WHEN IS IT OPTIMAL TO ABANDON A FIXED EXCHANGE RATE?

Sergio Rebelo
Carlos A. Vegh

When Is It Optimal to Abandon a Fixed Exchange Rate?
Sergio Rebelo and Carlos A. Vegh
NBER Working Paper No. 12793
December 2006
JEL No. F31

**ABSTRACT**

The influential Krugman-Flood-Garber (KFG) model of balance of payment crises assumes that a fixed exchange rate is abandoned if and only if international reserves reach a critical threshold value. From a positive standpoint, the KFG rule is at odds with many episodes in which the central bank has plenty of international reserves at the time of abandonment. We study the optimal exit policy and show that, from a normative standpoint, the KFG rule is suboptimal. We consider a model in which the fixed exchange rate regime has become unsustainable due to an unexpected increase in government spending. We show that, when there are no exit costs, it is optimal to abandon immediately. When there are exit costs, the optimal abandonment time is a decreasing function of the size of the fiscal shock. For large fiscal shocks immediate abandonment is optimal. Our model is consistent with the evidence that many countries exit fixed exchange rate regimes with plenty of international reserves in the central bank's vault.

Sergio Rebelo
Kellogg Graduate School of Management
Northwestern University
Leverone Hall
Evanston, IL  60208-2001
and NBER
s-rebelo@northwestern.edu

Carlos A. Vegh
Department of Economics
Tydings Hall, Office 4118G
University of Maryland
College Park, MD 20742-7211
and NBER
cvegh@ucla.edu

# 1. Introduction

Consider an open economy with a fixed exchange rate that suffers an unexpected fiscal shock. This shock consists of an increase in government expenditures that has to be financed with seignorage. When, if at all, should the fixed exchange rate regime be abandoned? Further, suppose that, with some probability, a future fiscal reform or a financial package from the International Monetary Fund (IMF) can restore the sustainability of the fixed exchange rate regime. For how long should policy makers wait for this scenario to materialize?

The decision to exit a fixed exchange rate regime is one of the most important policy questions in open-economy macroeconomics. This importance was recently illustrated by Argentina's abandonment in early 2002 of its 10-year old "Convertibility plan" that had tied the peso to the U.S. dollar at a one-to-one rate since April 1991. Most analysts agree that fixing the exchange rate was an effective strategy to eliminate runaway inflation. However, in the mid 1990s, as the fiscal situation began to deteriorate, the question of whether Argentina should abandon the fixed exchange rate began to surface with increasing frequency.[1] The IMF rescue packages in December 2000 and August 2001 bought Argentina some time. But, in the end, the fixed exchange rate was abandoned in January 2002.

Economic theory offers surprisingly little guidance as to the optimal time to exit a fixed exchange rate regime. The dominant paradigm for understanding this exit is the model proposed by Krugman (1979) and Flood and Garber (1984), which we refer to as the KFG model.[2] This model makes two central assumptions. The first assumption is that the root cause of the eventual abandonment of the

---

[1] See Mussa (2002) for a detailed analysis of Argentina's lax fiscal policy during the mid 1990s.

[2] The original KFG model does not have microfoundations. However, several authors have extended the KFG framework to models populated by optimizing representative agents. See, for example, Obstfeld (1986a), Calvo (1987), Drazen and Helpman (1987), Wijnbergen (1991), Burnside, Eichenbaum, and Rebelo (2001), and Lahiri and Végh (2003).

fixed exchange rate is an unsustainable fiscal policy. The second assumption is that the central bank follows an ad-hoc exit rule whereby the fixed exchange rate regime is abandoned only when the central bank exhausts its foreign exchange reserves and its ability to borrow.

To study the empirical plausibility of these two hypotheses, we collect in Table 1 data for 51 episodes in which regimes with fixed exchange rates were abandoned. These abandonments are often called "currency crises." Our episodes were selected from an updated version of Kaminsky and Reinhart's (1999) list of crisis episodes according to the criteria outlined in Appendix 7.1. Table 1 reports the change in the exchange rate in the month in which the fixed exchange rate regime was abandoned, as well as the change in the exchange rate in the 12 months before and after the abandonment.[3] Table 1 also reports the rate of change in real government spending in the three years prior to the crisis and the reserve losses that occurred in the 12 months prior to the crisis.

We view the fiscal data in Table 1 as lending empirical support to the first KFG assumption. There were increases in real government spending in the three years prior to the abandonment of the peg in 80 percent (37 out of 46) of the episodes for which we have fiscal data. Therefore, fiscal shocks are plausible suspects as the root cause of the decision to abandon a fixed exchange rate.

We think that the reserve-loss data in Table 1 implies that the second KFG assumption is empirically implausible. While the KFG model is not explicit about the critical lower bound for international reserves (is it zero? is it negative?), it is clearly in the spirit of the model that the monetary authority holds on to the peg for as long as it can. So we would expect to see central banks exhaust their international reserves before the fixed exchange rate is abandoned. Figure

---

[3]In some of the episodes included in Table 1 the exchange rate was not literally fixed, but followed a crawling peg or fluctuated within a narrow band.

2

1 depicts a histogram of the fraction of initial reserves lost during the 12 months prior to the crisis. In 12 out of 51 episodes countries have non-positive reserve losses (i.e., they gained reserves). In 38 out of the 51 episodes (or roughly 75 percent), reserve losses were less than 40 percent of initial reserves. While there were cases in which the monetary authority was willing to lose a large amount of reserves before devaluing, in most cases the peg was abandoned with plenty of ammunition left in the central bank's coffers. In other words, the monetary authority *chooses* to devalue as opposed to *being forced* to devalue by literally exhausting its reserves and its ability to borrow. We conclude that the KFG exit rule, a critical component of the KFG model, is inconsistent with the empirical behavior of reserves in countries that have abandoned fixed exchange rates. In addition, and given that it assumes an exogenous exit rule, the KFG model is unsuitable for understanding the decision to abandon a fixed exchange rate regime.

In this paper, we study the optimal exit from a fixed exchange rate regime.[4] Our analysis is in the spirit of the literature on optimal monetary and fiscal policy pioneered by Lucas and Stokey (1983). We argue that the assumption that central bankers choose the optimal time to abandon the peg generates empirical implications that are more plausible than those associated with the KFG exit rule.[5]

Our analysis is based on a standard cash-in-advance small-open-economy model, extended to incorporate rational policy makers. We first consider the case where

---

[4] Authors such as Buiter (1987), Flood, Garber, and Kramer (1996), Lahiri and Végh (2003), and Flood and Jeanne (2005) have studied whether it is feasible and/or optimal to delay the abandonment of the fixed exchange rate regime (i.e., "defend the peg") by borrowing or by raising interest rates. While these models give the central bank a more active role than in the original KFG model, they continue to assume that abandonment of the peg is governed by the KFG rule.

[5] Second-generation models of speculative attacks introduce an optimizing central banker (Obstfeld (1986b, 1996)). However, they assume that currency crises do not have a fiscal origin. Instead, the crises are caused by the incentive to increase output via unexpected inflation in Barro-Gordon type formulations.

there are no costs of abandoning the peg. In this case it is optimal to abandon the peg as soon as the fiscal shock occurs and without incurring any reserve losses. This policy is optimal independently of the level of international reserves and of whether the central bank faces a borrowing constraint.

We then consider the case in which there are costs of abandoning the peg. These exit costs can reflect, for instance, output losses or the cost of bailing out the banking system.[6] We choose to abstract from the source of these costs and simply assume that devaluing entails some fiscal and social cost. In this case there is a certain threshold value for the fiscal shock beyond which it is optimal to abandon immediately, incurring no reserve losses. For fiscal shocks lower than this threshold, the optimal exit time is a decreasing function of the size of the fiscal shock. In other words, the smaller the fiscal shock, the longer is the optimal delay.

Intuitively, the optimal exit time results from the trade off between two factors. For a given fiscal shock, delaying the abandonment of the peg reduces the present discounted value of the cost of abandoning. However, a longer delay requires a permanently higher level of inflation once the peg is abandoned. This increase in the post-abandonment rate of inflation produces a larger intertemporal distortion in consumption decisions. For large fiscal shocks, the cost of delaying (i.e., the larger intertemporal distortion) dominates because the gain from delaying is bounded by the economy's resources.

Some back-of-the-envelope calculations – based on our model, the fiscal data in Table 1, and on empirical estimates of the cost of balance of payment crises – suggest that an immediate abandonment should be at least as common as delayed abandonment. Hence, unlike the KFG model, our model is capable of explaining

---

[6]See Kaminsky and Reinhart (1999) and Gupta, Mishra, and Sahay (2003) for evidence on output and banking crises during currency crises.

4

the many episodes illustrated in Table 1 in which pegs were abandoned with still plenty of international reserves at the Central Banks' disposal.

To study the theoretical robustness of our results, we then consider four extensions of the basic model: (i) time-varying exit costs, (ii) social, but non-fiscal, costs of abandoning the peg, (iii) more general preferences, and (iv) the case in which the exist cost depends positively on the fiscal shock itself. In every single case, our main results go through which attests to the theoretical robustness of the paper's main message.

We then consider a stochastic version of our model in which the costs of abandoning arise endogenously. There are no fiscal or social exit costs but fiscal fundamentals are random. These fundamentals are governed by a stochastic process that captures the idea that a fiscal reform is more likely to occur while the economy has a fixed exchange rate. In particular we assume that, while the exchange rate is fixed, there may be a fiscal reform that restores the sustainability of the fixed exchange rate.[7] This reform arrives according to a Poisson process. Once the economy abandons the fixed exchange rate regime, there is no hope of a fiscal reform and the initial fiscal shock must be financed with seignorage revenues. There is thus an *option value* to maintaining the peg. In this context, the cost of abandoning the peg consists in giving up this option value. We show that there is a close connection, both formally and in terms of the properties of the optimal exit time, between this model and our benchmark model. In the stochastic model there is also a threshold value of the fiscal shock above which it is optimal to abandon as soon as the fiscal shock occurs. For shocks with values below this threshold, there is a negative relation between the size of the shock and the optimal exit time.

---

[7] See Flood, Bhandari, and Horne (1989) and Rigobon (2002) for analyses that also emphasize the link between fixed exchange rates and fiscal discipline.

The paper proceeds as follows. In Section 2 we introduce the model. In Section 3 we derive the basic results for the deterministic case. In Section 4 we examine the theoretical robustness of our results. In Section 5 we develop and solve the stochastic version of the model. Section 6 concludes.

## 2. The Basic Model

Consider a standard optimizing small-open-economy model in which money is introduced via a cash-in-advance constraint on consumption. All agents, including the government, can borrow and lend in international capital markets at a constant real interest rate $r$. There is a single consumption good in the economy and no barriers to trade, therefore the law of one price holds, $P_t = S_t P_t^*$, where $P_t$ and $P_t^*$ denote the domestic and foreign price level, respectively. The exchange rate, $S_t$, is defined as units of domestic currency per unit of foreign currency. For convenience we assume that $P_t^* = 1$, therefore $P_t = S_t$.

Before the fiscal shock occurs at time $t = 0^-$, the exchange rate is fixed at a level $S$. For $t < 0$ the economy has a sustainable fixed exchange rate regime and the government can satisfy its intertemporal budget constraint without resorting to seignorage. At $t = 0$ the economy suffers a 'fiscal shock': an increase in government spending that must be financed with seignorage revenues. Generating these revenues requires abandoning the fixed exchange rate regime at some point in time. Denote by $T$ the time at which the fixed exchange rate regime is abandoned. We wish to solve for the optimal value of $T$, which we denote by $T^*$.

### 2.1. Households

The representative household maximizes its lifetime utility, $V$, which depends on its consumption path, $c_t$:

$$V \equiv \int_0^\infty \ln(c_t)e^{-\rho t}dt. \tag{2.1}$$

The discount factor is denoted by $\rho$. The household's flow budget constraint is:

$$\begin{aligned}
\Delta b_t &= -(M_t - M_{t-})/S_t, &&\text{if } t \in J, \\
\dot{b}_t &= rb_t + y - c_t - \dot{m}_t - \varepsilon_t m_t, &&\text{if } t \notin J.
\end{aligned} \qquad (2.2)$$

Throughout the paper a dot over a variable represents the derivative of that variable with respect to time. Here $b_t$ denotes the household's holdings of foreign bonds that yield a real rate of return of $r$, and $y$ is a constant, exogenous, flow of output. The variable $m_t$ represents real money balances, defined as $m_t = M_t/P_t$, where $M_t$ denotes nominal money holdings. The variable $\varepsilon_t$ denotes the rate of devaluation, which coincides with the inflation rate, $\varepsilon_t = \dot{P}_t/P_t = \dot{S}_t/S_t$. To simplify, we assume that $r = \rho$.

As in Drazen and Helpman (1987), equation (2.2) takes into account the possibility of discrete changes in $b_t$ and $M_t$ at a finite set of points in time, $J$. Below we see that this set contains $t = 0$ and the time at which the peg is abandoned, $T$. These jumps are defined as $\Delta b_t \equiv b_t - b_{t-}$, where $b_{t-}$ represents the limit from the left. Since at any point in time after $t = 0$, the total level of real financial assets cannot change discretely, $b_{t-} + m_{t-} = b_t + m_t$.[8] At time $t = 0^-$, just before the household's time zero decisions are made, agents hold an amount $b_{0-}$ in real bonds. Their holdings of nominal money balances are $M_{0-}$ and their real money balances are therefore $m_{0-} = M_{0-}/S$.

Consumption is subject to a cash-in-advance constraint:

$$m_t \geq c_t. \qquad (2.3)$$

Since we only consider environments in which the nominal interest rate is positive, equation (2.3) always holds with equality.

---

[8] At $t = 0$ the total level of real financial assets may change discretely due to an unanticipated jump in the exchange rate, which changes the value of real money balances from $M_{0-}/S$ to $M_{0-}/S_0$.

