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SCHOOL FINANCE REFORM:
ASSESSING GENERAL EQUILIBRIUM EFFECTS

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ABSTRACT

In 1994 the state of Michigan implemented one of the most comprehensive school finance reforms undertaken to date in any of the states. Understanding the effects of the reform is thus of value in informing other potential reform initiatives. In addition, the reform and associated changes in the economic environment provide an opportunity to assess whether a simple general equilibrium model can be of value in framing the study of such reform initiatives. In this paper, we present and use such a model to derive predictions about the effects of the reform on housing prices and neighborhood demographic compositions. Broadly, our analysis implies that the effects of the reform and changes in the economic environment are likely to have been reflected primarily in housing prices and only modestly on neighborhood demographics. We find that evidence for the Detroit metropolitan area from the decade encompassing the reform is largely consistent with the predictions of the model.

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1. Introduction

The last thirty years have witnessed intense activity in school finance reform. Numerous states have centralized, in varying degrees, the funding of their public schools. While most states have done so prompted by their state courts, in 1993 Michigan centralized public school funding through the decision of its Legislature. Instead of being determined by individual school districts, school revenues are now determined by the state through a foundation grant system. The new system has given large absolute and relative revenue increases to low-revenue districts and has capped revenues of high-revenue districts. In addition, property taxes have been dramatically reduced and state taxes increased.

The reform had two clear goals: to lower property tax burdens and to reduce variation in revenues across districts. Both goals were accomplished. However, a reform of this kind and reach has the potential of leading to other effects as well. In metropolitan areas where public schools have residence requirements, households choose locations and schools jointly given the housing prices, property taxes and public school qualities that prevail in the different jurisdictions. By affecting property tax rates and public school revenues, this type of reform can alter housing prices and public school quality. In addition, the changes in relative school funding levels (and possibly qualities) across districts might induce households to move from one district to another.

We examine whether these general equilibrium effects took place in the Detroit metropolitan area,[2] which is the largest metropolitan area in Michigan and comprises about 41 percent of the state population. We use a multi-community equilibrium model to develop qualitative predictions about the effects of the reform, and investigate whether these predictions hold empirically. For analytical purposes we decompose the reform into two elements: a tax reform, and a change in the level and distribution of revenue. Furthermore, over the nineties the metropolitan area experienced a significant change in the shape of the income distribution. Although all segments of the distribution experienced an increase in real income, the lower and upper segments benefited from greater proportional increases.[3] To gain insight into the effects of these income changes,

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[2] Throughout we define the Detroit metropolitan area as the counties of Wayne, Macomb and Oakland.
[3] Using national data, Autor et al (2005) document similar changes in the U.S. income distribution.

we decompose them into a proportional increase in all incomes, and a mean-preserving income change. Controlling for the effects of changes in the metropolitan income distribution proves to be important for empirically assessing the effects of the reform.

We investigate the predictions of the model using data from the Detroit metropolitan area before and after the Michigan school reform, and find empirical support for them. This, in turn, has important policy implications. A central insight from the model is that, unless the revenue component of the reform alters the preexisting ordering of districts by revenue, the primary effects of the reform will be reflected in property values with relatively little impact on household location. Changes in property value may be accompanied by changes in school quality to the extent that expenditures impact quality. However, absent household relocation, such expenditure changes will not be accompanied by demographic changes that might affect peer qualities. This prediction rests on assumptions about the geographic distribution of housing, assumptions that we argue are likely to hold at least approximately in many metropolitan areas.

The Michigan reform created little incentive for relocation in the Detroit metropolitan area because it did not alter the pre-reform ranking of revenue across districts. In all fairness, the expenditure changes mostly aimed at small, rural districts (Courant and Loeb (1997)), so it should not come as a surprise that the Detroit metropolitan area did not see greater revenue gains. However, the very design of the reform limited the kind of general equilibrium relocation effects described above, which, via peer quality changes, might have further helped low-achieving urban districts such as Detroit Public Schools. Although the district's fourth grade pass rate in math tests rose from 16 to 50 percent between 1991 and 1999, and the seventh grade pass rate in math tests rose from 8.6 to 34.5 percent, this only meant going from the sixth to the thirteenth lowest place in fourth grade math among the 83 districts in the metropolitan area, and from the third to the fourth lowest place in seventh grade math. Moreover, as we discuss later, there is reason for caution in taking these measured gains at face value.

This paper makes several contributions to the analysis of school finance reform. First, we build on the work of researchers who have examined the general equilibrium effects of school finance reform using calibrated models (see, for instance, Nechyba

(2004), Fernandez and Rogerson (1998, 2003)), by empirically evaluating these effects.[4] Second, we extend the work of researchers who have focused on specific types of effects emerging from the Michigan reform (achievement in Cullen and Loeb (2004), Papke (2005) and Roy (2003), capitalization in Guilfoyle (1998a, 1998b)), by studying general equilibrium effects and providing an analytical framework to this end. Third, we study a specific metropolitan area, Detroit, rather than the entire state (Roy (2004)) in order to focus on a geographic context in which households have access to roughly the same set of residential and school choices. Finally, our analysis provides insights that are relevant not only to Michigan but also to other states, given that a number of recent state aid reforms have reduced property taxes while increasing state funding, and have embraced a state aid system with some similarities to the one adopted in Michigan (Yinger (2004)).

Furthermore, this paper makes a contribution to the analysis of the effects of changes in the metropolitan area income distribution on house prices, an issue which has not been, to our knowledge, systematically explored thus far. This framework may prove helpful in shedding light on the dynamics of house value appreciation and its variation across metropolitan areas. While being of interest in its own right, this analysis also provides a method for studying the effects of school reform while controlling empirically for changes in the income distribution.

The rest of this paper is organized as follows: section 2 describes public school funding in Michigan. Section 3 presents our theoretical model and Section 4 presents some stylized evidence that support our theoretical predictions. Section 5 concludes.

2. Public School Funding in Michigan

In 1990,[5] the Detroit metropolitan area, which comprises eighty-three school districts, had a population of about 3.93 million. The largest district is Detroit Public Schools, which overlaps with the city of Detroit and had a population of about a million and a K-

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[4] Loeb (2001) calibrates her model to Michigan data in a partial equilibrium framework.

[5] In what follows, demographic data refer to Census year, and school-related data to the Fall of the corresponding school year. Since demographic data are only available for Census years, 1990 data are the closest available measure for the pre-reform period. For consistency, pre-reform revenues are also measured as of 1989, and pre-reform achievement is measured as of 1991, since the series of comparable achievement data begin in 1991. "Revenue", "spending" and "aid" are per-student measures. Throughout this paper, all dollar figures are expressed in dollars of 2000, unless otherwise indicated.

12 enrollment of about 182,000 in 1990. As Figures 1a and 1b show, there was considerable variation in income and housing value across districts, with Detroit Public Schools ranking almost at the bottom. Similarly, local and state revenues differed widely across districts (see Figure 1c), as did school achievement measured by the pass rate for the fourth grade math test.[6] District average income, housing value, per-pupil revenue, and pass rates were highly and positively correlated. Furthermore, the districts with the highest property values had the lowest millages[7] (see Figure 1d).

The school finance reform implemented in the Fall of 1994 represented a drastic departure from the previous funding system, a district power equalization program that had been in effect since 1973.[8] Sales and use taxes rose from 4 to 6 percent, and homestead property taxes fell from a state average of 34 mills to a statewide uniform rate of 6 mills on all property. The Michigan School Aid Fund now includes revenues from the sales tax, a 6-mill state uniform property tax on homestead and non-homestead property, revenues from the state income tax, and other revenues. The state share of funding rose from 35 to 80 percent. A reduction of local property taxes and concomitant increase of state funding was also a feature of the recent school finance reforms in Kentucky, Texas and Vermont. Furthermore, voter dissatisfaction with high property taxes was a crucial motivation for the reforms in Michigan and Vermont (Yinger (2004)).

Proposal A implemented a foundation grant system guaranteeing each district a per-student revenue equal to the district's foundation allowance.[9] Districts are not allowed to supplement their foundation allowance. As Yinger (2004) reports, foundation aid formulas were employed in forty-one states as of 2004. A district's foundation allowance is based on its local and state revenue prior to Proposal A ("base revenue"). In 1994, foundation allowances were determined according to the following formula:

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[6] The pass rate is computed as the percent of students who obtain a grade of "satisfactory" in the state's math test, which is graded according to three categories: "low", "moderate" and "satisfactory." Although we report results for fourth grade math exams, the results for seventh grade are qualitatively similar, except that pass rates are consistently lower for 7th grade.

[7] Property tax rates are expressed in mills (dollars paid per $1,000 of assessed value). We only consider mills raised for operational purposes ("operating mills") given that Proposal A refers exclusively to them.

[8] For background on alternative state aid mechanisms, see Yinger (2004). For further details on the Michigan reform, see Addonizio et al (1995) and Cullen and Loeb (2004).

[9] To districts with foundation allowances above a certain threshold (equal to $6,500 in 1994), the state guarantees a per-student revenue equal only to that threshold, yet allows them to raise additional property taxes in order to reach their full foundation allowances. The revenue of these districts, however, continues to be capped by the magnitude of the foundation allowance.

$$fa = \begin{cases} 4{,}200 & \text{if } x \le 3{,}950 \\ x + 250 & \text{if } x \in (3{,}950, 4{,}200) \\ .961x + 414.35 & \text{if } x \in (4{,}200, 6{,}500) \\ x + 160 & \text{if } x \ge 6{,}500 \end{cases} \qquad (1)$$

where *fa* is the foundation allowance and *x* is base revenue in 1993 dollars. The state requires each district to levy 18 mills on non-homestead property, and covers the difference between this local revenue and the foundation allowance through state taxes. As (1) shows, Proposal A reduced the variation in revenue by raising revenue at the bottom of the distribution and limiting it at the top. Although foundation allowances were adjusted every year between 1994 and 2000, the weak ordering of districts by revenue remained the same as before the reform (see Figure 2). Furthermore, real revenues at the end of the decade differed only slightly from those before the reform.[10]

3. Model

The model takes as its point of departure the framework of Nechyba (1999), extended to encompass the preference and achievement structure of Epple and Romano (2003). The metropolitan area is populated by a continuum of households. Each household has one child. Household endowed income is denoted y_e and child ability is denoted b. The population is normalized to one. The joint distribution of ability and endowed income, $f(b, y_e)$, is continuous and strictly positive on support $[b_m, b_x] \times [y_m, y_x]$.

There is a continuum of houses partitioned into school districts, with the population of houses normalized to one. Every district *i* is in turn partitioned into neighborhoods, indexed by *j*. There are *K* neighborhoods in the economy. Houses may

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[10] The foundation allowance is the state's mechanism to deliver basic aid, the use of which is discretionary for the districts, as opposed to categorical aid which can only be used for specific purposes. A district's foundation allowance is a function of its 1993 base revenue, which includes, among others, property tax revenues, state basic aid, and most categorical aid components (Addonizio et al (1995)). Categorical aid was also overhauled in 1994, and a large number of categoricals previoulsy included in the foundation allowance were eliminated. The largest was the state's contribution to teachers' social security and retirement. These funds were transferred to the local districts via the foundation allowance to shift this rising burden away from the state, and districts must now make the corresponding contributions. In our empirical analysis we quantify the increase in revenue between 1989 and 1999 as the difference between the 1999 foundation allowance and the 1989 local and state revenue. Although the former is for basic aid and the latter includes basic and categorical aid, the bulk of these categoricals correspond to expenses that are not discretionary from the point of view of the districts, as explained above.

differ in quality h (units of housing services involving features such as size, age, etc.) across neighborhoods, but within a given neighborhood they have the same quality. The housing stock cannot be varied in quantity or quality. The exogenously determined housing service provided by a house located in neighborhood j and district i is denoted h_{ij}, and the fraction of the population that can be housed in that neighborhood is n_{ij}.

Each child must attend public school. Education is provided locally by school districts. There is one public school per neighborhood. A child may attend only the public school located in the neighborhood where the child's household resides.

