NBER WORKING PAPER SERIES

PRIVATE INFORMATION, WAGE BARGAINING AND EMPLOYMENT FLUCTUATIONS

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Working Paper 11967
http://www.nber.org/papers/w11967

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 2006

I thank Björn Brügemann, Bob Hall, Giuseppe Moscarini, Rob Shimer, and the participants in seminars at Northwestern, Yale and Duke for useful comments. I am particularly grateful to Rob Shimer for pointing out a serious mistake in early versions of the paper. The National Science Foundation provided research support. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 11967
January 2006, Revised June 2006
JEL No. E3, J6, D8

ABSTRACT

Shimer (2003) pointed out that although we have a satisfactory theory of why some workers are unemployed at any given time, we don't know why the number of unemployed workers varies so much over time. The basic Mortensen-Pissarides (1994) model does not generate nearly enough volatility in unemployment, for plausible parameter values. This paper extends the Mortensen-Pissarides model to allow for informational rents. Productivity is subject to publicly observed aggregate shocks, and to idiosyncratic shocks that are seen only by the employer. It is shown that there is a unique equilibrium, provided that the idiosyncratic shocks are not too large. The main result is that small fluctuations in productivity that are privately observed by employers can give rise to a kind of wage stickiness in equilibrium, and the informational rents associated with this stickiness are sufficient to generate relatively large unemployment fluctuations.

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1. Introduction

The standard view of unemployment is that it takes time for workers to find the right job, and for employers to find the right worker. Fluctuations in the productivity of jobs naturally give rise to fluctuations in the number of workers looking for jobs, and in the number of employers looking for workers. High productivity is associated with a tight labor market in which more workers have jobs and fewer workers are looking for jobs, while employers are keen to hire more workers, so vacancies are plentiful; conversely, when productivity is low, unemployment is high and there are few vacancies.

This simple description of the source of unemployment fluctuations suggests that it should be possible to measure the variability of productivity and use this to explain the variability of unemployment, to a rough approximation. The Mortensen-Pissarides (1994) model is the natural framework for such a calculation, since it gives a precise account of the relationship between productivity and search on each side of the labor market. Shimer (2003) showed that the basic Mortensen-Pissarides model in fact translates fluctuations in labor productivity into unemployment fluctuations that are very much smaller than those seen in U.S. data. Thus although we have a satisfactory theory of why some workers are unemployed at any given time, we don't know why the number of unemployed workers varies so much over time. To a substantial extent the number of unemployed workers varies because of movements into and out of the labor force, which are not included in the Mortensen-Pissarides model. But even for people who are firmly attached to the labor force, the variations are large. For example, in the U.S. over the period 1967-2005, the median annual unemployment rate of white men aged 35-39 was 3.7%; in 10 of these 39 years, the rate was 4.4% or higher, while there were also 10 years with a rate of 2.6% or lower. The basic reason for unemployment in this group is that no two workers are the same, and no two jobs are the same. Given that job separation rates are relatively stable, the unemployment rate is a measure of how long it takes to match workers and jobs. The question then is why the matching process should be so much slower in some years than in others.

Hall (2005) argued that this volatility problem can be fixed if the Nash bargaining component of the Mortensen-Pissarides model is replaced by a "sticky" wage-setting process. Brügemann and Moscarini (2005) showed that the volatility of unemployment remains implausibly low for a broad class of surplus-sharing rules: the Nash bargaining rule is not the source of the problem. On the other hand if there is some stickiness in wages, the employers' incentive to create vacancies is magnified when the economy improves, and this increases unemployment volatility.

As Rotemberg (2006) points out, the basic Mortensen-Pissarides model also predicts procyclical wages, which are not seen in the data, and this problem persists in the more general model developed by Yashiv (2006). Wage stickiness helps to resolve this discrepancy as well, but of course this is useful only if we understand why wages are sticky. Hall (2005) assumed that the wage level in a previous contract establishes a "social norm" that largely determines the wage in the next contract. In the absence of a theory of social norms, this explanation is incomplete. Similarly, Gertler and Trigari (2006) showed that staggered wage contracts magnify the incentive to create vacancies, but did not try to explain why workers and employers who are interested only in the present value of income would negotiate contracts that constrain the division of the surplus in matches that have not yet been made.

This paper shows that an extension of the Mortensen-Pissarides model in which some productivity fluctuations are privately observed by employers can explain the volatility of unemployment in a more parsimonious way. The introduction of private information precludes the Nash bargaining rule; instead, the surplus is divided by a simple "random dictator" mechanism that is a natural generalization of the Nash mechanism.[2] There are two main results. First, the extended model has a unique equilibrium. Second, this equilibrium exhibits a kind of wage stickiness, and the informational rents associated with this stickiness are sufficient to translate small fluctuations in productivity into relatively large unemployment fluctuations.[3]

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[2]This is a special case of the Neutral Bargaining Solution introduced by Myerson (1984).

[3]Brügemann and Moscarini (2005) rule out wage stickiness by assuming that the division of the surplus should be invariant to a change in the location of the productivity distribution. This assumption is appealing in the case of complete information. But when the employer has private information, it is optimal for workers to ignore small changes in the productivity distribution, and this gives rise to a kind of wage stickiness. Menzio (2005) develops this idea in great detail, and derives a bargaining equilibrium in which transient productivity fluctuations that are privately observed by employers are not transmitted to wages.

