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# MORTGAGE TIMING

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## ABSTRACT

The fraction of newly-originated mortgages that are of the adjustable-rate (ARM) versus the fixed-rate (FRM) type exhibits a surprising amount of time variation. A simple utility framework of mortgage choice points to the bond risk premium as theoretical determinant: when the bond risk premium is high, FRM payments are high, making ARMs more attractive. We confirm empirically that the bulk of the time variation in household mortgage choice can be explained by time variation in the bond risk premium. This is true regardless of whether bond risk premia are measured using forecasters' data, a VAR term structure model, or a simple rule-of-thumb based on adaptive expectations. This simple rule-of-thumb moves in lock-step with mortgage choice, thereby lending further credibility to a theory of strategic mortgage timing by households.

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One of the most important financial decisions any household has to make during its lifetime is whether to own a house and, if so, how to finance it. There are two broad categories of housing finance: adjustable-rate mortgages (ARMs) and fixed-rate mortgages (FRMs). Figure 1 plots the share of newly-originated mortgages that is of the ARM-type in the US economy between January 1985 and June 2006. This ARM share shows a surprisingly large variation; it varies between 10% and 70% over time. This paper seeks to explain this variation in households' mortgage choice.

[Figure 1 about here.]

Our premise is that the time variation in the ARM share is driven by time variation in bond risk premia, defined as the difference between the long-term interest rate and the expected future short-term interest rates. By now there is abundant evidence that the expectations hypothesis of the term structure of interest rates fails to hold empirically.[1] Time variation in bond risk premia affects the FRM rate, which is linked to the long-term interest rate, but not the ARM rate. A simple utility framework formalizes that when the risk premium on long-term bonds is high, the expected payments on the FRM are large relative to those on the ARM, making the ARM more attractive.

Empirically we test this prediction using three alternative methods to determine expected future short rates, which are needed to compute the bond risk premium: (i) using professional forecasters' data, (ii) constructing a term-structure model, and (iii) employing an adaptive expectations scheme that uses a short history of short rates. We show that a large fraction of the time variation in the ARM share can be attributed to time variation in bond risk premia. All three measures deliver the same economic effect: a one standard deviation increase in bond risk premia increases the ARM share by 8%.

Figure 2 illustrates our main result. It plots the ARM share (solid line, measured against the left axis) alongside the five-year bond risk premium (dashed line, measured against the right axis). We construct the bond risk premium as the difference between the five-year nominal Treasury bond yield and the forecasters' consensus expectation about the average nominal one-year rate over the next five years. The nominal yield data are from the Federal Reserve Bank of New York and forecaster data from Blue Chip. The correlation between the two series is 64%.

[Figure 2 about here.]

In Section 1, we formalize the utility-based mortgage choice argument. The model extends the work of Campbell (2006) by allowing for time variation in bond risk premia. It strips out some of

[1] Fama and French (1989), Campbell and Shiller (1991), Dai and Singleton (2002), Buraschi and Jiltsov (2005), Ang and Piazzesi (2003), Cochrane and Piazzesi (2005), and Ang, Bekaert, and Wei (2006), among others, document and study time variation in bond risk premia.

the rich life-cycle dynamics of Cocco (2005) and Campbell and Cocco (2003) in order to focus on the role of time-varying risk premia. Risk-averse borrowers not only care about expected mortgage payments, but also about the variability of these payments. The ARM payments vary with the real short rate, while the presence of inflation uncertainty makes the real FRM payments variable. This analysis points to three term structure determinants of mortgage choice: (i) the nominal risk premium, (ii) the variability of the real rate, and (iii) the variability of expected inflation.

In Section 2, we turn to the data, and regress the ARM share on various measures of the nominal bond risk premium. The first such analysis is based on the forecasters' data shown in Figure 2 (Section 2.2). Section 2.3 develops a vector auto-regressive (VAR) term structure model, which provides a second way to compute expectations of future nominal interest rates. In addition, the VAR framework provides a way to compute the variability of the real rate and expected inflation. The VAR-based bond risk premium is strongly related to the ARM share, and the variability of the real rate and the expected inflation enter with the sign predicted by the model.

Section 2.4 considers a third way to measure the bond risk premium, which we label the rule-of-thumb. This rule-of-thumb approximates the theoretical bond risk premium, which contains forward-looking expectations of future short rates, as the difference between the current long-term nominal interest rate and a *backward-looking* average of nominal short rates. Our motivation for this simple rule is a suspicion that households may not have the required financial sophistication to solve complex investment problems (Campbell (2006)).[2] This proxy for bond risk premia is much easier to compute; it only requires calculation of an average short rate over the recent past. Yet, it captures the dynamics of the bond risk premia that we extract from the VAR model. Figure 3 displays the ARM share alongside the rule-of-thumb for five-year bond risk premia, which uses three years of past short rate data. The figure documents a striking co-movement between the ARM share (solid line, left axis) and the rule-of-thumb for bond risk premia (dashed line, right axis), and suggests that making a close-to-optimal mortgage choice may be within reach of the average household.

[Figure 3 about here.]

Section 2.5 studies predictors of the ARM share proposed in the literature, such as the slope of the yield curve, the spread between an FRM rate and an ARM rate, or the long yield. In our empirical work, we find lower explanatory power for these variables than for the bond risk premium. Our model suggests an explanation for the yield spread and the long yield. The yield spread not only measures the nominal bond risk premium but also deviations of expected future

[2]One branch of the real estate finance literature documents slow prepayment behavior (e.g., Schwartz and Torous (1989), Stanton (1995), Boudoukh, Whitelaw, Richardson, and Stanton (1997), and Schwartz (2007)). Other relevant papers in real estate are Brunnermeier and Julliard (2006), who study the effect of money illusion on house prices, and Gabaix, Krishnamurthy, and Vigneron (2006), who study limits to arbitrage in mortgage-backed securities markets.

nominal short rates from the current nominal short rate. Intuitively, it ignores the rollover aspect of an ARM mortgage: its interest rate resets when the short-term interest rate changes. The VAR model shows that the two components of the yield spread are negatively correlated. When expected inflation is high, the inflation risk premium tends to be high as well. At the same time, expected future short rates are below the current short rate because inflation is expected to revert back to its long-term mean. Hence, the correlation is negative. Likewise, the long yield only measures the expected payments on the FRM, but does not account for the expected payments on the ARM, the expected future short rates. From a theoretical perspective, the yield spread and the long yield are imperfect predictors of mortgage choice, which is confirmed empirically.

While our three measures of the bond risk premium deliver similar results over the full sample, their performance diverges in the last ten years of the sample. This is mostly due to the increase in the ARM share in 2003-04, which is predicted correctly by the rule-of-thumb measure, but not by the other two forward-looking measures of the bond risk premium. Section 3 explains this divergence. Part of the explanation lies in product innovation in the ARM mortgage segment. But most of the divergence is due to large forecast errors in future short rates in this episode. This motivates us to consider the *inflation* risk premium component of the nominal risk premium, for which any forecast error that is common to nominal and real rates cancels out. We construct the inflation risk premium using real yield (TIPS) data and either Blue Chip forecasters' data or a VAR model for inflation expectations, and show that both measures have a strong positive correlation with the ARM share and deliver a similar economic effect.

In Section 4, we study the robustness of these results. First, we analyze the impact of the prepayment option, which is typically embedded in US FRM contracts, on the utility difference between the ARM and FRM.[3] We show that the prepayment option reduces the exposures to the underlying risk factors. However, it continues to hold that higher bond risk premia favor ARMs. In sum, we find that the presence of the option does not materially alter the results. Second, we show that the bond risk premium is also an important explanatory variable in a large cross-section of loan-level data. The loan-level analysis also allows us to investigate the importance of measures of financial constraints, such as the loan-to-value ratio or the FICO credit score. While they are statistically significant predictors of mortgage choice, they do not add much to the explanatory power of the bond risk premium, nor significantly reduce it. Third, we discuss statistical inference, and conduct a bootstrap exercise to calculate standard errors. Finally, we discuss liquidity issues in the TIPS markets and how they may affect our results on the inflation risk premium. We use real interest rate data generated by the term structure model of Ang, Bekaert, and Wei (2007) as an alternative to the TIPS data, and show that our results strengthen. We conclude that bond risk premia are a robust determinant of mortgage choice.

[3]We contribute to the large literature on rational prepayment models, e.g., Dunn and McConnell (1981), Stanton and Wallace (1998), Longstaff (2005), and Pliska (2006), by adding time variation in risk premia.

Our findings resonate with recent work in the portfolio literature by Brandt and Santa-Clara (2006), Campbell, Chan, and Viceira (2003), Sangvinatsos and Wachter (2005), and Koijen, Nijman, and Werker (2007). This literature emphasizes that forming portfolios that take into account time-varying risk premia can substantially improve performance for long-term investors.[4] Because the mortgage is a key component of the typical household's portfolio, and because an ARM exposes that portfolio to different interest rate risk than an FRM, choosing the wrong mortgage may have adverse welfare consequences (Campbell and Cocco (2003) and Van Hemert (2006)). In contrast to these studies, our exercise suggests that mortgage choice is an important financial decision where the use of bond risk premia is not only valuable from a normative point of view. Time variation in risk premia is also important from a positive point of view, to explain observed variation in mortgage choice.

Finally, our paper also relates to the corporate finance literature on the timing of capital structure decisions. The firm's problem of maturity choice of debt is akin to the household's choice between an ARM and an FRM. Baker, Greenwood, and Wurgler (2003) show that firms are able to time bond markets. The maturity of debt decreases in periods of high bond risk premia.[5] Our findings suggest that households also have the ability to incorporate information on bond risk premia in their long-term financing decision.

# 1 Determinants of Mortgage Choice

This section explores the choice between a fixed-rate mortgage (FRM) and an adjustable-rate mortgage (ARM) in a simple theoretical model. Rather than developing a full-fledged life-cycle model, our focus is on the role of bond risk premia in a tractable two-period analytical framework. The model we consider can be viewed as an extension of Campbell (2006). In Section 1.1, we set up the individual's mortgage choice problem. Section 1.2 discusses how bond prices are set, and Section 1.3 how mortgage rates are determined. Section 1.4 works out the risk-return tradeoff that households face when choosing a mortgage.

## 1.1 The Household's Problem

At time 0, the household purchases a house and uses a mortgage to finance it. The house has a nominal value $H_t^{\$}$ at time $t$. For simplicity, the loan is non-amortizing. We assume a loan-to-value ratio equal to 100%, so that the mortgage balance is given by $B = H_0^{\$}$. The investment horizon

[4] Campbell and Viceira (2001) and Brennan and Xia (2002) derive the optimal portfolio strategy for long-term investors in the presence of stochastic real interest rates and inflation, but assume risk premia to be constant.

[5] See Butler, Grullon, and Weston (2006) and Baker, Taliaferro, and Wurgler (2006) for a recent discussion. In ongoing work, Greenwood and Vayanos (2007) study the the relationship between government bond supply and excess bond returns.

and the maturity of the mortgage contract equal 2 periods. Interest payments on the mortgage are made at times 1 and 2. At time $t = 2$, the household sells the house at a price $H_2^\$$ and pays down the mortgage. The household chooses to finance the house using either an ARM or an FRM, with associated nominal interest rates $q^i$, $i \in \{ARM, FRM\}$. In each period, the household receives nominal income $L_t^\$$.

We postulate that the household is borrowing constrained: In each period, she consumes what is left over from the income she receives after making the mortgage payment (equation (2)). Because the constrained household cannot invest in the bond market, she cannot undo the position taken in the mortgage market.[6] Terminal consumption equals income after the mortgage payment plus the difference between the value of the house and the mortgage balance (equation (3)).

The household maximizes lifetime utility over real consumption streams $\{C/\Pi\}$, where $\Pi$ is the price index and $\Pi_0 = 1$. Preferences in (1) are of the CARA type with risk aversion parameter $\gamma$, except for a log transformation. The subjective time discount factor is $\exp(-\beta)$.[7]

$$\max_{i\in\{ARM,FRM\}} -\log\left(\mathbb{E}_0\left[e^{-\beta-\gamma\frac{C_1}{\Pi_1}}\right]\right) - \log\left(\mathbb{E}_0\left[e^{-2\beta-\gamma\frac{C_2}{\Pi_2}}\right]\right) \tag{1}$$

$$\text{s.t.} \quad C_1 = L_1^\$ - q_1^i B, \tag{2}$$

$$\text{and} \quad C_2 = L_2^\$ - q_2^i B + H_2^\$ - B, \tag{3}$$

We assume that real labor income, $L_t = L_t^\$/\Pi_t$, is stochastic and persistent:

$$L_{t+1} = \mu_L + \rho_L\left(L_t - \mu_L\right) + \sigma_L \varepsilon_{t+1}^L, \varepsilon_{t+1}^L \sim \mathcal{N}(0,1).$$

In addition, we assume that the real house value is constant and let $H_t = H_t^\$/\Pi_t$.[8]

## 1.2 Bond Pricing

The one-period nominal short rate at time $t$, $y_t^\$(1)$, is the sum of the real rate $y$ and expected inflation $x$:

$$y_t^\$(1) = y_t(1) + x_t. \tag{4}$$

[6] We can extend the model to allow for saving in one-period bonds. For realism, we then impose borrowing constraints along the lines of the life-cycle literature (Cocco, Gomes, and Maenhout (2005)).

[7] This transformation is reminiscent of an Epstein and Zin (1989) aggregator which introduces a small preference for early resolution of uncertainty (see also Van Nieuwerburgh and Veldkamp (2006)). While this modification is made for analytical convenience, it implies that $\beta$ does not affect mortgage choice. In Section 4.2, we investigate the role of the subjective discount rate in a power utility framework and show that the risk-return tradeoff is unaffected for standard values of $\beta$.

[8] It would be straightforward to extend the model to stochastic real house prices and to allow for a temporary and a permanent component in labor income, as in Campbell and Cocco (2003).

Denote the corresponding price of the one-period nominal bond $P_t^{\$}(1)$. Following Campbell and Cocco (2003), we assume that realized inflation and expected inflation coincide:

$$\pi_{t+1} = \log \Pi_{t+1} - \log \Pi_t = x_t, \tag{5}$$

so that there is no unexpected inflation risk.[9] To accommodate the persistence in the real rate and expected inflation, we model both processes to be first-order autoregressive:

$$\begin{aligned} y_{t+1}(1) &= \mu_y + \rho_y \left(y_t(1) - \mu_y\right) + \sigma_y \varepsilon_{t+1}^y, \\ x_{t+1} &= \mu_x + \rho_x \left(x_t - \mu_x\right) + \sigma_x \varepsilon_{t+1}^x. \end{aligned}$$

Their innovations are jointly Gaussian with correlation matrix $R$:

$$\begin{pmatrix} \varepsilon_{t+1}^y \\ \varepsilon_{t+1}^x \end{pmatrix} \sim \mathcal{N}\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{xy} \\ \rho_{xy} & 1 \end{bmatrix} \right) = \mathcal{N}\left(0_{2\times 1}, R\right).$$

We assume that labor income risk is uncorrelated with real rate and expected inflation innovations, an assumption that can be relaxed.

This structure delivers a familiar conditionally Gaussian term structure model. The important innovation in this model relative to the literature on mortgage choice is that the market prices of risk $\lambda_t$ are *time varying*. The nominal pricing kernel $M^{\$}$ takes the form:

$$\log M_{t+1}^{\$} = -y_t^{\$}(1) - \frac{1}{2}\lambda_t' R \lambda_t - \lambda_t' \varepsilon_{t+1},$$

with $\varepsilon_{t+1} = \left[\varepsilon_{t+1}^y, \varepsilon_{t+1}^x\right]'$ and $\lambda_t = [\lambda_t^y, \lambda_t^x]'$. If we were to restrict the prices of risk to be affine, our model would fall in the class of affine term structure models (see Dai and Singleton (2000)), but no such restriction is necessary.

The no-arbitrage price of a two-period zero-coupon bond is:

$$e^{-2y_0^{\$}(2)} = \mathbb{E}_0\left[M_{t+1}^{\$} M_{t+2}^{\$}\right] = e^{-y_0^{\$}(1) - \mathbb{E}_0\left(y_1^{\$}(1)\right) + \lambda_0' R\sigma + \frac{1}{2}\sigma' R \sigma},$$

with $\sigma = [\sigma_y, \sigma_x]'$. This equation implies that the long rate equals the average expected future short rate plus a time-varying nominal bond risk premium $\phi^{\$}$:

$$y_0^{\$}(2) = \frac{y_0^{\$}(1) + \mathbb{E}_0\left(y_1^{\$}(1)\right)}{2} - \frac{\lambda_0' R\sigma}{2} - \frac{1}{4}\sigma' R \sigma = \frac{y_0^{\$}(1) + \mathbb{E}_0\left(y_1^{\$}(1)\right)}{2} + \phi_0^{\$}(2). \tag{6}$$

[9] Brennan and Xia (2002) show that the utility costs induced by incompleteness of the financial market due to unexpected inflation are small. In a previous version of this paper, we have done a numerical, multi-period mortgage choice analysis. We found that unexpected inflation risk did not affect the household's risk-return tradeoff in any meaningful way.

The long-term nominal bond risk premium $\phi_0^{\$}(2)$ contains the market price of risk $\lambda_0$ and also absorbs the Jensen correction term.[10]

## 1.3 Mortgage Pricing

A competitive fringe of mortgage lenders prices ARM and FRM contracts to maximize profit, taking as given the term structure of Treasury interest rates generated by $M^{\$}$.

