NBER WORKING PAPER SERIES

THE MYTH OF LONG-HORIZON PREDICTABILITY

Jacob Boudoukh
Matthew Richardson
Robert Whitelaw

Working Paper 11841
http://www.nber.org/papers/w11841

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 2005

a Arison School of Business, IDC and NBER; b Stern School of Business, New York University and NBER. We would like to thank Jeff Wurgler, John Cochrane (the discussant), and seminar participants at Yale University, New York University and the NBER asset pricing program for helpful comments. Contact: Prof. M. Richardson, NYU, Stern School of Business, 44 W. 4th St., New York, NY 10012; mrichar0@stern.nyu.edu.

The Myth of Long-Horizon Predictability
Jacob Boudoukh, Matthew Richardson, and Robert Whitelaw
NBER Working Paper No. 11841
December 2005
JEL No. G12, G10, C32

ABSTRACT

The prevailing view in finance is that the evidence for long-horizon stock return predictability is significantly stronger than that for short horizons. We show that for persistent regressors, a characteristic of most of the predictive variables used in the literature, the estimators are almost perfectly correlated across horizons under the null hypothesis of no predictability. For example, for the persistence levels of dividend yields, the analytical correlation is 99% between the 1- and 2-year horizon estimators and 94% between the 1- and 5-year horizons, due to the combined effects of overlapping returns and the persistence of the predictive variable. Common sampling error across equations leads to ordinary least squares coefficient estimates and R2s that are roughly proportional to the horizon under the null hypothesis. This is the precise pattern found in the data. The asymptotic theory is corroborated, and the analysis extended by extensive simulation evidence. We perform joint tests across horizons for a variety of explanatory variables, and provide an alternative view of the existing evidence.

Jacob Boudoukh
34 Burla St #19
Tel Aviv 69364
Israel
and NBER
jboudouk@idc.il

Matthew Richardson
NYU, Stern School of Business
44 W. 4th St.
New York, NY 10012
and NBER
mrichar0@stern.nyu.edu

Robert Whitelaw
NYU, Stern School of Business
44 W. 4th St.
New York, NY 10012
and NBER
rwhitela@stern.nyu.edu

I. Introduction

Over the last two decades, the finance literature has produced growing evidence of stock return predictability, though not without substantive debate. The strongest evidence cited so far comes from long-horizon stock returns regressed on variables such as dividend yields, term structure slopes, and credit spreads, among others. A typical view is expressed in Campbell, Lo, and MacKinlay's standard textbook for empirical financial economics, *The Econometrics of Financial Markets* (1997, p.268):

> At a horizon of one month, the regression results are rather unimpressive: The R^2 statistics never exceed 2%, and the t-statistics exceed 2 only in the post-World War II subsample. The striking fact about the table is how much stronger the results become when one increases the horizon. At a two-year horizon the R^2 statistic is 14% for the full sample … at a four-year horizon the R^2 statistic is 26% for the full sample.

However, there is an alternative interpretation of this evidence: Researchers should be equally impressed by the short- and long-horizon evidence for the simple reason that the regressions are almost perfectly correlated. For an autocorrelation of 0.953 for annual dividend yields, we show analytically that the 1-year and 2-year predictive estimators are 98.8% correlated under the null hypothesis of no predictability. For longer horizons, the correlations are even higher, reaching 99.6% between the 4- and 5-year horizon estimators. This degree of correlation manifests itself in multiple-horizon regressions in a particularly unfortunate way.

Since the sampling error that is almost surely present in small samples shows up in each regression, both the estimator and R^2 are proportional to the horizon.

This paper provides analytical expressions for the correlations across multiple-horizon estimators, and then shows, through simulations, that these expressions are relevant in small samples. The analytical expressions relate the correlations across these estimators to both the degree of overlap across the horizons and the level of persistence of the predictive variable. Our findings relate to an earlier literature looking at joint tests of the random walk hypothesis for stock prices using multiple-horizon variance ratios and autocorrelations, among other estimators (see, e.g., Richardson and Smith (1991, 1994) and Richardson (1993)). This earlier literature stresses accounting for the degree of overlap. The problem here is much more severe. In the univariate framework, the predictive variable—past stock returns—is approximately independently and identically distributed (IID). In this paper's framework, the predictive variable, e.g., dividend yields, is highly persistent.

Our simulations show that any sampling error in the data under the null hypothesis of no predictability appears in the same manner in every multiple-horizon regression when the predictive variable is highly persistent. Using box plots and tables describing the relation across the multiple horizon estimates and R^2s, we show the exact pattern one should expect under the null hypothesis: The multiple-horizon estimates are monotonic in the horizon approximately two-thirds of the time, and the mean ratios of the 2- to 5-year estimators to the 1-year estimator are 1.93, 2.80, 3.59, and 4.32, respectively. Consider the actual estimated coefficients for the regression of 1- to 5-year stock returns on dividend yields over the 1926–2004 sample period: 0.131, 0.257, 0.390, 0.461, and 0.521. These correspond to monotonically increasing estimates with corresponding ratios of 1.96, 2.98, 3.53, and 3.99. We show that these estimates lie in the middle of the distribution of possible outcomes under the null hypothesis.

The theoretical and simulation analyses stress the importance of interpreting the evidence jointly across horizons. We develop an analytical expression for a joint test based on the Wald statistic. While a high level of persistence means that it can be dangerous to interpret regressions over multiple horizons, the joint tests show that this persistence may lead to powerful tests for economies in which predictability exists. Such predictability may take a particular form, in which the multiple-horizon coefficients are much less tied together than the null hypothesis implies. Applying the joint tests to commonly used predictive variables, we point out various anomalies and contrast our results with the conclusions of the existing literature.

Among the standard set of variables, none generate joint test statistics that are significant at the 10% level under the simulated distribution. Interestingly, the only variable that is significant at the 10% level under the asymptotic distribution is the risk-free rate, despite the fact that the associated horizon-by-horizon p-values are larger and the R^2s are smaller than for many of the other variables, including the dividend yield and the book-to-market ratio. Among more recently developed variables, joint tests confirm the ability of both the net payout yield (Boudoukh, Michaely, Richardson, and Roberts (2005)) and the equity share of new issuances (Baker and Wurgler (2000)) to forecast stock returns across all horizons.

The paper proceeds as follows. In Section 2, we provide the expressions for analyzing multiple-horizon regressions and show that the basic findings carry through to small samples. The small sample results are especially alarming in the context of the existing literature.

Section 3 applies the results to a number of data series and evaluates existing evidence using joint tests of predictability. Section 4 concludes.

II. Multiple Horizon Regressions

A. The Existing Literature

Fama and French (1988) is the first paper to document evidence of multivariate stock return predictability over multiple horizons.[1] In brief, they regress overlapping stock returns of one month to four years on dividend yields, reporting coefficients and R^2s that increase somewhat proportionately with the horizon. As it documents what has become one of the dominant stylized facts in empirical finance, the paper has over 250 citations to date. To illustrate the consensus view, consider part of John Cochrane's description of the three most important facts in finance in his survey, "New Facts in Finance" (1999, p.37).

> Now, we know that …
>
> [Fact] 2. Returns are predictable. In particular: Variables including the dividend/price (d/p) ratio and term premium can predict substantial amounts of stock return variation. This phenomenon occurs over business cycle and longer horizons. Daily, weekly, and monthly stock returns are still close to unpredictable…

This fact is emphasized repeatedly in other surveys (see, e.g., Fama (1998, p.1578), Campbell (2000, p.1522 and 2003, p.5) and Barberis and Thaler (2003, p.21), among others),

[1] See also Campbell and Shiller (1988).

is often used to calibrate theoretical models (see, among others, Campbell and Cochrane (1999, p.206), Campbell and Viceira (1999, p.434), Barberis (2000, p.225), Menzly, Santos, and Veronesi (2004, p.2), and Lettau and Ludvigson (2005, p.584)), and in motivation for new empirical tests (e.g., Ferson & Korajczyk (1995, p.309), Patelis (1997, p.1951), Lettau & Ludvigson (2001, p.815), and Ait-Sahalia and Brandt (2001, p.1297)).

It is fairly well known since Fama and French (1988), and in particular from Campbell (2001), that the key determinants of long-horizon predictability are

(i) The extent of predictability at short horizons, and

(ii) The persistence of the regressor.

The R^2s at long horizons relative to a single-period R^2 are a function of (ii). Holding everything else constant—single-period predictability in particular—higher persistence results in a higher fraction of explainable long-horizon returns. As a function of the horizon, the R^2 first rises with the horizon, but eventually decays, due to the exponential decline in the informativeness of the predictive variable. As we show below, (ii) also matters in the case of no predictability, but in the presence of sampling error. Nevertheless, this important fact has not been used as the main line of attack against evidence supporting the multivariate predictability of stock returns.

Three principal alternative lines of criticism have been put forward in the literature. The first involves data snooping, which is perhaps best described by Foster, Smith, and Whaley (1997). The idea is that the levels of predictability found at short horizons are not surprising, given the number of variables from which researchers can choose. A variety of papers support these findings somewhat, including Bossaerts and Hillion (1999), Cremers (2002), and Goyal and Welch (2003).

A second approach looks at the small sample biases of the estimators. Stambaugh (1999) shows that the bias can be quite severe, given the negative correlation between contemporaneous shocks to returns and the predictive variable, which usually involves some type of stock price deflator. His findings suggest much less predictability once the estimators are adjusted for this bias. However, Lewellen (2004) argues that the effect of the bias may be much smaller if one takes the persistence of the predictive variable into account. Lewellen's approach is similar to Stambaugh's (1999) Bayesian analysis of the predictability problem. While both of these papers certainly question the magnitude of the predictability, they do not address long-horizon predictability per se.

The third line of criticism, first explored by Richardson and Stock (1989) in a univariate setting, uses an alternative asymptotic theory, in which the horizon increases with the sample size. Valkanov (2003) argues that long-horizon regressions have poor properties relative to standard asymptotics.[2] He shows that the estimators may no longer be consistent, and have limiting distributions that are functionals of Brownian motions; in fact, the distributions are not normal, and are not centered on the true coefficient. Valkanov then shows that this alternative asymptotic theory works better in small samples. His results can be viewed as the theoretical foundation for earlier simulated distributions by Kim and Nelson (1993) and Goetzmann and Jorion (1993), and for the intuition put forward by Kirby (1997), who uses standard asymptotics.

