NBER WORKING PAPER SERIES

# A MULTIPLIER APPROACH TO UNDERSTANDING THE MACRO IMPLICATIONS OF HOUSEHOLD FINANCE

YiLi Chien
Harold Cole
Hanno Lustig

Working Paper 13555
http://www.nber.org/papers/w13555

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 2007

We would like to thank Mark Hugget, Urban Jermann, Narayana Kocherlakota, Dirk Krueger, Joseph Ostroy, Nikolai Roussanov, Viktor Tsyrennikov and Amir Yaron for comments. Andrew Hollenhurst provided excellent research assistance. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

A Multiplier Approach to Understanding the Macro Implications of Household Finance
YiLi Chien, Harold Cole, and Hanno Lustig
NBER Working Paper No. 13555
October 2007
JEL No. G12

## ABSTRACT

Our paper examines the impact of heterogeneous investment opportunities on the distribution of asset shares and wealth in an equilibrium model. We develop a new method for computing equilibria in this class of economies. This method relies on an optimal consumption sharing rule and an aggregation result for state prices that allows us to solve for equilibrium prices and allocations without having to search for market-clearing prices in each asset market. In a calibrated version of our model, we show that the heterogeneity in trading opportunities allows for a closer match of the wealth and asset share distribution as well as the moments of asset prices. We distinguish between "passive" traders who hold fixed portfolios of equity and bonds, and "active" traders who adjust their portfolios to changes in the investment opportunity set. In the presence of non-participants, the fraction of total wealth held by active traders is critical for asset prices, because only these traders respond to variation in state prices and hence help to clear the market, not the fraction of wealth held by all agents that participate in asset markets.

YiLi Chien
403 West State Street
Krannert School of Management
Purdue University
West Lafayette
IN 47907
yilichien@gmail.com

Hanno Lustig
Department of Economics
UCLA
Box 951477
Los Angeles, CA 90095-1477
and NBER
hlustig@econ.ucla.edu

Harold Cole
Economics Department
University of Pennsylvania
3718 Locust Walk
160 McNeil Building
Philadelphia, PA 19104
and NBER
colehl@sas.upenn.edu

# A Multiplier Approach to Understanding the Macro Implications of Household Finance *

YiLi Chien
Purdue University

Harold Cole
University of Pennsylvania

Hanno Lustig
UCLA and NBER

October 25, 2007

**Abstract**

Our paper examines the impact of heterogeneous investment opportunities on the distribution of asset shares and wealth in an equilibrium model. We develop a new method for computing equilibria in this class of economies. This method relies on an optimal consumption sharing rule and an aggregation result for state prices that allows us to solve for equilibrium prices and allocations without having to search for market-clearing prices in each asset market. In a calibrated version of our model, we show that the heterogeneity in trading opportunities allows for a closer match of the wealth and asset share distribution as well as the moments of asset prices. We distinguish between "passive" traders who hold fixed portfolios of equity and bonds, and "active" traders who adjust their portfolios to changes in the investment opportunity set. In the presence of non-participants, the fraction of total wealth held by active traders is critical for asset prices, because only these traders respond to variation in state prices and hence help to clear the market, not the fraction of wealth held by all agents that participate in asset markets.

Keywords: Asset Pricing, Risk Sharing, Limited Participation (JEL code G12)

## 1 Introduction

The correlation of household consumption and household income in the data presents a challenge for models with unlimited investment opportunities. This observation started the work on incomplete market models, which impose exogenous restrictions on trading opportunities. Recently, new

*We would like to thank Mark Hugget, Urban Jermann, Narayana Kocherlakota, Dirk Krueger, Joseph Ostroy, Nikolai Roussanov, Viktor Tsyrennikov and Amir Yaron for comments. Andrew Hollenhurst provided excellent research assistance.

evidence has emerged about the positive correlation of household wealth and household participation in asset markets, particularly in equity markets. Even among those households who participate in asset markets, there are substantial differences in their investment strategies and portfolio returns that are not easily explained by preference heterogeneity or differences in non-tradable risk exposure.[1] To explore this dimension of the household portfolio data, the canonical incomplete markets model needs to be augmented with heterogeneity in trading opportunities. We introduce heterogeneous trading technologies in an otherwise standard calibrated model, and we explore its quantitative implications.

Incomplete market economies with a large number of agents who trade in multiple assets are hard to analyze, even more so when different agents can trade different menus of assets. Our paper develops a new method for computing equilibria in a class incomplete market economies with heterogeneous investment opportunities. We then apply this method to solve a version of the model that is calibrated to match asset prices. The calibrated model's equilibrium distribution of wealth and asset holdings is closer to the data.

Our paper introduces heterogeneity in trading opportunities in an otherwise standard endowment economy with a large number of agents who are subject to both aggregate and idiosyncratic shocks, and who have constant relative risk aversion (CRRA) preferences with coefficient $\alpha$. We distinguish between four different trading technologies; each household has access to only one of these: (i) *complete* traders who trade a complete menu of assets, (ii) *z-complete* traders who trade claims whose payoffs are contingent on aggregate shocks (e.g. bonds of different maturities, equity etc.) but not idiosyncratic shocks, (iii) *diversified* investors who trade claim to diversifiable income, i.e. a fixed portfolio of bonds and stocks, and (iv) *non-participants* who only have access to a savings account. The last two only trade fixed portfolios of riskless and risky assets, but the first two do not.

Instead of directly imposing the trading restrictions on the recursive representation of the household's consumption and portfolio choice problem, we impose measurability restrictions on the household's time zero trading problem. These restrictions govern how net wealth is allowed to vary across different states of the world, similar to the measurability constraints in Aiyagari, Marcet, Sargent, and Seppala (2002) and Lustig, Sleet, and Yeltekin (2006). We use the multipliers on these constraints to derive a consumption sharing rule for households and an analytical expression for state prices. Importantly, the household's consumption sharing rule does not depend on the trading technology, only the dynamics of the multipliers do. State prices only depend on a weighted average of these multipliers –the $-1/\alpha$-th moment. We refer to this simply as the aggregate multiplier. It summarizes the aggregate shadow cost of the binding measurability and solvency constraints. This

[1] Campbell (2006) refers to the body of literature that documents this heterogeneity as "household finance". See Campbell (2006)'s AFA presidential address for a comprehensive discussion of these and other issues related to household finance.

extends the aggregation result by Lustig (2006), who considers a complete markets environment.

To implement our algorithm, we use a recursive net savings function as an accounting device. This function allows us to determine the individual's multiplier updating rule as a function of the updating rule for the aggregate multiplier and the restrictions implied by our asset structure. These two updating rules – the aggregate multiplier updating rule and the individual's multiplier updating rule – completely determine the equilibrium of our economy. Different trading technologies only change the individual and aggregate multiplier updating rules, but they do not change our aggregation result. In the computational section, we compute the individual household's multiplier rule, taking as given some initial aggregate multiplier updating rule. Next, we solve for a new aggregate multiplier updating rule by simulating a process for the aggregate multiplier given the conjectured rule. Finally, we iterate on the aggregate multiplier updating rule until convergence is achieved.

Quantitatively, our approach has several major advantages. First, our aggregation result implies that we only need to forecast a single moment of the multiplier distribution, regardless of the number and the nature of the different trading technologies. Also, our aggregation result allows us to directly compute the pricing kernel as a function of this moment. There is no need to search for the vector of prices that clears the various asset markets, as in the standard methods (Krusell and Smith (1997)). Searching for market-clearing prices is hard because, in general, we do not know the mapping from the wealth distribution to prices. In addition, the updating rule for the multipliers involves solving a simple system of equations.

A key distinction that emerges in our analysis is between "passive" traders who trade a fixed portfolio of safe and risky assets and "active" traders who adjust their portfolio in response to variation in the state prices. In doing so, active traders can reallocate consumption across aggregates states of natures. On the other hand, passive traders only respond to changes in average state prices that show up in the risk-free rate or the expected return on the market by reallocating consumption over time (i.e. by saving less or more). At the micro level, this distinction helps us to match the heterogeneity in portfolio composition and returns that was documented in the data, but this also affects outcomes at the macro level.

At the aggregate level, the non-participants create residual aggregate risk, because they consume "too much" in low aggregate consumption growth states and "too little" in high aggregate consumption growth states. In our economy, only the active traders bear the residual aggregate risk, not the diversified traders: The diversified trader's share of aggregate wealth cannot depend on the aggregate state of the economy, because they only trade a claim to diversifiable income. On the other hand, the active traders concentrate their consumption in "cheap" aggregate states (states with low state prices for aggregate consumption). Hence, to clear the goods market, state prices have to be much higher in recessions to induce a small segment of active traders to consume

less, and much lower in expansions. The non-participants and diversified traders are being forced to take the other side of these trades, consuming more in "expensive" aggregate states.

The presence of non-participants is critical. As long as all households can trade a claim to the market, regardless of the composition of the different trading groups, the risk premia are the same as in the representative agent economy, i.e. small and constant. This being the case, everyone bears the same amount of aggregate risk in equilibrium, the ability to reallocate consumption across different aggregate states of the world is redundant and the distinction between active and passive traders is moot: the aggregate multiplier adjustment to state prices is constant and there is no spread between the prices in different states. However, if we exclude some households from actively trading shares in total financial wealth or the market, this irrelevance result of Krueger and Lustig (2006), disappears and the distinction between active and passive traders starts to matter.[2] Non-participants matter for asset prices even though they do not accumulate much financial wealth; what matters is the size of their claim to labor income.

In the quantitative section of the paper, we show that the interaction between a small segment of active traders and a larger segment of passive traders improves the model's match with asset prices in the data along two dimensions. First, due to this interaction, equilibrium state prices are highly volatile and counter-cyclical but their conditional expectation –and hence the risk-free rate– is not. Passive traders consume too much in low aggregate consumption growth states (recessions) and too little in high aggregate consumption growth states (expansions). Since there is no predictability in aggregate consumption growth, changes in the risk-free rate do nothing to clear the market in each aggregate state tomorrow. Instead, changes in the average state price and hence the risk-free rate change the average consumption growth path of non-participants, a large fraction of the population, by the *same amount* in all aggregate states tomorrow, thus creating even more aggregate risk in the economy. Instead, the equilibrium state prices are highly volatile *across* aggregate states to induce the small segment of active traders to adjust their consumption growth in different aggregate states of the world by enough to clear the market. The active traders consume less in low aggregate consumption growth states when state prices are high and more in high growth states when state prices are low. Second, the share of total wealth owned by the active traders declines in low aggregate consumption growth states, because these take highly leveraged equity positions. As a result, the conditional volatility of state prices increases after each bad aggregate shock: a larger adjustment in state prices is needed to clear the goods markets. The model endogenously generates counter-cyclical Sharpe ratios, even though the aggregate consumption growth shocks are i.i.d. However, the model-implied correlation of returns and aggregate consumption growth is too large relative to the data.

[2]One of the key assumptions for this result is that aggregate shocks are i.i.d. and that the idiosyncratic shocks are independent of the aggregate shocks.

**Related Literature** In continuous-time finance, the Cox-Huang martingale approach has been applied to incomplete market environments, starting with Cuoco and He (1994) and Basak and Cuoco (1998a). These authors also rely on stochastic weighting schemes. Our approach differs because it provides a tractable and computationally efficient algorithm for computing equilibria in environments with a large number of agents subject to idiosyncratic risk and heterogeneity in trading opportunities. In that sense, this paper is more closely related to Krusell and Smith (1997) and (1998). KS developed a computational method that solves for approximate pricing functions that use the mean of the wealth distribution as the state variable. The KS method can approximate prices using only the mean of the wealth distribution because of approximate aggregation. In contrast to KS, we can express state prices as a function of the growth rates of aggregate consumption and a single moment of the multiplier distribution. The algorithm consists of a search for the optimal forecasting function for this single moment of the multiplier distribution rather than a search for a menu of pricing functions. Moreover, as we show in our example, our approach works even when approximate aggregation does not hold.

Standard incomplete market models cannot match the dispersion of the wealth distribution in the data. In the literature, preference heterogeneity (Krusell and Smith (1997)) or more recently concern for status Roussanov (2007), have been explored to generate more dispersion. Our paper focuses exclusively on heterogeneity in trading technologies; we show that this mechanism alone can generate the same skewness and kurtosis as in the data. However, the middle class in our model still accumulates too much wealth relative to the rich.

There is a growing literature on the asset pricing impact of limited stock market participation, starting with Saito (1996) and Basak and Cuoco (1998b). Our paper is the first to our knowledge to document the importance of distinguishing between active and passive traders for understanding asset prices and the wealth distribution. Other papers have focussed mostly on heterogeneity in *preferences* (e.g. see Krusell and Smith (1998) for heterogeneity in the rate of time preference and Vissing-Jorgensen (2002), Guvenen (2003) and Gomez and Michaelides (2007) for heterogeneity in the willingness of households to substitute intertemporally) and the heterogeneity in *participation decisions* (e.g. see Guvenen (2003) and Vissing-Jorgensen (2002)), rather than trading opportunities. There has been substantial progress on the empirical front in carefully documenting the heterogeneity of household investment decisions. In a comprehensive dataset of Swedish households, Calvet, Campbell, and Sodini (2006) find that sophisticated investors realize higher Sharpe ratios, but at the cost of incurring more volatility. Indeed, the active traders in our model realize much higher returns, but they adopt a sophisticated trading strategy that exploits the time variation in the risk premium to do so. Campbell (2006) argues that some households voluntarily limit the set of assets they decide to trade for fear of making mistakes, at the cost of forgoing higher returns. To capture this, we introduce "diversified investors", who simply trade a claim to the

market.

There is an active debate about the effects of limited participation on asset prices. Guvenen (2003) argues that limited participation goes a long way towards explaining the equity premium in a model with a bond- and a stockholder. In his model, investors do not face idiosyncratic risk. The model matches risk premia at the cost of too much volatility in the risk-free rate. In more recent work, Gomez and Michaelides (2007) also consider a model with bond-and stockholders, but they add idiosyncratic risk. Their model produces a large risk premium, which they attribute to imperfect risk sharing among stockholders, not to the exclusion of households from equity markets. In our benchmark model, we show analytically that market segmentation only affects the risk-free rate, but not risk premia, as long as there is no predictability in aggregate consumption growth and all traders can trade the market –a claim to all diversifiable income. We do not model the participation decision, but we show that the costs of non-participation are too large in a model with volatile state prices to be simply explained by standard cost arguments. Instead, one might have to appeal to differences in cognitive ability.[3] In our model, this seems plausible given the complexity of the trading strategies that fully realize the welfare gains of asset market participation.

This paper is organized as follows. Section 2 describes the environment, the preferences and trading technologies for all households. Section 3 characterizes the equilibrium allocations and prices using cumulative multipliers that record all the binding measurability and solvency constraints. Section 4 describes a recursive version of this problem that we can actually solve. This section also describes conditions under which market segmentation does not affect the risk premium. Finally, in section 5 we study a calibrated version of our economy. All the proofs are in the appendix. A separate appendix with auxiliary results is available from the authors' web sites[4].

# 2 Model

In this section we describe the environment, and we describe the household problem for each of different asset trading technologies. We also define an equilibrium for this economy.

## 2.1 Environment

This is an endowment economy with a unit measure of households who are subject to both aggregate and idiosyncratic income shocks. Households are ex ante identical, except for the access to trading technologies. Ex post, the households differ in terms of their idiosyncratic income shock realizations. Some of the households will be able to trade a complete set of securities, but others will trade a more limited set of securities. All of the households face the same stochastic process

[3] In the data, education is a strong predictor of equity ownership (see Table I in Campbell (2006)).

[4] `http://www.econ.ucla.edu/people/faculty/Lustig.html`

for idiosyncratic income shocks, and all households start with the same present value of tradeable wealth.

In the model time is discrete, infinite, and indexed by $t = 0, 1, 2, ...$ The first period, $t = 0$, is a planning period in which financial contracting takes place. We use $z_t \in Z$ to denote the aggregate shock in period $t$ and $\eta_t \in N$ to denote the idiosyncratic shock in period $t$. $z^t$ denotes the history of aggregate shocks, and, similarly, $\eta^t$, denotes the history of idiosyncratic shocks for a household. The idiosyncratic events $\eta$ are i.i.d. across households. We use $\pi(z^t, \eta^t)$ to denote the unconditional probability of state $(z^t, \eta^t)$ being realized. The events are first-order Markov, and we assume that

$$\pi(z^{t+1}, \eta^{t+1}|z^t, \eta^t) = \pi(z_{t+1}|z_t)\pi(\eta_{t+1}|z_{t+1}, \eta_t).$$

Since we can appeal to a law of large number, $\pi(z^t, \eta^t)/\pi(z^t)$ also denotes the fraction of agents in state $z^t$ that have drawn a history $\eta^t$. We use $\pi(\eta^t|z^t)$ to denote that fraction. We introduce some additional notation: $z^{t+1} \succ z^t$ or $y^{t+1} \succ y^t$ means that the left hand side node is a successor node to the right hand side node. We denote by $\{z^\tau \succ z^t\}$ the set of successor aggregate histories for $z^t$ including those many periods in the future; ditto for $\{\eta^\tau \succ \eta^t\}$. When we use $\succeq$, we include the current nodes $z^t$ or $\eta^t$ in the summation.

There is a single final good in each period, and the amount of it is given by $Y(z^t)$, which evolves according to

$$Y(z^t) = \exp\{z_t\}Y(z^{t-1}), \tag{2.1}$$

with $Y(z^1) = \exp\{z_1\}$. This endowment good comes in two forms. The first form is diversifiable income, which is not subject to the idiosyncratic shock, and is given by $(1-\gamma)Y(z^t)$. The other form is non-diversifiable income which is subject to idiosyncratic risk and is given by $\gamma Y(z^t)\eta_t$; hence $\gamma$ is the share of income that is non-diversifiable.

All households are infinitely lived and rank stochastic consumption streams $\{c(z^t, \eta^t)\}$ according to the following criterion

$$U(c) = E\left\{\sum_{t\geq 1}^{\infty} \beta^t \pi(z^t, \eta^t)\frac{c(z^t, \eta^t)^{1-\alpha}}{1-\alpha}\right\}, \tag{2.2}$$

where $\alpha > 0$ denotes the coefficient of relative risk aversion, and $c(z^t, \eta^t)$ denotes the household's consumption in state $(z^t, \eta^t)$.

## 2.2 Asset Trading Technologies

All of the households have access to *only* one of four asset trading technologies. We assume households cannot switch between technologies. It is straightforward to extend the methodology

we develop to allow for exogenous transitions between trading technologies. The probability of these transitions could even be contingent on the household's realized shocks. The first technology we consider gives households access to a complete menu of assets.

Households trade assets in securities markets and they trade the final good in spot markets that re-open in every period. A fraction $\mu_1$ of households can trade claims that are contingent on both their aggregate and their idiosyncratic state $(z^t, \eta^t)$, a fraction $\mu_2$ can trade claims contingent on the aggregate state $z^t$, a fraction $\mu_3$ can only trade claims to a share of diversifiable income, and a fraction $\mu_4$ can only trade non-contingent contracts to deliver units of the final good in the next time the spot market reopens.

