NBER WORKING PAPER SERIES

OPTIMAL STOCK TRADING WITH PERSONAL TAXES: IMPLICATIONS FOR PRICES AND THE ABNORMAL JANUARY RETURNS

George M. Constantinides

Working Paper No. 1176

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

August 1983

This paper is a substantially revised version of my 1980 paper titled "Short Term versus Long Term Capital Gains and Losses." I wish to thank my colleagues at the University of Chicago and workshop participants at the 1982 Summer Institute in Boston, the University of Pennsylvania, Cornell University, Northwestern University, Columbia University, and the University of Texas at Dallas. In particular, I wish to thank Eugene Fama, Philippe Jorion, Allan Kleidon, Merton Miller, Richard Roll and an anonymous referee. Aid from the Center for Research in Security Prices of the University of Chicago is gratefully acknowledged. The research reported here is part of the NBER's research program in Financial Markets and Monetary Economics and program in Taxation. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #1176
August 1983

Optimal Stock Trading with Personal Taxes: Implications For Prices and the Abnormal January Returns

ABSTRACT

The tax law confers upon the investor a timing option--to realize capital losses and defer capital gains. With the tax rate on long term capital gains and losses being about half the short term rate, the tax law provides a second timing option--to realize capital losses short term and realize capital gains long term, if at all. Our theory and simulation with actual stock prices over the 1962-1977 period establish that the second timing option is extremely valuable: Taxable investors should realize their long term capital gains in high variance stocks and repurchase the same or similar stock, in order to reestablish the short-term status and realize potential future losses short term.

Tax trading does not explain the positive abnormal returns of small firms. In the presence of transactions costs, tax trading predicts that the volume of tax-loss selling increases from January to December and ceases in the first few days of January. The trading volume seasonal maps into a stock price seasonal only if tax-loss sellers are assumed irrational or ignorant of the price seasonality.

Prof. George M. Constantinides
Graduate School of Business
University of Chicago
1101 East 58th Street
Chicago, IL 60637

1. INTRODUCTION

Capital gains and losses are taxed when the investor sells the stock--not when gains and losses actually occur. Suppressing the distinction between the short term tax rate on capital gains and losses and the long term rate, the optimal trading policy is to realize capital losses immediately and defer capital gains, thereby reducing the present value of the stream of tax payments on capital gains net of tax credits on capital losses. Constantinides (1983) formally derives the optimal trading policy, estimates the value of this timing option as a fraction of the stock price, and finds the effective tax rate on capital gains and on dividends. He also explains how the capital gains tax influences the investor's optimal consumption and investment program and derives the equilibrium pricing implications--how the capital gains tax modifies the asset pricing models of Breeden (1979), Brennan (1973) and Merton (1973).

With the tax rate on long term capital gains and losses being about half the rate on short term capital gains and losses, the tax law confers upon the investor a second timing option--to realize losses short term and realize gains long term, if at all. For those investors who can draw the distinction between the two tax rates, the second option is substantially more valuable than the first one. Suppose that the investor bought the stock exactly one year ago. If the stock price has declined, he optimally sells the stock and repurchases it, realizing a short term capital loss immediately. If the stock price has increased instead, the investor optimally defers the realization of a short term capital gain and one day later faces two alternatives. First, he may defer the realization of the long term gain. Second, he may sell the stock and repurchase it, realizing a long term gain and reestablishing the favorable short term status, in order to realize future capital losses at the short term rate.

Our theory, based on the option pricing argument of Black and Scholes (1973), confirms that under broad conditions the investor ought to realize a long term gain in order to reestablish the short term status. These conditions apply to high variance stocks and even to medium variance stocks, if transactions costs are low and the interest rate is low (see, Tables 1 and 2). The results are intuitively appealing. The timing option to realize losses short term and gains long term is more valuable the higher the stock variance, for essentially the same reason that a call option is more valuable the higher the stock variance. Thus the higher the stock variance the stronger the incentive to reestablish the short term status even at the expense of the tax on long term gains. Also, the lower the interest rate the higher the present value of the future tax benefit of this option. Transactions costs inhibit trading but the order of magnitude of the potential tax benefit is large relative to even substantial transactions costs.

We simulate three active tax trading policies and the buy-and-hold policy for a large sample of NYSE- and AMEX-listed stocks over the period 1962-1977. The performance under optimal trading far exceeds the performance under the buy-and-hold policy (see, Table 3 and Figures 1, 2, and 3). The tax trading benefit is large even when we allow for long term gains to offset short term losses dollar for dollar. The results readily translate into an annual tax subsidy which the government provides to taxable investors under various provisions of the tax law.

We relate our findings to the empirical evidence on positive abnormal returns by small firms, and positive abnormal returns in the month of January. We argue that tax trading not only fails to explain, but exacerbates the small firm anomaly, and cast doubts on a proposed explanation based on transactions costs. Regarding the January anomaly, we find that tax-loss selling predicts a seasonal pattern in trading volume, provided transactions costs are present. Tax-loss selling predicts a seasonal pattern in stock prices, only if we further assume irrationality or ignorance of the stock price seasonality on behalf of investors.

The paper is organized as follows: The tax environment is discussed in Section 2. In Section 3 we derive those properties of the optimal trading policy which do not depend on detailed assumptions on the stock price distribution (see, Propositions 1 and 2). In Section 4 we assume that the stock price follows a binomial process and provide conditions under which it is optimal to incur the cost of realizing a long term gain in order to reestablish the tax-advantageous short term status (see, Proposition 3). The conclusions are summarized in Tables 1 and 2 for a range of parameter values. Section 5 reports the simulation of the trading policies. In Section 6 we relate our findings to the empirical evidence on the small firm anomaly and the January anomaly. Concluding remarks are offered in Section 7.

2. THE TAX ENVIRONMENT

Unrealized capital gains and losses are not taxed. Realized capital gains and losses are short term if the asset has been held for one year or less, and long term otherwise.[1/] Net short term capital gains (or losses), X_S, are defined as the total short term capital gains net of total short term capital losses, including unused short term carryovers. Net long term capital gains (or losses), X_L, are defined as the total long term capital gains net of total long term capital losses, including unused long term carryovers. If

---

[1/] From 1942 to 1976 the holding period was six months. In 1977 it was increased to nine months, and in 1978 it was increased to one year.

both X_S and X_L are nonnegative, short term capital gains are taxed at the individual's marginal tax rate on ordinary income; and long term capital gains are taxed at 40% (until October 1979, 50%) of the individual's marginal tax rate on ordinary income.[2,3] If both X_S and X_L are nonpositive, then net short term capital losses and 50% of net long term capital losses are deductible from ordinary income. These deductions may jointly decrease the taxable ordinary income by a maximum of \$3,000 per tax year.[4] Unused capital losses are carried forward indefinitely.

A complication of the tax code is that net short term losses offset net long term gains one to one. Thus, if $X_S < 0 < X_L$ and $X_S + X_L < 0$, the loss $X_S + X_L$ is taxed as short term; if $X_S < 0 < X_L$ and $X_S + X_L > 0$, the gain $X_S + X_L$ is taxed as long term. Likewise, if $X_L < 0 < X_S$ and $X_S + X_L < 0$, the loss $X_S + X_L$ is taxed as long term; if $X_L < 0 < X_S$ and $X_S + X_L > 0$, the gain $X_S + X_L$ is taxed as short term.

If an asset is sold at a loss and repurchased within thirty days, the IRS terms the transaction a "wash sale" and disallows the loss deduction. The investor has a high probability to circumvent this rule by waiting thirty days before repurchasing the same asset. A safer way to circumvent the rule is to repurchase a different asset with the same risk and return characteristics. In any case, the wash sale provision does not apply to dealers or individuals who are in the business of trading stocks.

---

[2] Prior to 1969 the maximum rate on long term capital gains was 25% even to investors in tax brackets above 50%. In 1969-1976 (1976-1979) the marginal tax rate on capital gains above the first \$50,000 was as high as $42\frac{1}{2}$% (49%).

[3] Upon the investor's death the assets' basis is adjusted to market. Effectively realized and unrealized capital gains and losses remain untaxed.

[4] Until 1976 the deduction limit was \$1,000. It was increased to \$2,000 in 1977 and to \$3,000 thereafter.

3. PROPERTIES OF THE OPTIMAL TRADING POLICY

An investor may sell stock to invest the proceeds in relatively underpriced assets, consume, or rebalance his portfolio. We consider those times at which the investor is not motivated by any of the above reasons to sell stock and investigate the conditions under which he would optimally sell stock and repurchase it for tax reasons.[5/] We defer the discussion of transactions costs until Section 4.3.

There are at least three representative tax scenarios. In the first one the investor is a tax-exempt institution or an individual who continually carries forward large capital losses and expects the deduction limit to remain binding for many years. The marginal tax rate on capital gains and losses is zero and the investor pays no attention to the realization of capital gains and losses in pursuing his optimal investment policy.

In the second scenario the deduction limit is not binding and short term and long term capital gains and losses are taxed at the same rate. This scenario is plausible if (1) the individual investor is periodically forced to sell some of his assets by factors beyond his control and, on average, realizes large long term gains; and (2) he can defer the realization of short term capital gains until the holding period exceeds one year and then realize the capital gains long term. Then short term and long term capital losses simply offset some of the long term capital gains. The optimal trading policy is derived in Constantinides (1983) and is restated here for completeness.

PROPOSITION 0: Assume that transactions costs are zero and the tax rate on long term capital gains and losses equals the tax rate on short term capital

[5/] Holt and Shelton (1962) discuss the investor's trade-off between the benefit of switching to relatively underpriced assets and the cost of the capital gains tax.

<u>gains and losses. Then at any time that the investor is not forced to sell the stock, he optimally realizes losses and repurchases the stock; and defers the realization of gains.</u>

In the third scenario we focus on the distinction between the short term and the long term tax status. We assume that short term capital gains and losses are taxed at the rate τ, and long term capital gains and losses are taxed at the lower rate τ_L.[6,7] We examine the optimal trading policy under the simplifying assumption that trading occurs in discrete, one-year intervals. The length of the trading period conveniently coincides with the length of the holding period beyond which short term capital gains and losses become long term. If an asset is sold one year after purchase, the capital gain or loss is short term or long term at the investor's discretion. Obviously the investor delays the sale by at least one day if he has a short term capital gain, but not if he has a short term capital loss. Propositions 1 and 2 state some properties of the optimal trading policy which are free from distributional assumptions on the stock price.

PROPOSITION 1: <u>Assume that trading occurs in one-year intervals only, transactions costs are zero, and the tax rate on long term capital gains and</u>

---

[6] If the investor does not have short term gains, he offsets short term losses against long term gains one to one and, effectively, cannot use to his advantage the distinction between the short term and the long term tax status. This complication is suppressed here and discussed later in Section 5.4.

[7] We suppress the distinction between the tax rate on net long term gains, $\tau_L(\text{net gains}) = .4\tau$, and the tax rate on net long term losses, $\tau_L(\text{net losses}) = .5\tau$. This is justified because long term losses offset some of the long term gains and the marginal tax rate on long term losses equals the marginal tax rate on long term gains.

