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EQUILIBRIUM WAGE DISTRIBUTIONS

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Equilibrium Wage Distributions

ABSTRACT

This paper analyzes equilibrium in labor markets with costly search. Even in steady state equilibrium, identical labor may receive different wages; this may be the case even when the only source of imperfect information is the inequality of wages which the market is perpetuating. When there are information imperfections arising from (symmetric) differences in non-pecuniary characteristics of jobs and preferences of individuals, there will not in general exist a full employment, zero profit single wage equilibrium.

There are, in general, a multiplicity of equilbria. Equilibrium may be characterized by unemployment; in spite of the presence of an excess supply of labor, no firm is willing to hire workers at a lower wage. It knows that if it does so, the quit rate will be higher, and hence turnover costs (training costs) will be higher, so much so that profits will actually be lower. The model thus provides a rationale for real wage rigidity. The model also provides a theory of equilibrium frictional unemployment.

Though the constrained optimality (taking explicitly into account the costs associated with obtaining information and search) may entail unemployment and wage dispersion, the levels of unemployment and wage dispersion in the market equilibrium will not, in general, be (constrained) optimal.

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EQUILIBRIUM WAGE DISTRIBUTIONS*

by

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1. Introduction

The observation that different firms pay different prices for what appears to be the same commodity or pay different wages for what appears to be equivalent labor has long been explained in economics by a reference to "imperfect information." This paper is concerned with characterizing market equilibrium with imperfect information. We do not present a general theory; rather, we develop in some detail an example of importance in its own right--imperfect information in the labor market. Several properties of our example, in particular, the existence of equilibria with price (wage) dispersion, unemployment (excess supply of labor), positive profits (excess demand for labor), multiple equilibria and the non-optimality of some or all of the equilibria, we believe are of more general validity; other results may not be.

This paper is concerned with two kinds of imperfect information:

(a) Individuals may not know the wage paid in any particular firm, or whether there is a vacancy in any particular firm, until they apply for a job.[1]

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(b) There are a number of characteristics besides the wage rate associated with any job which are important to the individual. Some, like the normal length of a work week, become known when the individual applies for the job; information about other characteristics (e.g., personalities of colleagues) is acquired only gradually.

There is one important difference between the two kinds of imperfect information: there is a return to matching individuals up with jobs that suit them in any economic system. There will be imperfect information of this sort so long as individuals and jobs differ. On the other hand, the imperfect information resulting from wage dispersion is a kind of imperfect information generated by the market itself. In particular, a socialist economy could, if it chose, simply pay a uniform wage for identical labor. We are used to thinking of markets as serving a useful function in conveying information, e.g., about demands and supplies. Our analysis suggests that, under certain conditions, the market may, in effect, unnecessarily "create" imperfections of information which, not surprisingly, may turn out to be quite costly.

There are two problems in constructing an equilibrium model with imperfect information: first, how do we prevent the eventual accumulation of information? If individuals were infinitely lived, and jobs never changed, eventually, through search,[2] everyone would find the job which most suited him. "Imperfect information" would only characterize the market in the short run. In the discussion below, we maintain a "continual flow of ignorance" through a continual flow of new entrants into the labor force and a flow of deaths out of it.[3] This "flow of ignorance" is just large enough to offset the "flow of knowledge" resulting from search behavior, and an equilibrium with imperfect information is sustained.

The second problem is, how do we induce different firms to pay different wages? That is, imperfect information may explain why individuals pay a price for a commodity which is in excess of the lowest price being charged in the market, or why they accept a wage which is below the maximum being paid in the market. But we still must explain why some profit maximizing firms charge one price or pay one wage while other firms charge another price or pay another wage.

We argue that by paying a higher wage, the firm lowers labor turnover and hence its expected returns on specific training. A low wage firm has higher training costs. Under certain circumstances, the increase in training costs is just sufficient to offset the low wages. Profits viewed as a function of the wage paid must have more than one relative maxima, with the value of profits at the different relative maxima identical. The somewhat surprising result of this paper is that this can be the case under very simple conditions, even when all firms have the same training costs.

There is an important interaction between the search for higher wage jobs and the search for jobs which "match" one's preferences. The existence of wage dispersion clearly affects the profitability of searching for a "better match." What is not so obvious, however, is that when there is imperfect information of the second type ("matching individuals to jobs") there cannot, in general, exist a single wage, zero profit equilibrium.

Previous studies have focused on firm behavior when facing individuals whose turnover rates are affected by the wage rate (Mortenson (1970), Salop

(1973b), and with individual behavior in markets with wage dispersions (Salop (1973a). But there have been no attempts to link the two sides of the market together: the quit rate of the individuals is affected by the wage distribution; the wage paid by the firm with a given training cost is determined by the quit rate function, so the wage distribution in turn is determined by the distribution of training costs. Thus, corresponding to any distribution of training costs there is an equilibrium distribution of wages. But the wage rates and quit rates associated with any given level of training costs must be such that these firms just break even (assuming free entry).

Our analysis can thus be viewed as an attempt at a simple general equilibrium formulation of the conventional search models. Such a formulation is required because in its absence we are likely to be misled into formulating models in which there is not in fact wage or price dispersion, because such an analysis is required if we are to make any valid welfare economic statements about the behavior of markets with imperfect information, and because it can provide the basis of a macro-economic model with unemployment, explaining why wages do not fall even in the presence of unemployment.

The exact characteristics of the equilibrium turn out to depend rather sensitively on the exact assumptions one makes about production, search, tastes, labor supply, and information. The simplest version of the model is presented in the next section; this is then extended to a number of different directions in subsequent sections.[3a]

2. The Basic Model

2.1 Introduction

The basic model presented in this section is a simplified version of the conventional search model. The basic ingredients (described more fully below and modified in a number of ways in subsequent sections) are the following:

(a) Individuals are continually searching for a better, i.e., in our simplified model, higher paid, job. They quit when they successfully find a better job. (In the basic model, the intensity of search will be an exogenous parameter.)

(b) For simplicity, we assume only one commodity which we choose as our numeraire. All production processes are characterized by constant returns to scale, and there is free entry. Firms face specific training costs. By paying higher wages, they reduce their turnover rate. The optimal level of wages depends on their training costs.

The assumptions of free entry and the constant returns to scale property of the technology make the zero profit condition seem natural. This in turn has the implication that the scale of each firm is indeterminate; what is determinate, however, is the wage distribution. Later, we shall show some instances in which the zero profit condition may not be satisfied.

Implicitly, the model with which we are dealing here is a dynamic one. Firms hire workers and pay the training costs one period, receiving the returns in subsequent periods. In making their decisions, they must form expectations of future interest rates, prices, and other market conditions. We shall avoid these problems by focusing on the long-run equilibrium, where the wage distribution, prices, etc., are invariant.[4]

Each firm in equilibrium is then characterized by a wage and a level of employment and output; the market equilibrium is characterized by a wage distribution. In equilibrium,[5]

(a) given the quit rate function generated by the wage distribution, each firm has chosen the wage which maximizes its profits on each job (position);

(b) all job positions make zero profits; and

(c) the markets for goods and labor clear.

In the remainder of this section, we set up and analyze the equilibrium of our simplified economy. We proceed in several stages: in Section 2.2 we describe the behavior of individuals in the economy, while in Section 2.3 we describe the behavior of firms as they face a given quit rate function. Section 2.4 derives the quit rate function. Sections 2.5, 2.6, 2.7, and 2.8 construct equilibria with a single wage, with two wages, with multiple wages, and finally, with a continuum of wages. Sections 2.9, 2.10, and 2.11 discuss the welfare, comparative static, and stability properties of the equilibria.

2.2 Individual Behavior

We assume all individuals are identical.[6] They die exponentially at the rate μ , and they are replaced by an equal number of new workers, so that the labor supply remains constant at $\bar{L}$. Each laborer supplies one unit of labor. When individuals enter the labor force, they randomly apply for a job, which they accept. Meanwhile, they continue to search for a better, i.e., a higher paying job. The length of time to go from one firm to another to find out its wage is a random variable described by a Poisson process; the average number of searches per unit of time is s , which is fixed. There is no cost to search or to changing jobs, up to the search intensity s ; more intensive search is prohibitively expensive. Individuals do not know the wage paid by any particular firm; since s is determined in effect exogenously, we need make no assumption concerning whether the individual knows the wage distribution. Later (Section 4), however, we shall have to be more explicit on this point.

2.3 The Behavior of the Firm: The Determination of the Wage, Given the Quit Rate Function

For simplicity, we assume production processes require only labor, no capital,[7] and have constant returns to scale. Thus, a production process is characterized by a fixed training cost per worker T , which is assumed to occur instantaneously upon hiring the worker, and by a level of output per man-year $a(T)'$. In the model of this section we assume that the firm has no choice of technique.[7a]

Thus, the only decision of the firm is to choose a wage. Clearly, if at that wage rate it is making a profit, it will attempt to expand; if it makes a loss, it will contract or change its wage. We shall be concerned with characterizing equilibria; accordingly, under the assumption of free entry, each firm makes zero profits, and so is indifferent about the scale of production. Thus, having determined a wage policy, the firm simply accepts for employment all individuals who apply.

The wage is chosen so that it maximizes the present discounted value of its profits on each worker hired. Our restriction to stochastic equilibrium analysis allows us to greatly simplify the problem, for then the wage is constant over time, and is chosen to maximize:

(2.1) $$p_i \equiv a(T_i) - [w_i + (q(w_i) + r)T_i]$$

where

r is the rate of discount (rate of interest

$\frac{p}{r}$ is the present discounted value of profits

$q(w)$ is the quit rate function

The labor costs consist of the direct wage payments (w), training costs to replace workers who quit (q), and interest on previous expenditures for training costs (r). q, the quit rate, acts essentially like a depreciation factor on "human capital" expenditures of the firm. By increasing w, the direct labor costs are increased but the turnover costs (the "depreciation rate") is reduced. The profits are maximized (labor costs per worker minimized) when

(2.2) $$1 + q'(w)T = 0.$$ [8]

Normally, the quit rate function is drawn as in Figure 1a, as a convex function. The iso-cost curve is a straight line with slope $-1/T$, and its tangency with the quit rate function gives the optimal wage. As we increase training costs, we increase w smoothly.