We refer to the first set of households as the complete traders since they are able to trade a complete set of Arrow securities. We refer to the second set as the $z$-complete traders since they can only offset aggregate risk but not idiosyncratic risk through their asset trading. We refer to the third set of households as the diversified investors since they are trading a claim to total financial wealth or equivalently a claim to all diversifiable income. We will refer to the fourth set of households as non-participants, since they only have a savings account. All traders face exogenous debt constraints.

Since the return on the diversifiable income claim is measurable with respect to the asset trading structures of the complete and $z$-complete traders, we assume w.l.o.g. that the households in the first two partitions can also trade the claim to diversifiable income.

$\varpi(z^t)$ denotes the price of a claim to diversifiable income in aggregate state $z^t$. In each node, total diversifiable income is given by $(1-\gamma)Y(z^t)$. We use $q\left[(z^{t+1}, \eta^{t+1}), (z^t, \eta^t)\right]$ to denote the price of a unit claim to the final good in state $(z^{t+1}, \eta^{t+1})$ acquired in state $(z^t, \eta^t)$. The absence of arbitrage implies that there exist aggregate state prices $q(z_{t+1}, z^t)$ such that

$$q\left[\left(z^{t+1}, \eta^{t+1}\right), \left(z^t, \eta^t\right)\right] = \pi(\eta^{t+1}|z^{t+1}, \eta^t) q(z_{t+1}, z^t),$$

where $q(z_{t+1}, z^t)$ denotes the price of a unit of the final good in aggregate state $z^{t+1}$ given that we are in aggregate history $z^t$. From these, we can back out the present-value state prices recursively as follows:

$$\pi(z^t, \eta^t) P(z^t, \eta^t) = q(z_t, z^{t-1}) q(z_{t-1}, z^{t-2}) \cdots q(z_1, z^0) q(z_0).$$

We use $\widetilde{P}(z^t, \eta^t)$ to denote the Arrow-Debreu prices $P(z^t)\pi(z^t, \eta^t)$. Let $m(z^{t+1}|z^t) = P(z^{t+1})/P(z^t)$ denote the stochastic discount factor that prices any random payoffs. We assume there is always a non-zero measure of z-complete or complete traders to guarantee the uniqueness of the stochastic discount factor.

All households are endowed with a claim to their per capita share of both diversifiable and non-diversifiable income. Households cannot directly trade their claim to non-diversifiable risk,

though households can hedge this risk to the extent that they can trade a sufficiently rich menu of securities. For example, the complete households can hedge both their idiosyncratic and their aggregate risk. We assume that the non-participants cannot hold the claim to equity. During the initial trading period, they sell their claim to diversifiable income in exchange for non-contingent discount bonds since claim implicity includes a claim to equity.

Finally, the households face exogenous limits on their net asset positions. The value of the household's net assets must always be greater than $-\psi$ times the value of their non-diversifiable income, where $\psi \in (0, 1]$. We allow households to trade away or borrow up to 100% of the value of their claims to diversifiable capital.

**Complete Traders** We start with the household in the first asset partition who can trade both a complete set of contingent bonds as well as claims to diversifiable income. The budget constraint for this trader in the spot market in state $(z^t, \eta^t)$ as

$$\begin{aligned} & \gamma Y(z^t)\eta_t + a_{t-1}(z^t, \eta^t) + \sigma(z^{t-1}, \eta^{t-1}) \left[(1-\gamma)Y(z^t) + \varpi(z^t)\right] - c(z^t, \eta^t) \\ \geq & \sum_{z^{t+1} \succ z^t} q(z_{t+1}, z^t) \sum_{\eta^{t+1} \succ \eta^t} a(z^{t+1}, \eta^{t+1}) \pi(\eta^{t+1} | z^{t+1}, \eta^t) + \sigma(z^t, \eta^t)\varpi(z^t) \ \forall (z^t, \eta^t), \end{aligned} \tag{2.3}$$

where $a_{t-1}(z^t, \eta^t)$ denotes the number of unit claims to the final good purchased at $t-1$ for state $(z^t, \eta^t)$, $\sigma(z^{t-1}, \eta^{t-1})$ denotes the number of claims on diversifiable income acquired in state $(z^{t-1}, \eta^{t-1})$, where $(z^t, \eta^t) \succ (z^{t-1}, \eta^{t-1})$. The period 0 spot budget constraint is given by

$$\varpi(z^0)\left[1 - \sigma(z_0, \eta_0)\right] \geq \sum_{z_1} q(z_1, z^0) \sum_{\eta_1} a_0(z^1, \eta^1) \pi(\eta^{t+1} | z^{t+1}, \eta^t), \tag{2.4}$$

where $z^0$ and $\eta^0$ are degenerate states representing the initial position in the planning state at time 0 before any of the shocks have been realized, and where $\varpi(z^0)$ denotes the price of capital in the planning stage and $q(z_1, z^0)$ denotes the price in this stage of a claim to consumption in period 1. In addition to their spot budget constraint, these traders also face a lower bound on the value of their net asset position. Let $\underline{M}(\eta^t, z^t)$ be defined as

$$\underline{M}(\eta^t, z^t) = -\psi \sum_{\tau \geq t} \sum_{\{z^\tau \succeq z^t, \eta^\tau \succeq \eta^t\}} \gamma Y(z^\tau)\eta_\tau \frac{\pi(z^\tau, \eta^\tau) P(z^\tau, \eta^\tau)}{\pi(z^t, \eta^t) P(z^t, \eta^t)} \tag{2.5}$$

The lower bound is given by:

$$a_t(z^{t+1}, \eta^{t+1}) + \sigma(z^t, \eta^t)\left[d(z^{t+1}) + \varpi(z^{t+1})\right] \geq \underline{M}(\eta^{t+1}, z^{t+1}). \tag{2.6}$$

The complete trader's problem is to choose $\{c(z^t, \eta^t), a_t(z^{t+1}, \eta^{t+1}), \sigma(z^t, \eta^t)\}$, $a_0(z^1, \eta^1)$ and $\sigma(z^0, \eta^0)$

so as to maximize (2.2) subject (2.3-2.6).

**z-complete Traders** The households in the second asset partition have a budget constraint in the spot market in state $(z^t, \eta^t)$ given by

$$\begin{aligned} & \gamma Y(z^t)\eta_t + a_{t-1}(z^t, \eta^{t-1}) + \sigma(z^{t-1}, \eta^{t-1}) \left[(1-\gamma)Y(z^t) + \varpi(z^t)\right] - c(z^t, \eta^t) \\ \geq & \sum_{z^{t+1} \succ z^t} q(z_{t+1}, z^t) a_t(z^{t+1}, \eta^t) + \sigma(z^t, \eta^t)\varpi(z^t) \ \forall (z^t, \eta^t), \end{aligned} \tag{2.7}$$

where $a_t(z^{t+1}, \eta^t)$ denotes the number of claims acquired in period $t$ that payoff one unit if the aggregate state tomorrow is $z^{t+1}$, and where $\eta^t \succ \eta^{t-1}$. The period 0 spot budget constraint is given by

$$\varpi(z^0)\left[1 - \sigma(z_0, \eta_0)\right] \geq \sum_{z_1} q(z_1, z^0) a_0(z^1, \eta^0). \tag{2.8}$$

The $z$-complete traders face bounds on their net asset position which is given by:

$$a_t(z^{t+1}, \eta^t) + \sigma(z^t, \eta^t)\left[d(z^{t+1}) + \varpi(z^{t+1})\right] \geq \underline{M}(\eta^{t+1}, z^{t+1}) \tag{2.9}$$

for each $(z^{t+1}, \eta^{t+1}) \succ (z^t, \eta^t)$. Note here that for each aggregate state tomorrow, $z^{t+1}$, the magnitude of the bound is determined by the idiosyncratic state $\eta^{t+1}$ in which the present value of non-diversifiable income is smallest.

The $z$-complete trader's problem is to choose $\{c(z^t, \eta^t), a(z^{t+1}, \eta^t), \sigma(z^t, \eta^t)\}$, $a(z^1, \eta^0)$ and $\sigma(z^0, \eta^0)$ so as to maximize (2.2) subject (2.7-2.9).

**Diversified investors** We think of diversified investors as trading a claim to all of the diversifiable income. The diversified traders effectively hold a *fixed portfolio of equity and bonds*. Following Abel (1999), we define equity as a leveraged claim to consumption. Let $\phi$ denote the leverage parameter, let $b_t(z^t)$ denote the supply of one-period risk-free bonds, and let $R_t^f$ denote the risk-free rate. We can decompose the aggregate payout that flows from the diversifiable income claim $(1-\gamma)Y(z^t)$ into a dividend component $d_t(z^t)$ from *equity* and a bond component $R_t^f(z^{t-1})b(z^{t-1}) - b(z^t)$. The bond supply adjusts in each node $z^t$ to ensure that the bond/equity ratio equals $\phi$:

$$b(z^t) \ = \ \phi\left[\varpi(z^t) - b(z^t)\right]$$

for all $z^t$. The diversified trader invests a fraction $\phi/(1+\phi)$ in bonds and the remainder in equity. This is a natural benchmark, because we show this portfolio is the optimal one (and it is constant) in the case without non-participants. Hence, the diversified traders are rational only if their model

of the world is the one without non-participants, and hence without time-varying risk premia.

These households in the third asset partition have a budget constraint in the spot market in state $(z^t, \eta^t)$ given by

$$\gamma Y(z^t)\eta_t + \sigma(z^{t-1}, \eta^{t-1}) \left[(1-\gamma)Y(z^t) + \varpi(z^t)\right] - c(z^t, \eta^t) \geq \sigma(z^t, \eta^t)\varpi(z^t) \ \forall (z^t, \eta^t), \quad (2.10)$$

a degenerate period 0 constraint

$$\varpi(z^0)\left[1 - \sigma(z_0, \eta_0)\right] \geq 0, \quad (2.11)$$

and a net asset position bound

$$\sigma(z^t, \eta^t) \left[(1-\gamma)Y(z^{t+1}) + \varpi(z^{t+1})\right] \geq \underline{M}(\eta^{t+1}, z^{t+1}), \quad (2.12)$$

for each $(z^{t+1}, \eta^{t+1}) \succ (z^t, \eta^t)$. The diversified trader's problem is to choose $\{c(z^t, \eta^t), \sigma(z^t, \eta^t)\}$ and $\sigma(z^0, \eta^0)$ so as to maximize (2.2) subject (2.10-2.12).

**Non-participants** The households in the fourth and final partition have a spot budget constraint in state $(z^t, \eta^t)$ given by

$$\gamma Y(z^t)\eta_t + a_{t-1}(z^{t-1}, \eta^{t-1}) - c(z^t, \eta^t) \geq \sum_{z^{t+1} \succ z^t} q(z_{t+1}, z^t) a_t(z^t, \eta^t), \quad (2.13)$$

where $z^t \succ z^{t-1}$ and $\eta^t \succ \eta^{t-1}$, for states other than the first, and a first period budget constraint given by

$$\varpi(z^0) \geq a_0(z^0, \eta^0) \sum_{z_1} q(z_1, z^0)\pi(\eta^1 | z^1, \eta^0), \quad (2.14)$$

because they cannot hold the claim to diversified wealth. The asset bound for non-participants is given by

$$a_t(z^t, \eta^t) \geq \underline{M}(\eta^{t+1}, z^{t+1}) \quad (2.15)$$

for each $(z^{t+1}, \eta^{t+1}) \succ (z^t, \eta^t)$.The non-participant's problem is to choose $\{c(z^t, \eta^t), a_t(z^t, \eta^t)\}$ and $a_0(z^0, \eta^0)$ so as to maximize (2.2) subject to (2.13-2.15).

## 2.3 Equilibrium

For the sake of clarity, we use (e.g.) $\eta^{t-1}(\eta^t)$ to denote the history from zero to $t-1$ contained in $\eta^t$. We use the same convention for the aggregate histories. Using this notation, the market

clearing condition in the bond market is given by:

$$\sum_{\eta^t} \left[\mu_1 a^c_{t-1}(z^t, \eta^t) + \mu_2 a^z_{t-1}(z^t, \eta^{t-1}(\eta^t)) + \mu_4 a^{np}_{t-1}(z^{t-1}(z^t), \eta^{t-1}(\eta^t))\right] \pi(\eta^t|z^t) = 0,$$

where $a^c$, $a^z$, $a^{div}$, and $a^{np}$ denote the bond holdings of the complete-markets, $z$-complete, equity-only, and bonds-only traders respectively. The market clearing condition in the output claim market is given by

$$\sum_{\eta^t} \left[\mu_1 \sigma^c(z^t, \eta^t) + \mu_2 \sigma^z(z^t, \eta^t) + \mu_3 \sigma^{div}(z^t, \eta^t)\right] \pi(\eta^t|z^t) = 1.$$

An equilibrium for this economy is defined in the standard way. It consists of a list of bond and output claim holdings, a consumption allocation and a list of bond and tradeable output claim prices such that: (i) given these prices, a trader's asset and consumption choices maximizer her expected utility subject to the budget constraints, the solvency constraints and the measurability constraints, and (ii) the asset markets clear.

The next section analytically characterizes the household consumption function and the equilibrium pricing kernel in terms of the distribution of the household's stochastic multipliers.

# 3 Solving for Equilibrium Allocations and Prices

This section reformulates the household's problem in terms of a present-value budget constraint, and sequences of measurability constraints and solvency constraints. These measurability constraints capture the restrictions imposed by the different trading technologies of households. We show how to use the cumulative multipliers on these constraints as stochastic weights that fully characterize equilibrium allocations and prices.

## 3.1 Measurability Conditions

We begin by recursively substituting into the spot budget constraints, in order to derive an expression in terms of future consumption sequences and the initial asset position in state $(z^t, \eta^t)$.

**Complete Traders** For example, start from the complete traders constraint (2.3), and assume it holds with equality. Then we can substitute for future $a(z^{t+i}, \eta^{t+i})$, while using the equity no-arbitrage condition

$$\varpi(z^t) = \sum_{z_{t+1}} \left[d(z^{t+1}) + \varpi(z^{t+1})\right] q(z_{t+1}, z^t),$$

to obtain the following budget constraint in terms of present value prices:

$$a_{t-1}(z^t,\eta^t)+\sigma(z^{t-1},\eta^{t-1})\left[(1-\gamma)Y(z^t)+\varpi(z^t)\right]=\sum_{\{z^\tau\succeq z^t,\eta^\tau\succeq\eta^t\}}\left[c(z^\tau,\eta^\tau)-\gamma Y(z^\tau)\eta_\tau\right]\frac{\pi(z^\tau,\eta^\tau)P(z^\tau,\eta^\tau)}{\pi(z^t,\eta^t)P(z^t,\eta^t)}.$$

Rather than carry around both $a$ and $\sigma$, we will find it convenient to define net wealth as

$$\hat{a}_{t-1}(z^t,\eta^t)\equiv a_{t-1}(z^t,\eta^t)+\sigma(z^{t-1},\eta^{t-1})\left[(1-\gamma)Y(z^t)+\varpi(z^t)\right].$$

The borrowing constraint in terms of $\hat{a}$ is given by

$$\hat{a}_{t-1}(z^t,\eta^t)\geq\underline{M}(\eta^t,z^t). \tag{3.1}$$

Requiring that condition (3.1) hold for each $(z^t,\eta^t)$ is equivalent to imposing the spot budget constraints (2.3) and borrowing constraints (2.6) for the complete traders for all $t\geq 1$. In addition we have the period 0 budget constraint:

$$\varpi(z^0)=\sum_{t>0}\sum_{(z^t,\eta^t)}\left[c(z^t,\eta^t)-\gamma Y(z^t)\eta_t\right]\pi(z^t,\eta^t)P(z^t,\eta^t). \tag{3.2}$$

It is straightforward to show that the spot budget and debt bound constraints for the other types of traders imply that condition (3.1) hold for each $(z^t,\eta^t)$ and that condition (3.2) holds.

However, the limits on the menu of traded assets also imply additional measurability constraints which reflect the extent to which their net asset position can vary with the realized state $(z^t,\eta^t)$.

**z-complete Traders** The $z$-complete traders face the additional constraint that $a_{t-1}(z^t,\eta^t)$ is measurable with respect to $(z^t,\eta^{t-1})$. Since the payoff of the stock $\sigma(z^{t-1},\eta^{t-1})\left[(1-\gamma)Y(z^t)+\varpi(z^t)\right]$ is measurable with respect to$(z^t,\eta^{t-1})$, requiring that $a_{t-1}(z^t,\eta^t)=a_{t-1}(z^t,\tilde{\eta}^t)$ for all $z^t$, and $\tilde{\eta}^t,\eta^t$ such that $\eta^{t-1}(\tilde{\eta}^t)=\eta^{t-1}(\eta^t)$ is equivalent to requiring that

$$\hat{a}_{t-1}(z^t,\left[\eta^{t-1},\eta_t\right])=\hat{a}_{t-1}(z^t,\left[\eta^{t-1},\tilde{\eta}_t\right]), \tag{3.3}$$

for all $z^t$, $\eta^{t-1}$, and $\eta_t$, $\tilde{\eta}_t\in N$.

**Diversified investors** For the diversified investors, $a_{t-1}(z^t,\eta^t)=0$ and hence the present value of net borrowing in (3.1) is equal to $\sigma(z^{t-1},\eta^{t-1})\left[(1-\gamma)Y(z^t)+\varpi(z^t)\right]$. Thus their additional measurability constraints take the form:

$$\frac{\hat{a}_{t-1}([z^{t-1},z_t],[\eta^{t-1},\eta_t])}{(1-\gamma)Y(z^{t-1},z_t)+\varpi(z^{t-1},z_t)}=\frac{\hat{a}_{t-1}([z^{t-1},\tilde{z}_t],[\eta^{t-1},\tilde{\eta}_t])}{(1-\gamma)Y(z^{t-1},\tilde{z}_t)+\varpi(z^{t-1},\tilde{z}_t)}, \tag{3.4}$$

for all $z^{t-1}$, $\eta^{t-1}$, $z_t$, $\tilde{z}_t \in Z$, and $\eta_t$, $\tilde{\eta}_t \in N$.

**Non-participants** For the non-participants, the payoff in state $(z^t, \eta^t)$ is supposed to be measurable with respect to $(z^{t-1}, \eta^{t-1})$, and hence their additional measurability constraints take the form:

$$\hat{a}_{t-1}(\left[z^{t-1}, z_t\right], \left[\eta^{t-1}, \eta_t\right]) = \hat{a}_{t-1}(\left[z^{t-1}, \tilde{z}_t\right], \left[\eta^{t-1}, \tilde{\eta}_t\right]), \tag{3.5}$$

for all $z^{t-1}$, $\eta^{t-1}$, $z_t$, $\tilde{z}_t \in Z$, and $\eta_t$, $\tilde{\eta}_t \in N$.

