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ABSTRACT

We model and calibrate the arguments in favor and against short-term and long-term debt. These arguments broadly include: maturity premium, sustainability, and service smoothing. We use a dynamic equilibrium model with tax distortions and government outlays uncertainty, and model maturity as the fraction of debt that needs to be rolled over every period. In the model, the benefits of defaulting are tempered by higher future interest rates. We then calibrate our artificial economy and solve for the optimal debt maturity for Brazil as an example of a developing country and the U.S. as an example of a mature economy. We obtain that the calibrated costs from defaulting on long-term debt more than offset costs associated with short-term debt. Therefore, short-term debt implies higher welfare levels.

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1 Introduction

Prior to many of the major emerging market financial crises of the past decade—Mexico, Russia, Brazil, Argentina—governments borrowed large amounts of short-maturity liabilities. Each of these countries subsequently had to roll over large amounts of short-term debt to meet its payment obligations.[1] Scholars have argued that short-term liabilities render an economy particularly vulnerable as the shorter and more concentrated the debt maturity the more likely debt crises are to occur.[2] In addition, short-term debt may increase a country's exposure to sharp increases in interest rates, which may have additional negative consequences, as governments may need to increase taxes in order to service the debt.[3]

Reacting to the apparent link between the maturity structure of government debt and these financial crises, many academics and policy makers have urged governments to increase the maturity of the debt. However, governments usually have to pay a higher premium on long-term bonds, a premium that may reflect uncertainties about governments' ability (including issues of taxation and inflation) but also willingness to repay. The short-term maturity structure of emerging market debt simply might be a market response to deeper problems associated with uncertainty and enforcement of contracts.[4] Hence, at least from a theoretical point of view, it is not clear that the optimal maturity of government debt is characterized by large quantities of long-term debt.

In this paper, we quantitatively compare the different rationales in favor of and against short-term debt in order to evaluate the optimal debt maturity of a government. Those rationales broadly include: *maturity premium*, *sustainability,* and *service smoothing.* We do this by modeling the different arguments and calibrating the model in order to assess their quantitative importance.

Our model has the following main features. First, we construct a dynamic equilibrium infinite period economy. Second, we model maturity as the amount of debt that becomes due every period.[5] Thus, in our model, a government that attempts to lengthen maturity will structure the debt so that similar and small quantities of government bonds are rolled over each period. Third, following Grossman and Van Huyck (1988), we treat debt as a contingent claim.[6] That is, sovereign defaults occur as bad outcomes of

[1] See Borensztein et al. (2005) for an overview of the main stylized facts related to the composition and structure of sovereign debt and the role of short- term debt.
[2] See, for example, Alesina, Prati and Tabellini (1990), Chang and Velasco (2000), Cole and Kehoe (1996, 2000), Giaviazzi and Pagano (1990), and Rodrik and Velasco (1999).
[3] See Barro (1997) for a discussion.
[4] See discussions in Barro (1995, 1997), Chari and Kehoe (2003), Jeanne (2004), Rodrik and Velasco (1999), and Tirole (2002, 2003).
[5] Our modeling of debt maturity is close to Cole and Kehoe (1996, 2000) and Alesina et al. (1990).
[6] In the sovereign debt literature, two main assumptions are employed to smooth consumption. In contingent debt models such as that of Eaton and Gersovitz (1981), consumption smoothing is achieved by making debt issuance contingent on the realization of income: the sovereign issues (retires) new debt when realized income is low (high). In contingent service models such as that of Grossman and Van Huyck (1988), consumption smoothing is achieved by making debt servicing contingent on the realization of income: the sovereign services its debt fully only when

debt-servicing obligations that are implicitly contingent on the realized state of the world.[7] This assumption follows from the observation of two of the main stylized facts behind sovereign borrowing and defaulting: (i) defaults are usually partial and associated with identifiable bad states of nature; and (ii) sovereign states often are able to borrow soon after default. In their model, lenders sharply differentiate excusable defaults, which are justifiable when associated with implicitly understood contingencies from debt repudiations, which are inexcusable. Hence, in the model debt allows accommodating negative shocks to the government budget.

However, our model also recognizes that contingent debt is associated with incentive problems: when the method of financing is too convenient, the government is likely to abuse it. To capture this trade-off, we propose an information structure similar to the one used by Cole, Dow and English (1995) and Alfaro and Kanczuk (2005), in which lenders signal-extract the type of government. We assume there are two types of sovereigns. "Bad" sovereigns are extremely impatient and always default, independently of the state of the economy; "good" sovereigns default optimally (excusably) in order to smooth tax distortions. The assumption of different type of sovereign is meant to capture the uncertainty lenders face about the borrowers' preferences.[8] The equilibrium interest rate is determined by lenders, who signal-extract the type of government. When a government defaults, lenders charge high interest rates in the following period. Consequently, the government's financing problem might get worse. This assumption is consistent with another stylized fact of sovereign borrowing; namely, that countries with histories of defaults are charged higher interest rates than countries with no repayment difficulties.[9]

Within this framework of contingent service, we study three channels through which the choice between short- or long-term maturities might affect an economy. We name these three channels *maturity premium*, *sustainability,* and *service smoothing.*

M*aturity premium* captures the observation that long-maturity debt tends to be more expensive than short-maturity debt. As Barro (1997) notes, this stylized fact is not so obviously rationalized,

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realized income is high and defaults, either partially or fully, when realized income is low; see Grossman and Han (1999) for a taxonomy of debt models.

[7] As Obstfeld and Rogoff (1996) note, although much lending takes the form of non-contingent loans, debt crises have shown that payments can be rescheduled, renegotiated, or changed unilaterally if the borrower's economy falters; lenders have the option of making new loans or cutting off existing credit lines. That loans incorporate a premium to compensate for states of nature in which scheduled payments are not made in full shows that such eventualities are anticipated. Similarly, Eaton et al. (1986, p. 482) conclude: "[T]hough indeed the borrower is required to service a debt, there is no way that, in general, the borrower can be forced to do so under all contingencies. Debt and equity are both contingent claims, although they clearly differ in the nature of the contingencies involved."

[8] In Brazil, the 2002 election of Luis Ignacio "Lula" Da Silva serves as just one example of the difficulty lenders face in assessing a sovereign's type.

[9] Eichengreen and Portes (1989) and Ozler (1993) find evidence that lenders base their country risk assessment on past debt-servicing behavior. However, this is not the only view prevailing in the literature, see Jorgensen and Sachs (1989) and Eichengreen and Lindert (1989).

because, in principle, governments can default on both short- and long-term debt. However, this can be understood within the context of our modeling framework if one considers that the stochastic process that underlies the "type" of government (or constraints a government faces) might display some persistence. In other words, if today's government is of the good "type," the likelihood that one will observe a "good" government tomorrow is also high. Hence, in this case, a good outcome today has stronger implications for the near horizon: a government that is a good debtor today is more likely to be a good debtor next year than ten years out. What is not usually noticed is that the converse also is true. A government that defaults (or is expected to default) today and is, hence, recognized to be of the "bad type," is more likely to remain of the "bad type" next year than ten years from now. In this situation, short-term debt is more expensive than long-term debt.

Figure 1, which reports the debt premium (a measure of risk) for Brazil during 2002, illustrates this second phenomenon. The dotted line corresponds to the yield of 20-year duration bonds, and the continuous line corresponds to the yield of one-year duration bonds. Both lines refer to dollar denominated bonds. Although for most of the period considered, the one-year bond yield was considerably smaller than the twenty-year bond yield, for a few days in 2002, the order of the two yields inverted, implying a negative maturity premium. During this time, investors considered the possibility that the Workers' Party candidate, Luis Inácio Lula da Silva, who had once argued in favor of defaulting on the external debt, would win the election. When that scenario became most likely (at the beginning of August), fear of default dominated the markets. As rationalized in the model, the government was believed to be of the "bad" type today and likely to be of the "bad" type tomorrow. In this case, because investors attributed a higher probability of default on debt due in the short term than on debt due in 20 years (when the likelihood that the government would be of the bad type would be lower), long-term debt had a smaller premium than short-term debt.

Note that our analysis is consistent with the view that the sovereign may decide to default on, or renegotiate, the terms of certain bonds but not of others (in our specific case, certain maturities and not others).[10] As Duffie, Pedersen and Singleton (2003), note, there are a number of justifications for this assumption. (i) Bond covenants may not include cross-default clauses that would force, upon the default of one bond, the simultaneous default of other bonds of the same type, but of a different maturity. (ii) Even when bonds include cross-default clauses, creditors to sovereign bonds might have little or no incentives to exercise their rights. Sovereign bonds holders' incentives to invoke such clauses differ from those holding bonds to firms because there are no legal procedures whereby government assets can be compulsory reposed and distributed among the creditors. (iii) Sovereigns issue bonds under different legal

[10] Countries have executed "selective" defaults. See, for example, the case of the Russian default in 1998 analyzed in detail in Duffie, Pedersen, and Singleton (2003).

systems (domestic versus foreign such as those of the United States, United Kingdom, and other European countries) that have different rules such as collective-action clauses or "rights to accelerate." This strategy affords governments "legal" discretion over which bonds to default. Finally, although recent decades have seen changes in the laws as well as the developments of new legal strategies, surveying the literature Sturzenegger and Zettelmeyer (2005) conclude that the effect of legal mechanisms in enforcing sovereign debts remains limited by the fact that few sovereign assets (including future income streams) are in fact located outside its national borders, and that a sovereign cannot credibly commit to hand over assets within its borders in the event of a default. Even were an investor to invoke an acceleration clause, there is no evidence of a mechanism that could force a sovereign to honor it.

Sustainability refers to how the maturity structure of the debt makes a sovereign more or less prone to default. Alesina, Prati and Tabellini (1990), for example, conclude from an analysis of Italy's debt management policies during the 1980s, that a shorter and more concentrated debt structure increases the likelihood of a debt crisis. Cole and Kehoe's (1996) study of Mexico's 1994-1995 Tequila Crisis reach similar conclusions. Other researchers share the authors' conclusions and recommendations.[11]

Note that in Alesina, Prati and Tabellini (1990) and Kehoe and Cole (2000), longer maturity implies smaller quantities of debt to be rolled over in every period. Hence, the gains from defaulting depend on the size of the debt that needs to rolled-over: the smaller the amount to be rolled-over, the smaller the gains from defaulting. Consequently, long-term debt is more sustainable (or less vulnerable to roll over risk). It is important to note also that these studies model default as a once-and-for-all-time phenomenon. After defaulting a government is not able to borrow or lend again. In contrast, in a framework of repeated defaults such as ours, and consistent with stylized facts, defaulting does *not* imply permanent exclusion from the financial markets. Instead, defaulting countries are penalized with higher future interest rates. Therefore, it is not obvious anymore that longer maturity debt is more sustainable. Longer maturity and smaller amounts of debt to roll over imply lower gains from defaulting, but the costs associated with defaulting also may be lower.

The third channel we considered is *service smoothing* which is associated with a government's desire to smooth taxes over different states of nature in order to maximize welfare. Long-term debt allows accommodating negative shocks to the government expenditure, the tax base, and in particular the interest rate. Short-term debt, because it needs to be rolled over every period, is, in contrast, subject to current shocks. A debt structure that implies the same amount of service every period regardless of external

[11] For example, Rodrik and Velasco (1999) argue that countries should lengthen the average maturity of debt to reduce their vulnerability to crises. They note, however, that restraining short-term borrowing is "no free lunch" inasmuch as both government and private borrowers might have good reasons to assume some short-term debt.

shocks also implies fewer tax distortions.[12] In our environment, the interest rate is an endogenous variable, but there are shocks to the government expenditure relative to the tax base. Different maturity structures imply that the debt is more or less sensitive to these oscillations, and thus imply greater or lesser tax distortions.

We then calibrate our model and solve for the optimal debt maturity. We first calibrate our artificial economy to match that of a developing country. We choose the Brazilian case as virtually all public debt is linked to short-term interest rates (overnight). We find that the optimal structure, given the present volatility of fiscal finance, is to have exclusively short-term debt.[13] That is, for a given set of parameters, our calibrated results suggest that sustainability decreases with debt maturity. This result is contrary to that obtained by Cole and Kehoe (2000), among others, as well as the emerging conventional wisdom among scholars and policy makers alike.

