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AND EMPLOYMENT FLUCTUATIONS

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### **ABSTRACT**

Shimer (2003) pointed out that although we have a satisfactory theory of why some workers are unemployed at any given time, we don't know why the number of unemployed workers varies so much over time. The basic Mortensen-Pissarides (1994) model does not generate nearly enough volatility in unemployment, for plausible parameter values. This paper extends the Mortensen-Pissarides model to allow for informational rents. Productivity is subject to publicly observed aggregate shocks, and to idiosyncratic shocks that are seen only by the employer. It is shown that there is a unique equilibrium, provided that the idiosyncratic shocks are not too large. The main result is that small fluctuations in productivity that are privately observed by employers can give rise to a kind of wage stickiness in equilibrium, and the informational rents associated with this stickiness are sufficient to generate relatively large unemployment fluctuations.

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## **1. Introduction**

Shimer (2003) pointed out that although we have a satisfactory theory of why some workers are unemployed at any given time, we don't know why the number of unemployed workers varies so much over time. To a substantial extent these numbers vary because of movements into and out of the labor force. But even in the case of demographic groups who are firmly attached to the labor force, the variations are large. For example, over the period 1967-2004, the median annual unemployment rate of white men aged 35-39 was 3.7%. In 10 of these 38 years, the unemployment rate for this group was 4.4% or higher, while there were also 10 years with a rate of 2.6% or lower. The basic reason for unemployment is that no two workers are the same, and no two jobs are the same. But if the unemployment rate is a measure of how long it takes to match workers and jobs, why should the matching process be so much slower in some years than in others?

The basic Mortensen-Pissarides (1994) model does not generate nearly enough volatility in unemployment and vacancies, for plausible parameter values. Hall (2005) argued that this problem can be fixed if the Nash bargaining component of the model is dropped: Hall assumed that wages are sticky in the sense that the wage level in a previous contract establishes a "social norm" that largely determines the wage in the next contract. In the absence of a theory of social norms, this effectively requires the introduction of a free parameter. The question in this paper is whether an extension of the Mortensen-Pissarides model to allow for informational rents can explain the volatility of unemployment in a more parsimonious way. The main result is that relatively small fluctuations in productivity that are privately observed by employers can give rise to a kind of wage stickiness as an equilibrium phenomenon, and the informational rents associated with this stickiness are sufficient to generate relatively large unemployment fluctuations.

## **2. A Model of Sticky Wages with Private Information and Aggregate Shocks**

The model is a simplified version of the model analyzed in Kennan (2003). A successful job match generates a surplus to be divided between the worker and the employer. The value of the worker's output is modeled as a binary random variable whose realization ("L" for low or "H" for high) is observed privately by the employer when the match is made. The probability of

drawing a high surplus,  $p_s$ , is a publicly observed Markov pure jump process with two states ( $s = 1$  in the bad state and  $s = 2$  in the good state), and exit hazards  $\lambda_1$  and  $\lambda_2$ . The probability of the high surplus is assumed to be higher in the good state.<sup>2</sup> Job and worker flows are modeled in the standard way, following Mortensen and Pissarides (1994). When the joint continuation value from a match falls below the joint opportunity cost, the match is destroyed. The job destruction hazard rate is a constant,  $\delta$ , and there is a constant returns matching function that generates a flow of new matches  $M(N_U, N_V)$  from unemployment and vacancy stocks  $N_U$  and  $N_V$ . There is an infinitely elastic supply of potential vacancies, and the actual number of vacancies posted is such that the expected profit from a vacancy is zero.

The match surplus is divided in the following way. Either the employer or the worker is randomly selected to make an offer, and if this offer is rejected the match dissolves. Clearly, the employer's offer will just match the worker's reservation level, which is the value of searching for another match. The worker effectively has two choices: an offer that exhausts the low surplus, with a sure acceptance, or an offer that exhausts the high surplus, with acceptance only if the high surplus has actually been realized. It is assumed that the parameters are such that the worker always finds it optimal to demand the low surplus. Brügemann and Moscarini (2005) show that the volatility of unemployment remains implausibly low for a broad class of surplus-sharing rules: the Nash Bargaining rule is not the source of the problem. On the other hand if there is some stickiness in wages, the employers' incentive to create vacancies is magnified when the economy improves, and this increases unemployment volatility, as Hall pointed out. Brügemann and Moscarini (2005) rule out wage stickiness by assuming that the division of the surplus should be invariant to a change in the location of the productivity distribution. This assumption is very appealing in the case of complete information. But when the employer has private information, it is optimal for workers to ignore small changes in the productivity distribution, and this gives rise to a kind of wage stickiness. Menzio (2005) develops this idea in great detail, and derives a bargaining equilibrium in which transient productivity fluctuations that are privately observed by employers are not transmitted to wages.

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<sup>2</sup>In the numerical examples, the probability of the high surplus is assumed to be zero in the bad state. Tawara (2005) considers the quantitative implications of allowing for informational rents in both aggregate states.

The match surplus depends on whether the employer draws a high or low value from the output distribution, and it also depends on the aggregate state. Let  $y_s^L$  and  $S_s^L$  be the flow surplus and the continuation value of the match when the output value is low, and the aggregate state is  $s$ , and similarly when the output value is high. For simplicity, it is assumed that the difference between the low and high output values does not depend on the aggregate state. That is,  $y_2^H - y_2^L = y_1^H - y_1^L = \Delta y$ .

