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ABSTRACT

We propose a model that solves the crucial disconnect between business cycle models that treat default risk as an exogenous interest rate on working capital, and sovereign default models that treat output fluctuations as an exogenous process with ad-hoc default costs. The model replicates observed output dynamics around defaults, countercyclical spreads, high debt ratios, and key business cycle moments. Three features of the model are central for these results: working capital loans pay for imported inputs; default triggers an efficiency loss as imported inputs are replaced by imperfect domestic substitutes; and default on public and private foreign obligations occurs simultaneously.

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1 Introduction

Three key empirical regularities characterize the relationship between sovereign debt and economic activity in emerging economies:

(1) *Output displays V-shaped dynamics around defaults.* Arellano (2007) reports that in recent default episodes GDP deviations from a linear trend in the quarter in which default occurred were -14 percent in Argentina, -13 percent in Russia and -7 percent in Ecuador. Using quarterly data for 39 developing countries over the 1970-2005 period, Levy-Yeyati and Panizza (2006) show that the recessions associated with defaults tend to begin prior to the defaults and generally hit their through when the defaults take place. Tomz and Wright (2007) study the history of defaults in industrial and developing countries over the period 1820-2004 and find that the frequency of defaults is at its maximum when output is at least 7 percent below trend. They also found, however, that some defaults occurred with less severe recessions, or when output is not below trend in annual data.

(2) *Interest rates on sovereign debt and GDP are negatively correlated.* Neumeyer and Perri (2005) report that the cyclical correlations between the two rates range from -0.38 to -0.7 in five emerging economies, with an average correlation of -0.55. Uribe and Yue (2006) report correlations for seven emerging economies ranging from zero to -0.8, with an average of -0.42.[1]

(3) *External debt as a share of GDP is high on average, and higher when countries default.* Foreign debt was about a third of GDP on average over the 1998-2005 period for the group of emerging and developing countries as defined in IMF (2006). Within this group, the highly indebted poor countries had the highest average debt ratio at about 100 percent of GDP, followed by the Eastern European and Western Hemisphere countries, with averages of about 50 and 40 percent of GDP respectively. Reinhart et al. (2003) report that the external debt ratio during default episodes averaged 71 percent of GDP for all developing countries that defaulted at least once in the 1824-1999 period. The default episodes of recent years are in line with this estimate: Argentina defaulted in 2001 with a 64 percent debt ratio, and Ecuador and Russia defaulted in 1998 with debt ratios of 85 and 66 percent of GDP respectively.

These empirical regularities have proven difficult to explain. On one hand, quantitative business cycle models can account for the negative correlation between country interest rates and output *if* the interest rate on sovereign debt is introduced as the exogenous interest rate faced by a small open economy in which firms require working capital to pay the wages bill.[2] On the other hand, quantitative models of sovereign default based on the classic setup of Eaton and Gersovitz (1981) can generate countercyclical sovereign spreads *if* the sovereign country faces stochastic shocks to an exogenous output endowment and default

[1] Neumeyer and Perri used data for Argentina, Brazil, Korea, Mexico and the Philippines. Uribe and Yue added Ecuador, Peru and South Africa, but excluded Korea.

[2] See Neumeyer and Perri (2005), Uribe and Yue (2006) and Oviedo (2005).

entails exogenous output costs with special features.[3] The latter are also needed in order to obtain equilibria that feature non-trivial levels of debt at the observed default frequencies, but still the predicted mean debt ratios are under 10 percent of GDP (or when they are higher the models underestimate default probabilities by a wide margin).[4] Thus, there is a crucial disconnect between business cycle models and sovereign default models: the former lack an explanation for the default risk premia that drive their findings, while the latter lack an explanation for the output dynamics that are critical for their results.

This paper offers a solution to the country risk-business cycle disconnect by proposing a model of sovereign default with endogenous output fluctuations. The model borrows from the sovereign default literature the classic Eaton-Gersovitz recursive formulation of strategic default in which a sovereign borrower makes optimal default choices by comparing the payoffs of repayment and default. In addition, the model borrows from the business cycle literature a transmission mechanism that links default risk with economic activity via the financing cost of working capital. We extend the two classes of models by developing a framework in which the equilibrium dynamics of output and default risk are determined jointly, and influence each other via the interaction between foreign lenders, the domestic sovereign borrower, and domestic firms. In particular, a fall in productivity in our setup increases the likelihood of default and hence sovereign spreads, and this in turn increases the firms' financing costs leading to a further fall in output, which in turn feeds back into default incentives and sovereign spreads.

We demonstrate via numerical analysis that the model can explain the three empirical regularities of sovereign debt mentioned earlier: The model mimics the V-shaped pattern of output dynamics around defaults with large recessions that hit bottom during defaults, yields countercyclical interest rates on sovereign debt, and supports high debt-GDP ratios on average and in default episodes. Moreover, default triggers a credit amplification mechanism that amplifies the effect of TFP shocks on output by 80 percent, and the model also matches salient features of emerging markets business cycles such as the high variability of consumption and the countercyclical behavior of net exports.

These results hinge on three key features of the model: First, producers of final goods obtain working capital loans from abroad to finance imports of intermediate goods. Second, default causes an efficiency loss because it induces final goods producers to switch to domestic inputs that are imperfect substitutes for imported inputs, and causes labor to reallocate from production of final goods to production of intermediate goods. Third, the government diverts

[3]See, for example, Aguiar and Gopinath (2006), Arellano (2007), Bai and Zhang (2005) and Yue (2006).

[4]Arellano (2007) obtained a mean debt ratio of 6 percent of GDP assuming an output cost of default such that income is the maximum of actual output or 0.97 of average output while the economy is in financial autarky. Aguiar and Gopinath (2006) obtained a mean debt ratio of 27 percent assuming a cost of 2 percent of output per quarter, but the default frequency is only 0.02 percent (in their model without trend shocks and debt bailouts). Yue (2006) assumed the same output cost in a model with renegotiation calibrated to observed default frequencies, but obtained a mean debt ratio of 9.7 percent of output.

the firms' repayment of working capital loans when it defaults on its own debt, so that both agents default on their foreign obligations at the same time, and hence the interest rates they face are equal at equilibrium.

The transmission mechanism that connects country risk and business cycles in our model operates as follows: Final goods producers use labor and a combination of imported and domestic inputs determined by an Armington aggregator as factors of production. Foreign and domestic inputs are imperfect substitutes. Imported inputs need to be paid in advance using working capital loans from abroad, while domestic inputs require labor to be produced. Under these assumptions, the optimal mix of imported and domestic inputs depends on the country interest rate (inclusive of default risk), which drives the financing cost of working capital, and on the state of total factor productivity (TFP). When the country has access to world financial markets, final goods producers use both imported and domestic inputs, and fluctuations in default risk affect the cost of working capital and thus induce fluctuations in factor demands and output. In contrast, when the country does not have access to world financial markets, final goods producers switch to use domestic inputs only because of the prohibitive financing cost of imported inputs. Hence, financial autarky reduces production efficiency in two ways: it forces final goods producers to operate using domestic inputs (which are imperfect substitutes for imported inputs), and induces labor reallocation away from final goods production, so that the supply of domestic inputs can match the increased demand.

When the economy defaults, both the government and firms are excluded from world credit markets for some time, with an exogenous probability of re-entry as is common in the recent quantitative studies of sovereign default. Since the probability of default depends on whether the country's value of default is higher than that of repayment, there is feedback between the economic fluctuations induced by changes in interest rate premia, default probabilities, and country risk. In particular, rising country risk in the periods leading to a default causes a decline in economic activity as the firms' financing costs increase. In turn, the expectation of lower output at higher levels of country risk alters repayment incentives for the sovereign, affecting the equilibrium determination of default risk premia.

A central feature of our model is that the efficiency loss caused by sovereign default generates an endogenous output cost that is larger in "better" states of nature (i.e., *increasing* in the state of TFP). This is the case as long as the elasticity of substitution between foreign and domestic inputs is higher than 1 but less than infinite. The assumption that the two inputs are imperfect substitutes is crucial for this result. If the inputs are perfect substitutes there is no output cost of default. If their elasticity is unitary or less than unitary, the output cost does not rise with TFP (the costs are larger at lower elasticities for a given TFP level, but they do not increase as TFP rises).

The elasticity of labor supply also influences the output costs of default. In particular, the costs are larger the higher this elasticity because default triggers a reduction in total labor usage, thereby magnifying the efficiency loss caused by the exclusion from credit markets. However, output costs of default, and the efficiency loss that drives them, are still present even if total labor supply is inelastic. Final goods producers still have to shift from imported to domestic inputs, and labor still reallocates from final goods to intermediate goods production.

The increasing output cost of default is important because it implies that the option to default brings more "state contingency" into the asset market, allowing the model to produce equilibria that support significantly higher mean debt ratios than those obtained in existing models of sovereign default. The increasing output cost of default also implies that output can fall sharply when the economy defaults, and that, because this output drop is driven by an efficiency loss, part of the output collapse will appear as a drop in the Solow residual (i.e. the fraction of *aggregate* GDP not accounted for by capital and labor). This is consistent with the data of emerging economies in crisis showing that a large fraction of the output collapse is attributed to the Solow residual (see Meza and Quintin (2006) and Mendoza (2007)). Moreover, Benjamin and Meza (2007) show that in Korea's 1997 crisis, the productivity drop followed in part from a sectoral reallocation of labor.

Our treatment of the financing cost of working capital differs from the treatment in Neumeyer and Perri (2005) and Uribe and Yue (2006), both of which treat the interest rate on working capital as an exogenous variable calibrated to match the interest rate on sovereign debt. In contrast, in our setup both interest rates are driven by endogenous sovereign risk. In addition, in the Neumeyer-Perri and Uribe-Yue models, working capital loans pay the wages bill in full, while in our model firms use working capital to pay only for a small fraction of imported intermediate goods (about 1/10). This lower working capital requirement is desirable because, at standard labor income shares, working capital loans would need to be about 2/3rds of GDP to cover the wages bill, and this is difficult to reconcile with observed ratios of bank credit to the private sector as a share of output in emerging economies, which hover around 50 percent (including *all* credit to households and firms at all maturities, in addition to short-term revolving loans to firms).

The rest of the paper proceeds as follows: Section 2 presents the model. Section 3 explores the model's quantitative implications for a benchmark calibration. Section 4 conducts sensitivity analysis. Section 4 concludes.

2 A Model of Sovereign Default and Business Cycles

We study a dynamic stochastic general equilibrium model of sovereign default and business cycles. There are four groups of agents in the model, three in the "domestic" small open economy (households, firms, and the sovereign government) and one abroad (foreign lenders). There are also two sets of producers in the domestic economy: a sector f of final goods producers and a sector m of intermediate goods producers.

2.1 Households

Households choose consumption and labor supply so as to maximize a standard time-separable utility function $E\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t - h(L_t)\right)\right]$, where $0 < \beta < 1$ is the discount factor, and c_t and L_t denote consumption and labor supplied in period t respectively. $u(\cdot)$ is the period utility function, which is continuous, strictly increasing, strictly concave, and satisfies the Inada conditions. Following Greenwood, Hercowitz and Huffman (1988), we remove the wealth effect on labor supply by specifying period utility as a function of consumption net of the disutility of labor $h(L_t)$, where $h(\cdot)$ is increasing, continuously differentiable and convex. This GHH formulation of preferences plays an important role in allowing international real business cycle models to explain observed business cycle facts, and it also simplifies the "supply side" of the model by removing intertemporal considerations from the labor supply choice.

Households take as given the wage rate w_t, profits paid by firms in the f and m sectors $\left(\pi_t^f, \pi_t^m\right)$ and government transfers (T_t). Households do not borrow directly from abroad, but this is without loss of generality because the government borrows, pays transfers, and makes default decisions internalizing their utility function. This assumption implies that the households' optimization problem reduces to the following static problem:

$$\max_{c_t, L_t} E\left[\sum \beta^t u\left(c_t - h\left(L_t\right)\right)\right], \tag{1}$$

$$s.t. \quad c_t = w_t L_t + \pi_t^f + \pi_t^m + T_t. \tag{2}$$

Since the GHH utility function implies that the marginal rate of substitution between consumption and labor is equal to the marginal disutility of labor (and independent of consumption), the optimality condition for labor supply is:

$$h'\left(L_t\right) = w_t. \tag{3}$$

For purposes of the quantitative analysis, we define the labor disutility function in isoelastic form $h(L) = \frac{L^\omega}{\omega}$ with $\omega > 1$. Hence, the Frisch elasticity of labor supply will be given by $1/(\omega - 1)$. The period utility function takes the standard constant-relative-risk-aversion form $u\left(c, L\right) = \frac{(c - L^\omega/\omega)^{1-\sigma} - 1}{1-\sigma}$ with $\sigma > 0$.

2.2 Final Goods Producers

Firms in the f sector use two variable factors, labor L_t^f and intermediate goods M_t, and a time-invariant capital stock k. They face Markov TFP shocks ε_t, with transition probability distribution function $\mu\left(\varepsilon_t | \varepsilon_{t-1}\right)$. The production function is Cobb-Douglas:

$$y_t = \varepsilon_t \left(M\left(m_t^d, m_t^*\right)\right)^{\alpha_M} (L_t^f)^{\alpha_L} k^{\alpha_k} \tag{4}$$

with $0 < \alpha_L, \alpha_M, \alpha_k < 1$ and $\alpha_L + \alpha_M + \alpha_k = 1$.

