macroeconomics: whether nominal spending shocks have persistent real effects. In particular, recent monetary models that have tried to match the highly volatile movements in individual prices observed in U.S. data (such as Golosov and Lucas, 2007) and conclude that nominal shocks have only transient effects on real activity at any level of inflation. In our model, nominal shocks can have high persistent real effects, especially at low inflation rates, since downward wage inflexibility is accompanied by a high degree of upward wage rigidity; as inflation increases, rigidity decreases and so does persistence. This suggests that a menu-cost model á la Golosov and Lucas (2007) would have different implications with regards the real effects of nominal shocks if it were to encompass downward wage inflexibility.

Of course the trade-off between inflation and unemployment is bound to be much more complex that what illustrated through our stylized model. But there is no presumption that a more complicated model would eliminate the trade-off, as long as downward rigidities are included. Adding standard symmetric goods-price rigidities would introduce an argument for inflation as "sand" as in modern monetary models (see for example Woodford, 2003), as it would introduce price dispersion. Allowing for heterogeneity of sectorial shocks would strengthen the argument for inflation as "grease" as it would increase the need for relative price adjustments. Including a game-theoretic interaction between price setters and monetary authorities would unleash the comparison of discretionary versus commitment equilibria. Extending the model to an open economy framework would allow for features which are more realistic for many countries, especially emerging markets. Overall, an optimal inflation rate for policymakers of different countries can only be assessed through more complicated models encompassing the above features among many others (such as persistence of shocks, additional effects of inflation on the economy, and so on)[27], which are left for future work.

[27] See for example Friedman (1976) for a discussion of the costs of high inflation rates.

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A Appendix

A.1 Derivation of conditions (14)

Let $\mathcal{W}$ the space of non-decreasing non-negative stochastic processes $\{w_t(j)\}$. This is the space of processes that satisfy the constraint (13). First we show that the objective function is concave over a convex set. To show that the set is convex, note that if $\mathbf{x} \in \mathcal{W}$ and $\mathbf{y} \in \mathcal{W}$ then $\tau\mathbf{x} + (1-\tau)\mathbf{y} \in \mathcal{W}$ for each $\tau \in [0,1]$. Since the objective function is

$$E_{t_0}\left\{\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\pi(w_t(j), W_t, \tilde{Y}_t)dt\right\}$$

and $\pi(\cdot)$ is concave in the first-argument, the objective function is concave in $\{w_t(j)\}$ since it is the integral of concave functions.

Let $\{w_t^*(j)\}$ be a process belonging to $\mathcal{W}$ that maximizes (11) and $V(\cdot)$ the associated value function defined by

$$V(w_{t_0}(j), W_{t_0}, \tilde{Y}_{t_0}) = \max_{\{w_t(j)\}\in\mathcal{W}} E_{t_0}\left\{\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\pi(w_t(j), W_t, \tilde{Y}_t)dt\right\}.$$

We now characterize the properties of the optimal process $\{w_t^*(j)\}$. The Bellman equation for the wage-setter problem can be written as

$$\rho V(w_t(j), W_t, \tilde{Y}_t)dt = \max_{dw_t(j)} \pi(w_t(j), W_t, \tilde{Y}_t)dt + E_t\{dV(w_t(j), W_t, \tilde{Y}_t)\} \tag{A.1}$$

subject to

$$dw_t(j) \geq 0 \tag{A.2}$$

From Ito's Lemma we obtain that

$$\begin{aligned} E_t\{dV(w_t(j), W_t, \tilde{Y}_t)\} &= E_t\{V_w(w_t(j), W_t, \tilde{Y}_t)dw_t(j) + V_W(w_t(j), W_t, \tilde{Y}_t)dW_t + \\ &\quad +V_y(w_t(j), W_t, \tilde{Y}_t)d\tilde{Y}_t + \frac{1}{2}V_{yy}(w_t(j), W_t, \tilde{Y}_t)(d\tilde{Y}_t)^2 + \\ &\quad +\frac{1}{2}V_{WW}(w_t(j), W_t, \tilde{Y}_t)(dW_t)^2 + V_{yW}(w_t(j), W_t, \tilde{Y}_t)dW_t d\tilde{Y}_t\} \end{aligned}$$

$$\begin{aligned} E_t\{dV(w_t(j), W_t, \tilde{Y}_t)\} &= V_w(w_t(j), W_t, \tilde{Y}_t)dw_t(j) + V_W(w_t(j), W_t, \tilde{Y}_t)E_t dW_t + \\ &\quad +V_y(w_t(j), W_t, \tilde{Y}_t)\tilde{Y}_t\theta' dt + \frac{1}{2}V_{yy}(w_t(j), W_t, \tilde{Y}_t)\tilde{Y}_t^2\sigma_y^2 + \\ &\quad +\frac{1}{2}V_{WW}(w_t(j), W_t, \tilde{Y}_t)E_t(dW_t)^2 + V_{yW}(w_t(j), W_t, \tilde{Y}_t)E_t dW_t d\tilde{Y}_t \end{aligned} \tag{A.3}$$