Let  $U$  denote the state-dependent continuation value of an unmatched worker, and let  $G$  denote the joint continuation value of a matched worker-employer pair. In the low-output state, the joint match values are determined by the following asset pricing equations

$$\begin{aligned} rG_1^L &= y_1^L - \delta G_1^L + \delta U_1 + \lambda_1 (G_2^L - G_1^L) \\ rG_2^L &= y_2^L - \delta G_2^L + \delta U_2 - \lambda_2 (G_2^L - G_1^L) \end{aligned} \tag{1}$$

This specification assumes that the flow value of a match depends only on the aggregate state. This rules out two interesting alternatives. First, there is no possibility of switching from low to high output, once the match has been made. Even in the absence of informational rents, this would tend to increase unemployment volatility, by strengthening the incentive to create vacancies when the aggregate state is good. Brügemann (2005) analyzes the magnitude of this “vintage productivity” effect. Second, the flow value is the same for all workers. Nágypál (2006) shows that heterogeneity in workers’ (private) evaluations of nonpecuniary job characteristics can substantially increase the volatility of unemployment.

It is assumed that there is free entry of employers, so that the continuation value of an unmatched employer is zero in all states, so the (state-dependent) match surplus is the difference between the gross continuation value  $G$  and the joint continuation value of an unmatched worker,  $U$ . Thus

$$\begin{aligned} (r + \delta)(S_1^L + U_1) &= y_1^L + \delta U_1 + \lambda_1 (S_2^L - S_1^L + \Delta U) \\ (r + \delta)(S_2^L + U_2) &= y_2^L + \delta U_2 - \lambda_2 (S_2^L - S_1^L + \Delta U) \end{aligned} \tag{2}$$

where  $\Delta U = U_2 - U_1$ . This implies

$$S_2^L - S_1^L + \Delta U = \frac{y_2^L - y_1^L + \delta \Delta U}{r + \delta + \Lambda} \quad (3)$$

where  $\Lambda = \lambda_1 + \lambda_2$ . Substituting this in (2) gives

$$\begin{aligned} (r + \delta)S_1^L &= y_1^L - rU_1 + \frac{\lambda_1(y_2^L - y_1^L + \delta \Delta U)}{r + \delta + \Lambda} \\ (r + \delta)S_2^L &= y_2^L - rU_2 - \frac{\lambda_2(y_2^L - y_1^L + \delta \Delta U)}{r + \delta + \Lambda} \end{aligned} \quad (4)$$

Similarly, for a high-output match, the surplus values are given by

$$\begin{aligned} (r + \delta)S_1^H &= y_1^H - rU_1 + \frac{\lambda_1(y_2^H - y_1^H + \delta \Delta U)}{r + \delta + \Lambda} \\ (r + \delta)S_2^H &= y_2^H - rU_2 - \frac{\lambda_2(y_2^H - y_1^H + \delta \Delta U)}{r + \delta + \Lambda} \end{aligned} \quad (5)$$

The effect of the aggregate state on the match surplus is given by

$$S_2^H - S_1^H = S_2^L - S_1^L = \frac{y_2^L - y_1^L - (r + \Lambda)\Delta U}{r + \delta + \Lambda} \quad (6)$$

Thus if an unmatched worker has better prospects when the aggregate state is good, the match surplus might be lower when the aggregate state is good, for a given output draw. On the other hand there is a higher probability of drawing a high output value in the good aggregate state.

The effect of the output draw on the match surplus is given by

$$S_2^H - S_2^L = S_1^H - S_1^L = \frac{\Delta y}{r + \delta} \quad (7)$$

The rate at which unemployed workers find new matches is  $M(N_U, N_V)/N_U = m(\theta)$ , where  $\theta = N_V/N_U$  represents market tightness, and  $m(\theta) = M(1, \theta)$ . The job-finding rate function  $m(\theta)$  is assumed to be strictly increasing, and concave. When a match is made, the worker is selected to make an offer with probability  $v$ . In this case, the worker gets the low-output surplus, and the employer gets an informational rent if the realized match value is high. If the employer is selected to make an offer, the worker gets the reservation level  $U$  and the employer gets the whole surplus. Thus an unmatched worker's continuation values are determined by the asset pricing equations

$$\begin{aligned} rU_1 &= y_0 + m(\theta_1)vS_1^L + \lambda_1(U_2 - U_1) \\ rU_2 &= y_0 + m(\theta_2)vS_2^L - \lambda_2(U_2 - U_1) \end{aligned} \quad (8)$$

where  $y_0$  is the flow value of unemployment (including unemployment benefits and the value of leisure). Thus

$$\begin{aligned} rU_1 &= y_0 + \frac{r + \lambda_2}{r + \Lambda} m(\theta_1)vS_1^L + \frac{\lambda_1}{r + \Lambda} m(\theta_2)vS_2^L \\ rU_2 &= y_0 + \frac{r + \lambda_1}{r + \Lambda} m(\theta_2)vS_2^L + \frac{\lambda_2}{r + \Lambda} m(\theta_1)vS_1^L \end{aligned} \quad (9)$$

Employers post new vacancies to the point where the net profit from doing so is zero. When a match is made, the employer gets an informational rent if the match value is high, and also gets a

fraction  $1-\nu$  of the low-output surplus (in expectation). Thus the zero-profit conditions implied by free entry are

$$\begin{aligned} 0 &= -c + \frac{m(\theta_1)}{\theta_1} \left( (1-\nu)S_1^L + p_1(S_1^H - S_1^L) \right) \\ 0 &= -c + \frac{m(\theta_2)}{\theta_2} \left( (1-\nu)S_2^L + p_2(S_2^H - S_2^L) \right) \end{aligned} \quad (10)$$

where  $c$  is the flow cost of maintaining a vacancy, and  $p_s$  is the probability of drawing the high match value, for  $s = 1, 2$ .