All households are renters, and rental proceeds in the metropolitan area are paid to an investment fund. Shares in the investment fund are owned by metropolitan residents, with ownership shares proportional to income endowment. We let ry_e denote the income received from the housing investment fund by a household with endowed income y_e, where r, equal to the ratio between the metropolitan area's total value of the housing stock and aggregate endowed income, is determined in equilibrium.[11]

Households' preferences are represented by the utility function $u(g,h,c;b)$, where h is consumption of housing services, c is consumption of the numeraire, g is the quality of the school attended by the child, and b is the ability of the child in the household. Following Epple and Romano (2003), we interpret b broadly as school capability, which encompasses contributions of the student's home environment to school readiness.

Educational expenditures are financed by state income and sales taxes at rates τ and ω respectively, and by local property taxes at rates $\hat{t}$, with the mix of funding differing before and after the reform. We assume for simplicity that proceeds from the housing investment fund are not subject to state taxes. The income tax is imposed on endowed income, y_e, and the sales tax is applied to after-tax endowed income inclusive of earnings from the investment fund and net of housing expenditure, implying budget constraint: $y_e(1-\tau+r)=c(1+\omega)+\hat{p}(1+\hat{t})h$ where c is numeraire consumption and $\hat{p}$ is the

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[11] In this type of model, properties could either be owned by an absentee landlord or by the households in the economy. We adopt the latter option because it is better suited to the study of capitalization. Some initial allocation of houses to households is then needed. One alternative is to endow each household with a house whose value is independent of the household endowed income, as in Nechyba (1999). Another is to endow each household with a share of the total housing stock whose value is proportional to the household endowed income, as is done here. The income effects derived from the capitalization of Michigan's tax reform are small in the first case (see below), whereas they are zero in the second case (see Proposition 1).

net-of-tax per price per unit of housing. We can then write the budget constraint

as $y_e \frac{(1-\tau+r)}{(1+\omega)} = c + \frac{\hat{p}(1+\hat{t})}{(1+\omega)} h$. Let p be the tax-inclusive price of housing: $p = \frac{\hat{p}(1+\hat{t})}{(1+\omega)}$. Let

$v=ph$ be the tax-inclusive value of a house with h units of housing services, and $t = \hat{p}\hat{t}$ be

the property tax bill per unit of housing. Notice that p, $\hat{p}$, t and h will generally differ

across neighborhoods, and hence v will vary across neighborhoods as well. The

household budget constraint is then:

$$c + v = y \tag{2}$$

where y is net-of-tax endowed income inclusive of earnings from the housing investment

fund: $y = y_e \frac{(1-\tau+r)}{(1+\omega)}$. In what follows we refer to y as income. Let the CDF's for the

marginal distribution of y in the initial (1990) and final (2000) periods be $F(y)$ and $G(y)$.

It is convenient to define the utility function derived from substituting the budget

constraint into the utility function as:

$$U(v, g, h; y, b) = u(g, h, y - v; b) \tag{3}$$

Schools produce school quality g according to the quasi-concave production

function $g=g(\theta,s)$, where θ is peer quality, defined as the average ability of students

attending the school, and s is spending per student in the school's district. Spending per

student is the same for all schools within a district.

When choosing locations (and hence schools) households take public school

qualities $g_{ij} = g(\theta_{ij},s_i)$ and housing prices in all neighborhoods p_{ij} as given. Migrating

among locations is costless. Thus, a household with income y chooses a location (i,j) to:

$$Max_{ij}U(v_{ij}, g_{ij}, h_{ij}; y, b) \tag{4}$$

where, recall, h_{ij} is the quality of house type j in district i, and v_{ij} is the tax-inclusive

market value of house type j in district i.

The analysis that follows does not require modeling of the political process that

determines local property tax rates, state income tax rates, or the decision to reform the

system of school finance. Instead, the initial tax rates and expenditure levels in each

district are taken as given as is the state educational reform. The analysis focuses on

developing predictions about the effects of the reform and other factors (e.g., changes in

the income distribution) that occurred over the time interval for which outcome measures are available. The latter is the decade between the 1990 and 2000 decennial Census years.

Given tax rates and expenditures per student in each district, an *equilibrium* is a partition of the population into districts and neighborhoods, school qualities $g_{ij}=g(\theta_{ij},s_i)$, and tax-inclusive house value v_{ij} such that every house is occupied and no household can gain utility by moving. We prove that equilibrium exists before and after the reform, and characterize the equilibrium and equilibrium changes that are predicted by the model.

It is useful to characterize the Michigan reform as having two distinct elements- the change in financing, and the change in expenditures across districts. The following neutrality result characterizes the effects of changing the financing of public schools from district property taxes to state income and sales taxes while holding constant spending per pupil in all districts.

Proposition 1: (Tax-instrument neutrality) An allocation that is an equilibrium before school finance reform is also an equilibrium after the school finance reform if state revenues are generated by a combination of income and sales taxes. All property tax changes are capitalized into housing prices with no change in tax-inclusive housing prices.

Proof: Let t^* and s^* be respectively the vector of residential property tax bills and per student expenditures in all locations prior to the reform, and let τ^* and ω^* be respectively the income and sales tax rates before the reform. Similarly, let $\hat{t}'$ and t' be the property tax rates and bills after the reform, and τ' and ω' the income and sales tax rates after the tax reform. Let $\hat{p}^*$ and $\hat{p}'$ be the net-of-tax housing prices before and after the reform respectively. The gross-of-tax housing price in the equilibrium preceding the tax change is then $p^* = \dfrac{\hat{p}^*(1+\hat{t}^*)}{(1+\omega^*)}$, and $p' = \dfrac{\hat{p}'(1+\hat{t}')}{(1+\omega')}$ is the gross-of-tax price following the reform. Finally, let r^* and r' be the return from the housing investment fund before and after the reform. These are the ratios of the metropolitan area's total value of the housing stock to aggregate income before and after the reform, respectively.

By hypothesis, $\hat{t}'$, τ', and ω' are chosen to keep s^* unchanged. Thus, the change in income, sales, and property tax revenues must be equal to zero:

$$\left(\tau' - \tau^*\right)Y_e + Y_e \left[\omega' \frac{(1-\tau'+r')}{(1+\omega')} - \omega^* \frac{(1-\tau^*+r^*)}{(1+\omega^*)}\right]$$
$$-\sum_i \sum_j \left(\omega' p_{ij}' - \omega^* p_{ij}^*\right) H_{ij} + \sum_i \sum_j \left(t_{ij}' - t_{ij}^*\right) H_{ij} = 0 \tag{5}$$

where Y_e is aggregate endowed income and $H_{ij} = n_{ij} h_{ij}$.

Suppose gross-of-tax prices are unchanged by the reform: $p' = p^*$. Substituting this equality into (5) and simplifying, we obtain:

$$\left(\tau' - \tau^*\right)Y_e + Y_e \left[\omega' \frac{(1-\tau'+r')}{(1+\omega')} - \omega^* \frac{(1-\tau^*+r^*)}{(1+\omega^*)}\right]$$
$$-\left(\omega' - \omega^*\right)\sum_i \sum_j p_{ij} H_{ij} + \sum_i \sum_j \left(t_{ij}' - t_{ij}^*\right) H_{ij} = 0 \tag{6}$$

The change in payments from the investment fund must equal the change in property rentals. Hence:

$$(r' - r^*)Y_e = \sum_i \sum_j \left(\hat{p}_{ij}' - \hat{p}_{ij}^*\right) H_{ij} \tag{7}$$

Now $p = \frac{\hat{p}(1+\hat{t})}{(1+\omega)}$ implies $p(1+\omega) = \hat{p} + t$, or $p(1+\omega) - t = \hat{p}$. This result in (7) implies:

$$(r' - r^*)Y_e = \sum_i \sum_j \left(p_{ij}'(1+\omega') - t_{ij}' - p_{ij}^*(1+\omega^*) + t_{ij}^*\right) H_{ij} \tag{8}$$

Substituting (8) into (6) and simplifying, we obtain:

$$\frac{\omega^* + \tau^* - r^*}{(1+\omega^*)} = \frac{\omega' + \tau' - r'}{(1+\omega')} \tag{9}$$

Recall that the household budget constraint is: $y_e \frac{(1-\tau+r)}{(1+\omega)} = c + ph$. Now (9) and $p_{ij}' = p_{ij}^* = p$ imply that the household budget constraint is unchanged by the reform.

We have thus shown that if gross-of-tax housing prices do not change in response to the finance reform, then incomes of all households, net of tax payments and inclusive of investment fund receipts, are also unaffected by the tax reform, and government budgets are balanced. Hence, the choice that was optimal for each household before the reform will be optimal after the reform. Thus, the equilibrium prevailing before the reform will also be an equilibrium after the reform. The tax policy change is irrelevant in the sense that per se it has no effect on household choices. Tax "reform" only has effects if the proceeds of the tax are distributed differently after than before the reform. ∎

Remarks:

1. The preceding result applies regardless of how property tax rates in individual districts are changed. There is no necessity that such changes be uniform. They can be completely arbitrary, even random. Furthermore, the assumption that property revenues accrue to an investment fund is a convenient device for illustrating that, at most, property tax reform has income effects that would arise to the extent that household ownership of the returns to property is not exactly proportional to income. Such idiosyncratic income effects are surely of second order.[12] Moreover, some initial allocation of properties to households is required. Endowing individuals with property shares whose value is proportional to their endowed incomes seems quite natural.

2. The assumption of inelastic housing supply is key to the capitalization results. Hence, we may view the proposition as characterizing short-run implications of the tax reform. In the longer term, reduced property tax rates would induce an increase in housing supply, which would in turn lead to lower housing prices in such locations.[13] The assumption of inelastic supply may apply over a longer time frame in metropolitan areas that have stable or declining populations. The average metropolitan area in the United States grew at a rate of 16 percent over the decade, with construction and destruction rates of 22 and 6 percent respectively. Among the twenty largest metropolitan areas, the fastest-growing were Phoenix and Atlanta, which grew by approximately 40 percent. In contrast, Detroit's net growth was 6.5 percent, with construction and destruction rates of 11.9 and 5.4 percent respectively. Thus, while the size of the housing stock in Detroit did change over the decade, the change was relatively small. Furthermore, as Glaeser and Gyourko (2005) have noted, housing is a durable good that grows fast but declines slowly, and whose supply is hence rather inelastic in areas that have stable or declining

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[12] For example, consider a household with income y owning a house worth 1.6 times the household's income—our sample's average. Suppose the property tax reduction for this dwelling is fully capitalized, with no offsetting loss from increased sales or income taxes. Given the observed reduction in property tax rates under Proposal A, the household then has a capital gain of about 3 percent of house value, or $.048y$. While this is a non-trivial wealth increase, it will give rise to modest income effects. If this capital gain is annuitized at a real rate on the order of 5 to 10 percent, it translates to an annual income increase of .2 to .5 percent of y. Other households will have income losses of similar order. The income effects on housing demand and location choice of idiosyncratic income changes of this order of magnitude will be small.

[13] This type of long-run result would be captured, for example, in the Epple and Sieg (1999) framework.

populations. Therefore, the assumption of inelastic housing supply over the decade seems a relatively good approximation for Detroit.

3. Of particular importance in urban districts is the presence of non-residential property. The capitalization results extend when non-residential property is included in the model as a fixed stock owned by metropolitan residents.[14]

From an empirical perspective, Proposition 1 implies that the only effect of the change in financing is on housing prices. Hence, only the other component, the change in the level and distribution of expenditures, could potentially give rise to either demographic changes or school quality effects.[15] To investigate such effects, we adopt the following additional assumptions on the preference function $U(v,g,h,y;b)$.

A1. Separability: The utility function can be written $U(w(g,h),y-v;b)$,[16] where w is the "local bundle" of goods accessed by living in a particular location.