2. A Model of Sticky Wages with Private Information and Aggregate Shocks

The model is a simplified version of the model analyzed in Kennan (2003). A successful job match generates a surplus to be divided between the worker and the employer. The value of the worker's output is modeled as a binary random variable whose realization ("L" for low or "H" for high) is observed privately by the employer when the match is made. The probability of drawing the high surplus, p_s, is a publicly observed Markov pure jump process with two states ($s = 1$ in the bad state and $s = 2$ in the good state), and exit hazards λ_1 and λ_2. The expectation of the surplus is assumed to be higher in the good state. When the joint continuation value from a match falls below the joint opportunity cost, the match is destroyed. The job destruction hazard rate is a constant, δ, and there is a constant returns matching function that generates a flow of new matches $M(N_U,N_V)$ from unemployment and vacancy stocks N_U and N_V. There is an infinitely elastic supply of potential vacancies, and the actual number of vacancies posted is such that the expected profit from a vacancy is zero. Workers and employers maximize the present value of net income, using the interest rate r.

The match surplus is divided in the following way. Either the employer or the worker is randomly selected to make an offer, and if this offer is rejected the match dissolves. Clearly, the employer's offer will just match the worker's reservation level, which is the value of searching for another match. The worker effectively has two choices: an offer that exhausts the low surplus, with a sure acceptance, or an offer that exhausts the high surplus, with acceptance only if the high surplus has actually been realized. It is assumed that the parameters are such that the worker always finds it optimal to demand the low surplus.

Match Surplus

The match surplus depends on whether the employer draws a high or low value from the output distribution, and it also depends on the aggregate state. Let y_s^L and S_s^L be the flow surplus and the continuation value of the match when the output value is low, and the aggregate state is s, and similarly when the output value is high. For simplicity, it is assumed that the difference between the low and high output values does not depend on the aggregate state. That is, $y_2^H - y_2^L = y_1^H - y_1^L = \Delta y$.

Let U denote the state-dependent continuation value of an unmatched worker, and let G denote the joint continuation value of a matched worker-employer pair. In the low-output state, the joint match values are determined by the following asset pricing equations

$$\begin{aligned} rG_1^L &= y_1^L - \delta\left(G_1^L - U_1\right) + \lambda_1\left(G_2^L - G_1^L\right) \\ rG_2^L &= y_2^L - \delta\left(G_2^L - U_2\right) - \lambda_2\left(G_2^L - G_1^L\right) \end{aligned} \tag{1}$$

Thus the flow value of a match depends only on the aggregate state. This rules out two interesting alternatives. First, the flow value is the same for all workers. Nágypál (2006) shows that heterogeneity in workers' (private) evaluations of nonpecuniary job characteristics can substantially increase the volatility of unemployment. Second, there is no possibility of switching from low to high output, once the match has been made. Even in the absence of informational rents, this tends to increase unemployment volatility, by strengthening the incentive to create vacancies when a high-output match is more likely because the aggregate state is good. Costain and Reiter (2005) show that this vintage productivity effect can potentially explain the volatility of unemployment, but Brügemann (2005) shows that this effect is quite weak in the model considered in this paper.

It is assumed that there is free entry of employers, so that the continuation value of an unmatched employer is zero in all states. Thus the (state-dependent) match surplus S is the difference between G and U, and the match value equations can be rewritten as

$$\begin{aligned} (r+\delta)\left(S_1^L + U_1\right) &= y_1^L + \delta U_1 + \lambda_1\left(S_2^L - S_1^L + \Delta U\right) \\ (r+\delta)\left(S_2^L + U_2\right) &= y_2^L + \delta U_2 - \lambda_2\left(S_2^L - S_1^L + \Delta U\right) \end{aligned} \tag{2}$$

where $\Delta U = U_2 - U_1$. This implies

$$S_2^L - S_1^L + \Delta U = \frac{y_2^L - y_1^L + \delta \Delta U}{r + \delta + \Lambda} \tag{3}$$

where $\Lambda = \lambda_1 + \lambda_2$. Substituting this in (2) gives

$$
\begin{aligned}
(r+\delta)S_1^L &= y_1^L - rU_1 + \frac{\lambda_1\left(y_2^L - y_1^L + \delta\Delta U\right)}{r+\delta+\Lambda} \\
(r+\delta)S_2^L &= y_2^L - rU_2 - \frac{\lambda_2\left(y_2^L - y_1^L + \delta\Delta U\right)}{r+\delta+\Lambda}
\end{aligned}
\qquad \textbf{(4)}
$$

Using these equations, and the analogous equations for a high-output match, the effect of the aggregate state on the match surplus is given by

$$
S_2^H - S_1^H = S_2^L - S_1^L = \frac{y_2^L - y_1^L - (r+\Lambda)\Delta U}{r+\delta+\Lambda} \qquad \textbf{(5)}
$$

Thus even if an unmatched worker has better prospects when the aggregate state is good, the match surplus might be lower when the aggregate state is good, for a given output draw. On the other hand there is a higher probability of drawing a high output value in the good aggregate state.

The effect of the output draw on the match surplus is given by

$$
S_2^H - S_2^L = S_1^H - S_1^L = \frac{\Delta y}{r+\delta} \qquad \textbf{(6)}
$$

Unemployment Continuation Values

The rate at which unemployed workers find new matches is $M(N_U,N_V)/N_U = m(\theta)$, where $\theta = N_V/N_U$ represents market tightness, and $m(\theta) = M(1,\theta)$. The job-finding rate function $m(\theta)$ is assumed to be increasing, and strictly concave, with $m(0) = 0$.