To understand our result one has to analyze how the benefits and costs of defaulting change with debt maturity. In our model, in line with Cole and Kehoe (2000), as maturity increases, the amount of debt rolled over each period decreases, and thus the benefits from defaulting also decrease. However, in contrast with the work of these authors, in our model the costs of defaulting also decrease with an increase in maturity. This is so because, in our model, the sovereign is not (perpetually) excluded from the market, but rather faces a higher interest rate after defaulting. And, as we show, the increase in interest rates due to a default is smaller for long debt maturities. It turns out that for our calibrated experiments, this effect dominates, that is, the costs of defaulting increase more than the benefits of defaulting as maturity increases. As a consequence, our calibrated results suggest that in many cases, given fundamentals, it is not possible to lengthen an economy's maturity structure.

We also study the case of the United States, as an example of a mature economy. In this case, we find that that although the long maturity currently observed in the U.S. debt is not optimal, the potential gains from shortening the debt maturity seem almost irrelevant in terms of welfare. We thus conjecture that some form of "transaction costs", which we do not model, could explain why the U.S. government does not change its debt structure.

The closest related papers to our work are Cole and Kehoe's (1996, 2000) study of optimal debt policy. In their model, however, as we mentioned, debt services are not contingent, and defaulting governments lose access to international borrowing and lending. Using a similar framework, Arellano and Ramanarayanan (2006) advance a positive analysis of sovereign bonds maturities. They match various

[12] Excessive amounts of short-term debt can leave a government vulnerable to fluctuations in the refinancing costs. A large share of long-term debt and a long average maturity can reduce the fiscal budget sensitivity to fluctuations in the interest rate and, in particular, to a steep hike in interest rates; see Barro (1997).

[13] In terms of the stylized facts, compared with advanced countries, most emerging markets find it difficult to issue long-term debt even in their domestic markets; see Borensztein et al. (2005).

features of the Brazilian data, and their rationalization of the yield curve in defaults episodes is very similar to ours. Our paper is also related to the literature that emphasizes the role of agency costs on the side of the sovereign in explaining the use of short-term debt.[14] In environments with information asymmetries, Rodrik and Velasco (1999), Tirole (2003) and Jeanne (2004) study the role of short-term debt as a solution to a commitment problem. In these papers, shorter debt maturities enhance the countries' incentives to implement creditor-friendly policies, albeit leaving the countries more vulnerable to bad shocks. In this paper, in contrast, we quantify the implications of the different theoretical arguments.

The rest of the paper is organized as follows. The model is developed in section 2. Section 3 presents the computational implementation and equilibria. Section 4 defines the data and calibration. The results are discussed in sections 5 and 6. Section 7 concludes.

2 Model

We consider two economies that differ according to the maturity of the sovereign debt. In the first economy, the government can issue only one-period maturity debt. In the second, government debt is composed exclusively of two-period maturity bonds.[15] Both economies are populated by governments (sovereigns) that borrow funds and tax labor income from a *continuum* of identical, infinitely lived consumers (private sector) to finance exogenous paths of public expenditure. Sovereign type, which evolves over time, can be "good" or "bad" reflecting the particular sovereign's level of impatience. "Bad" sovereigns are extremely impatient and choose to default at any time independently of the state of the economy; "good" sovereigns may or may not choose optimally to default on their commitments.

In each period, t, the private sector supplies labor and capital to produce a single output good according to a Cobb-Douglas production function. The output can be used for private or government consumption or for private investment.

Let c^j_t, h^j_t, i^j_t, g^j_t, k^j_t, A^j_t denote the per capita levels of consumption, labor, investment, government spending, capital, and technology parameters for economies j=1, 2, respectively.

Feasibility requires

$$c_t^j + i_t^j + g_t^j = A_t^j \left(k_t^j\right)^{\alpha} \left(h_t^j\right)^{1-\alpha} \qquad (1)$$

The preferences of each consumer in each economy are given by

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[14] Other papers rely on borrower's liquidity problems to explain the existence of a positive risk premium on long-term debt, see Chang and Velasco (2000), and Broner, Lorenzoni and Schmukler (2004).

[15] This assumption simplifies our analysis. However, our main results should also hold for an economy that an issue both types of debt.

$$U^j = E\sum_{t=0}^{\infty}\beta^t u(c_t^j, h_t^j) \quad (2)$$

with $$u(c_t^j, h_t^j) = c^j - \varphi\frac{\left(h^j\right)^{1+1/\xi}}{(1+1/\xi)} \quad (3)$$

where φ, $\xi > 0$, and $\beta \in (0, 1)$. As discussed below, the assumption that consumption enters linearly in the period utility function has the advantage of simplifying the determination of the equilibrium interest rates without affecting the tax-smoothing motive of the model. The parameter ξ is referred to as the Frisch's intertemporal labor elasticity.

The per capita level of government consumption in each period, denoted by g^j_t, is exogenously specified. We assume that g^j_t can take a finite number of values and evolve over time according to a Markov transition matrix with elements $\pi_1(g^j_m, g^j_n)$. That is, the probability that $g^j_{t+1} = g^j_n$ given that $g^j_t = g^j_m$ is given by the matrix π_1 element of row *m* and column *n*. For future reference, a matrix π_2, representing the transition probabilities for two periods in advance can be derived from matrix π_1. In particular, $\pi_2 = \pi_1^2$, where the element $\pi_2(g^j_m, g^j_n)$ denotes the probability that $g^j_{t+2} = g^j_n$ given that $g^j_t = g^j_m$.[16]

Government consumption is financed through a proportional tax on labor income and with debt. Let τ^j_t denote the tax on labor income in period *t* for economy *j*. Government debt represents claims to future consumption units. We consider two types of such claims that differ with respect to maturity. In our first economy, we assume government debt to represent a claim to consumption units in the next period. In our second economy, government debt is assumed to be a claim to consumption two periods ahead.[17] We denote the amount of one-period contingent debt and two-period contingent debt, respectively, by $b^j_{1,t+1}$ and $b^j_{2,t+2}$. Note that maturity usually is understood to be the date on which a debt becomes due for payment. Cole and Kehoe (1996, 2000) and Alesina et al. (1990), for example, define maturity as the amount of debt that matures every period. A similar approach is taken in this paper.[18]

[16] Our assumption that government expenditure is exogenous is suitable for two different reasons. First, the objective of the paper is normative. Our goal is to study the optimal debt maturity structure given a budget constraint. Second, even if one were concerned with positive economics, it would still be the case that the volatility of debt would be (relatively) exogenous. Political economy models in which the government objective involves spending resources are usually useful for determining the *average* level of government expenditure. But even in these models the volatility of government expenditures tends to be a direct function of exogenous preferences or technology parameters; see Persson and Tabellini (2000).

[17] Note even if each economy had both types of debt, signaling is not possible in our environment because the bad type would always imitate the good type since it is costly to imitate the good type. This is so because bad sovereigns only care about current utility and an increase in debt services (resultant from a separating equilibrium) would reduce current consumption.

[18] The literature, however, has used other definitions. Missale and Blanchard's (1994) concept of maturity is related to the effect of a change in inflation on the value of debt. Hence, very short-term maturity is analogous to indexed debt by their definition. Barro (1997) argues that the relevant concept of short term is not the stated maturity of debt

Let $(1 + r^j_t)$ denote the (inverse of the) one period prices of these claims, $(1 + \rho_t)$ denote the riskless interest rate, and v^j_t denote the marginal product of capital. Let $\theta^j_t \in [0, 1]$ denote the default rate on contingent government debt outstanding in period t (one can also think of θ as a tax on debt).

In our first economy the private sector's budget constraint is given by

$$c_t^1 + i_t^1 + b_{1,t+1}^1 = w_t^1(1-\tau_t^1)h_t^1 + v_t^1 k_t^1 + (1-\theta_t^1)(1+r_t^1)b_{1,t}^1 \tag{4a}$$

In our second economy the private sector's budget constraint is given by

$$c_t^2 + i_t^2 + b_{2,t+2}^2 = w_t^2(1-\tau_t^2)h_t^2 + v_t^2 k_t^2 + [(1-\theta_t^2)(1+r_t^2)]^2 b_{2,t}^2 \tag{4b}$$

We assume that θ^j_t can take only two values, $\theta^j_t \in \{0, \chi\}$, where $\chi \in [0, 1]$ correspond respectively to the cases of not defaulting and defaulting.[19]

As in Grossman and Van Huyck (1988), we consider a case in which the levels of capital and debt are fixed and equal to k^i, b^1_1, and b^2_2. These assumptions, which imply that debt and capital cannot be used to smooth taxes, greatly reduces the calculation burden and is a necessary step to making equilibrium computable.[20] As Grossman and Han (1999) point out, this assumption is much less restrictive than it seems. They show that, when government can save after defaulting, contingent debt (or capital) does not allow for any additional tax smoothing. In contrast, contingent service might allow for additional tax smoothing than the one already attained through savings. In other words, varying b and k do not provide any additional smoothing from that already considered in the model.[21]

Because the capital level is constant, investment level must be equal to $i^j = \delta k^j$. Hence, the private sector's choices are to decide how much to work and how much to charge for government bonds.

The sovereign's preferences are given by

$$U = E\sum_{t=0}^{\infty}\beta_{sov}^t u(C_t^j, H_t^j) \tag{5}$$

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but rather the degree of sensitivity of debt payments to fluctuations in short-term market real interest rates. In the context of corporate finance, Diamond (1991) measures maturity relative to the timing of the arrival of cash flows. Rodrik and Velasco (1999) and Broner, Lorenzoni and Schmukler (2004) use a similar definition.

[19] Note that in our second economy the default factor (1 - θ) is squared, in order to make the two economies comparable. As we shall see, this is necessary, for example, for the interests to be the same in both economies if there is default in all the periods.

[20] As Grossman and Van Huyck (1988) note, this assumption simplifies the analysis without sacrificing qualitative generality. In an alternative set up, Cole and Kehoe (2000) assume that risk neutral households with no access to international bonds choose the capital level. Hence in their case, like in ours, i. debt is not used in production and ii. capital is not used to smooth consumption. These simplifications allow one to focus on the role of sovereign debt as a way to smooth consumption. There is an important sense in which it is consumption smoothing rather than the investing motive that underpins reputation for repayment in models of sovereign debt. A country with enough output could self-finance investment with no loss of utility if it does not care about smoothing consumption across periods, see Obstfeld and Rogoff (1996).

[21] More generally, one can think of shocks to the net variability of resources available for consumption and debt service after allowing for net current savings.

where $u(\,.)$ is as in the consumers' preferences, but C^j_t and H^j_t denote the aggregate per capita levels of consumption and labor, respectively. In our first economy, the sovereign budget constraint is

$$b_1^1 = g_t^1 - \tau_t^1 w_t^1 H_t^1 + (1-\theta_t^1)(1+r_t^1)b_1^1 \qquad (6a)$$

where we have already suppressed the time subscripts for the debt levels. Similarly, for our second economy, the government budget constraint is given by

$$b_2^2 = g_t^2 - \tau_t^2 w_t^2 H_t^2 + [(1-\theta_t^2)(1+r_t^2)]^2 b_2^2 \qquad (6b)$$

Within each economy there are two types of sovereigns that differ in the parameter β_{sov}. The "good" sovereign has the same discount factor of the private agents, $\beta_{sov} = \beta$. The "bad" sovereign, in contrast, fully discounts the future: $\beta_{sov} = 0$. A direct consequence of this assumption is that the bad sovereign always defaults.[22] A more subtle consequence of this assumption is that the good sovereign would not be able to separate herself from a bad sovereign, even if she had more instruments for trying doing so. In particular, this is true if a sovereign could issue different debt amounts, or vary debt maturity. Given her preferences, the bad sovereign would always imitate the good government actions. This would allow her to issue positive amounts of debt, and then defaulting in the following period, avoiding tax distortions.

