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CASH-FLOW RISK, DISCOUNT RISK, AND THE VALUE PREMIUM

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ABSTRACT

A habit persistence, general equilibrium model with multiple assets matches both the time series properties of the market portfolio and the cross-sectional predictability of returns on price sorted portfolios, the value premium. Consistent with empirical evidence, the model shows that (a) value stocks are those with higher cash-flow risk; (b) the size of the value premium is larger in "bad times," due to time variation in risk preferences; (c) the unconditional CAPM fails, because of general equilibrium restrictions on the market portfolio. The dynamic nature of the value premium rationalizes why the conditional CAPM and a Fama and French (1993) HML factor outperform the unconditional CAPM.

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I. INTRODUCTION

Historically, stocks with high book-to-market ratios, value stocks, have yielded higher average returns than stocks with low book-to-market ratios, growth stocks. The CAPM's major failure is its inability to price book-to-market sorted portfolios. A large collection of explanations – both rational and behavioral – have been proposed to address this value premium puzzle.[1] These explanations though are surprisingly detached from the voluminous literature that focuses on the properties of the aggregate market portfolio, such as the large equity premium and the high volatility and predictability of aggregate returns. In this paper we argue that the time series behavior of the market portfolio imposes general equilibrium restrictions on the behavior of the cross-section of average returns of price sorted portfolios. These restrictions are important as they provide tight implications about the cash-flow characteristics of value and growth stocks as well as about the variation over time of the value premium itself. Our predictions are broadly consistent with empirical evidence.

Specifically, ours is a representative agent economy where preferences are of the external habit persistence type introduced by Campbell and Cochrane (1999). This model generates plausible quantitative implications for the market portfolio through the time variation of the market price of consumption risk. We follow Menzly, Santos and Veronesi (2004, MSV henceforth), and embed these preferences in a general equilibrium setting with multiple risky assets. These assets have time varying expected dividend growth and differ from each other in their cash-flow risk, that is, in the covariance of their cash-flow with the aggregate economy. By generalizing the model of MSV, we are able to obtain numerous predictions about the cross-section of stock returns. In particular, we show that (a) value stocks are those with higher cash-flow risk and that cross-sectional differences in fundamentals cash-flow risk generate a value premium; (b) the time variation in risk preferences, due to habits, induces fluctuations in the value premium, which is high whenever the market premium is also high; (c) because of general equilibrium restrictions on the total wealth portfolio, the unconditional CAPM fails and thus a value premium *puzzle* obtains; and (d) an HML factor lines up returns as it captures aggregate differences in cash-flow risk in the economy. In addition, our model sheds light on the performance of the recently proposed conditional CAPM models.

[1]For the value premium see Rosenberg, Reid, and Lanstein (1985) and Fama and French (1992) and Fama and French (1998) for the international evidence. For behavioral explanations see for example Rosenberg, Reid, and Lanstein (1985), DeBondt and Thaler (1987) and Lakonishok, Shleifer, and Vishny (1994). For the rational ones see Fama and French (1993), Lettau and Ludvigson (2001), Gomes, Kogan and Zhang (2003) among others.

To understand the intuition of our results consider first the case where all assets have identical cash-flow risk and cross-sectional differences in expected returns arise only because of differences in the timing of their cash-flows, that is, in their "durations." We show that assets with high expected cash-flow growth are relatively more sensitive to shocks in risk preferences than otherwise identical assets with low expected cash-flow growth.[2] Can these *discount effects* alone generate the value premium? No, rather they generate a "growth premium." Indeed, assets with strong expected cash-flow growth have high price-dividend ratios and, as just mentioned, a high sensitivity to changes in the aggregate discount. As a consequence they command a higher premium and a counterfactual positive relation obtains between price-dividend ratios and average excess returns.

Suppose now that instead an asset has low duration and cash-flows that are positively correlated with aggregate consumption. In this case, and due to its low expected dividend growth, the total value of this asset is mainly determined by the current level of cash-flows, rather than by those in the future. The price of the asset is then mostly driven by cash-flow shocks and the fundamental risk embedded in these cash-flows drives also the risk of the asset. Thus, when cash-flows display substantial fundamental risk, the asset's premium is higher when the duration is lower. Can these *cash-flow effects* generate the value premium? Yes. Assets with high cash-flow risk and low duration have low price-dividend ratios. This is due to both the fact that they are risky, and thus prices have to be low to compensate agents for the risk they take, and because they have low expected dividend growth. Thus, potentially, the value premium can now arise, and whether it does or not depends on how the tension between "discount effects" (*high* risk when the asset has a *high* duration) and "cash-flow effects" (*high* risk when the asset has *low* duration) resolves quantitatively. An important objective of this paper is to analyze and assess this tension.

A second important contribution of our paper is to obtain predictions for the dynamics of the value premium. In particular, variation in risk preferences interacts with the cross-sectional dispersion in cash-flow risk to make value stocks particularly risky during "bad" times: Agents demand a relatively higher compensation for holding assets with cash-flows that covary positively with consumption growth when faced with adverse consumption shocks.

To evaluate the model's ability to yield quantitatively plausible implications we perform

[2]This point, which is standard in the fixed income literature, has been emphasized by Cornell (1999) who builds on Campbell and Mei (1993) to note that "pure technology bets that produce cash flows that are uncorrelated with the market, but which have long durations, will have high systematic risk."

an extensive simulation exercise. We choose preferences and cash-flow parameters to match the time series properties of the aggregate market portfolio and the return moments in the cross-section respectively. Throughout we mimic the procedure employed in the literature of sorting assets into decile portfolios formed on the basis of price-dividend ratios.[3]

Our simulations show that our consumption based general equilibrium model not only captures the properties of the aggregate market portfolio, as in Campbell and Cochrane (1999), but also many stylized facts observed in the cross-section of stock returns. First, a substantial value premium obtains, with value stocks earning about 5.16% more than growth stocks. This compares well with the 5.5% premium observed in the data. Second, the model produces a value premium that is higher in "bad times" than in "good times." In particular, in the model, the value premium increases to about 10% whenever the price dividend ratio of the market portfolio is in the lowest quintile of its distribution. This compares well with the 11% value premium that obtains when we perform the same exercise in the empirical data. Finally, the variation over time of the value premium rationalizes also why the conditional CAPM and a Fama and French (1993) HML factor perform much better than the unconditional CAPM, as observed in the data as well as in our simulations. Intuitively, conditioning information variables that are related to risk preferences, such as the consumption-to-wealth ratio of Lettau and Ludvigson (2001), capture the increase in the relative riskiness of value stocks in "bad times." Similarly, the loadings on the HML factor capture cross-sectional differences in cash flow risk across portfolios, while the variation over time of the premium on HML captures the dynamics of the relative riskiness of value versus growth stocks. Indeed, in our simulations the Fama-French model matches to a remarkable degree its empirical counterpart.

One important prediction of our model is that the sorting procedure naturally selects as value stocks those with high cash-flow risk, an implication empirically supported by a recent collection of papers.[4] These papers put forward some empirical measure of cash-flow risk and, invariably, show that value stocks have more cash-flow risk than growth stocks. Our paper differs markedly from most of the previous literature in that by proposing a theoretical model

[3]In our model, a notion of "book value" is not well defined and so we use price-dividend ratios in lieu of market-to-book ratios throughout (see Santos and Veronesi (2005) and Lettau and Wachter (2005)). Fama and French (1996, Table II) and Lettau and Wachter (2005, Table I) show that sorting by earnings-to-price or cash-flow to price generates as sizable a "'value" premium as sorting by book-to-market.

[4]See Cohen, Polk and Vuolteenaho (2003), Campbell, Polk and Vuolteenaho (2005), Bansal, Dittmar and Lundblad (2005), Parker and Julliard (2005), and Hansen, Heaton and Li (2005). Also Liew and Vassalou (2000) and Vassalou (2003) show that news about forecasts of GDP growth correlate with value stock returns.

we can address whether value stocks have "enough" cash flow risk to explain the magnitude of the value premium.[5] We perform an extensive sensitivity analysis on the parameters of the cash-flow model and show that "too large" a cross-sectional dispersion in cash-flow risk is needed to match cross-sectional properties of stock returns. We argue that this result is partially due to some restrictive assumptions in our model and discuss possible extensions to obtain more plausible magnitudes for the cross sectional dispersion in cash-flow risk.

The present paper is obviously related to MSV but there are several differences with that paper. First, our model is more general than the one in MSV and the additional flexibility is instrumental in the empirical performance of the model. Second, and most importantly, we focus on entirely different issues. In particular, whereas MSV are concerned with the time series predictability of industry portfolios, the present paper focuses on the cross sectional predictability of value sorted portfolio. The focus on the value premium allows us also to shed light on the vast literature on cross sectional predictability, something MSV did not touch upon. Finally, as already noted, the present paper is after a quantitative assessment of the cash-flow risk effects needed to generate a plausible value premium.

Our work is also related to three recent articles. A first paper is Campbell and Vuolteenaho (2004) who decompose shocks to market returns into shocks to expected discount rates and shocks to expected dividend growth rates. They show that value and growth load on these shocks differently and this, combined with the market price of risk associated with these shocks, generates a value premium and its corresponding puzzle. Santos and Veronesi (2005) put forward a general equilibrium model with labor income and multiple financial assets and show that the variation in the labor income-financial income mix affects the cross-section of stock returns. Financial assets have identical cash-flow risk and differ solely in the timing of their cash flows but a growth premium does not arise because they assume constant risk preferences. The value premium arises in their model because low duration assets (value stocks) are also those that contribute more to total dividends and therefore are riskier, thus having lower (normalized) prices and commanding a higher premium relative to growth. Their model, however, misses the time series properties of the aggregate market portfolio. Lettau and Wachter (2005) solve this shortcoming by adding to a cash-flow model similar to that of Santos and Veronesi (2005) an exogenous stochastic discount factor. They assume that the variation in the discount

[5] A notable exception is Hansen, Heaton and Li (2005). These authors propose a theoretical characterization of the long run trade-off between risk and return. They model the cash-flow processes of book-to-market sorted portfolios and estimate the parameters governing the long-run cash-flow covariation with consumption. They find that growth has low long-run covariation relative to value.

rate is subject to investor sentiment shocks that are uncorrelated to shocks to the aggregate economy. As a consequence, although growth stocks, which pay far in the future, are more sensitive to shocks in investor sentiment, they do not command a premium because discount risk is unpriced. The value premium in Lettau and Wachter (2005) arises through the same mechanism as in Santos and Veronesi (2005).[6] Our approach in this paper is very different. In our framework, the value premium arises because of differences in cash-flow risk across individual firms. We show that value stocks are, endogenously, those with high cash-flow risk and we measure the amount of cash-flow risk needed to generate a quantitatively plausible value premium. We also show that the variation in risk preferences and the cross-sectional dispersion in cash-flow risk interact to generate rich dynamics in the value premium. Finally, in our general equilibrium model the CAPM fails precisely because of general equilibrium restrictions, rather than from the variation of labor income or from exogenously specified sentiment shocks.

The paper proceeds as follows. Section II introduces the model and III the results. Section IV evaluates the model's ability to match basic moments of the returns data, both in the time series and the cross section. Section V analyzes existing asset pricing models through the lens of our model. Section VI contains the sensitivity analysis and quantifies the magnitudes of the cash-flow risk effects that are needed to generate the value premium. Section VII concludes. All proofs are in the Appendix.

II. THE MODEL

II.A Preferences

There is a representative investor who maximizes

$$E\left[\int_0^\infty u\left(C_t, X_t, t\right) dt\right], \tag{1}$$

where the instantaneous utility function is give by

$$u\left(C_t, X_t, t\right) = \begin{cases} e^{-\rho t}\frac{\left(C_t - X_t\right)^{1-\gamma}}{1-\gamma} & \text{if} \quad \gamma > 1 \\ e^{-\rho t}\log\left(C_t - X_t\right) & \text{if} \quad \gamma = 1 \end{cases} \tag{2}$$

[6] See also Brennan, Wang, and Xia (2004) and Brennan and Xia (2005) for a partial equilibrium model that ties the time series to the cross-section of stock returns. An investment-based general equilibrium model of the cross-section is also put forward by Gomes, Kogan, and Zhang (2003) who build on the partial equilibrium model of Berk, Green and Naik (1999). See also Zhang (2005).

In (2), the variable X_t denotes an *external* habit level and ρ denotes the subjective discount rate.[7] The exact specification of the external habit X_t is described below.

II.B Cash-flows

We consider an endowment economy with n financial assets. Each asset has an instantaneous dividend stream denoted by D_t^i, for $i = 1, .., n$. The aggregate endowment available for consumption at any time t is then equal to the sum of dividends.[8] The consumption good is immediately perishable and non-storable, which yields the equilibrium restriction

$$C_t = \sum_{i=1}^{n} D_t^i \tag{3}$$

and thus specific assumptions made on the dividend processes immediately translate into particular dynamics for aggregate consumption. Unfortunately, even relatively simple processes for D_t^i imply aggregate consumption processes that are difficult to work with and restrictive assumptions need to be made for tractability.[9] To better understand these restrictions and the nature of our assumptions below[10] define $\mathbf{D}_t = \left(D_t^1, ..., D_t^n\right)'$ and assume that

$$\frac{dD_t^i}{D_t^i} = \mu_D^i\left(\mathbf{D}_t\right)dt + \boldsymbol{\nu}_i' d\mathbf{B}_t \tag{4}$$

for some drifts $\mu_D^i\left(\mathbf{D}_t\right)$, $\boldsymbol{\nu}_i$ is a $n \times 1$ constant vector, and $d\mathbf{B}_t$ is a $n \times 1$ vector of Brownian motions. From equation (3) and Ito's lemma, the process for aggregate consumption is

$$\frac{dC_t}{C_t} = \mu_c\left(\mathbf{s}_t\right)dt + \boldsymbol{\sigma}_c\left(\mathbf{s}_t\right)' d\mathbf{B}_t \tag{5}$$

where $\mathbf{s}_t = \left(s_t^1, ..., s_t^n\right)' = \left(D_t^1/C_t, , ..., D_t^n/C_t\right)$ are shares of consumption produced by dividends, and

$$\mu_c\left(\mathbf{s}_t\right) = \sum_{i=1}^{n} s_t^i \mu_D^i \qquad \text{and} \qquad \boldsymbol{\sigma}_c\left(\mathbf{s}_t\right) = \sum_{i=1}^{n} s_t^i \boldsymbol{\nu}_i \tag{6}$$

[7] On habit persistence and asset pricing see Sundaresan (1989), Constantinides (1990), Abel (1990), Ferson and Constantinides (1991), Detemple and Zapatero (1991), Daniel and Marshall (1997), Campbell and Cochrane (1999), Heaton (1993 and 1995) Li (2001), and Wachter (2000). These papers only deal with the time series properties of the market portfolio and have no implications for the risk and return properties of individual securities. For recent supportive empirical evidence on external habit preferences see Luttmer (2005).

[8] For consistency with the data, we should consider also other forms of income such as labor income. Doing so, however, introduces an additional state variable and thus makes the results less transparent. See Santos and Veronesi (2005) for a discussion of the role of labor income in asset pricing.

[9] Recently, Cochrane, Longstaff and Santa Clara (2004) managed to solve in closed form the case where $n = 2$, dividends are log-normally distributed, and agents are endowed with log utility.

[10] See also Santos and Veronesi (2005).

The main difficulty in obtaining tractable expressions for asset prices lies in the dependence of $\mu_c(\mathbf{s}_t)$ and $\boldsymbol{\sigma}_c(\mathbf{s}_t)$ on the shares $\mathbf{s}_t$. Still, analytical formulas for asset prices can be obtained by making economically plausible assumptions on the joint processes of consumption C_t and shares $\mathbf{s}_t$, as advanced in MSV and Santos and Veronesi (2005). Here we follow Santos and Veronesi (2005) and assume:

Assumption 1: Aggregate consumption is given by

$$\frac{dC_t}{C_t} = \mu_c(\mathbf{s}_t)\,dt + \boldsymbol{\sigma}_c'\,d\mathbf{B}_t$$

where

$$\mu_c(\mathbf{s}_t) = \overline{\mu}_c + \mu_{c,1}(\mathbf{s}_t) \qquad \text{and} \qquad \mu_{c,1}(\mathbf{s}_t) = \mathbf{s}_t'\,\boldsymbol{\theta}_{CF}. \tag{7}$$

Above, $\boldsymbol{\theta}_{CF} = \left(\theta^1_{CF}, ..., \theta^n_{CF}\right)'$, and $\boldsymbol{\sigma}_c = (\sigma_c, 0, ..., 0)'$. The specification of θ^i_{CF} is explained below.

Assumption 2: For each i, the share s^i_t follows the mean reverting process

$$ds^i_t = \phi\left(\overline{s}^i - s^i_t\right)dt + s^i_t\boldsymbol{\sigma}^i(\mathbf{s}_t)\cdot d\mathbf{B}_t \tag{8}$$

where

$$\boldsymbol{\sigma}^i(\mathbf{s}_t) = \boldsymbol{\nu}_i' - \sum_{j=1}^n s^j_t\boldsymbol{\nu}_j' \tag{9}$$

The cash-flow model (8) imposes a structure on the relative size of firms, where "size" is measured as the fraction of total output produced by a given firm. In particular, it imposes the economically plausible assumption that no firm will take over the economy, as $s^i_t > 0$ for all i. In addition, the volatility $\boldsymbol{\sigma}^i(\mathbf{s}_t)$ in (9) ensures that $\sum_{i=1}^n s^i_t = 1$ for all t. It is worth noting that although the form of the volatility $\boldsymbol{\sigma}^i(\mathbf{s}_t)$ in (9) seems ad-hoc, it actually stems from the model (4) - (5), as it is possible to verify by Ito's lemma.

II.C Cash-flow risk

Given Assumptions 1 and 2, we can apply Ito's Lemma to $D^i_t = s^i_t C_t$ and obtain:

$$\frac{dD^i_t}{D^i_t} = \mu^i_{D,t}dt + \boldsymbol{\sigma}^i_D(\mathbf{s}_t)\,d\mathbf{B}_t \tag{10}$$

where

$$\mu^i_{D,t} = \overline{\mu}_c + \theta^i_{CF} + \phi\left(\frac{\overline{s}^i}{s^i_t} - 1\right) \tag{11}$$

$$\boldsymbol{\sigma}^i_D(\mathbf{s}_t) = \boldsymbol{\sigma}_c + \boldsymbol{\sigma}^i(\mathbf{s}_t) \tag{12}$$

In these formulas,

$$\theta^i_{CF} = \boldsymbol{\nu}'_i \cdot \boldsymbol{\sigma}_c$$

First, note that when the asset's *relative share*, $\overline{s}^i/s^i_t$, is low the asset's relative contribution to total consumption is below its long term average and the asset has a higher expected dividend growth.[11] Also, the long term dividend growth of this asset is given by $\overline{\mu}_c$, the unconditional expected return of consumption growth, as well as a parameter θ^i_{CF}, which is asset specific and it depends on the correlation of the stock shares with consumption growth.