When a match is made, the worker is selected to make an offer with probability ν. In this case, the worker gets the low-output surplus, and the employer gets an informational rent if the realized match value is high. If the employer is selected to make an offer, the worker gets the reservation level U and the employer gets the whole surplus. Thus an unmatched worker's continuation values are determined by the asset pricing equations

$$\begin{aligned} rU_1 &= y_0 + m(\theta_1)\nu S_1^L + \lambda_1\left(U_2 - U_1\right) \\ rU_2 &= y_0 + m(\theta_2)\nu S_2^L - \lambda_2\left(U_2 - U_1\right) \end{aligned} \tag{7}$$

where y_0 is the flow value of unemployment (including unemployment benefits and the value of leisure). Thus

$$\begin{aligned} rU_1 &= y_0 + \frac{r+\lambda_2}{r+\Lambda} m(\theta_1)\nu S_1^L + \frac{\lambda_1}{r+\Lambda} m(\theta_2)\nu S_2^L \\ rU_2 &= y_0 + \frac{r+\lambda_1}{r+\Lambda} m(\theta_2)\nu S_2^L + \frac{\lambda_2}{r+\Lambda} m(\theta_1)\nu S_1^L \end{aligned} \tag{8}$$

Vacancy Creation

Employers post new vacancies to the point where the net profit from doing so is zero. When a match is made, the employer gets an informational rent if the match value is high, and also gets a fraction 1-ν of the low-output surplus (in expectation). Thus the zero-profit conditions implied by free entry are

$$\begin{aligned} 0 &= -c + \frac{m(\theta_1)}{\theta_1}\left((1-\nu)S_1^L + p_1\left(S_1^H - S_1^L\right)\right) \\ 0 &= -c + \frac{m(\theta_2)}{\theta_2}\left((1-\nu)S_2^L + p_2\left(S_2^H - S_2^L\right)\right) \end{aligned} \tag{9}$$

where c is the flow cost of maintaining a vacancy, and p_s is the probability of drawing the high match value, for s = 1,2 .

It is convenient to let d = θ/m(θ) denote the expected duration of a vacancy. Then the free-entry conditions can be written as

$$\begin{aligned} cd_1 &= (1-\nu)S_1^L + \frac{p_1 \Delta y}{r+\delta} \\ cd_2 &= (1-\nu)S_2^L + \frac{p_2 \Delta y}{r+\delta} \end{aligned} \quad \textbf{(10)}$$

Solution

The model can be solved as follows. For given values of d_1 and d_2, the free entry conditions determine the low-state surplus values:

$$S_s^L = \frac{c\left(d_s - \alpha_s\right)}{1-\nu} \quad \textbf{(11)}$$

where

$$\rho_s = \frac{p_s \Delta y}{c\left(r+\delta\right)} \quad \textbf{(12)}$$

for s = 1, 2. Equation (2) can be rearranged to give U_1 and U_2 as linear functions of S_1^L and S_2^L, and U_1 and U_2 can then be expressed in terms of d_1 and d_2 as

$$\begin{aligned} rU_1 &= \mathrm{y}_1^L + \frac{c\left(\delta+r\right)\left(d_1-\rho_1\right)}{1-\nu} + \frac{\lambda_1\left(\mathrm{y}_2^L - \mathrm{y}_1^L\right)}{\Lambda + \mathrm{r}} + \frac{\lambda_1 \delta c\left(d_2 - \rho_2 - \left(d_1 - \rho_1\right)\right)}{\left(1-\nu\right)\left(r+\delta\right)} \\ rU_2 &= \mathrm{y}_2^L + \frac{c\left(\delta+r\right)\left(d_2-\rho_2\right)}{1-\nu} - \frac{\lambda_2\left(\mathrm{y}_2^L - \mathrm{y}_1^L\right)}{\Lambda + \mathrm{r}} + \frac{\lambda_2 \delta c\left(d_1 - \rho_1 - \left(d_2 - \rho_2\right)\right)}{\left(1-\nu\right)\left(r+\delta\right)} \end{aligned} \quad \textbf{(13)}$$

Next (11) can be substituted in (8), giving

$$rU_1 = y_0 + \frac{r+\lambda_2}{r+\Lambda}\frac{\nu cm(\theta_1)}{1-\nu}(d_1 - \rho_1) + \frac{\lambda_1}{r+\Lambda}\frac{\nu cm(\theta_2)}{1-\nu}(d_2 - \rho_2)$$
$$rU_2 = y_0 + \frac{r+\lambda_1}{r+\Lambda}\frac{\nu cm(\theta_2)}{1-\nu}(d_2 - \rho_2) + \frac{\lambda_2}{r+\Lambda}\frac{\nu cm(\theta_1)}{1-\nu}(d_1 - \rho_1) \tag{14}$$

After eliminating U_1 and U_2, this gives the following equations determining d_1 and d_2

$$\psi_1(d) = Z_1 + \left(\frac{\rho_1}{d_1} - 1\right)\nu H(d_1) - (r+\delta+\lambda_1)(d_1 - \rho_1) + \lambda_1(d_2 - \rho_2) = 0$$
$$\psi_2(d) = Z_2 + \left(\frac{\rho_2}{d_2} - 1\right)\nu H(d_2) - (r+\delta+\lambda_2)(d_2 - \rho_2) + \lambda_2(d_1 - \rho_1) = 0 \tag{15}$$

where $H(d) = \theta$, and

$$Z_s = \frac{(1-\nu)(y_s^L - y_0)}{c}, s = 1,2 \tag{16}$$

3. **Existence and Uniqueness of Equilibrium**

Since $m(\theta)$ is strictly concave, with $m(0) = 0$, the ratio $m(\theta)/\theta$ is strictly decreasing, so the function $d = \theta/m(\theta)$ is invertible. It is assumed that the inverse function $\theta = H(d)$ is convex, with $H(0) = 0$.[4] Under this assumption, it will be shown that an equilibrium with informational rents exists, and that it is unique.