Aside from these three methodology-based lines of criticism of the stock return predictability literature, there is scant evidence of empirical-based critique of long horizon predictability, one recent exception being Ang and Bekaert (2005). Our paper focuses on a different methodological aspect of predictability, examining the joint properties of the regression estimators across horizons. The conclusions here closely resemble those of Richardson and Smith (1991) and Richardson (1993) regarding long-horizon evidence against the random walk in Fama and French (1998) and Poterba and Summers (1988). In many ways, the arguments here are more damaging, because we show that the degree of correlation across the multiple-horizon estimators is much higher than it is in the case of long-horizon tests for the random walk. In fact, the null hypothesis of no predictability implies the exact pattern in coefficients and R^2s found in papers presenting evidence in favor of predictability. We show these results in the next two subsections.

B. Multiple Horizon Regressions: Statistical Properties

We consider regression systems of the following type:

$$
\begin{aligned}
R_{t,t+1} &= \alpha_1 + \beta_1 X_t + \varepsilon_{t,t+1} \\
&\vdots \\
R_{t,t+J} &= \alpha_J + \beta_J X_t + \varepsilon_{t,t+J} \\
&\vdots \\
R_{t,t+K} &= \alpha_K + \beta_K X_t + \varepsilon_{t,t+K},
\end{aligned}
\tag{1}
$$

where $R_{t,t+J}$ is the J-period stock return, X_t is the predictor, e.g., the dividend yield, and $\varepsilon_{t,t+J}$ is the error term over J periods. As is well known from Hansen and Hodrick (1980) and Hansen (1982), among others, the error terms are serially correlated due to overlapping observations. Using the standard generalized method of moments calculations under the null hypothesis of no predictability and under conditional homoskedasticity (e.g., Richardson and

[2] Ang and Bekeart (2005) show that the statistical significance of long horizon regressions is overstated once the researcher adjusts for heteroskedasticity and the overlapping errors by imposing the null in estimation.

Smith (1991)), Appendix A derives the covariance matrix for any two horizons, J and J^*, of $\hat{\beta}_J$ and $\hat{\beta}_{J^*}$:

$$T\,\mathrm{Var}(\hat{\beta}_J,\hat{\beta}_{J^*}) = \frac{\sigma_R^2}{\sigma_X^2}\begin{pmatrix} J+2\sum_{l=1}^{J-1}(J-l)\rho_l & J+\left[\sum_{l=1}^{J-1}(J-l)(\rho_l+\rho_{l+(J^*-J)})\right]+\sum_{l=1}^{J^*-J}J\rho_l \\ \cdots & J^*+2\sum_{l=1}^{J^*-1}(J^*-l)\rho_l \end{pmatrix}, \quad (2)$$

where $J^*>J$ and ρ_l is the l^{th}-order autocorrelation of X_t. The above expression for the covariance matrix of the estimators is not particularly intuitive, though it is immediately apparent that for J close to J^* the estimators are almost perfectly correlated. Less obvious is the fact that for $\mathrm{cov}(X_t,X_{t-l})\approx\sigma_X^2$ the estimators are also almost perfectly correlated irrespective of the horizon. Intuitively, the persistence of X_t acts in much the same way overlapping horizons do regarding independent information across multiple horizons.

A popular simplification is to assume that X_t follows an AR(1) (see, among others, Campbell (2001), Boudoukh and Richardson (1994), Stambaugh (1993), and Cochrane (2001)). Under the AR(1) model, $\mathrm{cov}(X_t,X_{t-l})=\rho_l\sigma_X^2=\rho^l\sigma_X^2$ where ρ is the autoregressive parameter for X_t. The covariance matrix in (2) reduces to a much simpler form:

$$T\,\mathrm{Var}(\hat{\beta}_J,\hat{\beta}_{J^*}) = \frac{\sigma_R^2}{\sigma_X^2}\begin{pmatrix} J+\frac{2\rho}{(1-\rho)^2}\left[(J-1)-\rho(J-\rho^{J-1})\right] & J+\frac{\rho}{(1-\rho)^2}\left\{2\left[(J-1)-\rho(J-\rho^{J-1})\right]+(1-\rho^J)(1-\rho^{J^*-J})\right\} \\ \cdots & J^*+\frac{2\rho}{(1-\rho)^2}\left[(J^*-1)-\rho(J^*-\rho^{J^*-1})\right] \end{pmatrix}.$$

(3)

For the special case $J = 1$, the correlation between J and J^* is

$$\frac{(1-\rho)^2+\rho(1-\rho)(1-\rho^{J^*-1})}{\sqrt{(1-\rho)^2}\sqrt{J^*(1-\rho)^2+2\rho\left[(J^*-1)-\rho(J^*-\rho^{J^*-1})\right]}}. \quad (4)$$

For example, for $J = 1$ and $J^* = 2$, we get $\sqrt{\frac{1+\rho}{2}}$. In our sample, the autocorrelation of the dividend yield is 0.953, which yields a correlation of 0.988 between the 1- and 2-year estimators. It does not get much better as the horizon J^* increases to 3, 4, and 5 years, producing correlations of 0.974, 0.959, and 0.945, respectively.[3] Even at a 10-year horizon, the correlation is over 87%. With the types of sample sizes faced by researchers in the field of empirical finance, these results suggest that one has to be extremely cautious in interpreting the coefficients separately (as has been the case in the existing literature).

For the typical 1- through 5-year horizons examined in the literature, we provide the analytical covariance matrix of the estimators under the null hypothesis of no predictability and the dividend yield's ρ of 0.953:

$$T\,\mathrm{Var}\left(\hat{\beta}_{1-5}\right)=\frac{\sigma_R^2}{\sigma_X^2}\begin{pmatrix}1 & 0.988 & 0.974 & 0.959 & 0.945\\ & 1 & 0.993 & 0.982 & 0.970\\ & & 1 & 0.995 & 0.986\\ & & & 1 & 0.996\\ & & & & 1\end{pmatrix}. \qquad (5)$$

Several observations are in order. First, the high degree of correlation across the multi-period estimators implies that, under the null hypothesis, the regressions are essentially redundant. This is important because there is little doubt that the literature has not taken this view. Second, under the null hypothesis, the estimators are asymptotically distributed as multivariate normal with a mean of zero. While this is clearly not true in small samples,[4] consider using this distribution to understand the effect of sampling error across the equations.

[3] Of course, these correlations increase even further as J increases for a fixed J*.

[4] See Stambaugh (1999) for small sample bias and Valkanov (2003) for non-normality of the distributions of the estimators.

Specifically, conditional on $\hat{\beta}_1$ being some given or estimated $\overline{\beta}_1$, what do we expect $\hat{\beta}_{J*}$ to be equal to under the null? Using properties of a bivariate normal, we can write

$$E[\hat{\beta}_{J*} \mid \hat{\beta}_1 = \overline{\beta}_1] = \left(1 + \frac{\rho(1-\rho^{J*-1})}{1-\rho}\right)\overline{\beta}_1 . \tag{6}$$

For ρ close to 1, the coefficients should basically be proportional to the horizon. As an example, for ρ=0.953, the $\hat{\beta}_{J*}$'s you would expect in terms of $\overline{\beta}_1$ are 1.953$\overline{\beta}_1$, 2.861$\overline{\beta}_1$, 3.727$\overline{\beta}_1$, and 4.552$\overline{\beta}_1$ for the 2-, 3-, 4-, and 5-year horizon regressions, respectively. Similarly, for the R^2 of the regression,

$$E[R^2_{J*} \mid R^2_1 = \overline{R}^2_1] = \frac{\left(1 + \frac{\rho(1-\rho^{J*-1})}{1-\rho}\right)^2}{J*}\overline{R}^2_1 . \tag{7}$$

For ρ close to 1, the R^2s also increase significantly with the horizon. The ratios of the R^2s are 1.907, 2.729, 3.472, and 4.143 for the 2-, 3-, 4-, and 5-year horizon regressions, respectively.

The intuition is straightforward. Compare the regression of $R_{t,t+1}$ on X_t to that of $R_{t,t+K}$ on X_t. The former regression involves summing the cross product of the sequence of $R_{t,t+1}$ and X_t for all t observations. Note that for a persistent series X_t, there is very little information across the sequence of X_t values. Thus, when an unusual draw from $R_{t,t+1}$ occurs (denote it $R_{t*,t*+1}$), and this observation happens to coincide with the most recent value of the predictive variable, X_{t*}, it will also coincide with all the surrounding X_t variables such as X_{t*-1}, X_{t*-2}, and X_{t*-3}. Since $R_{t*,t*+1}$ shows up in K of the long-horizon returns $R_{t,t+K}$ within the sample period (i.e., in $R_{t^*+1-K,t^*+1}, R_{t^*+2-K,t^*+2}, \ldots, R_{t^*,t^*+K}$), the impact of the unusual draw will be roughly K times larger in the long-horizon regression than in the one-period regression.

C. Multiple Horizon Regressions: Joint Tests

At first glance, the results in Section II.B provide a fairly devastating critique of the strategy of running multiple long-horizon regressions. However, this view is not necessarily accurate. Because the regressions are linked so closely under the null hypothesis, joint tests may have considerable power under alternative models.

What are these alternatives? The models must be such that the long horizons pick up information not contained in short horizons. The standard model, in which short-horizon returns are linear in the current predictor and that predictor follows a persistent ARMA process, is clearly not a good candidate. It would be better to focus on estimating the short-horizon and the ARMA process directly in this case (see, e.g., Campbell (2001), Hodrick (1992), and Boudoukh and Richardson (1994), among others). It should be noted, though, that the standard model is often chosen for reasons of parsimony rather than on an underlying theoretical basis.

Consider testing the null of no predictability in the regression system given in equation (1), i.e., $\beta_1 = \cdots = \beta_J = \cdots = \beta_K = 0$. The corresponding Wald Test statistic for this hypothesis is $T\hat{\beta}' V(\hat{\beta})^{-1}\hat{\beta}$ where $\hat{\beta}' = \left(\hat{\beta}_1 \cdots \hat{\beta}_J \cdots \hat{\beta}_K\right)$ and $V(\hat{\beta})$ is the covariance matrix of the $\hat{\beta}$ estimators with typical element of $\hat{\beta}_J$ and $\hat{\beta}_{J^*}$ given by $\text{Var}(\hat{\beta}_J, \hat{\beta}_{J^*})$ as described in equation (2).[5] The statistic follows an asymptotic chi-squared distribution with degrees of freedom given by the number of horizons used in estimation. Note that $V(\hat{\beta})$ is a function of the autocorrelation structure of the X_t variable (i.e., its persistence) as well as the degree of overlap between horizons, i.e., J versus J^*. Aside from the magnitude of the $\hat{\beta}$ estimators, what matters is whether the pattern in $\hat{\beta}$ across horizons is consistent with the correlation implied by $V(\hat{\beta})$.