The flow budget constraint, (2.2), together with (2.3) and the transversality condition, $\lim\limits_{t \to \infty} e^{-rt} b_t = 0$, implies the following intertemporal budget constraint:

$$b_{0^-} + y/r = \int_0^\infty c_t e^{-rt} dt + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt + \sum_{j \in J} e^{-rj} (M_j - M_{j^-})/S_j. \quad (2.4)$$

This budget constraint can be further simplified by using the cash-in-advance constraint and imposing the condition that $\lim\limits_{t \to \infty} e^{-rt} m_t = 0$:[9]

$$b_{0^-} + \frac{M_{0^-}}{S_0} + y/r = \int_0^\infty c_t (1 + r + \varepsilon_t) e^{-rt} dt. \quad (2.5)$$

This expression makes clear that, as is typical of cash-in-advance models, the *effective* price of consumption is given by $1 + r + \varepsilon_t$.

The first-order condition for the household's problem is:

$$1/c_t = \lambda (1 + r + \varepsilon_t), \quad (2.6)$$

where $\lambda$ is the Lagrange multiplier associated with (2.5).

## 2.2. Government

The government collects seignorage revenues and carries out expenditures, $g_t$. To simplify, we assume that government spending yields no utility to the representative household. The government's flow budget constraint is given by:

$$\begin{aligned} \Delta f_t &= (M_t - M_{t^-})/S_t, & \text{if } t \in J, \\ \dot{f}_t &= r f_t - g_t + \dot{m}_t + \varepsilon_t m_t, & \text{if } t \notin J, \end{aligned}$$

where $f_t$ denotes the government's net foreign assets. This flow budget constraint, together with the condition $\lim\limits_{t \to \infty} e^{-rt} f_t = 0$, implies the following intertemporal budget constraint for the government:

$$f_{0^-} + \int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt + \sum_{j \in J} e^{-rj} (M_j - M_{j^-})/S_j = \Gamma_{0^-}, \quad (2.7)$$

---

[9] This condition is always satisfied in equilibrium since (2.3) holds as an equality.

where, by definition, $\Gamma_{0-}$ is the present value of government spending:

$$\Gamma_{0-} \equiv \int_0^\infty g_t e^{-rt} dt.$$

If the peg is abandoned at time zero the jump in the money supply $(M_0 - M_{0-})$ is controlled by the central bank through its choice of $M_0$. In contrast, if the peg is abandoned at $T > 0$, the jump in the money supply $(M_T - M_{T-})$ is endogenously determined. Under perfect foresight the path for the exchange rate has to be continuous for all $t > 0$ to rule out arbitrage opportunities. This requirement implies that in equilibrium the household reduce its money holdings at time $T$ in anticipation of the higher inflation rate for $t \geq T$.

## 2.3. Equilibrium Consumption

Combining the household's and government's intertemporal constraints (equations (2.5) and (2.7), respectively), we obtain the economy's aggregate resource constraint:

$$b_{0-} + f_{0-} + y/r = \int_0^\infty c_t e^{-rt} dt + \Gamma_{0-}. \tag{2.8}$$

This constraint implies that the present value of output plus the total net foreign assets in the economy must equal the present value of consumption and government expenditures.

## 2.4. A Sustainable Fixed Exchange Rate Regime

Before time zero, the economy is in a sustainable fixed exchange rate regime, so agents expect $\varepsilon$ to be permanently zero. Sustainability of the peg requires that the government's net foreign assets be sufficient to finance the present value of government expenditures. This requirement condition for $t = 0^-$ is:

$$f_{0-} = \Gamma_{0-}.$$

9

In the fixed exchange rate regime, equations (2.3) and (2.8) imply that consumption and real balances are given by:

$$c_{0-} = y + rb_{0-}, \tag{2.9}$$
$$m_{0-} = c_{0-}.$$

Using the household's intertemporal constraint we can write consumption before time zero as:

$$c_{0-} = \frac{ra_{0-} + y}{1 + r}, \tag{2.10}$$

where $a_{0-} \equiv b_{0-} + M_{0-}/S_{0-}$.

## 2.5. Optimal Monetary Policy

Suppose that at time zero there is an unanticipated increase in the present value of government expenditures from $\Gamma_{0-}$ to $\Gamma_0$ and that this increase in expenditure must be financed with seignorage. Clearly, the peg has to be abandoned at some point because $\Gamma_0$ cannot be intertemporally financed with $\varepsilon = 0$. When is the optimal exit time? Throughout the paper we focus on the perfect commitment solution to this question.

After the fiscal shock takes place the aggregate constraint for the economy is:

$$b_{0-} + y/r = \int_0^\infty c_t e^{-rt} dt + \Delta\Gamma, \tag{2.11}$$

where $\Delta\Gamma = \Gamma_0 - \Gamma_{0-}$ represents the increase in the present value of government expenditures. Suppose that the government could finance this extra expenditure with lump sum taxes. Consumption would be constant over time at a level:

$$c_0 = c_{0-} - r\Delta\Gamma.$$

Since $\Delta\Gamma > 0$, the new level of consumption is lower than before. The economy has the same resources as before the fiscal shock, so the rise in government spending

10

has to be accommodated by a fall in private consumption. The corresponding fall in real money balances occurs through a fall in the nominal money supply at $t = 0$.

The government can replicate the lump sum taxes outcome by either expanding the money supply at a constant rate from $t = 0$ on, by printing money at $t = 0$, or by combining these two strategies. Suppose that the government abandons the fixed exchange rate regime at time zero, keeps $M_0 = M_{0-}$, and expands the money supply at a constant rate $\varepsilon$ such that the government budget constraint is satisfied:

$$\int_0^\infty (\dot{m}_t + \varepsilon_t m_t) e^{-rt} dt = \Delta\Gamma.$$

Since the central bank abandons the fixed exchange rate regime as soon as news about the fiscal shock arrives, there are no losses of reserves. Private agents are not given a chance to trade their money balances for foreign reserves at the fixed exchange rate $S$ before the devaluation occurs. The adjustment in the level of real balances occurs through a jump in the exchange rate, rather than through a discrete fall in the nominal money supply at time zero. The aggregate resource constraint (2.11) implies that consumption is equal to $c_0$. The cash-in-advance constraint implies that the new level of real balances is $m_0 = c_0$. This monetary policy is optimal since it replicates the outcome that can be achieved under lump sum taxes.

The fall in real balances from $c_{0-}$ to $c_0$ is associated with a jump in the exchange rate from $S$ to:

$$S_0 = S c_{0-}/c_0. \tag{2.12}$$

The constant level of money growth is given by: $\varepsilon = r\Delta\Gamma/c_0 > 0$. So from time zero on the currency depreciates at rate $\varepsilon$.

There is another optimal policy which consists of abandoning the peg at time zero and printing enough money to finance the new government spending. In this

case the resource constraint of the government is given by: $(M_0 - M_{0-})/S_0 = \Delta\Gamma$. Printing money at time zero amounts to taxing existing real balances and is therefore equivalent to lump sum taxes. Since all the seignorage revenue is collected at time zero, this policy implies a higher rate of instantaneous depreciation at time zero than that given by (2.12):

$$S_0 = \frac{S c_{0-}}{c_{0-} - (1+r)\Delta\Gamma}.$$

Any combination of the two policies discussed above, expanding the money supply at a constant rate from time zero on and printing money at time zero, is also optimal. Thus, there are multiple ways for monetary policy to achieve the optimal outcome but all these policies require that the fixed exchange rate be abandoned at time zero.

Abandoning the peg at time $T > 0$ yields a lower level of welfare than the policies just discussed. To show this result we use the following proposition.[10]

**Proposition 2.1.** *Once the fixed exchange rate regime is abandoned at time $T > 0$, it is optimal to expand the money supply at a constant rate, $\varepsilon$. So the optimal path for money growth, conditional on abandonment at time $T$, is:*

$$\varepsilon_t = 0, \quad \text{for } 0 \leq t < T , \tag{2.13}$$

$$\varepsilon_t = \varepsilon, \quad \text{for } t \geq T .$$

We now show that any positive $\varepsilon$ generates an intertemporal distortion on consumption. The value of $\varepsilon$ has to satisfy the government's intertemporal budget constraint, (2.7), which can be written as:

$$\frac{e^{-rT}\varepsilon}{r}\frac{M_T}{S_T} = \Delta\Gamma + \frac{M_{0-} - M_0}{S} + \frac{M_0 - M_T}{S}e^{-rT}. \tag{2.14}$$

---

[10]To prove this proposition solve the planner's problem for an economy with no cash-in-advance constraint. Then it is possible to show that the cash-in-advance economy with constant $\varepsilon$ can replicate the solution to the planner's problem. See Rebelo and Xie (1999) for details of a closed economy version of this result.

The term $(M_{0^-} - M_0)/S + [(M_0 - M_T)/S]e^{-rT}$ represents the net reserve losses incurred by the government as the household rearranges its money balances while the exchange rate is fixed in response to the changes in the path for inflation.

The first-order condition for the household's problem, (2.6), implies that consumption is constant within the subperiods $0 < t < T$ and $t \geq T$. Let us denote by $c^1$ and $c^2$ the level of consumption in the periods $0 < t < T$ and $t \geq T$, respectively. Using equations (2.9), (2.11), and the cash-in-advance constraint, (2.3), we can show that independently of the form of the momentary utility function and the value of $T$, the net reserve loss incurred by the government is given by:

$$(M_{0^-} - M_0)/S + [(M_0 - M_T)/S]e^{-rT} = r\Delta\Gamma. \tag{2.15}$$

Using this result, we can rewrite the government budget constraint (2.14) as:

$$\frac{e^{-rT}\varepsilon}{r}\frac{M_T}{S_T} = \Delta\Gamma(1+r). \tag{2.16}$$

This equation implies that $\varepsilon > 0$. The first-order condition for the household's problem, (2.6), implies that $c^2 < c^1$. Since the present value of resources that are available for consumption is independent of $T$, this non-flat path of consumption results in lower welfare compared to the case where the peg is abandoned at time zero.

The net reserve loss described in (2.15) is a cost that the government incurs when the abandonment of the fixed exchange rate regime is delayed. However, since this cost represents a transfer from the government to households it is not a cost to the economy as a whole. As a result this cost does not affect the optimal exit time. The next section considers the case in which there are social costs associated with the abandonment of the peg.

# 3. Exit Costs

In this section we introduce costs of abandoning the fixed exchange regime into our model. We assume that when the fixed exchange rate is abandoned the government incurs a fiscal cost of $\phi$ which also represents a social loss for the economy as a whole. This exit cost can be given several interpretations. First, it can reflect a fall in output and tax revenues following the abandonment of the peg. Second, since banking crises are typically a by-product of currency crises (see Kaminsky and Reinhart (1999)), these costs can stem from bailing out domestic banks.[11] Third, these costs can result from bailing out foreign creditors. A devaluation can make it optimal for domestic firms to default on foreign loans that had been guaranteed by the government. In these circumstances a devaluation creates a fiscal liability for the government.[12]

We proceed by setting up the Ramsey problem, starting with the condition that guarantees that the Ramsey solution is implementable as a competitive equilibrium.

## 3.1. The Implementability Condition

We need to distinguish between two cases: $T = 0$ and $T > 0$. If the fixed exchange rate is abandoned at $T = 0$, the government sets a constant, positive rate of devaluation $\varepsilon_0$ from time zero onwards. Given this policy, consumption is constant over time at a level we denote by $\bar{c}$,

$$\bar{c} = \frac{ra_0 + y}{1 + r + \varepsilon_0}, \tag{3.1}$$

---

[11] In the model proposed by Burnside, Eichenbaum and Rebelo (2004) the fact that banks are guaranteed by the government makes it optimal for them to use forward currency markets to increase their exposure to exchange rate risk. As a result, these banks go bankrupt when a devaluation occurs, creating a fiscal liability for the government.

[12] We develop this interpretation in a previous version of this paper, available upon request.

where $a_0 \equiv b_{0-} + M_{0-}/S_0$. We assume, without loss of generality, that the exchange rate remains constant $(S_0 = S)$ when the abandonment occurs at time zero. This assumption implies that $a_0 = a_{0-}$. Solving for $\varepsilon_0$, we obtain:

$$\varepsilon_0 = \frac{ra_0 + y}{\bar{c}} - (1 + r). \tag{3.2}$$

This equation is the implementability condition when $T = 0$.

The optimal path for $\varepsilon_t$, given that the peg is abandoned at $T > 0$, is given by equation (2.13) in Proposition 2.1. The consumer's first-order condition (2.6) implies that the levels of consumption within each subperiod ($0 < t < T$ and $t \geq T$) are constant. We denote these constant levels of consumption by $c^1$ for $0 < t < T$ and $c^2$, for $t \geq T$. Using this notation, we can rewrite the household's intertemporal constraint (2.5) as:

$$a_0 + y/r = \frac{c^1(1 + r)}{r}(1 - e^{-rT}) + \frac{c^2(1 + r + \varepsilon)}{r}e^{-rT}. \tag{3.3}$$

Since equation (2.6) implies that $c^1(1 + r) = c^2(1 + r + \varepsilon)$, the values of $c^1$ and $c^2$ are given by:

$$c^1 = \frac{ra_0 + y}{1 + r}, \tag{3.4}$$

$$c^2 = \frac{ra_0 + y}{1 + r + \varepsilon_T}. \tag{3.5}$$

Equation (3.4) has two implications. First, $c^1$ is determined by the household's problem (recall that $a_0 = a_{0-}$) so it is *not* a choice variable for the Ramsey planner. Second, $c^1$ is equal to $c_{0-}$ (see (2.10)).

Equation (3.5) is the implementability condition for the case of $T > 0$, which can be re-written as:

$$\varepsilon_T = \frac{ra_0 + y}{c^2} - (1 + r). \tag{3.6}$$

Equation (3.5) implies that, as $T$ tends to zero, $c^2$ converges to $\bar{c}$ (recall (3.1)).

15

## 3.2. Government's Budget Constraint

We can write the government's budget constraint, (2.7), as:

$$\varepsilon_0 \frac{M_T}{S_T} \frac{1}{r} = \Delta\Gamma + \phi + \frac{M_{0-} - M_0}{S}, \qquad \text{if } T = 0, \tag{3.7}$$

$$\varepsilon_T \frac{M_T}{S_T} \frac{e^{-rT}}{r} = \Delta\Gamma + \phi e^{-rT} + \frac{M_{0-} - M_0}{S_0} + \frac{M_0 - M_T}{S_T} e^{-rT}, \qquad \text{if } T > 0. \tag{3.8}$$

The exit cost, $\phi$, is included in this constraint since it is a fiscal cost that the government incurs at time $T$. Using the cash-in-advance constraint, (3.2), and (3.6) we can rewrite the government's budget constraint as:

$$\frac{c_{0-} - \bar{c}}{r} = \Delta\Gamma + \phi, \qquad T = 0, \tag{3.9}$$

$$\frac{e^{-rT}}{r} (c_{0-} - c^2) = \Delta\Gamma + \phi e^{-rT}. \qquad T > 0. \tag{3.10}$$

Since, as $T \to 0$, $c^2 \to \bar{c}$, constraint (3.9) converges to (3.10) as $T$ tends to zero.