Second, the stochastic discount factor is only driven by shocks to consumption growth. Thus, *cash-flow risk* is measured by the covariance of dividends with consumption growth

$$\sigma^i_{CF,t} \equiv Cov_t\left(\frac{dD^i_t}{D^i_t}, \frac{dC_t}{C_t}\right) = \boldsymbol{\sigma}_c\boldsymbol{\sigma}'_c + \theta^i_{CF} - \mathbf{s}'_t\ \boldsymbol{\theta}_{CF} \tag{13}$$

The conditional cash-flow risk of asset i, $\sigma^i_{CF,t}$, will play a prominent role in this paper. The term $\theta^i_{CF} - \mathbf{s}'_t \cdot \boldsymbol{\theta}_{CF}$ is parametrically indeterminate, that is, adding a constant to all θ^i_{CF} leaves this term unaffected, as $\sum_{i=1}^n s^i_t = 1$. Thus we are free to impose the identifiability restriction

$$\sum_{j=1}^n \overline{s}^j \theta^j_{CF} = 0, \tag{14}$$

and the expected covariance between asset i's cash-flow growth and consumption growth is

$$\overline{\sigma}^i_{CF} = E\left[\sigma^i_{CF,t}\right] = E\left[Cov_t\left(\frac{dD^i_t}{D^i_t}, \frac{dC_t}{C_t}\right)\right] = \boldsymbol{\sigma}_c\boldsymbol{\sigma}'_c + \theta^i_{CF}. \tag{15}$$

The parameter θ^i_{CF} then regulates the relative cash-flow risk of individual assets. Notice that the benchmark level of risk of an asset is the riskiness of aggregate consumption: An asset is risky (safe) if its cash-flows are more (less) risky than aggregate consumption. This is a general equilibrium restriction as, by definition, the variance of consumption growth must be a weighted average of its covariances with individual dividend growth. Throughout we refer to either $\overline{\sigma}^i_{CF}$ or θ^i_{CF} as "cash-flow risk" as there is a one to one mapping between them.

Finally note that the model is internally consistent: If we apply the general equilibrium restriction on the drift of the consumption process, (6), to the dividend process (10)

$$E_t\left[\frac{dC_t}{C_t}\right] = \sum_{i=1}^n s^i_t \mu^i_{D,t} = \overline{\mu}_c + \mathbf{s}'_t\ \boldsymbol{\theta}_{CF}, \tag{16}$$

which equals (7) in Assumption 1. Consumption growth then is not i.i.d. but rather has some predictable components which are linked to variation in the vector of shares, $\mathbf{s}_t$. Still, as we show below there is little predictability in practice as the parameters θ^i_{CF} are small.

[11] MSV test this prediction in a set of industry portfolios and find strong support for it.

II.D Habit Dynamics

In Campbell and Cochrane's (1999) habit model the fundamental state variable driving the attitudes towards risk is the *surplus consumption ratio,* $S_t = (C_t - X_t)\, C_t^{-1}$. To obtain closed form solutions for prices when there are multiple securities MSV use a log habit model and specify instead the inverse surplus S_t^{-1} as a mean reverting process. MSV's modelling device though cannot be applied when $\gamma > 1$ and, moreover, they only obtain approximate formulas for the case $\theta_{CF}^i \neq 0$. Thus here we opt for a different strategy and model the process

$$G_t = \left(\frac{C_t}{C_t - X_t}\right)^{\gamma} = S_t^{-\gamma}. \tag{17}$$

To obtain a plausible, yet tractable, model for the dynamics of G_t, consider first the implications for G_t under the standard assumption that X_t is an exponentially weighted average of past consumption levels, as in Constantinides (1990) and Detemple and Zapatero (1991),

$$X_t = \lambda \int_{-\infty}^{t} e^{-\lambda(t-\tau)} C_\tau d\tau.$$

An application of Ito's Lemma to (17) yields the process

$$dG_t = \left[\mu_G(G_t) - \sigma_G(G_t)\,\mu_{c,1}(\mathbf{s}_t)\right] dt - \sigma_G(G_t)\,\sigma_c dB_t^1, \tag{18}$$

where $\mu_G(G_t)$ and $\sigma_G(G_t) > 0$ are complicated functions of G_t, provided in equations (29) and (30) in the Appendix. Equation (18) shows that a higher expected consumption growth $\mu_{c,1}(\mathbf{s}_t)$ implies a lower drift rate of G_t. Intuitively, an increase in the expected growth rate of consumption implies a high future level of consumption relative to the current habit X_t and thus a higher surplus consumption ratio S_t and, given (17), a lower expected G_t. As in MSV and Campbell and Cochrane (1999), we make specific assumptions on $\mu_G(G_t)$ and $\sigma_G(G_t)$ in (18) to obtain a more manageable process. In particular, we assume

$$\mu_G(G_t) = k\left(\overline{G} - G_t\right) \qquad \text{and} \qquad \sigma_G(G_t) = \alpha\,(G_t - \lambda). \tag{19}$$

The first component of the drift of G_t is a mean reversion component and captures the basic idea of habit persistence models, namely that the habit X_t eventually "catches up" with C_t. The second component, as discussed above, links the drift rate of G_t to $\mu_{c,1}(\mathbf{s}_t)$. As for the diffusion component, and as in MSV, $\lambda \geq 1$ bounds G_t from below at λ and $\alpha > 0$ transmits the innovations in consumption growth, dB_t^1, to the convexity of the utility function. Note that MSV's model is a special case of (18) and (19) and obtains when $\gamma = 1$ and consumption growth is i.i.d., which is achieved by setting $\mu_{c,1}(\mathbf{s}_t) = 0$.

III. EQUILIBRIUM ASSET PRICES AND RETURNS

III.A The total wealth portfolio

We start by characterizing some basic properties of the total wealth portfolio as the intuition for some of these results becomes useful later.

Proposition 1: The price-consumption ratio, the expected excess return and diffusion terms of the total wealth portfolio are, respectively:

$$\frac{P_t^{TW}}{C_t} = \alpha_0^{TW}(\mathbf{s}_t) + \alpha_1^{TW}(\mathbf{s}_t) S_t^{\gamma} \tag{20}$$

$$E_t\left[dR_t^{TW}\right] = \left(\gamma + \alpha\left(1 - \lambda S_t^{\gamma}\right)\right)\left\{\frac{S_t^{\gamma}\alpha\left(1-\lambda S_t^{\gamma}\right)}{f_1^{TW}(\mathbf{s}_t) + S_t^{\gamma}}\sigma_c^2 + \sum_{j=1}^{n} w_{jt}^{TW}\sigma_{CF,t}^{j}\right\} \tag{21}$$

$$\boldsymbol{\sigma}_{R,t}^{TW} = \frac{S_t^{\gamma}\alpha\left(1-\lambda S_t^{\gamma}\right)}{f_1^{TW}(\mathbf{s}_t) + S_t^{\gamma}}\boldsymbol{\sigma}_c + \sum_{j=1}^{n} w_{jt}^{TW}\boldsymbol{\sigma}_D^{j}(\mathbf{s}_t), \tag{22}$$

where $\alpha_0^{TW}(\mathbf{s}_t)$, $\alpha_1^{TW}(\mathbf{s}_t)$, $f_1^{TW}(\mathbf{s}_t)$ and $\left\{w_{jt}^{TW}\right\}$ are given in the Appendix.

As in Campbell and Cochrane (1999) and MSV the price-consumption ratio of the total wealth portfolio is increasing in the surplus consumption ratio S_t: A high S_t implies a low local curvature of the utility function, a "less risk averse" attitude of the representative agent, and thus a higher price-consumption ratio. Unlike Campbell and Cochrane (1999) and MSV, the price-consumption ratio now depends on the entire vector of shares $\mathbf{s}_t$. The reason is that the general equilibrium restriction (5) generates a mild predictability in consumption growth (see equation (6)). The functions $\alpha_0^{TW}(\mathbf{s}_t)$ and $\alpha_1^{TW}(\mathbf{s}_t)$ are typically decreasing in expected consumption growth, because in our set up the elasticity of intertemporal substitution is less than one. Thus, this component implies that an increase in $\mu_c(\mathbf{s}_t)$ results in lower prices.[12]

As for the expected excess returns, (21), the term in parenthesis captures the fact that, intuitively, a high curvature parameter, γ, or a low surplus, S_t, imply high expected returns. The first term of the expression in brackets is linked to discount effects: As shown in the pricing function, changes in S_t induce a volatility of stock returns which is perfectly correlated with the stochastic discount factor, and thus it is priced. MSV discuss this effect more thoroughly.

[12] To review the economic reasoning, a low elasticity of intertemporal substitution implies a desire for consumption smoothing. Thus, an increase in expected consumption growth yields a higher desire of current consumption, and thus lower savings. The consumer then sell stocks and bonds, resulting in a decrease of the price-consumption ratio of the total wealth portfolio.

The second term in the bracket is the premium investors require because of changes in expected consumption growth. This term is typically *negative.* The reason is that our modelling device induces a mild positive correlation between shocks to consumption growth and shocks to *expected* consumption growth. Thus, a negative shock to consumption growth decreases the expected consumption growth which, as explained earlier, induces a positive impulse to the price. As a result, this component carries a negative premium.

III.B. Prices and returns for individual securities

Proposition 2: The price of asset i is given by

$$\frac{P_t^i}{D_t^i} = \alpha_0^i + \alpha_1^i S_t^\gamma + \alpha_2^i (\mathbf{s}_t) \left(\frac{\overline{s}^i}{s_t^i}\right) + \alpha_3^i (\mathbf{s}_t) S_t^\gamma \left(\frac{\overline{s}^i}{s_t^i}\right) \tag{23}$$

where α_0^i, α_1^i are positive constants and $\alpha_2^i (\mathbf{s}_t)$ and $\alpha_3^i (\mathbf{s}_t)$ are positive linear functions of the share vector $\mathbf{s}_t$ given in the Appendix.

As before, a higher surplus consumption ratio S_t, which implies lower "risk aversion," or a higher expected dividend growth, as measured by the relative share $\overline{s}^i/s_t^i$ (see (11)), result naturally in higher price-dividend ratios. The last term in (23) shows that shocks to the surplus consumption ratio have a stronger effect on the price-dividend ratio the higher the asset's expected dividend growth. This is linked to the duration effect that so prominent a role plays in what follows. Finally, as it was true for the total wealth portfolio, the price of each individual asset also depends on functions of the vectors of shares $\alpha_2^i (\mathbf{s}_t)$ and $\alpha_3^i (\mathbf{s}_t)$ and the intuition for the effect of changes in $\mathbf{s}_t$ on prices is identical to the one discussed above.

Proposition 3: The expected excess return of asset i is given by

$$E_t \left[dR_t^i\right] = \mu_{i,t}^{DISC} + \mu_{i,t}^{CF}$$

where

$$\mu_{i,t}^{DISC} = (\gamma + \alpha (1 - \lambda S_t^\gamma)) \left(\frac{S_t^\gamma}{f_1^i \left(\frac{\overline{s}^i}{s_t^i}, \mathbf{s}_t\right) + S_t^\gamma}\right) \alpha (1 - \lambda S_t^\gamma) \sigma_c^2 \tag{24}$$

$$\mu_{i,t}^{CF} = (\gamma + \alpha (1 - \lambda S_t^\gamma)) \left[\left(\frac{1}{1 + f_2^i (S_t, \mathbf{s}_t) \left(\frac{\overline{s}^i}{s_t^i}\right)} + \eta_{it}^i\right) \sigma_{CF,t}^i + \sum_{j \neq i} \eta_{jt}^i \sigma_{CF,t}^j\right] \tag{25}$$

with

$$f_1^i \left(\overline{s}^i/s_t^i, \mathbf{s}_t\right) = \frac{\alpha_0^i + \alpha_2^i (\mathbf{s}_t) \left(\overline{s}^i/s_t^i\right)}{\alpha_1^i + \alpha_3^i (\mathbf{s}_t) \left(\overline{s}^i/s_t^i\right)} > 0 \qquad \text{and} \qquad f_2^i (S_t, \mathbf{s}_t) = \frac{\alpha_2^i (\mathbf{s}_t) + \alpha_3^i (\mathbf{s}_t) S_t^\gamma}{\alpha_0^i + \alpha_1^i S_t^\gamma} > 0,$$

and η^i_{jt} are given in Appendix.

Proposition 3 shows that the expected excess return of individual stocks can be divided in two components. These two terms correspond to the two sources of shocks to returns: discount shocks and cash-flow shocks. We elaborate on them in detail next.

III.B.1 Discount risk effects

The source of this component of the risk premium, $\mu^{DISC}_{i,t}$, is the variation of the aggregate discount – proxied by S^γ_t. To interpret further this term notice first that

$$\frac{\partial P^i_t/P^i_t}{\partial S^\gamma_t/S^\gamma_t} = \frac{S^\gamma_t}{f_1\left(\overline{s}^i/s^i_t, \mathbf{s}_t\right) + S^\gamma_t}. \tag{26}$$

is the elasticity of prices to shocks in the variable driving the aggregate discount, which is S^γ_t. The volatility of these discount shocks is

$$\alpha\left(1 - \lambda S^\gamma_t\right)\boldsymbol{\sigma}_c,$$

which is the diffusion component of dS^γ_t/S^γ_t, the inverse of our state variable G_t, as it follows from a basic application of Ito's Lemma to (18). Clearly, only the component of these shocks that covaries with the shocks to the stochastic discount factor is priced which, given (31) in the Appendix, is

$$\left[\gamma + \alpha\left(1 - \lambda S^\gamma_t\right)\right]\alpha\left(1 - \lambda S^\gamma_t\right)\sigma^2_c. \tag{27}$$

The component of the asset's premium that is linked to discount effects is then the product of (26) and (27).

Cross-sectional variation in the discount effects can only be driven by differences in the price elasticity (26), which is in turn driven by the behavior of the function $f_1\left(\overline{s}^i/s^i_t, \mathbf{s}_t\right)$. We have been unable to obtain a general characterization of this function, but for parameter values that are empirically relevant we find that

$$\frac{\partial f_1\left(\overline{s}^i/s^i_t, \mathbf{s}_t\right)}{\partial\left(\overline{s}^i/s^i_t\right)} < 0,$$

and thus assets with a higher expected dividend growth, as measured by the relative share $\overline{s}^i/s^i_t$, display stronger discount effects. The intuition is straightforward: stocks with a high expected dividend growth pay the bulk of their proceeds far in the future. Thus, minor variations in the aggregate discount rate – through the risk aversion of the representative investor – result in large percentage variations of the price of the asset. This variation is naturally priced and thus the higher required premium of assets with high relative shares.

III.B.2 Cash-flow risk effects

The source of premia related to cash-flow shocks, $\mu_{i,t}^{CF}$, has two components to it, see equation (25). The first is related to shocks in the asset's dividends and the second is related to shocks in the dividends of the rest of the assets in the economy, which, as shown in (23), affect the price of asset i as well. The logic for the sources of the premia linked to cash-flow shocks is the same as in the discount effects case. First it can be easily shown that the elasticity of the price with respect to shocks to its own dividends is,

$$\frac{\partial P_t^i / P_t^i}{\partial D_t^i / D_t^i} = \frac{1}{1 + f_2^i\left(S_t, \mathbf{s}_t\right)\left(\frac{\overline{s}^i}{s_t^i}\right)} + \eta_{it}^i.$$

Recall also that we denote $\sigma_{CF,t}^i = cov_t\left(dD_t^i / D_t^i, dC_t / C_t\right)$ (see equation (13)). The first term of $\mu_{i,t}^{CF}$ is then the component of the dividend shocks that covaries with shocks to the stochastic discount factor multiplied by the effect that these shocks have on the price of asset i, as measured by the price elasticity. A similar logic applies to the second term in $\mu_{i,t}^{CF}$. Indeed it can be shown that

$$\frac{\partial P_t^i / P_t^i}{\partial D_t^j / D_t^j} = \eta_{jt}^i \quad \text{for} \quad j \neq i.$$

As before this component of the premium results from the product of this (cross) elasticity and the priced component of the shock to asset j's dividends, $\sigma_{CF,t}^j$.

How does the current level expected dividend growth, as measured by $\overline{s}^i / s_t^i$, affect the cash-flow risk component of expected stock returns? Given the conditional covariance of the dividend of asset i with aggregate consumption, $\sigma_{CF,t}^i$, the first term of (25) is unambiguous: Since $f_2^i\left(S_t, \mathbf{s}_t\right) > 0$, if the asset is "risky", that is, if $\sigma_{CF,t}^i > 0$, then a high expected dividend growth translates in a lower premium stemming from current dividend volatility. The intuition is also clear: a stock that pays more in the future than today has a relatively low dividend compared to the future. Thus, the risk embedded in current dividends, $\sigma_{CF,t}^i$, has a relatively low impact on the total risk of stock. In the limit, if the stocks does not pay *any* dividend today, it cannot have any "cash-flow risk", as there is zero current covariance of dividends with consumption. If instead the asset's dividends covary negatively with consumption growth ($\sigma_{CF,t}^i < 0$), then a high expected dividend growth increases the risk premium. The argument, of course, is the converse of the previous one.

The effect that the current expected dividend growth of asset i has on the second term of the cash-flow risk component of stock return (25) is more difficult to tell. To quantify these effects, the top panel of Figure 1 plots the quantity $\mu_{i,t}^{CF}$ as a function of the unconditional cash-flow risk $\overline{\sigma}_{CF}^{i} = E\left[\sigma_{CF,t}^{i}\right]$ at the steady state, that is, for the case where $S_t = \overline{S}$ and $\mathbf{s}_t = \overline{s}$. As it can be seen, the cash-flow component of expected return is increasing in $\overline{\sigma}_{CF}^{i}$. Note however, that there is a negative "bias" in this component of expected excess return. Indeed the case $\overline{\sigma}_{CF}^{i} = 0$ still implies a negative expected excess return stemming from cash-flow risk effects. This is due to the second component in (25), which is related to the time variation in the aggregate expected consumption growth. As we discussed in the case of the total wealth portfolio, this component carries typically a negative risk premium. Finally, the bottom panel of Figure 1 plots $\mu_{i,t}^{CF}$ as a function of $\overline{\sigma}_{CF}^{i}$ for the case where $S_t = \overline{S}$ but for a random draw of shares $\mathbf{s}_t$. Although an increasing pattern in σ_{CF}^{i} can be easily seen, cross-sectional differences in $\overline{s}^i/s_t^i$ may make the component $\mu_{i,t}^{CF}$ of an asset with high unconditional cash-flow risk $\overline{\sigma}_{CF}^{i}$ temporarily lower than that of an asset with lower cash-flow risk $\overline{\sigma}_{CF}^{i}$.

III.C The value premium

In order to gauge the source of the value premium in our model it is convenient to turn to Figure 2. Panels A, B, and C plot $\mu_{i,t}^{DISC}$, $\mu_{i,t}^{CF}$, and the total $E_t\left[dR_t^i\right]$ respectively against the relative share $\overline{s}^i/s_t^i$ for various levels of the asset's unconditional cash-flow risk $\overline{\sigma}_{CF}^{i}$, which correspond to different values of θ_{CF}^{i} (see expression (15)). In all cases, the level of surplus S_t is set to its steady state value $\overline{S}$. The parameters used are those of the calibration exercise discussed in detail in the next section.

Start with Panel A. As discussed in Section III.B.2, the discount risk component of expected return is increasing in the relative share $\overline{s}^i/s_t^i$, that is, with expected dividend growth (see (11)). The reason is that assets with high relative shares are more sensitive to shocks in the stochastic discount factor. These shocks are naturally priced and thus the higher required premia of assets with high relative shares. In addition, the discount risk component of expected returns does depend as well on the asset's unconditional cash-flow risk $\overline{\sigma}_{CF}^{i}$: Stocks with higher cash-flow risk $\overline{\sigma}_{CF}^{i}$ have a larger discount risk component in expected returns. The intuition is that stocks with a higher $\overline{\sigma}_{CF}^{i}$ are riskier and as a consequence have lower prices. It follows that changes in the stochastic discount factor have a larger impact, in *percentages*, on the prices of assets with higher levels of cash-flow risk. Notice though that the higher the level of the cash-flow risk the lower the effect of a change in the relative share on the discount risk component of expected returns.

