

NBER WORKING PAPER SERIES

ON OVERBORROWING

Martin Uribe

Working Paper 11913

<http://www.nber.org/papers/w11913>

NATIONAL BUREAU OF ECONOMIC RESEARCH

1050 Massachusetts Avenue

Cambridge, MA 02138

January 2006

I would like to thank Stephanie Schmitt-Grohé for comments and discussions and Arthur X. Liu and Javier Garcia-Cicco for research assistance. Newer versions of this paper are maintained at [www.econ.duke.edu/~uribe](http://www.econ.duke.edu/~uribe). The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

©2006 by Martin Uribe. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

On Overborrowing  
Martin Uribe  
NBER Working Paper No. 11913  
January 2006  
JEL No. F4

**ABSTRACT**

This paper characterizes the equilibrium dynamics in an economy facing an aggregate debt ceiling. This borrowing limit is intended to capture an environment in which foreign investors base their lending decisions predominantly upon macro indicators. Individual agents do not internalize the borrowing constraint. Instead, a country interest-rate premium emerges to clear the financial market. The implied equilibrium dynamics are compared to those arising from a model in which the debt ceiling is imposed at the level of each individual agent. The central finding of the paper is that the economy with the aggregate borrowing limit does not generate higher levels of debt than the economy with the individual borrowing limit. That is, there is no overborrowing in equilibrium.

Martin Uribe  
Department of Economics  
Duke University  
Durham, NC 27708-0097  
and NBER  
uribe@duke.edu

# 1 Introduction

A central question in emerging-market macroeconomics is what factors lead countries to accumulate excessive levels of external debt. It is often argued by economic observers and policymakers that emerging markets tend to overborrow when the lending decisions of foreign financial institutions are guided by rough indicators of the emerging country's macroeconomic performance and not by careful assessment of individual borrowers' abilities to repay. This is because individual agents fail to internalize the effect their own borrowing decisions have on the country's aggregate credit conditions. Overborrowing, it is argued, makes emerging countries prone to balance-of-payments crises, or sudden stops, and calls for government policy aimed at putting sand in the wheels of external finance. The contribution of this paper is to investigate whether the type of lending practices described above indeed lead emerging countries to overborrow.

To this end, I characterize the equilibrium dynamics of a small open economy subject to an aggregate borrowing constraint. I have in mind a situation in which foreign lenders lack the ability or the incentives to monitor individual investment projects in the emerging country and instead base their lending decisions on observation of a few macroeconomic indicators, such as total external debt or output growth. Individual agents do not internalize the credit constraint. I assume that in this economy credit rationing is implemented through a market mechanism. Specifically, when the aggregate debt limit is reached, an interest-rate premium emerges in the domestic economy that ensures that individual borrowing decisions are collectively compatible with the aggregate credit constraint. I compare the equilibrium dynamics of this economy to those of an economy in which the borrowing limit is imposed at the level of each individual agent.

The specific question that my investigation aims to address is whether the economy with the aggregate debt limit tends to overborrow relative to the economy with debt limits imposed at the level of each individual agent. I find that there is no overborrowing in equilibrium. The reason is that in the economy with the aggregate credit constraint, market incentives, conveyed by the interest rate, induce individual saving decisions that are identical to those caused by the imposition of agent-specific debt limits.

Section 2 presents a simple model of a small open economy facing an aggregate borrowing ceiling. Section 3 presents an economy where the debt limit is imposed at the individual level. Section 4 establishes analytically the central result of no overborrowing. It shows analytically that when the rents from financial rationing accrue to domestic residents, the equilibrium dynamics in the economy with the aggregate debt limit and in the economy with the individual debt limit are identical. Section 5 studies the case in which rents from financial

rationing accrue to the foreign lenders. In this case, the economies with an individual and an aggregate debt limit can no longer be compared analytically. However, I establish numerically the absence of overborrowing. Section 6 discusses the robustness of the central result and suggests relevant extensions.