Denote the ARM rate at time $t$ by $q_t^{ARM}$. This is the rate applied to the mortgage payment due in period $t+1$. In each period, the zero-profit condition for the ARM rate satisfies:

$$B = \mathbb{E}_t \left[ M_{t+1}^{\$} \left( q_t^{ARM} + 1 \right) B \right] = \left( q_t^{ARM} + 1 \right) B P_t^{\$}(1).$$

This implies that the ARM rate is equal to the one-period nominal short rate, up to an approximation:

$$q_t^{ARM} = P_t^{\$}(1)^{-1} - 1 \simeq y_t^{\$}(1).$$

Similarly, the zero-profit condition for the FRM contract stipulates that the present discounted value of the FRM payments must equal the initial loan balance:

$$B = \mathbb{E}_0 \left[ M_1^{\$} q_0^{FRM} B + M_1^{\$} M_2^{\$} q_0^{FRM} B + M_1^{\$} M_2^{\$} B \right] = q_0^{FRM} P_0^{\$}(1) B + \left[ q_0^{FRM} + 1 \right] P_0^{\$}(2) B.$$

Per definition, the nominal interest rate on the FRM is fixed for the duration of the contract. We abstract from the prepayment option for now, but examine the role it plays in Section 4.1. The FRM rate, which is a 2-period coupon-bearing bond yield, is then equal to:

$$q_0^{FRM} = \frac{1 - P_0^{\$}(2)}{P_0^{\$}(1) + P_0^{\$}(2)} \simeq \frac{2 y_0^{\$}(2)}{2 - y_0^{\$}(1) - 2 y_0^{\$}(2)} \simeq y_0^{\$}(2).$$

The FRM rate is approximately equal to the two-period nominal bond rate.

Our setup embeds two assumptions that merit discussion. The first assumption is that the stochastic discount factor $M^{\$}$ that prices the term structure of interest rates is different from the inter-temporal marginal rate of substitution of the households in section 1.1. Without this assumption, mortgage choice would be indeterminate.[11] The second assumption is that we price

[10] Our nominal bond risk premium is the risk premium on a strategy that holds a $\tau$-period bond until maturity and finances it by rolling over the 1-year bond. This definition is different from the one-period bond risk premium in which the long-term bond is held for one period only. Cochrane and Piazzesi (2006) study various definitions of bond risk premia, including ours.

[11] Any equilibrium model of the mortgage market requires a second group of unconstrained investors. Time variation in risk premia could then arise from time-varying risk-sharing opportunities between the constrained and the unconstrained agents, as in Lustig and Van Nieuwerburgh (2006). In their model, the unconstrained agents price the assets at each date and state. Such an environment justifies taking bond prices as given when studying the problem of the constrained investors. Lustig and Van Nieuwerburgh (2006) consider agents with (identical)

mortgages as derivatives contracts on the Treasury yield curve. Hence, the same sources that drive time variation in the Treasury yield curve will govern time variation in mortgage rates.

## 1.4 The Risk-Return Tradeoff

We now derive the optimal mortgage choice for the household of Section 1.1. The crucial difference between an FRM investor and an ARM investor is that the former knows the value of all nominal mortgage payments at time 0, while the latter knows the value of the nominal payments only one period in advance. The risk-averse investor trades off lower expected payments on the ARM against higher variability of the payments. Appendix A computes the life-time utility under the ARM and the FRM contract. It shows that the investor prefers the ARM contract over the FRM contract if and only if

$$\begin{aligned} & q_0^{FRM} - q_0^{ARM} + \left(q_0^{FRM} - \mathbb{E}_0\left[q_1^{ARM}\right]\right) e^{-\mathbb{E}_0[x_1]} > \\ & \frac{\gamma}{2} B e^{-x_0 - 2\mathbb{E}_0[x_1]} \left[\sigma' R \sigma + \left(\mathbb{E}_0\left[q_1^{ARM}\right] + 1\right)^2 \sigma_x^2 - 2\left(\mathbb{E}_0\left[q_1^{ARM}\right] + 1\right)\left(\sigma_x e_2' R \sigma\right)\right] \\ & -\frac{\gamma}{2} B e^{-x_0 - 2\mathbb{E}_0[x_1]} \left(q_0^{FRM} + 1\right)^2 \sigma_x^2. \end{aligned} \tag{7}$$

The left-hand side measures the difference in expected payments on the FRM and the ARM. All else equal, a household prefers an ARM when the expected payments on the FRM are higher than those on the ARM. Appendix A shows that the difference between the expected mortgage payments on the FRM and ARM contracts approximately equals the two-period bond risk premium $\phi_0^{\$}(2)$. This leads to the main empirical prediction of the model: the ARM share is positively related to the nominal bond risk premium.

The right-hand side of (7) measures the risk in the payments, where we recall that $\gamma$ controls risk aversion. The first line arises from the variability of the ARM payments, the second line represents the variability of the FRM payments. All else equal, a risk-averse household prefers the ARM when the payments on the ARM are less variable than those on the FRM. The risk in the FRM contract is inflation risk ($\sigma_x^2$). The balance and the interest payments erode with inflation. We relabel expected inflation risk as $V^x$. The risk in the ARM contract consists of three terms. ARMs are risky because the nominal contract rate adjusts to the nominal short rate each period. The variance of the nominal short rate is $\sigma' R \sigma$. The second term is expected inflation risk, which enters in the same form as in the FRM contract. However, inflation risk is offset by the third term which arises from the positive covariance between expected inflation and the nominal short rate ($\sigma_x e_2' R \sigma$). In low inflation states the mortgage balance erodes only slowly, but the low nominal

CRRA preferences. In numerical work, presented in Section 4.2, we verify that the same risk-return tradeoff that the constrained households face also hold for CRRA preferences. A full-fledged equilibrium analysis of the mortgage market is beyond the scope of the current paper.

short rates and ARM payments provide a hedge. The appendix shows that the risk in the ARM is approximately equal to the variability of the real rate. We relabel real rate risk as $V^y$. In summary, the second empirical prediction of the model is that the ARM share should be decreasing in the real rate variability and increasing in the expected inflation variability.

The risk-return tradeoff in (7) can be written concisely as:

$$\phi_t^{\$}(2) - \frac{\gamma}{2} BV_t^y + \frac{\gamma}{2} BV_t^x > 0, \tag{8}$$

for some generic period $t$.

# 2 Empirical Results

We are interested in explaining time variation in the fraction of all newly-originated mortgages that is of the adjustable-rate type. The main task to render the theory testable is to measure the nominal bond risk premium. It is the difference between the current nominal long interest rate and the average expected future nominal short rate (see (6)):

$$\phi_t^{\$}(\tau) = y_t^{\$}(\tau) - \frac{1}{\tau} \sum_{j=1}^{\tau} \mathbb{E}_t \left[ y_{t+j-1}^{\$}(1) \right]. \tag{9}$$

We propose three alternative ways to compute expected future short rates: using forecaster data (Section 2.2), using a VAR model (Section 2.3), and using adaptive expectations (Section 2.4). Throughout, our benchmark results are for $\tau = 5$ years. We also study results for $\tau = 10$ years. Combined with the current 5-year (10-year) nominal bond yield, the three alternatives deliver three time series for the 5-year (10-year) nominal bond risk premium. With these measures of the risk premium in hand, we turn to a regression of the ARM share, defined in Section 2.1, on the bond risk premia.

## 2.1 Data on the ARM Share in the U.S.

Our baseline data series is from the Federal Housing Financing Board. It is based on the Monthly Interest Rate Survey (MIRS), a survey sent out to mortgage lenders.[12] These MIRS data include only new house purchases (for both newly-constructed homes and existing homes), not refinancings.

[12]Major lenders are asked to report the terms and conditions on all conventional, single-family, fully-amortizing, purchase-money loans closed the last five working days of the month. The data thus excludes FHA-insured and VA-guaranteed mortgages, refinancing loans, and balloon loans. The data for our last sample month, June 2006, are based on 21,801 reported loans from 74 lenders, representing savings associations, mortgage companies, commercial banks, and mutual savings banks. The data are weighted to reflect the shares of mortgage lending by lender size and lender type as reported in the latest release of the Federal Reserve Board's Home Mortgage Disclosure Act data.

Purchase-money loans accounts for approximately 60% of the mortgage flow.[13] The sample consists predominantly of conforming loans, only a very small fraction is jumbo mortgages. The ARM share for jumbos in the MIRS sample is much higher on average, but has a 70% correlation with the conforming loans in the sample. While the data do not permit precise statements about the representativeness of the MIRS sample, its ARM share has a correlation of 94% with the ARM share in the Inside Mortgage Finance data.[14] The monthly data start in 1985.1 and run until 2006.6, and we label this series $\{ARM_t\}$. There is an alternative source of monthly ARM share data available from Freddie-Mac, based on the Primary Mortgage Market Survey.[15] This series is available from 1995.1 and has a correlation with our benchmark measure of 90%.

## 2.2 Forecaster Data

Our forecaster data come from Blue Chip Economic Indicators. Twice per year (March and October), a panel of around 40 forecasters predict the average three-month T-bill rate for the next calendar year, and each of the following four calendar years. They also forecast the average T-bill rate over the ensuing five years. We average the consensus forecast data over the first five, or all ten, years to construct the expected future nominal short rate in (9). This delivers a semi-annual time-series from 1985 until 2006 for $\tau = 5$ and one for $\tau = 10$. We use linear interpolation of the forecasts to construct monthly series (1985.1-2006.6).[16]

Monthly nominal yield data are obtained from the Federal Reserve Bank of New York.[17] Combining the 5-year (10-year) T-bond yield with the 5-year (10-year) expected future short rate from Blue Chip delivers the 5-year (10-year) nominal bond risk premium. Panel A of Figure 4 shows the 5-year (solid line) and 10-year time-series (dashed line); they have a correlation of 94%.

[Figure 4 about here.]

We then regress the ARM share on the nominal bond risk premium. We lag the predictor variable for one month in order to study what changes in this month's risk premium imply for next month's mortgage choice. In addition, the use of lagged regressors mitigates potential endogeneity

[13] Freddie Mac publishes a monthly index of the share of refinancings in mortgage originations. The average refi share over the 1987.1-2007.1 period is 39.3%.

[14] We thank Nancy Wallace for making these data available to us. This comparison is for annual data between 1990 and 2006, the longest available sample.

[15] This survey goes out to 125 lenders. The share is constructed based on the dollar volume of conventional mortgage originations within the 1-unit Freddie Mac loan limit as reported under the Home Mortgage Disclosure Act (HMDA) for 2004. Given that Freddie Mac also publishes the aforementioned refinancing share of originations based on the same Primary Mortgage Market Survey, it appears that this series includes not only purchase mortgages but also refinancings.

[16] The correlations with the ARM share are similar if we use either semi-annual or monthly data.

[17] The nominal yield data are available at `http://www.federalreserve.gov/pubs/feds/2006`.

problems that would arise if mortgage choice affected the term structure of interest rates.[18] The first two rows of Table 1 shows the slope coefficient, its Newey-West t-statistic using 12 lags, and the regression $R^2$ for these regressions. Throughout the paper, all regressors are normalized by their standard deviation for ease of interpretation. The 5-year bond risk premium is a highly significant predictor of the ARM share. It has a t-statistic of 3.9, and explains 40% of the variation in the ARM share. A one-standard deviation, or one percentage point, increase in the nominal bond risk premium increases the ARM share by 8.6 percentage points. This is a large effect since the average ARM share is 28.7%. Intuitively, an FRM holder pays the bond risk premium. An increase in the risk premium increases the expected payments on the FRM relative to the ARM, and makes the ARM more attractive. The results with the 10-year risk premium (Row 2) are comparable. The coefficient has the same magnitude, a t-statistic of 4.2, and an $R^2$ of 43%.

[Table 1 about here.]

## 2.3 VAR Model

The second way to implement equation (9) is to use a vector auto-regressive (VAR) term structure model, as in Ang and Piazzesi (2003). The state vector $Y$ contains the 1-year ($y_t^{\$}(1)$), the 5-year ($y_t^{\$}(5)$), and the 10-year nominal yields ($y_t^{\$}(10)$), as well as realized 1-year log inflation ($\pi_t = \log \Pi_t - \log \Pi_{t-1}$).[19] We start the model in 1985, near the end of the Volcker period. Our stationary, one-regime model would be unfit to estimate the entire post-war history (see Ang, Bekaert, and Wei (2007) and Fama (2006)). Estimating the model at monthly frequency gives us a sufficiently many observations (1985.1-2006.6 or 258 months). The VAR(1) structure with the 12-month lag on the right-hand side is parsimonious and delivers plausible long-term expectations.[20] We use the letter $u$ to denote time in months, while $t$ continues to denote time in years. The law of motion for the state is

$$Y_{u+12} = \mu + \Gamma Y_u + \eta_{u+12}, \quad \text{with} \quad \eta_{u+12} \mid \mathcal{I}_u \sim D(0, \Sigma_t), \tag{10}$$

with $\mathcal{I}_u$ representing the information at time $u$. Appendix B discusses our model for the conditional covariance matrix $\Sigma_t$.

[18] As a robustness check, we have tested for Granger causality. First, we regress the ARM share on its own lag and the lagged bond risk premium; the lagged bond risk premium is statistically significant. Second, we regress the bond risk premium on its lag and the lagged ARM share; the lagged ARM share is statistically insignificant. Therefore, the bond risk premium Granges causes the ARM share, but the reverse is not true.

[19] The inflation rate is based on the monthly Consumer Price Index for for all urban consumers from the Bureau of Labor Statistics. The inflation data are available at `http://www.bls.gov`.

[20] As a robustness check, we considered a VAR(2) model and estimated the model on the basis of quarterly instead of monthly data. The results become even somewhat stronger for a second-order VAR model and we found similar results for quarterly data as for monthly data.

The VAR structure immediately delivers average expected future nominal short rates:

$$\frac{1}{\tau}\mathbb{E}_u\left[\sum_{j=1}^{\tau} y_{u+(12\times(j-1))}(1)\right] = \frac{1}{\tau}e_1'\sum_{j=1}^{\tau}\left\{\left(\sum_{i=1}^{j-1}\Gamma^{i-1}\right)\mu+\Gamma^{j-1}Y_u\right\}. \quad (11)$$

Together with the nominal long yield, this delivers our second measure of the nominal bond risk premium in (9). Panel B of Figure 4 shows the 5-year and 10-year time series; they have a correlation of 96%.

Rows 3 and 4 of Table 1 show the ARM regression results using the VAR-based 5-year and 10-year bond risk premium. Again, both bond risk premia are highly significant predictors of the ARM share. The t-statistics are 4.2 and 3.9. They explain 32% and 35% of the variation in the ARM share, respectively. Interestingly, the economic magnitude of the coefficients is very close to the one obtained from forecasters: A one-standard deviation increase in the risk premium increases the ARM share by about 8 percentage points. The next step is to include the 1-year ahead conditional variances of the real rate ($V_t^y$) and inflation ($V_t^x$) in the ARM share regression. Rows 5 and 6 show that both enter with the predicted sign. That is, the ARM share increases in periods of high inflation uncertainty, and decreases when the real rate volatility is high. However, they are not significant and add relatively little value beyond the nominal bond risk premium.

## 2.4 Rule-of-Thumb

Section 1 developed a model of rational mortgage choice where time variation in mortgage choice was driven by time variation in bond risk premia. Sections 2.2 and 2.3 then used two different ways of computing forward-looking expectations of future nominal short rates that entered the nominal bond risk premium. The empirical evidence supported the claim that these bond risk premia are related to the ARM share variation. One potential concern with this explanation for mortgage choice is that it requires substantial "financial sophistication" on the part of the households to choose the "right mortgage at the right time". Campbell (2006) expresses scepticism about such sophistication, and presents examples of investment mistakes. Even though mortgage choice is one of the most important financial decisions, and even though households may obtain advice from financial professionals or mortgage lenders, we take such scepticism seriously. After all, estimating a VAR model to form conditional expectations may be beyond reach for the average household. In this section, we address this concern and show that a simple rule-of-thumb captures most of the variation in mortgage choice. The rule-of-thumb is strongly related to our measures of bond risk premia. It also nests two previously-proposed predictors of mortgage choice: the yield spread and the long-term interest rate.

In particular, we assume that households approximate conditional expectations of future short

rates in (9) by forming simple averages of past short rates, going back $\rho$ months in time:

$$\begin{aligned}\phi_t^{\$}(\tau) &\simeq y_t^{\$}(\tau) - \frac{1}{12\times\tau}\sum_{s=1}^{\tau\times 12}\left\{\frac{1}{\rho}\sum_{u=0}^{\rho-1} y_{t-u}^{\$}(1)\right\} \\ &= y_t^{\$}(\tau) - \frac{1}{\rho}\sum_{u=0}^{\rho-1} y_{t-u}^{\$}(12) \equiv \kappa_t(\rho;\tau). \end{aligned} \tag{12}$$

Equation (12) is a model of adaptive expectations that only requires knowledge of the current long bond rate, a history of recent short rates, and the ability to calculate a simple average. Our third measure for bond risk premia is the rule-of-thumb $\kappa_t(\rho;\tau)$, computed off Treasury interest rates. Panel C of Figure 4 shows the $\tau = 5$- and $\tau = 10$-year time-series with a three year look-back ($\rho = 36$ months). They have a correlation of 92%. Since we consider look-back periods up to 5 years, we loose the first 5 years of observations, and the series start in 1989.12.[21]

Rows 7 and 8 of Table 1 show the ARM regression results using $\kappa_t(36;5)$ and $\kappa_t(36;10)$. The rule-of-thumb gives the strongest results among the three measures of the bond risk premium. The 5-year (10-year) bond risk premium has a t-statistics of 7.1 (7.5) and explains 71% (68%) of the variation in the ARM share! The economic magnitude of the coefficients is very close to the one from the previous two measures: A one-standard deviation increase in the risk premium increases the ARM share by about 8 percentage points. Figure 3 in the introduction illustrates the striking co-movement between the ARM share and the rule-of-thumb for $\rho = 36$ months.

The left panel of Figure 5 shows the correlation of $\kappa_t(\rho, 5)$ with the ARM share for different values of $\rho$ (blue bars). The bars correspond to $\rho$ = 12, 24, 36, 48, and 60 months look-back. The results are shown for the period 1989.12-2006.6, the longest sample for which all measures are available. The rule-of-thumb measure of bond risk premia has the strongest association with the ARM share for intermediate values of the horizon over which average short rates are computed. The correlation is hump-shaped in $\rho$ in both panels. The highest correlation with observed mortgage choice is obtained when households use 3 years of short rate data in their computation. The correlation peaks around 80%.

