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ABSTRACT

This paper considers the risk management problem of an investor who holds a diversified portfolio of global equities or bonds and chooses long or short positions in currencies to manage the risk of the total portfolio. Over the period 1975-2005, we find that a risk-minimizing global equity investor should short the Australian dollar, Canadian dollar, Japanese yen, and British pound but should hold long positions in the US dollar, the euro, and the Swiss franc. The resulting currency position tends to rise in value when equity markets fall. This strategy works well for investment horizons of one month to one year. In the past 15 years the risk-minimizing demand for the dollar appears to have weakened slightly, while demands for the euro and Swiss franc have strengthened. These changes may reflect the growing role for the euro as a reserve currency in the international financial system. The risk-minimizing currency strategy for a global bond investor is close to a full currency hedge, with a modest long position in the US dollar. Risk-reducing currencies have had lower average returns during our sample period, but the difference in average returns is smaller than would be implied by the global CAPM given the historical equity premium.

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1 Introduction

What role should foreign currency play in a diversified investment portfolio? In practice, many investors appear reluctant to hold foreign currency directly, perhaps because they see currency as an investment with high volatility and low average return. At the same time, many investors hold indirect positions in foreign currency when they buy foreign equities and fail to hedge the currency exposure implied by the equity holding. Such investors receive the excess return on foreign equities over foreign bills—the foreign-currency excess return on foreign equity—plus the return on foreign bills, that is, the return on foreign currency.

The academic finance literature has explored a number of reasons why investors might want to hold foreign currency.[2] These can be divided into speculative demands, resulting from positive expected excess returns on foreign currency over the minimum-variance portfolio, and risk management demands, resulting from covariances of foreign currency with other assets that investors may wish to hold.[3]

Obviously it is possible that a particular currency may have a high expected return at a particular time, generating a speculative demand for that currency. For example, the literature on the forward premium puzzle (Hansen and Hodrick 1980, Fama 1984, Hodrick 1987, Engel 1996) shows that currencies with high short-term interest rates deliver high returns on average. This type of speculative demand is inherently asymmetric. For every currency with a high expected return, there must be another with a low expected return, and investors will tend to short currencies with low expected returns just as they go long those currencies with high expected returns. Investors whose domestic currency has a low expected return will tend to go long all foreign currencies and short their own, but investors whose domestic currency has a high expected return will tend to short foreign currencies.

A unique feature of currencies, however, is that investors in each country can simultaneously perceive positive expected excess returns on foreign currencies over their own domestic currencies. That is, a US investor can perceive a positive expected excess return on euros over dollars, while a European investor can at the same time

[2]For a discussion of currency hedging from a practitioner's perspective, see Thomas (1990).

[3]Risk management demands are more commonly called hedging demands, but this can create confusion in the context of foreign currency because hedging a foreign currency corresponds to taking a short position to cancel out an implicit long position in that currency. In this paper we use foreign currency terminology and avoid the use of the term hedging demand for assets.

perceive a positive expected excess return on dollars over euros. This possibility arises from Jensen's inequality and is known as the Siegel paradox (Siegel 1972). It can explain symmetric speculative demand for foreign currency by investors based in all countries. In practice, however, the currency demand generated by this effect is quite modest. If currency movements are lognormally distributed and the expected excess *log* return on foreign currency over domestic currency is zero (a condition that can be satisfied for all currency pairs simultaneously), then the expected excess *simple* return on foreign currency is one-half the variance of the foreign currency return. With a foreign currency standard deviation of about 10% per year, the expected excess foreign currency return is 50 basis points and the corresponding Sharpe ratio is only 5%. If no other risky investments were available, an investor with log utility would put half her portfolio in foreign currency, but a conservative investor with relative risk aversion of 5 would have only a 10% portfolio weight on foreign currency.

Since conservative investors have small speculative currency demands, their foreign currency holdings are primarily explained by their desire to manage portfolio risks. One type of risk management demand arises if there is no domestic asset that is riskless in real terms, for example because only nominal bills are available and there is uncertainty about the rate of inflation. In this case, the minimum-variance portfolio may contain foreign currency (Adler and Dumas 1983). This effect can be substantial in countries with extremely volatile inflation, such as some emerging markets, but is quite small in developed countries over short time intervals. Campbell, Viceira, and White (2003) show that it can be more important for investors with long time horizons, because nominal bills subject investors to fluctuations in real interest rates, while nominal bonds subject them to inflation uncertainty which is relatively more important at longer horizons. If domestic inflation-indexed bonds are available, however, they are riskless in real terms if held to maturity and thus drive out foreign currency from the minimum-variance portfolio.

Another type of risk management demand for foreign currency arises if an investor holds other assets for speculative reasons, and foreign currency is correlated with those assets. For example, an investor may wish to hold a globally diversified equity portfolio. If the foreign-currency excess return on foreign equities is negatively correlated with the return on the foreign currency (as would be the case, for example, if stocks are real assets and the shocks to foreign currency are primarily related to foreign inflation), then an investor holding foreign equities can reduce portfolio risk by holding a long position in foreign currency.

In this paper we explore the particular demand for foreign currency that results from the desire to manage equity and bond risks. We assume that a domestic asset exists that is riskless in real terms, so that an infinitely conservative investor would hold only this asset and would hold neither equity, bonds, nor foreign currency. We consider an investor with a given portfolio of equities or bonds, and we ask what foreign currency positions this investor should hold in order to minimize the risk of the total portfolio. We consider seven major currencies, the dollar, euro, Japanese yen, Swiss franc, pound sterling, Canadian dollar and Australian dollar, over the period 1975–2005. We consider investment horizons ranging from one month to a year.

We find that our seven currencies fall along a spectrum. At one extreme, the Australian dollar and Canadian dollar are positively correlated with local-currency returns on equity markets around the world, including their own domestic markets. At the other extreme, the euro and the Swiss franc are negatively correlated with world stock returns and their own domestic stock returns. The Japanese yen, British pound, and US dollar fall in the middle with the yen and pound more similar to the Australian and Canadian dollars, and the US dollar more similar to the euro and the Swiss franc.

When we consider currencies in pairs, we find that risk-minimizing equity investors should short those currencies that are more positively correlated with equity returns and should hold long positions in those currencies that are more negatively correlated with returns. When we consider all seven currencies as a group, we find that optimal currency positions tend to be long the US dollar, the Swiss franc, and the euro, and short the other currencies. A long position in the US-Canadian exchange rate is a particularly effective hedge against equity risk.

It is striking that the dollar, the Swiss franc, and the euro are widely used as reserve currencies by central banks, and more generally as stores of value by corporations and individuals around the world. The correlations we observe in the data are consistent with the idea that shocks to risk aversion drive down equity prices and drive up the values of the major reserve currencies as investors "flee to quality". The resulting movements of the reserve currencies generate additional demand for these currencies by risk-minimizing investors with diversified international equity positions. Consistent with this story, we find that the correlation of the euro and Swiss franc with world equity markets has become more negative in the second half of our sample, as the euro has come to play a more important role in the international financial system.

Many international equity investors think not about the foreign currency positions they would like to hold, but about the currency hedging strategy they should follow. An unhedged position in international equity corresponds to a long position in foreign currency equal to the equity holding. A fully hedged position corresponds to a net zero position in foreign currency. When currencies and equities are uncorrelated, full hedging is optimal (Solnik 1974). Empirically, Perold and Schulman (1988) find that US investors can substantially reduce volatility by fully hedging the currency exposure implicit in internationally diversified equity and bond portfolios. Our empirical results imply that equity investors should more than fully hedge their exposures to the yen, pound, and Australian and Canadian dollars to achieve net short positions, but should less than fully hedge the dollar, euro, and Swiss franc to maintain net long positions in these currencies. For all base currencies, these optimal strategies deliver substantially lower total volatility than unhedged or fully hedged strategies.

When we consider the risk-minimization problem of global bond investors, we find that currency returns are only weakly correlated with bond returns. The US dollar, however, does tend to appreciate when bond prices fall, that is when interest rates rise, around the world. This generates a modest demand for dollars by bond investors. US investors in fixed-income securities should at least fully hedge their international bond positions, consistent with common institutional practice.

Finally, we consider the equilibrium implications of our results. If reserve currencies are attractive to risk-minimizing global equity investors, these currencies may offer lower returns in equilibrium. We analyze the historical average returns on currency pairs and find that high-risk pairs have delivered higher average returns. However the historical reward for taking equity beta risk in currencies has been quite modest, and much smaller than the historical average excess return on a global stock index.

The organization of the paper is as follows. Section 2 lays out the analytical framework we use for our empirical analysis. We begin by defining returns on internationally diversified portfolios of equities and currencies, then show how to work with log (continuously compounded) returns over short time intervals. We state and solve the problem of choosing currency positions to minimize portfolio variance, given a set of equity holdings. Importantly, we show conditions under which variance-minimizing currency positions do not depend on the base currency of the investor. Section 3 describes our data and conducts preliminary statistical analysis of stock returns, bond returns, and currency returns. Section 4 presents our empirical results for different equity portfolios, sets of available currencies, investment horizons, and sample periods. Section 5 repeats the analysis for bond portfolios. Section 6 relates the historical average returns on currencies to their betas with a global equity index, and section 7 concludes.

2 Mean-Variance Analysis for Currencies

We consider the problem of a domestic investor who invests in assets from n foreign countries as well as in domestic assets, and must decide how much currency risk she wants to hedge or, equivalently, her currency exposure. The investor adjusts her exposure to foreign currencies by entering into forward exchange rate contracts or, equivalently, by borrowing and lending in her own currency and in foreign currencies. For convenience, throughout this section we set the domestic country to be the US, and hence refer to the domestic investor as a US investor, and to the domestic currency as the dollar. We also use stocks when we refer to the assets held by the investor. In our empirical analysis we consider each base currency in turn, and look at both equity portfolios and bond portfolios.

In our analysis, we assume that the investor has one-period mean-variance preferences over the currency composition of her portfolio, and that she chooses her optimal exposure to foreign currencies taking as given the composition of her equity portfolio. We make these assumptions both for tractability and because they reflect the common practice of institutional investors. In future research we would like to allow for simultaneous choice of equity portfolio weights and currency ratios under more general preferences, along the lines of the models in Campbell, Chan, and Viceira (2003) and Jurek and Viceira (2005).

2.1 Portfolio returns with currency hedging

Let $R_{c,t+1}$ denote the gross return in currency c from holding country c stocks from the beginning to the end of period $t+1$, and let $S_{c,t+1}$ denote the spot exchange rate in dollars per foreign currency c at the end of period $t+1$. By convention, we index the domestic country by $c = 1$ and the n foreign countries by $c = 2, ..., n+1$. Of course, the domestic exchange rate is constant over time and equal to 1: $S_{1,t+1} = 1$ for all t.

At time t, the investor exchanges a dollar for $1/S_{c,t}$ units of currency c in the spot market which she then invests in the stock market of country c. After one period, stocks from country c return $R_{c,t+1}$, which the US investor can exchange for $S_{c,t+1}$ dollars, to earn an unhedged gross return of $R_{c,t+1}S_{c,t+1}/S_{c,t}$. For an arbitrarily weighted portfolio, the unhedged gross portfolio return is given by

$$R^{uh}_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right),$$

where $\boldsymbol{\omega}_t = \text{diag}\left(\omega_{1,t}, \omega_{2,t}, ..., \omega_{n+1,t}\right)$ is the $(n+1 \times n+1)$ diagonal matrix of weights on domestic and foreign stocks at time t, $\mathbf{R}_{t+1}$ is the $(n+1\times 1)$ vector of gross nominal stock returns in local currencies, $\mathbf{S}_{t+1}$ is the $(n+1 \times 1)$ vector of spot exchange rates, and $\div$ denotes the element-by-element ratio operator, so that the c-th element of $(\mathbf{S}_{t+1} \div \mathbf{S}_t)$ is $S_{c,t+1}/S_{c,t}$. The weights add up to 1 in each period t:

$$\sum_{c=1}^{n+1} \omega_{c,t} = 1 \qquad \forall t. \tag{1}$$

We next consider the hedged portfolio. Let $F_{c,t}$ denote the one-period forward exchange rate in dollars per foreign currency c,[4] and $\theta_{c,t}$ the dollar value of the amount of forward exchange rate contracts for currency c the investor enters into at time t per dollar invested in her stock portfolio. At the end of period $t+1$, the investor gets to exchange $\theta_{c,t}/S_{c,t}$ units of the foreign-currency denominated return $R_{c,t+1}\omega_{c,t}/S_{c,t}$ back into dollars at an exchange rate $F_{c,t}$. She then exchanges the rest, which amounts to $(R_{c,t+1}\omega_{c,t}/S_{c,t} - \theta_{c,t}/S_{c,t})$ units of foreign currency c, at the spot exchange rate $S_{c,t+1}$. Collecting returns for all countries leads to a hedged portfolio return $R^h_{p,t+1}$ of

$$R^h_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}'_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) + \boldsymbol{\Theta}'_t \left(\mathbf{F}_t \div \mathbf{S}_t\right), \tag{2}$$

where $\mathbf{F}_t$ is the $(n+1\times 1)$ vector of forward exchange rates, and $\boldsymbol{\Theta}_t = (\theta_{1,t}, \theta_{2,t}, ..., \theta_{n,t}, \theta_{n+1,t})'$. Of course, since $S_{1t} = F_{1,t} = 1$ for all t, the choice of domestic hedge ratio $\theta_{1,t}$ is arbitrary. For convenience, we set it so that all hedge ratios add up to 1:

$$\theta_{1,t} = 1 - \sum_{c=2}^{n+1} \theta_{c,t}. \tag{3}$$

[4] That is, at the end of month t, the investor can enter into a forward contract to sell one unit of currency c at the end of month $t+1$ for a forward price of $F_{c,t}$ dollars.

Under covered interest parity, the forward contract for currency c trades at $F_{c,t} = S_{c,t}(1+I_{1,t})/(1+I_{c,t})$, where $I_{1,t}$ denotes the domestic nominal short-term riskless interest rate available at the end of period t, and $I_{c,t}$ is the corresponding country c nominal short-term interest rate. Thus the hedged dollar portfolio return (2) can be written as

$$R^h_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}'_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) + \boldsymbol{\Theta}'_t\left[\left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right], \tag{4}$$

where $\mathbf{I}_t = (I_{1,t}, I_{2,t}..., I_{n+1,t})$ is the $(n+1 \times 1)$ vector of nominal short-term interest rates and $\mathbf{I}^d_t = I_{1,t}\mathbf{1}$.

Equation (4) shows that selling currency forward—i.e., setting $\theta_{c,t} > 0$—is analogous to a strategy of shorting foreign bonds and holding domestic bonds, i.e. borrowing in foreign currency and lending in domestic currency.[5] That the hedged portfolio includes long and short positions in domestic and foreign bonds is intuitive. A long foreign stock position implies a long position in the currency of that country; thus an investor can hedge this currency exposure by simultaneously shorting bonds denominated in that currency and investing the proceeds in bonds denominated in her domestic currency.

By convention, an investor is said to fully hedge the currency risk exposure in her foreign stock portfolio when she sets $\theta_{c,t} = \omega_{c,t}$. Note that when $\omega_{c,t} > 0$, full currency hedging of the stock position implies that the investor shorts currency c one for one with the currency position implicit in her long stock market investment in country c at time t. Of course, the investor has not literally fully hedged all currency risk in her foreign stock investment, because this position will fluctuate with the realized return at time $t+1$. For example, if the stock return is positive, the units of currency c held by the investor at time $t+1$ will exceed $\omega_{c,t}/S_{c,t}$. The investor then benefits if the exchange rate has increased, and loses otherwise. It is also important to note that currency hedging instruments, whether bonds or forward contracts, are imperfect because they imply an exposure to the foreign risk-free interest rate that

[5]Note, however, that the two strategies are not completely equivalent except in the continuous time limit. Let us write the hedged return for an investor borrowing $\boldsymbol{\Theta}_{c,t}$ dollars (i.e. shorting bonds) in foreign currency c and lending $\boldsymbol{\Theta}_{c,t}$ dollars in domestic currency (i.e. holding domestic bonds) for each dollar invested in her stock portfolio. The return on this strategy is

$$R^{BL}_{p,t+1} = \mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \boldsymbol{\Theta}'_t\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right)\left(1 + \mathbf{I}_t\right) + \boldsymbol{\Theta}'_t\left(1 + \mathbf{I}^d_t\right),$$

which is slightly different from that of an investor hedging through forward contracts. We show in the appendix that, in continuous time, the two strategies are exactly equivalent.

cannot be separated from the pure exchange rate risk. Similarly, the investor is said to under-hedge currency risk when $\theta_{c,t} < \omega_{c,t}$, and to over-hedge when $\theta_{c,t} > \omega_{c,t}$.

To capture the fact that the investor can alter the currency exposure implicit in her foreign stock position using forward contracts or lending and borrowing, we now define a new variable $\psi_{c,t}$ as $\psi_{c,t} \equiv \omega_{c,t} - \theta_{c,t}$. A fully hedged portfolio, in which the investor does not hold any exposure to currency c, corresponds to $\psi_{c,t} = 0$. A positive value of $\psi_{c,t}$ means that the investor wants to hold exposure to currency c, or equivalently that the investor does not want to fully hedge the currency exposure implicit in her stock position in country c. Of course, a completely unhedged portfolio corresponds to $\psi_{c,t} = \omega_{c,t}$. Thus $\psi_{c,t}$ is a measure of currency demand or currency exposure. Accordingly we refer to $\psi_{c,t}$ as currency demand or currency exposure indistinctly.

For convenience, we now rewrite equation (4) in terms of currency demands:

$$\begin{aligned} R^h_{p,t+1} &= \mathbf{R}'_{t+1}\boldsymbol{\omega}_t \left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \mathbf{1}'\boldsymbol{\omega}_t \left[\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right] \\ &\quad + \boldsymbol{\Psi}'_t \left[\left(\mathbf{S}_{t+1} \div \mathbf{S}_t\right) - \left(\mathbf{1} + \mathbf{I}^d_t\right) \div \left(\mathbf{1} + \mathbf{I}_t\right)\right], \end{aligned}$$

where $\boldsymbol{\Psi}_t = \left(\psi_{1,t}, \psi_{2,t}, ..., \psi_{n+1,t}\right)'$.

Note that $\boldsymbol{\Psi}_t = \boldsymbol{\omega}_t\mathbf{1} - \boldsymbol{\Theta}_t$. Given the definition of $\psi_{c,t}$, equations (1) and (3) imply that

$$\psi_{1,t} = -\sum_{c=2}^{n+1} \psi_{c,t}. \tag{5}$$

or $\Psi'_t\mathbf{1} = \mathbf{0}$, so that $\psi_{1,t}$ indeed represents the domestic currency exposure. That currency demands must add to zero is intuitive. Since the investor is fully invested in stocks, she can achieve a long position in a particular currency c only by borrowing—or equivalently, by shorting bonds—in her own domestic currency, and investing the proceeds in bonds denominated in that currency. Thus the currency portfolio is a zero investment portfolio. Section 2.2 next develops this point in more detail.

2.2 Log portfolio returns over short time intervals

For convenience, we work with log (or continuously compounded) returns, interest rates, and exchange rates, which we denote with lower case letters. To this end, we compute a log version of equation (4) which holds exactly in the continuous time limit where investors adjust their hedge ratios continuously, and it is approximate otherwise.

We show in the appendix that the continuously compounded (or log) hedged portfolio excess return over the domestic interest rate is approximately equal to

$$r_{p,t+1}^{h} - i_{1,t} = \mathbf{1}'\boldsymbol{\omega}_t \left(\mathbf{r}_{t+1} - \mathbf{i}_t\right) + \boldsymbol{\Psi}_t' \left(\Delta\mathbf{s}_{t+1} + \mathbf{i}_t - \mathbf{i}_t^d\right) + \frac{1}{2}\Sigma_t^h, \tag{6}$$

where bold case letters denote the column vector of $(n+1)$ country observations, and small case letters denote logs. Thus $\mathbf{r}_{t+1} = \log\left(\mathbf{R}_{,t+1}\right)$, $\Delta\mathbf{s}_{t+1} = \log\left(\mathbf{S}_{t+1}\right) - \log\left(\mathbf{S}_t\right)$, and $\mathbf{i}_t = \log\left(1 + \mathbf{I}_t\right)$ and $\mathbf{i}_t^d = \log\left(1 + I_{1,t}\right)\mathbf{1}$.