The mix of intermediate goods used in production of final goods is determined by a standard Armington aggregator of imported inputs m_t^* and inputs produced at home m_t^d:

$$M\left(m_t^d, m_t^*\right) = \left(\lambda \left(m_t^d\right)^\mu + (1-\lambda)\left(m_t^*\right)^\mu\right)^{\frac{1}{\mu}}, \tag{5}$$

The elasticity of substitution between m_t^* and m_t^d is equal to $|1/(\mu-1)|$ and λ is the Armington weight of domestic inputs. The typical parameter restrictions on the Armington aggregator are $-\infty \leq \mu \leq 1$, $0 \leq \lambda \leq 1$. In our model, however, we will show later that $0 < \mu < 1$ is necessary in order to support equilibria in which y_t at default is positive and well defined, and the output cost of default increases with ε.

Imported inputs are sold in a competitive world market at the exogenous relative price p_m^*.[5] A fraction θ of the cost of these inputs needs to be paid in advance using working capital loans κ_t. These are intraperiod loans repaid at the end of the period that are offered by foreign creditors at the interest rate r_t. This interest rate is linked to the sovereign interest rate at equilibrium, as shown in the next section. Working capital loans satisfy the standard payment-in-advance condition:

$$\frac{\kappa_t}{1+r_t} \geq \theta p_m^* m_t^*. \tag{6}$$

Profit-maximizing producers of final goods choose κ_t so that this condition holds with equality.

The profits of final goods producers are given by:

$$\pi_t^f = \varepsilon_t \left(M\left(m_t^d, m_t^*\right)\right)^{\alpha_M} \left(L_t^f\right)^{\alpha_L} k^{\alpha_k} - p_m^*(1+\theta r_t) m_t^* - p_t^m m_t^d - w_t L_t^f, \tag{7}$$

where p_t^m is the endogenous price of domestic intermediate goods. As noted earlier, domestic inputs do not require working capital financing. This assumption is just for simplicity, the key element for the analysis is that at high levels of country risk (including periods without access to foreign credit markets) the financing cost of foreign inputs is higher than that of domestic inputs.

Final goods producers choose factor demands so as to maximize profits taking w_t, r_t, p_m^*, and p_t^m as given. The first-order conditions of their optimization problem are:

$$\alpha_M \varepsilon_t k^{\alpha_k} \left(M\left(m_t^d, m_t^*\right)\right)^{\alpha_M - \mu} \left(L_t^f\right)^{\alpha_L} (1-\lambda)\left(m_t^*\right)^{\mu-1} = p_m^* \left(1+\theta r_t\right) \tag{8}$$

$$\alpha_M \varepsilon_t k^{\alpha_k} \left(M\left(m_t^d, m_t^*\right)\right)^{\alpha_M - \mu} \left(L_t^f\right)^{\alpha_L} \lambda \left(m_t^d\right)^{\mu-1} = p_t^m \tag{9}$$

$$\alpha_L \varepsilon_t k^{\alpha_k} M_t^{\alpha_M} \left(L_t^f\right)^{\alpha_L - 1} = w_t. \tag{10}$$

These are standard optimality conditions equating marginal products of factors of pro-

[5] This price can also be modeled as a terms-of-trade shock with a given stochastic process.

duction (i.e. factor demands) to the corresponding marginal costs.

2.3 Intermediate Goods Producers

Domestic inputs do not require advance payment, but labor L_t^m is needed in order to produce them. Producers in the m sector operate with a production function given by $A(L_t^m)^\gamma$, with $0 \leq \gamma \leq 1$ and $A > 0$. A represents both the role of a fixed factor and an invariant state of TFP in the m sector. Given p_t^m and w_t, these producers choose L_t^m so as to solve the following profit maximization problem:

$$\max_{L_t^m} \ \pi_t^m = p_t^m A(L_t^m)^\gamma - w_t L_t^m. \tag{11}$$

Their optimal labor demand satisfies this standard condition:

$$\gamma p_t^m A(L_t^m)^{\gamma-1} = w_t. \tag{12}$$

2.4 Competitive Equilibrium of the Private Sector

Definition 1 *A competitive equilibrium for the private sector of the economy is given by sequences of allocations* $\left[c_t, L_t, L_t^f, L_t^m, m_t^d, m_t^*, \kappa_t\right]_{t=0}^\infty$ *and prices* $\left[w_t, p_t^m, \pi_t^f, \pi_t^m\right]_{t=0}^\infty$ *such that:*

1. The allocations $[c_t, L_t]_{t=0}^\infty$ *solve the households' utility maximization problem.*

2. The allocations $\left[L_t^f, m_t^d, m_t^*, \kappa_t\right]_{t=0}^\infty$ *solve the profit maximization problem of sector f producers.*

3. The allocations $[L_t^m]_{t=0}^\infty$ *solve the profit maximization problem of sector m producers.*

4. The market-clearing conditions for the labor market ($L_t^f + L_t^m = L_t$) and the domestic intermediate goods market ($A(L_t^m)^\gamma = m_t^d$) hold.

In this economy, GDP at factor costs is given by $w_t L_t^f + w_t L_t^m + \pi_t^f + \pi_t^m$. Using this definition together with the definitions of profits and the optimality conditions of the f and m sectors, it follows that GDP can be expressed as $gdp = (1-\alpha_m)\varepsilon_t(M_t)^{\alpha_m}(L_t^f)^{\alpha_L}k^{\alpha_k} + p_t^m A(L_t^m)^\gamma$. The first and second terms in the right-hand-side of this expression represent value added in the f and m sectors respectively (note that given the CES formulation of M_t it can be shown that $\alpha_m \varepsilon_t(M_t)^{\alpha_m}(L_t^f)^{\alpha_L}k^{\alpha_k} = p_m^*(1+\theta r_t)m_t^* + p_t^m m_t^d$).

A key constraint on the problem of the sovereign borrower making the default decision will be that private-sector allocations must be a competitive equilibrium. Since the sovereign government's problem and the equilibrium of the credit market will be characterized in recursive form as functions defined in the state space domain, it is useful to also characterize the above competitive equilibrium in terms of functions of state variables, and to distinguish private sector allocations in states in which the economy has credit market access from those in which it does not.

We start by expressing the private sector equilibrium when sector f has access to credit markets in terms of functions of r and ε that solve the following nonlinear system of equations:

$$\begin{aligned}
\alpha_M \varepsilon k^{\alpha_k}\left(M\left(m^d, m^*\right)\right)^{\alpha_M-\mu}\left(L^f\right)^{\alpha_L}(1-\lambda)\left(m^*\right)^{\mu-1} &= p_m^*(1+\theta r) \quad (13)\\
\alpha_M \varepsilon k^{\alpha_k}\left(M\left(m^d, m^*\right)\right)^{\alpha_M-\mu}\left(L^f\right)^{\alpha_L}\lambda\left(m^d\right)^{\mu-1} &= p^m \quad (14)\\
\alpha_L \varepsilon k^{\alpha_k}\left(M\left(m^d, m^*\right)\right)^{\alpha_M}\left(L^f\right)^{\alpha_L-1} &= w \quad (15)\\
\gamma p^m A (L^m)^{\gamma-1} &= w \quad (16)\\
h'(L) &= w \quad (17)\\
L^f + L^m &= L \quad (18)\\
A(L^m)^{\gamma} &= m^d \quad (19)\\
\kappa(r,\varepsilon) &= \theta p_m^* m^*(1+r). \quad (20)
\end{aligned}$$

We define the functions $m^*(r,\varepsilon)$, $m^d(r,\varepsilon)$, $L^f(r,\varepsilon)$, $L^m(r,\varepsilon)$, $L(r,\varepsilon)$, $p^m(r,\varepsilon), w(r,\varepsilon)$, and $\kappa(r,\varepsilon)$ to represent the solutions to the above equation system for a given pair (r,ε). Notice in particular that equation (13) implies that increases in the interest rate increase the marginal cost of imported inputs, and thus the demand for m^* falls as r rises. Moreover, since as we show later the interest rate faced by firms is directly influenced by sovereign default risk, it follows that increases in country risk have "supply side effects." One of these effects is the direct negative effect of r on m^*, but there are also indirect general equilibrium effects that operate via the substitution of foreign for domestic inputs and the labor market. We study these effects in detail in the next subsection.

During periods of exclusion from world credit markets, sector f does not have access to foreign working capital financing, and hence it only uses domestic inputs. The equilibrium allocations for this scenario can be approximated as the limiting case of the above nonlinear system as $r \to \infty$. If $0 < \mu < 1$ (which is the relevant range for obtaining an output cost of default increasing in TFP), the nonlinear system has closed-form solutions as $r \to \infty$, and these can be expressed as the following functions of ε:

$$\begin{aligned}
\tilde{L}(\varepsilon) &= \left[(\alpha_L+\gamma\alpha_m)\varepsilon k^{\alpha_k}\eta^{\frac{\alpha_m}{\mu}}A^{\alpha_m}\left(z_{L_m}\right)^{\alpha_m\gamma}\left(z_{L_f}\right)^{\alpha_L}\right]^{1/(\omega-\alpha_L-\alpha_m\gamma)} \quad (21)\\
\tilde{L}^f(\varepsilon) &= z_{L_f}\tilde{L}(\varepsilon) \quad (22)\\
\tilde{L}^m(\varepsilon) &= z_{L_m}\tilde{L}(\varepsilon) \quad (23)\\
\tilde{m}(\varepsilon) &= A\left(\tilde{L}^m(\varepsilon)\right)^{\gamma} \quad (24)\\
\tilde{p}^m(\varepsilon) &= \alpha_m\eta^{\frac{\alpha_m}{\mu}}\varepsilon\left(\tilde{m}(\varepsilon)\right)^{\alpha_m-1}\left(\tilde{L}^f(\varepsilon)\right)^{\alpha_L}k^{\alpha_k}, \quad (25)
\end{aligned}$$

where $z_{L_m} = \gamma\alpha_m / (\gamma\alpha_m + \alpha_L)$ and $z_{L_f} = \alpha_L / (\gamma\alpha_m + \alpha_L)$.

2.5 The "Credit Channel" and the Output Cost of Default

The effects of interest rate changes on the private sector equilibrium play an important role in our analysis because they drive output dynamics and the output cost of default, which are two key determinants of the default/repayment decision of the sovereign. Since these interest rate effects include direct and indirect general equilibrium effects that cannot be solved for analytically, it is easier to provide an economic intuition for them by means of a numerical example. For this example, we start from a baseline that uses the parameter values set for α_m, α_L, α_k, A, γ, ω, λ and μ in the calibration exercise of Section 3.

Figure 1 shows six charts with the allocations of L, L^f, L^m, M, m^d, and m^* for values of r ranging from 0 to 80 percent. Each chart includes results for the baseline value of μ (0.69), which corresponds to an Armington elasticity of substitution between foreign and domestic inputs of 3.23, and for two lower elasticities (1.93, which is the threshold below which m^d and m^* switch from gross substitutes to gross complements, and the Cobb-Douglas case of unitary elasticity of substitution).[6] We also show results for the baseline value of μ but assuming that labor supply is inelastic. To facilitate comparison across the charts, the allocations are plotted in ratios relative to the allocations when $r = 0$.

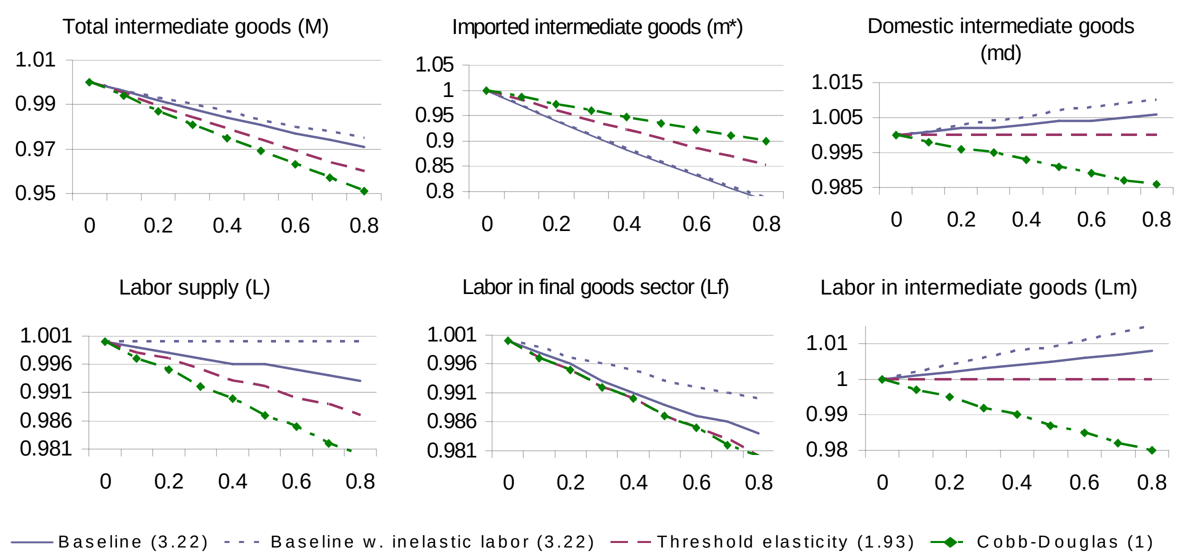


Figure 1: Effects of interest rate shocks on intermediate goods and labor allocations at different Armington elasticities of substitution
(ratios relative to allocations at zero interest rate)

[6]Note that the threshold would be at the unitary elasticity of substitution if labor supply were inelastic.