To see this, consider performing a Wald Test of the hypothesis $\beta_1 = \beta_2 = 0$. The corresponding Wald statistic is given by

$$T\,\frac{\sigma_X^2}{\sigma_R^2}\left[\frac{2\beta_1^2 + \beta_2^2/(1+\rho) - 2\beta_1\beta_2}{1-\rho}\right]. \tag{8}$$

For a given sample size T and estimated coefficient $\hat{\beta}_1 = \overline{\beta}_1$, this statistic is minimized at $\hat{\beta}_2 = (1+\rho)\overline{\beta}_1$. Since a low value of the statistic implies less evidence against the null, this result means that we not only expect a nonzero $\hat{\beta}_2$ under the null but that it should be of a magnitude greater than the $\hat{\beta}_1$ estimate. In fact, for a highly persistent regressor, the Wald statistic is minimized when the 2-period coefficient is almost double the one-period coefficient. Of course, the denominator of the test statistic goes to zero as the autocorrelation approaches one, so even small deviations from the predicted pattern under the null may generate rejections if the regressor is sufficiently persistent.

These results provide important clues in searching for powerful tests against the null of no predictability. If the alternative hypothesis does not imply coefficient estimates that increase at

[5] For other examples of joint tests in the predictability framework, see, for example, Richardson and Smith

the same rate across horizons or that are not as heavily tied to the predictive variable's persistence, one can find evidence of predictability even with modestly sized coefficients. But the fact that the no predictability null and the standard ARMA predictive model imply similar coefficient patterns (and thus low power) does not mean the null is false.

Treating the individual coefficient estimates separately in a joint setting can lead to very misleading conclusions. The null hypothesis of no predictability as described by the Wald Test is most supported in the data when we observe monotonically increasing/decreasing coefficient estimates that can be described by the horizon and persistence of the predictive variable. This is the exact pattern documented in the original Fama and French (1988) and Campbell and Shiller (1988) papers. One wonders how the finance literature would have treated these papers if armed with this fact, especially given the weak evidence of predictability at short horizons and also in the context of the previously mentioned data snooping arguments (e.g., Foster, Smith, and Whaley (1997)) and small sample bias (Stambaugh (1999)), both of which suggest that short-horizon significance is overstated.

D. Multiple Horizon Regressions: Simulation Evidence

The theoretical results in Sections II.B and II.C are based on asymptotic properties of fixed-horizon estimators. A priori there is reason to be wary of these results in small samples, particularly because of the considerable evidence of a bias in the coefficient estimators and of non-normality as discussed in Section II.A. Therefore, it is useful to evaluate the small sample properties of the estimators in general, and the patterns in sampling error across equations in

(1991), Hodrick (1992) and Ang and Bekeart (2005), among others.

particular. Previewing the results to come, the basic tenet of equations (2) and (3), namely, the dependence across equations, carries through to small samples.

We simulate the model in equation (1) under the assumption of no predictability, an AR(1) process on X_t, and 75 years of annual data. The analysis is performed over 1- to 5-year horizons with the AR parameter ρ, the standard deviation of X_t and $\varepsilon_{t,t+1}$, and the correlation between $\varepsilon_{t,t+1}$ and $u_{t,t+1} \equiv (X_{t+1} - \rho X_t)$ chosen to match the data.[6] The simulations involve 100,000 replications each.

Table 1A reports the simulated correlation matrix of the multiple-horizon estimators. Consistent with the theoretical analytical calculations in Section II.B, the correlations tend to be high, even for the most distant horizons. The simulated correlations between the 1-year and 2- to 5-year horizon estimators are 0.966, 0.926, 0.885, and 0.843, respectively, showing that the correlation calculations under the fixed-horizon asymptotics hold in small samples. Thus, the estimators' almost perfect cross-correlation leads to little independent information across equations, and the sampling error that is surely present in small samples shows up in every equation in (1).

As shown in Section II.B, persistence (i.e., ρ) is an important determinant of the magnitude of the correlation matrix of the multiple-horizon estimators. Figure 1A graphs the correlation between the 1-year and 2- to 5-year horizon estimators for values ρ = 0.953, 0.750, 0.500, 0.250, and 0.000. Consistent with the asymptotic theory, the correlations decrease as ρ falls. The drop-off can be quite large as the horizon increases. As a function of the above ρ values,

---

[6] Specifically, for the regression of annual stock returns on the most commonly used predictive variable, namely, dividend yields, we estimate $\rho = 0.953$, $\sigma_{\varepsilon} = 0.202$, $\sigma_{u} = 0.154$ and $\sigma_{\varepsilon u} = -0.712$. While the magnitudes of

the 1- and 2-year estimators have correlations of 0.966, 0.917, 0.849, 0.776, and 0.698, respectively, and the 1- and 5-year estimators have correlations of 0.843, 0.684, 0.544, 0.465, and 0.429. Even when the predictive variable has no persistence, the correlation can still be quite high due to the overlapping information across the multiple-horizon returns.

However, the staggering result in Table 1A is that 66% of all the replications produce estimates that are monotonic in the horizon. That is, almost two-thirds of the time, the data produce coefficients increasing or decreasing with the horizon, coinciding with the predictions from the asymptotic theory. To understand how unlikely monotonicity is, suppose that the five different multiple-horizon estimators were IID. In this setting, the probability of a monotonic relation is 0.83%, approximately $1/78^{th}$ of the true probability for the multiple-horizon estimators. Even with overlapping horizons, monotonicity drops sharply as ρ falls, i.e., from 66% to 37%, 20%, 11%, and 6% for $\rho = 0.750$, 0.500, 0.250, and 0.000, respectively. This result further highlights the importance of persistence in the predictive variable for generating these patterns.

One possible explanation for this finding is that the small sample bias increases with the horizon (e.g., Stambaugh (1999), Goetzmann and Jorion (1993), and Kim and Nelson (1993)). Table 1A confirms that the small sample bias increases with the horizon, with the means of the 1- to 5-year coefficients equal to 0.055, 0.106, 0.153, 0.196, and 0.235, respectively. To investigate whether the monotonicity is due to this bias, Table 1B duplicates Table 1A under the assumption that $\sigma_{\varepsilon u} = 0$. For this value, the small sample bias is theoretically zero, and the estimates are unbiased in our simulations. Interestingly, the monotonicity falls only slightly, to

---

σ_u and σ_ε do not matter, this is not true for either the persistence variable ρ (Boudoukh and Richardson (1994)) or the correlation $\sigma_{\varepsilon u}$ (e.g., Stambaugh (1999)). Thus, we also investigate different values for these parameters.

57%. Furthermore, Table 1B shows that the correlation matrix across the multiple-horizon estimators is virtually identical to that in Table 1A. Thus, the monotonicity is driven by the almost perfect correlation across the estimators and the increasing horizon, not by the small sample bias.

As described in Section II.A, much of the literature has argued for predictability by focusing on the increase in the coefficient estimates as a function of the horizon. Both theoretically and in simulation, we show that this result is expected under the null hypothesis of no predictability. An alternative measure of predictability also considered in the literature is the magnitude and pattern of R^2s across horizons. While the R^2 is linked directly to the coefficient estimate, it is nonetheless a different statistic of the data. Table 2A reports the simulated correlation matrix of the multiple-horizon R^2s as well as their means, medians, standard deviations, and monotonicity properties.

Similar to Table 1A, the R^2s are all highly correlated across horizons. For example, the simulated correlations between the 1-year and 2- to 5-year horizon R^2s are 0.949, 0.889, 0.828, and 0.767, respectively. This degree of correlation leads to R^2s that are monotonic in the horizon 52% of the time under the null hypothesis—the exact pattern documented in the literature. This result is not due to the Stambaugh (1999) small sample bias, as both the degree of correlation and monotonicity also appear in the simulated data without the bias (see Table 2B, where the cross-equation correlation is zero). Also, analogous to the evidence for the multiple-horizon coefficient estimators, the degree of cross-correlation and monotonicity depends crucially on the level of persistence ρ of the predictive variable.

Figures 1A and 1B show the correlation coefficients between the 1- and the J-period β estimates and R^2s. The correlations are plotted for different persistence parameters, and the figures illustrate both the monotonicity and near linearity one would expect and the dependence of this effect on the persistence parameter.

The theoretical calculations of Section II.B imply an even stronger condition than monotonicity. For ρ close to 1, the coefficients and R^2s should increase one-for-one with the horizon under the null hypothesis. Because this is the typical pattern found in US data, it seems worthwhile to investigate this implication through a simulation under the null hypothesis of no predictability. We compare the ratio of the 2- to 5-year coefficient and R^2 estimates to the 1-year estimates. Since there are numerical issues when using denominators close to zero, we run the analysis under the condition that the 1-year estimate have an absolute value greater than 0.01, or an R^2 greater than 0.5%. Approximately 88% and 62% of the simulations respectively satisfy these criteria.

Table 3A contains the results. As predicted by the theory, the mean ratios of the estimates are 1.93, 2.80, 3.59, and 4.32 for the 2-, 3-, 4-, and 5-year horizons, respectively. The R^2s are equally dramatic, with corresponding ratios of 1.96, 2.88, 3.77, and 4.61.[7] Note that these simulations are performed under the null hypothesis of no predictability. The βs are zero, but the other parameters are calibrated to match the joint distribution of stock returns and dividend yields in the data. How do these results compare with the estimated coefficients and R^2s from the actual data? In the data, the ratios for the 2-, 3-, 4-, and 5-year horizons are 1.96, 2.98, 3.53, and 3.99 for the β estimates, and 1.85, 3.07, 3.51, and 4.02 for the R^2s. The similarities are startling.

[7] Similar to the earlier tables, Table 3b shows that these findings are not due to the Stambaugh bias and hold equally well for $\sigma_{\varepsilon u} = 0$.