## 2 An Economy With An Aggregate Borrowing Ceiling

Consider an economy populated by a large number of identical households with preferences defined over consumption of a perishable good,  $c_t$ , and labor effort,  $h_t$ , and described by the utility function

$$E_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t), \quad (1)$$

where  $U$  denotes the period utility function, which is assumed to be increasing in the first argument, decreasing in the second argument, strictly concave, and twice continuously differentiable,  $\theta_t/\theta_{t-1}$  denotes the subjective discount factor. In modeling small open economies, it is typically assumed, as a way to ensure stationary equilibrium dynamics, that the subjective rate of discount is a function of endogenous variables (see, for instance, Schmitt-Grohé and Uribe, 2003, and the references cited therein). Here, I assume that  $\theta_0 = 1$  and that  $\theta_t/\theta_{t-1} = \beta(C_t, H_t)$ , where  $C_t$  and  $H_t$  denote, respectively, aggregate consumption and hours worked, and  $\beta$  is a function assumed to be decreasing in its first argument and increasing in its second argument. The household takes the evolution of  $C_t$  and  $H_t$  as given. The choice of aggregate variables as arguments of the discount factor simplifies the household's optimality conditions. It will become clear later, however, that the central result of this paper is robust to assuming that the discount factor is a function of the individual levels of consumption and effort.

Output, denoted  $y_t$ , is produced with a technology that takes labor as the only input. Production is subject to an aggregate stochastic stationary productivity shock denoted by  $z_t$ . Formally,  $y_t = e^{z_t} F(h_t)$ . The production function  $F$  is assumed to be positive, strictly increasing, and strictly concave.

The only financial asset available to households is a risk-free international bond. Letting  $a_t$  denote agent's debt due in period  $t$ , his sequential budget constraint is given by

$$\frac{a_{t+1}}{R_t} = a_t + c_t - e^{z_t} F(h_t), \quad (2)$$

where  $R_t$  denotes the gross interest rate on assets held between periods  $t$  and  $t+1$ . Households

are assumed to be subject to a no-Ponzi-game constraint of the form  $\lim_{j \rightarrow \infty} E_t \frac{a_{t+j+1}}{\prod_{s=0}^j R_{t+s}} \geq 0$ .

The household's problem consists in choosing contingent plans  $c_t$ ,  $h_t$ , and  $a_{t+1}$  so as to maximize (1) subject to (2) and the no-Ponzi-game constraint, given the processes  $R_t$  and  $z_t$  and the initial condition  $a_0$ . The first-order conditions associated with this problem are (2), the no-Ponzi-game constraint holding with equality, and

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{z_t} F'(h_t), \quad (3)$$

$$U_c(c_t, h_t) = \beta(C_t, H_t) R_t E_t U_c(c_{t+1}, h_{t+1}).$$

Foreign lenders impose an aggregate borrowing limit on the domestic economy, which stipulates that the aggregate per capita level of external liabilities assumed by the country in any period  $t \geq 0$ , which I denote by  $A_{t+1}$ , be no greater than a ceiling  $\kappa > 0$ . That is,

$$A_{t+1} \leq \kappa.$$

Individual households take the evolution of  $A_t$  as exogenous. At the same time, because all agents are identical, in equilibrium  $A_t = a_t$  for all  $t$ . In periods in which the aggregate borrowing ceiling is not binding, foreign investors lend to domestic residents at the world interest rate, which is assumed to be constant and equal to  $R^* > 1$ . When the aggregate borrowing limit is binding, the domestic interest rate may adjust upward to ensure market clearing in the domestic financial market. In this case the economy faces a country interest-rate premium, equal to  $R_t - R^*$ . It follows that  $R_t$  must satisfy  $R_t \geq R^*$  and  $(R_t - R^*)(A_{t+1} - \kappa) = 0$ .

## 2.1 The Rents From Financial Rationing

When the domestic interest rate,  $R_t$ , is above the world interest rate,  $R^*$ , a financial rent is generated. Values of  $R_t$  above  $R^*$  create pure rents because in this economy there no default by assumption. The precise way in which these rents are allocated will in general have consequences for aggregate dynamics. Here, I consider two polar cases. In one case, all financial rents accrue to the foreign lenders. In the other case, financial rents accrue entirely to domestic residents.

When financial rents are appropriated by nonresidents, increases in the domestic interest rate entail a resource cost to the domestic economy as a whole. This cost is reflected in an aggregate resource constraint of the form  $A_{t+1}/R_t = A_t + C_t - e^{z_t} F(H_t)$ . Note that this expression features the domestic interest rate,  $R_t$ , instead of the world interest rate  $R^*$ .