An increase in the rate of interest reduces the demand for m^* because of the direct effect by which the hike in r increases the effective price of imported inputs $p_m^*(1+\theta r)$. At equilibrium, this also reduces the demand for total intermediate goods M. The magnitude of these two effects, for a given change in r, depends on whether $m^*$and m^d are gross substitutes or gross complements. If they are gross substitutes (i.e. if m^d increases as $p_m^*(1+\theta r)$ rises), then m^d rises as m^* falls, and M falls less than it would if the two inputs are gross complements. For the same reason, however, m^* falls more when the inputs are gross substitutes. Moreover, the increase in r also reduces both L and L^f, and again the reduction in L is smaller and the decline in L^f larger when foreign and domestic inputs are gross substitutes (i.e. the higher is μ, or the higher is the elasticity of substitution between $m^*$and m^d). In addition, if foreign and domestic inputs are gross substitutes, L^f and L^m also behave as gross substitutes, so that L^m rises as L^f falls.

The charts in Figure 1 also illustrate how the elasticity of labor supply affects the responses of factor allocations to interest rate changes. Keeping μ at its baseline value, the effect on m^* is nearly unchanged if we make labor supply inelastic. M falls less with inelastic labor because m^d rises more, and this is possible because with inelastic labor supply L cannot fall in response to interest rate hikes, and this results in a larger increase in L^m and a smaller decline in L^f. Thus, even with inelastic labor supply, increases in r affect the efficiency of production by inducing a shift from foreign to domestic inputs, and by reallocating labor from production of final goods to production of intermediate goods.

The left-side plot of Figure 2 shows the output cost of default as a function of TFP in final goods for different values of the Armington elasticity of substitution between m^* and m^d. This output cost corresponds to the percent fall in output that occurs at each level of TFP as $r \to \infty$, relative to a baseline computed with an interest rate of 1 percent, and expressed as a ratio relative to the output cost when $\varepsilon = 1$ to facilitate comparisons across different parameterizations. The plot illustrates two important properties of the model: First, the output cost of default is increasing in the size of the TFP shock that hits the economy when it defaults. Second, this relationship is steeper at higher Armington elasticities. As the two inputs become gross complements (when the elasticity is less or equal than the 1.93 threshold) the relation is nearly flat, and for an elasticity of substitution close to 1 the output cost is independent of TFP.

It is straightforward now to explain why the restriction $0 < \mu < 1$ is needed. If $\mu = 1$, there is no output cost of default because the two inputs are perfect substitutes, and hence there is no efficiency loss in changing one input for the other (which is a result that can be proved analytically). If $\mu \leq 0$, the two inputs are complements and, since $m^* = 0$ in default states, $y_t = 0$ and the output cost of default is 100 percent with $\mu = 0$ (which is the case with a Cobb-Douglas Armington aggregator) and y_t is undefined with $\mu < 0$.

The right-side plot of Figure 2 shows a third important property of the output cost of default: While at higher Armington elasticities the cost is a steeper positive function of TFP, the actual output drop at default (shown in the plot for the scenario with $\varepsilon = 1$) is smaller. At elasticities higher than 20 the numerical example shows virtually no output loss. The plot also shows that adjusting the model's GDP measure to make it compatible with actual GDP data computed at constant prices (by keeping p^m constant) makes little difference for the size of the output costs of default.

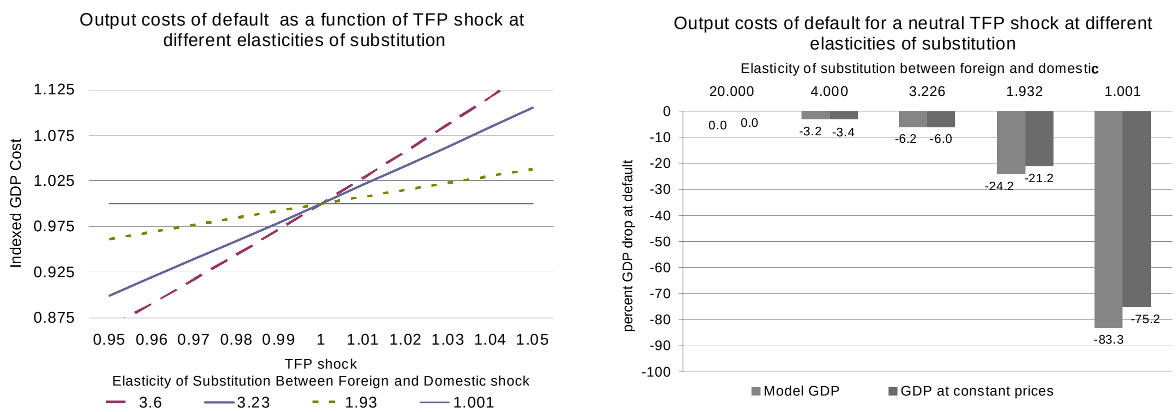


Figure 2: Output Costs of Default at Different Elasticities of Substitution Between Foreign and Domestic Inputs

The fact that the output cost of default increases with the size of the TFP shock implies that default is more painful at higher levels of TFP. This result plays a key role in enabling the model to support high debt levels together with observed default frequencies, because it makes the default option more attractive at lower states of productivity, and works as a desirable implicit hedging mechanism given the incompleteness of asset markets. This finding is in line with Arellano's (2007) result showing that an exogenous default cost with similar features can allow the Eaton-Gersovitz model to support non-trivial levels of debt together with observed default frequencies. In particular, she proposed an exogenous default cost function such that below a threshold level of an output endowment default does not entail an output cost, but above that threshold default reduces the endowment to a state-invariant fraction of its long-run average. In this second range, the default cost is increasing in the endowment realization at the time of default. Still, the mean debt ratio in her baseline calibration was only about 6 percent of GDP (assuming output at default is 3 percent below mean output). In contrast, we show later that our model with endogenous output cost of default yields a mean debt ratio that is about four times larger.

A similar analysis of the output costs of default as the one illustrated in Figure 2 but for different values of ω (instead of μ) shows that higher labor supply elasticities (i.e. lower ω) increase the output cost of default, converging to a cost of about 11.5 percent for infinitely elastic labor supply. The output cost of default is increasing in TFP for any value of ω, but, in contrast with what we found for μ, the slope of the relationship does not change as ω changes. We also found that adjusting the productivity parameter in production of intermediate goods (A) has qualitatively similar effects as changing ω.

The intuition behind the result that higher labor supply elasticity produces larger output costs of default can be explained by examining the labor market equilibrium using Figure 3. For simplicity, we plot labor demands and supply as linear functions. The labor demand functions are given by the marginal products in the left-hand-side of (10) and (12), and the labor supply is given by the marginal disutility of labor in the left-hand-side of (3). Since labor is homogenous across sectors, total labor demand is just the sum of the sectoral labor demands. The initial labor market equilibrium is at point A with wage w^*, total labor L^* and sectoral allocations L_m^* and L_f^*.

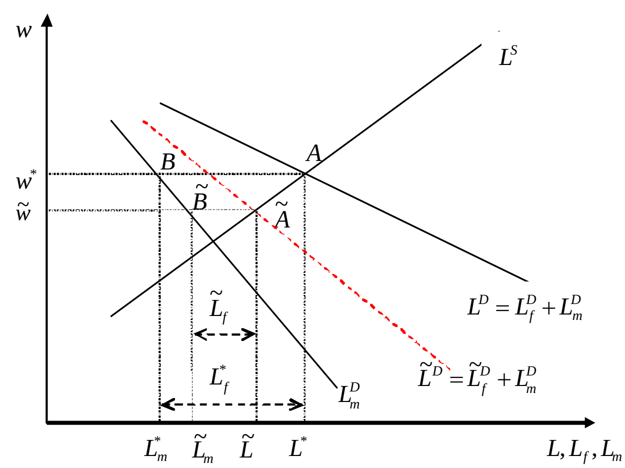


Figure 3: Interest Rate Shocks and the Labor Market Equilibrium

Consider now a positive interest rate shock that leads to a reduction in labor demand in final goods from L_f^D to $\tilde{L}_f^D$. This occurs because higher r causes a reduction in M, as explained earlier, and the marginal product of L_f is a negative function of M (since the production function of final goods is Cobb-Douglas). As a result, total labor demand shifts from L^D to $\widetilde{L}^D$.[7] The new labor market equilibrium is at point $\widetilde{A}$. The wage is lower than before, and so are the total labor allocation and the labor allocated to final goods, while labor allocated to production of domestic inputs rises.

It follows from visual examination of Figure 3 that if labor is infinitely elastic (if L^s is an horizontal line at the level of w^*), the interest rate hike leaves w unchanged instead of

[7]In Figure 3, we hold constant p_m for simplicity. At equilibrium, the relative price of domestic inputs changes, and this alters the value of the marginal product of Ld, and hence labor demand by the m sector. The results of the numerical example do take this into account and still are roughly in line with the intuition derived from Figure 3.

reducing it, L falls more, L_m is unchanged instead of rising, and L_f falls less.[8] Hence, the adverse effect on output is stronger. Similarly, going to the other extreme, if labor is inelastic (if L^s is a vertical line at the level of L^*), L cannot change, but w falls more than in the scenario plotted in Figure 3, L_m rises more, and L_f falls more. Hence, the decline in output is smaller.

2.6 The Sovereign Government

The sovereign government trades with foreign lenders one-period, zero-coupon discount bonds, so markets of contingent claims are incomplete. The face value of these bonds specifies the amount to be repaid next period and is denoted as b_{t+1}. When the country purchases bonds $b_{t+1} > 0$, and when it borrows $b_{t+1} < 0$. The set of bond face values is $B = [b_{\min}, b_{\max}] \subset R$, where $b_{\min} \leq 0 \leq b_{\max}$. We set the lower bound $b_{\min} < -\frac{\overline{y}}{r}$, which is the largest debt that the country could repay with full commitment. The upper bound $b_{\max}$ is the highest level of assets that the country may accumulate.[9]

The sovereign cannot commit to repay its debt. As in the Eaton-Gersovitz model, we assume that when the country defaults it does not repay at date t and the punishment is exclusion from the world credit market in the same period. The country re-enters the credit market with an exogenous probability η, and when it does it starts with a fresh record and zero debt.[10] Also as in the Eaton-Gersovitz setup, the country cannot hold positive international assets during the exclusion period, otherwise the model cannot support equilibria with debt.

We add to the Eaton-Gersovitz setup an explicit link between default risk and private financing costs. This is done by assuming that a defaulting sovereign can divert the repayment of the firms' working capital loans to foreign lenders. Hence, both firms and government default together. This is perhaps an extreme formulation of the link between private and public borrowing costs, but we provide later some evidence in favor of this view.

The sovereign government solves a problem akin to a Ramsey problem.[11] It chooses a debt policy (amounts and default or repayment) that maximizes the households' welfare subject to the constraints that: (a) the private sector allocations must be a competitive equilibrium; and (b) the government budget constraint must hold. The state variables are the bond position, working capital loans as of the end of last period, and the state of TFP, denoted by the triplet $(b_t, \kappa_{t-1}, \varepsilon_t)$. The price of sovereign bonds is given by the bond pricing function $q_t(b_{t+1}, \varepsilon_t)$. Since at equilibrium the default risk premium on sovereign debt will be

[8] The last effect hinges on the fact that the gap between L_m^D and L^D widens as the wage falls. This is a property of factor demands with Cobb-Douglas production.

[9] $b_{\max}$ exists when the interest rates on a country's saving are sufficiently small compared to the discount factor, which is satisfied in our paper since $(1 + r^*)\beta < 1$.

[10] We asbtract from debt renegotiation. See Yue (2006) for a quantitative analysis of sovereign default with renegotiation in which the length of financial exclusion is endogenous.

[11] See Cuadra and Sapriza (2007) for an analysis of optimal fiscal policy as a Ramsey problem in the presence of sovereign default in an endowment economy.

the same as on working capital loans, it follows that the interest rate on working capital is a function of $q_t(b_{t+1}, \varepsilon_t)$. Hence, the recursive functions that represent the competitive equilibrium of the private sector when the economy has access to world credit markets can be re-written as $\kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $M(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $m^*(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $m^d(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $L^f(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $L^m(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, and $L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$.

The recursive optimization problem of the government is summarized by the following value function:

$$V(b_t, \kappa_{t-1}, \varepsilon_t) = \begin{cases} \max\left\{v^{nd}(b_t, \varepsilon_t), v^d(\kappa_{t-1,}\varepsilon_t)\right\} & \text{for } b_t < 0 \\ v^{nd}(b_t, \varepsilon_t) & \text{for } b_t \geq 0 \end{cases}. \tag{26}$$

If the country has access to the world credit market at date t, the value function is the maximum of the value of continuing in the credit relationship with foreign lenders (i.e., "no default"), $v^{nd}(b_t, \varepsilon_t)$, and the value of default, $v^d(\kappa_{t-1,}\varepsilon_t)$. If the economy holds a non-negative bond position, the value function is simply the continuation value because in this case the economy is using the credit market to save, receiving a return equal to the world's risk free rate r^*.