Preferences satisfy the following single-crossing conditions:

A2. Single-crossing in income: $\dfrac{\partial\left(\frac{\partial v}{\partial w}\right)_{U=\bar{U}}}{\partial y} > 0$ (SCI)

A3. Neutral crossing in ability: $\dfrac{\partial\left(\frac{\partial v}{\partial w}\right)_{U=\bar{U}}}{\partial b} = 0$ (NCB)

Condition SCI implies that, holding housing quantity fixed, high-income households are willing to pay a higher increment for housing than a low-income household to obtain an increase in the quality of education. The SCI condition also implies that, holding the quality of education constant, a high income household will pay more for a larger house than will a low income household. Condition NCB implies neutral crossing with respect to ability for education and for housing.[17]

Following Epple and Romano (2003), we adopt the following assumption regarding the joint distribution of ability and income:

A4. $E(b|y)$ is strictly increasing in y.

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[14] Proof is available on request.
[15] If there are multiple equilibrium, it is possible that a switch to another equilibrium might occur. Of course, this possibility is also present absent the reform.
[16] This is satisfied by many commonly used utility functions including, for example, the Cobb-Douglas function, a utility function that is additively separable, or a nested CES function.
[17] As discussed further in Epple and Romano (1998), there is limited evidence regarding the effect of own ability on the willingness to pay for education quality, and we know of no evidence linking own ability and willingness to pay for housing. Hence, neutral assumptions in this regard are quite natural, and they greatly simplify the analysis that follows.

A4 carries the realistic implication that "school readiness" is increasing in income.

Proposition 2: Necessary conditions for equilibrium. Index all neighborhoods in the metropolitan area in order of ascending local bundles w_k, $k=1,\ldots,K$. Then equilibrium exhibits the following properties:

  a. Ascending (w_k,v_k) pairs: House value ascends in the same order as w_k across neighborhoods.

  b. Income Stratification: Given two households with equal student ability, if household with income y_2 resides in a higher-numbered neighborhood than household with income y_1, then $y_2 \geq y_1$ with equality for at most one income level.

  c. Ascending peer quality: Peer quality θ_k ascends in the same order as w_k across neighborhoods.

  d. Boundary indifference and strict preference for non-boundary households: For each pair of adjacent neighborhoods k and $k+1$, a household with income level y^k exists that is indifferent between residing in neighborhood k and $k+1$. All other households in the district strictly prefer their residential choice.

Proof: Results a, b, and d follow from proof of the analogous results in Epple and Romano (2003). While Epple and Romano take housing to be homogeneous and divisible with each individual consuming exactly one unit of housing, their proofs of the above results nonetheless generalize to the environment studied here. Result c follows from b (income stratification) and Assumption A4. ∎

  For the following proposition, we employ the following additional assumption:

  A5: Given any s_i, s_j, h_i, h_j, and θ such that $w(g(\theta,s_i),h_i) \geq w(g(\theta,s_j),h_j)$, if $\theta_i > \theta_j$, then $w(g(\theta_i,s_i),h_i) > w(g(\theta_j,s_j),h_j)$.

To interpret A5, consider two local bundles i and j with a common peer quality and with bundle j at least weakly preferred to bundle i. Holding unchanged other components of the bundles, replace the common peer quality with any pair of peer qualities that ascend between i and j. Then the dominance of i by j is preserved and becomes strict. A5 holds trivially when both $s_j \geq s_i$ and $h_j \geq h_i$. The value of the assumption is for cases when s and h do not ascend in the same order.

Proposition 3: Equilibrium exists.[18]

Proof: We present a constructive proof. Exploiting Proposition 1, we can, without loss of generality, treat expenditures on education as financed by a proportional income tax. Hence, let τ be the rate required to finance the combined educational expenditures of all districts. Recall that income y is endowed income net of the income tax plus revenues from housing: $y=y_e(1-\tau+R/Y_e)$ where R is aggregate revenue from housing and Y_e is aggregate endowed income of all households in the metropolitan area. Index neighborhoods by ascending $w(g(\bar{\theta},s_k),h_k)$ where $\bar{\theta}$ is mean ability of the population of students in the metropolitan area, and s_k is expenditure per student in the district in which neighborhood k is located. Next, stratify the population across neighborhoods by income, with boundary incomes, y^k, ascending in the same order as k. A4 then implies that peer average abilities, θ_k, ascend in the same order as k. From A5 it then follows that $w(g(\theta_k,s_k),h_k)$ ascends in the same order as k. Since the size of the metropolitan population equals the size of the metropolitan housing stock, a price normalization is required. We set the price, v_1, in community $k=1$ so that $0<v_1<(1-\tau)y_m$. This assures that it is feasible for the poorest household in the metropolitan area to purchase housing, and hence it is feasible for all households in the metropolitan area to purchase housing.

The boundary-indifference condition between communities k and $k+1$ is:

$$U(w(g(\theta_k,s_k),h_k),y^k-v_k)= U(w(g(\theta_{k+1},s_{k+1}),h_{k+1}),y^k-v_{k+1}) \qquad (10)$$

For $k=1$, the only unknown in the above condition is v_2. Hence, solve this equation for v_2. Proceeding recursively, solve the boundary-indifferences conditions for $v_3,\ldots,v_K$. By construction, then, housing markets clear in all neighborhoods, district budgets are balanced, and the necessary conditions of Proposition 2 hold. The latter necessary conditions embody optimal school and housing choice by all households.

It only remains to show that these conditions are satisfied when the revenues distributed from housing, R, equal revenues collected from housing. Let revenues collected from housing be denoted $\phi(R)$ where:

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[18] Nechyba (1999) proves existence in his framework while also allowing voting over local public good levels. We cannot simply invoke his proof because of extensions of the preference and achievement discussed earlier in this section.

$$\phi(R) = \sum_{k=1}^{K} n_k v_k(R) \tag{11}$$

The dependence of house prices on R, $v_j(R)$, arises via the distribution of house rents that households receive. Appendix A establishes that there is a fixed point R^* such that:

$$R^* = \phi(R^*) \tag{12} \qquad \blacksquare$$

Remarks: The proof provides a computationally simple method of computing equilibrium. Array the $w(g(\overline{\theta}, s_k), h_k)$ in ascending order. Allocate households to neighborhoods in the same order. For a given R, recursive application of the boundary indifference conditions determines all housing prices. A line search on R, calculating prices recursively on each trial value of R, can be used to find R^*.

Corollary 1: If there are no peer effects, the equilibrium is unique.

Proof: This result follows directly from the argument in Proposition 2. ∎

Corollary 2: There is an equilibrium before the reform and an equilibrium after the reform such that the ordering of incomes across neighborhoods within each district is the same before and after the reform. If peer effects operate at the district level, then incomes ascend across neighborhoods in the same order as house qualities in all equilibria.

Proof: In the equilibrium constructed in Proposition 3, incomes ascend in the same order as housing qualities within districts. Since the reform does not alter the ordering of house qualities, the first claim then follows. The second claim follows from observing that the order of $w(g(\overline{\theta}, s_i), h_{ij})$ within district i depends only on the ordering of house qualities, h_{ij} in the district. ∎

We next present two further results which suggest that demographic effects are likely modest. Coupled with Proposition 1, this suggests that the effects of expenditure equalization will be reflected primarily in property value and school quality changes, with little impact on the demographic composition of district populations. Absent change in population demographics, there will be no changes in peer qualities. Hence, school quality changes will be due to expenditure changes. We employ the following definition.

Definition: Index districts in order of ascending mean house quality. Housing is *stratified* across districts if the highest-quality house in district i has quality no higher than the lowest-quality house in $i+1$.

Proposition 4: (Across-District Income and Ability Stratification) If housing stocks are stratified across districts and school expenditure ascends in the same order as house quality, there exists an equilibrium in which households are stratified by income and peer average ability ascends in the same order as income.

Proof: This follows immediately from Proposition 3.

Remarks:

1. Proposition 4 implies that, following a school finance equalization that preserves the ordering of school expenditures, there is an equilibrium in which household demographics are unchanged by the reform.

2. Proposition 4 relies on the strong assumption of housing stratification across districts. In reality, there is generally incomplete stratification of housing across districts. Indeed, an attractive feature of the Nechyba (1999) framework is that incomplete income stratification across districts can arise because of imperfect housing stratification. Nonetheless, the proposition is of interest since school districts with higher spending per student also typically have higher quality housing.

3. If housing is imperfectly stratified, then the school reform may induce demographic changes, though it need not.

We have assumed thus far that peer effects operate at the school level, which is a natural characterization with a neighborhood school system. However, with frictionless open enrollment within each district, peer qualities would be equalized across schools in a district. When peer qualities equalize across schools in a district, the following holds.

Corollary 3: Suppose peer qualities equalize across all schools in a district and housing is stratified across districts. Then, for all equilibria in which school qualities ascend in the same order as housing qualities, there is a unique allocation of households to districts and to housing qualities.

Proof: Order all neighborhoods in the metropolitan area in order of ascending housing quality, and index neighborhoods by $k = 1,\ldots,K$. With school qualities ascending in the same order as housing qualities, households stratify by income in the same order as k. Uniqueness of the equilibrium then follows from the uniqueness of the ordering of k. ∎

Remark: This corollary does not establish that there is a unique equilibrium. If house qualities and school expenditures are similar in two districts, peer effects may be large

enough to offset those differences creating an additional equilibrium in which the district with the lower housing and school expenditure is occupied by higher income households.[19] In what follows, we focus on the case in which higher income households occupy the higher quality houses.

Proposition 4's implication of unchanging household demographics follows from the relatively strong assumptions that housing stratification and housing quality ascending in the same order as school spending. We note and emphasize that the assumption of housing stratification is used only in the proof of Proposition 4 and Corollary 3. No other results invoke this strong assumption. Furthermore, conditions we have shown to rule out demographic changes are sufficient but not necessary. Even absent housing stratification, school reform may not provide incentives that induce demographic changes. Also, as a practical matter, locations with higher spending tend to have higher housing quality as well. Hence, we believe our assumptions are likely to be a relatively good approximation in many metropolitan areas.

Another important change that took place over the decade under consideration in Detroit is the growth of household income in real terms, with proportionately greater gains at the lower and upper ends of the income distribution (see Figure 3).[20] It is useful to analyze this change in two steps. First we consider a mean-preserving change in the income distribution that reflects the relative increase in incomes by lower- and upper-income households. Then we consider a proportionate increase in all incomes.

Figure 4 shows the mean-adjusted cumulative distributions of income for 1990 and 2000 in Detroit. As the figure shows, the 2000 distribution crosses the 1990 distribution first from below and then from above, although the second crossing is less pronounced than the first. Recall that $F(.)$ and $G(.)$ denote respectively the distributions in the initial (1990) and final (2000) periods. Figures 3 and 4 motivate our focus, in the

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[19] See also Rothstein (2006) for a model in which high quality peers may offset low school effectiveness.

[20] In comparing the 1990 and 2000 income distributions, we faced the challenge that the income bins differed across Census years and were expressed in current rather than constant dollars. To create Figure 4, we increased the bounds of all 1990 income bins by 50.43 percent (the amount by which average metropolitan current income increased from 1990 to 2000). We then fitted each CDF using a high order polynomial, obtaining an essentially perfect fit for each. The fitted CDF's, which have the same mean, are plotted in Figure 4 and were inverted to construct Figure 3.

development that follows, on understanding the effect on housing prices of a shift in the income distribution in which $F(.)$ and $G(.)$ cross at least once.

To set the stage for this analysis, we strengthen assumption A2 by requiring not only that indifference curves in the (w,v) plane of households with different incomes cross only once, but also that the vertical difference between two such indifference curves is monotone increasing in the local bundle w. Hence, consider households with incomes y' and y'' where $y' < y''$. Let (w_0,v_0) be the point at which an indifference curve of y' and an indifference of y'' cross, and let (w,v') and (w,v'') be defined by:

$$U(w,y''-v'';b) = U(w_0,y''-v_0;b) \qquad\qquad (13)$$
$$U(w,y'-v';b) = U(w_0,y'-v_0;b) \qquad\qquad (14)$$

A2S. Strong Single Crossing: $\dfrac{\partial (v''-v')}{\partial w} > 0$ (SSC)

Common parametric utility functions that satisfy single-crossing (e.g., Cobb-Douglas, CES) also satisfy Strong Single Crossing (SSC).