Equation (6) provides an intuitive decomposition of the hedged portfolio excess return. The first term represents the excess return on a fully hedged stock portfolio. The second term involves only the vector of excess returns on currencies, $\Delta\mathbf{s}_{t+1} + \mathbf{i}_t - \mathbf{i}_t^d$, and thus represents pure currency exposure. Recall that $\psi_{c,t}$ is the position taken in currency c in excess of perfect hedging, for $c = 1, 2..., n+1$. Of course, this term vanishes when the investor chooses to avoid currency exposure and sets $\boldsymbol{\Psi}_t$ to a vector of zeroes. Finally, the third term in equation (6) is a Jensen's variance correction equal to

$$\begin{aligned} \Sigma_t^h = {} & \mathbf{1}'\boldsymbol{\omega}_t \operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{r}_{t+1} + \boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right) - \left(-\boldsymbol{\Psi}_t + \boldsymbol{\omega}_t \mathbf{1}\right)' \operatorname{diag}\left(\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right) \\ & - \operatorname{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} + \mathbf{i}_t^d - \mathbf{i}_t\right) + \boldsymbol{\Psi}_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1} - \mathbf{i}_t^d + \mathbf{i}_t\right)\right). \end{aligned} \tag{7}$$

Note that in continuous time investors can exactly hedge the currency positions implied by their stock portfolios as long as stock prices have continuous paths. We can see this by setting $\boldsymbol{\Psi}_t$ to a vector of zeroes in equation (6). Over discrete intervals, hedging is only approximate, but it can be highly accurate at reasonably rebalancing frequencies such as weekly or monthly. We will assume a monthly rebalancing frequency in our empirical analysis.

2.3 Mean-variance optimization

We consider the optimal currency exposure for a given stock portfolio. In terms of the expression for log hedged portfolio return (6), we assume that the vector $\boldsymbol{\omega}_t$ of portfolio weights is given, and that the choice variable is Ψ_t, the vector of currency demands. More specifically, we assume that the investor optimally chooses each period t a vector of currency demands

$$\widetilde{\mathbf{\Psi}}_t = \left(\psi_{2,t}, ..., \psi_{n+1,t}\right)'$$

to minimize the conditional variance of the log excess return on the hedged portfolio over that period, subject to a constraint on the expected return. Note that the demand for domestic currency $\psi_{1,t}$ is not included because it is given once the other currency demands are determined.

Formally, the investor solves the following mean-variance problem:

$$\begin{aligned} &\min_{\widetilde{\Psi}_t} \frac{1}{2} \operatorname{Var}_t \left(r^h_{p,t+1} - i_{1,t}\right) \\ \text{s.t. } &\operatorname{E}_t \left(r^h_{p,t+1} - i_{1,t}\right) + \frac{1}{2} \operatorname{Var}_t \left(r^h_{p,t+1} - i_{1,t}\right) = \mu^h_p. \end{aligned}$$

The Lagrangian associated with this problem is

$$\begin{aligned} \pounds\left(\widetilde{\Psi}_t\right) &= \frac{1}{2} \operatorname{Var}_t \left(r^h_{p,t+1}\right) + \lambda \left[\mu^h_p - \operatorname{E}_t \left(r^h_{p,t+1} - i_{1,t}\right) - \frac{1}{2} \operatorname{Var}_t \left(r^h_{p,t+1}\right)\right] \\ &= \frac{1}{2} (1-\lambda) \operatorname{Var}_t \left(r^h_{p,t+1}\right) + \lambda \left[\mu^h_p - \operatorname{E}_t \left(r^h_{p,t+1} - i_{1,t}\right)\right], \end{aligned}$$

where the multiplier λ is typically interpreted as a measure of the investor's risk tolerance.

Simple algebraic manipulation of the problem shown in the appendix leads to the following vector of optimal mean-variance currency demands:

$$\begin{aligned} \widetilde{\mathbf{\Psi}}^*_t(\lambda) &= \lambda \operatorname{Var}_t \left(\widetilde{\Delta\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right)^{-1} \left[\operatorname{E}_t \left(\widetilde{\Delta\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right) + \frac{1}{2} \operatorname{diag}\left(\operatorname{Var}_t \widetilde{\Delta\mathbf{s}}_{t+1}\right)\right] \\ &\quad - \operatorname{Var}_t \left(\widetilde{\Delta\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right)^{-1} \left[\operatorname{Cov}_t \left(\mathbf{1}'\boldsymbol{\omega}_t \left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \left(\widetilde{\Delta\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^d_t\right)\right)\right] \end{aligned} \tag{8}$$

where we denote by $\widetilde{M}$ the $(n \times m)$ submatrix that selects rows 2 to $n+1$ of the corresponding $(n+1 \times m)$ matrix M, i.e., $\widetilde{M}$ includes the values of M corresponding to foreign countries only.

Equation (8) shows that the optimal mean-variance demand for currency has two components that correspond to two possible motives to take on currency risk. The first component is a speculative demand that is proportional to the expected excess currency return. The investor wants to hold currency risk in proportion to the Sharpe ratio of the excess return on foreign currency over the domestic interest rate, and in proportion to her risk tolerance λ.

The speculative component of currency demand is zero when the expected excess return on foreign currency over domestic bonds is zero or, equivalently, when uncovered interest parity (UIP) holds. To see this, note that UIP implies that the forward rate $F_{c,t}$ is an unbiased predictor of the spot rate $S_{c,t+1}$,

$$\mathrm{E}_t\left(S_{c,t+1}\right) = F_{c,t} = S_{c,t}\left(1 + I_{1,t}\right) / \left(1 + I_{c,t}\right), \qquad c = 1, ..., n+1, \tag{9}$$

which we can rewrite in logs and in vector form as

$$\mathrm{E}_t\left(\mathbf{s}_{t+1}\right) = \mathbf{f}_t = \mathbf{s}_t + \mathbf{i}_t^d - \mathbf{i}_t - \frac{1}{2}\,\mathrm{diag}\left(\mathrm{Var}_t\left(\mathbf{s}_{t+1}\right)\right). \tag{10}$$

When equation (10) holds, the term in brackets in (8) is zero.

It is important to note that UIP as we have defined it in (9) cannot hold simultaneously for all base currencies. This is known as Siegel's paradox (Siegel 1972); it results from the facts that an exchange rate is a ratio of two prices, and that the expectation of the inverse of a ratio differs from the inverse of the expectation of that ratio when there is uncertainty. Thus speculative demand cannot be zero for all base currencies.

The second component of currency demand corresponds to a risk management (RM) demand for currency aimed at minimizing total portfolio return volatility regardless of expected return. For convenience, we rewrite this component of currency demand separately as

$$\widetilde{\boldsymbol{\Psi}}^*_{RM,t} = -\,\mathrm{Var}_t\left(\widetilde{\Delta\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}_t^d\right)^{-1}\left[\mathrm{Cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \left(\widetilde{\Delta\mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}_t^d\right)\right)\right]. \tag{11}$$

Equation (11) shows that, for given portfolio weights, $\widetilde{\Psi}^*_{RM,t}$ is proportional to the negative of the covariance between portfolio stock returns and exchange rates or, since portfolio weights are predetermined, between the returns on the stocks held in the portfolio and exchange rates. If stock returns and exchange rates are uncorrelated, the RM component of currency demand is zero. In this case holding currency exposure adds volatility to the investor's portfolio and, unless this volatility is compensated, the investor is better off holding no currency exposure at all or, equivalently, fully hedging her portfolio.

If stock returns and exchange rates are positively correlated, the domestic currency tends to appreciate when the foreign stock market falls. Thus the investor can reduce portfolio return volatility by over-hedging, that is, by shorting foreign currency in excess of what would be required to fully hedge the currency exposure implicit in her stock portfolio. Conversely, a negative correlation between stock returns and exchange rates implies that the foreign currency appreciates when the foreign stock market falls. Then the investor can reduce portfolio return volatility by under-hedging, that is, by holding foreign currency.

In our subsequent empirical analysis, we ignore the speculative component of currency demand, and instead focus exclusively on the risk management component of currency demand (11). We ignore the speculative component of currency demand for two reasons. First, this demand depends on expected excess returns on currencies, which are notoriously difficult to estimate. Second, many institutional investors do not have a strong opinion about the expected excess return on currencies, and instead are primarily interested in determining the degree of currency exposure that minimizes portfolio return volatility. That is, they are exclusively interested in the RM component of currency demand. In the rest of the paper we will refer to the RM component of currency demand simply as optimal currency demand or currency exposure.

2.4 Estimating optimal currency demands

Our empirical analysis is based on the estimation of optimal currency demands for a set of stock portfolios and currencies. To facilitate estimation, we make some additional assumptions about the conditional moments of stock returns and exchange rates that allow us to move from conditional moments to unconditional moments. First, we assume that the risk premia on stock returns over the local risk-free rate are constant over time; second, we assume that expected excess currency returns are also constant; third, we assume that the second moments of currency excess returns and the covariances of portfolio returns with currency excess returns are constant.

Under these assumptions, we can rewrite optimal currency demands (11) in terms of unconditional moments of returns and exchange rates as follows:

$$\widetilde{\mathbf{\Psi}}^{*}_{RM,t} = -\operatorname{Var}\left(\widetilde{\Delta \mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^{d}_t\right)^{-1} \operatorname{Cov}\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1} - \mathbf{i}_t\right), \widetilde{\Delta \mathbf{s}}_{t+1} + \widetilde{\mathbf{i}}_t - \widetilde{\mathbf{i}}^{d}_t\right). \quad (12)$$

Equation (12) shows that we can compute optimal currency exposures by estimating simple regression coefficients of portfolio excess returns $\mathbf{1}'\boldsymbol{\omega}_t(\mathbf{r}_{t+1} - \mathbf{i}_t)$, where returns are measured as local excess stock returns $r_{c,t+1} - i_{c,t}$, onto a constant and the vector of currency excess returns $\widetilde{\Delta \mathbf{s}}_{t+1} - \widetilde{\mathbf{i}}^{d}_t + \widetilde{\mathbf{i}}_t$, and switching the sign of the slopes.

A useful property of these optimal currency demands, proven in the appendix, is that for a given stock portfolio, they are invariant to changes in the base currency, provided that the set of available currencies (which always includes an investor's own domestic currency) does not change. If we restrict the set of available currencies to a pair, for example the US dollar and the euro, this means that residents of both the US and Germany will have the same optimal demands for dollars and euros corresponding to a given equity portfolio. Residents of a third country, however, have another domestic currency available to them and so they will not necessarily have the same demands for dollars and euros even if they hold the same equity portfolio. If we allow a larger set of available currencies, then residents of all the countries in the set will have the same vector of optimal currency demands for a given equity portfolio.

In our empirical analysis we consider several particular cases of (12) of practical relevance. First, we consider the case of an investor who is fully invested in a single-country stock portfolio and optimally decides how much exposure to a single currency c to hold in order to minimize total portfolio return volatility. In that case (12) reduces to

$$\psi^{*}_{RM,t} = -\frac{\operatorname{Cov}\left(r_{1,t+1} - i_{1,t}, \Delta s_{c,t+1} + i_{c,t} - i_{1,t}\right)}{\operatorname{Var}\left(\Delta s_{c,t+1} - i_{1,t} + i_{c,t}\right)}, \quad (13)$$

where for simplicity we assume that the stock market is the investor's own domestic stock market.

Thus the optimal currency demand is given by the negative of the slope coefficient estimated by a regression of the local excess stock return on the domestic market onto a constant and the excess return on currency c. A positive value of $\psi^{*}_{RM,t}$ means that the investor can reduce the volatility of her single-country stock portfolio by simultaneously borrowing $\psi^{*}_{RM,t}$ units of her own domestic currency per dollar invested in the domestic stock market, and investing them in bills denominated in currency c. We label this case as "single-country stock portfolio, single foreign currency."

Second, we consider the case of an investor who is fully invested in a single-country stock portfolio, so that $\mathbf{r}_{t+1} - \mathbf{i}_t$ in (12) is unidimensional, and uses the whole range of available currencies to minimize total portfolio return volatility. In that case the vector of optimal currency demands is given by the negative of the slopes of a multiple regression of the excess stock return on the domestic market onto a constant and the vector of currency excess returns. We label this case as "single-country stock portfolio, multiple currencies."

Third, we consider a case where the investor holds a global portfolio of stocks with equal or value weights, using the whole vector of available currencies to minimize total portfolio return volatility. We label this case as "world portfolio, multiple currencies." In the case of the regression analysis with the value-weighted portfolio, we assume that the covariance of the returns on this portfolio with the vector of currency excess reutrns is constant. Note that this assumption does not follow automatically from the assumption we use to justify the analysis with single-country stock portfolios—i.e., that the conditional covariances of single-country stock portfolios with the vector of currency excess returns are constant. However, our subsequent empirical analysis shows that in practice both types of assumptions are not incompatible, because our empirical results for equally-weighted and value-weigthed portfolios are fairly similar.

3 Data and Summary Statistics

Our empirical analysis uses data on exchange rates, short-term interest rates and yields on long-term bonds from the International Financial Statistics database published by the International Monetary Fund, and stock return data from Morgan Stanley Capital International.[6] We calculate log bond returns from yields on long-term

[6]In the case of the Swiss short-term interest rate, our data source is the OECD. We use euromoney rates up to 1989, and LIBOR rates afterwards, as published by the OECD.

bonds using the approximation suggested in Campbell, Lo and MacKinlay (1997).[7] These data series are available at a monthly frequency, but we consider several different investment horizons. Our basic analysis uses a one-quarter horizon and therefore runs monthly regressions of overlapping quarterly excess returns. We report results for seven countries: Australia, Canada, Euroland, Japan, Switzerland, the UK and the US. The sample period is 1975:7-2005:12, the longest sample period for which we have data available for all variables and for all seven markets.

We define "Euroland" as a value-weighted stock basket that includes Germany, France, Italy, and the Netherlands. These are the countries in the euro zone for which we have the longest record of stock total returns, interest rates, and exchange rates. For simplicity, we will refer to the Euroland stock portfolio as a "country" stock portfolio when describing our empirical results, even though this is not literally correct. With regard to currencies, prior to 1999 we refer to a basket of currencies from those countries, with weights given by their relative stock market capitalization, as euro. Of course, our definition of Euroland implies some look-ahead bias, since in 1975 it would not have been obvious whether a European monetary union would occur, and which countries from the region would have been part of that union. However, one can reasonably argue that these countries would have been candidates, and that from the perspective of today's investors, it probably makes sense to consider these markets as a single market. We have also conducted our analysis including only Germany in Euroland, and using the deutschmark to proxy for the euro before 1999; this procedure gives very similar results.

Table 1 reports the full-sample annualized mean and standard deviation of short-term nominal interest rates, log stock and bond returns in excess of their local short-term interest rates, changes in log exchange rates with respect to the US dollar, and

[7] This approximation to the log return on a coupon bond is

$$r_{c,n,t+1} = D_{cn} y_{cnt} - (D_{cn} - 1)\, y_{c,n-1,t+1},$$

where $r_{c,n,t+1}$ denotes the log return on a coupon bond with coupon rate c and n periods to maturity, $y_{cnt} \equiv \log(1 + Y_{cnt})$ denotes the log yield on this bond at time t, and D_{cn} is its duration, which we approximate as

$$D_{cn} = \frac{1 - (1 + Y_{cnt})^{-n}}{1 - (1 + Y_{cnt})^{-1}}.$$

In our computations we treat all bonds as having a maturity of 10 years, and assume that bonds are issued at par, so that the coupon rate equals the yield on the bond. We also assume that the yield spread between a 9 years and 11 months bond and a 10-year bond is zero.

currency excess returns with respect to the dollar. Annualized average nominal short-term interest rates differ across countries. They are lowest for Switzerland and Japan, and highest for Australia, Canada, and the UK.[8] But all short-term rates exhibit very low annualized volatility, 1% or less for all countries.

Annualized average stock excess returns are around 7% per annum for most countries except Canada and Japan, with a 5% annualized equity premium, and Switzerland, with 8.4%. Annual stock excess return volatilities are in the range 15%-20%, with the US market showing the lowest volatility, and Australia the largest. Annualized average bond excess returns are small compared to average stock excess returns, and very similar across all countries except Euroland at about 2.7% per annum. The average bond excess return in Euroland is considerably lower at 1%. By contrast, the volatilities of these bond excess returns vary widely across countries. Euroland and Switzerland exhibit the lowest volatilities at about 4% and 5% per annum. Australian bonds had a volatility of almost 10% per annum over this period. All other countries exhibit volatilities around 7%.

Average changes in exchange rates with respect to the US dollar over this period are negative for the Australian dollar, the Canadian dollar and the British pound, reflecting an appreciation of the US dollar with respect to these currencies over this period, essentially zero for the euro, and positive for the the Swiss franc and the yen. Exchange rate volatility relative to the dollar is around 11% for all currencies except the Canadian dollar, which moves closely with the US dollar giving a bilateral volatility of only 5.4%. Excess returns to currencies are small on average and exhibit annual volatility similar to that of exchange rates, a result of the stability of short-term interest rates. Using the usual formula for the mean of a serially uncorrelated random variable, it is easy to verify that average excess returns to currencies are insignificantly different from zero.

Table 2 reports the full-sample monthly correlations of foreign currency excess returns, $\Delta \mathbf{s}_{t+1} + \mathbf{i}_t - \mathbf{i}_t^d$ in our notation. We report currency return correlations for each base currency. Table 2 shows that all currency returns are positively cross-correlated. These correlations are large—almost all correlation coefficients are above 30%—but they are far from perfect, implying that we have significant cross-sectional variation in the dynamics of exchange rates. Three correlations stand out as unusually large. The Canadian dollar exhibits a very high correlation with the US dollar (85-

[8]If we include only Germany in Euroland, this region also exhibits one of the lowest average short-term nominal interest rates, similar to those of Japan and Switzerland.

91%) regardless of the base currency used to measure exchange rates. It also exhibits a high degree of correlation with the Australian dollar (70-77%), except when the base country used to measure exchange rates is the US. The high correlation of the Canadian dollar with both the US dollar and the Australian dollar reflects the dual role of the Canadian economy as a resource dependent economy that is simultaneously highly integrated with the US. The third high correlation is between the Swiss franc and the euro (84-92%), reflecting the integration of the Swiss economy with the rest of Euroland.

Table 3 reports full-sample quarterly correlations of stock market returns (Panel A) and bond returns (Panel B), both of them denominated in local currency. The correlation coefficients between stock market returns shown in Panel A are all between 30% and 60%, again with two important exceptions. The Canadian stock market is highly correlated with the US stock market (73%), and the Swiss stock market is highly correlated with the Euroland stock market (75%). The Canadian stock market also exhibits a large correlation of almost 60% with the Australian stock market. These correlations demonstrate again the dual role of the Canadian economy and the integration of the Swiss economy with the European economy.

While significant, the stock market correlations are still small enough to suggest the presence of substantial benefits of international diversification in this sample period. Not surprisingly, the Japanese stock market exhibits the lowest cross-sectional correlation with all other markets. This is a reflection of the prolonged period of low or negative stock market returns in Japan during the 1990's, at a time when most other markets delivered large positive returns.

Long-term bond market correlations are smaller than stock market correlations. Panel B in Table 3 shows that, with some important exceptions, these correlations are all in the range of 15%-40%. The exceptions are the Euroland bond market, which is highly correlated with both the Swiss bond market (50%) and the US bond market (55%), and the Canadian bond market, which is highly correlated with the US bond market (62%). Nevertheless, even these correlations are not as large as the corresponding stock market correlations. These results imply that there are large benefits to international diversification in bond market investing.

4 Optimal Currency Demands for Equity Investors

4.1 Single-country equity portfolios

We start our empirical analysis of optimal currency demand by examining the case of an investor who is fully invested in a single-country equity portfolio and is considering whether exposure to other currencies would help reduce the volatility of her portfolio return. We assume that the investor has a horizon of one quarter.

Table 4 reports optimal currency exposures for the case in which the investor is considering one currency at time (Panel A), and that in which she is considering multiple currencies simultaneously (Panel B). That is, Panel A reports the regression coefficient (13) while Panel B reports the vector (12) with the stock portfolio comprising a single stock market. In both panels, the reference stock market is reported at the left of each row, while the currency under consideration is reported at the top of each column. In all tables we report Newey-West heteroskedasticity and autocorrelation consistent standard errors in parentheses below each optimal currency exposure. Starred coefficients are those for which we reject the null of zero at a 5% significance level.

Panel A in Table 4 considers the case of an investor who is interested in finding how much to hedge of the currency exposure implicit in an investment in a specific stock market, in isolation of other investments and currencies this investor might hold. To facilitate the interpretation of this table and the remaining tables in the empirical sections, it is useful to recapitulate the exact interpretation of the coefficients shown in this table using a specific example. The first non-empty cell in the first column of the table, which corresponds to the Australian stock market and the euro, has a value of 0.39. This means that a risk-minimizing investor who is fully invested in the Australian stock market and has access to the Australian dollar and the euro should short (or borrow) 0.39 Australian dollars worth of Australian bills for each dollar of stock market exposure, and invest the proceeds of this short position in euro-denominated bills—for example, German bills. That is, the portfolio return minimizing strategy for this investor implies that she should optimally over-hedge the Australian dollar exposure implicit in her Australian stock market investment, and hold a net long 39% exposure to the euro.