As in Cole, Dow and English (1995) and Alfaro and Kanczuk (2005), the government type[23] evolves according to a Markov process (of common knowledge) with the transition probabilities given by[24]

$$\Psi_1 = \begin{bmatrix} 1-\psi & \psi \\ \psi & 1-\psi \end{bmatrix} \qquad (7a)$$

That is, a "good" type at t remains a good type at $(t + 1)$ with probability $1 - \psi$, and transitions to a bad type with probability ψ. Similarly, a "bad" type at t remains a bad type at $(t + 1)$ with probability $(1 - \psi)$ and transitions to a "good" type with probability ψ.[25] For this process to display persistence, we assume $\psi < 1/2$. We name ψ the *agency cost parameter*. Higher values of ψ imply that the current

[22] This assumption captures the flavor of Grossman and Van Huyck's (1988) "excusable defaults." A bad type always defaults (even in good times), which is not excusable. The good type might default, but only in relatively bad times ("excusable default").

[23] One can also consider that the same leader stays in power, but there is turnover among key advisors. An alternative interpretation is that the economy is occasionally subject to a confidence crisis. Such a crisis would imply in an interest rate hike, which is mathematically equivalent and could be interpreted as an increase in the prior probability that the sovereign is of the bad type.

[24] Note, however, that decision makers care about their consumption independently of whether or not they are in power. That is, they do not discount future consumption by the likelihood of their type changing, nor do they perceive their discount rate to be time varying.

[25] The symmetry of the transition matrix implies that the (invariant) frequency of good and bad government is equal for any agency cost ψ. Note that this simplifying assumption is immaterial for our results as we study the economy welfare conditional on having good governments in power.

government type carries less information about the future sovereign type. Thus, when making decision, consumers are more uncertain about the sovereign's type. We assume both economies to have the same transitional probability matrix. For future reference, an analogous matrix can be used to represent the transition probabilities for two periods in advance.

$$\Psi_2 = \begin{bmatrix} (1-\psi)^2 + \psi^2 & 2\psi(1-\psi) \\ 2\psi(1-\psi) & (1-\psi)^2 + \psi^2 \end{bmatrix} \tag{7b}$$

In each period, the timing is as follows (see figure 3). At the beginning of period *t*, the country inherits an amount of debt equal to b^j which bears an interest rate of r^j_t. In the two-period debt economy, these values were defined two periods before. Then, nature reveals the sovereign type and the public expenditure level. Only the public expenditure level can be observed by the consumers. After observing the public expenditure level, the sovereign decides whether or not to default, θ^j_t,. This determines the level of labor tax, τ^j_t, according to the sovereign budget constraint (equations 6a and 6b). Based on the sovereign decision, consumers decide how much to work, h^j_t. They also update their information about the sovereign's type and decide how much to charge for the next period contingent debt, r^1_{t+1} or r^2_{t+2}, in the two-period economy.

The assumption regarding the utility functional form greatly simplifies the solution. The Euler equation for labor (labor supply) jointly with the usual firms' maximization problem (labor demand)[26] imply

$$H^j_t = [(1-\tau^j_t)(1-\alpha)A^j_t k^\alpha / \varphi]^{1/(1/\xi+\alpha)} \tag{8}$$

Additionally, the Euler equation for consumption implies that the private sector behaves as risk-neutral lenders, with an opportunity cost given by the riskless asset, ρ. And, as usual, this price is a direct function of preference parameters, $\rho = 1/\beta - 1$.

As mentioned earlier, we assume that the private sector cannot directly observe the government's type. The lending rate r^i_t , therefore, depends on the perceived likelihood of default. We find it convenient to express lenders' information about the likelihood of default by defining two probabilities, p^j_t and q^j_t. Let $p^j_t \in [0, 1]$ be the probability that the *j* sovereign in period *t*, at the time of choosing whether or not to default, is of the "good" type. Let $q^j_t \in [0, 1]$ be the probability that a sovereign will default at time *t* given that the sovereign is of the "good" type. The perceived probability of default at *t* is then given by $1 - p^j_t(1 - q^j_t)$. Worth noticing, these probabilities refer to the perception of the lenders in period $(t - 1)$.

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[26] As usual, we assume there to be a continuum of firms (or a single price taker firm) and the demand for labor to equate wages with the marginal product of labor: $w_t = A_t(1 - \alpha)(k/H_t)^\alpha$. Similarly, the marginal product of capital is given by $\nu_t = A_t \alpha(H_t/k)^{1-\alpha}$.

In our first economy, for lenders to be indifferent between the riskless asset and the contingent debt, it must be that $1+\rho^{j} = p_{t}^{j}(1-q_{t}^{j})\left(1+r_{t}^{j}\right)+\left[1-p_{t}^{j}(1-q_{t}^{j})\right]\left(1+r_{t}^{j}\right)\left(1-\chi\right)$, which implies that the interest rate is given by

$$1+r_{t}^{1} = (1+\rho)/[p_{t}^{1}(1-q_{t}^{1})+(1-\chi)(1-p_{t}^{1}(1-q_{t}^{1}))] \qquad (9a)$$

Similarly, for our second economy, for lenders to be indifferent between the riskless asset and the contingent debt, the interest rate must be given by

$$(1+r_{t}^{2})^{2} = (1+\rho)^{2}/[p_{t}^{2}(1-q_{t}^{2})+(1-\chi)^{2}(1-p_{t}^{2}(1-q_{t}^{2}))] \qquad (9b)$$ [27]

A final assumption concerns the technology parameter A^{i}_{t}. We assume:

$$A_{t}^{j} = \exp[-\lambda(r_{t}^{j}-\rho)] \qquad (10)$$

The term $-\lambda(r^{j}_{t} - \rho)$, a departure from standard modeling, is meant to capture the output reduction due to contractual interest rate increases that have been documented in the literature. We add this term to consider the possibility that default causes an additional drop in output through its consequential interest rate boost.[28] As Calvo (2000) puts it, the literature has not yet paid sufficient attention to this stylized fact. He advances some explanations based on either halting the investment in a time to build a model or a credit crunch amplified by a financial accelerator. The term $-\lambda(r^{j}_{t} - \rho)$, the productivity factor that captures this stylized fact, should be seen as a shortcut that deserves further study. But, as in Alfaro and Kanczuk (2005), positive values for λ seem to be an important factor for the qualitative nature of the equilibria.

To understand how the model works, consider a good sovereign that chooses to default. If a sovereign defaults, choosing $\theta^{j}_{t} = \chi$, expression (6) indicates that the country will enjoy a lower tax distortion today, τ^{j}_{t}. But this decision might affect the future interest rate lenders charge and, thus, future taxes. Indeed, when lenders extract the information from the default in order to set the next period's interest rate, they will most likely consider the possibility that this period sovereign was of the "bad" type. This in turn, given (7a) or (7b), implies a higher probability that the sovereign country also will be of the "bad" type next period. Consequently, the private sector will choose to charge a higher interest rate (expressions (9a) and (9b)). As a consequence of defaulting, there are lower tax distortions today in exchange for higher tax distortions in the future. Following the usual assumptions regarding preferences and technology, welfare is higher for smoother tax profiles. Thus, default is a more likely outcome when the state of the economy is such that, *for a constant θ,* the government expenditure today is higher than the expected government expenditure in the future.

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[27] Notice that the probabilities p_t and q_t are set by the lenders at $(t-2)$.

[28] In fact, these additional output costs are an important characteristic of debt crises which are well documented, but not well understood. The term $\lambda(r_t - \rho)$ is meant to capture as well the stylized fact that these output costs tend to

3 Computational Implementation and Equilibrium

The model described is a stochastic dynamic game. For both economies we restrict our attention to the Markov perfect equilibria, which we discuss next.

For our first economy, we define the *state of the economy* at period *t* as the ordered set $(g^1_{t-1}, g^1_t, p^1_t)$, and the *excusable default set*, D[1], as

$$D^1 = \{((g^1_{t-1}, g^1_t, p^1_t) \textit{ such that lenders believe that the good type will default}\}$$

where, for any period, g^1_{-1} denotes the government expenditure in the previous period, g^1 this period government expenditure and p^1 lenders' assessment of the probability that the sovereign is of the good type. The excusable default set, part of the lenders' strategy, corresponds to all states of the economy in which lenders believe that the sovereign will default. In Grossman and Van Huyck's (1988) language, D^1 corresponds to the states of the economy in which defaults are excusable.

Given D^1, we can write the lenders' future probabilities assessments as a function of the state and of the sovereign's action as

$$p^1_{t+1} = 1-\psi \text{, if } \theta^1_t = 0,$$

$$p^1_{t+1} = \psi \text{, if } \theta^1_t = \chi \text{ and } (g^1_{t-1}, g^1_t, p^1_t) \notin D^1, \tag{11a}$$

$$p^1_{t+1} = p^1_t(1-\psi) + (1-p^1_t)\psi \text{, otherwise}$$

which correspond to simple Bayesian updating (see (7a)), and

$$q^1_{t+1} = \sum_{(g^1_t, g^1_{t+1}, p^1_{t+1}) \in D} \pi_1(g^1_{t+1}, g^1_t) \tag{12a}$$

which comes straight from the definition of the excusable default set.

Note that because the lenders' strategy is completely determined by the set D^1 and the expressions (9a), (11a), and (12a), given D^1 we can write the sovereign's problem as

$$V(g^1_{t-1}, g^1_t, p^1_t) = Max_{\theta^1_t}\{u(C^1_t, H^1_t) + \beta_{sov} EV(g^1_t, g^1_{t+1}, p^1_{t+1})\} \tag{13a}$$

such that (3), (6a), (8), (9a), (10), (11a), and (12a) hold.

Now we are ready to define the equilibrium for our first economy.

A *Markov perfect equilibrium* is an excusable default set D^1, a value function V^1, and a policy function θ^1such that, given D^1, θ^1 *is* a solution for the problem (13), and

$$\theta^1(g^1_{-1}, g^1_1, p^1_1) = \chi^1 \text{, for all } (g^1_{-1}, g^1_1, p^1_1) \in D^1 \text{, and}$$

$$\theta^1(g^1_{-1}, g^1_1, p^1_1) = 0 \text{, otherwise.}$$

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increase with higher interest premiums. Cole and Kehoe (2000) also assume that default implies additional costs of output. But in their formulation, these costs are constant and perpetual.

Although in a slightly different format much in line with one of “recursive competitive equilibrium,” this definition is not different from the usual Markov perfect equilibrium definition. Given the private sector’s strategy (the set D^1 and the expressions for p^1, q^1, H^1 and r^1) and the state of the economy, the sovereign maximizes utility. Given the sovereign strategy (θ^1) and the state of the economy, the private sector optimally chooses how much to work, and it is indifferent between buying contingent debt and earning the riskless rate. Hence, its strategy also is optimal.

Analogously, for our second economy, the *state of the economy* at period t is the ordered set $(g_{t-2}^2, g_{t-1}^2, g_t^2, p_t^2, p_{t+1}^2)$, and the *excusable default set*, is $D^2 = \{(g_{t-2}^2, g_{t-1}^2, g_t^2, p_t^2, p_{t+1}^2)$ *such that lenders believe that the good type will default*$\}$.