It is convenient to let  $d = \theta/m(\theta)$  denote the expected duration of a vacancy. Then the free-entry conditions can be written as

$$\begin{aligned} cd_1 &= (1-\nu)S_1^L + p_1(S_1^H - S_1^L) \\ cd_2 &= (1-\nu)S_2^L + p_2(S_2^H - S_2^L) \end{aligned} \quad (11)$$

The model can be solved as follows. For given values of  $d_1$  and  $d_2$ , the free entry conditions determine the low-state surplus values:

$$S_s^L = \frac{c(d_s - \alpha_s)}{1-\nu} \quad (12)$$

where

$$\alpha_s = \frac{p_s \Delta y}{c(r + \delta)} \quad (13)$$

for  $s = 1, 2$ .



Equation (2) can be rearranged to give  $U_1$  and  $U_2$  as linear functions of  $S_1^L$  and  $S_2^L$ , and  $U_1$  and  $U_2$  can then be expressed in terms of  $d_1$  and  $d_2$  as

$$\begin{aligned} rU_1 &= y_1^L + \frac{c(\delta + r)(d_1 - \alpha_1)}{1 - \nu} + \frac{\lambda_1(y_2^L - y_1^L)}{\Lambda + r} + \frac{\lambda_1 \delta c(d_2 - \alpha_2 - (d_1 - \alpha_1))}{(1 - \nu)(r + \delta)} \\ rU_2 &= y_2^L + \frac{c(\delta + r)(d_2 - \alpha_2)}{1 - \nu} - \frac{\lambda_2(y_2^L - y_1^L)}{\Lambda + r} + \frac{\lambda_2 \delta c(d_1 - \alpha_1 - (d_2 - \alpha_2))}{(1 - \nu)(r + \delta)} \end{aligned} \quad (14)$$

Next (12) can be substituted in (9), giving

$$\begin{aligned} rU_1 &= y_0 + \frac{r + \lambda_2}{r + \Lambda} \frac{\nu c m(\theta_1)}{1 - \nu} (d_1 - \alpha_1) + \frac{\lambda_1}{r + \Lambda} \frac{\nu c m(\theta_2)}{1 - \nu} (d_2 - \alpha_2) \\ rU_2 &= y_0 + \frac{r + \lambda_1}{r + \Lambda} \frac{\nu c m(\theta_2)}{1 - \nu} (d_2 - \alpha_2) + \frac{\lambda_2}{r + \Lambda} \frac{\nu c m(\theta_1)}{1 - \nu} (d_1 - \alpha_1) \end{aligned} \quad (15)$$

After eliminating  $U_1$  and  $U_2$ , this gives the following equations determining  $d_1$  and  $d_2$

$$\begin{aligned} \psi_1(d) &= Z_1 + \left( \frac{\alpha_1}{d_1} - 1 \right) \nu H(d_1) - (r + \delta + \lambda_1)(d_1 - \alpha_1) + \lambda_1(d_2 - \alpha_2) = 0 \\ \psi_2(d) &= Z_2 + \left( \frac{\alpha_2}{d_2} - 1 \right) \nu H(d_2) - (r + \delta + \lambda_2)(d_2 - \alpha_2) + \lambda_2(d_1 - \alpha_1) = 0 \end{aligned} \quad (16)$$

where  $H(d) = \theta$ , and

$$Z_s = \frac{(1 - \nu)(y_s^L - y_0)}{c} \quad (17)$$

for  $s = 1, 2$ .

It is assumed that the function  $d = \theta/m(\theta)$  is invertible, for any nonnegative  $d$ , and that the inverse function  $\theta = H(d)$  is convex.<sup>3</sup>

### Proposition 1

If the function  $\theta = H(d)$  is convex, and if  $H(0) = 0$ , then there is a unique vector  $d^* = (d_1^*, d_2^*)$  such that  $\psi(d^*) = 0$ .

The proof uses the following result.

### Lemma

Suppose  $\alpha$  is a positive number, and  $H$  is a twice differentiable function, with  $H(0) = 0$ ,  $H'(x) > 0$  and  $H''(x) > 0$ , for  $x > \alpha$ . Define the function  $h$ , on the domain  $[\alpha, \infty)$ , as

$$h(x) = \left( \frac{\alpha}{x} - 1 \right) H(x) \quad (18)$$

Then  $h'(x) < 0$  and  $h''(x) < 0$ .

### Proof

The first and second derivatives of  $h$  are as follows

$$\begin{aligned} h'(x) &= \left( \frac{\alpha}{x} - 1 \right) H'(x) - \frac{\alpha}{x^2} H(x) \\ h''(x) &= \left( \frac{\alpha}{x} - 1 \right) H''(x) - 2 \frac{\alpha}{x^2} H'(x) + 2 \frac{\alpha}{x^3} H(x) \\ &= \left( \frac{\alpha}{x} - 1 \right) H''(x) + 2 \frac{\alpha}{x^3} (H(x) - xH'(x)) \end{aligned} \quad (19)$$

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<sup>3</sup>Invertibility is implied by strict concavity of  $m$ , with  $m(0) = 0$ . Indeed if a given value of  $d$  were consistent with two different values of  $\theta$ , say  $\theta_1 = \gamma\theta_2$ , with  $0 < \gamma < 1$ , then we would have  $dm(\theta_1) - \theta_1 = dm(\theta_2) - \theta_2 = 0$ , but  $dm(\theta_1) - \theta_1 > \gamma dm(\theta_2) - \gamma\theta_2$ , since  $\theta_1$  is a convex combination of  $\theta_2$  and 0, and  $m(0) = 0$ . This gives a contradiction. In general, a given value of  $d$  might not be consistent with any value of  $\theta$ . For example, if  $d = 1$  and  $m(\theta) = \theta + \log(1+\theta)$  the equation  $d = \theta/m(\theta)$  has no solution, since  $m(\theta)/\theta > 1$ .