It should perhaps not come as a surprise that $\kappa_t(\rho;\tau)$ explains the variation in the ARM share better for the optimal value of $\rho$ than using the bond risk premium measure that we derived from the forecaster data or from the VAR model. After all, we now use a simpler model of expectations that can easily be implemented by households. If this model accurately describes households'

[21] We do not extend the sample before 1985.1 for two reasons. First, the interest rates in the early 1980s were dramatically different from those in the period we analyze. As such, we do not consider it to be plausible that households use adaptive expectations and data from the "Volcker regime" to form $\kappa$ in the first years of our sample. A second and related reason is that Butler, Grullon, and Weston (2006) argue that there is a structural break in bond risk premia in the early 1980s. To avoid any spurious results due to structural breaks, we restrict attention to the period 1985.1-2006.6.

behavior, we expect it to explain more of the variation in households' mortgage choice. In sum, this simple way of computing bond risk premia explains most of the variation in the ARM share. Section 3 is devoted to understanding the difference between the three risk premium measures in more depth.

[Figure 5 about here.]

## 2.5 Alternative Interest Rate Measures

The rule-of-thumb has the appealing feature that it nests two commonly-used predictors of mortgage choice as special cases (Campbell and Cocco (2003), Campbell (2006), and Vickery (2006)). First, when $\rho = 1$, we recover the yield spread:

$$\kappa_t(1;\tau) = y_t^{\$}(\tau) - y_t^{\$}(1).$$

The yield spread is the optimal predictor of mortgage choice in our model only if the conditional expectation of future short rates equals the current short rate. This is the case only when short rates follow a random walk. Second, when $\rho \to \infty$, then $\kappa_t(\rho;T)$ converges to the long-term yield in excess of the unconditional expectation of the short rate:

$$\lim_{\rho\to\infty} \kappa_t(\rho;T) = y_t^{\$}(T) - \mathbb{E}\left[y_t^{\$}(12)\right], \tag{13}$$

by the law of large numbers.[22] Because the second term is constant, all variation in financial incentives to choose a particular mortgage originates from variation in the long-term yield. This rule is optimal when short rates are constant. For all cases in between the two extremes, the simple model of adaptive expectations puts some positive and finite weight on average recent short-term yields to form conditional expectations.

**Yield Spread** The solid line in left panel of Figure 5 depicts the correlation between the yield spread and the ARM share ($\rho = 1$). It shows that the yield spread has a weak contemporaneous correlation with the ARM share (1989.12-2006.6). Rows 9 and 10 of Table 1 confirm that the lagged yield spread explains very little of the variation in the ARM share in the full sample (1985.1-2006.6); the $R^2$ is less than 1 percent.

Equation (9) allows us to decompose the nominal yield spread into the nominal bond risk premium and the deviations of average expected future short rates and the current nominal short

[22] This requires a stationarity assumption on the short rates.

rate:

$$y_t^{\$}(\tau) - y_t^{\$}(1) = \phi_t^{\$}(\tau) + \left( \frac{1}{\tau} \sum_{j=1}^{\tau} \mathbb{E}_t \left[ y_{t+j-1}^{\$}(1) \right] - y_t^{\$}(1) \right). \tag{14}$$

This condition is useful in understanding the difference between the slope of the yield curve and the long-term bond risk premium. In a homoscedastic world with zero risk premia ($\phi_t^{\$}(\tau) = 0$), the yield spread equals the difference between the average expected future short rates and the current short rate. Since long-term bond rates are the average of current and expected future short rates, both the FRM and the ARM investor will face the same expected payment stream in this world. The yield spread is completely uninformative about mortgage choice. Likewise, in a world with constant risk premia, *variations* in the yield spread capture *variations* in deviations between expected future short rates and the current short rate. But again, these variations are priced into both the ARM and the FRM contract. It is only the bond risk premium which affects the mortgage choice for a risk-averse investor. In our model with time-varying risk premia, estimated above, it turns out that the two terms on the right-hand side of (14) are *negatively* correlated. This makes the yield spread a noisy proxy for the nominal bond risk premium, and is responsible for the low $R^2$ in the regression of the ARM share on the yield spread.

**Long Yields** The dashed line in the left panel of Figure 5 shows that the correlation between the ARM share and the long rate is much higher than the correlation with the slope of the yield curve, but it is dominated by the rule-of-thumb. Rows 11 and 12 of Table 1 show that one standard deviation increase in the 10-year yield increases the ARM share by 8.5% in the full sample, a similar magnitude as for the risk premium. As we show in Section 3, the long yield performs much worse in recent times.

**Mortgage Rates** An alternative source of interest rate data comes from the mortgage market. We use the 1-year ARM rate as our measure of the short rate and the 30-year FRM rate as our measure of the long rate.[23] The right panel of Figure 5 shows the correlation of $\kappa_t(\rho, 5)$ with the ARM share for different values of $\rho$ (blue bars), computed using mortgage rates. The rule-of-thumb achieves its highest correlation of for an intermediate horizon of two years, comparable to what we find for Treasury yields.

As we did for Treasury yields, we regress the ARM share on the slope of the yield curve (30-year FRM rate minus 1-year ARM rate) and the long yield (30-year FRM rate). Row 13 of Table 1 shows that the FRM-ARM spread has much higher explanatory power than the Treasury yield spread. However, the improvement occurs only because it contains additional information that is

[23] We use the effective rate data from the Federal Housing Financing Board, Table 23. The effective rate adjusts the contractual rate for the discounted value of initial fees and charges. The FRM-ARM spreads with and without fees have a correlation of .998.

not in the Treasury yield spread.[24] The explanatory power of the FRM rate is similar to that of the long Treasury yield (Row 15 and right panel of Figure 5).

**Other Rules-of-Thumb** The rule-of-thumb that we introduce in Section 2.4 is motivated by the theoretical model in Section 1 and provides a way to compute the expectations of future short rates in (9). We now study two additional interest rate-based variables which implement alternative, more ad-hoc, rules-of-thumb. The first rule takes the current FRM rate minus the three-year moving average of FRM rate (row 16 of Table 1). The second rule does the same, but for the ARM rate (row 17). The first rule captures the idea behind the popular investment advice of "locking in a low long-term rate while you can". The slope coefficients in the FRM and ARM rule are smaller than what we find for the bond risk premium (6.0 and 3.1) and less precisely measured (t-stats of 3.7 and 2.4). The $R^2$ in the two regressions are 22% and 6%, respectively. Both alternative rules perform worse than the rule-of-thumb of Section 2.4, which is guided by the theory.

# 3 The Recent Episode and the Inflation Risk Premium

The previous sections show that various measures of the bond risk premium are positively and significantly related to the choice between an ARM and FRM mortgage. In this section, we investigate the difference between the rule-of-thumb measure, which shows the strongest relationship and is based on adaptive expectations, and the forecasters- and VAR-based measures, which show a somewhat weaker relationship and are based on forward-looking expectations.

Figure 6 shows that this difference in performance is especially pronounced after 2004. The figure displays the 10-year rolling-window correlation for each of the three measures with the ARM share. While the rule-of-thumb measure has a stable correlation across sub-samples, the performance of the forecasters-based measure as well as the VAR-based measure drop off steeply in 2004 and beyond.

[Figure 6 about here.]

The reason for this failure is that the ARM share increased substantially between June 2003 and December 2004 with no commensurate increase in the Blue Chip or VAR risk premia measures.

[24] The correlation between the FRM-ARM spread and the 10-1-year government bond yield spread is only 32%. This spread also captures the value of the prepayment option, as well as the lenders' profit margin differential on the FRM and ARM contracts. To get at this additional information, we orthogonalize the FRM-ARM spread to the 10-1 yield spread, and regress the ARM share on the orthogonal component (Row 14). For the full sample, we find a strongly significant effect on the ARM share. Partially this is due to the fact that this orthogonal spread component has a correlation of 60% with the fee differential between an FRM and an ARM contract. It only has a correlation of 16% with the rule-of-thumb risk premium.

Figure 2 illustrates this breakdown in comovement for the Blue Chip data. A similarly steep drop-off in correlation occurs for the long yield and for the FRM-ARM rate differential, both of which also performed well in the full sample. We explore two possible explanations for why the ARM share was high in 2004 when the forward-looking bond risk premia were low.

## 3.1 Product Innovation in the ARM Segment

A first potential explanation for the increase in the ARM share between June 2003 and December 2004 is product innovation in the ARM segment of the mortgage markets. An important development was the increased popularity of hybrid mortgages: adjustable-rate mortgages with an initial fixed-rate period.[25] Figure 7 shows our benchmark measure of the ARM share (solid line) alongside a measure of the ARM share that excludes all hybrid contracts with initial fixed-rate period longer than three years. We label this measure $\widetilde{ARM}$. A substantial fraction of the increase in the ARM share in 2003-05 was due to the rise of hybrids. Under this hypothesis the ARM share went up despite the low bond risk premium because new types of ARM mortgage contracts became available that unlocked the dream of home ownership.[26]

[Figure 7 about here.]

To test this hypothesis, we recompute the rolling correlations for $\widetilde{ARM}$, which excludes the hybrids. The correlation with the forecasters-based measure over the last 10-year window improves from 23% to 48%. The correlation over the longest available sample (since 1992) improves from 44% to 67%. In sum, the recently increased prevalence of the hybrids is part of the explanation. However, it cannot account for the entire story.

## 3.2 Forecast Errors

A second potential explanation is that the forecasters made substantial errors in their predictions of future short rates in recent times. We recall that nominal short rates came down substantially from 6% in 2000 to 1% in June 2003. Our Blue Chip data show that forecasters expected short rates to increase substantially from their 1% level in June 2003. Instead, nominal short rates increased only moderately to 2.2% by December 2004. Forecasters substantially over-estimated future short rates starting in the 2003.6-2004.12 period. As a result, the Blue Chip measure of bond risk premia is too low in that episode, and underestimates the desirability of ARMs.

[25] Starting in 1992, we know the decomposition of the ARM by initial fixed-rate period. We are grateful to James Vickery for making these detailed data available to us.

[26] In addition to the hybrid segment, the sub-prime market segment, which predominantly offers ARM contracts, also grew strongly over that period. However, our ARM sample does not contain this market segment.

Forecast errors in nominal rates translate in forecast errors for real rates. This is in particular the case when inflation is relatively stable and therefore easier to forecast. Figure 8 shows that the Blue Chip consensus forecast for the average real short rate over the next two years shows large disparities with its realized counterpart. We calculate the average expected future real short rate as the difference between the Blue Chip consensus average expected future nominal short rate and the Blue Chip consensus average expected future inflation rate. We calculate the realized real rate as the difference between the realized nominal rate and expected inflation, which we measure as the one-quarter ahead inflation forecast. The realized average future real short rates are calculated from the realized real rates. Finally, the forecast errors are scaled by the nominal short rate to obtain relative forecasting errors. The figure shows huge forecast errors in the 2000-2003 period, relative to the earlier period. The forecast errors are on the order of 1.25 percentage point per year, about 50-75% of the value of the nominal short rate. These large forecast errors motivate the use of the inflation risk premium, as explained below.

[Figure 8 about here.]

A similar problem arises with the VAR-based bond risk premium. The VAR system also fails to pick up the declining short rates in the 2000-2004 period. It therefore also over-predicts the short rate and underestimates the desirability of ARM contracts.

**Filtering Out Forecast Errors** Forecast errors in the real rate not only help us identify the problem, they also offer the key to the solution. The nominal bond risk premium in the model of Sections 1.1 and 1.2 contains compensation for both real rate risk and expected inflation risk:

$$\phi_t^{\$}(\tau) = \phi_t^y(\tau) + \phi_t^x(\tau). \tag{15}$$

Similar to the nominal risk premium in (9), the real rate risk premium, $\phi_t^y$, is the difference between the observed real long rate and the average expected future real short rate:

$$\phi_t^y(\tau) \equiv y_t(\tau) - \frac{1}{\tau}\sum_{j=1}^{\tau} \mathbb{E}_t\left[y_{t+j-1}(1)\right], \tag{16}$$

where $y_t(\tau)$ is the real yield of a $\tau$-month real bond at time $t$. Following Ang, Bekaert, and Wei (2007), we define the inflation premium at time $t$, $\phi_t^x$, as the difference between long-term nominal yields, long-term real yields, and long-term expected inflation:

$$\phi_t^x(\tau) \equiv y_t^{\$}(\tau) - y_t(\tau) - x_t(\tau). \tag{17}$$

where long-term expected inflation is given by:

$$x_t(\tau) \equiv \frac{1}{\tau}\mathbb{E}_t\left[\log \Pi_{t+\tau} - \log \Pi_t\right].$$

A key insight is that both the nominal long yield $y_t^{\$}(\tau)$ and the real long yield $y_t(\tau)$ contain expected future real short rates. Thus, their difference does not. Therefore, their difference zeroes out any forecast errors in expected future real short rates. Equation (17) shows that the inflation-risk premium, $\phi_t^x(\tau)$, contains the difference between $y_t^{\$}(\tau) - y_t(\tau)$, and therefore does not suffer from the forecast error problem.[27] In short, one way to correct the nominal bond risk premium for the forecast error is to only use the inflation risk premium component.

**Measuring the Inflation Risk Premium** To implement equation (17), we need a measure of long real yields and a measure of expected future inflation rates. Real yield data are available as of January 1997 when the US Treasury introduced Treasury Inflation-Protected Securities (TIPS).[28] We omit the first six months when liquidity was low, and only a 5-year bond was trading. In what follows, we consider two empirical measures for expected inflation.

Our first measure for expected inflation is computed from the same semi-annual Blue Chip long-range consensus forecast data we used for the nominal short rate, using the same method, but using the series for the CPI forecast instead of the nominal short rate.[29] The inflation-risk premium is then obtained by subtracting the real long yield and long-term expected inflation from the nominal long yield, as in (17).

Alternatively, we can use the VAR to form expected future inflation rates and thereby the inflation risk premium. We start by constructing the 1-year expected inflation series as a function of the state vector

$$x_t(1) \;=\; \mathbb{E}_t\left[\pi_{t+1}\right] = e_4'\mu + e_4'\Gamma Y_t, \tag{18}$$

where $e_4$ denotes the fourth unit vector. Next, we use the VAR structure to determine the $\tau$-year

[27] The same Blue Chip forecast data, as well as data from the Survey of Professional Forecasters, indeed show that inflation forecasts do not suffer from the same problem as nominal interest rate forecasts. This is consistent with Ang, Bekaert, and Wei (2007), who argue that inflation forecasts provide the best predictors of future inflation among a wide set of alternatives.

[28] The real yield data are available from McCulloch at `http://www.econ.ohio-state.edu/jhm/ts/ts.html`.

[29] We have compared the inflation forecasts from Blue Chip with those from the Survey of Professional Forecasters, the Livingston Survey, and the Michigan Survey, and found them to be very close. Ang, Bekaert, and Wei (2006) argue that such survey data provides the best inflation forecasts among a wide array of methods.

expectations of the average inflation rate in terms of the state variables:

$$\frac{1}{\tau}\mathbb{E}_t\left[\sum_{j=1}^{\tau} e_4' Y_{t+j-1}\right] = \frac{1}{\tau} e_4' \sum_{j=1}^{\tau}\left\{\left(\sum_{i=0}^{j-1}\Gamma^{i-1}\right)\mu + \Gamma^{j-1}Y_t\right\}. \tag{19}$$

With the long-term expected inflation from (19) in hand, we form the inflation risk premium as the difference between the observed nominal yield, the observed real yield, and expected inflation.

**Results** Figure 9 shows the inflation risk premium (dashed line) alongside the ARM share (solid line). The inflation risk premium is based on Blue Chip forecast data. Between March 2003 and March 2005 (closest survey dates), the inflation risk premium increased by 1.2 percentage points, or two standard deviations. The nominal bond risk premium, in contrast, only increased only by one standard deviation.

[Figure 9 about here.]

Over the period 1997.7-2006.6, the raw correlation between the ARM share and the 5-year (10-year) inflation risk premium is 84% (82%) for the Blue Chip measure and 80% (78%) for the VAR measure. Finally, we regress the ARM share on the 5-year and 10-year inflation risk premium for the period 1997.7-2006.6. For the Blue Chip measure, we find a point estimate of 6.95 (6.97) for the 5-year (10-year) inflation risk premium. The economic effect is therefore comparable to what we find for the nominal bond risk premium (Section 2.2). The coefficient is measured precisely; the t-statistic is 8.0 (7.9). The 5-year (10-year) inflation-risk premium alone explains 66% of the variation (67%) in the ARM share. Likewise, for the VAR-based measure, we find a point estimate of 6.80 (6.40) for the 5-year (10-year) inflation risk premium. The coefficient is measured precisely; the t-statistic equals 8.5 (6.8). The inflation risk premium alone explains 64% of the variation (56%) in the ARM share. We conclude that the inflation risk premium has been a very strong determinant of the ARM share in the last ten years.

In conclusion, in 2003 and 2004, the forward-looking expectations measures of the bond risk premium suffered from large differences between realized average short rates, and what forecasters or a VAR predicted for these same average short rates. The adaptive expectations scheme of the rule-of-thumb did not suffer from the same problem. This explains why it performed much better in predicting the ARM share in the last part of the sample. The inflation risk premium component of the bond risk premium successfully purges that forecast error from the forward-looking bond risk premium measures. We showed that it is a strong predictor of the ARM share in the 1997.7-2006.6 sample.

# 4 Robustness

In this section we discuss several alternative model assumptions, variable definitions, and a new loan-level data set. We find that our main finding is robust to these alternative specifications; the bond risk premium remains an important determinant of mortgage choice.

## 4.1 Prepayment Option

Sofar we have ignored one other potentially important determinant of mortgage choice: the prepayment option. In the US, an FRM contract typically has an embedded option which allows the mortgage borrower to pay off the loan at will. We show how the presence of the prepayment option affects mortgage choice within the utility framework of Section 1.