**Summary** Let $R^{port}(z^t)$ denote the return on the passive trader's total portfolio. In general, for "passive" traders, we can state the measurability condition as:

$$\frac{\hat{a}_{t-1}([z^{t-1}, z_t], [\eta^{t-1}, \eta_t])}{R^{port}(z^{t-1}, z_t)} = \frac{\hat{a}_{t-1}([z^{t-1}, \tilde{z}_t], [\eta^{t-1}, \tilde{\eta}_t])}{R^{port}(z^{t-1}, \tilde{z}_t)}, \tag{3.6}$$

for all $z^{t-1}$, $\eta^{t-1}$, $z_t$, $\tilde{z}_t \in Z$, and $\eta_t$, $\tilde{\eta}_t \in N$. For the non-participant, $R^{port}(z^t) = R^f(z^{t-1})$ is the risk-free rate, for the diversified trader, $R^{port}(z^t) = R(z^t)$ is the return on the market –the diversifiable income claim. Of course, a similar condition holds for any investor with fixed portfolios in the riskless and risky assets.

Given these results, we can restate the household's problem as one of choosing an entire consumption plan from a restricted budget set. To formally show the equivalence between the time zero trading equilibrium and the sequential trading equilibrium, we need to assume that interest rates are high enough.

**Condition 1.** *Interest rates are said to be high enough iff*

$$\sum_{t>0} \sum_{(z^t, \eta^t)} \left[Y(z^t)\eta_{max}\right] \pi(z^t, \eta^t) P(z^t, \eta^t) << \infty$$

If condition (1) is satisfied, we can appeal to proposition (4.6) in Alvarez and Jermann (2000) which establishes the equivalence of the time zero trading and the sequential trading equilibrium.[5]

Next, we turn to examining a household's problem given this reformulation. Because the complete traders do not face any measurability constraints, we start with the $z$-complete trader's problem. The central result is a martingale condition for the stochastic multipliers. We also discuss the same problem for the other traders, and we derive an aggregation result. Finally, we conclude this section by providing an overview.

[5]Our environment is somewhat different, because (i) we add measurability constraints and (ii) we have a large number of agents. (ii) is why we require that a claim to the maximum labor income realizations (rather than a claim to the aggregate endowment) is finitely valued.

## 3.2 Martingale Conditions

To derive the martingale conditions that govern household consumption, we consider the household problem in a time zero trading setup. Markets open only once at time zero. The household chooses a consumption plan and a net wealth plan subject to a single budget constraint at time zero, as well as an infinite number of solvency constraints and measurability constraints. These measurability constraints act as direct restrictions on the household budget set. We start off by considering the active traders.

### 3.2.1 Active Traders

Let $\chi$ denote the multiplier on the present-value budget constraint, let $\nu(z^t, \eta^t)$ denote the multiplier on the measurability constraint in node $(z^t, \eta^t)$, and, finally, let $\varphi(z^t, \eta^t)$ denote the multiplier on the debt constraint. The saddle point problem of a **z-complete trader** can be stated as:

$$\begin{aligned} L = \quad & \min_{\{\chi,\nu,\varphi\}} \max_{\{c,\hat{a}\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} u(c(z^t,\eta^t))\pi(z^t,\eta^t) \\ & + \chi \left\{ \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \widetilde{P}(z^t,\eta^t) \left[\gamma Y(z^t)\eta_t - c(z^t,\eta^t)\right] + \varpi(z^0) \right\} \\ & + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \nu(z^t,\eta^t) \left\{ \sum_{\tau\geq t} \sum_{(z^\tau,\eta^\tau)\succeq(z^t,\eta^t)} \widetilde{P}(z^\tau,\eta^\tau) \left[\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau)\right] + \widetilde{P}(z^t,\eta^t)\hat{a}_{t-1}(z^t,\eta^{t-1}) \right\} \\ & + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \varphi(z^t,\eta^t) \left\{ -\underline{M}_t(z^t,\eta^t)\widetilde{P}(z^t,\eta^t) - \sum_{\tau\geq t} \sum_{(z^\tau,\eta^\tau)\succeq(z^t,\eta^t)} \widetilde{P}(z^\tau,\eta^\tau) \left[\gamma Y(z^\tau)\eta_\tau - c(z^\tau,\eta^\tau)\right] \right\}, \end{aligned}$$

where $\widetilde{P}(z^t, \eta^t) = \pi(z^t, \eta^t)P(z^t, \eta^t)$. Following Marcet and Marimon (1999), we can construct new weights for this Lagrangian as follows. First, we define the initial cumulative multiplier to be equal to the multiplier on the budget constraint: $\zeta_0 = \chi$. Second, the multiplier evolves over time as follows for all $t \geq 1$:

$$\zeta(z^t, \eta^t) = \zeta(z^{t-1}, \eta^{t-1}) + \nu\left(z^t, \eta^t\right) - \varphi(z^t, \eta^t). \tag{3.7}$$

Substituting for these cumulative multipliers in the Lagrangian, we recover the following expression for the constraints component of the Lagrangian:

$$\begin{aligned} & + \sum_{t\geq 1} \sum_{z^t,\eta^t} \widetilde{P}(z^t,\eta^t) \left\{ \zeta(z^t,\eta^t) \left(\gamma\eta^t Y(z^t) - c(z^t,\eta^t)\right) + \nu\left(z^t,\eta^t\right) \hat{a}_{t-1}(z^t,\eta^{t-1}) - \varphi(z^t,\eta^t)\underline{M}(z^t,\eta^t) \right\} \\ & + \gamma\varpi(z^0). \end{aligned}$$

This is a standard convex programming problem –the constraint set is still convex, even with the measurability conditions and the solvency constraints. The first order conditions are necessary and sufficient.

The first-order condition with respect to consumption is given by:

$$\beta^t u'(c(z^t,\eta^t))\pi(z^t,\eta^t) - \left[\zeta(z^{t-1},\eta^{t-1}) + \nu\left(z^t,\eta^t\right) - \varphi\left(z^t,\eta^t\right)\right] P(z^t)\pi(z^t,\eta^t) = 0. \tag{3.8}$$

The first order condition for consumption in (3.8) implies that the cumulative multiplier measures the household's discounted marginal utility relative to the state price $P(z^t)$:

$$\frac{\beta^t u'(c(z^t,\eta^t))}{P(z^t)} = \zeta(z^t,\eta^t). \tag{3.9}$$

This condition is common to all of our traders irrespective of their trading technology because differences in their trading technology does not effect the way in which $c(z^t,\eta^t)$ enters the objective function or the constraint. This implies that the marginal utility of households is proportional to their cumulative multiplier, regardless of their trading technology.

The first order condition with respect to net wealth $\widehat{a}_t(z^{t+1},\eta^t)$ is given by:

$$\sum_{\eta^{t+1}\succ\eta^t} \nu\left(z^{t+1},\eta^{t+1}\right)\pi(z^{t+1},\eta^{t+1})P(z^{t+1}) = 0. \tag{3.10}$$

We refer to this as the martingale condition. This condition is specific to the trading technology. For the z-complete trader, it implies that the average measurability multiplier across idiosyncratic states $\eta^{t+1}$ is zero since $P(z^{t+1})$ is independent of $\eta^{t+1}$. In each aggregate node $z^{t+1}$, the household's marginal utility innovations not driven by the solvency constraints $\nu_{t+1}$ have to be white noise. The trader has high marginal utility growth in low $\eta$ states and low marginal utility growth in high $\eta$ states, but these innovations to marginal utility growth average out to zero in each node $(z^t, z_{t+1})$. If the solvency constraints do bind, then the cumulative multipliers decrease on average for any given $z$-complete trader:

$$E\{\zeta(z^{t+1},\eta^{t+1})|z^{t+1}\} \leq \zeta(z^t,\eta^t),$$

which we obtained by substituting (3.7) into the first-order condition (3.10). Hence our recursive multipliers are a bounded super-martingale, and we have the following lemma.

**Lemma 3.1.** *The z-complete trader's cumulative multiplier is a super-martingale:*

$$\zeta(z^t,\eta^t) \geq \sum_{\eta^{t+1}\succ\eta^t} \zeta(z^{t+1},\eta^{t+1})\pi(\eta^{t+1}|z^{t+1},\eta^t). \tag{3.11}$$

*The cumulative multiplier is a martingale if the solvency constraints do not bind for any $\eta^{t+1} \succ \eta^t$ given $z^{t+1}$.*

For the **complete traders**, there is no measurability constraint, and hence the constraints portion of the recursive Lagrangian is given simply by:

$$+\sum_{t\geq 1}\sum_{z^t,\eta^t}\widetilde{P}(z^t,\eta^t)\left\{\zeta(z^t,\eta^t)\left(\gamma\eta_t Y(z^t)-c(z^t,\eta^t)\right)+\nu\left(z^t,\eta^t\right)\hat{a}_{t-1}(z^t,\eta^t)-\varphi(z^t,\eta^t)\underline{M}(z^t,\eta^t)\right\}$$
$$+\gamma\varpi(z^0).$$

The first order condition with respect to $\widehat{a}_t(z^{t+1},\eta^{t+1})$ is given by:

$$\nu\left(z^{t+1},\eta^{t+1}\right)\pi(z^{t+1},\eta^{t+1})P(z^{t+1})=0, \tag{3.12}$$

which implies that $\nu\left(z^{t+1},\eta^{t+1}\right)$ is equal to zero for all $z^{t+1},\eta^{t+1}$. All of the other conditions, including the first-order condition with respect to consumption (3.8) and the recursive multiplier condition (3.7) are unchanged. This leads to the following recursive formulation of the cumulative multipliers:

$$\zeta(z^t,\eta^t)=\zeta(z^{t-1},\eta^{t-1})-\varphi(z^t,\eta^t),$$

The multipliers decrease if the solvency constraint binds in node $(z^t,\eta^t)$; if not, they remain unchanged. The history of a complete household $\eta^t$ only affects today's consumption and asset accumulation, as summarized in $\zeta$, through the binding solvency constraints. As a result, when state prices are high, the consumption share of the complete trader decreases if the solvency constraint does not bind, not only on average, across $\eta'$ states, but state-by-state.

The common characteristic for all active traders is that their marginal utility innovations are orthogonal to any aggregate variables, because we know that $E[\nu_{t+1}|z^{t+1}]=0$ in each node $z^{t+1}$. Below, we explore the implications of this finding, but first, we show that diversified traders and non-participants satisfy the same martingale condition, but with respect to a different measure.

The next section derives the martingale condition for the passive traders.

### 3.2.2 Passive Traders

We start by looking at the diversified traders. For the **diversified** investors, the constraints portion of the Lagrangian looks somewhat different:

$$+\sum_{t\geq 1}\sum_{z^t,\eta^t}\widetilde{P}(z^t,\eta^t)\left[\begin{array}{c}\zeta(z^t,\eta^t)\left(\gamma\eta^t Y(z^t)-c(z^t,\eta^t)\right)+\nu\left(z^t,\eta^t\right)\sigma(z^{t-1},\eta^{t-1})\\ \left[(1-\gamma)Y(z^t)+\varpi(z^t)\right]-\varphi(z^t,\eta^t)\underline{M}(z^t,\eta^t)\end{array}\right]+\gamma\varpi(z^0).$$

The other components of the Lagrangian are unchanged. The first order condition with respect to $\sigma(z^t, \eta^t)$ is given by:

$$\sum_{z^{t+1} \succ z^t, \eta^{t+1} \succ \eta^t} \nu\left(z^{t+1}, \eta^{t+1}\right) \left[(1-\gamma) Y(z^{t+1}) + \varpi(z^{t+1})\right] \pi(z^{t+1}, \eta^{t+1}) P(z^{t+1}) = 0. \tag{3.13}$$

The other conditions are identical. Using the recursive definition of the multipliers, the first order condition in (3.13) can be stated as:

$$\zeta(z^t, \eta^t) \geq \sum_{z^{t+1} \succ z^t, \eta^{t+1} \succ \eta^t} \zeta(z^{t+1}, \eta^{t+1}) \widetilde{\pi}(z^{t+1}, \eta^{t+1} | z^t, \eta^t), \tag{3.14}$$

where $R(z^{t+1})$ is the return on the tradeable income claim and the twisted probabilities are defined as:

$$\widetilde{\pi}(z^{t+1}, \eta^{t+1} | z^t, \eta^t) = \frac{m(z^{t+1}|z^t) R(z^{t+1})}{E\left\{m(z^{t+1}|z^t) R(z^{t+1}) | z^t\right\}} \pi(z^{t+1}, \eta^{t+1} | z^t, \eta^t),$$

So, the diversified traders' multipliers satisfy the martingale condition with respect to the these "risk-neutral" probabilities, whenever the borrowing constraints do not bind. Moreover, when ever the debt constraints do bind, their multipliers are pushed downwards in order to satisfy the constraint. So, relative to these twisted probabilities, the equity traders multipliers are a super-martingale.

When $z$ and $\eta$ are independent, only the aggregate transition probabilities are twisted:

$$\widetilde{\pi}(z^{t+1}, \eta^{t+1} | z^t, \eta^t) = \widetilde{\phi}(z^{t+1} | z^t) \varphi(\eta^{t+1} | \eta^t) \tag{3.15}$$

The same is true of the non-participant's multipliers, however the twisting factor is different.

**Non-participants** Finally, for the non-participants, the constraints portion of the recursive Lagrangian is given by

$$+\sum_{t \geq 1} \sum_{z^t, \eta^t} \widetilde{P}(z^t, \eta^t) \left\{\zeta(z^t, \eta^t) \left(\gamma \eta^t Y(z^t) - c(z^t, \eta^t)\right) - \nu\left(z^t, \eta^t\right) \hat{a}_{t-1}(z^{t-1}, \eta^{t-1}) - \varphi(z^t, \eta^t) \underline{M}(z^t, \eta^t)\right\}$$
$$+\gamma \varpi(z^0).$$

The first order condition with respect to $\widehat{a}_t(z^{t+1}, \eta^{t+1})$ is given by:

$$\sum_{z^{t+1} \succ z^t, \eta^{t+1} \succ \eta^t} \nu\left(z^{t+1}, \eta^{t+1}\right) \pi(z^{t+1}, \eta^{t+1}) P(z^{t+1}) = 0. \tag{3.16}$$

This implies that non-participants' multipliers have the super-martingale property:

$$\zeta(z^t,\eta^t)E\left\{m(z^{t+1}|z^t)|z^t\right\} \geq \sum_{z^{t+1}\succ z^t,\eta^{t+1}\succ\eta^t} \zeta(z^{t+1},\eta^{t+1})\widetilde{\pi}(z^{t+1},\eta^{t+1}|z^t,\eta^t) \tag{3.17}$$

with respect to the twisted probabilities

$$\widetilde{\pi}(z^{t+1},\eta^{t+1}|z^t,\eta^t) = \frac{m(z^{t+1}|z^t)}{E\left\{m(z^{t+1}|z^t))|z^t\right\}}\pi(z^{t+1},\eta^{t+1}|z^t,\eta^t),$$

whenever the borrowing constraints do not bind.

The martingale conditions are specific to the trading technology. These conditions enforce the Euler inequalities for the different traders: (i) the non-participants:

$$u'(c_t) \geq R_t^f \beta E_t\left\{u'(c_{t+1})\right\},$$

(ii) the diversified traders :

$$u'(c_t) \geq \beta E_t\left\{R_{t+1}u'(c_{t+1})\right\},$$

(iii) the $z-$*complete* traders :

$$u'(c_t) \geq \beta E_t\left\{u'(c_{t+1})\frac{P(z^t)}{P(z^{t+1})}|z^{t+1}\right\},$$

and (iv) the *complete* market traders:

$$u'(c_t) \geq \beta\left\{u'(c_{t+1})\frac{P(z^t)}{P(z^{t+1})}\right\}.$$

This follows directly from the martingale conditions and the first order condition for consumption. On the other hand, all households share the same first order condition for consumption, regardless of their trading technology. This implies that we can derive a consumption sharing rule and an aggregation result for prices.

## 3.3 Aggregate Multiplier

We can characterize equilibrium prices and allocations using the household's multipliers and the aggregate multipliers.

**Proposition 3.1.** *The household consumption share, for all traders is given by*

$$\frac{c(z^t,\eta^t)}{C(z^t)} = \frac{\zeta(z^t,\eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}, \text{ where } h(z^t) = \sum_{\eta^t}\zeta(z^t,\eta^t)^{\frac{-1}{\alpha}}\pi(\eta^t|z^t). \tag{3.18}$$

*The SDF is given by the Breeden-Lucas SDF and a multiplicative adjustment:*

$$m(z^{t+1}|z^t) \equiv \beta \left( \frac{C(z^{t+1})}{C(z^t)} \right)^{-\alpha} \left( \frac{h(z^{t+1})}{h(z^t)} \right)^{\alpha}. \tag{3.19}$$

The consumption sharing rule follows directly from the ratio of the first order conditions and the market clearing condition. Condition (3.9) implies that

$$c(z^t, \eta^t) = u'^{-1} \left[ \frac{\zeta(z^t, \eta^t) P(z^t)}{\beta^t} \right].$$

In addition, the sum of individual consumptions aggregate up to aggregate consumption:

$$C(z^t) = \sum_{\eta^t} c(z^t, \eta^t) \pi(\eta^t | z^t).$$

This implies that the consumption share of the individual with history $(z^t, \eta^t)$ is

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{u'^{-1} \left[ \frac{\zeta(z^t, \eta^t) P(z^t)}{\beta^t} \right]}{\sum_{\eta^t} u'^{-1} \left[ \frac{\zeta(z^t, \eta^t) P(z^t)}{\beta^t} \right] \pi(\eta^t | z^t)}.$$

With CRRA preferences, this implies that the consumption share is given by

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{\zeta(z^t, \eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}, \text{ where } h(z^t) = \sum_{\eta^t} \zeta(z^t, \eta^t)^{\frac{-1}{\alpha}} \pi(\eta^t | z^t).$$

Hence, the $-1/\alpha^{\text{th}}$ moment of the multipliers summarizes risk sharing within this economy. We refer to this moment of the multipliers simply as **the aggregate multiplier**. The equilibrium SDF is the standard Breeden-Lucas SDF times the growth rate of the aggregate multiplier. This aggregate multiplier reflects the aggregate shadow cost of the measurability and the borrowing constraints faced by households.

The expression for the SDF can be recovered directly by substituting for the consumption sharing rule in the household's first order condition for consumption (3.9). This aggregation result extends the complete market result in Lustig (2006) to the case of incomplete markets and heterogeneous trading technologies.

This proposition directly implies that an equilibrium for this class of incomplete market economies can be completely characterized by a process for these cumulative multipliers $\{\zeta(\eta^t, z^t)\}$, and by the associated aggregate multiplier process $\{h_t(z^t)\}$. Section 4 describes a method to solve for these multipliers. In the next subsection, we use the consumption sharing rule and the martingale

condition to highlight the effect of the heterogeneity in trading strategies on savings and investment behavior.

**Consumption Distribution** How is our SDF related to how the consumption distribution evolves over time? There is a tight connection between the aggregate weight growth rate and the growth rate of the $-\alpha$-th moment of the consumption distribution. We define $C_i^*$ as the $-\alpha^{\text{th}}$ moment of the consumption distribution in trader segment $i$.