## 3.3. The Ramsey problem

The Ramsey planner chooses $\{c^2, T\}$ to maximize the household's lifetime utility, (2.1), which can be rewritten as,

$$V = \frac{\log(c^1)}{r} (1 - e^{-rT}) + \frac{\log(c^2)}{r} e^{-rT}.$$

This maximization is subject to the household's intertemporal constraint, (3.3), the government's intertemporal constraint, (3.10), the implementability condition, (3.6), and a non-negativity constraint on $T$.

The Ramsey problem is continuous in $T$. This continuity property holds at $T = 0$, since both the objective and the constraints converge to the $T = 0$ case as $T \to 0$.

16

For conceptual clarity, we distinguish between $c^2$ and $\bar{c}$. However, $\bar{c}$ is simply the value of $c^2$ when $T = 0$, so the planner only chooses $c^2$. The first-order condition with respect to $c^2$ can be written as:[13]

$$\frac{1}{c^2} = \lambda. \tag{3.11}$$

The Kuhn-Tucker condition with respect to $T$ is given by:

$$\log(c^1) - \log(c^2) + \lambda[r\phi - (c^1 - c^2)] \leq 0, \; T \geq 0, \tag{3.12}$$
$$\left\{ \log(c^1) - \log(c^2) + \lambda[r\phi - (c^1 - c^2)] \right\} T = 0.$$

The optimal exit time is characterized by the following proposition, which includes the case of $\phi = 0$ discussed in the previous section as a special case.

**Proposition 3.1.** *The optimal exit time, $T^*$, is given by:*

| *Optimal Exit Time* | *Low $\Delta\Gamma$* $\Delta\Gamma < c^1/er$ | *High $\Delta\Gamma$* $\Delta\Gamma \geq c^1/er$ |
|---|---|---|
| *Low $\phi$ $(0 \leq \phi \leq \phi^*)$* | $T^* = 0$ | $T^* = 0$ |
| *Intermediate $\phi$ $(\phi^* < \phi < \phi^{**})$* | $T^* > 0$ | $T^* = 0$ |
| *High $\phi$ $(\phi \geq \phi^{**})$* | $T^* = 0$ | $T^* = 0$ |

*Proof. See Appendix 7.2.*

According to this proposition, delaying is optimal only when the fiscal shock is low ($\Delta\Gamma < c^1/er$) and $\phi$ takes on an intermediate value. In all other cases, it

---

[13]To obtain this condition we divide both sides of the first-order condition with respect to $c^2$ by $e^{-rT}$. This term is different from zero since $T$ must be finite. Otherwise, given that $\Delta\Gamma > 0$, the government's intertemporal constraint is violated.

is optimal to abandon immediately. To understand the intuition underlying this result it is useful to rewrite the Kuhn-Tucker condition, (3.12), using (3.11), as:

$$\underbrace{\log\left(\frac{c^1}{c^2}\right) - \left(\frac{c^1}{c^2} - 1\right)}_{\text{cost of delaying}} + \underbrace{\frac{r\phi}{c^2}}_{\text{benefit of delaying}} \leq 0. \tag{3.13}$$

Since $c^2 < c^1$ the term labeled "cost of delaying" is always negative. This cost results from the intertemporal distortion introduced by delaying the abandonment of the peg: inflation is zero until time $T > 0$ and positive from time $T$ on. The benefit of delaying is the flow saving, $r\phi$, relative to post-crisis consumption, $c^2$. This benefit results from reducing the present discounted value of the cost of abandonment. It is important to emphasize that admissible values of $\phi$ are bounded from above since $c^2$ must be positive. Formally, around $T = 0$, $\phi < c^1/r - \Delta\Gamma$ (see Appendix 7.2 for details).

When $\Delta\Gamma \geq c^1/er$, the intertemporal distortion introduced by delaying is so large that it dominates the benefit of delaying for any admissible $\phi$. To understand this result intuitively, rewrite equation (3.13) as:

$$\log\left(\frac{c^1}{c^2}\right) + 1 + \frac{c^1}{c^2}\left(\frac{r\phi}{c^1} - 1\right) \leq 0. \tag{3.14}$$

Since $\phi < c^1/r$, the coefficient on $c^1/c^2$ is negative. For a given $\phi$, as $\Delta\Gamma$ increases $c^2$ converges to zero, while $c^1$ remains constant. As a result, $c^1/c^2$ becomes arbitrarily large and the left hand side of (3.14) converges to $-\infty$. Put differently, the fact that the flow saving of delaying is bounded relative to $c^1$ explains why the distortion associated with delaying dominates and immediate abandonment is optimal.

When $\Delta\Gamma < c^1/er$, the intertemporal distortion imposed by abandoning the peg for some $T > 0$ is smaller and therefore the decision comes down to a trade-off between the costs and benefits of delaying. Clearly, if $\phi = 0$, there are no benefits

18

from delaying and immediate abandonment is optimal. For small values of $\phi$, the benefits are small relative to the intertemporal distortion that needs to be imposed and immediate exit is still optimal. There is some threshold level of $\phi$, $\phi^*$, beyond which the benefit of delaying becomes large enough to warrant a delayed exit. For values of $\phi$ larger than $\phi^{**}$, that is values of $\phi$ close to the maximum admissible value, both the cost of delaying and the benefit of delaying become arbitrarily large but the cost of delaying dominates. The intuition is the same as the one just discussed. As $\phi$ increases, $c^1$ remains constant and $c^2$ converges to zero. As a result, $c^1/c^2$ becomes arbitrarily large and the right hand side of (3.14) converges to $-\infty$. Hence, there is a value of $\phi$, $\phi^{**}$ beyond which the right hand side of (3.14) is negative and it is optimal to abandon at $T = 0$.

The optimal time to exit the peg is independent of the initial level of foreign reserves. Hence, the KFG rule of exiting only when reserves are exhausted is in general suboptimal. Our model is thus consistent with the evidence provided in the introduction that many countries exit the fixed exchange regime without having exhausted their reserves.

The table in proposition 3.1 shows that delaying the abandonment and incurring some reserve losses is optimal in only one out of six possible cases. However, the proposition says nothing about the empirical relevance of each of the six cases. We now provide some back-of-the-envelope calculations to illustrate the predictions of our model for $T^*$ using empirically-plausible values of the cost of abandoning ($\phi$) and the fiscal shock ($\Delta\Gamma$). While admittedly crude, these calculations shed light on how often it is optimal to abandon immediately. The choice of values for for $\phi$ and $\Delta\Gamma$ is admittedly difficult, forcing us to make some stark connections between the model and reality. For the fiscal shock we focus our attention on the 37 episodes underlying Table 1 for which there was a positive increase in government spending during the three years before the crisis. We compute $\Delta\Gamma$

19

by assuming that there is a once-and-and-for-all increase in (annual) government spending equivalent to the (geometric) average of the increase in the three year period before the crisis. For example, for the Argentinean crisis of June 1970, the increase in real government spending during the three years prior to the crisis was 15.9 percent. The corresponding geometric average is 5.0 percent per year. Assuming that the annual real interest rate is four percent, a once-and-for-all increase of 5.0 percent in government spending implies a present discounted value of 131.1 percent. Hence $\Delta\Gamma$ takes the value 1.311. We follow the same procedure for each of the other 36 episodes.

We choose values of $\phi$ based on the existing literature. Using a sample of 195 crises in 91 countries, Gupta, Mishra, and Sahay (2003) compute empirical estimates of the output costs entailed by currency crises. They report that the average output fall that can be attributed to crisis episodes in the 1970s, 1980s, and 1990s is, respectively, 3.0, 1.1, and 0.8 percent. Hutchinson and Neuberger (2001) focus exclusively on emerging markets and examine 51 crises in 24 countries over the period 1975-1997. They conclude, controlling for other factors, that a crisis leads to output falls of between five and eight percent. Based on these studies, we consider values of $\phi$ ranging from one to eight percent. Table 2 reports the percentage of cases (among the 37 episodes of Table 1) for which $T^* = 0$. For example, for $\phi = 0.01$, $T^* = 0$ for 59 percent of the cases and $T^* > 0$ for the remaining 41 percent. For $\phi = 0.08$ immediate abandonment is optimal in eight percent of the cases.[14]

---

[14]In these calculations we assume that the elasticity of intertemporal substitution is 0.30 to be consistent with the estimates in Reinhart and Vegh (1995).

**Table 2**

$T^*$ for various values of $\phi$

|            |      |      | $\phi$ |      |
|------------|------|------|--------|------|
|            | 0.01 | 0.03 | 0.05   | 0.08 |
| $T^* = 0$  | 59   | 27   | 14     | 8    |
| $T^* > 0$  | 41   | 73   | 86     | 92   |

We thus conclude that the model predicts immediate abandonment for a broad range of values of $\phi$, which is consistent with the evidence shown above indicating that many countries have abandoned exchange rate pegs with still plenty of international reserves in their coffers.[15]

### 3.4. Properties of the Optimal Policy

We can now analyze how the values of $T^*$, post-abandonment inflation, and reserve losses depend on $\phi$ and $\Delta\Gamma$ for the admissible range of parameter values. Formal proofs are relegated to Appendices 7.3 and 7.4.

Figure 2 shows the behavior of the optimal values of $T$, $\varepsilon$, and the reserve loss as a function of $\phi$ for a *small* fiscal shock ($\Delta\Gamma < c^1/er$). Panel A shows the behavior of the optimal exit time. Up to $\phi = \phi^*$, the optimal solution is to abandon immediately. In the region in which $T^* > 0$, the value of $T^*$ is a non-monotonic function of $\phi$. For values of $\phi$ larger than $\phi^{**}$, it again becomes optimal to abandon immediately.

Panel B of Figure 2 shows the behavior of the optimal inflation rate. For $T = 0$ we proceed as in (2.16) and use (2.9) and (3.9) to write $\varepsilon_0$ and $\bar{c}$ as

$$\varepsilon = \frac{r(1+r)}{c^2}(\Delta\Gamma + \phi), \tag{3.15}$$

$$\bar{c} = rb_{0^-} + y - r(\Delta\Gamma + \phi). \tag{3.16}$$

---

[15]Of course, even when $T^* > 0$ there are many cases in which the reserve losses are small. Hence, the cases of immediate abandonment constitute a lower bound for instances of abandonment with small or no reserve losses.

It follows from (3.15) and (3.16) that, for $T = 0$, the rate of inflation is an increasing function of $\phi$. This property also holds for $T^* > 0$. As Panel B illustrates, the rate of inflation is an increasing function of $\phi$ for all admissible $\phi$ values.

Finally, Panel C in Figure 2 illustrates the behavior of the loss of reserves at the time of abandonment. This loss is equal to $c^1 - c^2$. However, since $c^1$ is independent of $T$ and $\phi$, the reserve loss when $T^* > 0$ depends only on the behavior of $c^2$. The reserve loss is an increasing function of $\phi$. When $T^* = 0$ there are no reserve losses.

Figure 3 illustrates the behavior of $T$, $\varepsilon$, and the loss of reserves as a function of the fiscal shock for a given value of $\phi$.[16] Panel A shows that, when $T^* > 0$, the optimal exit time is a decreasing function of the fiscal shock. In other words, the larger the fiscal shock the sooner it is optimal to abandon the peg. Intuitively, a larger fiscal shock requires a higher inflation rate once the peg is abandoned, which imposes a larger intertemporal distortion. As a result, it is optimal to abandon earlier to reduce the intertemporal distortion. When the value of the fiscal shock reaches $\Delta\Gamma^*(\equiv c^1/er)$, it becomes optimal to abandon immediately.

The rate of inflation after the regime is abandoned does not depend on $\Delta\Gamma$ whenever $T^* > 0$. This property reflects two opposing forces that cancel each other out. First, for a given $T$, a larger fiscal shock tends to increase the inflation rate. Second, since $T^*$ falls as the fiscal shocks increases, the inflation rate falls. In this case of logarithmic preferences, these two effects exactly cancel each other out. When it is optimal to abandon immediately (i.e., for $\Delta\Gamma \geq \Delta\Gamma^*$), the inflation rate is an increasing function of the fiscal shock. This property follows from (3.15) and (3.16). When $\Delta\Gamma = 0$, it is not optimal to abandon the peg and hence the

---

[16]The given value of $\phi$ is $(c^1/r)(1 - 2/e)$. As shown in Appendix 7.4, the Kuhn-Tucker condition is exactly equal to zero for this value of $\phi$.

optimal inflation rate is zero.

We conclude by discussing the effects of introducing a borrowing constraint on the government. To simplify we consider the case in which government expenditure is constant at a level $g_{0-}$ before the fiscal shock and at a level $g_0 > g_{0-}$ after the fiscal shock. Suppose that there is a binding borrowing constraint that dictates that $f_t \geq \bar{f}$. It can be shown that lifetime utility, $V$, is an increasing function of $T$ for values of $T$ below the optimal. Once the regime is abandoned, $f_t$ becomes constant, $f_t = f_T$ for $t \geq T$. The value of $f_T$ is a decreasing function of $T$. Thus, whenever $T^* > 0$, a borrowing constraint forces the economy to abandon the fixed exchange rate regime before $T^*$. In this situation we can think of central bankers as following the KFG model, since they maintain the regime for as long as possible and, at the time of abandonment, exhaust their ability to borrow. However, in general, appealing to the presence of a borrowing constraint does not justify the KFG exit rule since, when $T^* = 0$, borrowing constraints have no impact on the decision to exit the fixed exchange rate.

## 4. Model Extensions

In order to assess the theoretical robustness of our key results, we explore in this section several extensions of the basic model analyzed in Section 3.