Since  $x \geq \alpha$ , and  $H'(x) > 0$ , it is clear that  $h$  is decreasing. The function  $H(x)/x$  is increasing. In fact if  $x_1 < x_2$  then  $x_1 = \gamma x_2$  and  $H(x_1) \leq \gamma H(x_2) + (1-\gamma)H(0) = \gamma H(x_2)$ , because  $H$  is convex and  $H(0) = 0$ , so  $H(x_1)/x_1 \leq H(x_2)/x_2$ . The derivative of  $H(x)/x$  is  $(xH'(x) - H(x))/x^2$ , so  $H(x) - xH'(x) \leq 0$ . Thus  $h$  is concave.

### Proof of Proposition 1

First it will be shown that  $\psi(d^*) = 0$  implies  $d^* > \alpha$ . If  $d_1 \leq \alpha_1$  and  $d_2 \geq \alpha_2$  then  $\psi_1(d) > 0$ ; and if  $d_1 \geq \alpha_1$  and  $d_2 \leq \alpha_2$  then  $\psi_2(d) > 0$ . If  $d \leq \alpha$ , write  $\psi(d)$  as

$$\begin{aligned}\psi_1(d) &= Z_1 + (\alpha_1 - d_1) \nu \frac{H(d_1)}{d_1} - (r + \delta)(d_1 - \alpha_1) + \lambda_1[(d_2 - \alpha_2) - (d_1 - \alpha_1)] \\ \psi_2(d) &= Z_2 + (\alpha_2 - d_2) \nu \frac{H(d_2)}{d_2} - (r + \delta)(d_2 - \alpha_2) - \lambda_2[(d_2 - \alpha_2) - (d_1 - \alpha_1)]\end{aligned}\tag{20}$$

These equations show that either  $\psi_1(d)$  or  $\psi_2(d)$  is a sum of four positive terms: the first three terms are positive in both equations, and if the last term is negative in the first equation, it is positive in the second, and vice versa. Thus  $\psi(d) \neq 0$  if  $d \leq \alpha$ .

Next it will be shown that a solution exists. Note that  $\psi(\alpha) = Z > 0$ . Define  $A$  as the solution of the linear equations obtained by setting  $H = 0$ . Then

$$A_s = \alpha_s + \frac{\bar{Z}_s}{r + \delta}\tag{21}$$

where

$$\begin{aligned}\bar{Z}_1 &= \frac{(r + \delta + \lambda_2)Z_1 + \lambda_1 Z_2}{r + \delta + \lambda_1 + \lambda_2} \\ \bar{Z}_2 &= \frac{(r + \delta + \lambda_1)Z_2 + \lambda_2 Z_1}{r + \delta + \lambda_1 + \lambda_2}\end{aligned}\tag{22}$$

Thus  $A > \alpha$  and  $\psi(A) < 0$ .

Since  $\psi_1$  is increasing in  $d_2$  and decreasing in  $d_1$ , the equation  $\psi_1(d) = 0$  can be solved to obtain  $d_2$  as an increasing function of  $d_1$ . Write this as  $d_2 = \Upsilon_1(d_1)$ . Since  $\psi_2$  is increasing in  $d_1$  and decreasing in  $d_2$ , the equation  $\psi_2(d) = 0$  can also be solved to obtain  $d_2$  as an increasing function of  $d_1$ . Write this as  $d_2 = \Upsilon_2(d_1)$ . Define the function  $\Upsilon_0(x) = \Upsilon_2(x) - \Upsilon_1(x)$ . Since  $\psi_1(\alpha_1, \Upsilon_1(\alpha_1)) = 0$ , and  $\psi_1(\alpha_1, \alpha_2) > 0$ , and  $\psi_1$  is increasing in  $d_2$ , it follows that  $\Upsilon_1(\alpha_1) < \alpha_2$ . Also, since  $\psi_2(\alpha_1, \Upsilon_2(\alpha_1)) = 0$ , and  $\psi_2(\alpha_1, \alpha_2) > 0$ , and  $\psi_2$  is decreasing in  $d_2$ , it follows that  $\Upsilon_2(\alpha_1) > \alpha_2$ . Therefore  $\Upsilon_0(\alpha_1)$  is positive. By a similar argument,  $\Upsilon_0(A_1)$  is negative. Also,  $\Upsilon_0$  is continuous (since  $\psi_1$  is linear in  $d_2$  and  $\psi_2$  is linear in  $d_1$ ). So by the intermediate value theorem  $\Upsilon_2(x) = \Upsilon_1(x)$  for some  $x \in [\alpha_1, A_1]$ . This means that  $\psi(x, \Upsilon_1(x)) = 0$ , showing that a solution exists.

To show uniqueness, define the function  $g(z) = \psi(\alpha + z)$ . Then  $g_1$  is increasing in  $z_2$  and  $g_2$  is increasing in  $z_1$ , and both  $g_1$  and  $g_2$  are concave, and  $g(0) > 0$ . Therefore, by Theorem 1 in Kennan (2001),  $g$  has at most one positive root, meaning that  $\psi$  has at most one root above  $\alpha$ . Since it has already been shown that  $\psi$  does have a root above  $\alpha$ , and no roots anywhere else, the proof is complete.