Panel B of Figure 2 plots the cash-flow risk component of expected returns which, as discussed in Section III.B.2, is decreasing in expected dividend growth for stocks with high cash-flow risk. Finally, Panel C reports the total expected return for each asset that is obtained by adding to the discount risk component the cash-flow risk component of stock returns.

III.C.1 Discount risk effects and the "growth premium"

In our framework, and given expression (23), sorting assets according to their price-dividend ratio is akin to sorting them on both cash-flow risk, $\overline{\sigma}^i_{CF}$, and expected dividend growth, $\overline{s}^i/s^i_t$. In particular, value stocks (assets with low P/D ratios) are, on average, associated with high $\overline{\sigma}^i_{CF}$ and low expected dividend growth $\overline{s}^i/s^i_t$. Consider now the case where cross-sectional differences in cash-flow risk are "small" (e.g. $\theta^i_{CF} \approx 0$ for all i). Then, $\overline{\sigma}^i_{CF}$ are roughly the same across all assets and the sorting procedure selects assets according to expected dividend growth. In this case, discount effects dominate and the total expected excess return are as in the lower line of Panel A. Since low price-dividend ratio stocks are those with low relative shares $\overline{s}^i/s^i_t$, value stocks are found on the left-hand side of the panel and thus have low expected excess returns. Similarly, high price-dividend ratio stocks are those with high $\overline{s}^i/s^i_t$ and growth stocks are on the right-hand side of the panel and have high expected excess returns. Thus, if cross-sectional differences in cash-flow risk are "small,"then growth stocks have higher expected excess returns than value stocks and a "growth premium" obtains.[13]

III.C.2 Cash-flow risk effects

It follows from the discussion above that for a value premium to obtain there must be sufficiently large cross-sectional differences in cash-flow risk. Indeed, consider now Panel C, which reports the total expected return when both discount effects (Panel A) and cash-flow effects (Panel B) are present. Value stocks (assets with low P/D ratio) have on average high risk ($\overline{\sigma}^i_{CF}$) and low expected dividend growth ($\overline{s}^i/s^i_t$). This combination corresponds to the area around the top-left corner of the plot, that is, to high expected excess return. Conversely, growth stocks (assets with high P/D ratios) must have a combination of low $\overline{\sigma}^i_{CF}$ and high $\overline{s}^i/s^i_t$. This combination can be found on the bottom-right corner of the plot. As it can be seen then value stocks will command a high premium and growth stocks a low (and even negative) premium. Thus, if cross-sectional differences in cash-flow risk are "large", then value stocks have higher expected excess returns than growth stocks and a "value premium" obtains.

[13]This result is in contrast with Lettau and Wachter (2005) who find a value premium with homogeneous cash flow risk. In their partial equilibrium setting, variation in the market price of risk is due to "investor sentiment" and it is not priced. Thus differences in expected future cash flows do not yield differences in expected returns.

III.C.3 The dynamics of the value premium

The presence of discount risk effects which are associated with the time series variation in risk preferences have implications for the dynamics of the value premium. Essentially, discount risk effects interact with the cross-sectional dispersion in cash-flow risk to induce fluctuations in the value premium, as shown in Figure 3. This figure plots the expected excess returns of three assets against the surplus consumption ratio, S_t. The dotted line shows the expected excess return of the market portfolio; the solid line corresponds to the expected excess return of a representative value stock with high cash-flow risk and low expected dividend growth; finally the dash line corresponds to the premium of a representative growth stock with low cash-flow risk and high expected dividend growth. As it can be seen, when the surplus consumption ratio is low (high), the value premium is high (low): Assets with a high value of θ^i_{CF} are particularly *riskier* when the representative agent's is highly risk averse which occurs whenever adverse consumption growth shocks depress the surplus consumption ratio, increasing in turn the market premium and its dividend yield. Thus in our model the value premium has a strong predictable component, being high (low) when the market premium is high (low).

IV. EMPIRICAL PREDICTIONS

In this section we conduct a simulation study to evaluate the extent to which the model can match the standard return moments both in the time series and the cross-section, which can be found in Table I. The data set is standard and it is very briefly described in the Notes to Table I. Panel A shows mean and standard deviation for the returns on the market portfolio and the risk free rate. Panel B shows the predictability regressions of Fama and French (1988) and Campbell and Shiller (1988) for two different sample periods, which are meant to emphasize the sensitivity of these results to the particular period under consideration. Panel C shows the value premium and its corresponding puzzle, the failure of the CAPM to generate the large cross-sectional dispersion in average returns across book-to-market sorted portfolios.

IV.A Details of the simulation

We simulate the model presented in Section II.B with 10,000 years of quarterly data for 200 firms. We sort these assets into ten portfolios according to their price-dividend ratio[14] in an effort to mimic the standard procedure used in the cross-sectional literature and focus our

[14]Our model does not have "book" so we normalize prices by our theoretical cash-flow measure. The "value premium" obtains when either earnings or cash-flows are used to normalize prices. See, for instance, Fama and French (1996, Table II) and Fama and French (1998, Table III), which also includes international evidence.

analysis on these ten portfolios. Table II contains the parameter values that are going to be used throughout and which were chosen to generate moments in simulated data close to their empirical counterparts in Table I. We set the average and standard deviation of consumption growth at 2% and 1.5% respectively. This latter value should be measured against the value in the postwar sample of 1.22% and the one for the longer sample starting in 1889, which is 3.32%.[15] We choose $\gamma = 1.5$, which is between the values used by MSV, $\gamma = 1$, and Campbell and Cochrane (1999), $\gamma = 2$. This choice implies a steady state value of the local curvature of the utility function of $\gamma\overline{S}^{-1} = 48$, higher than the already high value of Campbell and Cochrane (1999) which is 35. The minimum value of this local curvature is 27.75. Finally the parameter k and α are similar to the values chosen by MSV.

As for the share process, we assume that all of the 200 simulated assets have the same steady state contribution to overall consumption, $\overline{s}^i = 1/200 = .005$. Also the speed of mean reversion is set at $\phi = .07$, which is the value estimated by MSV for the market portfolio. The key parameter of interest in our model is the one that controls differences in cash-flow risk, θ^i_{CF}. Our general equilibrium setting requires that this parameter is symmetrically distributed around zero (see (14)). Then we assume

$$\theta^i_{CF} \in \left[-\overline{\theta}_{CF}, \overline{\theta}_{CF}\right],$$

where $\overline{\theta}_{CF} > 0$. Throughout, and with some abuse of terminology, we refer to $\overline{\theta}_{CF}$ as the cash-flow risk parameter but the reader should keep in mind that it is the *support* of the cash-flow risk parameters of individual assets.

Finally we choose the vector $\boldsymbol{\nu}_i$ in (9) so that for each i it only has two non-zero entries: $\boldsymbol{\nu}_i = (\nu_{i,0}, 0, ..., 0, \nu_{i,i}, 0, ..)$. Given θ^i_{CF}, the first entry by definition must be $\nu_{i,0} = \theta^i_{CF}/\sigma_c$. To avoid parameter proliferation, the second entry – the idiosyncratic part – is chosen constant across all assets according to the formula, $\nu^2_{i,i} = \overline{\nu}^2 - \max(\nu^2_{i,0})$, where $\overline{\nu}$ is a chosen parameter. In words, $\overline{\nu}$ is the maximum share volatility across assets.

We start by discussing a baseline case with $\overline{\theta}_{CF} = .00345$ and $\overline{\nu} = .55$ for it generates a quantitatively plausible value premium. We investigate this case in detail and then, in Section VI, we study the behavior of the model under different values for $\overline{\theta}_{CF}$ and $\overline{\nu}$. We also postpone a discussion of the size of the cash-flow risk effects until that section.

[15] See Campbell and Cochrane (1999) Table 2.

IV.B The time series properties of the market and the value premium

Table III is the analog to Table I but in simulated data. As shown in Panel A, the model generates a sizable, if slightly low, equity premium and volatility of stock returns, and the risk free rate moments are reasonable. Panel B of Table III shows the predictability regressions for all the standard horizons. As already mentioned the model does well in this dimension: The coefficients all have positive signs and increase with the forecasting horizon as do the $t-$ statistics. The R^2s are relatively lower than their empirical counterparts but not far off the mark for the case of the 1948-2001 sample. These results simply reproduce the good performance of Campbell and Cochrane (1999) and MSV for the market portfolio.

Panel C of Table III contains the average excess returns for the ten sorted portfolios. The value premium obtains nicely in our setup. Indeed the value premium is a healthy 5.16%, only slightly below the empirically observed one of 5.50%. Notice though that the average excess returns for each portfolio are below their empirical counterparts. The reason is that, as mentioned above, the model misses the equity premium by about 3%. This low premium also affects the Sharpe ratio, which is low relative to its empirical counterpart but, importantly, they decrease with the price-dividend ratio, an important feature of the data (see Table I.)

The line denoted $\text{Avge}\left(\theta^i_{CF}\right) \times 100$ reports the average cash-flow risk parameter for each of the ten portfolios. As discussed in Section III.C, the sorting procedure picks cross-sectional variation in the cash-flow risk parameter, θ^i_{CF} : Stocks in the value portfolio, portfolio 10, have, on average, a high cash-flow risk parameter whereas the opposite is true for the growth portfolio, portfolio 1. In our framework, and in line with much of the recent empirical research on this issue (see Section VI.A), value stocks are indeed riskier in the cash-flow sense and the strength of this effect is enough to undo the natural "discount riskiness" of growth stocks.

IV.C The dynamics of the value premium

To ascertain the time series variation of the value premium, Table IV Panel A shows the average excess return of the first and tenth decile portfolio as a function of whether the market-to-book ratio of the market portfolio is above or below a certain percentile, denoted by $\bar{c}$. For instance, the first line shows that the average excess rate of return of the first decile (growth) portfolio is 13.18% if the market-to-book of the market portfolio is *below* the 15th percentile of its empirical distribution and that of the tenth decile (value) portfolio is 23.57%. The value premium is then 10.38%. Instead when the market-to-book is *above* the 15th percentile the first decile portfolio has an average excess return of 5.73% and the tenth portfolio has one of 10.35% for a total value premium of 4.62%, which is considerably lower than the previous one. This pattern holds for any cut-off point: The value premium is higher whenever the market-to-book of the market portfolio is low which are also periods where the average excess return of the market is high, as shown in the columns headed by $\overline{R}^M$.

Panel B of Table IV reports the same calculations as in Panel A but in simulated data. The only difference is that, naturally, instead of using the market-to-book we use the price-dividend ratio of the market portfolio to identify the state. The pattern is indeed very similar with the only exception of the level of the premia which is, as already discussed, lower than in the data. The value premium is higher when the price-dividend ratio of the market portfolio is low than when it is high. For instance, when the price-dividend ratio of the market portfolio is below the 15th percentile the value premium is 10.90% whereas when it is above is only 4.15%, very close to their empirical counterparts. In summary then, the discount risk effects needed to replicate the time series properties of the market portfolio interact with the cross-sectional dispersion in cash-flow risk to generate variation in the value premium. Value stocks are particularly risky during bad times, periods when the aggregate market premium and its dividend yield are high relative to their unconditional mean, an effect that is present both in the data and the model.

V. THE CAPM AND OTHER ASSET PRICING MODELS

A central finding of the empirical asset pricing literature is the inability of CAPM of Sharpe (1964) and Lintner (1965) to explain the value premium. In our setup the CAPM does not hold but the question remains as to whether it performs well in simulated data. We address this issue in Section V.A. In Sections V.B and V.C we investigate the extent to which our framework is consistent with two popular and successful models designed to address the value premium puzzle: The Fama and French (1993) model and the conditional asset pricing models proposed of late of which Lettau and Ludvigson (2001) is the foremost example. In particular, given that in our set up *all* these models are misspecified, what is the feature of the data that these models capture that generates the "good fit" relative to the CAPM?

V.A The CAPM

V.A.1 The CAPM and the value premium puzzle

The value premium puzzle can be seen in the last line of Table I Panel C (CAPM β). The beta of the sorted portfolios is flat if not slightly decreasing in the market-to-book, at odds with the strong increasing pattern in average returns.[16] The CAPM produces no cross-sectional dispersion in its measure of risk when confronted with substantial variation in average returns. To do this more formally we turn to Table V Panel A where we report the results of time series regressions of the excess returns on each of the ten portfolios on the excess returns on the market portfolio,

$$R_t^p = \alpha + \beta^M R_t^M + \epsilon_t^p \qquad \text{for} \qquad p = 1, 2, \cdots, 10.$$

We do this for both empirical (Panel A-1) and simulated data (Panel A-2). The panel shows the intercepts in the time series, α, and its corresponding $t-$statistic, $t(\alpha)$. It also reports the beta on the market portfolio, β^M and its $t-$statistic, $t\left(\beta^M\right)$. We have omitted the $t-$statistic on the loading for the case of simulated data because, as in the empirical data, they are all strongly significant (well above 100).

Start with the case of the empirical data, Panel A-1. The intercepts, "alphas" of the CAPM time series regressions are large and statistically significant. Growth stocks have large negative intercepts whereas value stocks have large positive ones. The poor performance of the CAPM can also be seen in line 1 of Panel A in Table VI, where we report the standard Fama-MacBeth cross-sectional regressions. The coefficient is not statistically significant, enters with the wrong sign and the R^2 is just 11%.

Turn next to the time series regressions in simulated data, Panel A-2 of Table V. Unlike the case in the empirical data, the betas cross-sectionally correlate positively with average excess returns, an important issue on which more below. Still the cross-sectional dispersion in betas is not enough to match the cross-sectional dispersion in average returns generated by the model. Indeed the pattern and statistical significance of the intercepts in simulated data is similar to its counterpart in empirical data. A visual impression of this result can be obtained by looking at the bottom panel of Figure 4, which shows the average excess returns for the ten decile simulated portfolios plotted against the CAPM fitted returns. As it can be seen, while average returns range between 3.07% for high price-dividend ratio stocks and 8.23% for low price-dividend ratio stocks, the "fitted" returns only range between 3.67% and 5.50%. That is, the model not only generates the value premium but also the value premium puzzle.

[16] The inability of the CAPM to explain the cross section of average returns is pronounced in the postwar sample used in this paper. Recently though Ang and Chen (2005) and Fama and French (2005) show that the behavior of the CAPM in the earlier sample covering 1927-1963 is much better. Still Daniel and Titman (2005, Table 3) and Fama and French (2005) perform triple sorts, on ME, BE/ME and (preformation) market beta to find variation in average returns unrelated to beta thus rejecting the CAPM also in the long sample.

V.A.2 Fama-MacBeth regressions in simulated data

In our simulated data, the CAPM betas correlate positively with average excess returns. Thus cross-sectional regressions that impose no constraints on the level of estimated market premium may immediately induce a good fit as measured by the R^2. This can be seen in line 5 of Panel B in Table VI, where we run the Fama-MacBeth regression in artificial data: The CAPM produces a good fit with an R^2 of 91%. Moreover the market enters significantly and with the right sign. The estimated quarterly market premium though is 2.56%, which corresponds to an annualized value above 10%. This number should be compared to the market premium in our model which is 4.35% (see Table III Panel A.) Thus the CAPM "works" in our model at the expense of an unreasonable level in the market premium.[17]

V.B The Fama and French (1993) model

V.B.1 Cash-flow risk effects, discount risk effects, and HML

The Fama and French (1993) model has become a standard benchmark in asset pricing tests. How well does it work in our set up? To answer this question we construct an HML factor in artificial data that is long the three top decile portfolios and short the bottom three shown in Table III Panel C. This panel also reports the average cash-flow risk parameter θ_{CF}^i for each of the decile portfolios. There is a clear ordering of the average cash-flow risk across decile portfolios: Value stocks have a much larger value of θ_{CF}^i than growth stocks. HML then captures cross-sectional variation in θ_{CF}^i across price-dividend sorted portfolios. In addition, as shown in Figure 3, it is important to emphasize that differences in cash-flow risk θ_{CF}^i also yield differences in the impact that discount effects have on expected returns. HML then captures both cash-flow risk and, partly, discount risk.

V.B.2. Time series and cross-sectional regressions evidence

Table V Panel B presents the results of time series regressions,

$$R_t^p = \alpha + \beta_M R_t^M + \beta_{HML} R_t^{HML} + \epsilon_t^p \qquad \text{for} \qquad p = 1, 2, \cdots, 10.$$

Panel B-1 shows the results in the case of the empirical data. The results are well known. The intercepts go down considerably and only one of them is statistically significant; value

[17] This message has recently been emphasized by Lewellen and Nagel (2005) and Daniel and Titman (2005): A small but slightly positive cross-sectional covariation between betas and average returns can result in the unwarranted support of asset pricing models that fail to impose economically based restrictions on the size of the premia of the proposed factors.

(growth) stocks have a large (small) loading on HML and the inclusion of HML in the time series regression collapses the betas on the market portfolio around 1 (see Fama and French (1993, page 21-26)).

Panel B-2 shows the time series regression in simulated data. Again we do not report the $t-$statistic on the loadings on the market and HML as they are all above 100. Turning first to the loadings on the market portfolio, notice that, as it was the case in the empirical sample, adding HML to the time series regressions has the effect of *reducing* the spread in the estimates of β_M^i and collapse them around 1. As Fama and French (1993) note this pattern is related to the negative correlation between the market and the returns on HML.

As for the loading on the HML portfolio notice that it has a strong cross-sectional variation which reflects the cross-sectional variation in the underlying cash-flow risk of the different portfolios. Indeed the loading on HML of the growth portfolio is $-.28$ whereas that of the value portfolio is 1.07. Also the size of the intercepts of the time series regressions drop considerably relative to the size of the intercepts when only the market portfolio is present.[18] Moreover there is no longer any pattern in the variation of the intercept across decile portfolios, and their t-stats are much lower than in Panel A, which shows that HML is capturing the systematic pattern of misspricing documented in Panel A.

The evidence in the Fama-MacBeth regression confirms the time series evidence. Line 2 of Table VI Panel A shows that HML enters significantly and the estimated size of the premium on HML is very close to the average excess return of the HML portfolio. This is also the case in our simulated regression, which is shown in line 6 of Panel B in Table VI. The coefficient on the loading on HML is very similar to its empirical counterpart and, once annualized, close to our estimated average excess return on the HML portfolio, which is 3.21%. The only caveat is that the market portfolio is significant in our simulated Fama-MacBeth regressions whereas it is not in the empirical data. Yet, this table shows that the inclusion of HML in the cross-sectional regression aligns the portfolios correctly, as the intercept is now close to zero (with $t-$statistics equal to -1.64 even with 40,000 observations) and the (quarterly) market premium equals 1.31%, which annualized is 5.24%, still higher than the average market return in simulation (4.35%), but much smaller than the one obtained for the CAPM case.

[18] Notice that the value-weighted sum of the alphas should be equal to zero. Given that the only negative alpha is that of the growth portfolio, it must be the case that some of the assets in the growth portfolio must have extreme prices. We thank Gene Fama for pointing out this to us.

V.C Conditional asset pricing models

Conditional asset pricing models have been proposed recently to address the inability of the CAPM to explain the value premium. The idea, as advanced by Hansen and Richard (1987), is that the CAPM may fail unconditionally but may hold conditionally and thus tests of the CAPM that ignore conditioning information are misspecified. Researchers have reacted to this observations by using as a proxy for investors' information set variables that are known to forecast returns in the time series.[19] Typically this has led to tests of multifactor model where the additional factor, other than the market, is the market itself interacted with the proposed conditioning variables.