No argument, however, has been given why the quit rate function should have the given shape, rather than that of Figure 1b. There, $w \leqq \hat{w}$ for $T < \hat{T}$. As T increases above $\hat{T}$, there is a jump in the wage, and thereafter it increases smoothly with T.

In either case, however, <u>the wage is a monotonically increasing function of training costs</u>.

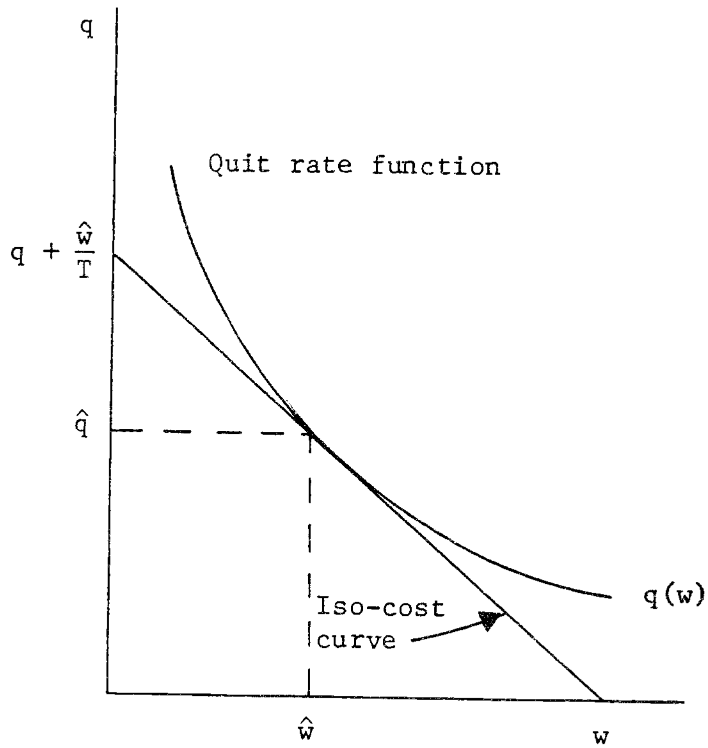

Figure 1a

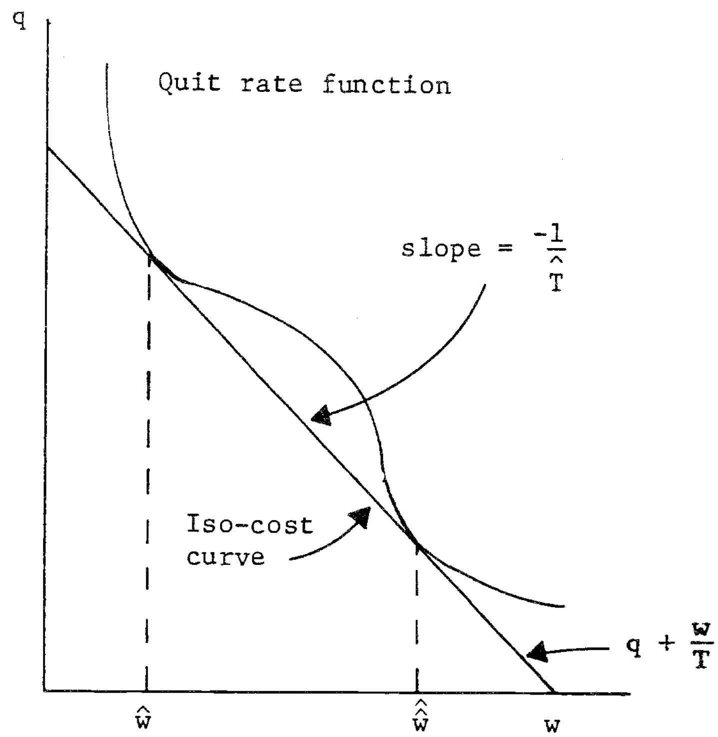

Figure 1b

2.4 The Determination of the Quit Rate Function

The crucial question, then, is the determination of the quit rate function. Under our assumptions, the quit rate is just the death rate, plus the probability of an individual finding a better job. The latter is equal to the average number of searches per unit of time, times the probability that on one of these searches the individual finds a wage greater than the wage he is presently receiving. For simplicity, we assume the probability of finding a job with a wage higher than w is equal to the percentage of jobs paying wages higher than w.[9] If $F(w)$ is the percentage of firms who pay a wage less than or equal to w, then the quit rate is[10, 11]

$$q(w) = \mu + s(1 - F(w)) \quad (2.3)$$

Hence, if $F(w)$ is differentiable at w,

$$q'(w) = -sf(w) \quad (2.4)$$

where $f(w)$ is the density function of w. Thus, for the quit rate function to be convex, the density function must be monotonically declining; i.e., <u>the only continuous wage density functions are those in which the density function of wage is monotonically decreasing</u>. In particular, continuous unimodal distributions, such as the normal distribution, imply that $f' > 0$ for wages below the mode, and hence are not consistent with equilibrium.[12]

If all other firms were to pay a wage w_o, then the quit rate function would look as in Figure 2a; if a fraction π of the firms were to pay a wage w_1 and the remainder were to pay a wage of w_o, the quit rate function would look as in Figure 2b. A firm with training cost

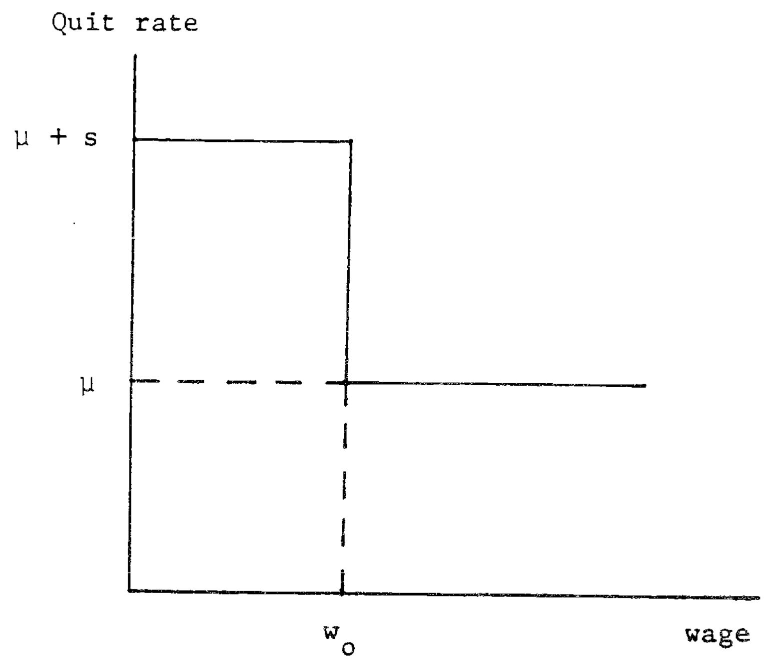

Figure 2a

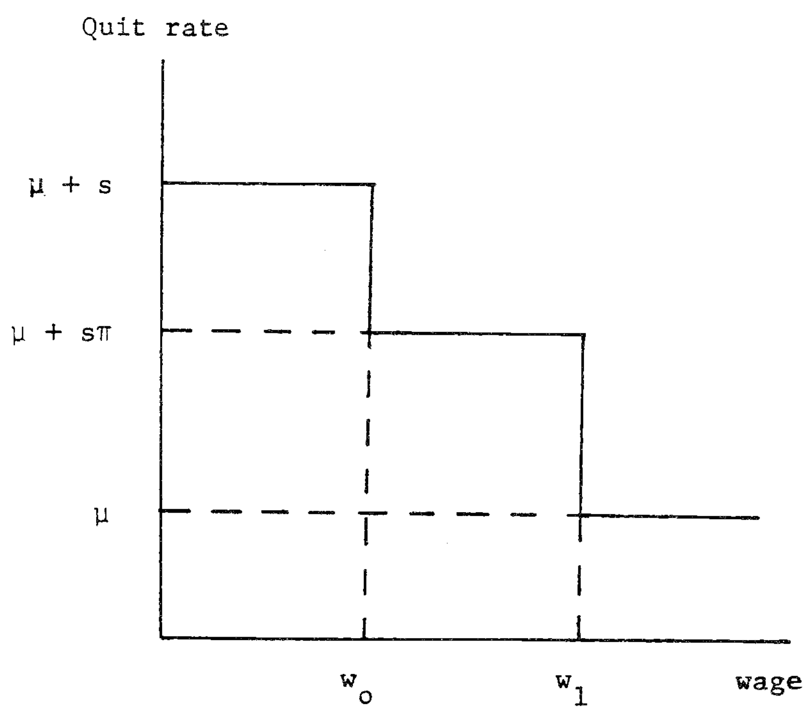

Figure 2b

(2.5a) $$T = \frac{w_1 - w_o}{s\pi} \equiv \hat{T}$$

would be indifferent to paying a wage of w_o and a wage of w_1 ; firms with higher training costs pay w_1 , lower training costs w_o . Similarly, if a fraction π_2 of the firms were to pay a wage w_2 , a fraction π_1 were to pay a wage w_1 , then the quit rate function would look as in Figure 2c. Again, firms with training costs

(2.5b) $$T = \frac{w_2 - w_1}{s\pi_2} = \hat{T}_2$$

would be indifferent to paying wage w_2 or wage w_1 ; firms with training costs

(2.5c) $$T = \frac{w_1 - w_o}{s\pi_1} = \hat{T}_1$$

would be indifferent to paying wage w_o and w_1 . Firms with training costs greater than max $(\hat{T}_2, w_2 - w_o/s(\pi_1 + \pi_2))$ pay wage w_2 ; those with training costs less than min $(\hat{T}_2, w_2 - w_o/s(\pi_1 + \pi_2))$ pay wage w_o .