### Optimality of Pooling Offers

It is assumed that when a match is made in the good aggregate state, and the worker is selected to make an offer, it is optimal to demand the low surplus, rather than demand the high surplus at the risk of destroying the match. Thus the equilibrium surplus values must satisfy the following no-screening conditions

$$\begin{aligned} S_1^L &\geq p_1 S_1^H = p_1 \left( S_1^L + \frac{\Delta y}{r + \delta} \right) \\ S_2^L &\geq p_2 S_2^H = p_2 \left( S_2^L + \frac{\Delta y}{r + \delta} \right) \end{aligned} \tag{23}$$

which can be written as

$$S_s^L \geq \frac{p_s}{1-p_s} \frac{\Delta y}{c(r+\delta)} \quad (24)$$

for  $s = 1, 2$ . Using the free entry conditions, this reduces to

$$d_s \geq \bar{\alpha}_s \quad (25)$$

where

$$\bar{\alpha}_s = \left(1 + \frac{1-\nu}{1-p_s}\right) \left(\frac{p_s \Delta y}{c(r+\delta)}\right) = \left(1 + \frac{1-\nu}{1-p_s}\right) \alpha_s \quad (26)$$

Since  $\alpha_s = 0$  for  $p_s = 0$ , Proposition 1 implies that a unique equilibrium satisfying the no-screening conditions exists if  $p_s$  is small enough. Conversely, the no-screening condition fails as  $p_s$  approaches 1, as of course it should.

### Theorem 1

If  $H(d)$  is a convex function, with  $H(0) = 0$ , and if  $\psi(\bar{\alpha}) \geq 0$ , then a unique equilibrium exists.

### Proof

By Proposition 1, there is a unique vector  $d^*$  such that  $\psi(d^*) = 0$ . Since  $\psi(\bar{\alpha}) \geq 0$  and  $\psi(A) < 0$ , the argument used in the proof of Proposition 1 can be used to show that  $\psi$  has a root in the rectangle  $[\bar{\alpha}, A]$ , and since there is only one root above  $\bar{\alpha}$ , this root is  $d^*$ . The no-screening conditions are satisfied because  $d^* \geq \bar{\alpha}$ . Therefore  $d^*$  is the unique equilibrium.

Theorem 1 characterizes a set of parameter values for which an equilibrium exists, and it shows that if the parameters lie in this set, the equilibrium is unique.

### 3. Unemployment Volatility

The volatility of unemployment can be analyzed by comparing the steady-state levels of unemployment associated with each aggregate state (rather than measuring standard deviations in simulated data). Although this ignores movements along the transition paths from one steady state to the other, these transitions occur very rapidly, since the job-finding rate in the data is about 50% per month.

Standard parameter values are used as far as possible, following Shimer (2005) and Hall (2005). The interest rate is set at 5% per annum, and the job destruction rate  $\delta$  is set at .35 per annum, so that the monthly rate is about 3%. The flow value of nonemployment is set initially at 40% of the flow value of employment.

The simplest choice for the matching technology is a constant-returns Cobb-Douglas function,  $M = \mu U^\gamma V^{1-\gamma}$ , with  $m(\theta) = \mu \theta^{1-\gamma}$ . In this case the equilibrium conditions (16) can be stated as

$$\begin{aligned}\zeta_1 &= (\nu\phi_1 + r + \delta)\xi_1 + \lambda_1(\xi_1 - \xi_2) \\ \zeta_2 &= (\nu\phi_2 + r + \delta)\xi_2 + \lambda_2(\xi_2 - \xi_1)\end{aligned}\tag{27}$$

where  $\phi_s = m(\theta_s)$  is the job-finding rate, and where

$$\begin{aligned}\xi_s &= \left(\phi_s\right)^{\frac{\gamma}{1-\gamma}} - \frac{\mu^{\frac{1}{1-\gamma}}}{c} \frac{p_s \Delta y}{r + \delta} \\ \zeta_s &= \frac{\mu^{\frac{1}{1-\gamma}}}{c} (1 - \nu)(y_s^L - y_0)\end{aligned}\tag{28}$$

Thus, as Shimer (2005) noted, the parameters  $c$  and  $\mu$  enter only through the ratio  $\mu_0 = \frac{\mu^{\frac{1}{1-\gamma}}}{c}$ .

This ratio is set so that the average job-finding rate matches the data. The exit rate from unemployment is about 50% per month in the data, so  $\mu_0$  is chosen to solve the equilibrium equations with  $\phi_1\lambda_2 + \phi_2\lambda_1 = 6(\lambda_2 + \lambda_1)$ , meaning that the average job-finding rate is 6 per

annum, the average being taken with respect to the invariant distribution of the Markov process.

The expected cost of filling a vacancy in state  $s$  is given by  $cd_s = \frac{(\phi_s)^{\frac{\gamma}{1-\gamma}}}{\mu_0}$ .

In the NBER postwar data, the average duration of a recession is about a year, and the average duration of an expansion is about 5 years. This implies that the exit hazards are  $\lambda_2 = 1/5$  and  $\lambda_1 = 1$ . Shimer (2005) reports summary statistics for detrended labor productivity (output per person), using an HP filter with smoothing parameter 100,000: the standard deviation is .02 log points. Since the model in this paper assumes that productivity is a two-state process, it is perhaps better to measure volatility as the difference between the average levels of productivity during recessions and expansions. Using the same detrended productivity series, this difference is .028 log points. Letting  $Y_1$  and  $Y_2$  denote aggregate state-contingent productivity levels, this implies that  $Y_2$  should be about 3% above  $Y_1$ , so  $Y_2$  is set to 1.03, with  $Y_1$  normalized at one.<sup>4</sup>

The magnitude of the informational rent is chosen so as to match the fluctuations in productivity. The simplest way to do this is to set  $p_1\Delta y = 0$  and  $p_2\Delta y = .03$ , meaning that there is no informational rent in the bad state, and the rent in the good state is enough to account for the observed variation in aggregate productivity levels.

