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# HOME PRODUCTION, MARKET PRODUCTION AND THE GENDER WAGE GAP: INCENTIVES AND EXPECTATIONS 

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#### Abstract

The purpose of this paper is to study the joint determination of gender differentials in labor market outcomes and in the household division of labor. Specifically, we explore the hypothesis that incentive problems in the labor market amplify differences in earnings due to gender differentials in home hours. In turn, earnings differentials across genders reinforce the division of labor within the household. This gives rise to a potentially self-fulfilling feedback mechanism. As a consequence, gender differentials in earnings will be larger than any initial difference in relative productivity across genders. Even if productivity in home and market work is the same for female and male workers, both gendered and ungendered equilibria are possible and equally likely. If women $s$ comparative advantage in home production is large enough, there exists a unique equilibrium in which they have higher home hours and lower earnings than men. Our model delivers predictions on the relation between earnings ratios, incentive pay and home hours. First, gender earnings differentials should be higher for married workers in occupations in which the incentive problem is more severe. This effect should be stronger when the gender difference in home hours is greater. Moreover, the difference in the fraction of incentive pay across genders should be smaller for higher values of the female/male earnings ratio. Second, the husband/wife ratio of home hours should be negatively related with both the husband/wife earnings ratio and the difference in the fraction of incentive pay. We use the Census and the PSID to study these predictions and find that they are amply supported by the data.


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## 1 Introduction

The purpose of this paper is to study the joint determination of gender differentials in labor market outcomes and in the household division of labor. Specifically, we explore the hypothesis that incentive problems in the labor market amplify differences in earnings due to gender differentials in home hours. In turn, earnings differentials across genders reinforce the division of labor within the household. This gives rise to a potentially self-fulfilling feedback mechanism.

Our theoretical analysis is based on two components. We model households according to Chiappori's $(1988,1997)$ "collective labor supply" approach. Households value a public home good produced with time of both spouses. The intra-household allocation of home hours is efficient, and thus depends on the spouses' relative earnings. On the labor market, firms and workers contract over earnings. Workers' effort is private information. This generates a moral hazard problem. Following Becker (1985), we assume that the utility cost of effort is increasing in home hours. In addition, we assume that home hours are private information. This introduces adverse selection. In an extension of Holmstrom and Milgrom (1991), firms offer incentive compatible labor contracts that are constrained-efficient. Optimal contracts imply that workers' earnings and effort are inversely related to home hours.

The interaction between the optimal intra-household allocation process and the incentive problem in the labor market serves as an amplification mechanism, so that the gender differential in earnings is larger than any initial difference in relative productivity across genders. Even if productivity in home and market work is the same for female and male workers, two classes of equilibria are possible, gendered and ungendered. Gendered equilibria arise when firms believe that home hours are different for female and male workers. In particular, if firms believe that home hours are higher for women, they will offer them labor contracts with lower earnings and effort. This implies that the opportunity cost of home hours is lower for women. Then, it is efficient for wives to allocate more time to home production, thus confirming firms' beliefs. If instead firms believe that home hours are higher for men, by a similar logic, husbands' opportunity cost of home hours will be lower, leading households to optimally validate firms' beliefs. In ungendered equilibria, firms perceive home hours to be the same across genders and they offer the same contracts to female and male workers. As a consequence, spouses will face the same earning opportunities, the efficient intra-household allocation of labor will not be related to gender, and firms expectations will be confirmed.

Gendered and ungendered equilibria are equally likely, when there are no ex ante differences across genders. To capture biological differences, such as women's ability to bear children, we also allow for higher relative productivity of women in home production. In this case, there always exists an equilibrium in which women devote more time to home production and therefore have lower earnings than men. There are no ungendered equilibria. Interestingly, if women's comparative advantage in home production is small enough, there also exists an equilibrium in which women's home hours are lower than men's and they receive higher earnings. This result underscores the importance of the feedback between labor market outcomes and household decisions in generating gender differentials.

Our model delivers several predictions about differentials in earnings and the structure
of compensation across genders in relation to home hours. First, gender earning differentials should be higher for married workers in occupations in which the incentive problem is more severe. The severity of the incentive problem is related, in our model, to the variance, conditional on worker's effort, of observable performance measures used to provide incentives. This effect is stronger when the differences in home hours between women and men is greater. Relatedly, the difference in the fraction of incentive pay across male and female workers should be smaller for higher values of the female/male earnings ratio.

The dependence of labor market outcomes on home hours delivers additional predictions on the relation between earnings ratios, incentive pay and home hours across spouses. The husband/wife ratio of home hours should display a negative correlation with both the husband/wife earnings ratio and the difference in the fraction of incentive pay. The first property is an implication of the households' optimal choice of home hours. While this prediction is common to other efficient models of intra-household allocation, the second property is specific to the feedback mechanism between home hours and the incentive problem in the labor market that we highlight in our model.

We use a variety of data sources to support these predictions. We use Census data for year 2000 to study gender earnings differentials by marital status across industries and for three broad occupational categories: management, sales and production. We argue that incentive problems are most stringent in management and sales relative to production occupations. Managers have a wide range of responsibilities, hence, the uncertainty associated with their performance, given their effort should be greater than for workers in production occupations. Similarly, sales volumes depend to a large degree on variables that are not directly related to sales personnel's effort. These considerations are less important for production workers. We find that, for the sample of married workers, gender differentials in earnings are greatest in management and sales occupations. Our model does not have any predictions for single workers. However, we consider single workers in the empirical analysis, since the difference in home hours by gender is much smaller for this sample. This enables us to evaluate the prediction that earnings differentials are related to differences in home hours. We find that for single workers gender earnings differential are smallest in management and sales occupations across all industries. As a consequence, we observe a large married-single difference in gender earnings ratios in sales and management occupations. For production occupations, gender earnings ratios do not differ substantially by marital status.

Since the Census does not include information on the structure of earnings, we use PSID data from the late 1990s to document the negative relation between the male/female difference in the fraction of incentive pay and the female/male earnings ratio. We find a negative and significant correlation between the two ratios. Moreover, we also find that there is a strong negative correlation between the aggregate fraction of incentive pay and the female/male earnings ratio across occupations. This confirms our Census findings, since incentive pay is used more in those occupations where the incentive problem is more severe, as discussed in MacLeod and Parent (2003). In a cross-section of married couples from the PSID, we also find a negative correlation between the wife/husband ratio of home hours and the wife/husband ratio of earnings, and a positive correlation between the hours ratio and the husband-wife difference
in the fraction of incentive pay. This confirms our model's prediction.
Our model bridges three literatures: the literature on the sexual division of labor in the Beckerian tradition; the one on incentive contracts and job design, as in Holmstrom and Milgrom (1991); and finally the literature on statistical discrimination, as in Coate and Loury (1993) and Lundberg and Starz (1983). As argued by Becker (1985), what distinguishes gender and racial discrimination is that the feedback on the optimal intra-household division of labor generates a larger impact on earnings. The centerpiece of our model is to identify the source of statistical discrimination with the incentive problem on the labor market.

Our model emphasizes the importance of incentives for gender differences in earnings and the structure of compensation. In this we build on Goldin's (1986) pioneering study. She explores the role of supervisory and monitoring costs in rationalizing aspects of occupational segregation by gender. She argues that the prevalence of piece-rate compensation in manufacturing and of "career tracks" in the clerical sector can both be understood in the context of a labor market model with private information and costly monitoring, where firms use gender as a signal of labor market attachment. Goldin (1990) concludes that "... By segregating workers by sex into job ladders (and some dead-end positions), firms may have been better able to use the effort-inducing and ability-revealing mechanisms of the wage structure." This prediction also resonates with current debates on gender discrimination in personnel policy. For example, in June 2004 a federal judge ruled in favor of class-action status for the Dukes vs Wal-Mart gender discrimination lawsuit. The ruling was based on extensive evidence presented by the plaintiffs, Drogin (2003), showing that women working at Wal-Mart stores face pay disparities in most job categories, and take longer to enter management positions. ${ }^{1}$ Finally, it is also interesting to note how expectations of a gender wage gap characterize both male and female workers. As documented by Babcock and Laschever (2003): "Women report salary expectations between 3 and 32 percent lower than those of men for the same jobs; men expect to earn 13 percent more than women during their first year of full-time work and 32 percent more at their career peaks."

Our paper is organized as follows. Section 2 presents the model and discusses the results of numerical simulations. Section 3 reports evidence supporting the model's predictions. Finally, Section 4 concludes.

## 2 The Model

The economy is populated by a continuum of adult agents, ex ante identical except for gender, and a continuum of identical firms. The agents are equally divided by gender, they are all married and belong to a household. Households are made up of two agents of different gender. There are two types of goods in this economy- a private market good and public home goods. Households combine market goods and home hours of each spouse to produce the public home good, which is household specific. Each agent is employed by a firm. Firms produce the

[^0]market good using the agents' labor as the only input. Hence, there are two components of our model economy, the labor market and households. On the labor market, firms and individual agents negotiate the terms of labor contracts. Households efficiently choose the allocation of home hours across spouses. We assume that individual utility is increasing in consumption of the market good and decreasing in the number of hours worked at home and in the effort applied to market work. Following Becker (1985), we posit that an agent's utility cost of effort is increasing in home hours. We also assume that agents' home hours and effort are not observed by firms. Then, firms face adverse selection and moral hazard when contracting with workers. Firms will offer incentive compatible labor contracts that maximize the surplus from the employment relationship subject to incentive compatibility constraints stemming from the private information. Individual agents' labor market outcomes will depend on their home hours, which are chosen at the household level. On the other hand, an household's efficient choice of home hours will depend on the spouses relative earnings, which are determined on the labor market. Hence, there is a feedback from household decisions to labor market outcomes.

We now describe the labor contracts and the household decision problem in detail, and present our definition of equilibrium.

### 2.1 Labor Contracts

On the labor market, each firm hires agents to produce output. The output of one agent is related to her effort, according to:

$$
\begin{equation*}
y=f(e)+\omega, \tag{1}
\end{equation*}
$$

The function $f(e)$ denotes expected output, where $f$ is strictly increasing, twice continuously differentiable and weakly concave. The random variable $\omega$ is distributed normally with zero mean and variance $\Sigma^{2}>0$.

Each agent has a utility function:

$$
\begin{equation*}
U(c, h, e)=-\exp (-\sigma[c-v(h, e)]), \tag{2}
\end{equation*}
$$

where $c$ is individual consumption of the market good, $h$ denotes home hours, and $e$ denotes effort applied to market work. We adopt a CARA specification, where the coefficient of relative risk aversion, $\sigma$, is strictly greater than zero, and $v(\cdot)$ denotes the disutility of market and home work, where $h \in \mathbb{R}_{+}$and $e \in[0,1]$. The function $v$ is increasing in both its arguments, twice continuously differentiable and satisfies:

$$
\begin{equation*}
v_{h e}>0 . \tag{3}
\end{equation*}
$$

Hence, the marginal cost of effort is increasing in home hours ${ }^{2}$.
Firms choose labor contracts to maximize the surplus from the employment relationship. We assume that effort, $e$, and home hours, $h$, are not observed by firms, while output, $y$, is observable. Hence, labor contracts will be constrained-efficient, since firms will be subject to incentive compatibility constraints. Since home hours do not influence agents' output directly,

[^1]they can be interpreted as an agent's type from the standpoint of firms. Labor contracts will specify an earnings function $w$, and effort to be implemented for each type of agent, $h$, in the population. Earnings will depend on output. This property is required to implement strictly positive effort, given that it is private information. Moreover, since home hours are also unobserved, the optimal menu of contracts will depend on the firms' belief over the distribution of home hours. We characterize this distribution with its density $\pi$, and we represent labor contracts as a mapping, $\mathcal{C}(\pi)=\{w, e\}(h)$, where $h$ is understood to belong to the support of $\pi$. Condition (3) is the analogue of a single crossing condition for this model. This ensures that, given that contracts are incentive compatible, agents with home hours $h$ will self-select into the appropriate contract in the menu implied by $\mathcal{C}(\pi)$.

It is important to note that gender is observable, so firms can offer different contracts to female and male workers. However, since the contract space is unrestricted, firms will find it optimal to do so if and only if they believe that the distribution of home hours differs across genders.

To elucidate the role of our informational assumptions in the determination of labor market outcomes, we derive the properties of constrained-efficient labor contracts when home hours are observable first, and then consider the case in which home hours are also private information.

If firms observe home hours but effort is not observable, they face a moral hazard problem. Firms will choose labor contracts to solve:

$$
\max _{\{w(y), e\}, e \in[0,1]} S(e ; h)
$$

(Problem F1)
subject to

$$
\begin{equation*}
e=\arg \max _{e \in[0,1]} E[u(e)] \tag{4}
\end{equation*}
$$

where the objective function is the expected surplus from the employment relationship, and (4) is the incentive compatibility constraint. As shown in Holmstrom and Milgrom (1991), CARA utility implies that, without loss of generality, we can restrict attention to earnings functions of the form: $w(y)=\bar{w}+\tilde{w} y$. We refer to $\bar{w}$ and $\tilde{w} y$ as salary and incentive pay, respectively. This implies that under CARA, the expected surplus from the employment relationship can be written as:

$$
\begin{equation*}
S(e ; h)=f(e)-v(h, e)-\sigma \Sigma^{2}(\tilde{w})^{2} / 2 \tag{5}
\end{equation*}
$$

The first term is expected output, the second term is the utility cost of working, given home hours $h$. The last term stems from the need to provide incentives by making earnings depend on output, $y$. To implement $e>0$, firms must set $\tilde{w}>0$, which implies that earnings are stochastic and reduces the surplus from the employment relationship, since workers are risk averse. Given the CARA assumption on preferences, the incentive compatibility constraint simplifies to:

$$
\begin{equation*}
e=\arg \max _{e \in[0,1]} \tilde{w} f(e)-v(h, e) \tag{6}
\end{equation*}
$$

We can use the first order approach and replace (6) with the following:

$$
\begin{align*}
\tilde{w} f^{\prime}(e) & =v_{e}(h, e)  \tag{7}\\
\tilde{w} f^{\prime \prime}(e)-v_{e e}(h, e) & \leq 0 \tag{8}
\end{align*}
$$

Since we assume $f^{\prime \prime} \leq 0$ and $v_{e e}>0$, (8) will automatically be satisfied. The salary component of earnings does not influence workers' incentives to exert effort. We impose a zero profit condition on firms, which implies $\bar{w}=y(1-\tilde{w})$ and $w=y$.