### 4.1. Time-varying Exit Costs

Here we consider the case in which the exit cost, $\phi$, varies over time. On one hand, the exit cost can decline over time if postponing the abandonment of the peg gives firms time to prepare for the change in regime by changing prices or hedging exchange rate risk. On the other hand, the costs associated with a currency crisis can increase with the post-crisis rate of inflation. To simplify we assume that $\phi_t$

grows at a constant rate $\delta$ that can be positive or negative:

$$\phi_t = \phi e^{\delta t}. \tag{4.1}$$

Here $\phi_t$ is the cost of abandoning the peg at time $t$, $\phi$ is a positive constant, and $\delta$ is the rate at which the cost changes over time.

The consumer's problem remains the same as in Section 3. The government's new budget constraint is:

$$\frac{e^{-rT}}{r}(c_{0-} - c^2) = \Delta\Gamma + \phi e^{-(r-\delta)T}, \qquad T \geq 0. \tag{4.2}$$

The first-order condition for the Ramsey problem, (3.11), remains valid and the Kuhn-Tucker condition is given by:

$$\log\left(\frac{c^1}{c^2}\right) - \left[\frac{c^1}{c^2} - 1 - \frac{(r-\delta)\phi e^{\delta T}}{c^2}\right] \leq 0. \tag{4.3}$$

The following proposition generalizes the results in Proposition 3.1.

**Proposition 4.1.** *If $\delta \geq r$, it is optimal to abandon at $T = 0$ for any value of $\Delta\Gamma > 0$. If $\delta < r$ the optimal exit time, $T^*$, is given by:*

| *Optimal Exit Time* | *Low $\Delta\Gamma$* <br> $\Delta\Gamma < \frac{c^1}{r}\left(e^{\frac{\delta-r}{r}} - \frac{\delta}{r}\right) - \delta$ | *High $\Delta\Gamma$* <br> $\Delta\Gamma \geq \frac{c^1}{r}\left(e^{\frac{\delta-r}{r}} - \frac{\delta}{r}\right) - \delta$ |
|---|---|---|
| *Low $\phi$ $(0 \leq \phi \leq \phi^*)$* | $T^* = 0$ | $T^* = 0$ |
| *Intermediate $\phi$ $(\phi^* < \phi < \phi^{**})$* | $T^* > 0$ | $T^* = 0$ |
| *High $\phi$ $(\phi \geq \phi^{**})$* | $T^* = 0$ | $T^* = 0$ |

*Proof. See Appendix .*

When $\delta = r$, the present discounted value of the exit cost is independent of $T$, so there is no benefit from delaying. If $\delta > r$, delaying increases the present

discounted value of the exit cost. In both of these cases it is optimal to abandon right away. When $\delta < r$, delaying reduces the present discounted value of the exit costs, so delaying can be optimal.

To illustrate the effect of different values of $\delta$, Figure 4 plots $T^*$ as a function of the fiscal shock ($\Delta\Gamma$) for three values of $\delta$ ($\delta = 0$, $\delta = 0.03$, and $\delta = -0.03$).[17] The $\delta = 0$ case is the case studied in Section 3. When $\delta$ is negative ($\delta = -0.03$), the threshold value of $\Delta\Gamma$ beyond which it is optimal to abandon right away is larger than that for the $\delta = 0$ case. The opposite is true when $\delta$ is positive ($\delta = 0.03$).

The intuition for these results can be explained using equation (4.3), which can be rewritten as:

$$\underbrace{\log\left(\frac{c^1}{c^2}\right) - \left(\frac{c^1}{c^2} - 1\right)}_{\text{cost of delaying}} + \underbrace{\frac{(r - \delta)\phi e^{\delta T}}{c^2}}_{\text{benefit of delaying}} \leq 0. \tag{4.4}$$

For $\delta = 0$, this condition reduces to (3.13). The cost of delaying is the same as before. The benefit of delaying captures the flow saving, now given by $(r - \delta)\phi e^{\delta T}$ which, in light of (4.1), can also be written as $(r - \delta)\phi_t$. Compare the $\delta = 0$ case with the $\delta < 0$ case. When $\delta < 0$ – and for a *given* $\phi_t$ – the flow saving from delaying for an additional moment is higher for the $\delta = 0$ case. This effect calls for an additional delay. However, a negative $\delta$ implies that, all else equal, the *current* cost of devaluing, $\phi e^{\delta T}$, is smaller than it would be in the $\delta = 0$ case. By making the flow saving smaller, this effect calls for a smaller delay. For small values of $T$ (and in particular around $T = 0$), the first effect dominates, which implies that the threshold value beyond which it is optimal to abandon immediately is a decreasing function of $\delta$, as illustrated in Figure 4 (notice that the threshold is the largest for $\delta = -0.03$ and the smallest for $\delta = 0.03$). When comparing $\delta = -0.03$ and $\delta = 0$, this first effect continues to dominate up to the value of $\Delta\Gamma$ corresponding

---

[17]Parameter values for Figure 4 are $r = 0.037$, $a_0 = -0.35$, $y = 1$, and $\phi = 0.002$.

to point A in Figure 4. Below this point the second effect dominates and, for a given fiscal shock, $T$ is smaller for $\delta = -0.03$ than for $\delta = 0$.

Figure 5 illustrates the effects of the parameter $\delta$ on $T^*$ as a function of the cost of abandonment, $\phi$.[18] The threshold of $\phi$ below which it is optimal to abandon immediately is the largest for $\delta = 0.03$ and the lowest for $\delta = -0.03$. Intuitively, around $T = 0$, the higher is $\delta$ the lower is $T$ for a given $\phi$ since the benefits of delaying – given by $(r - \delta)\phi$ – are lower.[19]

## 4.2. The Exit Cost is Not a Fiscal Cost

So far we have assumed that the exit cost $\phi$ is both a fiscal cost and a social cost. One could think that the fiscal nature of the exit cost drives our main results since delaying the abandonment reduces the exit cost that has to be financed by the fiscal authority. To show that our results do not depend on the exit cost being a fiscal cost, we briefly discuss a version of the model presented in Section 3 in which the cost of abandonment, $\theta$, is a reduction in the endowment of the economy, but does not enter the government's budget constraint. This cost can be interpreted as a loss in output that occurs when the peg is abandoned.

The only modification to the model in Section 3 is in the household's intertemporal constraint which is now:

$$b_{0-} + \frac{M_{0-}}{S_0} + \frac{y}{r} - \theta e^{-rT} = \int_0^\infty c_t(1 + r + \varepsilon_t)e^{-rt}dt. \tag{4.5}$$

Here $y/r - \theta e^{-rT}$ is the present discounted value of the endowment *net* of the cost of abandoning the peg. Households take $T$ as given, so they view $\theta e^{-rT}$ as exogenous to their decisions. However, the Ramsey planner takes the exit cost into account.

---

[18]Parameter values are the same as for Figure 4 except that now the fiscal shock, $\Delta\Gamma$, is set to 0.3.

[19]Although not shown in Figure 5, $T^*$ eventually begins to fall and becomes zero for larger values of $\phi$, as in Figure 2, Panel A.

The main complication introduced by this formulation is that the Ramsey planner's problem is not continuous in $T$ at time zero. This property results from the fact that $c^1$ depends on $T$ since the term $\theta e^{-rT}$ affects the household budget constraint. Therefore, $c^1$ is a choice variable for the Ramsey planner when $T > 0$ but not when $T = 0$. This difficulty forces us to solve the model numerically. We compute $T^*$ analytically assuming that $T^* > 0$ and comparing the value of $V$ associated with this solution with the value of $V$ for $T^* = 0$.

We verified that our main results hold for a wide range of parameters. Panel A of Figure 6 shows that $T^*$ falls with the fiscal shock until, once the fiscal shock is large, it is optimal to abandon immediately. Panel B shows $T^*$ as a function of the cost of abandonment, $\theta$. As in the model of Section 3, it is optimal to abandon immediately for small values of $\theta$. Beyond a certain threshold, the optimal time of abandonment is an increasing function of $\theta$.[20]

Intuitively, even though $\theta$ is not a fiscal cost, it has fiscal repercussions. An increase in $\phi$ reduces $c^2$, which is the tax base for the post-crisis inflation tax. Increasing $T$ raises household wealth, and hence $c^2$, which increases tax revenues for a given post-crisis inflation rate. This effect needs to be traded-off against the fact that delaying implies a higher post-crisis devaluation rate and hence a larger intertemporal distortion.

### 4.3. Non-unitary Elasticity of Intertemporal Substitution

Up to this point, we have assumed that momentary utility is logarithmic. Here we consider the more general case in which utility is given by:

$$V \equiv \int_0^\infty \frac{c_t^{1-1/\sigma} - 1}{1 - 1/\sigma} e^{-\rho t} dt, \tag{4.6}$$

---

[20] Although not shown in the figure, for higher values of $\theta$, $T^*$ falls because the intertemporal distortion becomes arbitrarily large and dominates the benefit of delaying.

27

where $\sigma > 0$ denotes the elasticity of intertemporal substitution. When $\sigma$ is different from one the Ramsey planner's problem is discontinuous at $T = 0$ because the level of consumption before abandonment $(c^1)$ differs from the initial level of consumption $(c_{0-})$. Therefore we solved the model numerically.

The results that we obtain for a very wide range of parameters are consistent with the results discussed in Section 3 for the logarithmic case. As an illustration, Figure 7 depicts the value of $T^*$ as a function of the fiscal shock for $\sigma = 0.5$, $\sigma = 1$, and $\sigma = 2$.[21] As in the logarithmic case, there is a a threshold value of the fiscal shock beyond which $T^* = 0$. When $T^* > 0$ the value of $T^*$ is a decreasing function of the fiscal shock. In terms of comparing the effects of the intertemporal elasticity of substitution on $T^*$, note that when all solutions are interior, the lower the intertemporal elasticity of substitution, the larger the $T^*$ for a given fiscal shock. However, beyond a certain threshold, $T^* = 0$ for any of the three cases.

The intuition behind the results illustrated in Figure 7 is as follows. When the intertemporal elasticity of substitution is low the fact that the rate of inflation is higher after $T$ introduces a smaller distortion into the consumer's intertemporal consumption choice. For this reason $T^*$ is higher than in the logarithmic case. Conversely, a higher intertemporal elasticity of substitution implies that consumers are more sensitive to a given distortion, so it is optimal to delay less.

Figure 8 illustrates the model's results for the optimal time of abandonment as a function of the exit cost, $\phi$. We can see that the qualitative nature of our previous results remains unchanged. There is a certain threshold value of $\phi$ below which $T^* = 0$. Above this threshold, $T^*$ is an increasing function of $\phi$. Although not shown in the figure, the non-monotonicity discussed in the previous section holds for very large values of $\phi$. Finally, notice that the value of $T^*$ depends on

---

[21]The parameters are identical to those we use in the construction of Figure 4.

the elasticity of intertemporal substitution. For a given value of $\phi$, the smaller is the intertemporal elasticity of substitution, the higher is $T^*$. Intuitively, for a given $\phi$, the lower is the intertemporal elasticity of substitution, the smaller the impact on utility of a given rate of post-crisis inflation. Hence, it is optimal to delay more.

In sum, allowing for non-unitary elasticity of intertemporal substitution does not alter the qualitative nature of the results. The quantitative differences that emerge are due to the fact that the lower is the willingness of consumers to substitute over time, the smaller are the effects of intertemporal distortions. Therefore, lower elasticities of intertemporal substitution imply higher values of $T^*$.

### 4.4. The Exit Cost Increases with the Fiscal Shock

So far we have assumed that the exit cost is independent of the fiscal shock. However, one could imagine scenarios in which the exit cost depends positively on the fiscal shock. We now analyze this case and show that our main results continue to hold.

Suppose that the cost of abandoning the peg is given by:

$$\phi_t = \phi_0 + \alpha \Delta \Gamma,$$

for some $\alpha > 0$. For simplicity, we analyze the case in which $\phi_0 = 0$. When $\phi_0 > 0$, the same results go through since the presence of a positive cost of exiting when $\Delta \Gamma = 0$ reinforces the results described below.

The consumer's problem remains the same as in Section 3. The Kuhn-Tucker condition for the Ramsey planner becomes:

$$\underbrace{\log\left(\frac{c_1}{c_2}\right) - \left(\frac{c_1}{c_2} - 1\right)}_{\text{cost of delaying}} + \underbrace{\frac{r\alpha\Delta\Gamma}{c^2}}_{\text{benefit of delaying}} \leq 0. \tag{4.7}$$

This equation is analogous to equation (3.13). We now show that, as in the case discussed in Section 3, there is a threshold value of the fiscal shock beyond which it is optimal to abandon immediately.

**Proposition 4.2.** *For any given $\alpha > 0$, there is a threshold value of $\Delta\Gamma$, $\Delta\Gamma^*$, such that for any $\Delta\Gamma \geq \Delta\Gamma^*$, it is optimal to abandon immediately.*

Proof. See Appendix 7.6.

For a sufficiently large fiscal shock, both the cost and benefit of delaying become arbitrary large, but the former dominates and hence it is optimal to abandon right away. The intuition parallels the discussion following equation (3.13) in Section 3. In other words, the key is that the flow saving is bounded relative to $c^1$ and, hence, for large fiscal shocks, the cost of delaying dominates and immediate abandonment is optimal.

We also show in Appendix 7.7 that when the solution is interior the optimal $T$ is a decreasing function of the fiscal shock. In sum, even in this case in which the cost of exiting increases with the fiscal cost, we obtain the same qualitative results as in Section 3.

## 5. Stochastic Fiscal Reform

In sections 3 and 4 we study the optimal monetary policy in models where there are both fiscal and social costs of abandoning the fixed exchange rate regime. We now consider an economy where these costs are absent but where government spending is stochastic. As in the previous sections, we assume that before time zero the fixed exchange rate regime was sustainable, so the government's net foreign assets were sufficient to finance the present value of government spending. At time zero the economy learns that the present value of government spending has increased by $\Delta\Gamma$. The new element introduced in this section is that while the exchange

rate is fixed (after time zero but before the peg is abandoned) there can be a reduction in government spending that makes the peg, once again, sustainable. This expenditure reduction occurs according to a Poisson process with arrival rate $\lambda$. If the peg is abandoned, the increase in government spending becomes permanent and has to be financed with seignorage revenues. There is thus an option value of holding on to the peg. This formulation captures in a simple way the idea that a fixed exchange rate regime exerts pressure on the fiscal authorities to enact reforms to make the peg sustainable. This pressure disappears once the exchange rate floats. An alternative interpretation is that the country can receive a bailout transfer from abroad that pays for the increase in government spending and renders the peg sustainable. This external bailout arrives according to a Poisson process.