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[4]This assumption holds in the Cobb-Douglas case. The condition $H(0) = 0$ means that the expected vacancy duration shrinks to zero as the number of vacancies per unemployed worker shrinks to zero. Although this is a reasonable condition, it effectively rules out any constant returns CES matching technology except for the Cobb-Douglas case. Indeed if the matching function is defined by $(M/\mu)^\varrho = \alpha U^\varrho + (1-\alpha)V^\varrho$, then a positive value of ϱ is ruled out because it implies that matches can be made even if there are no vacancies. On the other hand a negative value of ϱ is ruled out by the condition that $\theta/m(\theta)$ shrinks to zero as θ decreases to zero. This is a case in which local behavior around $\theta = 0$ has global implications because the CES parametric family is inflexible. It is not difficult to stitch together a Cobb-Douglas and a CES with negative ϱ, so that the function is Cobb-Douglas near zero, with $H(0) = 0$. Then if $\varrho < -1$, the function $H(d)$ is not convex.

Proposition 1

If the function $\theta = H(d)$ is convex, and if $H(0) = 0$, then there is a unique vector $d^* = (d_1^*, d_2^*)$ such that $\psi(d^*) = 0$.

The proof uses the following result.

Lemma 1

Suppose a is a positive number, and H is a twice differentiable function, with $H(0) = 0$, $H'(x) > 0$ and $H''(x) > 0$, for $x > a$. Define the function h, on the domain $[a,\infty)$, as

$$h(x) = \left(\frac{a}{x} - 1\right) H(x) \qquad \textbf{(17)}$$

Then $h'(x) < 0$ and $h''(x) < 0$.

Proof

The first and second derivatives of h are as follows

$$\begin{aligned} h'(x) &= \left(\frac{a}{x} - 1\right) H'(x) - \frac{a}{x^2} H(x) \\ h''(x) &= \left(\frac{a}{x} - 1\right) H''(x) - 2\frac{a}{x^2} H'(x) + 2\frac{a}{x^3} H(x) \\ &= \left(\frac{a}{x} - 1\right) H''(x) + 2\frac{a}{x^3}\big(H(x) - xH'(x)\big) \end{aligned} \qquad \textbf{(18)}$$

Since $x \geq a$, and $H'(x) > 0$, it is clear that h is decreasing. Any convex (differentiable) function H that passes through the origin has the property that $xH'(x) \geq H(x)$. Thus $h''(x) \leq 0$.

Proof of Proposition 1

First it will be shown that $\psi(d^*) = 0$ implies $d^* > \rho$. Indeed if $d_1 \le \rho_1$ and $d_2 \ge \rho_2$ then $\psi_1(d) > 0$; and if $d_1 \ge \rho_1$ and $d_2 \le \rho_2$ then $\psi_2(d) > 0$. If $d \le \rho$, write $\psi(d)$ as

$$\begin{aligned}\psi_1(d) &= Z_1 + (\rho_1 - d_1)\nu\frac{H(d_1)}{d_1} - (r+\delta)(d_1-\rho_1) + \lambda_1[(d_2-\rho_2)-(d_1-\rho_1)] \\ \psi_2(d) &= Z_2 + (\rho_2 - d_2)\nu\frac{H(d_2)}{d_2} - (r+\delta)(d_2-\rho_2) - \lambda_2[(d_2-\rho_2)-(d_1-\rho_1)]\end{aligned} \qquad \textbf{(19)}$$

These equations show that either $\psi_1(d)$ or $\psi_2(d)$ is a sum of four positive terms: the first three terms are positive in both equations, and if the last term is negative in one equation, it is positive in the other. Thus $\psi(d) \ne 0$ if $d \le \rho$.

Next it will be shown that a solution exists. Note that $\psi(\rho) = Z > 0$. Define b as the solution of the linear equations obtained by setting $H = 0$. Then

$$b_s = \rho_s + \frac{\bar{Z}_s}{r+\delta} \quad , s = 1,2 \qquad \textbf{(20)}$$

where

$$\begin{aligned}\bar{Z}_1 &= \frac{(r+\delta+\lambda_2)Z_1 + \lambda_1 Z_2}{r+\delta+\lambda_1+\lambda_2} \\ \bar{Z}_2 &= \frac{(r+\delta+\lambda_1)Z_2 + \lambda_2 Z_1}{r+\delta+\lambda_1+\lambda_2}\end{aligned} \qquad \textbf{(21)}$$

Thus $b > \rho$ and $\psi(b) < 0$.

Since ψ_1 is increasing in d_2 and decreasing in d_1, the equation $\psi_1(d) = 0$ can be solved to obtain d_2 as an increasing function of d_1. Write this as $d_2 = \Upsilon_1(d_1)$. Since ψ_2 is increasing in d_1 and decreasing in d_2, the equation $\psi_2(d) = 0$ can also be solved to obtain d_2 as an increasing function of d_1. Write this as $d_2 = \Upsilon_2(d_1)$. Define the function $\Upsilon_0(x) = \Upsilon_2(x) - \Upsilon_1(x)$. Since $\psi_1(\rho_1, \Upsilon_1(\rho_1)) = 0$, and $\psi_1(\rho_1, \rho_2) > 0$, and ψ_1 is increasing in d_2, it follows that $\Upsilon_1(\rho_1) < \rho_2$.

Also, since $\psi_2(\rho_1, \Upsilon_2(\rho_1)) = 0$, and $\psi_2(\rho_1, \rho_2) > 0$, and ψ_2 is decreasing in d_2, it follows that $\Upsilon_2(\rho_1) > \rho_2$. Therefore $\Upsilon_0(\rho_1)$ is positive. By a similar argument, $\Upsilon_0(b_1)$ is negative. Also, Υ_0 is continuous (since ψ_1 is linear in d_2 and ψ_2 is linear in d_1). So by the intermediate value theorem $\Upsilon_2(x) = \Upsilon_1(x)$ for some $x \in (\rho_1, b_1)$. This means that $\psi(x, \Upsilon_1(x)) = 0$, showing that a solution $d^* = (x, \Upsilon_1(x))$ exists (with $d^* > \rho$).