The continuation value $v^{nd}(b_t, \varepsilon_t)$ is defined as follows:

$$v^{nd}(b_t, \varepsilon_t) = \max_{c_t, b_{t+1}} \left\{ \begin{array}{l} u\left(c_t - h(L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t))\right) \\ \qquad +\beta E\left[V\left(b_{t+1}, \kappa\left(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t\right), \varepsilon_{t+1}\right)\right] \end{array} \right\}, \tag{27}$$

subject to

$$\begin{aligned} c_t + q_t(b_{t+1}, \varepsilon_t) b_{t+1} - b_t \leq \varepsilon_t f\Big(M(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), L^f(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), k\Big) \\ - m^*(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t) p_m^* \left(1 + \frac{\theta}{q_t(b_{t+1}, \varepsilon_t)} - \theta\right), \end{aligned} \tag{28}$$

where $f(\cdot) = M^{\alpha_M}(L_t^f)^{\alpha_L} k^{\alpha_k}$. The constraint of this problem is the resource constraint of the economy at a competitive equilibrium. The left-hand-side is the sum of consumption and net exports, and the right-hand-side is GDP. This constraint is obtained by combining the households' budget constraint (2) with the government budget constraint, $T_t = b_t - q_t(b_{t+1}, \varepsilon_t) b_{t+1}$, and recalling that total factor payments $w_t^f L_t^f + w_t^m L_t^m + \pi_t^f + \pi_t^m$ equal GDP.

The resource constraint captures three important features of the model: First, the government internalizes how interest rates and working capital affect the competitive equilibrium allocations of output and factor demands. Second, the households cannot borrow from abroad, but the government internalizes their desire to smooth consumption and transfers to them an amount equal to the negative of the balance of trade (i.e. it gives the private sector the flow of resources it needs to finance the gap between GDP and consumption). Third, the working capital loans κ_{t-1} and κ_t do not enter explicitly in the continuation value or in the resource constraint. Still, we need to keep track of the state variable κ_t because the amount of working capital loans taken by final goods producers at date t affects the sovereign's incentive to default at $t+1$, as explained below.

The value of default $v^d(\kappa_{t-1}, \varepsilon_t)$ is:

$$v^d(\kappa_{t-1}, \varepsilon_t) = \max_{c_t} \left\{ \begin{array}{c} u\left(c_t - h(\tilde{L}(\varepsilon_t))\right) \\ +\beta(1-\eta) Ev^d(0, \varepsilon_{t+1}) + \beta\eta EV(0, 0, \varepsilon_{t+1}) \end{array} \right\}, \tag{29}$$

subject to:

$$c_t = \varepsilon_t f\left(\lambda^{\frac{1}{\mu}} \tilde{m}(\varepsilon), \tilde{L}^f(\varepsilon), k\right) + \kappa_{t-1}. \tag{30}$$

Note that $v^d(\kappa_{t-1}, \varepsilon_t)$ takes into account the fact that in case of default at date t, the country has no access to financial markets this period, and hence the country consumes the total income given by the resource constraint in the default scenario. In this case, since firms cannot borrow to finance purchases of imported inputs, $\tilde{m}(\varepsilon)$, $\tilde{L}(\varepsilon)$ and $\tilde{L}^f(\varepsilon)$ are competitive equilibrium allocations that correspond to the case when the f sector operates with domestic inputs only (notice in this case, if $0 < \mu < 1$, $M = \lambda^{\frac{1}{\mu}} \tilde{m}(\varepsilon)$). Moreover, because the defaulting government diverts the repayment of last period's working capital loans, total household income includes government transfers equal to the appropriated repayment for the amount κ_{t-1} (i.e., on the date of default, the government budget constraint is $T_t = \kappa_{t-1}$). The value of default at t also takes into account that at $t+1$ the economy may re-enter world capital markets with probability η and associated value $V(0, 0, \varepsilon_{t+1})$, or remain in financial autarky with probability $1-\eta$ and associated value $v^d(0, \varepsilon_{t+1})$.

For a debt position $b_t < 0$ and given a level of working capital κ_{t-1}, default is optimal for the set of realizations of the TFP shock for which $v^d(\kappa_{t-1}, \varepsilon_t)$ is at least as high as $v^{nd}(b_t, \varepsilon_t)$:

$$D(b_t, \kappa_{t-1}) = \left\{\varepsilon_t : v^{nd}(b_t, \varepsilon_t) \leq v^d(\kappa_{t-1}, \varepsilon_t)\right\}. \tag{31}$$

It is critical to note that this default set has a different specification than in the typical Eaton-Gersovitz model of sovereign default (see Arellano (2007)), because the state of working capital affects the gap between the values of default and repayment. This results in a two-dimensional default set that depends on b_t and κ_{t-1}, instead of just b_t.

Despite the fact that the default set depends on κ_{t-1}, the probability of default remains a function of b_{t+1} and ε_t only. This is because the f sector's optimality conditions imply that the next period's working capital loan κ_t depends on ε_t and the interest rate, which is a function of b_{t+1} and ε_t. Thus the probability of default at $t+1$ perceived as of date t for a country with a productivity ε_t and debt b_{t+1}, $p_t(b_{t+1}, \varepsilon_t)$, can be induced from the default set, the decision rule for working capital, and the transition probability function of productivity shocks $\mu\left(\varepsilon_{t+1}|\varepsilon_t\right)$ as follows:

$$p_t\left(b_{t+1},\varepsilon_t\right)=\int_{D\left(b_{t+1},\kappa_t\right)} d\mu\left(\varepsilon_{t+1}|\varepsilon_t\right), \text{ where } \kappa_t=\kappa\left(q_t\left(b_{t+1},\varepsilon_t\right),\varepsilon_t\right). \tag{32}$$

The economy is considered to be in financial autarky when it has been in default for at least one period and remains without access to world credit markets as of date t. As noted above, the economy can exit this exclusion stage at date $t+1$ with probability η. We assume that during the exclusion stage the economy cannot build up its own stock of savings to supply working capital loans to firms, which could be used to purchase imported inputs.[12] This assumption ensures that, as long as the economy remains in financial autarky, the optimization problem of the sovereign is the same as the problem in the default period but evaluated at $\kappa_{t-1}=0$ (i.e. $v^d\left(\varepsilon_t,0\right)$).

The model preserves a standard feature of the Eaton-Gersovitz model: Given ε_t, the value of defaulting is independent of the level of debt, while the value of not defaulting increases with b_{t+1}, and consequently the default set and the equilibrium default probability grow with the country's debt. The following theorem formalizes this result:

Theorem 1 *Given a productivity shock ε and level of working capital loan κ, for $b^0<b^1\leq 0$, if default is optimal for b^1, then default is also optimal for b^0. That is $D\left(b^1,\kappa\right)\subseteq D\left(b^0,\kappa\right)$. The country agent's probability of default in equilibrium satisfies $p^*\left(b^0,\varepsilon\right)\geq p^*\left(b^1,\varepsilon\right)$.*

Proof. See Appendix. ■

2.7 Foreign Lenders

International creditors are risk-neutral and have complete information. They invest in sovereign bonds and in private working capital loans. Foreign lenders behave competitively and face an opportunity cost of funds equal to the world risk-free interest rate. Competition implies that they expect zero profits at equilibrium, and that the returns on sovereign debt and the world's risk-free asset are fully arbitraged:

$$q_t\left(b_{t+1},\varepsilon_t\right)=\begin{cases}\frac{1}{1+r^*} & \text{if } b_{t+1}\geq 0\\ \frac{\left[1-p_t\left(b_{t+1},\varepsilon_t\right)\right]}{1+r^*} & \text{if } b_{t+1}<0\end{cases}. \tag{33}$$

This condition implies that at equilibrium bond prices depend on the risk of default. For a high level of debt, the default probability is higher. Therefore, equilibrium bond prices decrease with indebtedness. This result, formalized in Theorem 2 below, is again in line with the Eaton-Gersovitz model and is also consistent with the empirical evidence documented by Edwards (1984).

[12] Alternatively, we could assume that the default punishment includes exclusion from world capital markets and from the world market of intermediate goods.

Theorem 2 *Given a productivity shock ε and level of working capital loan κ, for $b^0 < b^1 \leq 0$, the equilibrium bond price satisfies $q^*\left(b^0, \varepsilon\right) \leq q^*\left(b^1, \varepsilon\right)$.*

Proof. See Appendix. ■

The returns on sovereign bonds and working capital loans are also fully arbitraged. Because the sovereign government diverts the repayment of working capital loans when it defaults, foreign lenders assign the same risk of default to private working capital loans as to sovereign debt, and hence the no-arbitrage condition between sovereign lending and working capital loans implies:

$$r_t\left(b_{t+1}, \varepsilon_t\right) = \frac{1}{q_t\left(b_{t+1}, \varepsilon_t\right)} - 1, \text{ if } \kappa_t > 0. \tag{34}$$

2.8 Country Risk & Private Interest Rates: Some Empirical Evidence

The result that the interest rates on sovereign debt and private working capital are the same raises a key empirical question: Are sovereign interest rates and the rates of interest faced by private firms closely related in emerging economies?

Providing a complete answer to this question is beyond the scope of this paper, but we do provide empirical evidence suggesting that corporate and sovereign interest rates do move together. To study this issue, we constructed country estimates of firms' financing costs that aggregate measures derived from firm-level data. We constructed a measure of firm-level effective interest rates as the ratio of a firm's total debt service divided by its total debt obligations using the *Worldscope* database, which provides the main lines of balance-sheet and cash-flow statements of publicly listed corporations. We then constructed the corresponding aggregate country measure as the median across firms. Table 1 reports these estimates of corporate interest rates together with the standard EMBI+ measure of interest rates on sovereign debt and the correlations between the two.

Table 1 shows that the two interest rates are positively correlated in most countries, with a median correlation of 0.7, and in some countries the relationship is very strong (see Figure 4).[13] The Table also shows that the effective financing cost of firms is generally *higher* than the sovereign interest rates. This fact indicates that the common conjecture that firms (particularly the large corporations covered in our data) may pay lower rates than governments with default risk is incorrect.

Arteta and Hale (2007) and Kohlscheen and O'Connell (2008) provide further evidence of significant adverse effects of sovereign default on private access to foreign credit in emerging economies. Arteta and Hale show that there are strong, systematic negative effects on private corporate bond issuance during and after default episodes. Kohlscheen and O'Connell document that the volume of trade credit provided by commercial banks falls sharply when

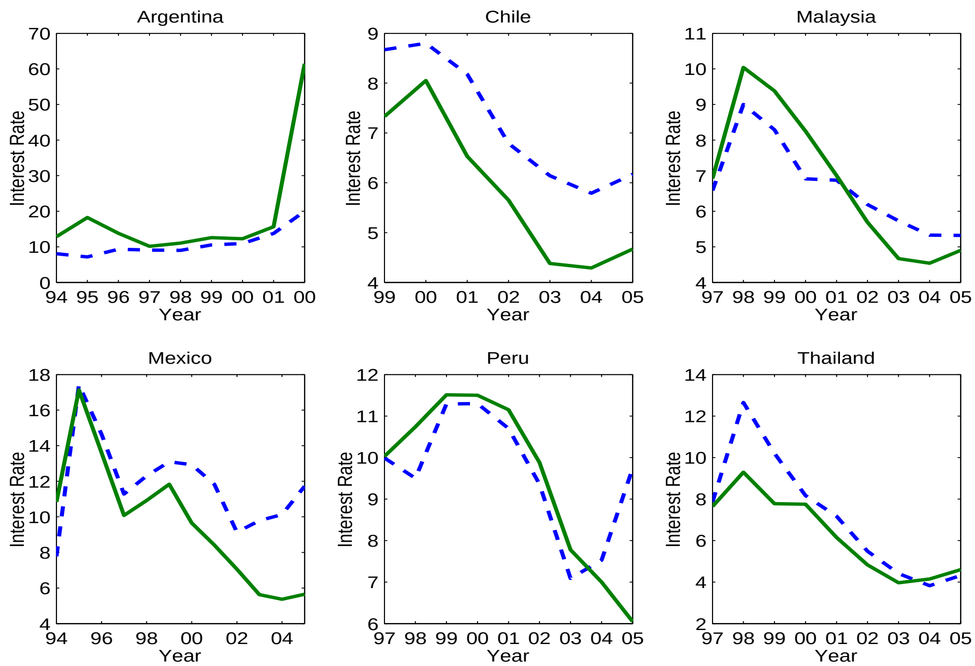
[13] Arellano and Kocherlakota (2007) also document a positive correlation between private domestic lending rates and sovereign spreads using the domestic lending-deposit spread data from the Global Financial Data.

countries default. The median drops in trade credit are about 35 and 51 percent two and four years after default events.

