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Pairwise-core monetary trade in the Lagos-Wright model*

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Abstract

The Lagos-Wright model has been analyzed using particular trading protocols. Here, *weakly* and *strongly* implementable allocations are studied, where *weak* and *strong* are used in the sense of (weak) Nash (immune to individual defection) and strong Nash (immune to individual and cooperative pairwise defection). It is shown that the first-best allocation is strongly implementable without intervention for all sufficiently high discount factors. And, if people are free to skip the centralized meeting, then Friedman-rule intervention that uses lump-sum taxation in the centralized meeting to raise the return on money does not enlarge even the set of weakly implementable allocations. (99 words)

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1 Introduction

In Lagos-Wright [7], pairwise meetings (decentralized trade) alternate in time with a centralized meeting (with competitive trade). The crucial feature of

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the model is quasi-linear preferences in the centralized meeting. Such preferences imply that the dispersed holdings of money that emerge from decentralized trade are not state variables. The model and many variants of it have been analyzed for particular trading protocols in pairwise meetings—Nash bargaining between traders, take-it-or-leave-it offers by buyers, and price-taking. Here two notions of implementability are applied to the model: an allocation is weakly (strongly) implementable if it is immune to individual (individual and cooperative defection among those in pairwise meetings).[1] The quasi-linearity allows us to fully characterize both kinds of implementable allocations.

According to all the previously studied trading protocols, the first-best allocation is not achievable without intervention. Our characterization implies that the first-best is strongly implementable when people are sufficiently patient. Previous expositions of the model also show that the first-best is achievable if buyers make take-it-or-leave-it offers and if there is lump-sum-tax financed payment of interest on money at the Friedman-rule rate. We show that if preferences in the centralized meeting are linear and people are free to skip the centralized meeting (an implication of no-commitment ignored in the existing literature), then such intervention does not help: it does not enlarge the set of weakly implementable allocations.

2 The environment

Time is discrete, preferences are additively separable over dates, and there is a nonatomic unit measure of people who maximize expected discounted utility with discount factor $\delta \in (0,1)$. Odd dates have pairwise meetings and even dates have centralized meetings. There is a single good at each even date. At odd dates, there is specialization: a person is a buyer at each odd date with probability $1/N$, a seller with probability $1/N$, and neither (is in a no-coincidence meeting) with probability $1-(2/N)$, where $N \geq 2$. The period utility of someone who becomes a seller at an odd date and produces $y \in \mathbb{R}_+$ is $-c(y)$, while that of someone who becomes a buyer at an odd date and who consumes y is $u(y)$, where $c(0)=u(0)=0$, c and u are strictly

[1] Earlier applications of weak implementability in monetary models are Kocherlakota [4], Kocherlakota and Wallace [5], Cavalcanti and Wallace [1], and Katzman et al [3]. The only application of strong implementability in monetary models is Deviatov [2], who studies optimal inflation numerically.

increasing and differentiable with c convex and u strictly concave.[2] Moreover, there exists $\tilde{y} > 0$ such that $c(\tilde{y}) = u(\tilde{y})$. There are special preferences for the even-date good. The utility of "consuming" z amount of that good is z. (As in [7], $z < 0$ should be interpreted as production.)[3]

All goods are perishable, people cannot commit to future actions, and there is no monitoring (histories are private information)—assumptions that serve to make money essential. In addition, we permit people to hide money.

We consider allocations of the form $(y, z) \in \mathbb{R}^2_+$, where y is production and consumption in any single-coincidence pairwise meeting and z is production (of the even-date good) of any person who *was* a single-coincidence consumer at the previous date and consumption of any person who *was* a single-coincidence producer at the previous date. Moreover, associated with such (y, z) is zero production and consumption by those who were in no-coincidence meetings at the previous date.

Let $h(y) \equiv [u(y) - c(y)]/N(1 - \delta^2)$, the ex ante payoff from the allocation (y, z) prior to pairwise meetings. Notice that because the person is as likely to produce z as to consume z, the magnitude of z does not appear in this expression. If there is perfect monitoring, then the following set can be implemented.

$$P = \{(y, z) \in \mathbb{R}^2_+ : c(y) \leq \delta z + \delta^2 h(y) \text{ and } z \leq \delta h(y)\}. \tag{1}$$

That is, under perfect monitoring and its threat of permanent autarky, it is sufficient to satisfy the two participation constraints for the producer: one for production of y in a pairwise meeting; and one for production of z in a centralized meeting.