We continue to choose $k=1,\ldots,K$ such that all neighborhoods in the metropolitan area are indexed in ascending order of w_k. For ease of reference, we will use "left" and "right" to indicate the relative positions of neighborhoods in this ordering, i.e., neighborhood a is left of b if a has a lower index than b. Neighborhoods that are next to each other in this ordering will be termed "adjacent."

Proposition 5: Suppose $G(y)$ crosses $F(y)$ twice, once from below at y_{c1} and once from above at y_{c2} where $y_{c1} < y_{c2}$. Suppose, in addition, that peer quality changes associated with a change in the income distribution do not change the relative desirability of the housing-local public good pairs across neighborhoods.[21] Then the change in housing prices will exhibit two peaks.

Proof: Let k_{c1} and k_{c2} be the indices of communities occupied by y_{c1} and y_{c2} respectively. Let y_k and y_k' be the incomes of the households indifferent between adjacent neighborhoods k-1 and k before and after the change in income distribution. Now

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[21] If b and y were independent, peer quality changes would not accompany income changes. However, we have assumed (A4) that b and y are positively correlated. Hence, changes in peer quality may arise via this correlation when the income distribution changes. We assume for Proposition 5 that such changes are not large enough to change the relative desirability of local bundles.

$y_k' > y_k$ for all $k \leq k_{c1}$ and for all $k > k_{c2}$. SSC then implies that $\Delta v_{k-1} < \Delta v_k$ for all $k \leq k_{c1}$ and for all $k > k_{c2}$. Similarly, SSC implies that $\Delta v_{k-1} > \Delta v_k$ for all $k \in (k_{c1}, k_{c2}]$. It follows that there is a peak at k_{c1} and a "valley" at k_{c2}, i.e. $\Delta v_{k_{c2}-1} > \Delta v_{k_{c2}} < \Delta v_{k_{c2}+1}$. ∎

Corollary 4: The change in housing prices will exhibit as many peaks as there are crossings of the mean-adjusted income distributions.

Proof: The proof extends the approach to the proof of Proposition 5.

For our final result, we specialize to Cobb-Douglas utility and production functions. Hence, the utility function is: $u(g,h,c;b)=g^\alpha c^\gamma h^\delta b^\beta$, and the school production function is $g = s^{1-\rho}\theta^\rho$. In addition, we assume that the distribution of $[ln(y), ln(b)]$ prior to the policy change is bivariate normal.[22]

Proposition 6: An equilibrium of this model is also an equilibrium if all incomes increase by the same proportion, m, all house prices increase by proportion m, and spending on education is unchanged in all districts.

Proof: Consider a household n with income y_n and ability b_n for which (i,j) is its preferred location in the initial equilibrium. Then:

$$\left(\theta_i^\rho s_i^{1-\rho}\right)^\alpha [y_n - p_{ij}h_{ij}]^\gamma h_{ij}^\delta b_n^\beta \geq \left(\theta_i^\rho s_{i'}^{1-\rho}\right)^\alpha [y_n - p_{i'j'}h_{i'j'}]^\gamma h_{i'j'}^\delta b_n^\beta \quad \forall i' \neq i \; and \; \forall j' \neq j \qquad (15)$$

Now, suppose all incomes and house prices increase by proportion m, which implies that all abilities and peer qualities increase by proportion m'.[23] If no household other than n relocates, then for household n, the preceding expression implies:

$$\begin{aligned}
&\left((m'\theta_i)^\rho s_i^{1-\rho}\right)^\alpha \left[my_n - mp_{ij}h_{ij}\right]^\gamma h_{ij}^\delta (m'b)^\beta \geq \\
&\left((m'\theta_{i'})^\rho s_{i'}^{1-\rho}\right)^\alpha \left[my_n - mp_{i'j'}h_{i'j'}\right]^\gamma h_{i'j'}^\delta (m'b)^\beta \; \forall i' \neq i \; and \; j' \neq j
\end{aligned} \qquad (16)$$

Thus (i,j) continues to be household n's most preferred location. Since this argument applies for all n, the initial equilibrium is also an equilibrium when all incomes and house prices increase by proportion m. ∎

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[22] See Nechyba (2000) and Epple and Romano (1998) for computational models with induced Cobb-Douglas utilities. Epple and Romano (1998) also use the above joint distribution of income and ability.

[23] Under the assumption that $[ln(y), ln(b)]$ is normally distributed, it can be shown that in response to a proportionate increase in all incomes, the increase in abilities is proportional to the increase in incomes, and mean peer ability increases by the same proportion in all neighborhoods. Details are available on request.

Remark: The preceding proof also applies if expenditure on education increases by proportion *m* everywhere, which would be the case if each district's education expenditure were financed entirely by the district's property tax revenue. An important implication of this proposition is that, absent other changes, house prices rise by a given proportion when all incomes increase by the same proportion even though the equilibrium allocations do not change. In particular, then, if all incomes increase by the same proportion between two successive time periods and housing stocks do not change, house prices will increase by the same proportion.

To summarize, our analysis implies that the reform will lead to limited changes in demographic compositions across districts and neighborhoods, and capitalization of property tax and expenditure changes. The model also implies that changes in the level and distribution of income will result in house value changes that incorporate two components. One component is appreciation associated with the increase in the level of income (Proposition 6). The other component is comprised of relative price changes induced by changes in the income distribution (Proposition 5). When neighborhoods are ordered by 1990 income rank, changes in house values will exhibit as many peaks as there are crossings of the pre- and post-reform mean-adjusted income distributions. In the next section we present empirical evidence related to these predictions.

4. Empirical Evidence

In this section we begin by examining evidence related to the model's predictions on ordering, stratification and demographic compositions. Then we analyze whether the predictions for property values hold in the data. We conclude by exploring the implications of our findings for school quality.

Evidence on Ordering and Stratification

Our model delivers predictions for ordering and stratification across neighborhoods and districts as well as changes in property values. To assess these predictions, we use data at both geographic levels. In the Census Bureau's geography, the closest approximation to a neighborhood is a Census tract. Hence, we collected 1990 and 2000 data at the tract level.

Since 36 percent of Census tracts in Detroit changed boundaries over the decade, all our data are normalized to the 2000 boundaries as we use Census data from the 1990 Long Form in 2000 Boundaries and the 2000 Long Form. Our data include 1,122 tracts with at least fifty specified owner-occupied housing units in 1990 and 2000. We used Geographic Information Software to associate Census tracts and school districts. As for the district-level data, demographics and house values come from the School District Data Book; achievement comes from the Michigan Department of Education and is measured by the fourth-grade average pass rate in the math exam, and financial data come from the Bulletin 1014 published by the Michigan Department of Treasury. All monetary values are expressed in 2000 dollars. In what follows, "income", "house value" and "rental value" refer to averages at the corresponding geographic level.

In our theoretical, one-period model the value of a house, v, equals its per-period tax-inclusive rental price p times the units of housing services h. Since a house is actually a durable good whose price equals the net present value of its services net of property tax payments (Yinger et al (1988)), we need to derive our theoretical value, v, from observed house prices, V. Following Poterba's (1992) relationship between implicit rent v and house value V for an owner who itemizes income tax deductions, and adjusting for the sales tax to conform to our theoretical definition of rental price, we obtain the following:

$$v = \left[\frac{\left(1-\tau_f\right)\left(i+\hat{t}\right)+\eta+\mu+\chi-\pi}{1+\omega} \right] V \qquad (17)$$

where the term in brackets is the user cost rate, τ_f is the marginal federal income tax rate, i is the nominal mortgage interest rate, $\hat{t}$ is the property tax rate, η is risk premium for housing investments, μ and χ are maintenance and depreciation respectively, π is the rate of expected house appreciation, and ω is the sales tax rate.[24] In the Detroit metropolitan area, the average marginal federal income tax rate τ_f for a household was equal to 19.16 percent and 18.13 percent in 1990 and 2000 respectively; the mortgage rate i was equal to 10.22 percent and 7.26 percent respectively; the five-year average appreciation rate π was equal to 6.8 percent and 8.62 percent respectively, and the sales

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[24] Note that state income taxes do not affect the relationship between house value and implicit rent unless housing expenses are deducted for the state income tax, which is not the case in Michigan.

tax ω was equal to 4 and 6 percent respectively. Local property tax rates ranged between 1.7 and 3.9 percent in 1990, and were equal to 0.6 percent in 2000. Following Poterba, we use $\eta = .04$ and $\chi = \mu = .02$. Finally, we compute 1990 and 2000 tract average rental values v by applying (17) to the corresponding tract-level average house values V and the user cost rate. [25]

Since we do not observe house quality h or local bundles w, we construct measures for them as follows. Following Ferreyra (2007), we construct a tract-level housing quality index that captures housing physical characteristics and neighborhood amenities, *excluding* public school quality. We do so by regressing the logarithm of tract average rental price on a set of neighborhood characteristics and school district fixed effects, then making each tract's neighborhood quality index equal to the tract's fitted rental value net of school district fixed effects.[26] The motivation for this regression is that, broadly speaking, rental prices reflect housing characteristics, neighborhood amenities, and public school quality. To construct the tract-level local index w we adopt the following parameterization: $w = g^{\alpha} h^{\delta}$, with $\alpha = 0.12$ and $\delta = 0.16$.[27] In this index, g (achievement) is constant within a district while h varies across tracts and districts.

According to Proposition 2, income, rental value and the local bundle ascend in the same order across neighborhoods in equilibrium. Hence, panel (a) of Table 1 shows tract-level correlations of income, rental value and the local index, w, for 1990 and 2000. Corollary 2 of Proposition 3 implies that the within-district ordering of income, rental value and the local index is the same before and after the reform. Thus, panel (b) in Table 1 shows the partial correlation of each of these variables across the years 1990 and 2000

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[25] We calculated the average marginal federal income tax rate using Taxsim from www.nber.org along with the 1990 and 2000 distributions of household income for Detroit. Mortgage rates are from the Federal Housing Board. We calculated the five-year average appreciation rates using data from the Office of Federal Housing Enterprise Oversight. The property tax rates used in these calculations equal half of the actual rates because property is assessed at half of its market value in Michigan, although the results do not vary substantially when using the actual rates.

[26] For 1990, we use 1990 tract-level data from the 1990 Long Form in 2000 Boundaries. As neighborhood characteristics in the regression, we use tract average housing characteristics from the Census, and linear and quadratic terms in tract distance to the metropolitan area center. See Ferreyra (2002) for more details on the computation of the housing qualities.

[27] These parameter values come from column (1) of Table 3 in Ferreyra (2007), who structurally estimates a general equilibrium model of residential and school choice closely related to the one presented in this paper. To construct h for 2000, we use the 1990 coefficients from the rental value regression described above and apply them to 2000 data. Results are robust to the use of spending per student rather than achievement in the w index.

controlling for district fixed effects. The high correlations of incomes and rental values in panels (a) and (b) lend support to Propositions 2 and 3.

Proposition 4 establishes conditions such that house quality, income, achievement and rental values ascend in the same order across districts. Hence, panel (c) of Table 1 displays 1990 and 2000 district-level correlations among these variables. Since Proposition 4 also establishes conditions such that the ordering of each of these variables across districts is the same in both years, Panel (d) of Table 1 shows the correlation between the 1990 and 2000 values for each of these variables. The high correlations in panels (c) and (d) provide evidence in favor of Proposition 4. The correlations with respect to achievement test scores are less strong.[28] Provided the conditions established in Proposition 4 hold, the across-neighborhood ordering of income, rental value and the local index should be the same in 1990 and 2000. To assess this implication, panel (e) of Table 1 displays the tract-level correlation for these variables between 1990 and 2000. The high correlations in panel (e) support the implication.