Panel A of Table 4 shows that optimal demands for foreign currency are large, positive and statistically significant for two stock markets (rows of the table), those of Australia and Canada. Investors in the Australian and Canadian stock markets are keen to hold foreign currency, regardless of the particular currency under consideration, because the Australian and Canadian dollars tend to depreciate against all currencies when their stock markets fall; thus any foreign currency serves as a hedge against fluctuations in these stock markets. At the opposite extreme, it is optimal for investors in the Swiss stock market to hold economically and statistically large short positions in all currencies, implying that the Swiss franc tends to appreciate against all currencies when the Swiss stock market falls. Results are similar for the Euroland stock market, except that this market is hedged by a long position in the Swiss franc. The Japanese and UK stock markets generate large positive demands for the Swiss franc and the euro, and negative or small positive demands for all other currencies. The Brisith stock market generates significant negative demands for the Australian dollar and the Canadian dollar.

The last row of this panel describes individual optimal currency demands for a portfolio fully invested in US stocks. Most of these demands are economically small and statistically insignificant, but there are two important exceptions to this pattern. The first exception is a modest positive demand for the Swiss franc, which tends to appreciate when the US stock market falls. (The euro generates a similar demand, but it is not statistically significant.) The second exception is a large negative demand for the Canadian dollar, reflecting the fact that the Canadian dollar tends to depreciate when the US stock market falls.

Panel B of Table 4 reports optimal currency demands for single-country stock portfolios considering all currencies simultaneously. That is, each row of Panel B reports (12) when $\mathbf{r}_{t+1}$ is unidimensional and equal to the stock market shown on the leftmost column. Panel B shows that, when single-country stock market investors consider investing in all currencies simultaneously, they almost always choose positive exposures to the US dollar, the euro and the Swiss franc, and negative exposures to the Australian dollar, Canadian dollar, British pound, and Japanese yen. By contrast with Panel A, the optimal currency demands are largest, both economically and statistically, for the US dollar (on the long side) and the Canadian dollar (on the short side), while they are considerably smaller for the euro and the Swiss franc. This reflects two features of the multiple-currency analysis. First, a position that is long the US dollar and short the Canadian dollar is a highly effective hedge against movements in stock markets around the world. Second, the euro and Swiss franc are both good hedges but they are highly correlated; thus the demand for each currency is smaller, and less statistically significant, when investors are allowed to take positions in both currencies. This result is consistent with the notion that investors see the euro and the Swiss franc as substitutes for one another.

4.2 Global equity portfolios

Thus far we have considered only investors who are fully invested in a single country stock market, and use currencies to hedge the risk of that stock market. In this section we consider investors who hold internationally diversified stock portfolios, and optimally choose their currency exposure in order to minimize their portfolio return variance.

We start our analysis considering an investor who is equally invested in the seven stock markets included in our analysis: Euroland, Australia, Canada, Japan, Switzerland, the UK, and the US. Table 5 reports optimal currency demands for such an investor optimizing at different investment horizons ranging from 1 month to 12 months. We have already noted in Section 2.4 that, in the multiple-currency case, optimal currency demands generated by a given global stock portfolio are the same regardless of the currency base. Accordingly, we only need to report one set of currency demands for each investment horizon. Note that the identity (5) implies that the numbers in each row add up to zero.

Panel A of Table 5 considers the case in which investors have access to all seven currencies from the countries included in the equally-weighted stock portfolio. Panel B excludes Canada and Switzerland from the analysis because the Canadian stock market is highly correlated with the US stock market, and the Canadian dollar is also highly correlated with the US dollar; similarly, there is a very high positive correlation between the Swiss stock market and the Euroland market, and between the Swiss franc and the euro. Thus Panel B considers a case in which investors do not have close currency substitutes available for investment. This helps us understand the role of the Canadian dollar and the US dollar, and the euro and the Swiss franc in investors' portfolios.

Panel A of Table 5 shows that the optimal currency exposure associated with the equally-weighted world portfolio implies a large, statistically significant exposure to the dollar at horizons up to a quarter, and smaller, not statistically significant exposures at longer horizons. The dollar exposure is highest at a one-month horizon at 60% of the value of the equity portfolio, but is still very large at a one quarter horizon at 40%. Simultaneously, the equally-weighted portfolio also implies a large statistically significant negative exposure to the Canadian dollar at horizons up to a quarter. These two positions are not independent of each other: Panel B shows that, once we exclude the Canadian dollar from the menu of currencies available to the investor, the optimal exposure to the US dollar becomes small and statistically insignificant. These results are consistent with those implied by single country stock portfolios, and once again suggest that a position that is long the US dollar and short the Canadian dollar helps investors hedge against global stock market movements.

Panel A also shows that the equally-weighted stock portfolio implies positive exposures to the euro and the Swiss franc. These exposures are not large or statistically significant individually (except for the Swiss franc at a 12-month horizon), because the euro and the Swiss franc are close substitutes. Panel B shows that when the Swiss franc is excluded from the menu of currencies, the demand for the other currency in the pair, the euro, increases dramatically and is statistically significant at all horizons.

Both Panel A and Panel B show that, in addition to the optimal negative exposure to the Canadian dollar already discussed, the optimal exposures to the Australian dollar, the Japanese yen, and the British pound are also negative. These short positions are small and statistically insignificant for the Australian dollar and the pound, but they become larger and more significant as the investment horizon increases for the yen.

Once again, it is useful to recapitulate the exact meaning of the numbers we report to facilitate the discussion of the results. The numbers shown in Table 5 are optimal currency exposures. If it is optimal for all investors to fully hedge the currency exposure implicit in their stock portfolios or, equivalently, to hold no currency exposure, the optimal currency demands shown in Table 5 should be equal to zero everywhere. To obtain optimal currency hedging demands from optimal currency exposures, we need only compute the difference between portfolio weights—which in this case are 14.3% for each country stock market—and the optimal currency exposure corresponding to that country.

If we focus on a one-month horizon, the results in Panel A imply that, say, a Euroland investor holding our equally-weighted seven-country portfolio would borrow in other currencies an amount worth 100 euro cents per euro invested in the stock portfolio, and use the proceeds to buy US T-bills worth 60 euro cents, Euroland (say, German) bills worth 17 euro cents, and Swiss bills worth 23 euro cents. These purchases would be financed with proceeds from borrowing Australian dollars (16 euro cents per euro invested in the stock portfolio), Canadian dollars (61 cents), yen (11 cents) and British pounds (11 cents).

We can easily restate these results in terms of hedging demands. For each dollar invested in the stock portfolio, this Euroland investor would underhedge her exposure to the dollar, and overhedge her exposure to the Australian dollar, the Canadian dollar, the yen and the British pound. More precisely, this Euroland investor would not only not hedge the 14% dollar exposure implied by the stock portfolio, but she would also enter into forward contracts to buy dollars worth today 46 euro cents. She would simultaneously enter into forward contracts to sell Australian dollars, Canadian dollars, yen and British pounds worth today, respectively, 30, 75, 25, and 25 euro cents per euro invested in the stock portfolio.

Overall, Table 5 shows that for all horizons the optimal currency exposure associated with the equally-weighted world portfolio implies long exposure to the US dollar and the euro (or a combination of the euro and the Swiss franc), a large short position in the Canadian dollar, and smaller short positions in all other major currencies.

It is also interesting to examine the variance-minimizing currency exposures implied by a value-weighted portfolio of international stocks. Table 6, whose structure is identical to that of Table 5, reports optimal currency exposures implied by this value-weighted world portfolio. Optimal currency exposures for the value-weighted portfolio are qualitatively and quantitatively similar to those for the equally-weighted portfolio. Investors want economically and statistically significant long exposures to the dollar, the euro, and the Swiss franc, and negative exposures to the yen, the Australian dollar, the Canadian dollar and the British pound. That is, they want to underhedge their exposure to the dollar, the euro, and the Swiss franc, and overhedge their exposure to the other currencies.

The similarity between the results for the equally weighted portfolio shown in Table 5 and the results for the value weighted portfolio shown in Table 6 derives from the fact that, with the exception of the US stock market, no single stock market overwhelmingly dominates the market capitalization of the overall portfolio. On average, the US stock market represents 49.3% of total market capitalization (and 54.5% at the end of our sample period). The Japanese, Euroland and British stock markets follow with weights of 20.6% (and 11.3% at the end of our smaple period), 12.9% (13.7%), and 9.7% (11.3%), respectively. The Australian, Canadian and Swiss markets are much smaller, respectively representing 1.7% (2.5%), 3.2% (3.5%) and

2.6% (3.3%) of our seven countries' market capitalization.[9]

Our analysis so far has been focused on portfolios that are either completely invested in a single-country stock market, or fully diversified internationally. In practice, it is common for many institutional investors to hold equity portfolios which are heavily biased toward their own local stock market which nonetheless have a significant component of international diversification. Thus it is relevant to look at a case that captures this practice. As an illustration, Table A2 in the Appendix examines the optimal currency exposures at a one-quarter horizon of "home biased world portfolios" which are 75% invested in the stock market indicated on the leftmost column of the table, and 25% in a value-weighted world portfolio that excludes this market. The results are qualitatively similar to those in Tables 4 and 5.

The main conclusion that emerges from our discussion is that global stock market investors find it optimal to hold economically significant exposures to the US dollar, the euro, and the Swiss franc. These exposures minimize the volatility of their portfolio returns, because these three currencies tend to appreciate when international stock markets fall.

Table 7 quantifies the variance reduction that investors can achieve by combining their international stock market portfolios with optimally chosen currency exposures. We report the annualized volatility of the 3-month return on the equally-weighted world portfolio, the value-weighted world portfolio, and the single-country (or 100% home-biased) portfolios. For comparison, we also report the volatility of the currency unhedged portfolio—which of course depends on the base currency—and of a portfolio that is fully currency hedged—so currency demands are set to zero. Finally, we report the volatility of a half-hedged portfolio, a strategy that is popular among some institutional investors.

Table 7 shows that the benefit of full currency hedging depends sensitively on an investor's base currency. It is particularly large for Euroland and Swiss investors, because these investors have a risk-reducing base currency so they gain by hedging back to that currency and out of foreign currencies. The volatility reduction from full currency hedging is particularly small for Australian and Canadian investors,

[9] These weights are remarkably stable over our sample period, with the exception of the Japanese stock market and the US stock market in the late 1980's. In that period, the relative market capitalization of the Japanese stock makret grew rapidly to represent 45% of total market capitalization, at the expense of the US market, whose weight in the portfolio decreased to an overall sample minimum of 30%.

because the home currency for these investors is risky in the sense that it is positively correlated with their equity positions. In fact, full currency hedging actually increases risk for a Canadian investor.

Optimal hedging, however, reduces risk for all investors, including Australians and Canadians. Relative to full hedging, optimal hedging reduces the standard deviation of an equally-weighted world portfolio by 1.35 percentage points, and the standard deviation of a value-weighted world portfolio by 1.34 percentage points.

4.3 Stability across subperiods

This section examines whether our empirical results are sample specific or whether they capture stable relations between excess returns on stocks and currencies. The sample period for which we have estimated optimal currency exposures includes an early period of global high inflation and interest rates, with exceptional performance of the Japanese stock market relative to other stock markets, followed by another subperiod of global lower inflation and interest rates, with extremely poor performance of the Japanese stock market. It is reasonable to examine if the results we have shown for the full sample hold across these two markedly different subperiods. Accordingly, we divide our sample period into the periods 1975–1989 and 1990–2005.

Figures 1 to 5 show the time series of 18-month moving averages of the annualized return on our value-weighted, currency-hedged global stock market portfolio, and the annualized excess return on a base currency relative to an equally weighted average of foreign currencies. A vertical line divides each graph between the first and the second subperiod. We report results for five base currencies, the Australian dollar (Figure 1), the Japanese yen (Figure 2), the euro (Figure 3), the Swiss franc (Figure 4) and the US dollar (Figure 5). We omit plots for the Canadian dollar and the British pound, because they look broadly similar to the plots for the Australian dollar and the yen.

It is striking to observe that, throughout our sample period, the local currency and world stock market lines tend to move together for Australia and Japan—and similarly for Canada and the UK. This pattern reflects the strong positive correlations between these countries' currencies and world stock markets that we have already discussed. The figures show that these correlations are stable over time.

In the early part of the sample Euroland and Switzerland look like Australia, Canada, Japan, and the UK with positive correlations between the world currency and stock markets, but this pattern weakens toward the end of the subperiod, and clearly reverses in the second subperiod. The US has episodes of both positive and negative comovement in both subperiods, but there is a general tendency for the dollar to move against world markets particularly in the first subperiod.

These patterns determine the optimal currency positions that we find when we split the sample into two subsamples divided at December 1989. Table 8 reports results for an investor holding an equally-weighted global stock portfolio, and using the vector of available currencies to manage risk. We report results at three horizons (1, 3 and 12 months) in a fashion analogous to Table 5. Panel A considers stock markets and currencies from our seven countries, and Panel B excludes Canada and Switzerland.

The results in Table 8 are generally familiar, with long positions for the US dollar, Swiss franc, and euro, and short positions for other currencies. It is striking, however, that US dollar positions tend to fall between the first subperiod and the second, while the sum of euro and Swiss franc positions (in Panel A) or the euro position (in Panel B) strongly increase. These changes are visible at a one-month horizon, but are more dramatic at 3- and 12-month horizons. Results are qualitatively similar if we use a value-weighted rather than an equal-weighted world equity portfolio.

Overall, this subperiod analysis suggests one major change occurring between the periods 1975-1989 and 1990-2005. In the 1990's the Swiss franc and the euro became more competitive with the US dollar as desirable currencies for risk-minimizing global equity investors.

5 Optimal Currency Demands for Bond Investors

5.1 Single-country bond portfolios

We now consider optimal currency exposures generated by bond portfolios. We first consider the case of single-country bond portfolios. This is probably the case that is relevant to most individual investors and many institutional investors, since "home bond bias" is even more prevalent among investors than "home equity bias." In most countries there are very few mutual funds focused on international bonds.

Table 9, whose structure is identical to Table 4, reports optimal currency demands generated by single-country bond portfolios. The table reports results for two cases: Panel A considers the case of an investor who can invest only in one currency at a time; Panel B considers the multiple currency case.

The optimal currency demands shown in Panel A are in general small, and most of them are not statistically significant from zero. There are a few exceptions where they are more significant, both economically and statistically. The first exception is the demand for the dollar. The column of the panel corresponding to the dollar shows that, as in the case of single-country stock portfolios, most single-country bond portfolios generate positive demands for the dollar. These demands result from a negative correlation between local bond excess returns and excess returns on the dollar with respect to the domestic currency. They indicate that when domestic long-term bond returns fall, the domestic currency tends to depreciate with respect to the dollar, thus making the dollar an attractive currency to hedge domestic bond risk. However, these correlations are weak, and the implied risk management demands for the dollar are considerably smaller than those generated by local equity returns. The column corresponding to the Canadian dollar also shows positive, statistically significant demands for this currency, with the exception of the US bond portfolio. However, the Canadian dollar demand switches sign in Panel B when both the Canadian dollar and the US dollar are available.

An examination of the rows in Panel A of Table 9 shows that there are two bond markets that tend to generate statistically significant, though small, demands for foreign currencies. The first market is the UK bond market. All of the currency demands shown in the row corresponding to UK bonds are positive and, with the sole exception of the yen, statistically significant, implying that the pound tends to depreciate with respect to all currencies when UK bonds fall. The second market is the US bond market, which generates statistically significant negative demands for all currencies with the exception of the Australian dollar. These results imply that the dollar tends to appreciate when US bonds fall.

Panel B shows optimal currency exposures generated by single-country bond portfolios when investors can invest in all currencies simultaneously. Multiple currency demands exhibit a pattern similar to single currency demands. They are small in magnitude and not statistically significant in most cases. Once again, the US dollar is a partial exception in that it has significant positive demands from Canadian, Japanese, and US bond investors.

5.2 Global bond portfolios

We now consider the case of internationally diversified bond portfolios. To keep things simple, we consider the case of an equally-weighted global bond portfolio. Table 10 reports optimal currency exposures at horizons between one month and one year in the multiple currency case. The structure of Table 10 is identical to the structure of Table 5.

Table 10 shows that, in the case of international bond portfolios, optimal currency demands are generally very small and not statistically significant. The US dollar once again represents an exception. At short horizons up to a quarter, the optimal demand for the US dollar is positive and statistically significant, regardless of whether the dollar is the only currency available for investment, or just one of many. But these dollar exposures are even smaller than in the case of single-country bond portfolios.

Table 8 and Figures 1 through 5 have explored the stability of optimal currency demands generated by equity portfolios. A similar analysis, shown in Table A3 and Table A4 in the appendix, shows that optimal currency demands for all currencies except the US dollar generated by bond portfolios are consistently zero across different subperiods. In the case of the US dollar, the positive demand for the US dollar generated by both single-country bond portfolios and international bond portfolios in the full sample appears to be driven mainly by the first half of the sample. The demand for US dollars, though still statistically significant at some horizons, is much smaller in the second subperiod. To illustrate these results, Figure 6 and Figure 7 plot the time series of rolling 18-month averages of the return on an equally weighted, currency-hedged global bond portfolio and the excess returns on the euro and the US dollar relative to an equally weighted basket of currencies. A vertical line divides each graph between the first and the second half of our sample period.

Figure 6 shows no clear pattern in the comovement of world bond returns and returns on the euro. They move together at times, and in opposite directions at other times. These alternate patterns are equally distributed across both subperiods of the sample. For the US dollar, however, Figure 7 shows some evidence of a negative correlation, particularly in the first half of the sample.

Overall, our results imply that international bond investors should fully hedge the currency exposure implicit in their bond portfolios, with possibly a small long bias towards the US dollar. In fact, Table A5 in the appendix shows that, unlike in the case of equity portfolios, the reduction in the total portfolio return variance (relative to a policy of no currency hedging) associated with the optimal hedging policy is indistinguishable for all practical purposes from the variance reduction associated with a policy of full currency hedging. Interestingly, full currency hedging is much more common among international bond funds than among international equity funds.

6 Currency Returns in Equilibrium

In section 4 we showed that currencies systematically differ in their comovements with global stock markets. Excess returns on "reserve" currencies such as the US dollar, the euro, and Swiss franc covary negatively with global stock market returns, while excess returns on "normal" currencies covary positively. These correlations generate positive risk management demands for reserve currencies, and negative risk management demands for normal currencies.

However, in equilibrium investors must be willing to hold all currencies (Black 1990). This suggests that average excess returns on currencies might adjust to generate speculative currency demands that offset the risk management demands we have identified. In global capital market equilibrium, investors may be willing to receive lower compensation for holding US dollar, euro, and Swiss franc denominated bills because of the hedging properties of these currencies, while they may demand higher compensation for holding bills denominated in other currencies. In fact, we saw in Table 1 that the US, Euroland, and Switzerland have had the lowest currency returns in our sample, and the lowest interest rates with the exception of Japan. If this is a systematic phenomenon, it suggests that a country benefits from having a reserve currency not only because international demand for its monetary base generates seigniorage revenue, but also because international demand for its Treasury bills reduces the interest cost of financing the government debt.[10]

We now explore the equilibrium consequences of risk management demand for

[10]In a similar spirit, Lustig and Verdelhan (2007) show that currencies with high interest rates have high covariances with US consumption growth. The connection between liquidity preference (the demand for safe assets with low returns) and risk was first made explicitly by Tobin (1958).

currencies by looking at the relation between average currency excess returns, and the betas of currencies with a global stock index. We consider all possible non-redundant pairs (or exchange rates) in our cross section of currencies, and treat each one as a long-short portfolio of bills. For example, the excess return on the Canadian dollar with respect to the US dollar is the return on a portfolio long Canadian bills and short US Treasury bills. For each of these portfolios, we compute the average log currency excess return and its beta with respect to the currency-hedged excess return on a value-weighted global stock portfolio, and we plot all these mean returns and betas together in a single figure.[11] To simplify our plot, we choose the ordering of the pairs so that their betas are all positive.

Figure 8 shows the mean-beta diagram based on our full sample. This figure plots full-sample annualized average excess currency returns on the vertical axis, and currency betas in the horizontal axis. The points marked with a "+" refer to long-short currency portfolios with euro-denominated bills on the short side of the portfolio. The square corresponds to the portfolio long Canadian dollars and short US dollars, and the circles correspond to all other non-redundant currency pairs.

The figure also plots a regression line of currency excess returns on currency betas, with the intercept restricted to equal zero. We can interpret this line as the security market line generated from a global CAPM using currencies as assets. The slope of this line is 3.2%, and the R^2 is reasonably large at 48%; adding a free intercept has little effect on these estimates. The slope of the security market line reflects the equilibrium world market premium implied by currency returns. At 3.2% per annum, this premium is smaller than the ex-post average excess return on world stock markets over this period, which Table 1 shows is about 7%. However, this estimate is close to the ex-ante equity risk premium that others have estimated from US equity returns over periods in which the ex-post equity premium has also been very large (Fama and French, 2002).