We show below that the set of allocations that can be implemented with monetary trade and without intervention is

$$V = \{(y, z) \in \mathbb{R}^2_+ : c(y) \leq \delta z \leq \frac{\delta^2}{N(1 - \delta^2) + \delta^2} u(y)\}. \tag{2}$$

To compare V and P, it is helpful to consider the set

$$V_y = \{y \in R_+ : c(y) \leq \delta^2 h(y)\}. \tag{3}$$

[2] The assumption $c(0) = u(0) = 0$ is without loss of generality. If it does not hold, then in all the expressions that follow we replace $u(y)$ by $u(y) - u(0)$ and $c(y)$ by $c(y) - c(0)$.

[3] In [7], the authors assume quasi-linearity and a net gain from produciing z and consuming z for some z. The results in the next section also hold for that version.

Lemma 1 *The set V_y is the projection of V on its first component and $V \subset P$.*[4]

The set V_y has a simple interpretation; if the centralized meeting is dropped from the model, then it is the set that can be implemented with perfect monitoring and no commitment.

As regards welfare, given that $h(y)$ is ex ante utility and that V_y is the projection of V on its first component, the optimum subject to $(y, z) \in V$ is the solution to the following problem.

Problem 1 *Choose $y \in V_y$ to maximize $h(y)$.*

We have

Lemma 2 *Let $y_{\max}$ be the maximal solution to*

$$c(y) = Ru(y),$$

where

$$R = \frac{\delta^2}{N(1-\delta^2)+\delta^2} < 1$$

and let $y^* = \arg\max[u(y) - c(y)]$. Then, $V_y = [0, y_{\max}]$ and the solution to problem 1 is $\min\{y_{\max}, y^*\}$.

Parameters—in particular, δ—determine whether $y_{\max} \geq y^*$ or $y_{\max} < y^*$.

3 Monetary trade without intervention

Money is divisible, in fixed supply, and the per capita amount is normalized to be 1. We begin by defining the *monetary outcome* associated with (y, z).

Definition 1 (The monetary outcome associated with (y, z)) *Prior to pairwise meetings, each person has 1 unit of money. In each single-coincidence pairwise meeting, the producer produces y and acquires 1 unit of money, and the consumer consumes y and surrenders 1 unit of money. At the next date, in the centralized meeting, those who were producers at the previous date consume z and surrender 1 unit of money, and those who were consumers produce z and acquire 1 unit of money.*

[4] All proofs are in the appendix.

Now we define weak and strong implementability of (y, z).

Definition 2 *The outcome (y, z) is weakly (strongly) implementable if there exist off-equilibrium trades such that those trades and the monetary outcome associated with (y, z) satisfy the following conditions:*

(1) the pairwise trades are immune to individual (individual and cooperative pairwise) defection;

(2) when each agent faces the price of money z in the centralized meeting, the agent chooses to leave with 1 unit of money.

Notice that the only centralized-meeting requirement is the competitive trade assumed by Lagos and Wright. That is because competitive trade is known to be equivalent to the meeting-specific core in such meetings.

Proposition 1 *A necessary and sufficient condition for weak implementation of (y, z) is $(y, z) \in V$.*

The sufficiency part of the proof uses the following off-equilibrium trades. In single-coincidence pairwise meetings, a buyer with at least 1 unit of money trades 1 unit for y, while one with less than 1 unit does not trade. A seller with less than 1 unit of money produces y, while one with more money produces 0. In the centralized meeting, each person trades competitively at the price z. If $(y, z) \in V$, then these trades weakly implement the monetary outcome in definition 2.

Proposition 2 *Let $y^{**} = \min\{y_{\max}, y^*\}$, the solution to problem 1. There exists z such that (y^{**}, z) is strongly implementable.*

The proof uses the following off-equilibrium trades. For pairwise meetings in which the seller has off-equilibrium money holdings, the proposed trade is the equilibrium trade. If the buyer in pairwise trade has more than 1 unit of money, then the proposed trade maximizes the seller's payoff subject to giving the buyer no less than the equilibrium payoff. (If $y^{**} = y_{\max}$, then this trade implies output in $(y_{\max}, y^*]$; otherwise, output is y^*.) If the buyer has less than 1 unit, then the proposed trade maximizes the seller's payoff subject to giving the buyer no less than no trade. These trades are depicted in figure 1 for the case $y^{**} = y_{\max}$.