Figures 2A and 2B plot the ratios as a function of the persistence parameter ρ. For large ρ, both the coefficient estimates and R^2s increase linearly with the horizon, with fairly steep slopes (albeit not quite one-for-one). As persistence drops off, the slope diminishes dramatically. For $\rho = 0$, the ratio plot is actually flat. Nevertheless, given the high persistence of the predictive variables used in practice, the more relevant ratios would be those corresponding to steep slopes. These graphs show the mean of the ratio; however, understanding the full distribution allows us to examine whether the actual estimates fall within the empirical null distributions.

To better understand the statistical likelihood of the observed evidence in light of the distribution of the various relevant coefficients under the null hypothesis, Figures 3A and 3B show box plots of the distribution of the multiple-horizon coefficient estimates and R^2s conditional on the 1-year coefficient estimate and R^2 being close to the actual values (i.e., $\hat{\beta} =$ 0.131 and $R^2 = 5.16\%$). The box plots show the median, the 25th and 75th percentiles, and the more extreme 10th and 90th percentiles of the distribution. Several observations are in order. First, consistent with Figures 2A and 2B, the percentiles linearly increase at a fairly steep rate. Second, the actual values of the coefficients and R^2s from the data (marked as diamonds in the graph) lie uniformly between the 25th and 75th percentiles. Given some amount of sampling error, the hypothesis of no predictability produces precisely the pattern one would expect in the coefficients under the alternative hypothesis. Because the sample sizes are relatively small, the presence of sampling error is almost guaranteed. Third, the plots show that what matters is the magnitude of the coefficient at short horizons. In the voluminous literature on stock return predictability in finance, no researcher has ever considered the short-horizon evidence to be remarkable.

III. Empirical Evidence

The theory and corresponding simulation evidence in Section II suggests that it will be very difficult to distinguish between the null hypothesis of no predictability and alternative models of time-varying expected returns that involve persistent autoregressive processes. The reason is that sampling error produces virtually identical patterns in both R^2s and coefficients across horizons. However, this finding does not necessarily imply that joint tests will not distinguish the null from other alternatives. Recall that the null hypothesis implies highly correlated regression coefficient estimators, which induce the coefficient pattern. Even with unremarkable coefficient estimators, yet nonconforming coefficient patterns, one might find strong evidence against the null hypothesis of no predictability.

In this section, we look at a number of commonly used variables to test the predictability of stock returns. For stock returns, we use the excess return on the value-weighted (VW) CRSP portfolio, where excess returns are calculated at a monthly frequency using the 1-month T-bill rate from the CRSP Fama risk-free rate file. For predictors, we use the log dividend yield on the CRSP VW index, three other payout yields adjusted for repurchases and new equity issues, the log earnings yield on the S&P500, the default spread between Baa and Aaa yields, the term spread between long-term government bond yields and T-bill yields, the log book-to-market ratio, the aggregate equity share of new issuances, and the 1-month T-bill yield.[8]

---

[8] See Boudoukh, Michaely, Richardson, and Roberts (2005) for a detailed description of the various measures of payout yield. The data for the first 4 variables are available on Michael Roberts' website *http://www.finance.wharton.upenn.edu\mrrobert\public_html\Research\Data*. See Goyal and Welch (2003) for details on variables 5 to 8. We thank Amit Goyal for graciously providing the data. See Baker and Wurgler

The regression analysis corresponds to equation (1), and covers return horizons of 1 to 5 years over the period 1926–2004. We use the same number of observations for each horizon; therefore, the predictor variables span the period 1925–1999 (75 observations) when available.[9] For each set of multiple-horizon regressions, we calculate the coefficient, its analytical standard error (using equation (2)), its asymptotic p-value, and its simulated p-value under an AR(1) with matching parameters.[10] The AR(1) coefficient used in the simulations is the estimated first-order autocorrelation, corrected for the small sample bias (Kendall (1954)),

$$\hat{\rho}_{1C} = \hat{\rho}_1 + \frac{1+3\hat{\rho}_1}{T}. \quad (9)$$

In addition, we conduct a joint Wald test across the equations, using both asymptotic and simulated p-values. Throughout, asymptotic standard errors, p-values, and test statistics are calculated using the uncorrected sample autocorrelation function. The results are reported in Table 4.

Most of the series show the much-cited pattern of increasing coefficient estimates and corresponding R^2s. For the dividend yield, the payout yield including total repurchases, the payout yield including treasury stock-adjusted repurchases (all on the CRSP VW index), the earnings yields on the S&P500, the default spread, the term spread, the book-to-market ratio, and the risk-free rate the increases in R^2 from the 1-year to the 5-year horizon are 5.16% to 20.76%, 8.66% to 25.83%, 7.73% to 25.83%, 3.33% to 11.80%, 0.31% to 6.42%, 3.04% to

---

(2000) for a description of the equity share of new issuances. The data are available on Jeff Wurgler's website http://pages.stern.nyu.edu/~jwurgler/. The 1-month T-bill yield comes from the CRSP Fama risk-free rate file.

[9] The four payout yield series start in 1926 (74 observations) and the equity share series starts in 1927 (73 observations).

[10] Because the equations involve overlapping observations across multiple horizons, small sample adjustments for coefficient estimators and standard errors (e.g., Amihud and Hurvich (2004) and Amihud, Hurvich, and Wang (2005)) are no longer strictly valid. As developing methods for our particular regression system lies outside the scope of this paper, we rely on simulated p-values as a correction for both the correlation (e.g., Stambaugh (1999)) and long-horizon (e.g., Valkanov (2003)) biases.

13.90%, 3.66% to 18.52%, and 3.11% to 12.65%, respectively. However, the (corrected) persistence levels of the associated variables are 95.3%, 88.7%, 91.9%, 79.1%, 83.8%, 64.2%, 93.4%, and 95.7%, respectively (see Table 5). It should not be surprising that many of the series have significant coefficients using asymptotic p-values across most of the horizons. Under the null hypothesis, the regressions at each horizon are virtually the same.

Table 5 is an alternative representation of the results in Table 4, i.e., the ratios of the coefficient estimates and R^2s across horizons. For the series cited above (except for the default spread), the ratios for both quantities are similar to the simulated ratios under the null hypothesis of no predictability. In all cases, the ratios (and therefore the underlying coefficient estimates and R^2s) increase with the horizon. Thus, the finding that some of the 1-year regressions are significant, and that the same variables produce virtually identical patterns at longer horizons, is actually evidence that the annual regression results are due to sampling error.[11] The joint tests confirm this phenomenon by generally producing higher p-values, e.g., 0.18, 0.16 and 0.20 for the three payout yield variables on the CRSP VW index, 0.39 for the earnings yield on the S&P500, 0.65 for the default spread, 0.32 for the term spread, 0.32 for the book-to-market ratio, and 0.08 for the risk-free rate, the only variable significant at the 10% level.

Several observations illustrate the nature of the joint tests. First, consider the regression results for the dividend yield versus the two payout yield measures on the CRSP VW index. By almost any eyeball measure, the evidence for the payout yield appears to be stronger. All of the horizons produce larger coefficient estimates and R^2s and lower p-values. While four of the five p-values are less than 0.02 for the payout yield, none of the coefficients satisfy this criterion for the dividend yield. Nevertheless, the p-value of the joint test for the payout yield is similar to that for the dividend yield.

Second, the individual coefficient p-values and corresponding R^2s of the one marginally significant variable (out of series cited above) under the joint cross-horizon test, i.e., the risk-free rate, look less impressive if anything than those for the other series. Yet the significance level of the joint test is much higher.[12] Why? The pattern in the coefficients, while monotonic, is much less linear and one-to-one than implied by the estimated autocorrelation function. This result illustrates the power of the joint test to uncover seemingly innocuous differences across horizons.

Third, the simulated p-values in general show much less significance for both the individual and joint tests. For example, the risk-free rate is no longer significant at the 10% level. This mirrors the small sample findings of Goetzmann and Jorion (1993), Kim and Nelson (1993), and Valkanov (2003). As Tables 1A and 1B show, the correlation pattern across multiple-horizon estimators is robust to small sample considerations.

Finally, two variables, the net payout yield (i.e., payout yield plus net issuance) and the equity share of new issuances, are strongly significant across horizons as evidenced by Wald statistics with *p*-values of 0.00 and 0.01, respectively. These results are consistent with the short-horizon findings of Boudoukh, Michaely, Richardson, and Roberts (2005) and Baker and Wurgler (2000), and show that this evidence continues to long horizons. Of some interest, while the coefficients and R^2s are large across all horizons, the pattern is no longer monotonic. This finding provides even sharper evidence against the null since the series are positively

[11] This conclusion has even greater support once the researcher takes into account the data-snooping arguments of Foster, Smith, and Whaley (1997).

autocorrelated at the relevant horizons. With this degree of autocorrelation and the overlapping horizons, one would have expected a pattern similar to the other predictive variables.

IV. Conclusion

Long-horizon stock return predictability is considered to be one of the more important pieces of evidence in the empirical asset pricing literature over the last couple of decades (see, e.g., the textbooks of Campbell, Lo, and MacKinlay (1997) and Cochrane (2001)). The evidence is set forth as a yardstick for theoretical asset pricing models and is slowly penetrating the practitioner community (for two recent examples, see Brennan and Xia (2005) and Asness (2003)).

Long-horizon predictability has also been documented in other markets, which is perhaps not surprising, given our analysis. The highly cited work of Fama and Bliss (1987) and Mark (1995) document results similar in spirit to the ones discussed in this paper for bond returns and exchange rates, respectively. Both papers involve highly persistent regressors and document nearly linearly increasing βs and R^2s.

In this paper, we show that stronger long-horizon results, in the form of higher βs and increasing R^2s, present little if any independent evidence over and above the short-horizon results for persistent regressors. Under the null hypothesis of no predictability, sampling variation can generate small levels of predictability at short horizons. This result is well known. Our research shows that higher levels of predictability at longer horizons are to be expected as well.

---

[12] In a multivariate regression framework that includes both dividend yields and the short rate, Ang and Bekeart (2005) find that the short rate has predictive power across multiple horizons.