Alternatively, rents from credit rationing could accrue entirely to domestic residents. This case arises when, possibly because of competition among foreign lenders, domestic financial institutions borrow in the world financial market at the rate  $R^*$ . Thus, the country interest-rate premium represents a net rent to domestic financial intermediaries. I assume that these rents are distributed in a lump-sum fashion among domestic households, who own the domestic financial institutions in equal shares. In this case, the existence of an interest-rate premium does not introduce a resource cost to the domestic economy. The aggregate resource constraint is therefore given by  $A_{t+1}/R^* = A_t + C_t - e^{z_t}F(H_t)$ . Note that this expression features the world interest rate,  $R^*$ , instead of the domestic interest rate,  $R_t$ .

In equilibrium we have that individual and aggregate variables are identical; thus  $C_t = c_t$ ,  $H_t = h_t$ , and  $A_t = a_t$ . We are ready to provide definitions of competitive equilibria when financial rents accrue to foreign lenders and when financial rents accrue to domestic residents:

**Definition 1 (Equilibrium When Rents Accrue Domestically)** *A stationary competitive equilibrium under an aggregate borrowing ceiling when rents from financial rationing accrue to domestic residents is a set of stationary stochastic processes  $\{c_t, h_t, a_{t+1}, R_t\}_{t=0}^{\infty}$  satisfying*

$$U_c(c_t, h_t) = \beta(c_t, h_t)R_t E_t U_c(c_{t+1}, h_{t+1}), \quad (4)$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{z_t} F'(h_t), \quad (5)$$

$$R_t \geq R^*, \quad (6)$$

$$a_{t+1} \leq \kappa, \quad (7)$$

$$(R_t - R^*)(a_{t+1} - \kappa) = 0, \quad (8)$$

$$\frac{a_{t+1}}{R_t} = a_t + c_t - e^{z_t} F(h_t), \quad (9)$$

given the process  $\{z_t\}_{t=0}^{\infty}$  and the initial condition  $a_0$ .

**Definition 2 (Equilibrium When Rents Accrue to Foreigners)** *A stationary competitive equilibrium under an aggregate borrowing ceiling when rents from financial rationing accrue to foreign lenders is a set of stationary stochastic processes  $\{c_t, h_t, a_{t+1}, R_t\}_{t=0}^{\infty}$  satisfying conditions (4)-(8) and the resource constraint*

$$\frac{a_{t+1}}{R_t} = a_t + c_t - e^{z_t} F(h_t), \quad (10)$$

given the process  $\{z_t\}_{t=0}^{\infty}$  and the initial condition  $a_0$ .

I postpone the characterization of equilibrium in these economies until I describe equilibrium in an economy with an internalized borrowing limit.

### 3 An Economy With An Individual Borrowing Ceiling

Suppose now that lenders impose a debt ceiling at the level of each individual household. That is,

$$a_{t+1} \leq \kappa. \tag{11}$$

Unlike in the economy described in the previous section, in this economy domestic agents internalize the borrowing constraint. Therefore, they will take this constraint into account in solving their intertemporal optimization problem. Accordingly, the household problem consists in maximizing (1) subject to (2) and (11). The optimality conditions of this problem consist of (2), (3), (11), and

$$U_c(c_t, h_t) \left[ \frac{1}{R_t} - \xi_t \right] = \beta(c_t, h_t) E_t U_c(c_{t+1}, h_{t+1}),$$

$$\xi_t \geq 0,$$

$$(a_{t+1} - \kappa) \xi_t = 0,$$

where  $\xi_t$  denotes the Lagrange multiplier associated with the debt constraint (11) divided by the marginal utility of consumption. When the debt ceiling is binding,  $\xi_t$  is strictly positive, and the household faces an effective (shadow) interest rate given by  $\tilde{R}_t \equiv R_t / (1 - R_t \xi_t)$ , which is greater than the market interest rate  $R_t$ . This effective interest rate reflects the fact that at the market interest rate the household would like to borrow beyond the limit  $\kappa$ .