Note that if w_1 is to be chosen by any firm, $\hat{T}_1 < \hat{T}_2$, i.e.,

(2.5d) $$\frac{\pi_1}{w_1 - w_o} > \frac{\pi_2}{w_2 - w_1} \quad .$$

This is the discrete version of the result noted above that continuous wage density functions must be a monotonically declining function of w .

2.5 The Single Wage Equilibrium

There is a unique single wage equilibrium, characterized by the zero profit condition

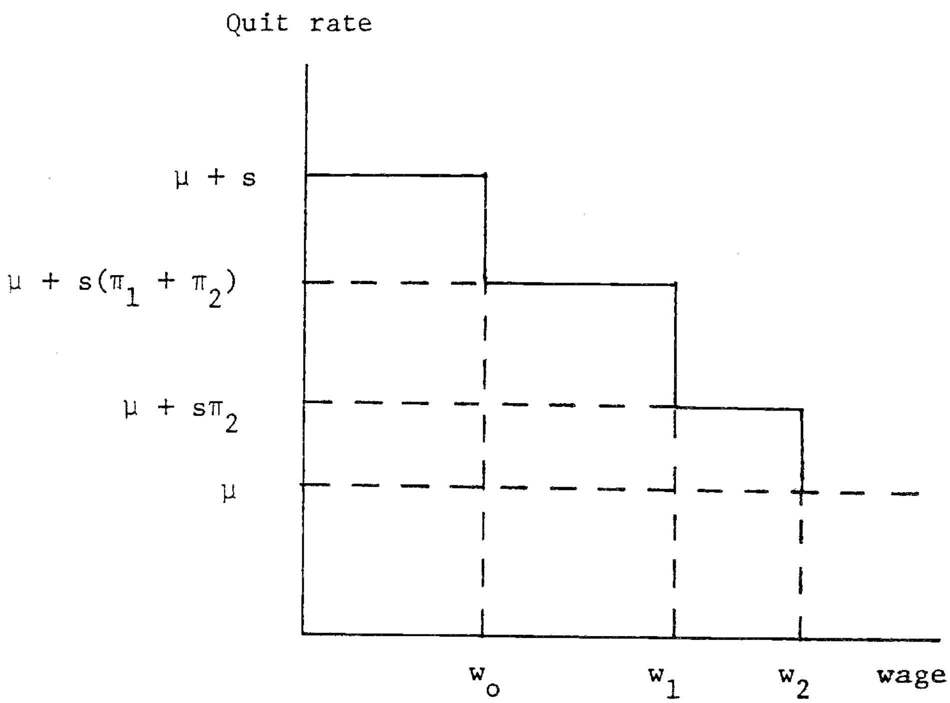

Figure 2c

(2.6) $$a(T) = w^* + (\mu + r)T \quad .$$

We also need to assume that, if $\underline{w}$ is the minimum wage at which workers can be obtained,[13]

$$\underline{w} + (\mu + r + s)T > w^* + (\mu + r)T \quad ,$$

i.e.,

(2.7) $$s > \frac{w^* - \underline{w}}{T} \quad .$$

The search intensity is sufficiently high that a firm finds it unprofitable to pay the minimum wage. For the remainder of this section, we assume (2.7) is satisfied, leaving until the next section a discussion of what happens if it is not.

2.6 A Two-Wage Equilibrium

There is a continuum of two-wage equilibria, characterized by

(2.8) $$\begin{aligned} w_1 &= w^* \\ w_o &= w_1 - s\pi T \quad . \end{aligned}$$

The wage paid by the high wage firms is always the same as that paid in the single wage equilibria. But the low wage firm pays a lower wage; however, because of the higher turnover rate, its labor costs are the same. Any pair of (π, w_o) satisfying (2.8) is an equilibrium. The quit rate function for one such equilibrium is depicted in Figure 3a.

2.7 A Three-Wage Equilibrium

There is also a continuum of three-wage equilibria, characterized by

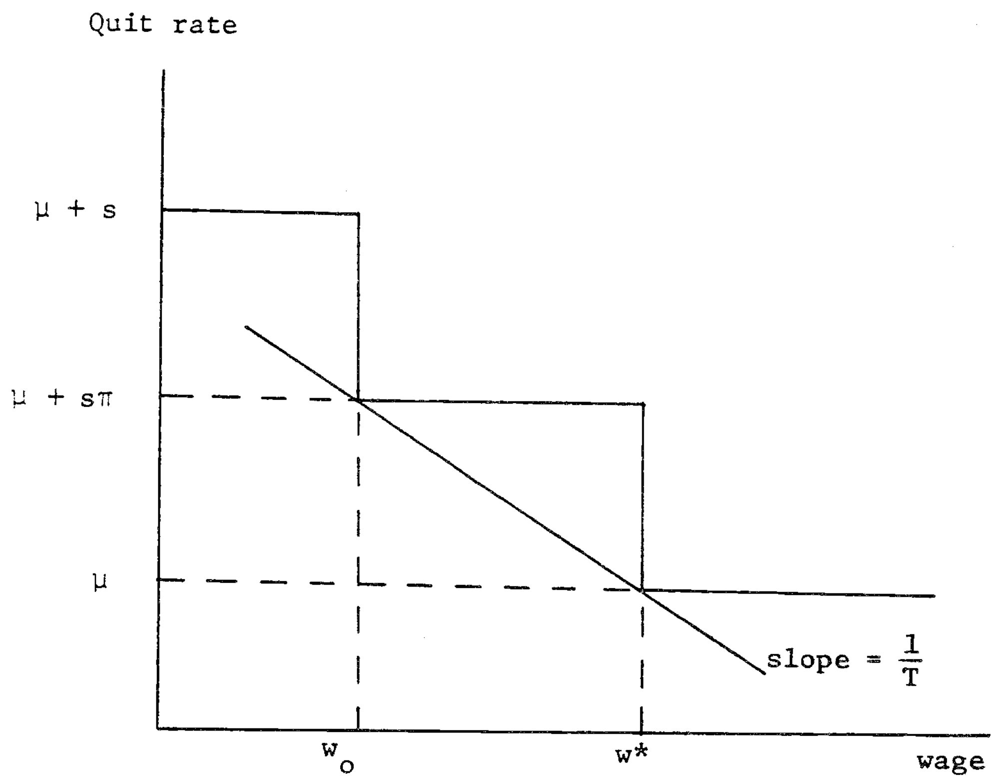

Figure 3a

Two-Wage Equilibrium

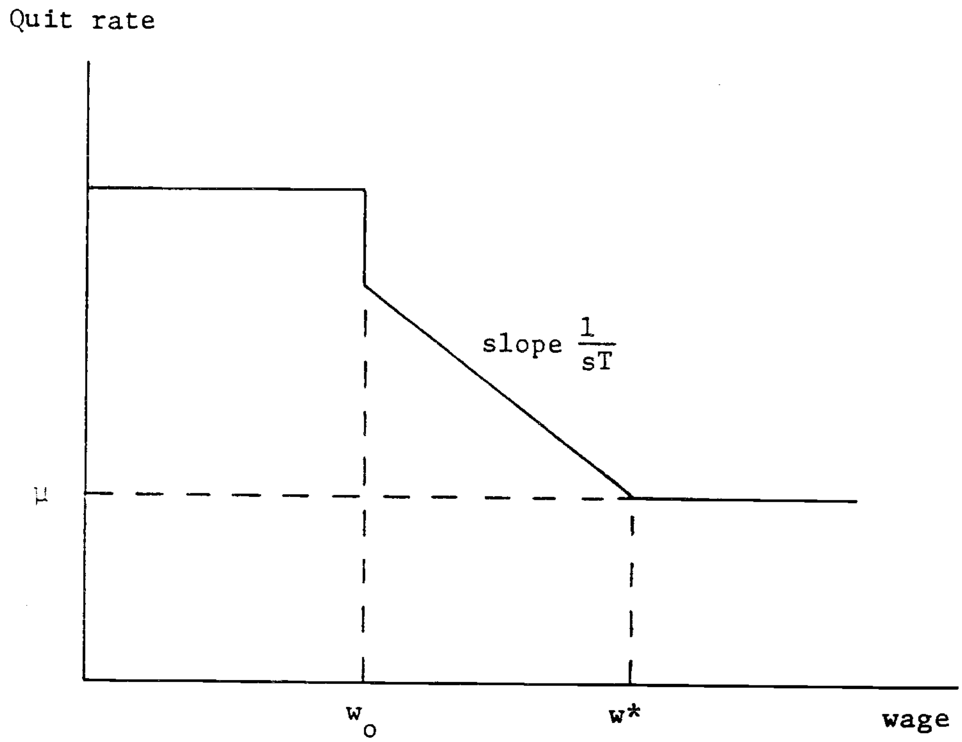

Figure 3b

Quit rate function for continuous wage distribution with mass at w_o .

$$\begin{aligned} w_2 &= w^* \\ w_1 &= w_2 - s\pi_2 T = w^* - s\pi_2 T \qquad (2.9) \\ w_o &= w_1 - s\pi_1 T = w^* - s(\pi_1 + \pi_2)T \quad . \end{aligned}$$

Any set of $\{w_1, w_o, \pi_1, \pi_2\}$, with $\pi_1 < 1$, $qp_1 + \pi_2 < 1$, satisfying (2.9) and the inequality (2.5d) is an equilibrium.

2.8 Many Wage Equilibria

It is clear that equilibria with any finite number of wage levels can be constructed. Indeed, there may be a continuum of wage levels with a uniform density between w* and w_o of 1/sT, and a mass point at w_o, with $w_o > w^* - sT$. (Figure 3b).