To obtain analytical solutions, we will restrict attention to the following functional forms:

$$
\begin{align*}
f(e) & =e,  \tag{9}\\
v(h, e) & =(\psi+h) \frac{e^{2}}{2} . \tag{10}
\end{align*}
$$

The parameter $\psi>0$ can be interpreted as a fixed cost of working on the market.
Proposition 1 The optimal labor contract with observed home hours satisfies:

$$
\begin{align*}
e^{*}(h) & =\frac{1}{(\psi+h)\left(1+\sigma \Sigma^{2}(\psi+h)\right)}  \tag{11}\\
\tilde{w}^{*}(h) & =(\psi+h) e^{*} \tag{12}
\end{align*}
$$

In addition, expected earnings are given by $E w^{*}(h)=f\left(e^{*}(h)\right)$, with $E w^{* \prime}(h)<0$ and $E w^{* \prime \prime}(h)>0$.

Proof. In Appendix.
The optimal effort level and the fraction of incentive pay are decreasing in $h$, since the marginal utility cost of effort is increasing in home hours. Hence, expected total earnings, $w$, will also be decreasing in home hours. Effort and the fraction of incentive pay also decrease with risk aversion, $\sigma$, and with the parameter $\Sigma$, which represents the variance of an worker's output for given effort. High values of $\Sigma$ make it harder for firms to provide incentives for high effort.

If both home hours and effort are unobserved, this introduces additional constraints on the optimal contracts, which we refer to as adverse selection incentive compatibility constraints. Adverse selection implies that workers can extract an informational rent $T_{j}, j=L, H$, which reduces the surplus generated from the employment relation and may reduce the level of effort that can be implemented. The incentive compatibility constraints imply that workers will self-select the contract on the menu appropriate to their level of home hours.

We describe the firms' problem under the assumption that home hours can only take on two values and $h \in\left\{h_{L}, h_{H}\right\}$ with $h_{L}<h_{H}$, respectively, with $\pi\left(h_{j}\right)=0.5$ for $j=L, H$, since this is the only distribution of home hours that can occur in equilibrium in our model, as we prove in section 2.3. Firms take $h_{L}, h_{H}$ and $\pi(\cdot)$ as given, but they will determined in equilibrium from the optimal behavior of households.

The contracting problem in the case in which home hours are unknown is given by:

$$
\begin{equation*}
\max _{\left\{e_{j}, \tilde{w}_{j}\right\}_{j=L, H}, T_{L}, T_{H}} 0.5 \sum_{j}\left(f\left(e_{j}\right)-v\left(h_{j}, e_{j}\right)-\sigma \Sigma^{2} \frac{\tilde{w}_{j}^{2}}{2}-T_{j}\right) \tag{ProblemF2}
\end{equation*}
$$

subject to

$$
\begin{align*}
\tilde{w}_{j} f^{\prime}\left(e_{j}\right) & =v_{e}\left(h_{j}, e_{j}\right)  \tag{13}\\
f\left(\hat{e}_{i}\right) \tilde{w}_{i}-v\left(h_{j}, \hat{e}_{i}\right)-\sigma \Sigma^{2} \frac{\tilde{w}_{i}^{2}}{2}+T_{i} & \leq f\left(e_{j}\right) \tilde{w}_{j}-v\left(h_{j}, e_{j}\right)-\sigma \Sigma^{2} \frac{\tilde{w}_{j}^{2}}{2}+T_{j}  \tag{14}\\
f^{\prime}\left(\hat{e}_{i}\right) \tilde{w}_{i} & =v_{e}\left(h_{j}, \hat{e}_{i}\right), \tag{15}
\end{align*}
$$

for $j=L, H$, where $\hat{e}_{i}$ denotes the level of effort chosen by an agent of type $j$ when she untruthfully reports to be of type $i$. If the distribution of home hours is degenerate so that $\pi\left(h_{L}\right)=1$ or $\pi\left(h_{H}\right)=1$, then this problem collapses to Problem F1

The properties of the optimal labor contracts depends on the pattern of binding adverse selection incentive compatibility constraints and are summarized in the following proposition.

Proposition 2 A) For $1<\sigma \Sigma^{2}\left(\psi+h_{L}\right)<\left(\frac{\psi+h_{H}}{\psi+h_{L}}+1\right) 0.5$, the adverse selection incentive compatibility constraint is binding for workers with low home hours. Then:

$$
\begin{gather*}
\tilde{w}_{L}=\frac{1}{\left(\psi+h_{L}\right) 2 \sigma \Sigma^{2}}, e_{L}=\frac{\tilde{w}_{L}}{\left(\psi+h_{L}\right)},  \tag{16}\\
\tilde{w}_{H}=\frac{\left(\psi+h_{L}\right)}{\left(2 \psi+h_{H}+h_{L}\right)}, e_{H}=\frac{\tilde{w}_{H}}{\left(\psi+h_{H}\right)},  \tag{17}\\
T_{L}=0.5\left(\tilde{w}_{H}^{2}-\tilde{w}_{L}^{2}\right)\left(\frac{1}{\left(\psi+h_{L}\right)}-\sigma \Sigma^{2}\right), T_{H}=0 . \tag{18}
\end{gather*}
$$

B) For $1>\sigma \Sigma^{2}\left(\psi+h_{H}\right)>0.5\left(1+\frac{\psi+h_{L}}{\psi+h_{H}}\right)$, the adverse selection incentive compatibility constraint will be binding for workers with high home hours. Then:

$$
\begin{gather*}
\tilde{w}_{L}=\frac{\psi+h_{H}}{2 \psi+h_{H}+h_{L}}, e_{L}=\frac{\tilde{w}_{L}}{\left(\psi+h_{L}\right)},  \tag{19}\\
\tilde{w}_{H}=\frac{1}{2 \sigma \Sigma^{2}\left(\psi+h_{H}\right)}, e_{H}=\frac{\tilde{w}_{H}}{\left(\psi+h_{H}\right)},  \tag{20}\\
T_{L}=0, T_{H}=0.5\left(\frac{1}{\left(\psi+h_{H}\right)}-\sigma \Sigma^{2}\right)\left(\tilde{w}_{L}^{2}-\tilde{w}_{H}^{2}\right) . \tag{21}
\end{gather*}
$$

C) For $1 \geq \sigma \Sigma^{2}\left(\psi+h_{L}\right)$ and $1 \leq \sigma \Sigma^{2}\left(\psi+h_{H}\right)$, the adverse selection incentive compatibility constraint will not be binding. Then:

$$
\begin{equation*}
\tilde{w}_{j}=\tilde{w}^{*}\left(h_{j}\right), e_{j}=e^{*}\left(h_{j}\right), T_{j}=0, \text { for } j=L, H, \tag{22}
\end{equation*}
$$

where $\tilde{w}^{*}(\cdot)$ and $e^{*}(\cdot)$ are defined in (12) and (11), respectively.
Proof. In Appendix.
This proposition illustrates that three possible scenarios can arise. If utility is decreasing in $\tilde{w}_{j}$ for both $j$, which corresponds to case A), the adverse selection incentive compatibility constraint is binding for workers with low home hours. Then, $T_{L}>0$ and $\tilde{w}_{H}>\tilde{w}_{L}$. In case B), utility for both types of workers is increasing in $\tilde{w}_{j}$ and the adverse selection incentive
compatibility constraint is binding for workers with high home hours. This leads to $T_{H}>0$ and $\tilde{w}_{L}>\tilde{w}_{H}$. In case C), utility is increasing in $\tilde{w}_{L}$ for types with low home hours and decreasing in $\tilde{w}_{H}$ for types with high home hours. Hence, both the adverse selection incentive compatibility constraints will not be binding. Hence, the optimal menu of labor contracts corresponds to the one in which home hours are observed.

Cases A) and B) can only arise if the difference between high and low home hours, $h_{H}-h_{L}$, is large enough. They feature an additional inefficiency due to the binding adverse selection incentive compatibility constraint. It can be easily verified that in both case A) and B), $e_{L}<e^{*}\left(h_{L}\right)$ and $\tilde{w}_{L}<\tilde{w}^{*}\left(h_{L}\right)$, while $e_{H}>e^{*}\left(h_{H}\right)$ and $\tilde{w}^{*}\left(h_{H}\right)<\tilde{w}_{H}$, where $e^{*}(\cdot)$ and $\tilde{w}(\cdot)$ are the optimal effort and fraction of incentive pay when home hours are observed. Hence, private information on home hours reduces effort for the worker with low home hours and increases effort for the worker with high home hours. This enables $\tilde{w}_{L}-\tilde{w}_{H}$ to be lower than $\tilde{w}^{*}\left(h_{L}\right)-\tilde{w}^{*}\left(h_{H}\right)$ and relaxes the adverse selection incentive compatibility constraint and the corresponding informational rent. While in both case A) and B ), it is the case that $e_{L}>e_{H}$, there is a misallocation with respect to levels of effort implemented by the optimal contract when home hours are known.

The labor contracting environment described above elegantly embeds elements of job design and of optimal compensation policy for a wide class of occupations. The incentive pay component in the optimal earnings schedule is consistent with a variety of widely used compensation schemes, since the variable $y$ can be interpreted as an observable measure of performance. For example, for sales workers, $y$ corresponds to volume of sales, and $\tilde{w}$ represents the optimal commission rate. For management position, $y$ may stand for profits corresponding to a unit or division under a manager's supervision. Then, $\tilde{w}$ captures the dependence of the manager's total earnings on this observable measure of performance. As discussed in Milgrom and Roberts (1992), bonuses received by workers in addition to their basic salary are most often implicitly or explicitly linked to observable performance. Hence, $\tilde{w} y$ can be interpreted as a bonus, the size of which, depends on output. In addition, a menu of contracts in which one specifies high effort and one specifies low effort can be interpreted as two different jobs or positions within a firm.

### 2.2 Households

We model households according to the "collective labor supply" approach developed by Chiappori $(1988,1997)^{3}$. Three ingredients of this paradigm are crucial from our standpoint. Each spouse individually chooses consumption of the market good and effort. Spouses jointly choose home hours, the level of production of the home public good and a sharing rule for household wealth. Individual and joint decisions occur simultaneously.

Each household is endowed with wealth $a$. We denote the amount of household wealth at-

[^2]tributed to each spouse with $s_{i}$, for $i=f, m$, where $f, m$ stand for female and male, respectively. The production function for the home public good is
\[

$$
\begin{equation*}
G=g\left(h_{f}, h_{m}, k\right), \tag{23}
\end{equation*}
$$

\]

where $k$ is the amount of market good used in home production. We restrict attention to specifications in which $h_{f}$ and $h_{m}$ are substitutes. We assume that $g$ is increasing in each argument and concave.

Households and individual agents take as given the price of the market good and the mapping between individual home hours, earnings and effort, conditional on gender, implied by the labor contracts offered by firms. We denote the set of labor contracts offered with $\mathcal{C}_{i}\left(\pi_{i}\right)=\left\{w_{i}^{*}, e_{i}^{*}\right\}(h), i=f, m$, where the functions $w_{i}^{*}$ and $e_{i}^{*}$ satisfy Problem F2. The incentive compatibility constraints in the firms' problem and imply that individual optimality of market consumption and effort for given home hours is satisfied for each spouse for given $h_{i}$ and also $s_{i}$, due to the CARA specification of preferences. We can then define the following individual value function:

$$
\begin{equation*}
V_{i}\left(s_{i}, h_{i} ; \mathcal{C}\right)=E U\left(s_{i}+w_{i}^{*}\left(h_{i}\right), h_{i}, e_{i}^{*}\left(h_{i}\right)\right), \tag{24}
\end{equation*}
$$

for $i=f, m$, from the solution of Problem F2. It follows that the households' problem is to choose $G, k h_{i}$ and $s_{i}$ to maximize:

$$
\begin{equation*}
\sum_{i=f, m} \lambda_{i} V_{i}\left(s_{i}, h_{i} ; \mathcal{C}\right)+\theta \log (G), \tag{ProblemH}
\end{equation*}
$$

subject to (23), $h_{f}, h_{m} \geq 0$, and $\sum_{i} s_{i}+k=a+\Pi$. The parameters, $\lambda_{i}$, for $i=f, m$, represent the weight of each spouse in household decisions.

### 2.2.1 Choice of Home Hours

We adopt the following functional form for $g$ :

$$
\begin{gather*}
g\left(h_{f}, h_{m}, k\right)=H\left(h_{f}, h_{m}\right)^{\delta} k^{1-\delta},  \tag{25}\\
H\left(h_{f}, h_{m}\right)=\left[h_{m}^{\zeta}+h_{f}^{\zeta}\right]^{1 / \zeta}, \tag{26}
\end{gather*}
$$

with $\delta, \zeta \in(0,1)$. The function $H(\cdot)$ aggregates the contribution of spousal home hours to the production of the home public good. The parameter $\delta$ denotes the contribution of market goods to the production of the public home good, while $\zeta$ determines the substitutability of spousal home hours in home production.

The optimal choice of $h_{f}, h_{m}, k$ and $G$ is independent of the weights $\lambda_{i}$ and can be analyzed as a sequence of cost minimization problems. The solution to this problem depends on the spouses' opportunity cost of home hours, determined by labor contracts. The substitutability of spousal hours in the production of the public home good implies that marginal differences in market earnings will give rise to an allocation of home hours in which the spouse with lower
earning potential in market work devotes more time to home production. We interpret the intra-household allocation of home hours as a long term arrangement of the spouses, that may be costly to reverse in the short run.