Foreign lenders supply funds to domestic residents at the world interest rate. Therefore,  $R_t$  equals  $R^*$  at all dates and states. The following definition of a competitive equilibrium then applies:

**Definition 3 (Equilibrium With An Individual Debt Ceiling)** *A stationary competitive equilibrium under an individual debt ceiling is a set of stationary stochastic processes  $\{c_t, h_t, a_{t+1}, \xi_t\}_{t=0}^\infty$  satisfying*

$$U_c(c_t, h_t) \left[ \frac{1}{R^*} - \xi_t \right] = \beta(c_t, h_t) E_t U_c(c_{t+1}, h_{t+1}), \tag{12}$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{z_t} F'(h_t), \quad (13)$$

$$\xi_t \geq 0, \quad (14)$$

$$a_{t+1} \leq \kappa, \quad (15)$$

$$(a_{t+1} - \kappa)\xi_t = 0. \quad (16)$$

$$\frac{a_{t+1}}{R^*} = a_t + c_t - e^{z_t} F(h_t), \quad (17)$$

given the process  $\{z_t\}_{t=0}^{\infty}$  and the initial condition  $a_0$ .

We are ready to compare equilibrium dynamics under aggregate and individual debt limits.

## 4 An Equivalence Result

In this section, I show that the equilibrium processes for debt, consumption, hours, and output in the economy with an individual debt ceiling are identical to those induced by the economy with an aggregate debt ceiling with rents from financial rationing accruing to domestic households. To see this, consider the economy with an individual debt constraint. Definition 3 lists the equilibrium conditions corresponding to this economy. Equations (12) and (14) and the fact that  $U_c(c_t, h_t) > 0$  imply that  $\xi_t \in [0, 1/R^*]$ . Define  $R_t = R^*/(1 - R^*\xi_t)$ . Use this definition to eliminate  $\xi_t$  from the equilibrium conditions (12)-(17). It follows immediately that the resulting expressions are identical to the equilibrium conditions pertaining to the economy with an aggregate debt limit and rents accruing to domestic households, equations (4)-(9).

We conclude that in the simple economic environment studied here, the practice by foreign investors of basing their lending decisions on macroeconomic indicators—as opposed to individual solvency indicators—does not induce overborrowing. The individual incentives created by the market (i.e., by  $R_t$ ) in the economy with the aggregate debt limit are exactly the same as those emerging from an individual debt limit. The following proposition summarizes this result:

**Proposition 1** *The equilibrium dynamics of  $c_t$ ,  $h_t$ ,  $y_t$ , and  $a_t$  are identical in the economy with an individual debt limit and in the economy with an aggregate debt limit with rents from financial rationing accruing to domestic households.*

The no-overborrowing result contained in this proposition contrasts sharply with the findings of Fernández-Arias and Lombardo (1998). These authors conclude that when agents fail to internalize the debt limit, the economy tends to overborrow. The structure of the



model economy used by Fernández-Arias and Lombardo is similar to the one presented here, with two minor differences. Namely, their model is cast in perfect foresight and in continuous time, and output is assumed to take the form of an exogenous endowment. The central difference between the Fernández-Arias and Lombardo model and the one studied here has to do with the mechanism through which credit rationing is brought about in the economy with an aggregate debt limit. In the formulation I adopt in this paper, credit rationing is implemented through a market mechanism. The interest rate,  $R_t$ , adjusts to induce agents to borrow an amount that in the aggregate is in line with the credit limit imposed on the country as a whole. In the Fernández-Arias and Lombardo model, credit rationing is not implemented through the price system. Indeed, they assume that the domestic interest rate is always equal to the world interest rate ( $R_t = R^*, \forall t$ ). Instead they impose a credit constraint of the type  $a_t \leq a_\tau, t \geq \tau$ , at the level of each individual household, where  $\tau$  is the date at which the aggregate borrowing constraint becomes binding, which is known under perfect foresight. Agents do not internalize the fact that in equilibrium  $a_\tau$  must equal  $\kappa$ . Note that in the Fernández-Arias and Lombardo model agents internalize a substantial part of the credit limit, namely the fact that individual debts cannot grow beyond  $a_\tau$  after time  $\tau$ . The only aspect of the debt ceiling agents do not internalize is the ceiling  $\kappa$  itself. In the formulation adopted in the present paper, by contrast, agents do not internalize any component of the credit limit. They borrow and lend freely at the interest rate  $R_t$  (subject, of course, to the standard no-Ponzi-game constraint).