The size of the fiscal reform or of the external bailout that has to occur to make the fixed exchange rate regime sustainable depends, naturally, on the path of government spending. If the reform occurs at time $t$ the present value of government spending from time $t$ on has to be reduced to a value $\Gamma_t$ given by:

$$\Gamma_t = f_0 e^{rt} - e^{rt} \int_0^t g_s e^{-rs} ds. \tag{5.1}$$

Expression (5.1) implies that if there has been no new spending between time zero and time $t$ all that is necessary to make the peg sustainable is to cancel the plans for new government spending in the future. However, if new spending has already taken place in the time interval up to time $t$ the government needs to reduce the present value of government spending below its level before the fiscal shock.

The optimal policy reduces to choosing the time $T$ at which the fixed exchange rate regime is abandoned, if a fiscal reform has not in the meantime materialized. A higher value of $T$ makes a fiscal reform more likely. However, the longer the horizon $T$, the larger the intertemporal consumption distortion that the government

31

has to introduce if reform does not occur.

**The Time When Reform Occurs**   We start by characterizing the case in which a fiscal reform has just occurred making the fixed exchange rate sustainable. Consumption is constant and its level, which we denote by $c^*$, can be computed using the household's budget constraint:

$$b + y/r = c^*/r + (c^* - m).$$

Here $b$ and $m$ denote the levels of net foreign assets and real balances that households had in the period where the reform took place. The term $(c^* - m)$ represents the jump in real balances that occurs when agents learn that the fixed exchange rate regime has become sustainable. Lifetime utility is given by:

$$V^*(b + m) = \frac{\log\left[(rb + rm + y)/(1 + r)\right]}{r}.$$

**The $t \geq T$ Regime**   Suppose that we have reached time $T$ and a reform has not occurred. The fixed exchange rate regime is abandoned and the growth rate of money rises to a level $\varepsilon$ such that the government's intertemporal resource constraint is satisfied. Consumption is constant at a level which we denote by $c^2$. This level can be computed using the household's budget constraint:

$$b + y/r = c^2(1 + \varepsilon)/r + (c^2 - m), \tag{5.2}$$

where $(c^2 - m)$ represents the jump in real balances that takes place at time $T$ in response to a permanent increase in inflation from zero to $\varepsilon$. Using (5.2) to solve for $c^2$, we can compute lifetime utility at time $T$:

$$V(b + m, T) = \frac{\log\left[(rb + rm + y)/(1 + r + \varepsilon)\right]}{r}. \tag{5.3}$$

The value function (5.3) bears a simple relation with the value function associated with the reform regime:

$$V(b+m,T) = V^*(b+m) - \frac{\log(p)}{r},$$

where $p$ is given by

$$p \equiv \frac{1+r+\varepsilon}{1+r}. \tag{5.4}$$

The fact that $r = \rho$ and that inflation is constant means that for any time period $t \geq T$ the value function coincides with $V(b+m,T)$:

$$V(b+m,t) = V(b+m,T) \text{ for } t \geq T.$$

**The Regime for $t \leq T$ and No Reform** The optimality equation for the household's problem during this period is:

$$
\begin{aligned}
rV(b+m,t) &= \max_{c^1}\{\log(c^1) + V_2(b+m,t) + \\
&\quad [r(b+m) + y - c^1(1+r)]V_1(b+m,t) + \\
&\quad \lambda[V^*(b+m) - V(b+m,t)]\}.
\end{aligned}
$$

The first order condition with respect to consumption $(c^1)$ is:

$$1/c^1 = V_1(b+m,t)(1+r).$$

It is easy to verify that the value function has the form:

$$V(b+m,t) = \frac{\log\left[(rb+rm+y)/(1+r)\right]}{r} - \frac{e^{-(\lambda+r)(T-t)}\log(p)}{r}. \tag{5.5}$$

This equation has a simple interpretation. Consider first an economy in which a fiscal reform has no chance of occurring ($\lambda = 0$) and which will switch to the floating regime with certainty at time $T$. Since utility declines by $\log(p)/r$ at time $T$ lifetime utility at time $t$ would be:

$$\frac{\log\left[(rb+rm+y)/(1+r)\right]}{r} - \frac{e^{-r(T-t)}\log(p)}{r}. \tag{5.6}$$

33

Our value function is similar to (5.6) but the discount factor applied to $\log(p)/r$ incorporates $\lambda$ to reflect the fact that there is an ongoing probability of a fiscal reform until time $T$.

## 5.1. Optimal Monetary Policy

At time zero, when the economy learns that there has been an increase in the present value of government spending, the lifetime utility of the household declines from $V^*(b+m)$ to $V(b+m,0)$ (given by equation (5.5)).

The central bank chooses $T$, the maximum length of time that it is optimal to wait for a fiscal reform to occur. If the economy reaches time $T > 0$ without a fiscal reform, the central bank has to print money to satisfy the government's intertemporal budget constraint. Since it is optimal to choose a constant growth rate of money, the government's present value resource constraint is:

$$\frac{\varepsilon c^2}{r}e^{-rT} + (c^1 - m_{0-}) + (c^2 - c^1)e^{-rT} = \Delta\Gamma. \tag{5.7}$$

There are no stochastic elements in this equation. This constraint is only relevant when the economy reaches time $T$ without a fiscal reform, in which case all uncertainty has been resolved. Since the economy is in a sustainable fixed exchange rate regime at $t = 0^-$, $m_{0-} = c^1$. Using this fact, and the equation $c^2 = c^1/p$ together with (5.4) we can rewrite (5.7) as:

$$p = \frac{c^1/r}{c^1/r - \Delta\Gamma e^{rT}}. \tag{5.8}$$

This equation defines $p$ as a function of $T$ .

The optimal policy can be characterized by maximizing $V(b+m,0)$, given by (5.5), subject to (5.8). $T^*$ is given by:

$$\log\left(\frac{c^1}{c^2}\right) - \left(\frac{c^1}{c^2} - 1\right) + \frac{\lambda}{r+\lambda}\frac{r\Delta\Gamma e^{rT}}{c^2} \leq 0, \tag{5.9}$$

34

which holds with equality whenever $T^* > 0$. Equation (5.9) is similar to the one that characterizes the case in which the exit cost is increasing with the fiscal shock (see equation (4.7). The term $e^{rT}$ reflects the fact that as time passes the size of the fiscal reform has to increase in order to restore the sustainability of the fixed exchange rate.

The optimal abandonment time is characterized by the following proposition.

**Proposition 5.1.** *For every finite positive value of $\lambda$ there is a threshold value for the present value of government spending, $\Gamma^*$, such that for $\Gamma_0 > \Gamma^*$ it is optimal to abandon the peg at time zero ($T = 0$), while for $\Gamma_0 \leq \Gamma^*$ it is optimal to delay abandoning the peg ($T \geq 0$). The value of $\Gamma^*$ is increasing in $\lambda$.*

Proof: See Appendix 7.8.

The intuition for this proposition is similar to that of the case in which the exit cost is increasing with the fiscal shock. Take $\phi$ as given and evaluate (5.9) for $T = 0$:

$$\log\left(\frac{c^1}{c^2}\right) + 1 + \frac{c^1}{c^2}\left(\frac{\lambda}{r+\lambda}\frac{r\Delta\Gamma}{c^1} - 1\right) \leq 0, \tag{5.10}$$

If (5.10) takes on a negative value it is optimal to choose $T = 0$. Since the fiscal cost cannot exceed the wealth of the economy, $r\Delta\Gamma < c^1$, and the coefficient on $c^1/c^2$ is negative. For a given $\phi$, as $\Delta\Gamma$ increases $c^2$ converges to zero, while $c^1$ remains constant. As a result, $c^1/c^2$ becomes arbitrarily large and the left-hand side of (5.10) converges to $-\infty$.[22] Since the flow saving of delaying is bounded relative to $c^1$, the cost of delaying dominates and immediate abandonment is optimal for large values of $\Delta\Gamma$. The fact that $\Gamma^*$ is increasing in $\lambda$ is also intuitive: it means that when the reform arrival rate is higher the range of fiscal shocks for which it is optimal to delay abandoning the peg is larger.

---

[22]It should be clear that in this case $T = 0$ is the global optimum. In order for $c^2$ to be positive it must be the case that $\Delta\Gamma e^{rT} < c^1/r$, so for any $T > 0$ the left-hand side of (5.9) converges to $-\infty$.

## 6. Conclusion

Versions of the Krugman-Flood-Garber currency crisis model are widely used to study the abandonment of fixed exchange rate regimes. This class of models assumes that the central bank follows a mechanical exit rule: a peg is abandoned if and only if international reserves reach a critical lower bound. From a positive standpoint the KFG rule is at odds with many episodes in which the central bank has plenty of international reserves at the time of abandonment. From a normative standpoint our analysis suggests that the KFG rule is suboptimal.

We characterize the optimal exit strategy in a model in which the fixed exchange rate regime has become unsustainable due to an unexpected increase in the present value of government spending. We show that when there are no exit costs it is optimal to abandon immediately. When there are exit costs, the optimal abandonment date is a decreasing function of the size of the fiscal shock. For large fiscal shocks immediate abandonment is optimal.

So far we have studied a basic monetary model where the only impact of inflation is that it distorts intertemporal consumption allocations. This analysis provides us with a point of departure to study richer environments in which tax revenue and the cost of financing public debt are endogenous and where monetary policy affects the level of economic activity.

# References

[1] Buiter, Willem "Borrowing to Defend the Exchange Rate and the Timing and Magnitude of Speculative Attacks," *Journal of International Economics*, 23: 221-239, 1987.

[2] Burnside, Craig, Martin Eichenbaum and Sergio Rebelo "Government Guarantees and Self-Fulfilling Speculative Attacks," *Journal of Economic Theory*, 119: 31-63, 2004.

[3] Burnside, Craig, Martin Eichenbaum and Sergio Rebelo "Prospective Deficits and the Asian Currency Crisis," *Journal of Political Economy*, 109: 1155-1198, 2001.

[4] Calvo, Guillermo "Balance of Payments Crises in a Cash-in-Advance Economy," *Journal of Money Credit and Banking*, 19: 19-32, 1987.

[5] Drazen, Allan and Elhanan Helpman "Stabilization with Exchange Rate Management," *The Quarterly Journal of Economics*, 835-855, 1987.

[6] Flood, Robert P., and Peter Garber "Collapsing Exchange Rate Regimes: Some Linear Examples," *Journal of International Economics*, 17: 1–13, 1984.

[7] Flood, Robert., P. Peter Garber, and Charles Kramer, "Collapsing Exchange-Rate Regimes: Another Linear Example," *Journal of International Economics*, 41, 223-234, 1996.

[8] Flood, Robert P., and Olivier Jeanne, "An Interest Rate Defense of a Fixed Exchange Rate?," *Journal of International Economics*, 66: 471-484; 2005.

[9] Flood, Robert P., Jagdeep S. Bhandari, and Jocelyn Horne, "Evolution of Exchange Rate Regimes," *IMF Staff Papers*, 36: 810-835, 1989.

[10] Gupta, Poonam, Deepak Mishra, and Ratna Sahay, "Output Response to Currency Crises," IMF Working Paper 03/230, *International Monetary Fund*, 2003.

[11] Hutchison, Michael M., and Ilan Neuberger, "Output Costs of Currency and Balance of Payments Crises in Emerging Markets," unpublished manuscript (UCSC, 2001).

[12] Kaminsky, Graciela and Carmen Reinhart, "The Twin Crises: The Causes of Banking and Balance-of-Payments," *American Economic Review*, 89: 473-500, 1999.

[13] Krugman, Paul, "A Model of Balance of Payments Crises," *Journal of Money, Credit and Banking*, 11: 311–25, 1979.

[14] Lahiri, Amartya and Carlos Végh "Delaying the Inevitable: Interest Rate Defense and BOP Crises," *Journal of Political Economy* 111: 404-424, 2003.

[15] Lucas, Robert, and Nancy Stokey, "Optimal Fiscal and Monetary Policy in an Economy without Capital," *Journal of Monetary Economics,* 12: 55-93, 1983.

[16] Mussa, Michael, *Argentina and the Fund: From Triumph to Tragedy* (Washington, DC: Institute for International Economics, 2002).

[17] Obstfeld, Maurice "Speculative Attack and the External Constraint in a Maximizing Model of the Balance of Payments," *Canadian Journal of Economics*, 29: 1-20, 1986a.

[18] Obstfeld, Maurice "Rational and Self-Fulfilling Balance-of-Payments Crises," *American Economic Review*, 76: 72–81, 1986b.

[19] Obstfeld, Maurice "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review*, 40: 1037-48, 1996.

[20] Rebelo, Sergio and Danyang Xie "On the Optimality of Interest Rate Smoothing," *Journal of Monetary Economics*, 43: 263-282, 1999.

[21] Reinhart, Carmen and Carlos Vegh "Nominal Interest Rates, Consumption Booms, and Lack of Credibility: A Quantitative Examination," *Journal of Development Economics*, 46: 357-378, 1995.

[22] Rigobon, Roberto "Disinflation and Fiscal Reform: A Neoclassical Perspective," *Journal of International Economics* 58: 265-297, 2002.

[23] Wijnbergen, Sweder Van "Fiscal Deficits, Exchange Rate Crises and Inflation," *Review of Economic Studies*, 58: 81-92, 1991.

# 7. Appendices

## 7.1. Episode selection

Our original sample consists of the 96 currency crisis episodes identified in Kaminsky and Reinhart (1999) and updates. Since the selection criteria used by Kaminsky and Reinhart is based on a weighted average of reserve losses and changes in the exchange rate, we choose a sub-sample based exclusively on changes in the exchange rate. We choose those episodes in which the devaluation in the month of abandonment is at least 10 percent and that meet one of the following criteria:

1. There was a fixed exchange rate (or a negative rate of change in the exchange rate) for at least 12 months before the devaluation.

2. Devaluation in the 12 months following and including the month of abandonment is at least twice as large as the devaluation in the previous 12 months.

These criteria are used to exclude countries where 10 percent devaluation are recurrent events.