**Corrolary 3.1.** *If there are only complete and z-complete traders, then the SDF is bounded below by the growth rate of the $-\alpha^{th}$ moment of the consumption distribution:*

$$\beta \left( C_i^*(z^{t+1})/C_i^*(z^t) \right) \leq m(z^{t+1}|z^t).$$

*This follows directly from the martingale condition and the consumption sharing rule. If the borrowing limits never bind in equilibrium (e.g. in the case of natural borrowing limits), then these two SDF's coincide:*

$$\beta \left( C^*(z^{t+1})/C^*(z^t) \right) = m(z^{t+1}|z^t).$$

*Finally, in the case of diversified traders, then the following inequality holds for the return on a claim to tradeable output:*

$$E_t \left[ \beta \left( C_{div}^*(z^{t+1})/C_{div}^*(z^t) \right) R(z^{t+1}) \right] \leq E_t \left[ m(z^{t+1}|z^t) R(z^{t+1}) \right] = 1.$$

Kocherlakota and Pistaferri (2005) derive this exact aggregation result with respect to the $-\alpha^{\text{th}}$ moment of the consumption distribution directly from the household's Euler equation in an environment where all agents trade the same assets. We show this is a lower bound in the case of binding borrowing or solvency constraints. Before we conclude this section, we derive conditions under which the aggregate weight growth rate is constant and/or equal to one.

## 3.4 Savings Behavior of Active Traders

Complete traders do not have a precautionary motive to save, while z-complete traders do. As a result, when interest rates are low, complete traders invariably de-cumulate assets, while z-complete traders may not choose to do so.

**Corrolary 3.2.** *The unconstrained complete trader's consumption share changes at a rate $-h(z^{t+1})$ $/h(z^t)$ in each $\eta^{t+1}, z^{t+1}$ state in the next period.*

If $h(z^{t+1})/h(z^t) > 1$ on average, and hence the risk-free rate is lower than in a representative agent economy, the complete trader's consumption share decreases on average, because he is dissaving. Complete traders have no precautionary motive to save – as reflected in the absence of measurability constraints–, and hence they run down their assets in each $(\eta^{t+1}, z^{t+1})$ state, when state prices are high, until they hit the binding solvency constraints. This is an "aggressive" trading strategy. This is not true for the z-complete trader.

**Corrolary 3.3.** *If the state price is low and $h(z^{t+1})/h(z^t) \leq 1$, the unconstrained z-complete trader's consumption share increases on average across $\eta^{t+1}$ states in the next period. If the state price is high and $h(z^{t+1})/h(z^t) > 1$, her consumption share can increase or decrease.*

Because of the market incompleteness, the $z$-complete trader may still accumulate assets in equilibrium even if the state price is high (or expected returns are low), and choose an increasing consumption path over time, as long as his borrowing constraint does not bind. This reflects his precautionary motive to save.

## 3.5 Investment Behavior

The martingale condition for active traders puts tight restrictions on the joint distribution of returns and consumption growth. Using the SDF expression in (3.19), we can state the martingale condition as $E_t[m_{t+1}\nu_{t+1}] = 0$ for non-participants, z-complete traders and complete traders. This gives rise to the following expression for marginal utility growth of an unconstrained trader:

$$E_t\left[\frac{\zeta_{t+1}}{\zeta_t}\right] = 1 - E_t[m_{t+1}]^{-1} cov_t\left[\frac{\zeta_{t+1}}{\zeta_t}, m_{t+1}\right] \tag{3.20}$$

The *covariance term* drops out for active traders (complete and z-complete traders) because $E[\nu_{t+1}|z^{t+1}] = 0$ in each node $z^{t+1}$. This orthogonality condition is the hallmark of an "active trading" strategy. Using the consumption sharing rule, this implies that:

$$cov_t\left[\Delta \log c_{t+1} - \Delta \log C_{t+1} + \Delta \log h_{t+1}, X(z^{t+1})\right] \simeq 0$$

where $X(z^{t+1})$ is any random payoff (including $m$ itself). This condition is trivially satisfied for the complete trader, whose consumption growth is $\Delta \log c_{t+1} = \Delta \log C_{t+1} - \Delta \log h_{t+1}$ in each node. Active traders increase their consumption growth when state prices are lower than in the representative agent model, and they decrease consumption growth when state prices are higher than in the representative agent model.

In a separate appendix, we show that these results go through even if agents have non-expected utility as in Epstein and Zin (1989). The next section derives a recursive set of updating rules for these multipliers, and we show under what conditions this separation result obtains.

# 4 Computation

This section describes a computational method that builds on the recursive saddle point problem.

## 4.1 Updating function for household multipliers

To allow us to compute equilibrium allocations and prices for a calibrated version of this economy, we recast our optimality conditions in recursive form. To do so, we define a new accounting variable: the promised savings function. Making use of the consumption sharing rule, we can express the household's present discounted value of future savings or "promised savings" as a function of the individual's multiplier:

$$\begin{aligned} S(\zeta(z^t,\eta^t);z^t,\eta^t) &= \left[\gamma\eta_t - \frac{\zeta(z^t,\eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}\right] C(z^t) \\ &+ \sum_{z^{t+1},\eta^{t+1}} \frac{\pi(z^{t+1},\eta^{t+1})P(z^{t+1})}{\pi(z^t,\eta^t)P(z^t)} S(\zeta(z^{t+1},\eta^{t+1});z^{t+1},\eta^{t+1}). \end{aligned} \tag{4.1}$$

This recursive expression for promised savings holds for all of our different asset traders.

Since the present-value budget constraint implies that

$$S(\zeta(z^{t+1},\eta^{t+1});z^{t+1},\eta^{t+1}) = -\hat{a}_t(z^{t+1},\eta^{t+1}),$$

we can simply restate the solvency constraint, and all of our measurability conditions in terms of the promised savings function. The Kuhn-Tucker condition on the *borrowing constraint* reads as:

$$\varphi(\eta^{t+1},z^{t+1})\left[S(\zeta(z^{t+1},\eta^{t+1});z^{t+1},\eta^{t+1}) + \underline{M}(z^{t+1},\eta^{t+1})\right] = 0. \tag{4.2}$$

This condition is common to all traders, regardless of the trading technology. However, the measurability and optimality conditions depend upon the trading technology.

For example, let $S^z(\cdot)$ denote the **z-complete trader**'s savings function. Our *measurability constraint* requires that the discounted value of the future surpluses be equal for each future $\eta^{t+1}$, or

$$S^z(\zeta(z^{t+1},\eta^{t+1});z^{t+1},\eta^{t+1}) = S^z(\zeta(z^{t+1},\tilde{\eta}^{t+1});z^{t+1},\tilde{\eta}^{t+1}) \text{ for all } \eta^{t+1},\ \tilde{\eta}^{t+1} \text{ and } z^{t+1}.$$

This implies the following Kuhn-Tucker condition for the measurability constraints:

$$\left[S^z(\zeta(z^{t+1},\eta^{t+1});z^{t+1},\eta^{t+1}) - S^z(\zeta(z^{t+1},\tilde{\eta}^{t+1});z^{t+1},\tilde{\eta}^{t+1})\ \right]\nu(\eta^{t+1},z^{t+1}) = 0 \text{ for all } \eta^{t+1}, \tag{4.3}$$

for all $\eta^{t+1}$, $\tilde{\eta}^{t+1}$ and $z^{t+1}$. Conditions (4.2-4.3) and the martingale condition (3.10 ), reproduced

here,

$$\sum_{\eta^{t+1} \succ \eta^t} \nu\left(z^{t+1}, \eta^{t+1}\right) \pi(z^{t+1}, \eta^{t+1}) P(z^{t+1}) = 0$$

determine the *multiplier updating function*:

$$T^z(z^{t+1}, \eta^{t+1} | z^t, \eta^t)(\zeta(z^t, \eta^t)) = \zeta(z^{t+1}, \eta^{t+1}).$$

$T^z$ is determined by solving a simple set of simultaneous equations. Let # denote the cardinality of a set. Using the martingale condition, note that in each node $z_{t+1}$, we have $\#Y - 1$ measurability equations to be solved for $\#Y - 1$ multipliers $\nu(\eta^t, \eta_{t+1}, z^{t+1})$, one for each $\eta_{t+1}$. In addition, in each node $z_{t+1}$, we have $\#Y - 1$ Kuhn-Tucker conditions to be solved for $\#Y - 1$ multipliers $\varphi(\eta^t, \eta_{t+1}, z^{t+1})$, one for each $\eta_{t+1}$. Finally, the law of motion for the cumulative multiplier $\zeta$ is given in (3.7).

In the recursive formulation, the **complete trader**'s *measurability condition* collapses to:

$$S^c(\zeta(z^{t+1}, \eta^{t+1}); z^{t+1}, \eta^{t+1}) = S^c(\zeta(z^{t+1}, \eta^{t+1}); z^{t+1}, \eta^{t+1}) \text{ for all } z^{t+1} \text{ and } \eta^{t+1}. \tag{4.4}$$

This implies the following Kuhn-Tucker condition for the measurability constraints:

$$\left[S^c(\zeta(z^{t+1}, \eta^{t+1}); z^{t+1}, \eta^{t+1}) - S^c(\zeta(z^{t+1}, \eta^{t+1}); z^{t+1}, \eta^{t+1}) \right] \nu(\eta^{t+1}, z^{t+1}) = 0 \text{ for all } \eta^{t+1}, \tag{4.5}$$

for all $\eta^{t+1}$ and $z^{t+1}$, which is vacuous and hence can be dropped. Also in the recursive formulation, we know that the martingale condition reduces to:

$$\nu(z^{t+1}, \eta^{t+1}) = 0 \tag{4.6}$$

for all $z^{t+1} \succ z^t$ and $\eta^{t+1} \succ \eta^t$. This condition implies that this trader's multipliers are a degenerate bounded super-Martingale.

Conditions (4.2), (4.5) and (4.6) determine the complete trader's multiplier updating function

$$T^c(z^{t+1}, \eta^{t+1} | z^t, \eta^t)(\zeta(z^t, \eta^t)) = \zeta(z^{t+1}, \eta^{t+1}).$$

Again, even though the form of the promised savings function, $S^c(\zeta(z^{t+1}, \eta^{t+1}); z^{t+1}, \eta^{t+1})$, is the same, because the multipliers are being updated differently with the complete structure, the value of function will in general be different from that of the $z$-complete securities market promised savings function.

For fixed portfolio traders, which includes both the diversified investors and the nonparticipants

as special cases, the *measurability condition* is given by:

$$\frac{S^{port}(\zeta(z^{t+1},\eta^{t+1});z^{t+1},\eta^{t+1})}{R^{port}(z^t,z_{t+1})}=\frac{S^{port}(\zeta(z^t,\tilde{z}_{t+1},\eta^t,\tilde{\eta}_{t+1});z^t,\tilde{z}_{t+1},\eta^t,\tilde{\eta}_{t+1})}{R^{port}(z^t,\tilde{z}_{t+1})} \tag{4.7}$$

for all $\eta^{t+1},\eta^t,\tilde{\eta}_{t+1},z^{t+1}$ and $z^t,\tilde{z}_{t+1}$ and, the *martingale condition* becomes:

$$\sum_{z^{t+1}\succ z^t,\eta^{t+1}\succ\eta^t}\nu\left(z^{t+1},\eta^{t+1}\right)R^{port}(z^t,\tilde{z}_{t+1})\pi(z^{t+1},\eta^{t+1})P(z^{t+1})=0. \tag{4.8}$$

The updating functions for the passive traders,

$$T^{div}(z^{t+1},\eta^{t+1}|z^t,\eta^t)(\zeta(z^t,\eta^t))\ \textit{and}\ T^{np}(z^{t+1},\eta^{t+1}|z^t,\eta^t)(\zeta(z^t,\eta^t)),$$

are the solution to conditions (4.2), the Kuhn-Tucker condition associated with the measurability condition (4.7) and their martingale condition (4.8). For the diversified traders, the appropriate return is the return on diversified wealth, and for the nonparticipants it is the risk-free rate.[6]

## 4.2 Aggregate multiplier updating operator

To summarize, the updating function $T^j(\cdot), j\in\{c,z,div,np\}$ is a solution to a system of equations defined by:

1. **measurability** conditions using recursive expression for $S$
2. **martingale** conditions
3. **borrowing constraint** using recursive expression for $S$

Finally, these updating functions for each of the trading technologies $T^j(\cdot), j\in\{c,z,div,np\}$ determine the law of motion for the aggregate multiplier:

$$h(z^{t+1})=\sum_{j\in T}\int\sum_{\eta^{t+1}\succ\eta^t}\left\{\left[T^j(z^{t+1},\eta^{t+1}|z^t,\eta^t)(\zeta(z^t,\eta^t))\right]^{\frac{-1}{\alpha}}\frac{\pi(\eta^{t+1},z^{t+1}|\eta^t,z^t)}{\pi(z^{t+1}|z^t)}\right\}d\Phi_t^j,$$

where $\Phi_t^j$ is the joint distribution of multipliers and endowments and $j\in\{c,z,div,np\}$. These aggregate multiplier dynamics govern the dynamics of the SDF, and hence of risk premia and asset prices. Clearly, this defines an aggregate multiplier updating operator $\{h_t^1(z^t)\}=T^h\{h_t^0(z^t)\}$ that maps the initial multiplier function $\{h_t(z^t)\}$ into a new aggregate multiplier function. We are looking for a fixed point of this operator.

[6]Rather than simply have the traders hold a fixed weighted portfolio, we can allow for optimum portfolio choice among a finite set of assets. The separate appendix provides a detailed discussion.

For certain configurations of the trading segments, we can establish that $h_t$ is either non-decreasing over time or increases on average. These results are in a separate appendix.[7]

## 4.3 Algorithm

In the next section, we develop some conditions under which aggregate and idiosyncratic risk separate. In the case of separation, $h(z^{t+1})/h(z^t)$ is deterministic, independent of the aggregate history $z^t$. However in general, the growth rate of the aggregate multiplier process $\{g_t(z^t)\}$ depends on the entire history. Of course, in an infinite horizon economy, we cannot record the entire aggregate history of shocks in the state space. To actually compute equilibria in a calibrated version of this economy, we propose an algorithm that only uses the last $n$ shocks, following Verarcierto (1998), and we use $s$ to denote a truncated aggregate history in $Z^n$. We define $g(s, s') = h(z^{t+1})/h(z^t)$, conditional on the last $n$ elements of $z^{t+1}$ equaling $s'$ and the last $n$ elements of $z^t$ equaling $s$. The algorithm we apply is:

1. conjecture a function $g_0(s, s') = 1$.
2. solve for the equilibrium updating functions $T_0^j(s', \eta'|s, \eta)(\zeta)$ for all trader groups $j \in \{c, z, div, np\}$.
3. By simulating for a panel of $N$ households for $T$ time periods, we compute a new aggregate weight forecasting function $g_1(s, s')$.
4. We continue iterating until $g_k(s, s')$ converges.

The computational algorithm is discussed in detail in the separate appendix (section B.1). Using the recursive savings function, we can characterize the aggregate multiplier dynamics analytically under some assumptions. First, we derive some bounds on the growth rate of $\{h_t\}$. Second, we conditions under which the growth rate is constant and hence the aggregate risk premium is not affected by limited participation.

## 4.4 The Separability of Aggregate and Idiosyncratic Risk

In this section, we show that the equilibrium distribution of the household multipliers does not depend on the realization of the aggregate shocks provided that all agents can trade a claim to all diversifiable income, and provided that

**Condition 2.** *The aggregate shocks are i.i.d. :* $\phi(z_{t+1}|z_t) = \phi(z_{t+1})$.

[7] see B.1 proposition and B.2 in the separate appendix.

**Condition 3.** *The idiosyncratic shocks are independent of the aggregate shocks:*

$$\pi(\eta_{t+1}, z_{t+1}|\eta_t, z_t) = \varphi(\eta_{t+1}|\eta_t)\phi(z_{t+1}|z_t).$$

This result is an extension of Krueger and Lustig (2006) to the case of segmented markets. In the absence of non-participants, the degree of consumption smoothing within and among different trading groups only affects the risk-free rate, not the risk premium. To prove this result, all we need to show is that the multiplier updating functions $T^i$ do not depend on the aggregate history $z^t$.

We start out by noting the borrowing constraints are proportional to aggregate income. From our definition (2.5) and our asset pricing result (3.19), it follows that

$$\underline{M}(\eta^t, z^t) = -\psi \sum \tau \geq t \sum_{\{z^\tau \succeq z^t, \eta^\tau \succeq \eta^t\}} \gamma Y(z^\tau)\eta_\tau \frac{\pi(z^\tau, \eta^\tau)\beta^{\tau-t}Y(z^\tau)^{-\alpha}h(z^\tau)^\alpha}{\pi(z^t, \eta^t)\beta^t Y(z^t)^{-\alpha}h(z^t)^\alpha}.$$

Since the growth rate of $Y(z^t)$ is i.i.d. by assumption, it follows that $\underline{M}(\eta^t, z^t)/Y(z^t)$ is independent of $z^t$, and hence

$$\underline{M}(z^t, \eta^t) = \underline{M}(\eta^t)Y(z^t).$$

Then, we define the ratio of savings to aggregate consumption $\tilde{S}$ as follows:

$$S(\zeta(z^t, \eta^t); z^t, \eta^t) = Y(z^t)\tilde{S}(\zeta(z^t, \eta^t); z^t, \eta^t). \tag{4.9}$$

Our recursive relationship for $S(\zeta(z^t, \eta^t); z^t, \eta^t)$ implies that

$$\tilde{S}(\zeta(z^t, \eta^t); z^t, \eta^t) = \gamma\eta_t - \frac{\zeta(z^t, \eta^t)^{\frac{-1}{\alpha}}}{h(z^t)} + \beta \sum_{z_{t+1}} \widehat{\phi}(z_{t+1}|z^t) \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta_t)\tilde{S}(\zeta(z^{t+1}, \eta^{t+1}); z^{t+1}, \eta^{t+1}).$$

where

$$\widehat{\phi}(z_{t+1}|z^t) = \phi(z_{t+1}) \left[\frac{h(z^{t+1})}{h(z^t)}\right]^\gamma e^{(1-\gamma)z_{t+1}}.$$

In addition, our debt constraint in terms of the savings/consumption ratio $\tilde{S}$ is simply given by:

$$\tilde{S}(\zeta(z^{t+1}, \eta^{t+1}); z^t, \eta^t) \leq \underline{M}(\eta^{t+1}). \tag{4.10}$$

**Proposition 4.1.** *If condition (3) and (2) are satisfied, in any economy without non-participants the equilibrium values of the multipliers $\zeta$ and the equilibrium consumption shares are independent of $z^t$.*

The reason behind the independence result is straightforward. Start by conjecturing that $h(z^{t+1})/h(z^t)$ does not depend on $z^{t+1}$, and conjecture that the savings/consumption ratio $\tilde{S}(\zeta(z^t,\eta^t);z^t,\eta^t)$ does not depend on $z^t$. This being the case, nothing else in the recursive equation depends on the realization of the aggregate shock $z_t$ , because $\widehat{\phi}(z_{t+1})$ does not depend on $z^t$, in the measurability constraints z-complete traders or in the debt constraint. That versifies our conjecture about the savings consumption ratio So, the measurability constraint for the z-complete traders is independent of $z_t$:

$$\tilde{S}^z(\zeta(\eta^{t+1}),\eta^{t+1}) = \tilde{S}^z(\zeta(\tilde{\eta}^{t+1});\tilde{\eta}^{t+1}) \text{ for all } \eta^{t+1},\ \tilde{\eta}^{t+1} \text{ and } z^{t+1}, \tag{4.11}$$

and this implies that the updating function does not depend on $z^t$ either:

$$T^z(\eta^{t+1}|\eta^t)(\zeta(\eta^t)) = \zeta(\eta^{t+1}).$$

What about the diversified investors? Let $pd_t$ denote the price/dividend ratio on a claim to consumption. For the diversified investors, the measurability constraint reads as:

$$\frac{\tilde{S}^{div}(\zeta(\eta^{t+1}),\eta^{t+1})}{[(1-\gamma)+pd_{t+1})]} = \frac{\tilde{S}^{div}(\zeta(\eta^t,\tilde{\eta}_{t+1});\eta^t,\tilde{\eta}_{t+1})}{[(1-\gamma)+pd_{t+1}]}$$

for all $\eta^{t+1},\eta^t,\tilde{\eta}_{t+1},z^{t+1}$ and $z^t,\tilde{z}_{t+1}$. Since the $pd_t$ can only evolve deterministically, given the i.i.d. shocks and the conjecture about $h_{t+1}/h_t$, the diversified trader faces the same measurability constraints as the $z$-complete traders. Hence, the diversified investor's updating function does not depend on $z^{t+1}$:

$$T^{div}(\eta^{t+1}|\eta^t)(\zeta(\eta^t)) = \zeta(\eta^{t+1}).$$

This being the case, it easy to show that $h_{t+1}/h_t$ does not depend on $z^{t+1}$ either, as long as there are no non-participants, simply because nothing on the right hand side depends on $z^{t+1}$:

$$h_{t+1}-h_t = \sum_{j\in T}\int \sum_{\eta^{t+1}\succ\eta^t} \left\{ \left[T^j(\eta^{t+1}|\eta^t)(\zeta(\eta^t))\right]^{\frac{-1}{\alpha}} \varphi(\eta_{t+1}|\eta_t) - \zeta(\eta^t)^{\frac{-1}{\alpha}} \right\} d\Phi_t^j \tag{4.12}$$

where $T=\{c,z,div\}$.