3. Implications of the Basic Model

Our model has a number of important implications, which we take up in this section. First, we show that most of the equilibria considered are not Pareto efficient. Next, we show that a slight extension of our model generates equilibria with unemployment and with positive profits. Finally, we consider the implications of our results for the analysis of comparative statics for search equilibria.

3.1 Welfare Economics

By the very nature of search and information there are important externalities which firms will not take into account.

First, as has long been recognized (see, e.g., Arrow (1959)), imperfect information means that all firms have, as it were, some degree of monopoly power; they can obtain workers even though they pay less than the "market wage." They quickly lose workers to other firms, but in the meanwhile, they are able to exploit the absence of information, provided the costs of turnover are not too high. But what has not been sufficiently recognized is that this exercise of monopoly power is, in some sense, itself the cause of imperfect information; that is, it is the exercise of this monopoly power which results in the wage differentials, in the absence of which search would be unnecessary. There is thus a cost, in addition to the direct loss of consumer surplus usually associated with the exercise of monopoly power, in the additional search and turnover costs.

There is the further externality imposed on the low wage firm by an increase in the wage of a high wage firm: although it reduces its own quit rate--which at equilibrium results in a reduction in turnover costs just sufficient to compensate for the increased wage--it increases the quit rate of other firms and the firm fails to take account of the additional turnover costs of the other firms.

Note that all of the equilibria with wage distributions are Pareto inferior to that with a single wage. The low wage individuals are unambiguously worse off and resources are wasted in training that otherwise would not need to have been spent.

The low wage firms may claim that they have to pay low wages because of their high turnover; but it is equally true that they have a high turnover because they pay low wages.

3.2 Unemployment Equilibria

The rigidity of real wages has long been recognized to play a central role in generating unemployment equilibria. While older discussions ascribed this rigidity to institutional factors, it has more recently been recognized that if the wage affects the net productivity of the worker,[14] then the firm may not lower its wage, even in the face of excess supply.

The usual story for why there cannot exist unemployment equilibria is that if workers are unemployed, they offer to work for less; if the wage is bid down, demand for labor is increased and the supply is decreased. The process stops only when demand equals supply. In efficiency wage models, this argument is not valid: firms may not hire workers at lower wages if to do so lowers their profits.

Here, lowering the wage increases the turnover rate, and hence firms may be reluctant to lower the wage, even though there are workers available willing to work for less.[15]

To show the nature of the unemployment equilibrium we replace our assumption of a constant returns to scale technology with one with diminishing returns; there is an aggregate production function $F(L)$, and the full employment wage is defined by

$$F'(\overline{L}) = w^f + (\mu + r)T \tag{3.1}$$

where $\overline{L}$ is the total available labor supply. The value of the marginal product of labor at full employment equals the full employment real wage plus the turnover costs. Now assume all firms are paying a wage in excess of w^f , say

$$w^* > w^f \quad . \tag{3.2}$$

Then they will wish to hire workers only to the point where

$$F'(L^*) = w^* + (\mu + r)T \tag{3.3}$$

$$L^* < \overline{L} \tag{3.4}$$

that is, there will be unemployment. Moreover, provided condition (2.7) is satisfied, there is no wage which is acceptable to workers which yields a higher profit than w^* . Hence firms have no incentives either to change the wage they pay or the number of workers they hire.

Note that if all firms could simultaneously change their wages, lowering them to w^f , then full employment could be attained. Thus, unemployment is caused by too high wages, but the excessively high wages come about not because of union pressure; rather, the interactions of the wage policies of the different firms, and the inability of the different firms to coordinate their wage policies, leads to an inefficient Nash equilibrium.

Indeed, it is easy to see in this model how a disturbance to the economy can move it from a position of full employment to one of unemployment. Assume, initially, that the economy were at full employment but then the technology is disturbed in such a way as to decrease the marginal product of labor. So long as all other firms continue to pay w^* , it pays each firm to continue to do so: all adjustments take place in the number of workers hired.[16]

Though in the model formulated here, there exists a full employment single wage equilibrium, in the model presented in Section 5 the only single wage equilibrium may entail unemployment.

3.3 Positive Profit Equilibrium

Though it seems natural enough to impose the zero profit condition in a model with free entry and constant returns to scale, in models with costly search, it is possible that there exist positive profit equilibria. The usual story for why in equilibrium there must be zero profits is that if profits are positive, some firm will attempt to recruit workers away from the other firms, bidding up the wage; the process continues until there are zero profits. But offering a slightly higher wage does not instantaneously recruit all the workers, as it would if search costs were zero. By raising its wage a little bit, the firm succeeds in recruiting a few more workers--those who arrive at its doors with current wages higher than the old wage but below the new wage.[17] The firm balances off the increased probability of getting a worker with the decreased profits it gets per worker. Equilibrium is characterized by the wages paid being profit maximizing wages, but not necessarily zero profit wages. In equilibrium, all wages paid must make the same expected profits, and these must exceed the expected profits earned by paying any other wage. Thus, the wage distribution must satisfy

$$(3.5)\qquad \frac{[a - w - T(\mu + r + s(1-F))\ ]\ F}{r + \mu + s(1-F)} = k \text{ for wages paid}$$

$$\frac{[a-w-T(\mu+r+s(1-F))]}{r + \mu + s(1-F)}F \leq k \text{ for wages not paid}$$

since the probability of having a worker who applies accept a job is F , and the expected present discounted value of profits earned if he accepts is

$$(3.6)\qquad R \equiv \frac{a - w - (\mu+s(1-F) + r)T}{r + \mu + s(1-F)} \quad .$$

This gives a wage distribution of the form

$$(3.7)\qquad F = \begin{cases} \dfrac{w - a+(\mu+r+s)T - sk+\sqrt{(w - a + (\mu+r+s)T - sk)^2 + 4sTk(r+\mu+s)}}{2sT} & \text{for } w \,\_\, \hat{w} \geq \underline{w} \\ 0 \text{ for } w > \hat{w} & \end{cases}$$

depicted in Figure 4. There is a mass point at $\hat{w}$.

Even though at the highest wage there are positive profits, increasing the wage further recruits no additional workers, and does not lower the quit rate; hence firms have no incentive to raise wages. (Notice that all equilibria must be of the form (3.7); there is still a multiplicity of equilibria, caused by the indeterminancy of $\hat{w}$ and k.) But in a positive profit equilibrium there cannot be a mass point at the highest wage, since then increasing the wage slightly further would have a discrete effect on the probability of acceptance, and hence on profits.[18]

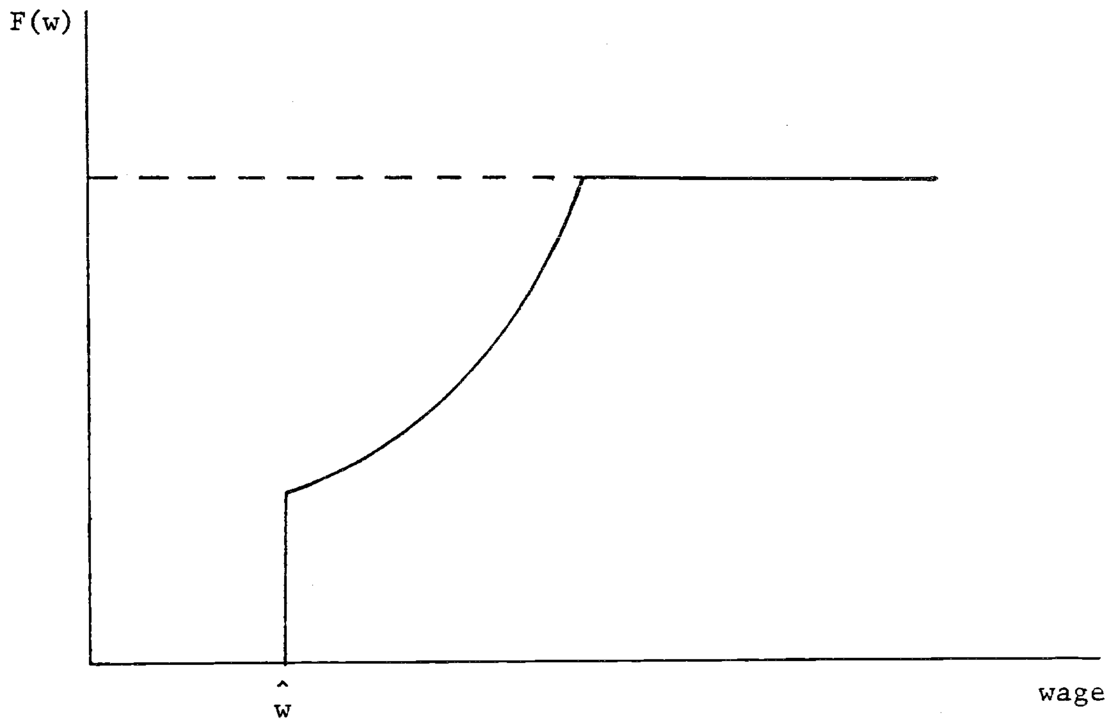

Figure 4

With positive profits, there are incentives for firms to enter, but no entering firms would have an incentive to offer a higher wage.

Though in the model presented so far, there is no determinate number of firms (job-positions), a slight modification of the model allows for us to determine this. Assume that each job requires a machine that costs \$K. The profits described above (Eq. 3.6) are thus the expected present discounted value of the quasi-rents. As entry occurs, the probability of any worker arriving at a firm declines, and hence following the departure of a worker, the expected time until the vacancy is filled increases; entry occurs until the expected present discounted value of quasi-rents is equal to the price of the machine. For simplicity, we assume a firm consists of one machine (one job). Since the number of searches per worker per unit time is s, the average number of arrivals per unit time at a firm is sL/N, where L is the number of workers and N the number of jobs. Hence a firm that pays a wage w has a probability of $\gamma = \frac{sL}{N} F$ of having a vacancy filled per unit of time. Straightforward calculations establish that zero profits entails[19]

$$(3.8) \qquad K = \frac{\gamma}{\gamma+r}\frac{p}{r+q} \Big/ 1 - \frac{\gamma}{\gamma+r}\frac{q}{r+q}$$

where $q = s(1-F) + \mu$ and $p = a - w - (q+r)T$.