Given D, we can write the lenders’ future probabilities assessments as a function of the state and of the sovereign’s action as,

$$p_{t+1}^2 = (1-\psi)^2 + \psi^2, \text{ if } \theta_t^2 = 0$$

$$p_{t+2}^2 = 2\psi(1-\psi), \text{ if } \theta_t^2 = \chi \text{ and } (g_{t-2}^2, g_{t-1}^2, g_t^2, p_t^2, p_{t+1}^2) \notin D^2 \qquad (11b)$$

$$p_{t+1}^2 = p_{t+1}^2(1-\psi) + (1-p_{t+1}^2)\psi \text{ otherwise,}$$

which corresponds to simple Bayesian updating (see (7a) and (7b)), and

$$q_{t+2}^2 = \sum_{(g_{t-1}^2, g_t^2, g_{t+1}^2, p_{t+1}^2, p_{t+2}^2) \in D^2} \pi_2(g_{t+2}^2, g_t^2) \qquad (12b)$$

which comes straight from the definition of the excusable default set. Given D, we can write the sovereign’s problem as

$$V(g_{t-2}^2, g_{t-1}^2, g_t^2, p_{t+1}^2, p_{t+2}^2) = Max_{\theta_t}\{u(C_t^2, H_t^2) + \beta_{sov} EV(g_{t-1}^2, g_t^2, g_{t+1}^2, p_{t+1}^2, p_{t+2}^2)\} \qquad (13b)$$

such that (3), (6b), (8), (9b), (10), (11b), and (12b) hold. As before, a *Markov perfect equilibrium* is an excusable default set D^2, a value function V^2, and a policy function θ^2 such that, given D^2, θ^2 *is* a solution for the problem (13b), and

$$\theta(g_{-2}^2, g_{-1}^2, g^2, p^2, p_{+1}^2) = \chi,, \text{ for all } \theta(g_{-2}^2, g_{-1}^2, g^2, p^2, p_{+1}^2) \in D^2, \text{ and}$$

$$\theta(g_{-2}^2, g_{-1}^2, g^2, p^2, p_{+1}^2) \text{ otherwise.}$$

4 Data and Calibration

We calibrate our model to match case of Brazil as an example of an emerging market.[29] We also study the case of the United Sates as an example of a mature economy. We focus first on the Brazilian case, the results of which are quantitatively more interesting. We then present results for the U.S. economy.

We used data from 1957 to 2003 and calibrated our model such that each period corresponds to one year. To calibrate the government expenditure, we used data on the ratio of government expenditure to GDP. This choice reflects an attempt to capture the tax smoothing motive, that is, the variation of government expenditures relative to that of tax revenues. Note that our model includes no technology shocks, which have a direct impact on GDP and tax revenues. We "internalize" tax revenue shocks into the government expenditure shocks by studying the ratio g_t/y_t. We assume that g can be at one of two possible levels, g_H and g_L, and that the transition probability matrix that defines the Markov process is symmetric. To calibrate the necessary parameters, we first detrend the g_t/y_t series. We then calculate their mean, standard deviation, and autoregressive coefficients. For both the United States and Brazil, the mean is about g/y = 20%, and the autoregressive coefficient is about 0.97. The standard deviation, in contrast, is quite different between the two countries: government expenditure is about ten times more volatile in Brazil ($\sigma_{g/y\text{-}BRAZIL} = 0.40$) than in the United States ($\sigma_{g/y\text{-}U.S.} = 0.04$). To match these facts, we set

$$\pi = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}$$

for both countries. For the United States, we set $g_H/y = 0.208$ and $g_L/y = 0.192$. And for Brazil, we set $g_H/y = 0.28$ and $g_L/y = 0.12$.

We calibrate most of the technology and preferences parameters following the real business cycle literature. The (the inverse of the) price of the non contingent debt is given by $\rho = 0.05$. This implies a discount parameter of $\beta = 0.95$. The depreciation level is $\delta = 0.05$ and $\alpha = 0.33$. We also use the Frisch elasticity proposed by Domeij and Floden (2003), $\xi = 0.5$, and take the average hours of work to be H=0.3.

We conduct experiments for different values of the debt level. In order to make our two economies comparables, there must be a mapping between b_1 and b_2. For a given b_1 we set b_2, such that the amount of debt service per period is the same in the two economies *if there is never a default*. That is, when $\theta_t = 0$ and $r_t = \rho$ for any t, it must be the case that $[(1 + \rho) - 1]b_1 = [(1 + \rho)^2 - 1]b_2$. This implies $b_2 = b_1/(2 + \rho)$. In the equilibria descriptions in the following sections, we always refer to values of b_1 and consider b_2 to be implicitly defined in this manner.

For calibration purposes only, we set the debt level equal to b_1 = 60% of output. This enables us to determine the amount of tax revenue required to equilibrate the government budget constraint for a given level of government consumption, g. It follows that the mean value of the tax rate on labor is τ = 35.1%. With this value and the Euler equation for labor, we can calibrate $\varphi = 8.71$.

[29] For the calibration of the Brazilian economy, we follow Kanczuk (2004).

Because the transition probabilities of the government type are unobservable, we experiment with many different agency cost parameters (ψ). Recall that higher values of ψ are associated with more information asymmetry (less information about the type) and hence with a less severe punishment for not honoring the debt (i.e., defaulting). In this sense, we expect developed economies (such as the U.S.) to display smaller values of ψ than the developing economies (Brazil).

The parameter χ, which reflects the default rate, is difficult to calibrate given the complexity of information in each debt renegotiation. Some researchers attempt to calculate how much is paid in each renegotiation by calculating the ratio of the present value of the payments corrected by the Libor and amount lent. Eichengreen and Portes (1989) calculate that Chile, Bolivia, and Venezuela defaulted on about 35% of their debt during the 1930s. Cohen (1992) finds that in the 1980s, when the residual debt at the secondary markets is considered, default was almost nil, that is, debtor countries paid roughly a return of close to the Libor rate over their debts.[30] Bulow (1992) calculates that if one considers that investors hedged against the dollar depreciation, defaults in the 1980s amount to about 30%. We opted to use as our benchmark χ = 20%, but also experimented with χ = 10% and χ = 30%, obtaining similar results.

Finally, for the parameter λ, which corresponds approximately to the output elasticity to contractual interest rates, we use as a benchmark λ = 10%. We also experimented with higher values, obtaining similar results. An interest rate premium of 20% with a value of λ = 10% implies an additional output drop of about 2%, which is smaller than the stylized facts suggested by Cohen (1992).

Table 1 presents the full list of parameter calibrations.

5 Results for an Emerging Market: The Case of Brazil

5.1 Equilibrium Selection

Our model is set up so that for any parameter values there is always an equilibrium that corresponds to the case in which the sovereign defaults in all possible states. To observe this, consider that the excusable default set D^1 contains all possible states. One sees from expression (12a) that $q^1 = 1$ and from expression (9a) that $1 + r_t^1 = (1+\rho)/(1-\tau_t^1)$ for all states. These expressions indicate that punishment is independent of what a sovereign does or, better, that investors are not drawing any information from a sovereign's actions. The sovereign consequently has no incentive not to default and chooses to default in any state. This strategy validates the equilibrium. An analogous argument works for the two-period debt economy.

This trivial equilibrium of "defaulting in all states" does not, however, capture the flavor of contingent-debt. Hence, we focus on the other equilibria. It turns out that for the observed debt levels in the Brazilian and U.S. economies, if we assume no defaulting costs ($\lambda = 0$), these other equilibria do not exist. This is the case for a wide range of different combinations of parameter values: different debt levels (b = 5%, 10%, 15%, …), and agency costs (ψ = 10%, 1%, 0.1%, …). It is important to note that this result is *not* a reincarnation of Bulow and Rogoff's (1989) theorem. Grossman and Han (1999) proved there were positive amounts of sustainable debt in an environment such as ours. Contingent debt servicing permits more consumption smoothing than would saving and dissaving alone and thus, the sovereign would resist the temptation to repudiate its debt.[31] Our result is a quantitative one—for our economy, the *calibrated* value of debt is not sustainable if we assume λ to be zero.

If instead we consider output costs and set $\lambda = 0.1$, the model displays many equilibria. Here, as in any repeated game, one lacks a good way to choose among the many possible equilibria. These different equilibria display different default patterns, corresponding to different contingent claims and, in this sense, are not really comparable.

In order to make comparisons, we choose to focus on three equilibria, which are the natural choices in terms of simplicity. The first of these equilibria, which we described already, is the equilibrium in which government defaults in every state. We name this equilibrium the *always*-default-equilibrium. The second equilibrium we consider is the case in which government does not default, regardless of the state of the economy. We name this one the *never*-default-equilibrium. Note that due to the existence of governments of the bad type, even in this equilibrium there is a premium to compensate for the possibility of default. In the third equilibrium we focus, government debt corresponds to a claim that is contingent on the current state of government expenditure level. This claim determines that there should be default if $g_t = g_H$, and that there should not be a default if $g_t = g_L$. This is so regardless of the last period's government expenditure and of any beliefs about the type of the sovereign. We call this the *sometimes*-default-equilibrium.

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[30] Cohen (1992) finds that despite arrears and rescheduling creditor countries managed to recover an important part of their claims. He further shows that if the secondary market price is taken as the liquidative value of liquidation all major debtors (except Brazil) delivered a market return to the commercial banks.

[31] As Grossman and Han (1999) explain, with contingent debt issuance once the sovereign reaches any positive debt ceiling, repudiation would not reduce the possibilities for future consumption smoothing; whereas with debt contingent services, debt servicing repudiation would always reduce the possibilities for future consumption smoothing. Because there is a finite limit to accumulated savings and debt, the possibility of contingent saving or contingent debt issuance would not accommodate complete smoothing of consumption. Thus, even if the sovereign could save and dissave it would still be valuable to issue debt with contingent servicing in order to achieve more consumption smoothing.

5.2 Maturity Premium

Figure 1, as mentioned earlier, reports the interest rate premium for Brazil during 2002. The dotted line corresponds to the yield of 20-year duration bonds, the continuous line to the yield of one-year duration bonds. Both are dollar denominated bonds. Note that for most of the period considered the one-year bond yield is considerably smaller than the 20-year bond yield. This difference is usually referred to as the maturity premium. But during a few days in 2002 the order of the yields inverted implying a negative maturity premium. In anticipation of the October 6th presidential election in Brazil, investors were considering the possibility that the Workers' Party candidate, Luis Inácio Lula da Silva would, if elected, carry out earlier plans to default on the national debt. Particularly at the beginning of August, when it seemed most likely that Lula might be the next president, fear of default dominated the markets and the one-year bond yield reached 35%.[32] Figure 2, which complements figure 1, shows the yield curve on two particular days during that period. The continuous line represents April 19th, when the one-year maturity yield reached its minimum, the dotted line August 1st, when the one-year maturity yield reached its maximum, value. Note that the yields of bonds of all maturities were higher on the crisis day than on a tranquil day. More important, note that the slopes of the two curves are opposite: for the longest bond duration, on April 19th the yield was highest and on August 1st it was lowest. After winning the election Lula proved the investors wrong. When, among other things, he appointed a conservative Ministry of Finance who guaranteed that there would be no default on debt, bond yields slowly returned to their previous levels.

Understanding this pattern of short- and long-duration bond yields is not a trivial matter. As Barro (1997) points out, a government could, in principle, default both on long- and short-maturity bonds. However, this stylized fact can be easily rationalized in our framework. As mentioned, we assumed that the government type is determined by a Markov process with some persistence. Hence, the government type today contains a lot of information about the government type tomorrow, but less information about the government type two periods from now. Thus, if a government is believed to be of the good type today, it is also believed to be of the good type tomorrow, but less is known about the government type two periods from now. Thus, short-term debt yields a smaller return than long-term debt. Conversely, if a government is believed to be of the bad type today, which was the case in Brazil on August 1st, it is also believed to be likely to be of the bad type tomorrow, but less is known about its type two periods from now. Thus, long-term debt yields a smaller premium than short-term debt.

Figure 4 shows this stylized fact in our model. It corresponds to the never-default-equilibrium for various agency costs values when debt is held constant at 60% of GDP. Similar results can be obtained for other equilibria. For the never-default-equilibrium, as for other equilibria, the interest rate depends on the state of the economy. Thus, for each combination of parameters, there are many interest rates, each one associated with each state. Thus, to keep things simple, we chose to depict only the two extremes of the possible interest rates. These are the interest rates associated with states of the economy in which the probability that the government is of the good type today are the highest or the lowest possible.

Four curves are plotted in figure 4. The two continuous curves correspond to the one-period debt economy. The upper continuous curves are associated with the state with the lowest probability, the lower continuous curve with the state with the highest probability, that the government is of the good type. That is, the upper curves correspond to the most turbulent period, and the lower curves to the most tranquil period. The interest rates associated with other states would lie on curves somewhere between these two. The dotted-line curves in figure 4 correspond to the two-period debt economy, the upper curve being associated with lowest probability (turbulence), the lower curve with the highest probability (tranquility), that the government is of the good type. As above, interest rates associated with other states would lie on curves between these two. Three interesting observations can be made about figure 4.