**Corrolary 4.1.** *Independent of the market segmentation, if all households can trade a claim to diversifiable income, the (conditional) equity risk premium is the Breeden-Lucas one.*

When $\{h_{t+1}/h_t\}$ is non-random, market incompleteness only affects the risk-free rate, not the risk premium. The consumption shares of all households do not depend on the aggregate shocks. There is no time variation in expected returns, and households only want to trade a claim to aggregate consumption to hedge against aggregate risk. All the asset market participants face the same measurability condition if $\{h_{t+1}/h_t\}$ is non-random. The distinction between active and

passive traders is irrelevant, because there is no spread between state prices other than that in a representative agent model. Households all hold fixed portfolios (i.e. the market) in equilibrium, and there exists a stationary equilibrium with an invariant wealth distribution. This result implies that the multipliers are not affected by the aggregate shocks.

**Non-participants** This independence with respect to the value of $z_{t+1}$ is not true for the non-participants, since the measurability condition in terms of $\tilde{S}$ is given by

$$\frac{\tilde{S}_{t+1}(\zeta(z^{t+1},\eta^{t+1});z^{t+1},\eta^{t+1})}{e^{z_{t+1}}}=\frac{\tilde{S}_{t+1}(\zeta(\tilde{z}^{t+1},\tilde{\eta}^{t+1});\tilde{z}_{t+1},\tilde{\eta}^{t+1})}{e^{\tilde{z}_{t+1}}}, \tag{4.13}$$

for all $(\eta^{t+1}),(\eta^t,\tilde{\eta}_{t+1}),(z^{t+1})$ and $(z^t,\tilde{z}_{t+1})$. Clearly, this household's multiplier updating function will depend on the aggregate history. This measurability condition implies that the ratio of non-participant household net wealth to aggregate consumption needs to be counter-cyclical.

The inclusion of a positive measure of non-participants causes a breakdown in the separation of aggregate and idiosyncratic risk. There no longer is an equilibrium with a stationary distribution of wealth; $\{h_{t+1}/h_t\}$ depends on the entire history of aggregate shocks. This drives a wedge between the martingale condition of the active investors and the diversified investors. We explore the quantitative importance of this in the rest of the paper.

## 4.5 Shifting Aggregate Risk

We can define the aggregate promised savings function for each group of traders $j\in\{c,z,div,np\}$:

$$S_a^j(z^t)=\left[\gamma\mu^j-\frac{h^j(z^t)}{h(z^t)}\right]C(z^t)+\sum_{z^{t+1}}\frac{\pi(z^{t+1})P(z^{t+1})}{\pi(z^t)P(z^t)}S_a^j(z^{t+1}),$$

by aggregating across all the households in segment $j$, and exploiting the linearity of the pricing functional. Finally, the sum of the aggregate savings functions is (minus) a claim to diversifiable income:

$$\sum_j S_a^j(z^t)=-[\varpi(z^t)+(1-\gamma)Y(z^t)]$$

This follows directly from market clearing. The measurability restrictions on the household savings function in turn imply restrictions on the aggregate savings share of each trader group.

The **diversified traders** do not bear any of the residual aggregate risk, (in terms of their savings share) created by non-participants.

**Proposition 4.2.** *The aggregate savings share* $\frac{S_a^{div}(z^t)}{[\varpi(z^t)+(1-\gamma)Y(z^t)]}$ *of diversified traders cannot depend on* $z_t$.

Since the measurability constraints are satisfied for the individual household's savings function, they also need to be satisfied for the aggregate savings function. So by the LLN:

$$\frac{S_a^{div}(z^t, z_{t+1})}{[(1-\gamma)Y(z^t, z_t) + \varpi(z^t, z_t)]} = \frac{S_a^{div}(z^t, \tilde{z}_{t+1})}{[(1-\gamma)Y(z^t, \tilde{z}_{t+1}) + \varpi(z^t, \tilde{z}_{t+1})]}$$

where we have used the fact that the denominator is measurable w.r.t. $z^t$ . The household measurability condition implies that the aggregate savings of the diversified traders be proportional to the diversifiable income claim in all the aggregate states.

By the same logic,

**Proposition 4.3.** *The aggregate savings share of non-participants* $\frac{S_a^{np}(z^t)}{[\varpi(z^t)+(1-\gamma)Y(z^t)]}$ *is inversely proportional to the aggregate endowment growth rate*

This follows directly from the measurability condition of the non-participant households, which implies that their individual, and hence their aggregate, saving level cannot depend upon $z_{t+1}$.

Since the diversified traders have (conditionally) constant savings shares, and the non-participant traders have counter-cyclical savings shares, regardless of the $\{h\}$ process, there cannot be an equilibrium without active traders. The market simply cannot be cleared without active traders, if there are non-participants.

Table 1 summarizes the main effects of heterogeneity in trading technologies on asset prices and portfolio composition. These results rely on the absence of predictability of aggregate consumption growth and the independence of idiosyncratic and aggregate shocks. In the first panel, we summarize the effect on the equity premium. In the absence of non-participants, the composition of the other trader segments has no effect on the equity premium; the Breeden-Lucas risk premium obtains. However, as soon as there is a positive fraction of non-participants, this irrelevance result disappears. In the second panel, we look at the portfolio effects. All traders hold the market portfolio in the absence of non-participants. However, when there are non-participants, the active traders decide to increase their exposure to market risk.

[Table 1 about here.]

Next, we solve a calibrated version of this economy numerically, to examine the quantitative importance of heterogeneous trading opportunities for asset prices.

# 5 Quantitative Results

This section evaluates a calibrated version of the model. The first subsection discusses the calibration of the parameters and the endowment processes. The benchmark model has no aggregate

consumption growth predictability (IID economy). Hence, all of the dynamics are generated by the heterogeneity of trading technologies. In the second subsection, we show that the model with heterogeneous trading opportunities manages to reconcile the low volatility of the risk free rate with the large and counter-cyclical volatility of the stochastic discount factor. We use this economy to explore the impact of changes in the active trader's segment composition. The last subsection explores the model's implications for the distribution of wealth and asset shares across households.

We choose the distribution of trading technologies to generate asset prices that provide a reasonable match to the data. In the first part, we focus on the calibration with 5% of each of the complete and z-complete traders, 20% of the diversified traders and 70% of the nonparticipants since this calibration included all of our trading types. However, since the complete market traders do not accumulate much wealth because they are able to hedge their idiosyncratic risk, we later focus on a calibration that does not include them. The calibration with 10% z-complete traders, 20% of the diversified traders and 70% of the nonparticipants does almost as well in terms of asset prices while providing a better fit on the wealth and asset share distributions.

## 5.1 Calibration

The model is calibrated to annual data. We choose a coefficient of relative risk aversion $\alpha$ of five and a time discount factor $\beta$ of .95. These preference parameters allow us to match the collaterizable wealth to income ratio in the data when the tradeable or collateralizable income share $1-\gamma$ is 10%, as discussed below. Non-diversifiable income includes both labor income and entrepreneurial income, among other forms.

**IID Economy** In the benchmark calibration, there is no predictability in aggregate consumption growth, as in Campbell and Cochrane (1999) –we impose condition (2). We refer to this as the IID economy. This is a natural benchmark case because the statistical evidence for consumption growth predictability is weak. Moreover, in the IID experiment, all of the equilibrium dynamics in risk premia flow from the binding borrowing and measurability constraints, not from the dynamics of the aggregate consumption growth process itself. [8] The other moments for aggregate consumption growth are taken from Mehra and Prescott (1985b). The average consumption growth rate is 1.8%. The standard deviation is 3.15%. Recessions are less frequent: 27% of realizations are low aggregate consumption growth states.

In addition, we impose independence of the idiosyncratic risk from aggregate shocks on the labor income process –condition (3) holds. By ruling out counter-cyclical cross-sectional variance of labor income shocks, we want to focus on the effects of concentrating aggregate risk among a

[8] Our IID experiment is designed to show that the heterogeneous trading technologies also generate similar dynamics endogenously. Campbell and Cochrane (1999)'s model is designed to demonstrate that the external habit process endogenously generates the right dynamics in risk premia without creating risk-free rate volatility.

small section of households, as opposed to concentrating income risk in recessions. The Markov process for $\log \eta(y, z)$ is taken from Storesletten, Telmer, and Yaron (2003) (see page 28). The standard deviation is .60, and the autocorrelation is 0.89. We use a 4-state discretization. The elements of the process for $\log \eta$ are $\{0.38, 1.61\}$.

Finally, given conditions 2 and 3, the risk premium and portfolio irrelevance result that we derived for the case without non-participants applies. This will provide us with a natural benchmark for the asset pricing and wealth distribution results.

**Collateralizable Wealth** The average ratio of household wealth to aggregate income in the US is 4.30 between 1950 and 2005. The wealth measure is total net wealth of households and non-profit organizations (Flow of Funds Tables). We choose a collateralizable income ratio $\alpha$ of 10%. The implied ratio is 4.88 in the model's benchmark calibration. Finally, we set the solvency constraint equal to zero: $\underline{M} = 0$.

**Assets Traded** Equity in our model is simply a leveraged claim to diversifiable income. In the Flow of Funds, the ratio of corporate debt-to-net worth is around 0.65, suggesting a leverage parameter $\psi$ of 2. However, Cecchetti, Lam, and Mark (1990) report that standard deviation of the growth rate of dividends is at least 3.6 times that of aggregate consumption, suggesting that the appropriate leverage level is over 3. Following Abel (1999) and Bansal and Yaron (2004) , we choose to set the leverage parameter $\psi$ to 3. The returns on this security are denoted $R_{lc}$. We also consider the returns on a perpetuity (denoted $R_b$).

**Composition** In our benchmark model, 70% of households only trade the riskless asset. The remaining 30% is split between diversified investors, z-complete traders and complete traders. We begin by discussing the asset pricing implications of heterogeneous trading opportunities in the IID version of our economy. In the next subsection, we show that this composition of traders allows for a close match of the wealth and asset share distribution.

**Accuracy** To assess the accuracy of the approximation method, we report the highest coefficient of variation for the actual simulated realizations of $[h'/h]$, conditioning on the truncated history of length 5. These are reported in the upper panel of 2. If the method were completely accurate, this statistic would be zero because the actual realizations would not vary in a truncated history. This coefficient (CV) varies between .57% and .28%. So, the forecasting errors are small. The truncated aggregate history explains approximately all of the variation in $[h'/h]_t$.[9] In addition, we checked how well we would have done simply by conditioning on the first moment of the wealth distribution. In the lower panel of 2, we report the $R^2$ in a regression of the $\log SDF$ on the first

[9]The implied $R^2$ is approximately $1 - CV^2$.

moment of the wealth distribution. The $R^2$ are vary between 3% and 60 %. Clearly, approximate aggregation does not hold, in the sense that more moments of the wealth distribution are necessary to forecast the SDF.

[Table 2 about here.]

We use the IID economy as a laboratory for understanding the interaction between active and passive traders and its effect on asset prices. This interaction generates counter-cyclical state price volatility without risk-free rate volatility, unlike other heterogeneous agent models (see e.g. Lustig (2006), Alvarez and Jermann (2001), and Guvenen (2003)).

## 5.2 Risk and Return

The asset pricing statistics for the IID economy were generated by drawing 10,000 realizations from the model, simulated with 3000 agents. Table 3 reports the asset pricing results in our baseline experiment. As a benchmark, the first column in the table also reports the corresponding numbers for the RA (representative agent) economy. We consider three cases in the HTT economy. In all cases the fractions of active traders (10%), diversified traders (20%) and non-participants are constant (70%), but we change the composition of the active trader segment. The first column in table 3 reports the results for 10 % z-complete traders (case 1). In this case, there are no complete traders. The second column has 5% z-complete and 5% complete traders (case 2), and the last column has 10 % complete traders (case 3). The fractions of traders can be interpreted as fractions of human wealth (or labor income), rather than fractions of the population. Finally, the last column reports the moments in the data.

[Table 3 about here.]

**Representative Agent Economy** We start by listing some properties of returns in the RA economy. In the **RA economy**, the maximum Sharpe ratio is .19 and the equity risk premium ($E\left[R_{lc} - R_f\right]$) is 2.3%. The conditional market price of risk $[\sigma_t[m]/E_t[m]]$ is constant, because the shocks are i.i.d. Hence, the risk premia are constant as well. Finally, the risk-free rate in the RA economy is 12% and it is also constant. As a result, there is no risk in bond returns ($E[R_b - R_f] = 0$).

All of the moments of risk *premia* reported in column 1 are identical in the HTT economy without non-participants, regardless of the composition of the pool of participants.[10] As long as all households can trade a claim to diversifiable income, the lack of consumption smoothing has no bearing on risk premia, and its only effect is to lower the equilibrium risk-free rate (not reported in the table).

[10] see Proposition 4.1.

**Heterogeneous Trading Technologies Economy** In the **HTT economy**, the interaction between active and passive traders generates volatile state prices and a stable risk-free rate. We start by considering case 1 –no complete traders. We adopt this case with only z-complete traders in the active traders segment as our benchmark. These make up 10 % of the population. The remaining 90% is split between diversified traders (20%) and non-participants (70%). The model's market segmentation was calibrated to match asset prices. As an out-of-sample check of the model, the next subsection compares the implications of these choices for the wealth distribution and the asset class share distribution against the data.

In case 1 of the HTT economy, the maximum Sharpe ratio, $(\sigma[m]/E[m])$, is .44. The risk premium on the leveraged consumption claim is 6.7% $(E[R_{lc} - R_f])$, while the standard deviation of returns $(std[R_{lc} - R_f])$ is 15.2%. But, it is still well below the average realized excess return in post-war US data of 7.5%. However, the average price/dividend ratio $(E[PD_{lc}])$ in the data is 33, substantially higher than that in the model. As others have suggested (Fama and French (2002)), a decrease in the risk premium over the last part of the sample may have causes higher realized returns.

The risk-free rate $R_f$ is low (1.73%) and essentially constant. The standard deviation of the risk-free rate is .06%. There is also substantial time variation in expected excess returns; the standard deviation of the conditional market price of risk $Std[\sigma_t[m]/E_t[m]]$ is 3.3%, comparable to that in Campbell and Cochrane (1999) 's model. The conditional market price of risk varies between .4 and .8. Since the risk-free rate is essentially constant in the IID economy, bond returns (a perpetuity in the model) are essentially equal to the risk-free rate $(E[R_b - R_f])$. In the data, long-run bonds yielded an average excess return of 1% with a Sharpe ratio of .09.

We also look at the autocorrelation of stock returns $(\rho[R_{lc}(t), R_{lc}(t-1)])$. This is close to zero in the model, as a result of the IID aggregate shocks, while the autocorrelation is around -.2 in the data. The correlation of returns with the risk-free rate in the data is around .2, compared to zero in the model $(\rho[R_{lc}, R_f])$. Introducing some moderate autocorrelation in aggregate consumption growth allows for a better match of the time-series properties of returns in the data.[11]

Finally, the correlation between stock returns and aggregate consumption growth is much too large in our model. In the HTT version of our model, the correlation between stock returns and aggregate consumption growth is much too high (97 %) compared to only 15 % in the post-war data (1945-2004). Using Shiller's longer sample, we obtain a correlation of 18 % (1890-2004).[12] This shortcoming of the model is due to the simple 2-shock structure we chose for aggregate consumption growth.

[11]Results are in section B.6 of the separate appendix.

[12]This correlation can be lowered by allowing for richer consumption and divided processes.

**Complete Traders** As we increase the fraction of complete traders in the active traders segment, the market price of risk increases from .44 to .51, but more significantly, the standard deviation of the conditional market price of risk $Std\left[\sigma_t[m]/E_t[m]\right]$ increases from 3.3% to 5.8 %. These complete traders adopt a more aggressive trading strategy and are more levered in equity. This creates more counter-cyclical variation in the market price of risk. However, this does not come at the cost of introducing more volatility in the risk-free rate. The standard deviation of the risk-free rate increases from 3 to 29 basis points, still well below the standard deviation in the data.

**Time Variation** To understand the time variation, we focus on a specific case–the one with 5% complete and 5% z-complete traders. Figure 1 plots a simulated path of 100 years for the $\{h'/h\}$ shocks to the aggregate multiplier process in the top panel, the conditional risk premium on equity in the middle panel and the conditional market price of risk in the bottom panel. The shaded areas in the graph indicate low aggregate consumption growth states. As is clear from the top panel in figure 1, $[h'/h]$ is large in recessions -low aggregate consumption growth states- to induce the active traders to consume less in that state of the world, because the passive traders consume "too much" in those states. Similarly, $[h'/h]$ needs to be small in high aggregate consumption growth states, to induce the active traders to consume more in those states. The volatility in state prices induces the small segment of active traders to reallocate consumption across aggregate states and absorb the residual aggregate risk from the non-participants.