The parameter values are summarized in Table [1](#).

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<sup>4</sup>Productivity could alternatively be measured as output per hour, and smaller smoothing parameters could also be justified. Since output per hour varies less than output per person, and smaller smoothing parameters (like the conventional choice of 1,600) attribute more of the variance to the trend component, these alternatives would give smaller volatility estimates. The point is that by any reasonable measure, labor productivity is not very volatile.

Table 1: Parameter Values			
Parameter	Notation	Value	Comments
matching function	$m(\theta)$	$\mu\theta^{1-\gamma}$	see text
recession exit hazard	$\lambda_1$	1	recession duration (1year)
expansion exit hazard	$\lambda_2$	1/5	expansion duration (5 years)
unmatched flow payoff	$y_0$	0.4	Shimer
low output	$y_1^L$	1	normalization
informational rent	$p_2\Delta y$	0.030	volatility of labor productivity
separation rate	$\delta$	.35	Shimer
interest rate	$r$	.05	

The steady-state unemployment levels are determined in the usual way as

$$u_s^* = \frac{1}{1 + \frac{m(\theta_s)}{\delta}}$$

In the case of a (Cobb-Douglas) matching function that is symmetric in unemployment and vacancies ( $\gamma = 1/2$ ), the equilibrium values of  $\phi_1$  and  $\phi_2$  for the parameters in Table 1 can be obtained from the following equations:

$$\begin{aligned} 9\mu_0 &= 20\phi_1^2 + 56\phi_1 - 40\phi_2 \\ 138\mu_0 &= 200\phi_2^2 + 240\phi_2 - 80\phi_1 - 15\mu_0\phi_2 \end{aligned}$$

When  $\mu_0$  is chosen so as to give an average job-finding rate of 6, the solution is ( $\phi_1 = 4.295536223$ ,  $\phi_2 = 6.340892756$ ,  $\mu_0 = 39.54966078$ ). In this example,  $\alpha$  and  $\bar{\alpha}$  are given by



$$\begin{aligned}
\bar{\alpha}_1 &= \alpha_1 = 0 \\
\alpha_2 &= \frac{3}{40c} \\
\bar{\alpha}_2 &= \left(1 + \frac{1}{2(1-p_2)}\right) \alpha_2
\end{aligned} \tag{29}$$

Since there is no informational rent in the bad state, the no-screening condition is irrelevant in that state. In the good state the no-screening condition holds if  $d_2 = \phi_2/\mu^2 \geq \bar{\alpha}_2$ . The equilibrium depends on  $p_s$  only through the effect of  $p_s$  on  $\alpha_s$  (provided that the no-screening condition holds), and with  $p_1 = 0$ ,  $\alpha_2$  depends on  $p_2$  only through the product  $p_2\Delta y$ , which is set to 0.03. The no-screening condition then holds provided that  $p_2 \leq 0.5605$ .

Table 2 shows that informational rents can generate realistic variations in the unemployment rate. Even though the informational rent is only 3% of the productivity level, it moves the unemployment rate by about 40%. To put this in context, the table also shows the unemployment rates for a baseline parameter set that matches the variance of aggregate productivity by letting the match surplus depend on the aggregate state, with no idiosyncratic variation. These baseline parameter values are as in Table 1, but with  $y_1^L = 1$ ,  $y_2^L = 1.03$ , and  $p_2\Delta y = 0$ . In this case, the unemployment rate is virtually constant. The table includes results for a symmetric Cobb-Douglas matching function, with  $v = 1/2$ , and also for the labor share and matching elasticity parameters used by Shimer ( $\gamma = v = 0.72$ ). Although these parameters affect the level of unemployment, they have little effect on volatility.

Table 2: Unemployment Volatility					
		Baseline		Informational Rent	
Productivity Variation	$y_2^L$	1.03		1.0	
	$p_2 \Delta y$	0		.03	
	$v = \gamma$	0.50	0.72	0.50	0.72
Steady State Unemployment Rates	$u_1^*$	5.61%	5.55%	7.53%	6.61%
	$u_2^*$	5.49%	5.48%	5.23%	4.73%

Hagedorn and Manovskii (2005) have argued that the Mortensen-Pissarides model can generate realistic unemployment fluctuations if the value of the worker's outside option is close to the value of production. In the model considered here, this means setting  $y_0$  near 1. Hagedorn and Manovskii calibrated  $y_0$  as .943, with  $v = .061$ . Table 3 explores the implications of these parameter values, in the model with no informational rents.