The point in the figure that lies furthest to the right corresponds to the portfolio long Canadian dollars and short US dollars. We have shown already that a position long US dollars and short Canadian dollars is a particularly effective hedge against fluctuations in global equity markets. Conversely, a portfolio long Canadian dollars

[11] There are no meaningful differences if we use the log of average currency excess returns, or calculate betas with respect to a value-weighted global stock portfolio. Also, note that currency betas with respect to the global stock market portfolio are proportional to the negative of the currency demands that we find in section 4 for the case with a global stock portfolio and a currency pair at a time.

and short US dollars is particularly risky, because it is highly positively correlated with global stock markets. Figure 8 shows that this is the portfolio with the largest full-sample beta, above 0.6. It provides investors with an average positive return of about 1.2% per annum (see Table 1) which, though positive, is located below the fitted security market line and is well below the average return on other portfolios with lower betas.

We have shown that portfolios which are long euro-denominated bills also help investors attenuate fluctuations in global stock portfolios, because the euro tends to covary negatively with global equity returns. Conversely, portfolios which are short euros and long other currencies are positively correlated with global equities. These are the points corresponding to the euro pairs shown in Figure 8. As expected, these portfolios all exhibit positive betas. While their average excess returns exhibit a positive relation with betas, they tend to lie below the fitted security market line.

Overall, we do see differences in average realized currency returns that correlate with currency risks. However, in the case of the US dollar and the euro, these average realized returns have been modest. Investors who expect average currency returns of this magnitude will still tilt their currency portfolios in the directions identified earlier in the paper.

Figure 9 repeats the exercise shown in Figure 8, except that it treats all currency pairs in each of the subperiods we considered in section 4.3 (1975–1989 and 1990–2005) as separate assets. Consistent with the results of section 4.3, portfolios which are short the euro tend to be significantly riskier in the second subsample, reflecting the increasing tendency of the euro to move as a reserve currency. Also consistent with our earlier results, the portfolio which is long the Canadian dollar and short the US dollar is somewhat less risky in the second subperiod. In general, currency pairs show a much wider dispersion in betas in the second subperiod. The security market lines have modest positive slopes in both subperiods. Again this implies that investors who expect average currency returns similar to those that have been realized historically will tend to hold reserve currencies and short normal currencies in the manner discussed earlier.

7 Conclusion

In this paper we have studied the correlations of foreign exchange rates with stock returns and bond returns over the period 1975–2005 and have drawn out the implications for risk management by international equity and bond investors. We have found that many currencies—in particular the Australian dollar, Canadian dollar, Japanese yen, and British pound—are positively correlated with world stock markets. The euro, the Swiss franc, and the bilateral US-Canadian exchange rate, however, are negatively correlated with the world equity market. These patterns imply that international equity investors can minimize their equity risk by taking short positions in the Australian and Canadian dollars, Japanese yen, and British pound, and long positions in the US dollar, euro, and Swiss franc. For US investors, the implication is that the currency exposures of international equity portfolios should be at least fully hedged, and probably overhedged, with the exception of the euro and Swiss franc which should be partially hedged. These results are robust to variation in the investment horizon between one month and one year.[12] We obtain similar results when we consider the 1970's and 1980's in one subsample and the 1990's and 2000's in another, except that risk-minimizing equity investors should hold more euros and Swiss francs in the later period and slightly fewer dollars.

We have also found that risk management demands for currencies by bond investors are small or zero, regardless of the home country of these investors, and regardless of whether these investors hold only domestic bonds or an international bond portfolio. These optimal zero currency demands reflect a very weak correlation between bond excess returns and currency excess returns. The only exception is a weak negative correlation of bond returns with excess returns on the dollar relative to other currencies. This correlation implies a small positive allocation to the dollar by most bond investors. Our results thus provide support for the practice prevalent among international bond investors to hedge the currency exposures implicit in their international bond holdings.

Campbell, Viceira, and White (2003) show that long-term investors interested in minimizing real interest rate risk using international portfolios of bills—or equivalently, currency exposures—also have large demands for bills denominated in euros

[12] Froot (1993) studies the dollar and the pound over a longer sample period and finds that risk-minimizing foreign currency positions increase with the investment horizon, implying that long-horizon equity investors should not hedge their currency risk. We do not find this horizon effect in our post-1975 dataset.

and US dollars, because these two currencies have had relatively stable interest rates. Their results suggest that these two currencies are attractive stores of value for international money market investors. Our results add to this evidence, by showing that the US dollar and the euro tend to appreciate when international stock markets fall. This negative correlation generates demands for US dollar and euro denominated bills as a way to reduce the volatility of international stock portfolios. In other words, the US dollar and the euro are attractive stores of value for international equity investors.

One might expect that in equilibrium, those currencies that are attractive for risk management purposes would offer lower average returns. Indeed, there is a positive relation between average currency returns in our sample and the betas of currencies with a currency-hedged world stock index. However the reward for taking beta exposure through currencies has been quite modest in our sample, certainly well below the historical equity premium. To the extent that international investors are willing to receive lower compensation for holding US dollar and euro denominated bills because of the hedging properties of these currencies, a country benefits from having a reserve currency not only because international demand for its monetary base generates seigniorage revenue, but also because international demand for its Treasury bills reduces the interest cost of financing the government debt.

These findings raise the interesting question why currencies are so heterogeneous in their correlations with equity markets. Correlations between currencies and domestic equity markets could result from shocks to fundamentals that affect both the profitability of corporations and the fiscal positions of governments; or from capital flows, driven by investor sentiment, that move equity markets jointly with currency markets; or from the effects of exchange rate movements on the costs and output prices of corporations (Pavlova and Rigobon 2003). However we need a mechanism to explain why the US dollar, euro, and Swiss franc behave differently from other currencies. One possible explanation is that they attract flows of capital at times when bad news arrives about the world economy, or when investors become more risk averse. This "flight to quality" drives up the dollar, euro, and Swiss franc at times when the prices of risky financial assets decline. This explanation takes as given that these currencies are regarded as safe assets and therefore benefit from a flight to quality. It is consistent with the role of the dollar, and increasingly the euro, as reserve currencies in the international financial system. Our finding that the risk-minimizing demand for euros has increased over time suggests that the euro has partially displaced the dollar as a reserve currency.

A natural extension of this paper is to consider the risk management demand of long-term investors not by using long-term mean-variance analysis, as in this paper, but using the long-term portfolio choice framework of Merton (1971), as implemented for example by Campbell, Chan, and Viceira (2003) and Jurek and Viceira (2006). Long-horizon mean-variance analysis ignores the fact that investors can rebalance their portfolios over time, and the alternative framework takes this into account.

References

Adler, Michael and Bernard Dumas, 1983, International portfolio choice and corporation finance: A synthesis, *Journal of Finance* 38, 925–984.

Black, Fischer, 1972, Capital market equilibrium with restricted borrowing, *Journal of Business* 45, 444–454.

Black, Fischer, 1990, Equilibrium exchange rate hedging, *Journal of Finance* 45, 899–907.

Campbell, John Y., Yeung Lewis Chan, and Luis M. Viceira, 2003, A multivariate model of strategic asset allocation, *Journal of Financial Economics* 67, 41–80.

Campbell, John Y., Andrew W. Lo and A. Craig MacKinlay, 1997, *The Econometrics of Financial Markets*, Princeton University Press, Princeton, NJ.

Campbell, John Y., Luis M. Viceira, and Joshua S. White, 2003, Foreign currency for long-term investors, *Economic Journal* 113, C1–C25.

Engel, Charles, 1996, The forward discount anomaly and the risk premium: a survey of recent evidence, *Journal of Empirical Finance* 3, 123–192.

Fama, Eugene F., 1984, Forward and spot exchange rates, *Journal of Monetary Economics* 14, 319–338.

Fama, Eugene F. and Kenneth R. French, 2002, The equity premium, *Journal of Finance* 57, 637–659.

Froot, Kenneth A., 1993, Currency hedging over long horizons, NBER Working Paper No. 4355.

Hansen, Lars Peter and Robert J. Hodrick, 1980, Forward rates as optimal predictors of future spot rates: an econometric analysis, *Journal of Political Economy* 88, 829–853.

Hodrick, Robert J., 1987, *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets*, Harwood Academic Publishers, Chur, Switzerland.

Jurek, Jakub and Luis M. Viceira, 2006, Optimal value and growth tilts in long-horizon portfolios, NBER Working Paper No. 12017.

Lustig, Hanno N. and Adrien Verdelhan, 2007, The cross-section of foreign currency risk premia and consumption growth risk, *American Economic Review* 97, 89–117.

Merton, Robert C., 1971, Optimum consumption and portfolio rules in a continuous-time model, *Journal of Economic Theory* 3, 373–413.

Pavlova, Anna and Roberto Rigobon, 2003, Asset prices and exchange rates, NBER Working Paper No. 9834.

Perold, André F. and Evan C. Schulman, 1988, The free lunch in currency hedging: Implications for investment policy and performance standards, *Financial Analysts Journal* 44 (3), 45-50.

Siegel, Jeremy J., 1972, Risk, interest rates, and the forward exchange, *Quarterly Journal of Economics* 86, 303–309.

Solnik, Bruno H., 1974, An equilibrium model of the international capital market, *Journal of Economic Theory* 8, 500–524.

Stulz, René M., 1981, A model of international asset pricing, *Journal of Financial Economics* 383–406.

Thomas, Lee R., 1990, *The Currency-Hedging Debate*, IFR Books, London, United Kingdom.

Tobin, James, 1958, Liquidity preference as behavior towards risk, *Review of Economic Studies* 25, 68–85.

Appendix to Global Currency Hedging

A1. Log portfolio returns over short time intervals

Assuming log-normality of the hedge returns, the derivation of the optimal Ψ requires an expression for the log-return on the hedged portfolio, $r_{p,t+1}^{hedge}$. We compute this log hedged return as a discrete-time approximation to its continuous-time counterpart. In order to do this, we need to specify, in continuous time, the return processes for stocks $P_{c,t}$, for currencies $X_{c,t}$ and for interest rates $B_{c,t}$.We assume that they all follow a geometric brownian motions:

$$\begin{aligned}
\frac{dP_{c,t}}{P_{c,t}} &= \mu_{P_c}dt + (\sigma_{P_c})_t\, dW_t^{P_c}, \qquad c = 1...n+1 && (1)\\
\frac{dB_{c,t}}{B_{c,t}} &= \mu_{B_c}dt, \qquad\qquad\qquad\qquad c = 1...n+1 && (2)\\
\frac{dX_{c,t}}{X_{c,t}} &= \mu_{X_c}dt + (\sigma_{X_c})_t\, dW_t^{X_c}, \qquad c = 1...n+1, && (3)
\end{aligned}$$

where $W_t^{P_c}$, $W_t^{B_c}$ and $W_t^{X_c}$ are diffusion processes. $\frac{dP_{c,t}}{P_{c,t}}$ represents the stock return, $\frac{dB_{c,t}}{B_{c,t}}$ the nominal return to holding a riskless bond from country and $\frac{dX_{c,t}}{X_{c,t}}$ the return to holding foreign currency c.

For notational simplicity, in what follows, we are momentarily dropping time subscripts for the standard deviations.

Using Ito's lemma, the log returns on each asset are given by:

$$\begin{aligned}
d\log P_{c,t} &= \frac{dP_{c,t}}{P_{c,t}} - \frac{1}{2}\sigma_{P_c}^2 dt\\
d\log B_{c,t} &= \frac{dB_{c,t}}{B_{c,t}} - \frac{1}{2}\sigma_{B_c}^2 dt\\
d\log X_{c,t} &= \frac{dX_{c,t}}{X_{c,t}} - \frac{1}{2}\sigma_{X_c}^2 dt.
\end{aligned}$$

Note that, because country 1 is the domestic country, which has a fixed exchange rate of 1, we have $d\log X_{1,t} = 0$. This implies $\mu_{X_1} = \sigma_{X_1} = 0$.

The domestic currency return on foreign stock is then given by $\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}}$. To derive an expression for this return, we will note that the return dynamics above, by standard calculations, imply :

$$\begin{aligned}\log P_{c,t}X_{c,t} &= \log P_{c,0}X_{c,0} + \left(\mu_{P_c} + \mu_{X_c} - \frac{1}{2}\sigma_{P_c}^2 - \frac{1}{2}\sigma_{X_c}^2\right)t \\ &\quad + \sigma_{P_c}\left(W_t^{P_c} - W_0^{P_c}\right) + \sigma_{X_c}\left(W_t^{X_c} - W_0^{X_c}\right)\end{aligned}$$

Differentiating, and then applying Ito's lemma, yields :

$$\begin{aligned}\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}} &= \frac{dP_{c,t}}{P_{c,t}} + \frac{dX_{c,t}}{X_{c,t}} + \sigma_{P_c}\sigma_{X_c}\rho_{P_c,X_c}dt \\ \frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}} &= d\log P_{c,t} + d\log X_{c,t} + \frac{1}{2}\operatorname{Var}_t\left(p_{c,t} + x_{c,t}\right)dt, \qquad (4)\end{aligned}$$

where $x_{c,t} = d\log X_{c,t}$ and $p_{c,t} = dlogP_{c,t}$. Note that for c=1, the formula does yield the simple stock return as $\frac{dP_{1,t}X_{1,t}}{P_{1,t}X_{1,t}} = \frac{dP_{1,t}}{P_{1,t}} + \frac{dX_{1,t}}{X_{1,t}} + \sigma_{P_1}\sigma_{X_1}\rho_{P_1,X_1}dt = \frac{dP_{1,t}}{P_{1,t}}$.

A similar calculation yields the following dynamics for the return of the strategy consisting in holding the domestic bond and shorting the foreign one :

$$\frac{d\left(B_{1,t}/B_{c,t}\right)}{B_{1,t}/B_{c,t}} = d\log B_{1,t} - d\log B_{c,t} \qquad (5)$$

We note V_t the value of the portfolio.The log return on the portfolio, by Ito's lemma, is :

$$d\log V_t = \frac{dV_t}{V_t} - \frac{1}{2}\left(\frac{dV_t}{V_t}\right)^2.$$

We can now derive each of the right-hand side terms:

$$\frac{dV_t}{V_t} = \sum_{c=1}^{n+1}\omega_{c,t}\left(\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}}\right) + \sum_{c=1}^{n+1}\theta_c\omega_{c,t}\frac{d\left(B_{1,t}/B_{c,t}\right)}{B_{1,t}/B_{c,t}} - \sum_{c=1}^{n+1}\theta_c\omega_{c,t}\frac{dX_{c,t}}{X_{c,t}},$$

which follows from our convention regarding the domestic country.

Using expressions (3) , (4) , and (5) to substitute and moving to matrix notation, we get :

$$\begin{aligned}\frac{dV_t}{V_t} &= \mathbf{1}'\boldsymbol{\omega}\left(\mathbf{p}_{t+1} + \mathbf{x}_{t+1}\right) - \boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1} - \mathbf{b}_t^d + \mathbf{b}_t\right) \\ &\quad + \frac{1}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t diag\left(\operatorname{Var}_t\left(\mathbf{p}_{t+1} + \mathbf{x}_{t+1}\right)\right) - \boldsymbol{\Theta}_t' diag\left(\operatorname{Var}_t \mathbf{x}_{t+1}\right)\right]dt \quad ,\end{aligned}$$

where $\mathbf{p}_{t+1} = (d\log P_{1,t},\ d\log P_{2,t}...,d\log P_{n+1,t})'$, $\mathbf{x}_{t+1} = (d\log X_{1,t},\ d\log X_{2,t}...,d\log X_{n+1,t})'$, $\mathbf{b}_t^d = (d\log B_{1,t})\,\mathbf{1},\ \mathbf{b}_t = (d\log B_{1,t},\ d\log B_{2,t}...,d\log B_{n+1,t})'$ and $diag\,(X)$ denotes, for a symmetric $(n\times n)$ matrix X, the $(n\times 1)$ vector of its diagonal terms.

Then,

$$
\begin{aligned}
\left(\frac{dV_t}{V_t}\right)^2 &= \mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\right]dt+o\left(dt\right)\\
&= \left[\begin{array}{c} \mathbf{1}'\boldsymbol{\omega}_t\,\mathrm{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)\boldsymbol{\omega}_t\iota \\ -2\mathbf{1}'\boldsymbol{\omega}_t\,\mathrm{cov}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1},\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\boldsymbol{\Theta}_t \\ +\boldsymbol{\Theta}'\boldsymbol{\omega}_t\,\mathrm{Var}_t\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\boldsymbol{\Theta}_t \end{array}\right]dt+o\left(dt\right).
\end{aligned}
$$

So, finally,

$$
\begin{aligned}
d\log V_t &= \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right) \\
&\quad +\frac{1}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t diag\left(\mathrm{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)\right)-\boldsymbol{\Theta}_t' diag\left(\mathrm{Var}_t\,\mathbf{x}_{t+1}\right)\right]dt \\
&\quad -\frac{1}{2}\,\mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\mathbf{x}_{t+1}-\mathbf{b}_t^d+\mathbf{b}_t\right)\right]dt+\mathrm{o}\left(dt\right) \quad .
\end{aligned} \tag{6}
$$

Now, we get the approximation for $r_{p,t+1}^h$ by computing the previous expression for $dt=1$, replacing $d\log X_{c,t} = \Delta s_{c,t+1}$, $dlogP_{c,t} = r_{c,t+1}$, and $dlogB_{c,t} = i_{c,t}$ and neglecting the higher order terms. Noting, for any variable, $\mathbf{z}_t$, the $(n+1\times 1)$ vector $(z_{1,t},z_{2,t}..z_{n+1,t})$, this is equivalent to replacing in equation (6) $\mathbf{p}_{t+1}$ by $\mathbf{r}_{t+1}$, $\mathbf{x}_{t+1}$ by $\boldsymbol{\Delta}\mathbf{s}_{t+1}$, $\mathbf{b}_t^d$ by $\mathbf{i}_t^d$ and $\mathbf{b}_t$ by $\mathbf{i}_t$.

$$
r_{p,t+1}^h \simeq \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)+\frac{1}{2}\Sigma_t^h
$$

where Σ_{t+1}^h is equal to :

$$
\begin{aligned}
\Sigma_t^h &= \mathbf{1}'\boldsymbol{\omega}_t diag\left(\mathrm{Var}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)-\boldsymbol{\Theta}_t' diag\left(\mathrm{Var}_t\,\boldsymbol{\Delta}\mathbf{s}_{t+1}\right) \\
&\quad -\mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)-\boldsymbol{\Theta}_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right]
\end{aligned}
$$

where, for any variable z, $\mathbf{z}_t$ denotes the vector of country observations $(z_{1,t},z_{2,t}...z_{n+1,t})'$ and small case letters denote logs in the following fashion : $r_{c,t+1} = \log\left(R_{c,t+1}\right)$, $s_{t+1} = \log\left(S_{t+1}\right)$, $i_t^d = \log\left(1+I_{1,t}\right)\mathbf{1}$ and $i_{c,t} = \log\left(1+I_{c,t}\right)$.