since $dw_t(j)$ has finite variation implying $(dw_t(j))^2 = dw_t(j)dW_t = dw_t(j)d\tilde{Y}_t = 0$. We have defined $\theta' \equiv \theta + \frac{1}{2}\sigma_y^2$. Substituting (A.3) into (A.1) and maximizing over $dw_t(j)$ we obtain the complementary slackness condition:

$$V_w(w_t(j), W_t, \tilde{Y}_t) \leq 0$$

for each t and

$$V_w(w_t(j), W_t, \tilde{Y}_t) = 0$$

for each t when $dw_t(j) > 0$. We can write (A.1) as

$$\begin{aligned}
\rho V(w_t(j), W_t, \tilde{Y}_t)dt \;=\;& \pi(w_t(j), W_t, \tilde{Y}_t)dt + V_W(w_t(j), W_t, \tilde{Y}_t)E_t dW_t + \\
& + V_y(w_t(j), W_t, \tilde{Y}_t)\tilde{Y}_t\theta' dt + \frac{1}{2}V_{yy}(w_t(j), W_t, \tilde{Y}_t)\tilde{Y}_t^2\sigma_y^2 + \\
& + \frac{1}{2}V_{WW}(w_t(j), W_t, \tilde{Y}_t)E_t(dW_t)^2 + V_{yW}(w_t(j), W_t, \tilde{Y}_t)E_t dW_t d\tilde{Y}_t
\end{aligned}$$

which can be differentiated with respect to $w_t(j)$ to obtain

$$\begin{aligned}
\rho V_w(w_t(j), W_t, \tilde{Y}_t)dt \;=\;& \pi_w(w_t(j), W_t, \tilde{Y}_t)dt + V_{Ww}(w_t(j), W_t, \tilde{Y}_t)E_t dW_t + \\
& + V_{yw}(w_t(j), W_t, \tilde{Y}_t)\tilde{Y}_t\theta' dt + \frac{1}{2}V_{yyw}(w_t(j), W_t, \tilde{Y}_t)\tilde{Y}_t^2\sigma_y^2 + \\
& + \frac{1}{2}V_{WWw}(w_t(j), W_t, \tilde{Y}_t)E_t(dW_t)^2 + V_{yWw}(w_t(j), W_t, \tilde{Y}_t)E_t dW_t d\tilde{Y}_t.
\end{aligned} \tag{A.4}$$

Since the objective is concave and the set of constraints is convex, the optimal choice for $w_t(j)$ is unique. It follows that $w_t(j) = W_t$ for each j. Thus $dw_t(j) = dW_t$ and dW_t has also finite variation. Moreover, super-contact conditions (see Dixit, 1991, and Dumas, 1991) require that when $dW_t > 0$

$$V_{ww}(W_t, W_t, \tilde{Y}_t) = 0,$$

$$V_{wW}(W_t, W_t, \tilde{Y}_t) = 0,$$

$$V_{wy}(W_t, W_t, \tilde{Y}_t) = 0.$$

It follows that we can write (A.4) as

$$\rho v(W_t, \tilde{Y}_t) = \tilde{\pi}_w(W_t, \tilde{Y}_t) + v_y(W_t, \tilde{Y}_t)\tilde{Y}_t\theta' + \frac{1}{2}v_{yy}(W_t, \tilde{Y}_t)\tilde{Y}_t^2\sigma^2 \tag{A.5}$$

where we have defined $v(W_t, \tilde{Y}_t) \equiv V_w(W_t, W_t, \tilde{Y}_t)$ and

$$\tilde{\pi}_w(W_t, \tilde{Y}_t) \equiv k_w \left[\frac{1}{W_t}\frac{1}{\mu_p} - \frac{\mu_w}{\mu_p}\left(\frac{\tilde{Y}_t}{W_t}\right)^{1+\eta}\frac{1}{W_t}\right],$$

with $k_w \equiv 1-\theta_w$. In particular we can define the function $W(\tilde{Y}_t)$ such that