In any case, Propositions 1, 2, and 3 remain valid if the condition $\tau_L(\text{marginal gain}) = \tau_L(\text{marginal loss}) < \tau$ is replaced by the pair of weaker conditions, $\tau_L(\text{marginal gain}) < \tau$ and $\tau_L(\text{marginal loss}) < \tau$.

losses is less than the tax rate on short term capital gains and losses, i.e., $\tau_L < \tau$. Then, at any time that the investor is not forced to sell the stock, he optimally:

a. Defers the realization of a short term gain.

b. Sells the stock and repurchases it to realize a loss, short term if possible.

c. Sells the stock and repurchases it to change the long term status to short term, whenever the stock price is equal to the basis and the status is long term.

Proof:

a. Realizing the gain long term one year and one day after purchase dominates the policy of realizing the gain short term one year after purchase. This proves (a).

b. Suppose that the stock was purchased longer than one year ago with cost basis P_0, and the stock price now is P_t, $P_t < P_0$. If the investor does nothing now and sells the stock at time T, $T > t$, the after-tax proceeds are $(1 - \tau_L)P_T + \tau_L P_0$.

We show that the active policy of realizing the loss at time t and repurchasing the stock dominates the above policy. In selling the stock at time t the investor receive a tax rebate $\tau_L(P_0 - P_t)$ which he invests in a riskless bond with after-tax annual return R, $R > 1$. The basis of the stock is P_t and the status is short term. At time T he sells the stock and the bond. The after-tax proceeds are at least $(1 - \tau_L)P_T + \tau_L P_t + R^{T-t}\tau_L(P_0 - P_t)$ and exceed the after-tax proceeds of the passive policy by at least

$(R^{T-t} - 1)\tau_L(P_0 - P_t) > 0.$ [8/] If the investor dies at time T and the effective tax rate becomes zero, the proceeds of the optimal policy exceed the proceeds of the passive policy by at least $R^{T-t}\tau_L(P_t - P_0) > 0.$

Next suppose that the stock was purchased just one year ago. By the above argument, realizing a long term loss (by waiting one day before selling the stock) dominates the passive policy. Also realizing the loss short term dominates the policy of realizing it long term. This proves (b).

c. The short term status dominates the long term status. With the short term status, if capital gains are realized in the future, they are realized long term irrespective of the current status; if capital losses are realized one period hence, they are realized short term. Therefore the investor switches to the short term status whenever he can do so costlessly. This proves (c).

An investor holds a share of stock with price P, purchased t periods ago at cost basis $\hat{P}$. We introduce the concept of the value of a stock position, $V(P, \hat{P}, t)$. At the time that the investor purchases the stock, the value of the position is $\hat{P}$, i.e., $V(\hat{P}, \hat{P}, 0) = \hat{P}$. If the stock price falls one year later, the investor optimally realizes a short term loss (see, Proposition 1) and the value of the position is $V(P, \hat{P}, 1) = (1 - \tau_s)P + \tau_s\hat{P}$, $P < \hat{P}$. If the stock price rises t years after purchase, the optimal policy may be to realize the long term gain, in which case $V(P, \hat{P}, t) = (1 - \tau_L)P + \tau_L\hat{P}$, or to defer it, in which case $V(P, \hat{P}, t) > (1 - \tau_L)P + \tau_L\hat{P}$. At those times that $V(P, \hat{P}, t) > (1 - \tau_L)P + \tau_L\hat{P}$, the value of a stock position is not a market price because the investor may not sell the stock and transfer the unrealized capital gain to the buyer.

[8/] We say at least, because if $T = t + 1$ and $P_T < P_t$, the loss realized at time T is short term and $(1 - \tau)P_T + \tau P_t > (1 - \tau_L)P_T + \tau_L P_t$.

We define the value of a stock position, $V(P, \hat{P}, t)$, as the <u>after-tax shadow price</u>, such that the investor is indifferent between having the stock with basis $\hat{P}$ and age t, or having $V(P, \hat{P}, t)$ after-tax dollars.

The function $V(P, \hat{P}, t)$ is convex in $\hat{P}$. It is also homogeneous of degree one in $(P, \hat{P})$, under the additional assumption that the distribution of the stock return, $\tilde{P}_T/P_t$, at time t is independent of the price P_t. These two properties imply that $V(P, \hat{P}, t)$ is convex in P. (The above three statements are proved in Appendix 1.) The latter property leads to a partial characterization of the optimal trading policy when the price exceeds the basis and the stock has been held for at least one year.

PROPOSITION 2: <u>Under the assumptions of Proposition 1, if it is optimal to realize a long term gain when</u> $P = h\hat{P}$, $h > 1$, <u>then it is also optimal to realize a long term gain for all</u> P, $\hat{P} \leq P \leq h\hat{P}$.

Proof: If it is optimal to realize a long term gain when $P = h\hat{P}$, then

$$V(h\hat{P}, \hat{P}, t) = (1 - \tau_L)h\hat{P} + \tau_L\hat{P}, \qquad t \geq 1 . \tag{1}$$

By Proposition 1,

$$V(\hat{P}, \hat{P}, t) = \hat{P}, \qquad t \geq 1 . \tag{2}$$

Since V is convex in its first argument (see, Appendix 1, Lemma 3), equations (1) and (2) imply

$$V(P, \hat{P}, t) \leq (1 - \tau_L)P + \tau_L\hat{P} , \qquad \hat{P} \leq P \leq h\hat{P} , \qquad t \geq 1 . \tag{3}$$

But

$$V(P, \hat{P}, t) \geq (1 - \tau_L)P + \tau_L\hat{P} , \qquad t \geq 1 \tag{4}$$

since the investor has the option to sell the stock and realize a long term gain. Combining equations (3) and (4) we obtain

$$V(P, \hat{P}, t) = (1 - \tau_L)P + \tau_L \hat{P}, \qquad \hat{P} \leq P \leq h\hat{P},\ t \geq 1 \ . \tag{5}$$

This proves the claim and completes the proof.

In the next section we assume that the stock price follows a binomial process and provide conditions under which it is optimal to realize a long term gain. When these conditions are violated, it is optimal to defer all long term (and short term) gains.

4. THE OPTIMAL TRADING POLICY IN A BINOMIAL MODEL OF STOCK PRICES

4.1 Introduction

We assume that the annual stock prices, P_t, P_{t+1}, ..., are generated by a binomial process as follows:

$$P_t \begin{cases} \nearrow \ P_{t+1} = uP_t, & \text{with probability } 1 - q \\ \searrow \ P_{t+1} = u^{-1}P_t, & \text{with probability } q \end{cases}$$

where u is constant over time and is greater than one. No dividends are paid on the stock. Transactions costs are zero. The investor is infinitely lived and he is never forced to realize a capital gain or loss.[9/]

There exists a single-period riskless asset with after-tax interest rate $R - 1$. Irrespective of the tax rates τ, τ_L, the following condition

---

[9/] The conclusion in this section is that the investor should optimally realize a long term capital gain or a short term capital loss in every period on medium and high variance stocks. This conclusion remains valid if we allow for forced realizations, provided that the investor is not forced to realize capital gains short term.

is necessary, if the after-tax cash flows of the risky and the riskless asset do not dominate one another:

$$u^{-1} < R < u \quad (6)$$

In the remainder of the introduction we assume that the tax rate on short term capital gains and losses, τ, equals the tax rate on long term capital gains and losses. We state some known results which we employ in Section 4.2 to characterize the optimal trading policy when there is a distinction between the short term and long term status.

With the two tax rates being equal, the value of a stock position is independent of the time period that the stock has been held, and we write $V(P, \hat{P})$ without the argument t. The optimal policy, as stated in Proposition 0, is to realize losses and defer gains, i.e.,

$$\begin{aligned} V(P, \hat{P}) &= (1 - \tau)P + \tau\hat{P}, & P \leqslant \hat{P} \\ &> (1 - \tau)P + \tau\hat{P}, & P > \hat{P}. \end{aligned} \quad (7)$$

The functional form of $V(P, \hat{P})$ is derived in Constantinides (1983) under the assumption that the logarithm of the stock price follows a Wiener process with drift. A similar derivation (see, Appendix 2) under the assumption that the stock price follows a binomial process, yields the following:

$$V(P, \hat{P}) = \left[1 - \frac{(1 - u^{-1})(u - R)\tau}{uR - 2 + u^{-1}R}\right]P + \frac{(1 - u^{-1})(u - R)\tau}{uR - 2 + u^{-1}R} P^m \hat{P}^{1-m}, \quad P \geqslant u^{-1}\hat{P} \quad (8)$$

where

$$m = [\ell n(u - R) - \ell n(uR - 1)]/\ell nu < 0 . \quad (9)$$

4.2 The Optimal Trading Policy

The following proposition completes the characterization of an optimal policy.

PROPOSITION 3: Assume that trading occurs in one-year intervals only; there are no forced realizations; the stock price is generated by a binomial process with $u^{-1} < R < u$; there are no dividends; transactions costs are zero; and the tax rate on long term capital gains and losses is less than or equal to the tax rate on short term capital gains and losses, i.e., $\tau_L \leq \tau$.

a. If

$$\frac{u - R}{uR - 1} \leq \frac{\tau_L}{\tau}, \tag{10}$$

at any time that $P \geq u\hat{P}$, it is optimal to defer the gain.

b. If

$$\frac{\tau_L}{\tau} \leq \frac{u - R}{uR - 1}, \tag{11}$$

at any time that $P = u\hat{P}$, it is optimal to realize the gain, and realize it long term.[10/]

[10/] Under this policy we leave unspecified the action when $P > u\hat{P}$ because this contingency never arises: Before $P > u\hat{P}$, the price will have equalled $u\hat{P}$ at some earlier period, a long term capital gain will have been realized, and the basis will have been updated.

This argument hinges on our earlier assumption that the stock price can increase by only one step each year. If, instead, the stock price were generated by a trinomial process, $P_{t+1} = u^{-1}P_t$ or uP_t or u^2P_t, then we would have to consider separately three candidate policies:

(a) Defer a gain if $P \geq u\hat{P}$.

(b) Realize a gain if $P = u\hat{P}$ but defer a gain if $P > u^2\hat{P}$.

(c) Realize a gain if $P \geq u\hat{P}$.

Our discussion following Table 1 in this section suggests that our conclusions do not critically depend on the binomial process.

Proof: By Proposition 2, when the stock price exceeds the basis, an optimal policy is either to defer all gains, or to defer short term gains and realize long term gains.

Assume, for the sake of the argument, that the optimal policy is to defer all gains. Since only losses are realized, and are always realized short term, the value of a position, $V(P, \hat{P}, t)$, is independent of the long term rate. In particular, the value of a position is given by equation (8), which was obtained under the assumption that short term and long term gains and losses are taxed at the same rate, τ. Since, by assumption, it is optimal to defer the gain when $P = u\hat{P}$, we impose the condition

$$(12) \qquad V(u\hat{P}, \hat{P}, t) > (1 - \tau_L)u\hat{P} + \tau_L\hat{P}, \qquad t \geq 1.$$

Equations (8), (9) and (12) imply, after tedious manipulations, equation (10). If equation (10) is violated the optimal policy is to realize a long term gain when $P = u\hat{P}$. This completes the proof.