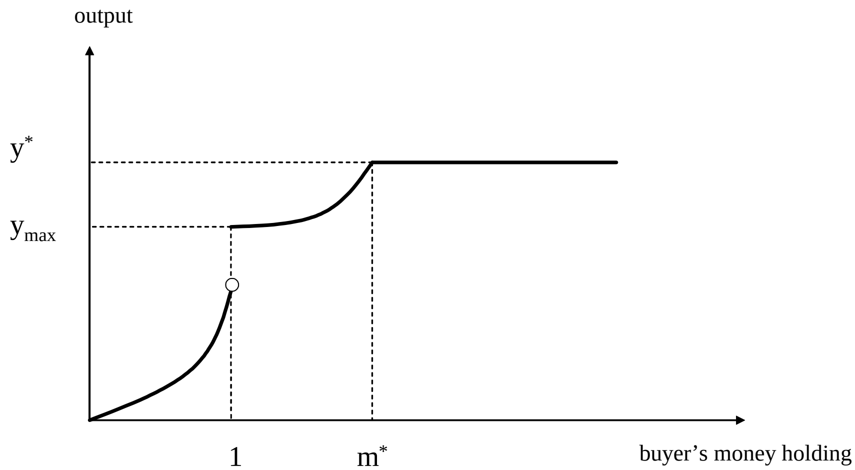

Figure 1. Off-equilibrium output as a function of the buyer's money

They are constructed as follows. Let m^* be the solution to $u(y^*) - u(y^{**}) = \delta z(m^* - 1)$, so that the buyer is indifferent between the trades (m^*, y^*) and $(1, y^{**})$. If the money holding of the buyer, x, satisfies $x \geq m^*$, then the proposed trade is (m^*, y^*). If $x \in [1, m^*]$, then the buyer surenders x for some $y \in [y^{**}, y^*]$ that is determined to give the seller all the surplus that exceeds the surplus of the equilibrium trade. If $x < 1$, then the buyer surrenders x for an amount of output that gives the seller all the surplus from the trade. As the figure suggests, the share of the surplus received by each trader varies with off-equilibrium money holdings.

These trades generate the same value functions that appear in the proof of weak implementability. In particular, they imply an affine value function for money at the start of the centralized meeting. That implies that buyers with more than 1 unit of money are indifferent among the proposed trades for $x \geq 1$, including the equilibrium trade. Thus, they have no incentive to hide their money holding. Moreover, because the trade $(1, y^{**})$ is weakly implementable, no buyer with 1 unit wants to hide money.

4 Interest on money

A standard exercise in the Lagos-Wright model is to study the effects of lump-sum taxes used to finance interest on money (see [7], [6], and [8]). And, it is standard to make the payment of that tax mandatory, not subject to any no-commitment restriction. Here we impose no-commitment by making participation in the centralized meeting voluntary. In particular, any agent can choose not to participate in the centralized meeting, while continuing on into the next pairwise meeting. Someone who chooses not to participate in the centralized meeting does not pay the tax, does not receive the tax-financed dividend on money, and, of course, does not trade in the centralized meeting. Because trade in the pairwise meeting is, by assumption, not monitored, a person cannot be punished in pairwise meetings for skipping the centralized meeting. We show that under this participation constraint, nothing outside the set V can be weakly implemented.

Mainly for notational ease, we formulate the tax as a tax in the form of the centralized good that is used to pay a real dividend on money. In what follows, the meaning of the allocation (y, z) is unchanged, as is the description of the monetary outcome associated with (y, z). However, we need an amended definition of implementability.