Appendix A

Under the null hypothesis of no predictability, $\beta_1 \cdots = \beta_J = \cdots = \beta_{J*} = \cdots = 0$, we can rewrite regression equation system (1) for any J and $J*$ horizons as:

$$E[f_t(\cdot)] \equiv E\begin{pmatrix} (R_{t,t+1} - \alpha_1 + \beta_1 X_t) \\ (R_{t,t+1} - \alpha_1 + \beta_1 X_t) X_t \\ \vdots \\ (R_{t,t+J} - J\alpha_1 + \beta_J X_t) X_t \\ \vdots \\ (R_{t,t+J*} - J^*\alpha_1 + \beta_{J*} X_t) X_t \\ \vdots \end{pmatrix} = 0 . \tag{10}$$

Under the assumption of conditional homoskedasticity of the error terms above, one can apply the approach of Richardson and Smith (1991) (see also Boudoukh and Richardson (1994)) to analytically derive the asymptotic distribution of the estimators $\hat{\beta}' = (\cdots \hat{\beta}_J \cdots \hat{\beta}_{J*} \cdots)$. Applying results from Hansen (1982), the vector of regression coefficients $\hat{\theta} = (\hat{\alpha}_1 \; \hat{\beta})'$ has an asymptotic normal distribution with mean $(\alpha_1 \; 0)'$ and variance-covariance matrix $\left[D_0' S_0^{-1} D_0\right]^{-1}$, where $D_0 = E\left[\partial f_t / \partial \theta\right]$ and $S_0 = \sum_{l=-\infty}^{+\infty} E\left[f_t f_{t-l}\right]$. Under these assumptions, it is possible to calculate D_0 and S_0 analytically. Specifically,

$$D_0 = \begin{pmatrix} 1 & \mu_X & \cdots & 0 & \cdots & 0 & \vdots \\ \mu_X & \sigma_X^2 + \mu_X^2 & \cdots & 0 & \cdots & 0 & \vdots \\ \vdots & \cdots & \ddots & \cdots & \cdots & \cdots & \vdots \\ J\mu_X & 0 & \cdots & \sigma_X^2 + \mu_X^2 & \cdots & 0 & \vdots \\ \vdots & \cdots & \cdots & \cdots & \ddots & \cdots & \vdots \\ J^*\mu_X & 0 & \cdots & 0 & \cdots & \sigma_X^2 + \mu_X^2 & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \ddots \end{pmatrix}, \tag{11}$$

and

$$S_0 = \begin{pmatrix} \sigma_R^2 & \sigma_R^2\mu_X & \cdots & J\sigma_R^2\mu_X & \cdots & J^*\sigma_R^2\mu_X & \vdots \\ \sigma_R^2\mu_X & \sigma_R^2(\mu_X^2 + \sigma_X^2) & \cdots & \sigma_R^2\left(J\mu_X^2 + \sigma_X^2\left[1 + \sum_{l=1}^{J-1}\rho_l\right]\right) & \cdots & \sigma_R^2\left(J^*\mu_X^2 + \sigma_X^2\left[1 + \sum_{l=1}^{J^*-1}\rho_l\right]\right) & \vdots \\ \vdots & \cdots & \ddots & \cdots & \cdots & \cdots & \vdots \\ J\sigma_R^2\mu_X & \sigma_R^2\left(J\mu_X^2 + \sigma_X^2\left[1 + \sum_{l=1}^{J-1}\rho_l\right]\right) & \cdots & \sigma_R^2\left(J^2\mu_X^2 + \sigma_X^2\left[J + 2\sum_{l=1}^{J-1}(J-l)\rho_l\right]\right) & \cdots & \sigma_R^2\left(JJ^*\mu_X^2 + \sigma_X^2\left[J + \sum_{l=1}^{J-1}(J-l)[\rho_l + \rho_{J*-J+l}] + \sum_{l=1}^{J*-J}J\rho_l\right]\right) & \vdots \\ \vdots & \cdots & \cdots & \cdots & \ddots & \cdots & \vdots \\ J^*\sigma_R^2\mu_X & \sigma_R^2\left(J^*\mu_X^2 + \sigma_X^2\left[1 + \sum_{l=1}^{J^*-1}\rho_l\right]\right) & \cdots & \sigma_R^2\left(JJ^*\mu_X^2 + \sigma_X^2\left[J + \sum_{l=1}^{J-1}(J-l)[\rho_l + \rho_{J*-J+l}] + \sum_{l=1}^{J*-J}J\rho_l\right]\right) & \cdots & \sigma_R^2\left(J^{*2}\mu_X^2 + \sigma_X^2\left[J^* + 2\sum_{l=1}^{J^*-1}(J^*-l)\rho_l\right]\right) & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \ddots \end{pmatrix}, \tag{12}$$

where μ_X is the mean of X_t, σ_X^2 is the unconditional variance of X_t, $\square_l$ is the l^{th} order autocorrelation of X_t, σ_R^2 is the variance of single period returns R_t, and $J^* > J$. Using D_0 and S_0 above, and performing the relevant matrix calculations, one gets the desired result given in equation (2).

References

Ait-Sahalia, Y., and M. Brandt, 2001, "Variable Selection for Portfolio Choice," *Journal of Finance*, 56 (4), 1297–1351.

Amihud, Y., and C. Hurvich, 2004, "Predictive Regression: A Reduced-Bias Estimation Method," *Journal of Financial and Quantitative Analysis*, 39 (4), 813–841.

Amihud, Y., C. Hurvich, and Y. Wang, 2005, "Hypothesis Testing in Predictive Regressions," NYU working paper.

Ang, A., and G. Bekaert, 2005, "Stock Return Predictability: Is it There?" Columbia University working paper.

Asness, C., 2003, "Fight the Fed Model," *Journal of Portfolio Management*, 30 (1), 11–24.

Baker, M. and J. Wurgler, 2000, "The Equity Share in New Issues and Aggregate Stock Returns," *Journal of Finance*, 55 (5), 2219-2257.

Barberis, N., 2000, "Investing for the Long Run when Returns Are Predictable," *Journal of Finance*, 55 (1), 225–264

Barberis, N., and R. Thaler, 2003, "A Survey of Behavioral Finance," *The Handbook of the Economics of Finance*, ed. North Holland: Amsterdam.

Bossaerts, P., and P. Hillion, 1999, "Implementing Statistical Criteria to Implement Return Forecasting Models: What Do We Learn?" *Review of Financial Studies*, 12 (2), 405–428.

Boudoukh, J., R. Michaely, M. Richardson, and M. Roberts, 2005, "On the Importance of Measuring Payout Yield: Implications for Empirical Asset Pricing," *Journal of Finance*, forthcoming.

Boudoukh, J., and M. Richardson, 1994, "The Statistics of Long-Horizon Regressions Revisited," *Mathematical Finance*, 4 (2), 103–120.

Brennan, M., and Y. Xia, 2005, "Persistence, Predictability, and Portfolio Planning," Wharton working paper.

Campbell, J., 2000, "Asset Pricing at the Millennium," *Journal of Finance*, 55 (4), 1515–1567.

Campbell, J., 2001, "Why Long Horizons? A Study of Power Against Persistent Alternatives," *Journal of Empirical Finance*, 8, 459–491.

Campbell, J., 2003, "Consumption-Based Asset Pricing" in Chapter 13 of *The Handbook of the Economics of Finance*, ed. North Holland: Amsterdam, 803–881.

Campbell, J., and J. Cochrane, 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107 (2), 205–251.

Campbell, J., A. Lo, and C. MacKinlay, 1997, *The Econometrics of Financial Markets*, Princeton: Princeton University Press.

Campbell, J., and R. Shiller, 1988, "The Dividend–Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, 1 (3), 195–228.

Campbell, J., and L. Viceira, 1999, "Consumption and Portfolio Decisions When Expected Returns Are Time Varying," *Quarterly Journal of Economics*, 114 (2), 433–495

Cochrane, J., 1999, "New Facts in Finance," *Economic Perspectives*, 23 (3), 36–58.

Cochrane, J., 2001, *Asset Pricing*, Princeton: Princeton University Press.

Cremers, M., 2002, "Stock Return Predictability: A Bayesian Model Selection Perspective," *Review of Financial Studies*, 15 (4), 1223–1249.

Fama, E., 1998, "Market Efficiency, Long-Term Returns, and Behavioral Finance," *Journal of Financial Economics*, 49, 283–306.

Fama, E., and R. Bliss, 1987, "The Information in Long Maturity Forward Rates," *American Economic Review,* 77, 680–92.

Fama, E., and K. French, 1988, "Dividend Yields and Expected Stock Returns," *Journal of Financial Economics*, 22, 3–25.

Ferson, W., and R. Korajczyk, 1995, "Do Arbitrage Pricing Models Explain the Predictability of Stock Returns?" *Journal of Business*, 68 (3), 309–349

Foster, D., T. Smith, and R. Whaley, 1997, "Assessing Goodness-of-Fit of Asset Pricing Models: The Distribution of the Maximal R-Squared," *Journal of Finance*, 52 (2), 591–607.

Goetzmann, W., and P. Jorion, 1993, "Testing The Predictive Power of Dividend Yields," *Journal of Finance*, 48, 663–679.

Goyal, A., and Welch I., 2003, "Predicting the Equity Premium with Dividend Ratios," *Management Science*, 49 (5), 639–654.

Hansen, L., 1982, "Large Sample Properties of Generalized Method of Moments," *Econometrica*, 50 (4), 1029–1054.

Hansen, L., and R. Hodrick, 1980, "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," *Journal of Political Economy*, 88 (5), 829–853.

Hodrick, R., 1992, "Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement," *Review of Financial Studies*, 5(3), 357–386.

Kendall, M.G., 1954, "Note on Bias in the Estimation of Autocorrelation," *Biometrika*, 41, 403–404.

Kim, M.J., and C.R. Nelson, 1993, "Predictable Returns – the Role of Small Sample Bias," *Journal of Finance*, 48 (2), 641–661.

Kirby, C., 1997, "Measuring the Predictable Variation in Stock and Bond Returns," *Review of Financial Studies*, 10 (3), 579–630.

Lettau, M., and S. Ludvigson, 2001, "Consumption, Aggregate Wealth, and Expected Stock Returns," *Journal of Finance*, 56 (3), 815–849.

Lettau, M., and S. Ludvigson, 2005, "Expected Returns and Expected Dividend Growth," *Journal of Financial Economics*, 76 (3), 583–626.