Table 3: Unemployment Volatility (no informational rent)						
		Baseline	High $y_0$	Low $v$	High $y_0$ low $v$	Higher $y_0$ low $v$
Variant		$y_0 = .40$ $v = .5$	$y_0 = .943$ $v = .5$	$y_0 = .40$ $v = .061$	$y_0 = .943$ $v = .061$	$y_0 = y_1^L = 1$ $v = .061$
Steady State	$u_1^*$	5.61%	6.38%	5.58%	6.10%	8.21%
Unemployment Rates	$u_2^*$	5.49%	5.37%	5.50%	5.41%	5.17%

When the workers' outside opportunities are almost as good as their market production opportunities, unemployment is indeed more volatile. Mortensen and Nagypál (2005) argue that setting  $y_0 = .943$  is quite unrealistic, since it implies that the *average* worker has little to gain from employment. Moreover, Hornstein, Krusell and Violante point out that it also implies implausibly large changes in unemployment rates in response to small changes in unemployment benefits. And even this rather extreme value of  $y_0$  generates only about a 20% difference in the

unemployment rates in the two states. The last column of the table shows that volatility increases sharply as  $y_0$  approaches 1. It might seem that everyone should be unemployed in the bad state if  $y_0 = 1$ , since this means that jobs produce no surplus, and in order to move workers into jobs, it is necessary to expend resources on vacancy costs. But in fact the bad state is not expected to last very long, and jobs generate a (small) surplus in the good state. Moving some workers into jobs in the bad state reduces congestion in the matching process when the economy switches to the good state. If the transition to the good state is unlikely, the unemployment rate in the bad state will be high. But in the data, recessions are relatively short-lived, so the Hagedorn and Manovskii calibration yields a relatively small difference between the unemployment rates in the good and bad states. Table 4 shows that in a comparison of steady states with no transitions, the Hagedorn and Manovskii calibration gives much more volatility.<sup>5</sup> But this is largely beside the point, since the volatility in the data is generated by a single economy with transitions between states, rather than the steady states of two different economies.<sup>6</sup>

<b>Table 4: Unemployment Volatility</b> with no transitions ( $\lambda_1 = \lambda_2 = 0$ )						
		Baseline	High $y_0$	Low $v$	High $y_0$ low $v$	Informational Rent
Variant		$y_0 = .40$ $v = .5$	$y_0 = .943$ $v = .5$	$y_0 = .40$ $v = .061$	$y_0 = .943$ $v = .061$	$y_0 = .40$ $v = .5$
Steady State	$u_1^*$	5.65%	6.82%	5.69%	7.25%	7.51%
Unemployment Rates	$u_2^*$	5.51%	5.51%	5.51%	5.51%	5.51%

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<sup>5</sup>Here  $\mu_0$  cannot be chosen so as to equate the average job-finding rate in the model with the empirical value, because each realization of the aggregate is permanent, so there is no invariant distribution that can be used to take an average. Instead,  $\mu_0$  is chosen so that the job-finding rate in the good state matches the data ( $\phi_2 = 6$ ).

<sup>6</sup>Hagedorn and Manovskii used a very low value for the labor share parameter ( $v = .061$ ). Although this generates additional volatility in the comparison of two unrelated economies shown in Table 4, it actually reduces volatility in the more relevant comparison of steady states of a single stochastic economy, as shown in Table 3.

#### 4. The Effects of Informational Rents

Suppose that there are no transitions, and that the wage rate is fixed, as in Hall (2005). Then the free entry condition is

$$cd = \frac{y^L - w + p\Delta y}{r + \delta} \quad (30)$$

The right side of this equation represents the expected present value of profits from a filled vacancy, and the left side is the expected cost of filling the vacancy. A change in productivity, with a fixed wage, is offset by an increase in the vacancy duration. If the profit flow is small (because the wage is high), a small productivity change implies a large proportional change in profits, and therefore a large proportional change in the vacancy duration, which translates directly into the unemployment rate.

In the standard Mortensen-Pissarides model, the wage is a nested weighted average of the productivity levels while employed ( $y$ ) and while unemployed ( $y_0$ ). In the informational rents model, the wage is determined in exactly the same way, assuming the low realization of the productivity shock ( $y_L$ ). That is,

$$w = \frac{\phi}{\phi + A} y + \frac{A}{\phi + A} w_0 \quad (31)$$

where

$$w_0 = \nu y + (1 - \nu) y_0 \quad (32)$$

The free entry condition then be written as

$$cd = \frac{1}{\phi + A} \frac{1 - \nu}{\nu} (y - y_0) + \frac{p\Delta y}{r + \delta} \quad (33)$$

The result for the standard model (with  $p\Delta y = 0$ ) differs from the fixed wage result in two respects. First, for a given job-finding rate, a large proportional change in  $d$  requires a large proportional change in the flow surplus from employment (rather than in the flow profit). This means that small productivity shocks do not cause large unemployment movements unless the

flow surplus is small (as in the Hagedorn-Manovskii calibration). Second, an increase in  $d$  implies an increase in  $\phi$ , and this dampens the relationship between productivity and unemployment: workers receive a larger share of the flow surplus when an increase in the job-finding rate increases the continuation value of being unemployed, and this diminishes the incentive to create vacancies.

When the wage is fixed, small productivity changes lead to large proportional changes in profits, which are transmitted directly to the unemployment rate. Informational rents act in much the same way, even when wages are not fixed, because small productivity changes that are observed privately by employers do not affect wages. The Nash wage is close to the productivity level, for standard parameter values, so the profit flow  $y^L - w$  is small in equation (30). Changes in  $p\Delta y$  therefore give rise to large proportional changes in profits, and in the unemployment rate.