First, the upper curves are decreasing, the lower curves increasing. This is the case because, regardless of the maturity of the debt, higher agency costs (ψ) impute to current beliefs less information about the future type of government. Investors who believe the government to be of the good type today (the lower curves) are less sure it will be of the good type in the future as ψ increases. Hence, the interest rate premium also increases with ψ. Analogously, investors who believe the government to be of the bad type today (the upper curves) are less sure it also will be of the bad type in the future for higher values of ψ. In this case, the interest rate premium decreases as ψ increases. This result is consistent with the stylized facts highlighted for Brazil.

Second, the distance between the interest rate extremes is greater for the one-period than for the two-period debt. That is, the range of possible interest rates for the two-period economy is contained in the range of possible interest rates for the one-period debt economy. This is so because the current belief is less important in the two-period than in the one-period debt economy. Investors worried about defaults two periods from now are less sure about the type of the government by then whether they believe the government to be of the good type or the bad type today. In the one-period economy the current state is a more important factor for either government type.

Third, the distances between the one-period and the two-period debt curves are greater for intermediate values of ψ. That is, for either small or big values of ψ, the one-period debt curves seem to converge to the two-period debt curves. The explanation for this lies in comparing the elements of

[32] Brazil has a complex composition of debt with maturities ranging from 1 day to 30 years. The average duration (maturity) of debt in Brazil is about 35 months in 2004 (see Alfaro, Di Tella and Vogel (2004)).

matrices (7a) and (7b), which converge for $\psi\rightarrow0$ and $\psi\rightarrow1/2$. That is, as agency costs disappear the information is the same regardless of the numbers of periods ahead. The same is true as agency costs tend to reach their maximum.

5.3 Sustainability

To analyze sustainability, it is necessary to assess the benefits and costs of a government default. In most sovereign debt analyses, including those of Alesina, Prati and Tabellini (1990) and Kehoe and Cole (2000), the cost of defaulting includes permanent exclusion from the capital markets and, therefore, forfeiture of tax smoothing. The benefit of defaulting, according to these authors, is increased present consumption, which is directly linked to the size of the debt: the higher the debt level, the higher the consumption increase in the event of default. Consequently, there is a maximum amount of debt that is sustainable. Beyond some threshold of debt level, the benefits of defaulting surpass the costs of defaulting and debt ceases to be sustainable. In these models, the effect of maturity on sustainability can be easily analyzed. An increase in maturity implies that the benefit from defaulting will apply to a smaller amount of debt. Or, as Cole and Kehoe (2000) put it: “[A]s the maturity of the debt gets longer, the amount that the government needs to borrow every period gets smaller. This decreases the government’s incentive to default whether or not it can roll over its debt.”

In a framework such as ours, in which default is a repeated phenomenon and governments can continue to smooth taxes after default, the analysis of sustainability and its relationship with debt maturity is more complicated. The benefit from defaulting is (as in the other type of models) associated with a higher consumption level today. However, since the country does face not permanent exclusion from the international capital markets, the costs from defaulting are not associated with the impossibility of smoothing taxes. Rather, they are associated with two other factors.

One is lower future consumption levels due to higher future interest rates. Because investors signal-extract the type of government, a default leads to higher future interest rates, which, in turn, imply higher debt services and lower consumption. The other relevant cost of defaulting is that associated with output, modeled earlier via an ad-hoc relationship between productivity and interest rates. A rise in interest rates post default begets a drop in productivity. Recall that, as discussed in subsection 5.1, the output costs from defaulting are an important determinant of the qualitative features of equilibrium; without them the presently observed levels of debt in Brazil and the United States would not be sustainable.

Before considering the effect of debt maturity, it is convenient to consider the effects of agency costs and debt level on sustainability. As agency costs increase, the amount of information about the future type of government decreases and thus the punishment for a defaulting government decreases.

Consequently, as agency costs increase, sustainability decreases. Regarding debt level, a default implies a bigger increase in consumption when debt is greater, hence, the benefit of defaulting increases with the amount of debt. The costs of default also increase with debt, but less so. The first factor we discussed, the increase in debt service, is directly proportional to debt. But the second factor, the output drop due to interest rate increases, is not affected by the debt level. Consequently, the costs of defaulting are relatively inelastic to the debt and there is hence a threshold of the debt level after which debt ceases to be sustainable.

Figures 5 and 6 depict the region of sustainability for the one-period and two-period debt economies, respectively. The horizontal axis represents the agency cost (ψ), the vertical axis the debt level (as a fraction of GDP). As just discussed, both higher debt level and higher agency costs decrease sustainability.[33] Following this logic, figures 5 and 6 depict frontiers in the (debt) X (agency costs) plane. The region below and to the left of the frontiers corresponds to situations in which debt is sustainable. In the region on the top and to the right of the frontier, debt is not sustainable. Each figure depicts the frontiers of the two equilibria that we analyzed: the never-default-equilibrium and the sometimes-default-equilibrium. As explained earlier, the always-default-equilibrium is always sustainable regardless of agency cost and debt level. Note that in both economies (figures 5 and 6) the locus for which the never-default-equilibrium is sustainable is contained in the locus for which the sometimes-default-equilibrium is sustainable. This is as expected since as agency cost or debt level increases it becomes increasingly difficult not to default in some states of the economy.

Now we are ready to analyze the effect of debt maturity on sustainability. Figures 7 and 8 refer, respectively, to the never-default and sometimes-default equilibria. Each figure contrasts the frontiers associated with each economy. Note that the sustainability locus of the one-period debt economy contains the sustainability locus of the two-period debt economy. That is, for a given set of parameters sustainability decreases with debt maturity. This result is contrary to the result obtained by Cole and Kehoe (2000), among others.

To understand our result recall first that the logic proposed by Cole and Kehoe does not apply to our framework. As maturity increases, the amount of debt in each period decreases as does, the benefit from defaulting. But an increase in maturity also decreases the cost of defaulting. This is so because maturity affects the interest rate and thus the output drop after a default. This can be seen in figure 4. Recall from our previous analysis of that figure that there is a smaller dispersion of interest rates in the two-period debt economy. That is, for longer maturities there is a smaller difference between the interest rates of good and bad states (before and after a default). Consequently, for longer maturities, the output

[33] This result can be related to the "debt intolerance" finding of Reinhart, Rogoff and Savastano (2003) which links debt sustainability levels with past histories of default and inflation.

drop due to a default is smaller. Our results indicate that this second effect predominates in the determination of sustainability. In other words, debt is less sustainable in our two-period debt economy because the impact of a default on the interest rate is smaller, hence, the output drop consequent to, and cost of, a default are also smaller. Figures 9 and 10 give a schematic view of this argument.

Given that our results contrast with those of Alesina, Prati and Tabellini (1990) and Cole and Kehoe (2000), it becomes important to study which of them is more supportive of the stylized facts. Unfortunately, this is a non-trivial task as it involves counterfactual experiments. We limit ourselves to two casual anecdotes, one supportive of each result.

The first anecdote, which supports the view of Cole and Kehoe, is to imagine a country hit by an international confidence crisis. A country with debt with very short maturity is much more vulnerable to such a shock. In this case, one can say that longer maturity should make the debt less vulnerable or more sustainable. The second anecdote, supportive of our view, is to imagine a country with a very short maturity structure, say, one month, that attempts to (slowly) lengthen the maturity of its debt to, say, 20 years. As a response, investors would likely react by not buying the debt and thus triggering a default.[34] In this case one can say that in this country only short-maturity debt is sustainable or, equivalently, that longer maturity makes debt less sustainable in this country. This was certainly the case of Brazil in many instances when the country attempted, unsuccessfully, to sell longer maturity debt.

To better compare these two views consider a country in crisis. According to Cole and Kehoe's (2000) model, the crisis would not have occurred if the country's debt had been of longer maturity. According to our model, given its fundamentals, it was impossible for the country to have had longer maturity debt. In other words, long-term debt simply was not sustainable from the outset.

5.4 Service Smoothing

According to Barro (1997), in order to minimize tax distortions, the optimal debt structure implies the same amount of service every period regardless of shocks. Based on this argument, he proposes that debt should be long term (a coupon). In our framework, the economy is hit both by government expenditure shocks and government type shocks (which can also be interpreted as external shocks). Figure 4 shows the effects of these shocks on interest rates. In short, long-maturity debt implies less dispersion in interest rate values. It follows that one should expect long-maturity debt to also imply less tax distortion.

Figure 11 depicts for the never-default-equilibrium, the standard deviation of the taxes for both economies when debt level is 60% of GDP. Our results suggest that, in fact, longer term maturity implies

[34] Note that this would not occur if the country decided to shorten its debt maturities; investors would have no problem buying the shorter maturity debt.

more tax smoothing. However, this result must be contrasted with the results reported in figure 12, which depicts the mean of taxes for both economies, again assuming a 60% debt level. Figure 10 shows taxes to be lower in the longer-maturity economy. This is so because in our model, good states of the economy (those with low taxes) occur more often than bad states of the economy (those with high taxes). This is not an artificial artifact of our model but is rather perfectly consistent with the observed stylized facts (see figure 2).

As a conclusion, we find that even as it implies more tax smoothing (reduction of the standard deviation), longer maturity may imply more tax distortion (higher mean). Thus, it is not clear that longer maturity should imply higher welfare levels.

5.5 Welfare

Figures 13 and 14 depict welfare levels for the one-period and two-period debt economies, respectively, when agency costs are assumed to be ψ = 10%. As in Lucas (1987), welfare is measured as a percentage of consumption. We normalize our measures with respect to the hypothetical case in which taxes are assumed to be *lump-sum*, thus implying no distortion. That is, a *lump-sum* taxes economy model (first best) has a welfare loss of zero.[35]

Each of these two figures reports welfare levels for the three equilibria concepts for different debt levels. As the never-default and sometimes-default equilibria are sustainable only for relatively low levels of debt, their welfare levels are not reported for economies with high indebtedness.

Note that for all equilibria, higher debt levels are associated with lower welfare. Higher debt levels imply higher debt service and thus lower consumption. The welfare drop due to an increase in debt seems, however, quantitatively small. In contrast, the welfare difference between two different equilibria (for the same debt level) is quantitatively more important.

In order to make welfare comparisons, we assume the economy to always be in its best possible equilibrium for each level of debt. That is, as long as the never-default-equilibrium is sustainable, it will be chosen to measure the welfare loss. The same logic applies to the sometimes-default-equilibrium when the never-default-equilibrium is not sustainable, in such a way that the final welfare curve is given by the envelope of each equilibrium welfare curve. This analysis should thus be seen as referring to the maximum welfare level an economy can attain.

Note that in figure 15, in which we compare the welfare of the one- and two-period debt economies, the one-period economy always dominates the two-period economy. For some debt levels, the difference between the two economies' welfare is quite small. This is the case when the same equilibrium

[35] It is noteworthy that with lump-sum taxes the welfare level is the same regardless of debt level and debt maturity.

is sustainable in both economies. For other debt levels, when some equilibrium ceases to be sustainable in only one of the economies, the welfare gap between the two economies is much more expressive.

As a final analysis we consider how the three channels represented in our model–maturity premium, sustainability, and service smoothing–interact to determine how debt maturity affects welfare. Taking *service smoothing* first, as discussed earlier, longer debt maturity reduces the variance of the taxes and, in this sense, improves welfare. But this seems to be a second order effect. The first channel, *maturity premium*, associated with the fact that good periods (low interest rates and taxes) occur more often than bad periods (high interest rates and taxes) implies that the mean of taxes increases with debt maturity. Thus, when we compare welfare levels for the same equilibrium, we find that short-debt maturity is better than long-term maturity. In other words, the *maturity premium* channel seems to dominate the *service smoothing* channel. Numerically, the gain from lengthening the maturity is in the order of 0.05% of consumption.

The *sustainability* channel becomes an issue when we assume that the economy is always in its best equilibrium. Short-maturity debt sustains more and better equilibria than long-term debt. Quantitatively, our results suggest that the welfare gap between different equilibria is much more important than the welfare gap for the same equilibrium. In some situations, the one-period debt economy implies a welfare gain of 0.3% of consumption relative to the two-period debt economy.