Lewellen, J., 2004, "Predicting Returns with Financial Ratios," *Journal of Financial Economics*, 74 (2), 209–235.

Mark, N. C., 1995, "Exchange Rates and Fundamentals, Evidence on Long Horizon Predictability," *American Economic Review*, 85 (1), 201–218.

Menzly, L., T. Santos and P. Veronesi, 2004, "Understanding Predictability," *Journal of Political Economy*, 112, 1–47.

Patelis, A.D., 1997, "Stock Return Predictability and the Role of Monetary Policy," *Journal of Finance*, 52 (5), 1951–1972

Poterba, J., and L. Summers, 1988, "Mean Reversion in Stock Prices: Evidence and Implications," *Journal of Financial Economics*, 22, 27–60.

Richardson, M., 1993, "Temporary Components of Stock Prices: A Skeptic's View," *Journal of Business and Economics Statistics*, 11 (2), 199–207.

Richardson, M., and T. Smith, 1991, "Tests of Financial Models in the Presence of Overlapping Observations," *Review of Financial Studies*, 4 (2), 227–254.

Richardson, M., and T. Smith, 1994, "A Unified Approach to Testing for Serial Correlation in Stock Returns," *Journal of Business*, 67 (3), 371–399.

Richardson, M., and J. Stock, 1989, "Drawing Inferences from Statistics Based on Multi-Year Asset Returns," *Journal of Financial Economics*, 25, 323–348.

Stambaugh, R.F., 1993, "Estimating Conditional Expectations When Volatility Fluctuates," NBER Technical Paper 140.

Stambaugh, R.F., 1999, "Predictive Regressions," *Journal of Financial Economics*, 54 (3), 375–421.

Valkanov, R., 2003, "Long-Horizon Regressions: Theoretical Results and Applications," *Journal of Financial Economics*, 68 (2), 201–232.

Panel A: $\sigma_{\varepsilon u} = -0.712$

Coefficient estimates

				Correlations			
				Horizon			
Horizon	Mean	SD	Median	2	3	4	5
1	0.055	0.076	0.043	0.966	0.926	0.885	0.843
2	0.106	0.143	0.085		0.980	0.946	0.909
3	0.153	0.203	0.126			0.985	0.957
4	0.196	0.257	0.165				0.988
5	0.235	0.307	0.203				
% monotonic		66.02					

Test statistics

				Size		
	Mean	SD	Median	10%	5%	1%
Wald	6.227	3.901	5.469	18.507	10.645	3.015
P-value	0.403	0.288	0.361			

Panel B: $\sigma_{\varepsilon u} = 0$

Coefficient estimates

				Correlations			
				Horizon			
Horizon	Mean	SD	Median	2	3	4	5
1	0.000	0.070	0.000	0.960	0.913	0.867	0.823
2	0.000	0.133	0.001		0.977	0.940	0.900
3	0.001	0.194	0.001			0.984	0.954
4	0.001	0.251	0.002				0.988
5	0.000	0.305	0.001				
% monotonic		57.30					

Test statistics

				Size		
	Mean	SD	Median	10%	5%	1%
Wald	5.949	3.876	5.142	16.813	9.684	2.773
P-value	0.429	0.294	0.399			

Table 1: Distribution of Coefficient Estimates and Test Statistics

Panel A reports the mean, standard deviation, and median of the coefficient estimates from the predictive regression (equation (1)), and the correlations between these estimates for horizons of 1 to 5 years across 100,000 simulations. "% monotonic" is the percentage of the simulations that produce coefficients that are monotonic in the horizon. Panel A also reports the mean, standard deviation, and median of the joint Wald test statistic (across horizons), the associated p-values, and the percentage of statistics that reject the null hypothesis of no predictability at the 10%, 5%, and 1% levels. There are 75 observations for each simulation, and simulations are performed under the null hypothesis of no predictability using the parameters $\rho = 0.953$, $\sigma_{\varepsilon} = 0.212$, $\sigma_u = 0.154$, $\sigma_{\varepsilon u} = -0.712$. Panel B reports the same statistics for $\sigma_{\varepsilon u} = 0$ (all other simulation parameters are the same as in Panel A).

Panel A: $\sigma_{\varepsilon u}$ = –0.712

				Correlations			
				Horizon			
Horizon	Mean	SD	Median	2	3	4	5
1	1.833	2.378	0.918	0.949	0.889	0.828	0.767
2	3.469	4.348	1.816		0.969	0.918	0.861
3	4.966	6.041	2.665			0.977	0.933
4	6.337	7.525	3.454				0.981
5	7.600	8.837	4.259				
% monotonic		52.21					

Panel B: $\sigma_{\varepsilon u}$ = 0

				Correlations			
				Horizon			
Horizon	Mean	SD	Median	2	3	4	5
1	1.345	1.861	0.618	0.927	0.846	0.768	0.696
2	2.525	3.400	1.203		0.957	0.892	0.821
3	3.614	4.771	1.746			0.969	0.914
4	4.626	5.995	2.280				0.976
5	5.574	7.099	2.790				
% monotonic		42.58					

Table 2: Distribution of R^2s

Panel A reports the mean, standard deviation, and median of the R^2s from the predictive regression (equation (1)) and the correlations between them for horizons of 1 to 5 years across 100,000 simulations. "% monotonic" is the percentage of the simulations that produce R^2s that are monotonic in the horizon. There are 75 observations for each simulation, and simulations are performed under the null hypothesis of no predictability using the parameters ρ = 0.953, σ_ε = 0.212, σ_u = 0.154, $\sigma_{\varepsilon u}$ = –0.712. Panel B reports the same statistics for $\sigma_{\varepsilon u}$ = 0 (all other simulation parameters are the same as in Panel A).

Panel A: $\sigma_{\varepsilon u} = -0.712$

	Coefficient estimate ratios				R^2 ratios			
Horizon	Mean	SD	Median	# of sim.	Mean	SD	Median	# of sim.
2	1.934	0.874	1.919	88,495	1.957	0.813	1.875	62,126
3	2.798	1.894	2.739	88,495	2.880	1.726	2.612	62,126
4	3.592	3.056	3.472	88,495	3.766	2.744	3.237	62,126
5	4.318	4.326	4.139	88,495	4.612	3.821	3.785	62,126
% monotonic		70.38			% monotonic		60.40	

Panel B: $\sigma_{\varepsilon u} = 0$

	Coefficient estimate ratios				R^2 ratios			
Horizon	Mean	SD	Median	# of sim.	Mean	SD	Median	# of sim.
2	1.872	1.154	1.887	87,058	1.905	0.959	1.783	54,599
3	2.620	2.426	2.665	87,058	2.756	1.987	2.400	54,599
4	3.269	3.846	3.350	87,058	3.552	3.082	2.887	54,599
5	3.835	5.357	3.945	87,058	4.299	4.206	3.241	54,599
% monotonic		61.27			% monotonic		49.00	

Table 3: Distribution of Coefficient Estimate and R^2 Cross-Horizon Ratios

Panel A reports the mean, standard deviation, and median of the coefficient estimate and R^2 ratios (i.e., $\hat{\beta}_i / \hat{\beta}_1$ and R^2_i/R^2_1, i = 2,…,5) from the predictive regression (equation (1)) across the simulations out of the 100,000 for which $\hat{\beta}_1 > 0.01$ or $R^2_1 > 0.5\%$, respectively. "% monotonic" is the percentage of these simulations that produce coefficient estimates and R^2s that are monotonic in the horizon. There are 75 observations for each simulation, and simulations are performed under the null hypothesis of no predictability using the parameters $\rho = 0.953$, $\sigma_\varepsilon = 0.212$, $\sigma_u = 0.154$, $\sigma_{\varepsilon u} = -0.712$. Panel B reports the same statistics for $\sigma_{\varepsilon u} = 0$ (all other simulation parameters are the same as in Panel A).

	Horizon					
	1	2	3	4	5	Wald
	Log dividend yield, CRSP VW					
$\hat{\beta}$	0.131	0.257	0.390	0.461	0.521	7.576
Std. err.	0.067	0.130	0.191	0.249	0.306	
Asym. p-value	0.025	0.025	0.021	0.032	0.044	0.181
Sim. p-value	0.148	0.142	0.125	0.150	0.172	0.293
R^2	5.164	9.551	15.836	18.143	20.756	
	Log payout yield, CRSP VW, cash flow					
$\hat{\beta}$	0.214	0.401	0.567	0.657	0.736	7.840
Std. err.	0.085	0.162	0.235	0.304	0.370	
Asym. p-value	0.006	0.007	0.008	0.015	0.023	0.165
Sim. p-value	0.046	0.045	0.044	0.059	0.072	0.211
R^2	8.664	14.530	20.912	22.997	25.829	
	Log payout yield, CRSP VW, Treasury stock					
$\hat{\beta}$	0.188	0.354	0.510	0.601	0.682	7.309
Std. err.	0.078	0.152	0.221	0.286	0.350	
Asym. p-value	0.008	0.010	0.010	0.018	0.026	0.199
Sim. p-value	0.072	0.069	0.066	0.082	0.096	0.270
R^2	7.729	13.213	19.714	22.387	25.827	
	Log net payout yield, CRSP VW, cash flow					
$\hat{\beta}$	0.718	1.321	1.536	1.537	1.512	19.024
Std. err.	0.173	0.315	0.431	0.528	0.616	
Asym. p-value	0.000	0.000	0.000	0.002	0.007	0.002
Sim. p-value	0.000	0.000	0.001	0.004	0.012	0.001
R^2	23.399	37.990	36.887	30.253	26.247	
	Log earnings yield, S&P500					
$\hat{\beta}$	0.101	0.228	0.328	0.383	0.380	5.175
Std. err.	0.064	0.120	0.171	0.218	0.262	
Asym. p-value	0.057	0.028	0.028	0.040	0.073	0.395
Sim. p-value	0.125	0.073	0.066	0.081	0.129	0.430
R^2	3.334	8.065	11.757	13.373	11.798	

Table 4: Coefficient Estimates and R^2s from Predictive Regressions

The table reports results from the regression of 1- to 5-year CRSP value-weighted returns on various lagged predictor variables (equation (1)) for the period 1926–2004 (75 observations). $\hat{\beta}$ is the estimated coefficient, with associated asymptotic standard error (equation (3)), p-value under the null hypothesis of no predictability, and the asymptotic Wald test and p-value for the joint hypothesis of no predictability across horizons. The table also reports simulated p-values (100,000 simulations) for both the individual coefficients and the Wald test. There are 75 observations for each simulation, and simulations are performed under the null hypothesis of no predictability using parameters estimated from the data.