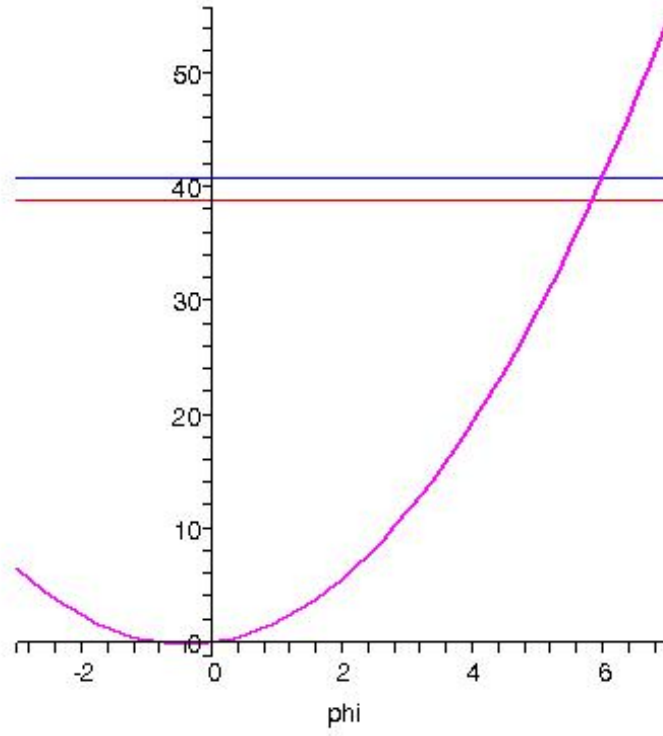
The equilibrium relationships between productivity, informational rents and the unemployment rate can be characterized more explicitly in the case of a Cobb-Douglas matching function. In this case, the main equilibrium condition (16) with no transitions reduces to (two copies of) the following equation:

$$\zeta = \left( \phi^{\frac{\gamma}{1-\gamma}} - R \right) (\phi + A) \quad (34)$$

where  $\phi$  is the job-finding rate, and

$$R = \frac{\mu_0 p \Delta y}{r + \delta}, \zeta = \frac{1 - \nu}{\nu} \mu_0 (y - y_0), A = \frac{r + \delta}{\nu} \quad (35)$$

The effect of productivity fluctuations with a square-root matching function ( $\gamma = 1/2$ ) and no informational rents is illustrated in Figure 1, which plots the quadratic function on the right side of equation (34) against the constant on the left side, with  $R$  set to zero. Productivity shocks move the horizontal line up and down in this figure, and the equilibrium job-finding rate adjusts along the quadratic curve. Since this curve is steep at the baseline equilibrium, small productivity



**Figure 1**

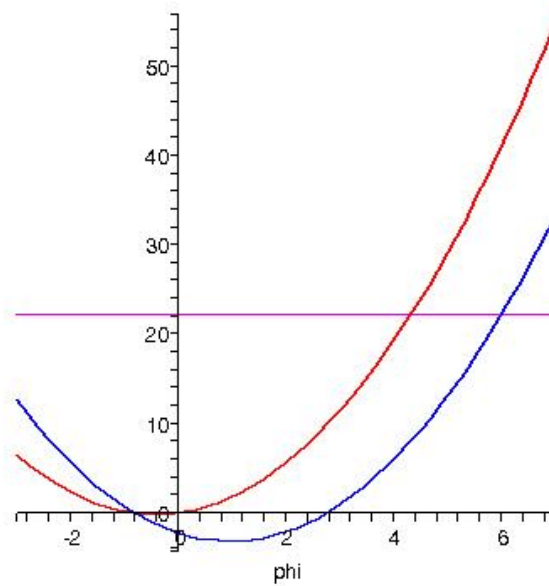
shocks have little effect on the job-finding rate.<sup>7</sup>

The elasticity of the job-finding rate with respect to productivity in this case is

$$\frac{\partial \log(\phi)}{\partial \log(y)} = \frac{y}{y - y_0} \frac{1}{\frac{\gamma}{1 - \gamma} + \frac{\phi}{\phi + A}} \quad (36)$$

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<sup>7</sup>In this figure,  $\mu_0$  is chosen so that the job-finding rate in the good steady state matches the data. Using the baseline parameters from Table 1, with  $\Delta_y = 0$  and  $y_2^L = 1.03$ , setting  $\mu_0 = 1360/21$  implies  $\phi_2 = 6$ . The horizontal lines are drawn for  $y_1^L = 1$  and  $y_2^L = 1.03$  (with  $\mu_0$  fixed at  $1360/21$ ).



**Figure 2**

The effect of informational rents is shown in Figure 2. When  $R$  is positive, the quadratic curve shifts to the right, and a small informational rent has a large effect on the equilibrium job-finding rate. On the other hand, the effect of (publicly observed) productivity movements remains small.<sup>8</sup>

## 5. Conclusion

Rent is a powerful economic force, and private information is a pervasive rent source, so it is plausible that private information can help to explain features of the economy that are otherwise difficult to understand. It has been shown here that the introduction of private information in an otherwise standard model of unemployment fluctuations provides a reasonable explanation for the volatility of unemployment. In the standard Mortensen-Pissarides model, unemployment fluctuations are driven by labor productivity shocks. In the data, these shocks are small, and the implied fluctuations in unemployment are also small, and much smaller than the fluctuations in

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<sup>8</sup>Here  $\mu_0$  is again chosen so that the job-finding rate in the good steady state matches the data. Using the baseline parameters from Table 1, with  $p_2\Delta y = 3/100$  and  $y_2^L = 1$ , setting  $\mu_0 = 1360/37$  implies  $\phi_2 = 6$ .

the data. But if the productivity realizations are privately observed by employers, the implications for unemployment fluctuations are quite different. Small productivity shocks generate informational rents for employers, and small rents are a powerful job creation force. Thus privately observed productivity shocks of the magnitude seen in the data can generate realistic unemployment fluctuations.



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