## 7.2. Proof of Proposition 3.1

This appendix contains the proofs of Proposition 3.1. We first outline some preliminary steps. The starting point is the Kuhn-Tucker condition (**??**), reproduced below for convenience:

$$\log\left(\frac{c^1}{c^2}\right) - \left[\left(\frac{c^1}{c^2} - 1\right) - \frac{r\phi}{c^2}\right] \leq 0. \tag{7.1}$$

We now express this condition as a function solely of parameters, using the government budget constraint, (3.10):

$$\log\left(\frac{c^1}{c^1 - r(e^{rT}\Delta\Gamma + \phi)}\right) - \left[\left(\frac{c^1}{c^1 - r(e^{rT}\Delta\Gamma + \phi)} - 1\right) - \frac{r\phi}{c^1 - r(e^{rT}\Delta\Gamma + \phi)}\right] \leq 0.$$

To simplify notation, let:

$$p(T, \phi, \Delta\Gamma) \equiv \frac{c^1}{c^1 - r(e^{rT}\Delta\Gamma + \phi)},$$

$$p_\phi = \frac{rp^2}{c^1}.$$

Then define:

$$\Psi(T, \phi, \Delta\Gamma) \equiv \log(p) - (p - 1) + \frac{r\phi p}{c^1}. \qquad (7.2)$$

Hence,

$$\Psi_\phi = \frac{p_\phi}{c^1}(c^1 - \phi r - 2re^{rT}\Delta\Gamma).$$

We need to impose bounds on $\phi$ and $\Delta\Gamma$ to ensure that $c^2$ is positive (recall that $c^1$ is exogenous). Equation (3.10) implies:

$$c^2 = c^1 - r(e^{rT}\Delta\Gamma + \phi) > 0.$$

For a given $\Delta\Gamma$, $\phi$ is bounded by:

$$\phi < \frac{c^1}{r} - e^{rT}\Delta\Gamma. \qquad (7.3)$$

For a given $\phi$, $\Delta\Gamma$ is bounded by:

$$\Delta\Gamma < e^{-rT}\left(\frac{c^1}{r} - \phi\right).$$

We consider two sub-cases:

1. $\Delta\Gamma > c^1/2r$

2. $c^1/er \leq \Delta\Gamma \leq c^1/2r$

**Case 1**: $\Delta\Gamma > c^1/2r$. Notice that:

$$\Psi_\phi = \frac{p_\phi}{c^1}\left(c^1 - \phi r - 2re^{rT}\Delta\Gamma\right).$$

Evaluate this expression at $T = 0$ to obtain:

$$\Psi_\phi = \frac{p_\phi}{c^1}\left(c^1 - \phi r - 2r\Delta\Gamma\right) < 0,$$

since $\Delta\Gamma > c^1/2r$. Hence, $\Delta\Gamma > c^1/2r$ is a sufficient condition for $\Psi_\phi < 0$.

**Case 2:** $c^1/er \le \Delta\Gamma \le c^1/2r$. In this case, notice that $\Psi_\phi$ becomes zero for a value of $\phi$ which we denote by $\phi^{\max}$. This value is given by:

$$\phi^{\max} = \frac{c^1}{r} - 2\Delta\Gamma.$$

For further reference, we evaluate $p$ at $\phi^{\max}$:

$$p(0, \phi^{\max}, \Delta\Gamma) \;\equiv\; \frac{c^1}{c^1 - r\left(\Delta\Gamma + \phi^{\max}\right)},$$
$$p(0, \phi^{\max}, \Delta\Gamma) \;\equiv\; \frac{c^1}{r\Delta\Gamma}.$$

Using this expression we evaluate $\Psi$ at $\phi^{\max}$:

$$\Psi(0, \phi^{\max}, \Delta\Gamma) \;=\; \log(p) - (p-1) + \frac{r\phi^{\max}p}{c^1},$$
$$\Psi(0, \phi^{\max}, \Delta\Gamma) \;=\; \log[p(0, \phi^{\max}, \Delta\Gamma)] - 1.$$

Hence:

$$\Psi(0, \phi^{\max}, \Delta\Gamma \;=\; c^1/er) = 0$$
$$\Psi(0, \phi^{\max}, \Delta\Gamma \;>\; c^1/er) < 0$$

The solution is $T^* = 0$. It is a boundary solution for $\Delta\Gamma = c^1/er$ and a corner solution for $\Delta\Gamma > c^1/er$.

We now consider the case of small fiscal shocks.

We denote the two roots by $\phi^*$ and $\phi^{**}$, with $\phi^* < \phi^{**}$.

Notice that if $\Delta\Gamma < c^1/er$, then:

$$p(0, \phi^{\max}, \Delta\Gamma) \equiv \frac{c^1}{r\Delta\Gamma} > e.$$

This inequality implies that:

$$\Psi(0, \phi^{\max}, \Delta\Gamma < c^1/er) > 0.$$

Since we know that $\Psi(0, 0, \Delta\Gamma < c^1/er) < 0$, by continuity it follows that $\phi^*$ exists. To establish existence of the second root, $\phi^{**}$, we now show that the limit of $\Psi(T, \phi, \Delta\Gamma)$ as $\phi$ approaches the upper bound given in (7.3) is $-\infty$. This limit is given by:

$$\lim_{\phi \longrightarrow \frac{c^1}{r} - \Delta\Gamma} \Psi(T, \phi, \Delta\Gamma) = \log(p) - (p - 1) + \frac{r\phi p}{c^1}.$$

Since $p \longrightarrow \infty$ as $\phi \longrightarrow \frac{c^1}{r} - \Delta\Gamma$, we need to collect terms in $p$ to evaluate the resulting coefficient:

$$\lim_{\phi \longrightarrow \frac{c^1}{r} - \Delta\Gamma} \Psi(T, \phi, \Delta\Gamma) = 1 + \log(p) + p\left(-1 + \frac{r\phi}{c^1}\right).$$

The coefficient on $p$ is always negative because, from (7.3), $\phi < c^1/r$. Hence, the limit is $-\infty$.

### 7.3. Behavior of $T$, $\varepsilon$, and Reserve Loss as a Function of $\phi$.

**Behavior of $T$.** Take as given $\Delta\Gamma \in (0, c^1/er)$ and consider the ranges for $\phi$ (established above) for which the solution for $T$ is interior. In that case, $\Psi(T^*, \phi, \Delta\Gamma) = 0$ implicitly defines $T^*$ as a function of $\phi$:

$$\phi = \frac{c^1}{rp}(p - 1 - \log p). \tag{7.4}$$

Hence:

$$\frac{dT}{d\phi} = \frac{\Psi_\phi c^1}{p_T r \Delta \Gamma e^{rT}},$$

where the behavior of $\Psi_\phi$ has been derived above. Hence, $T$ is an increasing function of $\phi$ for $\phi \in [\phi^*, \phi^{\max})$ and a decreasing function for $\phi \in [\phi^{\max}, \phi^{**})$. For all other values of $\phi$, the value of $T^* = 0$, as established above. Figure 2, Panel A, shows $T^*$ as a function of $\phi$.

**Behavior of $\varepsilon$**   For the range of interior solutions, it follows from (7.4) that:

$$\frac{d\varepsilon}{d\phi} = \frac{r(1+r)p^2}{c^1 \log(p)} > 0.$$

When $T^* = 0$, $\varepsilon$ is also an increasing function of $\phi$, as follows from (3.15) and (3.16). Figure 2, Panel B, illustrates the optimal $\varepsilon$ as a function of $\phi$. Clearly, at $\phi = \phi^* = \phi^{**}$, this function need not be differentiable.

**Behavior of loss of reserves**   By definition, the reserve loss at $T$ is equal to $c^1 - c^2$. Since $c^1$ is independent of both $T$ and $\phi$, we just need to check the behavior of $c^2$ as a function of $\phi$ for interior solutions (naturally, for $T = 0$, the reserve loss is zero). Since $c^2 = c^1/p$, it follows that:

$$\frac{dc^2}{d\phi} = -\frac{c^1}{p^2(1+r)}\frac{d\varepsilon}{d\phi} < 0.$$

Hence, the reserve loss is an increasing function of $\phi$ when the solution is interior (see Figure 2, Panel C).

## 7.4. Behavior of $T$, $\varepsilon$, and the Loss of Reserves as a Function of $\Delta \Gamma$

**Behavior of $T$**   We now derive the behavior of the optimal values of $T$, $\varepsilon$, and the loss of reserves as a function of $\Delta \Gamma$ for a given $\phi \in (\phi^*, \phi^{**})$. As shown

44

above, the solution is interior for $\Delta\Gamma \leq \frac{c^1}{er}$. In this range, setting (7.2) to zero yields:

$$\frac{dT}{d\Delta\Gamma} = -\frac{p_{\Delta\Gamma}}{p_T} < 0.$$
$$\lim_{\Delta\Gamma \to 0} T = \infty.$$

For any $\Delta\Gamma \geq \frac{c^1}{er}$, the solution is $T = 0$, as shown above. In Figure 3, and without loss of generality, the given value of $\phi$ has been taken to be $\phi = \frac{c^1}{r}(1 - \frac{2}{e})$. It can be checked that $\Psi[\frac{c^1}{r}(1 - \frac{2}{e}); \frac{c^1}{er}] = 0$ and hence in Panel A, $T(\Delta\Gamma^* = \frac{c^1}{er}) = 0$.

**Behavior of $\varepsilon$** Consider now the behavior of the optimal value of $\varepsilon$ as a function of $\Delta\Gamma$. Since the RHS of (7.4) is a strictly increasing function of $p$, it follows that, when the solution is interior, the optimal value of $p$ (and hence $\varepsilon$) is fully determined by $\phi$ and is therefore independent of $\Delta\Gamma$. Hence, for $0 < \Delta\Gamma < \frac{c^1}{er}$, the optimal $\varepsilon$ does not depend on $\Delta\Gamma$.[23] For $\Delta\Gamma \geq \frac{c^1}{er}$, $T^*$ is zero. It then follows from (3.15) and (3.16) that $\varepsilon$ is an increasing function of $\Delta\Gamma$. (See Panel B in Figure 3.)

**Behavior of the loss of reserves** Finally, consider the reserve loss ($\equiv c^1 - c^2$). Clearly, for $\Delta\Gamma \geq \frac{c^1}{er}$, the reserve loss is zero since the peg is abandoned right away. For $0 < \Delta\Gamma < \frac{c^1}{er}$, the reserve loss equals $c^1(p - 1)/p > 0$. Since $p$ is independent of $\Delta\Gamma$ when the solution is interior, then the reserve loss is also independent of $\Delta\Gamma$ in this range.

### 7.5. Proof of Proposition 4.1

We start with expression (4.3) and proceed in exactly the same way as in Proposition 3.1 (see Appendix 7.2).

---

[23]For $\Delta\Gamma = 0$, the optimal $\varepsilon$ is zero since $T^* = \infty$, i.e. the peg will never be abandoned.

### 7.6. Proof of Proposition 4.2

The starting point is the Kuhn-Tucker condition (4.7). Using $c^2 = c^1 - r\Delta\Gamma \left(e^{rT} + \alpha\right)$ and defining $p$ as

$$p(T, \Delta\Gamma) \equiv \frac{c^1}{c^1 - r\Delta\Gamma \left(e^{rT} + \alpha\right)}, \tag{7.5}$$

we can rewrite condition (4.7) as

$$\Psi(T, \Delta\Gamma) \equiv \log(p) - (p - 1) + \frac{r\alpha\Delta\Gamma p}{c^1}. \tag{7.6}$$

We will proceed by showing that $\Psi(0, \Delta\Gamma)$ takes an inverted-U shape form and has one zero root for some value of $\Delta\Gamma$, denoted by $\Delta\Gamma^*$. Hence, for any $\Delta\Gamma \geq \Delta\Gamma^*$, it will be optimal to abandon immediately.

It follows from (7.6) that $\Psi(0, 0) = 0$. Further, by differentiating (7.6) with respect to $\Delta\Gamma$, it follows that

$$\left.\frac{d\Psi}{d\Delta\Gamma}\right|_{\Delta\Gamma=0} = \frac{r\alpha}{c^1} > 0,$$

which indicates that $\Psi$ is an increasing value of $\Delta\Gamma$ for small values of $\Delta\Gamma$.

We now evaluate $\Psi(0, \Delta\Gamma)$ as $\Delta\Gamma$ approaches its maximum admissible value. For $c^2$ to be positive, we need to impose the condition that $c^1 > r\Delta\Gamma \left(e^{rT} + \alpha\right)$. For $T = 0$, this condition becomes

$$\Delta\Gamma < \frac{c^1}{r(1 + \alpha)}, \tag{7.7}$$

which, for a given $\alpha > 0$, imposes an upper bound on the value of $\Delta\Gamma$. We now take the limit of $\Psi(0, \Delta\Gamma)$ as $\Delta\Gamma$ tends to $c^1/[r(1 + \alpha)]$. From (7.5) evaluated at $T = 0$, it follows that, since $c^1$ is given, $p \to \infty$ as $\Delta\Gamma \to c^1/[r(1 + \alpha)]$. Hence:

$$\lim_{\Delta\Gamma \to \frac{c^1}{r}\frac{1}{(1+\alpha)}} \Psi(0, \Delta\Gamma) = \lim_{p \to \infty} p\left(\frac{r\alpha\Delta\Gamma}{c^1} - 1\right) = -\infty,$$

since, from (7.7), it follows that

$$\frac{r\alpha\Delta\Gamma}{c^1} < \frac{\alpha}{1+\alpha} < 1.$$

Since we know that $\Psi(0, \Delta\Gamma)$ increases for small values of $\Delta\Gamma$, it follows by continuity that $\Psi(0, \Delta\Gamma)$ reaches some maximum value and then decreases, crossing the horizontal axis for some value $\Delta\Gamma^*$. Hence, for any value of $\Delta\Gamma > \Delta\Gamma^*$, there is a corner solution (and, for $\Delta\Gamma = \Delta\Gamma^*$ we have a boundary solution).