6 The Case of the United States

As we pointed out earlier, the U.S. and Brazilian economies seem to be similar in most respects save the volatility of government expenditure, which for the United States is about one tenth what it is for Brazil. Once we recalibrate our model to match the U.S. economy we can analyze the effects of debt maturity on welfare. As one might expect, the results are quite similar from a qualitative point of view. But in quantitative terms we find the U.S. economy to be much less sensitive to changes in debt structure.

Figure 16 depicts the sustainability region of the never default equilibrium for the United States. When compared to figure 7, figure 16 indicates that the never default equilibrium is sustainable for a wider range of debt and agency costs parameters. That is, because the U.S. economy has lower volatility, and thus lower benefits from tax smoothing and defaulting, debt is more sustainable.

Figure 17 depicts the maximum welfare for the U.S. economy when agency costs are almost null (ψ = .1%). With this calibration, the model considers a government of the bad type to be an almost impossible event. As before, we normalize the welfare levels with respect to a case in which *lump-sum* taxes are available. That is, figure 17 measures the welfare cost with respect to a first-best situation with no tax distortion.

Note that the one-period debt economy displays higher welfare levels than the two-period debt economy and that the difference between them increases with debt level. This result is qualitatively the same as that already obtained for Brazil, and not surprising. But figure 15 is more informative if one is concerned with the quantitative results. First, note that welfare costs associated with the first-best are much smaller in the United States. When debt is 10% of GDP, the one-period Brazilian economy displays a welfare loss of 0.25% of consumption; the equivalent figure for the U.S. economy is only 0.002%. Second, note that the difference between one- and two-period debt is also much smaller in the United States. For debt equal to 60% of GDP, which corresponds to the presently observed debt level, the welfare gain from shortening debt maturity from two years to one year is 0.30% in Brazil and 0.001% in the United States.[36]

The general conclusion is thus that although the long maturity currently observed in U.S. debt is not optimal, the potential gains from shortening debt maturity seem to be almost irrelevant in terms of welfare. We conjecture that some form of "transaction costs", which we have not modeled, could explain why the U.S. government does not shorten its debt maturity.

7 Conclusions

In this paper we model and calibrate the arguments in favor and against short-term versus long-term debt. These arguments broadly include the risk of rolling over large quantities of debt, tax smoothing, and the maturity term premium associated with the incentive to default.

We use a dynamic equilibrium model with tax distortion and government outlays uncertainty and model maturity by changing the fraction of debt that needs to be rolled over every period. In the model the benefits of defaulting are tempered by higher future interest rates. We discuss the implications of the model for mature economies (the case of the United States) and emerging markets (the case of Brazil).

In a broad sense our results caution against policies aimed at lengthening debt maturity. Our calibrated results suggest that in many cases, given fundamentals, it is not possible to lengthen the maturity structure of the economy, and that even when long-maturity debt is sustainable it seems to be associated with equilibria in which welfare levels are lower. Similar conclusions are reached by Tirole (2003) who then notes that "forcing private borrowers to title the maturity structure toward the long term reduces welfare."

To obtain these results we resorted to many simplifications needed to keep the computational analysis manageable. We assumed that debt levels were constant, that preferences were linear on consumption, that the government expenditure process was exogenous, and that long-term debt duration

[36] In fact, the U.S. debt maturity is shorter than usually thought. In December 2004 it was 50 months (Bloomberg).

was only two years. We believe that relaxing these assumptions could improve the analysis by making the results quantitatively more precise. But we do not expect that doing so would change the results qualitatively. In particular, our finding that short-term debt is more sustainable and associated with higher welfare levels than long-term debt should be robust to these simplifications.

That said, our results are nevertheless a consequence of the model we chose. Our model (i) recognized debt to be a contingent claim implicitly understood by investors, (ii) treated default as a repeated phenomenon, (iii) considered the punishment from defaulting to be associated not with market exclusion but with higher interest rates and the effect thereof on government budget (higher tax distortions) and output (productivity drop). Understanding which are the best assumptions and most crucial features of the sovereign debt process remains the most important avenue for future research.

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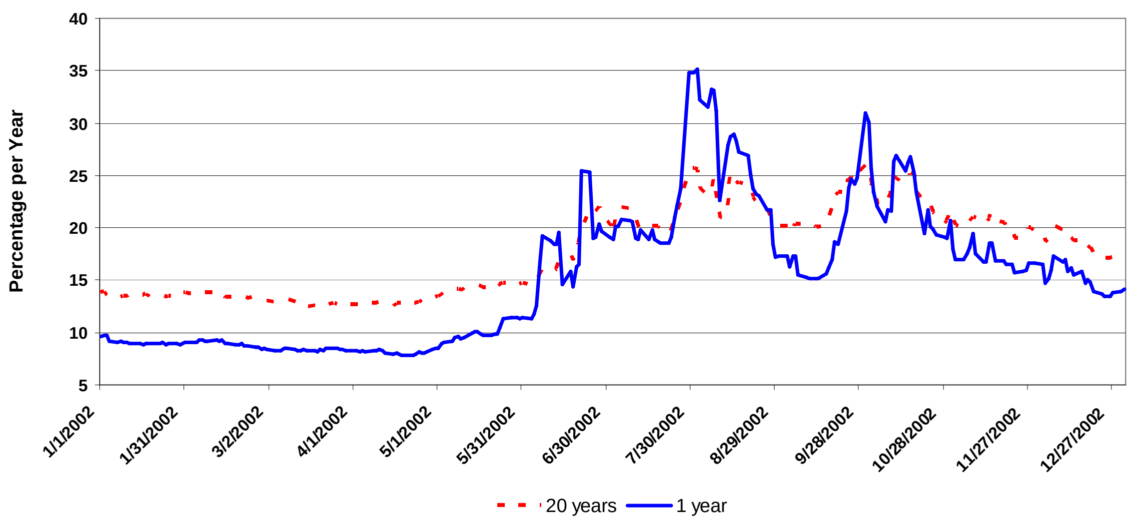
Table 1: Calibration

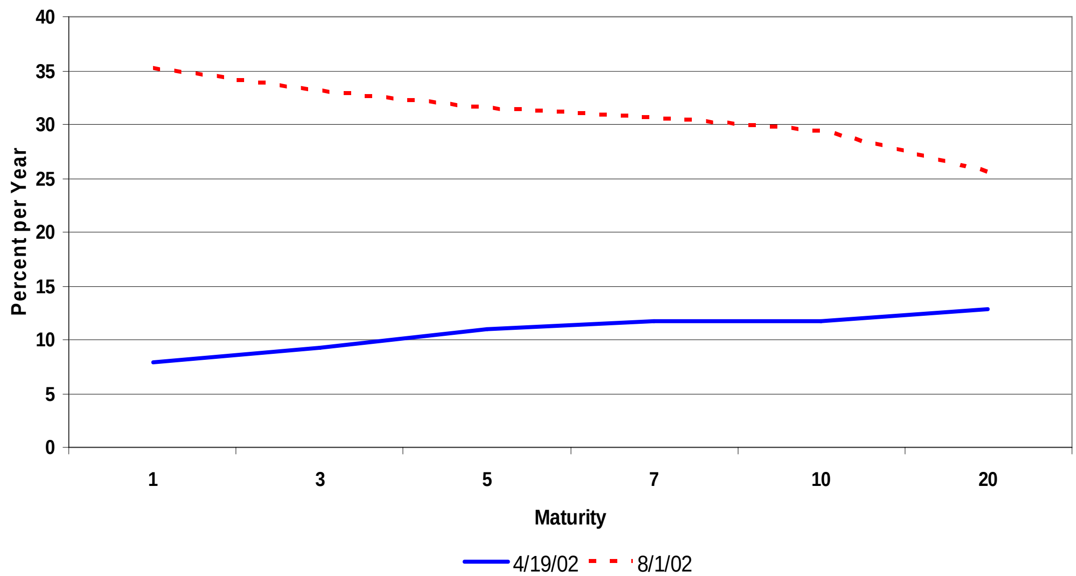
Preferences	
$\beta = .95$ (time discount parameter)	
$\xi = .5$ (Frisch intertemporal labor elasticity)	
$\varphi = 8.71$ (labor disutility parameter)	
Technology	
$\alpha = .33$ (capital share)	
$\delta = .05$ (depreciation rate)	
$\lambda = .1$ (output costs of default)	
Brazilian Government	
$g_H = .28$ (high government expenditure level)	
$g_L = .12$ (low government expenditure level)	
$\pi_{11} = \pi_{22} = .95$ (government expenditure persistence)	
$b_1 = 60\%$ (debt level)	
$\chi = .2$ (default rate)	
U.S. Government	
$g_H = .208$ (high government expenditure level)	
$g_L = .192$ (low government expenditure level)	

Figure 1: Brazil -- Government Bond Yields
Jan 1st, 2002--Dec 31st, 2002



Source: Bloomberg.

Figure 2: Brazil -- Yield Curve
Tranquile and Turbulent Periods



Source: Bloomberg

Figure 3: Time Line

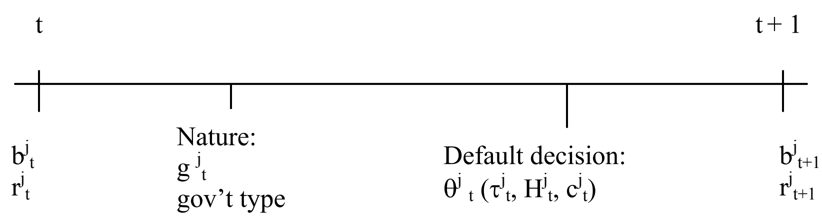


Figure 4: Brazil -- Interest Rates in the 'Never-Default-Equilibrium'