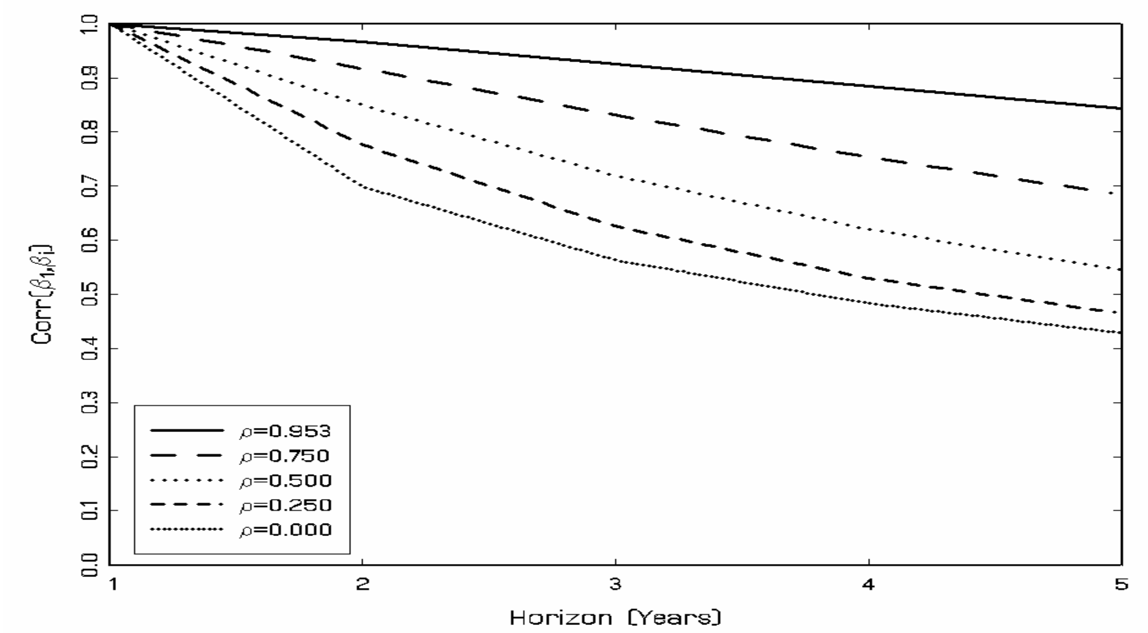
	Horizon 1	2	3	4	5	Wald
	Default yield spread					
$\hat{\beta}$	1.372	4.961	7.111	9.982	12.512	3.335
Std. err.	2.864	5.420	7.734	9.825	11.759	
Asym. p-value	0.316	0.180	0.179	0.155	0.144	0.648
Sim. p-value	0.429	0.278	0.280	0.251	0.237	0.690
R^2	0.306	1.911	2.770	4.559	6.417	
	Term yield spread					
$\hat{\beta}$	2.663	3.715	5.860	9.350	11.336	7.082
Std. err.	1.763	3.157	4.260	5.136	5.853	
Asym. p-value	0.065	0.120	0.084	0.034	0.026	0.215
Sim. p-value	0.077	0.133	0.100	0.048	0.043	0.230
R^2	3.041	2.829	4.966	10.560	13.905	
	Log book-to-market ratio					
$\hat{\beta}$	0.086	0.187	0.289	0.358	0.384	5.841
Std. err.	0.052	0.100	0.146	0.189	0.229	
Asym. p-value	0.049	0.031	0.024	0.029	0.047	0.322
Sim. p-value	0.225	0.180	0.155	0.164	0.208	0.441
R^2	3.665	8.295	13.988	18.023	18.520	
	Equity share of new issuances					
$\hat{\beta}$	-0.741	-1.181	-1.311	-1.351	-1.189	16.161
Std. err.	0.216	0.352	0.461	0.552	0.652	
Asym. p-value	0.000	0.000	0.002	0.007	0.034	0.006
Sim. p-value	0.001	0.001	0.005	0.012	0.039	0.005
R^2	16.126	20.103	17.271	14.976	10.831	
	Risk-free rate					
$\hat{\beta}$	-1.287	-1.812	-2.911	-4.234	-5.165	9.776
Std. err.	0.842	1.644	2.420	3.176	3.918	
Asym. p-value	0.063	0.135	0.114	0.091	0.094	0.082
Sim. p-value	0.075	0.141	0.119	0.094	0.094	0.145
R^2	3.112	2.946	5.367	9.485	12.646	

Table 4 Cont'd

		Horizon				$\hat{\rho}_{1C}$
		2	3	4	5	
ln (D/P) (CRSP VW)	$\hat{\rho}_{i-1}$	0.901	0.780	0.687	0.637	0.953
	$\hat{\beta}_i / \hat{\beta}_1$	1.962	2.982	3.527	3.986	
	R^2_i / R^2_1	1.850	3.067	3.514	4.020	
ln (payout/P) (CRSP VW, CF)	$\hat{\rho}_{i-1}$	0.837	0.674	0.574	0.508	0.887
	$\hat{\beta}_i / \hat{\beta}_1$	1.869	2.645	3.065	3.432	
	R^2_i / R^2_1	1.677	2.414	2.654	2.981	
Ln (payout/P) (CRSP VW, TS)	$\hat{\rho}_{i-1}$	0.867	0.721	0.618	0.567	0.919
	$\hat{\beta}_i / \hat{\beta}_1$	1.887	2.719	3.202	3.634	
	R^2_i / R^2_1	1.710	2.551	2.897	3.342	
ln (net payout/P) (CRSP VW, CF)	$\hat{\rho}_{i-1}$	0.670	0.280	0.108	0.128	0.713
	$\hat{\beta}_i / \hat{\beta}_1$	1.839	2.138	2.139	2.105	
	R^2_i / R^2_1	1.624	1.576	1.293	1.122	
ln (E/P) (S&P500)	$\hat{\rho}_{i-1}$	0.746	0.565	0.405	0.318	0.791
	$\hat{\beta}_i / \hat{\beta}_1$	2.250	3.234	3.774	3.746	
	R^2_i / R^2_1	2.419	3.527	4.011	3.539	
Default spread	$\hat{\rho}_{i-1}$	0.790	0.564	0.383	0.306	0.838
	$\hat{\beta}_i / \hat{\beta}_1$	3.617	5.184	7.277	9.122	
	R^2_i / R^2_1	6.250	9.060	14.912	20.990	
Term spread	$\hat{\rho}_{i-1}$	0.603	0.213	0.008	-0.058	0.642
	$\hat{\beta}_i / \hat{\beta}_1$	1.395	2.201	3.511	4.257	
	R^2_i / R^2_1	0.930	1.633	3.472	4.572	
ln (B/M)	$\hat{\rho}_{i-1}$	0.882	0.721	0.580	0.456	0.934
	$\hat{\beta}_i / \hat{\beta}_1$	2.177	3.364	4.179	4.476	
	R^2_i / R^2_1	2.263	3.816	4.917	5.053	
Equity share of new issuances	$\hat{\rho}_{i-1}$	0.332	0.116	0.046	0.291	0.360
	$\hat{\beta}_i / \hat{\beta}_1$	1.594	1.771	1.824	1.605	
	R^2_i / R^2_1	1.247	1.071	0.929	0.672	
Risk-free rate	$\hat{\rho}_{i-1}$	0.905	0.816	0.759	0.730	0.957
	$\hat{\beta}_i / \hat{\beta}_1$	1.408	2.262	3.290	4.014	
	R^2_i / R^2_1	0.947	1.725	3.048	4.064	

Table 5: Autocorrelation Estimates and Coefficient Estimate and R^2 Ratios from Predictive Regressions

The table reports the estimated autocorrelation function ($\hat{\rho}_{i-1}$), the corrected first-order autocorrelation ($\hat{\rho}_{1C}$), the coefficient estimate ratios ($\hat{\beta}_i / \hat{\beta}_1$, i = 2,…,5) and the R^2 ratios (R^2_i / R^2_1, i = 2,…,5) from the regression of 1- to 5-year CRSP value-weighted returns on various lagged predictor variables (equation (1)) for the period 1926–2004.



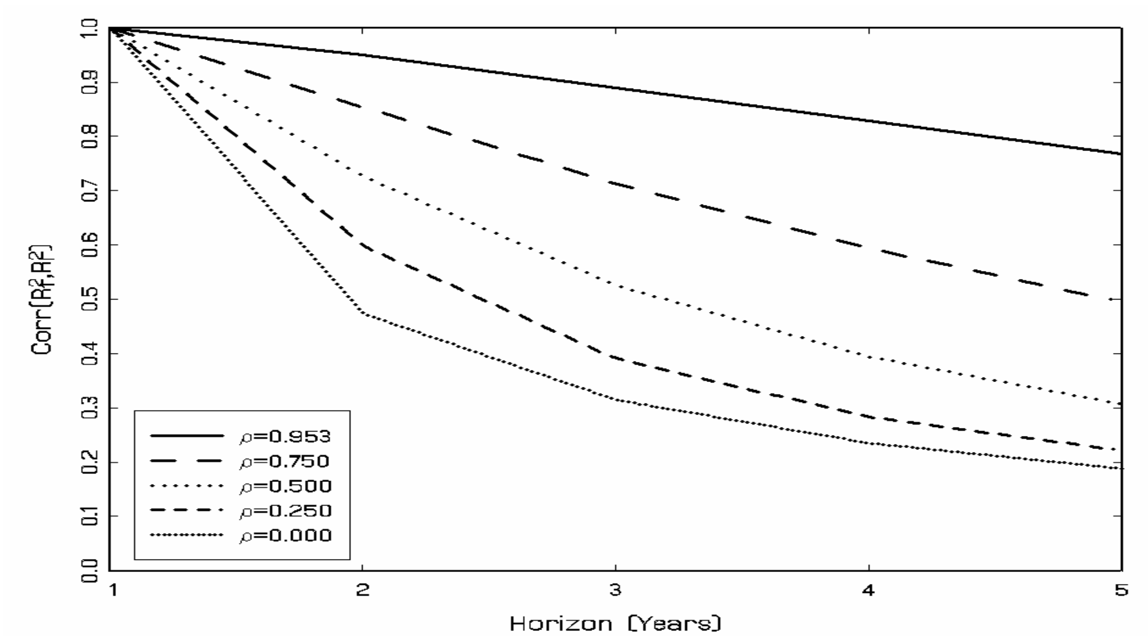


Figure 1: Cross-Horizon Correlations between Coefficient Estimates and R^2s

The top panel plots the correlation between the coefficient estimate at the 1-year horizon and those at the 2- to 5-year horizons from the predictive regression (equation (1)) across 100,000 simulations for different values of ρ (the autocorrelation of the predictor variable). There are 75 observations for each simulation, and simulations are performed under the null hypothesis of no predictability using the parameters $\sigma_\varepsilon = 0.212$, $\sigma_u = 0.154$, $\sigma_{\varepsilon u} = -0.712$. The bottom panel plots the analogous correlations for the predictive regression R^2s.