To show uniqueness, define the function $g(z) = \psi(\rho + z)$. Then $g(0) > 0$, g_1 is increasing in z_2 and g_2 is increasing in z_1, and both g_1 and g_2 are concave by Lemma 1. Therefore, by Theorem 1 in Kennan (2001), g has at most one positive root, meaning that ψ has at most one root above ρ. Since it has already been shown that ψ does have a root above ρ, and no roots anywhere else, the proof is complete.

Optimality of Pooling Offers

It has been assumed that when a match is made in the good aggregate state, and the worker is selected to make an offer, it is optimal to demand the low surplus, rather than demand the high surplus at the risk of destroying the match. Thus the equilibrium surplus values must satisfy the following no-screening conditions

$$
\begin{aligned}
S_1^L &\geq p_1 S_1^H = p_1\left(S_1^L + \frac{\Delta y}{r+\delta}\right) \\
S_2^L &\geq p_2 S_2^H = p_2\left(S_2^L + \frac{\Delta y}{r+\delta}\right)
\end{aligned}
\qquad \textbf{(22)}
$$

which can be written as

$$
S_s^L \geq \frac{p_s}{1-p_s}\frac{\Delta y}{c(r+\delta)} \qquad \textbf{(23)}
$$

for s = 1,2. Using the free entry conditions, this reduces to

$$d_s \geq \overline{\rho}_s \tag{24}$$

where

$$\overline{\rho}_s = \left(1 + \frac{1-\nu}{1-p_s}\right)\rho_s \tag{25}$$

Since $\rho_s = 0$ for $p_s = 0$, Proposition 1 implies that a unique equilibrium satisfying the no-screening conditions exists if p_1 and p_2 are small enough. Conversely, the no-screening condition fails as p_s approaches 1 (as of course it should).

The main theoretical result is Theorem 1, which characterizes a set of parameter values for which an equilibrium exists, and shows that if the parameters lie in this set, the equilibrium is unique.

Theorem 1

If H(d) is a convex function, with H(0) = 0, and if $\psi(\overline{\rho}) \geq 0$, then a unique equilibrium exists.

Proof

By Proposition 1, there is a unique vector d^* such that $\psi(d^*) = 0$. Since $\psi(\overline{\rho}) \geq 0$ and $\psi(b) < 0$, the argument used in the proof of Proposition 1 can be used to show that ψ has a root in the rectangle $[\overline{\rho}, A]$, and since there is only one root above ρ, this root is d^*. The no-screening conditions are satisfied because $d^* \geq \overline{\rho}$. Therefore d^* is the unique equilibrium.

4. **The Effects of Informational Rents**

Suppose that there are no transitions, and that the wage rate is fixed, as in Hall (2005). Then the free entry condition is

$$c = \frac{1}{d}\frac{y^L - w + p\Delta y}{r+\delta} \tag{26}$$

The right side of this equation is the capital gain from a filled vacancy, multiplied by the hazard rate, and the left side is the flow cost of maintaining the vacancy. A higher productivity level, with a fixed wage, is offset in equilibrium by a lower hazard rate. If the profit flow is small (because the wage is high), a small productivity change implies a large proportional change in profits, and therefore a large proportional change in the rate at which vacancies are filled, which implies a large change in the unemployment rate.

In the standard Mortensen-Pissarides model, the wage is a nested weighted average of the productivity levels while employed (y) and while unemployed (y_0). In the informational rents model, the wage is determined in exactly the same way, assuming the low realization of the productivity shock (y^L). That is,

$$w = \frac{\phi}{\phi + b} y^L + \frac{b}{\phi + b} w_0 \tag{27}$$

where $\phi = m(\theta)$ is the job-finding rate, and

$$\begin{aligned} b &= \frac{p\Delta y + (1-\nu)(y^L - y_0)}{c(r+\delta)} \\ w_0 &= \nu y + (1-\nu) y_0 \end{aligned} \tag{28}$$

The free entry condition can then be written as

$$cd = \frac{1}{\phi + b} \frac{1-\nu}{\nu} (y - y_0) + \frac{p\Delta y}{r+\delta} \tag{29}$$

The result for the standard model (with $p\Delta y = 0$) differs from the fixed wage result in two respects. First, if the job-finding rate is held constant, a large proportional change in d requires a large proportional change in the flow surplus from employment (rather than in the flow profit). This means that small productivity shocks do not cause large unemployment movements unless the flow surplus is small, as in Hagedorn and Manovskii (2005). Second, this exaggerates the relationship between productivity and unemployment, because the job-finding rate does not in fact stay constant when d increases. An increase in d implies an increase in ϕ, and this dampens the relationship between productivity and unemployment: workers receive a larger share of the flow surplus when an increase in the job-finding rate increases the continuation value of being unemployed, and this diminishes the incentive to create vacancies.

Informational rents affect unemployment in much the same way as fixed wages, because small productivity changes that are observed privately by employers do not affect wages. The wage is close to the low productivity level, for standard parameter values, so the profit flow y^L - w is small in equation (26). Changes in $p\Delta y$ therefore give rise to large proportional changes in profits, and in the unemployment rate.