$$v(W(\tilde{Y}_t), \tilde{Y}_t) = 0 \tag{A.6}$$

$$v_w(W(\tilde{Y}_t), \tilde{Y}_t) = 0, \tag{A.7}$$

$$v_y(W(\tilde{Y}_t), \tilde{Y}_t) = 0, \tag{A.8}$$

when $dW_t > 0$ while $v(W_t, \tilde{Y}_t) \leq 0$ when $dW_t = 0$. We now solve for the functions $W(\tilde{Y}_t)$ and $v(W_t, \tilde{Y}_t)$. Thus we seek for functions $W(\tilde{Y}_t)$ and $v(W_t, \tilde{Y}_t)$ that satisfies (A.5) and the boundary conditions (A.6)–(A.8). A particular solution to (A.5) is given by

$$v^p(W_t, \tilde{Y}_t) = \frac{k_w}{\rho}\frac{1}{W_t}\frac{1}{\mu_p} - \frac{k_w}{\rho - \theta'(1+\eta) - \frac{1}{2}(1+\eta)\eta\sigma_y^2}\frac{\mu_w}{\mu_p}\left(\frac{\tilde{Y}_t}{W_t}\right)^{1+\eta}\frac{1}{W_t}$$

while in this case the complementary solution has the form

$$v^c(W_t, \tilde{Y}_t) = W_t^{-1-\gamma}\tilde{Y}_t^{\gamma}$$

where γ is a root that satisfies the following characteristic equation

$$\frac{1}{2}\gamma^2\sigma^2 + \gamma\theta - \rho = 0 \tag{A.9}$$

i.e.

$$\gamma = \frac{-\theta + \sqrt{\theta^2 + 2\rho\sigma^2}}{\sigma^2}.$$

Since when $W_t \to \infty$ and/or $\tilde{Y}_t \to 0$, the length of time until the next wage adjustment can be made arbitrarily long with probability arbitrarily close to one (see Stokey, 2007), then it should be the case that

$$\lim_{W_t \to \infty}[v(W_t, \tilde{Y}_t) - v^P(W_t, \tilde{Y}_t)] = 0$$

$$\lim_{Y_t \to 0}[v(W_t, \tilde{Y}_t) - v^P(W_t, \tilde{Y}_t)] = 0$$

which both require that γ should be positive. The general solution is then given by the sum of the particular and the complementary solution, so that

$$v(W_t, \tilde{Y}_t) = \frac{k_w}{\rho}\frac{1}{W_t}\frac{1}{\mu_p} - \frac{k_w}{\rho - \theta'(1+\eta) - \frac{1}{2}(1+\eta)\eta\sigma_y^2}\frac{\mu_w}{\mu_p}\left(\frac{\tilde{Y}_t}{W_t}\right)^{1+\eta}\frac{1}{W_t} + kW_t^{-1-\gamma}\tilde{Y}_t^{\gamma} \tag{A.10}$$

for a constant k to be determined. Since

$$v_w(W_t,\tilde{Y}_t) = -\frac{k_w}{\rho}\frac{1}{W_t^2}\frac{1}{\mu_p}+\frac{k_w(2+\eta)}{\rho-\theta'(1+\eta)-\frac{1}{2}(1+\eta)\eta\sigma_y^2}\frac{\mu_w}{\mu_p}\left(\frac{\tilde{Y}_t}{W_t}\right)^{1+\eta}\frac{1}{W_t^2}-(1+\gamma)kW_t^{-2-\gamma}\tilde{Y}_t^{\gamma} \tag{A.11}$$

and

$$v_y(W_t,\tilde{Y}_t) = -k_w\frac{1+\eta}{\rho-\theta'}\frac{\mu_w}{\mu_p}\left(\frac{\tilde{Y}_t}{W_t}\right)^{1+\eta}\frac{1}{\tilde{Y}_tW_t}+\gamma kW_t^{-1-\gamma}\tilde{Y}_t^{\gamma-1}, \tag{A.12}$$

the boundary conditions (A.6)–(A.8) imply

$$\frac{k_w}{\rho}\frac{1}{\mu_p}-\frac{k_w}{\rho-\theta'(1+\eta)-\frac{1}{2}(1+\eta)\eta\sigma_y^2}\frac{\mu_w}{\mu_p}\left(\frac{\tilde{Y}_t}{W_t(\tilde{Y}_t)}\right)^{1+\eta}+k\left(\frac{\tilde{Y}_t}{W_t(\tilde{Y}_t)}\right)^{\gamma}=0, \tag{A.13}$$

$$-\frac{k_w}{\rho}\frac{1}{\mu_p}+\frac{k_w(2+\eta)}{\rho-\theta'(1+\eta)-\frac{1}{2}(1+\eta)\eta\sigma_y^2}\frac{\mu_w}{\mu_p}\left(\frac{\tilde{Y}_t}{W_t(\tilde{Y}_t)}\right)^{1+\eta}-(1+\gamma)k\left(\frac{\tilde{Y}_t}{W_t(\tilde{Y}_t)}\right)^{\gamma}=0, \tag{A.14}$$

$$-k_w\frac{1+\eta}{\rho-\theta'(1+\eta)-\frac{1}{2}(1+\eta)\eta\sigma_y^2}\frac{\mu_w}{\mu_p}\left(\frac{\tilde{Y}_t}{W_t}\right)^{\eta}+\gamma k\left(\frac{\tilde{Y}_t}{W_t(\tilde{Y}_t)}\right)^{\gamma-1}=0. \tag{A.15}$$