The middle panel plots the expected excess return on equity $E\left[R_{lc} - R_f\right]$. Clearly, the IID economy produces counter-cyclical variation in the risk premium. The underlying mechanism is shown in the bottom panel. As is clear from the bottom panel, the interaction between active and passive traders generates counter-cyclical variation in the conditional market price of risk $[\sigma_t[m]/E_t[m]]$. In high $[h'/h]$ states, active traders realize low portfolio returns. The wealth of active traders decreases as a fraction of total wealth. This means, that in order to clear the market, the future $[h'/h]$ -shocks need to be larger (in absolute value), and this in turn increases the conditional volatility of the stochastic discount factor. As a result, the conditional market price of risk $[\sigma_t[m]/E_t[m]]$ increases after each low aggregate consumption growth realization. The driving force behind the time variation is the time-varying exposure of active traders to equity risk. We explore this in the next subsection.

[Figure 1 about here.]

## 5.3 Portfolio and Consumption Choice

The reason for the heterogeneity in portfolio choice is not only the heterogeneity in trading technologies, but also the presence of non-participants. In the case without non-participants, all households, complete, z-complete and diversified traders would choose the same market portfolio: 25% equity

and 75% bonds! However, in the case of non-participation, the fraction active traders invest in equity varies over time and across traders. On average, the equity share is 93% for the z-complete trader and about 160% for the complete traders. These fractions are highly volatile as well. The standard deviation is 60% for the complete trader and 30% for the -complete trader.

Not surprisingly, the heterogeneity in portfolio choice shows up in portfolio returns. Table 4 reports the average portfolio returns realized by all traders in a segment. We take case 2 as our benchmark. We start with the complete investors. Their investment strategy delivers an average excess return on their portfolio of 10% ($E\left[R_c^W - R_f\right]$) or roughly twice the equity premium. The average excess return on the z-complete traders' portfolio ($E\left[R_z^W - R_f\right]$) is lower at 5.8%. The z-complete trader earns about the equity risk premium on his portfolio. Finally, the diversified investor earns excess returns of around 1.5% while the non-participants realize zero excess returns. As a result, these investors do not manage to accumulate wealth.

Figure 2 plots the wealth (top panel), the equity share (share of total portfolio invested in leveraged consumption claim's) and the conditional market price of risk (bottom panel) for the z-complete trader. The sequence of aggregate shocks (shaded area) is the same as in figure 1. These z-traders invest a much larger portfolio share in equity than the diversified trader, but more importantly, the share varies substantially over time, between 50 and 150%. Their equity exposure (middle panel) tracks the variation in the conditional market price of risk (bottom panel) and the equity premium perfectly.

Since the active traders are highly leveraged, their share of total wealth (see top panel) declines substantially after a low aggregate shock, and their "market share" declines. As a result, the conditional volatility of the aggregate multiplier shocks increases; larger shocks are needed to get the active traders to clear the markets. In response to the increase in the conditional market price of risk, the active traders increase their leverage. This also explains why increasing the size of the complete traders imputes more time variation to the conditional market price of risk, since these traders are more levered.

[Figure 2 about here.]

[Table 4 about here.]

**Consumption** This heterogeneity in portfolio choice shows up in household consumption and aggregate consumption for each trader segment as well. The left panel in table 5 reports the correlation of stock returns and household consumption growth as well as the standard deviation of household consumption growth. The hatted variables denote shares of aggregate consumption. The panel on the right report moments for consumption aggregated across all households in a trader segment. As a benchmark, consider the case without non-participants. Household consumption

shares do not depend on aggregate shocks $z^t$, regardless of their trading technology, and the correlation of consumption share growth with returns is zero $\rho\left[R_s, (\Delta \log(\widehat{c}_i)\right]$ for all participants.

However, in the economy with non-participants, the correlation of consumption share growth with stock returns is highest for complete traders (.64), and decreases to .58 for z-complete traders and 0 for diversified traders. The overall correlation for the participants $\rho\left[R_s, (\Delta \log(\widehat{c}_p)\right]$ is only about .20. The correlation of household consumption growth for all participants $\rho\left[R_s, (\Delta \log(c_i)\right]$ is .43. So an econometrician with data on all market participants would estimate the coefficient of relative risk aversion from the Euler equation for stock returns to be much higher than 5. The standard deviation of household consumption growth can be ranked according to the trading technology, from 5.6% for the complete traders to 12% for the non-participants. Note that the standard self-insurance mechanism breaks down for non-participants and diversified traders; they fail to accumulate enough assets.

Of course, the z-complete and complete traders absorb the residual of aggregate risk created by the passive traders. The second panel in table 5 reports the correlation of returns with *aggregate* consumption share growth and standard deviation of aggregate consumption growth for each group of traders. This is the growth rate of total consumption in each segment $\widehat{C}^j(z^t) = h^j(z^t)/h(z^t)$. The z-complete traders and the complete traders bear all of the aggregate risk. The aggregate consumption share growth of this trader segment has a correlation of .95 with stock returns. The same correlation for diversified investors is -.08, while the correlation for bond holders is -.9.

[Table 5 about here.]

**Portfolio Choice and Returns** The top panel of Table 6 reports the moments of portfolio returns and the wealth distribution. On average, the z-complete trader invests 69% in equity, but the fraction is highly volatile (19%). The z-trader realizes an average excess return of 5.6% ($E[R_i^W - R_f]$), compared to 1.5% for the diversified trader and 0 % for the non-participant.

In addition, the z-trader accumulates 3.2 times the average wealth level ($E[W_i/W]$), while the diversified trader is right at the average. Non-participants fail to accumulate wealth; on average, their holdings amount to only 69% of the average. This will severely limit the amount of self-insurance these non-participant households can achieve. On average, the z-trader accumulates 4.6 times more wealth than the non-participant. Because the z-trader invests a large fraction of his wealth in the risky asset, his wealth share is highly volatile. The coefficient of variation for the z-trader's wealth share is 45%. However, most of this reflects aggregate rather than idiosyncratic risk. On the other hand, these coefficient of variation for the passive traders are higher, but that reflects mostly idiosyncratic risk.

The welfare costs of being a passive trader are large. Figure 3 plots the fraction of lifetime consumption a fixed portfolio trader would be willing to give up to become a z-complete trader

against the fraction he invests in equity. The full line shows the welfare costs if the trader invested a fixed fraction in the dividend claim in the benchmark calibration with 10 % z-complete traders and 20 % diversified traders (case 1); the dashed line does the same for the calibration with 5% complete, 5% z-complete and 20 % diversified traders (case 2) and the dotted line for the case without z-complete traders but with 10 % complete market traders (case 3). In the benchmark calibration, the fixed portfolio trader needs leverage of around 100% (levered claim) to reduce the welfare cost to less than 1.5% of lifetime consumption. The remaining 1.5 % is the welfare cost of keeping fixed portfolios. The size of this cost depends on the extent of time variation. As we increase the fraction of complete market traders, the time variation in the market price of risk increases, which in turn pushes up the minimum welfare loss to 3% in case 3. In addition, the leverage required increases to 140 %.

[Figure 3 about here.]

The middle panel of Table 6 reports the moments of household consumption growth and aggregate consumption share growth. We report the ratio of the standard deviation of household consumption growth and the standard deviation of aggregate consumption growth to make the numbers comparable to recent studies of household consumption growth; the standard deviation of aggregate consumption growth in our model is much higher than the same standard deviation in recent decades.

The z-trader's consumption growth has the lowest volatility (7.7%) -2.8 times the volatility of aggregate consumption growth-, but most of this variation is common across z-traders; the volatility of their aggregate share growth rate is 4.6%. The z-traders exploit the variation in state prices. On the other hand, the diversified traders's volatility is 11.5% (3.4 times the volatility of aggregate consumption growth), and virtually none of this volatility is common (only .4%). This not surprising given the result in section 4.5. The non-participant's consumption growth, expressed in shares of aggregate consumption, is the highest at 13.2% (3.8 times the volatility of aggregate consumption growth), almost all of which is due to idiosyncratic risk. Their failure to accumulate enough assets after good idiosyncratic histories prevents them from self-insuring. As we discussed in section 4.5, the consumption share of active traders is highly pro-cyclical, while the consumption share of the non-participants is counter-cyclical.

Note that the overall correlation of consumption growth with returns for all participants is about .4 and .16 for non-participants. However, for the diversified traders, this correlation is .75. So, if an econometrician with access to data generated by our model were to limit his sample to wealthier households, the risk aversion estimate from the Euler equation for stocks would decrease, even though households have the same preferences, simply because their consumption growth

is more correlated with returns.[13] This is exactly what Mankiw and Zeldes (1991) and Brav, Constantinides, and Geczy (2002) have documented.

We estimated the EIS off the household Euler equation for bond returns and stock returns. We followed the procedure outlined by Vissing-Jorgensen (2002). We find similar evidence of preference heterogeneity. First, both the estimates obtained from the bond and stock Euler equation are biased upwards. All these households have EIS of .2, but we find estimates between $[1.5, 1.6]$ using the bond returns and between $[.32, .39]$ for stock returns. Vissing-Jorgensen (2002) reports estimates in the range $[.3, .4]$ for stock returns and $[.8, 1]$ for bond returns. Our $EIS$ estimates are highest for the most sophisticated investors, as has been documented in the data. Also note that the estimates are upward biased for all households.[14]

Finally, we also compared the equilibrium stochastic discount factor to the growth rate of the $-\alpha$-th moment of the consumption distribution for all the households $\beta\left(C_i^*(z^{t+1})/C_i^*(z^t)\right)$. In section 4.2, we showed this growth rate is a lower bound on the actual SDF. The standard deviation of this growth rate is less than half of the actual volatility of the SDF. This is consistent with the empirical findings of Kocherlakota and Pistaferri (2005) who tested $\beta\left(C_i^*(z^{t+1})/C_i^*(z^t)\right)$ on the Euler equation for stocks and bonds using household consumption data; they found large Euler equation errors.

[Table 6 about here.]

The next subsection considers the model's implications for the wealth distribution and the asset class share distribution. Since we calibrated the market segmentation to match asset prices, we regard these as over-identifying restrictions on our model.

## 5.4 Wealth and Asset Class Share Distribution

We consider two versions of the benchmark model. In the version labeled "standard", households are ex ante identical. In the version labeled "twisted", we introduce permanent income differences to match the income distribution, while keeping the fraction of human wealth in each trader segment constant. This way, the asset pricing implications of the model are not affected.Table 7 lists the percentile ratios in the twisted version of the model and the data. Essentially, the heterogeneity in trading opportunities makes the rich richer and the poor poorer. However, the middle class in our model accumulates too much assets.

[Table 7 about here.]

[13] From the Euler equation, it is clear that the Sharpe ratio is approximately equal to the coefficient of risk aversion times the correlation of returns and consumption growth times the standard deviation of consumption growth.

[14] The source of the bias is the time variation in the second moments of household consumption growth and its correlation with the instruments.

Table 10 reports the summary statistics and the percentile ratios for the standard and twisted version of the model in the first panel. We contrast these with the same ratios from the 2004 SCF for US households. The Gini coefficient in the data is .727 (SCF, 2001). Our model produces a Gini coefficient of .59. The model without heterogeneous trading opportunities produces a Gini coefficient of .48. So, the heterogeneity in trading opportunities bridges half of the gap with the data, by producing fatter tails and a more skewed distribution. The skewness of the wealth distribution increases from .8 to 2.7(compared to 3.6 in the data) while the kurtosis increases from 2.8 to 12.9. (compared to 15.9 in the data).

First, consider the standard version of the model (column 1). Households in the 75 -th percentile accumulate 5 times as much wealth as households in the 25-th percentile, while households in the 80-th percentile accumulate 8.7 times as much wealth as households in the 20-th. The effect of the heterogeneity in trading technologies is most visible in the tails. The 90/10 ratio is 182 in the standard model. This ratio is only 45 in a version of the model with only diversified traders.

The second column reports the same statistics for the version of the model that is calibrated to match the income distribution. The 75/25 ratio increase to 6.9 while the 80/20 ratio increases to 12.49. The 90/10 ratio increases to 240. The twisted version of the model still falls well short of the data. The poor households still accumulate too much wealth in the model compared to the data. This discrepancy is not surprising given that these households have no life-cycle motive for borrowing and saving. However, the model does quite well in matching the right tail of the wealth distribution in the data. The second panel focusses on the left tail of the wealth distribution. The 50/10 ratio in the twisted version of our model is 65, compared to 100 in the data. However, the 90/50 ratio is only 3.7 in our model, compared to 9.5 in the data. This discrepancy is partly due to the fact that the twisted income distribution in our model does not quite match that in the data in the highest income percentiles.

[Table 8 about here.]

Finally, we turn to the asset class share distribution, and we check whether our model can replicate the distribution of asset shares in the data. Table 9 shows the equity share (as a fraction of the household portfolio) at different percentiles of the wealth distribution in the model and the data. In the data, we rank households in terms of net worth and we backed out their equity holding as a fraction of net wealth less private business holdings – the latter is non-tradeable (like labor income). Because there is quite some time variation in these shares, we report the 2001 and 2004 numbers. Overall, the standard model tends to under-predict equity shares between the 50 and 80th percentile, but it does rather well in the left and the right tail.

[Table 9 about here.]

**Increase in the Volatility of Returns and the Equity Premium** Suppose we adopt the Mehra and Prescott (1985a) calibration instead. This means we drop the i.i.d. assumption for aggregate shocks. When we allowed for negative autocorrelation instead in the growth rate of aggregate consumption, as in Mehra and Prescott (1985a), the returns on the levered output claim become substantially more volatile (22%) and the equity premium increases to 10.8%. This is mainly the result of an increase in the volatility of the risk-free rate.[15]

As we pointed out, this contributes more volatility to stock returns and it raises the equity premium to 10.3%. This brings the HTT model closer to matching the tails of the wealth distribution. In particular, the kurtosis increases to 15.7 and the skewness increases to 3.18. And the 90/10 ratio increases to 472. Nonetheless, the middle class still accumulates too much wealth.

[Table 10 about here.]

## 5.5 Robustness

We examined the impact of relaxing the borrowing limits or increasing the tradeable income share. We find this mainly increases the risk-free rate, but has a small effect on risk premia. First, we increased the fraction of the present-value of labor income that households can borrow against, which is parameterized by $\phi$. Starting from our benchmark value of 0, risk premia fall by almost 1% for both our levered claim and the dividend security as we increase $\phi$ from 0 to 0.05. However, further increases in $\phi$ have no effect. At $\phi = .25$, the risk premium on the levered security is 1.1% lower than at $\phi = 0$. At the same time, the market price of risk, $\sigma[m]/E[m]$, falls from an average of 0.47 down to an average of 0.40, while the standard deviation of the conditional market price of risk $Std[\sigma_t[m]/E_t[m]]$ decreases from 0.05 to 0.03. However, the risk-free rate increases by 160 basis points. Thus, risk premia remained relatively high and volatile even in this extreme case; the tightness of the borrowing limits mainly impacts the risk-free rate. This points to the offloading of aggregate risk on active traders as the main driving force behind the volatile and counter-cyclical state prices, not the borrowing limits. Second, we also examined the impact of increasing the tradeable share of income. If we decrease $\gamma$ from 0.90 to 0.70, the average market price of risk dropped from 0.47 to 0.42, and the standard deviation of the conditional market price of risk decreases from 0.05 to 0.03. At the same time, the average risk premium on the levered output claim falls from 6.44% to 6.36%. However, the risk free rate increases from 1.92% to 6.53%.

Finally, we also computed a version of the economy with four aggregate states, 2 states with high average aggregate output growth and 2 states with low average aggregate output growth. We keep the average growth rate constant across regimes. The introduction of a high and a low growth regime allows us to break the tight link between aggregate consumption growth and returns in the

[15]These results are reported in the separate appendix.

benchmark model. The high growth regime has average growth that is 2 percentage points higher; in the low growth regime, it is 2 percentage point lower. With probability .3, there is a regime switch in each period. The asset pricing results we obtained in this case are similar: the risk premium on the levered output claim ($E[R_{lc}-R_f]$ ) is 8.1 % and the Sharpe ratio ($E[R_{lc}-R_f]/\sigma[R_{lc}-R_f]$) is 43 %, but the correlation between aggregate consumption growth and returns is only .43 (compared to one in the benchmark calibration). However, this comes at the cost of an increased volatility of the risk-free rate to 4%.

# 6 Conclusion

In the quantitative section of the paper, we calibrated a model with heterogeneity in trading technologies to match key moments of asset prices in the data. The heterogeneity in trading opportunities brings the standard model much closer to matching the asset class share and wealth distribution in the data. The passive traders in our model accumulate much less wealth than the active traders, even though they have identical preferences, simply because the latter are compensated for bearing the residual aggregate risk created by the non-participants. Hence, it is imperative to study the wealth and asset share distribution in a model that generates large and volatile risk premia. However, the heterogeneity in trading opportunities cannot fully account for the lack of wealth accumulation among US households that are part of the middle class. In addition, the correlation of aggregate consumption growth and returns in the model is much higher than in the data. To solve the model, we developed a new solution method that not only substantially simplifies the computations. Our multiplier approach also brings out the mechanism through which the offloading of aggregate risk on active traders affects asset prices.

# References


Abel, A. B. (1999): "Risk Premia and Term Premia in General Equilibrium," *Journal of Monetary Economics*, 43, 3–33.

Aiyagari, S. R., A. Marcet, T. J. Sargent, and J. Seppala (2002): "Optimal Taxation Without State-Contingent Debt," *Journal of Political Economy*, 110(6), 1220–1254.

Alvarez, F., and U. Jermann (2000): "Efficiency, Equilibrium, and Asset Pricing with Risk of Default.," *Econometrica*, 68(4), 775–798.

——— (2001): "Quantitative Asset Pricing Implications of Endogenous Solvency Constraints," *Review of Financial Studies*, 14, 1117–1152.

Bansal, R., and A. Yaron (2004): "Risks for the Long-Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59(4), 1481–1509.

Basak, S., and D. Cuoco (1998a): "An Equilibrium Model with Restricted Stock Market Participation," *Review of Financial Studies*, 11(2), 309–341.

——— (1998b): "An Equilibrium Model with Restricted Stock Market Participation," *Review of Financial Studies*, Vol. 11(2), pp. 309–341.

Brav, A., G. M. Constantinides, and C. C. Geczy (2002): "Asset Pricing with Heterogenous Consumers and Limited Participation: Empirical Evidence," *The Journal of Political Economy*, 110(4), 793–824.

Calvet, L., J. Y. Campbell, and P. Sodini (2006): "Down or Out: Assessing the Welfare Costs of Household Investment Mistakes," *NBER Working Paper 12030*.

Campbell, J. Y. (2006): "Household Finance," *Journal of Finance*, LXI(4), 1553–1607.

Campbell, J. Y., and J. H. Cochrane (1999): "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107(2), 205–251.

Cecchetti, S. G., P.-S. Lam, and N. C. Mark (1990): "Mean Reversion in Equilibrium Asset Prices," *American Economic Review*, Vol. 80, No. 3, 398–418.

Cuoco, D., and H. He (1994): "Dynamic Equilibrium in Infinite Dimensional Economies with Incomplete Financial Markets," Working Paper University of Pennsylvania.

Epstein, L. G., and S. Zin (1989): "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–969.

Fama, E. F., and K. R. French (2002): "The Equity Premium," *Journal of Finance*, 57(2), 637–659.