### 7.7. Proof that T is a decreasing function of $\Delta\Gamma$

To show this, set equation (7.6) equal to zero and differentiate with respect to $T$ and $\Delta\Gamma$ to obtain:

$$\frac{dT}{d\Delta\Gamma} = -\frac{p_{\Delta\Gamma}\left(\frac{1-p}{p} + \frac{r}{c^1}\alpha\Delta\Gamma\right) + \frac{r}{c^1}\alpha p}{p_T\left(\frac{1-p}{p} + \frac{r}{c^1}\alpha\Delta\Gamma\right)}, \tag{7.8}$$

where

$$p_T = p^2\frac{r\Delta\Gamma e^{rT}}{c^1} > 0, \tag{7.9}$$

$$p_{\Delta\Gamma} = p^2\frac{r(e^{rT} + \alpha)}{c^1} > 0. \tag{7.10}$$

Using (7.5), we can simplify (7.8) to read:

$$\frac{dT}{d\Delta\Gamma} = -\frac{p_{\Delta\Gamma}}{p_T} - \frac{\alpha p}{p_T\Delta\Gamma e^{rT}} < 0,$$

which shows that, for interior solutions, $T$ is a decreasing function of the fiscal shock.

### 7.8. Proof of Proposition 5.1

Equation (5.9) can be re-written as:

$$K(p) \equiv (1-p)r + (r+\lambda)\log(p) \leq 0.$$

It is useful to define the function $K(p)$ as:

$$K(p) \equiv (1-p)r + (r+\lambda)\log(p).$$

This function is concave and for $\lambda > 0$ it intersects the x-axis twice, at $p = 1$ and at a value of $p$ greater than 1 which we denote by $p^*$. The maximum value of $K$ is achieved for $p = (r+\lambda)/r$. To check whether $T = 0$ is optimal we can set $T = 0$ in (5.8) to compute the value of $p$ that would be consistent with the government budget constraint if the peg was abandoned immediately. We denote this value of $p$ by $p^0$:

$$p^0 = \frac{c^1/r}{c^1/r - (\Delta\Gamma + m_{0-} - c^1)}.$$

Using the fact that $b_{0-} + m_{0-} + y/r = c^1(1+r)/r$ we can rewrite this expression as:

$$p^0 = \frac{c^1/r}{b_{0-} + y/r - \Delta\Gamma}.$$

We can then use this expression for $p^0$ to evaluate the Kuhn-Tucker condition. If $K(p^0) < 0$, $T = 0$ is optimal, otherwise $T > 0$ is optimal. The variable $p^0$ is an increasing function of $\Delta\Gamma$ which takes the value 1 when $\Delta\Gamma = 0$ (in this case there is no expenditure shock at time zero and the regime continues to be sustainable). The value of $p^0$ converges to infinity as $\Delta\Gamma \to b_{0-} + y/r$. This limiting value of $\Delta\Gamma$ is such that government spending exhausts all the resources of the economy. Define $\Delta\Gamma^*$ as the value of $\Delta\Gamma$ such that $p^0 = p^*$. Then for $\Delta\Gamma > \Delta\Gamma^*$, $K(p^0) < 0$ so it is optimal to abandon immediately. For $\Delta\Gamma < \Delta\Gamma^*$, $K(p^0) > 0$ and $T^* > 0$. Finally, it is easy to see that $p^*$ is an increasing function of $\lambda$. This property implies that $\Delta\Gamma^*$ is also an increasing function of $\lambda$.

## Table 1. Currency Crises Episodes

| Country | Date | Change in exch. rate (in %) | Loss of reserves (in %) | Change in real gov spending (in %) | Change in exch. rate 12 months before (in %) | Change in exch. rate 12 months after (in %) |
|---|---|---|---|---|---|---|
| Argentina | Jun 70 | 14.3 | 7.3 | 15.9 | 0.0 | 17.7 |
| | Jun 75 | 160.0 | 75.6 | 3.9 | 100.0 | 1,301.7 |
| | Feb 81 | 11.3 | 53.3 | 35.7 | 22.1 | 393.6 |
| | Apr 89 | 386.7 | 15.8 | 34.0 | 200.0 | 29,324.8 |
| | Jan 02 | 40.1 | 45.9 | 13.7 | 0.0 | 232.2 |
| Bolivia | Nov 82 | 358.3 | -33.3 | 24.0 | 76.2 | 358.3 |
| | Nov 83 | 155.1 | -32.1 | 36.8 | 358.3 | 2,483.4 |
| | Sep 85 | 1421.3 | 24.1 | -29.9 | 1,282.2 | 2,605.7 |
| Brazil | Feb 83 | 38.6 | 50.8 | -16.4 | 104.8 | 292.3 |
| | Jul 89 | 42.6 | -19.1 | 106.8 | 680.4 | 3,917.2 |
| | Oct 91 | 38.7 | 11.9 | -17.1 | 452.0 | 1,276.6 |
| | Jan 99 | 64.1 | 34.1 | 24.4 | 8.3 | 48.0 |
| Chile | Set 72 | 66.6 | 60.4 | NA | 0.0 | 900.0 |
| | Dec 74 | 28.2 | 66.2 | NA | 100.0 | 400.0 |
| | Jun 82 | 19.1 | 15.6 | 20.4 | 0.0 | 62.4 |
| Finland | Oct 82 | 13.7 | 16.9 | 16.0 | 8.2 | 16.7 |
| | Sep 92 | 16.3 | 10.1 | 33.3 | -8.3 | 50.4 |
| Indonesia | Nov 78 | 50.6 | 3.6 | 12.6 | 0.0 | 51.1 |
| | April 83 | 37.8 | 54.1 | 27.3 | 7.8 | 42.3 |
| | Sep 86 | 44.3 | 8.9 | 9.5 | 1.2 | 44.9 |
| Israel | Nov 74 | 42.9 | 24.1 | NA | 0.0 | 66.7 |
| | Nov 77 | 47.6 | -11.1 | NA | 23.0 | 78.6 |
| | Oct 83 | 31.5 | 2.3 | NA | 119.2 | 530.2 |
| Malaysia | Aug 97 | 12.4 | 15.8 | 15.1 | 5.4 | 57.2 |
| Mexico | Sep 76 | 58.7 | 39.0 | 16.8 | 0.0 | 82.9 |
| | Feb 82 | 67.7 | 27.7 | 82.7 | 13.8 | 277.6 |
| | Dec 94 | 54.4 | 75.0 | 13.2 | 10.8 | 121.8 |
| Norway | May 86 | 12.2 | -3.5 | 20.5 | -22.1 | -3.6 |
| Peru | Jun 76 | 44.4 | 54.5 | 17.8 | 16.3 | 73.4 |
| | Oct 87 | 25.9 | 49.2 | 6.3 | 13.9 | 1,473.3 |
| | Sep 88 | 657.6 | 42.8 | -17.5 | 107.6 | 10,719.9 |
| Philippines | Feb 70 | 46.7 | -29.9 | 15.0 | 0.3 | 63.3 |
| | Oct 83 | 27.3 | 80.7 | 0.4 | 26.6 | 63.6 |
| | Jul 97 | 15.4 | 7.4 | 17.1 | 0.7 | 59.5 |
| Spain | Feb 76 | 11.3 | 0.4 | 2.3 | 6.3 | 15.1 |
| | Jul 77 | 22.0 | 15.7 | -9.4 | 2.5 | 13.2 |
| Sweden | Aug 77 | 11.5 | 13.4 | 20.0 | -2.0 | 3.7 |
| | Oct 82 | 18.2 | -5.7 | 13.9 | 12.3 | 24.3 |
| | Nov 82 | 18.8 | -49.7 | 13.5 | -5.4 | 41.1 |
| | Nov 84 | 17.7 | -38.8 | 21.2 | 0.0 | 15.3 |
| | July 97 | 24.3 | 23.2 | 33.9 | 1.7 | 64.1 |
| Turkey | Aug 70 | 65.1 | -84.1 | 17.8 | 0.0 | 65.1 |
| | Jan 80 | 100.0 | 28.0 | -9.4 | 40.0 | 155.0 |
| | Mar 84 | 21.2 | 53.3 | 3.7 | 98.8 | 126.3 |
| | Feb 01 | 36.1 | 6.4 | 58.7 | 21.2 | 93.0 |
| Uruguay | Mar 72 | 99.6 | -119.4 | 14.1 | 0.0 | 241.1 |
| | Nov 82 | 38.7 | 37.4 | 44.1 | 18.4 | 176.3 |
| Venezuela | Feb 84 | 74.4 | -40.0 | -19.0 | 0.2 | 74.4 |
| | Dec 86 | 93.3 | 37.2 | 13.5 | 0.0 | 93.3 |
| | Mar 89 | 154.3 | 35.7 | -9.8 | 0.0 | 197.3 |
| | Dec 95 | 70.6 | 22.1 | -3.2 | 0.0 | 178.1 |

Source: Authors' calculations based on data from International Financial Statistics.
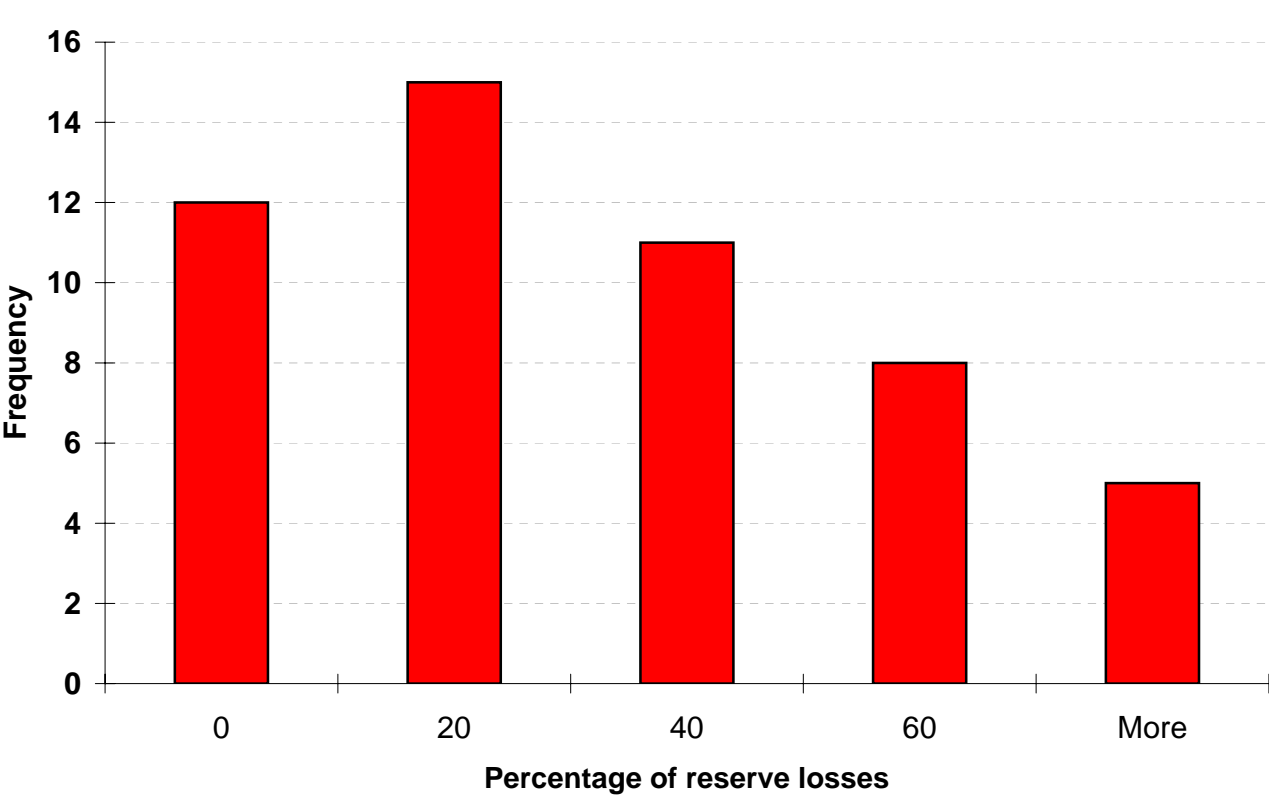
**Figure 1. Histogram of Reserve Losses**

# Figure 2. Optimal policy as a function of cost of abandoning
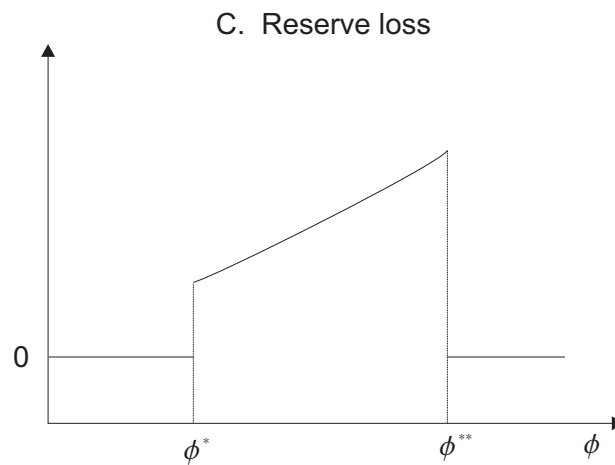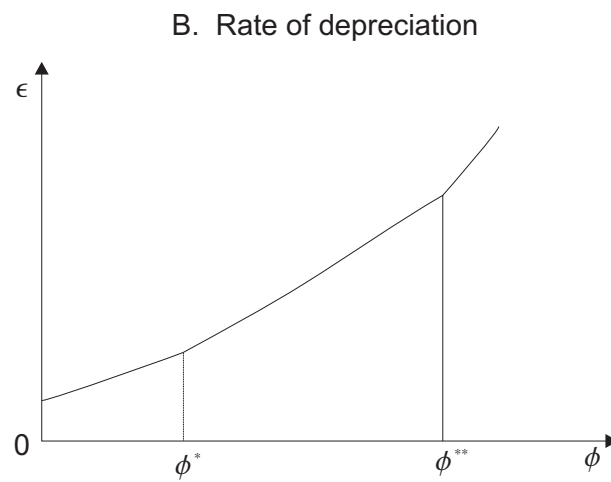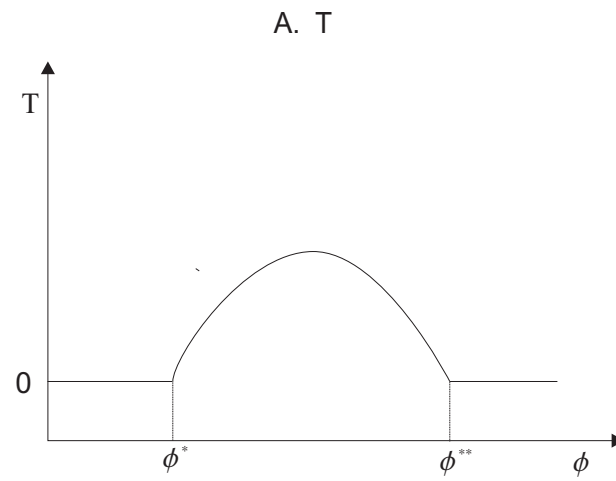
## A. T