If the short term and long term tax rates are equal, then $\tau_L/\tau = 1 > (u - R)/(uR - 1)$ and it is optimal to defer a long term gain. For sufficiently low long term tax rate relative to the short term rate, it is optimal to realize long term gains. The function $(u - R)/(uR - 1)$ is decreasing in R. For a sufficiently high interest rate condition (10) is satisfied and the optimal policy is to defer a long term gain because the future benefits associated with the short term tax status become less valuable. The function $(u - R)/(uR - 1)$ is increasing in u. For sufficiently high value of u condition (10) is violated and the optimal policy is to realize a long term gain. The higher u is, the larger the variance of the stock return. Thus it is optimal to realize long term gains on high variance stocks but not on low variance stocks.

For the assumed binomial process, the conditional mean of the annual logarithmic stock return, μ, is

$$\mu = E[\ln(P_{t+1}/P_t)|P_t] = (1 - 2q)\ln u \tag{13}$$

and the conditional variance, σ^2, is

$$\sigma^2 = \mathrm{var}[\ln(P_{t+1}/P_t)|P_t] = 4q(1 - q)(\ln u)^2 . \tag{13'}$$

We eliminate q from equations (13) and (13') and obtain

$$u = e^{\sqrt{\mu^2+\sigma^2}} . \tag{14}$$

In Table 1 we report u (in brackets) and the critical tax ratio, $(u - R)/(uR - 1)$, for a range of the parameters μ, σ and R. If the tax ratio, τ_L/τ, is below the critical ratio, the optimal policy is to realize long term capital gains; otherwise the optimal policy is to defer long term and short term gains. The range of σ is representative of the stocks listed on the NYSE and AMEX, with the median being about $\sigma = .40$ per year. With the ratio $\tau_L/\tau = .4$ and with the tax-exempt, annual interest rate taken to be 5%, the first panel states that the investor should sell the stock and repurchase it at the end of every year and realize a short term loss or a long term gain, whatever the case may be. The second panel, with the tax-exempt annual interest rate taken to be 10%, states that the investor should refrain from realizing long term gains in few cases, marked in the table with an asterisk. These cases refer to stocks with very low variance (the bottom 25% of all NYSE and AMEX listed stocks) and expected rate of return lower or equal to the expected rate of return on riskless tax-exempt bonds. We conclude that, at least in the absence of transactions costs, the investor optimally sells the stock and repurchases it every year.

Our conclusion may be criticized on the grounds that it is specific to the assumed binomial process. For example the entry in Table 1 with $R = 1.05$, $\mu = .05$ and $\sigma = .40$ is $u = 1.50$ and the critical ratio of tax rates is .78. That is, provided the ratio τ_L/τ is less than .78, if the stock price is 1.50 times the basis one year after purchase, the investor optimally realizes a long term gain. Suppose, however, that the stock price follows a different stochastic process so that one year after purchase it could be any multiple of the basis. It does not necessarily follow that the investor optimally realizes a long term gain, however large the gain may be. An investor realizes a long term gain, however large the gain may be (within the context of the binomial model), provided

(15) $$V(P, \hat{P}, t) \leq (1 - \tau_L)P + \tau_L \hat{P}, \quad \forall\, P/\hat{P} \geq 1 .$$

Equations (8) and (15) imply

(16) $$\frac{\tau_L}{\tau} \leq \frac{(1 - u^{-1})(u - R)}{uR - 2 + u^{-1}R} .$$

The entries in Table 1 with a dagger signify the stocks for which the investor optimally realizes a long term gain, <u>however large that gain may be</u>, in order to reestablish the short term status, when the ratio of the tax rates is $\tau_L/\tau = .40$. We conclude that, in the absence of transactions costs, the investor optimally realizes a long term gain on medium and high variance stocks, however large the gain may be.

4.3 Proportional Transactions Costs

With proportional transactions costs the optimal policy is complex. We simply assume that transactions costs are small so that it is plausible to limit our attention to the two policies which are optimal in the absence of transactions costs. As in Sections 4.1 and 4.2, we first assume that it is optimal to defer gains. We then state conditions on the parameters such that this policy dominates the policy of realizing long term gains.

The one-way proportional transactions costs rate is γ, where $0 < \gamma < 1$. When the stock price is $\hat{P}$, the investor pays $(1 + \gamma)\hat{P}$ to buy one share, i.e.,

$$V(\hat{P},\ (1 + \gamma)\hat{P},\ 0) = (1 + \gamma)\hat{P}\ . \tag{17}$$

If the price falls to $u^{-1}\hat{P}$, the investor realizes a short term loss, and receives, net of tax and transactions costs,

$$V(u^{-1}\hat{P},\ (1 + \gamma)\hat{P},\ 1) = (1 - \tau)(1 - \gamma)u^{-1}\hat{P} + \tau(1 + \gamma)\hat{P}\ . \tag{18}$$

If the price first rises, by assumption, the investor defers the gain; if the price first rises and then falls to the level of the purchase price, the investor sells the stock to reestablish the short term status, i.e.,

$$\begin{aligned} V(P,\ (1 + \gamma)\hat{P},\ t) &= (1 - \tau_L)(1 - \gamma)P + \tau_L(1 + \gamma)\hat{P},\ P = \hat{P},\ t \geq 1 \\ &\geq (1 - \tau_L)(1 - \gamma)P + \tau_L(1 + \gamma)\hat{P},\ P > \hat{P},\ t \geq 1\ . \end{aligned} \tag{19}$$

As in Appendix 2, but now with proportional transactions costs, we compare portfolios with different bases and obtain the following:

$$V(P,\ (1 + \gamma)\hat{P},\ t) = R^{-1}[(1 - k)V(uP,\ (1 + \gamma)\hat{P},\ t + 1)+kV(u^{-1}P,\ (1 + \gamma)\hat{P},\ t + 1)] \tag{20}$$

where

$$k = \frac{u - R}{u - u^{-1}}\ . \tag{21}$$

We combine equations (17), (18), (19) and (20) and obtain

$$(22)\qquad (1+\gamma)\hat{P} > R^{-1}[(1-k)\{(1-\tau_L)(1-\gamma)u\hat{P} + \tau_L(1+\gamma)\hat{P}\} + k\{(1-\tau)(1-\gamma)u^{-1}\hat{P} + \tau(1+\gamma)\hat{P}\}] \; .$$

Using equation (21) we eliminate k from equation (22) and obtain

$$(23)\qquad \frac{\tau_L}{\tau} > \frac{(u-R)\{(1+\gamma)u-(1-\gamma)\} - 2(u-u^{-1})uR\gamma/\tau}{(uR-1)\{1-\gamma)u-(1+\gamma)\}} \; .$$

Note that, in the absence of transactions costs, equation (23) becomes $\tau_L/\tau > (u-R)/(uR-1)$, which is the earlier equation (10).

In Table 2 we assume 4% round-trip transactions costs rate, i.e., $2\gamma = .04$ and 50% short term tax rate.[11/] We report u (in brackets) and the critical tax ratio (i.e., the right-hand side of equation (23)) for a range of the parameters μ, σ and R. If the tax ratio, τ_L/τ, is below the critical ratio, the optimal policy is to realize long term capital gains when $P = u\hat{P}$; otherwise, the optimal policy is to defer long term and short term gains. With the ratio $\tau_L/\tau = .40$, and with the tax-exempt annual interest rate taken to be either 5% or 10%, Table 2 indicates that the investor should sell the stock and repurchase it annually to realize a short term loss or long term gain on all high variance stocks, $\sigma > .60$. The investor should do the same annually on medium variance stocks $(\sigma = .40)$ also, if transactions costs are negligible but not if they are substantial.

11/ Stoll and Whaley (1983) estimate the round-trip transactions costs as the sum of the bid-asked spread and the minimum commission schedule provided in the NYSE Fact Book until 1974, the last year of fixed commissions. For 1960-1979, they find the average round-trip transactions costs to be 6.77% for the largest decile NYSE-listed stocks, and 2.71% for the smallest decile NYSE-listed stocks. These estimates exaggerate the transactions costs applicable to medium size and large investors: First, negotiated commissions are small. Second, sophisticated investors, who can wait a few days before they execute a trade, need not have the full bid-asked spread work against them. Analyzing 215,000 individual tickets on over $27 billion of equity trades by thirteen organizations in 1978 and 1979, Beebower and Surz (1980) conclude that the average round-trip transactions costs rate is less than 1% for each one of these organizations.

(Compare Tables 1 and 2.) These conclusions are supported by the simulation reported in the next section.

5. SIMULATING TRADING POLICIES WITH ACTUAL STOCK PRICES, 1962-1977

5.1 Introduction

We simulate three trading policies over the fifteen year period, 1962-1977, and compare their performance against the buy-and-hold policy. We start in 1962, the first year covered by the University of Chicago's CRSP Daily Stock Master file. The investment in each stock is made on December 3, 1962, and trading in future years occurs only once every year at the beginning or middle of December.[12/] (The description of the trading policies explains how the exact trading day is chosen.)

We assume that the 1982 tax law was in force over the simulation period. The simulation then predicts the future effectiveness of the active trading policies under the current tax law, if the price fluctuations in the 1962-1977 period are representative of the future. The holding period beyond which short term capital gains and losses become long term is taken to be one year instead of six or nine months (which was the statutory time interval in the 1962-1977 period). The marginal tax rate on ordinary income and on short term capital gains and losses is taken to be 50%. The marginal tax rate on long term capital gains and losses is taken to be 20%. The limit on the

---

[12/] With individual investors' capital gains and losses taxed only once every year, investors have an incentive to realize their losses in December instead of the following January and defer their gains from December to the following January. The implied trading volume seasonality is discussed in Section 6.2 in connection with the January stock return anomaly.

A referee suggested that we simulate a policy in which losses are realized in December and gains are realized in January. We discuss this policy in Section 5.4 as policy V and argue that it is practically indistinguishable from policy III which we simulate.

capital loss deduction from ordinary income is assumed nonbinding. This assumption overestimates the effectiveness of the active policies.

The sample includes all securities which were on the CRSP Daily Stock Market file as of December 3, 1962, and remained on file until December 1977. Of the 2,027 securities on file on December 3, 1962, only 1,147 remained on file until December 1977, and qualify for inclusion in the sample. In a merger, reorganization, or exchange, if a security on file is replaced by another security on file until December 1977, then the security is included in the sample. If a security is delisted from the NYSE and immediately relisted on the AMEX (or vice versa), the security is included in the sample, provided it is on file until December 1977. Finally, suspension of trading by the SEC or halting of trading by the exchange for more than one year disqualifies a security for inclusion in the sample.

The highest variance stocks are the ones most likely to be delisted from the NYSE or AMEX. Our exclusion from the sample of all stocks not continuously listed over the 15-year simulation period eliminates from the sample many high variance stocks.[13] Furthermore, the highest variance stocks are likely to be small firms traded over-the-counter and therefore excluded from the sample. The selection procedure eliminates many high variance stocks for which the active trading policies work best.