An alternative way to assess the ordering and stratification predictions is the analysis of rank order violations (see Table 2). Proposition 2, for instance, predicts that income, rental value and the local bundle have the same ordering across neighborhoods. If this prediction holds in the data, then the ranking of tracts by the local index should be the same as the ranking by income. Thus, counting the number of pair wise rank order violations between the local index and income provides an empirical assessment of the prediction. A similar reasoning can be applied to assess the remaining predictions. Since the metropolitan area contains 1,122 tracts and 83 districts, there are 628,881 possible pair wise rank order violations in tract-level data and 3,402 in district-level data.[29]

Columns (1) and (2) investigate rank order violations of income and rental value with respect to the local index at the tract level for 1990 and 2000, respectively. As

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[28] Although Table 1's correlations involving achievement are comparatively low, it should be kept in mind that the achievement measure is likely to be affected by substantial measurement error, as Kane and Staiger (2002) have documented. In principle, such error would tend to decline with district size. Indeed, when we compute weighted correlations taking into account district size, all the correlations involving achievement rise. Perhaps a more serious issue is the imperfect measure that achievement scores provide of the underlying construct of interest, discussed further in the section below on school quality.

[29] From a list of N elements, $N(N-1)$ pairs of distinct elements can be formed. In half of them the first element will be no smaller than the second, and in the other half the first element will be smaller than the second. Hence, $N(N-1)/2$ is the maximum number of pair wise rank order violations in a set of N elements.

column (1) indicates, in 1990 the income ordering of only 14.1 percent of tracts in 1990 differs from the local index ordering, and the rental value ordering of only 10.8 percent of tracts differs from the local index ordering. The results are very similar for 2000. Thus, columns (1) and (2) provide additional support for Proposition 2. Also consistent with our correlation analysis, column (3) provides evidence that income, rental value and the local index have approximately the same ranking in 1990 and 2000.

Since Proposition 4 relies on the assumption of housing stratification, columns (4) and (5) investigate the validity of this assumption through the following procedure. First, we sort districts by increasing order of district average housing quality. Then, within each district we sort tracts by increasing order of housing quality. If there were stratification across districts, the resulting list of tracts should be perfectly sorted by housing quality – equivalently, each tract's position in the list should be equal to its quality ranking in the metropolitan area. Thus, counting the number of pair wise rank order violations in the tract-level housing quality list offers an assessment of the stratification assumption. The small percent of pair wise rank order violations each year provides reasonable evidence that the assumption holds in the data. Addressing Proposition 4, columns (6) and (7) investigate rank order violations of income, rental value and achievement relative to housing quality at the district level. These results, illustrated in Figure 5 for 1990, indicate that the ordering of these variables is approximately the same across districts each year. Finally, column (8) provides evidence that housing quality, income, rental value and achievement have the same ordering across years.

The prediction that the income ordering across districts and neighborhoods is the same before and after the reform suggests that each period the demographic cross section will be a replica of the previous one. Given that people age, this does not rule out household relocation across neighborhoods and districts. Rather, it implies that arrivals of young households and departures of old households are such that neighborhood and district demographics are maintained. The evidence presented in Tables 1 and 2 indicates that district and neighborhood demographics are indeed highly stable.[30]

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[30] To our knowledge, the only other studies that examine the impact of school finance reform on community demographics are Aaronson (1999), who uses panel data from several states, Keely (2005), who focuses on the Kentucky school finance reform, and Roy (2004), who studies the Michigan reform. None of these studies finds evidence of demographic changes.

To summarize, our correlation and rank order violation analyses lend considerable support to our ordering and stratification predictions, particularly with respect to incomes and rental values. The correlations of the local index with income are also relatively supportive. The evidence with respect to achievement scores is less strong. However, as we noted earlier, achievement is the most difficult of the model's constructs to measure accurately. We now turn to the predictions for house values and capitalization.

Evidence on Property Values and Capitalization

Our model suggests that the following regression, run on neighborhood level data, should capture the changes in house values predicted by Propositions 1, 3, 5 and 6:

$$\hat{\Delta V} = aV + b_1\Delta\hat{t}V + b_2\Delta s + b_3 rank + b_4 rank^2 \qquad (18)$$

where Δ represents changes between 1990 and 2000, V stands for 1990 neighborhood average value, $\hat{t}$ is effective property tax rate, s is average local and state revenue, and *rank* is the 1990 tract rank by average household income.[31] Coefficients b_1 and b_2 capture the capitalization of the property tax and spending reform, respectively, and b_3 and b_4 capture the effect of the mean-preserving change in the income distribution. Parameter a is the percentage rate of growth of property values not associated with changes in property taxes, education spending, or income rank. We normalize (18) by dividing by V:

$$\frac{\hat{\Delta V}}{V} = a + b_1\Delta\hat{t} + b_2\frac{\Delta s}{V} + b_3\frac{rank}{V} + b_4\frac{rank^2}{V} \qquad (19)$$

where the constant term represents the proportionate change in housing price associated with a proportionate increase in income.[32,33]

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[31] The effective property tax rate is half of the nominal tax rate because property is assessed at half of its market value. Since average house value in 1990 is only reported for specified owner-occupied housing units (one-family houses on less than 10 acres without a business or medical office on the property), our 1990 and 2000 data on house value pertain to specified owner-occupied housing units. In the average tract in our sample, these account for about 87 percent of all owner-occupied housing units. Although an alternative summary of tract value is the median, we chose to focus on the average because the normalization of the 1990 data to 2000 boundaries did not have access to individual-level data but to block level data, which were then aggregated up to the tract level. Hence, the resulting tract medians are not the actual medians which would be obtained through access to the original individual-level data but rather the weighted averages of block-level medians.

[32] Hu and Yinger (2007) also exploit the difference in tract-level average house value between 1990 and 2000 to study the capitalization of school district consolidation in the state of New York.

Columns (1) and (3) of Table 3 present the results for regression (19) for the full sample. To interpret the coefficient on millage change, consider an unanticipated small change in $\hat{t}$ in a short run interval when the tax-inclusive implicit rent v does not change and the house value V changes only because of changes in $\hat{t}$. Then the rate of change in V with respect to $\hat{t}$ is:[34]

$$\frac{1}{V}\frac{\partial V}{\partial \hat{t}} = -\frac{\left(1-\tau_f\right)}{\left[\left(1-\tau_f\right)\left(i+\hat{t}\right)+\eta+\mu+\chi-\pi\right]} \tag{20}$$

Given our measures for the components of the user cost rate, the rate of change in V with respect to $\hat{t}$ implied by equation (20) is -7.49 and -14.70 for 1990 and 2000, respectively. Thus, other things constant, our regression should yield a value for b_1 on the order of -1500 to -700 if property taxes were fully capitalized. Our coefficients on millage change in columns (1) and (3) fall in this range and hence are consistent with full capitalization.[35] They imply an average increase in property values close to 10 percent.

To interpret the coefficient on the normalized revenue change, note that $b_2\frac{\Delta s}{V}$ can be written as $b_2\frac{s}{V}\frac{\Delta s}{s}$, implying that the elasticity of house value is given by $b_2\frac{s}{V}$. Since the district average spending in 1990 was equal to $6,323, and the tract average value was equal to $105,304, our estimates of the elasticity of house value with respect to revenue equal 0.48 and 0.41 respectively in columns (1) and (3) of Table 3.[36]

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[33] We do not conduct a similar analysis for renter-occupied housing because rental property is categorized as non-homestead in Michigan (Lockwood (2002)), and the tax reform was less favorable to non-homestead than homestead property (see Section 2). In our sample, for instance, the average district achieved a non-homestead property tax reduction of approximately 6 mills. The effect of such a small change would hardly be identified with any precision. Also the increase in the taxable value of homestead property was capped at the lower of the rate of inflation or 5% per year. Given the low post-reform millage rate, the effect of this tax cap on properties that appreciated faster than the cap is quite small.

[34] We believe this approach, employing the user cost of housing, has not previously been used to deriving implications for capitalization. Yinger et al (1998) presents the conventional derivation.

[35] In their review of capitalization studies, Yinger et al (1998) document modest rates of tax capitalization. In a study published shortly afterwards, Palmon and Smith (1998) find evidence of full capitalization. More recently, Fischel (2001) reviews capitalization studies and points to flaws and limitations in their design that drive the partial capitalization result.

[36] In judging this correlation, it is important to bear in mind that education expenditure is an annualized value whereas property value measures the present value of the flow of housing services. When the elasticity estimates are interpreted with this in mind, the estimates are quite plausible. For instance, based on Table's 3 column 1, the predicted increase in value for a tract with the district average increase in revenue ($650) is about $5,200. Annuitized with a rate of 5 or 10 percent, this increase in revenue is

Proposition 5 and the associated corollary establish that, relative to neighborhood income rank, the changes in house value should have the same number of peaks as there are crossings of the CDF's in Figure 4. There is a crossing of the CDF's toward the lower end of the income distribution, implying that there should be at least one peak. We find a second crossing at approximately the 90th percentile of the income distribution. When we employ a quadratic function of rank, the coefficients on the normalized income rank and rank squared in column (1) of Table 3 suggest that the change in house prices follows an inverted U-shaped form. The coefficients of the rank variables in column (3) indicate another peak at the very top end. The latter is consistent with the second crossing toward the high end of the income distribution. The model implies that the peaks should occur at the income levels at which the CDF's cross. The peaks implied by the regressions in columns (1) and (3) are broadly consistent with this prediction. Given that our polynomial function is an approximation to the unknown function characterizing the pattern of price changes, we view the shape of the polynomial as quite encouraging evidence in support of this prediction.[37] Finally, the constants indicate the estimated proportionate change in house values. According to Proposition 6, the estimated constant should be of similar magnitude to the increase in real net-of-tax average income, which was approximately 12 percent between 1990 and 2000. The estimated constant terms in columns (1) and (3) of Table 3 are not significantly different from 12.

Note that all coefficients in Table 3 are statistically significant at the 5 percent level when the errors are not corrected for clustering. Correcting the standard errors for arbitrary within-district correlation and heteroskedasticity causes our standard errors rise, particularly for the two district-level variables and the constant (millage and expenditure changes are invariant across neighborhoods in a district). While the coefficients on expenditure change remain significant when standard errors are cluster corrected, the coefficients on the millage change and the constant terms do not. Nonetheless, we view the correct magnitude of these coefficients as encouraging, while recognizing the lack of

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between $6,500 and $13,000. Hence, the increase in revenue is capitalized at a rate between 40 and 80 percent ($5,200/$13,000 and $5,200/$6,500 respectively). Dee (2000) conducts a similar calculation to study the capitalization of school finance reform. Using discount rates of 5 or 10 percent, his results imply capitalization rates between 50 and 100 percent, which are comparable to ours.

[37] When we estimate the regression in column (1) of Table 3 without the functions of income rank, the coefficients on tax and revenue change rise by about 50 percent. These results highlight the importance of controlling for the effects of changes in the income distribution.

precision of the estimated effects of tax changes that arises due to the intra-district correlation of the observations.

The investigation of the robustness of our findings proves to be reassuring. There are seven Census tracts with appreciation rates above 200 percent, which is higher than the 99th percentile of appreciation rates. To verify that these outliers do not dominate our regression results, columns (2) and (4) present estimates for the dataset that excludes them. The results differ little from those in columns (1) and (3).

In our experimentation with higher order polynomials of rank, we find, not surprisingly, that the estimate of the constant term is sensitive to the order of the polynomial (compare columns (1) and (3), and (2) and (4)) of Table 3). For instance, Figure 6 displays the predicted increases in house values associated with estimating a cubic regression while imposing alternative values for the constant term. Since Proposition 6 implies that a value of 12 would be consistent with the observed increase in average household income, we use estimates from a regression including a cubic function of income rank and an intercept of 12 to calculate the predicted effects of income rank on house value. These effects imply median and mean changes in house prices of 6.6 percent and 9.6 percent respectively. Hence, the effects of changes in the shape of the income distribution are non-trivial in magnitude.