**FRM Rate With Prepayment** A household prefers to prepay at time 1 if the utility derived from the ARM contract exceeds that of the FRM contract. Prepayment entails no costs, but this assumption is easy to relax in our framework. It then immediately follows from comparing the time-1 value function that prepayment is optimal if and only if:

$$q_0^{FRMP} > q_1^{ARM},$$

where the superscript $P$ in $q_0^{FRMP}$ indicates the FRM contract with prepayment. The FRM rate with prepayment satisfies the following zero-profit condition. It stipulates that the present value of mortgage payments the lender receives must equate the initial mortgage balance $B$:

$$\begin{aligned} B &= \mathbb{E}_0\left[M_1^{\$} q_0^{FRMP} B + I_{\left(q_0^{FRMP} > q_1^{ARM}\right)} M_1^{\$} M_2^{\$} q_1^{ARM} B + I_{\left(q_0^{FRMP} \leq q_1^{ARM}\right)} M_1^{\$} M_2^{\$} q_0^{FRMP} B + M_1^{\$} M_2^{\$} B\right] \\ &= q_0^{FRMP} P_0^{\$}(1) B + \left[q_0^{FRMP} + 1\right] P_0^{\$}(2) B - B\mathbb{E}_0\left[M_1^{\$} M_2^{\$} \max\left\{q_0^{FRMP} - q_1^{ARM}, 0\right\}\right], \end{aligned}$$

where the last term represents the value of the embedded prepayment option held by the household. $I_{(x<y)}$ denotes an indicator function that takes a value of one when $x < y$. This option value satisfies:

$$B\mathbb{E}_0\left[M_1^{\$} M_2^{\$} \max\left\{q_0^{FRMP} - q_1^{ARM}, 0\right\}\right] = B\left(1 + q_0^{FRMP}\right)\left[P_0^{\$}(2)\,\Phi(d_1) - \frac{1}{1 + q_0^{FRMP}} P_0^{\$}(1)\,\Phi(d_2)\right],$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution, and the expressions for $d_1$ and $d_2$ are provided in Appendix C. The second step is an application of the Black and Scholes (1973) formula and is spelled out in Appendix C as well (See also Merton (1973) and Jamshidian (1989)). The household has $B\left(1 + q_0^{FRMP}\right)$ European call options on a two-period bond with expiration date $t = 1$ (when it becomes a one-year bond with price $P_1^{\$}(1) = \frac{1}{1+q_1^{ARM}}$), and with an exercise price of

$\frac{1}{1+q_0^{FRMP}}$. Substituting the option value into the zero-profit condition we get:

$$B = \left(q_0^{FRMP} + \Phi\left(d_2\right)\right) P_0^{\$}\left(1\right) B + \left[q_0^{FRMP} + 1\right] P_0^{\$}\left(2\right) B \left(1 - \Phi\left(d_1\right)\right).$$

The mortgage balance equals the sum of (i) the (discounted) payments at time $t = 1$, a certain interest payment and a principal payment with risk-adjusted probability $\Phi\left(d_2\right)$, and (ii) the (discounted) payments at time $t = 2$, when both interest and principal payments are received with risk-adjusted probability $1 - \Phi\left(d_1\right)$. The no-arbitrage rate $q_0^{FRMP}$ on an FRM with prepayment solves the fixed-point problem:

$$q_0^{FRMP} = \frac{1 - \left(1 - \Phi\left(d_1\right)\right) P_0^{\$}\left(2\right) - \Phi\left(d_2\right) P_0^{\$}\left(1\right)}{P_0^{\$}\left(1\right) + \left(1 - \Phi\left(d_1\right)\right) P_0^{\$}\left(2\right)},$$

which cannot be solved for analytically as $q_0^{FRMP}$ appears in $d_1$ and $d_2$ on the right-hand side. For $\Phi\left(d_1\right) = \Phi\left(d_2\right) = 1$, prepayment is certain, and we retrieve the expression for the year-one ARM rate, $q_0^{ARM}$. For $\Phi\left(d_1\right) = \Phi\left(d_2\right) = 0$, prepayment occurs with zero probability, and we obtain the expression for the FRM without prepayment, $q_0^{FRM}$.

This framework clarifies the relationship between time-varying bond risk premia and the price of the prepayment option. The bond risk premium goes up when the price of interest rate risk goes down. But a decrease in the price of risk also makes prepayment less likely under the risk-neutral distribution. Therefore, the price of the prepayment option is decreasing in the bond risk premium.

**Reduced Sensitivity** A fixed-rate mortgage *without* prepayment option is a coupon-bearing nominal bond, issued by the borrower and held by the lender.[30] An FRM *with* prepayment option resembles a callable bond: the borrower has the right to prepay the outstanding mortgage debt at any point in time. The price sensitivity of a callable bond to interest rate shocks differs from that of a regular bond. This is illustrated in Figure 10. We use the bond pricing setup of Section 1.2 and set $\mu_y = \mu_x = 2\%$, $\rho_y = \rho_x = 0.5$, $\rho_{xy} = 0$, $\sigma_y = \sigma_x = 2\%$, and $\lambda_0 = [-0.4, -0.4]'$. These values imply a two-period nominal bond risk premium of $\phi_0^{\$}\left(2\right) = 0.78\%$. We vary the short rate at time zero, $y_0^{\$}\left(1\right) = y_0\left(1\right) + x_0$, assuming $y_0(1) = x_0$. The callable bond can be called at time one with exercise price of 0.96 (per dollar face value). The non-callable bond price is decreasing and convex in the nominal interest rate. The callable bond price is also decreasing in the nominal interest rate, but, the relationship becomes concave when the call option is in the money ("negative convexity"). This means that the callable bond has positive, but diminished exposure to nominal interest-rate risk.

[30] This analogy is exact for an interest-only mortgage. When the mortgage balance is paid off during the contractual period (amortizing), the loan can be thought of as a portfolio of bonds with maturities equal to the dates on which the down-payments occur. Acharya and Carpenter (2002) discuss the valuation of callable, defaultable bonds.

[Figure 10 about here.]

**Utility Implications of the Prepayment Option** Next, we study how the prepayment option affects the relationship between the bond risk premium and the ARM-FRM utility differential. We use the same term-structure variables as in Figure 10, but vary the market prices of risk $\lambda_0$. We maintain the assumption of equal prices of inflation risk and real interest rate risk, and fix the initial real interest and inflation rate at their unconditional means, i.e. $y_0(1) = \mu_y$ and $x_0 = \mu_x$. We assume the investor has a mortgage balance and house size normalized to 1, constant real labor income of 0.41, and a risk aversion coefficient $\gamma = 10$. Figure 11 plots the difference between the lifetime utility from the ARM contract and the lifetime utility from the FRM contract. The solid line depicts the case without prepayment option; the dashed line plots the utility difference when the FRM has the prepayment option. No approximations are used for this exercise. The utility difference is increasing in the bond risk premium, both with and without prepayment option. However, the sensitivity of the utility difference to changes in the bond risk premium is somewhat reduced in presence of a prepayment option. This is consistent with the fact that a callable bond has diminished interest rate exposure and therefore contains a lower bond risk premium than a non-callable bond. This shows that our main result, a positive relationship between the utility difference of an ARM and an FRM contract and the nominal bond risk premium, goes through.

[Figure 11 about here.]

## 4.2 Impatience and Moving

To investigate the role of the subjective discount factor and the moving rate, we solve a multi-period extension of the model in Section 1. In contrast to the household's preferences in (1), we use a time-separable CRRA utility function over real consumption with subjective discount factor $\beta$. We allow for an exogenous moving probability. Section 2.5 showed that the yield spread did not display a strong co-movement with the ARM share. We argued that it not only captures the bond risk premium, but also deviations of expected future short rates from current short rates. However, when a household is perfectly impatient and only cares about consumption in the current period ($\beta = 0$), only the current period's differential between the long-term and the short-term interest rate matters. The same is true if a household plans to move in the current year.[31] The model features 360-month mortgage contracts, and the term structure dynamics are taken from Koijen, Nijman, and Werker (2007). As was the case in the data, the correlation between the yield

[31]Mobility in and of itself is an unlikely candidate to explain variation in the ARM share. Current Population Survey data for 1948-2004 from the US Census show that the average annual (monthly) moving rate is 18.1% (1.27%), and the out-of-county moving rate is 6.2% (1.16%). Moreover, these moving rates show no systematic variation over time.

spread and the bond risk premium is low in the model (-7%). Figure 12 plots the $R^2$ of regressing the model's certainty-equivalent consumption differential between the ARM and FRM contracts on the model's bond risk premium (solid line) or on the model's yield spread (dashed line). Each point corresponds to a different value of the (annual) subjective time discount factor $\beta$, between 0.5 and 1. For low values of the subjective discount factor ($\beta < .70$), the slope of the yield curve has a stronger relationship to the relative desirability of the ARM. However, for higher values of $\beta$ (e.g., conventional values such as $\beta = .95$), only the bond risk premium matters. We have also experimented with an upward sloping labor income profile, as in Cocco, Gomes, and Maenhout (2005), and found a similar cut-off rule. Finally, a similar result hold for mobility rates: below 10% per month, the risk premium is the more important predictor. For empirically relevant moving rates below 2%, the risk premium is the only relevant predictor.

[Figure 12 about here.]

Finally, we have investigated the extent to which the yield spread affects mortgage choice in the data, over and above the risk premium. In a multiple regression of the ARM share on the risk premium and the yield spread, the latter was typically not significant. Its sign flips across specifications, its t-statistic is low, and it does not contribute to the $R^2$ of the regression, beyond the effect of the risk premium.

## 4.3 Financial Constraints

One alternative hypothesis is that there is a group of financially-constrained households which postpones the purchase of a house until the ARM rate is sufficiently low to qualify for a mortgage loan. Under this alternative hypothesis, the time-series variation in the dollar volume of ARMs would drive the variation in the ARM share. Figure 13 plots the dollar volume of ARM and FRM mortgage originations for the entire U.S. market, scaled by the overall size of the mortgage market. The data are compiled by OFHEO. It shows that there are large year-on-year fluctuations in both the ARM and the FRM market segment. This dispels the hypothesis that the variation in the ARM share over the last 20 years is driven by fluctuations in participation in the ARM segment.

[Figure 13 about here.]

**Loan-level Data** Individual-level loan data provide a more direct vehicle to investigate the importance of financial constraints. We explore a new data set which contains information on 911,000 loans from a large mortgage trustee for mortgage-backed security special purpose vehicles.[32]

[32]It contains data from many of the largest mortgage lenders (Aames Capital, Bank of America, Citi Mortgage, Countrywide, Indymac, Option One, Ownit, Wells Fargo, Washington Mutual). We use information on the loan

While the sample spans 1994-2007, 95% of mortgage contracts are originated between 2000 and 2005, a period for which we have argued mortgage choice is more difficult to explain. Nevertheless, it would be reassuring to find that the same variable that explains the aggregate time series of mortgage choice also explains mortgage choice in the cross-section. Furthermore, we can investigate the extent to which variables that capture credit constraints, such as the loan's balance, FICO score, and loan-to-value ratio, affect mortgage choice. Finally, the data allow us to investigate the regional dimension of mortgage choice.

Table 2 reports loan-level results of probit regressions of an ARM dummy. All right-hand side variables have been scaled by their standard deviation. We report the coefficient estimate, a robust t-statistic, and the faction of loans that is correctly classified by the probit model.[33] We keep the 654,368 loans for which we have all variables of interest available. The first row shows that the rule-of-thumb bond risk premium is a strong predictor of loan-level mortgage choice. It has the right sign, a t-statistic of 253, and it -alone- classifies 69.4% of loans correctly. Its coefficient indicates that a one standard deviation increase in the bond risk premium increases the probability of an ARM choice from 39% to 56%, an increase of more than one-third. Row 2 shows the probit results without bond risk premium, but with three prominent indicators of financial constraints, loan balance, FICO, and LTV, and with four regional dummies for the biggest mortgage markets (California, Florida, New York, and Texas). A lower balance, lower FICO scores, and especially higher LTV ratio increase the probability of the ARM. However, the (scaled) coefficients on the loan characteristics are smaller than the bond risk premium coefficient, suggesting a smaller economic effect. Furthermore, the three financial constraint variables classify only 59% of loans correctly; adding four state dummies increases correct classifications to 61.7%. Relative to row 1, adding the same three characteristics and four regional dummies to the bond risk premium does not increase the probability of classified loans (Row 3). The number of classified loans is 68.8%, no bigger of what is explained by the bond risk premium alone.[34] Moreover, the bond risk premium remains the largest and by far the most significant regressor. Its marginal effect on the probability of choosing an ARM is unaffected. The fourth row adds the 5-year bond yield to the set of regressors. The effect of the bond risk premium increases and is estimated more precisely. The long yield is significant, but comes in with the wrong sign (the same happens if the only regressor is the long yield). The fifth row adds the yield spread, the difference between the 5-year and the 1-year bond yields. It enters with the correct sign and is highly significant. However, it further strengthens the

type, the loan origination year and month, the balance, the loan-to-value ratio, the FICO score, and the contract rate at origination. We also have geographic information on the region of origination. We merge these data with our bond risk premium and interest rate variables, with matching based on month of origination. We thank Nancy Wallace for graciously making these data available to us.

[33]By pure chance, one would classify 50% of the contracts correctly.

[34]Note that the maximum likelihood estimation does not maximize correct classifications, so that adding regressors does not necessarily increase correct classifications.

effect of the risk premium. The coefficient is 50% higher than in the third row, and the t-statistic is 308 compared to 253. Correct loan classifications increase slightly to 70.9%. It does not seem to be the case that the effect of the yield spread solely captures binding borrowing constraints. Adding the yield spread to the bond risk premium as an explanatory variable has a much stronger effect than adding the financial constraint variables plus regional dummies.

We conclude that, first, the bond risk premium is a powerful predictor of mortgage choice in these loan-level data. Second, while measures of financial constraints certainly enter significantly in these regressions, both their economic and statistical effect on mortgage choice is smaller.[35]

[Table 2 about here.]

**Regional Variation** There is substantial cross-state variation in mortgage choice in the US. In 2006, the ARM share was above 40% in California, but less than 10% in Connecticut. The loan-level data set is large enough to investigate the relationship between the ARM share on the bond risk premium state by state. We find strong positive co-movement for all states. Interestingly, the size of the probit coefficient and its t-statistic are rather similar across states. This suggests that the cross-state variation in ARM share is a level-effect, which does not interfere with our model of time variation in the ARM share.

## 4.4 Persistence of Regressor

In contrast to the bond risk premium, most term structure variables do not explain much of the variation in the ARM share (Table 1). This is especially true in the last ten years of our sample, when the inflation risk premium has strong explanatory power (Section 3), but the real yield or the FRM-ARM rate differential do not. This suggests that our results for the risk premium are not simply an artifact of regressing a persistent regressand on a persistent regressor, because many of the other term structure variables are at least as persistent.[36] To further investigate this issue, we conduct a block-bootstrap exercise, drawing 10,000 times with replacement 12-month blocks of innovations from an augmented VAR. The latter consists of the four equations of the VAR of Section 2.3, and is augmented with an equation for the ARM share. The ARM share equation is allowed to depend on the four lagged VAR elements, as well as on its own lag. The lagged ARM share itself does not affect the VAR elements. The bootstrap estimate recovers the point estimate (no bias), and it leads to a confidence interval that is narrower (6.40) than the Newey-West confidence interval we use in the main text (8.24), but wider than an OLS confidence interval (3.73). We conclude that the Newey-West standard errors we report are conservative.

[35] We also investigated the effect of the aggregate loan-to-value ratio, aggregate house price-income, and house price-rent ratios on the ARM share, but found no relationship.

[36] The ARM share itself is not that persistent. Its annual autocorrelation is 30%, compared to 76% for the one-year nominal interest rate. An AR(1) at an annual frequency only explains 8.8% of the variation in the ARM share.

One further robustness check we performed is to regress quarterly changes in the ARM share (between periods $t$ and $t+3$) on changes in the term structure variables of the benchmark regression specification (between periods $t-1$ and $t$). We continue to find a positive and strongly significant effect of the risk premium on the ARM share (t-statistic around 5). The effect of a change in the bond risk premium is similar to the one estimated from the level regressions: a one percentage point increase in the bond risk premium leads to a 10 percentage point increase in the ARM share over the next quarter. The $R^2$ of the regression in changes is obviously lower, but still substantial. For the 5-year (10-year) risk premium based on the VAR, it is 12% (18%), for the forecaster measure it is 25% (30%), and for the rule-of-thumb it is 26% (27%).

## 4.5 Liquidity and the TIPS Market

The results in Section 3, which use the inflation risk premium, are based on TIPS data. The TIPS markets suffered from liquidity problems during the first years of operation, which may have introduced a liquidity premium in TIPS yields (see Shen and Corning (2001) and Jarrow and Yildirim (2003)). A liquidity premium is likely to induce a downward bias in the inflation risk premium. As long as this bias does not systematically covary with the ARM share, it operates as an innocuous level effect and adds measurement error.

To rule out the possibility that our inflation risk premium results are driven by liquidity premia, we use real yield data backed-out from the term structure model of Ang, Bekaert, and Wei (2007) instead of the TIPS yields. We treat the real yields as observed, and use them to construct the inflation risk premium.[37] Since the Ang-Bekaert-Wei data are quarterly (1985.IV-2004.IV), we construct the quarterly ARM share as the simple average of the three monthly ARM share observations in that quarter. We then regress the quarterly ARM share on the one-quarter lagged inflation and real rate risk premium. We find that both components of the nominal bond risk premium, the inflation-risk premium, and the real rate risk premium, enter with a positive sign. This is consistent with the theoretical model developed in Section 1. Both coefficients are statistically significant: The Newey-West t-statistic on the inflation risk premium is 3.90 and the t-statistic on the real rate risk premium is 2.12. The regression R-squared is 53%.

As a final robustness check, we repeated our regressions using only TIPS data after 1999.1, after the initial period of illiquidity. We found very similar results to those based on data starting in 1997.7. This suggests that liquidity problems in TIPS markets may have affected the inflation-risk premium, but this does not significantly affect our results. We conclude that our results are robust to using alternative real yield data.

[37] We thank Andrew Ang for making these data available to us.