By eliminating stocks which the investor would be forced to sell before the completion of the 15-year period, the selection procedure overestimates the effectiveness of the policy of deferring gains throughout the 15-year period (policy I) but not of the policy of realizing all gains and losses in

---

[13] Before exclusion, 40% of the stocks are classified as high variance, 30% as medium variance, and 30% as low variance. After eliminating the stocks not continuously listed over the 15-year period, 32% of the remaining stocks are in the high variance group, 32% in the medium variance group, and 37% in the low variance group.

every year (policy II). The selection procedure only slightly overestimates the effectiveness of the policy of realizing gains in alternate years (policy III).

All securities on the CRSP file are classified by CRSP into ten portfolios with equal number of securities, based on the variance of the daily excess return in 1962. Portfolio one includes the securities with the highest variance and portfolio ten includes those with the lowest. We classify the sample of 1,147 securities into three groups as follows: 367 high variance stocks with CRSP variance classification 1-4; 361 medium variance stocks with CRSP variance classification 5-7; and 419 low variance stocks with CRSP variance classification 8-10. We do not reclassify the stocks into groups in subsequent years although their CRSP variance classification may change annually.[14/]

One hundred dollars are invested in each stock on December 3, 1962. The number of purchased shares is maintained constant throughout the 15-year period, adjusted only for stock splits, stock dividends, mergers, reorganizations and exchanges. Cash dividends, partial liquidations, and cash proceeds from selling rights are not used to repurchase stock. These cash distributions are taxed at the marginal tax rate of 50%, if appropriate, and are deposited each December in a cash fund that we associate with each stock and

---

[14/] Annual reclassification of stocks by variance would be meaningless in our simulation because we compare the performance of the active policies against a policy of buying the stock and holding it over 15 years. One of the conclusions derived from the simulation is that the active trading policies work best for high variance stocks. This conclusion would be reinforced if the simulation was done with stocks which not only had high variance in 1962 but which maintained high variance throughout the 15-year period.

for each policy.[15/] The cash fund is invested each December in one-year Treasury bills, with the interest earned on the Treasury bills being taxed at 50% and reinvested until December 1977. If a capital loss is realized on the stock, the tax rebate is also deposited in this fund. If a capital gain is realized on the stock, the tax due is subtracted from the fund. The balance of the cash fund may be positive or negative. In December 1977 this balance is added to the after-tax proceeds from selling the stock.

5.2 Policy I: Realize Losses in Every December and Defer Gains

One hundred dollars are invested in the stock on December 3, 1962. One year later, on December 3, 1963, we observe the stock price.[16/] If the stock has a capital gain, we defer it. If the stock has a capital loss, we sell the stock, realize the loss short term and deposit the tax rebate in the cash fund associated with the stock. On the following trading day we repurchase the same number of shares.[17/] We repeat the procedure each December until 1977. In Decembers after 1963 a capital loss is realized short term only if the stock was last sold and repurchased in the previous December; otherwise any loss realized is long term. In December 1977 the stock is sold and a capital gain or loss is realized. (The capital gain can always be realized long term by waiting one more day before selling the stock.) The cash fund, which has

---

15/ For example, the cash distributions between December 3, 1962, and the next trading date in December 1963 are held, earning no interest, until the December 1963 trading date. At that time they are deposited in one-year Treasury bills.

16/ If this date is not a valid trading date, we observe the stock price on the last trading date on which capital losses qualify for short term status.

17/ If the number of shares in the active trading policy were not kept constant, the comparison of after tax cash values of the active and buy-and-hold policies would be meaningless because a different amount of risk would be associated with each policy.

the after-tax cash distributions and tax rebates with interest, is added to the after-tax proceeds from selling the stock. The sum is the net proceeds in year 1977 under policy I.

Under the buy-and-hold, one hundred dollars are invested in the stock on December 3, 1962. On the same date in December 1977 that the stock is sold under policy I, the stock is sold under the buy-and-hold. Capital gains or losses are realized long term. The cash fund associated with this policy and which has the after-tax cash distributions with interest, is added to the after-tax proceeds from selling the stock. The sum is the net proceeds in year 1977 under the buy-and-hold.

The ratio X_I/X_{BH} of the net proceeds of policy I and the buy-and-hold is a measure of their relative performance.[18/] Adjustment for risk is unnecessary in this comparison because the number of shares is held constant in both policies.[19/] In Table 3 and in Figure 1 we report this ratio for the three variance groups of stocks with zero transactions costs and with 4% round-trip transactions costs. Referring to Figure 1, in the absence of transactions costs the wealth relatives are never less than one because policy I dominates the buy-and-hold. Policy I outperforms the buy-and-hold by a greater margin for high variance stocks than for medium or low variance stocks. The option to realize losses and defer gains is more valuable for high variance stocks than for medium or low variance stocks.

[18/] An alternative measure of their relative performance, $(X_I - 100)/(X_{BH} - 100)$, exaggerates the benefit of the active policy for stocks that have low realized return over the simulation period, i.e., $X_{BH} \approx 100$.

[19/] This statement is correct only to a first approximation. By analogy, a portfolio consisting of a call option and a bond paying off the exercise price at the option's maturity, has the same risk as a share of stock only to a first approximation. There is no practical way to refine the adjustment for risk in our simulation or even estimate the direction of bias.

The superiority of policy I over the buy-and-hold is modest. For the group of high variance stocks the median wealth ratio of policy I and the buy-and-hold is 1.043 with zero transactions costs, and 1.016 with 4% round-trip transactions costs. In policy II the investor realizes even long term gains in order to reestablish the tax-advantageous short term status. As we shall see, this policy pays off handsomely.

5.3 Policy II: Realize Gains and Losses in Every December

One hundred dollars are invested in the stock on December 3, 1962. One year later, on December 3, 1963, we observe the stock price. If the stock has a capital loss we realize it short term and repurchase the same number of shares on the following trading day. We deposit the tax rebate in the cash fund associated with the stock. If, instead, the stock has a capital gain, we wait until the following trading day. If the stock still has a capital gain, we realize it long term and repurchase the same number of shares on the following trading day. The capital gains tax is paid out of the cash fund associated with the stock. We repeat the procedure until 1977. In December 1977 the stock is sold. The after-tax proceeds plus the balance of the cash fund is the net proceeds under policy II. Under the buy-and-hold, one hundred dollars are invested in the stock on December 3, 1962. On the same date in December 1977 that the stock is sold under policy II, the stock is sold under the buy-and-hold. The after-tax proceeds plus the balance of the cash fund is the net proceeds under the buy-and-hold.

The ratio of the net proceeds under policy II and the buy-and-hold is reported in Table 3 and Figure 2. The performance of policy II is spectacular for high and medium variance stocks, even with 4% round-trip transactions costs. Updating the basis yearly and reestablishing the short term status is far more profitable than the policy of realizing losses and deferring gains.

The results are in agreement with the theoretical prediction of Section 4 and Tables 1 and 2.

The optimal policy, as discussed in Section 4, is to realize gains long term, provided the ratio of the stock price to the basis is below some critical number. It is only for stocks which satisfy equation (16) and which are marked with a dagger in Table 1 that the optimal policy is to realize a gain, however large it may be. In policy II gains are realized however large they may be. Therefore, the results underestimate the effectiveness of a more sophisticated policy of deferring large gains on low variance stocks.

Under the assumption that the holding period for long term capital gains and losses is six months (as was indeed the case in the years 1962-1976) instead of one year, policy II is dominated by the policy of realizing all capital gains long term and all capital losses short term every December and June. (This statement is true even though short term losses and long term gains incurred in the same year offset each other one to one.) Therefore policy II underestimates the effectiveness of a more sophisticated policy applicable in the years 1962-1976.

In a different sense policy II overestimates the benefits of tax trading. If the investor applies policy II to a portfolio of assets (instead of a single asset), each year he typically realizes short term losses on some stocks and long term gains on others. The offsetting of gains and losses undermines the effectiveness of policy II. The next policy is designed to overcome this problem.

5.4 Policy III: Realize Gains in Alternate Decembers and Realize Losses in Every December

One hundred dollars are invested in the stock on December 3, 1962. In Decembers of odd years gains are deferred and losses are realized, short term if possible. In odd years the problem of one to one offsetting between short term losses and long term gains does not arise because no gains are realized. In Decembers of even years both gains and losses are realized. Gains are realized long term. The investor may be able to realize the losses short term provided the asset was last purchased one year ago, and there are no realized long term gains on other assets to offset the short term gain one to one. We cannot ascertain whether the investor has realized long term gains on other assets in the same year without having observed the realized returns on all of the portfolio assets. In policy III we simply assume that all losses realized in even years (but not in odd years) are realized long term. This assumption being conservative, policy III underestimates the effectiveness of the optimal policy.

On December 1977 the stock is sold. The after-tax proceeds plus the balance of the cash fund is the net proceeds under policy III. On the same date in December 1977 that the stock is sold under policy III, the stock is sold under the buy-and-hold. The after-tax proceeds plus the balance of the cash fund is the proceeds under the buy-and-hold.

The ratio of the net proceeds under policy III and the buy-and-hold is reported in Table 3 and Figure 3. Even though policy III exaggerates the adverse effect of tax offsetting between short term losses and long term gains, it substantially outperforms the buy-and-hold for all categories of stocks. Even with 4% round-trip transactions costs, policy III outperforms the buy-and-hold for high variance and some medium variance stocks.

A variant of policy III, which we may call policy IV, is to defer gains and realize losses, preferably short term, in odd years. So far the policy is the same as policy III. But in policy IV, in even years gains are realized long term and losses are deferred. This differs from policy III in that in policy III losses are realized long term in even years. A moment's reflection should convince the reader that policy III dominates policy IV in the absence of transactions costs. In the presence of transactions costs policy III does not dominate policy IV because it involves more frequent transactions. Simulations not reported here confirm that, in the absence of transactions costs, policy III dominates policy IV. With 4% round-trip transaction costs policy III still outperforms policy IV.

Yet another variant of policy III, which we may call policy V, is to realize losses, preferably short term, every December; and realize long term gains only on the Januaries of odd years (instead of realizing them on the Decembers of even years). An upper bound on the performance of this policy is obtained by replacing the tax rate on long term gains in policy III by $\tau_L/(1 + r)$, where r is the interest rate; and leaving the tax rates on long term and short term losses unchanged. We conclude that policy V is only marginally superior to policy III.

5.5 The Timing option

In buying stock, a taxable investor obtains a timing option on the realization of capital gains and losses. The option is utilized under the optimal trading policy but is wasted under the naive buy-and-hold policy. Suppose that one hundred dollars invested in stock become X_{BH} dollars after tax in fifteen years under the buy-and-hold and X_0 under the optimal policy. If $X_0/X_{BH} = 2$ we say that the timing option represents fifty dollars of the initial investment. More generally, we say that the timing option represents fraction $1 - X_{BH}/X_0$ of the initial investment. The timing option provides an alternative interpretation of Table 3. For example, for high variance stocks and trading policy III, the timing option represents, on average, fraction $1 - 1/1.747 = .43$ of the original investment without transactions costs, or fraction $1 - 1/1.359 = .26$ with 4% round-trip transactions costs.