Lines 3 and 4 in Panel A of Table VI shows that conditioning by the dividend yield of the market portfolio and the *cay* variable of Lettau and Ludvigson (2001) results also in a coefficient for the instrumented market that is strongly significant. In addition the R^2 is an impressive 83% and 81% respectively. Panel B, line 7 shows that our model does also well in this dimension. When we interact the returns of the market portfolio with the simulated dividend yield of the market portfolio we obtain a strongly significant coefficient and, once again, of similar magnitude to its empirical counterpart.[20] The intuition behind these conditional asset pricing models is that they capture the fact that value stocks become relatively riskier in bad times, as shown in Table IV and Figure 3. In our setup the conditional CAPM does not hold but is mechanically bound to do better than its unconditional counterpart because it captures the conditional effects that arise out of the interaction of discount effects with the cross-sectional dispersion in θ^i_{CF}.

VI. DISCUSSION

VI.A Do value stocks have larger cash-flow risk?

An important prediction of our model is that value stocks have larger cash-flow risk than growth stocks. Is this the case? A flurry of recent papers argues that this is indeed the case. For instance, Cohen, Polk and Vuolteenaho (2003) obtain cash-flow betas by regressing different measures of firms' cash-flows on the corresponding measures of market cash-flows, such as

$$\sum_{j=0}^{R-1} \rho^j_{CPV} \Delta d^p_{t+j,j+1} = \beta^p_{CF,0} + \beta^p_{CF,1} \sum_{j=0}^{R-1} \rho^j_{CPV} \Delta d^{mkt}_{t+j} + \varepsilon^p_{t+R-1} \qquad (28)$$

[19]See, among others, the conditional asset pricing models of Jagannathan and Wang (1996), Ferson and Harvey (1999), Lettau and Ludvigson (2001), and Santos and Veronesi (2005).

[20]We do not report the results for *cay* as in our setting, *cay* is perfectly correlated with $\log(D/P)$.

for each time t and each portfolio $p = 1, ..., 10$. Here, $\Delta d^p_{t+j,j+1}$ is the dividend growth at time $t + j$ of the portfolio p which was formed $j + 1$ years earlier, that is, at $t - 1$. Similarly, Δd^{mkt}_{t+j} is the dividend growth of the market at time $t + j$. Finally, $\rho^j_{CPV} = .95$ is a discount, and R is the number of years over which the average growth rate is computed. They call the regression coefficient $\beta^p_{CF,1}$ the cash-flow beta. Their results are in Table VII.

Notice first that, irrespective of the cash-flow measure used, value stocks have higher cash-flow betas than growth stocks, though magnitudes differ across measures. If either (accumulated) return on equity, $\sum_{j=0}^4 \rho^j ROE^p_{t+j,j+1}$, or (accumulated) dividend growth is used as a measure of cash-flow growth, the regression coefficients roughly double when we go from growth to value stocks. If instead we use (accumulated) earnings growth relative to market value, $\left(X^p_{t+4,4} - X^p_{t-1,0}\right)/ME^p_{t-1,0}$, the coefficients increase by a factor of 10. Finally if (accumulated) earnings relative to market, $\sum_{j=0}^4 \rho^j \left(X^p_{t+j,j+1}/ME^p_{t+j-1,j}\right)$, is used they increase almost by a factor of 20.

In a recent study, Campbell, Polk and Vuolteenaho (2005) confirm these findings and extend them to different sample periods. These authors show that value stocks's profitability covaries with the aggregate market cash-flow news more than growth stocks (see their Tables 6 and 7). A similar exercise is performed by Bansal, Dittmar, and Lundblad (2005) who regress market-to-book sorted portfolios' dividend growth on a moving average of consumption growth rates, and find that indeed cash-flow betas are larger for value sorted portfolios (see Table 1, Panel A). Finally, Hansen, Heaton and Li (2005) show that growth stocks have low long-run cash-flow covariation with consumption relative to value.

In summary, there is substantial empirical evidence that indeed value stocks have "more" cash-flow risk than growth stocks. We turn next to the question of whether they have "enough of it" to generate a quantitatively plausible value premium.

VI.B Sensitivity analysis: Asset Pricing

The simulations performed in Section IV and V are based on the particular set of parameters for the share process reported in Table II. We study next the impact that different values for $\overline{\nu}$ and $\overline{\theta}_{CF}$ have on the time series and the cross-section of stock returns. In Section VI.C we analyze what these different values imply for the properties of individual dividends.

Table VIII reports results in simulations under three values of the share volatility, $\overline{\nu}$, and five values of the cash-flow parameter, $\overline{\theta}_{CF}$. Recall that the latter parameter defines the interval $[-\overline{\theta}_{CF}, \overline{\theta}_{CF}]$ in which individual firms' cash-flow risk are uniformly distributed.

VI.B.1 Sensitivity analysis: The market portfolio

For each level of $\overline{\nu}$, the average market premium and volatility decline as we move from low to high values of $\overline{\theta}_{CF}$. Instead the properties of the market portfolio are largely unaffected as we vary $\overline{\nu}$. For instance, when $\overline{\nu} = .25$, the average market premium and the volatility decline from 9.90% and 24.16% to 3.97% and 10.23% respectively as we increase the cross sectional dispersion in cash-flow risk from $\overline{\theta}_{CF} = 0$ to $\overline{\theta}_{CF} = .00345$. Similarly, the level of interest rate and its volatility also decline, although the difference is much less striking and, in all cases, rather reasonable (see Table I for comparison). Finally, the predictability of aggregate excess returns weakens as we increase $\overline{\theta}_{CF}$. For instance, the R^2 of the three year return regression declines from 23% when $\overline{\theta}_{CF} = 0$ to 4.4% when $\overline{\theta}_{CF} = .00345$.

To understand why changes in $\overline{\theta}_{CF}$ affect market returns in our model, recall first that our framework implies a mild predictability of consumption growth.[21] Moreover, it can be shown that consumption growth and *expected* consumption growth are positively correlated. Since in our model the representative agent has a low elasticity of intertemporal substitution and thus a preference for consumption smoothing, we obtain an effect on prices that is absent from the habit persistence models of Campbell and Cochrane (1999) and MSV, which assume that consumption growth is i.i.d. Assume for instance that a negative shock to consumption growth occurs. The intuition of habit persistence models is that this negative shock induces an increase in the representative agent risk aversion and thus a decline in the stock price that is sharper than in the case without habit. This effect is mitigated in our present model, however, because a negative consumption shock is associated, on average, with a drop in expected consumption growth as well. Preferences for intertemporal consumption smoothing imply that the representative agent will attempt to save more when expected consumption growth decreases, increasing his demand for stocks and bonds. This additional demand for assets thus reduces the initial drop in prices, its corresponding volatility and effectively reduces both the equity premium and the predictability. In our model, the size of this counterbalancing effect depends on the "size" of the term $\mu_{c,1}(\mathbf{s}_t) = \mathbf{s}_t' \, \boldsymbol{\theta}_{CF}$, which governs the variation in expected consumption growth, see Assumption 1 and the general equilibrium restrictions (6) and (16). If all θ_{CF}^i are close to zero, as it is the case when $\overline{\theta}_{CF}$ is low, then this effect is negligible, but if they are large – as it is necessary to obtain substantial cash-flow effects – then the intertemporal substitution effect will be large.

[21] The predictability is indeed mild: Regressing the $\log(D/P)$ on future consumption growth in artificial data, we obtain R^2 between 0.4% and 0.6%. No other predictor improves upon this one in our model.

VI.B.2 Sensitivity analysis: The value premium and the performance of the CAPM

The last two columns of Table VIII report the implications of various levels of $\overline{\nu}$ and $\overline{\theta}_{CF}$ for the value premium and the corresponding CAPM fit. For any value of $\overline{\nu}$, a low level of $\overline{\theta}_{CF}$ tends to generate a growth premium, rather than a value premium. For instance, when $\overline{\nu} = .25$ and $\overline{\theta}_{CF} = 0$, the column $10 - 1$ shows a value premium of -2.44%, that is, a growth premium. This effect can also be seen in Panel A of Figure 5, which plots the average log of the price-dividend ratio of the ten sorted portfolios versus their corresponding average excess returns.[22] As discussed in earlier sections, an increase in the dispersion of cash-flow risk generates a value premium: For $\overline{\nu} = .25$ the value premium goes from -2.44% for $\overline{\theta}_{CF} = 0$ to 7.10% for $\overline{\theta}_{CF} = .00345$.

We saw above that the properties of the market portfolio are largely unaffected by changes in $\overline{\nu}$. The most striking effect of a higher level of volatility is in the inability of the CAPM to price our set of test portfolios. To understand why is this the case, notice that in our model the CAPM with respect to the total wealth portfolio holds neither conditionally nor unconditionally as the total wealth portfolio is not perfectly correlated with the stochastic discount factor. Indeed, the time variation in expected consumption growth induces a variation in prices of the total wealth portfolio that is uncorrelated with consumption shocks. Assumption 1, which follows the general equilibrium restriction (6), implies that a higher *idiosyncratic* volatility of shares would generate a higher volatility of expected consumption growth that is not correlated with consumption shocks and thus a worse CAPM performance.[23] This is exactly what the last panel of Table VIII shows: When $\overline{\nu} = .55$ the model can replicate the bad performance of the CAPM when the value premium is quantitatively plausible ($\overline{\theta}_{CF} = .00345$).

The value premium puzzle though is not a robust feature of the data. For instance Ang and Chen (2005) and Fama and French (2005) show that the CAPM performs much better in a similar set of test portfolios when using a long sample that starts in 1927.[24] We chose $\overline{\nu} = .55$ and $\overline{\theta}_{CF} = .00345$ to illustrate the model's ability to replicate both the value premium and its corresponding puzzle. But if the CAPM's performance is not an issue, the volatility

[22] This figure corresponds to the parameter choice $\overline{\nu} = .55$, to make it comparable with Figure 4.

[23] More generally, the CAPM is violated in our setting whenever expected consumption growth is (mildly) time varying, and this variation is uncorrelated with consumption shocks. It is possible to extend the model in this direction by simply assuming that $\overline{\mu}_c$ in Assumption 1 is time varying. We do not pursue this extension here, as the model becomes significantly more complicated but the intuition of the results would be the same in this case.

[24] See also footnote 16.

parameter can be lowered to $\overline{\nu} = .25$ and the cash-flow risk parameter, $\overline{\theta}_{CF}$ can now be between .002 and .003. Notice also that this would also improve notably the model's performance in what refers to the market portfolio. The reason is that lower values of the cash-flow risk parameter attenuate the intertemporal substitution effects discussed above. Thus lower values of $\overline{\nu}$, which are feasible if the value premium puzzle is not a concern, improve considerable the overall performance of the model and lower, if only slightly, the cash-flow risk parameters needed to generate the value premium.

VI.C Sensitivity analysis: Dividend Growth

As shown in Table VIII, the model can generate plausible quantitative properties for both the market portfolio and the cross-section of stock returns. But what do the specific parameter choices mean for the properties of the individual dividend processes?

To answer this question Table IX reports the range of correlation coefficients between the dividend growth of individual assets and consumption growth and the average dividend growth volatility across the simulated assets for each value of $\overline{\nu}$ and $\overline{\theta}_{CF}$. The next two columns report the cash-flow betas of Cohen, Polk and Vuolteenaho (2003), described in equation (28) in Section VI.A, for the growth portfolio (portfolio 1) and the value portfolio (portfolio 10); the remaining coefficients simply grow linearly from the minimum to the maximum. Finally, the last column in Table VIII reports the average volatility of individual firms' stock returns.

Consider first the case where $\overline{\nu} = .25$ and $\overline{\theta}_{CF} = 0$. In this case the range of correlation coefficients between individual dividend and consumption growth is very low, between .04 and .07. Recall that in this case all assets have, by construction, the same cash-flow risk as consumption itself. This is also apparent in the next two columns: The cash-flow betas are, naturally, very close to 1. Finally, the volatility of dividend growth is reasonable, about 24%, while the volatility of returns for individual stocks is about 27.6%. But, as shown in Table VIII, the case $\overline{\theta}_{CF} = 0$ is one that generates a growth premium rather than a value premium. As we increase $\overline{\theta}_{CF}$ in order to obtain the value premium, the range of correlation coefficients between dividend growth and consumption growth widens substantially, to reach the range $[-.89, .91]$ for the case $\overline{\nu} = .25$ and $\overline{\theta}_{CF} = .00345$. In addition, the volatility of both dividend growth and stock returns decline to about 16% for both and the cash-flow betas range from -9.6 for growth stocks to 7.94 for value stocks.

As we increase the volatility of shares $\overline{\nu}$, as one would need to do if the value premium puzzle is to obtain, some features of the cash-flow dividend growth improve but at the expense of others. For instance, the range of correlations of dividend growth and consumption growth when $\overline{\nu} = .55$ and $\overline{\theta}_{CF} = .00345$ is now $[-.37, .42]$, still large, but better than for the case where $\overline{\nu} = .25$. The cash-flow betas marginally improve as well, although their spread is still too large compared to the results obtained by Cohen et al (2003). The most salient effect of increasing the volatility of shares from $\overline{\nu} = .25$ to .55, however, is the large increase in the volatility of dividend growth, which now reaches 52.60%. Instead the average volatility of individual stock returns increases slightly, from 16.68% to 22.96%. These last results are obviously hard to reconcile with the empirical evidence.

VI.D But is it all bad news? Some intuition on the magnitude of $\overline{\theta}_{CF}$

The previous section suggests that in order to generate a quantitatively plausible value premium and the observed poor performance of the CAPM (see Table VIII), we have to assume a large cross-sectional dispersion of cash-flow risk (see the last row of Table IX). We discuss next some intuition on why does our model need these extreme parameters as well as some potential extensions to address these problems.

First notice that the sign of the cash-flow betas is negative for growth stocks and positive for value stocks in simulated data whereas Cohen et al. (2003) obtained positive numbers throughout. This is due to our counterfactual assumption that all the sources of consumption are financial, and assumption that it is easy to relax. Indeed dividends make up only about 10% of total consumption in the data. In Santos and Veronesi (2005) we explored the role of labor income in asset pricing tests and argued that it is less risky than consumption so that it has a negative θ_{CF}. In that model then all financial assets can have a positive θ_{CF}, and, as consequence, the cash-flow betas would be positive across the ten sorted portfolios. Here we abstract from adding labor income to the model as it would introduce one additional state variable, and the analysis and the intuition of the model would become substantially more complicated.

Focusing next on the the magnitude of the cash-flow risk dispersion, part of the difference between simulated and empirical data may be due to measurement error, which is of course absent in our simulations. This measurement error in the cash-flow properties of the market portfolio biases towards zero the cash-flow beta as defined by the regression in Cohen et al. (2003). For instance, in our simulations for the case $\overline{\theta}_{CF} = .00345$ and $\overline{\nu} = .55$, if we add a level of noise to the market dividend growth that is of the same magnitude as its actual volatility (=.03), we find that, in Table IX, the cash-flow betas are given by $\beta^1_{CF,1} = -3.79$ and $\beta^{10}_{CF,1} = 2.39$, that is, the spread between value and growth is cut by about half. If the noise is twice the value of its actual volatility then $\beta^1_{CF,1} = -1.48$ and $\beta^{10}_{CF,1} = 0.92$, which are much closer to their empirical counterparts.[25]

Relevant as they are, alternative sources of income and measurement problems are not likely to fully address the large magnitudes of cash-flow risk needed to generate a sizable value premium. Indeed these large magnitudes are required to "undo" the discount risk effects that are in turn important to generate quantitatively plausible properties for the market portfolio. To elaborate further, in Santos and Veronesi (2005) we used a similar cash-flow model as the one described in Section II, but the representative agent is assumed to have the standard CRRA preferences. Moreover, the agent receives income from both financial and non financial assets. As mentioned, in that model, labor income (in average, about 90% of total income) has a negative θ_{CF}^{j}, while all of the financial assets have identical positive cash-flow risk θ_{CF}^{i}. Thus, a value premium obtains even with identical cash-flow risk parameters across financial assets. The drawback of that model, though, is that it is unable to generate reasonable properties of the aggregate market portfolio: the predictability of stock returns – which is induced by the variation over time of the labor income-to-consumption ratio – is small compared to the data, and the volatility of stock returns is just 6.2%. That is, in Santos and Veronesi (2005) a sizable value premium obtains but the assumption of a standard CRRA utility function for the representative agent makes it impossible to generate enough predictability or volatility of the aggregate market.

Campbell and Cochrane's (1999) key contribution is precisely to show how a strong variation in risk preferences is able to generate the main time series properties of the market portfolio. But an unexpected drawback of this modelling device is to induce a growth premium in the cross-section, unless cross-sectional differences in cash-flow risk are "large enough." The "size" of these cash-flow risk effects then can only be assessed in a model where the strong discount risk effects required to generate the time series properties of the market portfolio are present, otherwise one would underestimate the magnitudes of the cash-flow risk effects that are in turn needed to obtain the value premium.

It follows from the previous discussion that one possible direction to generate more plausible magnitudes of the cash-flow risk parameter is to generate variation in the discount that is unpriced in the cross section. Thus growth stocks would comove more with the discount than value stocks but this does not result in a growth premium. This is exactly the route chosen by Lettau and Wachter (2005). The problem, of course, is the interpretation of this source of

[25] This is a "rough" calculation: To properly perform this exercise, we should keep the volatility of aggregate dividend (= consumption) constant across noise levels.

variation in the discount. They refer to this exogenous source of variation in the discount as "investor sentiment" though it is hard to assess quantitatively this effect.

Alternatively, the duration of the growth assets may not be as long as suggested by the model. Indeed, an important limiting feature of the model is that assets are infinitely lived. But growth stocks though can have shorter duration if they are more likely to disappear than value stocks, which, on average correspond to more established firms. In this case then the differences in duration between growth and value are less pronounced than implied by the model and thus there will less of a value premium. These two last alternatives offer fruitful venues for future research and can potentially relax the pressure on the cash-flow risk parameters.

VII. CONCLUSIONS

Two sources of risk combine to determine the time series properties of the market portfolio and the cross-sectional properties of stock returns: discount risk and cash-flow risk. Campbell and Cochrane (1999) argue that time variation of the market price of risk - i.e. discount risk - is important to reconcile many empirical facts about the aggregate market portfolio. We show that this channel though imposes tight restrictions on the cash-flow properties of value versus growth stocks. Specifically, value stocks are (endogenously) those with high cash flow risk relative to growth, that is, their dividends covary more with the aggregate economy than the dividends of growth stocks, a prediction consistent with recent empirical evidence. Our model is able not only to match the time series properties of the aggregate portfolio, as in Campbell and Cochrane (1999), but it also generates a large value premium and its corresponding value premium puzzle, that is, the documented inability of the CAPM to price value-sorted portfolios.

In addition, our model also generates a time variation of the value premium over the business cycle that lines up well with the data. This variation of the value premium stems from the fact that the discount risk effects that drive the time series properties of the market portfolio interact with the cross sectional dispersion of cash-flow risk to make value stock particularly riskier than growth stocks in bad times, that is, when the market premium is high. This dynamic aspect of the value premium allow us to explain the source of recent empirical "successes" in explaining the value premium puzzle, such as the multi factor model of Fama French (1993) and the conditional CAPM model of Lettau and Ludvigson (2001). Although these models are misspecified in our general equilibrium setting, they pick up this dynamic variation in cross-sectional risk due to the interaction of discount risk effects and the cross-sectional dispersion in cash flow risk.

We have shown that the model seems to require a large cross sectional dispersion of cash-flow risk to explain the value premium, one that seems at odds with the data. We have argued for possible extensions of the model to address this excessive magnitude. We view our model as a first step into understanding the sources of risk that explain both the time-series and the cross-section of stock returns. Indeed, an important message of this paper is that we cannot study one set of empirical facts independently of the other: any story that attempts to quantitatively explain the cross-section of stock returns must also be consistent with the time series properties of the market portfolios. Otherwise, the parametrization that is used to obtain quantitative predictions at the cross-sectional level may be quite misleading.