## 5 Resource Costs

When rents from financial rationing are appropriated by foreign lenders, it is no longer possible to compare analytically the dynamics of external debt in the economies with the aggregate debt limit and in the economy with the individual debt limit. I therefore resort to numerical methods to characterize competitive equilibria.

To this end, I adopt the following functional forms for preferences and technology:  $U(c, h) = [c - \omega^{-1}h^\omega]^{1-\sigma} / (1 - \sigma)$ .  $\beta(c, h) = [1 + c - \omega^{-1}h^\omega]^{-\psi}$ , and  $F(h) = k^\alpha h^{1-\alpha}$ , where  $\sigma, \omega, \psi, k$ , and  $\alpha$  are fixed parameters.

Table 1 displays the values I assign to the deep structural parameters of the model.

Table 1: Parameter Values

$\sigma$	$\omega$	$\psi$	$\alpha$	$R^*$	$\kappa$	$k^*$	$\pi_{HH} = \pi_{LL}$	$z^H = -z^L$
2	1.455	0.0222	0.32	1.04	7.83	78.3	0.71	0.0258

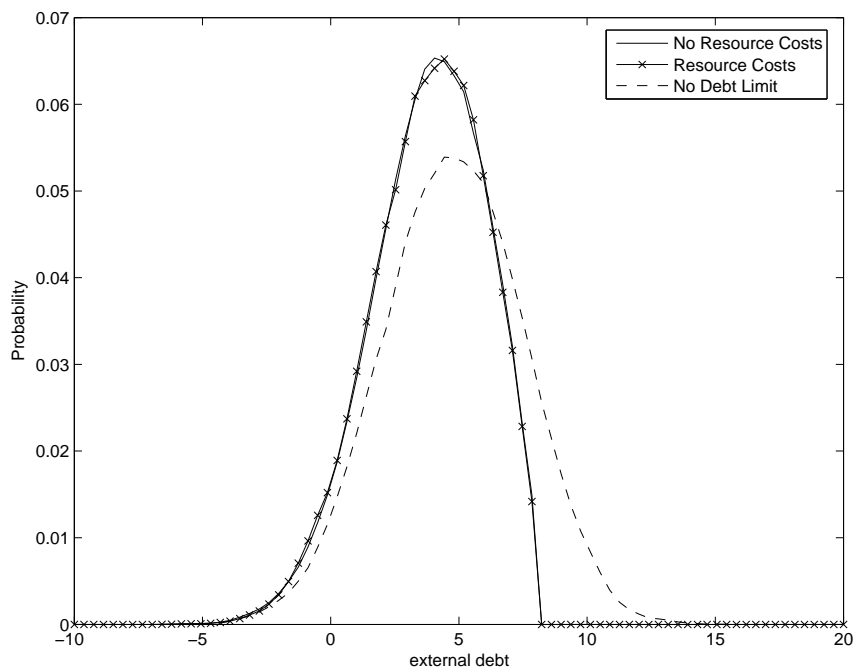
The time unit is meant to be one year. The values for  $\alpha$ ,  $\omega$ ,  $\sigma$ , and  $R^*$  are taken from Schmitt-Grohé and Uribe (2003). I set the parameter  $\psi$  so as to induce a debt-to-GDP ratio,  $a/y$ , of 50 percent in the deterministic steady state. The calibrated value of  $\kappa$  is such that in the economy without the debt limit, the probability that  $a_t$  is larger than  $\kappa$  is about 15 percent. The value assigned to the parameter  $k$  ensures that, if  $k$  is interpreted as a factor of production that is in fixed aggregate supply (such as land), then its market price in the deterministic steady state is unity. The productivity shock is assumed to follow a two-state symmetric Markov process with mean zero. Formally,  $z_t$  takes on values from the set  $\{z^1, z^2\}$  with transition probability matrix  $\pi$ , and  $z^1$ ,  $z^2$ , and  $\pi$  satisfy  $z^1 = -z^2$  and  $\pi_{11} = \pi_{22} = 1 - \pi_{21} = 1 - \pi_{12}$ . I set  $\pi_{11}$  equal to 0.71 and  $z^1$  equal to 0.0258. This process displays the same serial correlation (0.58) and twice as large a standard deviation (2.58 percent) as the one estimated for Canada by Mendoza (1991). My choice of a process for the productivity shock that is twice as volatile as the one observed in a developed small open economy like Canada reflects the view that to a first approximation what distinguishes business cycles in developed and developing countries is that the latter are about twice as volatile as the former (Kydland and Zarazaga, 1997).