Table 1: Sovereign and Corporate Interest Rates

Country	Sovereign Interest Rates	Median Firm Interest Rates	Correlation
Argentina	13.32	10.66	0.87
Brazil	12.67	24.60	0.14
Chile	5.81	7.95	0.72
China	6.11	5.89	0.52
Colombia	9.48	19.27	0.86
Egypt	5.94	8.62	0.58
Malaysia	5.16	6.56	0.96
Mexico	9.40	11.84	0.74
Morocco	9.78	13.66	0.32
Pakistan	9.71	12.13	0.84
Peru	9.23	11.42	0.72
Philippines	8.78	9.27	0.34
Poland	7.10	24.27	0.62
Russia	15.69	11.86	-0.21
South Africa	5.34	15.19	0.68
Thailand	6.15	7.30	0.94
Turkey	9.80	29.26	0.88
Venezuela	14.05	19.64	0.16

There is also evidence suggesting that our assumption that the government can divert the repayment of the firms' foreign obligations is realistic. In particular, it is not uncommon for governments to take over the foreign obligations of the corporate sector in actual default episodes. The following quote by the IMF Historian explains how this was done in Mexico's 1982-83 default, and notes that arrangements of this type have been commonly used since then: *"A simmering concern among Mexico's commercial bank creditors was the handling of private sector debts, a substantial portion of which was in arrears...the banks and some official agencies had pressured the Mexican government to assume these debts...Known as the FICORCA scheme, this program provided for firms to pay dollar-denominated commercial debts in pesos to the central bank. The creditor was required to reschedule the debts over several years, and the central bank would then guarantee to pay the creditor in dollars. Between March and November 1983, close to $12 billion in private sector debts were rescheduled under this program... FICORCA then became the prototype for similar schemes elsewhere."* (Boughton (2001), Ch. 9, pp. 360-361)



——— Sovereign Bond Interest Rates - - - - Median Firm Financing Cost

Figure 4: Sovereign Bond Interest Rates and Median Firm Financing Costs

2.9 Recursive equilibrium

Definition 2 *The model's recursive equilibrium is given by (i) a decision rule $b_{t+1}(b_t, \kappa_{t-1}, \varepsilon_t)$ for the sovereign government with associated value function $V(b_t, \kappa_{t-1}, \varepsilon_t)$, consumption and transfers rules $c(b_t, \kappa_{t-1}, \varepsilon_t)$ and $T(b_t, \kappa_{t-1}, \varepsilon_t)$, default set $D(b_t, \kappa_{t-1})$ and default probabilities $p^*(b_{t+1}, \varepsilon_t)$; and (ii) an equilibrium pricing function for sovereign bonds $q^*(b_{t+1}, \varepsilon_t)$ such that:*

1. *Given $q^*(b_{t+1}, \varepsilon_t)$, the decision rule $b_{t+1}(b_t, \kappa_{t-1}, \varepsilon_t)$ solves the recursive maximization problem of the sovereign government (26).*
2. *The consumption plan $c(b_t, \kappa_{t-1}, \varepsilon_t)$ satisfies the resource constraint of the economy*
3. *The transfers policy $T(b_t, \kappa_{t-1}, \varepsilon_t)$ satisfies the government budget constraint.*
4. *Given $D(b_t, \kappa_{t-1})$ and $p^*(b_{t+1}, \varepsilon_t)$, the bond pricing function $q^*(b_{t+1}, \varepsilon_t)$ satisfies the arbitrage condition of foreign lenders (33).*

Condition 1 requires that the sovereign government's default and saving/borrowing decisions be optimal given the interest rates on sovereign debt. Condition 2 requires that the private consumption allocations implied by these optimal borrowing and default choices be both feasible and consistent with a competitive equilibrium (recall that the resource constraint of the sovereign's optimization problem considers only private-sector allocations that are competitive equilibria). Condition 3 requires that the decision rule for government transfers shifts the appropriate amount of resources between the government and the private sector (i.e. an amount equivalent to net exports when the country has access to world credit markets, or the diverted repayment of working capital loans when a default occurs, or zero when the economy is in financial autarky beyond the date of default). Notice also that given conditions 2 and 3, the consumption plan satisfies the households' budget constraint. Finally, Condition 4 requires the equilibrium bond prices that determine country risk premia to be consistent with optimal lender behavior.

A solution for the above recursive equilibrium includes solutions for the private sector equilibrium allocations with and without credit market access. A solution for equilibrium interest rates on working capital as a function of b_{t+1} and ε_t follows from (34). Expressions for equilibrium wages, profits and the price of domestic inputs as functions of r_t and ε_t follow then from the firms' optimality conditions and the definitions of profits described earlier.

3 Quantitative analysis

3.1 Calibration

We study the quantitative implications of the model by conducting numerical simulations setting the model to a quarterly frequency and using a benchmark calibration based mostly on data for Argentina, as is typically done in the related literature on quantitative studies of sovereign default. Table 2 shows the parameter values of this benchmark calibration.

The risk aversion parameter σ is set to 2 and the quarterly world risk-free interest rate r^* is set to 1 percent, which are standard values in quantitative business cycle and sovereign default studies. The curvature of labor disutility in the utility function is set to $\omega = 1.455$, which implies a Frisch wage elasticity of labor supply of $1/(\omega - 1) = 2.2$. This is the value typically used in RBC models of the small open economy (e.g. Mendoza (1991) and Neumeyer and Perri (2005))), and is based on estimates for the U.S. quoted by Greenwood, Hercowitz and Huffman (1988). The probability of re-entry after default is 0.125, which implies that the country stays in exclusion for 2 years after default on average, in line with the estimates of Gelos et al. (2003).

The share of intermediate goods in gross output α_M is set to 0.43, which corresponds to the average ratio of intermediate goods purchases to gross production calculated using annual data for Argentina for the period 1993-2005 from the United Nation's *UNData*.[14] Given the value of α_M, we set $\alpha_k = 0.17$ so that the capital income share *in value added* of the f sector ($\alpha_k/(1 - \alpha_m)$) matches the standard 30 percent ($0.17/(1 - 0.43) = 0.3$). These factor shares imply a labor share in gross output of final goods of $\alpha_L = 1 - \alpha_m - \alpha_k = 0.40$, which yields a labor share *in value added* $\alpha_L/(1 - \alpha_m) = 0.7$ that matches the standard 70 percent. The

[14]Mendoza (2007) reports a very similar share for Mexico, and Gopinath, Itskhoki, and Rigobon (2007) show shares in the 40-45 percent range for several countries.

labor share in intermediate goods production γ is also set to 0.7, since this is also the share of labor in value added in the m sector.

Table 2: Benchmark Model Calibration

Calibrated Parameters		*Value*	*Target statistics*	
CRRA risk aversion	σ	2	Standard RBC value	
Risk-free interest rate	r^*	1%	Standard RBC value	
Capital share in final goods gross output	α_k	0.17	Standard capital share in GDP (0.3)	
Int. goods share in final goods gross output	α_m	0.43	Share of int. goods in gross output	
Labor share in final goods gross output	α_L	0.40	Standard labor share in GDP (0.7)	
Labor share in GDP of int. goods	γ	0.7	Standard labor share in GDP (0.7)	
Curvature parameter of labor supply	ω	1.455	Frisch wage elasticity (2.2)	
Re-entry probability	η	0.125	Length of exclusion (2 years)	
Parameters set with SMM		*Value*	*Targets from Argentina's data*	
Productivity persistence	ρ_ε	0.90	GDP autocorrelation	0.79
Productivity innovations std. dev.	σ_ϵ	2.14%	GDP std. deviation	4.70%
Intermediate goods TFP	A	0.20	Output drop in default	13%
Armington weight of domestic inputs	λ	0.61	Nominal imported-domestic inputs ratio	18%
Armington curvature parameter	μ	0.69	Real imported-domestic inputs ratio	16%
Subjective discount factor	β	0.87	Default frequency	0.69%
Working capital coefficient	θ	0.10	Trade balance volatility	2.88%

Productivity shocks in final goods production follow an AR(1) process:

$$\log \varepsilon_t = \rho_\varepsilon \log \varepsilon_{t-1} + \epsilon_t, \tag{35}$$

with $\epsilon_t \overset{iid}{\sim} N\left(0, \sigma_\epsilon^2\right)$. We use a Markov approximation to this process with 25 realizations constructed using the method proposed by Tauchen (1986). Data limitations prevent us from estimating (35) directly using actual TFP data, so we set σ_ϵ^2 and ρ_ε (together with other parameters to be discussed below) using the simulated method of moments (SMM) set to target a set of moment conditions from the data. The target moment conditions for σ_ϵ^2 and ρ_ε are Argentina's standard deviation and first-order autocorrelation of quarterly H-P detrended GDP. We use seasonally-adjusted quarterly real GDP from the Ministry of Finance (MECON) for the period 1980Q1 to 2005Q4. The standard deviation and autocorrelation of the cyclical component of GDP are 4.7 percent and 0.79 respectively. The process of productivity shocks derived using SMM features $\rho_\varepsilon = 0.90$ and $\sigma_\epsilon = 2.14$ percent. Note that, as mentioned earlier, GDP in the model is measured as $(1-\alpha_m)\, \varepsilon M^{\alpha_m} L_f^{\alpha_l} \overline{k}^{\alpha_k} + p_m m^d$, but in actual GDP data at constant prices the price of intermediate goods is fixed at a base year's price. Hence, we keep p_m fixed at the value implied by the private sector equilibrium at $r = 0.01$ and $\varepsilon = 1$ to compute a measure of the model's GDP comparable with the data. This adjustment does not affect our quantitative findings significantly.

The additional parameters calibrated using SMM are μ, λ, A, β, and θ . These parameters are targeted to match the average nominal and real expenditure ratios of imported to domestic inputs, the fraction of output loss at default, the frequency of default, and the volatility of the trade balance-GDP ratio.[15] Given serious limitations of the national accounts data for Argentina, the target statistics for the average input expenditure ratios are computed using Mexican data for the period 1988-2004. The average ratios of imported to domestic inputs at current and constant prices are 18 and 15.7 percent respectively. We assume that Argentina has similar ratios.[16] The default frequency is 0.69 percent because Argentina has defaulted five times on its external debt since 1824 (the average default frequency is 2.78 percent annually or 0.69 percent quarterly). The output loss at default is set to 13 percent. This corresponds to the deviation from the H-P trend in Argentina's quarterly GDP as of the first quarter of 2002.[17] The standard deviation of Argentina's quarterly trade balance-to-GDP ratio is 2.88 percent.

The SMM estimate of μ is 0.69 and the estimate of λ is 0.61, so the Armington elasticity of substitution between m^* and m^d is 3.22 and there is a small bias in favor of domestic inputs.[18] Interestingly, these values are very similar to econometric estimates we obtained using the 17 observations available from annual Mexican data for 1988-2004. We estimated μ and λ by applying non-linear least squares on the condition equating the marginal rate of technical substitution between m^* and m^d with the corresponding relative price (derived using conditions (8) and (9)), and we obtained estimates of $\mu = 0.66$ and $\lambda = 0.62$, both statistically significant (with standard errors of 0.11 and 0.12 respectively).

The subjective discount factor is 0.87, which is in the range of the values used in existing studies on sovereign default.[19] The estimate for A is 0.2. Finally, the estimate for θ implies

[15] A can be used to target the output drop at default because, as mentioned in Section 2, changes in A have similar effects as changes in ω. In particular, lower values of A yield larger output drops at default without altering the slope of the relationship between TFP and these output drops.

[16] Several industrial countries have input expenditure ratios similar to Mexico's, but the ratios vary widely across countries. Goldberg and Campa (2008) report ratios of imported inputs to total intermediate goods for 17 industrial countries that vary from 14 to 49 percent, with a median of 23 percent. This implies ratios of imported to domestic inputs in the 16 to 94 percent range, with a median of 30 percent.

[17] Argentina declared default in the last week of December in 2001, but it is reasonable to assume that, in quarterly data, the brunt of the real effects of the debt crisis were felt in the first quarter of 2002. Arellano (2007) also follows this convention to date the default as of the first quarter of 2002. She estimated the output cost at 14 percent, measured as a deviation from a linear trend.

[18] Empirical estimates of Armington elasticities vary widely. McDaniel and Balistreri (2002) review the literature and quote estimates ranging from 0.14 to 6.9. They explain that elasticities tend to be higher when estimated with disaggregated data, in cross-sectional instead of time-series samples, or when using long-run instead of short-run tests.

[19] The values of β in Aguiar and Gopinath (2006), Arellano (2007), and Yue (2006) range from 0.8 to 0.953.

that firms pay only 1/10 of the cost of imported inputs in advance.

In the subsections that follow we examine different aspects of the model's numerical solution based on this benchmark calibration. In order to obtain solutions that apply to the model's stochastic stationary state, we feed the TFP process to the model and conduct 2000 simulations, each with 500 periods and truncating the first 100 observations.

3.2 Cyclical Co-movements in the Benchmark Simulation

This subsection examines the ability of the benchmark model to account for observed statistical moments of business cycles and sovereign debt. We show that the model approximates well the countercyclical behavior of country interest rates and the high GDP ratios of external debt, and that it also accounts for two key emerging market business cycle facts: the high cyclical variability of consumption and the negative correlation of net exports with GDP. Table 3 compares the moments produced by the model with moments from Argentine data. The bond spreads data are quarterly EMBI+ spreads on Argentine foreign currency denominated bonds from 1994Q2 to 2001Q4, taken from J.P. Morgan's EMBI+ dataset.