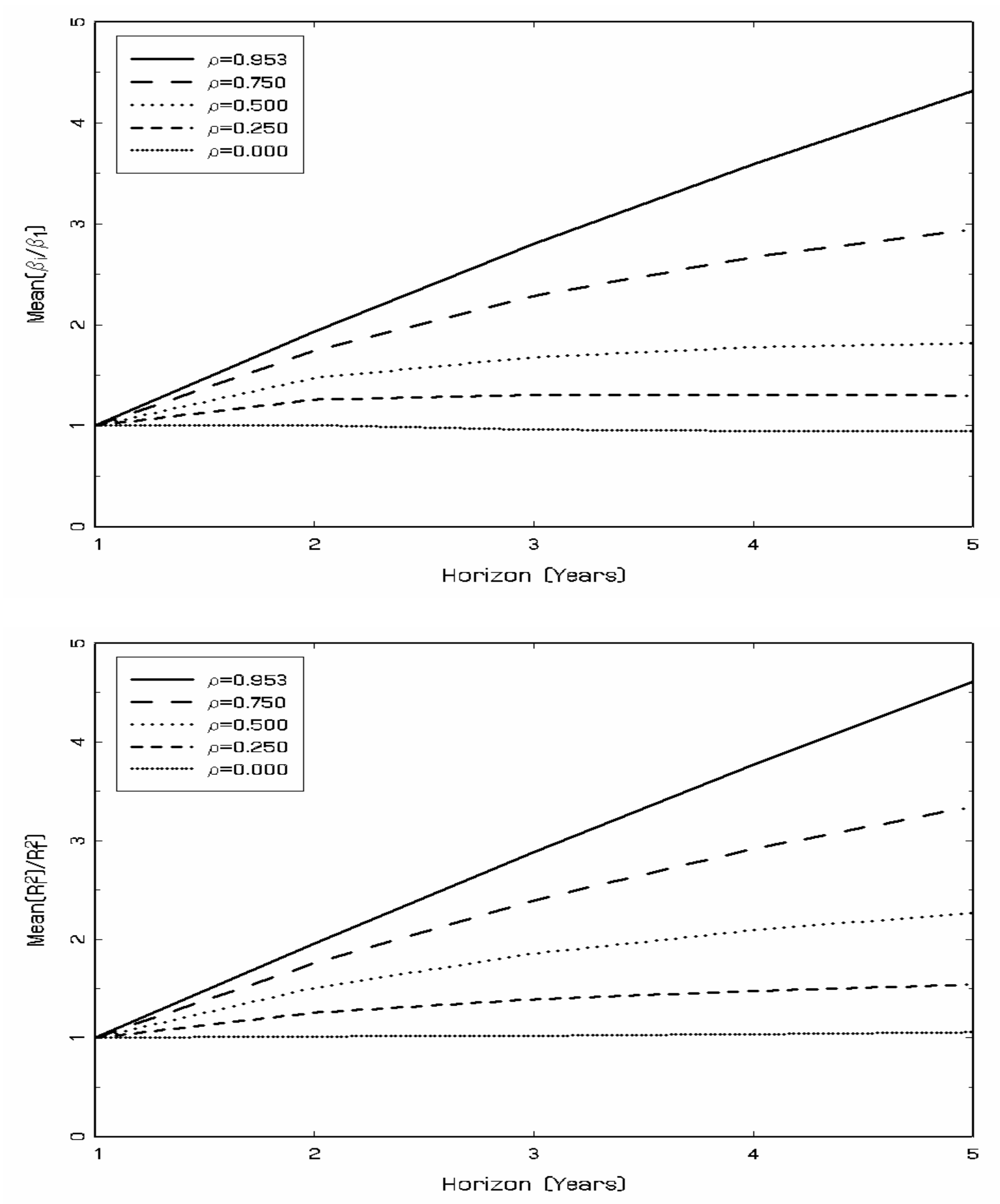


Figure 2: Mean Coefficient Estimate and R^2 Cross-Horizon Ratios

The top panel plots the mean coefficient estimate ratios (i.e., $\hat{\beta}_i / \hat{\beta}_1$, i = 2,…,5) from the predictive regression (equation (1)) across the simulations out of the 100,000 for which $|\hat{\beta}_1| > 0.01$ for different values of ρ (the autocorrelation of the predictor variable). There are 75 observations for each simulation, and simulations are performed under the null hypothesis of no predictability using the parameters $\sigma_\varepsilon = 0.212$, $\sigma_u = 0.154$, $\sigma_{\varepsilon u} = -0.712$. The bottom panel plots the means of the analogous R^2 ratios (i.e., R^2_i / R^2_1, i = 2,…,5) for simulations with or $R^2_1 > 0.5\%$.

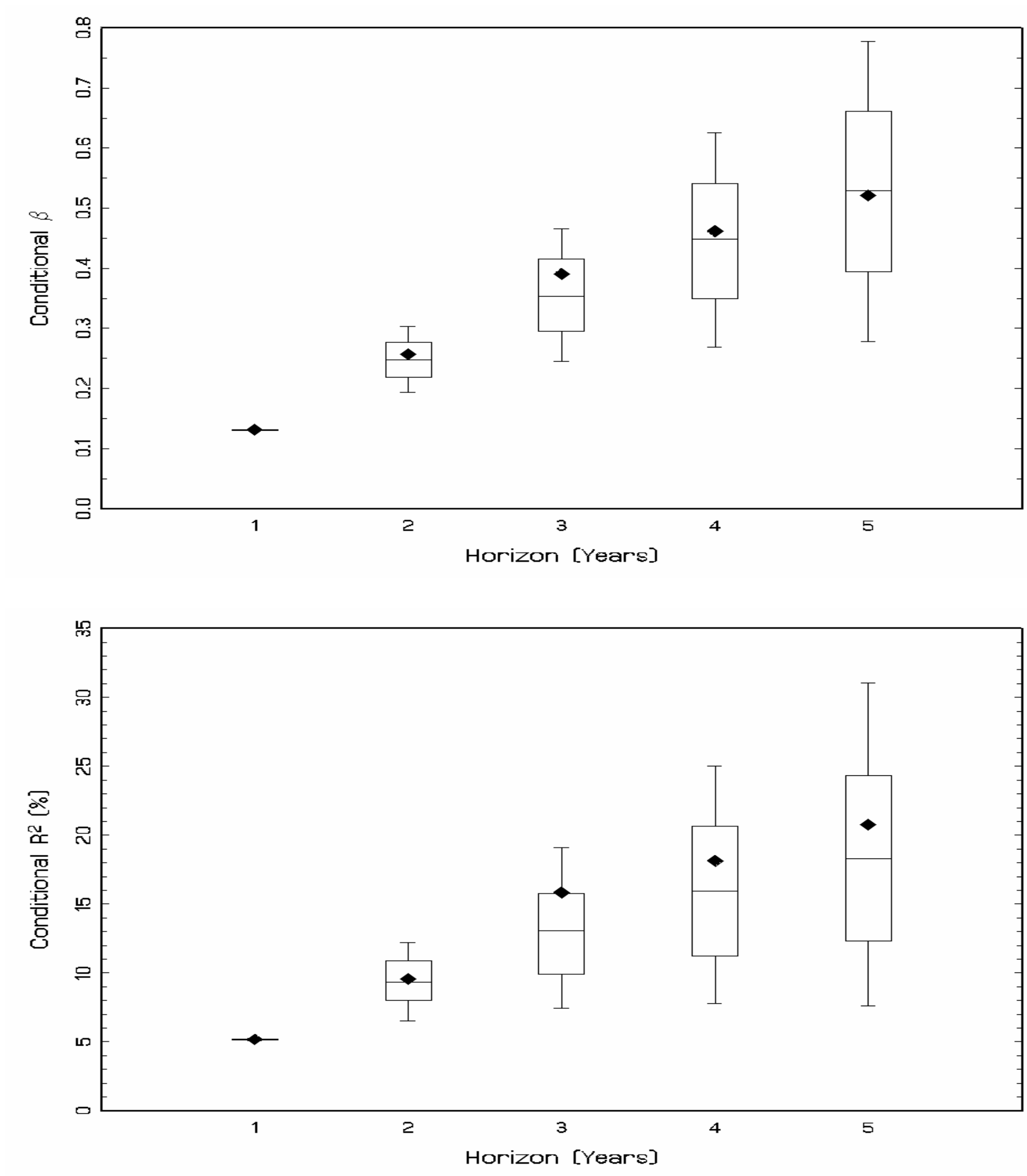


Figure 3: Conditional Distribution of Coefficient Estimates and R^2s

The top panel provides a box plot of the simulated distributions of the coefficient estimates for horizons 2- to 5-years from the predictive regression (equation (1)) for the 971 out of 100,000 simulations for which $0.115 < \hat{\beta}_1 < 0.119$. The boxes show the median, 25th/75th percentiles, and 10th/90th percentiles. The diamonds mark the actual coefficient estimates from the first regression in Table 4. There are 75 observations for each simulation, and simulations are performed under the null hypothesis of no predictability using the parameters $\rho = 0.953$, $\sigma_\varepsilon = 0.212$, $\sigma_u = 0.154$, $\sigma_{\varepsilon u} = -0.712$. The bottom panel plots the analogous simulated distributions for the predictive regression R^2s for the 899 simulations for which $4.215\% < R^2_1 < 4.414\%$ and the corresponding actual R^2s.