# 5 Conclusion

We have shown that the time variation in the nominal risk premium on a long-term nominal bond can explain a large fraction of the variation in the share of newly-originated mortgages that are of the adjustable-rate type. Thinking of fixed-rate mortgages as a short position in long-term bonds and adjustable-rate mortgages as rolling over a short position in short-term bonds implies that fixed-rate mortgage holders are paying a nominal bond risk premium. The higher the bond risk premium, the more expensive the FRM, and the higher the ARM share. Our results are consistent across three different methods of computing bond risk premia. We used forecasters' expectations, a VAR-model, and a simple adaptive expectation scheme, or "rule-of-thumb". This last measure explains 70% of the variation in the ARM share. Other, perhaps more straightforward, term structure variables such as the slope of the yield curve, have much lower explanatory power for the ARM share.

For all three measures of the bond risk premium, a one standard deviation increase leads to an eight percentage point increase in the ARM share. Studying these different risk premium measures also reveals interesting differences. In the last ten years of our sample, only the rule-of-thumb continues to predict the ARM share. We track the poorer performance of the forecasters-based measure down to large forecast errors in future short rates. We show that these forecast errors are not present in the inflation risk premium component of the bond risk premium. We use real yield data and inflation forecasts to construct the inflation risk premium and show that it has strong predictive power for the ARM share. This exercise lends further credibility to a theory of strategic mortgage timing by households.

In a previous version of the paper, we have also studied the UK. Fixed rate mortgages are a lot less prevalent in the UK than in the US, and only a recent addition to the market. So, while the maturity choice may be somewhat less relevant, we still found a similar positive covariation between the ARM share and the bond risk premium. This implies that the link that we document between bond risk premia and aggregate mortgage choice is not typical for the US mortgage market only.

Taken together, our findings suggest that households may be making close-to-optimal mortgage choice decisions. Capturing the relevant time variation in bond risk premia is feasible by using a simple rule-of-thumb. This paper contributes to the growing household finance literature (Campbell (2006)), which debates the extent to which households make rational investment decisions. Given the importance of the house in the median household's portfolio and the prevalence of mortgages to finance the house, the problem of mortgage origination deserves a prominent place in this debate.

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# A Risk-Return Tradeoff

This appendix computes the expected utility from time-1 and time-2 consumption for each of the contracts. We first compute the utility without log transformation, and only at the end, when comparing the two mortgage contracts, reintroduce this log transformation.

**Utility from time-1 consumption** The (exponent of) utility from time-1 consumption on the FRM contract is:

$$\begin{aligned}\mathbb{E}_0\left(e^{-\beta-\gamma\frac{C_1}{\Pi_1}}\right)=\mathbb{E}_0\left(e^{-\beta-\gamma\left[\frac{L_1^{\$}-q_0^{FRM}B}{\Pi_1}\right]}\right) &= \mathbb{E}_0\left(e^{-\beta-\gamma\left[L_1-\frac{q_0^{FRM}B}{\Pi_1}\right]}\right)\\ &= e^{-\beta-\gamma\left(\mathbb{E}_0(L_1)-\frac{\gamma\sigma_L^2}{2}-\frac{q_0^{FRM}B}{\Pi_1}\right)}.\end{aligned}$$

For the ARM contract it is:

$$\mathbb{E}_0\left(e^{-\beta-\gamma\frac{C_1}{\Pi_1}}\right)=\mathbb{E}_0\left(e^{-\beta-\gamma\left[\frac{L_1^{\$}-q_0^{ARM}B}{\Pi_1}\right]}\right)=e^{-\beta-\gamma\left(\mathbb{E}_0(L_1)-\frac{\gamma\sigma_L^2}{2}-\frac{q_0^{ARM}B}{\Pi_1}\right)}.$$

**Utility from time-2 consumption** Under the FRM, the time-1 value of the time-2 utility equals:

$$\mathbb{E}_1\left[e^{-2\beta-\gamma\frac{C_2}{\Pi_2}}\right]=e^{-2\beta-\gamma\left(H_2+\mathbb{E}_1[L_2]-\frac{\gamma\sigma_L^2}{2}-\frac{\left(q_0^{FRM}+1\right)B}{\Pi_2}\right)},$$

using the same argument as in the period-1 utility calculations.

Next, we calculate the time-0 utility of this time-2 utility:

$$\begin{aligned}\mathbb{E}_0\left[e^{-2\beta-\gamma\frac{C_2}{\Pi_2}}\right] &= \mathbb{E}_0\left[e^{-2\beta-\gamma\left(H_2+\mathbb{E}_0[L_2]+\rho_L\sigma_L\varepsilon_1^L-\frac{\gamma\sigma_L^2}{2}-\left(q_0^{FRM}+1\right)Be^{-x_0-x_1}\right)}\right]\\ &\simeq \mathbb{E}_0\left[e^{-2\beta-\gamma\left(H_2+\mathbb{E}_0[L_2]+\rho_L\sigma_L\varepsilon_1^L-\frac{\gamma\sigma_L^2}{2}-\left(q_0^{FRM}+1\right)Be^{-x_0-\mathbb{E}_0[x_1]}(1-(x_1-\mathbb{E}_0[x_1]))\right)}\right]\\ &= \mathbb{E}_0\left[e^{-2\beta-\gamma\left(H_2+\mathbb{E}_0[L_2]+\rho_L\sigma_L\varepsilon_1^L-\frac{\gamma\sigma_L^2}{2}-\left(q_0^{FRM}+1\right)Be^{-x_0-\mathbb{E}_0[x_1]}(1-\sigma_x\varepsilon_1^x)\right)}\right]\\ &= e^{-2\beta-\gamma\left(H_2+\mathbb{E}_0[L_2]-\left(q_0^{FRM}+1\right)Be^{-x_0-\mathbb{E}_0[x_1]}\right)+\frac{\gamma^2}{2}\left(\left(1+\rho_L^2\right)\sigma_L^2+\left(q_0^{FRM}+1\right)^2B^2e^{-2x_0-2\mathbb{E}_0[x_1]}\sigma_x^2\right)}.\end{aligned}$$

In these steps, we used:

$$\begin{aligned}\Pi_2 &= \Pi_1e^{x_1},\quad \Pi_1=e^{x_0},\\ \mathbb{E}_1(L_2) &= \mu_L+\rho_L(L_1-\mu_L)=\mu_L+\rho_L^2(L_0-\mu_L)+\rho_L\sigma_L\varepsilon_1^L=\mathbb{E}_0(L_2)+\rho_L\sigma_L\varepsilon_1^L,\\ e^{-x_1} &\simeq e^{-\mathbb{E}_0(x_1)}-e^{-\mathbb{E}_0(x_1)}\left[x_1-\mathbb{E}_0(x_1)\right].\end{aligned}$$

For the ARM contract, the time-1 value of the time-2 utility equals:

$$\mathbb{E}_1\left[e^{-2\beta-\gamma\frac{C_2}{\Pi_2}}\right]=e^{-2\beta-\gamma\left(H_2+\mathbb{E}_1(L_2)-\frac{\gamma\sigma_L^2}{2}-\frac{\left(1+q_1^{ARM}\right)B}{\Pi_2}\right)}.$$

Then for the time-0 value function, it holds:

$$\begin{aligned}
\mathbb{E}_0\left[e^{-2\beta-\gamma\frac{C_2}{\Pi_2}}\right] &\simeq \mathbb{E}_0\left[e^{-2\beta-\gamma\left(H_2+\mathbb{E}_0[L_2]+\rho_L\sigma_L\varepsilon_1^L-\frac{\gamma\sigma_L^2}{2}-\left(1+q_1^{ARM}\right)Be^{-x_0-\mathbb{E}_0[x_1]}(1-\sigma_x\varepsilon_1^x)\right)}\right] \\
&= \mathbb{E}_0\left[e^{-2\beta-\gamma\left(H_2+\mathbb{E}_0[L_2]+\rho_L\sigma_L\varepsilon_1^L-\frac{\gamma\sigma_L^2}{2}-B\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1+q_1^{ARM}-\mathbb{E}_0\left[q_1^{ARM}\right]\right)e^{-x_0-\mathbb{E}_0[x_1]}(1-\sigma_x\varepsilon_1^x)\right)}\right] \\
&\simeq \mathbb{E}_0\left[e^{-2\beta-\gamma\left(H_2+\mathbb{E}_0[L_2]+\rho_L\sigma_L\varepsilon_1^L-\frac{\gamma\sigma_L^2}{2}-B\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1+\sigma'\varepsilon_1\right)e^{-x_0-\mathbb{E}_0[x_1]}(1-\sigma_x\varepsilon_1^x)\right)}\right] \\
&= e^{-2\beta-\gamma\left(H_2+\mathbb{E}_0[L_2]-B\left(\mathbb{E}_0\left(q_1^{ARM}\right)+1\right)e^{-x_0-\mathbb{E}_0(x_1)}\right)+\frac{\gamma^2}{2}\left[\left(1+\rho_L^2\right)\sigma_L^2\right]}\times \\
&\quad \mathbb{E}_0\left[e^{-\gamma B\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)e^{-x_0-\mathbb{E}_0[x_1]}\sigma_x\varepsilon_1^x+\gamma Be^{-x_0-\mathbb{E}_0[x_1]}\sigma'\varepsilon_1-\gamma Be^{-x_0-\mathbb{E}_0[x_1]}\left(\sigma'\varepsilon_1\right)\sigma_x\varepsilon_1^x}\right] \\
&\simeq e^{-2\beta-\gamma\left(H_2+\mathbb{E}_0[L_2]-B\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)e^{-x_0-\mathbb{E}_0[x_1]}\right)}\times \\
&\quad e^{\frac{\gamma^2}{2}\left[\left(1+\rho_L^2\right)\sigma_L^2+B^2\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)^2e^{-2x_0-2\mathbb{E}_0[x_1]}\sigma_x^2+B^2e^{-2x_0-2\mathbb{E}_0[x_1]}\sigma'R\sigma-2B^2\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)e^{-2x_0-2\mathbb{E}_0[x_1]}\left(\sigma_xe_2'R\sigma\right)\right)}.
\end{aligned}$$

The last approximation assumes that $\gamma e^{-x_0-\mathbb{E}_0(x_1)}\left(\sigma'\varepsilon_1\right)\sigma_x\varepsilon_1^x$ is zero (a shock times a shock). $(\sigma_xe_2'R\sigma)$ is the covariance of $x$ and $y^{\$}$, where we defined $e_2=[0,1]'$. In the third line of the approximation, we use $q_1^{ARM}\simeq y_1^{\$}(1)$.

Now we reintroduce the log transformation to the exponential preferences. Households prefer the ARM if and only if the life-time utility of the ARM contract exceeds that of the FRM contract:

$$\begin{aligned}
&\beta+\gamma\left(\mathbb{E}_0\left(L_1\right)-\frac{\gamma\sigma_L^2}{2}-\frac{q_0^{ARM}B}{\Pi_1}\right) \\
&+2\beta+\gamma\left(H_2+\mathbb{E}_0\left[L_2\right]-B\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)e^{-x_0-\mathbb{E}_0[x_1]}\right) \\
&-\frac{\gamma^2}{2}\left[\begin{array}{l}\left(1+\rho_L^2\right)\sigma_L^2+B^2\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)^2e^{-2x_0-2\mathbb{E}_0[x_1]}\sigma_x^2 \\ +B^2e^{-2x_0-2\mathbb{E}_0[x_1]}\sigma'R\sigma-2B^2\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)e^{-2x_0-2\mathbb{E}_0[x_1]}\left(\sigma_xe_2'R\sigma\right)\end{array}\right) \\
>& \\
&\beta+\gamma\left(\mathbb{E}_0\left(L_1\right)-\frac{\gamma\sigma_L^2}{2}-\frac{q_0^{FRM}B}{\Pi_1}\right) \\
&+2\beta+\gamma\left(H_2+\mathbb{E}_0\left[L_2\right]-\left(q_0^{FRM}+1\right)Be^{-x_0-\mathbb{E}_0[x_1]}\right)-\frac{\gamma^2}{2}\left(\left(1+\rho_L^2\right)\sigma_L^2+\left(q_0^{FRM}+1\right)^2B^2e^{-2x_0-2\mathbb{E}_0[x_1]}\sigma_x^2\right).
\end{aligned}$$

This simplifies to:

$$\begin{aligned}
&q_0^{FRM}-q_0^{ARM}+\left(q_0^{FRM}-\mathbb{E}_0\left[q_1^{ARM}\right]\right)e^{-\mathbb{E}_0[x_1]} \\
>& \\
&\frac{\gamma}{2}Be^{-x_0-2\mathbb{E}_0[x_1]}\left[\sigma'R\sigma+\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)^2\sigma_x^2-2\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)\left(\sigma_xe_2'R\sigma\right)\right] \\
&-\frac{\gamma}{2}Be^{-x_0-2\mathbb{E}_0[x_1]}\left(q_0^{FRM}+1\right)^2\sigma_x^2.
\end{aligned}$$

**Simplifying Expressions** The first term on the right-hand side of the inequality, i.e., the risk induced by the ARM contract, can be rewritten as:

$$\begin{aligned}
&\frac{\gamma}{2}Be^{-x_0-2\mathbb{E}_0[x_1]}\left[\sigma'R\sigma+\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)^2\sigma_x^2-2\left(\mathbb{E}_0\left[q_1^{ARM}\right]+1\right)\left(\sigma_xe_2'R\sigma\right)\right] \\
=& \frac{\gamma}{2}Be^{-x_0-2\mathbb{E}_0[x_1]}\left[\sigma_y^2-2\sigma_x\sigma_y\rho_{xy}\mathbb{E}_0\left[q_1^{ARM}\right]+\mathbb{E}_0\left[q_1^{ARM}\right]^2\sigma_x^2\right] \\
\simeq& \frac{\gamma}{2}Be^{-x_0-2\mathbb{E}_0[x_1]}\sigma_y^2,
\end{aligned}$$

in which we use that $2\sigma_x\sigma_y\rho_{xy}\mathbb{E}_0\left[q_1^{ARM}\right]$ and $\mathbb{E}_0\left[q_1^{ARM}\right]^2\sigma_x^2$ are an order of magnitude smaller than $\sigma_y^2$, which motivates the approximation in the third line. This in turn implies that the ARM contract primarily carries real rate risk, while, in contrast, the FRM contract carries only inflation risk. This is the risk-return trade-off discussed in the main text.

Ignoring the $e^{-\mathbb{E}_0[x_1]}$ inflation term, the left-hand side of above inequality is the difference in expected nominal payments per dollar mortgage balance. We have:

$$2q_0^{FRM} - q_0^{ARM} - \mathbb{E}_0\left(q_1^{ARM}\right) \simeq 2y_0^{\$}(2) - y_0^{\$}(1) - \mathbb{E}_0\left[y_1^{\$}(1)\right] = 2\phi_0^{\$}(2)$$

where we use the approximations of Section 1.3.

# B VAR with Heteroscedasticity

We now extend the VAR model to allow for heteroscedastic innovations. In particular, we allow for time-varying volatility in the real interest rate ($y$) and expected inflation ($x$). Long-term expectations are unaffected by the switch from homoscedastic to heteroscedastic model, so that the term structure dynamics presented before remain identical.

We first estimate the innovations $(\hat{\eta}_t, t = 1, \ldots, T)$ from the VAR model and construct the implied innovations to the real rate and expected inflation according to (20) and (21),

$$\eta_{t+12}^x = x_{t+12}(12) - \mathbb{E}_t\left[x_{t+12}(12)\right] = e_4'\Gamma\eta_{t+12}, \tag{20}$$
$$\eta_{t+12}^y = y_{t+12}(12) - \mathbb{E}_t\left[y_{t+12}(12)\right] = \left(e_1' - e_4'\Gamma\right)\eta_{t+12}. \tag{21}$$

Next, we model both conditional variances as an exponentially affine function in their own level

$$V_t^x \equiv \mathrm{Var}_t\left[x_{t+12}(12)\right] = \mathrm{Var}_t\left[\eta_{t+12}^x\right] = \exp(\alpha_x + \beta_x x_t(12)), \tag{22}$$
$$V_t^y \equiv \mathrm{Var}_t\left[y_{t+12}(12)\right] = \mathrm{Var}_t\left[\eta_{t+12}^y\right] = \exp(\alpha_y + \beta_y y_t(12)). \tag{23}$$

The coefficients $\alpha_i$ and $\beta_i$, $i = x, y$, are estimated consistently via non-linear least squares

$$(\hat{\alpha}_i, \hat{\beta}_i) = \arg\min_{\alpha_i,\beta_i} \frac{1}{T}\sum_{t=1}^{T}\left(\left[\hat{\eta}_{t+12}^i\right]^2 - \exp(\alpha_i + \beta_i i_t(12))\right)^2.$$

The estimation delivers two time-series for 1-year ahead conditional variances for 1985-2006.6. Conditional real rate volatility is 1.06% per year on average, while expected inflation volatility is three times lower at 0.35% per year on average. There is some time variation in these one-year ahead conditional volatilities. The two conditional volatilities co-move strongly negatively; their correlation is -0.71.