Eq.(3.8) can be solved as before for the equilibrium wage distribution, which gives all firms the same profits; and the value of L/N which ensures that the level of profits is zero.[20]

Earlier, we noted that if condition (2.7) was not satisfied there would be no zero profit equilibrium: it would always pay a firm to lower its wages to the minimal acceptable wage, to exploit the monopoly power arising out of costly search. While when condition (2.7) is satisfied there exists positive profit and zero profit equilibria, when condition (2.7) is not satisfied the only equilibria have positive profits, and are of the form we have described in this section.

An Alternative Resolution. If there is more than one commodity, then there is an alternative way that, even when condition (2.7) is not satisfied, a full employment zero profit equilibrium may be attained. Assume that there is a second sector requiring no specific training; we let the output of that sector be our numeraire. As firms enter the industry requiring training in response to positive profits, price falls and profits are reduced.[21]

3.4 Comparative Statics

Because of the continuum of equilibria, it is difficult to do meaningful comparative static calculations. The question we are particularly interested in is, does an increase in the search intensity lead to a narrowing of the wage distribution? Although the partial equilibrium analyses have suggested that it would, the general equilibrium analysis of this paper suggests that the contrary may occur. Consider the two-wage equilibria. An increase in s either lowers the wage paid by the low wage firm--because of the higher turnover they cannot "afford" to pay as high wages as previously--or it reduces the proportion of firms paying the high wage (i.e., the turnover rate of low wage firms will be the same so long as $s\pi$ is constant.)

3.5 Stability

This paper is mainly concerned with equilibrium analysis; yet is is worth noting in a heuristic manner an apparent instability of the multi-wage equilibria.

Consider the two-wage equilibrium. If a single firm happened to switch from paying wage w_o to paying wage w_1 , w_1 would become more profitable; all the firms would switch. Conversely, if a single firm happened to switch from paying w_1 to paying w_o, w_o would become more profitable than w_1 (since now the turnover rate is lowered) and again it would pay all of the firms paying w_1 to switch. The only equilibrium which appears to be stable is the single wage equilibrium. Again, not too much emphasis should be put on this result, since, as we shall show in later sections, with slight elaborations on the model, there may not even exist a single wage equilibrium.

4. Robustness of the Model

In developing the model of Section 2, we introduced a number of simplifying assumptions. We have explored the consequences of loosening the various assumptions, and on the basis of these explorations, we believe our model is robust; if anything, it becomes easier to generate wage distributions and equilibria with unemployment or positive profits. In this section, we briefly describe two extensions of the model, to consider the possibility that firms have a choice of technique and that individuals have a choice of search intensity (with alternative specifications of the search technology). In the next section, we consider an important extension to the case where firms differ in their non-pecuniary characteristics.

4.1 Choice of Technique

Allowing firms to have a choice of technique does not seriously alter the analysis. It does, however, permit us to generate a richer class of wage distributions. We assume that output per man is an increasing concave function of training costs,

$$a'(T) > 0 , \qquad a''(T) < 0 \tag{4.1}$$

Two-Wage Distributions. With two wages, a zero profit equilibrium is characterized by the zero profit equations

$$\begin{aligned} a(T_1) &= w_1 + (\mu+r)T_1 \\ a(T_o) &= w_o + (\mu+r+s\pi)T_1 \quad , \end{aligned} \tag{4.2}$$

the optimal choice of technique equations,

$$a'(T_1) = \mu+r$$

(4.3)

$$a\,(T_o) = \mu+r+s\pi \quad ,$$

and the inequalities ensuring that the firm which chooses technique T_1 chooses to pay ways w_1 ,

$$\frac{w_1 - w_o}{s\pi} < T_1$$

(4.4)

$$\frac{w_1 - w_o}{s\pi} > T_o$$

Figure 5 shows diagrammatically the solution. (4.3) can be solved for the high training cost technology: it is the same in all equilibria. Then (4.2) can be solved for the wage which will yield zero profits for that technique. Consider a wage w_o . Draw the line through w_o tangent to $a(T)$. Let the proportion of high wage paying firms be such that $\mu + r + s\pi$ equals the slope of that line. Then, from (4.3) T_o will be the technique chosen by firms paying w_o , and from (4.2) these firms will just be breaking even.

Notice that in the example we have constructed differences in training costs are endogenous to the model; they are not an exogenously determined characteristic of firms.

Distributions with any finite number of wage levels may similarly be constructed.

Continuous Wage Distributions. If there is to be a continuous distribution of wages, the zero profit condition implies that for all $\{w,T\}$ observed,

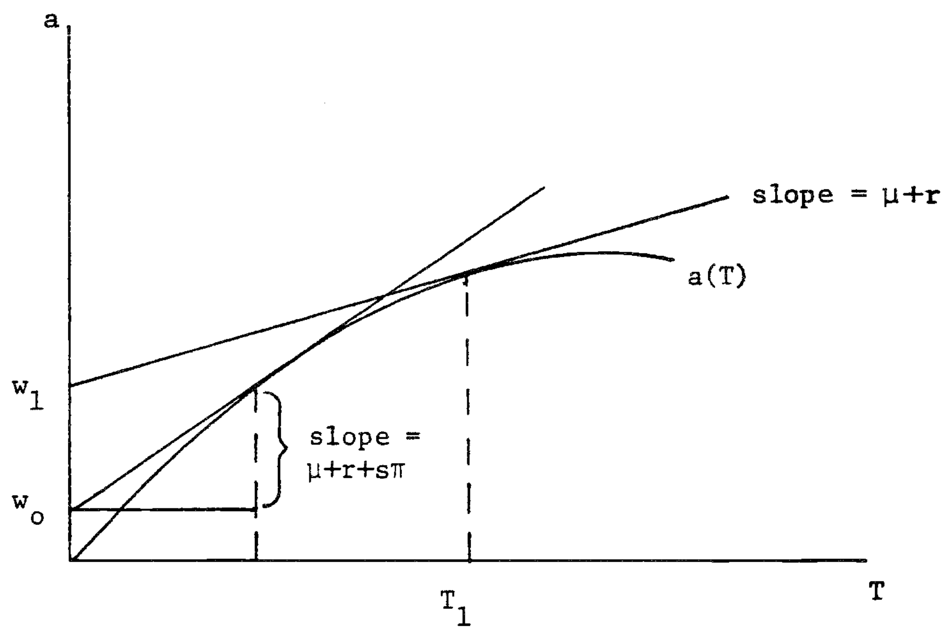

Figure 5

(4.5) $$a(T) = w + T(\mu+r+s(1-F(w)))$$

and the first order conditions for w and T give us

(4.6) $$a'(T) = \mu+r+s(1-F(w))$$

and

(4.7) $$1 - sfT = 0 \quad .$$

Substituting

(4.8) $$a'\left(\frac{1}{sf}\right) = \mu + r + s(1-F)$$

Thus, the density function of wages can be simply related to the production function $a(T)$.

4.2 Search Costs and the Determination of Search Intensity

If there are costs associated with undertaking search, then a rational individual, in deciding how much search he should undertake, would compare the expected benefits at each search intensity with the costs. It is natural to assume that the costs are an increasing, convex function of search intensity, i.e., if $C(s)$ is the cost per unit of time of search at intensity s ,

(4.9) $$C' \geq 0 \quad , \quad C'' \geq 0 \quad .$$

In effect, in our earlier analysis we assumed the cost function took on the special shape depicted in Figure 6b.

The correct calculation of the benefits of search is not a simple matter. We must know what the individual's knowledge about the distribution