The Cobb-Douglas Case

The equilibrium relationships between productivity, informational rents and the unemployment rate can be characterized more explicitly in the case of a constant-returns Cobb-Douglas matching function, $M = \mu U^{\alpha} V^{1-\alpha}$, with $m(\theta) = \mu\theta^{1-\alpha}$. In this case the equilibrium conditions (15) can be stated as

$$\begin{aligned} \zeta_1 &= (\phi_1 + A)\xi_1 + \frac{\lambda_1}{\nu}(\xi_1 - \xi_2) \\ \zeta_2 &= (\phi_2 + A)\xi_2 + \frac{\lambda_2}{\nu}(\xi_2 - \xi_1) \end{aligned} \qquad \textbf{(30)}$$

where $\phi_s = m(\theta_s)$, and

$$\begin{aligned} A &= \frac{r+\delta}{\nu} \\ \xi_s &= (\phi_s)^{\frac{\alpha}{1-\alpha}} - \frac{\mu^{\frac{1}{1-\alpha}}}{c}\frac{p_s\Delta y}{r+\delta} \\ \zeta_s &= \frac{\mu^{\frac{1}{1-\alpha}}}{c}\frac{1-\nu}{\nu}\left(y_s^L - y_0\right) \end{aligned} \qquad \textbf{(31)}$$

Thus, as Shimer (2005) noted, the parameters c and μ enter only through the ratio $\mu_0 = \frac{\mu^{\frac{1}{1-\alpha}}}{c}$.

If the aggregate state is permanent, equation (30) reduces to (two copies of) the following equation:

$$\zeta = \left(\phi^{\frac{\alpha}{1-\alpha}} - R \right) \left(\phi + A \right) \tag{32}$$

where

$$R = \frac{\mu_0 p \Delta y}{r + \delta}, \zeta = \frac{1-\nu}{\nu} \mu_0 \left(y^L - y_0 \right) \tag{33}$$

The effect of productivity variation with a square-root matching function ($\alpha = ½$) and no informational rents is illustrated in Figure 1, which plots the quadratic function on the right side of equation (32) against the constant on the left side, with R set to zero. Productivity differences move the horizontal line up and down in this figure, and the equilibrium job-finding rate adjusts along the quadratic curve. For standard parameter values, this curve is steep at the baseline equilibrium, and small productivity differences therefore have little effect on the job-finding

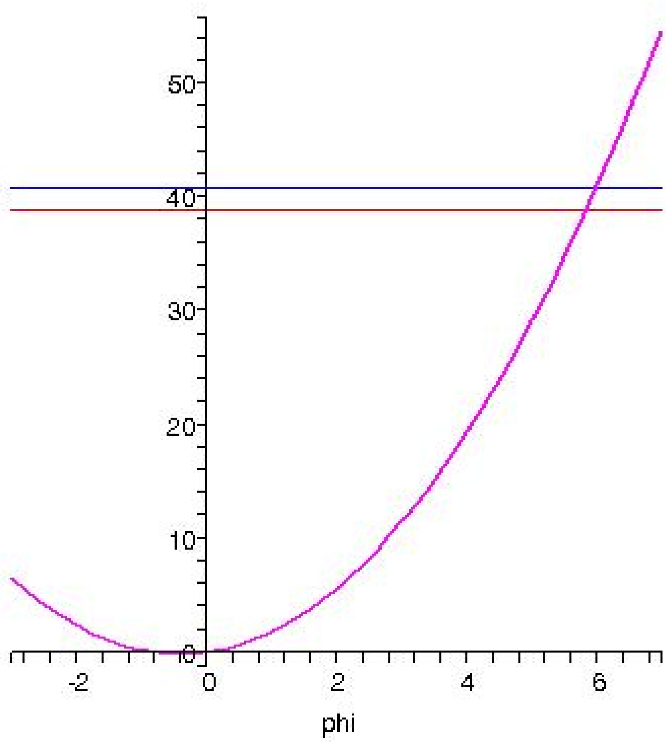

Figure 1

rate.[5]

The elasticity of the job-finding rate with respect to productivity with no informational rents is

$$\frac{\partial \log(\phi)}{\partial \log(y)} = \frac{\dfrac{\partial \log(\zeta)}{\partial \log(y)}}{\dfrac{\partial \log(\zeta)}{\partial \log(\phi)}} = \frac{y}{y - y_0} \frac{1}{\dfrac{\gamma}{1-\gamma} + \dfrac{\phi}{\phi + b}} \quad \textbf{(34)}$$

This elasticity is not large unless the match surplus is small.

[5]In this figure, μ_0 is chosen so that the job-finding rate in the good steady state matches the data. Using the baseline parameters from Table 1 below, with $\Delta y = 0$ and $y_2^L = 1.03$, setting $\mu_0 = 1360/21$ implies $\phi_2 = 6$. The horizontal lines are drawn for $y_1^L = 1$ and $y_2^L = 1.03$.

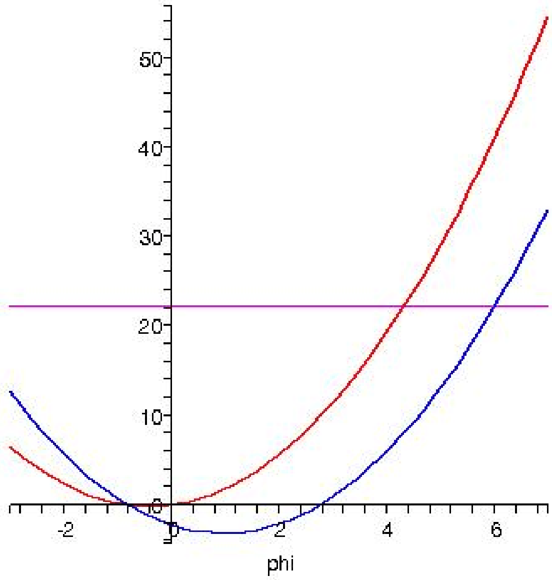

Figure 2

The effect of informational rents is shown in Figure 2. When R is positive, the quadratic curve shifts to the right (in the relevant region), and a comparison of the two curves shows that a small informational rent has a large effect on the equilibrium job-finding rate. On the other hand, the effect of (publicly observed) productivity movements remains small.[6]

5. **Unemployment Volatility**

The volatility of unemployment can be analyzed by comparing the steady-state levels of unemployment associated with each aggregate state (rather than measuring standard deviations in simulated data). Although this ignores movements along the transition paths from one steady state to the other, these transitions occur very rapidly, since the job-finding rate in the data is about 50% per month.