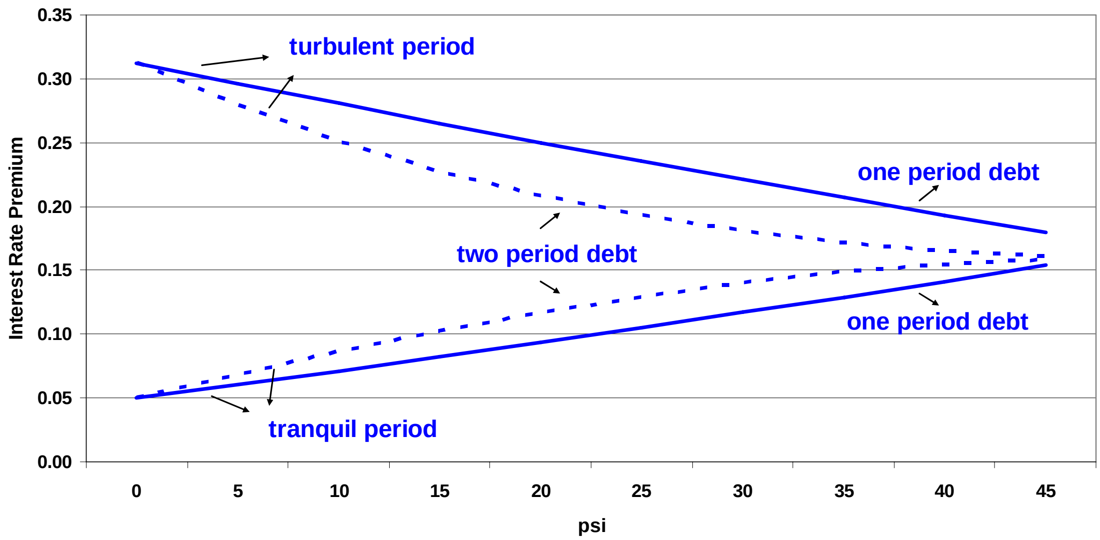


Figure 5: Brazil -- Sustainability in the One Period Debt Economy

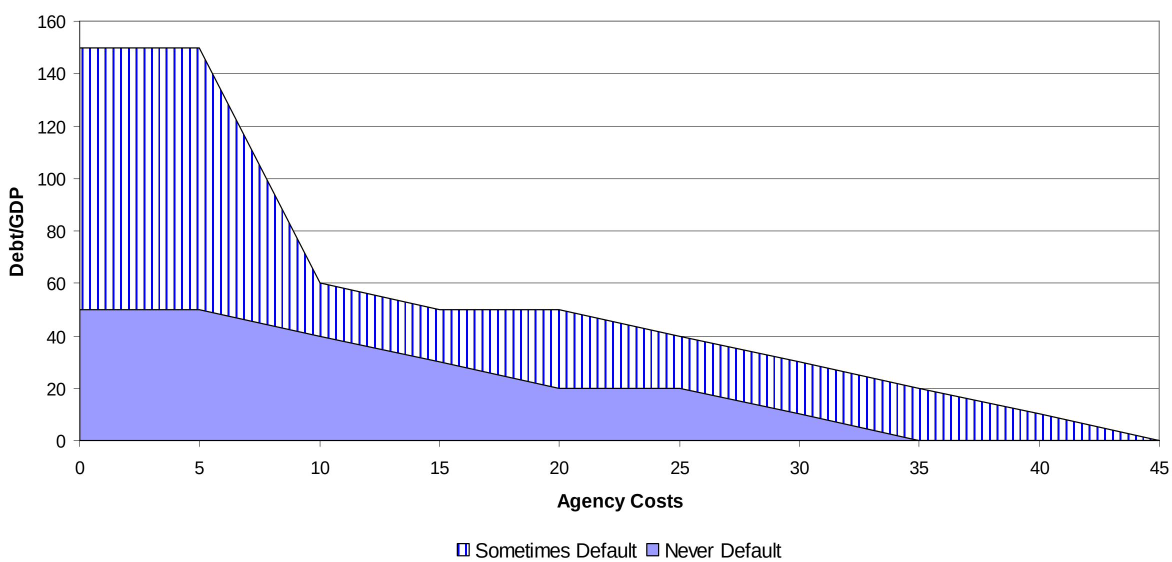


Figure 6: Brazil -- Sustainability in the Two Period Debt Economy

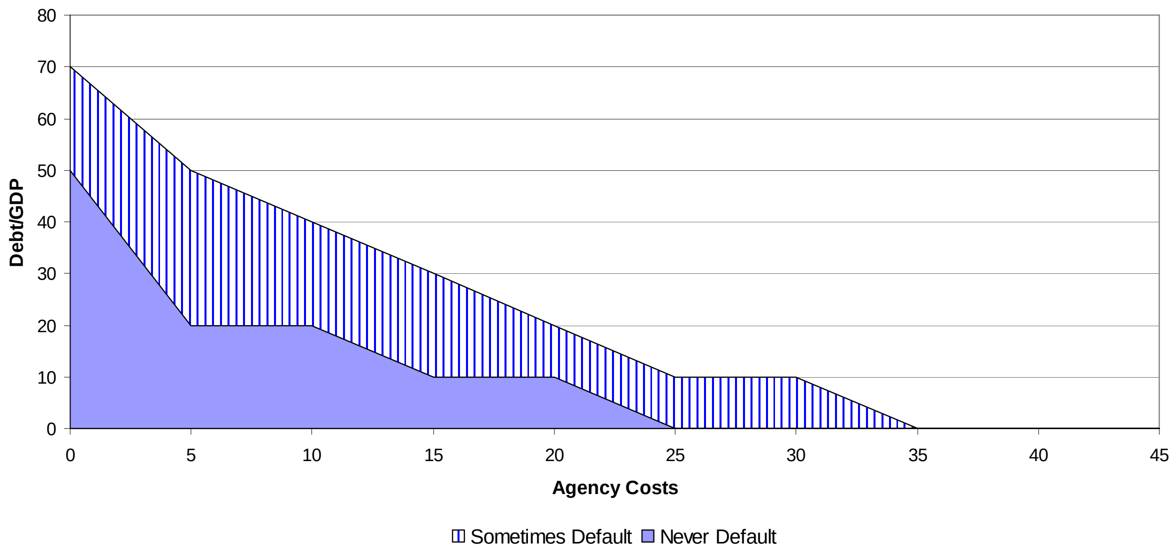


Figure 7: Brazil -- Sustainability for the 'Never-Default-Equilibrium'

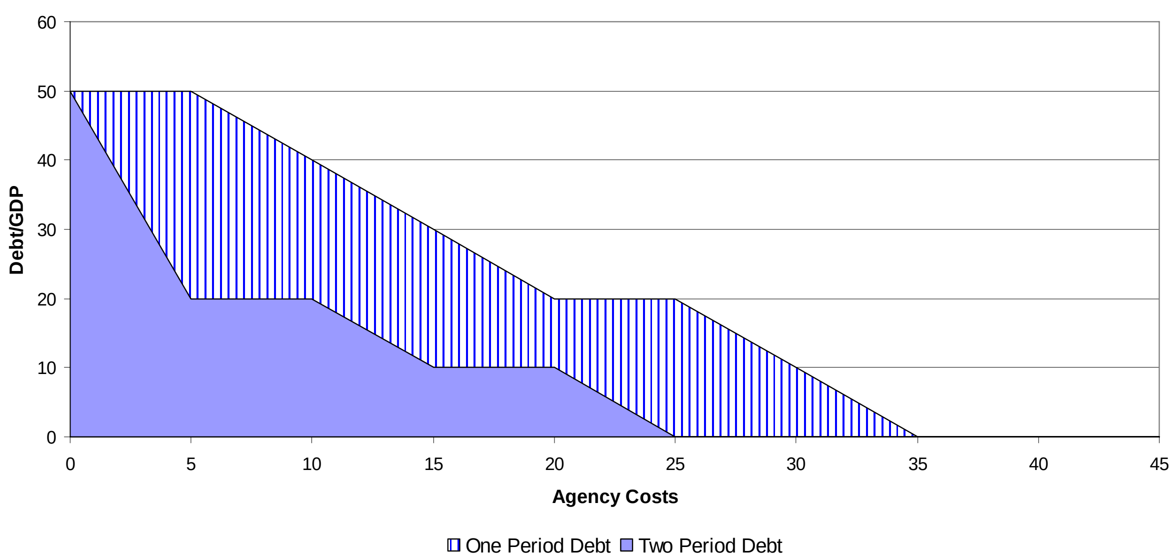


Figure 8: Brazil -- Sustainability for the 'Sometimes-Default-Equilibrium'

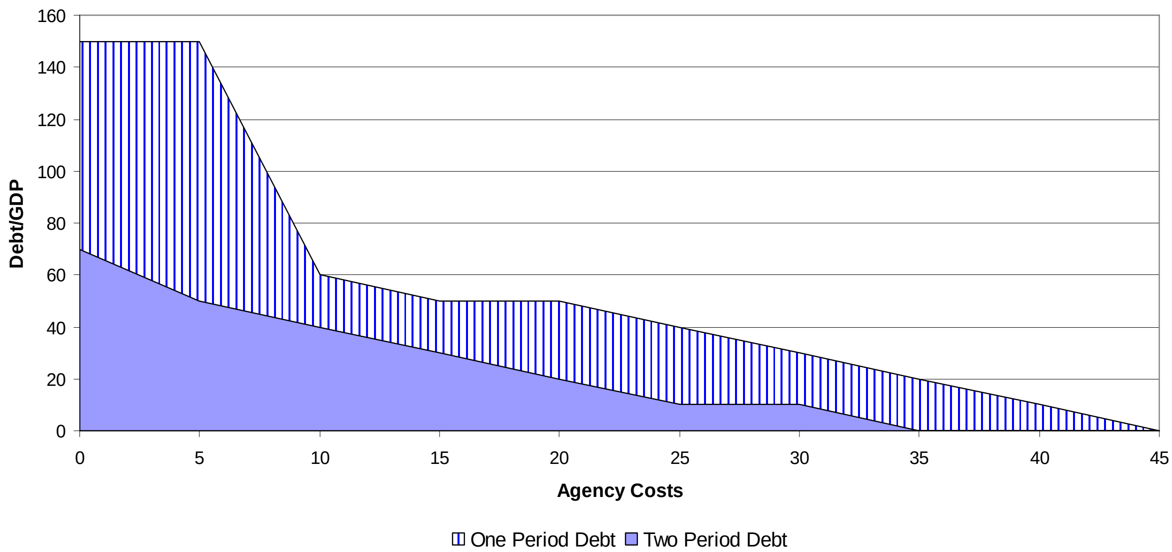


Figure 9: Debt Sustainability in Cole and Kehoe (2000)

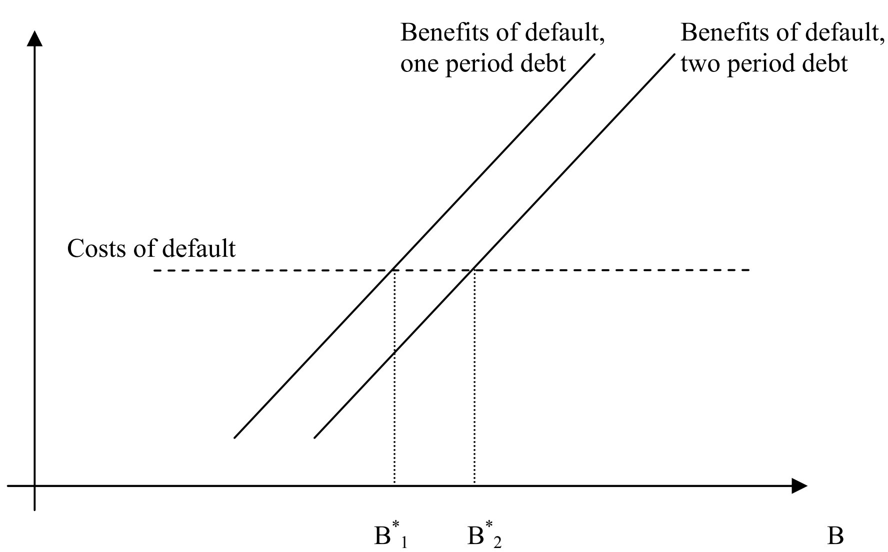


Figure 10: Debt Sustainability in this Paper

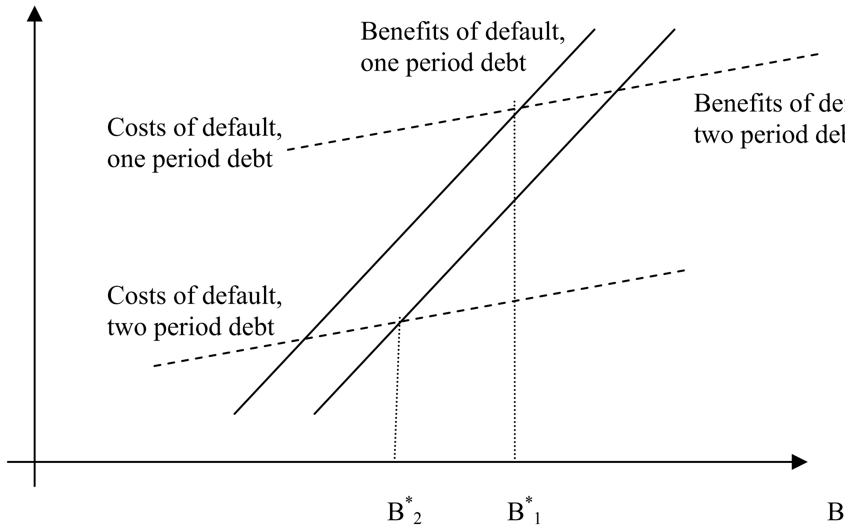


Figure 11: Brazil -- Standard Deviation of Taxes (% GDP)