We can now rewrite the portfolio return as a function of $\boldsymbol{\Psi}_t$ by substituting for $\boldsymbol{\Theta}_t$. This yields :

$$
\begin{aligned}
r^h_{p,t+1} &= \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\mathbf{i}^d_t-\mathbf{i}_t\right)+\boldsymbol{\Psi}'_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}^d_t+\mathbf{i}_t\right)+\frac{1}{2}\Sigma^h_t \\
&= i^d_{1,t}+\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\boldsymbol{\Psi}'_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}^d_t+\mathbf{i}_t\right)+\frac{1}{2}\Sigma^h_t,
\end{aligned}
$$

where:

$$
\begin{aligned}
\Sigma^h_t &= \mathbf{1}'\boldsymbol{\omega}_t\,\mathrm{diag}\left(\mathrm{Var}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)-\left(-\boldsymbol{\Psi}_t+\boldsymbol{\omega}_t\mathbf{1}\right)'\mathrm{diag}\left(\mathrm{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right) \qquad (7)\\
&\quad -\mathrm{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\mathbf{i}^d_t-\mathbf{i}_t\right)+\boldsymbol{\Psi}'_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}^d_t+\mathbf{i}_t\right)\right).
\end{aligned}
$$

A2. Equivalence between forward contracts and foreign currency borrowing and lending

With the same notations and assumptions as above, when the investor uses forward contracts to hedge currency risk, the portfolio return is:

$$
R^h_{p,t+1}=\mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1}\div\mathbf{S}_t\right)-\boldsymbol{\Theta}'_t\left[\left(\mathbf{S}_{t+1}\div\mathbf{S}_t\right)-\left(\mathbf{1}+\mathbf{I}^d_t\right)\div\left(\mathbf{1}+\mathbf{I}_t\right)\right]
$$

Another natural view is one in which the investor borrows in foreign currency and lends in domestic currency to hedge currency risk. Then, the portfolio return is:

$$
R^{BL}_{p,t+1}=\mathbf{R}'_{t+1}\boldsymbol{\omega}_t\left(\mathbf{S}_{t+1}\div\mathbf{S}_t\right)-\boldsymbol{\Theta}'\left(\mathbf{S}_{t+1}\div\mathbf{S}_t\right)\left(1+\mathbf{I}_t\right)+\boldsymbol{\Theta}'\left(1+\mathbf{I}^d_t\right)
$$

Then, with V^{BL}_t the value of the portfolio with borrowing and lending, we have in continuous time:

$$
\begin{aligned}
\frac{dV^{BL}_t}{V^{BL}_t} &= \sum_{c=1}^{n+1}\omega_{c,t}\left(\frac{dP_{c,t}X_{c,t}}{P_{c,t}X_{c,t}}\right)-\sum_{c=1}^{n+1}\Theta_{c,t}\frac{dX_{c,t}B_{c,t}}{X_{c,t}B_{c,t}}+\sum_{c=1}^{n+1}\Theta_{c,t}\frac{dB_{1,t}}{B_{1,t}} \\
&= \sum_{c=1}^{n+1}\omega_{c,t}\left(\log P_{c,t}+\log X_{c,t}+\frac{1}{2}\,\mathrm{Var}_t\left(p_{c,t}+x_{c,t}\right)dt\right) \\
&\quad -\sum_{c=1}^{n+1}\Theta_{c,t}\left(\log\left(X_{c,t}\right)+\log\left(B_{c,t}\right)+\frac{1}{2}\,\mathrm{Var}_t\left(x_{c,t}\right)dt\right) \\
&\quad +\sum_{c=2}^{n+1}\Theta_{c,t}\log\left(B_{1,t}\right) \\
&= \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\Theta'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}^d_t\right)+\frac{1}{2}\mathbf{1}'\boldsymbol{\omega}_t\,\mathrm{diag}\,\mathrm{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)dt \\
&\quad -\frac{1}{2}\Theta'\,\mathrm{diag}\,\mathrm{Var}_t\left(\mathbf{x}_{t+1}\right)dt
\end{aligned}
$$

and

$$\left(\frac{dV_t^{BL}}{V_t^{BL}}\right)^2 = \mathrm{Var}_t\left(\boldsymbol{\omega}_t'\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\Theta'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}_t^d\right)\right)dt+o\left(dt\right).$$

So

$$\begin{aligned}
d\log V_t^{BL} &= \frac{dV_t^{BL}}{V_t^{BL}}-\frac{1}{2}\left(\frac{dV_t^{BL}}{V_t^{BL}}\right)^2\\
&= \boldsymbol{\omega}_t'\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}_t^d\right)+\frac{1}{2}\boldsymbol{\omega}_t'\,\mathrm{diag}\,\mathrm{Var}_t\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)dt\\
&\quad -\frac{1}{2}\boldsymbol{\Theta}'\,\mathrm{diag}\,\mathrm{Var}_t\left(\mathbf{x}_{t+1}\right)dt\\
&\quad -\frac{1}{2}\,\mathrm{Var}_t\left(\boldsymbol{\omega}_t'\left(\mathbf{p}_{t+1}+\mathbf{x}_{t+1}\right)-\boldsymbol{\Theta}'\left(\mathbf{x}_{t+1}+\mathbf{b}_t-\mathbf{b}_t^d\right)\right)dt+o\left(dt\right)
\end{aligned}$$

We now go to the limit of $dt=1$ and get :

$$\begin{aligned}
r_{p,t+1}^{BL} &\simeq \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)-\boldsymbol{\Theta}'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}+\mathbf{i}_t-\mathbf{i}_t^d\right)+\frac{1}{2}\Sigma_t^h\\
&= r_{p,t+1}^h
\end{aligned}$$

A3. Mean-variance problem optimization

A3.1. Unconstrained hedge ratio

In the general case, $r_{p,t+1}^h-i_{1,t}^d=\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)+\frac{1}{2}\Sigma_t^h$, and the Lagrangian is:

$$\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\left(1-\lambda\right)\mathrm{Var}_t\left[\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right]\\
&\quad +\lambda\left[\mu_H-\mathrm{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)-\frac{1}{2}\Sigma_t^h\right]
\end{aligned}$$

Substituting for Σ_t^h using equation (7), this expression is equivalent to :

$$\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\,\mathrm{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)+\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)\\
&\quad +\lambda\left[\mu_H-\mathrm{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)-\Psi_t'\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)\right]\\
&\quad -\frac{\lambda}{2}\left[\mathbf{1}'\boldsymbol{\omega}_t\,\mathrm{diag}\left(\mathrm{Var}_t\left(\mathbf{r}_{t+1}+\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)-\left(\boldsymbol{\omega}_t\mathbf{1}-\Psi_t\right)'\mathrm{diag}\left(\mathrm{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)\right]
\end{aligned}$$

$$
\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\operatorname{Var}_t\left(\Psi_t'\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)-\lambda\operatorname{E}_t\left(\Psi_t'\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)\\
&\quad -\frac{\lambda}{2}\Psi_t'\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{\Delta s}_{t+1}\right)\right)\\
&\quad +\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\Psi_t'\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)\\
&\quad +\frac{1}{2}\operatorname{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right)-\lambda\operatorname{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right)\\
&\quad +\frac{\lambda}{2}\mathbf{1}'\boldsymbol{\omega}_t\left[\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{\Delta s}_{t+1}\right)\right)-\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{r}_{t+1}+\mathbf{\Delta s}_{t+1}\right)\right)\right]\\
&\quad +\lambda\mu_H
\end{aligned}
$$

$$
\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\Psi_t'\operatorname{Var}_t\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\Psi_t-\lambda\Psi_t'\begin{bmatrix}\operatorname{E}_t\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\\ +\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{\Delta s}_{t+1}\right)\right)\end{bmatrix}\\
&\quad +\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\mathbf{\Delta s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)\Psi_t\\
&\quad +K\left(\lambda\right)
\end{aligned}
$$

where

$$
\begin{aligned}
K\left(\lambda\right) &= \lambda\mu_H+\frac{1}{2}\operatorname{Var}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right)-\lambda\operatorname{E}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right)\right)\\
&\quad +\frac{\lambda}{2}\mathbf{1}'\boldsymbol{\omega}_t\left[\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{\Delta s}_{t+1}\right)\right)-\operatorname{diag}\left(\operatorname{Var}_t\left(\mathbf{r}_{t+1}+\mathbf{\Delta s}_{t+1}\right)\right)\right]
\end{aligned}
$$

$K\left(\lambda\right)$ is independent of $\widetilde{\Psi}_t$.

Now, we need to solve only for $\widetilde{\Psi}_t$ as Ψ_1, the demand for domestic currency, is given once the other currency demands are determined. We rewrite the Lagrangian in terms of $\widetilde{\Psi}_t$:

$$
\begin{aligned}
\pounds\left(\widetilde{\Psi}\right) &= \frac{1}{2}\widetilde{\Psi}_t'\operatorname{Var}_t\left(\widetilde{\mathbf{\Delta s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\widetilde{\Psi}_t-\lambda\widetilde{\Psi}_t'\begin{bmatrix}\operatorname{E}_t\left(\widetilde{\mathbf{\Delta s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\\ +\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\mathbf{\Delta s}}_{t+1}\right)\right)\end{bmatrix}\\
&\quad +\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\mathbf{\Delta s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\right)\widetilde{\Psi}_t\\
&\quad +K\left(\lambda\right)
\end{aligned}
$$

The F.O.C. gives the following expression for the optimal $\widetilde{\Psi}_t$:

$$
\begin{aligned}
0 &= \operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\mathbf{\Delta s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\right)\\
&\quad +\operatorname{Var}_t\left(\widetilde{\mathbf{\Delta s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\widetilde{\mathbf{\Psi}}_t^*-\lambda\left[\operatorname{E}_t\left(\widetilde{\mathbf{\Delta s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)+\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\mathbf{\Delta s}}_{t+1}\right)\right)\right]
\end{aligned}
$$

Finally, the optimal vector of currency demands is :

$$
\begin{aligned}
\widetilde{\boldsymbol{\Psi}}_t^*(\lambda) \quad = \quad & \lambda \operatorname{Var}_t^{-1}\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\left[E_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)+\frac{1}{2} diag\left(\operatorname{Var}_t \widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right] \\
& -\operatorname{Var}_t^{-1}\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\left[\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\right)\right]
\end{aligned}
$$

A3.2. Constrained hedge ratio

In the case where $\widetilde{\Psi}_t=\psi_t\widetilde{\mathbf{1}}$ (where $\widetilde{\mathbf{1}}$ denotes an $n\times 1$ vector of ones), we note ψ_t^* the optimal scalar constrained hedge ratio and we have :

$$
\begin{aligned}
\pounds\left(\psi_t\right) \quad = \quad & \frac{1}{2}\psi_t^2\widetilde{\mathbf{1}}'\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\widetilde{\mathbf{1}}-\lambda\psi_t\widetilde{\mathbf{1}}'\left[\begin{array}{c} \mathrm{E}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right) \\ +\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right)\end{array}\right] \\
& +\psi_t\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\right)\widetilde{\mathbf{1}} \\
& +K(\lambda)
\end{aligned}
$$

and

$$
\begin{aligned}
\psi_t^* \quad = \quad & \frac{\lambda\widetilde{\mathbf{1}}'\left[\mathrm{E}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)+\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}\right)\right)\right]}{\widetilde{\mathbf{1}}'\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\widetilde{\mathbf{1}}} \\
& -\frac{\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\right)\widetilde{\mathbf{1}}}{\widetilde{\mathbf{1}}'\operatorname{Var}_t\left(\widetilde{\boldsymbol{\Delta}\mathbf{s}}_{t+1}-\widetilde{\mathbf{i}}_t^d+\widetilde{\mathbf{i}}_t\right)\widetilde{\mathbf{1}}}
\end{aligned}
$$

In this case, ψ_t^* can equivalently be written in terms of the full matrices :

$$
\begin{aligned}
\psi_t^* \quad = \quad & \frac{\lambda\mathbf{1}'\left[\mathrm{E}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)+\frac{1}{2}\operatorname{diag}\left(\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}\right)\right)\right]}{\mathbf{1}'\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\mathbf{1}} \\
& -\frac{\mathbf{1}'\operatorname{cov}_t\left(\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right),\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\right)\mathbf{1}}{\mathbf{1}'\operatorname{Var}_t\left(\boldsymbol{\Delta}\mathbf{s}_{t+1}-\mathbf{i}_t^d+\mathbf{i}_t\right)\mathbf{1}}
\end{aligned}
$$

This case corresponds to a domestic investor hedging the same ratio of his foreign stock holdings for all foreign currencies.

A4. Invariance of optimal currency demand with respect to base country

In the system of n^2 bilateral exchange rates, there are really only n free parameters as all exchange rates can be backed out of the n bilateral rates for one base domestic country. We use this fact to show that, for a portfolio of stocks from the $n+1$ countries in our model, the optimal hedge ratios on stocks from country c, Ψ_c^{j*} is the same for any base country j. Let us now use the subscript j to index the domestic country.

We assume for this derivation that weights on international stocks are the same for investors from all countries so that $\boldsymbol{\omega}_t^j = \boldsymbol{\omega}_t$. In terms of our empirical tests, this result will hence apply to the cases of an equally weighted or a value weighted world portfolios, in which weights do not vary with the base country. They do not hold for a home biased porftolio, in which weights by definition vary with base country.

Let us think of country 1 as our base country, and write the optimal vector of foreign currency demand assuming that $\lambda^j = 0$ for all values of j. We have :

$$\begin{aligned} \widetilde{\boldsymbol{\Psi}}_{RM}^{1*} &= -\operatorname{Var}_t\left(\widetilde{\Delta s}_{t+1}^{1} - \widetilde{i}_t^{1,d} + \widetilde{i}_t^{1}\right)^{-1}\left[\operatorname{cov}_t\left(\mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right), \widetilde{\Delta s}_{t+1}^{1} - \widetilde{i}_t^{1,d} + \widetilde{i}_t^{1}\right)\right] && (8) \\ &= -\operatorname{Var}_t\left(\widetilde{x}_{t+1}^1\right)^{-1}\left[\operatorname{cov}_t\left(\mathbf{y}_{t+1}^W, \widetilde{x}_{t+1}^1\right)\right] && (9) \end{aligned}$$

where $x_{t+1}^1 = \Delta s_{t+1}^1 - i_t^{1,d} + i_t^1$ and $\mathbf{y}_{t+1}^W = \mathbf{1}'\boldsymbol{\omega}_t\left(\mathbf{r}_{t+1}-\mathbf{i}_t\right).$

Now, let us consider exchange rates from the perspective of country 2. By definition of the exchange rate between countries 1 and 2, it follows that $s_{t+1,1}^2 = -s_{t+1,2}^1$.

Also, by definition of the exchange rates, $S_{t+1,3}^2$ units of currency 2 can be exchanged into one unit of currency 3. And one unit of currency 3 is equivalent to $S_{t+1,3}^1$ units of currency 1, which is equivalent to $S_{t+1,3}^1/S_{t+1,2}^1$ units of currency 2. So, the absence of arbitrage implies the equality: $S_{t+1,3}^2 = S_{t+1,3}^1/S_{t+1,2}^1$. In logs, $s_{t+1,3}^2 = s_{t+1,3}^1 - s_{t+1,2}^1$. More generally, the following equality can be derived from the absence of arbitrage:

$$s_{t+1,c}^2 = s_{t+1,c}^1 - s_{t+1,2}^1 \qquad c = 3 ... n+1$$

In matrix notation, this amounts to a linear relationship between $\widetilde{\mathbf{\Delta s}}^2_{t+1}$ and $\widetilde{\mathbf{\Delta s}}^1_{t+1}$:

$$\widetilde{\mathbf{\Delta s}}^2_{t+1} = A_2 \cdot \widetilde{\mathbf{\Delta s}}^1_{t+1}$$

where $A_2 = \begin{pmatrix} \text{-1} & 0 & .. & .. & 0 \\ \text{-1} & 1 & 0 & .. & .. \\ \text{-1} & 0 & 1 & 0 & .. \\ \text{-1} & 0 & 0 & .. & 0 \\ \text{-1} & 0 & .. & 0 & 1 \end{pmatrix}$.

Given our notations :

$$\widetilde{\mathbf{i}}^{1,d}_t - \widetilde{\mathbf{i}}^1_t = (i_{t,2} - i_{t,1}, i_{t,3} - i_{t,1}, ..i_{t,n+1} - i_{t,1})'$$

and

$$\widetilde{\mathbf{i}}^{2,d}_t - \widetilde{\mathbf{i}}^2_t = (i_{t,1} - i_{t,2}, i_{t,3} - i_{t,2}, .., i_{t,n+1} - i_{t,2})'.$$

It follows that: $\widetilde{\mathbf{i}}^{2,d}_t - \widetilde{\mathbf{i}}^2_t = A\left(\widetilde{\mathbf{i}}^{1,d}_t - \widetilde{\mathbf{i}}^1_t\right)$.

Similarly, we have the following linear relationship between $\widetilde{\mathbf{x}}^2_{t+1}$ and $\widetilde{\mathbf{x}}^1_{t+1}$:

$$\widetilde{\mathbf{x}}^2_{t+1} = A\widetilde{\mathbf{x}}^1_{t+1} \qquad , \tag{10}$$

Let us substitute equation (10), the formula for $\widetilde{\mathbf{x}}^2_{t+1}$, into equation (**??**), the formula for the optimal hedge ratio. We use the properties of matrix second moments that $\mathrm{Var}\,(AX) = A\,\mathrm{Var}\,(X)\,A'$, $\mathrm{cov}(AX,Y) = Acov\,(X,Y)$, and the property of inverse matrices that $(AB)^{-1} = B^{-1}A^{-1}$. Also, we note that $A_2 = (A_2)^{-1}$ and $(A_2')^{-1} = A_2'$. Substitution yields:

$$\begin{aligned} \widetilde{\mathbf{\Psi}}^{2*}_{RM} &= -\mathrm{Var}_t\left(\widetilde{\mathbf{x}}^2_{t+1}\right)^{-1}\left[\mathrm{cov}_t\left(\mathbf{y}^W_{t+1}, \widetilde{\mathbf{x}}^2_{t+1}\right)\right] \\ &= -\left(A_2'\right)^{-1}\mathrm{Var}_t\left(\widetilde{\mathbf{x}}^1_{t+1}\right)^{-1}(A_2)^{-1}\left[A_2\,\mathrm{cov}_t\left(\widetilde{\mathbf{x}}^1_{t+1}, \mathbf{y}^W_{t+1}\right)\right] \\ \widetilde{\mathbf{\Psi}}^{2*}_{RM}\left(\lambda^2\right) &= -\left(A_2'\right)^{-1}\mathrm{Var}_t\left(\widetilde{\mathbf{x}}^1_{t+1}\right)^{-1}\mathrm{cov}_t\left(\widetilde{\mathbf{x}}^1_{t+1}, \mathbf{y}^W_{t+1}\right) \end{aligned}$$

$$\widetilde{\mathbf{\Psi}}^{2*} = A_2'\widetilde{\mathbf{\Psi}}^{1*}$$

We write out the vector $\widetilde{\mathbf{\Psi}}^{2*}_{RM}$:

$$\widetilde{\mathbf{\Psi}}^{2*}_{RM} = \left(-\sum_{c=2}^{n+1}\Psi^{1*}_c, \Psi^{1*}_3, \Psi^{1*}_4, .., \Psi^{1*}_{n+1}\right)$$

Given the property that $\sum\limits_{c=1}^{n+1} \Psi_c^{j*} = 1$ for $j = 1..n+1$, $\Psi_1^{1*} = -\sum\limits_{c=2}^{n+1} \Psi_c^{1*}$ so that $\widetilde{\mathbf{\Psi}}_{RM}^{2*} = \left(\Psi_1^{1*}, \Psi_3^{1*}, \Psi_4^{1*}, .., \Psi_{n+1}^{1*}\right)$. Applying this same property twice, $\Psi_2^{2*} = -\sum\limits_{c\neq 2}^{n+1} \Psi_c^{2*} = -\sum\limits_{c\neq 2}^{n+1} \Psi_c^{1*} = \Psi_2^{1*}$, so that: $\Psi_{RM}^{2*} = \left(\Psi_1^{1*}, \Psi_2^{1*}, \Psi_3^{1*}, \Psi_4^{1*}, .., \Psi_{n+1}^{1*}\right) = \Psi_{RM}^{1*}$. Finally, the vector of optimal currency positions is the same for investors based in country 2 as that of country 1 investors.

Similar results hold for $j = 3...n+1$, where $A_3 = \begin{pmatrix} 1 & \text{-}1 & .. & .. & 0 \\ 0 & \text{-}1 & 0 & .. & .. \\ 0 & \text{-}1 & 1 & 0 & .. \\ 0 & .. & 0 & .. & 0 \\ 0 & \text{-}1 & .. & 0 & 1 \end{pmatrix}$, $A_4 = \begin{pmatrix} 1 & 0 & \text{-}1 & .. & 0 \\ 0 & 1 & .. & .. & .. \\ 0 & 0 & \text{-}1 & 0 & .. \\ 0 & .. & .. & 1 & 0 \\ 0 & \text{-}1 & \text{-}1 & 0 & 1 \end{pmatrix}$, etc...

This analysis justifies dropping the base-country subscript j and interpreting the $(n+1 \times 1)$ vector $\mathbf{\Psi}^* = \left(-\sum\limits_{c=2}^{n+1} \Psi_c^{1*}, \Psi_2^{1*}, \Psi_3^{1*}, .., \Psi_{n+1}^{1*}\right)'$ as a common vector of foreign currency demands that is independent of the country of origin.

A situation in which investors from all countries are hedged perfectly corresponds to $\mathbf{\Psi}^* = (0, 0, .., 0)'$.

A situation in which investors from country 1 are not hedged at all corresponds to $\mathbf{\Psi}^* = \left(-1, \omega_2^1, \omega_3^1.., \omega_{n+1}^1\right)'$. That is, investors from country i undo the hedge of the fully hedged portfolio by taking long positions in each foreign currency proportional to the weight of each foreign country in their stock portfolio. (The perfectly hedged portfolio obtains by shorting each foreign currency by that same amount.) They need to borrow one unit of domestic currency to finance that.

Finally, note that this proof relies on the fact that all relevant exchange rates for an investor in a given base country are linear combinations of the relevant exchange rates for each other base country. In other words, the assumption is that all investors optimize over the same set of currencies.