# C Derivation of the Prepayment Option Formula

The value of the prepayment option is given by:

$$\begin{aligned}
B\mathbb{E}_0\left[M_1^{\$}M_2^{\$}\max\left\{\left(q_0^{FRMP}-q_1^{ARM}\right),0\right\}\right] &= B\mathbb{E}_0\left[\mathbb{E}_1\left[M_1^{\$}M_2^{\$}\max\left\{\left(q_0^{FRMP}-q_1^{ARM}\right),0\right\}\right]\right] \\
&= B\mathbb{E}_0\left[M_1^{\$}\max\left\{\left(q_0^{FRMP}-q_1^{ARM}\right)P_1^{\$}(1),0\right\}\right] \\
&= B\mathbb{E}_0\left[M_1^{\$}\max\left\{\left(1+q_0^{FRMP}-P_1^{\$}(1)^{-1}\right)P_1^{\$}(1),0\right\}\right] \\
&= B\mathbb{E}_0\left[M_1^{\$}\max\left\{\left(\left(1+q_0^{FRMP}\right)P_1^{\$}(1)-1\right),0\right\}\right] \\
&= B\left(1+q_0^{FRMP}\right)\mathbb{E}_0\left[M_1^{\$}\max\left\{\left(P_1^{\$}(1)-\frac{1}{1+q_0^{FRMP}}\right),0\right\}\right]
\end{aligned}$$

where we use that $q_1^{ARM}=P_1^{\$}(1)^{-1}-1$. The pricing kernel and the one-year bond price at time $t=1$ are given by:

$$\begin{aligned}
M_1^{\$} &= e^{-y_0^{\$}(1)-\frac{1}{2}\lambda_0'R\lambda_0-\lambda_0'\varepsilon_1} \\
P_1^{\$}(1) &= e^{-y_1^{\$}(1)}=e^{-\mathbb{E}_0\left[y_1^{\$}(1)\right]-\sigma'\varepsilon_1}
\end{aligned}$$

We project the innovation to the pricing kernel on the innovation to the nominal short rate:

$$\begin{aligned}
\eta_1 &\equiv \sigma'\varepsilon_1 \\
\eta_2 &\equiv \lambda_0'\varepsilon_1-\frac{\mathrm{Cov}\left(\eta_1,\lambda_0'\varepsilon_1\right)}{\mathrm{Var}\left(\eta_1\right)}\eta_1=\lambda_0'\varepsilon_1-\frac{\sigma'R\lambda_0}{\sigma'R\sigma}\eta_1
\end{aligned}$$

with $\eta_1$ and $\eta_2$ orthogonal and variances given by:

$$\mathrm{Var}\left[\eta_1\right] = \sigma'R\sigma, \quad \mathrm{Var}\left[\eta_2\right]=\lambda_0'R\lambda_0-\frac{\left(\sigma'R\lambda_0\right)^2}{\sigma'R\sigma}$$

We first solve for the value of one call option for a general exercise price $K$, denoted by $C_0(K)$:

$$\begin{aligned}
C_0(K) &= \mathbb{E}_0\left[M_1^{\$}\max\left\{\left(P_1^{\$}(1)-K\right),0\right\}\right] \\
&= \mathbb{E}_0\left[e^{-y_0^{\$}(1)-\frac{1}{2}\lambda_0'R\lambda_0-\frac{\sigma'R\lambda_0}{\sigma'R\sigma}\eta_1-\eta_2}\max\left\{\left(e^{-\mathbb{E}_0\left[y_1^{\$}(1)\right]-\eta_1}-K\right),0\right\}\right] \\
&= \mathbb{E}_0\left[e^{-y_0^{\$}(1)-\frac{1}{2}\lambda_0'R\lambda_0-\frac{\sigma'R\lambda_0}{\sigma'R\sigma}\eta_1}\max\left\{\left(e^{-\mathbb{E}_0\left[y_1^{\$}(1)\right]-\eta_1}-K\right),0\right\}\right]e^{\frac{1}{2}\left(\lambda_0'R\lambda_0-\frac{\left(\sigma'R\lambda_0\right)^2}{\sigma'R\sigma}\right)}
\end{aligned}$$

The option will be exercised if and only if the following holds

$$\eta_1<-\log(K)-\mathbb{E}_0\left[y_1^{\$}(1)\right],$$

which occurs with probability

$$\Phi\left(\frac{-\log(K)-\mathbb{E}_0\left[y_1^{\$}(1)\right]}{\sqrt{\sigma'R\sigma}}\right)\equiv\Phi\left(x^{\star}\right).$$

We proceed:

$$\begin{aligned}
C_0(K) &= e^{\frac{1}{2}\left(\lambda_0' R\lambda_0 - \frac{(\sigma' R\lambda_0)^2}{\sigma' R\sigma}\right)} \mathbb{E}_0\left[e^{-y_0^{\$}(1)-\frac{1}{2}\lambda_0' R\lambda_0 - \frac{\sigma' R\lambda_0}{\sigma' R\sigma}\eta_1}\left(e^{-\mathbb{E}_0\left[y_1^{\$}(1)\right]-\eta_1} - K\right) I_{\left(\eta_1/\sqrt{\sigma' R\sigma} < x^\star\right)}\right] \\
&= e^{\frac{1}{2}\left(\lambda_0' R\lambda_0 - \frac{(\sigma' R\lambda_0)^2}{\sigma' R\sigma}\right)} \int_{-\infty}^{x^\star} e^{-y_0^{\$}(1)-\mathbb{E}_0\left[y_1^{\$}(1)\right]-\frac{1}{2}\lambda_0' R\lambda_0 - \frac{\sigma' R\lambda_0}{\sigma' R\sigma}\sqrt{\sigma' R\sigma}x - \sqrt{\sigma' R\sigma}x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\mathrm{d}x \\
&\quad -e^{\frac{1}{2}\left(\lambda_0' R\lambda_0 - \frac{(\sigma' R\lambda_0)^2}{\sigma' R\sigma}\right)} \int_{-\infty}^{x^\star} K e^{-y_0^{\$}(1)-\frac{1}{2}\lambda_0' R\lambda_0 - \frac{\sigma' R\lambda_0}{\sigma' R\sigma}\sqrt{\sigma' R\sigma}x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\mathrm{d}x,
\end{aligned}$$

where we use that $\eta_1/\sqrt{\sigma' R\sigma}$ is standard normally distributed. Rewriting and using that:

$$-2y_0^{\$}(2) = -y_0^{\$}(1) - \mathbb{E}_0\left[y_1^{\$}(1)\right] + \frac{1}{2}\sigma' R\sigma + \sigma' R\lambda_0,$$

we obtain:

$$\begin{aligned}
C_0(K) &= P_0^{\$}(2)\int_{-\infty}^{x^\star} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(x + \frac{\sigma' R\lambda_0}{\sigma' R\sigma}\sqrt{\sigma' R\sigma} + \sqrt{\sigma' R\sigma}\right)^2}\mathrm{d}x - KP_0^{\$}(1)\int_{-\infty}^{x^\star} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(x + \frac{\sigma' R\lambda_0}{\sigma' R\sigma}\sqrt{\sigma' R\sigma}\right)^2}\mathrm{d}x \\
&= P_0^{\$}(2)\,\Phi\left(x^\star + \frac{\sigma' R\lambda_0}{\sigma' R\sigma}\sqrt{\sigma' R\sigma} + \sqrt{\sigma' R\sigma}\right) - KP_0^{\$}(1)\,\Phi\left(x^\star + \frac{\sigma' R\lambda_0}{\sigma' R\sigma}\sqrt{\sigma' R\sigma}\right),
\end{aligned}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Using the definition of $x^\star$, we conclude that the option value is given by:

$$\begin{aligned}
C_0(K) &= P_0^{\$}(2)\,\Phi(d_1) - KP_0^{\$}(1)\,\Phi(d_2), \\
d_1 &\equiv \frac{-\log(K) - \mathbb{E}_0\left[y_1^{\$}(1)\right] + \sigma' R\sigma + \sigma' R\lambda_0}{\sqrt{\sigma' R\sigma}}, \\
&= \frac{\log\left(P_0^{\$}(2)/K\right) + y_0^{\$}(1) + \frac{1}{2}\sigma' R\sigma}{\sqrt{\sigma' R\sigma}}, \\
d_2 &\equiv d_1 - \sqrt{\sigma' R\sigma},
\end{aligned}$$

where the second line for $d_1$ uses the pricing formula of a two-period bond. Now using $K = 1/\left(1 + q_0^{FRMP}\right)$ and the fact that the investor has $B\left(1 + q_0^{FRMP}\right)$ of these options, yields the value of the prepayment option:

$$B\mathbb{E}_0\left[M_1^{\$} M_2^{\$} \max\left\{\left(q_0^{FRMP} - q_1^{ARM}\right), 0\right\}\right] = B\left(1 + q_0^{FRMP}\right) C_0\left(1/\left(1 + q_0^{FRMP}\right)\right). \tag{24}$$

## Table 1: The ARM Share and the Nominal Bond Risk Premium

This table reports slope coefficients, Newey-West t-statistics (12 lags), and $R^2$ statistics for regressions of the ARM share on a constant and the regressors reported in the first column. The regressors are the $\tau$-year nominal bond risk premium $\phi_t^{\$}(\tau)$, measured in three different ways. We consider $\tau = 5$ and $\tau = 10$ years. The first measure is based on Blue Chip forecast data (rows 1 and 2), the second measure is based on the VAR (rows 3-6), the third measure is based on the rule-of-thumb (rows 7-8) with a 3-year look-back period. For the VAR, we also show multiple regressions with the nominal bond risk premium, the conditional variance in the real rate, $V_t^y$, and the conditional variance of inflation, $V_t^x$, on the right-hand side (rows 5-6). Rows 9 and 10 show regressions of the ARM share on the $\tau$-one-year yield spread $y_t^{\$}(\tau) - y_t^{\$}(12)$. Rows 11 and 12 use the $\tau$-year nominal yield, $y_t^{\$}(\tau)$, as predictor. Row 13 uses the difference between the effective 30-year FRM rate $y_t^{\$}(FRM)$ and the effective ARM rate $y_t^{\$}(ARM)$, while row 15 uses $y_t^{\$}(FRM)$ as independent variable. Row 14 uses the component of the FRM-ARM spread that is orthogonal to the 10-1 Treasury bond spread. Rows 16 and 17 consider two other rules-of-thumb. Rule I takes the current FRM rate minus the three-year moving average of the FRM rate (row 16). Rule II does the same, but for the ARM rate (row 17). In all rows, the regressor is lagged by one period, relative to the ARM share. All independent variables have been normalized by their standard deviation. The sample is 1985.1-2006.6, except for rows 7 and 8, where we use 1989.12-2006.6.

| | | | slope | t-stat | slope | t-stat | slope | t-stat | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1. | Blue Chip | $\phi_t^{\$}(5)$ | 8.63 | 3.91 | | | | | 40.25 |
| 2. | | $\phi_t^{\$}(10)$ | 8.89 | 4.22 | | | | | 42.62 |
| 3. | VAR | $\phi_t^{\$}(5)$ | 7.73 | 4.16 | | | | | 32.21 |
| 4. | | $\phi_t^{\$}(10)$ | 8.07 | 3.91 | | | | | 35.13 |
| 5. | | $\phi_t^{\$}(5)$,$V_t^y$,$V_t^x$ | 5.40 | 2.72 | -1.13 | -0.45 | 3.45 | 1.16 | 39.41 |
| 6. | | $\phi_t^{\$}(10)$,$V_t^y$,$V_t^x$ | 5.72 | 2.77 | -1.65 | -0.69 | 2.54 | 0.92 | 40.36 |
| 7. | Rule-of-thumb | $\phi_t^{\$}(5)$ | 7.88 | 7.08 | | | | | 71.23 |
| 8. | | $\phi_t^{\$}(10)$ | 7.70 | 7.47 | | | | | 68.03 |
| 9. | Slope | $y_t^{\$}(5) - y_t^{\$}(1)$ | 0.46 | 0.21 | | | | | 0.11 |
| 10. | | $y_t^{\$}(10) - y_t^{\$}(1)$ | $-0.66$ | $-0.32$ | | | | | 0.23 |
| 11. | Long yield | $y_t^{\$}(5)$ | 8.37 | 3.76 | | | | | 37.76 |
| 12. | | $y_t^{\$}(10)$ | 8.53 | 3.85 | | | | | 39.26 |
| 13. | Mortgage rates | $y_t^{\$}(FRM) - y_t^{\$}(ARM)$ | 8.09 | 3.17 | | | | | 35.31 |
| 14. | | $y_t^{\$}(FRM) - y_t^{\$}(ARM)$ orth. | 8.75 | 3.86 | | | | | 41.28 |
| 15. | | $y_t^{\$}(FRM)$ | 7.81 | 3.71 | | | | | 32.87 |
| 16. | Other Rules-of-Thumb | Rule I | 6.00 | 3.74 | | | | | 22.54 |
| 17. | | Rule II | 3.13 | 2.42 | | | | | 6.12 |

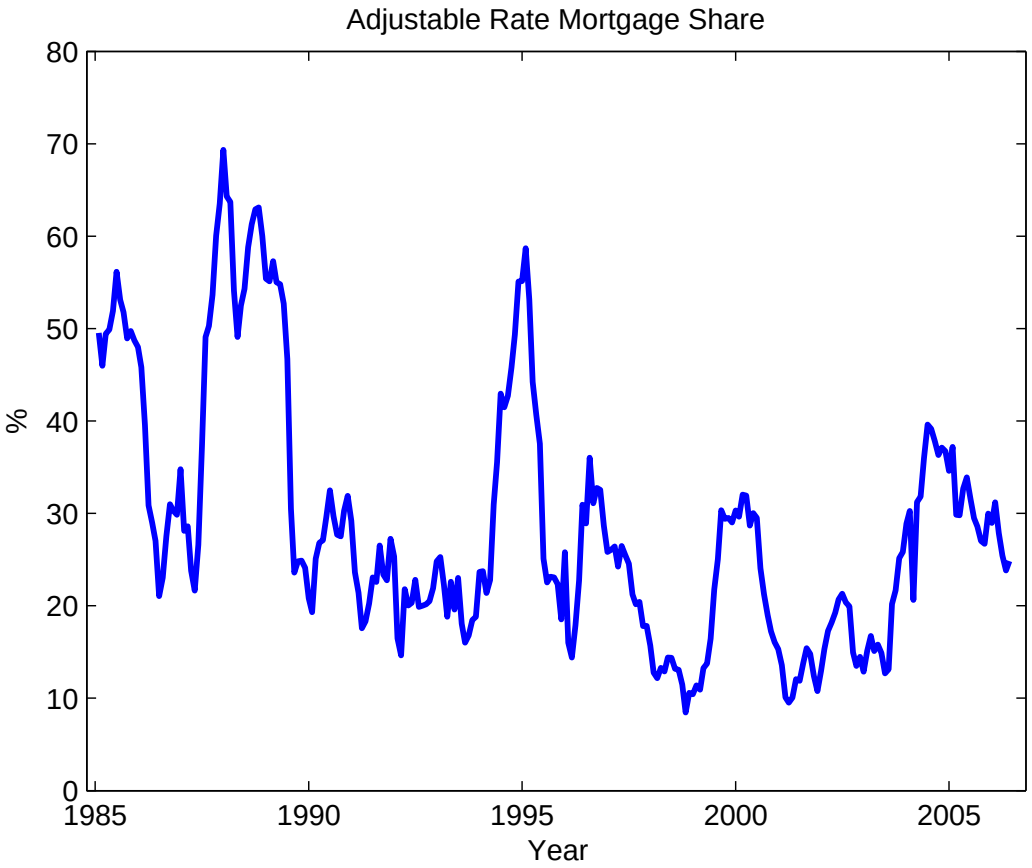
## Table 2: Probit Regressions of the ARM Share in Loan-Level Data

This table reports slope coefficients, robust t-statistics (in brackets), and $R^2$ statistics for probit regressions of an ARM dummy on a constant and one or more regressors, reported in the first column. The regressors are $\kappa$, the rule-of-thumb 5-year bond risk premium, the loan balance at origination (Bal), the loan's credit score at origination (FICO), the loan's loan-to-value ratio (LTV), the long-term interest rate (5-year Treasury yield), and the 5-1 year yield spread. The seventh column indicates when we include four regional dummies for the biggest mortgage markets (California, Florida, New York, and Texas). All independent variables have been normalized by their standard deviation. The sample consists of 654,368 mortgage loans originated between 1994-2006.6.

| $\kappa_t(36;5)$ | $y^{\$}(5)$ | $y^{\$}(5) - y^{\$}(1)$ | Bal | FICO | LTV | Regional dummies | % correctly classified |
|---|---|---|---|---|---|---|---|
| 0.43 | | | | | | No | 69.4 |
| [253] | | | | | | | |
| | | | -0.05 | -0.05 | 0.17 | Yes | 61.7 |
| | | | [21] | [28] | [100] | | |
| 0.42 | | | -0.01 | -0.08 | 0.13 | Yes | 68.8 |
| [244] | | | [4] | [45] | [72] | | |
| 0.54 | -0.47 | | -0.00 | -0.15 | 0.16 | Yes | 71.6 |
| [290] | [237] | | [1] | [71] | [80] | | |
| 0.65 | | 0.43 | -0.00 | -0.11 | 0.17 | Yes | 70.9 |
| [299] | | [206] | [2] | [58] | [90] | | |

Figure 1: The Share of Adjustable Rate Mortgages in the US.

The figure plots the fraction of all newly-originated mortgages that are of the adjustable-rate type between January 1985 and June 2006. The complementary fraction are fixed-rate mortgages. The data are from the Federal Housing Financing Board and are based on the Monthly Interest Rate Survey sent out to mortgage lenders. It covers purchase-money mortgages, but not refinancings. ARMs include hybrid mortgages that have an initial fixed-interest rate payment period.



## Figure 2: The Nominal Bond Risk Premium and the ARM Share

The figure plots the fraction of all mortgages that are of the adjustable-rate type against the left axis, and the nominal bond risk premium against the right axis. The bond risk premium is computed as the difference between the 5-year nominal bond yield and the average expected future nominal 1-year yield over the next five years. The forecast data are the consensus estimates of the average 3-month T-bill rate over the next year, one year from now, two, three, and four years from now. They are based on semi-annual data, interpolated to monthly frequency.