## B. Rate of depreciation



## C. Reserve loss

# Figure 3. Optimal policy as a function of the fiscal shock

## A. T



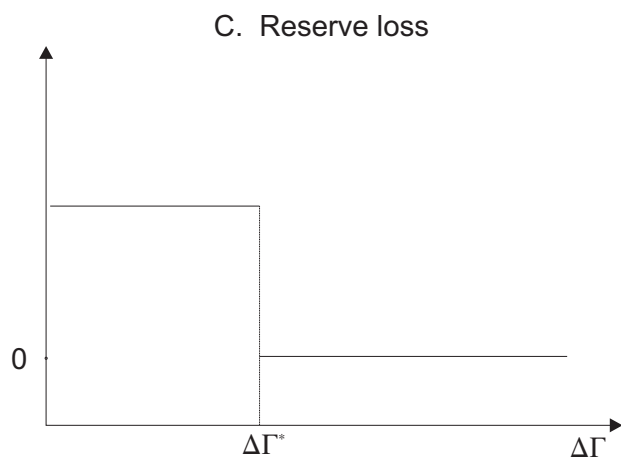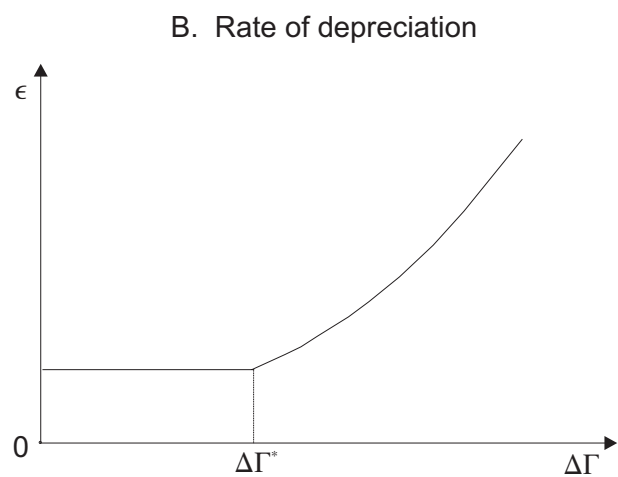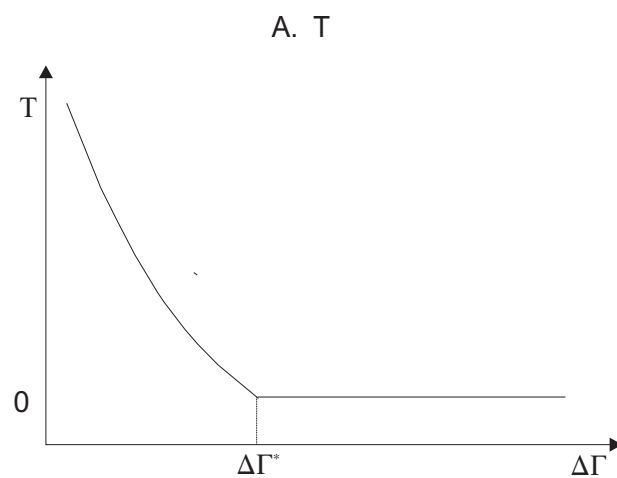## B. Rate of depreciation



## C. Reserve loss

Figure 4: Optimal T as a function of the fiscal shock for different values of δ



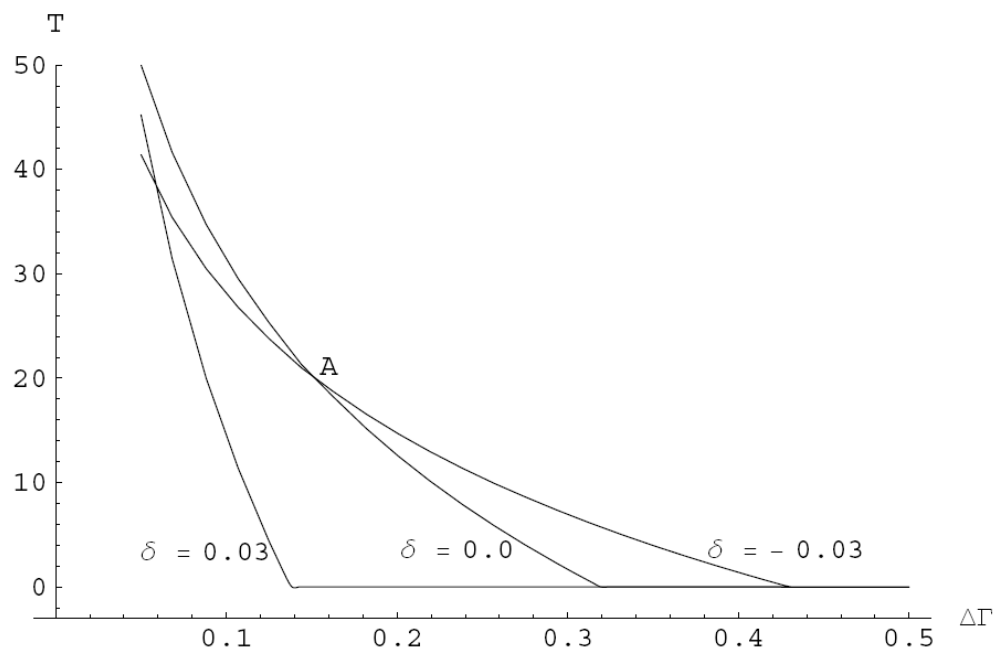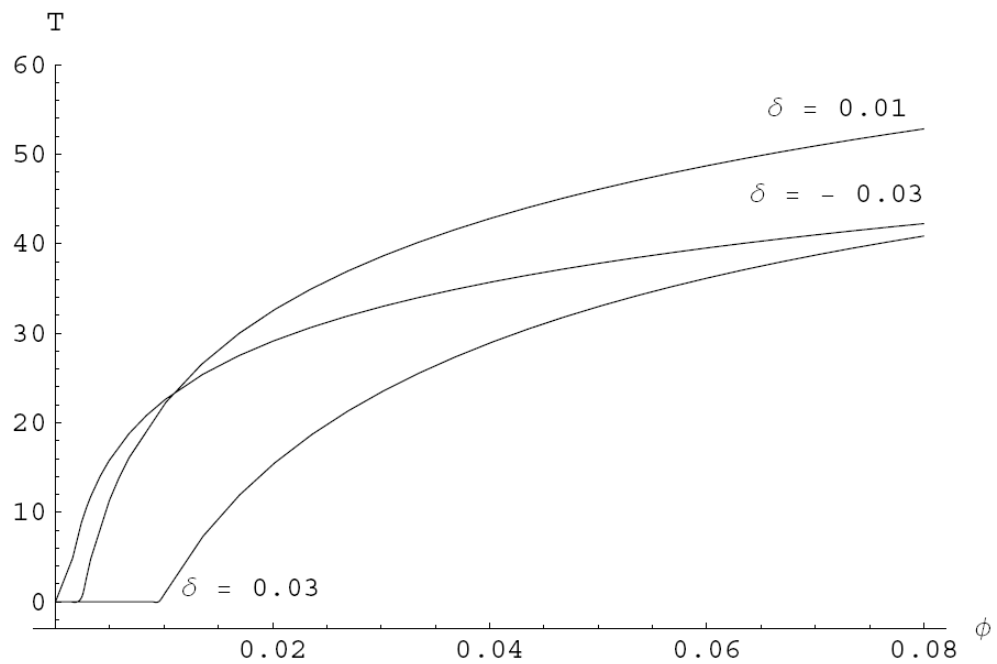Figure 5: Optimal T as a function of the exit cost for different values of δ

Figure 6: The exit cost is not a fiscal cost



A. Optimal T as a function
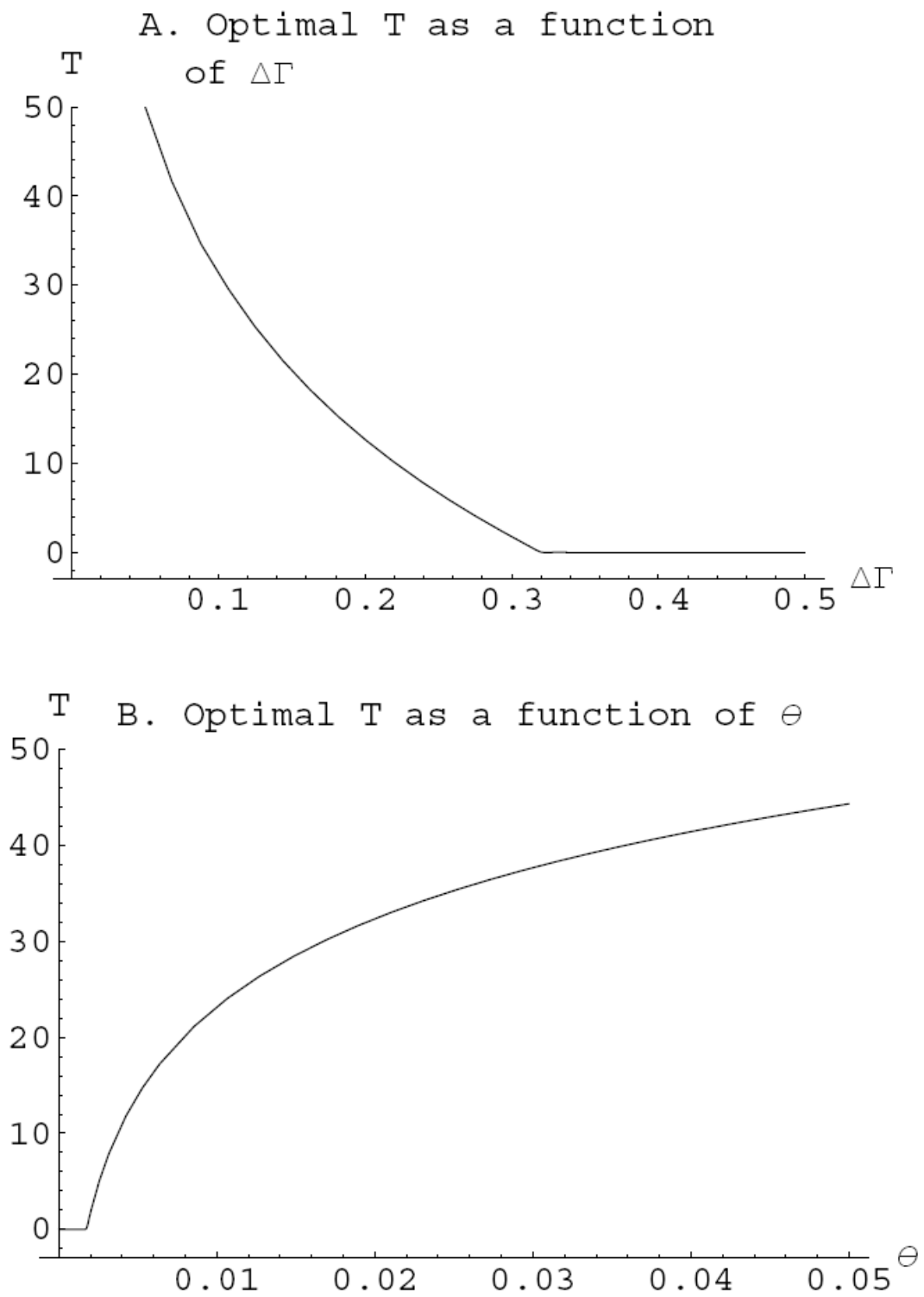of ΔΓ



B. Optimal T as a function of Θ

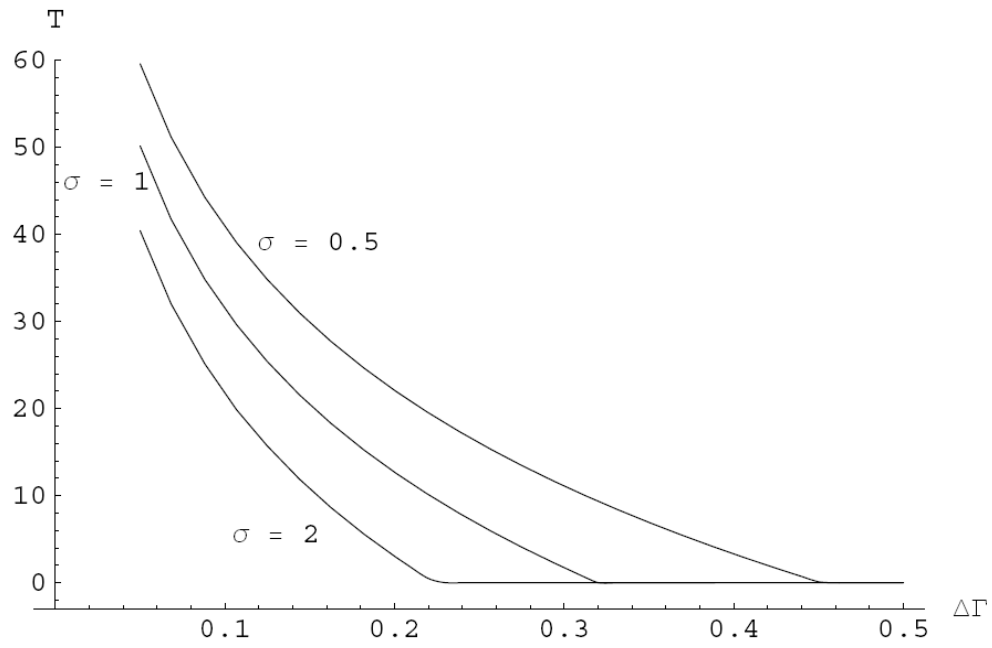Figure 7: Optimal T as a function of the fiscal shock for different values of σ



Figure 8: Optimal T as a function of the fiscal shock for different values of σ