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APPENDIX

The habit dynamics: If $X_t = \lambda \int_{-\infty}^{t} e^{-\lambda(\tau - t)} C_\tau d\tau$ we have $dX_t = \lambda (C_t - X_t) dt$. Define then $G_t = f(C_t, X_t) = (C_t / (C_t - X_t))^\gamma$. We then have

$$
\begin{aligned}
f_C &= -\gamma G_t \left(G_t^{\frac{1}{\gamma}} - 1 \right) C_t^{-1} \\
f_{CC} &= \left\{ \gamma (\gamma - 1) G \left(G_t^{\frac{1}{\gamma}} - 1 \right)^2 + 2\gamma \left(G_t^{\frac{1}{\gamma}} - 1 \right) G_t^{\frac{1}{\gamma}+1} \right\} C_t^{-2} \\
f_X &= \gamma G_t \frac{1}{(C_t - X_t)}
\end{aligned}
$$

where we used $G_t^{\frac{1}{\gamma}} = C_t / (C_t - X_t)$ and $G_t^{\frac{1}{\gamma}} - 1 = X_t / (C_t - X_t)$. Ito's Lemma then yields

$$dG_t = \left\{ \mu_G (G_t) - \sigma_G (G_t) \mu_{c,1} (\mathbf{s}_t) \right\} dt - \sigma_G (G_t) \sigma_c dB_t^1$$

where

$$
\begin{aligned}
\mu_G (G_t) &= \gamma \lambda G_t + \frac{1}{2} \gamma (\gamma - 1) G \left(G_t^{\frac{1}{\gamma}} - 1 \right)^2 \sigma_c^2 + \gamma \left(G_t^{\frac{1}{\gamma}} - 1 \right) G_t^{\frac{1}{\gamma}+1} \sigma_c^2 - \sigma_G (G_t) \overline{\mu}_c & (29) \\
\sigma_G (G_t) &= \gamma G_t \left(G_t^{\frac{1}{\gamma}} - 1 \right) & (30)
\end{aligned}
$$

Proof of Propositions

Our strategy to obtain prices and returns in our economy is standard. Given (2), the stochastic discount factor is given by

$$m_t = e^{-\rho t} (C_t - X_t)^{-\gamma} = e^{-\rho t} C_t^{-\gamma} G_t.$$

We use Ito's Lemma and our assumptions on the dynamics of C_t and $G_t = S_t^{-\gamma}$ to obtain

$$\frac{dm_t}{m_t} = -r_t^f dt + \boldsymbol{\sigma}_m' d\mathbf{B}_t,$$

where the first, and only non-zero, entry in the diffusion component vector, $\boldsymbol{\sigma}_m$, is given by

$$\sigma_m^1 = - \left[\gamma + \alpha \left(1 - \lambda S_t^\gamma \right) \right] \sigma_c. \quad (31)$$

Then we exploit our assumptions on the dynamics of C_t, $G_t = S_t^{-\gamma}$ and s_t^i to solve for

$$P_t^i = E_t \left[\int_t^\infty \left(\frac{m_\tau}{m_t} \right) D_\tau^i d\tau \right] = E_t \left[\int_t^\infty \left(\frac{m_\tau}{m_t} \right) s_\tau^i C_\tau d\tau \right] \quad (32)$$

in closed form. We then use (32) to compute returns and calculate the expected excess returns

$$E_t \left[dR_t^i \right] = -cov \left(\frac{dm_t}{m_t}, dR^i \right) = -\boldsymbol{\sigma}_m' \boldsymbol{\sigma}_R^i, \quad (33)$$

where $\boldsymbol{\sigma}_R^i$ is the diffusion component associated with the returns of asset i.

Proof of Proposition 1. This is a corrollary to Proposition 2, and it is proved below.

Proof of Proposition 2. **Part (a). Pricing Formula.** The pricing formula is

$$
\begin{aligned}
P_t^i &= E_t \left[\int_t^\infty e^{-\rho(\tau - t)} \frac{u_c (C_\tau, X_\tau)}{u_c (C_t, X_t)} D_\tau^i d\tau \right] \\
&= C_t^\gamma G_t^{-1} E_t \left[\int_t^\infty e^{-\rho(\tau - t)} C_\tau^{1-\gamma} G_\tau s_\tau^i d\tau \right]
\end{aligned}
$$

We divide the proof in two parts: First, we obtain a general pricing formula which depends on the state variables.

Second, we obtain analytical solutions for the coefficients of these state variables.

Part a.1: A pricing formula. For this proof, it is convenient to rewrite the share processes in its general form as

$$ds_t^i = \sum_{j=1}^{n} s_t^j \lambda_{ji} dt + s_t^i \left(\boldsymbol{\nu}^i - \mathbf{s}_t' \boldsymbol{\nu}\right) d\mathbf{B}_t$$

where $\lambda_{ji} = \phi \overline{s}^i$, for $i \neq j$, and $\lambda_{ii} = -\sum_{j\neq i} \lambda_{ij} = -\phi \sum_{j\neq i} \overline{s}^j = -\phi\left(1-\overline{s}^i\right) = \phi\overline{s}^i - \phi$. Define the two quantities

$$q_t^i = C_t^{1-\gamma} G_t s_t^i \quad \text{and} \quad p_t^i = C_t^{1-\gamma} s_t^i$$

and the $2n \times 1$ vector $\mathbf{y}_t = [\mathbf{q}_t, \mathbf{p}_t]$. An application of Ito's Lemma and tedious algebra shows

$$d\mathbf{y}_t = \widehat{\boldsymbol{\Lambda}}_y \mathbf{y}_t dt + \boldsymbol{\Sigma}_{y,t} d\mathbf{B}_t$$

where

$$\widehat{\boldsymbol{\Lambda}}_y = \begin{bmatrix} \boldsymbol{\Lambda}' + \widehat{\Theta}_q & \widehat{\Theta}_{qp} \\ 0 & \boldsymbol{\Lambda}' + \widehat{\Theta}_p \end{bmatrix},$$

$\boldsymbol{\Lambda} = \phi\left(\overline{\mathbf{s}} \times \mathbf{1}_n'\right)$, $\widehat{\Theta}_i$ for $i = q, p, qp$ are diagonal matrices with ii element given by

$$\begin{aligned}
\widehat{\theta}_q^i &= (1-\gamma)\overline{\mu}_c - \frac{1}{2}\gamma(1-\gamma)\sigma_c^2 - k - (1-\gamma)\sigma_c^2\alpha + (1-\gamma)\theta^i - \alpha\theta^i \\
\widehat{\theta}_{qp}^i &= k\overline{G} + (1-\gamma)\sigma_c^2\alpha\lambda + \alpha\lambda\theta^i \\
\widehat{\theta}_p^i &= (1-\gamma)\overline{\mu}_c - \frac{1}{2}\gamma(1-\gamma)\sigma_c^2 + (1-\gamma)\theta^i
\end{aligned}$$

and $\boldsymbol{\Sigma}_{y,t}$ is an appropriate matrix. Assuming existence of the expectation in the pricing function, we can apply Fubini's theorem

$$P_t^i = C_t^{\gamma} G_t^{-1} E_t\left[\int_t^{\infty} e^{-\rho(\tau-t)} y_\tau^i d\tau\right] = C_t^{\gamma} G_t^{-1} \int_t^{\infty} E_t\left[e^{-\rho(\tau-t)} y_\tau^i\right] d\tau$$

The expectation in the integral can be computed as follows: Let $\boldsymbol{\omega}$ be the vector of eigenvalues of $\widehat{\boldsymbol{\Lambda}}_y$, $\left[e^{\boldsymbol{\omega}(\tau-t)}\right]$ the diagonal matrix with ii element given by $e^{\omega_i(\tau-t)}$ and $\mathbf{U}$ the matrix of associated eigenvectors. Then, we can write

$$E_t\left[e^{-\rho(\tau-t)} y_\tau^i\right] = \boldsymbol{\iota}_i \cdot \mathbf{U} \cdot \left[e^{\boldsymbol{\omega}(\tau-t)}\right] \cdot \mathbf{U}^{-1} \cdot \mathbf{y}_t e^{-\rho(\tau-t)} = \sum_{k=1}^{2n} \sum_{j=1}^{2n} u_{ik} e^{(\omega_k-\rho)(\tau-t)} \left[u_{jk}^{-1}\right] y_{jt}$$

where $\left[u_{jk}^{-1}\right]$ is the jk element of U^{-1}. Substituting into the expectation, and taking the integral, we find

$$\int_t^{\infty} E_t\left[e^{-\rho(\tau-t)} y_\tau^i\right] d\tau = \sum_{k=1}^{2n} \sum_{j=1}^{2n} \frac{u_{ik}\left[u_{jk}^{-1}\right]}{\rho - \omega_k} y_{jt} = \sum_{j=1}^{2n} b_j^i y_{jt}$$

where

$$b_j^i = \sum_{k=1}^{2n} \frac{u_{ik}\left[u_{kj}^{-1}\right]}{\rho - \omega_k}$$

Below, we obtain these coefficients in closed form. Note, however, that by substituting $y_{jt} = q_{jt}$ for $j = 1, ..., n$

and $y_{jt} = p_{j-n,t}$ for $j = n+1,..,n$ we obtain

$$\begin{aligned} P_t^i &= C_t^\gamma G_t^{-1} E_t \left[\int_t^\infty e^{-\rho(\tau - t)} y_\tau^i d\tau \right] = C_t^\gamma G_t^{-1} \left(\sum_{j=1}^n b_{1j}^i q_{j,t} + \sum_{j=1}^n b_{2j}^i p_{j,t} \right) \\ &= C_t^\gamma G_t^{-1} \left(C_t^{1-\gamma} G_t \sum_{j=1}^n b_{1j}^i s_t^j + C_t^{1-\gamma} \sum_{j=1}^n b_{2j}^i s_t^j \right) \\ &= C_t \sum_{j=1}^n \left(b_{1j}^i + b_{2j}^i S_t^\gamma \right) s_t^j \end{aligned}$$

Part a.2: Analytical formulas for $b_{1,j}^i$ and $b_{2,j}^i$. We finally obtain a closed form formula for b_j^i's, and thus, of b_{1j}^i and b_{2j}^i. First, note that we can write

$$b_j^i = \boldsymbol{\iota}_i \cdot \mathbf{U} \cdot \left(\boldsymbol{\Omega}^{-1} \right) \cdot \mathbf{U}^{-1} \boldsymbol{\iota}_j$$

where $\boldsymbol{\Omega}$ is the matrix with the eigevalues of $\mathbf{I}\rho - \widehat{\boldsymbol{\Lambda}}_y$ on the principal diagonal. But then, since $\mathbf{U} \cdot \left(\boldsymbol{\Omega}^{-1} \right) \cdot \mathbf{U}^{-1} = \left(\mathbf{I}\rho - \widehat{\boldsymbol{\Lambda}}_y \right)^{-1}$ we have that for $i = 1, ..., n$ and $j = 1, ..., 2n$

$$b_j^i = \boldsymbol{\iota}_i \cdot \left(\mathbf{I}\rho - \widehat{\boldsymbol{\Lambda}}_y \right)^{-1} \cdot \boldsymbol{\iota}_j$$

We now explicitly compute these quantities. Define $\mathbf{B} = \left(\mathbf{I}\rho - \widehat{\boldsymbol{\Lambda}}_y \right)^{-1}$, so that

$$\mathbf{B} \left(\mathbf{I}\rho - \widehat{\boldsymbol{\Lambda}}_{\mathbf{y}} \right) = \mathbf{I}$$

Making this explicit, for every $i = 1, .., n$ (row) we have

$$\sum_{j=1}^{2n} b_j^i \left(\mathbf{I}\rho - \widehat{\boldsymbol{\Lambda}}_{\mathbf{y}} \right)_j = \boldsymbol{\iota}_i$$

where $\left(\mathbf{I}\rho - \widehat{\boldsymbol{\Lambda}}_{\mathbf{y}} \right)_j$ is the jth row of $\left(\mathbf{I}\rho - \widehat{\boldsymbol{\Lambda}}_{\mathbf{y}} \right)$ and $\boldsymbol{\iota}_i$ is a $(1 \times 2n)$ row vector with 1 in ith position, and zero elsewhere. For every i, we have a system of equations that pins down b_j^i for all $j = 1, .., 2n$. We now solve this system of equation. To limit the number of indices involved, we do this exercise for $i = 1$. Of course, the methodology works for every i. For $i = 1$ we have then the following two systems of equations. The first holds for $j = 1, .., n$ and the second for the remaining n rows:

$$\begin{aligned} b_1^1 \left(\rho - \phi \overline{s}^1 + \phi - \widehat{\theta}_q^1 \right) - \sum_{j=2}^n b_j^1 \phi \overline{s}_j &= 1 && \text{(row 1)} \\ -b_1^1 \phi \overline{s}^1 + b_2^1 \left(\rho - \phi \overline{s}^2 + \phi - \widehat{\theta}_q^2 \right) - \sum_{j=3}^n b_j^1 \phi \overline{s}_j &= 0 && \text{(row 2)} \\ &\vdots \\ -\sum_{j=1}^{n-1} b_j^1 \phi \overline{s}_j + b_n^1 \left(\rho - \phi \overline{s}^n + \phi - \widehat{\theta}_q^n \right) &= 0 && \text{(row n)} \end{aligned}$$

$$-b_1^1\theta_{qp}^1 + b_{n+1}^1\left(\rho - \phi\overline{s}^1 + \phi - \widehat{\theta}_p^1\right) - \sum_{j=2}^{n} b_{n+j}^1\phi\overline{s}_j = 0 \qquad \text{(row } n+1\text{)}$$

$$-b_2^1\theta_{qp}^2 - b_{n+1}^1\phi\overline{s}_1 + b_{n+2}^1\left(\rho - \phi\overline{s}^2 + \phi - \widehat{\theta}_p^2\right) - \sum_{j=3}^{n} b_{n+j}^1\phi\overline{s}_j = 0 \qquad \text{(row } n+2\text{)}$$

$$\vdots$$

$$-b_n^1\theta_{qp}^n - \sum_{j=1}^{n-1} b_{n+j}^1\phi\overline{s}_j + b_{2n}^1\left(\rho - \phi\overline{s}^n + \phi - \widehat{\theta}_p^n\right) = 0 \qquad \text{(row } 2n\text{)}$$

The first set of equation is readily solved. In fact, we can write

$$\begin{aligned} b_1^1 &= \alpha_q^1 + \alpha_q^1 \times \phi\sum_{j=1}^{n} b_j^1\overline{s}^j \\ b_k^1 &= \alpha_q^k \times \phi\sum_{j=1}^{n} b_j^1\overline{s}^j \quad \text{for} \quad k = 2,..,n \end{aligned}$$

where

$$\alpha_q^i = \frac{1}{\left(\rho + \phi - \widehat{\theta}_q^i\right)}$$

Multiply both sides of each row $k = 1,...,n$ by $\overline{s}^k$ and sum across rows to obtain

$$\sum_{j=1}^{n} b_j^1\overline{s}^j = \overline{s}^1\alpha_q^1 + \sum_{j=1}^{n}\overline{s}_t^j\alpha_q^j\left(\phi\sum_{j=1}^{n} b_j^1\overline{s}^j\right)$$

Define the constants

$$H_q = \sum_{j=1}^{n}\overline{s}^j\alpha_q^j \text{ and } K_q = \frac{1}{1-\phi H_q}$$

Solving for $\sum_{j=1}^{n} b_j^1\overline{s}^j$ we obtain the quantity

$$\sum_{j=1}^{n} b_j^1\overline{s}^j = \overline{s}^1\alpha_q^1 K_q$$

Thus

$$b_1^1 = \alpha_q^1 + \alpha_q^1 \times \phi\overline{s}^1\alpha_q^1 K_q \qquad (34)$$

$$b_k^1 = \alpha_q^k \times \phi\overline{s}^1\alpha_q^1 K_q \quad \text{for} \quad k = 2,..,n \qquad (35)$$

Hence, the first term in the P/C ratio obtained earlier, i.e.

$$\frac{P_t^1}{C_t} = \sum_{j=1}^{n} b_{1j}^1 s_t^j + \sum_{j=1}^{n} b_{2j}^1 s_t^j S_t^\gamma$$

is given by

$$\sum_{j=1}^{n} b_{1j}^1 s_t^j = \alpha_q^1 s_t^1 + \phi\overline{s}^1\alpha_q^1 K_q\sum_{j=1}^{n}\alpha_q^k s_t^j$$

where recall that for $j = 1,...,n$ we defined earlier $b_{1j}^1 = b_j^1$.

We now turn to the second system of equations, which for $k = 1,...,n$ can be rewritten as

$$b_{n+k}^1 = \alpha_p^k\phi\sum_{j=1}^{n} b_{n+j}^1\overline{s}^j + b_k^1\alpha_p^k\widehat{\theta}_{qp}^k$$

with

$$\alpha_p^k = \frac{1}{\left(\rho + \phi - \widehat{\theta}_p^k\right)}$$

and b_k^1 given in (34) - (35). Substitute b_k^1 first, to obtain

$$\begin{aligned} b_{n+1}^1 &= \alpha_p^1 \phi \sum_{j=1}^n b_{n+j}^1 \overline{s}^j + \alpha_{pq}^1 + \alpha_{pq}^1 \times \phi \overline{s}^1 \alpha_q^1 K_q \\ b_{n+k}^1 &= \alpha_p^k \phi \sum_{j=1}^n b_{n+j}^1 \overline{s}^j + \alpha_{pq}^k \phi \overline{s}^1 \alpha_q^1 K_q \end{aligned}$$

where

$$\alpha_{pq}^k = \alpha_p^k \widehat{\theta}_{qp}^k \alpha_q^k$$

As before, for $k = 1, .., n$ multiply both sides by $\overline{s}^k$ and sum across k's to obtain

$$\sum_{k=1}^n \overline{s}^k b_{n+k}^1 = \alpha_{pq}^1 \overline{s}^1 + \left(\sum_{k=1}^n \overline{s}^k \alpha_p^k\right) \phi \sum_{j=1}^n b_{n+j}^1 \overline{s}^j + \left(\sum_{k=1}^n \overline{s}^k \alpha_{pq}^k\right) \phi \overline{s}^1 \alpha_q^1 K_q$$

Let

$$H_p = \left(\sum_{k=1}^n \overline{s}^k \alpha_p^k\right)$$

and solve for $\sum_{k=1}^n \overline{s}^k b_{n+k}^1$ to find

$$\sum_{k=1}^n \overline{s}^k b_{n+k}^1 = \alpha_{pq}^1 \overline{s}^1 K_p + \left(\sum_{k=1}^n \overline{s}^k \alpha_{pq}^k\right) \phi \overline{s}^1 \alpha_q^1 K_q K_p$$

where

$$K_p = \frac{1}{(1 - \phi H_p)}$$

Substitute back into b_{n+1}^1 and b_{n+k}^1 and find

$$\begin{aligned} b_{n+1}^1 &= \alpha_{pq}^1 + \overline{s}^1 g_1^1 \\ b_{n+k}^1 &= \overline{s}^1 g_k^1 \end{aligned}$$

where for $k = 1, ..., n$

$$g_k^1 = \alpha_q^1 \phi \left\{ \alpha_p^k \left(\alpha_p^1 \theta_{pq}^1 K_p + \left(\sum_{j=1}^n \overline{s}^j \alpha_{pq}^j\right) \phi K_q K_p \right) + \alpha_{pq}^k K_q \right\}$$

Thus, the second part in the price-consumption ratio is given by

$$\sum_{j=1}^n b_{2j}^1 s_t^j = \alpha_{pq}^1 s_t^1 + \overline{s}^1 \sum_{k=1}^n g_k^1 s_t^k$$