Gomez, F., and A. Michaelides (2007): "Asset Pricing with Limited Risk Sharing and Heterogeneous Agents," *Journal of Finance*, forthcoming.

Guvenen, F. (2003): "A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-Sectional Heterogeneity," *Working Paper University of Rochester*.

Kocherlakota, N., and L. Pistaferri (2005): "Asset Pricing Implications of Pareto Optimality with Private Information," *Working Paper Stanford University*.

Krueger, D., and H. Lustig (2006): "The Irrelevance of Market Incompleteness for the Price of Aggregate Risk," Working Paper UCLA and University of Pennsylvania.

Krusell, P., and A. Smith (1997): "Income and Wealth Heterogeity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomic Dynamics*, 1, 387–422.

——— (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 6, 867–896.

Lustig, H. (2006): "The Market Price of Aggregate Risk and the Wealth Distribution," Mimeo UCLA.

Lustig, H., C. Sleet, and S. Yeltekin (2006): "Fiscal hedging and the Yield Curve," Working Paper Carnegie Mellon University.

Mankiw, N. G., and S. P. Zeldes (1991): "The Consumption of Stockholders and Nonstockholders," *Journal of Financial Economics*, 29(1), 97–112.

Marcet, A., and R. Marimon (1999): "Recursive Contracts," Working Paper Universitat Pompeu Fabra.

Mehra, and E. Prescott (1985a): "The Equity Premium: A Puzzle.," *Journal of Monetary Economics*, 15(2), 145–161.

Mehra, R., and E. C. Prescott (1985b): "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15., 145–161.

Roussanov, N. (2007): "Diversification and its Discontents: Idiosyncratic and Entrepreneurial Risk in the Quest for Social Status," Working Paper Wharton School of Business.

Saito, M. (1996): "Limited Participation and Asset Prices," *Working Paper University of British Columbia*.

Storesletten, K., C. Telmer, and A. Yaron (2003): "Asset Pricing with Idiosyncratic Risk and Overlapping Generations," Mimeo.

Verarcierto, M. (1998): "Plant Level Irreversible Investment and Equilibrium Business Cycles," *Working Paper Federal Reserve Bank of Chicago*.

Vissing-Jorgensen, A. (2002): "Limited Asset Market Participation and Elasticity of Intertemporal Substitution," *The Journal of Political Economy*, 110(4), 825–853.

# A Proofs

- Proof of Lemma 3.1:

  *Proof.* Our optimality conditions (3.7, 3.8, 3.10) imply that if the borrowing constraint does not bind, then
  $$\zeta(z^t, \eta^t) = \sum_{\eta^{t+1} \succ \eta^t} \zeta(z^{t+1}, \eta^{t+1}) \pi(\eta^{t+1} | z^{t+1}, \eta^t). \tag{A.1}$$
  Hence, when the borrowing constraint doesn't bind for any possible $\eta^{t+1}$ given $z^{t+1}$, the multipliers are a Martingale. □

- Proof of Corollary 3.3:

  *Proof.* We know that $E\{\zeta(z^{t+1}, \eta^{t+1}) | z^{t+1}\} \leq \zeta(z^t, \eta^t)$. This implies that
  $$E\{\zeta^{-1/\alpha}(z^{t+1}, \eta^{t+1}) | z^{t+1}\} \geq E\{\zeta(z^{t+1}, \eta^{t+1}) | z^{t+1}\}^{-1/\alpha} = \zeta(z^t, \eta^t)^{-1/\alpha}.$$
  Assume $h(z^{t+1}) \leq h(z^t)$. Then the risk-sharing rule in (A.3) implies the unconstrained z-complete trader's consumption share increases over time.

  □

- Proof of Corollary 3.2:

  *Proof.* Follows directly from the risk sharing rule. □

- Proof of Proposition 3.1:

*Proof.* Condition (3.8) implies that

$$c(z^t, \eta^t) = u'^{-1}\left[\beta^{-t}\zeta(z^t, \eta^t)P(z^t)\right].$$

In addition, the sum of individual consumptions aggregate up to aggregate consumption

$$C(z^t) = \sum_{\eta^t} c(z^t, \eta^t)\pi(\eta^t|z^t).$$

This implies that the consumption share of the individual with history $(z^t, \eta^t)$ is

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{u'^{-1}\left[\beta^{-t}\zeta(z^t, \eta^t)P(z^t)\right]}{\sum_{\eta^t} u'^{-1}\left[\beta^{-t}\zeta(z^t, \eta^t)P(z^t)\right]\pi(\eta^t|z^t)}. \tag{A.2}$$

With CRRA preferences, this implies that the consumption share is given by

$$\frac{c(z^t, \eta^t)}{C(z^t)} = \frac{\zeta(z^t, \eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}, \text{ where } h(z^t) = \sum_{\eta^t} \zeta(z^t, \eta^t)^{\frac{-1}{\alpha}}\pi(\eta^t|z^t). \tag{A.3}$$

Hence, the $-1/\alpha^{\text{th}}$ moment of the multipliers summaries risk sharing within this economy. And, with this moment we get a simple linear risk sharing rule with respect to aggregate consumption.

Making use of (A.2) and the individual first-order condition, we get that

$$\beta^t u'\left[\frac{u'^{-1}\left[\beta^{-t}\zeta(z^t, \eta^t)P(z^t)\right]}{\sum_{\eta^t} u'^{-1}\left[\beta^{-t}\zeta(z^t, \eta^t)P(z^t)\right]\pi(\eta^t|z^t)}C(z^t)\right] = P(z^t)\zeta(z^t, \eta^t).$$

If $u'^{-1}$ is homogeneous, which it is with CRRA preferences, then this expression simplifies to

$$\beta^t u'\left[\frac{C(z^t)}{\sum_{\eta^t} u'^{-1}\left[\zeta(z^t, \eta^t)\right]\pi(\eta^t|z^t)}\right] = P(z^t),$$

which implies that the ratio of the state prices is given by

$$\frac{\beta u'\left[\frac{C(z^{t+1})}{\sum_{\eta^t} u'^{-1}[\zeta(z^{t+1}, \eta^{t+1})]\pi(\eta^{t+1}|z^{t+1})}\right]}{u'\left[\frac{C(z^t)}{\sum_{\eta^t} u'^{-1}[\zeta(z^t, \eta^t)]\pi(\eta^t|z^t)}\right]} = \frac{P(z^{t+1})}{P(z^t)}. \tag{A.4}$$

Given that we are assuming CRRA preferences, this implies the following proposition. □

- Proof of Corollary 3.1:

*Proof.* To see this, note that if we use the risk sharing rule in equation (A.3), we obtain that the $-\alpha$-th power of consumption for an individual household is:

$$c(z^t, \eta^t)^{-\alpha} = \frac{\zeta(z^t, \eta^t)}{h(z^t)^{-\alpha}}C_t(z^t)^{-\alpha}.$$

Next, we define $C^*$ as the $-\alpha^{\text{th}}$ moment of the consumption distribution, or

$$C^*(z^t) = \sum_{\eta^t} c(z^t, \eta^t)^{-\alpha} \frac{\pi(z^t, \eta^t)}{\pi(z^t)} = \frac{C_t(z^t)^{-\alpha}}{h(z^t)^{-\alpha}} \sum_{\eta^t} \zeta(z^t, \eta^t) \frac{\pi(z^t, \eta^t)}{\pi(z^t)},$$

and, we compute the growth rate of the $-\alpha$-th power of consumption:

$$\beta \left( C^*(z^{t+1})/C^*(z^t) \right) = \frac{\beta \left( \frac{C(z^{t+1})}{h(z^{t+1})} \right)^{-\alpha}}{\left( \frac{C(z^t)}{h(z^t)} \right)^{-\alpha}} \left( \frac{\frac{\sum_{\eta^{t+1}} \pi(z^{t+1}, \eta^{t+1})}{\pi(z^{t+1})} \zeta_{t+1}}{\frac{\sum_{\eta^t} \pi(z^t, \eta^t)}{\pi(z^t)} \zeta_t} \right),$$

where the last term is equal to one if the borrowing constraints do not bind, and smaller than one otherwise. This follows from the martingale condition for z-complete and complete traders. For the diversified traders, we know that the last term is one if we sum across aggregate states and multiply by the diversifiable income claim return

$$\zeta(z^t, \eta^t) = \sum_{z^{t+1} \succ z^t, \eta^{t+1} \succ \eta^t} \zeta(z^{t+1}, \eta^{t+1}) \widetilde{\pi}(z^{t+1}, \eta^{t+1} | z^t, \eta^t)$$

This in turn implies that

$$\beta \left( C^*(z^{t+1})/C^*(z^t) \right) \leq m(z^{t+1}|z^t).$$

for complete and z-complete traders and that:

$$E_t \left[ \beta \left( C^*(z^{t+1})/C^*(z^t) \right) R(z^{t+1}) \right] \leq E_t \left[ m(z^{t+1}|z^t) R(z^{t+1}) \right] = 1.$$

for diversified traders. □

- Proof of Proposition 4.1:

*Proof.* Conjecture that $\frac{h(z^{t+1})}{h(z^t)} = g_{t+1}$ is a non-random sequence. Normalize $h_t$ to one. Conjecture that $S(\zeta(z^t, \eta^t); z^t, \eta^t)$ does not depend on $z^t$. Given conditions (2) and (3), we know that

$$\tilde{S}_t(\zeta(\eta^t); \eta^t) = \left[ \gamma \eta_t - \zeta(\eta^t)^{\frac{-1}{\alpha}} \right] + \widehat{\beta}_t \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta_t) \tilde{S}_{t+1}(\zeta(\eta^{t+1}); \eta^{t+1}), \tag{A.5}$$

where $\widehat{\beta}_t = \beta \sum_{z_{t+1}} \phi(z_{t+1}) g_{t+1}^{\gamma} \exp((1-\gamma) z_{t+1})$ and $\lambda(z_{t+1})$ is defined as the growth rate $\frac{Y_{t+1}}{Y_t}$. In addition, our debt constraint in terms of $\tilde{S}$ is simply

$$\tilde{S}_t(\zeta(\eta^t); \eta^t) \leq \underline{M}(\eta^t). \tag{A.6}$$

Note that neither the recursion (A.5) or the debt constraint (A.6) depend upon the value of the realization of $z_t$. For $z$-complete traders, the measurability condition is given by

$$\tilde{S}_t(\zeta(\eta^{t+1}); \eta^{t+1}) = \tilde{S}_t(\zeta(\tilde{\eta}^{t+1}); \tilde{\eta}^{t+1}) \tag{A.7}$$

for all $\eta^{t+1}$, $\tilde{\eta}^{t+1}$ and $z^{t+1}$ where $\eta^t(\eta^{t+1}) = \eta^t(\tilde{\eta}^{t+1})$. Their optimality condition is still (A.1). Hence, none of the equations determining either $\tilde{S}$ or the updating rule for $\zeta$ depend on $z_{t+1}$. This is also true for the complete traders, since their measurability condition is degenerate, and their optimality condition is (4.6). The dynamics of the multipliers on the measurability constraints and

the solvency constraints do not depend on $z^t$, only on $\eta^t$. This confirms that $\{h_t\}$ does not depend on the aggregate history of shocks $\{z^t\}$, and hence is a non-random sequence.

This independence is also true for the diversified investors. The reason is that their measurability condition is given by

$$\frac{\tilde{S}_{t+1}(\zeta(z^{t+1},\eta^{t+1});z^{t+1},\eta^{t+1})}{[(1-\gamma)+\varpi_{t+1}(z^{t+1})/Y(z^{t+1})]}=\frac{\tilde{S}_{t+1}(\zeta(\tilde{z}^{t+1},\tilde{\eta}^{t+1});\tilde{z}^{t+1},\tilde{\eta}^{t+1})}{[(1-\gamma)+\varpi_{t+1}(\tilde{z}^{t+1})/Y(\tilde{z}^{t+1})]}, \tag{A.8}$$

for all for all $\eta^{t+1}$ and $\tilde{\eta}^{t+1}$, $z^{t+1}$ and $\tilde{z}^{t+1}$ where $\eta^t(\eta^{t+1})=\eta^t(\tilde{\eta}^{t+1})$ and $z^t(z^{t+1})=z^t(\tilde{z}^{t+1})$. Hence, the independence holds iff $\varpi_{t+1}(z^{t+1})/Y(z^{t+1})$ is deterministic, i.e. does not depend on $z^{t+1}$. Given conditions (2) and (3), and given our conjecture that $\{h_t\}$ is deterministic, it is easy to show that $\tilde{\varpi}_t$ is deterministic as well, because no arbitrage implies that:$\tilde{\varpi}_t=1+\widehat{\beta}\tilde{\varpi}_{t+1}$.

□

- Proof of proposition 4.2:

*Proof.* First, since the measurability constraints are satisfied for the individual household's savings function, they also need to be satisfied for the aggregate savings function. So by the LLN:

$$\frac{S_a^{div}(z^{t+1})}{[(1-\gamma)Y(z^{t+1})+\varpi(z^{t+1})]} \;=\; \frac{S_a^{div}(z^t,\tilde{z}_{t+1})}{[(1-\gamma)Y(z^t,\tilde{z}_{t+1})+\varpi(z^t,\tilde{z}_{t+1})]}$$

where we have used the fact that the denominator is measurable w.r.t. $z^t$. Note that

$$\sum_k S_a^k(z^{t+1})=-\left[(1-\gamma)Y(z^t,\tilde{z}_{t+1})+\varpi(z^t,\tilde{z}_{t+1})\right].$$

Hence the ratio

$$S_a^{div}(z^{t+1})/\sum_k S_a^k(z^{t+1})=\kappa(z^t)$$

cannot not depend on $z_t$, because of the *measurability* condition. □

- Proof of proposition 4.3:

*Proof.* For non-participant traders $j=np$, $S_a^j(z^t)$ cannot not depend on $z_t$, because of the *measurability* condition. □

Table 1: Asset Pricing and Portfolio Implications

| | Market Segmentation | | |
|---|---|---|---|
| *complete* | $\mu_1$ | $\mu_1$ | $\mu_1$ |
| *z-complete* | $\mu_2$ | $\mu_2$ | $\mu_2$ |
| *diversified* | $\mu_3$ | 0 | $\mu_3$ |
| *non-part* | 0 | 0 | $\mu_4$ |
| | Asset Prices | | |
| $R^e$ | $RA$ | $RA$ | $\neq RA$ |
| $R^f$ | $< RA$ | $< RA$ | $< RA$ |
| | Portfolios | | |
| *complete* | $Market$ | $Market$ | $Levered$ |
| *z-complete* | $Market$ | $Market$ | $Levered$ |
| *diversified* | $Market$ | $Market$ | $Market$ |
| *non-part* | / | / | $Bonds$ |

Table 2: Approximation

| | *Case 1* | *Case 2* | *Case 3* |
|---|---|---|---|
| *complete* | 0% | 5% | 10% |
| *z-complete* | 10% | 5% | 0% |
| *diversified* | 20% | 20% | 20% |
| *non-part* | 70% | 70% | 70% |
| $z' = l, z = l$ | 31.5 | 57.5 | 3.1 |
| $z' = h, z = l$ | 32.2 | 53.1 | 1.0 |
| $z' = l, z = h$ | 15.7 | 22.5 | 4.5 |
| $z' = h, z = h$ | 27.9 | 18.3 | 9.5 |
| $\sup \frac{std([h'/h])}{E([h'/h])}(\%)$ | 0.579 | 0.309 | 0.287 |

*Notes:* Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 10000 draws from an economy with 3000 agents. Benchmark calibration idiosyncratic shocks and IID calibration of aggregate shocks. The first panel reports the $R^2$ in a regression of the $\log SDF_t$ on the mean of the wealth distribution $E(\log W)_t$. The second panel reports the maximal coefficient of variation across all aggregate truncated histories of the actual aggregate multiplier growth rate $[h'/h]$ in percentages.

Table 3: Asset Pricing

| | **RA Economy** | **HTT Economy** | | | **Data** |
|---|---|---|---|---|---|
| | | *Case 1* | *Case 2* | *Case 3* | |
| *complete* | | 0% | 5% | 10% | |
| *z-complete* | | 10% | 5% | 0% | |
| *diversified* | | 20% | 20% | 20% | |
| *non-part* | | 70% | 70% | 70% | |
| $E[R_f]$ | 12.96 | 1.737 | 1.922 | 2.185 | 1.049 |
| $\sigma[R_f]$ | 0.000 | 0.066 | 0.237 | 0.292 | 1.560 |
| $\sigma[m]/E[m]$ | 0.193 | 0.440 | 0.467 | 0.510 | |
| $Std[\sigma_t[m]/E_t[m]]$ | 0.000 | 0.033 | 0.045 | 0.058 | |
| $E[R_{lc} - R_f]$ | 3.081 | 6.702 | 6.435 | 6.874 | 7.531 |
| $\sigma[R_{lc} - R_f]$ | 15.94 | 15.27 | 13.89 | 13.69 | 16.94 |
| $E[R_{lc} - R_f]/\sigma[R_{lc} - R_f]$ | 0.193 | 0.438 | 0.463 | 0.502 | 0.444 |
| $E[W^{Coll}/C]$ | 0.855 | 5.960 | 4.889 | 6.458 | 3.870 |
| $E[PD_{lc}]$ | 7.936 | 20.98 | 18.02 | 23.16 | 33.87 |
| $\sigma[PD_{lc}]$ | 13.09 | 15.92 | 15.59 | 15.20 | 16.78 |
| $E[R_b - R_f]$ | 0.000 | −0.271 | −0.046 | −0.437 | 1.070 |
| $\sigma[R_b - R_f]$ | 0.000 | 0.604 | 0.143 | 0.935 | 9.366 |
| $E[R_b - R_f]/\sigma[R_b - R_f]$ | / | −0.324 | −0.467 | −0.449 | .1145 |
| $\rho[R_{lc}(t), R_{lc}(t-1)]$ | 0.000 | −0.015 | −0.010 | −0.010 | -0.191 |
| $\rho[R_{lc}(t), R_{lc}(t-1)]$ | 0.000 | 0.003 | 0.012 | −0.005 | -0.191 |
| $\rho[R_{lc}, R_f]$ | 0.000 | −0.024 | −0.014 | −0.020 | 0.272 |

*Notes:* Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 10000 draws from an economy with 3000 agents. Benchmark calibration idiosyncratic shocks and IID calibration of aggregate shocks. Reports the moments of asset prices for the RA (Representative Agent) economy, for the HTT (Heterogeneous Trading Technology) economy and for the data. We use post-war US annual data for 1946-2005. The market return is the CRSP value weighted return for NYSE/NASDAQ/AMEX. We use the Fama risk-free rate series from CRSP (average 3-month yield). To compute the standard deviation of the risk-free rate, we compute the annualized standard deviation of the ex post real monthly risk-free rate. The return on the long-run bond is measured using the Bond Total return index for 30-year US bonds from Global Financial Data.