Standard parameter values are used as far as possible, following Shimer (2005) and Hall (2005). The interest rate is set at 5% per annum, and the job destruction rate δ is set at .35 per annum, so that the monthly rate is about 3%. The flow value of nonemployment is set

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[6]Here μ_0 is again chosen so that the job-finding rate in the good steady state matches the data. Using the baseline parameters from Table 1, with $p_2\Delta y = 3/100$ and $y_2^L = 1$, setting $\mu_0 = 1360/37$ implies $\phi_2 = 6$..

initially at 40% of the flow value of employment. The matching function is Cobb-Douglas. The exit rate from unemployment is about 50% per month in the data, so μ_0 is chosen to solve the equilibrium equations with $\phi_1\lambda_2 + \phi_2\lambda_1 = 6\ (\lambda_2 + \lambda_1)$, meaning that the average job-finding rate is 6 per annum, the average being taken with respect to the invariant distribution of the Markov process. The expected cost of filling a vacancy in state s is given by $cd_s = \dfrac{(\phi_s)^{\frac{\gamma}{1-\gamma}}}{\mu_0}$.

In the NBER postwar data, the average duration of a recession is about a year, and the average duration of an expansion is about 5 years. This implies that the exit hazards are $\lambda_2 = 1/5$ and $\lambda_1 = 1$. Shimer (2005) reports summary statistics for detrended labor productivity (output per person), using an HP filter with smoothing parameter 100,000: the standard deviation is .02 log points. Since the model in this paper assumes that productivity is a two-state process, it is perhaps better to measure volatility as the difference between the average levels of productivity during recessions and expansions. Using the same detrended productivity series, this difference is .028 log points. Letting Y_1 and Y_2 denote aggregate state-contingent productivity levels, this implies that Y_2 should be about 3% above Y_1, so Y_2 is set to 1.03, with Y_1 normalized at one.[7]

The variation in the informational rent is chosen so as to match the fluctuations in productivity. A simple way to do this is to set $(p_2 - p_1)\Delta y = .03$, with $y_1^L = y_2^L$, so that there are just two possible realizations of the surplus regardless of the aggregate state, but the probability of drawing the higher surplus is higher in the good state. For example, if there is no informational rent in the bad state ($p_1 = 0$), the rent in the good state is enough to account for the observed variation in aggregate productivity levels.

The parameter values are summarized in Table 1.

[7]Productivity could alternatively be measured as output per hour, and smaller smoothing parameters could also be justified. Since output per hour varies less than output per person, and smaller smoothing parameters (like the conventional choice of 1,600) attribute more of the variance to the trend component, these alternatives would give smaller volatility estimates. The point is that by any reasonable measure, labor productivity is not very volatile.

Table 1: Parameter Values			
Parameter	**Notation**	**Value**	**Comments**
matching function	$m(\theta)$	$\mu\theta^{1-\gamma}$	see text
recession exit hazard	λ_1	1	recession duration (1year)
expansion exit hazard	λ_2	1/5	expansion duration (5 years)
unmatched flow payoff	y_0	0.4	Shimer
low output	$y_1^L = y_2^L$	1	
informational rent	$p_2\Delta y$	0.030	volatility of labor productivity ($p_1 = 0$)
separation rate	δ	.35	Shimer
interest rate	r	.05	

The steady-state unemployment levels are determined in the usual way as

$$u_s^* = \frac{1}{1 + \frac{m(\theta_s)}{\delta}}$$

In the case of a (Cobb-Douglas) matching function that is symmetric in unemployment and vacancies ($\alpha = ½$), the equilibrium values of ϕ_1 and ϕ_2 for the parameters in Table 1 can be obtained from the following equations:

$$\begin{aligned} 9\mu_0 &= 20\phi_1^2 + 56\phi_1 - 40\phi_2 \\ 138\mu_0 &= 200\phi_2^2 + 240\phi_2 - 80\phi_1 - 15\mu_0\phi_2 \end{aligned}$$

When μ_0 is chosen so as to give an average job-finding rate of 6, the solution is ($\phi_1 = 4.295536223$, $\phi_2 = 6.340892756$, $\mu_0 = 39.54966078$). In this example, ρ and $\bar{\alpha}$ are given by

$$
\begin{aligned}
\overline{\rho}_1 &= \rho_1 = 0 \\
\rho_2 &= \frac{3}{40c} \\
\overline{\rho}_2 &= \left(1 + \frac{1}{2(1-p_2)}\right)\rho_2
\end{aligned}
\tag{35}
$$

Since there is no informational rent in the bad state, the no-screening condition is irrelevant in that state. In the good state the no-screening condition holds if $d_2 = \phi_2/\mu^2 \geq \overline{\rho}_2$. The equilibrium depends on p_s only through the effect of p_s on ρ_s (provided that the no-screening condition holds), and with $p_1 = 0$, ρ_2 depends on p_2 only through the product $p_2\Delta y$, which is set to 0.03. The no-screening condition then holds provided that $p_2 \leq 0.5605$.