We repeat our analysis using Weighted Least Squares (see Table 4) with the square root of the number of specified owner-occupied housing units as our weighting variable. The WLS results tell qualitatively the same story as the OLS results, reinforcing the robustness of our findings. Nonetheless, since our analysis employs aggregate data, the concern exists that changes in the composition of the housing stock might be driving our results. Thus, we repeat our analysis only for tracts with low construction and destruction rates, low vacancy and high ownership, and small changes in vacancy and ownership. While we do not report the details in the interest of space, the results prove to be quite robust to these screenings of the data.

Although our evidence is consistent with full capitalization, this evidence faces two substantive limitations – namely, the use of aggregate rather than individual-level data, and a broad time window (1990-2000) around the reform. Ideally, we would circumvent these problems by observing housing units sold twice, once before and once

after the reform, during a small window of time around it. Such an analysis was carried out by Guilfoyle (1998a, 1998b), who used data on houses sold both in 1992 and 1996. His preferred specification using house-level data yields evidence of a capitalization rate below full capitalization. His estimates using mean sales prices for each community in Oakland County between 1990 and 1996, which are more comparable to ours because they rely on aggregate data, are closer to full capitalization.[38]

To summarize, our empirical results provide evidence of full capitalization of the property tax reduction, capitalization of the revenue reform, and changes in house values associated with changes in the income distribution over the decade. This evidence is consistent with the theoretical predictions emerging from our model.

Effects on School Quality

The evidence that indicates limited changes in district demographics implies that peer quality in public schools should not have changed much. Absent changes in peer quality, the observed changes in school quality should be solely related with changes in revenue according to our model. Hence, we now investigate changes in school achievement over the decade using fourth grade math pass rates in 1991 and 1999, depicted in Figure 7. Column (1) of Table 5 shows the regression of the change in pass rate on the change in revenues. According to this coefficient, an extra thousand dollars is associated with about six additional percentage points in the fourth grade pass rate. This is close to 40 percent of the standard deviation in 1991 test scores, similar to the 40 or 50 percent obtained by Roy (2004) and slightly above some of the estimates from Papke (2005).

Nonetheless, column (2) of Table 5 suggests some caution, as similar evidence presented by Cullen and Loeb (2004) does for the entire state. Once we control for the 1991 pass rate, the coefficient on revenue falls due to the negative correlation between the increase in revenue and the initial pass rate. Thus, at least part of the positive association between revenue and achievement gains might be driven by the fact that the

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[38] Since Guilfoyle's effective property tax rate is divided by a real interest rate of 3 percent, we divide his tax rate coefficients by 0.03 to compare to ours. Given that his dependent variable is the log of property values, these coefficients should be between -15 and -7 to be consistent with full capitalization. After dividing by 0.03, the resulting coefficients are between -5.5 and -3.5 for his dual sales sample, and between -9 and -8 for his aggregate data.

districts with the largest revenue gains had the highest potential for academic gains because they had the lowest initial achievement. Furthermore, the mere magnitude of the decade's improvement suggests that factors other than the revenue increase may have been associated with the achievement gains, not least of which might be the progressive learning about the test, first administered in 1991, on the part of students and teachers.[39]

To summarize, the data provide evidence of capitalization of the property tax and revenue reform, and of the house value changes associated with changes in the income distribution. Furthermore, the data suggest that although the reform favored the low-revenue districts, demographics in these (or other) districts did not change much. Nonetheless, low-revenue districts experienced the largest achievement gains, although these might not have been fully associated with revenue gains.

6. Concluding Remarks

We investigate whether the school funding and property tax reform implemented in Michigan in 1994 caused general equilibrium effects in the Detroit metropolitan area. Our stylized model of the reform implies that we should primarily expect changes in house values and revenues, capitalization of the tax and revenue reform, no demographic changes, and school quality improvement associated to the revenue increase. These predictions are broadly consistent with the data. While the predictions on demographics and school quality rely on the initial equilibrium satisfying certain conditions, these are not overly restrictive. A further contribution of the paper is the analysis of the effects of the shift of the income distribution in the Detroit metropolitan area. We have decomposed these changes into a change in the mean and a mean-preserving change for analytical purposes, and have found empirical support for the predicted effect of such changes. The analysis of the effects of changes in the income distribution is of interest in its own right, and further research might apply the theoretical framework presented here to shed light

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[39] Fuller et. al. (2006) provide extensive evidence that state achievement tests often register improvements when little or no improvement is found on national tests of the same student population, reporting "…yawning gaps between federal and state test results…". Roy (2003) provides evidence that Michigan's academic gains are much smaller when measured by federal rather than state tests. Whereas the adoption of public school choice, charter schools and accountability measures may have boosted achievement, Courant et al (2003) document little activity in this regard in the Detroit metropolitan area by 2000.

on some of the dramatic differences in house value appreciation across different metropolitan areas over the past decade.

Our analysis does not incorporate private schools. Some results such as Proposition 1 extend immediately when there are private schools. Thus, our model predicts that the tax instrument changes associated with the Michigan reforms would not affect the allocation between public and private schools. Other results can be extended when suitably amended. For example, the ascending bundles property applies if all neighborhood public schools continue to attract students, i.e., no neighborhood school is entirely supplanted by private schools. Other results do not extend. In particular, as Nechyba (1999, 2000) has emphasized, households attending private schools locate without regard to public school quality. Thus, the income stratification property in Proposition 2(b) need not hold in the presence of private schools. However, private schools serve a small proportion of students, and we would expect that the modest changes in public school expenditure from the Michigan reform would have little impact on the allocation of students between public and private schools. Indeed, the change in the proportion of students attending private schools in the Detroit metropolitan area was only 0.7 percentage points over the period from 1990 to 2000. Given the relatively small "market share" of private schools, we would not expect the effects of the Michigan public school reforms to be significantly altered by the presence of private schools.

Our analysis rests, *inter alia*, on the assumption that housing stocks are fixed. In the long run, this assumption is clearly untenable. Indeed, recent research (Brueckner and Rosenthal (2005)) suggests that, over the long term, redevelopment may have a transformative effect on urban areas. As we noted in the discussion of our empirical analysis, substantial redevelopment may have occurred in some Census tracts in the Detroit metropolitan area. Thus, the decade-long interval to which we are restricted by data may be stretching the limits of the fixed-stock assumption. Nonetheless, the broad congruence between the model's predictions and our empirical findings suggests that the framework can be helpful in assessing short- to medium-term effects of policy changes. Hence, we believe that our analytical perspective is useful in understanding the effects of the Michigan reform and that similar analyses can prove useful more generally in anticipating effects of actual and prospective school finance reforms.

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TABLE 1
Tract- and District-Level Correlations

a. 1990 and 2000 Tract-Level Correlations (2000 correlations in italics)

	Income	Rental Value	Local Index
Income	1.000		
Rental Value	0.939 *0.920*	1.000	
Local Index	0.842 *0.840*	0.881 *0.826*	1.000

b. Within-District Tract-Level Correlations Across 1990 and 2000

Income	Rental Value	Local Index
0.865	0.881	0.861

c. 1990 and 2000 District-Level Correlations (2000 correlations in italics)

	Housing Quality	Income	Rental Value	Achievement
Housing Quality	1.000			
Income	0.932 *0.879*	1.000		
Rental Value	0.901 *0.817*	0.969 *0.978*	1.000	
Achievement	0.674 *0.487*	0.709 *0.566*	0.771 *0.581*	1.000

d. District-Level Correlations Across 1990 and 2000

Housing Quality	Income	Rental Value	Achievement
0.951	0.976	0.981	0.739

e. Tract-Level Correlations Across 1990 and 2000

Income	Rental Value	Local Index
0.944	0.953	0.900

Number of observations: 1122 Census tracts, 83 schools districts. Source: 1990 Long Form in 2000 Boundaries, 2000 Long Form, School District Data Book, Michigan Department of Education, and authors' own calculations.

TABLE 2
Tract- and District-Level Rank Order Violations (in percents)

variables	1990 tracts (1)	2000 tracts (2)	1990 and 2000 tracts (3)	1990 tracts (4)	2000 tracts (5)	1990 districts (6)	2000 districts (7)	1990 and 2000 districts (8)
Income, Local Index	14.1	17.4						
Rental Value, Local Index	10.8	15.2						
2000 Income, 1990 Income			10.3					
2000 Rental Value, 1990 Rental Value			7.2					
2000 Local Index, 1990 Local Index			14.6					
Housing Quality[40]				16.9	17.7			
Income, Housing Quality						10.2	14.6	
Rental Value, Housing Quality						16.3	19.5	
Achievement, Housing Quality						25.8	32.6	
2000 Housing Quality, 1990 Housing Quality								9.5
2000 Income, 1990 Income								7.1
2000 Rental Value, 1990 Rental Value								8
2000 Achievement, 1990 Achievement								20.8

Source: authors' own calculations based on the sources for Table 1.

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[40] For an explanation of the rank order violations listed on this row, see the text.

TABLE 3
Percent Change in Tract Average House Value

Independent Variable	OLS Estimates (1)	OLS Estimates (2)	OLS Estimates (3)	OLS Estimates (4)
millage change between 1989 and 1999	-1459.745	-887.060	-1464.941	-890.613
	(547.294)	(424.592)	(546.017)	(422.227)
	[1359.745]	[1031.058]	[1369.592]	[1034.978]
normalized change in average revenue between 1989 and 1999	8.011	7.736	6.998	6.578
	(1.408)	(1.094)	(1.463)	(1.133)
	[2.300]	[2.471]	[2.395]	[2.559]
normalized 1989 average income rank	3.961	5.037	6.772	8.240
	(0.709)	(0.552)	(1.328)	(1.031)
	[0.587]	[0.562]	[2.071]	[1.486]
normalized 1989 average income rank squared	-0.005	-0.005	-0.014	-0.015
	(0.001)	(0.001)	(0.004)	(0.003)
	[0.001]	[0.001]	[0.006]	[0.005]
normalized 1989 average income rank cubic			7.00e-06	7.96e-06
			(2.80e-06)	(2.17e-06)
			[3.84e-06]	[3.17e-06]
Constant	22.012	21.465	19.153	18.212
	(6.829)	(5.291)	(6.908)	(5.336)
	[15.054]	[14.048]	[13.636]	[12.783]
number of observations	1122	1115	1122	1115
R^2	0.194	0.267	0.199	0.276
s.e. of regression	33.417	25.889	33.339	25.745
mean of dependent variable (in percentage points)	52.605	51.012	52.605	51.012
include tracts with percent change in tract average house value > 200 percent?	Yes	No	Yes	No

Unit of observation: Census tract with at least 50 specified owner-occupied housing units in 1990 and 2000.

Millage reduction computed as described in the text. "Normalized" means divided by 1989 tract average house value.

Ordinary Least Squares. Conventional standard errors in parentheses. Standard errors robust to arbitrary within-district correlation in brackets. Data: 1990 Long Form in 2000 Boundaries and 2000 Long Form

TABLE 4
Percent Change in Tract Average House Value

Independent Variable	WLS Estimates (1)	WLS Estimates (2)	WLS Estimates (3)	WLS Estimates (4)
millage change between 1989 and 1999	-1173.656	-996.087	-1101.840	-917.081
	(362.837)	(326.588)	(357.341)	(319.537)
	[922.480]	[821.129]	[908.548]	[801.012]
normalized change in average revenue between 1989 and 1999	6.899	6.308	5.046	4.329
	(1.042)	(0.939)	(1.070)	(0.958)
	[2.630]	[2.408]	[2.606]	[2.300]
normalized 1989 average income rank	5.839	6.407	11.312	12.227
	(0.532)	(0.480)	(1.042)	(0.934)
	[0.686]	[0.859]	[1.418]	[1.152]
normalized 1989 average income rank squared	-0.006	-0.006	-0.021	-0.022
	(0.001)	(0.001)	(0.003)	(0.002)
	[0.001]	[0.001]	[0.004]	[0.003]
normalized 1989 average income rank cubic			.0000121	.0000129
			(2.00e-06)	(1.79e-06)
			[2.58e-06]	[2.34e-06]
Constant	14.156	13.416	7.972	6.840
	(4.441)	(3.995)	(4.488)	(4.012)
	[12.974]	[12.868]	[11.342]	[11.394]
number of observations	1122	1115	1122	1115
R^2	0.329	0.374	0.350	0.402
s.e. of regression	23.289	20.903	22.924	20.439
mean of dependent variable (in percentage points)	47.69	47.27	47.69	47.27
include tracts with percent change in tract average house value > 200 percent?	Yes	no	Yes	No

Unit of observation: Census tract with at least 50 specified owner-occupied housing units in 1990 and 2000.