Definition 3 *The outcome (y, z) is weakly (strongly) implementable if there exist off-equilibrium trades and $\tau \in [0, z(1-\delta^2)]$ such that those trades and the monetary outcome associated with (y, z) satisfy the following conditions:*

(1) the pairwise trades are immune to individual (individual and cooperative pairwise) defection;

(2) when, in the centralized meeting, each agent pays a lump-sum tax τ, faces the price of money $z - \tau$, and receives a dividend τ per unit of money brought into the centralized meeting, the agent chooses not to skip the centralized meeting, and leaves it with 1 *unit of money.*

Notice that the degenerate distribution of money and the above trades are consistent with the allocation (y, z). That is, those who did not trade in a pairwise meeting pay a tax τ and receive a dividend τ, and, hence, neither produce nor consume in the centralized meeting; those who were producers in a pairwise meeting pay a tax τ, get dividends 2τ, and sell 1 unit of money at the price $z - \tau$ and, therefore, consume z; and those who were consumers in a pairwise meeting receive no dividends, pay a tax τ, and pay $z - \tau$ to buy 1 unit of money; and, hence, produce z.

At the price $z - \tau$, the net rate of return on money from one centralized meeting to the next is $\tau/(z - \tau)$. As τ varies between 0 and $z(1 - \delta^2)$, this rate varies between 0 and $(1 - \delta^2)/\delta^2$, the "Friedman-rule" rate.

Proposition 3 *If participation in the centralized meeting is voluntary (see definition* 3*), then* $(y, z) \in V$ *is necessary for weak implementability.*

The proof shows that those who consume in pairwise meetings, and, hence, would enter the centralized meeting with no money, choose to skip the centralized meeting. Therefore, freedom to skip the centralized meeting eliminates any beneficial role for the above taxation scheme.

In a literal sense, this proposition uses linearity of preferences in the centralized meeting and does not hold under quasi-linearity. But quasi-linearity does not, of course, justify making the tax mandatory. Instead, it would require a more detailed formulation of tax enforcement and how it is related to other activities in the centralized meeting.

5 Concluding remarks

The pairwise-core requirement (strong implementability), a form of coalition-proofness, is a reasonable requirement to impose in models of pairwise meetings. If it does not hold, then pairs leave meetings with unrealized gains from trade. In the Lagos-Wright model, imposing it, relative to weak implementability, is without cost in the sense that it never eliminates optima. More significantly relative to the trading protocols used in the literature, pairwise-core allocations can be viewed as allocations that a society can achieve. The society makes an ex ante choice of one such allocation and of the initial distribution of money that supports it. The pairwise-core property assures that those in meetings have no beneficial individual or cooperative defection.

Our results imply that entire classes of trading protocols that have been studied are missing good pairwise-core allocations. There is no reason to think that that result is limited to the Lagos-Wright model. Thus, much hinges on whether we study all pairwise-core allocations or choose a particular class of trading protocols. Existing work also overstates the beneficial role of tax schemes used to finance the payment of interest on money. It does so for two reasons. First, as just noted, that work understates what can be achieved without intervention. Second, most of it fails to subject tax schemes to no-commitment.

6 Appendix: Proofs

Lemma 1. The set V_y is the projection of V on its first component and $V \subset P$.

Proof. We first show that V_y is the projection of V on its first component. If $y \in V_y$, then $(y, c(y)/\delta) \in V$. Using the definition of $h(y)$, it is easy to show that the inequality $c(y) \leq \frac{\delta^2}{N(1-\delta^2)+\delta^2} u(y)$ is equivalent to $c(y) \leq \delta^2 h(y)$. Therefore, if $(y, z) \in V$, then $y \in V_y$.

Also,

$$c(y) - u(y) \leq \delta^2 h(y) - u(y) = \frac{\delta^2 \left(u(y) - c(y)\right)}{N(1-\delta^2)} - u(y). \tag{4}$$

So

$$u(y) \leq \left(\frac{\delta^2}{N(1-\delta^2)} + 1 \right) \left(u(y) - c(y)\right) \tag{5}$$

and

$$\frac{u(y)}{N(1-\delta^2)+\delta^2} \leq \frac{u(y) - c(y)}{N(1-\delta^2)} = \delta^2 h(y) \tag{6}$$

Thus, if (y, z) satisfies the second inequality in the definition of V, then it also satisfies the second inequality in the definition of P. Clearly, the same holds for the first inequality. Therefore, $V \subset P$. ■

Lemma 2. Let $y_{\max}$ be the maximal solution to

$$c(y) = Ru(y),$$

where

$$R = \frac{\delta^2}{N(1-\delta^2)+\delta^2} < 1$$

and let $y^* = \arg\max[u(y) - c(y)]$. Then, $V_y = [0, y_{\max}]$ and the solution to problem 1 is $\min\{y_{\max}, y^*\}$.