A5. Tables [Appendix tables follow main text tables at the end of the document]

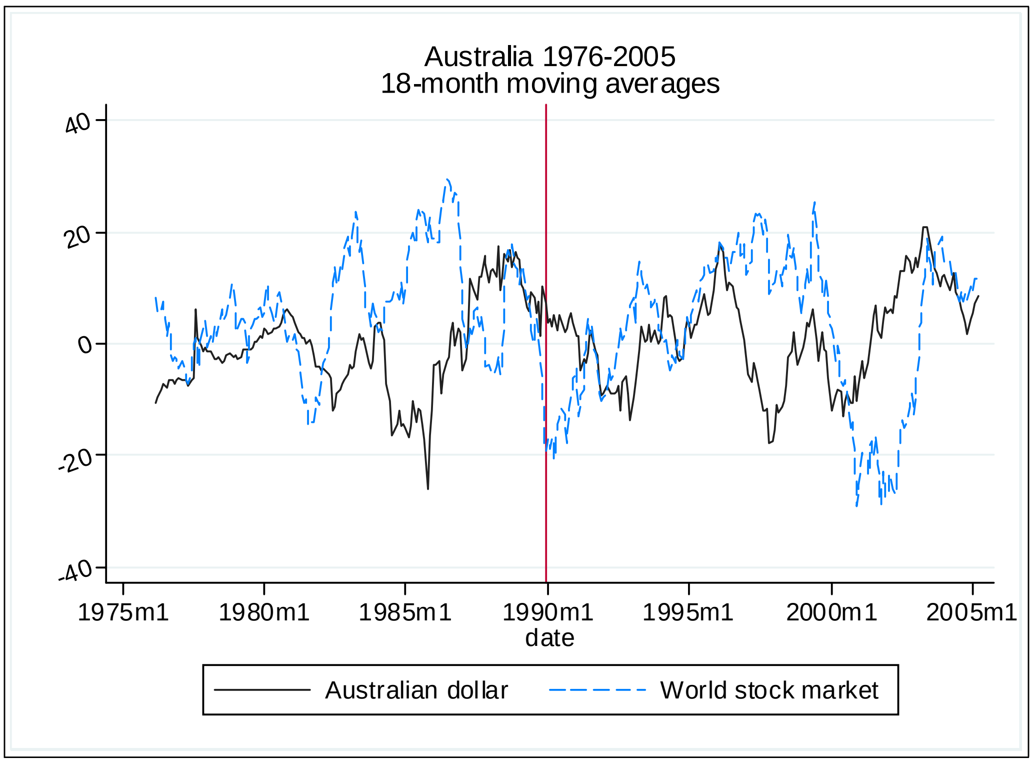

Figure 1: Currency and Hedged World Stock Market Excess Returns

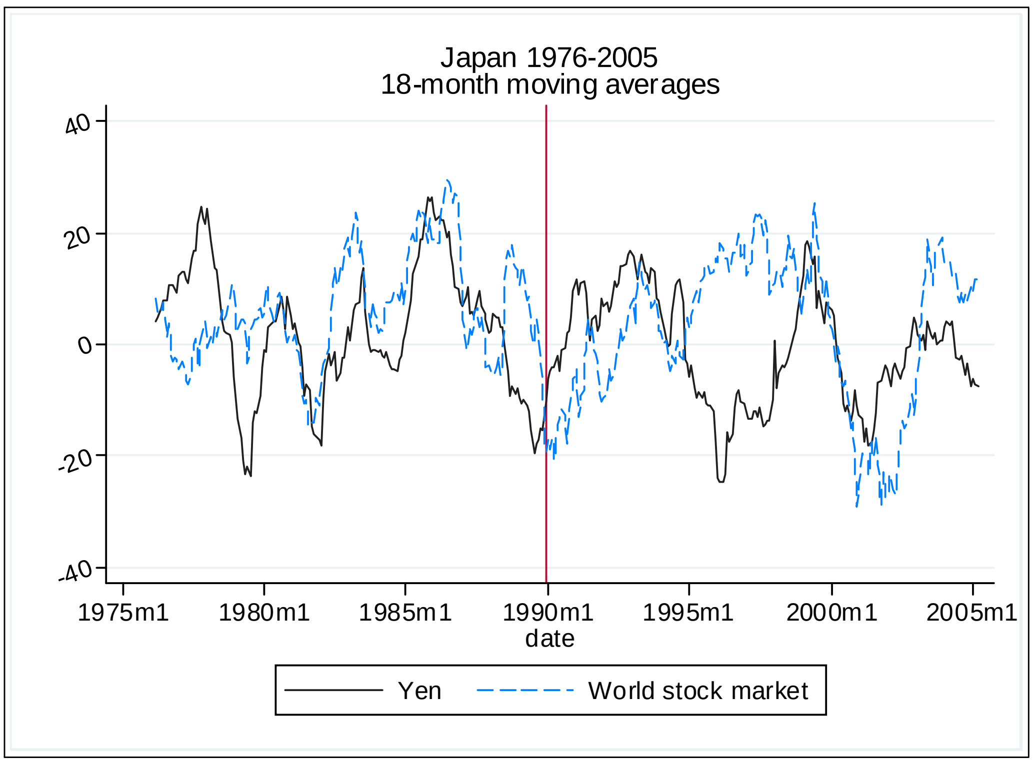

Figure 2: Currency and Hedged World Stock Market Excess Returns

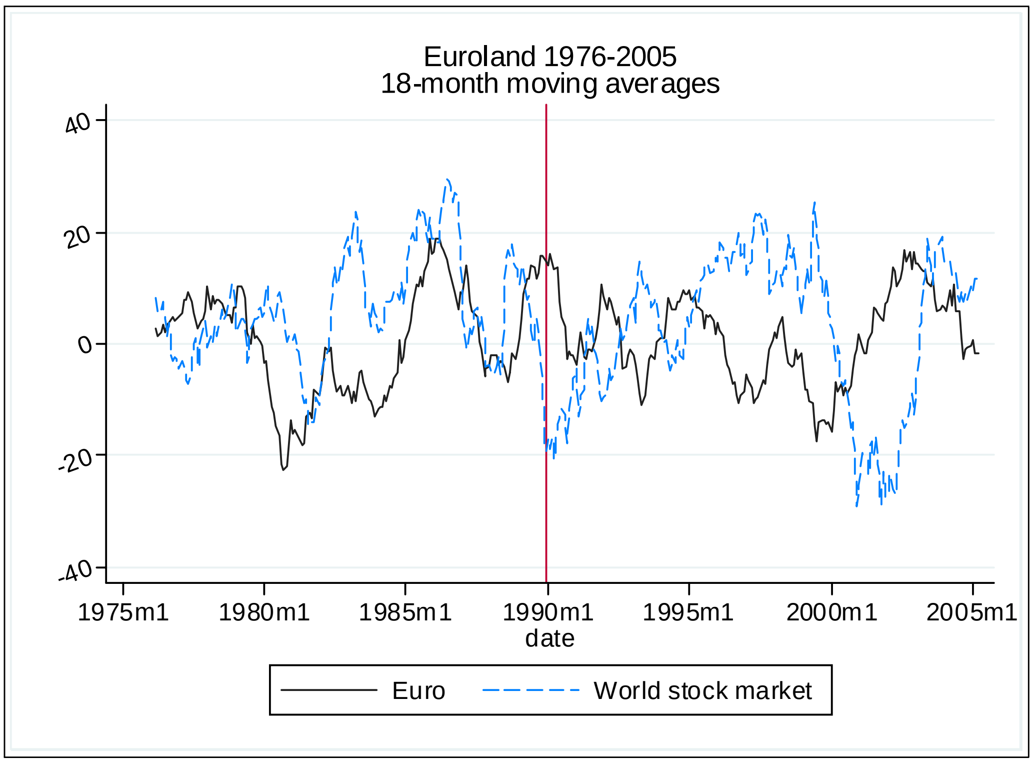

Figure 3: Currency and Hedged World Stock Market Excess Returns

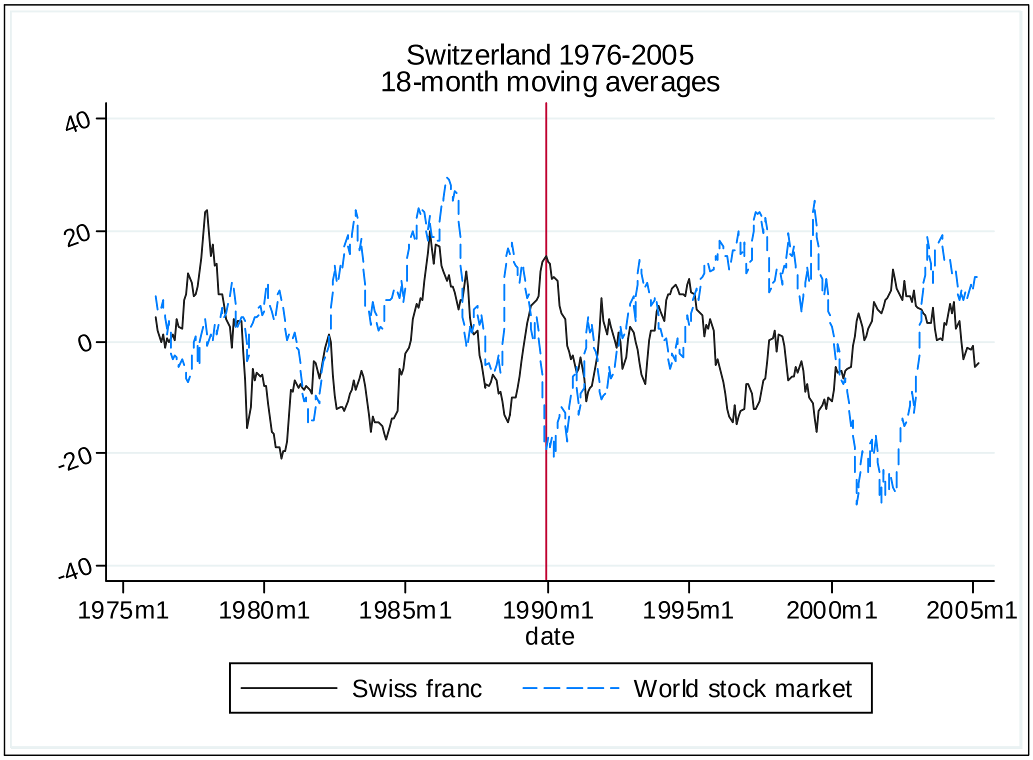

Figure 4: Currency and Hedged World Stock Market Excess Returns

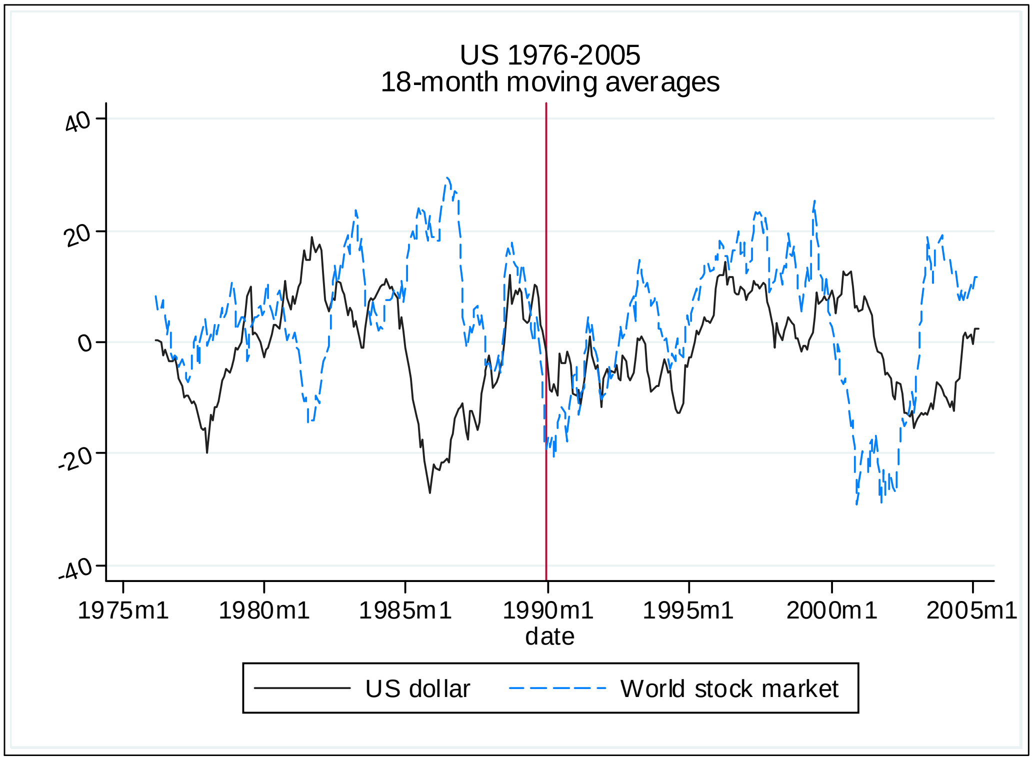

Figure 5: Currency and Hedged World Stock Market Excess Returns

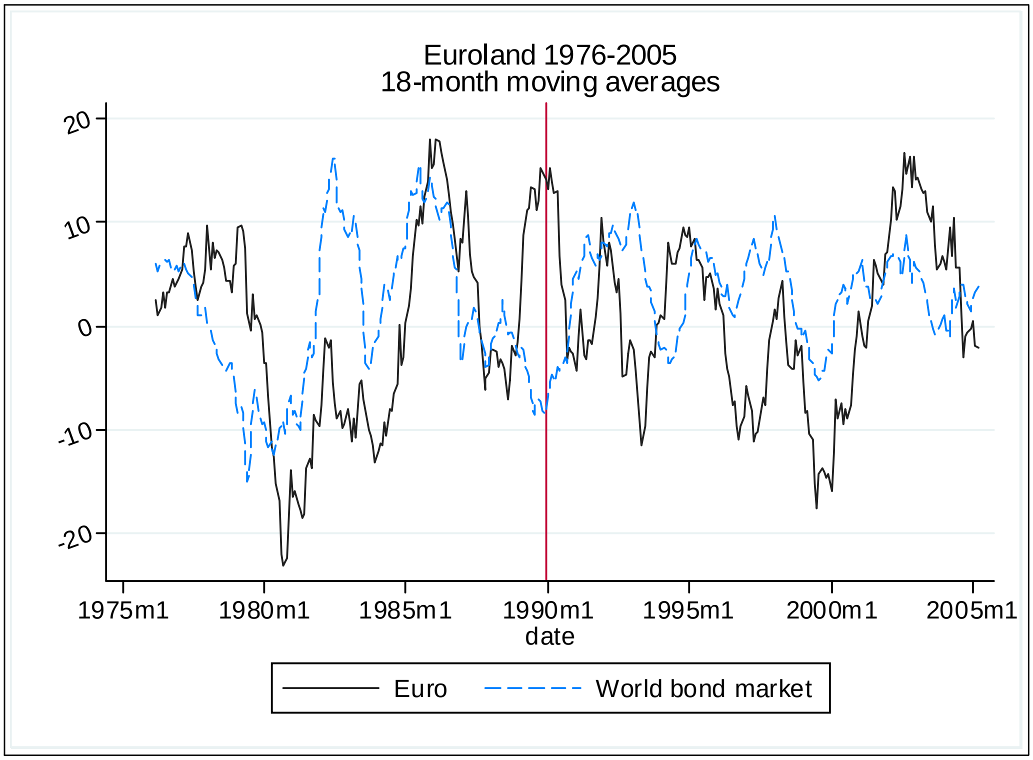

Figure 6: Currency and Hedged World Bond Market Excess Returns

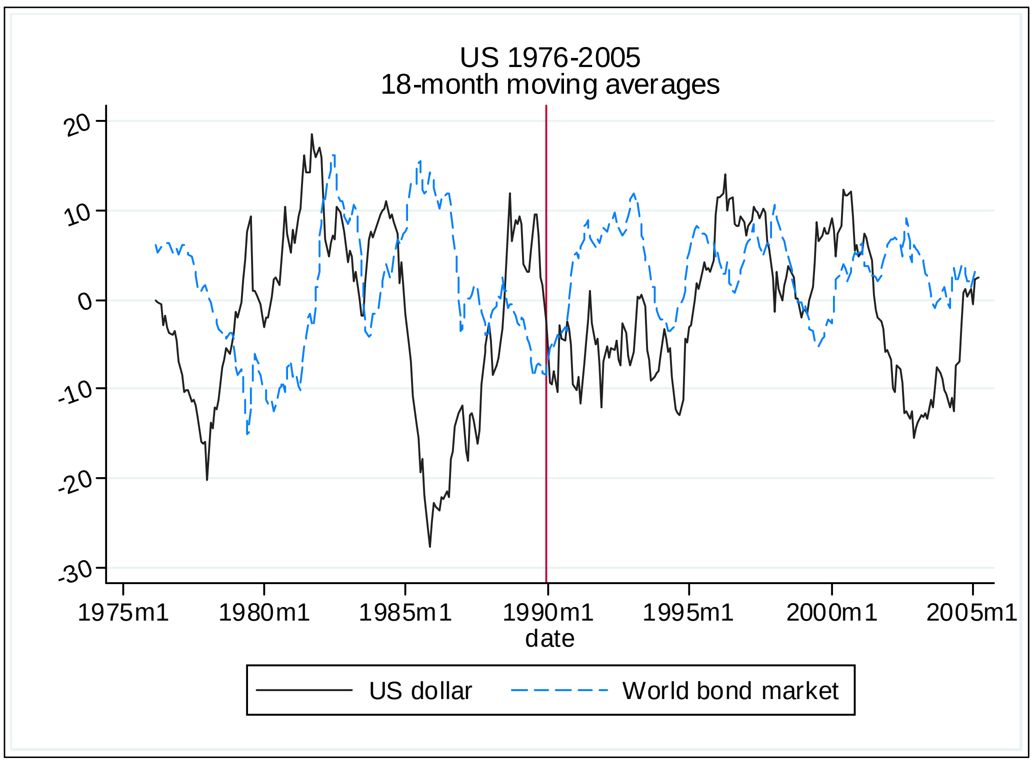

Figure 7: Currency and Hedged World Bond Market Excess Returns

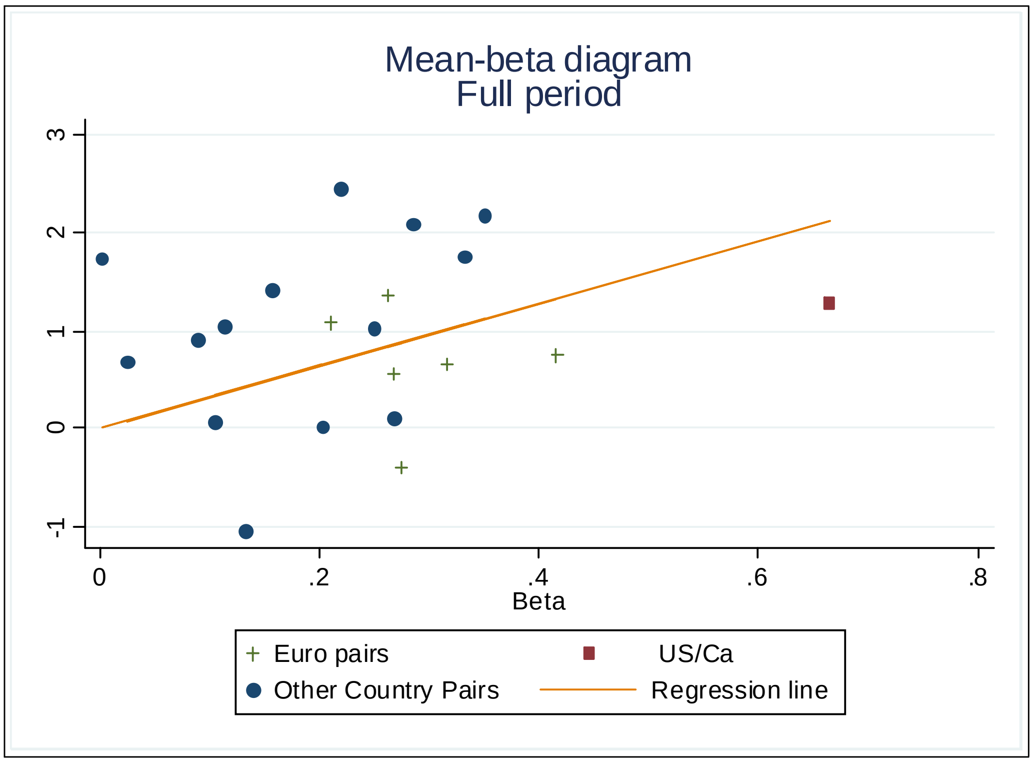

Figure 8: Beta with value-weighted, currency-hedged world portfolio and regression line with no intercept