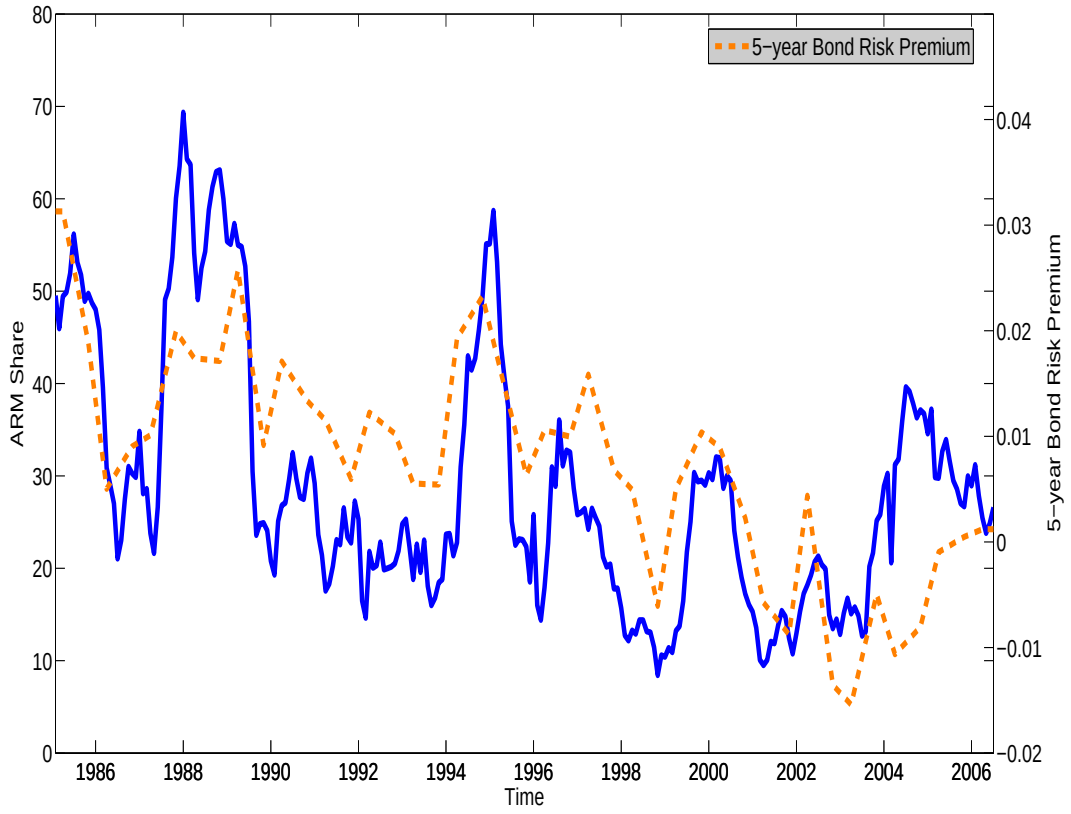
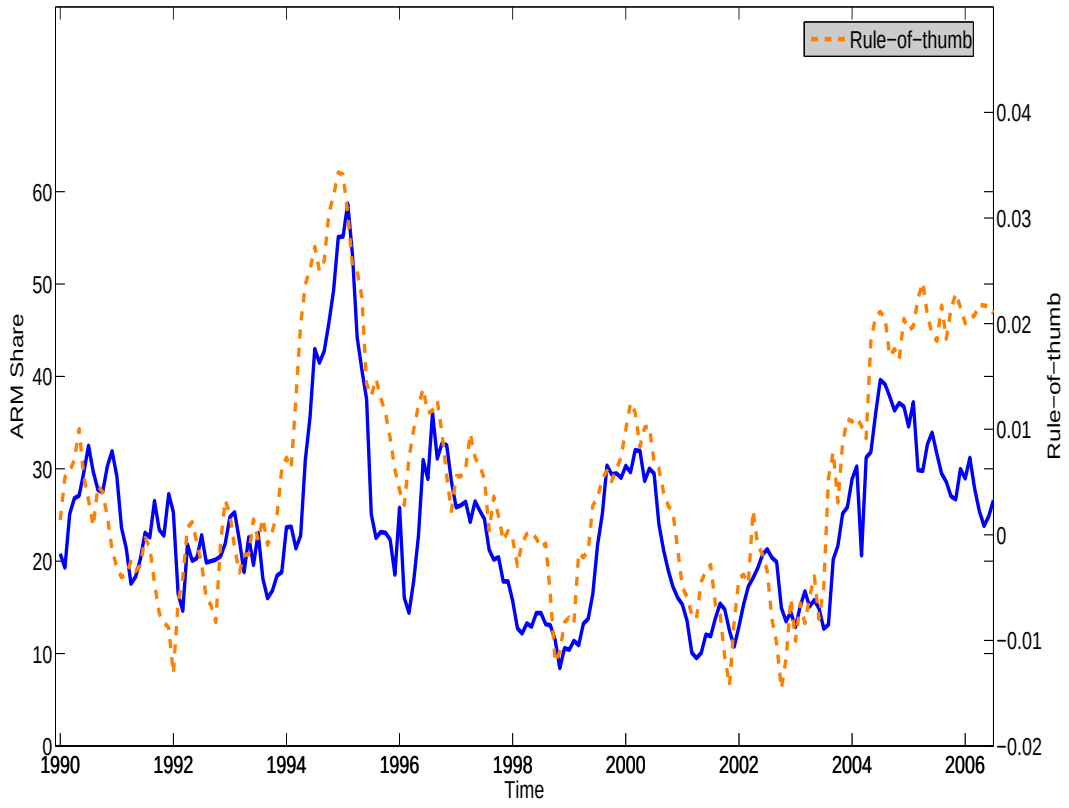


Figure 3: Rule-of-Thumb for the Bond Risk Premium and the ARM Share.

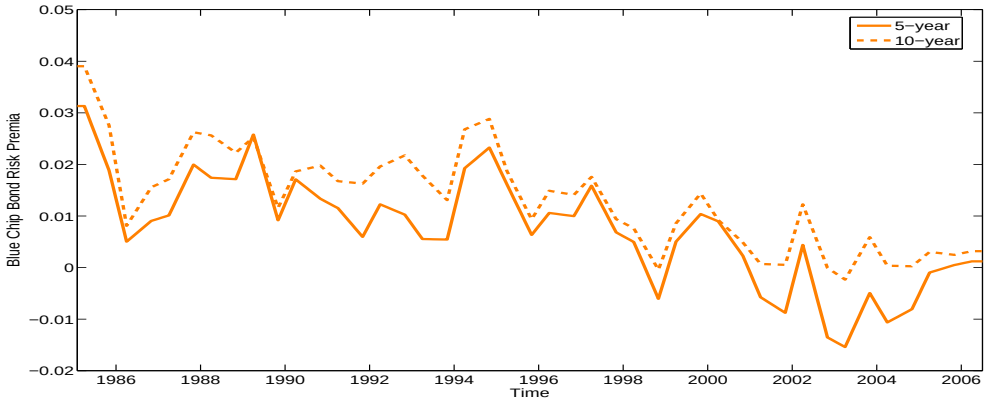
The solid line corresponds to the ARM share in the US, and its values are depicted on the left axis. The dashed line displays the time series of the bond risk premium that follows from the model in Section 2.4. It is computed as the difference between the 5-year yield and the 3-year moving average of the 1-year yield. The time series runs from 1989.12 to 2006.6.



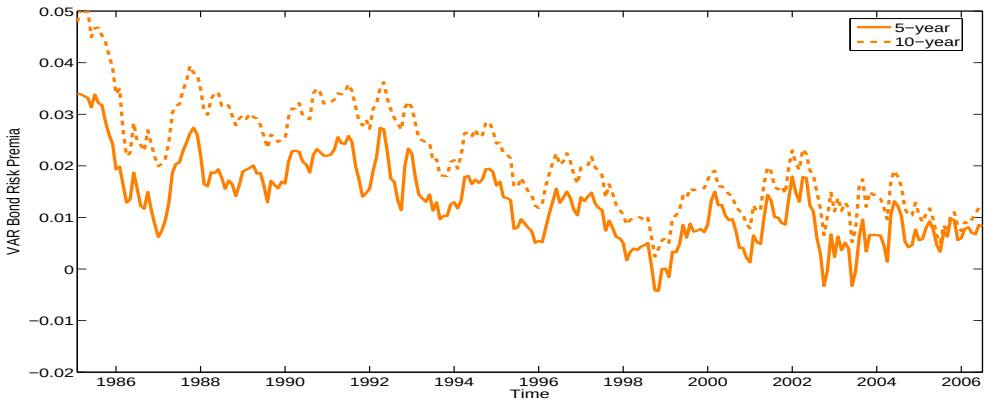
## Figure 4: Three Measures of the Nominal Bond Risk Premium

Each panel plots the 5-year and the 10-year nominal bond risk premium. The average expected future nominal short rates that go into this calculation differ in each panel. In the top panel we use Blue Chip forecasters data. In the middle panel we use forecasts formed from a VAR model. In the bottom panel we use adaptive expectations with a three-year look-back period.

Panel A: Using Blue Chip Data




Panel B: Using VAR Model




Panel C: Using Rule-of-Thumb

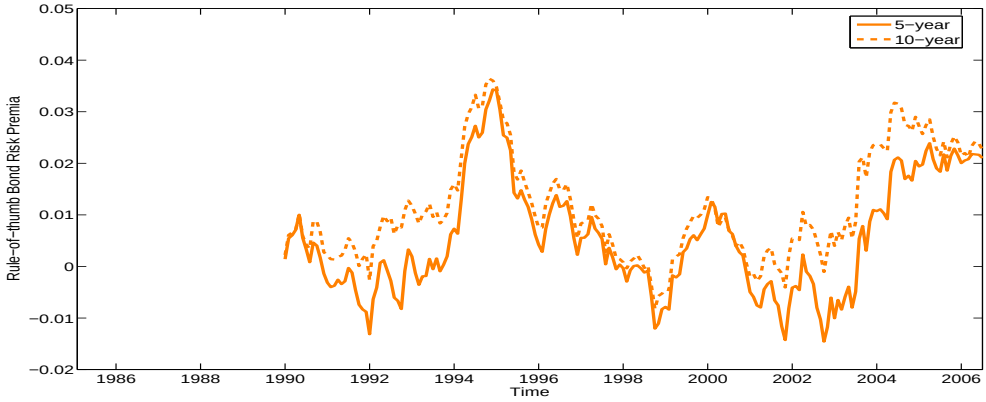
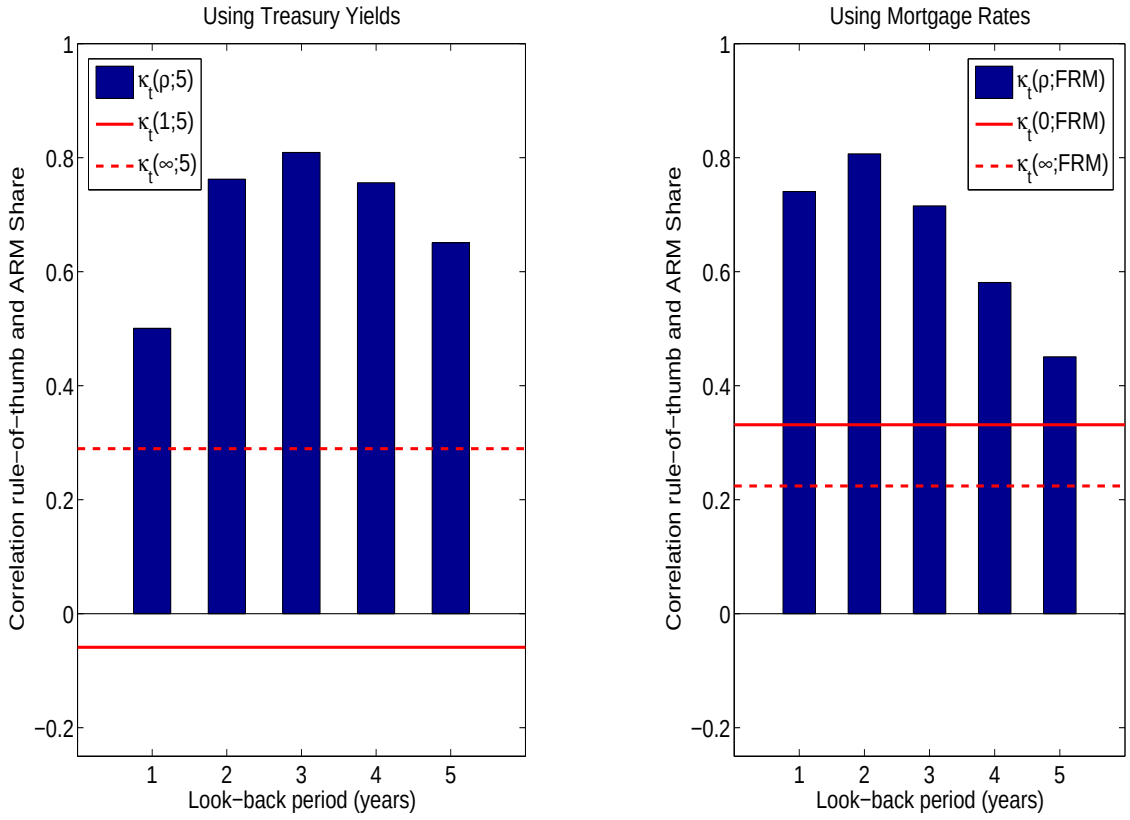


Figure 5: Correlation of Rule-of-Thumb and the ARM Share for Different Horizons $\rho$.

The figure plots the correlation of bond risk premia using the rule-of-thumb (red dashed line) with the ARM share. Bond risk premia are computed as the difference between the 10-year yield and the $\rho$-month moving average of short rates, i.e., $\kappa_t(\rho;5)$. The blue bars correspond to $\rho = 24$, 36, 48, 60. The red line corresponds to the correlation between the yield spread (i.e., $\rho = 1$) and the ARM share. The red dashed line depicts the correlation between the 10-year yield and the ARM share (i.e., $\rho = \infty$). The left panel uses Treasury yields as yield variable, while the right panel uses the effective ARM and effective 30-year FRM rates. The time series runs from 1989.12 to 2006.6.



## Figure 6: Rolling Window Correlations

The figure plots 10-year rolling window correlations of each of the three bond risk premium measures with the ARM share. The top line is for the rule-of-thumb measure, the middle line is for the measure based on Blue Chip forecasters data, and the bottom line is based on the VAR. The first window is based on the 1985-1995 data sample.

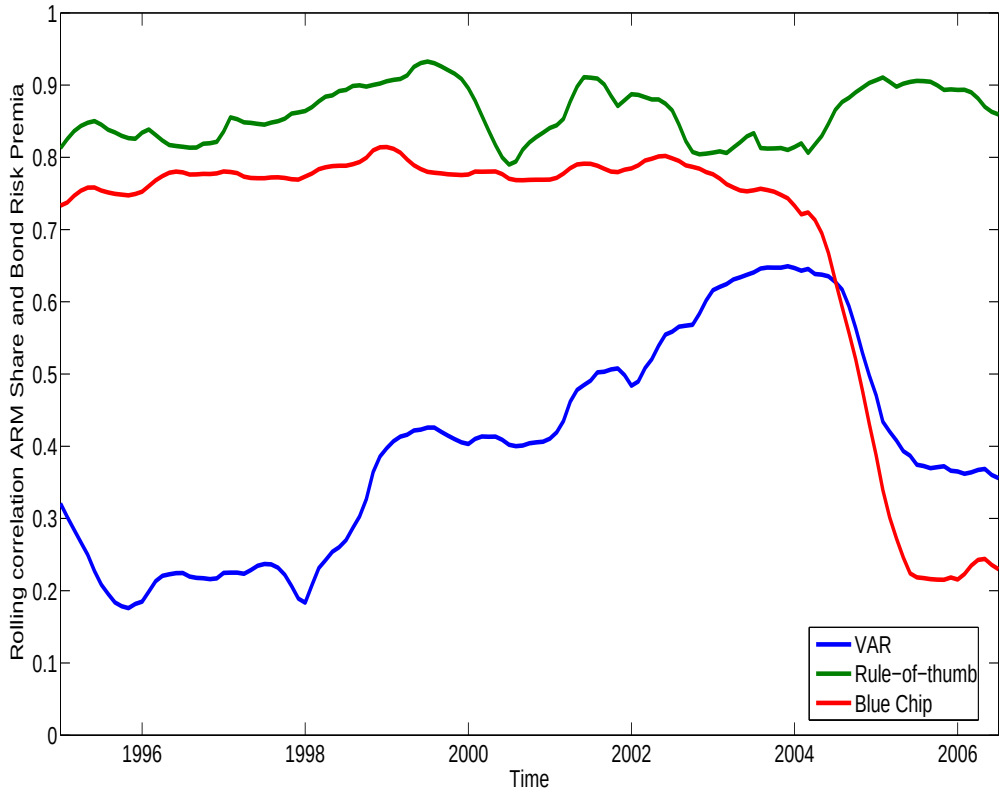
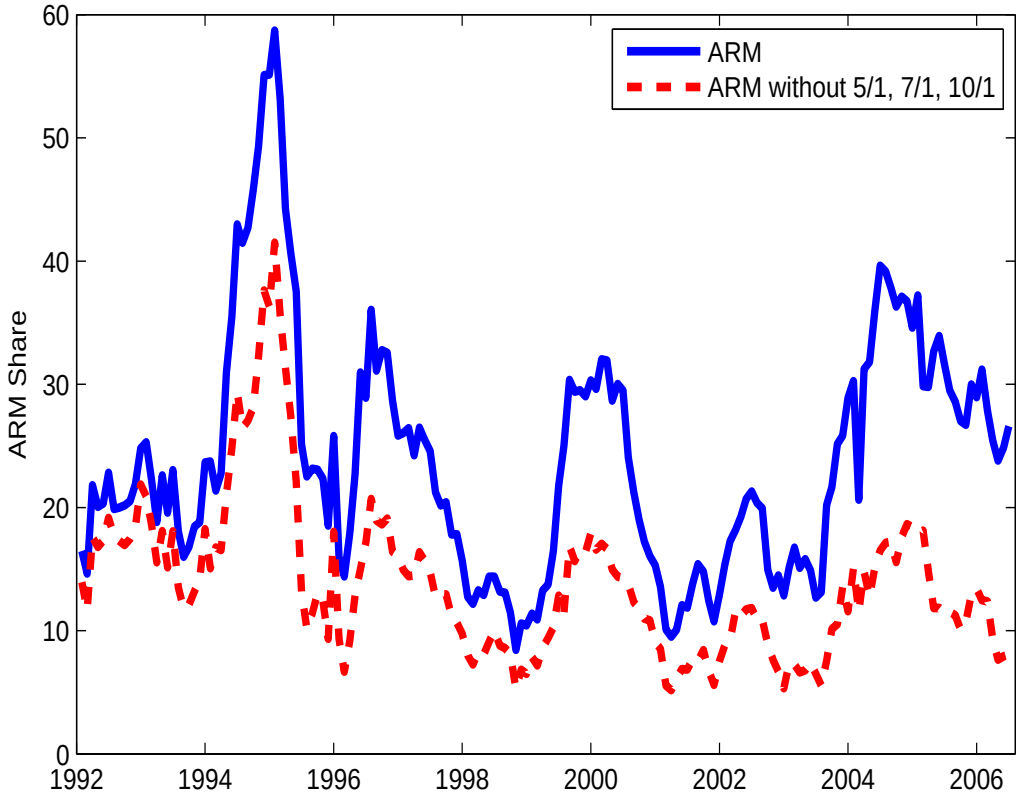