Table 3: Model Simulation and Statistics in the Data

Statistics	Data	Model
Corr. between Bond Spreads and GDP	-0.62	-0.19
Corr. between Bond Spreads and Trade Balance	0.68	0.18
Corr. between Trade Balance and GDP	-0.58	-0.24
Consumption Std. Dev./Output Std. Dev.	1.44	1.31
Average Debt/GDP	35%	23.11%
Bond Spreads Std. Dev.	0.78%	0.71%
Average Bond Spreads	1.86%	0.58%
Corr. between M and Spreads	-	-0.22
Corr. between M and GDP	-	0.70
Corr. between total labor and Spreads	-	-0.21
Corr. between total labor and GDP	-	0.72
Corr. between default and GDP	-	-0.12
Fraction of defaults with GDP below trend	-	100%
Fraction of defaults with GDP 2 std dev. below trend	-	76.62%

The model accounts for the negative correlation between spreads and GDP because sovereign bonds have higher default risk in bad states. Several quantitative models of sovereign debt (e.g. Arellano (2007), Aguiar and Gopinath (2005), Yue (2006)) and business cycle models of emerging economies (e.g. Neumeyer and Perri (2005), Uribe and Yue (2006)) also produce countercyclical spreads, but as we noted earlier the former treat output as an exogenous endowment and in the latter country risk is exogenous. In contrast, our model matches the negative correlation between GDP and spreads in a setting in which both output and country risk are endogenous, and influence each other because of the relationship between country risk and working capital financing. The countercyclical net exports follow from the fact that, when the country is in a bad state, it faces higher interest rates and tends to borrow less. The country's trade balance thus increases because of the lower borrowing, leading to a negative correlation between net exports and output.

Consumption variability exceeds output variability in Argentina, and this is a common feature across emerging economies. The model is able to mimic this stylized fact because the ability to use external debt to smooth consumption is negatively affected by the higher interest rates induced by increased default probabilities. The sovereign borrows less when the economy faces an adverse productivity shock, and thus households adjust consumption by more than in the absence of default risk. On the other hand, because agents are impatient, the benevolent government borrows more to increase private consumption when the productivity shock is good. Hence, the variability of consumption rises.

The model produces a debt-to-GDP ratio of 23 percent on average. This high debt ratio is mainly the result of two features of the model: the large output drop that occurs when the country defaults, and the increasing output cost of default as a function of the state of TFP. Although a 23 percent debt ratio is still below Argentina's 35 percent average debt-output ratio (based on data from the World Bank's WFD dataset for the 1980-2004 period), it is much larger than the debt ratios typically obtained in quantitative models of sovereign default with exogenous output costs already targeted to improve the models' quantitative performance. For instance, Yue's (2006) model with renegotiation and an exogenous 2 percent output cost at default yields an average debt ratio of 9.7 percent. Arellano (2007) obtains a mean debt ratio of 6 percent of GDP assuming that output when the economy defaults equals the maximum of actual output or 97 percent of average output.[20]

The model also matches closely the volatility of the Argentine bond spreads observed in the data. Yet the average bond spread is lower than in the data. Because we assume a zero recovery rate on defaulted debt and risk-neutral creditors, bond spreads are linked one-to-one with default probabilities (see eq. (33)). Since the quarterly default frequency is 0.7 percent (as in the data), the model can only generate an average bond spread of a similar magnitude, which is about 1/3 of the average spreads observed in the data.

Table 3 also shows that sovereign spreads and intermediate goods are negatively correlated. This is due to the credit transmission mechanism that operates via the working capital requirement (as explained in subsection 2.5). This mechanism is also behind the negative correlation between labor and spreads. In turn, intermediate goods and labor are positively correlated with GDP because of the standard real-business-cycle effects of TFP shocks and

[20] As mentioned earlier, Aguiar and Gopinath (2006) obtained a higher mean debt ratio (27 percent of GDP) assuming a cost of 2 percent of output, but with a default frequency of only 0.02 percent.

because of the reinforcing effect of the countercyclical spreads.

We also report in Table 3 the correlation between defaults and GDP and the fractions of default events that occur when GDP is below trend and two-standard-deviations or more below trend. The correlation between defaults and GDP is -0.12, very close to Tomz and Wright's (2007) cross-country historical estimate for the period 1820-2004.[21] Default in the model with the quarterly benchmark calibration only occurs in "bad times," since all default events occur when GDP is below trend. About 3/4s of them occur with very deep recessions in which GDP is at least two standard deviations below trend. These two statistics seem at odds with Tomz and Wright's findings indicating that not all defaults coincide with bad times in annual data. If we aggregate our quarterly simulation data into a comparable annual frequency, however, we find that 11 percent of defaults occur in "good times" (i.e. with GDP above trend) and 89 percent occur in bad times, and only about 30 percent of defaults occur when GDP is two standard deviations or more below trend.

3.3 Output Dynamics around Default Episodes

We illustrate the model's ability to match V-shaped output dynamics around default episodes by applying event study techniques to the simulated time series data. Figure 5 plots the model's average path of output around default events together with the data for Argentina's HP detrended GDP around the recent default (1999Q1 to 2004Q3). The event window covers 12 quarters before and 10 quarters after debt defaults, with the default events normalized to date 0. We plot the average for output in the model at each date $t = -12, ..., 10$ around default events in the 2000 simulations. Hence, the simulated GDP line represents the average behavior of output around defaults in the model's stochastic stationary state. Since Argentina's data is for a single default event, instead of a long-run average across Argentina's defaults, we add dashed lines with one-standard-error bands around the model simulation average. Note that the relative magnitudes of the recession and recovery match the data quite well. The output dynamics for Argentina before and after the debt crisis are mostly within the one-standard-error bands of the model simulations.

Figure 5 shows that the model produces a substantial output drop when the country defaults, equivalent on average to about 13 percent below the H-P trend of the country's output. Defaults are always triggered by adverse TFP shocks, but these shocks are not unusually large. The standard deviation of the calibrated TFP process (σ_ε) is 4.91 percent. By contrast, the average decline in TFP in default events (i.e. at $t = 0$ in Figure 5) is 5 percent, which is about the same size as the TFP standard deviation. This suggests that the model embodies a business cycle transmission mechanism that amplifies significantly the real effects of TFP shocks when these shocks trigger default. The magnitude of this amplification

[21] They report an unconditional correlation between default and output of -0.08, and a correlation conditional on countries that actually defaulted of -0.11.

effect can be quantified by computing the average output drop that the model produces in response to a 5 percent TFP shock when there is no default, and comparing it with the 13 percent mean output drop that the same shock produces in default episodes. Without default, a 5 percent TFP shock produces a mean output drop of about 7.2 percent. Thus, the amplification coefficient due to default is $13/7.2 = 1.8$. Default amplifies the output effect of TFP shocks by about 80 percent.

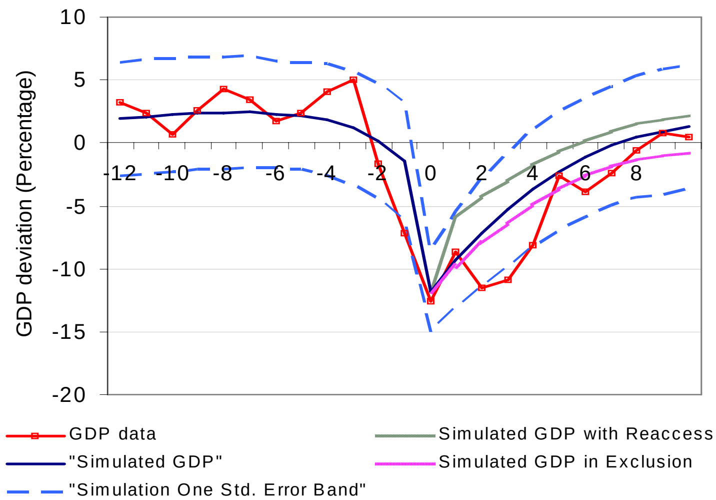


Figure 5: Output around Default Events

The model displays a V-shaped recovery after default. This recovery is driven by two effects. First, since the TFP shock is mean-reverting, TFP is likely to improve after default (on average, TFP rises by 1 percent at $t = 1$). Therefore, even though the country remains in financial autarky on average from dates 1 to 10, the economy recovers because TFP improves. The second effect is the surge in output that occurs when the country re-enters credit markets (as final goods producers switch back to a more efficient mix of imported and domestic inputs).

The two effects that induce the post-default recovery are illustrated in Figure 5 by the lines that show the simulated paths of GDP with continued exclusion for 10 quarters after default and with immediate re-entry one period after default. In the first scenario, the recovery reflects only the effect of the mean reversion of the TFP shock. GDP remains below that in the simulation average because the latter is also affected by the probability of credit-market re-entry. In contrast, the second scenario with immediate re-entry to international credit markets shows a big rebound in GDP at $t = 1$, because of the efficiency gain obtained as final goods producers regain access to imported inputs. The simulation average lies below this immediate re-entry line because re-entry to credit markets is stochastic with 12.5 percent probability. Since re-entry has a relatively low probability, the model simulation for average GDP weighs more the TFP recovery effect than the credit market re-entry effect.

These V-shaped output dynamics are qualitatively consistent with the data of many emerging markets that experienced Sudden Stops. Calvo, Izquierdo and Talvi (2006) conducted a cross-country empirical analysis of the recovery of emerging economies from Sudden Stops, and found that most recoveries are not associated with improvements in credit market access. In our model as well, recovery occurs (on average) even though the economy continues to be excluded from world credit markets.

The output dynamics also suggest that the model can account for the seemingly dominant role of productivity shocks in output collapses during financial crises. In particular, this can be the result of the efficiency loss caused by the change in the mix of intermediate goods and the sectoral reallocation of labor when the economy defaults. To demonstrate this point, we use the model's simulated data to compute Solow residuals in the standard way: We assume an aggregate Cobb-Douglas production function for economy-wide GDP, $gdp_t = s_t(L_t)^a k^{1-a}$, and compute the Solow residual s using the model's data for L and gdp, setting a to the model's average of the ratio of total wage payments to GDP, $w_t L_t / gdp_t$, which is about 0.7. By construction, however, the "true" TFP shock driving the model is ε_t in the production function of final goods.

Figure 6 compares the quarter-on-quarter average growth rates of the Solow residual, true TFP and GDP around default events in the baseline model simulations. There is little difference between the Solow residual and true TFP except when the economy defaults. In default events, the Solow residual overestimates the true adverse TFP shock by a large margin (on average, s falls by nearly twice as much as ε when the economy defaults).

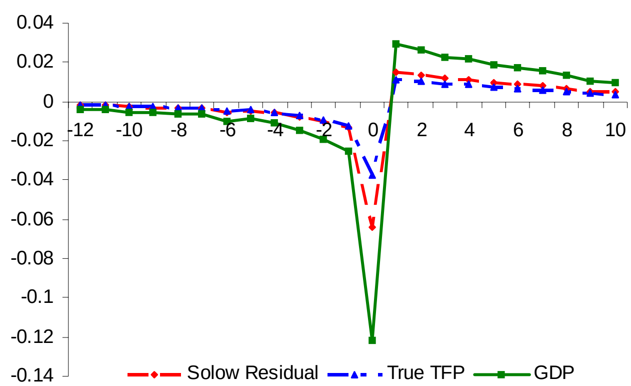


Figure 6: Growth Rates of GDP, True TFP and Solow Residual around Default

A standard decomposition of the contributions of changes in TFP and in factors of production to changes in GDP shows that the contribution of true TFP to the output collapse at default is about 31 percent. In contrast, the contribution of the Solow residual is nearly 53 percent, which would suggest misleadingly that the contribution of TFP shocks is 1.72 times larger than it actually is. The large difference between the two is due to the fact that the

Solow residual treats the efficiency loss induced by the default as a reduction in aggregate TFP.

The model also matches nicely the dynamics of sovereign bond spreads before a debt crisis. The left panel of Figure 7 presents event windows showing the mean of simulated output and bond spreads up to 12 quarters before default events in the stationary distribution of the model. This plot illustrates the negative correlation between output and bond spreads before a debt crisis. In particular, the spread increases as the country approaches a debt crisis. The average *quarterly* spread increases from 0.6 percent at $t = -12$ to almost 2.5 percent in the quarter before default. At the same time, HP detrended output starts to decline three quarters before default, and suffers a sharp drop when default occurs. These features match relatively well the Argentine experience. The right-side panel of Figure 7 shows the HP detrended real GDP and EMBI+ sovereign bond spreads for Argentina from 1999Q1 to 2001 Q4. The data show a relatively stable sovereign spread before 2000 and a sharp increase in 2001, and Argentina also experienced a relatively steady output performance and then a very deep recession starting in 2001.