Note that this is a set of three equations, two of which are independent.[28] They determine k and the function $W_t(\tilde{Y}_t)$. In particular, we obtain that

$$W_t(\tilde{Y}_t) = c\mu_w^{\frac{1}{1+\eta}}\tilde{Y}_t$$

where

$$c \equiv \left(\frac{\gamma-\eta-1}{\gamma}\frac{\rho}{\rho-\theta'(1+\eta)-\frac{1}{2}(1+\eta)\eta\sigma_y^2}\right)^{\frac{1}{1+\eta}}.$$

Using (A.9), we can write

$$c(\theta,\sigma_y^2,\eta,\rho) = \left(\frac{\theta+\frac{1}{2}\gamma(\theta,\sigma_y^2,\rho)\sigma_y^2}{\theta+\frac{1}{2}(\gamma(\theta,\sigma_y^2,\rho)+\eta+1)\sigma_y^2}\right)^{\frac{1}{1+\eta}}$$

which shows that $0 < c(\theta,\sigma_y^2,\eta,\rho) \leq 1$.

In the main text, we use the result that $c(\cdot)$ is non decreasing in η. Note that the derivative of $c(\cdot)$ with respect to η is

[28] In fact, the homogenous function has been chosen appropriately for this purpose.

$$-\frac{c(\theta,\sigma_y^2,\eta,\rho)}{(1+\eta)^2}\cdot\left(\ln\frac{\theta+\frac{1}{2}\gamma(\theta,\sigma_y^2,\rho)\sigma_y^2}{\theta+\frac{1}{2}(\gamma(\theta,\sigma_y^2,\rho)+\eta+1)\sigma_y^2}+\frac{\frac{1}{2}(1+\eta)\,\sigma_y^2}{\theta+\frac{1}{2}(\gamma(\theta,\sigma_y^2,\rho)+\eta+1)\sigma_y^2}\right)$$

which is always non-negative because the terms in the round bracket can be written as

$$\ln z+1-z$$

which is always non-positive for any z.

Moreover note that $c(\theta,\sigma_y^2,\eta,\rho)=c(\sigma_y^2/\theta,\eta,\rho/\theta)$ since $\gamma(\theta,\sigma_y^2,\rho)=\gamma(\sigma_y^2/\theta,\rho/\theta)$.

A.2 Adding the employment constraint $0\le l_t^j\le 1$

Having computed the optimum without the constraint $0\le l_t^j\le 1$, we can now study how the solution changes when employment is enforced not to exceed maximum employment. The optimization problem is still concave under a convex set. The solution will be unique, so it should be that $0\le l_t^j=L_t\le 1$. In the unconstrained optimum we have shown that

$$W_t\ge c\mu_w^{\frac{1}{1+\eta}}\tilde{Y}_t.$$

Combining it with

$$L_t=\frac{1}{\mu_p}\frac{\tilde{Y}_t}{W_t}$$

we obtain

$$L_t\le\frac{\mu_w^{-\frac{1}{1+\eta}}}{c\mu_p}=\frac{1-u_f}{c}.$$

Hence, c cannot be smaller than $1-u_f$, otherwise $L_t>1$. By the concavity of the optimization problem, it follows that if the desired c is below $1-u_f$, then $W_t=c^*\mu_w^{\frac{1}{1+\eta}}\tilde{Y}_t$ when $dW_t>0$ where $c^*=1-u^f$. In particular, we obtain now that

$$\begin{aligned}W(\tilde{Y}_t)&=c^*(\theta,\sigma_y^2,\eta,\delta,u^f)\cdot\mu_w^{\frac{1}{1+\eta}}\tilde{Y}_t\\&=c^*(\theta,\sigma_y^2,\eta,\delta,u^f)\cdot W_t^f\end{aligned}$$

where $c^*(\cdot)$ is a function of the model parameters as follows

$$c^*(\theta,\sigma_y^2,\eta,\delta,u^f)=\begin{cases}c(\theta,\sigma_y^2,\eta,\delta) & if\quad c\ge 1-u_f\\ 1-u_f & if\quad c<1-u_f\end{cases}.$$