The optimal choice of $h_{f}$ and $h_{m}$ for given $H$ solves the following cost minimization problem:

$$
C^{H}(\bar{H} ; \mathcal{C})=\min _{h_{f}, h_{m} \geq 0} w\left(h_{f}\right)+w\left(h_{m}\right)
$$

(Problem H1)
subject to

$$
\left[h_{m}^{\zeta}+h_{f}^{\zeta}\right]^{1 / \zeta} \geq \bar{H}
$$

for given $\bar{H}>0$ and given $\mathcal{C}_{j}\left(\pi_{i}\right)$ for $j=f, m$.
The first order necessary conditions for this problem are:

$$
\begin{align*}
& \left(\frac{h_{f}}{h_{m}}\right)^{1-\zeta}=\frac{E\left[w_{m}^{\prime}\left(h_{m}\right)\right]}{E\left[w_{f}^{\prime}\left(h_{f}\right)\right]},  \tag{27}\\
& \bar{H}=h_{m}\left[\left(\frac{h_{f}}{h_{m}}\right)^{\zeta}+1\right]^{1 / \zeta}, \tag{28}
\end{align*}
$$

where $w^{\prime}(h)$ denotes the derivative of total earnings with respect to home hours, which corresponds to the opportunity cost of home hours, and the expectation is taken with respect to $\omega$.

Equation (27) clarifies that the opportunity cost of home hours for each spouse depends on labor contracts and determines the optimal allocation of home hours. The substitutability of spousal hours in the production of the public home good implies that the spouse with lower opportunity cost, $E\left[w^{\prime}(h)\right]$, will devote more time to home production. The difference in spousal home hours for given labor contracts depends on the elasticity of substitution in $H$. If $w_{f}(h)=w_{m}(h)$ for all $h \geq 0$, households are indifferent over the allocation of home hours across spouses and they will randomize.

We describe the problems for the choice of $H, k$ and $G$ in Appendix. The solution to the household problem can be represented by the policy functions $s_{i}(a ; \mathcal{C}), h_{i}(a ; \mathcal{C}), k(a ; \mathcal{C})$, and $G(a ; \mathcal{C})$ for $i=f, m$.

### 2.3 Equilibrium

We now provide a definition of equilibrium for our economy.
Definition 3 An equilibrium is given by beliefs $\pi_{i}(h)$ for $i=f, m$, labor contracts $\mathcal{C}_{i}\left(\pi_{i}\right)=$ $\left\{w_{i}(y), e_{i}\right\}(h)$ for $i=f, m$, and policy functions for the household $\left\{G, k, h_{f}, h_{m}, s_{f}, s_{m}\right\}(a, \mathcal{C})$, such that:
i) Labor contracts solve Problem F2, given beliefs;
ii) Household policy functions solve the household problem, given labor contracts;
iii) The resulting distribution of home hours in the population is consistent with firms' beliefs.

We characterize the set of equilibria under the assumption that all households are homogeneous with respect to $\theta$ and $a$. We consider symmetric equilibria in which the allocation of home hours $\left\{h_{f}, h_{m}\right\}$ is the same for all households and beliefs are constant across firms.

Given that individuals of different gender are ex ante identical, the equilibrium distribution of home hours across genders depends on firms' self-fulfilling beliefs about this distribution ${ }^{4}$. We say that an equilibrium is gendered when firms believe that the distribution of home hours is different in the population of female and male workers. We say that it is ungendered otherwise. The same selection of labor contracts will be offered to female and male workers in ungendered equilibria. Households will be indifferent over which spouse should be assigned high home hours and they will randomize.

The following lemma shows that any equilibrium with a non-degenerate distribution of home hours must be ungendered, if all households are homogeneous.

Lemma 4 If households are homogeneous with respect to $\theta$ and $a$, in any symmetric equilibrium, there will at most be two values of home hours in the population, $\left\{h_{L}, h_{H}\right\}$, with $0<h_{L} \leq h_{H}$. If the distribution of home hours in the population is non-degenerate, that is $\pi_{f}\left(h_{j}\right) \in(0,1)$ and $\pi_{m}\left(h_{j}\right) \in(0,1)$ for $j=H, L$ with $h_{L}<h_{H}$, then the equilibrium is ungendered and $\pi_{f}\left(h_{j}\right)=\pi_{m}\left(h_{j}\right)=0.5$ for $j=L, H$.

The proof is in the Appendix. The first result is based on the observation that in a symmetric equilibrium, the optimal strategy for the allocation of home hours across spouses will be constant across households. This will result in two values of home hours in the population in a gendered equilibrium, while in an ungendered equilibrium the randomization optimal strategy for the allocation of home hours of an individual household will correspond to the equilibrium distribution of home hours by gender. For randomization to be optimal, households must be indifferent over the allocation of home hours across spouses, which requires the distribution of home hours to be the same for female and male workers. Moreover, if there are two values of home hours in the population, the only distribution consistent with an ungendered equilibrium is $\pi_{m}\left(h_{j}\right)=\pi_{f}\left(h_{j}\right)=0.5$ for $j=L, M$. Then, in an equilibrium with non-degenerate distribution of home hours, labor contracts solve Problem F2.

The following proposition characterizes equilibria with a degenerate distribution of home hours.

Proposition 5 If all households are homogeneous with respect to $\theta$ and $a$, the set of equilibria with degenerate distribution of home hours uniquely includes:
i) Two gendered equilibria, with distribution of home hours given by $\pi_{i}\left(h_{H}\right)=1$ and $\pi_{j}\left(h_{L}\right)=1$ for $i, j=f, m$ and $i \neq j$;

[^3]ii) One ungendered equilibria, $\pi_{f}(\bar{h})=\pi_{m}(\bar{h})=1$ for some $\bar{h}>0$.

In gendered equilibria, the distribution of home hours is different for male and female workers. By Lemma 4, all such equilibria have a degenerate distribution of home hours, with $\pi_{f}\left(h_{H}\right)=1$ and $\pi_{m}\left(h_{L}\right)=1$, or $\pi_{m}\left(h_{H}\right)=1$ and $\pi_{f}\left(h_{L}\right)=1$, where $h_{L}$ and $h_{H}$ are endogenously determined. Proposition 5 proves that two such equilibria exist, in addition to an ungendered equilibrium in which all workers have the same level of home hours.

We prove proposition 5 in the Appendix. Here, we describe the argument heuristically, since it clarifies the feedback mechanism between labor contracts and the households' problem. Firms' beliefs over the distribution of home hours shape the trade-off faced by households in the allocation of home hours, since they determine the spouses' relative earning potential by gender. Households take labor contracts as given and choose home hours based on this trade-off. This, in turn, induces the effective distribution of home hours in the population.

Given that by Lemma 4 there can be at most two values of home hours in the population, if firms believe that the distribution of home hours is different across genders, then such a distribution will be degenerate. Hence, there will be no adverse selection and labor contracts will solve Problem F1. To illustrate the argument, we focus on the equilibrium with distribution given by $\pi_{f}\left(h_{H}\right)=1$ and $\pi_{m}\left(h_{L}\right)=1$. While in equilibrium only one contract will be offered to female and male workers, to characterize the equilibrium, we need to allow households to contemplate their optimal choice of home hours for "out of equilibrium" menus of labor contracts that satisfy the restriction, $\max E w_{f}(h)<\max E w_{m}(h)$. By the properties of labor contracts derived in Propositions 1 and 2, this restriction would arise if firms believe that female workers have lower home hours than male workers. For such an equilibrium to exist, equation (27) must have a solution with $h_{m} / h_{f}<1$. Equation (27) is represented in figure 1 for a given value of $h_{f}$. The dashed line represents the right hand side of the equation while the solid line represents the left hand side.

We prove that, generically, there are two values of the ratio $h_{m} / h_{f}$ that solve this equation for given $h_{f}$. The first is $h_{m} / h_{f}=1$, the second is a value of this ratio strictly greater than zero and strictly smaller than 1 . Given that $\max E w_{f}(h)<\max E w_{m}(h), h_{m} / h_{f}=1$ is not optimal for Problem H1, because it corresponds to the maximum value of the objective. Therefore, the solution corresponds to the crossing with $h_{m} / h_{f}<1$. This pins down the equilibrium ratio $h_{m} / h_{f}=h_{L} / h_{H}$ and establishes that $\pi_{f}\left(h_{H}\right)=1$ and $\pi_{m}\left(h_{L}\right)=1$ is the equilibrium distribution of home hours. The equilibrium value of $h_{f}=h_{H}$ can then be derived from equation (28) and by solving the rest of the household problem. Since Problem H1 has a unique solution under restriction $\max E w_{f}(h)<\max E w_{m}(h)$, the resulting equilibrium is unique in its class.

A similar reasoning can be used to construct the equilibrium with distribution of home hours given by $\pi_{f}\left(h_{H}\right)=0$ and $\pi_{m}\left(h_{L}\right)=0$, which is characterized by the restriction on total earnings $\max E w_{f}(h)>\max E w_{m}(h)$. Equation (27) can be used to solve for $h_{f} / h_{m}$ for given $h_{m}$. Since women and men have identical home and market productivity, the equilibrium values of $h_{L}$ and $h_{H}$ will be the same in the previous equilibrium. Finally, the ungendered equilibrium can be constructed based on the restriction $E w_{f}(h)=E w_{m}(h)$, which implies that $h_{f}=h_{m}$


Figure 1: Solutions to equation (45) for $h_{f}=0.3, \zeta=0.8, \Sigma=1, \sigma=1, \psi=1$.
solves Problem H1, with resulting distribution of home hours $\pi_{f}(\bar{h})=\pi_{m}(\bar{h})=1$, for some $\bar{h}>0$.

An ungendered equilibrium with non-degenerate distribution of home hours may also exist. A non-degenerate distribution of home hours arises only if households find it optimal to randomize over the allocation of home hours across spouses, which requires that the same menu of contracts be offered to male and female workers. As shown in Lemma 4, this can only occur if the randomization strategy is the same for wives and husbands, which implies $\pi_{f}\left(h_{j}\right)=\pi_{m}\left(h_{j}\right)=0.5$ for $j=L, H$. Then, equilibrium labor contracts will solve Problem F2. The existence of this equilibrium requires that $E w_{H}^{\prime} / E w_{L}^{\prime}<1$ and that $E w_{L}^{\prime \prime}>0$, for $w_{j}, j=L, H$, that satisfy Proposition 2. This can be guaranteed by appropriately restricting the parameters. Rather than characterizing these restrictions, we concentrate on ungendered equilibria with a degenerate distribution of home hours, since the ungendered equilibrium with non-degenerate distribution is strictly Pareto-dominated by the ungendered equilibrium with degenerate distribution of home hours.

Proposition 5 identifies the set of possible equilibria for the model, either one of which could occur. However, the prevailing gender role distinction in most societies is one in which men specialize in market production and women in home production. Typically, gender differences in labor market outcomes, such as the earnings gap, have been ascribed to this division of labor, which is seen as the result of biological differences, in particular, women's ability to bear children. In the next section, we explore this argument in the context of our model.

### 2.3.1 Equilibrium with Ex-ante Differences Across Genders

We assume that female and male workers are equally productive in market work, but female workers are more productive in home work. Specifically, we posit that:

$$
\begin{equation*}
H\left(h_{f}, h_{m}\right)=\left[h_{m}^{\zeta}+(1+\varepsilon) h_{f}^{\zeta}\right]^{1 / \zeta} \tag{29}
\end{equation*}
$$

where $\varepsilon>0$. The strictly positive sign of $\varepsilon$ corresponds to women's higher relative productivity in home production, which we relate to their ability to bear children. The parameter $\varepsilon$ can be interpreted as a measure of the decreased relative market productivity of women during and after pregnancy. Alternatively, if children are viewed as a component of the public home good, $\varepsilon$ captures women's higher relative contribution to the nourishment of children via breast feeding. Technological advances, such as medical improvements reducing the physical stress associated with pregnancy and the introduction of baby formula, can be represented as a decrease in the value of $\varepsilon$.

The following result holds.
Proposition 6 Assume all households are homogeneous with respect to $\theta$ and $a$. There exists a unique value of $\varepsilon, \bar{\varepsilon}$, such that: i) For $0<\varepsilon \leq \bar{\varepsilon}$, there are two equilibria, one of which features $h_{f} / h_{m}<1$, with distribution of home hours $\pi_{f}\left(h_{H}\right)=0$ and $\pi_{m}\left(h_{L}\right)=0$, and one which features $h_{f} / h_{m}>1$, with distribution of home hours $\pi_{f}\left(h_{H}\right)=1$ and $\pi_{m}\left(h_{L}\right)=1$; ii)
for $\varepsilon>\bar{\varepsilon}$, there is one equilibrium with $h_{f} / h_{m}>1$ and distribution of home hours $\pi_{f}\left(h_{H}\right)=1$ and $\pi_{m}\left(h_{L}\right)=1$.

The proof is in the Appendix and we illustrate the argument graphically here. The first order necessary conditions for Problem H1 under (29) are given by:

$$
\begin{gather*}
\left(\frac{h_{f}}{h_{m}}\right)^{1-\zeta}=\frac{E\left[w_{m}^{\prime}\left(h_{m}\right)\right]}{E\left[w_{f}^{\prime}\left(h_{f}\right)\right] /(1+\varepsilon)},  \tag{30}\\
\bar{H}=h_{m}\left[(1+\varepsilon) h_{f}^{\zeta}+h_{m}^{\zeta}\right]^{1 / \zeta} \tag{31}
\end{gather*}
$$

If firms believe that female home hours are smaller than male home hours, max $E w_{f}(h)>$ $\max E w_{m}(h)$, where labor contracts solve Problem F1, by Lemma 4. To verify that $h_{f} / h_{m}<1$ is optimal for the household, we need to analyze the solutions to equation (30), which is represented in figure 2. The lower dashed line corresponds to the right hand side of (30) for $\varepsilon=0$, while the higher dashed line corresponds to the right hand side of (30) for $\varepsilon>0$. The properties of labor contracts imply that for $\varepsilon>0$ there are two zeros of (30), both with with $h_{f} / h_{m}<1$. However, by $\max E w_{f}(h)>\max E w_{m}(h)$ and since $E w(h)$ is decreasing and convex in $h$ by Proposition 1, the lowest value of $h_{f} / h_{m}$ that solves (30) is optimal for Problem H1. The optimal value of $h_{m}$ can be derived from (31) as a function of $H$, which is pinned down by the rest of the household problem. The resulting distribution of home hours is $\pi_{f}\left(h_{H}\right)=0$ and $\pi_{m}\left(h_{L}\right)=1$, consistent with firms' beliefs. Clearly, for $\varepsilon$ high enough, equation (30) does not have a solution and this equilibrium fails to exist.