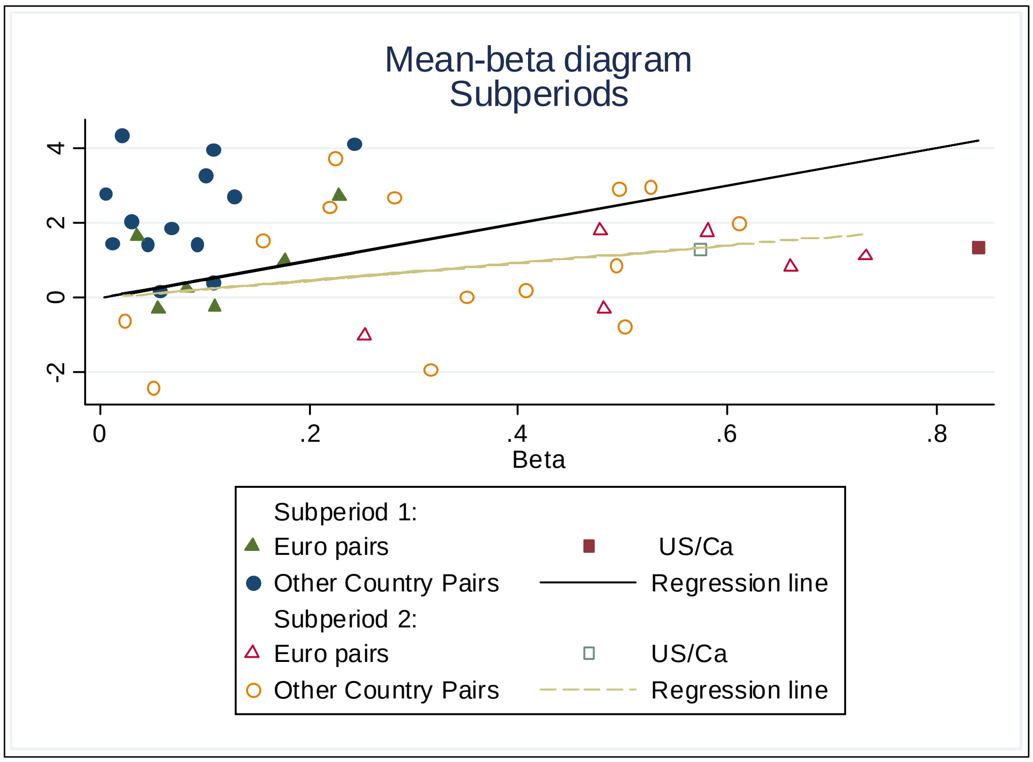

Figure 9: Beta with value-weighted, currency-hedged world portfolio and regression lines with no intercept

Table 1
Summary Statistics

	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Interest rates							
$E(i_{c,t})$	6.41	8.51	7.41	3.71	3.21	8.11	5.81
$\sigma(i_{c,t})$	0.81	1.01	1.01	0.91	0.71	0.91	0.81
Hedged stock excess returns							
$E(r_{c,t} - i_{c,t}) + ½\sigma^2$	7.31	7.21	5.21	4.91	8.41	7.31	7.01
$\sigma (r_{c,t} - i_{c,t})$	16.81	19.51	17.01	18.21	16.41	17.01	14.91
Hedged bond excess returns							
$E(r^b_{c,t} - i_{c,t}) + ½\sigma^2$	1.01	2.21	2.51	2.91	2.21	2.81	2.71
$\sigma (r^b_{c,t} - i_{c,t})$	3.91	9.51	7.71	7.21	4.81	6.71	7.31
Δ exchange rate							
$E(\Delta s_{c,t}) + ½\sigma^2$	0.71	-1.41	-0.21	3.71	3.11	-0.11	.
$\sigma(\Delta s_{c,t})$	10.71	10.21	5.41	11.51	12.21	10.71	.
Currency excess returns							
$E(\Delta s_{c,t} + i_{c,t} - i_{US,t}) + ½\sigma^2$	1.51	1.31	1.31	1.61	0.61	2.21	.
$\sigma(\Delta s_{c,t} + i_{c,t} - i_{US,t})$	10.61	10.21	5.41	11.61	12.31	10.81	.

Note. Stock market returns are from the Morgan Stanley Capital International database. All other variables are from the IMF's IFS database. Data are monthly. Coverage extends from 1975:7 to 2005:12. Unless otherwise specified, all following tables use data from the full period.

Variables i, r, r^b and s respectively denote log nominal short-term interest rates (returns on 3-month treasury bills), log stock return in local currency, log bond return (long-term bonds with a maturity of 10 years) and log exchange rates. All statistics reported are in percentage points. Averages and standard deviations are computed using monthly data and annualized.

Hedged stock excess returns are the returns on foreign stocks to a fully hedged investor, i.e. local currency returns, in excess of the local nominal interest rate.

Hedged bond excess returns are the returns on foreign bonds to a fully hedged investor, i.e. local currency returns, in excess of the local nominal interest rate.

Exchange rates are with respect to the dollar, in dollars per unit of foreign currency (i.e. the dollar depreciates when the exchange rate increases).

The currency excess return is the return to a US investor of borrowing in dollars to hold foreign currency.

Table 2
Currency return correlations

	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Base country: Euroland							
Euroland	.						
Australia	.	1.00					
Canada	.	0.70	1.00				
Japan	.	0.35	0.35	1.00			
Switzerland	.	-0.09	-0.11	0.20	1.00		
UK	.	0.31	0.34	0.24	-0.02	1.00	
US	.	0.63	0.87	0.40	-0.07	0.38	1.00
Base country: Australia							
Euroland	1.00						
Australia	.	.					
Canada	0.53	.	1.00				
Japan	0.67	.	0.46	1.00			
Switzerland	0.92	.	0.47	0.69	1.00		
UK	0.78	.	0.51	0.59	0.72	1.00	
US	0.59	.	0.85	0.55	0.54	0.58	1.00
Base country: Canada							
Euroland	1.00						
Australia	0.23	1.00					
Canada	.	.	.				
Japan	0.59	0.25	.	1.00			
Switzerland	0.91	0.20	.	0.62	1.00		
UK	0.71	0.24	.	0.50	0.65	1.00	
US	0.32	0.11	.	0.35	0.31	0.34	1.00
Base Country: Japan							
Euroland	1.00						
Australia	0.46	1.00					
Canada	0.55	0.74	1.00				
Japan	.	.	.	.			
Switzerland	0.87	0.33	0.39	.	1.00		
UK	0.72	0.49	0.56	.	0.60	1.00	
US	0.55	0.67	0.90	.	0.41	0.58	1.00
Base Country: Switzerland							
Euroland	1.00						
Australia	0.47	1.00					
Canada	0.52	0.77	1.00				
Japan	0.31	0.46	0.47	1.00			
Switzerland	.	.	.	.	.		
UK	0.56	0.49	0.53	0.37	.	1.00	
US	0.51	0.71	0.91	0.51	.	0.55	1.00
Base Country: UK							
Euroland	1.00						
Australia	0.35	1.00					
Canada	0.42	0.71	1.00				
Japan	0.50	0.42	0.44	1.00			
Switzerland	0.84	0.25	0.30	0.52	1.00		
UK	.	.	.	.	.	.	
US	0.42	0.64	0.88	0.48	0.32	.	1.00
Base Country: US							
Euroland	1.00						
Australia	0.26	1.00					
Canada	0.18	0.43	1.00				
Japan	0.55	0.25	0.10	1.00			
Switzerland	0.90	0.21	0.12	0.58	1.00		
UK	0.69	0.25	0.14	0.44	0.61	1.00	
US	.	.	.	.	.	.	.

Note. This table presents cross-country correlations of foreign currency log excess returns $s_{c,t}+i_{c,t}-i_{d,t}$, where d indexes the base country. Correlations are presented separately for investors from each base country. They are computed using monthly returns.

Table 3
Cross-country return correlations

Stock Markets	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : Stocks							
Euroland	1.00						
Australia	0.47	1.00					
Canada	0.56	0.58	1.00				
Japan	0.46	0.32	0.36	1.00			
Switzerland	0.75	0.44	0.53	0.41	1.00		
UK	0.66	0.50	0.55	0.37	0.61	1.00	
US	0.66	0.51	0.73	0.37	0.63	0.63	1.00
Panel B : Bonds							
Euroland	1.00						
Australia	0.32	1.00					
Canada	0.41	0.27	1.00				
Japan	0.32	0.17	0.31	1.00			
Switzerland	0.50	0.23	0.29	0.30	1.00		
UK	0.44	0.18	0.33	0.18	0.28	1.00	
US	0.55	0.24	0.62	0.32	0.37	0.37	1.00

Note. This table presents correlations of hedged stock market excess returns ($r_{c,t}$-$i_{c,t}$, see Table 1 note) and hedged bond excess return (r^bc,t-ic,t, see Table 1 note). They are computed using monthly returns.

Table 4

Optimal currency exposure for single-country stock portfolios: single and multiple currency cases

Stock market	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : Single currency							
Euroland		-0.43*	-0.57*	-0.32*	0.37	-0.37*	-0.52*
		(0.11)	(0.14)	(0.09)	(0.20)	(0.12)	(0.14)
Australia	0.39*		0.13	0.18	0.32*	0.16	0.29*
	(0.12)		(0.13)	(0.11)	(0.11)	(0.14)	(0.12)
Canada	0.42*	-0.02		0.13	0.35*	0.14	0.96*
	(0.11)	(0.12)		(0.10)	(0.10)	(0.12)	(0.22)
Japan	0.34*	-0.09	-0.06		0.36*	0.17	0.04
	(0.12)	(0.11)	(0.12)		(0.12)	(0.10)	(0.13)
Switzerland	-0.51*	-0.37*	-0.44*	-0.27*		-0.32*	-0.43*
	(0.17)	(0.08)	(0.10)	(0.09)		(0.09)	(0.11)
UK	0.26	-0.26*	-0.32*	-0.10	0.26*		-0.24
	(0.13)	(0.11)	(0.13)	(0.08)	(0.10)		(0.13)
US	0.19	-0.14	-0.77*	-0.03	0.19*	0.09	
	(0.11)	(0.09)	(0.17)	(0.10)	(0.09)	(0.11)	
Panel B : Multiple currencies							
Euroland	0.42	-0.10	-0.40	-0.19	0.34	-0.09	0.02
	(0.26)	(0.12)	(0.24)	(0.12)	(0.21)	(0.14)	(0.24)
Australia	0.55*	-0.20	-0.66*	-0.11	0.11	-0.31	0.62*
	(0.24)	(0.14)	(0.20)	(0.13)	(0.22)	(0.17)	(0.25)
Canada	0.34	-0.06	-1.00*	-0.21	0.32	-0.31*	0.92*
	(0.23)	(0.11)	(0.24)	(0.11)	(0.23)	(0.15)	(0.23)
Japan	0.38	-0.18	-0.58*	-0.27	0.16	0.03	0.46
	(0.21)	(0.16)	(0.26)	(0.15)	(0.20)	(0.13)	(0.25)
Switzerland	0.12	-0.15	-0.20	-0.02	0.40*	-0.01	-0.13
	(0.24)	(0.10)	(0.21)	(0.12)	(0.19)	(0.14)	(0.23)
UK	0.34	-0.13	-0.49*	-0.18	0.27	-0.01	0.20
	(0.25)	(0.11)	(0.22)	(0.10)	(0.22)	(0.16)	(0.22)
US	0.09	0.04	-0.91*	-0.23*	0.31	-0.01	0.71*
	(0.21)	(0.09)	(0.18)	(0.10)	(0.17)	(0.12)	(0.20)

Note. This table considers an investor holding a portfolio composed of equity from his own country, who chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the equity being held (as well as the base country), columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the hedged excess return to the row country stock market onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return to the row country stock market onto the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms in the same row and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table 5

Optimal currency exposure for an equally-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	0.17	-0.16	-0.61*	-0.11	0.23	-0.11	0.60*
	(0.15)	(0.11)	(0.14)	(0.07)	(0.12)	(0.08)	(0.15)
2 months	0.29	-0.13	-0.63*	-0.19*	0.26	-0.11	0.51*
	(0.15)	(0.09)	(0.15)	(0.07)	(0.13)	(0.09)	(0.15)
3 months	0.32	-0.11	-0.61*	-0.17	0.27	-0.10	0.40*
	(0.17)	(0.09)	(0.16)	(0.09)	(0.15)	(0.11)	(0.18)
6 months	0.20	-0.05	-0.38	-0.25*	0.35	-0.06	0.19
	(0.26)	(0.14)	(0.25)	(0.12)	(0.20)	(0.16)	(0.28)
12 months	-0.20	0.21	-0.22	-0.41*	0.67*	-0.20	0.15
	(0.40)	(0.20)	(0.36)	(0.17)	(0.30)	(0.21)	(0.37)
Panel B : 5 country optimization							
1 month	0.37*	-0.29*		-0.08		-0.10	0.11
	(0.11)	(0.11)		(0.07)		(0.08)	(0.08)
2 months	0.50*	-0.27*		-0.15*		-0.09	0.01
	(0.11)	(0.09)		(0.07)		(0.09)	(0.11)
3 months	0.56*	-0.27*		-0.14		-0.09	-0.06
	(0.11)	(0.10)		(0.08)		(0.11)	(0.14)
6 months	0.53*	-0.21		-0.21*		-0.02	-0.09
	(0.14)	(0.13)		(0.10)		(0.15)	(0.18)
12 months	0.44*	0.05		-0.34*		-0.16	0.01
	(0.19)	(0.17)		(0.15)		(0.19)	(0.22)

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping T-months returns, T varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table 6

Optimal currency exposure for a value-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	0.13	-0.09	-0.70*	-0.13	0.22	-0.09	0.66*
	(0.17)	(0.10)	(0.15)	(0.08)	(0.13)	(0.08)	(0.15)
2 months	0.22	-0.07	-0.73*	-0.22*	0.26	-0.06	0.60*
	(0.16)	(0.09)	(0.15)	(0.08)	(0.13)	(0.09)	(0.16)
3 months	0.22	-0.04	-0.76*	-0.23*	0.30*	-0.03	0.55*
	(0.17)	(0.09)	(0.17)	(0.10)	(0.15)	(0.11)	(0.19)
6 months	0.11	0.01	-0.60*	-0.32*	0.39*	0.03	0.39
	(0.24)	(0.14)	(0.22)	(0.12)	(0.19)	(0.15)	(0.26)
12 months	-0.29	0.25	-0.49	-0.46*	0.72*	-0.09	0.36
	(0.39)	(0.22)	(0.36)	(0.18)	(0.30)	(0.21)	(0.37)
Panel B : 5 country optimization							
1 month	0.29*	-0.25*		-0.08		-0.08	0.12
	(0.11)	(0.09)		(0.07)		(0.08)	(0.09)
2 months	0.42*	-0.25*		-0.16*		-0.05	0.04
	(0.11)	(0.10)		(0.08)		(0.09)	(0.11)
3 months	0.46*	-0.24*		-0.17		-0.03	-0.03
	(0.11)	(0.10)		(0.09)		(0.11)	(0.14)
6 months	0.45*	-0.20		-0.24*		0.06	-0.07
	(0.13)	(0.13)		(0.11)		(0.15)	(0.18)
12 months	0.39	0.01		-0.32*		-0.08	0.00
	(0.20)	(0.20)		(0.16)		(0.21)	(0.23)

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with constant value weights (reflecting the end-of-period 2005:12 weights as reported in Table 7), who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon *T* of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping *T*-months returns, *T* varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table 7

Variance Reduction: standard deviations of hedged portfolios

Base country	No hedge	Half hedge	Full hedge	Optimal hedge
Equally-weighted portfolio				
Euroland	16.99	15.20	13.86	12.51
Australia	14.41	13.45	13.86	12.51
Canada	13.48	13.24	13.86	12.51
Japan	16.26	14.44	13.86	12.51
Switzerland	18.24	15.71	13.86	12.51
UK	16.10	14.54	13.86	12.51
US	14.63	13.83	13.86	12.51
Value-weighted portfolio				
Euroland	17.28	15.18	13.82	12.48
Australia	15.43	13.77	13.82	12.48
Canada	13.26	13.15	13.82	12.48
Japan	15.81	14.18	13.82	12.48
Switzerland	18.64	15.72	13.82	12.48
UK	16.70	14.73	13.82	12.48
US	14.05	13.71	13.82	12.48
Single country portfolio				
Euroland	18.02	N/A	N/A	16.59
Australia	18.75	N/A	N/A	17.57
Canada	17.40	N/A	N/A	15.79
Japan	18.65	N/A	N/A	17.62
Switzerland	17.67	N/A	N/A	16.58
UK	16.82	N/A	N/A	15.62
US	14.63	N/A	N/A	13.46

Note. This table reports the variance of portfolios featuring different uses of currency for risk-management.

We present results for two types of global equity portfolios (equally-weighted and value-weighted) and for single-country portfolio as respectively described in Table 5, Table 6 and panel B of Table 4). Within each panel, rows represent base countries and columns represent the risk-management strategy.

"No hedge" refers to the simple equity portfolio. "Half hedge" refers to a portfolio in which half of the implicit currency risk is neutralized. "Full hedge" refers to a portfolio in which all of the implicit currency risk is neutralized. "Optimal hedge" refers to a portfolio in which the currency position is chosen optimally to minimize variance.

Reported standard deviations are annualized, and measured in percentage points.

All results presented are computed considering returns at a quarterly horizon.

Table 8

Subperiod analysis

Equally-weighted global equity portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
Subperiod I : 1975-1989							
1 month	0.15	-0.11	-0.73*	-0.06	0.08	-0.06	0.73*
	(0.20)	(0.16)	(0.23)	(0.12)	(0.13)	(0.11)	(0.24)
3 months	0.14	-0.05	-0.63*	-0.20	0.22	-0.09	0.62
	(0.21)	(0.12)	(0.26)	(0.14)	(0.18)	(0.15)	(0.35)
12 months	-0.62	0.23	-0.15	-0.31	0.57	-0.04	0.33
	(0.45)	(0.22)	(0.61)	(0.23)	(0.33)	(0.23)	(0.61)
Suberiod II : 1990-2005							
1 month	0.10	-0.25*	-0.49*	-0.15	0.51*	-0.20	0.48*
	(0.27)	(0.12)	(0.18)	(0.09)	(0.23)	(0.13)	(0.18)
3 months	0.44	-0.17	-0.65*	-0.08	0.37	-0.12	0.22
	(0.28)	(0.14)	(0.21)	(0.10)	(0.23)	(0.14)	(0.19)
12 months	0.56	-0.17	-0.31	-0.23	0.47	-0.22	-0.11
	(0.52)	(0.29)	(0.37)	(0.23)	(0.49)	(0.25)	(0.37)
Panel B : 5 country optimization							
Subperiod I : 1975-1989							
1 month	0.21	-0.22		-0.06		-0.06	0.13
	(0.19)	(0.16)		(0.12)		(0.11)	(0.10)
3 months	0.35*	-0.15		-0.15		-0.10	0.05
	(0.17)	(0.11)		(0.11)		(0.15)	(0.20)
12 months	-0.10	0.14		-0.20		-0.02	0.18
	(0.22)	(0.20)		(0.15)		(0.21)	(0.24)
Suberiod II : 1990-2005							
1 month	0.56*	-0.40*		-0.08		-0.20	0.12
	(0.12)	(0.12)		(0.08)		(0.11)	(0.13)
3 months	0.79*	-0.47*		-0.06		-0.11	-0.15
	(0.13)	(0.12)		(0.11)		(0.13)	(0.17)
12 months	1.02*	-0.40		-0.22		-0.20	-0.20
	(0.21)	(0.23)		(0.19)		(0.25)	(0.32)

Note. This table replicates Table 5 for two subperiods, respectively extending from 1975:7 to 1989:12 and from 1990:1 to 2005:12. Time horizons include 1, 3 and 12 months only.