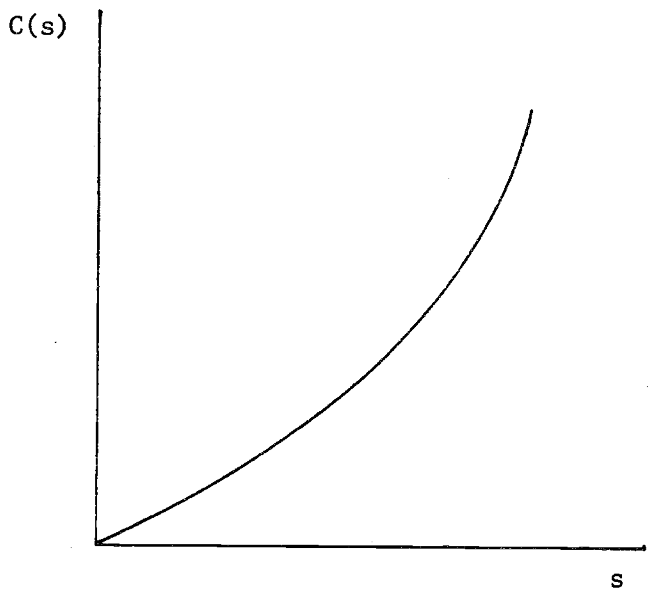

Figure 6a

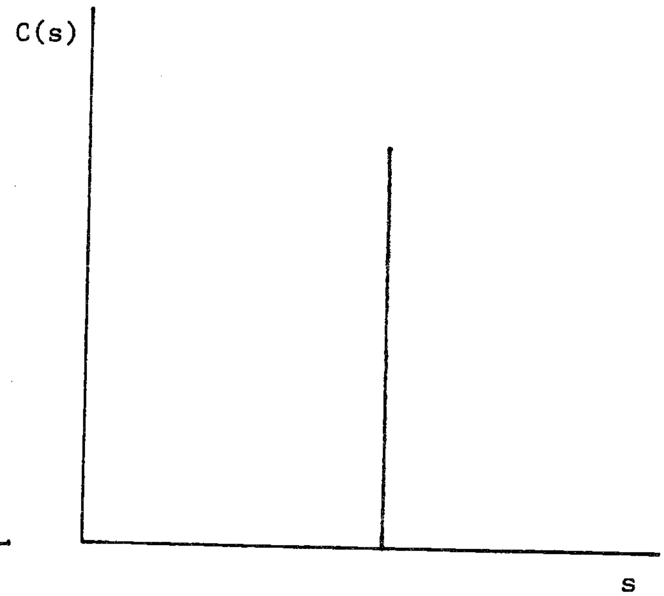

Figure 6b

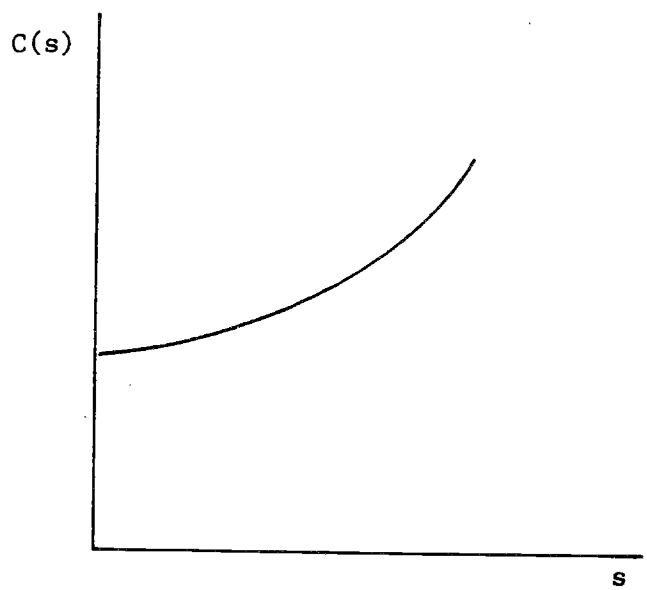

Figure 6c

of wages is, his attitudes towards risk, as well as his expectations concerning the duration which he will keep any job. For instance, if the individual does not know the wage distribution, not only does search yield a direct return in the possibility of finding a better job, but it yields an indirect return in enabling the individual to know better the wage distribution and hence to make a "better" decision with respect to search intensity. For simplicity, assume the individual is risk neutral, and has perfect information about the distribution of wages.[22] Straightforward calculations verify that the convexity of the quit rate function depends on the third derivative of the search-cost function; clearly there is no necessity for the quit rate function to be convex even when f is monotonically declining.

The extension of the analysis of Section 2 follows in a straightforward manner. There is one situation in which a problem does arise: if the search cost appears as in Figure 6c, then for the highest paying firm $\varepsilon = 0$, and the quit rate function is flat at $w = w_{max}$. Hence, no firm with positive training costs would pay w_{max}. Accordingly, if there exists an equilibrium wage distribution, it is the degenerate distribution where all firms pay the same wage equal to the minimum wage; but this in turn cannot be a zero profit equilibrium. There exists a non-zero profit equilibrium with all firms paying $w = \underline{w}$.[23]

Although the cost function depicted in Figure 6c may be considered to be pathological, and the problems generated are really those associated with the fixed cost of search, even in that case there may exist a multiple wage equilibrium if the non-pecuniary benefits resulting from search are taken into account, or if individuals differ with respect to their search costs, with some individuals having $C'(0) = 0$, or alternative information technologies (e.g., an individual buys a newspaper, which reports to him simultaneously all wage offers; see Salop and Stiglitz (1977)).

An Alternative Formulation. An alternative formulation of our model has been suggested by Dybvig and Jaynes. Everyone applies to every job, but obviously accepts only those jobs which offer a higher wage. The firm selects randomly among the interested applicants. Thus, a firm paying a wage of w has $(\mu+F(w))L$ interested applicants, where L is the total labor force. In the steady state equilibrium the number of vacancies at wage w is $\mu Lf(w)$. Hence, the probability of an individual at a firm paying wage $\hat{w}$ quitting to a firm paying wage w is[24] $\mu f/\mu+F$, and the total quite rate is thus

$$q(\hat{w}) = \int_{\hat{w}}^{\infty} \frac{\mu f(w)}{\mu + F(w)} dw \quad .$$

The analysis proceeds exactly as in Section 1.[25]

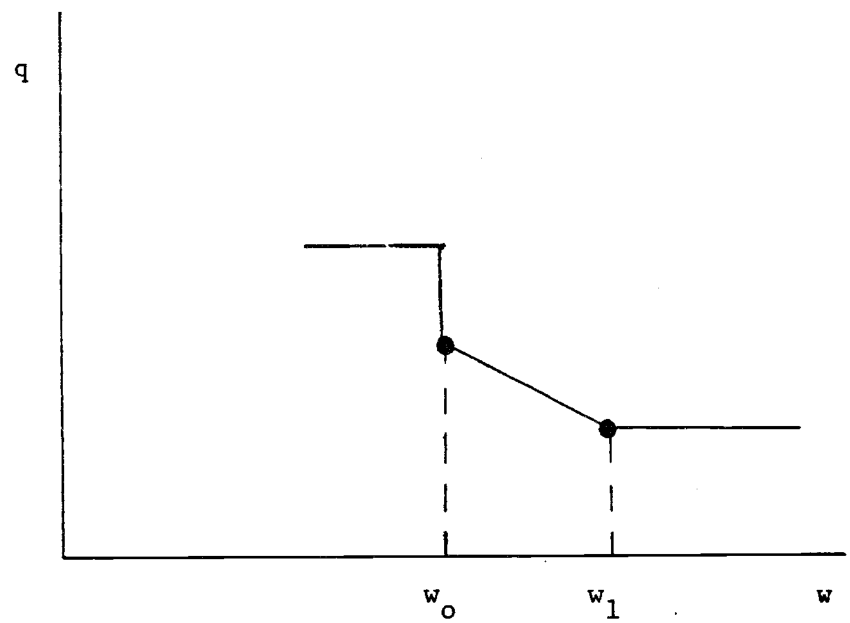

Figure 7

5. Non-Pecuniary Returns

Jobs differ not only in the wages they pay, but in certain non-pecuniary characteristics. Some individuals will prefer the characteristics associated with one firm, others those associated with another firm. We assume, for simplicity, that the individual does not know these characteristics until after completing training, thereupon he knows them fully.[26] The value, in "consumption equivalents," of the non-pecuniary characteristics we denote by θ . Without loss of generality, we let $E\theta = 0$.[27] The distribution of evaluations of the non-pecuniary characteristics of any firm we denote by $M(\theta)$; that is, in a random sample of individuals arriving at the given firm, $M(\theta)$ will discover that their evaluation of its non-pecuniary characteristics is less than or equal to θ . We assume, moreover, that the evaluation of firm i is independent of his evaluation of firm j ; and for simplicity, this distribution is the same for all firms. The density function we denote by $m(\theta)$. A risk neutral individual at a job with wage w and whose non-pecuniary characteristics he values at θ accepts a job if its wage, $\hat{w}$, satisfies

$$\hat{w} \geqq w + \theta \quad . \tag{5.1}$$

Thus the quit rate function is simply[28]

$$q(w) \;=\; \mu + \int s(1-F(w+\theta))\tilde{m}(\theta,w)\,d\theta \tag{5.2}$$

where $\tilde{m}(\theta,w)$ is the distribution of individuals by θ in a firm paying wage w (and can be related to $M(\theta)$ and $F(w)$), so

$$(5.3) \qquad q' = -\int sf(w+\theta)\tilde{m}(\theta,w)d\theta + \int s(1-F)\tilde{m}_w(\theta,w)d\theta \quad .$$

The important implication of (5.3) is that even the firm which pays the highest wage can, by further increments in its wage, reduce its turnover rate, and, if it attempted to reduce its wage, it would increase its turnover rate, even though there are costs of search. There are several interesting properties of the equilibrium.

5.1 Impossibility of a Full Employment Single Wage Equilibrium

If all firms paid the same wage w^*, then all individuals who discovered that $\theta < 0$ would search for a better job and indeed, since they do not know the characteristics of the job before accepting it and going through training, would accept the first offer (since all firms pay the same wage).

The quit rate of a firm (in steady state) which decides to pay, say, a wage $w > w^*$ can be easily calculated: all those with $\theta < w^* - w$ will quit.

Those who do not quit make up a proportionately larger fraction of each firm's labor force. Those with $\theta < w^* - w$ have an average duration on the job of $1/\mu + s$ while those with $\theta \geq w^* - w$ have an average duration of $1/\mu$. Thus the average quit rate is just

$$(5.5) \qquad q(w,w^*) = \frac{1}{\frac{M(w^* - w)}{\mu + s} + \frac{1 - M(w^* - w)}{\mu}}$$

with

$$(5.6) \qquad \frac{\partial q}{\partial w} = \frac{-q^2 sm(w^* - w)}{\mu(\mu + s)} \quad .$$

For there to be a single wage equilibrium (assuming $M(0) = 1/2$)

$$(5.7) \qquad \left.\frac{-\partial q(w,w^*)}{\partial w}\right|_{w=w^*} = \frac{4sm(0)\mu(\mu + s)}{(2\mu + s)^2} = \frac{1}{T}$$

which will not in general be the case.[29]

5.2 An Unemployment Equilibrium

If

$$(5.8) \qquad \left.\frac{-\partial q(w,w^*)}{\partial w}\right|_{w=w^*} \equiv \frac{4sm(0)\mu(\mu + s)}{(2\mu + s)^2} > \frac{1}{T}$$

there exists an unemployment equilibrium. The greater the unemployment rate, the smaller is the marginal effect of a decrease in the wage in increasing the turnover rate.