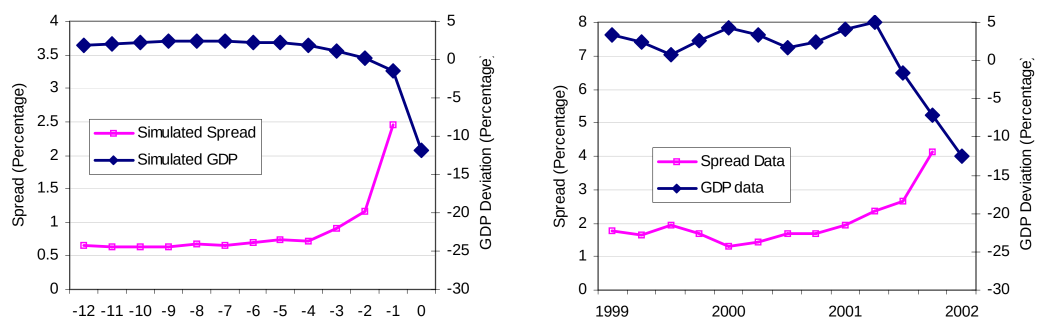


Figure 7: Dynamics of Output and Sovereign Spreads before a Debt Crisis

Figure 8 shows the event windows for the average of the model simulations of consumption, current account, foreign bond holdings, labor, intermediate goods, and sectoral labor allocations (along with the corresponding one-standard-error bands). Consumption drops sharply when the government defaults and in the period that follows, and then it recovers following the V-shaped dynamics of GDP. The debt-output ratio is over 23 percent on average before default, and it increases to about 32 percent in the period just before default. The model also generates a sharp reversal in the current account. The country runs a small current account deficit on average, but default, and the loss of credit market access that it entails, produce a large jump of about 30 percentage points of GDP in the current account. Labor and intermediate goods also fall sharply when the economy defaults. Moreover, since default triggers a shift from imported to domestic inputs in final goods production, labor is reallocated from the f sector to the m sector, and hence labor in the latter falls by less than in the former.

The sharp declines in GDP, consumption, labor and intermediate goods, together with the large reversal in the current account, indicate that the model yields predictions consistent with the Sudden Stops observed in emerging economies. In most of the Sudden Stops literature, however, the current account reversal is modeled as an exogenous shock, whereas in this model both the current account reversal and the economic collapse are endogenous.[22]

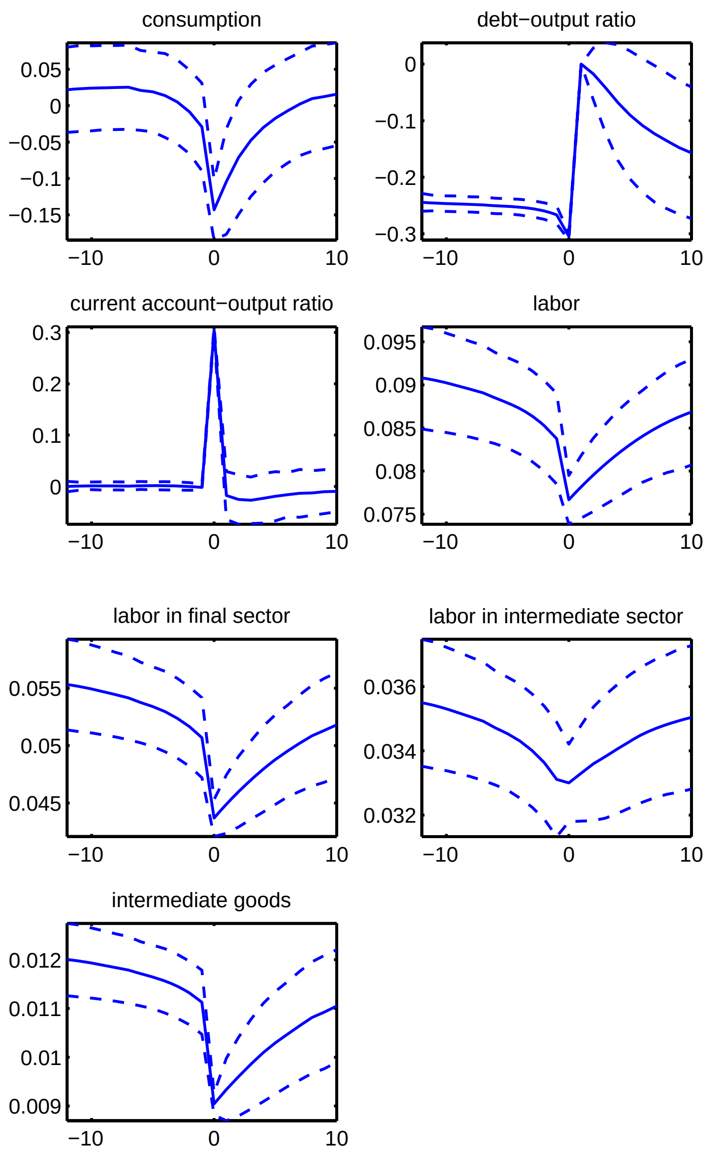


Figure 8: Macro Dynamics around Default Episodes

[22]Mendoza (2007) proposed an alternative model of endogenous Sudden Stops based on collateral constraints and Irving Fisher's debt-deflation mechanism instead of sovereign default risk.

4 Sensitivity Analysis

In this Section we conduct a sensitivity analysis to study how the model's quantitative predictions change when we vary the working capital requirement θ, the parameters of the Armington aggregator μ and λ, and the labor supply curvature parameter ω. In reviewing these results, it is important to keep in mind that θ, μ and λ were calibrated using SMM to target statistics from actual data. Hence, the sensitivity analysis helps us explain how the model's results depend on the values of these parameters, but the "best" values that the parameters can take (conditional on the statistics being targeted and on the use of SMM to target them) are the benchmark values.

4.1 Working Capital

Column (I) of Table 4 shows the results of simulating the model without working capital. In this case, there is no endogenous output cost of default. Output is low simply because TFP is low when default occurs, and there is no amplification of this effect due to default. To keep the results comparable with those reported in some of the existing quantitative studies of sovereign debt that assume that the output cost of default is a proportional drop in an exogenous endowment (e.g. Aguiar and Gopinath (2006), Yue (2006)), we introduce a similar exogenous output cost of default. This cost is set to match the observed output cost of default of 13 percent for Argentina.[23] The other parameters are kept unchanged.

Table 4: Changes in the Working Capital Constraint

Statistics	(I) No working capital $\theta = 0.0$	(II) Benchmark $\theta = 0.1$	(III) Larger WC $\theta = 0.15$
Output loss	13.0%	13.2%	9.1%
GDP std. dev.	4.76%	4.69%	5.11%
Default probability	0.12%	0.60%	4.19%
Debt/GDP	0.15%	23.11%	17.82%
Bond spreads std. dev.	0.16%	0.71%	2.98%
Average Bond Spreads	0.12%	0.58%	3.09%
Corr. between Spreads and GDP	-0.04	-0.19	-0.06
Corr. between Spreads and TB	-0.21	0.18	0.08
Corr. between TB and GDP	-0.28	-0.24	-0.03
Nominal ratio of imported m	17.9%	17.8%	12.3%
Real ratio of imported m	16.0%	15.6%	13.6%
Trade Balance std. dev.	0.01%	2.18%	2.54%

[23] We do this taking into account the output effect of the responses of M and L to TFP changes, which are absent from sovereign debt models that assume output is an exogenous endowment.

The model without working capital performs much worse than the benchmark in terms of its ability to match the important features of the data that the benchmark model approximated well (Column (II) of Table 4 reproduces the results of the benchmark model). The frequency of defaults falls from 0.6 percent to 0.12 percent. The mean debt ratio declines by nearly 23 percentage points of GDP, and the average and standard deviation of country spreads fall by 55 and 46 basis points respectively. In addition, the GDP correlation of sovereign spreads increases to -0.04, and the correlation between spreads and net exports falls sharply from 0.18 to -0.21. These results follow from two important differences in the model without working capital relative to the benchmark: First, the cost of default becomes independent of the state of nature, and second, bond spreads no longer have a direct impact on production. As a result, debt is not as good a hedging mechanism as in the benchmark model, making default more painful ex ante in the model without working capital, and thus reducing the average debt ratio.

The left-side panel of Figure 9 compares the simulation averages of GDP deviations from H-P trend around default events for the benchmark model and the model without working capital. The GDP drop at default is identical in the two cases by construction, since both models are calibrated to obtain the same 13 percent output loss at default. Before the default occurs, however, the two models produce significantly different dynamics. The benchmark model shows a gradual recession that starts 6 quarters before the output collapse when default occurs, while the model without working capital produces a sustained boom that peaks just before default hits, which is not in line with the data. This is because the model without working capital lacks an endogenous feedback mechanism from default risk to output, and default is preceded (on average) by positive TFP shocks. The recovery stage is again nearly identical in both models, but this is also by construction because in both cases we observe the mean reversion of shocks targeted to produce GDP stochastic processes with identical characteristics. The benchmark model shows slightly higher output because of the possibility (with low probability) of credit-market re-entry, and the surge in output associated with this event.

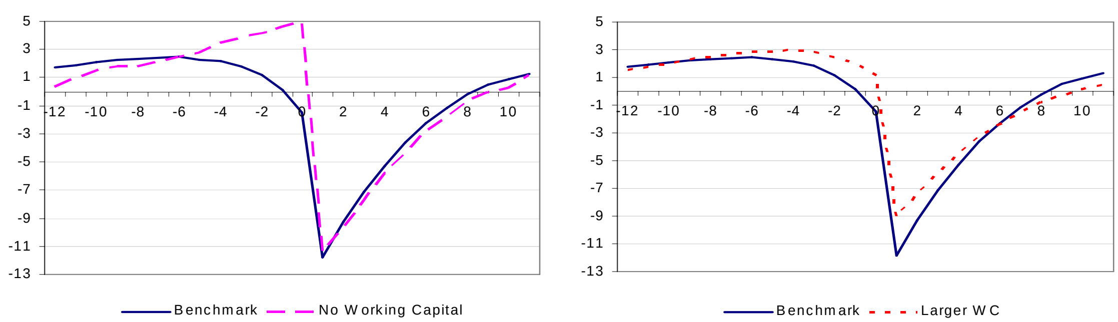


Figure 9: Comparison of Output Dynamics for Alternative Working Capital Specifications

How sensitive are the model's results to the value of the working capital requirement beyond the extreme case of $\theta = 0$? To answer this question, we report in Column (III) of Table 4 results for $\theta = 0.15$, instead of 0.1 as in the benchmark case. The higher working capital coefficient reduces the mean debt ratio by 6 percentage points of GDP and generates a smaller output cost of default. In contrast, the variability of GDP, the probability of default, and the mean and standard deviation of spreads all increase sharply as θ rises.

These changes reflect the fact that the higher θ has opposing effects on default incentives and production plans. On one hand, since a larger fraction of imported inputs requires foreign financing to be purchased, changes in sovereign interest rates have a larger impact on production. This amplifies the response of output to productivity shocks, making output more volatile. This result is complementary to the finding in Uribe and Yue (2006) showing that the impact of output on country interest rates magnifies business cycle volatility, and the result in Neumeyer and Perri (2005) showing that working capital loans that charge sovereign interest rates also amplify business cycle volatility. On the other hand, default leads to a lower output cost of default on average because the TFP shock that triggers default is smaller with $\theta = 0.15$ than in the benchmark case with $\theta = 0.1$. Thus, the output levels before and after default are closer, generating a smaller output loss. At the same time, this lower output cost of default and the higher GDP variability make the sovereign exercise the default option more often, increasing the default probability and the volatility of bond spreads, and reducing the mean debt/GDP ratio. The quantitative effects of tightening the working capital constraint on the debt/GDP ratio and the default frequency are particularly large, and we get these results even though average sovereign spreads, and hence the average interest rate on working capital, do not deviate sharply from the one-percent risk free rate.[24]

The right-side panel of Figure 9 shows the output dynamics around default events for the benchmark model and the model with $\theta = 0.15$. The model with the higher working capital coefficient still produces V-shaped dynamics around defaults, but with a milder recession before default and a smaller drop in output at default, as explained above.

The distribution of defaults across "bad times" and "good times" also changes with the value of θ. In particular, the higher value of θ shifts the distribution away from the states with larger output drops. At a quarterly frequency, the model with $\theta = 0.15$ continues to generate 100 percent of the default episodes when GDP is below trend, as in the benchmark, but the fraction of defaults that occur when output is two standard deviations or more below trend falls from 76 percent in the benchmark to 37.2 percent. Aggregating to an annual frequency, we find that with $\theta = 0.15$ about 12 percent of the defaults occur with output above trend, and 5 percent of them occur with GDP two standard deviations or more below

[24]Neumeyer and Perri (2005) and Uribe and Yue (2006) use average interest rates around 7 percent and set $\theta = 1$, and they find that the working capital constraint is important for business cycle dynamics. Oviedo (2005) also showed that obtaining significant effects of working capital in the small open economy RBC model requires high values of r^* and θ.

trend (compared with 30 percent of defaults in the benchmark case). The correlation between GDP and default is about -0.27 at both frequencies.

4.2 Imperfect Substitution between Foreign and Domestic Inputs

Column (I) of Table 5 reports simulation results reducing μ from 0.69 in the benchmark case to $\mu = 0.5$. This reduces the Armington elasticity of substitution between m and m^* from 3.22 to 2. Column (II) reproduces the results for the benchmark calibration. Column (III) reports results lowering the Armington weight λ from the benchmark value of 0.61 to $\lambda = 0.5$. All of the other parameters are the same as in the benchmark calibration.