I solve the model using the Chebyshev Parameterized Expectations method. The state space is discretized using 1000 points for the stock of debt,  $a_t$ . The parameterization of expectations uses 50 coefficients. I compute the equilibrium for three model economies. An economy with no debt limit, an economy with a debt limit and financial rents accruing to domestic residents, and an economy with a debt limit and financial rents flowing abroad. The procedure approximates the equilibrium with reasonable accuracy. The DenHaan-Marcet test for 5-percent left and right tails yields, respectively, (0.047,0.046), (0.043,0.056), and (0.048,0.056). In conducting this test, I use 1000 simulations of 5000 periods each, dropping the first 1000 realizations. Matlab code written to produce the numerical results reported in this section are available on the author's website.

Figure 1 displays with a solid line the equilibrium probability distribution of external debt in the economy with an aggregate debt limit and financial rents from rationing accruing to domestic agents. According to proposition 1, this line coincides with the debt distribution in the economy with a household-specific debt limit. The figure shows with a dash-crossed line the distribution of debt in the economy with an aggregate debt limit and financial rents accruing to foreign lenders. As a reference, the figure also displays, with a dashed line, the debt distribution in an economy without a debt limit. The main result conveyed by the figure is that the distribution of debt in the economy with a debt limit is virtually unaffected by whether financial rents are assumed to flow abroad or stay within the country's limits.

The resource costs due to financial rents remitted abroad in the economy in which this

Figure 1: Equilibrium Distribution of External Debt



rents belong to foreigners are fairly small. This implication is the result of two characteristics of the equilibrium. First, the economy seldom hits the debt limit. In effect, even though  $\kappa$  is calibrated to create a right tail of 15 percent probability in the debt distribution pertaining to the economy with no debt limit, in the economies with a debt limit this constraint binds less than once every one hundred years. Second, when the debt limit does bind, it produces a country interest-rate premium of less than 2 percent on average. Because the external debt is about 40 percent of GDP in the economies with a debt limit, it follows that the cost of remitting financial rents abroad is less than 0.008 percent of GDP per year on average.

## 6 Robustness and Extensions

The central result of this paper, namely the equivalence of aggregate and individual credit limits for the equilibrium behavior of external debt, is robust to a number of modifications of the basic model studied thus far. For instance, it can be shown that it continues to hold in the context of an economy with capital accumulation. The equivalence result can also be shown to be robust to alternative specifications of the discount factor. In particular, when the discount factor is assumed to depend on the individual levels of consumption and effort, as opposed to aggregate measures of these variables.

For analytical and computational simplicity, the model considered in this paper features

a constant debt limit. A more realistic formulation would incorporate a collateral constraint limiting the level of debt to the market value of the stocks of physical capital or land. Perhaps a more relevant modification would endogenize the emergence of the debt limit.

Another modification of the model that is in order is the inclusion of a working-capital constraints whereby payments to factors of production by firms require holding non-interest-bearing assets. Such constraints allow for output and employment drops in response to increases in the country-interest-rate premium.

## References

- Fernández-Arias, Eduardo and Davide Lombardo, “Private External Overborrowing in Undistorted Economies: Market Failure and Optimal Policy,” manuscript, March 1998.
- Kydland, Finn E. and Carlos E. J. M. Zarazaga, “Is the business cycle of Argentina “different”?,” *Economic & Financial Review*, Q-IV, 1997, 21-36.
- Mendoza E.G., “Real Business Cycles in a Small Open Economy,” *American Economic Review*, 81, 1991, 797-818.
- Schmitt-Grohé, Stephanie and Martín Uribe, “Closing Small Open Economy Models,” *Journal of International Economics*, 61, October 2003, 163-185.