Proof. The only thing to prove is the claim about V_y. Let $\psi(y) = Ru(y) - c(y)$, a strictly concave function, with $\psi(0) = 0$. Suppose there are two points y_1 and y_2 such that $\psi(y_2) = \psi(y_1) = 0$, with $y_1 < y_2$. Then $y_1 = 0$ (since otherwise $\psi(y_1) > \frac{y_1}{y_2}\psi(y_2) = 0$, a contradiction). Thus the set $\{y > 0 : \psi(y) = 0\}$ contains at most one element; this element is $y_{\max}$, if it exists, and otherwise $y_{\max} = 0$. Note that $V_y = \{y \geq 0 : \psi(y) \geq 0\}$.

If $y > y_{\max}$ then $\psi(y) < 0$ (since otherwise $\psi(y_{\max}) > \frac{y_{\max}}{y}\psi(y) > 0$, a contradiction). And if $y \in [0, y_{\max}]$ then $\psi(y) \geq 0$, by concavity. Thus $V_y = [0, y_{\max}]$. ■

Proposition 1. A necessary and sufficient condition for weak implementation of (y, z) is $(y, z) \in V$.

Proof. *Necessity.* If (y, z) is weakly implementable, then the value of entering the centralized market with x amount of money is

$$w_c(x) = z(x-1) + \delta w_p(1), \tag{7}$$

where

$$w_p(1) = \frac{1}{N(1-\delta^2)}[u(y) - c(y)] \tag{8}$$

is the equilibrium payoff of having 1 unit of money just prior to pairwise meetings.

In a single-coincidence meeting, defection to no-trade by a seller with 1 unit of money assures a payoff no less than $\delta w_c(1)$, while following the monetary trade (y, z) for the seller gives $-c(y) + \delta w_c(2)$. Thus, it must be the case that

$$-c(y) + \delta w_c(2) \geq \delta w_c(1) \tag{9}$$

which, by (7), implies the first inequality that defines V.

Next, consider an agent who enters the centralized market with 0 money. It is feasible for this agent to produce 0 and resume equilibrium actions starting at the next date. This gives the payoff $-\frac{\delta c(y)}{N} - \frac{(N-1)\delta^2 z}{N} + \delta^3 w_p(1)$. Thus, it must be the case that

$$-z + \delta w_p(1) \geq -\frac{\delta c(y)}{N} - \frac{(N-1)\delta^2 z}{N} + \delta^3 w_p(1), \tag{10}$$

which, by (8), is equivalent to the second inequality that defines V.

Sufficiency. Assume that everyone else follows the monetary trade outcome associated with (y, z) and, therefore, has 1 unit of money when entering pairwise trade. Consider the following off-equilibrium trades in single-coincidence meetings: a buyer with more money or a seller with less money makes the equilibrium trade; otherwise there is no-trade. In the centralized meeting, each person transacts so as to leave with 1 unit of money.

It is necessary and sufficient for weak implementability that there exist value functions and a price p such that the monetary trade outcome and the above trades satisfy

$$w_c(x) = \max_{q \leq px}(q + \delta w_p(x - q/p)), \tag{11}$$

$$\begin{aligned} Nw_p(x) &= (N-2)\delta w_c(x) + \max\{u(y) + \delta w_c(x-1), \delta w_c(x)\} + \\ &\quad \max\{-c(y) + \delta w_c(x+1), \delta w_c(x)\} \text{ for } x \geq 1, \end{aligned} \tag{12}$$

and

$$Nw_p(x) = (N-1)\delta w_c(x) + \max\{-c(y) + \delta w_c(x+1), \delta w_c(x)\}, \tag{13}$$

for $0 \leq x < 1$. Here, $w_p(x)$ (p for pairwise) represents the value of holding x units of money at the begining of an odd date and $w_c(x)$ (c for centralized) represents the value of holding x units of money at the begining of an even date.