If firms believe female home hours are greater than male home hours, max $E w_{f}(h)<$ $\max E w_{m}(h)$. To study whether $h_{m} / h_{f}>1$ is optimal for Problem H1 in this case, it is useful to rewrite equation (30) as:

$$
\begin{equation*}
\left(\frac{h_{m}}{h_{f}}\right)^{1-\zeta}=\frac{E\left[w_{f}^{\prime}\left(h_{f}\right)\right]}{E\left[w_{m}^{\prime}\left(h_{m}\right)\right](1+\varepsilon)}, \tag{32}
\end{equation*}
$$

and solve for $h_{m} / h_{f}$. This equation is represented in figure 3. The higher dashed line corresponds to the right hand side of this equation for $\varepsilon=0$, while the lower one corresponds to strictly positive value of $\varepsilon$. Generically, there are two values of $h_{m} / h_{f}$ that solve equation (32) for $\varepsilon>0$, one strictly smaller and the other strictly greater than 1 . However, $h_{m} / h_{f}>1$ is not optimal for Problem H1 under max $E w_{f}(h)<\max E w_{m}(h)$. Hence, the unique solution to Problem H1 features $h_{m} / h_{f}<1$. The optimal value of $h_{f}$ can be derived from equation (32) for given $H$. Solving the complete household problem determines the equilibrium distribution of home hours, which satisfies $\pi_{f}\left(h_{H}\right)=1$ and $\pi_{m}\left(h_{L}\right)=1$, consistent with firm beliefs. The existence of this equilibrium is guaranteed for any strictly positive value of $\varepsilon$.

Proposition 6 has several interesting implications. No ungendered equilibria are possible when there are ex ante differences across genders. There always exists an equilibrium in which wives devote more time to home production. In this equilibrium, $h_{f} / h_{m}$ is increasing in $\varepsilon$. Surprisingly, if relative productivity differences are small enough, an additional equilibrium exists


Figure 2: Solutions to equation (30) for $\varepsilon=0.2, h_{m}=0.3, \zeta=0.8, \Sigma=1, \sigma=1, \psi=1$.


Figure 3: Solutions to equation (32) for $h_{f}=0.3, \varepsilon=0.2, \zeta=0.8, \Sigma=1, \sigma=1, \psi=1$.
in which wives' home hours are lower than husbands'. The region of multiple equilibria can be characterized by a threshold value of $\varepsilon, \bar{\varepsilon}$. The intuition for the existence of this additional equilibrium is that women's higher relative home productivity reduces the extent to which they need to contribute to the production of the home public good. Such an equilibrium is more likely to exist, that is $\bar{\varepsilon}$ is higher, if the degree of complementarity in spouses' home hours in home production is high, which corresponds to low values of the parameter $\zeta$ in the aggregator $H\left(h_{f}, h_{m}\right)$. Small values of $\zeta$ increase the curvature of the left hand side of equation (30), thus raising the value of $\bar{\varepsilon}$. The threshold $\bar{\varepsilon}$ also depends on the utility cost of market work $\psi$. Specifically, higher values of $\psi$ raise the intercept of the right hand side of equations (30) and (32), thus reducing the equilibrium value of $\bar{\varepsilon}$.

Based on this result, we can interpret the prevailing pattern of gender specialization in the context of our model in the following way. Initially, high values of $\varepsilon$ due to poor medical technology imply that the only possible equilibrium is one in which women are mostly devoted to home production and men specialize in market work. Subsequent improvements in medical technology reduce the value of $\varepsilon$, thus making ungendered equilibria possible. However, the selffulfilling nature of equilibria for low $\varepsilon$, coupled with the gendered initial conditions, may have contributed to the persistence in gender differences in labor market outcomes and household roles, despite the lack of significant differences in relative productivities. ${ }^{5}$ Technological changes that reduce the complementarity between spouses' hours in the production of the public home good would actually reduce the region in which the equilibrium with lower home hours can occur for given $\varepsilon$. By contrast, a lower value of the utility cost of work would expand this region.

We explore theses issues in Albanesi and Olivetti (2005b).

### 2.4 The Feedback Between Home Hours and Labor Market Outcomes

In this section, we explore the relation between the allocation of home hours across genders and labor market outcomes as predicted by our model. Since our equilibrium analysis concentrates on equilibria with degenerate distribution of home hours, we restrict attention to labor contracts under moral hazard only that satisfy Proposition 1.

We first study the role of the parameter $\Sigma$, which corresponds to the standard deviation of output for given effort. An increase in this parameter makes it harder to infer effort from observed output and increases the severity of the incentive problem. Equation (11) makes clear that effort is decreasing in the value of this parameter, and that this effect is greater for higher levels of home hours. Given that higher $\Sigma$ reduces the optimal level of effort to be implemented, the fraction of incentive pay will also be declining in $\Sigma$. By equation (12), this effect will be stronger at higher home hours, since the marginal cost of effort for the worker is increasing in home hours.

[^4]

Figure 4: Properties of optimal labor contracts for $h_{f}=0.3$ and $h_{m}=0.1$.

Taken together, these properties of labor contracts imply that if women's home hours are higher than men's, the female/male earnings ratio will be declining in $\Sigma$, while the difference in the fraction of incentive pay across male and female workers will be increasing in $\Sigma$. This property is illustrated in figure 4 . The female/male earnings ratio corresponds to the red line (left axis) and the difference in the fraction of incentive pay between male and female workers corresponds to the black line (right axis). $\Sigma$ ranges between 0 and $70 \%$ of worker potential output. Home hours are set to $h_{f}=0.3$ and $h_{m}=0.1$. This corresponds to the average ratio of wives to husbands home hours observed in the PSID for the 1990's. ${ }^{6}$

For $\Sigma=0$, effort is equal to output, there is no moral hazard, and the fraction of incentive pay is zero for both female and male workers. However, since women have higher home hours, firms will offer them a labor contract in which they exert lower effort. Hence, earnings will be lower for female workers. In this example, the earnings ratio is $75 \%$. Positive values of $\Sigma$ exacerbate gender differentials in earnings for given differences in home hours. As $\Sigma$ increases, the earnings ratio drops quite rapidly, while the male-female fraction of incentive pay increases. For $\Sigma$ equal to $50 \%$, the earnings ratio is equal to $60 \%$, while male workers' fraction of incentive pay is 8 percentage points greater than for female workers.

In figure 5, we reproduce this graph for smaller differences in home hours across genders,

[^5]

Figure 5: Properties of optimal labor contracts for $h_{f}=0.15$ and $h_{m}=0.1$.
specifically $h_{f}=0.15$ and $h_{m}=0.10$. The resulting ratio of female to male home hours in this example corresponds to the average female/male ratio of home hours for never married workers in the PSID. The pattern of variation in relation to $\Sigma$ is analogous to that in figure 4. However, the earnings ratio is significantly higher, equal to $93 \%$ for $\Sigma=0$ and dropping to $89 \%$ for $\Sigma=50 \%$. The difference in the fraction of incentive pay across male and female workers only reaches $2 \%$ for $\Sigma=50 \%$.

Taken together these findings translate into the following predictions: i) the female/male earnings ratio should be lower when the incentive problem is more severe and the difference in the fraction of incentive pay across male and female workers should be negatively related to the female/male earnings ratio; ii) these effects are stronger when the differences in home hours between women and men is greater. In the next section, we provide an empirical evaluation of these predictions. We argue that the severity of the incentive problem as captured by $\Sigma$ exhibits substantial variation across occupations. We then explore the variation of the earnings ratio and of the difference in incentive pay across males and females for single and married workers across occupations using Census data. Our model does not have any predictions for single workers. However, we consider single workers in our empirical analysis, since the difference in home hours by gender is much smaller for this sample. This enables us to evaluate the prediction that earnings differentials are related to differences in home hours, given that the relation between earnings, the fraction of incentive pay and home hours is independent from


Figure 6: Properties of optimal labor contracts for $\Sigma=0.31$.
the solution of the household problem.
The dependence of labor market outcomes on home hours delivers specific predictions concerning the relation between earnings ratios, incentive pay and home hours across spouses. Specifically, the wife/husband ratio of home hours should display a negative correlation with the wife/husband earnings ratio and a positive correlation with the husband/wife difference in the fraction of incentive pay. The first property is a direct implication of Problem H1, the households' optimal choice of home hours across spouses. This prediction is common to other efficient models of intra-household allocation. The second property stems from the specific feedback mechanism between home hours and the incentive problem in the labor market that we highlight in our model.

We illustrate these predictions in figure 6. The red line corresponds to the wife/husband earnings ratio (left axis) and the black line to the husband/wife difference in the fraction of incentive pay (right axis). They are plotted against $h_{f} / h_{m}$ for $\Sigma=0.31$. Clearly, the earnings ratio is smaller than 1 only if the wife's home hours are greater than the husband's. Moreover, this ratio is decreasing in the difference in home hours across spouses, while the opposite is true for the fraction of incentive pay. For $h_{f} / h_{m}=3$, the wife/husband earnings ratio is equal to $70 \%$ in this example, while men's fraction of incentive pay is 5 percentage points greater than women's. In the next section, we evaluate these predictions using a sample of married couples from the PSID.

## 3 Connecting the Model with the Evidence

In this section, we draw on different data sources to evaluate the model's predictions.
We use Census 2000 to study gender earnings differentials by marital status across industries and for three broad occupational categories. We find that for married workers gender differentials in earnings are greatest in management and sales occupations across all industries. Evidence on job characteristics suggests that these occupations are likely to be the ones in which incentive problems are most stringent, as discussed below. For the sample of single workers, we do not observe the same pattern of gender differentials. Since the Census does not include information on the fraction of incentive pay, we use PSID data from the late 1990s to examine the variation in the fraction of incentive pay across occupations by gender. We find that there is a strong negative correlation between the male/female difference in the fraction of incentive pay and the female/male earnings ratio across occupations, as predicted by the model

Finally, we study the relation between home hours, earnings and the fraction of incentive pay for husbands and wives using the PSID. We find that the relation between the wife/husband ratio of home hours and the earnings ratio is negative, while the relation between the home hours ratio and the difference in the fraction of incentive pay across husbands and wives is positive as predicted by our model.

### 3.1 Earnings Gaps and Incentive Problems

We use data from the one-percent Integrated Public Use Microsample (IPUMS) of the decennial Census for the year 2000 to document differences in gender earnings differentials across industries for three broad occupational categories: management, sales and production. ${ }^{7}$ We consider this occupational classification based on the notion that incentive problems are more stringent in sales and management relative to production. In our model, the severity of the incentive problem is linked to the degree of uncertainty over the workers' effort for given observable measures of performance, which correspond, respectively, to the parameter $\Sigma$ and output ${ }^{8}$. As shown in MacLeod and Parent (2003), management and sales occupations are characterized by greater workers' autonomy and larger variety of tasks, which increase the severity of the incentive problem. More generally, for management occupations, the uncertainty associated with managers' effort given observable performance measures is related to the complexity of the job. For sales occupations, sales volumes are typically used as a benchmark measure of performance. Yet, these depend to a large degree on variables that are not directly related to

[^6]a sales personnel's effort and may be uncertain. ${ }^{9}$ These considerations are less important for production workers.

Following the Census classification we consider 16 industries: Construction, Manufacturing, Wholesale Trade, Retail Trade, Transportation and Warehousing, Information, Finance and Insurance, Real Estate and Rental/Leasing, Professional, Scientific and Technical Services, Administrative and Support and Waste Management and Remediation Services, Educational Services, Health Care and Social Assistance, Arts, Entertainment and Recreation, Accommodation and Food Services, Other Services (except Public Administration), and Public Administration. ${ }^{10}$ Our sample includes all white individuals between 25 and 54 years of age, who are not in school, do not reside on a farm or live in group quarters. We also exclude the armed forces and restrict attention to those individuals who worked at least 50 weeks in the previous year and who usually work at least 30 hours per week.

For each industry and for each of the three broad occupational categories, we compute the gender gap in earnings for married and never married workers by running median regressions that control for a gender dummy, as well as for human capital variables - age and its square term and education. Hence, the statistics that we report in this section are purged of systematic differences in observable characteristics across genders; they measure residual gender earnings differentials. For the education variable, we group individuals according to four broad educational categories: less than high school, high school completed, some college and college completed. We construct four education dummies based on this categorization. ${ }^{11}$ The omitted dummy variable corresponds to individuals who completed less than twelve years of schooling. The dependent variable is the log of annual earnings. We use total labor earnings in our analysis because this is the data counterpart of the measure of total labor compensation in our model. However, one could argue that this is not the appropriate measure of labor compensation when making gender comparisons, since women tend to work fewer hours than men on the labor market. Hence, our analysis could be confounding gender differences in market hours and in hourly compensation. This concern is attenuated by the fact that we only consider individuals that usually work at least 30 hours per week and who were employed for at least 50 weeks in the previous year. ${ }^{12}$ This sample selection criterion considerably reduces the

[^7]variation in the number of market hours within and between gender groups. ${ }^{13}$ The summary statistics for our sample are reported in Appendix.

Table 5 reports the female/male ratio of median earnings for full-time year-round workers for the three occupational categories by industry and by marital status. The first column refers to management occupations, the second to sales occupations and the third to production occupations. In each column, we report the statistics separately for married workers and for never married workers.