Generalizing the above derivations for every $i = 1, ...,$n, we can finally write

$$\frac{P_t^i}{D_t^i} = \alpha_0^i + \alpha_1^i S_t^\gamma + \alpha_2^i \left(\mathbf{s}_t\right) \left(\frac{\overline{s}^i}{s_t^i}\right) + \alpha_3^i \left(\mathbf{s}_t\right) \left(\frac{\overline{s}^i}{s_t^i}\right) S_t^\gamma$$

where

$$\begin{aligned}
\alpha_0^i &= \alpha_q^i = \frac{1}{\left(\rho+\phi-\widehat{\theta}_q^i\right)} \\
\alpha_1^i &= \alpha_{pq}^i = \frac{\widehat{\theta}_{pq}^i}{\left(\rho+\phi-\widehat{\theta}_q^i\right)\left(\rho+\phi-\widehat{\theta}_p^i\right)} \\
\alpha_2^i(\mathbf{s}_t) &= \phi\alpha_q^i K_q\left(\mathbf{s}_t'\ \boldsymbol{\alpha}_q\right) \\
\alpha_3^i(\mathbf{s}_t) &= \mathbf{s}_t'\ \mathbf{g}^i
\end{aligned}$$

where

$$g_k^i = \alpha_q^i\phi\left\{\alpha_p^k\left(\alpha_p^i\widehat{\theta}_{pq}^i K_p + \left(\overline{\mathbf{s}}'\ \boldsymbol{\alpha}_{pq}\right)\phi K_q K_p\right) + \alpha_{pq}^k K_q\right\}$$

and

$$\begin{aligned}
\widehat{\theta}_q^i &= (1-\gamma)\overline{\mu}_c - (1-\gamma)\left(\frac{1}{2}\gamma+\alpha\right)\sigma_c^2 - k + (1-\gamma-\alpha)\theta^i \\
\widehat{\theta}_{qp}^i &= k\overline{G} + (1-\gamma)\sigma_c^2\alpha\lambda + \alpha\lambda\theta^i \\
\widehat{\theta}_p^i &= (1-\gamma)\overline{\mu}_c - \frac{1}{2}\gamma(1-\gamma)\sigma_c^2 + (1-\gamma)\theta^i
\end{aligned}$$

Proposition 3: The diffusion component of stock returns is given by

$$\boldsymbol{\sigma}_{R,t}^i = \frac{S_t^\gamma\alpha\left(1-\lambda S_t^\gamma\right)}{f_1^i\left(\frac{\overline{s}^i}{s_t^i},\mathbf{s}_t\right)+S_t^\gamma}\boldsymbol{\sigma}_c + \left(\frac{1}{1+f_2^i\left(S_t,\mathbf{s}_t\right)\left(\frac{\overline{s}^i}{s_t^i}\right)} + \eta_{it}^i\right)\boldsymbol{\sigma}_D^i(\mathbf{s}_t) + \sum_{j\neq i}\eta_{jt}^i\boldsymbol{\sigma}_D^j(\mathbf{s}_t) \tag{36}$$

In fact, we can write

$$P_t^i = C_t\left(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t)\overline{s}^i + \left(\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t)\overline{s}^i\right)S_t^\gamma\right)$$

Define by $\widetilde{S}_t = S_t^\gamma = G_t^{-1}$. Using Ito's Lemma, it is immediate to see that the diffusion of $d\widetilde{S}$ is given by

$$\boldsymbol{\sigma}_S\left(S^\gamma\right) = S_t^\gamma\alpha\left(1-\lambda S_t^\gamma\right)\boldsymbol{\sigma}_c$$

Thus, an application of Ito's Lemma shows that the diffusion term of P_t^i is given by

$$\begin{aligned}
\boldsymbol{\sigma}_{R,t}^i &= \boldsymbol{\sigma}_c + \frac{\left(\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t)\overline{s}^i\right)S_t^\gamma\alpha\left(1-\lambda S_t^\gamma\right)}{\left(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t)\overline{s}^i + \left(\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t)\overline{s}^i\right)S_t^\gamma\right)}\boldsymbol{\sigma}_c \\
&\quad + \sum_{k=1}^n\left\{\frac{\left(\alpha_0^i + \alpha_1^i S_t^\gamma\right)1_{\{i\}} + \phi\alpha_q^i K_q\alpha_q^k + g_k^i}{\left(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t)\overline{s}^i + \left(\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t)\overline{s}^i\right)S_t^\gamma\right)}\right\}s_t^k\boldsymbol{\sigma}^k(\mathbf{s}_t)
\end{aligned}$$

where $1_{\{i\}}$ is the indicator function for $k=i$. Since $\boldsymbol{\sigma}_D^i(\mathbf{s}_t) = \boldsymbol{\sigma}_c + \boldsymbol{\sigma}^i(\mathbf{s}_t)$, and since by construction

$$\sum_{k=1}^n\left\{\frac{\left(\alpha_0^i + \alpha_1^i S_t^\gamma\right)1_{\{k=i\}} + \phi\alpha_q^i K_q\alpha_q^k + g_k^i}{\left(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t)\overline{s}^i + \left(\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t)\overline{s}^i\right)S_t^\gamma\right)}\right\}s_t^k = 1$$

we can rewrite

$$\begin{aligned}
\boldsymbol{\sigma}_{R,t}^i &= \frac{S_t^\gamma\alpha\left(1-\lambda S_t^\gamma\right)}{f_1^i\left(\overline{s}^i/s_t^i;\mathbf{s}_t\right)+S_t^\gamma}\boldsymbol{\sigma}_c + \sum_{k=1}^n\left\{\frac{\left(\alpha_0^i + \alpha_1^i S_t^\gamma\right)1_{\{k=1\}} + \phi\alpha_q^i K_q\alpha_q^k + g_k^i}{\left(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t)\overline{s}^i + \left(\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t)\overline{s}^i\right)S_t^\gamma\right)}\right\}s_t^k\boldsymbol{\sigma}_D^k(\mathbf{s}_t) \\
&= \frac{S_t^\gamma\alpha\left(1-\lambda S_t^\gamma\right)}{f_1^i\left(\overline{s}^i/s_t^i;\mathbf{s}_t\right)+S_t^\gamma}\boldsymbol{\sigma}_c + \left\{\frac{1}{1+f_2\left(S;\mathbf{s}_t\right)\left(\frac{\overline{s}^i}{s_t^i}\right)} + \eta_{i,t}^i\right\}\boldsymbol{\sigma}_D^i(\mathbf{s}_t) + \sum_{k\neq i}\eta_{k,t}^i\boldsymbol{\sigma}_D^k(\mathbf{s}_t)
\end{aligned}$$

where

$$f_1^i\left(\overline{s}^i/s_t^i;\mathbf{s}_t\right) = \frac{\alpha_0^i+\alpha_2^i(\mathbf{s}_t)\left(\overline{s}^i/s_t^i\right)}{\alpha_1^i+\alpha_3^i(\mathbf{s}_t)\left(\overline{s}^i/s_t^i\right)}$$

$$f_2(S;\mathbf{s}_t) = \frac{\alpha_2^i(\mathbf{s}_t)+\alpha_3^i(\mathbf{s}_t)S_t^\gamma}{\alpha_0^i+\alpha_1^iS_t^\gamma}$$

and

$$\eta_{k,t}^i = \frac{\left(\phi\alpha_q^iK_q\alpha_q^k+g_k^i\right)s_t^k}{\left(\alpha_0^is_t^i+\alpha_2^i(\mathbf{s}_t)\overline{s}^i+\left(\alpha_1^is_t^i+\alpha_3^i(\mathbf{s}_t)\overline{s}^i\right)S_t^\gamma\right)}$$

Note that also that

$$f_1'<0 \text{ if and only if } \frac{\alpha_2^i(\mathbf{s})}{\alpha_3^i(\mathbf{s})}<\frac{\alpha_0^i}{\alpha_1^i}=\frac{1}{\alpha_p^i\widehat{\theta}_{pq}}$$

Q.E.D.

Part (b) of Proposition 3: The Expected Return

The expected return is obtained immediately from $\boldsymbol{\sigma}_{R,t}$ by using the formula

$$E_t\left[dR_t^i\right]=-Cov_t\left(dR_t^i,\frac{dm}{m_t}\right)$$

Q.E.D.

Proof of Proposition 1: (a) The price consumption ratio of the total wealth portfolio can be obtained by simply adding the prices of individual securities. In particular, we find

$$\alpha_0^{TW}(\mathbf{s}_t) = \sum_{i=1}^n\alpha_q^is_t^i+\sum_{i=1}^n\phi\overline{s}^i\alpha_q^iK_q\sum_{j=1}^n\alpha_q^ks_t^j=\left(1+\phi K_q\overline{\mathbf{s}}'\ \boldsymbol{\alpha}_q\right)\boldsymbol{\alpha}_q'\ \mathbf{s}_t$$

$$\alpha_1^{TW}(\mathbf{s}_t) = \sum_{i=1}^n\alpha_{pq}^is_t^i+\sum_{i=1}^n\phi\overline{s}^i\sum_{k=1}^n\left\{\alpha_p^k\left(\alpha_{pq}^iK_p+\overline{\mathbf{s}}\boldsymbol{\alpha}_{pq}\phi\alpha_q^iK_qK_p\right)+\alpha_{pq}^k\alpha_q^iK_q\right\}s_t^k$$

Algebra shows

$$\alpha_0^{TW}(\mathbf{s}_t) = \frac{1}{1-\phi H_q}\boldsymbol{\alpha}_q'\ \mathbf{s}_t$$

$$\alpha_1^{TW}(\mathbf{s}_t) = \frac{1}{1-\phi H_q}\left(\left(\boldsymbol{\alpha}_p\mathbf{s}_t\right)K_p\phi\overline{\mathbf{s}}\boldsymbol{\alpha}_{pq}+\boldsymbol{\alpha}_{pq}\mathbf{s}_t\right)$$

Part (b). An application of Ito's Lemma to $P_t^{TW}=C_t\left(\alpha_0^{TW}(\mathbf{s}_t)+\alpha_1^{TW}(\mathbf{s}_t)S_t^\gamma\right)$ implies that the diffusion part of the TW portfolios is given by

$$\begin{aligned}
\boldsymbol{\sigma}_{P,t}^{TW} &= \boldsymbol{\sigma}_c+\frac{\alpha_1^{TW}(\mathbf{s}_t)}{\alpha_0^{TW}(\mathbf{s}_t)+S_t^\gamma\times\alpha_1^{TW}(\mathbf{s}_t)}S_t^\gamma\alpha\left(1-\lambda S_t^\gamma\right)\boldsymbol{\sigma}_c \\
&\quad+\frac{\boldsymbol{\alpha}_q+S_t^\gamma\left(K_p\phi\overline{\mathbf{s}}\boldsymbol{\alpha}_{pq}\boldsymbol{\alpha}_p+\boldsymbol{\alpha}_{pq}\right)}{\boldsymbol{\alpha}_q'\ \mathbf{s}_t+S_t^\gamma\times\left(\left(\boldsymbol{\alpha}_p\mathbf{s}_t\right)K_p\phi\overline{\mathbf{s}}\boldsymbol{\alpha}_{pq}+\boldsymbol{\alpha}_{pq}\mathbf{s}_t\right)}\mathbf{I}(\mathbf{s}_t)\boldsymbol{\sigma}(\mathbf{s}_t) \\
&= \boldsymbol{\sigma}_c+\frac{S_t^\gamma\alpha\left(1-\lambda S_t^\gamma\right)}{f_1^{TW}(\mathbf{s}_t)+S_t^\gamma}\boldsymbol{\sigma}_c+\sum_{j=1}^n\frac{\left\{\alpha_q^j+S_t^\gamma\left(K_p\phi\overline{\mathbf{s}}\boldsymbol{\alpha}_{pq}\alpha_p^j+\alpha_{pq}^j\right)\right\}s_t^j}{\sum_{k=1}^n\left\{\alpha_q^k+S_t^\gamma\times\left(K_p\phi\overline{\mathbf{s}}\boldsymbol{\alpha}_{pq}\alpha_p^k+\alpha_{pq}^k\right)\right\}s_t^k}\left(\boldsymbol{\nu}^j-\mathbf{s}'\cdot\boldsymbol{\nu}\right) \\
&= \frac{S_t^\gamma\alpha\left(1-\lambda S_t^\gamma\right)}{f_1^{TW}(\mathbf{s}_t)+S_t^\gamma}\boldsymbol{\sigma}_c+\sum_{j=1}^nw_{jt}^{TW}\boldsymbol{\sigma}_D(\mathbf{s}_t)
\end{aligned}$$

with

$$f_1^{TW}(\mathbf{s}_t)=\frac{\alpha_0^{TW}(\mathbf{s}_t)}{\alpha_1^{TW}(\mathbf{s}_t)}$$

and where

$$w_{jt}^{TW} = \frac{\left\{\alpha_q^j + S_t^\gamma \left(K_p \phi \bar{\mathbf{s}} \boldsymbol{\alpha}_{pq} \alpha_p^j + \alpha_{pq}^j\right)\right\} s_t^j}{\sum_{k=1}^n \left\{\alpha_q^k + S_t^\gamma \times \left(K_p \phi \bar{\mathbf{s}} \boldsymbol{\alpha}_{pq} \alpha_p^k + \alpha_{pq}^k\right)\right\} s_t^k}$$

are weights such that $\sum_j w_{jt}^{TW} = 1$. Given the form of the stochastic discount factor, we obtain

$$\begin{aligned} E_t\left[dR_t^{TW}\right] &= -Cov_t\left(dR_t^{TW}, \frac{dm_t}{m_t}\right) \\ &= \left(\gamma + \alpha\left(1 - \lambda S^\gamma\right)\right)\left\{\frac{S_t^\gamma \alpha\left(1 - \lambda S_t^\gamma\right)}{f_1^{TW}\left(\mathbf{s}_t\right) + S_t^\gamma} \boldsymbol{\sigma}_c + \sum_{j=1}^n w_{jt}^{TW} \sigma_{CF,t}^{jc}\right\} \end{aligned}$$

Q.E.D.

Table I
Basic moments in empirical data: 1948-2001

Panel A: Summary statistics for the market portfolio

$\overline{R}^M$	$\text{vol}(R^M)$	$\overline{r}^f$	$\text{vol}(r^f)$
7.71%	16.25%	1.44%	3.08%

Panel B: Predictability regressions

	Panel B-1: Sample 1948-2001				Panel B-2: Sample 1948-1995			
Horizon	4	8	12	16	4	8	12	16
$\ln\left(\frac{D}{P}\right)$	.13	.2	.26	.35	.28	.48	.63	.78
t−stat.	(2.13)	(1.65)	(1.34)	(1.29)	(4.04)	(4.00)	(4.49)	(5.41)
R^2	.09	.10	.11	.14	.19	.32	.43	.54

Panel C: The value premium

	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
$\overline{R}$ (%)	6.86	7.77	7.67	7.63	8.53	9.96	8.39	11.00	11.39	12.36
$\overline{ME/BE}$	5.05	2.68	2.00	1.63	1.38	1.18	1.01	.86	.70	.45
$\overline{P/D}$	43.47	31.38	26.87	24.65	22.65	21.62	20.64	19.95	20.00	21.77
Sharpe Ratio	.352	.450	.452	.461	.555	.640	.522	.657	.644	.600
CAPM β	1.13	1.02	1.01	.95	.88	.89	.88	.91	.92	.98

Notes to Table I. *Panel A*: Summary statistics for the market portfolio. $\overline{R}^M$ is the annualized average excess returns of the market portfolio over the three month Treasury Bill. $\text{vol}(R^M)$ is the annualized standard deviation of the returns on the market portfolio. $\overline{r}^f$ is the average risk free rate, as measured by three-month Treasury Bill rate, and $\text{vol}(r^f)$ is its annualized standard deviation. *Panel B*: Predictability quarterly regressions of excess returns at the 1, 2, 3, and 4 year horizon on the log of the price dividend ratio of the market portfolio. t−stat denotes the Newey-West t− statistic where the number of lags is the double of the forecasting horizon. *Panel C*: $\overline{R}$ is the annualized average excess returns of each of the decile portfolios, $\overline{ME/BE}$ is the average market-to-book and $\overline{P/D}$ the average price dividend ratio. CAPM β is obtined by running time series regressions of excess return on each of the ten decile portfolios sorted on ME/BE on the market excess return, where ME is the market equity and BE is the book value. Quarterly dividends, returns, market equity and other financial series are obtained from the CRSP-COMPUSTAT database. The sample period is 1948-2001. The construction of the BE/ME sorted portfolios follows the standard procedure of Fama and French (1992): Each year t portfolios are sorted into 10 BE/ME sorted portfolios using book-to-market ratios for year $t-1$. Returns on each of these portfolios are calculated from July of year t to June of year $t+1$.

Table II
Model parameters used in the simulation

Panel A: Consumption and preference parameters

μ_c	σ_c	γ	ρ	$\gamma/\overline{S}$	$\min\{\gamma/S_t\}$	α	k
.02	.015	1.5	.072	48	27.75	77	.13

Panel B: Share process parameter in the base line model

n	$\overline{\theta}_{CF}$	$\overline{s}^i$	ϕ	$\overline{\nu}$
200	.00345	.005	.07	0.55

Notes to Table II. *Panel A*: μ_c is the annual average growth rate of the consumption process, σ_c is the standard deviation of consumption growth, γ is the coefficient controlling the local curvature of the utility function, ρ is the subjective discount rate, $\overline{G}$, λ, α and k are the parameters controlling the dynamics of the process $G_t = S_t^{-\gamma}$, where $S_t = (C_t - X_t)C_t^{-1}$ is the surplus consumption ratio and the process for G_t is given by

$$dG_t = \left[k\left(\overline{G} - G_t\right) - \alpha\left(G_t - \lambda\right)\mu_{c,1}\left(\mathbf{s}_t\right)\right]dt - \alpha\left(G_t - \lambda\right)\sigma_c dB_t^1. \tag{37}$$

Panel B: The share process for $i = 1, 2\cdots, n$ is

$$ds_t^i = \phi\left(\overline{s}^i - s_t^i\right) + s_t^i\boldsymbol{\sigma}^i(\mathbf{s}_t)d\mathbf{B'}_t$$

$n = 200$ is the number of assets in our artificial economy. θ_{CF}^i is the parameter controlling the cash-flow risk. Each assets is assigned a value of θ_{CF}^i, which are distributed uniformly in the range above. $\overline{s}^i$ is the fraction that each assets constributes to consumption in the steady state and ϕ is the speed of mean reversion of the share process. Finally, $\boldsymbol{\sigma}^i(\mathbf{s}_t) = \boldsymbol{\nu}^i - \mathbf{s}_t'\boldsymbol{\nu}$ where $\boldsymbol{\nu}^i$ are vectors with $\nu_{i,0} = \theta_{CF}^i/\sigma_c$, $\nu_{i,i} = \sqrt{\overline{\nu}^2 - \nu_{0,i}^2}$, and the remaining entries equal to zero. The simulation consists of 10,000 years of daily data.