Consider $p = z$ and the trades described above. These imply the following value functions:

$$\hat{w}_c(x) = z(x-1) + \delta\hat{w}_p(1), \tag{14}$$

$$N\hat{w}_p(x) = u(y) + \delta\hat{w}_c(x-1) - c(y) + \delta\hat{w}_c(x+1) + (N-2)\delta\hat{w}_c(x) \tag{15}$$

for $x \geq 1$, and

$$N\hat{w}_p(x) = (N-1)\delta\hat{w}_c(x) - c(y) + \delta\hat{w}_c(x+1) \tag{16}$$

for $0 \leq x < 1$.

We claim that these satisfy (11)-(13). First, we need $u(y) + \delta\hat{w}_c(x-1) \geq \delta\hat{w}_c(x)$ for $x \geq 1$. This is equivalent to $u(y) \geq \delta(\hat{w}_c(x) - \hat{w}_c(x-1)) = \delta z$, which follows from the second inequality in (2). Similarly, for $x \geq 0$, we need $-c(y) + \delta\hat{w}_c(x+1) \geq \delta\hat{w}_c(x)$ or $c(y) \leq \delta(\hat{w}_c(x+1) - \hat{w}_c(x) = \delta z$, which is equivalent to the first inequality in (2).

It remains to show that $q = z(x-1)$ attains the maximum in (11) when $p = z$. We start by considering two exhaustive sets of alternatives for q.

Case (i): $q \leq z(x-1)$. By (15), we have

$$\hat{w}_p(x - \frac{q}{p}) = \frac{1}{N}[u(y) - c(y)] + \delta^2\hat{w}_p(1) + \frac{1}{N}\delta\{[-Nq + Nz(x-1)]\}, \tag{17}$$

and the maximization problem in (11) becomes

$$\max_{q\leq z(x-1)}(1-\delta^2)q+\delta\frac{1}{N}[u(y)-c(y)]+\delta^2 z(x-1)+\delta^3\hat{w}_p(1). \tag{18}$$

This is maximized at $q=z(x-1)$ and the maximized objective is

$$z(x-1)+\delta[\frac{u(y)-c(y)}{N}+\delta^2\hat{w}_p(1)]. \tag{19}$$

Case (ii): $q>z(x-1)$. By (16), we have

$$\hat{w}_p(x-\frac{q}{p})=\frac{1}{N}[-c(y)]+\delta^2\hat{w}_p(1)+\delta\{zx-q-\frac{N-1}{N}z\} \tag{20}$$

and the maximization problem in (11) becomes

$$\max[(1-\delta^2)q]+\delta[\frac{-c(y)}{N}+\delta zx-\frac{N-1}{N}\delta z+\delta^2\hat{w}_p(1)]. \tag{21}$$

This is maximized at $q=zx$, and the value is then

$$zx+\delta[\frac{-c(y)}{N}-\frac{N-1}{N}\delta z+\delta^2\hat{w}_p(1)]. \tag{22}$$

By (22) and (19), the necessary and sufficient condition for $q=z(x-1)$ to be the solution to (11), is

$$\delta\frac{u(y)}{N}\geq\frac{N(1-\delta^2)+\delta^2}{N}z, \tag{23}$$

which is equivalent to the second inequality in (2). ■

Proposition 2. Let $y^{**}=\min\{y_{\max},y^*\}$, the solution to problem 1. There exists z such that (y^{**},z) is strongly implementable.

Proof. Using the value functions in the proof of proposition 1 (see (7) and (8)), we define off -equilibrium trades for pairwise meetings as solutions to the following maximization problems. These problems are set out assuming that money is observable. Then we will show that observability plays no role. Let $x=(x_b,x_s)$ be money holdings of the buyer and the seller, respectively.

(i) Problem for $(x_b,x_s)=(x_b,1)\geq(1,1)$: $\max_{(m,y)\in[0,x_b]\times\mathbb{R}_+}[-c(y)+\delta w_c(m+1)]$ subject to

$$u(y)+\delta w_c(x_b-m)\geq u(y^{**})+\delta w_c(x_b-1). \tag{24}$$

(ii) Problem for $(x_b, x_s) = (x_b, 1) < (1,1)$: $\max_{(m,y)\in[0,x_b]\times\mathbb{R}_+}[-c(y) + \delta w_c(m+1)]$ subject to

$$u(y) + \delta w_c(x_b - m) \geq u(0) + \delta w_c(x_b). \tag{25}$$

(iii) Problem for $(x_b, x_s) = (1, x_s)$: $\max_{(m,y)\in[0,1]\times\mathbb{R}_+}[u(y) + \delta w_c(1-m)]$ subject to

$$-c(y) + \delta w_c(m + x_s) \geq -c(y^{**}) + \delta w_c(1 + x_s). \tag{26}$$

In each of these problems, the constraint set is not empty and the solution exists and is unique. Because w_c is affine, there is no need to consider lotteries. (Any lottery over the transfer of money can be replaced by its mean without changing the payoffs.) Moreover, in each case, the displayed constraint holds at equality at the solution.