Our findings suggest that there is a considerable variation in the female/male earnings ratio across industries, and across the three occupational categories within each industry, even after controlling for gender difference in human capital characteristics. Moreover, the patterns of variation differ substantially by marital status. The first row of the table, displays the average female/male ratio of median earnings across all industries for each occupational category. For married workers, the ratio is lowest in management and sales occupation, for the never married instead it is lowest in production occupations. The median married woman in sales earns, on average across all industries, just 69 percent of what the median married man earns, while in management occupations she earns 72 percent of the median married man's total earnings. The highest value of the gender earnings ratio for married workers is in production occupations, where the median woman earns 80 percent of median male earnings. For the sample of never married workers the ranking of earnings ratios across occupations is reversed and gender differentials are smaller especially for management and sales. The median single woman earns 92 percent in sales and 94 percent in management of the total labor compensation earned by the median man in the corresponding occupation. For this group, production occupations display the lowest ratio, equal to 83 percent. As a result, the difference in gender earnings ratio of married relative to never married workers is substantial in sales and management occupation, approximately 20 percentage points. By contrast, gender earnings ratios do not vary significantly by marital status for production workers. That is, as predicted by our theory, we observe the largest earnings penalty for married women in the occupations where the incentive problem is most severe.

As shown in the remaining of the table, the variation of female/male median earnings ratios across industries is striking. For married workers, it ranges from around 55 percent for sales occupations (in retail trade, the educational service industry and the accommodation and food service industry), to 83 percent for production occupations in a variety of industries. ${ }^{14}$ For
for total annual wages and salaries, WKSWORK1 for weeks worked and UHRSWORK for usual hours worked per week. The three labor market variables report information for the year preceding the Census survey.
${ }^{13}$ We have also conducted our analysis using the log of hourly earnings as a dependent variable. Our findings in this case are consistent with the ones reported in this section, and results are available upon request. However, we do not report them here, since the hourly wage does not have a model counterpart, given that in our framework, labor contracts are non-linear in hours. We have also performed the analysis for the the sample of workers with children. The pattern of ranking of the gender earnings ratios by marital status across occupational categories and industries is identical to the one reported in the paper for the overall sample.
We have also experimented with different sample inclusion rules by considering all racial groups and by expanding the sample to include all individuals aged 16 to 64 . In all the cases the results of our analysis are quantitatively similar to the ones reported in the paper.
${ }^{14}$ The industries are: Construction, Transportation and Warehousing, Finance and Insurance, Real Estate
never married workers, gender differences in earnings are much smaller. The gender earnings ratio ranges from 70 percent, for sales occupations in the Health Care and Social Assistance industry, to more than a 100 percent in management and sales occupations, in a number of industries.
and Rental/Leasing, Professional, Scientific and Technical Services, Educational Services, Arts, Entertainment and Public Administration. Interestingly, the ratio is also equal to 0.83 for sales workers in the information industry.

Table 5: Gender differences in earnings across industries, occupation, and marital status (Full-time, year-round workers, $\%$ female/male median earnings ratios)

|  | Management |  | Sales |  | Production |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | married | single | married | single | married | single |
| Average across all industries | $\mathbf{7 2}$ | $\mathbf{9 4}$ | $\mathbf{6 9}$ | $\mathbf{9 2}$ | $\mathbf{8 0}$ | $\mathbf{8 3}$ |
| Accommodation and Food |  |  |  |  |  |  |
| Administrative, Support, Waste mgmt | 76 | 90 | 65 | 99 | 80 | 84 |
| Arts, Entertainment \& Recreation | 77 | 104 | 78 | 87 | 81 | 85 |
| Construction | 67 | 75 | 75 | 77 | 83 | 87 |
| Educational Services | 81 | 93 | 82 | 95 | 83 | 87 |
| Finance and Insurance | 65 | 88 | 64 | 87 | 83 | 87 |
| Health Care \& Social Assistance | 70 | 91 | 57 | 70 | 77 | 80 |
| Information | 73 | 91 | 83 | 102 | 82 | 86 |
| Manufacturing | 76 | 90 | 71 | 93 | 66 | 67 |
| Other Services (no Public Adm.) | 72 | 101 | 64 | 78 | 76 | 79 |
| Profess,Scientific\&Tech. Services | 72 | 90 | 70 | 90 | 83 | 86 |
| Public Administration | 80 | 103 | 81 | 129 | 83 | 87 |
| Real Estate and Rental/Leasing | 66 | 103 | 64 | 113 | 83 | 87 |
| Retail Trade | 65 | 101 | 56 | 83 | 72 | 74 |
| Transportation and Warehousing | 71 | 94 | 64 | 85 | 83 | 87 |
| Wholesale Trade | 70 | 101 | 75 | 92 | 79 | 83 |
|  |  |  |  |  | 87 |  |

We also find interesting systematic patterns in the ranking of gender earnings ratios over the three occupational categories across all the industries. In particular, for married workers, the female/male median earnings ratio is lowest for sales occupations in 9 of the 16 industries - $56 \%$ of the cases. In 6 of the 7 remaining industries ( $38 \%$ of the cases) the occupation that display the lowest ratio of female/male median earnings is management. On the other end, when we look at the frequency with which the gender wage ratio is highest across all industries, we find the opposite pattern. Married workers in production occupations display the largest ratio in 14 industries - $88 \%$ of the cases. Instead, we find that for single workers the gender earnings ratio is largest in management occupations in 8 of the 16 industries, $50 \%$ of the cases, and in sales occupations in 7 of the remaining 8 industries, $44 \%$ of the cases. In most industries, the ratio is lowest in production occupations - $69 \%$ of the cases.

These patterns reinforce the observation that, for married workers, gender earnings ratios are systematically lower in occupations where the incentive problem is most stringent. This is not the case for single workers. We interpret this evidence as supportive of our mechanism. In the next section, we use PSID data to further our analysis.

### 3.2 Fraction of incentive pay, earnings gaps and home hours

We use PSID data for the late 1990s to evaluate our model's predictions that the male/female difference in the fraction of incentive pay should be negatively related to the female/male earnings ratio across industries/occupations. As we did with the Census data, we select our sample to include all white men and women between 25 and 54 years of age who are not in school, who are not in the armed forces, and who worked at least 30 hours per week and 50 weeks per year. Summary statistics for this sample are reported in Appendix. In our analysis, we study gender earnings ratios at the occupation/industry level. This level of disaggregation requires a larger sample size than the one available in each wave of the PSID. Hence, we do not exploit the panel dimension of the data but simply pool together all the individuals in the 1994 to 2001 waves. The resulting statistics can be interpreted as medium run averages of the relevant variables. ${ }^{15}$

To analyze the relation between the fraction of incentive pay and gender earnings differentials, we use the information on bonuses and commissions. We calculate the fraction of incentive pay as the ratio of bonuses and commissions to labor income, defined as wages and salaries, plus bonuses and commissions. ${ }^{16}$

The PSID coding of occupations differs from the one available from the Census 2000. In our analysis, we construct occupational categories that are similar to the ones used for our Census analysis. We consider the following categories: management positions in administration, management positions in banking, finance and in the clerical sector, lower level management occupations, professional occupations (engineers,architects, lawyers, and medical doctors), technical occupations (in the health sector, engineering, and social sciences), occupations in community/social services, social scientists and university professors, teachers other than college professors, occupations in arts and entertainment, design, sports and the media, sales occupations, clerical occupations, craftsmen, operatives, physical laborers, in services excluding private households. ${ }^{17}$

We find a strong negative correlation between the female/male earnings ratio and the male/female difference in the fraction of incentive pay. The correlation coefficient is -0.55 and it is significant at the five percent level. Figure 7 displays a scatter plot of these two variables. Consistent with our Census findings, sales and management occupations in banking, finance and in the clerical sector are characterized by the lowest female/male earnings ratio and the highest male/female difference in the fraction of incentive pay. ${ }^{18}$ According to our

[^8]

Figure 7: Correlation between the M-F difference in the fraction of incentive pay and the F/M earnings ratio.
model, this relation should be stronger for married workers. However, the PSID only reports information on bonuses and commissions for household heads, that are predominantly married males or single women. ${ }^{19}$ Since observations on married women are extremely limited, we cannot condition on marital status for this analysis.

We also use the PSID data on bonuses and commissions to corroborate our findings on the severity of the incentive problem and gender earnings differentials discussed in section 3.1.

[^9]

Figure 8: Correlation between the F/M earnings ratio and the aggregate fraction of incentive pay.

Figure 8 reports a scatter plot of the aggregate fraction of incentive pay and the female/male earnings ratio across occupations. The correlation between these two variables is -0.58 , significant at the five percent level. Consistent with MacLeod and Parent (2003), the occupations where the incentive problem is more severe exhibit a higher fraction of incentive pay. These same occupations also have low female/male earnings ratios.

It is important to note that, our female sample is disproportionately composed of single women. Hence, the average female shares of incentive pay by occupation are likely to provide an upper bound on the actual statistics for the entire female population. As a consequence, the male-female differences in incentive pay shares we report likely underestimate the actual difference observed in the data, especially so for sales and management occupations where the incentive problem is most severe.

There is a large variation in the percentage of female workers across the occupations consid-


Figure 9: Correlation between the percentage of female workers and the $\mathrm{F} / \mathrm{M}$ earnings ratio.
ered in this analysis. This raises the question of whether in fact it is occupational sorting that fully explains the gender differential in earnings and incentive pay across occupations. One could think of two reasons why occupational choice might depend on gender- comparative advantage or differences in preferences. ${ }^{20}$ Occupational sorting based on comparative advantage would predict that, if workers are paid their marginal product, the female/male earnings ratio should be higher in occupations in which women have a comparative advantage, thus giving rise to a positive correlation between the percentage of female workers and the female/male earnings ratio across occupations. This prediction is strongly rejected by the data. As shown in Figure 9, there is no clear relation between these two variables and their correlation is not significantly different from zero. ${ }^{21}$

[^10]Survey and experimental evidence seems to suggest that women are more risk averse than $m^{22}$. This observation could also provide the basis for occupational sorting of workers by gender. In our model, absent any differences in home hours, if risk aversion varies across workers and is not observed, firms will offer incentive compatible contracts that screen workers along this dimension. If women are systematically more risk averse, they will choose occupations with a lower fraction of incentive pay. ${ }^{23}$ This prediction is also strongly rejected by the data. In Figure 10, we plot the aggregate fraction of incentive pay against the percentage of female workers. There is no clear relation between these two variables and their correlation is not significantly different from zero.

Finally, our model is based on the assumption that households make efficient decisions on the allocation of home hours, which implies that the spouse with higher earnings will contribute fewer hours to the production of the home public good. This prediction is common to other efficient models of intra-household allocation. However, our model also delivers an additional prediction. The structure of the optimal labor contract implies that the spouse with highest earnings also receives the largest fraction of incentive pay. This gives rise to the prediction that the spouse with the highest fraction of incentive pay will contribute fewer hours to home production. In a cross-section of married couples, these predictions translate into a negative correlation between the wife/husband ratio of home hours and the wife/husband ratio of earnings, and a positive correlation between the hours ratio and the husband-wife difference in the fraction of incentive pay. We study these correlations across a sample of married couples using the PSID.

The ideal data set to test these implications would include information on home hours, market hours, labor earnings and the structure of compensation for both spouses for an ample cross-section of married couples. While being far from ideal, the PSID is one of the few data set that allows us to move in this direction. In particular, we have information on home hours, market hours and earnings of both spouses. ${ }^{24}$ However, we only have information on bonuses and commissions for household's heads. In order to recover the information for spouses, we use the available information on his/her occupation jointly with the gender-specific average shares of incentive pay by occupational categories. For each spouse we impute a value of $\tilde{w}$ that is equal to the fraction of incentive pay received by the average worker of the same gender in the

[^11]

Figure 10: Correlation between the aggregate fraction of incentive pay and the percentage of female workers.
same occupation. Since in our sample approximately 80 percent of household heads report a zero amount of bonuses and commissions, we also impute the heads' fraction of incentive pay by occupation. We then compute $\tilde{w}_{m}-\tilde{w}_{f}$ for each couple as the difference of the imputed incentive pay shares of husband and wife. ${ }^{25}$

We are interested in studying the aggregate correlation between spousal home hours, earnings and incentive pay shares across a cross-section of married couples that is as homogeneous as possible relative to their age, presence of young children in the household, and labor market attachment of both spouses. This sample inclusion rule aims to minimize the impact of additional factors, such as race, cohort and wife's labor market attachment, that could be driving the cross-sectional correlations while maintaining a reasonable sample size. For this reason, we consider a sample that includes male-headed married couples where both husband and wife are white, the head of the household is between 25 and 44 years old and both spouses are full-time year-round workers (they both work at least 30 hours per week and at least 50 weeks per year). We only include couples that report information on all the relevant variables for both partners.

For this sample of workers, there is a substantial variation in the number of repeated observations for each couple across waves of the PSID (for example only a third of the married couples that we observe in 1994 are still in the sample in 1995.) For each couple the entry/exit behavior from our sample can be due to a variety of reasons: divorce, attrition from the overall PSID sample, a change in the employment status of one of the two partners or lack of information on one of the variables of interest for our analysis. Hence, in this case, it is not meaningful to either pool together all the waves of the PSID or to take averages over the set of repeated observations across waves. Each cross-section of data that satisfies the sample selection criteria only includes a small number of observations in each wave. In order to maximize the size of the cross-section, we include one data point for each married couple, corresponding to the first year in which the couple satisfies our sample selection criteria for waves 1994-2001. ${ }^{26}$ Summary statistics for this sample are presented in Appendix. We report the results of our analysis in Table 6.

[^12]Table 6
Home hours, earnings and incentive pay across a sample of married couples

| $\operatorname{corr}\left(\frac{w_{f}}{w_{m}}, \frac{h_{f}}{h_{m}}\right)$ | all | with kids |
| :--- | :---: | :---: |
|  | $\mathbf{- 0 . 2 7}$ | $\mathbf{- 0 . 2 7}$ |
| $\operatorname{corr}\left(\tilde{w}_{m}-\tilde{w}_{f}, \frac{h_{f}}{h_{m}}\right)$ | 0.03 | $(0.000)$ |
|  | $(0.582)$ | $(0.007)$ |
|  |  |  |
| Number of couples | 300 | 167 |
| p-values in parentheses. |  |  |

Entries in column 1 refer to the sample of married couples. Column 2 reports correlation coefficients for the sample of married couples with children less than 13 years old. Our sample consists of 300 couples of which 167 have children. The data confirms the model's prediction that wife/husband earnings and home hours ratios are negatively correlated across all households. This is true irrespective of the presence of children. The correlation coefficient is -0.27 and significant at the 1 percent level in both samples. On the other hand, the intensity and significance of the positive relation between the difference in incentive pay shares across spouses and the wife/husband ratio of home hours depends on the presence of children in the household. We find that for the overall sample there is a positive but small and not significant correlation across these two variables, whereas for the sample of married couples with children, the correlation coefficient is equal to 0.21 and it is significant at the 1 percent level.