Table III
Basic moments in simulated data

Panel A: Summary statistics for the aggregate portfolio

$\overline{R}^M$	$\text{vol}(R^M)$	$\overline{r}^f$	$\text{vol}(r^f)$
4.35%	13.03%	.69%	4.36%

Panel B: Predictability regressions

Horizon	4	8	12	16
$\ln\left(\frac{D}{P}\right)$	.25	.38	.43	.47
t−stat.	(29.11)	(34.68)	(37.58)	(39.46)
R^2 (%)	5.74	7.82	7.57	7.06

Panel C: The value premium

	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
$\overline{R}$ (%)	3.07	3.58	4.37	4.77	5.27	5.45	5.84	6.00	6.43	8.23
$\overline{ln\,(P/D)}$	6.38	5.07	4.613	4.35	4.12	3.90	3.68	3.44	3.15	2.68
Avge(θ^i_{CF}) × 100	−.2858	−.1589	−.0665	−.0083	.0295	.0568	.0787	.0958	.1128	.1431
Sharpe Ratio	.260	.271	.307	.313	.331	.328	.336	.330	.334	.366
CAPM β	.84	.91	.98	1.05	1.10	1.13	1.16	1.20	1.22	1.26
CAPM fitt. ret. (%)	3.67	3.94	4.28	4.55	4.78	4.91	5.05	5.21	5.29	5.50

Notes to Table III. *Panel A*: Summary statistics for the market portfolio. $\overline{R}^M$ is the annualized average excess returns of the market portfolio over the three month Treasury Bill. $\text{vol}(R^M)$ is the annualized standard deviation of the returns on the market portfolio. $\overline{r}^f$ is the average risk free rate and $\text{vol}(r^f)$ is its annualized standard deviation. *Panel B*: Predictability quarterly regressions of excess returns at the 1, 2, 3, and 4 year horizon on the log of the price dividend ratio of the market portfolio. t−stat denotes the Newey-West $t-$statistic where the number of lags is the double of the forecasting horizon. *Panel C*: Annualized average returns $\overline{R}$, average log price-dividend ratio, $\overline{ln\,(P/D)}$, and CAPM β. CAPM fitted returns are the returns resulting from multiplying the CAPM betas from the previous line by the average excess return of the market portfolio reported in Panel A. Avge(θ^i_{CF}) × 100 refers to the average θ^i_{CF} (multiplied by 100) for the assets in the corresponding decile portfolio.

Table IV
The dynamics of the value premium

Panel A: Annualized average excess returns (%) in empirical data

Market-to-book of market portfolio < $\overline{c}$					Market-to-book of market portfolio > $\overline{c}$				
$\overline{c}$	1	10	10-1	$\overline{R}^M$	$\overline{c}$	1	10	10-1	$\overline{R}^M$
15%	13.18	23.57	10.38	15.40	15%	5.73	10.35	4.62	6.34
20%	10.57	21.70	11.14	13.41	20%	5.95	10.06	4.11	6.31
25%	5.51	19.16	13.64	9.89	25%	7.31	10.11	2.80	6.99
30%	6.97	19.49	12.51	10.50	30%	6.82	9.32	2.50	6.62
35%	8.19	18.65	10.45	11.14	35%	6.15	8.98	2.83	5.87

Panel B: Annualized average excess returns (%) in simulated data

Price-dividend of market portfolio < $\overline{c}$					Price-dividend of market portfolio > $\overline{c}$				
$\overline{c}$	1	10	10-1	$\overline{R}^M$	$\overline{c}$	1	10	10-1	$\overline{R}^M$
15%	7.37	18.27	10.90	10.43	15%	2.30	6.46	4.15	3.27
20%	6.56	16.07	9.51	9.22	20%	2.19	6.26	4.07	3.13
25%	5.96	14.60	8.64	8.36	25%	2.10	6.10	4.00	3.01
30%	5.50	13.46	7.96	7.67	30%	2.02	5.98	3.96	2.92
35%	5.13	12.60	7.47	7.18	35%	1.95	5.87	3.92	2.82

Notes to Table IV. *Panel A*: Annualized average excess returns in empirical data of the growth (portfolio 1) and value (portfolio 10) portfolios depending on whether the market-to-book of the market portfolio is below or above the $\overline{c}$ percentile of its empirical distribution. *Panel B*: Annualized average excess returns in simulated data of the growth (portfolio 1) and value (portfolio 10) portfolios depending on whether the simulated price-dividend ratio of the market portfolio is below or above the $\overline{c}$ percentile of its distribution in simulated data. $\overline{R}^M$ is the average excess return on the market portfolio in empirical data (Panel A) and simulated data (Panel B).

Table V
Asset pricing models: Time series regressions (quarterly)

Panel A: Time series regression $R_t^p = \alpha + \beta^M R_t^M + \epsilon_t^p$ for $p = 1, 2, \cdots, 10$

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Panel A-2: Empirical data

Portf.	Growth 1	2	3	4	5	6	7	8	9	Value 10
α	−.46	−.03	−.02	.07	.44	.78	.40	.99	1.07	1.20
$t(\alpha)$	(−2.00)	(−.18)	(−.14)	(.32)	(2.07)	(3.73)	(1.51)	(3.73)	(3.32)	(2.65)
β^M	1.13	1.02	1.01	.95	.88	.89	.88	.91	.92	.98
$t(\beta^M)$	(39.80)	(43.68)	(42.56)	(30.32)	(27.24)	(27.27)	(21.38)	(21.33)	(17.56)	(14.16)

Panel A-2: Simulated data

Portf.	Growth 1	2	3	4	5	6	7	8	9	Value 10
α	−.15	−.09	.02	.06	.12	.13	.20	.20	.29	.68
$t(\alpha)$	(−14.25)	(−5.95)	(1.52)	(3.27)	(6.99)	(6.87)	(9.12)	(8.35)	(10.32)	(17.56)
β^M	.84	.91	.98	1.05	1.10	1.13	1.16	1.20	1.22	1.26

Panel B: Time series regression $R_t^p = \alpha + \beta^M R_t^M + \beta^{HML} R_t^{HML} + \epsilon_t^p$ for $p = 1, 2, \cdots, 10$

Panel B-1: Empirical data

Portf.	Growth 1	2	3	4	5	6	7	8	9	Value 10
α	.20	.17	.02	−.12	.19	.28	−.40	.01	−.08	−.36
$t(\alpha)$	(1.13)	(1.05)	(.14)	(−.61)	(.87)	(1.58)	(−2.15)	(.09)	(−.43)	(−1.23)
β^M	1.04	.99	1.00	.98	.91	.96	.99	1.05	1.09	1.20
$t(\beta^M)$	(43.68)	(51.25)	(46.13)	(35.28)	(30.25)	(38.66)	(39.90)	(48.04)	(39.61)	(29.85)
β^{HML}	−.42	−.12	−.03	.12	.16	.31	.50	.61	.72	.97
$t(\beta^{HML})$	(−12.13)	(−2.37)	(−.68)	(1.88)	(3.62)	(8.85)	(10.35)	(15.52)	(21.04)	(14.14)

Panel B-2: Simulated data

Portf.	Growth 1	2	3	4	5	6	7	8	9	Value 10
α	−.01	.02	.07	.06	.09	.10	.11	.03	.07	.13
$t(\alpha)$	(−1.15)	(1.24)	(4.50)	(3.44)	(5.26)	(4.85)	(5.38)	(1.57)	(2.97)	(5.38)
β^M	.93	.97	1.01	1.05	1.08	1.11	1.11	1.10	1.09	.93
β^{HML}	−.28	−.21	−.09	−.01	.06	.08	.16	.31	.41	1.07

Notes to Table V. *Panel A*: Time series regressions in empirical (Panel A-1) and simulated (Panel A-2) data of returns on each of the book-to-market sorted portfolios on the market excess return. Simulation parameters are contained in Table II. α denotes the intercept of the time series regression and β^M the regression coefficient. $t(\alpha)$ and $t(\beta^M)$ denote the heteroskedasticity corrected $t-$statistic. *Panel B*: Time series regressions in empirical (Panel B-1) and simulated (Panel B-2) data of returns on each of the book-to-market sorted portfolios on the market excess return and the returns on HML, where β^{HML} is the regression coeffcient on HML. The $t-$statistics in simulated data have been omitted as they are all well above 100 for the case of the regression coefficients, β^M and β^{HML}.

Table VI
Asset pricing models: Fama-MacBeth regressions (quarterly)

Panel A: Empirical data

	Const.	Mkt.	SMB	HML	Mkt×log(D/P)	Mkt×*cay*	Adj. R^2
1.	4.69	−2.52					11%
	(3.21)	(−1.65)					
2.	.36	1.63	−.31	1.05			80%
	(.23)	(.99)	(−.31)	(2.16)			
3.	2.72	−.87			1.71		83%
	(2.24)	(−.65)			(2.46)		
4.	3.06	−1.37				.06	81%
	(2.48)	(−1.01)				(2.34)	

Panel B: Simulated data

	Const.	Mkt.	HML	Mkt×log(D/P)	Adj. R^2
5.	−1.45	2.56			91%
	(−19.93)	(32.45)			
6.	−.17	1.31	.94		99%
	(−1.64)	(11.85)	(28.69)		
7.	.63	.38		1.16	98%
	(3.56)	(2.00)		(10.11)	

Notes to Table VI. *Panel A*: Fama-MacBeth regressions in empirical data. Line 1, CAPM regressions where Mkt. represents the average excess return of the market portfolio. Line 2, Fama and French (1993) model, where SMB is the return on "small minus big" and HML is the return on "high minus low". Line 3, conditional CAPM regression where the dividend yield, log(D/P), of the market portfolio is used as a conditioning variable. Line 4 conditional CAPM regression where the variable *cay* of Lettau and Ludvigson (2001) is used as a conditioning variable. *Panel B*: Fama-MacBeth regressions in simulated data. $t-$statistic in parenthesis and Adj. R^2 is the adjusted R^2.

Table VII
Cash-flow betas: Cohen, Polk, and Vuolteenaho (2003)

Cash-flow definition	Growth 1	2	3	4	5	6	7	8	9	Value 10
$\sum_{j=0}^{4}\rho^j ROE^p_{t+j,j+1}$	.72	.91	.94	.95	.96	.97	.98	1.12	1.28	1.51
std. err.	(.52)	(.31)	(.12)	(.25)	(.14)	(.12)	(.14)	(.25)	(.32)	(.29)
$\sum_{j=0}^{4}\rho^j \frac{X^p_{t+j,j+1}}{ME^p_{t+j-1,j}}$	.35	.65	.92	1.17	1.26	1.63	1.93	2.97	4.15	11.26
std. err.	(.31)	(.31)	(.17)	(.16)	(.28)	(.69)	(1.01)	(2.26)	(3.26)	(10.76)
$\frac{X^p_{t+4,j+4}-X^p_{t-1,0}}{ME^p_{t-1,0}}$	.21	.66	1.46	1.61	.24	1.83	2.74	5.50	2.38	2.64
std. err.	(.19)	(.08)	(.52)	(.28)	(.61)	(.60)	(1.24)	(2.69)	(.60)	(1.65)
$\sum_{j=0}^{4}\rho^j \Delta d^p_{t+j,j+1}$	.79	.90	.96	1.03	1.34	1.44	1.14	1.44	1.39	1.28
std. err.	(.19)	(.13)	(.10)	(.13)	(.28)	(.46)	(.31)	(.88)	(.77)	(.91)

Notes to Table VII. This table reports the results of Cohen, Polk, and Vuolteenaho (2003, Table II Panel B) for the following regressions with annual data for each of the ten decile portfolios sorted on market-to-book,

$$\begin{aligned}
\sum_{j=0}^{4}\rho^j ROE^p_{t+j,j+1} &= \beta^p_{CF,0} + \beta^p_{CF,1}\sum_{j=0}^{4}\rho^j ROE^M_{t+j} + \epsilon^p_4 \\
\sum_{j=0}^{4}\rho^j \frac{X^p_{t+j,j+1}}{ME^p_{t+j-1,j}} &= \beta^p_{CF,0} + \beta^p_{CF,1}\sum_{j=0}^{4}\rho^j \frac{X^M_{t+j,j+1}}{ME^M_{t+j-1}} + \epsilon^p_4 \\
\frac{X^p_{t+4,j+4} - X^p_{t-1,0}}{ME^p_{t-1,0}} &= \beta^p_{CF,0} + \beta^p_{CF,1}\left(\frac{X^M_{t+4} - X^M_{t-1}}{ME^M_{t-1}}\right) + \epsilon^p_4 \\
\sum_{j=0}^{4}\rho^j \Delta d^p_{t+j,j+1} &= \beta^p_{CF,0} + \beta^p_{CF,1}\sum_{j=0}^{4}\rho^j \Delta d^M_{t+j} + \epsilon^p_4.
\end{aligned} \tag{38}$$

ROE denotes the ratio of clean surplus earning ($X_t = BE_t - BE_{t-1} + D_t$ where BE_{t-1} is the beginning of the period book equity and D_t are the dividends from CRSP) to BE_{t-1}. ME_{t-1} denotes the market value at the beginning of the period and $\Delta d^p_{t+j,j+1}$ is the log of dividend growth of decile portfolio p. The first subscript refers to the year of observation and the second to the number of years after the portfolio formation in the sorting procedure. Similar quantities are defined for the market portfolio. GMM standard errors computed using the Newey-West formula with four lags and leads are reported in parenthesis. ρ is a constant, linked to one minus the dividend yield, set at .95.

Table VIII
Cash-flow risk, the market portfolio and the value premium

Cash-flow risk		Market portfolio				Predictability				Value premium	
$\overline{\nu}$	$\overline{\theta}_{CF} \times 100$	$\overline{R}^M$	vol(R^M)	$\overline{r}^f$	vol(r^f)	b_{12}	R^2_{12}	b_{16}	R^2_{16}	$10-1$	CAPM $10-1$
.25	.0	9.90	24.16	1.16	5.44	.76	23.1	.78	22.4	−2.44	−2.53
	.1	9.69	23.62	1.12	5.32	.75	22.2	.77	21.6	−1.27	−1.42
	.2	8.95	21.79	1.00	4.97	.72	18.9	.74	18.1	2.40	2.34
	.3	7.02	17.22	.79	4.46	.58	9.6	.58	8.2	7.51	7.45
	.345	3.97	10.23	.67	4.20	.38	4.4	.43	4.1	7.10	6.70
.40	.0	9.90	24.16	1.16	5.44	.76	23.1	.78	22.4	−3.22	−3.37
	.1	9.69	23.63	1.12	5.33	.75	22.1	.77	21.4	−2.56	−2.77
	.2	8.96	21.83	1.00	4.99	.71	18.6	.73	17.9	−.07	−.25
	.3	7.06	17.37	.80	4.49	.58	9.8	.59	8.6	4.91	4.62
	.345	4.09	11.14	.68	4.23	.46	7.0	.51	6.5	6.29	4.57
.55	.0	9.90	24.16	1.16	5.44	.76	23.1	.78	22.4	−3.67	−3.86
	.1	9.70	23.66	1.13	5.34	.74	21.9	.76	21.2	−3.27	−3.48
	.2	8.99	21.95	1.01	5.05	.70	18.2	.72	17.3	−1.49	−1.70
	.3	7.15	17.85	.81	4.60	.58	10.1	.59	9.0	2.83	2.19
	.345	4.35	13.03	.69	4.36	.43	7.6	.47	7.1	5.16	1.83

Notes to Table VIII. This table reports basic moments of the returns for three different values of $\overline{\nu}$, which determines the maximum volatility of share process across assets, and the measure of cash-flow risk, $\overline{\theta}_{CF} \geq 0$, which determines the support on which the cash-flow risk parameters of individual firms are uniformly distributed, $\theta^i_{CF} \in [-\overline{\theta}_{CF}, \overline{\theta}_{CF}]$. $\overline{R}^M$ is the annualized average excess returns of the market portfolio over the three month Treasury Bill. vol(R^M) is the annualized standard deviation of the returns on the market portfolio. $\overline{r}^f$ is the average risk free rate and vol(r^f) is its annualized standard deviation. All these numbers are in percentages. b_{12} and b_{16} are the regressions coefficients of the quarterly predictability regressions of excess returns on the log of the price dividend ratio of the market portfolio for the three and four year horizons. R^2_{12} and R^2_{16} are the corresponding R^2s. The t−stats are omitted but they are all well above standard significance levels. 10-1 denotes the value premium, in percentages, defined as the difference between the average return on the value portfolio, portfolio 10, and the growth portfolio, portfolio 1. CAPM 10-1 is the fitted CAPM value premium, where the betas are calculated the standard way in simulated data and the market premium is the corresponding $\overline{R}^M$ in each line.

Table IX
The properties of the cash-flow process

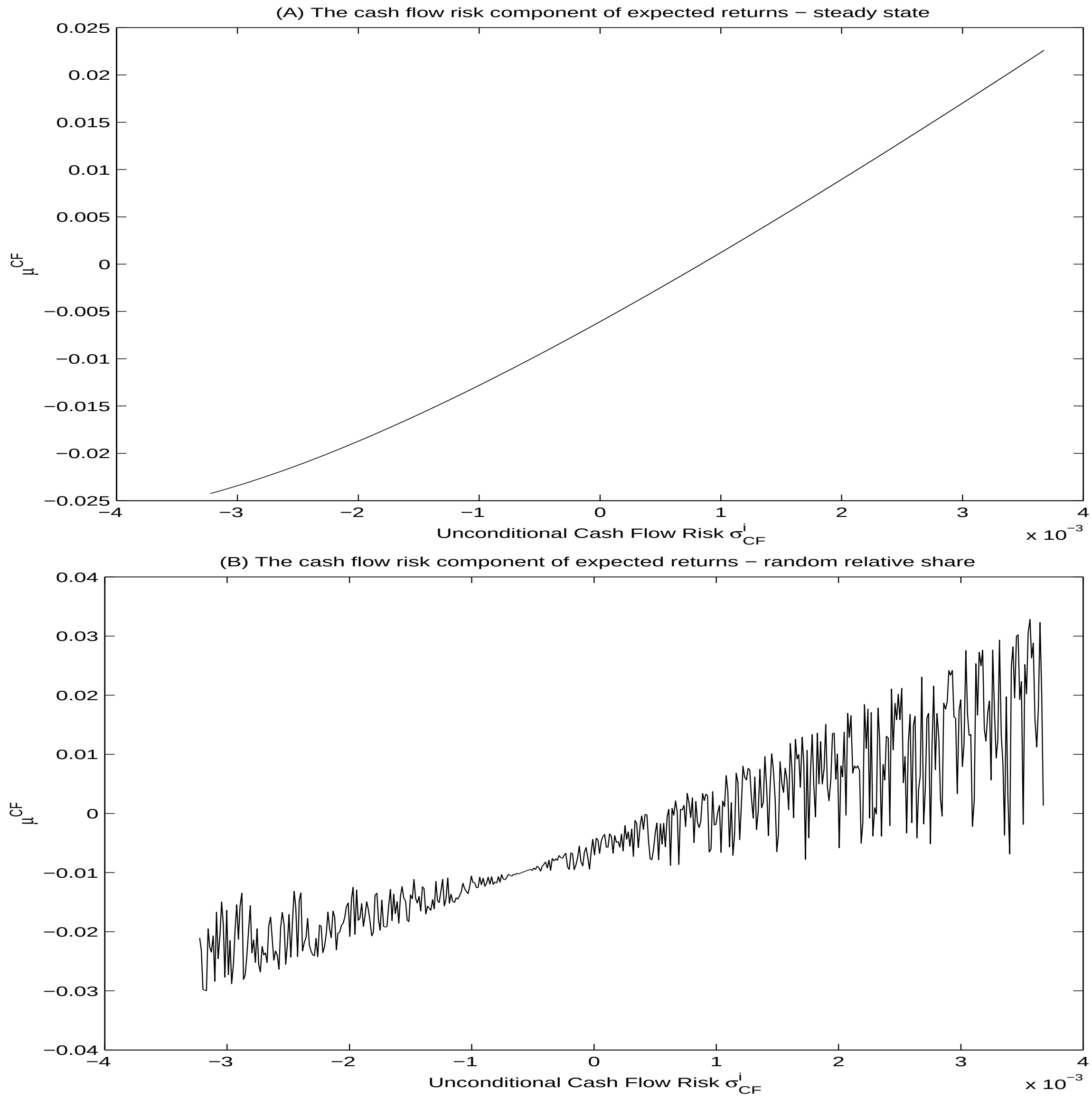
$\overline{\nu}$	$\overline{\theta}_{CF} \times 100$	$[\underline{\rho}, \overline{\rho}]$	Avge(σ_D^i)	$\beta_{CF,1}^1$	$\beta_{CF,1}^{10}$	Avge(σ_R^i)
.25	0	$[.04, .07]$	24.88	1.04	.96	27.67
	.1	$[-.21, .32]$	24.29	.04	1.89	27.33
	.2	$[-.48, .57]$	22.44	−3.30	4.15	26.11
	.3	$[-.76, .81]$	18.92	−8.14	6.70	22.88
	.345	$[-.89, .91]$	16.39	−9.62	7.94	16.68
.40	0	$[.02, .05]$	40.04	1.09	.96	31.33
	.1	$[-.13, .20]$	39.65	.43	1.49	31.02
	.2	$[-.29, .36]$	38.50	−1.80	3.10	29.88
	.3	$[-.46, .52]$	36.55	−6.37	5.22	26.66
	.345	$[-.53, .59]$	35.40	−8.63	5.73	19.83
.55	0	$[.01, .04]$	56.20	1.17	.99	34.86
	.1	$[-.10, .15]$	55.87	.69	1.28	34.55
	.2	$[-.21, .26]$	54.96	−1.01	2.40	33.41
	.3	$[-.32, .37]$	53.47	−4.79	4.28	30.10
	.345	$[-.37, .42]$	52.60	−7.40	4.73	22.96