Millage reduction computed as described in the text. "Normalized" means divided by 1989 tract average house value
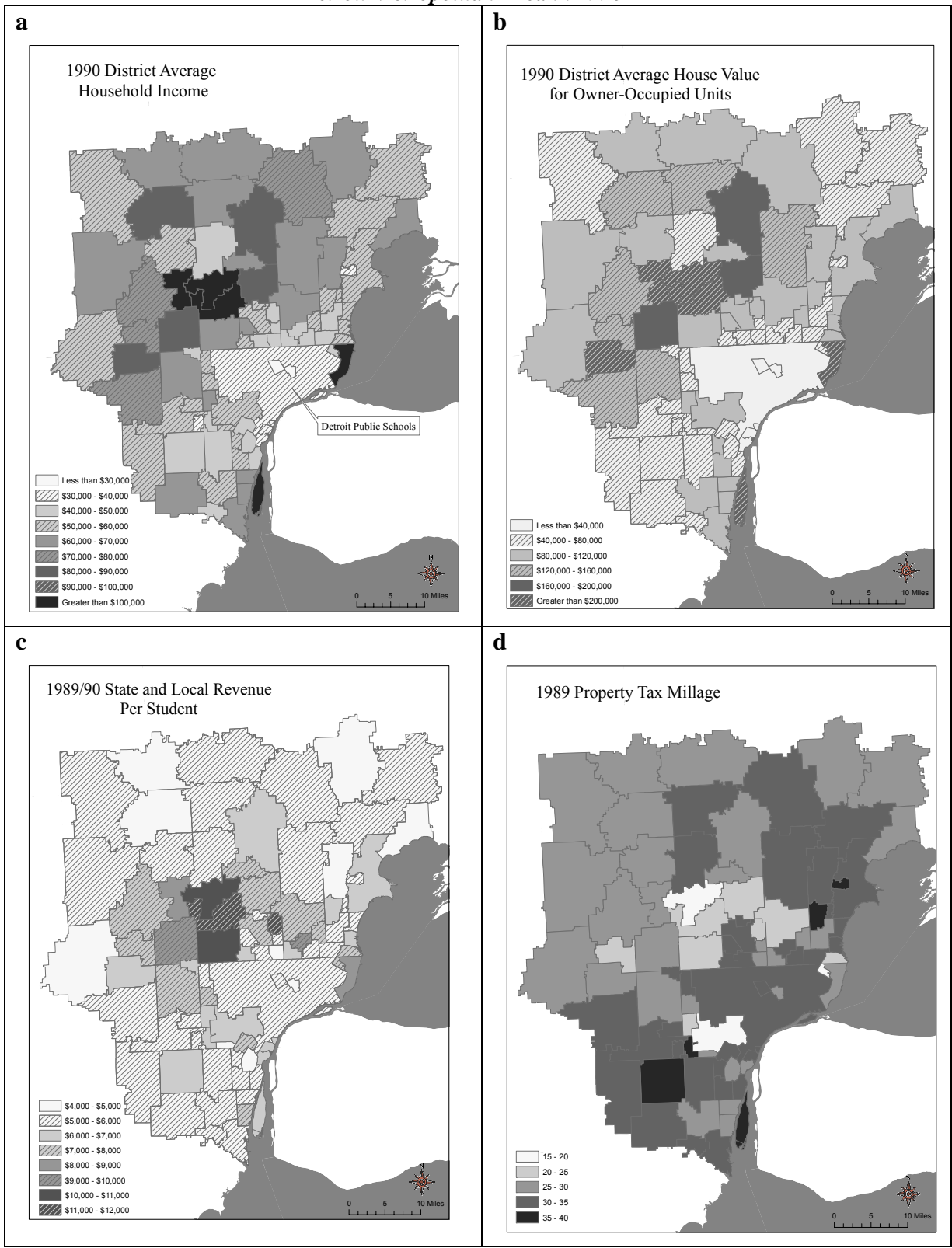
Weighted Least Squares. Conventional standard errors in parentheses. Standard errors robust to arbitrary within-district correlation in brackets. Data: 1990 Long Form in 2000 Boundaries and 2000 Long Form

TABLE 5
Change in Fourth Grade Math Passing Rate Between 1991 and 1999
(passing rate in percent)

| Independent Variable	Estimates (1)	Estimates (2)
change in average revenue between 1989 and 1999 (in thousands of dollars)	6.106 (1.859)	4.682 (1.854)
1991 fourth grade math passing rate		-0.211 (0.075)
Constant	35.050 (1.627)	44.147 (3.589)
number of observations	83	83
R^2	0.118	0.197
s.e. of regression	9.890	9.492
mean of dependent variable	39.028	39.028

Unit of observation: school district. Ordinary Least Squares. Standard errors in parentheses.
Data: Michigan Bulletin 1014 and Michigan Department of Education.

**FIGURE 1**
Detroit Metropolitan Area in 1990

a — 1990 District Average Household Income

Less than $30,000
$30,000 - $40,000
$40,000 - $50,000
$50,000 - $60,000
$60,000 - $70,000
$70,000 - $80,000
$80,000 - $90,000
$90,000 - $100,000
Greater than $100,000

Detroit Public Schools

b — 1990 District Average House Value for Owner-Occupied Units

Less than $40,000
$40,000 - $80,000
$80,000 - $120,000
$120,000 - $160,000
$160,000 - $200,000
Greater than $200,000

c — 1989/90 State and Local Revenue Per Student

$4,000 - $5,000
$5,000 - $6,000
$6,000 - $7,000
$7,000 - $8,000
$8,000 - $9,000
$9,000 - $10,000
$10,000 - $11,000
$11,000 - $12,000

d — 1989 Property Tax Millage

15 - 20
20 - 25
25 - 30
30 - 35
35 - 40

Source: SDDB, MI Bulletin 1014, and MI Dept. of Education. Property tax rates expressed in mills.

FIGURE 2
1999 Real Foundation Allowance and 1993 Base Revenue
Detroit Metropolitan Area
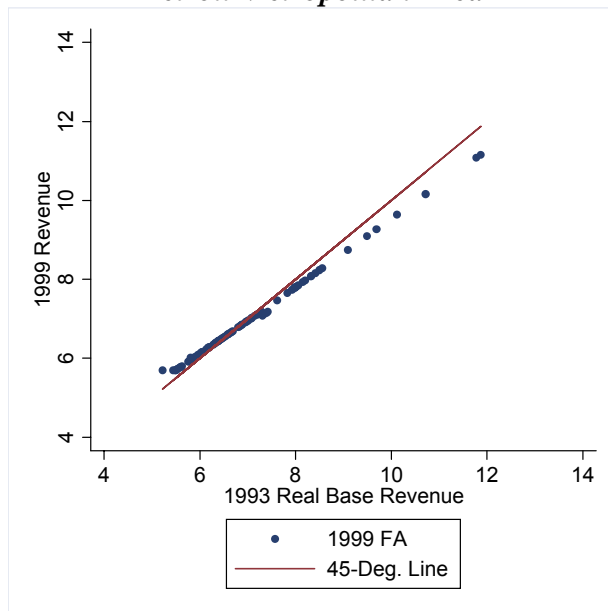
FA: Foundation Allowance. Revenue and foundation allowance expressed in thousands of 2000 dollars. Source: Michigan Bulletin 1014.
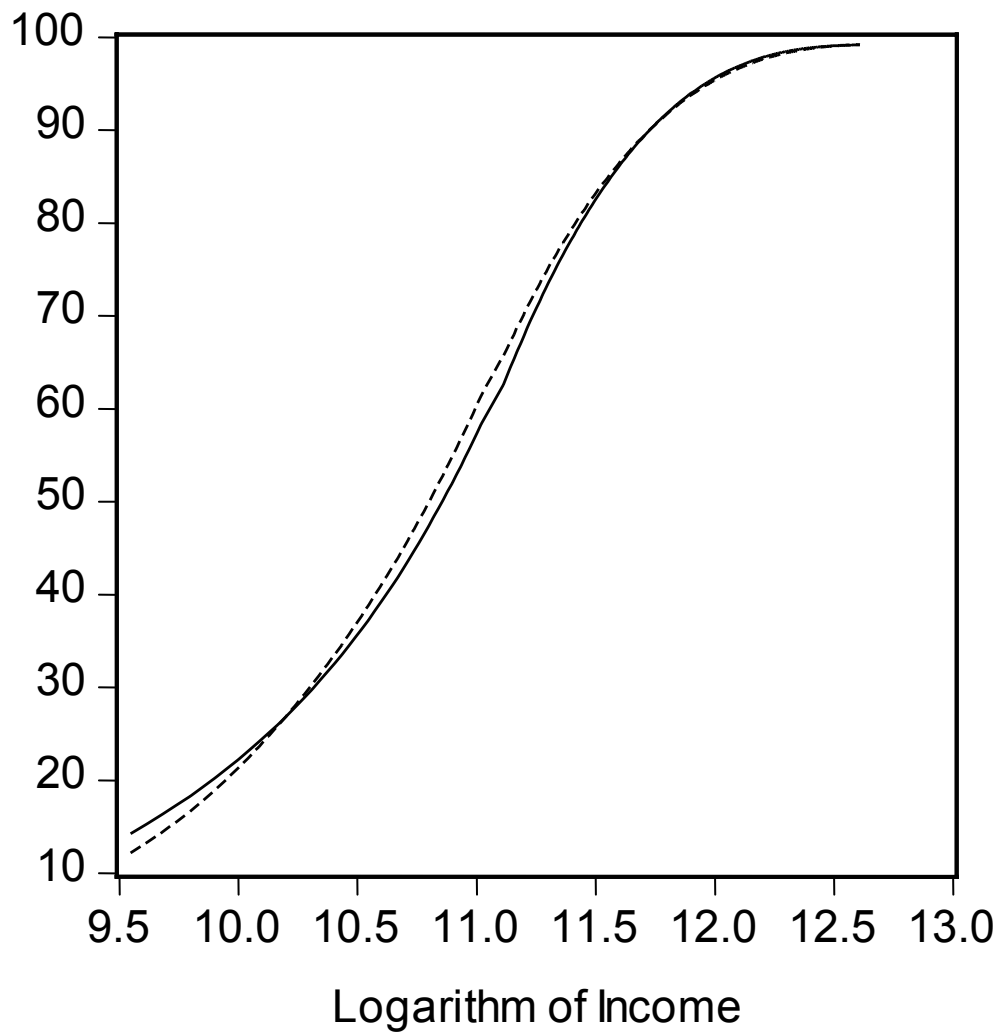
FIGURE 3
Proportional Change in Real Household Income by Income Percentile
between 1990 and 2000
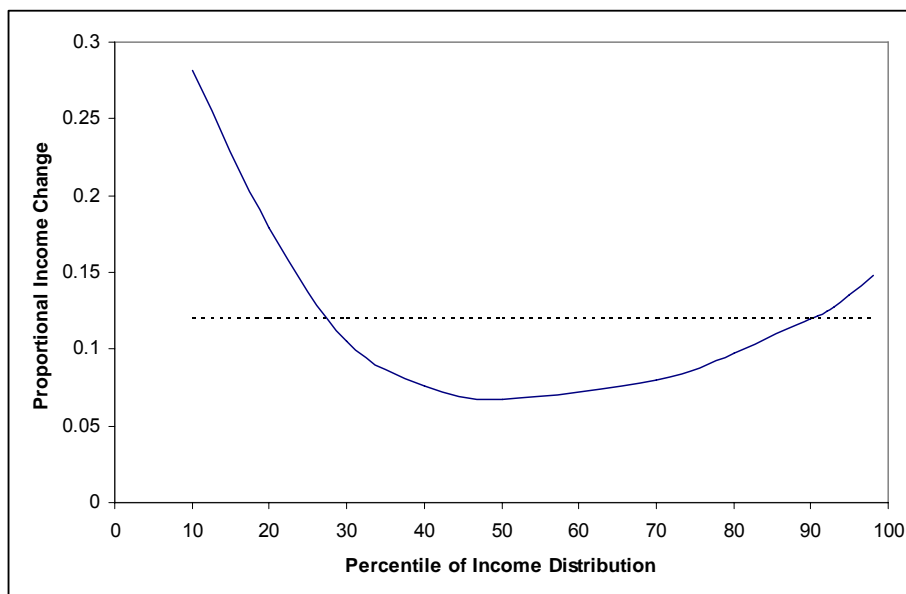
Solid line represents proportional change in real household income by income percentile between 1990 and 2000. Dashed line represents proportional change in real average household income between 1990 and 2000. Source: Authors' calculations using data from US Census for the Detroit metropolitan area.
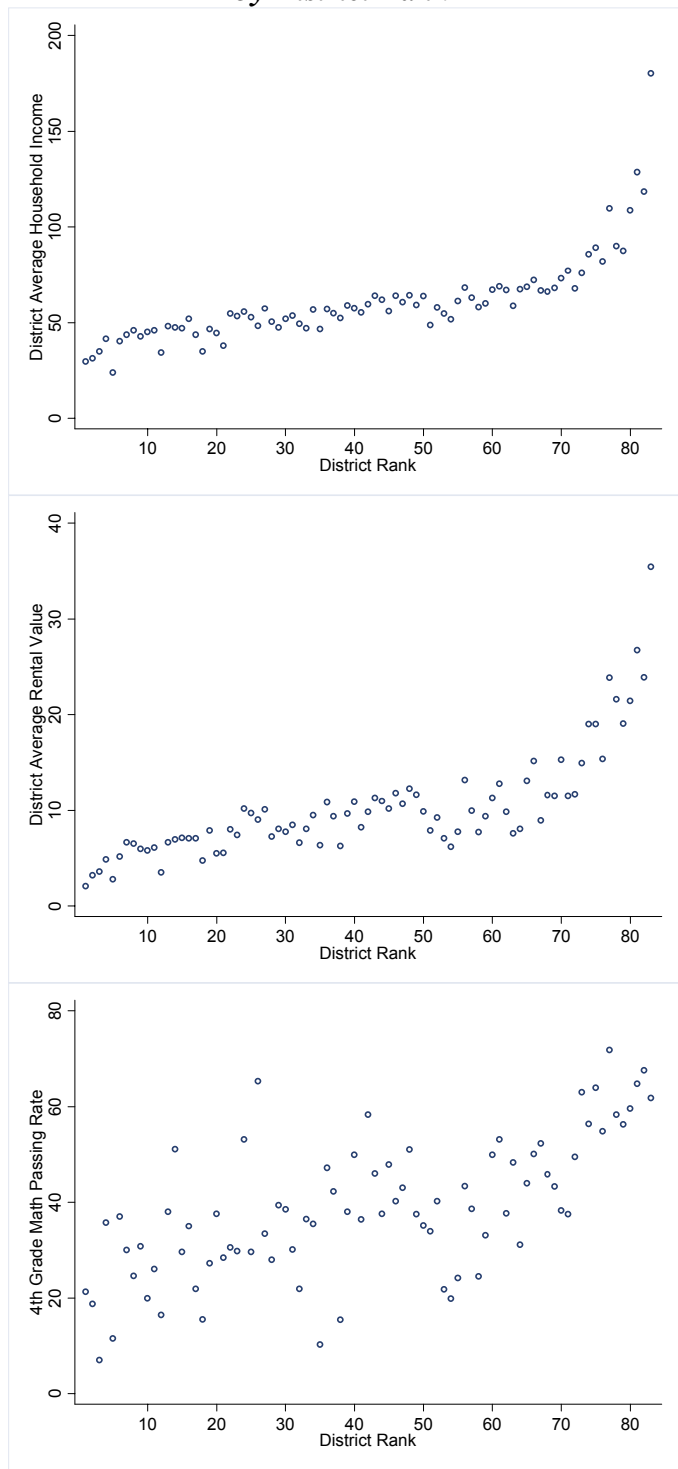
FIGURE 4
Cumulative Income Distributions in 1990 and 2000