Table 2 shows that informational rents can generate realistic variations in the unemployment rate. Even though the informational rent is only 3% of the productivity level, it moves the unemployment rate by about 40%. To put this in context, the table also shows the unemployment rates for a baseline parameter set that matches the variance of aggregate productivity by letting the match surplus depend on the aggregate state, with no idiosyncratic variation. These baseline parameter values are as in Table 1, but with $y_1^L = 1$, $y_2^L = 1.03$, and $p_2\Delta y = 0$. In this case, the unemployment rate is virtually constant. The table includes results for a symmetric Cobb-Douglas matching function, with $\nu = ½$, and also for the labor share and matching elasticity parameters used by Shimer ($\alpha = \nu = 0.72$). Although these parameters affect the level of unemployment, they have little effect on volatility.

Table 2: Unemployment Volatility					
		Baseline		Informational Rent	
Productivity Variation	y_2^L	1.03		1.0	
	$p_2\Delta y$	0		.03	
	$\nu = \alpha$	0.50	0.72	0.50	0.72
Steady State Unemployment Rates	u_1^*	5.61%	5.55%	**7.53%**	6.61%
	u_2^*	5.49%	5.48%	**5.23%**	4.73%

Hagedorn and Manovskii (2005) have argued that the Mortensen-Pissarides model can generate realistic unemployment fluctuations if the value of the worker's outside option is close to the value of production. In the model considered here, this means setting y_0 near 1. Hagedorn and Manovskii calibrated y_0 as .943, with $\nu = .061$. Table 3 explores the implications of these parameter values, in the model with no informational rents.

Table 3: Unemployment Volatility (no informational rent)						
		Baseline	High y_0	Low ν	High y_0 low ν	Higher y_0 low ν
Variant		$y_0 = .40$ $\nu = .5$	$y_0 = .943$ $\nu = .5$	$y_0 = .40$ $\nu = .061$	$y_0 = .943$ $\nu = .061$	$y_0 = y_1^L = 1$ $\nu = .061$
Steady State Unemployment Rates	u_1^*	5.61%	6.38%	5.58%	6.10%	8.21%
	u_2^*	5.49%	5.37%	5.50%	5.41%	5.17%

When the workers' outside opportunities are almost as good as their market production opportunities, unemployment is indeed more volatile. Mortensen and Nagypál (2005) argue that setting $y_0 = .943$ is quite unrealistic, since it implies that the *average* worker has little to gain from employment. Moreover, as Costain and Reiter (2006) and Hornstein, Krusell and Violante (2005) point out, this setting also implies implausibly large changes in unemployment rates in response to small changes in unemployment benefits. And even this rather extreme value of y_0 generates only about a 20% difference in the unemployment rates in the two states. The last column of the table shows that volatility increases sharply as y_0 approaches 1. It might seem that everyone should be unemployed in the bad state if $y_0 = 1$, since this means that jobs produce no surplus, and in order to move workers into jobs, it is necessary to expend resources on vacancy costs. But in fact the bad state is not expected to last very long, and jobs generate a (small) surplus in the good state. Moving some workers into jobs in the bad state reduces congestion in the matching process when the economy switches to the good state. If the transition to the good state is unlikely, the unemployment rate in the bad state will be high. But in the data, recessions are relatively short-lived, so the Hagedorn and Manovskii calibration yields a relatively small difference between the unemployment rates in the good and bad states. Table 4 shows that in a comparison of steady states with no transitions, the Hagedorn and Manovskii calibration gives much more volatility.[8] But this is largely beside the point, since the volatility in the data is generated by a single economy with transitions between states, while Table 4 compares the steady states of two different economies.[9]

Table 4: Unemployment Volatility with no transitions ($\lambda_1 = \lambda_2 = 0$)

		Baseline	High y_0	Low ν	High y_0 low ν	Informational Rent
Variant		$y_0 = .40$ $\nu = .5$	$y_0 = .943$ $\nu = .5$	$y_0 = .40$ $\nu = .061$	$y_0 = .943$ $\nu = .061$	$y_0 = .40$ $\nu = .5$
Steady State Unemployment Rates	u_1^*	5.65%	6.82%	5.69%	7.25%	7.51%
	u_2^*	5.51%	5.51%	5.51%	5.51%	5.51%

[8]Here μ_0 cannot be chosen so as to equate the average job-finding rate in the model with the empirical value, because each realization of the aggregate state is permanent, so there is no invariant distribution that can be used to take an average. Instead, μ_0 is chosen so that the job-finding rate in the good state matches the data ($\phi_2 = 6$).

[9]Hagedorn and Manovskii used a very low value for the labor share parameter ($\nu = .061$). Although this generates additional volatility in the comparison of two unrelated economies shown in Table 4, it actually reduces volatility in the more relevant comparison of steady states of a single stochastic economy, as shown in Table 3.

6. **Conclusion**

Rent is a powerful economic force, and private information is a pervasive rent source, so it is plausible that private information can help to explain features of the economy that are otherwise difficult to understand. It has been shown here that the introduction of private information in an otherwise standard model of unemployment fluctuations provides a reasonable explanation for the volatility of unemployment. In the standard Mortensen-Pissarides model, unemployment fluctuations are driven by labor productivity shocks. In the data, these shocks are small, and the implied fluctuations in unemployment are also small, and much smaller than the fluctuations in the data. But if the productivity realizations are privately observed by employers, the implications for unemployment fluctuations are quite different. Small productivity shocks generate informational rents for employers, and small rents are a powerful job creation force. Thus privately observed productivity shocks of the magnitude seen in the data can generate realistic unemployment fluctuations.

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