6. EMPIRICAL IMPLICATIONS

6.1 The Abnormal Returns of Small Firms

Banz (1981) and subsequently Reinganum (1981) and others document the small-firm anomaly: Classifying all NYSE-traded stocks into five portfolios based on stock market value, and using monthly returns over the period 1931-1978, Keim (1983b, Table 2) reports that the annual average excess return of the smallest quintile firms exceeds the annual average excess return of the largest quintile firms by 16%. We argue that tax trading, not only fails to explain the anomaly, but also casts doubts on Stoll and Whaley's (1983) explanation which is based on transactions costs.[20]

If tax trading is to explain the anomaly, we must identify some characteristic of large firms which makes them better candidates for tax trading than small firms. Variance of return is not the sought-after characteristic. Small firms have higher variance of return than large firms. Then the tax timing option is more valuable for small than for large firms and the prediction is that small firms have lower before-tax mean return

---

[20] Also there are at least three pieces of empirical evidence which cast doubts on a tax trading and/or transactions costs explanation of the size anomaly. First, Brown, Kleidon and Marsh (1983) find that, although ranking on firm size appears to produce excess returns inconsistent with the CAPM, the sign of the excess returns is unstable over time. Second, Keim (1983a) finds that nearly half of the excess returns are due to just the month of January. Third, Chen (1983) finds that APT factor risk premia partly explain the anomaly; and Chan, Chen and Hsieh (1983) find that macroeconomic variables suggested by factor analysis and the APT partly explain the anomaly.

than large firms. This argument not only fails to explain, but exacerbates the anomaly.[21/]

Relying on the differential transactions costs between small and large firms, Stoll and Whaley (1983) claim to explain the anomaly. They compare the before tax returns, net of transactions costs, of small and large firms listed on the NYSE. They find that the abnormal returns of small firms become statistically insignificant, provided that the round-trip transactions costs are incurred at least annually, and interpret their findings as a resolution of the anomaly. Whereas they explicitly incorporate transactions costs, they omit from their calculations the perceived benefit which prompts investors to trade despite the presence of these costs. If the perceived benefit from trading in small firms is larger than the benefit from trading in large firms, it may compensate the investor for the higher transactions costs associated with small firms, thereby leaving the anomaly unexplained.

[21/] The following (contrived) argument based on transactions costs may explain the direction, but not the magnitude, of the anomaly. For small firms transactions costs are large and investors follow the buy-and-hold policy. For large firms taxable investors follow an active policy, taking advantage of the distinction between the short term and the long term tax rates. With zero transactions costs, after fifteen years of tax trading, the mean wealth ratio under policy II and the buy-and-hold is 1.604 (see, Section 5.3 and Table 3). Effectively, the government subsidizes the investment in the stock of large firms at the annual rate of $(1/15)\ell n\ 1.604 = .032$. No such subsidy applies to the stock of small firms. Taxable investors bid up the stock prices of large firms and, in equilibrium, the annual, after-tax mean return of small firms exceeds the mean return of large firms by 3.2%. Assuming 20% capital gains tax, the before-tax difference in the mean returns is $3.2/.8 = 4.0\%$. The benefit disappears once we recognize that (1) for large firms the variance of stock return is below the average variance, (2) the differential transactions costs are exaggerated, and (3) we must follow a trading policy (e.g., policy III) to avoid the disadvantageous offsetting of short term losses and long term gains.

To illustrate the point, consider the subset of trades generated by taxable investors following trading policy III (see, Section 5.4).[22/] We capture the differential transactions costs between small and large firms by assuming that the round-trip transactions costs rate is 4% for small firms and zero for large firms. We also recognize the fact that the stock returns of small firms are more variable than the stock returns of large firms. For high variance stocks and 4% transactions costs, the mean wealth relative of policy III and the buy-and-hold is 1.359; for medium variance stocks and zero transactions costs, the mean wealth relative is very similar, being 1.383 (see, Table 3). For small firms, the high variance of stock return compensates for the high transactions costs. At least in this example, the differential transactions costs fail to explain the small-firm anomaly.

6.2 The Abnormal January Returns

Wachtel (1942) and subsequently Dyl (1973), Officer (1975), Rozeff and Kinney (1976), and others document the abnormally high January stock returns. Dyl (1973) and Branch (1977) find positive abnormal returns for stocks that have experienced losses during the previous year. Roll (1983) finds that most of the abnormal returns occur on the last trading day of December and the first four trading days of the following January. Also

---

[22/] We do not claim that capital gains tax is the most important reason for trading. In fact it is difficult to explain the observed volume of stock trading without explicitly recognizing that agents are asymmetrically informed. Glosten and Milgrom (1982) present a model of heterogeneously informed traders to explain the bid-asked spread and the trading volume. The price path is such that the uninformed traders' rate of return equals the discount rate, if they hold the stock for T periods. Our point remains valid in the context of their model also: The informed traders' rate of return exceeds the discount rate. Also, an uninformed trader has an incentive to hold the stock for longer than T periods and earn a rate of return higher than the discount rate. This is so because there is no endogenous reason in the model to sell the stock after T periods.

Givoly and Ovadia (1983), Reinganum (1983) and Roll (1983) find that the abnormal returns of stocks, that have experienced losses during the previous year, are substantially larger for small rather than large firms.

Several of these studies discuss tax-loss selling as a possible explanation of the anomaly. Dyl (1973, 1977) finds abnormally low (high) December trading volume for stocks that have appreciated (depreciated) during the year but correctly argues that the volume seasonality does not imply a price seasonality, if the demand for stocks is perfectly elastic. Gultekin and Gultekin (1982) and Korajczyk (1982) study the seasonality in stock index returns in several countries, with widely differing tax laws and tax year-end. With some reservations, Gultekin and Gultekin (1982) interpret their findings as providing support for the tax-loss selling hypothesis. Brown, Keim, Kleidon and Marsh (1983) find that Australian stocks have positive abnormal returns in January and July and, to a lesser degree, in December and August, although the tax year-end in Australia is June 30. Also Berges, McConnell and Schlarbaum (1982) find that Canadian stock have positive abnormal January returns even before 1972, although capital gains were not taxed in Canada before 1972.

Our contribution to this debate is to identify the critical assumptions in a tax-related explanation of the seasonality in trading volume and stock prices. We consider four scenarios with different assumptions on tax rates and transactions costs.

In the first scenario there is no distinction between the short term and long term tax rates, and transactions costs are zero. The investor optimally realizes a loss whenever it occurs, and defers gains. Even though taxes are paid only at year-end, the investor realizes a loss immediately, lest the stock price rises and the opportunity of taking the loss vanishes. This scenario does not predict an increase in tax-loss selling at year-end.

In the second scenario realized capital gains and losses are taxed at the lower long term rate; long term gains do not offset short term losses dollar for dollar, even if incurred in the same tax year; and transactions costs are zero. The investor optimally realizes short term losses immediately, lest they become long term or vanish. This scenario does not predict an increase in tax-loss selling at year-end. The investor has an incentive to realize long term gains on medium and high variance stocks and reinstate the short term status. He also has an incentive to defer the realization of capital gains from the end of one year to the beginning of the next one and save the interest on the tax. This senario predicts decreased tax-gain selling at the end of a year and increased tax-gain selling at the beginning of a year. If the selling pressure were to depress the price, this scenario would predict a negative abnormal January return for stocks that have had gains during the previous year.

In the third scenario realized capital gains and losses are taxed at the lower long term rate; long term gains offset short term losses dollar for dollar in the same year; and transactions costs are zero. The optimal policy is complex but we identify the relevant factors: The investor wishes to realize short term losses immediately, lest they become long term or vanish. However, if he has already realized large long term gains earlier in the year, he wishes to defer the short term losses to the following year, if possible. The investor wishes to realize long term gains on medium and high variance stocks, preferably at the beginning of the following year. The important message is that none of these incentives predicts increased tax-loss selling at year-end.

In the fourth scenario we assume that there is no distinction between the short term and long term tax rates, but there are transactions costs. If there is no price seasonality, the investor realizes his losses by following a control-limit policy, as in Figure 4. At the beginning of the year he is reluctant to incur the transactions costs and realize a small capital loss. Towards the end of the year his reluctance is overcome by his preference for a tax rebate this year rather than next year. This policy predicts that tax-loss selling gradually increases from January to December and suddenly ceases in the first few days of January; realizing the loss at the end of December dominates the realization of the same loss at the beginning of January.

An objection to the above argument is that, without the distinction between the short term and long term tax rates, transactions costs may dissipate the benefit of tax trading and the optimal policy may be to refrain from tax-loss selling. However, when we draw the distinction between the short term and long term tax rates, our simulation demonstrates that transactions costs do not significantly decrease the benefit from tax trading (see, Table 3). The optimal trading policy becomes complex, but the essential point remains that, with transactions costs, tax-loss selling gradually increases from January to December and suddenly ceases in the first few days of January.

The selling pressure may or may not affect the stock price. The crucial assumption is that, after selling the stock to realize a loss, investors do not repurchase the same stock or the stock sold by other investors to realize a loss. Then tax-loss selling depresses the price of stocks traded in illiquid markets, such as the stocks of small firms. With the tax-loss selling drying up at the beginning of January, the stocks experience positive abnormal returns in January.

This explanation is subject to the criticism that otherwise sophisticated tax-loss sellers are irrational or, at least, ignorant of the stock price seasonality. Rational tax-loss sellers should repurchase different stock sold by other tax-loss sellers at about the same time. Effectively, pairs of tax-loss sellers swap stocks and this activity does not depress the price of either one of the stocks.

If a tax-loss seller has a special reason (e.g., inside information) to repurchase the same stock, he can wait at least one month before the repurchase, and bypass the wash sale provision. Being rational, he can modify the trading policy illustrated in Figure 4, accelerate tax-loss sales from December to the previous October or November, and thereby repurchase the stock in time to receive the abnormal January returns. Alternatively, the investor can change his tax year to end in a month other than December.

We conclude that tax-loss selling in the presence of transactions costs predicts a seasonal pattern in trading volume. It predicts a seasonal pattern in stock prices only if we further assume irrationality or ignorance on behalf of investors.

7. CONCLUDING REMARKS

Tax trading does not explain the small-firm anomaly, but predicts a seasonal pattern in trading volume which maps into a seasonal pattern in stock prices, the January anomaly, only if we assume irrationality or ignorance on behalf of investors. These results in no way detract from the basic message of our theoretical calculations and simulation: For those investors who can draw the distinction between the short term and long term tax rates, the benefit of optimal tax trading on medium and high variance stocks outweighs even large transactions costs.