Let U be the unemployment rate; it is easy to see that, in general, the quit rate at any firm will depend on the unemployment rate as well as the wage w^* being paid by all other firms:[30]

$$q = q(w,w^*,U) \quad .$$

Then, the equilibrium unemployment rate is given by the solution of the pair of equations

$$(5.9) \qquad -T\, q_w(w^*,w^*,U) = 1$$

$$(5.10) \qquad a = w^* + (q(w,w,U) + r)T \quad .$$

5.3 Non-Optimality of the Natural Rate of Unemployment

The unemployment rate derived in the previous section can be thought of as the natural rate of unemployment. This unemployment rate has, however, no obvious optimality properties. Assume the government worked to maximize the expected utility of the representative worker; we assume, for simplicity, that the government can control wages and the number of jobs (hence U) directly, but that it is constrained to paying wages which break even, i.e., (5.10) must be satisfied. The only individuals who are unemployed are the young (all other individuals remain at their jobs until they find alternative employment);[31] once they obtain a job they receive a wage of w^* until they die. The average present discounted value of their non-pecuniary enjoyment depends on how frequently individuals quit; we write $\overline{\theta} = \overline{\theta}(q)$, $\overline{\theta}' > 0$. Thus, the expected present discounted value of utility of an individual is represented by

$$(5.11) \qquad g(U)\ [\frac{w^*}{r+\mu} + \overline{\theta}(q)] \quad ,$$

where the function $g(U)$ reflects the lowered discounted value of the life-time stream of utility resulting from the fact that the individual does not obtain employment immediately. The government wishes to maximize (5.11) subject to the constraint (5.10), which implies that

$$(5.12) \qquad \frac{dw^*}{dU} = -q_U T \ , \qquad \frac{dq}{dU} = q_u$$

where we have made use of the fact that in the single wage equilibrium the quit rate depends just on U , not on w^* .[32]

The government chooses U so that

$$(5.13) \qquad g'(\overline{U})\ [\frac{w^*}{r+\mu} + \overline{\theta}(q)] + g(U) q_U [\frac{-T}{r+\mu} + \overline{\theta}'] \ \leqq 0$$

with the inequality implying $U = 0$.

Since $g'(U) < 0$ (increasing the unemployment rate increases the expected time to obtain a job and hence decreases the present discounted value of lifetime income), and $q_U < 0$, it is apparent that if

$$\frac{T}{r+\mu} < \overline{\theta}', \quad U = 0 .$$

If training costs are relatively low, the optimal employment rate is zero; while if

$$\frac{T}{r+u} > \overline{\theta}' , \quad U \geqq 0 .$$

If training costs are high, the optimal unemployment rate may be positive.

There are several implicit differences between the market and the government solutions. First, each firm believes that it can reduce its turnover by raising its wage relative to others; but of course, when they all try to do this, this simply raises the wage, reduces the demand for labor, but has no direct effect on the turnover rate. The reduction in the demand for labor, that is, the increase in unemployment, has, however, an important externality effect: it reduces turnover at all firms and this lowers aggregate expenditure on training costs, enabling a consequent rise in the wage rate. Though the firm fails to take into account this benefit of the increased unemployment rate, it also fails to take into account the increased cost: the poorer matching of individuals with firms and the delay in young individuals obtaining employment. We suspect, but have not been able to show, under any general set of conditions, that there is a presumption that the natural unemployment rate is too high.

In this section, we have simply compared the optimal single wage equilibrium with the single wage market equilibrium. The government can choose a whole wage distribution; optimality may well entail wage inequality: workers who are ex ante identical receive different wages.[33]

5.4 Frictional Unemployment

There is a second, quite distinct kind of unemployment which can arise in a slight variation of the model presented so far. If we assume that search can only be undertaken while the individual is unemployed, or more generally, that search may be carried on less expensively while unemployed, then the equilibrium may be characterized by a certain level of frictional unemployment. All those who find that their evaluation of the non-pecuniary attributes of the firm are sufficiently negative quit, and (in the absence of the kind of unemployment described in 5.3 or 3.2) remain unemployed for an expected duration of 1/s, the time to arrive at the next firm.

5.5 Positive Profits Equilibrium

If

$$(5.14) \qquad \left.\frac{\partial q(w,w^*)}{\partial w}\right|_{w = w^*} < \frac{1}{T} \quad \text{for all } w \geq \underline{w} ,$$

there exists a positive profits equilibrium. (5.14) implies that firms will wish to lower their wages until the minimum acceptable wage $\underline{w}$. In general, at $\underline{w}$

$$a > w + (q+r)T ,$$

there are positive profits. The analysis of the positive profits equilibrium follows along the lines of Section 3.

5.6 Multiple Wage Equilibrium

Thus far, in this section we have shown that with non-pecuniary characteristics of jobs there does not exist a single wage zero profit full employment equilibrium, but there may exist a single wage equilibrium, either with positive profits or unemployment. A natural question to raise at this juncture is, can we have a multiple wage zero profit full employment equilibrium; in other words, do our difficulties arise from the restriction to all firms paying the same wage? The answer is no: there do not exist multiple wage full employment zero profit equilibria either.

As before, let π be the proportion of firms paying $w_1 > w_o$. The quit rate function can now be written as

$$\frac{1}{q} = \frac{M(w_o - w)}{\mu + s} + \frac{M(w_1 - w) - M(w_o - w)}{\mu + s\pi} + \frac{1 - M(w_1 - w)}{\mu}$$

so

$$\frac{\partial q(w^*, w_o, w_1, \pi)}{\partial w} = -q^2\{m(w_o - w)(\frac{1}{\mu + s\pi} - \frac{1}{\mu + s}) + m(w_1 - w)(\frac{1}{\mu} - \frac{1}{\mu + s\pi})\}$$

A zero profit equilibrium will in general not exist; we require, at $w = w_1$ and $w = w_2$,

$$\frac{\partial q(w, w_1, w_2, \pi)}{\partial w} = -\frac{1}{T}$$

and

$$w_1 + T[q(w_1;w_1,w_2,\pi) + r] = w_2 + T[q(w_2;w_1,w_2,\pi) + r] .$$

These provide four equations in three unknowns and a careful examination of the structure of the equations in the context of simple examples should readily convince the reader that there is no redundancy. Moreover, increasing the number of wage levels does not resolve the difficulty; for each wage level we add two unknowns, the wage rate, and the proportion of firms at that wage rate; and two equations, the zero profit equation and the profit maximization equation.

6. Concluding Comments

In this paper we have investigated the implications of imperfect information for the equilibrium wage distribution. We have shown how, even in a steady state equilibrium, identical labor may receive different wages; the competitive forces which we normally think of as eliminating such differences in the long run may not do so when there is imperfect information.[34] Indeed, this may be the case even when the only source of "imperfect information" is the inequality of wages which the market it perpetuating. When there are information imperfections arising from (symmetric) differences in non-pecuniary characteristics of jobs and preferences of individuals, there will not in general exist a full employment, zero profit single wage equilibrium.

Perhaps even more significant was the result that in equilibrium, there could be unemployment; in spite of the presence of an excess supply of labor, no firm is willing to hire workers at a lower wage, since it knows that if it does so, the quit rate will be higher, and hence turnover costs (training costs) will be higher, so much so that profits will actually be lower. Thus, this model provides a rationale for real wage rigidity. The model, in addition, provides a theory of frictional unemployment, as workers who are badly mismatched with firms (in terms of the non-pecuniary characteristics of the job) rejoin the unemployment pool to seek a better match.

Though the constrained optimality (that is, taking explicitly into account the costs associated with obtaining information and search) may entail unemployment and wage dispersion, the levels of unemployment and wage dispersion in the market equilibrium will not, in general, be optimal. The nature of the government intervention that is necessary to effect a Pareto improvement is, unfortunately, far from obvious.

FOOTNOTES

1. In addition, they may or may not know the wage distribution. In later sections, we shall need to be more explicit about what information individuals have prior to making their search decisions.

2. If search is costly, individuals will stop short of obtaining "perfect information," and so price (wage) dispersions might be maintained.

3. If there were exogenous sources of uncertainty, e.g., technical change, so that "jobs" have a finite life, then the analysis would be similar to that contained here.

3a. Since this paper was originally written, a number of other theories of price or wage distribution have been formulated. Without presenting a complete survey of what has become an extensive literature, it may be useful to note a few of the more important strands. In Salop (1977) and Salop and Stiglitz (1977), the price distribution is used to discriminate among individuals who have different search costs. In earlier versions of this paper, as well as in Reinganum (1979), firms with different technologies can be shown to pay different wages or charge different prices. In Mortenson (1973), costly search prevents labor markets from becoming fully arbitraged in response to disturbances in submarkets. (The arguments are analogous to those of Grossman and Stiglitz (1980) who show that with costly information, capital markets cannot become fully arbitraged.) Closest in spirit to this paper are those studies which have attempted to show that equilibrium may be characterized by a price distribution, even if individuals and firms are identical and even if there are no exogenous sources of noise (Salop and Stiglitz (1982), Butters (1977)). While in Butters, stores which charge lower prices are able to recruit more customers per dollar spent on advertising, in our model, firms which pay higher wages are able to retain workers for a longer duration.

4. Because our model is stochastic, to avoid random variations in the aggregate variables of interest, we assume a large economy.

5. In more general versions of this model, to characterize the equilibrium we also need to specify the choice of technique by the firm, the choice of search intensities by individuals, and the relative prices of different commodities. There are, correspondingly, some additional equilibrium conditions; e.g., each firm chooses its technique to maximize its profits; each individual chooses his search intensity (given the wage of the firm he is presently at, the costs of search, and the wage distribution) to maximize his expected utility.

6. In Section 5, we assume individuals differ with respect to their evaluation of the non-pecuniary characteristics associated with any firm. The assumption of identical individuals is made to avoid the possibility that the equilibrium wage distribution is generated by firms' attempting to act as discriminating monopolists. See Salop and Stiglitz (1977).