## 4 Concluding Remarks

The data available through the Census and the PSID enables us to provide evidence in support of the mechanisms for the determination of gender earnings differentials embedded in our model, as well as for its implications for the interaction with household variables. However, neither data set is ideal. Specifically, the Census does not include information on home hours or on the fraction of incentive pay. This information is included in the PSID, although this data set does not include information on the structure of earnings for both husbands and wives. To fully explore these predictions empirically, one would need a data set that includes information on the structure of earnings for a broad class of sectors and jobs, as well as, detailed household level information. To our knowledge, this information is not available for the U.S. ${ }^{27}$

Our model can also be used to think about how different sources of technological progress might impact gender differences in labor market outcomes and the household division of labor. For example, Galor and Weil (1996) develop a model in which skill bias technological change contributes to a transformation of women's role in home and market production by influencing their fertility and labor choices. On the other hand, Greenwood, Seshadri and Yorukoglu

[^13](2005) have emphasized that new consumer durables introduced in the twentieth century acted as "engines of liberation" for women's time. The consumer durable revolution obviously also influenced the home production technology, in particular the degree of complementarity in spousal home hours. Similarly, improvements in medical technology that reduced the physical stress associated with pregnancy, as well as the introduction of breast milk substitutes, would determine a reduction in women's comparative advantage in home production. ${ }^{28}$ In Albanesi and Olivetti (2005), we use a version of our model that allows for participation decisions to study the effect of these developments on women's employment and earnings and on the division of labor within the household.

Finally, in our framework, there are no efficiency losses associated with gender discrimination, given that all workers are equally productive in market work. An extension of the model that allows for a non-degenerate distribution of individual productivities, symmetric across genders, could address this issue. In a gendered equilibrium, female workers with high productivity may be offered contracts in which they exert low effort. This would generate misallocation costs associated with gender discrimination.

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## 5 Appendix

### 5.1 Labor Contracts

Proof of Proposition 1. The first order necessary conditions for Problem 1 at an interior solution are:

$$
\begin{gather*}
f^{\prime}(e)-v_{e}(h, e)+\mu\left(\tilde{w} f^{\prime \prime}(e)-v_{e e}(h, e)\right)=0,  \tag{33}\\
-\sigma \Sigma^{2} \tilde{w}+\mu f^{\prime}(e)=0 . \tag{34}
\end{gather*}
$$

To solve for effort, substitute $\mu=\frac{\sigma \Sigma^{2} \tilde{w}}{f^{\prime}(e)}$ and $\tilde{w}=\frac{v_{e}(h, e)}{f^{\prime}(e)}$, into (33) to obtain an equation in $e$ :

$$
\begin{equation*}
f^{\prime}(e)-v_{e}(h, e)+\frac{\sigma \Sigma^{2} \frac{v_{e}(h, e)}{f^{\prime}(e)}}{f^{\prime}(e)}\left(\frac{v_{e}(h, e)}{f^{\prime}(e)} f^{\prime \prime}(e)-v_{e e}(h, e)\right)=0 . \tag{35}
\end{equation*}
$$

Assuming (9)-(10), (35) simplifies to:

$$
1-(\psi+h) e-\sigma \Sigma^{2}(\psi+h)^{2} e=0
$$

which implies (11) and (12). Imposing zero profits on firms, delivers $E w^{*}(h)=f\left(e^{*}(h)\right)$. Then:

$$
\begin{aligned}
E w^{* \prime}(h) & =\frac{-\left(1+2 \sigma \Sigma^{2}(\psi+h)\right)}{\left((\psi+h)+\sigma \Sigma^{2}(\psi+h)^{2}\right)^{2}}<0, \\
E w^{* \prime \prime}(h) & =2 \frac{1+3 \sigma \Sigma^{2}(\psi+h)+3\left[\sigma \Sigma^{2}(\psi+h)\right]^{2}}{\left((\psi+h)+\sigma \Sigma^{2}(\psi+h)^{2}\right)^{3}}>0 .
\end{aligned}
$$

Proof of Proposition 2. The Lagrangian for Problem F2 is:

$$
\begin{align*}
& \max _{\left\{e_{j}, \tilde{w}_{j}\right\}_{j=L, H}, T_{L}, T_{H}} 0.5 \sum_{j}\left(f\left(e_{j}\right)-v\left(h_{j}, e_{j}\right)-\sigma \Sigma^{2} \frac{\tilde{w}_{j}^{2}}{2}-T_{j}\right)  \tag{36}\\
& -\sum_{j} \mu_{j}\left(\tilde{w}_{j} f^{\prime}\left(e_{j}\right)-v_{e}\left(h_{j}, e_{j}\right)\right) \\
& -\sum_{j, i \neq j} \lambda_{j}\left[f\left(\hat{e}_{i}\right) \tilde{w}_{i}-v\left(h_{j}, \hat{e}_{i}\right)-\sigma \Sigma^{2} \frac{\tilde{w}_{i}^{2}}{2}+T_{i}-f\left(e_{j}\right) \tilde{w}_{j}+v\left(h_{j}, e_{j}\right)+\sigma \Sigma^{2} \frac{\tilde{w}_{j}^{2}}{2}-T_{j}\right] \\
& -\sum_{j, i \neq j} \xi_{j}\left(f^{\prime}\left(\hat{e}_{i}\right) \tilde{w}_{i}-v_{e}\left(h_{j}, \hat{e}_{i}\right)\right),
\end{align*}
$$

The first order necessary conditions for problem 2, substituting in the specific function forms for $f$ and $v$, are:

$$
\begin{gather*}
0.5\left(1-\left(\psi+h_{j}\right) e_{j}\right)+\mu_{j}\left(\psi+h_{j}\right)-\lambda_{j}\left(-\tilde{w}_{j}+\left(\psi+h_{j}\right) e_{j}\right)=0  \tag{37}\\
-0.5 \sigma \Sigma^{2} \tilde{w}_{j}-\mu_{j}-\lambda_{j}\left(-e_{j}+\sigma \Sigma^{2} \tilde{w}_{j}\right)-\lambda_{i}\left(\hat{e}_{j}-\sigma \Sigma^{2} \tilde{w}_{j}\right)=0  \tag{38}\\
-\lambda_{j}\left(\tilde{w}_{i}-\left(\psi+h_{j}\right) \hat{e}_{i}\right)+\xi_{j}\left(\psi+h_{j}\right)=0  \tag{39}\\
-0.5+\lambda_{j}-\lambda_{i} \leq 0, \text { with equality for } T_{j}>0  \tag{40}\\
\tilde{w}_{j}-\left(\psi+h_{j}\right) e_{j}=0,  \tag{41}\\
\hat{e}_{i} \tilde{w}_{i}-\left(\psi+h_{j}\right) \frac{\hat{e}_{i}^{2}}{2}-\sigma \Sigma^{2} \frac{\tilde{w}_{i}^{2}}{2}+T_{i}-e_{j} \tilde{w}_{j}+\left(\psi+h_{j}\right) \frac{e_{j}^{2}}{2}+\sigma \Sigma^{2} \frac{\tilde{w}_{j}^{2}}{2}-T_{j} \leq 0,  \tag{42}\\
\lambda_{j}\left[\hat{e}_{i} \tilde{w}_{i}-\left(\psi+h_{j}\right) \frac{\hat{e}_{i}^{2}}{2}-\sigma \Sigma^{2} \frac{\tilde{w}_{i}^{2}}{2}+T_{i}-e_{j} \tilde{w}_{j}+\left(\psi+h_{j}\right) \frac{e_{j}^{2}}{2}+\sigma \Sigma^{2} \frac{\tilde{w}_{j}^{2}}{2}-T_{j}\right]=0, \lambda_{j} \geq(\mathbb{Q} 3) \\
\tilde{w}_{i}-\left(\psi+h_{j}\right) \hat{e}_{i}=0, \tag{44}
\end{gather*}
$$

for $j=L, H$ and $i \neq j$. By $\left(\hat{e}_{i}\right), \xi_{j}=0$ for $L, H$. Only one adverse selection incentive compatibility constraint can bind at any given time. There are three possible cases. A) $\lambda_{L}>0$, $T_{L}>0, T_{H}=0$ and $\lambda_{H}=0$. Then, solving equations (37)-(44) yields:

$$
\begin{equation*}
\mu_{L}=0.5\left(e_{L}-\frac{1}{\left(\psi+h_{L}\right)}\right), \mu_{H}=0.5\left(e_{H}-\frac{1}{\left(\psi+h_{H}\right)}\right), \tag{j}
\end{equation*}
$$

and (16)-(18). To verify that this is a solution, we check that $T_{L}$ is indeed strictly positive. Substituting:

$$
\begin{aligned}
\tilde{w}_{H}-\tilde{w}_{L} & =\frac{\left(\psi+h_{L}\right)}{\left(2 \psi+h_{H}+h_{L}\right)}-\frac{1}{\left(\psi+h_{L}\right) 2 \sigma \Sigma^{2}} \\
& =\frac{2 \sigma \Sigma^{2}\left(\psi+h_{L}\right)-\frac{\left(\psi+h_{H}\right)}{\left(\psi+h_{L}\right)}-1}{\left(2 \psi+h_{H}+h_{L}\right) 2 \sigma \Sigma^{2}}>0
\end{aligned}
$$

Hence, if $1<\sigma \Sigma^{2}\left(\psi+h_{L}\right)<\left(\frac{\left(\psi+h_{H}\right)}{\left(\psi+h_{L}\right)}+1\right) 0.5, T_{L}$ is positive.
B) $T_{H}>0, \lambda_{H}>0$ and $T_{L}=\lambda_{L}=0$. Solving equations (37)-(44) yields:

$$
\begin{equation*}
0.5 \frac{\left(\tilde{w}_{L}-1\right)}{\left(\psi+h_{L}\right)}=\mu_{L}, 0.5 \frac{\left(\tilde{w}_{H}-1\right)}{\left(\psi+h_{H}\right)}=\mu_{H}, \tag{j}
\end{equation*}
$$

and (19)-(21). Since:

$$
\begin{aligned}
\tilde{w}_{L}-\tilde{w}_{H} & =\frac{\psi+h_{H}}{2 \psi+h_{H}+h_{L}}-\frac{1}{2 \sigma \Sigma^{2}\left(\psi+h_{H}\right)} \\
& =\frac{2 \sigma \Sigma^{2}\left(\psi+h_{H}\right)-\frac{2 \psi+h_{H}+h_{L}}{\left(\psi+h_{H}\right)}}{\left(2 \psi+h_{H}+h_{L}\right) 2 \sigma \Sigma^{2}}>0
\end{aligned}
$$

if and only if $1>\sigma \Sigma^{2}\left(\psi+h_{H}\right)>0.5 \frac{2 \psi+h_{H}+h_{L}}{\left(\psi+h_{H}\right)}$, then $T_{H}>0$.
C) $\lambda_{L}=\lambda_{H}=0$ and $T_{L}=T_{H}=0$. When the adverse selection incentive compatibility constraint is not binding, the solution to the first order conditions is:

$$
\begin{gather*}
0.5 \frac{\left(1-\left(\psi+h_{j}\right) e_{j}\right)}{\left(\psi+h_{j}\right)}=-\mu_{j}  \tag{j}\\
\tilde{w}_{j}=\frac{1}{1+\left(\psi+h_{j}\right) \sigma \Sigma^{2}}, e_{j}=\frac{1}{\psi+h_{j}+\left(\psi+h_{j}\right)^{2} \sigma \Sigma^{2}} \tag{w}
\end{gather*}
$$

for $j=H, L$. This delivers (22). To verify that the adverse selection incentive compatibility constraints are indeed not binding, substitute the expressions for $\tilde{w}_{j}$ and $e_{j}$ above into the constraints, to yield:

$$
\begin{aligned}
{\left[\frac{1}{\left(\psi+h_{H}\right)}-\sigma \Sigma^{2}\right]\left(\tilde{w}_{L}^{2}-\tilde{w}_{H}^{2}\right) } & \leq 0 \\
{\left[\left(\tilde{w}_{H}\right)^{2}-\left(\tilde{w}_{L}\right)^{2}\right]\left(\frac{1}{\left(\psi+h_{L}\right)}-\sigma \Sigma^{2}\right) } & \leq 0
\end{aligned}
$$

Since $\tilde{w}_{L}>\tilde{w}_{H}$, the two inequalities are satified for $\frac{1}{\left(\psi+h_{H}\right)}-\sigma \Sigma^{2} \leq 0$ and $\frac{1}{\left(\psi+h_{L}\right)}-\sigma \Sigma^{2} \geq 0$.