Table 4: Household Portfolio Returns

| | *Case 1* | *Case 2* | *Case 3* |
|---|---|---|---|
| *complete* | 0% | 5% | 10% |
| *z-complete* | 10% | 5% | 0% |
| *diversified* | 20% | 20% | 20% |
| *non-part* | 70% | 70% | 70% |
| $E[R_c^W - R_f]$ | NA | 0.107 | 0.126 |
| $E[R_z^W - R_f]$ | 0.056 | 0.058 | NA |
| $E[R_{div}^W - R_f]$ | 0.015 | 0.015 | 0.016 |
| $E[R_{np}^W - R_f]$ | 0.000 | 0.000 | 0.000 |
| $E[R_c^W - R_f]/\sigma[R_c^W - R_f]$ | NA | 0.077 | 0.136 |
| $E[R_z^W - R_f])/\sigma[R_z^W - R_f]$ | 0.413 | 0.447 | NA |
| $E[R_{div}^W - R_f]/\sigma[R_{div}^W - R_f]$ | 0.413 | 0.436 | 0.471 |
| $E[R_{np}^W - R_f]/\sigma[R_{np}^W - R_f]$ | 0.000 | 0.000 | 0.000 |

*Notes:* Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 10000 draws from an economy with 3000 agents. Benchmark calibration of idiosyncratic shocks and IID calibration of aggregate shocks. Reports the moments of average returns on the portfolio of each trader. These are the moments of *average* portfolio returns for all the traders in a segment.

Table 5: Consumption

| | *Case 1* | *Case 2* | *Case 3* | | *Case 1* | *Case 2* | *Case 3* |
|---|---|---|---|---|---|---|---|
| *complete* | 0% | 5% | 10% | | 0% | 5% | 10% |
| *z-complete* | 10% | 5% | 0% | | 10% | 5% | 0% |
| *diversified* | 20% | 20% | 20% | | 20% | 20% | 20% |
| *non-part* | 70% | 70% | 70% | | 70% | 70% | 70% |
| | **Household Consumption** | | | | **Aggregate Consumption** | | |
| $\sigma\left[\Delta\log(c_c)\right]$ | NA | 5.641 | 5.417 | $\sigma[\Delta\log(\widehat{C}_c)]$ | NA | 3.840 | 4.873 |
| $\sigma\left[\Delta\log(c_z)\right]$ | 7.892 | 7.131 | NA | $\sigma[\Delta\log(\widehat{C}_z)]$ | 3.972 | 4.402 | NA |
| $\sigma\left[\Delta\log(c_{div})\right]$ | 11.44 | 11.35 | 11.07 | $\sigma[\Delta\log(\widehat{C}_{div})]$ | 0.334 | 0.329 | 0.368 |
| $\sigma\left[\Delta\log(c_{np})\right]$ | 12.62 | 12.50 | 12.35 | $\sigma[\Delta\log(\widehat{C}_{np})]$ | 1.062 | 1.037 | 1.071 |
| $\rho\left[R_s,(\Delta\log(\widehat{c}_p)\right]$ | 0.163 | 0.204 | 0.283 | | | | |
| $\rho\left[R_s,(\Delta\log(\widehat{c}_c)\right]$ | NA | 0.649 | 0.857 | $\rho[R_s,\Delta\log(\widehat{C}_c)]$ | NA | 0.949 | 0.951 |
| $\rho\left[R_s,(\Delta\log(\widehat{c}_z)\right]$ | 0.482 | 0.588 | NA | $\rho[R_s,\Delta\log(\widehat{C}_z)]$ | 0.965 | 0.956 | NA |
| $\rho\left[R_s,(\Delta\log(\widehat{c}_{div})\right]$ | 0.003 | −0.002 | −0.003 | $\rho[R_s,\Delta\log(\widehat{C}_{div})]$ | 0.119 | −0.083 | −0.117 |
| $\rho\left[R_s,(\Delta\log(\widehat{c}_{np})\right]$ | −0.071 | −0.070 | −0.073 | $\rho[R_s,\Delta\log(\widehat{C}_{np})]$ | −0.965 | −0.959 | −0.948 |
| $\sigma(\Delta\log(c_c))$ | 8.652 | 8.411 | NA | $\sigma(\Delta\log(C_c))$ | 8.310 | 7.280 | NA |
| $\sigma(\Delta\log(c_z))$ | NA | 9.72 | 10.17 | $\sigma(\Delta\log(C_z))$ | NA | 7.852 | 7.438 |
| $\sigma(\Delta\log(c_{div}))$ | 11.83 | 12.10 | 12.21 | $\sigma(\Delta\log(C_{div}))$ | 3.545 | 3.556 | 3.622 |
| $\sigma(\Delta\log(c_{np}))$ | 12.83 | 13.00 | 13.11 | $\sigma(\Delta\log(C_{np}))$ | 2.560 | 2.582 | 2.554 |
| $\rho\left[R_s,(\Delta\log(c_p)\right]$ | 0.431 | 0.463 | 0.503 | | | | |
| $\rho\left[R_s,(\Delta\log(c_c)\right]$ | NA | 0.843 | 0.920 | | | | |
| $\rho\left[R_s,(\Delta\log(c_z)\right]$ | 0.712 | 0.784 | NA | | | | |
| $\rho\left[R_s,(\Delta\log(c_{div})\right]$ | 0.291 | 0.287 | 0.291 | | | | |
| $\rho\left[R_s,(\Delta\log(c_{np})\right]$ | 0.200 | 0.203 | 0.209 | | | | |

*Notes:* Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 10000 draws from an economy with 3000 agents. Benchmark calibration of idiosyncratic shocks and IID calibration of aggregate shocks. The first panel reports the moments for average household consumption share (share of aggregate endowment) growth in each trader segment. The second panel reports the moments for the growth rate of the aggregate consumption share of each trader segment. The third panel reports the moments of average returns on the portfolio of each trader. These are the moments of *average* portfolio returns for all the traders in a segment. Hatted variables denote shares of the aggregate endowment.

Table 6: Household Wealth and Consumption

| | **Returns** | | **Wealth** | |
|---|---|---|---|---|
| | $E[R_i^W - R_f]$ | $E[R_i^W - R_f]/\sigma[R_i^W - R_f]$ | $E[W_i/W]$ | $\sigma[W_i/W]$ |
| $z-complete$ | 0.056 | 0.413 | 2.847 | 1.390 |
| $div$ | 0.015 | 0.413 | 0.966 | 0.686 |
| $np$ | 0.000 | 0.000 | 0.745 | 0.565 |
| | **Household Consumption** | | **Aggregate Consumption** | |
| | $\sigma[\Delta\log(\widehat{c}_i)]$ | $\rho[R_s,(\Delta\log(\widehat{c}_i)]$ | $\sigma[\Delta\log(\widehat{C}_i)]$ | $\rho[R_s,(\Delta\log(\widehat{C}_i)]$ |
| $p$ | / | 0.142 | | |
| $z-complete$ | 7.7641 | 0.424 | 4.616 | 0.964 |
| $div$ | 11.589 | 0.003 | 0.406 | 0.119 |
| $np$ | 13.299 | $-0.063$ | 1.471 | $-0.964$ |
| | $\sigma[\Delta\log(c_i)]/\sigma[\Delta\log(C)]$ | $\rho[R_s,(\Delta\log(c_i)]$ | $\sigma[\Delta\log(C_i)]$ | $\rho[R_s,(\Delta\log(C_i)]$ |
| $p$ | | 0.439 | | |
| $z-complete$ | 2.852 | 0.724 | 7.438 | 1.000 |
| $div$ | 3.423 | 0.296 | 3.622 | 1.000 |
| $np$ | 3.765 | 0.204 | 2.554 | 1.000 |
| | $EIS_{R_b}$ | $EIS_{R_{eq}}$ | | |
| $div$ | 1.520 | 0.320 | | |
| $z-complete$ | 1.593 | 0.390 | | |

*Notes:* Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 10000 draws from an economy with 3000 agents. Benchmark calibration of idiosyncratic shocks and aggregate shocks. Hatted variables denote shares of aggregate consumption. $EIS_{R_b}$ and $EIS_{R_{eq}}$ are the EIS estimates obtained by regressing average household consumption growth on bond returns and stock return using data generated by the MP model. We use the log dividend-price ratio, the lagged real stock return and the bond risk premium as instruments.

Table 7: Matching Income Distribution

| | **Model** | **US Data** |
|---|---|---|
| 75/50 | 2.739 | 1.785 |
| 80/50 | 2.893 | 2.041 |
| 85/50 | 3.062 | 2.414 |
| 90/50 | 3.353 | 2.908 |
| 75/25 | 4.136 | 3.449 |
| 80/25 | 4.369 | 3.943 |
| 85/25 | 4.624 | 4.663 |
| 90/25 | 5.063 | 5.618 |
| 80/20 | 4.613 | 4.710 |
| 85/15 | 6.537 | 7.024 |
| 90/10 | 11.42 | 11.64 |

*Notes:* Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 10000 draws from an economy with 3000 agents. Benchmark calibration of aggregate and idiosyncratic shocks. The income data are from the 2004 SCF.

Table 8: Household Wealth Distribution

| | **Bewley Model** | | **HTT Model** | | **US Data 2004** | |
|---|---|---|---|---|---|---|
| | **Wealth** | | **Wealth** | | **Net Worth** | **Total Assets** |
| | Standard | Twisted | Standard | Twisted | | |
| *kurtosis* | 1.96 | 2.84 | 6.97 | 12.92 | 15.87 | 48.85 |
| *skewness* | 0.23 | 0.88 | 1.80 | 2.73 | 3.616 | 6.250 |
| *Gini* | 0.40 | 0.48 | 0.53 | 0.57 | 0.793 | 0.697 |
| $W_{75}/W_{25}$ | 4.03 | 5.42 | 6.37 | 6.90 | 25.09 | 10.64 |
| $W_{80}/W_{20}$ | 6.38 | 9.09 | 11.28 | 12.49 | 65.41 | 33.42 |
| $W_{85}/W_{15}$ | 12.63 | 19.12 | 26.11 | 29.85 | 211.9 | 55.75 |
| $W_{90}/W_{10}$ | 48.14 | 82.20 | 182.64 | 240.5 | 999.1 | 580.5 |
| $W_{50}/W_{10}$ | 23.18 | 25.33 | 53.32 | 65.02 | 105.0 | 91.00 |
| $W_{90}/W_{50}$ | 2.077 | 3.22 | 3.42 | 3.69 | 9.510 | 6.378 |

*Notes:* Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 10000 draws from an economy with 3000 agents. Benchmark calibration of idiosyncratic shocks and aggregate shocks. The wealth data are from the 2004 SCF. The HTT model has 10% z-complete traders, 20% diversified traders and 70 % non-participants. The Bewley model has 100% diversified traders.

Table 9: Equity Share Distribution

| | **Data** | | **Model** | |
|---|---|---|---|---|
| Percentile | 2001 | 2004 | Standard | Twisted |
| 15% | 4.512 | 2.633 | 5.694 | 3.942 |
| 25% | 15.40 | 6.797 | 6.617 | 3.293 |
| 35% | 6.057 | 6.669 | 7.331 | 3.722 |
| 50% | 8.077 | 2.762 | 6.817 | 3.115 |
| 65% | 11.09 | 10.16 | 6.572 | 8.207 |
| 75% | 19.04 | 10.12 | 7.962 | 11.02 |
| 80% | 14.45 | 17.34 | 9.204 | 10.08 |
| 85% | 24.16 | 16.56 | 13.11 | 9.263 |
| 90% | 32.59 | 18.94 | 27.50 | 12.78 |
| 95% | 34.30 | 25.37 | 52.02 | 41.86 |
| 100% | 42.67 | 34.19 | 59.02 | 59.80 |

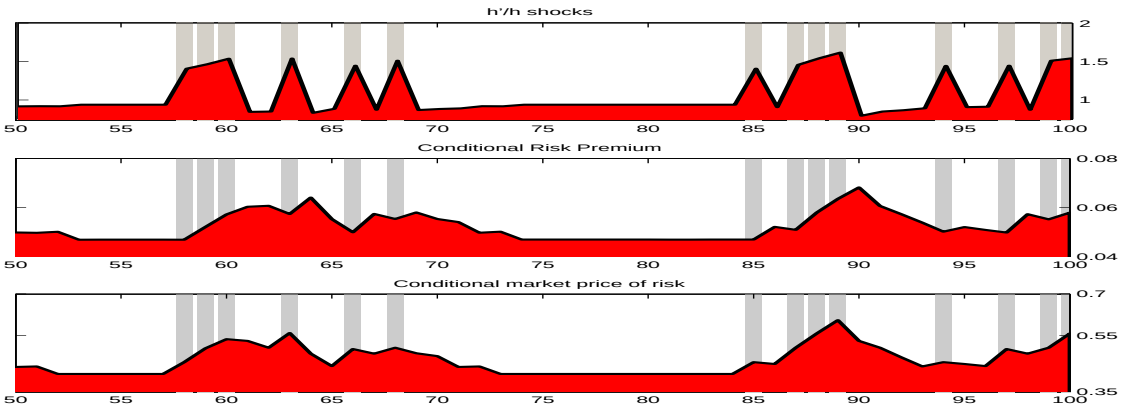
*Notes:* Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 10000 draws from an economy with 3000 agents. Benchmark calibration of idiosyncratic shocks and aggregate shocks. The wealth data are from the 2001 and 2004 SCF. The equity share reported is the share of equity as a fraction of net worth less private business holdings.

Table 10: Household Wealth Distribution in MP Economy

| | **Bewley Model** | | **HTT Model** | | **US Data 2004** | |
|---|---|---|---|---|---|---|
| | **Wealth** | | **Wealth** | | **Net Worth** | **Total Assets** |
| | Standard | Twisted | Standard | Twisted | | |
| *kurtosis* | 1.956 | 2.842 | 10.43 | 15.78 | 15.87 | 48.85 |
| *skewness* | 0.231 | 0.882 | 2.398 | 3.189 | 3.616 | 6.250 |
| *Gini* | 0.405 | 0.486 | 0.513 | 0.587 | 0.793 | 0.697 |
| $W_{75}/W_{25}$ | 4.124 | 5.529 | 5.030 | 6.967 | 25.09 | 10.64 |
| $W_{80}/W_{20}$ | 6.623 | 9.409 | 8.803 | 13.31 | 65.41 | 33.42 |
| $W_{85}/W_{15}$ | 13.34 | 19.12 | 21.78 | 36.15 | 211.9 | 55.75 |
| $W_{90}/W_{10}$ | 54.35 | 82.20 | 252.0 | 472.6 | 999.1 | 580.5 |
| $W_{50}/W_{10}$ | 26.10 | 25.33 | 103.4 | 134.8 | 105.0 | 91.00 |
| $W_{90}/W_{50}$ | 2.082 | 3.245 | 2.436 | 3.505 | 9.510 | 6.378 |

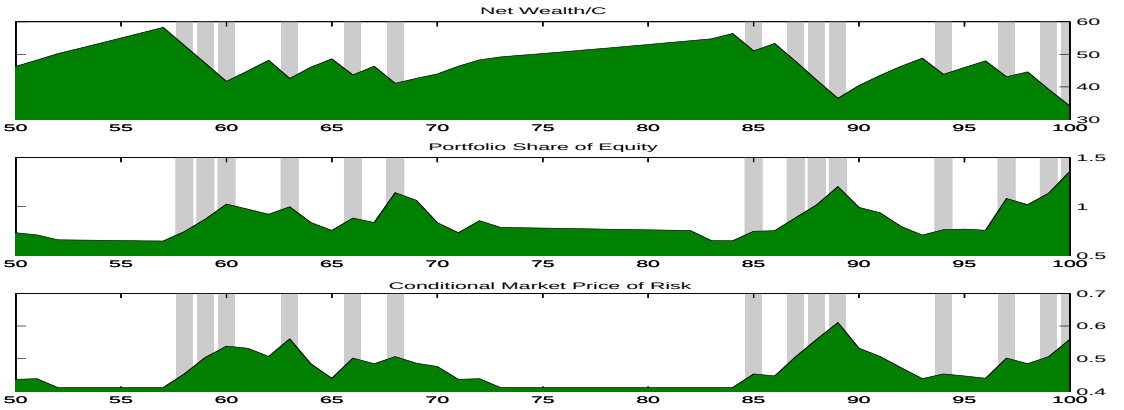
*Notes:* Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 10000 draws from an economy with 3000 agents. Benchmark calibration of idiosyncratic shocks and MP calibration of aggregate shocks. The wealth data are from the 2004 SCF. The HTT model has 10% z-complete traders, 20% diversified traders and 70% non-participants. The Bewley model has 100% diversified traders.

Figure 1: Conditional Risk Premium and Market Price of Risk



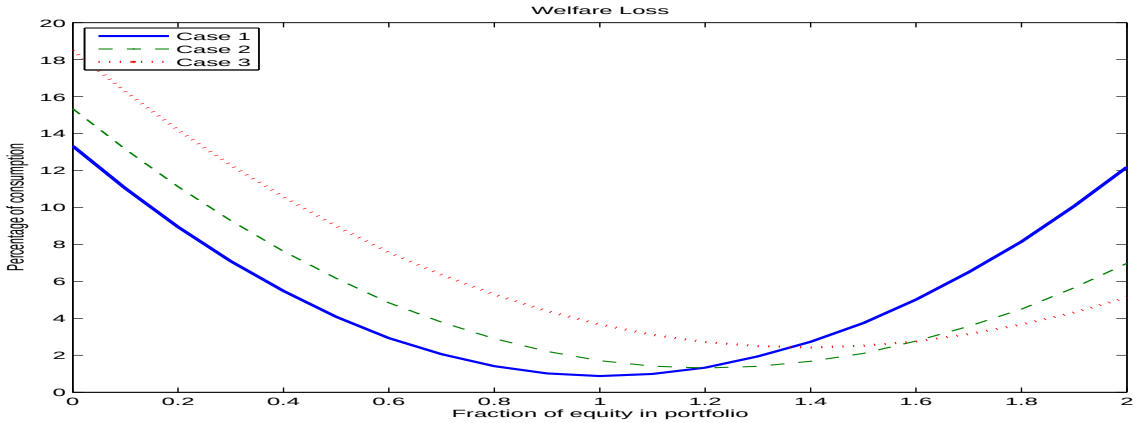

*Notes:* Market Segmentation: 5% complete, 5% in z-complete, 20% diversified and 70% non-participants. Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. Plot of 50 draws from an economy with 3000 agents. Benchmark calibration of idiosyncratic shocks and IID calibration of idiosyncratic shocks. The shaded are indicates low aggregate consumption growth states.

Figure 2: Equity Share




*Notes:* Market Segmentation: 5% complete, 5% in z-complete, 20% diversified and 70% non-participants. Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1. The simulation moments are generated by 100 draws from an economy with 3000 agents. Benchmark calibration of idiosyncratic and IID calibration of aggregate shocks. The shaded areas indicate low aggregate consumption growth states.

Figure 3: Equity Share




*Notes:* Parameters setting: $\gamma = 5$, $\beta = 0.95$, collateralized share of income is 0.1 . The simulation moments are generated by 100 draws from an economy with 3000 agents. Benchmark calibration of idiosyncratic shocks and aggregate shocks. Case 1: 0/10/20 (complete/z-complete/diversified) composition of trader segments. Case 2: 5/5/20 composition. Case 3: 10/0/20 composition.