TABLE 1

CRITICAL RATIO OF THE LONG TERM TO THE SHORT TERM TAX RATE BELOW WHICH THE OPTIMAL POLICY IS TO REALIZE LONG TERM CAPITAL GAINS: THE ZERO TRANSACTIONS COSTS CASE

Expected Annual Stock Return (μ)	Standard Deviation of Annual Stock Return					
	σ = .10	σ = .20	σ = .40	σ = .60	σ = .80	σ = 1.00
			ANNUAL RISKLESS RETURN R = 1.05			
.05	.40 (1.12)	.62 (1.23)	.78 (1.50)†	.85 (1.83)†	.88 (2.23)†	.90 (2.72)†
.10	.48 (1.15)	.64 (1.25)	.79 (1.51)†	.85 (1.84)†	.88 (2.24)†	.90 (2.73)†
.15	.58 (1.20)	.67 (1.28)	.79 (1.53)†	.85 (1.86)†	.88 (2.26)†	.90 (2.75)†
.20	.64 (1.25)	.71 (1.33)	.80 (1.56)†	.85 (1.88)†	.88 (2.28)†	.90 (2.77)†
			ANNUAL RISKLESS RETURN R = 1.10			
.05	.09* (1.12)	.37* (1.23)	.62 (1.50)†	.72 (1.83)†	.78 (2.23)†	.81 (2.72)†
.10	.19* (1.15)	.40 (1.25)	.62 (1.51)†	.72 (1.84)†	.78 (2.24)†	.81 (2.73)†
.15	.31* (1.20)	.44 (1.28)	.63 (1.53)†	.73 (1.86)†	.78 (2.26)†	.81 (2.75)†
.20	.40 (1.25)	.50 (1.33)	.64 (1.56)†	.73 (1.88)†	.78 (2.28)†	.82 (2.77)†

In parentheses the parameter u of the binomial process generating annual prices. The asterisk denotes stocks for which the optimal policy is to defer the realization of a long term gain when the price equals u times the basis and τ_L/τ = .40. The dagger denotes stocks for which the optimal policy is to realize a long term gain, however large the gain may be, where τ_L/τ = .40.

TABLE 2

CRITICAL RATIO OF THE LONG TERM TO THE SHORT TERM TAX RATE BELOW WHICH THE OPTIMAL POLICY IS TO REALIZE LONG TERM CAPITAL GAINS: THE CASE WITH FOUR PERCENT ROUND-TRIP TRANSACTIONS COSTS

Expected Annual Stock Return (μ)	Standard Deviation of Annual Stock Return					
	σ = .10	σ = .20	σ = .40	σ = .60	σ = .80	σ = 1.00
	ANNUAL RISKLESS RETURN R = 1.05					
.05	NEVER* (1.12)	.12* (1.23)	.55 (1.50)	.69 (1.83)	.76 (2.23)	.81 (2.72)
.10	NEVER* (1.15)	.18* (1.25)	.56 (1.51)	.70 (1.84)	.76 (2.24)	.81 (2.73)
.15	NEVER* (1.20)	.26* (1.28)	.57 (1.53)	.70 (1.86)	.77 (2.26)	.81 (2.75)
.20	.18* (1.25)	.36* (1.33)	.59 (1.56)	.71 (1.88)	.77 (2.28)	.81 (2.77)
	ANNUAL RISKLESS RETURN R = 1.10					
.05	NEVER* (1.12)	NEVER* (1.23)	.38* (1.50)	.56 (1.83)	.66 (2.23)	.72 (2.72)
.10	NEVER* (1.15)	NEVER* (1.25)	.39* (1.51)	.57 (1.84)	.66 (2.24)	.72 (2.73)
.15	NEVER* (1.20)	.03* (1.28)	.40 (1.53)	.57 (1.86)	.66 (2.26)	.72 (2.75)
.20	NEVER* (1.25)	.14* (1.33)	.43 (1.56)	.58 (1.88)	.67 (2.28)	.72 (2.77)

The short term tax rate is 50%. In parentheses the parameter u of the binomial process generating annual prices. The asterisk denotes stocks for which the optimal policy is to defer the realization of a long term gain when the price equals u times the basis and τ_L/τ = .40.

TABLE 3

WEALTH RELATIVES OF POLICIES I, II, III AND THE BUY-AND-HOLD

	HIGH VARIANCE (367 stocks)		MEDIUM VARIANCE (361 stocks)		LOW VARIANCE (419 stocks)	
	$2\gamma = 0$	$2\gamma = 4\%$[a]	$2\gamma = 0$	$2\gamma = 4\%$[a]	$2\gamma = 0$	$2\gamma = 4\%$[a]
POLICY I: REALIZE LOSSES IN EVERY DECEMBER AND DEFER GAINS. WEALTH RELATIVES OF POLICY I AND BUY-AND-HOLD.						
Mean	1.120	1.084	1.058	1.028	1.039	1.000
25 percentile	1.002	1.000	1.000	.990	1.000	.982
50 percentile	1.043	1.016	1.007	1.000	1.019	1.000
75 percentile	1.168	1.110	1.059	1.022	1.060	1.008
POLICY II: REALIZE GAINS AND LOSSES IN EVERY DECEMBER. WEALTH RELATIVES OF POLICY II AND BUY-AND-HOLD.						
Mean	2.242	1.720	1.604	1.224	1.381	1.050
25 percentile	1.521	1.188	1.296	.995	1.245	.957
50 percentile	1.966	1.521	1.468	1.109	1.323	1.008
75 percentile	2.651	2.050	1.694	1.293	1.445	1.095
POLICY III: REALIZE GAINS IN ALTERNATE DECEMBERS AND REALIZE LOSSES IN EVERY DECEMBER. WEALTH RELATIVES OF POLICY III AND BUY-AND-HOLD.						
Mean	1.747	1.359	1.383	1.094	1.275	1.011
25 percentile	1.308	1.040	1.173	.964	1.176	.930
50 percentile	1.547	1.214	1.300	1.034	1.238	.982
75 percentile	1.922	1.501	1.444	1.162	1.324	1.064

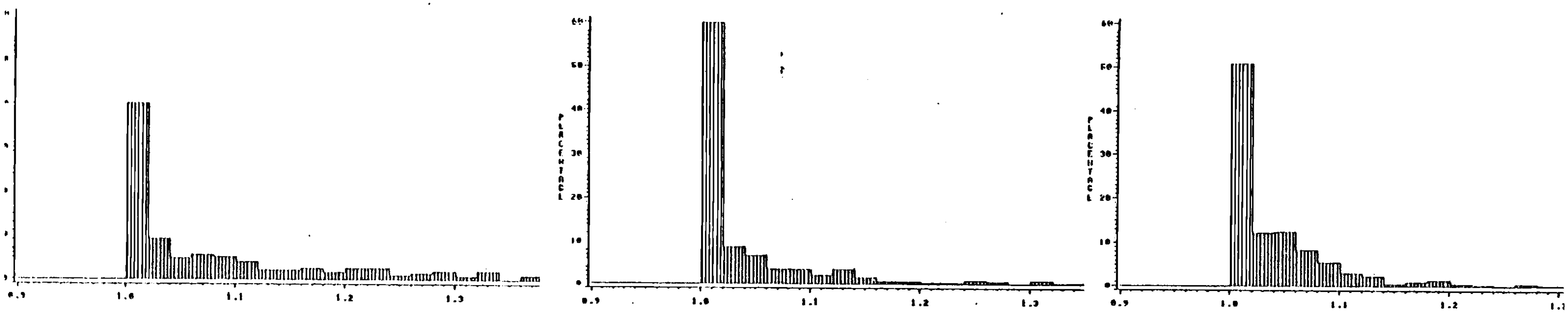
The marginal tax rate on ordinary income and on short term capital gains and losses is 50%. The marginal tax rate on long term capital gains and losses is 20%. The initial investment is made in December 1962 and the wealth relatives are compared in December 1977.

[a]The round-trip transactions costs rate is 4%.

FIGURE 1

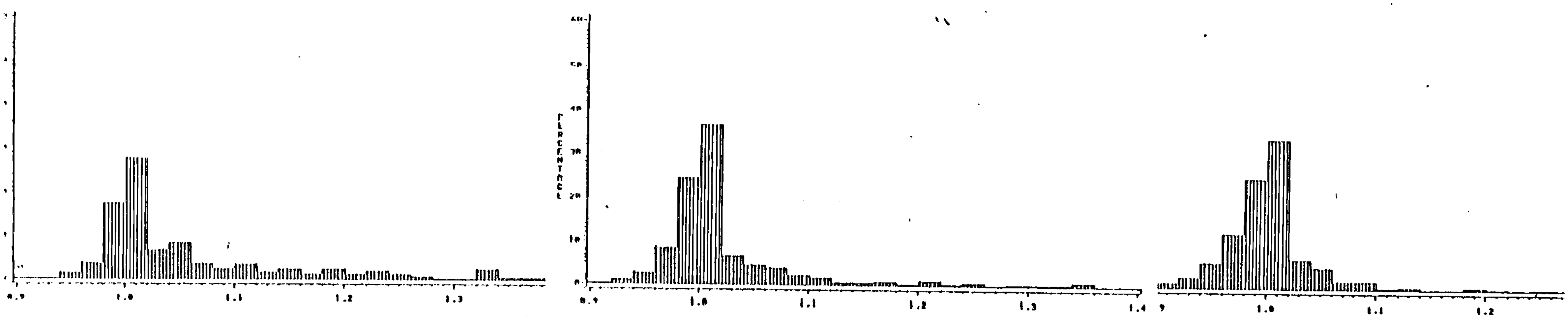
POLICY I: REALIZE LOSSES IN EVERY DECEMBER AND DEFER GAINS.
WEALTH RELATIVES OF POLICY I AND BUY-AND-HOLD.

ZERO TRANSACTIONS COSTS



HIGH VARIANCE | MEDIUM VARIANCE | LOW VARIANCE

4% ROUND TRIP TRANSACTIONS COSTS

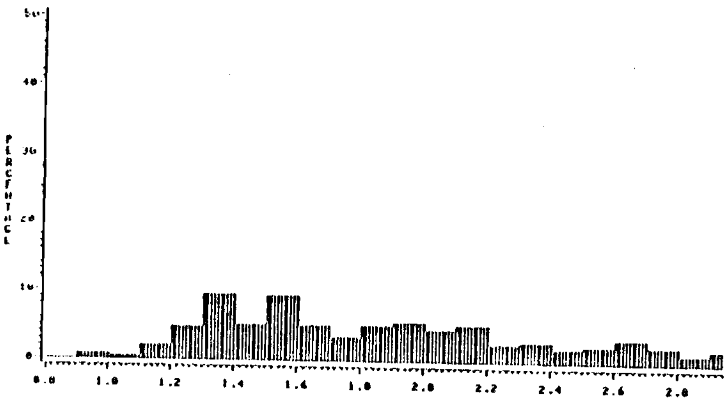


HIGH VARIANCE | MEDIUM VARIANCE | LOW VARIANCE

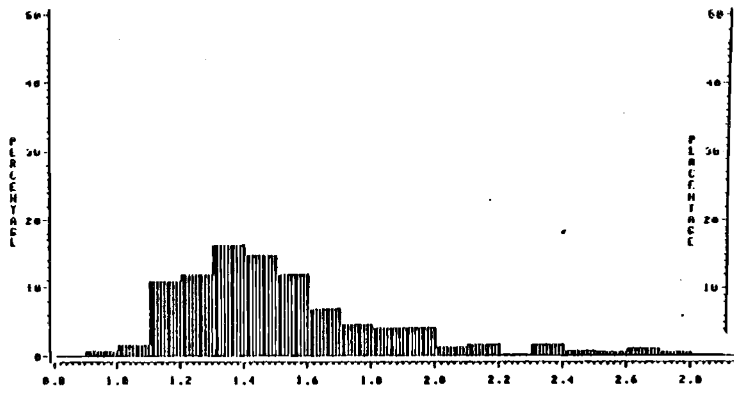
FIGURE 2

POLICY II: REALIZE GAINS AND LOSSES IN EVERY DECEMBER.
WEALTH RELATIVES OF POLICY II AND BUY-AND-HOLD.