7. Alternatively, if firms are large, then the number of applicants will equal the number of deaths; then the firm's production process may involve capital as well as labor. In other cases; that is, for small firms with capital, after each death or quit there may be a (random) period of idleness of the machine or the firm may carry an inventory of underemployed workers. This means that firms must worry about the percentage of time machines which are idle, and individuals may apply to firms with no vacancies. This complicates but does not basically change the analysis.

7a. We assume here (as in fact seems to be the case) that at least some part of the specific training and turnover costs are borne by the firm. For a model in which the contract design is endogenous (so that in particular, the fraction of the specific training and turnover costs borne by the firm is endogenously determined) see Arnott and Stiglitz (1983). They show that so long as workers are more risk asverse than firms, a fraction of the specific training costs will be borne by firms. (For a further discussion of the point, see also Stiglitz (1974).

8. The number of applicants who are willing to accept a job is, of course, a function of the wage offered, but because of the zero profit condition, this has no effect on the wage offered in equilibrium. But see below.

9. This is consistent, for instance, with each of the large firms having a number of places to apply, in proportion to the number of jobs which are available.

10. If instead of assuming that the time required to sample an additional firm is a random variable we had assumed the individual makes s searches per unit time, then

$$q(w) = \mu + (1 - (F(w))^s) , \quad q' = -sF^{s-1}f' .$$

The rest of the calculations must similarly be modified in a straightforward manner.

11. It is clear from (2.3) that the quit rate is a function not only of the wage paid by the firm in question, but by all other firms. Thus if there were m firms, we could have written the i^{th} firm's quit rate as

$$q_i = q_i(w_1, w_2, \ldots, W_i, \ldots, W_m)$$

Our notation suppresses the dependence of q_i on w_j , $j \neq i$, and $\partial q_i / \partial w_i$ is denoted by $q'(w)$.

12. Not too much emphasis should be placed on this result, since, as we shall see, it is not true in more complicated versions of the model.

13. Several different interpretations can be given to w_{min} . In an economy with effective minimum wage legislation, then w_{min} is the legislated minumum wage. Alternatively, w_{min} may be thought of as the reservation wage, below which individuals will not work. Or w_{min} may be thought of as the wage which individuals could earn in self-employment. Or there may be one sector in which there is an organized, competitive labor market of the conventional sort; individuals know they can obtain employment in that sector. Finally, the efficiency wage may provide the minimum wage. (See fn.

14. These theories are broadly referred to as efficiency wage theories; in the development literature, they were first discussed by Leibenstein (1956). While in his theory productivity depended on wages because of nutrition, subsequent developments by Stiglitz (1976, 1982a, 1982b), Weiss (1980), Shapiro and Stiglitz (1982), and Calvo (1979) related productivity to wages through selection (the quality of the applicant pool is affected by the wage paid) and incentive effects. For a more extensive development of the relationship between turnover and unemployment in a slightly different model, see Stiglitz (1974).

15. We assume throughout this paper--we would argue realistically--that there are some turnover costs borne by the firm. A natural question to raise at this juncture is, if the worker cannot induce the firm to hire him by lowering his wage, why can't he simply offer to pay a larger fraction (possibly all) of the turnover costs. There are at least three possible explanations. First, workers may not have the capital to pay for the training costs. Second, there is a moral hazard problem on the part of the firm; it could take the application fee, which is allegedly for training, and shortly thereafter fire him. Third, those workers who are willing to pay the most for the job may not be the most productive (there are quality-selection effects). The arguments are parallel to those presented in Stiglitz and Weiss for why an increase in collateral requirements may not be used to equilibrate the credit market, or in Shapiro-Stiglitz for why bonding may not eliminate the incentive-unemployment with which they were concerned.

In any case, all that matters for our analysis is that, for one reason or another, there are some turnover costs borne by the firm.

16. Similar arguments hold for any other change in technology, e.g., a change in s, T, r, or μ. Note that an increase in the real interest rate will, in this model, lead to unemployment.

17. We are explicitly ruling out other means by which firms may recruit workers, e.g., by advertising. These methods, too, are not costless. Our assumptions ensure that the number of applicants at a firm are independent of the wage it pays. This would not be the case under alternative hypotheses concerning the information structure.

18. This result is dependent on our assumption that the number of searches per unit of time is, effectively, exogenously determined. If all other firms pay the same wage, then if there is any marginal cost to doing any search, if any single firm raises its wage, it will not induce any search, and hence will not increase the speed with which it fills vacancies. It is easy to see, more generally, that there may exist equilibria in which at the highest wage there is a mass point.

19. The expected present discounted value of a worker, once he is hired, is $p/r+q$. The expected present discounted value of a vacancy at time t is Ve^{-rt}, and such a vacancy will occur at t with probability qe^{-qt}. The total return to a machine can thus be written

$$V = \underbrace{\int_0^\infty Ve^{-rt}}_{\substack{\text{Probability}\\ \text{of filling}\\ \text{a vacancy}\\ \text{at time } t}} \underbrace{\left[\frac{p}{r+q} + qV\int_0^\infty e^{-r\tau}e^{-q\tau}d\tau\right] e^{-rt}dt}_{\substack{\text{The present discounted value}\\ \text{of a vacancy filled at time } t}}$$

20. Implicitly, throughout the analysis, we are assuming that the machines cannot be rented out when they are idle.

21. For a more extensive elaboration of the two-sector model with equilibrium wage distributions, see the 1974 IMSSS version of "Equilibrium Wage Distributions."

The analysis requires that the training costs not entail simply output of the training goods sector.

22. The maximum expected present discounted value of net income of the individual who is presently receiving a wage w is

$$(4.10)\qquad V(w) = \max_s \left\{ \frac{w - C(s) + s\int_w^{w_{max}} V(\tilde{w})dF(\tilde{w})}{\mu + r + s(1 - F(w))} \right\} .$$

Hence s is chosen so that

$$(4.11)\qquad C'(s) = \int_w^{w_{max}} V(\tilde{w})dF(\tilde{w}) - (1 - F(w))V(w)$$

where the second order condition implies that $C'' > 0$. From (4.11) we obtain the search intensity as a function of the wage paid (given the distribution $F(w)$).

23. If the amount of labor each individual supplies is a function of w (rather than fixed, as in the analysis of the rest of this paper), then the equilibrium wage will be the monopoly wage, i.e., if $\hat{L}(w)$ is the amount of labor an individual supplies as a function of the wage, the equilibrium wage will be given by the solution to

$$\hat{L}(w^*) + w\hat{L}'(w^*) = 0 .$$

This corresponds to the result of Diamond (1971), who shows that the equilibrium in his search model involves a single price at the monopoly level. But note how dependent it is on the particular assumptions made.

24. We have ignored the possibility that in any period the individual gets offered simultaneously two jobs. If the periods are short, so that the number of vacancies is small relative to the number of applicants, this is a small probability event; in our continuous time formulation, this is a zero probability event.

25. There is a two-wage distribution, with $w_1 = w^*$ and $w_o = w_1 - \frac{\pi\mu T}{1-\pi+\mu}$. Since all low wage individuals and those newly entering the labor force apply for the $\mu\pi L$ vacancies at high wage firms, their chance of getting one of the higher wage jobs is $\mu\pi/(1-\pi) + \mu$.

Any combination of (w_o,π) satisfying $w_o = w_1 - \pi\mu T/1-\pi+\mu$ will do. For the continuum, $-q'T = 1$, or $\frac{\mu f}{\mu+F} T = 1$, i.e., $F = ke^{w/\mu T} - \mu$ for $w_{min} \leq \hat{w} < \mu T \ln (1+\mu)/k,\ k < \mu$. If $\hat{w} > w_{min}$, there is a mass point at $w\ \hat{w}$; there is never a mass point at w_{max}.

26. There are, of course, some non-pecuniary characteristics which are known before accepting the job; there will affect the acceptance rate, but not the quit rate. Similarly, there are pecuniary characteristics, such as promotion policy, that may become known only gradually to the individual.

27. If most individuals agree that one firm has more desirable characteristics than another, we can simply add the mean value of θ onto the wage.

28. For simplicity, we revert to our search model of Section 2 where s is fixed. For the more general case, see the 1974 IMSSS version of "Equilibrium Wage Distributions."

29. While in the earlier case, a policy of a constant wage was optimal, it will not necessarily be so here, since those who remain longer with the firm have revealed information about their evaluation of the non-pecuniary characteristics of the firm; namely, that $\theta > 0$. The firm can exploit that information in designing a seniority wage structure. This is a difficult problem, the solution to which would not alter the basic qualitative propositions put forth in this section.

Similar considerations suggest that our first order condition (5.7) is only approximately correct; if the firm chooses a fixed wage rate, the probability that any individual quits will be a function of the length of time he has been on the job, i.e., $q = q(w,t)$. The present discounted value of profits of the firm are thus

$$P = \int q[a - w]e^{-[r + \bar{q}(w,t)]t}dt - T$$

where

$$\bar{q}(w,t) = \frac{\int_0^t q(w,t)dt}{t} .$$

In our analysis we have replaced $\bar{q}(w,t)$ with $\bar{q}(w,\infty)$. Again, the central result, that for all $w \geq w_{min}$

$$\frac{\partial P}{\partial w} \neq 0 \quad \text{when } w = w^* \text{ for all firms}$$

is still valid.

30. The derivation of the quit rate function from the underlying assumption on search and non-pecuniary evaluations is tedious; all that is crucial for our result is that $q_{wU}(w,w^*,U) < 0$ at $w = w^*$.

31. This assumes that $-\theta_{min} < w^*$, i.e., the non-pecuniary disutility does not exceed the wage; the modification for the case where some individuals quit to join the unemployment pool is straightforward.

32. This follows from our strong assumption that individuals value jobs by $w + \theta$, so there are no income effects in the valuation of non-pecuniary attributes.

33. For an example with this property, see the IMSSS version of "Equilibrium Wage Distributions."

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