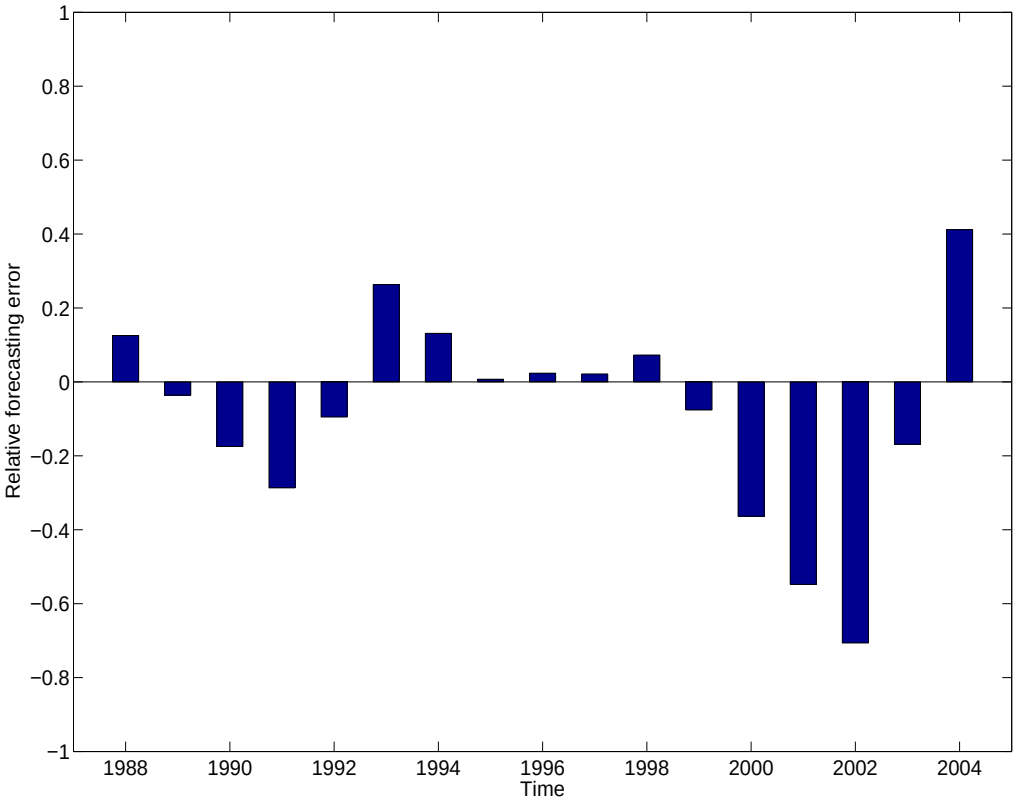
Figure 7: Product Innovation in the Mortgage Market

The solid line plots our benchmark ARM share, which includes all hybrid mortgage contracts, between 1992.1 and 2006.6. The dashed line excludes all hybrids with an initial fixed-rate period of more than three years. The data are from the Monthly Interest Rate Survey compiled by the Federal Housing Financing Board.



## Figure 8: Errors in Predicting Future Real Rates

The figure plots forecast errors in expected future real short rates. The forecast error is computed using Blue Chip forecast data. The average expected future real short rate is calculated as the difference between the Blue Chip consensus average expected future nominal short rate and the Blue Chip consensus average expected future inflation rate. The realized real rate is computed as the difference between the realized nominal rate and the realized expected inflation, which are measured as the one-quarter ahead inflation forecast. The realized average future real short rates are calculated from the realized real rates. The forecast errors are scaled by the nominal short rate to obtain relative forecasting errors. The forecast errors are based on two-year ahead forecasts.



## Figure 9: The Inflation Risk Premium and the ARM Share.

The figure plots the fraction of all mortgages that are of the adjustable-rate type against the left axis (solid line), and the inflation risk premium (dashed line) against the right axis. The inflation risk premium is computed as the difference between the 5-year nominal bond yield, the 5-year real bond yield and the expected inflation. The real 5-year bond yield data are from McCulloch and start in January 1997. The inflation expectation is the Blue Chip consensus average future inflation rate over the next 5 years.

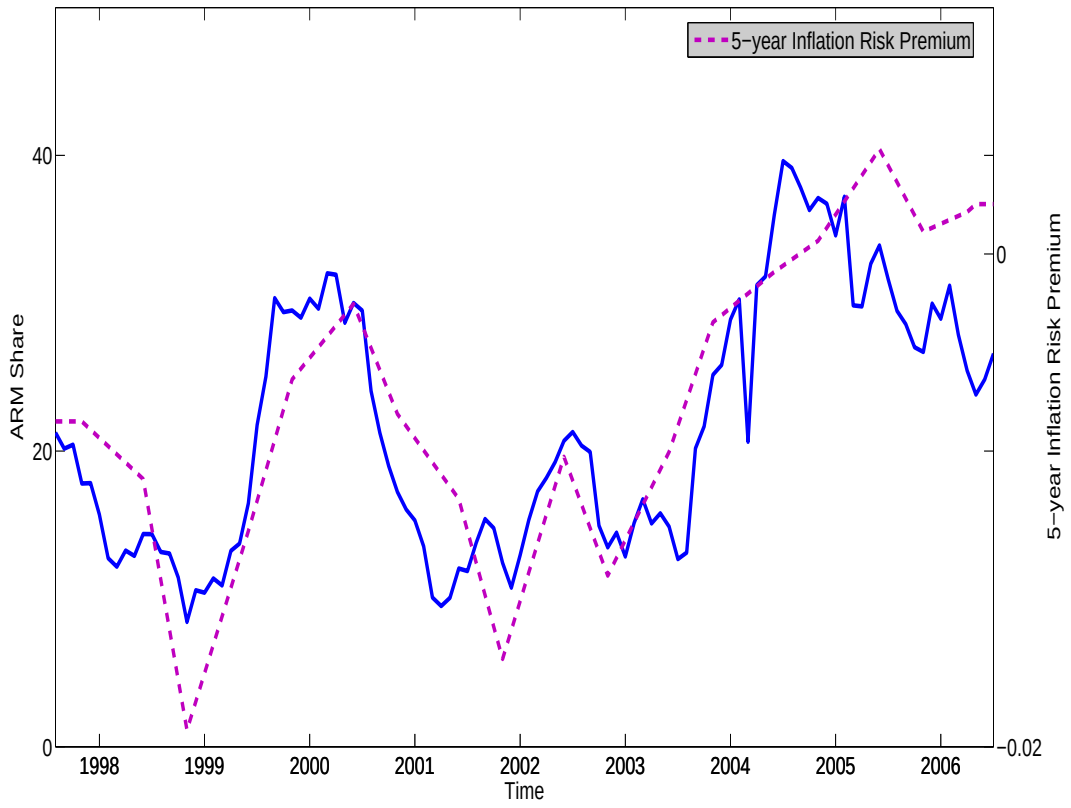


Figure 10: Price Sensitivity to Changes in the Nominal Interest Rate.

The figure plots the price sensitivities of the FRM contract with and without prepayment to the nominal interest rate, $y_0^{\$}(1)$. The mortgage values are determined within the model of Section 4.1. The analogous fixed-income securities are a regular bond (FRM without prepayment) and a callable bond (FRM with prepayment).

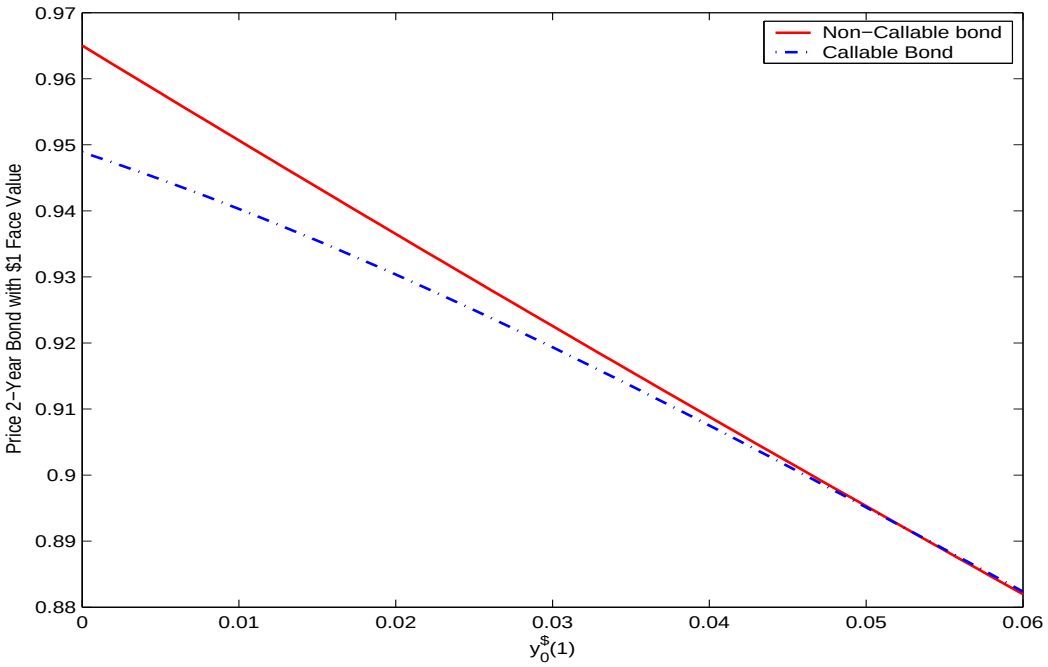
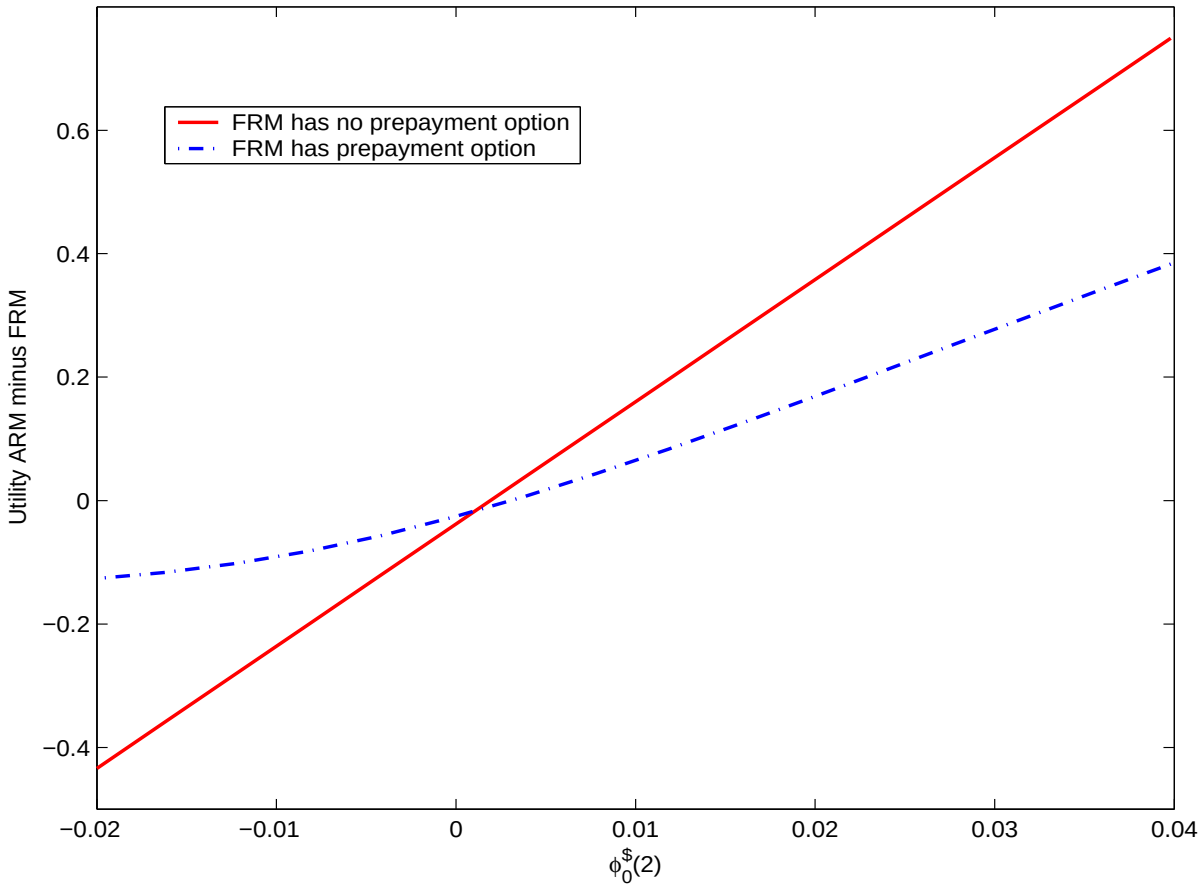


Figure 11: Utility Difference Between ARM and FRM - Prepayment

The figure plots the utility difference between an ARM contract and an FRM contract without prepayment as well as the utility difference between an ARM contract and an FRM contract with prepayment.



## Figure 12: Effect of the Rate of Time Preference

Each point in the figure corresponds to the $R^2$ of a regression of the certainty equivalent consumption difference between an ARM contract and an FRM contract on either the bond risk premium (solid line) or one the yield spread (dashed line). The annualized subjective discount factor $\beta$, on the horizontal axis, is varied between 0.5 and 1. The time-series are generated from a model, which is a multi-period extension of the model in Section 1. The coefficient of relative risk aversion is $\gamma = 4$. The exogenous moving probability is held constant at 1% per month.

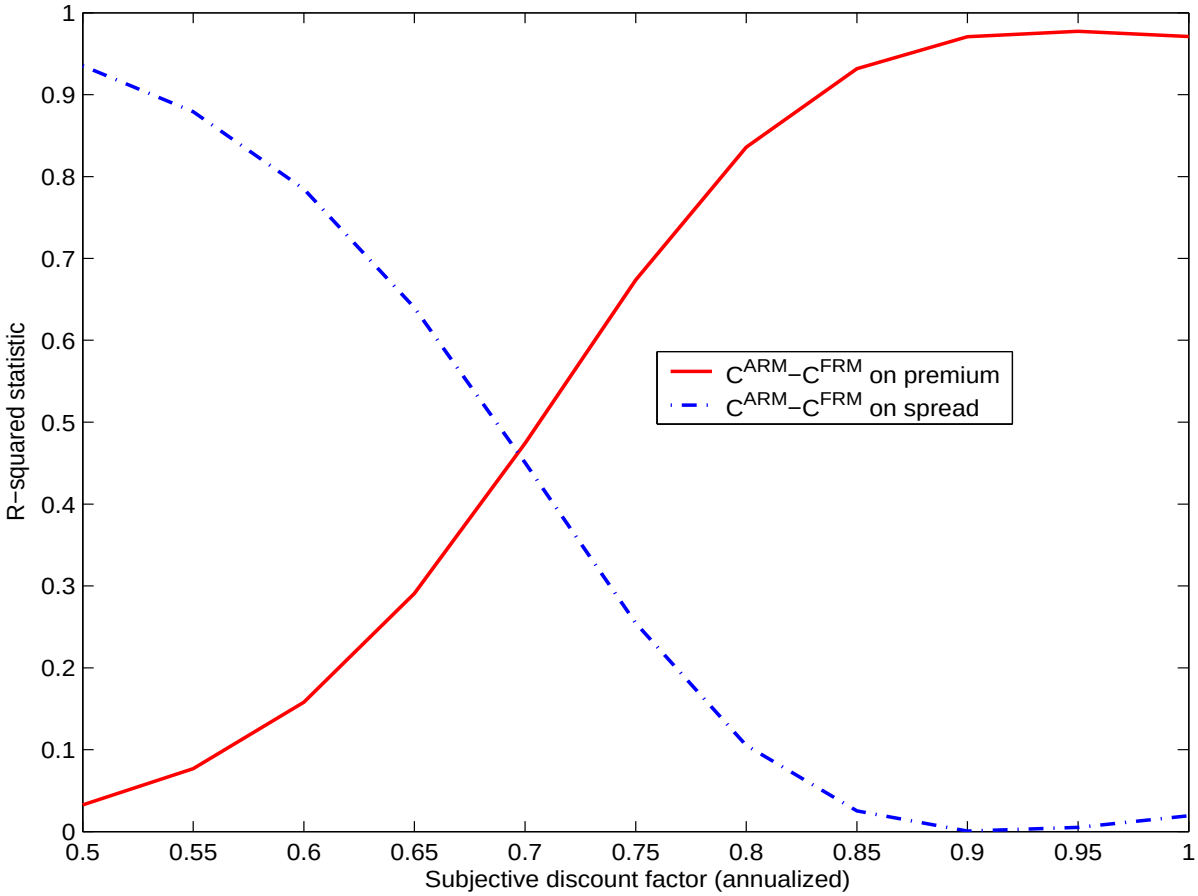


Figure 13: Mortgage Originations in the US.

The figure plots the volume of conventional ARM and FRM mortgage originations in the US between 1990 and 2005, scaled by the overall size of the mortgage market. Data are from the Office of Federal Housing Finance Enterprise Oversight (OFHEO).