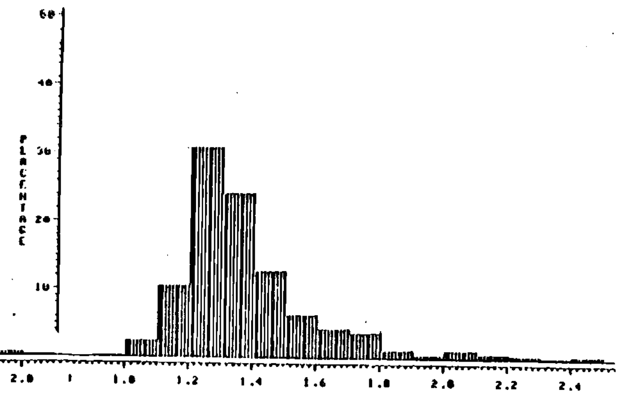
ZERO TRANSACTIONS COSTS



HIGH VARIANCE

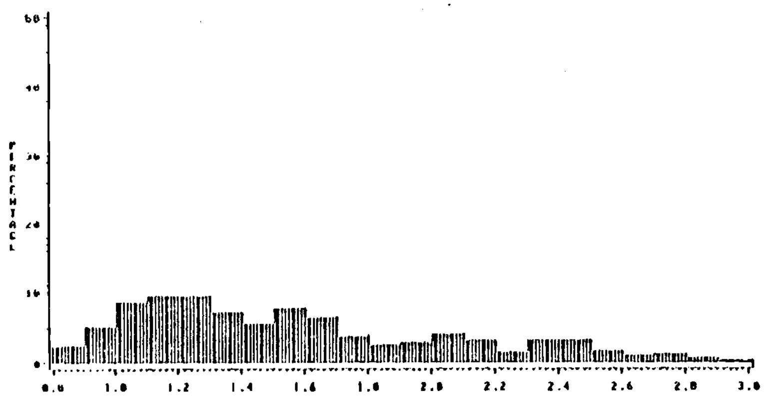


MEDIUM VARIANCE

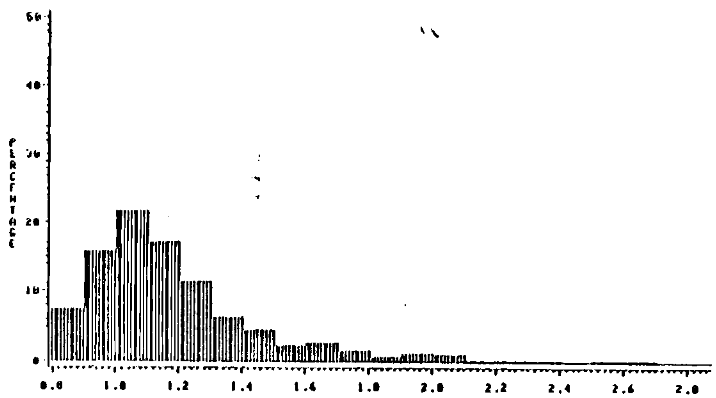


LOW VARIANCE

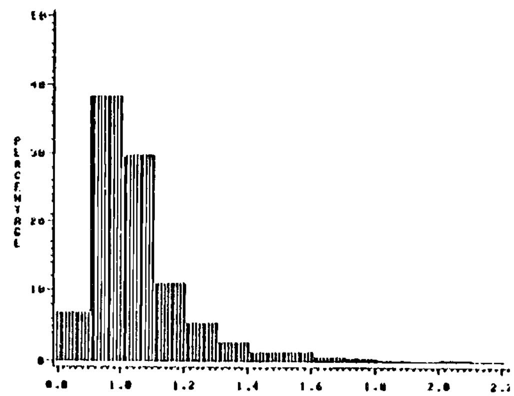
4% ROUND TRIP TRANSACTIONS COSTS



HIGH VARIANCE



MEDIUM VARIANCE

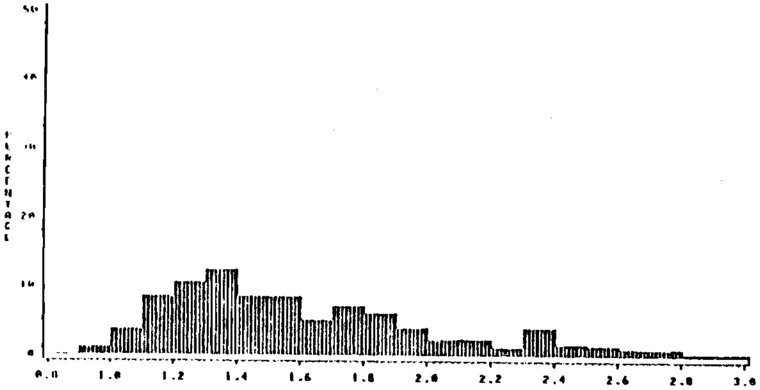


LOW VARIANCE

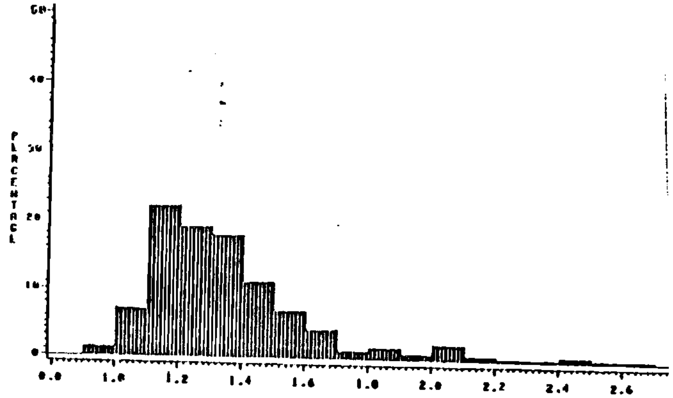
FIGURE 3

POLICY III: REALIZE GAINS IN ALTERNATE DECEMBERS AND REALIZE LOSSES IN EVERY DECEMBER. WEALTH RELATIVES OF POLICY III AND BUY-AND-HOLD.