### 5.2 Household Problem

Let $M C^{H}(\mathcal{C})=\partial C^{H}(H ; \mathcal{C}) / \partial H$ be the marginal cost of $H$, which is independent of $H$ given that $H(\cdot)$ is homogeneous of degree 1 . Specifically, by (27)-(28):

$$
M C^{H}(\mathcal{C})=\left[(1+\varepsilon)\left(\frac{E w^{\prime}\left(h_{f}\right)}{(1+\varepsilon)}\right)^{\zeta /(\zeta-1)}+\left(E w^{\prime}\left(h_{m}\right)\right)^{\zeta /(\zeta-1)}\right]^{(\zeta-1) / \zeta}
$$

The second cost minimization problem for the household can be written as:

$$
\begin{equation*}
C^{G}(\bar{G} ; \mathcal{C})=\min _{k, H \geq 0} k+M C^{H}(\mathcal{C}) H \tag{ProblemH2}
\end{equation*}
$$

subject to

$$
H^{\delta} k^{1-\delta} \geq \bar{G}
$$

for $\bar{G}>0$. The first order necessary conditions for this problem imply:

$$
\begin{aligned}
k & =\left(\frac{1}{M C^{H}(\mathcal{C})} \frac{\delta}{1-\delta}\right)^{-\delta} G \\
\frac{H}{k} & =\left(\frac{1}{M C^{H}(\mathcal{C})} \frac{\delta}{1-\delta}\right)
\end{aligned}
$$

These equations define $k$ and $H$ as a function of $G$. We can then define $M C^{G}(\mathcal{C})=\partial C^{G}(G ; \mathcal{C}) / \partial G$, with:

$$
M C^{G}(\mathcal{C})=(1-\delta)^{-(1-\delta)}\left(\frac{\delta}{M C^{H}(\mathcal{C})}\right)^{-\delta}
$$

Problems H1 and H2 are convex minimization problems. Hence, first order necessary conditions are sufficient and the optima will be attained by the respective policy functions. Combining the solutions to problem H 1 and H 2 , we can define the functions $\hat{h}_{f}(G ; \mathcal{C}), \hat{h}_{m}(G ; \mathcal{C})$ that express the optimal intra-household allocation of home hours as a function of the level of public home consumption. The last step of the household problem is to optimize (??) by choice of $G, s_{f}$ and $s_{m}$ subject to $s_{f}+s_{m}+M C^{G}(\mathcal{C}) G \leq a$, since we consider equilibria with $\Pi=0$. The solution to this problem gives rise to the policy functions: $s_{i}(a ; \mathcal{C})$, and $G(a ; \mathcal{C})$, and recursively to $h_{i}(a ; \mathcal{C})=\hat{h}_{m}(G(a ; \mathcal{C}) ; \mathcal{C})$ for $i=f, m$.

### 5.3 Equilibrium

## Proof of Lemma 4

To prove the first result, note that if all households are homogeneous with respect to $\theta$ and $a$, the allocation of home hours is the same for all households and all firms have the same beleifs in a symmetric equilibrium. Hence, in a gendered equilibrium, home hours will be constant across wifes and husbands, leading to two values of home hours in the population with $0<h_{L}<h_{H}$. In an ungendered equilibrium, households are indifferent over the distribution of home home hours across spouses and they randomize. The randomization strategy will be the same across all households leading to at most two values of home hours in the population. To prove the second result, note that a non-degenerate distribution of home hours occurs when households are indifferent over the allocation of home hours across spouses. Suppose that the distribution of home hours is non-degenerate and the equilibrium is gendered, so that $\pi_{m}\left(h_{j}\right) \neq \pi_{f}\left(h_{j}\right)$ for $j=L, M$ for some $0<h_{L}<h_{H}$. Then, wives and husbands will not be facing the same menu of labor contracts and randomization will not be optimal and the distribution of home hours will be degenerate. Contradiction. Hence, if the distribution of home hours is non-degenerate, the equilibrium is ungendered. Now, suppose that in an ungendered equilibrium, the households' randomization strategy does not assign $h_{H}$ and $h_{L}$ with equal probability to the wife and the husband, so that $\operatorname{Pr}\left(h_{L}=h_{f}\right) \neq \operatorname{Pr}\left(h_{L}=h_{m}\right)$. By the law of conditional expectations, $\operatorname{Pr}\left(h_{L}=h_{i}\right)=\operatorname{Pr}(i) \pi_{i}\left(h_{L}\right)$ for $i=f, m$. Since $\operatorname{Pr}(i)=0.5$ for $i=f, m, \operatorname{Pr}\left(h_{L}=h_{f}\right) \neq \operatorname{Pr}\left(h_{L}=h_{m}\right)$ implies $\pi_{f}\left(h_{L}\right) \neq \pi_{m}\left(h_{L}\right)$. Contradiction. Then, in any equilibrium with a non-degenerate distribution of home hours, $\pi_{f}\left(h_{L}\right)=\pi_{m}\left(h_{L}\right)=0.5$ for $j=L, H$.

## Proof of Proposition 5

If firms' beleifs satisfy $\operatorname{Pr}\left(h_{f}<h_{m}\right)=1$, then $\max E w_{f}(h)>\max E w_{m}(h)$. If such an equilibrium exists, $h_{f}<h_{m}$ and the distribution of home hours will be given by $\pi_{f}\left(h_{H}\right)=0$ and $\pi_{m}\left(h_{L}\right)=0$, by Lemma 4 . Hence, equilibrium labor contracts will satisfy proposition 1 . Such an equilibrium exists, if Problem H1 has a solution with $h_{f} / h_{m}<1$. Such an equilibrium is unique, if this is the unique solution to Problem H1. Note that (27) and (28) can be rewritten as:

$$
\begin{gather*}
(x)^{1-\zeta}=\frac{E w^{\prime}\left(h_{m}\right)}{E w^{\prime}\left(x h_{m}\right)},  \tag{45}\\
\frac{H}{h_{m}}=\left[x^{\zeta}+1\right]^{1 / \zeta} \tag{46}
\end{gather*}
$$

where $x=h_{f} / h_{m}$. Equation (45) implicitely defines $x$ as a function of $h_{m}$, while (46) defines $h_{m}$ as a function of $H$. The following lemma characterizes the solutions to (45).

Lemma 7 If labor contracts satisfy proposition 1, equation (45) generically has two solutions, $x_{1}\left(h_{m}\right)=1$ and $x_{2}\left(h_{m}\right)<1$, with $\lim _{h_{m} \rightarrow 0} x_{2}\left(h_{m}\right)=1$ and $\lim _{h_{m} \rightarrow \infty} x_{2}\left(h_{m}\right)=0$. Moreover, equation (46) has a unique finite solution $h_{m}^{i}$ for each branch $x_{i}\left(h_{m}\right)$ for $i=1,2$, with $h_{m}^{1}>h_{m}^{2}$ for given $H$.

Proof. The left hand side of equation (45) is increasing and concave in $x$ and crosses the fourtyfive degree line at $x=0$ and $x=1$. Given that firms' beleifs over the distribution of home hours in the population are degenerate, the contracts offered to female and male workers are described by proposition 1. It follows that $\frac{E w^{\prime}\left(h_{m}\right)}{E w^{\prime}\left(h_{m}\right)}=1$, so that one solution to (45) is $x_{1}\left(h_{m}\right)=1$. Since, $E^{\prime} w^{* \prime}(h)<0$ and $E w^{* \prime \prime}(h)>0$, for all $0<x<1, \frac{E w^{\prime}\left(h_{m}\right)}{E w^{\prime}\left(x h_{m}\right)}<$ 1. Moreover, the right hand side of (45) is continuous and increasing in $x$, since the slope of this expression as a function of $x$, given by $h_{m} \frac{E w^{\prime}\left(h_{m}\right)}{E w^{\prime}\left(x h_{m}\right)} \frac{-E w^{\prime \prime}\left(x h_{m}\right)}{E w^{\prime}\left(x h_{m}\right)}$, is positive. Since by (12) and (11), $\lim _{x \rightarrow 0} E w^{\prime}\left(x h_{m}\right)<0$ and $E w^{\prime}\left(h_{m}\right) / \lim _{x \rightarrow 0} E w^{\prime}\left(x h_{m}\right)<1$, there must be another crossing at $x_{2}\left(h_{m}\right)<1$. The convexity of $E w^{\prime}(h)$, implies that $x_{2}\left(h_{m}\right)$ is decreasing in $h_{m}$. In addition, proposition 1 implies $w^{* \prime}(0)$ is finite and $\lim _{h_{m} \rightarrow \infty} E w^{\prime}(h)=0$. Then, $\lim _{h_{m} \rightarrow 0} x_{2}\left(h_{m}\right)=1$ and $\lim _{h_{m} \rightarrow \infty} x_{2}\left(h_{m}\right)=0$ follows. By $(46), h_{m}^{1}(H)=H 2^{-1 / \zeta}$. Since $x_{2}\left(h_{m}\right)$ is decreasing in $h_{m}, \lim _{h_{m} \rightarrow 0} x_{2}\left(h_{m}\right)=1$ and $\lim _{h_{m} \rightarrow \infty} x_{2}\left(h_{m}\right)=0$, the right hand side of equation (46) evaluated at $x_{2}\left(h_{m}\right)$ is bounded below 1 , and bounded above by $2^{-1 / \zeta}$. Since $\lim _{h_{m} \rightarrow 0} H / h_{m}=\infty$ and $\lim _{h_{m} \rightarrow \infty} H / h_{m}=0$, (45) has a unique finite solution when evaluated at $x_{2}\left(h_{m}\right), h_{m}^{2}(H)>0$.

By lemma 7, generically there exist two zeros for equation (45), $x_{1}=1$ and $x_{2} \in(0,1)$. However, under max $E w_{f}(h)<\max E w_{m}(h), x_{1}=1$ is not optimal for Problem H1. Hence, the unique solution to problem H1 is $0<h_{L}=h_{f}=x_{2} h_{m}=x_{2} h_{H}$ for $h_{m}$ that solves (28) and $H$ that solves Problem H2. This solution is constant for all households. Hence, the resulting distribution of home hours is $\pi_{f}\left(h_{H}\right)=0$ and $\pi_{m}\left(h_{L}\right)=0$, consistent with firms' beliefs.

If firms' beleifs satisfy $\operatorname{Pr}\left(h_{f}>h_{m}\right)=0$, then $\max E w_{f}(h)<\max E w_{m}(h)$. If such an equilibrium exists, $h_{f}>h_{m}$ and the distribution of home hours will be given by $\pi_{f}\left(h_{H}\right)=1$
and $\pi_{m}\left(h_{L}\right)=1$. By (27) and (28), we can write:

$$
\begin{gather*}
(y)^{1-\zeta}=\frac{E w^{\prime}\left(h_{f}\right)}{E w^{\prime}\left(y h_{f}\right)},  \tag{47}\\
\frac{H}{h_{f}}=\left[1+y^{\zeta}\right]^{1 / \zeta} \tag{48}
\end{gather*}
$$

where $y=h_{m} / h_{f}$. Applying lemma 7 to (47)-(48) implies that there are two zeros for (47): $y_{1}=1$ and $y_{2}\left(h_{f}\right)<1$. But under $\max E w_{f}(h)<\max E w_{m}(h), y_{1}=1$ is not optimal for Problem H1. Hence, the unique solution to Problem H1 is $0<h_{L}=h_{m}=y_{2} h_{f}=y_{2} h_{H}$ for all households, resulting in the distribution of home hours $\pi_{m}\left(h_{H}\right)=0$ and $\pi_{f}\left(h_{L}\right)=0$, consistent with firms' beliefs. This proves result i) in proposition 5. Note that $y_{2}(h)=x_{2}(h)$ and $h_{m}^{2}(H)=h_{f}^{2}(H)$.

If firms' beliefs satisfy $\operatorname{Pr}\left(h_{f}=h_{m}\right)=1$, then $E w_{f}(h)=E w_{m}(h)$ for all possible values of $h$. By Lemma 7, $x_{1}=1$ is a zero for equation (45). Moreover, under $E w_{f}(h)=E w_{m}(h)$, the ratio $h_{f} / h_{m}=1$ solves Problem H1 and induces distribution of home hours $\pi_{i}(\bar{h})=1$ for $i=f, m$ with $\bar{h}=H 2^{-1 / \zeta}$, by (46), consistent with firms' beliefs. By contrast the zero $x_{2}<1$ for equation (45) would induce a distribution of home hours inconsistent with firms' beliefs. Since there is a unique value of $\bar{h}$ which solves Problem H1, this equilibrium is unique. This proves result ii) in proposition 5 .

### 5.4 Ex Ante Differences

## Proof of Proposition 6

Assume that firms believe that female home hours are smaller than male home hours, so that $\max E w_{f}(h)>\max E w_{m}(h)$. To see if $h_{f} / h_{m}<1$ is optimal for the household, we need verify whether the system:

$$
\begin{align*}
(x)^{1-\zeta} & =\frac{E\left[w_{m}^{\prime}\left(h_{m}\right)\right]}{E\left[w_{f}^{\prime}\left(x h_{m}\right)\right] /(1+\varepsilon)},  \tag{49}\\
\frac{H}{h_{m}} & =\left[(1+\varepsilon) x^{\zeta}+1\right]^{1 / \zeta} \tag{50}
\end{align*}
$$

has a solution with $x<1$ when, by Lemma 4, labor contracts solve Problem F1. By Lemma 7, for $\varepsilon>0$ (49) has two zeros, with $0<x_{2}<x_{1}<1$. By max $E w_{f}(h)>\max E w_{m}(h)$ and since $E w(h)$ is decreasing and convex in $h$ by Proposition $1, x_{1}$ will not be optimal for Problem H1. Hence, households will choose $h_{L}=h_{f}=x_{2} h_{m}=h_{H}$ and the resulting distribution of home hours will be $\pi_{f}\left(h_{H}\right)=0$ and $\pi_{m}\left(h_{L}\right)=0$, consistent with firms' beliefs. If $\varepsilon$ is high enough, however, equation (49) fails to have a solution so that this equilibrium fails to exist.

If firms believe female home hours are greater than male home hours, max $E w_{f}(h)<$ $\max E w_{m}(h)$. This outcome can be an equilibrium if $h_{m} / h_{f}>1$ solves Problem H1. To verify
this, consider the system of equations:

$$
\begin{align*}
(y)^{1-\zeta} & =\frac{E\left[w_{f}^{\prime}\left(h_{f}\right)\right]}{E\left[w_{m}^{\prime}\left(y h_{f}\right)\right](1+\varepsilon)},  \tag{51}\\
\frac{H}{h_{f}} & =\left[(1+\varepsilon)+y^{\zeta}\right]^{1 / \zeta} . \tag{52}
\end{align*}
$$

By Lemma 7, for $\varepsilon>0$, generically, there are two zeros for equation (51), $0<y_{1}<1<y_{2}$. However, $y_{2}$ is not optimal for Problem H1 under $\max E w_{f}(h)<\max E w_{m}(h)$. Hence, the unique solution to Problem H1 is $y_{2}>1$. Then, the equilibrium distribution of home hours will satisfy $\pi_{f}\left(h_{H}\right)=1$ and $\pi_{m}\left(h_{L}\right)=1$, with $0<h_{L}=h_{m}=y_{2} h_{f}=h_{H}$, consistent with firm beliefs.