Table 5: Changes in Parameters of Armington Aggregator of Intermediate Goods

Statistics	(I) Lower Elasticity $\mu = 0.5$	(II) Benchmark $\mu = 0.69$ $\lambda = 0.61$	(III) Lower Weight $\lambda = 0.5$
Output loss	25.1%	13.2%	16.9%
GDP std. dev.	4.95%	4.69%	5.43%
Default probability	0.02%	0.60%	0.19%
Debt/GDP	168.59%	23.11%	92.36%
Bond spreads std. dev.	0.07%	0.71%	2.28%
Average Bond Spreads	0.02%	0.58%	0.12%
Corr. between Spreads and GDP	-0.03	-0.19	-0.33
Corr. between Spreads and TB	0.01	0.18	0.09
Corr. between TB and GDP	-0.29	-0.24	-0.37
Nominal ratio of imported m	34.3%	17.8%	58.2%
Real ratio of imported m	29.2%	15.6%	46.2%

The results reported in Table 5 show that the values of μ and λ affect significantly the magnitude of the output loss at default, as would be expected given the analysis of Section 2. With a lower Armington elasticity or a lower Armington weight on domestic inputs, imported inputs are "more important" in final goods production.[25] When the elasticity of substitution is lower, domestic inputs are poorer substitutes for imported inputs, and hence the efficiency loss of the f sector when the economy defaults is bigger. Similarly, if domestic (imported) inputs have a lower (higher) weight in the Armington aggregator, default is more painful when access to foreign inputs is lost. Accordingly, the output costs of default in the scenarios

[25] This is also reflected in the nominal and real expenditure ratios of imported to domestic inputs. These ratios are much higher with lower μ or lower λ than in the benchmark case or in the Mexican data reported by Mendoza (2007). On the other hand, they are in line with some of the ratios implied by the data reported by Goldberg and Campa (2006).

with lower μ and lower λ reach about 25 and 17 percent respectively, compared with 13 percent in the benchmark. Because of these higher output costs of default, the probabilities of default are lower (0.02 and 0.19 percent respectively) and the mean debt ratios are much higher (169 and 92 percent respectively).

Changes in μ and λ also affect business cycle comovements. Lower μ and lower λ produce higher output variability. The standard deviations of GDP increase to 4.9 and 5.4 percent respectively, compared with 4.7 in the benchmark. The correlations of GDP with spreads and net exports remain negative, as in the benchmark case, but they are significantly more negative with $\lambda = 0.5$.

Figure 10 shows the output dynamics around default episodes for the benchmark case and the cases with $\mu = 0.5$ and $\lambda = 0.5$. These alternative specifications preserve the V-shaped output dynamics. However, the scenario with $\mu = 0.5$ produces a period of sustained expansion before defaults, instead of the gradual recession in the benchmark case, and a larger collapse in output when default occurs. The case with $\lambda = 0.5$ yields a milder recession before default but again a larger output collapse at the time of default.

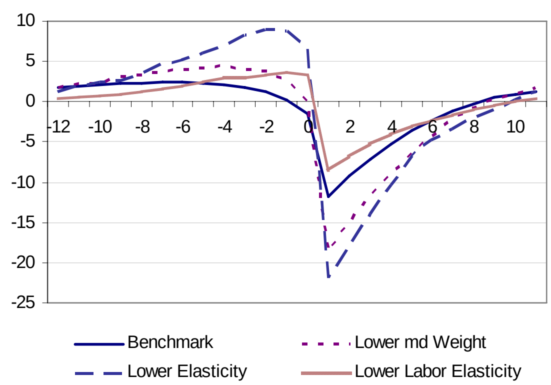


Figure 10: Comparison of Output Dynamics for Armington Parameters and Labor Elasticity

Increasing μ and λ increases the dispersion of the distribution of default events across output realizations at a quarterly frequency relative to the benchmark case. It is still the case that 100 percent of the defaults occur with output below trend as in the benchmark, but the fraction of defaults that occurs with output 2 percent below trend or more increases from about 3/4 in the benchmark to 99 and 96 percent with $\mu = 0.5$ and $\lambda = 0.5$ respectively. At an annual frequency, however, we do not observe an unambiguous increase in the dispersion of default events. Relative to the benchmark, the fraction of defaults that occurs with output two standard deviations or more below trend falls to 16 percent with $\mu = 0.5$ but rises to 37 percent with $\lambda = 0.5$. The correlations between GDP and default events are -0.03 and -0.07 with $\mu = 0.5$ and $\lambda = 0.5$ respectively, compared with -0.12 in the benchmark case.

4.3 Labor Supply Elasticity

Column (I) of Table 6 presents the results of a simulation rising the labor supply curvature parameter ω from 1.45 in the benchmark to $\omega = 2.0$. Hence the labor supply elasticity falls from 2.2 to 1 (i.e. unitary elasticity). As demonstrated in Section 2, a lower labor supply elasticity generates a lower output cost of default, without altering the slope of the relationship between default costs and TFP. Hence, the model with a less elastic labor supply supports less debt. At the same time, however, output variability declines, since labor variability is lower with the lower elasticity. As a result, the default probability does not necessarily increase, despite the lower default penalty. This is because the two effects push in opposite directions: The lower output cost tends to increase the default frequency, but the lower output variability tends to reduce it. The results in Column (I) of Table 6 show that in the simulation with $\omega = 2.0$, the net result of the two effects yields a lower default frequency than in the benchmark case. The bond spreads are lower and less volatile as well. The correlation between GDP and spreads rises, while net exports become more countercyclical. Also, in line with the results shown in Section 2, the expenditure ratios of imported to domestic inputs are not affected by changes in the labor supply elasticity.

Table 6: Changes in Elasticity of Labor Supply

Statistics	(I) Lower Elasticity ω= 2.0	(II) Benchmark $\omega = 1.45$
Output loss	11.5%	13.2%
GDP std. dev.	3.45%	4.69%
Default probability	0.14%	0.60%
Debt/GDP	22.87%	23.11%
Bond spreads std. dev.	0.18%	0.71%
Average Bond Spreads	0.15%	0.58%
Corr. between Spreads and GDP	-0.11	-0.19
Corr. between Spreads and TB	0.01	0.18
Corr. between TB and GDP	-0.34	-0.24
Nominal ratio of imported m	17.8%	17.8%
Real ratio of imported m	15.6%	15.6%

The output dynamics around default in the scenario with $\omega = 2$ are again qualitatively consistent with the V-shaped pattern of the other scenarios (see Figure 10). Quantitatively, however, this scenario yields a period with a mild expansion before default, instead of recession, and a smaller output collapse when default takes place.

The change in labor elasticity also affects the distribution of default events across output realizations, but the effects are not as large as those caused by changing the Armington parameters. The fractions of default with output below trend or two standard deviations below trend are about the same as in the benchmark. Aggregating to annual data, however, the fraction of defaults with output below trend fell from 90 percent in the benchmark to 73 percent with $\omega = 2$, and the fraction with output two standard deviations below trend fell from 30 to 6 percent. The correlation between output and defaults fell from -0.12 to -0.04.

5 Conclusions

This paper proposed a model of strategic sovereign default with endogenous output dynamics and examined its quantitative predictions. In the model, producers of final goods choose an optimal mix of imported and domestic inputs. Purchases of foreign inputs require foreign working capital financing. Purchases of domestic inputs do not require credit, but these inputs are imperfect substitutes for foreign inputs, and producing them requires reallocation of labor away from final goods production. As a result, default causes an efficiency loss by forcing final goods producers to operate using only domestic inputs, and inducing labor to reallocate from the final goods sector to the sector producing domestic inputs. Lenders charge the same default risk premium on working capital loans as on sovereign debt because the sovereign diverts the repayment of working capital loans when the country defaults. This is in line with empirical evidence showing that corporate and sovereign interest rates are strongly correlated, and that in sovereign defaults since the 1980s Debt Crisis we often observe governments taking over the foreign obligations of private firms.

The model is consistent with three key stylized facts of sovereign debt: (1) the V-shaped dynamics of output around default episodes, (2) the negative correlation between interest rates on sovereign debt and output, and (3) high debt-output ratios on average and when defaults take place. The model also replicates the observed countercyclical dynamics of net exports, the positive correlation between country spreads and GDP, and the high variability of private consumption, and it is calibrated to be consistent with observed default frequencies.

The model produces an endogenous output cost of default that is increasing in the state of productivity. This result follows from the fact that the financing cost of working capital when default occurs rises too much for firms to find it profitable to use imported inputs, and hence they optimally switch to domestic inputs and suffer the corresponding efficiency loss. In turn, this efficiency loss is larger the higher TFP was before the switch. This increasing endogenous output cost of default is consistent with the shape of exogenous output costs that Arellano (2007) identified as necessary in order to obtain default incentives that trigger default in bad states of nature, at non-negligible debt ratios and at realistic default frequencies. However, the endogenous feedback between production and default in our model produces a mean debt ratio four times larger than in Arellano's endowment economy model.

Our results also show that the model embodies a powerful credit-driven amplification mechanism that can provide an explanation for the seemingly large contribution of productivity shocks to output collapses during financial crises. In default episodes, this mechanism amplifies the output effect of TFP shocks by 80 percent. Solow residuals computed in the standard way overestimate the contribution of true TFP to the collapse of output when the economy defaults by about 70 percent.

Three features of the model are critical for the results: imported inputs require working capital, the government diverts the firms' working capital repayment when it defaults, and default induces an efficiency loss in production. Without the first two features, output would not respond to changes in country risk. On the other hand, the model would also fail if we rely "too much" on those two features: If exclusion from world credit markets implies that firms cannot buy foreign inputs and there are no domestic inputs available, the output collapse and the associated cost of default would be unrealistically large (infinitely large if 100 percent of the cost of imported inputs requires payment in advance). In reality, firms in emerging economies facing financial crisis substitute foreign inputs with high financing costs for domestic inputs that can be employed at permissible financial terms, and/or look for alternative forms of credit, including inter-enterprise credit and internal financing using retained earnings or redirecting capital expenditures. The efficiency loss is also critical. Without it the working capital channel would not produce a sharp and sudden drop in output during periods of financial turmoil.

Our findings suggest that the model we proposed can provide a solution to the disconnect between sovereign debt models (which rely on exogenous output dynamics with particular properties to explain the stylized facts of sovereign debt) and models of emerging markets' business cycles (which assume an exogenous financing cost of working capital calibrated to match the interest rate on sovereign debt). We acknowledge, however, that the linkages between sovereign default and private sector borrowers, and the mechanisms by which default induces economy-wide efficiency losses, should be the subject of further research. For instance, the studies by Cuadra and Sapriza (2008) and D'Erasmo (2008) show that political uncertainty may help endowment economy models in the Eaton-Gersovitz class to generate higher debt ratios at observed default frequencies. This suggests that introducing a mechanism to link political uncertainty with private sector decisions in a model with sovereign risk can be a promising line of research. Similarly, the findings of Bi (2008a and 2008b) on debt dilution effects and dynamic renegotiation in endowment economy models suggest that adding these features to default models with endogenous output dynamics can also be important. Finally, results obtained by Arellano (2007), Lizarazo (2005) and Volkan (2008) suggest that adding risk-averse foreign lenders can also contribute to produce higher debt rations and break the one-to-one link between spreads and default probabilities, so that bond spreads include an additional risk premium and can get closer to the data.

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Appendix

PROOF of THEOREM 1

Given a productivity shock ε and level of working capital loan κ, the utility from defaulting $v^d(\kappa, \varepsilon')$ is independent of b. We can also show that the utility from not defaulting $v^{nd}(b, \varepsilon')$ is increasing in b_{t+1}. Therefore, if $V\left(b^1, \kappa, \varepsilon'\right) = v^d(\kappa, \varepsilon')$, then it must be the case that $V\left(b^0, \kappa, \varepsilon'\right) = v^d(\kappa, \varepsilon')$. Hence, any ε' that belongs in $D\left(b^1, \varepsilon\right)$ must also belong in $D\left(b^0, \varepsilon\right)$.

Let $d^*(b, \varepsilon')$ be the equilibrium default decision rule. The equilibrium default probability is then given by

$$p(b, \varepsilon) = \int d^*\left(b, \varepsilon'\right) d\mu\left(\varepsilon'|\varepsilon\right).$$

From $D\left(b^1, \varepsilon'\right) \subseteq D\left(b^0, \varepsilon'\right)$, if $d^*\left(b^1, \varepsilon'\right) = 1$, then $d^*\left(b^0, \varepsilon'\right) = 1$. Therefore,

$$p\left(b^0, \varepsilon\right) \geq p\left(b^1, \varepsilon\right).$$

PROOF of THEOREM 2

From Theorem 1, given a productivity shock ε and level of working capital loan κ, for $b^0 < b^1 \leq 0$, $p^*\left(b^0, \varepsilon\right) \geq p^*\left(b^1, \varepsilon\right)$. The equilibrium bond price is given by

$$q\left(b', \varepsilon\right) = \frac{1 - p(b', \varepsilon)}{1 + r}.$$

Hence, using Theorem 1, we obtain that:

$$q\left(b^0, \varepsilon\right) \leq q\left(b^1, \varepsilon\right).$$