Estimated Cumulative Density Functions in 1990 and 2000

Source: Authors' calculations using data from US Census. Solid line represents the 1990 CDF; dashed line represents the 2000 CDF.

FIGURE 5
1990 District Average Household Income, Rental Value and Passing Rate
by District Rank

Income and rental value are expressed in thousands of dollars; 4th grade math passing rate is in percent. "District rank" refers to rank by district average housing quality. Data source: 1990 Long Form in 2000 Boundaries and Michigan Department of Education.

FIGURE 6

Predicted Patterns of Housing Price Change for
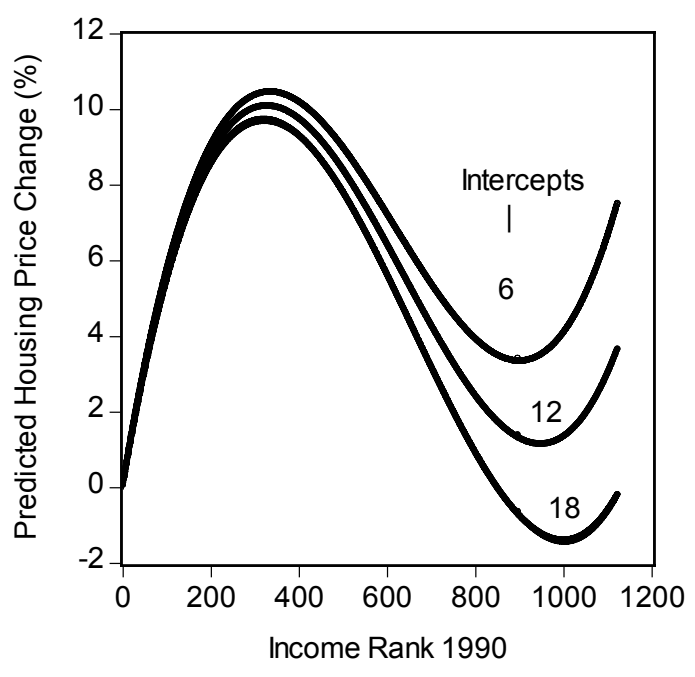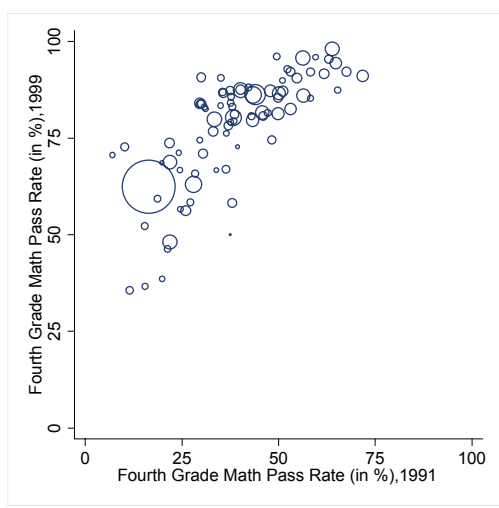Three Different Intercepts

FIGURE 7
District Average Pass Rate for Fourth Grade Math Exam in 1991 and 1999

Districts weighted by number students taking math test in 1991.
Source: Michigan Department of Education.

Appendix A

Proposition A-1: There exists a fixed point R* such that R*=ϕ(R*)

Proof: Rental income as a function of R is:

A-1. $\phi(R) = \sum\limits_{j=1}^{J} n_j v_j(R)$

where n_j is the fraction of metropolitan housing located in neighborhood j. Now

A-2. $\phi'(R) = \sum\limits_{j=1}^{J} n_j v_j'(R)$

where $\phi'(R)$ and $v_j'(R)$ denote the derivatives of ϕ and v_j with respect to R. To calculate the derivative in the summation in (A-2), we use the boundary-indifference conditions:

A-3. $U(w_j, (1+\tau+\frac{R}{y_e})y_e^j - v_j) = U(w_{j+1}, (1+\tau+\frac{R}{y_e})y_e^j - v_{j+1})$ $j = 1,...,J-1$

where $\bar{y}_e$ is mean endowed income in the metropolitan area.[41] Differentiating (A-3) with respect to R we obtain:

A-4. $U^j(\frac{y_e^j}{y_e} - v_j') = U^{j+1}(\frac{y_e^j}{y_e} - v_{j+1}')$

Here, $U^j(\cdot)$ denotes the derivative of $U(\cdot)$ with respect to its second argument. Solving (A-4) we obtain:

A-5. $v_{j+1}' = \frac{y_e^j}{y_e}(1 - \frac{U^j}{U^{j+1}}) + \frac{U^j}{U^{j+1}}v_j'$

A price normalization is required. We fix v_1, such that $0 < v_1 < y_m$. Hence $v_1' = 0$. We now prove by induction that $v_{j+1}' < \frac{y_e^j}{y_e}$ for all $j = 2,...,J-1$. Evaluating (A-5) for $j = 1$, we obtain

A-6. $v_2' = \frac{y_e^1}{y_e}(1 - \frac{U^1}{U^2}) < \frac{y_e^1}{y_e}$

The inequality follows from $U^j > 0$ for all j. Continuing with the induction, we assume

A-7. $v_j' < \frac{y_e^{j-1}}{y_e}$

and demonstrate that the result then holds for $j+1$. Equations (A-5) and (A-7) imply the first inequality below. The second inequality follows from income stratification: $y_e^j > y_e^{j-1}$.

A-8. $v_{j+1}' < \frac{y_e^j}{y_e}(1 - \frac{U^j}{U^{j+1}}) + \frac{U^j}{U^{j+1}}\frac{y_e^{j-1}}{y_e} < \frac{y_e^j}{y_e}(1 - \frac{U^j}{U^{j+1}}) + \frac{U^j}{U^{j+1}}\frac{y_e^j}{y_e} = \frac{y_e^j}{y_e}$

This concludes the proof that:

A-9. $v_{j+1}' < \frac{y_e^j}{y_e}$ for all $j = 1,...,J-1$

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[41] The metropolitan population is normalized to one, hence aggregate endowed income equals mean endowed income: $Y_e = \bar{y}_e$.

Now (A-9) and (A-2) imply the first inequality below. The second inequality follows from observing that the n_j sum to one, and the lowest endowed income in each community j, y_e^{j-1}, is less than the average endowed income in each community: $y_e^{j-1} < \bar{y}_e^j$. The third inequality follows from addition of a positive term to the preceding sum:

A-10. $\phi'(R) = \displaystyle\sum_{j=1}^{J} n_j v'_j < \sum_{j=2}^{J} n_j \frac{y_e^{j-1}}{\bar{y}_e} < \sum_{j=2}^{J} n_j \frac{\bar{y}_e^j}{\bar{y}_e} < \sum_{j=1}^{J} n_j \frac{\bar{y}_e^j}{\bar{y}_e} = 1$

Define $\Delta(R)$ as follows:

A-11. $\Delta(R) = R - \phi(R)$

Single-crossing in y implies that the v_j ascend in order j. This and $v_1 > 0$ imply that $\phi(R) > 0$ when $R=0$, implying:

A-12. $\Delta(0) = -\phi(0) < 0$

Differentiating (A-11) with respect to R and using (A-10), we have:

A-13. $\Delta'(R) = 1 - \phi'(R) > 0$

Moreover, $\phi'(R)$ is strictly less than a constant, $\displaystyle\sum_{j=2}^{J} n_j \frac{y_e^{j-1}}{\bar{y}_e}$, that is less than one. This and (A-12) imply that there is an R^* such that $\Delta(R^*) = 0$.