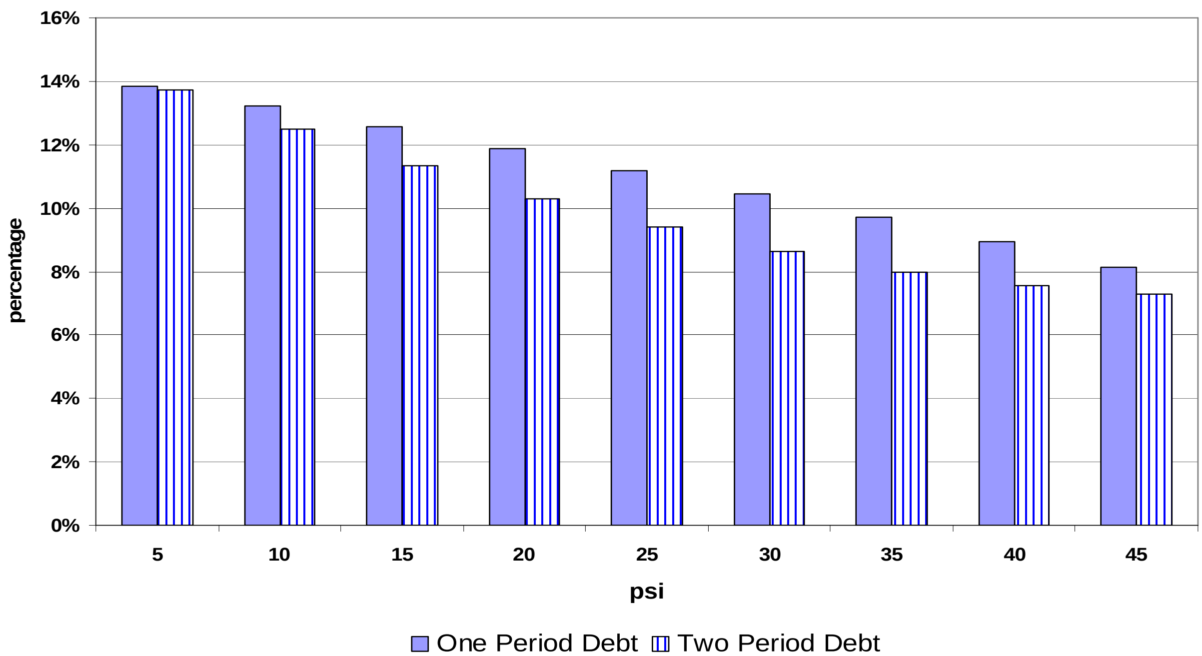


Figure 12: Brazil -- Mean of Taxes (% of GDP)

Figure 13: Brazil -- Welfare in One Period Debt Economy
Agency Costs = 10%

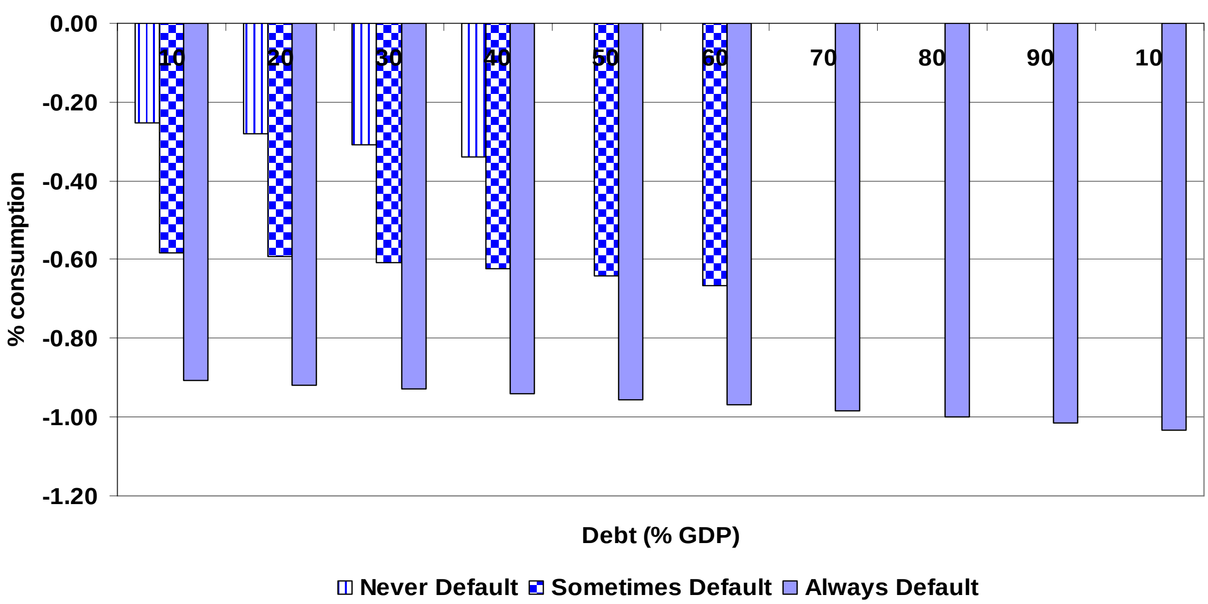


Figure 14: Brazil -- Welfare in Two Period Debt Economy
Agency Costs = 10%

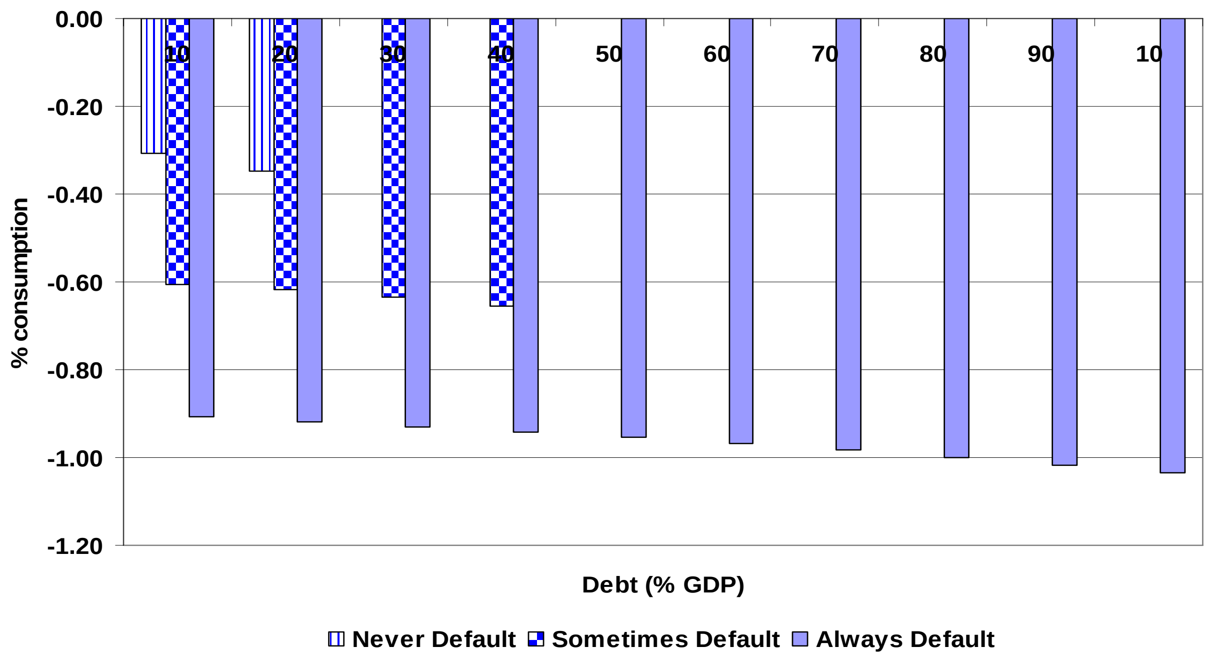


Figure 15: Brazil -- Welfare for Best Equilibrium

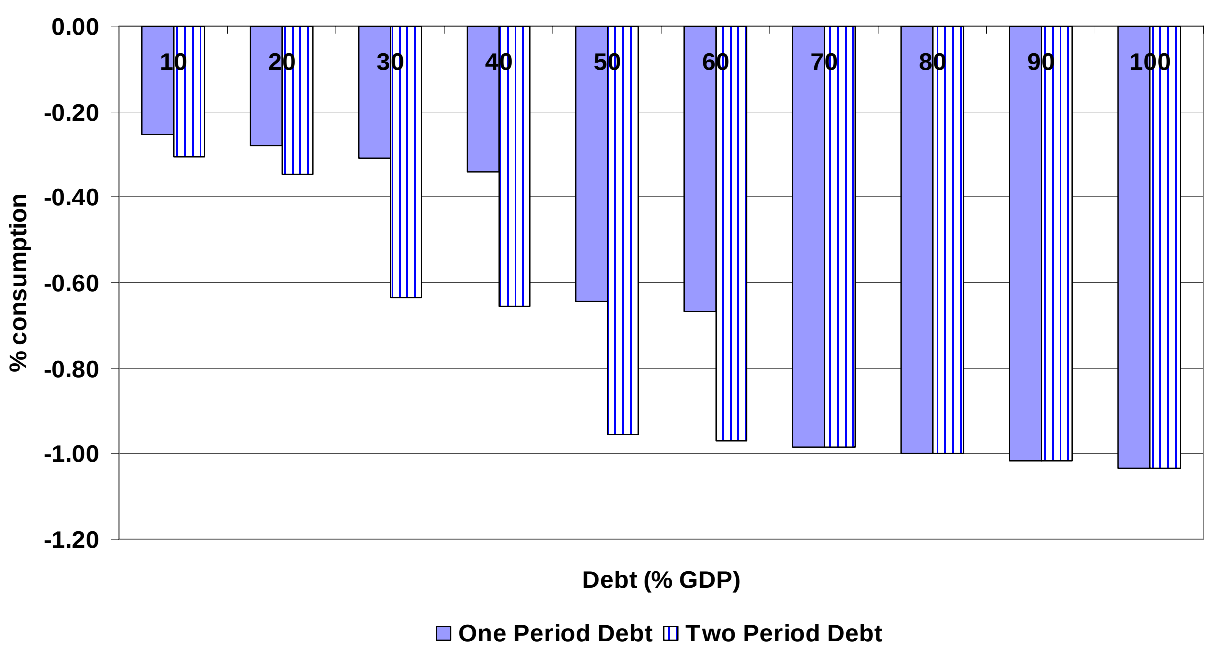


Figure 16: U.S. - Sustainability for the N*ever* Default Equilibria

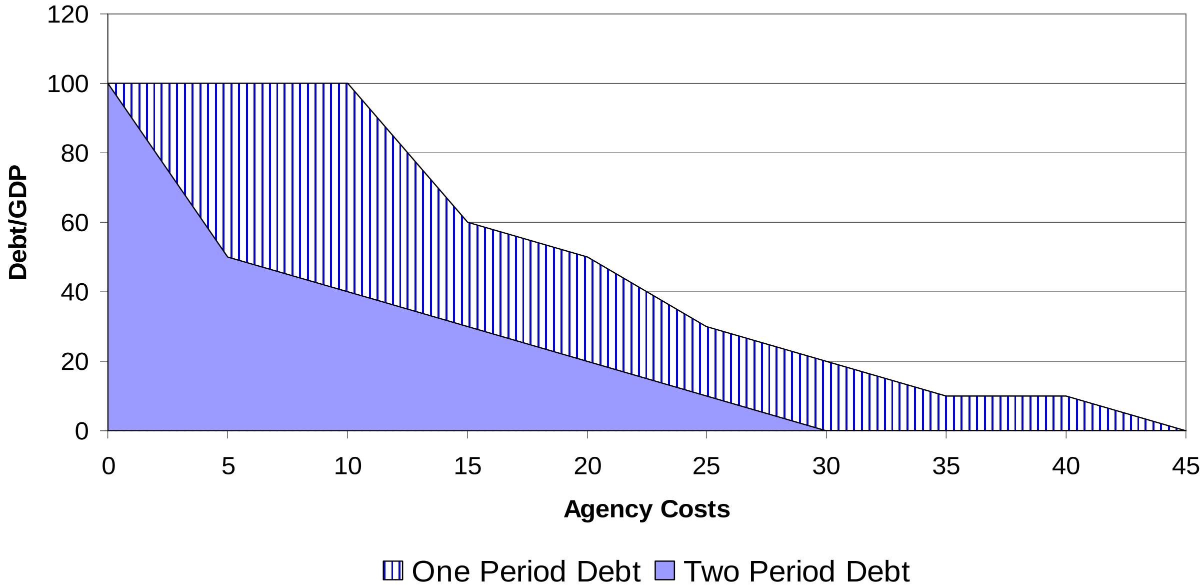


Figure 17: U.S. -- Welfare