Summary Statistics for the Census sample

|  | Males |  | Females |  |
| :---: | :---: | :---: | :---: | :---: |
|  | mean | st. dev. | mean | st. dev. |
| Age | 40.03 | 8.06 | 40.14 | 8.19 |
| Less thanHS | 0.07 | 0.25 | 0.04 | 0.20 |
| HS | 0.30 | 0.46 | 0.30 | 0.46 |
| Some college | 0.30 | 0.46 | 0.35 | 0.48 |
| College+ | 0.33 | 0.47 | 0.30 | 0.46 |
| Married spouse present | 0.70 | 0.46 | 0.62 | 0.49 |
| Married spouse absent | 0.01 | 0.10 | 0.01 | 0.09 |
| Separated | 0.02 | 0.12 | 0.03 | 0.16 |
| Divorced | 0.11 | 0.31 | 0.17 | 0.38 |
| Widowed | 0.00 | 0.06 | 0.02 | 0.12 |
| Never married | 0.16 | 0.37 | 0.16 | 0.36 |
| Number of children | 1.09 | 1.20 | 0.94 | 1.08 |
| Salary (annual) | 49552 | 49929 | 33240 | 29358 |
| Market Hours (annual) | 2405 | 477 | 2185 | 387 |
| Log hourly earnings | 2.86 | 0.65 | 2.59 | 0.58 |
| Management | 0.07 | 0.25 | 0.05 | 0.22 |
| Business and financial operations | 0.04 | 0.21 | 0.07 | 0.25 |
| Computer and math | 0.04 | 0.19 | 0.02 | 0.15 |
| Architecture and engineering | 0.04 | 0.20 | 0.01 | 0.09 |
| Life, physical, and social science | 0.01 | 0.11 | 0.01 | 0.10 |
| Community and social services | 0.01 | 0.10 | 0.02 | 0.14 |
| Legal occupations | 0.01 | 0.12 | 0.02 | 0.13 |
| Education, training and library | 0.02 | 0.13 | 0.05 | 0.22 |
| Arts, design, ent, sports and media | 0.02 | 0.14 | 0.02 | 0.14 |
| Healthcare practitioner and techn. | 0.03 | 0.16 | 0.09 | 0.28 |
| Healthcare support | 0.00 | 0.05 | 0.03 | 0.17 |
| Protective services | 0.03 | 0.18 | 0.01 | 0.09 |
| Food preparation and serving | 0.02 | 0.12 | 0.03 | 0.17 |
| Building, ground cleaning/maintenance | 0.03 | 0.16 | 0.02 | 0.13 |
| Personal care services | 0.01 | 0.08 | 0.03 | 0.18 |
| Sales | 0.11 | 0.32 | 0.11 | 0.31 |
| Office and administrative support | 0.06 | 0.24 | 0.27 | 0.45 |
| Farming, fishing and forestry | 0.01 | 0.08 | 0.00 | 0.04 |
| Construction and extraction | 0.10 | 0.30 | 0.00 | 0.06 |
| Installation, maintenance and repair | 0.08 | 0.27 | 0.01 | 0.07 |
| Production | 0.11 | 0.32 | 0.06 | 0.24 |
| Transportation and material moving | 0.08 | 0.28 | 0.02 | 0.13 |
| Agriculture, forestry, fishing, hunting | 0.01 | 0.11 | 0.00 | 0.06 |
| Mining | 0.01 | 0.09 | 0.00 | 0.04 |
| Utilities | 0.02 | 0.13 | 0.01 | 0.08 |
| Construction | 0.12 | 0.32 | 0.02 | 0.14 |
| Manufacturing | 0.21 | 0.41 | 0.12 | 0.32 |
| Wholesale trade | 0.06 | 0.23 | 0.03 | 0.17 |
| Retail Trade | 0.10 | 0.30 | 0.11 | 0.32 |
| Transportation and Warehousing | 0.06 | 0.24 | 0.03 | 0.16 |
| Information | 0.03 | 0.18 | 0.04 | 0.19 |
| Finance and Insurance | 0.04 | 0.20 | 0.09 | 0.28 |


| Real Estate and Rental and Leasing | 0.02 | 0.13 | 0.02 | 0.14 |
| :--- | :--- | :--- | :--- | :--- |
| Professional, Scientific, and Technical | 0.07 | 0.26 | 0.07 | 0.26 |
| Administrative and Support and Waste Maı | 0.03 | 0.16 | 0.03 | 0.16 |
| Educational Services | 0.04 | 0.18 | 0.08 | 0.27 |
| Health care and social assistance | 0.04 | 0.20 | 0.20 | 0.40 |
| Arts, Entertainment and Recreation | 0.01 | 0.12 | 0.01 | 0.12 |
| Accomodation and Food Services | 0.03 | 0.16 | 0.04 | 0.20 |
| Other Services (exclude Public Administr | 0.04 | 0.20 | 0.04 | 0.20 |
| Public Administration | 0.06 | 0.24 | 0.06 | 0.23 |
| Number of Observations | 31489615 |  |  |  |
|  |  |  |  |  |

Note: Our sample includes $25-54$ white men and women, who are not in school, not in the armed forces, do not reside on a farm or live in group quarters. We include individuals who worked at least 50 weeks in the previous year and who usually work at least 30 hours per week.

## Summary Statistics for the PSID sample

| Overall Sample |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Males |  | Females |  |
|  | mean | st. dev. | mean | st. dev. |
| Age | 37.88 | 7.94 | 38.10 | 8.09 |
| Years of education | 13.13 | 2.92 | 12.91 | 3.88 |
| Married | 0.80 | 0.40 | 0.69 | 0.46 |
| Never married | 0.09 | 0.29 | 0.10 | 0.31 |
| Widowed | 0.00 | 0.05 | 0.01 | 0.12 |
| Divorced | 0.09 | 0.28 | 0.16 | 0.37 |
| Separated | 0.02 | 0.13 | 0.03 | 0.17 |
| Number of children | 1.11 | 1.14 | 0.90 | 1.04 |
| Salary (annual) | 45601 | 40303 | 28104 | 20889 |
| Log hourly earnings | 2.74 | 0.62 | 2.40 | 0.57 |
| Fraction of incentive pay | 0.01 | 0.05 | 0.01 | 0.04 |
| Market Hours (annual) | 2,453 | 513 | 2,159 | 410 |
| Weekly hours worked | 46.47 | 8.70 | 41.39 | 7.18 |
| Weeks worked | 50.82 | 0.75 | 50.72 | 0.73 |
| Number of Observations | 5452 |  | 3046 |  |


| Sample of Married couples |  |  |
| :--- | :---: | :---: |
|  | mean | st. dev. |
| Age of husband | 34.15 | 5.87 |
| Age of wife | 32.46 | 6.30 |
| Home hours of husband | 8.26 | 9.73 |
| Home hours of wife | 16.15 | 10.49 |
| Labor income of husband | 39615 | 28083 |
| Labor income of wife | 25224 | 21040 |
| Wife/husband ratio of home hours | 2.32 | 2.62 |
| Wife/husband earnings ratio | 0.63 | 0.49 |
| Husband/Wife difference incentive share | -0.002 | 0.01 |
| Number of children | 1.03 | 1.13 |
|  |  |  |
| Number of Observations | 300 |  |

Note: The overall sample includes 25-54 white men and women, who are not in school or in the armed forces. We include individuals who worked at least 50 weeks in the previous year and who usually work at least 30 hours per week.
The sample of married couples refer to male-headed household where the head of the household is $25-44$, both spouses are white and they both work full-time year-round.


[^0]:    ${ }^{1}$ Discrimination lawsuits based on analogous complaints where filed by a team of women brokers at Merrill Lynch and by women researchers working at Rand corporation during the summer of 2004. See The New York Times, August 22, 2004 and The New York Times, September 5, 2004, respectively.

[^1]:    ${ }^{2}$ See Albanesi and Olivetti (2005a) for a version of the model with variable market hours.

[^2]:    ${ }^{3}$ This paradigm does not focus on a particular model of spousal interaction, rather it merely restricts household decisions to be Pareto efficient. This framework is consistent with a variety of "household bargaining" models, as in McElroy and Horney (1981) and Manser and Brown (1980). See also Bergstrom (1997) for a review.

[^3]:    ${ }^{4}$ Francois (1998) presents a model in which equilibria with gender wage differentials are self-fulfilling. His result relies on exogenously given job heterogeneity. One class of jobs operate under an efficiency wage setting while a second class of jobs operate under piece rate wage setting. Earnings are higher in the efficiency wage jobs. In an equilibrium with female wage discrimination, the first class of jobs is assigned to men, the second to women. The female wage differential stems from job segregation. If all workers were to operate under the same job, the gender wage gap would be reversed. Hence, this model cannot account for gender differentials within the same occupational categories.

[^4]:    ${ }^{5}$ Gender differentials in earnings and home hours have proved to be very persistent. O'Neill (2003) shows that there is still a $10 \%$ differential in female and male wages in the U.S. in 2000 that remains unexplained by gender differences in schooling, actual experience and job characteristics. Moreover, PSID data for the period 1976-2001 show that husbands' home hours are roughly one third of wives' and that this difference is stable over time.

[^5]:    ${ }^{6}$ Other parameter values are $\psi=0.1$ and $\sigma=1$.

[^6]:    ${ }^{7}$ We use the industry variable, INDNAICS, that reports the type of establishment in which a person worked in terms of the good or service produced. Industries are coded according to the North American Industrial Classification System developed in 1997. We use the variable OCCSOC for occupation. OCCSOC classifies occupations according to the 1998 Standard Occupational Classification (SOC) system. The Census also provide an aggregation of all the occupations in 23 broader categories that include the three categories considered in the analysis. The definition of production occupations also includes construction and extraction workers.
    ${ }^{8}$ See sections 2.1 and 2.4.

[^7]:    ${ }^{9}$ For example, sales workers are typically assigned to specific territories or products. Hence, sales volumes will fluctuate with shocks to local demand. See Catalyst (1995) for a description of the sales occupation, especially in relation to gender.
    ${ }^{10}$ We have excluded from our sample workers in Agriculture; Forestry Fishing and Hunting, Mining and Utilities. This is because for these three industries we are unable to compute adjusted gender earnings gaps for the sample of never married workers in sales and management occupations. That is, once we control for age and education in each of the occupation/industry cells there is not enough variation left to estimate the coefficient on the female dummy.
    ${ }^{11}$ The first dummy is equal to one if an individual has completed less than twelve years of schooling and is equal to zero otherwise. The second dummy variable is equal to one if he or she has completed twelve years of schooling, and is equal to zero otherwise. The third dummy variable equals one if the individual has completed between twelve and fifteen years of schooling and it is equal to zero otherwise. Finally, the fourth dummy variable is equal to one if an individual has completed at least sixteen years of education and it equals zero otherwise.
    ${ }^{12}$ We use the following Census variables in our analysis: EDUCREC for educational attainment, INCWAGE

[^8]:    ${ }^{15}$ In the PSID, data on hours worked, total labor earnings, bonuses and commision income, are reported for the previous calendar year. Hence, our data covers the time period 1993-2000. In 1997 the PSID started collecting information bi-annually. Hence, our sample includes 6 waves of PSID data.
    ${ }^{16}$ Disaggregated data on bonuses are available from 1984 to 1992. However, data on commissions were made available only since the 1994 wave (in the Income Plus PSID files). Hence, we restrict attention to the most recent waves. The majority of workers report either bonus or commission pay. We exclude workers with real weekly earnings below $\$ 67$ in 1982 dollars from the sample. We deflate nominal variables using the CPI with base year 2000 .
    ${ }^{17}$ We exclude from the analysis the category of laborers working in private households because no male reports to be employed in this occupation.
    ${ }^{18}$ To account for the role of differences in hours worked in determining gender earnings differentials, we also

[^9]:    conduct this analysis for hourly wages. We find that the correlation between the female/male difference in log hourly wages and the male/female difference in the fraction of incentive pay is -0.54 and also significant at the five percent level.
    ${ }^{19}$ Information on bonuses and commission is only available for 824 women out of approximately 3,000 women in the sample. Of these, 270 are never married, 426 are divorced and only 13 are married. Information on incentive pay is available for most of the men in the sample $(5,427$ out of 5,452 observations of which 4,349 are married.)

[^10]:    ${ }^{20}$ In our theoretical analysis, workers are identical in market productivity and talents. Hence, we do not derive any predictions on occupational sorting.
    ${ }^{21}$ The same is true for the difference in log hourly wages.

[^11]:    ${ }^{22}$ See for example Niederle and Vesterlund (2005).
    ${ }^{23}$ Jullien, Salanie and Salanie (2003) analyze a model with hidden effort and private risk aversion. They show that in a separating equilibrium, agents with higher risk aversion will have lower fraction of incentive pay, under the analogue of a single-crossing condition. However, this does not imply that they exert lower effort, since a given level of earnings variability has a stronger incentive effects for more risk-averse agents.
    ${ }^{24}$ The variable that reports home hours in the PSID poses a measurement problem. The survey respondent is asked to provide a measure of weekly hours worked at home by him- or herself and by the spouse (if married.) No time diaries are used. This could be problematic if respondents tend to overestimate their own home hours and to underestimate their spouses' home hours. In particular, if respondents are disproportionately women we would tend to overestimate the wife/husband ratio of home hours. However, this concern seems to be of secondary importance since we are interested in a measure of total hours spent in home production. Evidence from time-use surveys for the late-1990s (Freeman (2000)) confirms the PSID evidence that wives spend, on average, at least twice as much time than their husbands in home production activities irrespective of their labor market status.

[^12]:    ${ }^{25}$ We have also constructed the husband-wife differences in $\tilde{w}$ by using the reported incentive pay shares for the household's head. We obtain similar results to the ones discussed in the paper.
    ${ }^{26}$ We experimented with different sample selection criteria. For example, we considered all the observations in one wave, say 1999, and then added married couples from adjacent waves. The results obtained for these alternative samples are consistent with the ones reported in the paper.

[^13]:    ${ }^{27}$ Suitable data is available for Norway.

[^14]:    ${ }^{28}$ In a similar vein, Guner and Greenwood (2004) analyze the role of technological progress on household formation since World War II.