By definition, these trades are in the pairwise core. And because the displayed constraints hold at equality, the payoffs imply the proposition 1 value functions. It remains to show that no one wants to hide money and that the equilibrium trade solves the first problem when $x_b = 1$.

No hiding of money. Consider the first problem for $x'_b > x_b \geq 1$ and the respective solutions, (m', y') and (m, y). Letting $\Delta = x'_b - x_b$, the payoff to x'_b from misrepresenting himself as x_b is

$$\begin{aligned} u(y) + \delta w_c(x'_b - m) &= u(y) + \delta w_c(x_b - m) + \delta z \Delta = \\ u(y^{**}) + \delta w_c(x_b - 1) + \delta z \Delta &= u(y^{**}) + \delta w_c(x'_b - 1) = \\ &\quad u(y') + \delta w_c(x'_b - m'), \end{aligned} \tag{27}$$

where the first and third equalities follow from the fact that w_c is affine, and the second and fourth from constraint (24) at equality. The fact that $x_b = 1$ does not misrepresent himself as $x_b < 1$ follows from weak implementability and the fact that the displayed constraints hold at equality. And the result that there is no hiding of money for the second and third problems follows from exactly the kind of argument used for the first problem.

The solution to the first problem when $x_b = 1$. We have to show that the solution is $(1, y^{**})$, the equilibrium trade. Using constraint (24) at equality, the objective in that problem is $u(y) - c(y) + \alpha$, where α is a constant. There are two cases. If the constraint $m \leq 1$ does not restrict the choice of y, then the solution is $y = y^*$. In this case, $y^{**} = y^*$ because $y^{**} < y^*$ and $m \leq 1$ are inconsistent with constraint (24) at equality. Finally, $y^{**} = y^*$ and constraint (24) at equality imply $m = 1$. If the constraint $m \leq 1$ is binding, then $m = 1$. This and constraint (24) at equality imply $y = y^{**}$. ∎

Proposition 3. If participation in the centralized meeting is voluntary (see definition 3), then $(y, z) \in V$ is necessary for weak implementability.

Proof. Suppose that (y, z) is weakly implementable. Then, as noted in the text,

$$w_c(x) = (x-1)z + \delta w_p(1), \text{ for } x \in \{0, 1, 2\}, \tag{28}$$

where

$$w_p(1) = \frac{1}{N(1-\delta^2)}[u(y) - c(y)]. \tag{29}$$

The requirement that agents not skip the centralized market is

$$w_c(x) \geq \delta w_p(x) \text{ for } x \in \{0, 1, 2\}. \tag{30}$$

Now, someone who enters a pairwise meeting without money has with probability $1/N$ the option to produce, acquire a unit of money, and enter the centralized market; and otherwise skip the centralized market. Therefore,

$$w_p(0) \geq \frac{-c(y) + \delta w_c(1)}{N} + \frac{N-1}{N}\delta^2 w_p(0), \tag{31}$$

or, equivalently,

$$w_p(0) \geq \frac{-c(y) + \delta w_c(1)}{N - (N-1)\delta^2}. \tag{32}$$

Then, by (28), the requirement that someone with 0 not skip the centralized market is

$$-z + \delta w_p(1) \geq \frac{-c(y) + \delta^2 w_p(1)}{N - (N-1)\delta^2}. \tag{33}$$

By (29), this is easily seen to be equivalent to the second inequality that defines V. Moreover, because a producer with 1 unit of money in a pairwise meeting has the option to not produce and enter the pairwise meeting with 1, we require $-c(y) + \delta w_c(2) \geq \delta w_c(1)$. According to (28), this is the first inequality that defines V. ∎

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