Table 9

Optimal currency exposure for single-country bond portfolios: single and multiple currency cases

Bond market	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : Single currency							
Euroland		0.04	0.05	-0.03	-0.02	0.09*	0.06
		(0.03)	(0.03)	(0.04)	(0.06)	(0.04)	(0.04)
Australia	-0.02		0.04	0.00	-0.01	0.02	0.06
	(0.05)		(0.07)	(0.05)	(0.05)	(0.04)	(0.05)
Canada	-0.07	0.12*		-0.07	-0.08	-0.07	0.24*
	(0.05)	(0.05)		(0.05)	(0.04)	(0.06)	(0.08)
Japan	0.05	0.09*	0.14*		0.01	0.10*	0.16*
	(0.05)	(0.04)	(0.05)		(0.05)	(0.05)	(0.05)
Switzerland	0.08	0.02	0.05	-0.02		0.09*	0.05
	(0.07)	(0.02)	(0.03)	(0.04)		(0.04)	(0.03)
UK	0.22*	0.07*	0.11*	0.02	0.13*		0.12*
	(0.05)	(0.04)	(0.05)	(0.04)	(0.05)		(0.04)
US	-0.21*	0.03	-0.21*	-0.15*	-0.18*	-0.09	
	(0.05)	(0.05)	(0.09)	(0.05)	(0.04)	(0.05)	
Panel B : Multiple currencies							
Euroland	-0.10	0.01	-0.01	-0.07	0.03	0.08	0.07
	(0.08)	(0.03)	(0.07)	(0.04)	(0.07)	(0.05)	(0.06)
Australia	-0.13	-0.02	-0.07	0.00	0.03	0.06	0.14
	(0.15)	(0.08)	(0.13)	(0.06)	(0.12)	(0.07)	(0.10)
Canada	0.03	0.18*	-0.35*	-0.08	-0.08	-0.07	0.36*
	(0.12)	(0.05)	(0.11)	(0.06)	(0.11)	(0.08)	(0.08)
Japan	-0.05	-0.02	0.00	-0.12	-0.07	0.07	0.18
	(0.10)	(0.05)	(0.10)	(0.06)	(0.08)	(0.05)	(0.09)
Switzerland	-0.03	-0.03	0.05	-0.06	-0.04	0.10*	0.01
	(0.08)	(0.04)	(0.08)	(0.05)	(0.08)	(0.05)	(0.07)
UK	0.28*	0.01	-0.05	-0.10	-0.04	-0.23*	0.13
	(0.13)	(0.06)	(0.14)	(0.06)	(0.11)	(0.06)	(0.10)
US	-0.22	0.19*	-0.30*	-0.10	-0.02	0.09	0.36*
	(0.11)	(0.06)	(0.12)	(0.06)	(0.09)	(0.07)	(0.09)

Note. This table considers an investor holding a portfolio composed of long-term bonds from his own country, who chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the bond being held (as well as the base country), columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the hedged excess return to the row country bond onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return to the row country stock bond onto the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms in the same row and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table 10

Optimal currency exposure for an equally-weighted global bond portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	0.02	0.00	-0.12*	-0.06*	-0.04	-0.01	0.22*
	(0.05)	(0.02)	(0.05)	(0.03)	(0.04)	(0.03)	(0.05)
2 months	-0.01	0.03	-0.14*	-0.08*	-0.03	0.00	0.23*
	(0.07)	(0.03)	(0.06)	(0.03)	(0.05)	(0.04)	(0.05)
3 months	-0.03	0.04	-0.10	-0.07	-0.03	0.01	0.18*
	(0.07)	(0.04)	(0.08)	(0.04)	(0.07)	(0.05)	(0.06)
6 months	-0.08	0.13*	-0.05	-0.10	0.00	0.06	0.05
	(0.11)	(0.05)	(0.10)	(0.06)	(0.10)	(0.07)	(0.08)
12 months	-0.26	0.17	0.03	-0.11	0.11	0.14	-0.08
	(0.17)	(0.09)	(0.16)	(0.08)	(0.13)	(0.11)	(0.11)
Panel B : 5 country optimization							
1 month	-0.02	-0.03		-0.07*		-0.01	0.13*
	(0.03)	(0.02)		(0.03)		(0.03)	(0.03)
2 months	-0.04	-0.01		-0.09*		-0.01	0.15*
	(0.04)	(0.03)		(0.03)		(0.03)	(0.04)
3 months	-0.06	0.01		-0.08		0.01	0.11*
	(0.05)	(0.03)		(0.04)		(0.05)	(0.04)
6 months	-0.08	0.11*		-0.10		0.06	0.01
	(0.08)	(0.04)		(0.06)		(0.07)	(0.06)
12 months	-0.14	0.18*		-0.10		0.12	-0.06
	(0.12)	(0.06)		(0.07)		(0.11)	(0.07)

Note. This table considers an investor holding a portfolio composed of bonds from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global bond portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping T-months returns, T varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A1

Optimal currency exposure for an equally-weighted global equity portfolio: single-currency case

Base country	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Euroland		-0.37*	-0.45*	-0.25*	0.28	-0.30*	-0.33*
		(0.09)	(0.10)	(0.07)	(0.15)	(0.09)	(0.11)
Australia	0.37*		0.02	0.14	0.33*	0.21*	0.16*
	(0.09)		(0.08)	(0.07)	(0.07)	(0.09)	(0.08)
Canada	0.45*	-0.02		0.15	0.38*	0.25*	0.55*
	(0.10)	(0.08)		(0.09)	(0.09)	(0.11)	(0.16)
Japan	0.25*	-0.14	-0.15		0.32*	0.05	-0.06
	(0.07)	(0.07)	(0.09)		(0.08)	(0.06)	(0.09)
Switzerland	-0.28	-0.33*	-0.38*	-0.32*		-0.29*	-0.30*
	(0.15)	(0.07)	(0.09)	(0.08)		(0.07)	(0.09)
UK	0.30*	-0.21*	-0.25*	-0.05	0.29*		-0.13
	(0.09)	(0.09)	(0.11)	(0.06)	(0.07)		(0.11)
US	0.33*	-0.16*	-0.55*	0.06	0.30*	0.13	
	(0.11)	(0.08)	(0.16)	(0.09)	(0.09)	(0.11)	

Note. This table considers an investor holding a portfolio composed of stocks from all countries, with equal weights, who chooses a position in one foreign currency at a time to minimize the variance of his portfolio. Rows indicate the base country of the investor, columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the excess return to the global equity portfolio onto the excess return of the column country currency to an investor based in the row country. All regressions include an intercept.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A2

Optimal currency exposure for a home-biased global equity portfolio: single and multiple currency cases

Base country	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
PANEL A : Single currency							
Euroland		-0.40*	-0.52*	-0.31*	0.34	-0.32*	-0.45*
		(0.10)	(0.12)	(0.08)	(0.18)	(0.11)	(0.13)
Australia	0.37*		0.09	0.16	0.31*	0.17	0.25*
	(0.10)		(0.11)	(0.09)	(0.09)	(0.12)	(0.10)
Canada	0.42*	-0.01		0.12	0.35*	0.17	0.88*
	(0.10)	(0.10)		(0.09)	(0.09)	(0.11)	(0.19)
Japan	0.31*	-0.09	-0.08		0.35*	0.15	0.02
	(0.10)	(0.09)	(0.10)		(0.10)	(0.08)	(0.11)
Switzerland	-0.45*	-0.35*	-0.42*	-0.29*		-0.29*	-0.38*
	(0.15)	(0.08)	(0.09)	(0.09)		(0.08)	(0.10)
UK	0.25*	-0.24*	-0.30*	-0.10	0.25*		-0.21
	(0.11)	(0.10)	(0.12)	(0.07)	(0.09)		(0.12)
US	0.23*	-0.14	-0.71*	-0.01	0.22*	0.11	
	(0.11)	(0.08)	(0.16)	(0.09)	(0.09)	(0.11)	
Panel B : Multiple currencies at once							
Euroland	0.36	-0.08	-0.50*	-0.20	0.33	-0.08	0.17
	(0.23)	(0.11)	(0.21)	(0.11)	(0.18)	(0.13)	(0.22)
Australia	0.47*	-0.16	-0.68*	-0.14	0.15	-0.24	0.60*
	(0.20)	(0.12)	(0.17)	(0.11)	(0.19)	(0.15)	(0.22)
Canada	0.30	-0.05	-0.94*	-0.22*	0.31	-0.23	0.83*
	(0.20)	(0.10)	(0.21)	(0.10)	(0.20)	(0.13)	(0.21)
Japan	0.34	-0.14	-0.63*	-0.25*	0.20	0.01	0.48*
	(0.17)	(0.13)	(0.21)	(0.12)	(0.16)	(0.12)	(0.21)
Switzerland	0.14	-0.12	-0.35	-0.07	0.37*	-0.02	0.04
	(0.21)	(0.09)	(0.19)	(0.11)	(0.17)	(0.13)	(0.21)
UK	0.30	-0.11	-0.56*	-0.20*	0.28	-0.02	0.30
	(0.21)	(0.10)	(0.19)	(0.09)	(0.18)	(0.13)	(0.20)
US	0.15	0.00	-0.83*	-0.22*	0.30	-0.03	0.62*
	(0.18)	(0.09)	(0.17)	(0.09)	(0.15)	(0.11)	(0.19)

Note. This table considers an investor holding a home-biased portfolio of global equity. The portfolio is constructed by assigning a 75% weight to the home country of the investor, and distributing the remaining 25% over the four other countries according to their value weights. The investor chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the base country of the investor, columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the excess return on the row country home biased global equity portfolio onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return on the row country portfolio on the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A3 - Subperiod I

Optimal currency exposure for single-country bond portfolios: single and multiple currency cases

Bond market	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : Single currency							
Euroland		0.09*	0.13*	-0.07	-0.10	0.12*	0.16*
		(0.04)	(0.05)	(0.06)	(0.08)	(0.05)	(0.05)
Australia	0.08		0.15	0.10	0.07	0.12*	0.17*
	(0.06)		(0.08)	(0.07)	(0.06)	(0.05)	(0.07)
Canada	-0.10	0.13		-0.12	-0.11	-0.06	0.50*
	(0.08)	(0.07)		(0.07)	(0.06)	(0.09)	(0.17)
Japan	0.22*	0.14*	0.27*		0.09	0.20*	0.32*
	(0.07)	(0.06)	(0.06)		(0.08)	(0.06)	(0.06)
Switzerland	0.11	0.08*	0.11*	0.00		0.09	0.11*
	(0.09)	(0.03)	(0.04)	(0.06)		(0.05)	(0.04)
UK	0.32*	0.10	0.16*	0.09	0.17*		0.20*
	(0.07)	(0.05)	(0.07)	(0.08)	(0.07)		(0.06)
US	-0.25*	0.00	-0.60*	-0.20*	-0.20*	-0.09	
	(0.08)	(0.07)	(0.20)	(0.09)	(0.07)	(0.08)	
Panel B : Multiple currencies							
Euroland	-0.14	0.07	-0.19	-0.19*	0.03	0.10	0.31*
	(0.11)	(0.04)	(0.12)	(0.06)	(0.09)	(0.05)	(0.13)
Australia	-0.17	-0.17*	-0.13	0.02	0.08	0.10	0.27
	(0.17)	(0.08)	(0.22)	(0.10)	(0.15)	(0.09)	(0.21)
Canada	-0.01	0.24*	-0.68*	-0.23*	-0.03	0.00	0.71*
	(0.15)	(0.08)	(0.19)	(0.09)	(0.13)	(0.10)	(0.17)
Japan	0.05	-0.08	-0.10	-0.30*	-0.08	0.07	0.43*
	(0.14)	(0.05)	(0.16)	(0.08)	(0.10)	(0.06)	(0.16)
Switzerland	-0.07	0.03	0.01	-0.11*	-0.03	0.05	0.12
	(0.10)	(0.05)	(0.14)	(0.05)	(0.09)	(0.05)	(0.11)
UK	0.42*	0.01	-0.24	-0.14	-0.06	-0.31*	0.32
	(0.17)	(0.08)	(0.29)	(0.12)	(0.16)	(0.08)	(0.24)
US	-0.25	0.20*	-0.75*	-0.19	0.02	0.13	0.84*
	(0.15)	(0.08)	(0.21)	(0.12)	(0.12)	(0.10)	(0.20)

Note. This table considers an investor holding a portfolio composed of long-term bonds from his own country, who chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the bond being held (as well as the base country), columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the hedged excess return to the row country bond onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return to the row country stock bond onto the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms in the same row and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A3 - Subperiod II

Optimal currency exposure for single-country bond portfolios: single and multiple currency cases

Bond market	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : Single currency							
Euroland		-0.02	-0.02	0.02	0.16*	0.03	-0.03
		(0.03)	(0.04)	(0.03)	(0.08)	(0.06)	(0.03)
Australia	-0.17*		-0.20*	-0.10	-0.14*	-0.15*	-0.11
	(0.05)		(0.10)	(0.05)	(0.04)	(0.05)	(0.06)
Canada	-0.06	0.14		-0.01	-0.05	-0.08	0.11
	(0.06)	(0.08)		(0.05)	(0.05)	(0.05)	(0.07)
Japan	-0.09	0.04	-0.01		-0.08	-0.01	-0.01
	(0.06)	(0.06)	(0.06)		(0.06)	(0.07)	(0.07)
Switzerland	0.04	-0.03	-0.02	-0.01		0.10	-0.03
	(0.10)	(0.04)	(0.04)	(0.06)		(0.07)	(0.04)
UK	0.05	0.06	0.05	-0.01	0.05		0.03
	(0.06)	(0.03)	(0.04)	(0.04)	(0.05)		(0.03)
US	-0.18*	0.10	-0.01	-0.11*	-0.17*	-0.11*	
	(0.05)	(0.06)	(0.09)	(0.05)	(0.04)	(0.04)	
Panel B : Multiple currencies							
Euroland	-0.20*	0.00	0.03	0.02	0.16	0.08	-0.09
	(0.10)	(0.05)	(0.09)	(0.04)	(0.09)	(0.05)	(0.06)
Australia	-0.15	0.26*	-0.21	-0.02	0.03	-0.02	0.12
	(0.21)	(0.09)	(0.17)	(0.08)	(0.17)	(0.09)	(0.10)
Canada	0.01	0.19*	-0.23	0.01	-0.04	-0.14	0.20*
	(0.15)	(0.07)	(0.12)	(0.06)	(0.14)	(0.08)	(0.08)
Japan	-0.46*	0.22*	-0.14	0.04	0.18	0.15	0.00
	(0.16)	(0.10)	(0.14)	(0.07)	(0.15)	(0.08)	(0.10)
Switzerland	-0.03	-0.09	0.06	0.00	-0.05	0.21*	-0.10
	(0.13)	(0.06)	(0.11)	(0.06)	(0.12)	(0.08)	(0.09)
UK	-0.04	0.06	0.02	-0.05	0.09	-0.07	-0.01
	(0.16)	(0.07)	(0.13)	(0.05)	(0.13)	(0.06)	(0.08)
US	-0.33*	0.25*	-0.13	-0.05	0.07	0.06	0.12
	(0.15)	(0.08)	(0.13)	(0.06)	(0.13)	(0.07)	(0.07)

Note. This table considers an investor holding a portfolio composed of long-term bonds from his own country, who chooses a foreign currency position to minimize the variance of his portfolio. Panel A allows the investor to use only one foreign currency. Panel B allows her to choose a vector of positions in all available foreign currencies. Rows indicate the bond being held (as well as the base country), columns the currencies used to manage risk.

Cells of Panel A are obtained by regressing the hedged excess return to the row country bond onto the excess return on the column country currency. Rows of Panel B (excluding diagonal terms) are obtained by regressing the excess return to the row country stock bond onto the vector of all foreign currency excess returns. All regressions include an intercept. Diagonal terms in Panel B are obtained by computing the opposite of the sum of other terms in the same row and the corresponding standard deviation.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping quarterly returns. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A4- Subperiod I

Optimal currency exposure for an equally-weighted global bond portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	0.08	0.01	-0.24*	-0.11*	-0.06	0.00	0.33*
	(0.07)	(0.04)	(0.10)	(0.04)	(0.05)	(0.03)	(0.09)
2 months	0.01	0.03	-0.30*	-0.14*	-0.04	0.00	0.43*
	(0.08)	(0.04)	(0.10)	(0.05)	(0.06)	(0.04)	(0.10)
3 months	-0.02	0.04	-0.30*	-0.16*	-0.01	0.02	0.43*
	(0.10)	(0.05)	(0.14)	(0.06)	(0.08)	(0.06)	(0.12)
6 months	-0.11	0.13*	-0.22	-0.22*	0.04	0.09	0.29*
	(0.13)	(0.06)	(0.15)	(0.08)	(0.11)	(0.08)	(0.15)
12 months	-0.29	0.24*	-0.14	-0.25*	0.12	0.20*	0.13
	(0.16)	(0.11)	(0.19)	(0.08)	(0.10)	(0.09)	(0.14)
Panel B : 5 country optimization							
1 month	0.02	-0.04		-0.11*		-0.01	0.15*
	(0.06)	(0.03)		(0.04)		(0.03)	(0.04)
2 months	-0.03	-0.04		-0.13*		-0.01	0.21*
	(0.06)	(0.03)		(0.05)		(0.04)	(0.05)
3 months	-0.03	-0.04		-0.14*		0.01	0.20*
	(0.08)	(0.04)		(0.07)		(0.05)	(0.06)
6 months	-0.07	0.05		-0.19*		0.09	0.12
	(0.10)	(0.04)		(0.08)		(0.08)	(0.08)
12 months	-0.18	0.16		-0.21*		0.20*	0.03
	(0.12)	(0.09)		(0.07)		(0.09)	(0.11)

Note. This table considers an investor holding a portfolio composed of bonds from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon T of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping T-months returns, T varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A4- Subperiod II

Optimal currency exposure for an equally-weighted global bond portfolio: multiple-currency case

Time horizon	Currency						
	Euroland	Australia	Canada	Japan	Switzerland	UK	US
Panel A : 7 country optimization							
1 month	-0.07	0.00	-0.05	-0.04	0.04	0.00	0.12*
	(0.09)	(0.03)	(0.06)	(0.03)	(0.07)	(0.04)	(0.06)
2 months	-0.10	0.07	-0.07	-0.04	0.04	0.01	0.08
	(0.12)	(0.04)	(0.08)	(0.04)	(0.10)	(0.05)	(0.06)
3 months	-0.17	0.13*	-0.08	-0.01	0.06	0.04	0.04
	(0.12)	(0.06)	(0.11)	(0.05)	(0.11)	(0.06)	(0.06)
6 months	-0.26	0.23*	-0.11	0.02	0.12	0.07	-0.07
	(0.18)	(0.09)	(0.14)	(0.07)	(0.15)	(0.08)	(0.08)
12 months	-0.42*	0.24*	-0.14	0.04	0.25	0.12	-0.10
	(0.21)	(0.12)	(0.22)	(0.09)	(0.15)	(0.11)	(0.10)
Panel B : 5 country optimization							
1 month	-0.05	-0.01		-0.04		0.00	0.10*
	(0.04)	(0.03)		(0.03)		(0.04)	(0.04)
2 months	-0.07	0.06		-0.04		0.01	0.04
	(0.06)	(0.03)		(0.04)		(0.05)	(0.05)
3 months	-0.13*	0.12*		0.00		0.03	-0.02
	(0.06)	(0.04)		(0.05)		(0.06)	(0.05)
6 months	-0.15	0.21*		0.02		0.05	-0.13*
	(0.09)	(0.05)		(0.06)		(0.08)	(0.07)
12 months	-0.16	0.20*		0.04		0.08	-0.16
	(0.13)	(0.05)		(0.08)		(0.11)	(0.10)

Note. This table considers an investor holding a portfolio composed of bonds from all countries, with equal weights, who chooses a vector of positions in all available foreign currencies to minimize the variance of his portfolio. In this case, the optimal currency positions do not depend on the investor's base country.

Rows indicate the time-horizon *T* of the investor, columns the currencies used to manage risk.

Rows are obtained by regressing the excess return on the global equity portfolio onto the vector of all foreign currency excess returns. All regressions include an intercept. All returns considered are at the row time-horizon.

Reported currency positions are the amount of dollars invested in foreign currency per dollar in the portfolio.

We run monthly regressions on overlapping *T*-months returns, *T* varying from 1 month to 12 months. Standard errors are corrected for auto-correlation due to overlapping intervals using the Newey-West procedure.

Table A5

Variance Reduction: standard deviations of hedged bond portfolios

Base country	No hedge	Half hedge	Full hedge	Optimal hedge
Equally-weighted portfolio				
Euroland	7.64	5.87	5.40	5.21
Australia	10.79	7.34	5.40	5.21
Canada	9.24	6.76	5.40	5.21
Japan	9.60	6.41	5.40	5.21
Switzerland	8.85	6.17	5.40	5.21
UK	9.30	6.60	5.40	5.21
US	9.56	6.98	5.40	5.21
Value-weighted portfolio				
Euroland	9.04	6.97	6.63	6.24
Australia	12.74	8.83	6.63	6.24
Canada	9.85	7.75	6.63	6.24
Japan	9.81	7.12	6.63	6.24
Switzerland	10.31	7.27	6.63	6.24
UK	10.67	7.81	6.63	6.24
US	9.39	7.74	6.63	6.24
Single country portfolio				
Euroland	5.05	N/A	N/A	4.92
Australia	8.80	N/A	N/A	8.73
Canada	7.89	N/A	N/A	7.46
Japan	7.97	N/A	N/A	7.63
Switzerland	5.58	N/A	N/A	5.44
UK	7.84	N/A	N/A	7.47
US	8.59	N/A	N/A	7.95

Note. This table reports the variance of portfolios featuring different uses of currency for risk-management.

We present results for two types of global bond portfolios (equally-weighted and value-weighted) and for single-country portfolios as described in Tables 9 and 10). Within each panel, rows represent base countries and columns represent the risk-management strategy.

"No hedge" refers to the simple bond portfolio. "Half hedge" refers to a portfolio in which half of the implicit currency risk is neutralized. "Full hedge" refers to a portfolio in which all of the implicit currency risk is neutralized. "Optimal hedge" refers to a portfolio in which the currency position is chosen optimally to minimize variance.

Reported standard deviations are annualized, and measured in percentage points.

All results presented are computed considering returns at a quarterly horizon.