Notes to Table IX. For each value of $\overline{\nu}$ and $\overline{\theta}_{CF}$ the table reports several moments of the cash-flow process in simulated data. $[\underline{\rho}, \overline{\rho}]$ stands for the range of the correlation coefficients between individual dividend growth and consumption growth; Avge(σ_D^i) stands for the average standard deviation of dividend growth across the 200 individual assets in percentages. $\beta_{CF,1}^1$ and $\beta_{CF,1}^{10}$ correspond to the regression coefficients of the time series regression in simulated data

$$\sum_{j=0}^{4} \rho^j \Delta d_{t+j,j+1}^p = \beta_{CF,0}^p + \beta_{CF,1}^p \sum_{j=0}^{4} \rho^j \Delta d_{t+j}^M + \epsilon_4^p \quad \text{for} \quad p = 1, 10$$

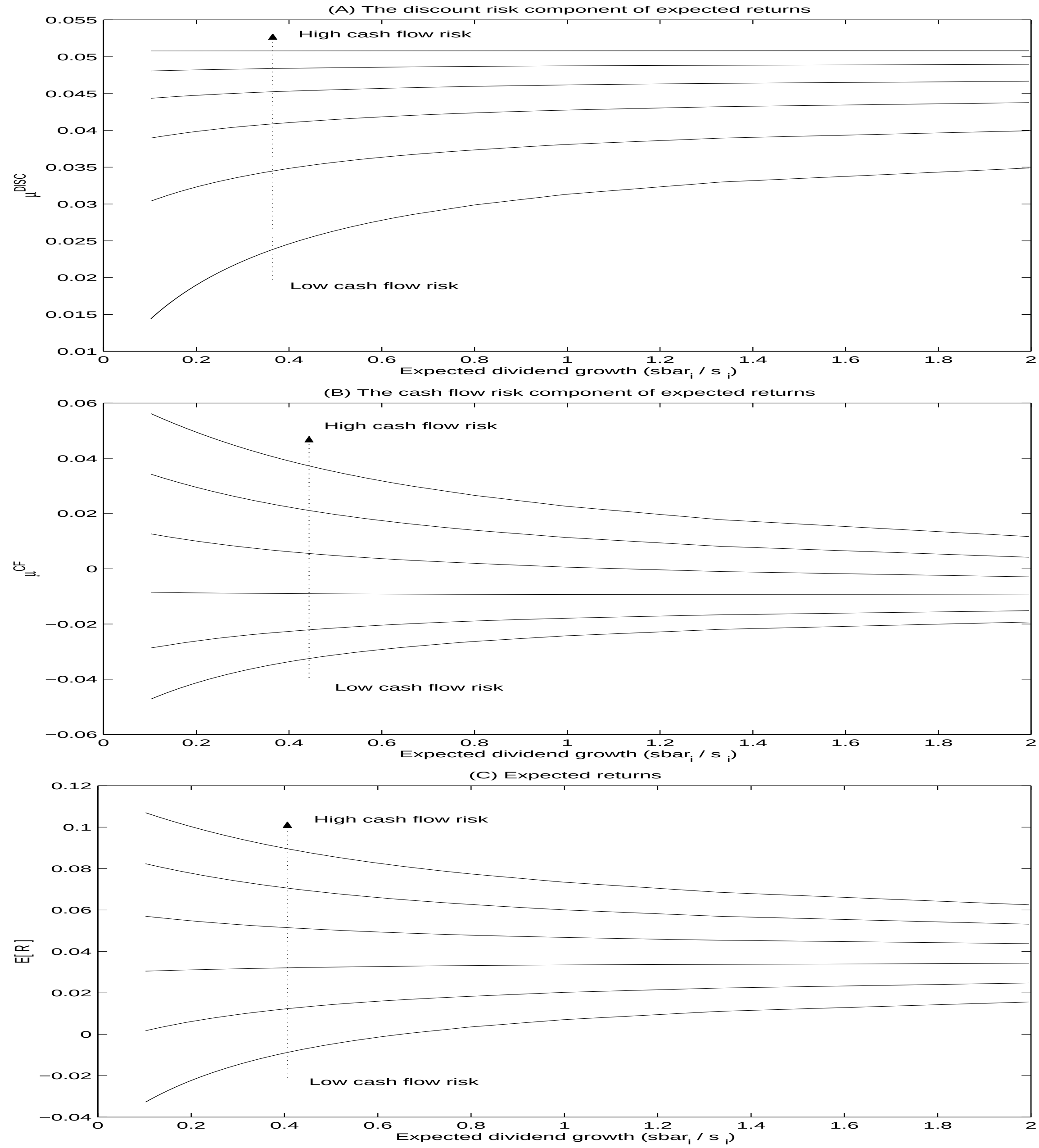
for the Growth ($p = 1$) and Value ($p = 10$) portfolios and should be compared to the coefficients in the corresponding regression run by Cohen, Polk, and Vuolteenaho (2003) in empirical data (see equation (38) in the Notes to Table V.) Avge(σ_R^i) stands for the average standard deviation of returns across the 200 individual assets in percentages.

Figure 1: The Cash Flow Component of Expected Return



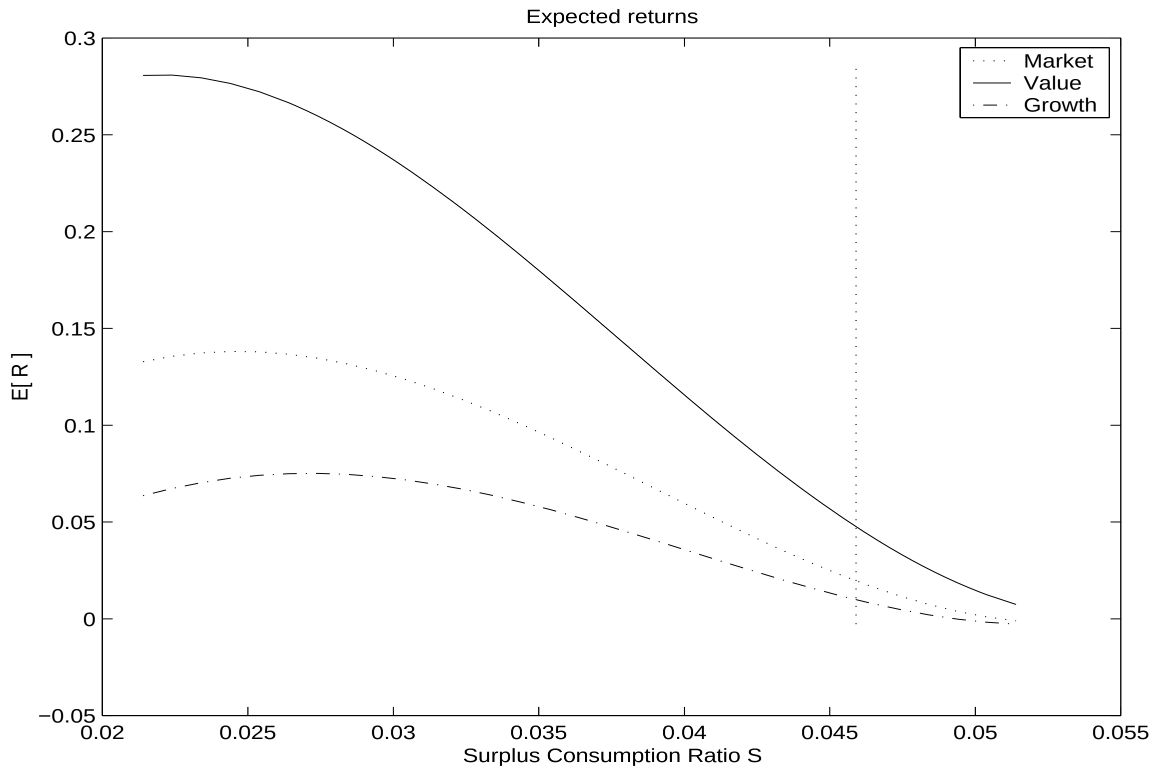
The top panel plots the steady-state cash flow component of individual assets' expected return against the unconditional cash flow risk parameter $\sigma^i_{CF} = E\left[cov\left(dD^i/D^i, dC/C\right)\right]$. For each asset, relative share is assumed equal to one, $\overline{s}^i/s^i_t = 1$, and suplus consumption ratio is assumed equal to its steady state value $S_t = \overline{S}$. The bottom panel reports the same quantities, but under a random selecetion for relative shares $\overline{s}^i/s^i_t$.

Figure 2: Expected Returns and Expected Dividend Growth



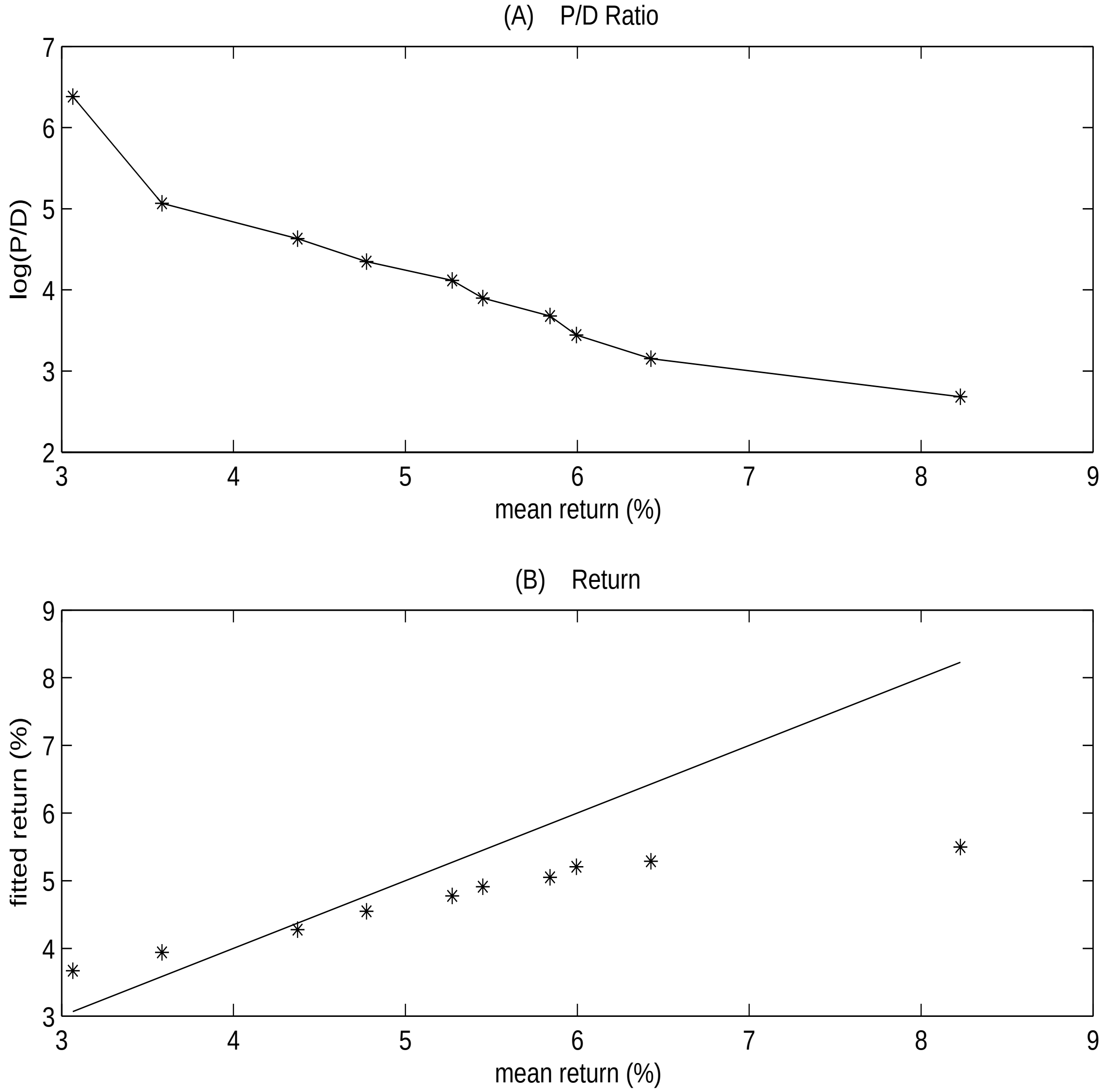
The top panel plots the theoretical discount component of individual stock returns plotted against the relative share $\overline{s}^i/s_t^i$, which proxies for expected dividend growth. This quantity is computed for various levels of the asset unconditional cash flow risk $\sigma_{CF}^i = E\left[cov\left(dD^i/D^i, dC/C\right)\right]$. The middle panel plots the cash flow risk component of stock returns, plotted against the relative share $\overline{s}^i/s_t^i$, again for various levels of unconditional cash flow risk. The bottom panel reports the total conditional expected return for individual assets.

Figure 3: Expected Returns and Surplus Consumption Ratio



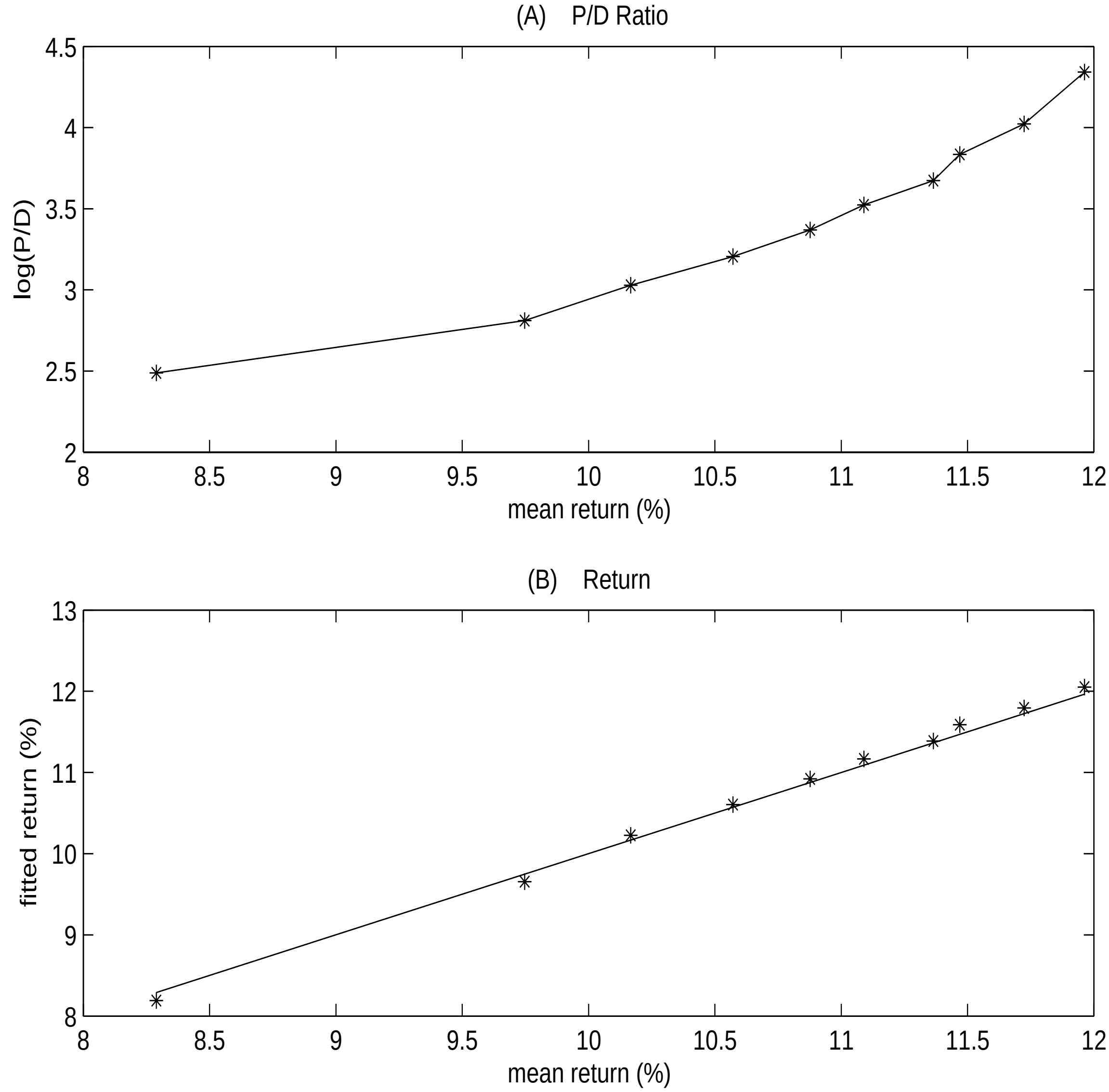
This figure shows the theoretical expected return for the market portfolio (dotted line), a representative value stock (solid line), and a representative growth stock (dash dotted line), plotted against values of the surplus consumption ration S_t. The vertical dotted line is the median value of the surplus consumption ratio S_t. The representative value (growth) stock is chosen with low (high) expected dividend growth and high (low) cash flow risk θ_{CF}.

Figure 4: The Cross-Section of Stock Returns in Simulated Data



The top panel plots the average log price-dividend ratio (y-axis) of P/D sorted portfolios versus their unconditional average return (x-axis) in artificial data, under the assumption that assets differ cross-sectionally in their cash flow risk parameter θ^i_{CF}. Under the same assumptions, the bottom left panel plots the "fitted" average return according to the CAPM, i.e. $E[\text{ Return}^i\ ] = \beta^i_{CAPM}\ E[\text{ Return}^{mkt}\ ]$, on the y-axis against the average return on the x-axis.

Figure 5: The Cross-Section of Stock Returns with only Discount Risk Effects



The top panel plots the average log price-dividend ratio (y-axis) of P/D sorted portfolios versus their unconditional average return (x-axis) in artificial data, under the assumption that assets have no cross-sectional differences in cash flow risk, $\sigma^i_{CF} = \sigma^j_{CF} = \sigma^2_c$. Under the same assumptions, the bottom panel plots the "fitted" average return according to the CAPM, i.e. $E[\text{ Return}^i\ ] = \beta^i_{CAPM}\ E[\text{ Return}^{mkt}\ ]$, on the y-axis against the average return on the x-axis.