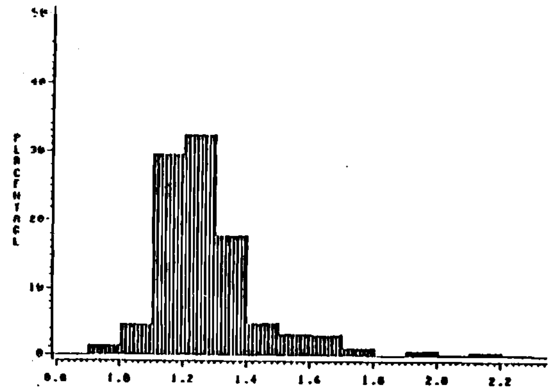
ZERO TRANSACTIONS COSTS



HIGH VARIANCE

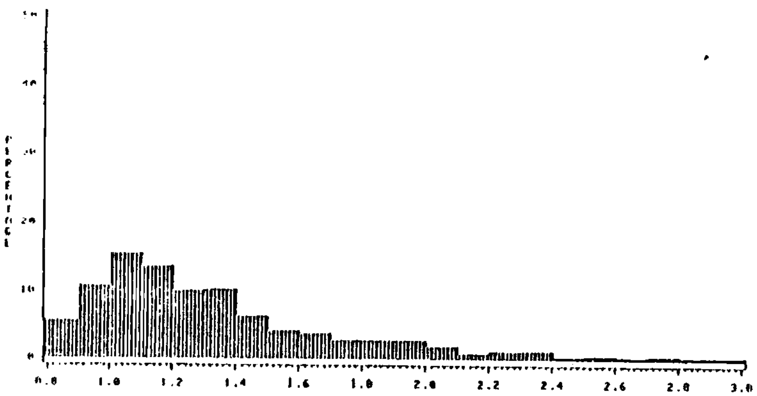


MEDIUM VARIANCE

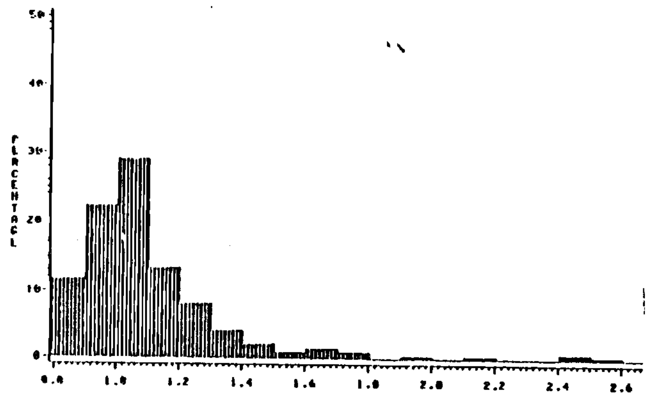


LOW VARIANCE

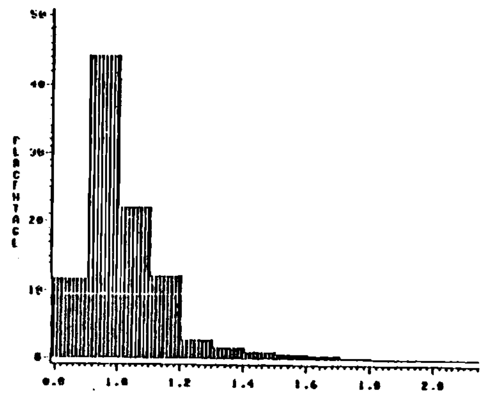
4% ROUND TRIP TRANSACTIONS COSTS



HIGH VARIANCE



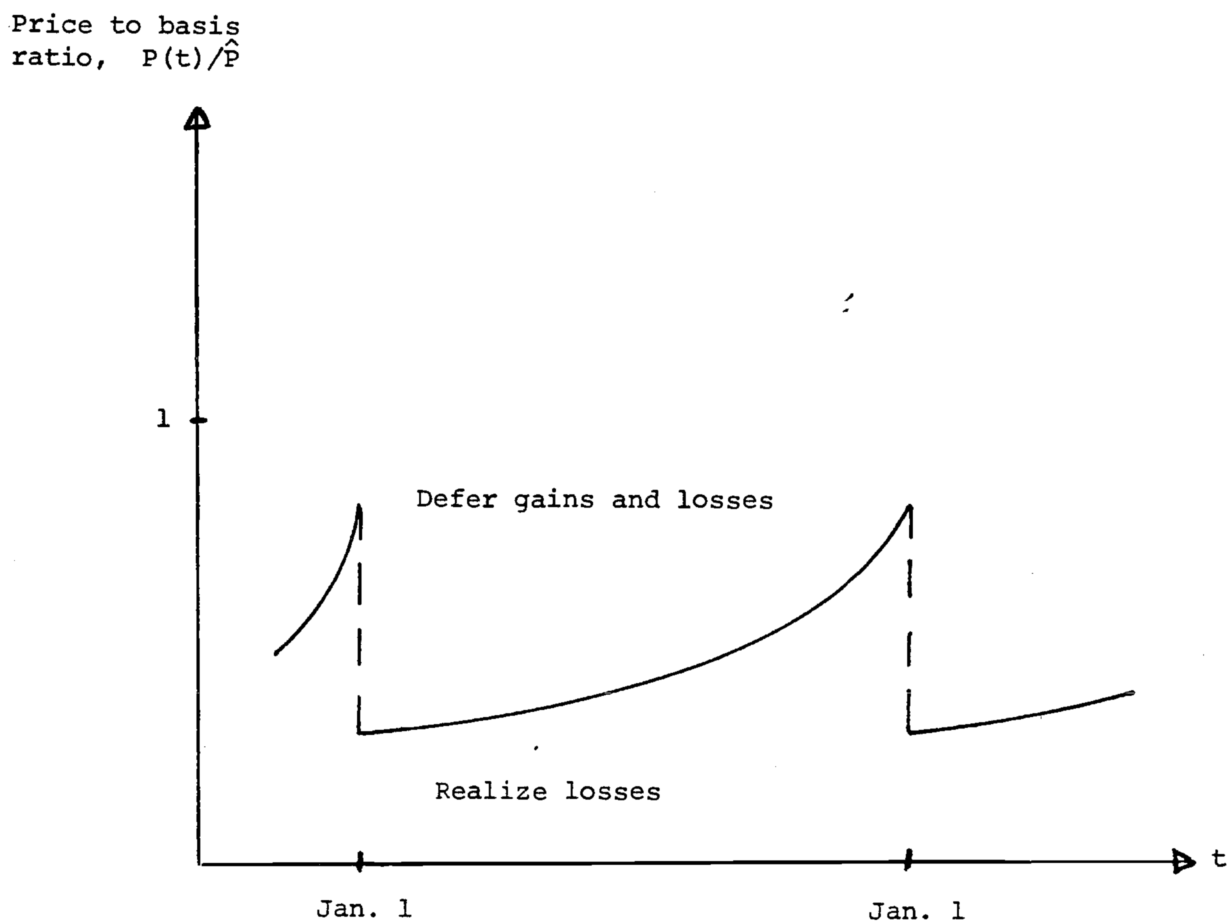
MEDIUM VARIANCE



LOW VARIANCE

Figure 4

THE OPTIMAL TRADING POLICY WITH ANNUAL TAXATION OF CAPITAL GAINS AND LOSSES, TRANSACTIONS COSTS AND WITHOUT DISTINCTION BETWEEN SHORT AND LONG TERM TAX STATUS



APPENDIX 1

Proof that $V(P, \hat{P}, t)$ is convex in P

Lemmas 1 and 2 lead to the main result, Lemma 3. Lemmas 1, 2, and 3 correspond to Merton's (1973b) Theorems 4, 9, and 10.

LEMMA 1: $V(P, \hat{P}, t)$ is convex in $\hat{P}$.

Proof: Consider two portfolios. Portfolio A consists of α shares with price P per share, purchased at price $\hat{P}$ per share t periods ago; and $1 - \alpha$ shares with price P per share, purchased at price $\hat{\hat{P}}$ per share t periods ago. We restrict α by $0 \leq \alpha \leq 1$. The value of portfolio A is $\alpha V(P, \hat{P}, t) + (1 - \alpha)V(P, \hat{\hat{P}}, t)$.

Portfolio B is simply one share with price P per share, purchased at price $\alpha\hat{P} + (1 - \alpha)\hat{\hat{P}}$ per share t periods ago.

If portfolio B is optimally liquidated at time T when the stock price is P_T, the after-tax proceeds are $(1 - \tau_L)P_T + \tau_L\{\alpha\hat{P} + (1 - \alpha)\hat{\hat{P}}\}$. At time T, the after-tax proceeds in liquidating portfolio A are $\alpha\{(1 - \tau_L)P_T + \tau_L\hat{P}\} + (1 - \alpha)\{(1 - \tau_L)P_T + \tau_L\hat{\hat{P}}\}$ and equal the after-tax proceeds in liquidating portfolio B. Therefore the value of portfolio A at time t is at least as large as the value of portfolio B, i.e.,

$$\alpha V(P, \hat{P}, t) + (1 - \alpha)V(P, \hat{\hat{P}}, t) \geq V(P, \alpha\hat{P} + (1 - \alpha)\hat{\hat{P}}, t) .$$

LEMMA 2: Assume that the distribution of the stock return, P_T/P_t, at time t is independent of the price P_t. Then, $V(P, \hat{P}, t)$ is homogeneous of degree one in $(P, \hat{P})$, i.e., $V(hP, h\hat{P}, t) = hV(P, \hat{P}, t)$, $h > 0$.

The proof is left to the reader.

LEMMA 3: Assume that the distribution of the stock return is price-independent. Then $V(P, \hat{P}, t)$ is convex in P.

Proof: For any α, $0 \leq \alpha \leq 1$,

$$V(\alpha P + (1 - \alpha)P', \hat{P}, t)$$

$$\leq \frac{\alpha P}{\alpha P + (1 - \alpha)P'} V\left(\alpha P + (1 - \alpha)P', \left(\frac{\alpha P + (1 - \alpha)P'}{P}\right) \hat{P}, t\right)$$

$$+ \frac{(1 - \alpha)P'}{\alpha P + (1 - \alpha)P'} V\left(\alpha P + (1 - \alpha)P', \left(\frac{\alpha P + (1 - \alpha)P'}{P'}\right) \hat{P}, t\right)$$

by the convexity of V in its second argument (see, Lemma 1). We simplify the right-hand side using the property that V is homogeneous of degree one in its first and second arguments (see, Lemma 2) and obtain

$$V(\alpha P + (1 - \alpha)P', \hat{P}, t) \leq \alpha V(P, \hat{P}, t) + (1 - \alpha)V(P', \hat{P}, t) .$$

APPENDIX 2

<u>Determination of the function</u> $V(P, \hat{P})$ <u>when the short term and long term tax rates are equal.</u>

We determine the functional form of $V(P, \hat{P})$ in the case $P \geq \hat{P}$ by comparing two positions in the same stock but with different bases. Consider two portfolios with the following composition:

<u>First portfolio</u>

(i) $V(uP, \hat{\hat{P}}) - V(u^{-1}P, \hat{\hat{P}})$ shares with basis $\hat{P}$, $\hat{P} \leq P$.

(ii) A riskless, one-period bond with price $R^{-1}[V(uP, \hat{P}) - V(u^{-1}P, \hat{P})]V(uP, \hat{\hat{P}})$.

<u>Second portfolio</u>

(i) $V(uP, \hat{P}) - V(u^{-1}P, \hat{P})$ shares with basis $\hat{\hat{P}}$, $\hat{\hat{P}} \leq P$.

(ii) A riskless, one-period bond with price $R^{-1}[V(uP, \hat{\hat{P}}) - V(u^{-1}P, \hat{\hat{P}})]V(uP, \hat{P})$.

One period later, the stock price is either uP or $u^{-1}P$. If the stock price is uP, the first portfolio's value is

$$[V(uP, \hat{\hat{P}}) - V(u^{-1}P, \hat{\hat{P}})]V(uP, \hat{P}) + [V(uP, \hat{P}) - V(u^{-1}P, \hat{P})]V(uP, \hat{\hat{P}})$$

and equals the second portfolio's value. If the stock price is $u^{-1}P$, the first portfolio's value is

$$[V(uP, \hat{\hat{P}}) - V(u^{-1}P, \hat{\hat{P}})]V(u^{-1}P, \hat{P}) + [V(uP, \hat{P}) - V(u^{-1}P, \hat{P})]V(uP, \hat{\hat{P}})$$

and again equals the second portfolio's value. Therefore, the investor is indifferent between the two portfolios at the beginning of the period and we obtain

$$[V(uP, \hat{\hat{P}}) - V(u^{-1}P, \hat{\hat{P}})]V(P, \hat{P}) + R^{-1}[V(uP, \hat{P}) - V(u^{-1}P, \hat{P})]V(uP, \hat{\hat{P}})$$

$$= [V(uP, \hat{P}) - V(u^{-1}P, \hat{P})]V(P, \hat{\hat{P}}) + R^{-1}[V(uP, \hat{\hat{P}}) - V(u^{-1}P, \hat{\hat{P}})]V(uP, \hat{P}) .$$

Rearranging, we obtain

$$(2.1)\qquad \frac{V(P,\,\hat{P}) - R^{-1}V(uP,\,\hat{P})}{V(uP,\,\hat{P}) - V(u^{-1}P,\,\hat{P})} = \frac{V(P,\,\hat{\hat{P}}) - R^{-1}V(uP,\,\hat{\hat{P}})}{V(uP,\,\hat{\hat{P}}) - V(u^{-1}P,\,\hat{\hat{P}})} \equiv -R^{-1}k$$

where k is a constant to be determined. Since the left-hand side is independent of $\hat{\hat{P}}$ and the right-hand side is independent of $\hat{P}$, k is independent of $\hat{P}$, $\hat{\hat{P}}$. Since $V(P,\,\hat{P})$ is homogeneous of degree one in $(P,\,\hat{P})$, it follows that $V(P,\,0)$ is homogeneous of degree one in P. Setting $\hat{P} = 0$ in (2.1) we obtain

$$(2.2)\qquad k = \frac{u - R}{u - u^{-1}}\ .$$

Equations (6) and (2.2) imply $0 < k < 1$.

We rewrite equation (2.1) as

$$(2.3)\qquad V(P,\,\hat{P}) = R^{-1}[(1 - k)V(uP,\,\hat{P}) + kV(u^{-1}P,\,\hat{P})], \qquad P \geq \hat{P}\ .$$

We interpret $1 - k$, k as the pseudoprobabilities of the two states which occur in the binomial process, as in Cox, Ross and Rubinstein (1979). Equation (2.3) has the following interpretation: $V(P,\,\hat{P})$ is the expected value of positions $(uP,\,\hat{P})$ and $(u^{-1}P,\,\hat{P})$ discounted by R, where the expectation is with respect to the pseudoprobabilities $1 - k$ and k. The actual probabilities $1 - q$, q do not enter equation (2.3). The corresponding result in the option pricing theory of Black and Scholes (1973) is that the option price is independent of the expected rate of return of the underlying security.

We eliminate k from equations (2.2) and (2.3) and obtain the difference equation in P

$$(2.4)\qquad V(P,\,\hat{P}) = R^{-1}\Big[\Big(\frac{R - u^{-1}}{u - u^{-1}}\Big)\,V(uP,\,\hat{P}) + \Big(\frac{u - R}{u - u^{-1}}\Big)\,V(u^{-1}P,\,\hat{P})\Big]\ , \qquad P \geq \hat{P}$$

with general solution

$$V(P, \hat{P}) = AP + BP^{m}\,\hat{P}^{1-m}, \qquad P > u^{-1}\hat{P} \tag{2.5}$$

where

$$m \equiv [\ell n(u - R) - \ell n(uR - 1)]/\ell nu < 0 . \tag{2.6}$$

It remains to determine the constants A and B. Using equation (7), continuity of the function V at $P = \hat{P}$ requires

$$V(\hat{P}, \hat{P}) = \hat{P} \tag{2.7}$$

and continuity at $P = u^{-1}\hat{P}$ requires

$$V(u^{-1}\hat{P}, \hat{P}) = (1 - \tau)u^{-1}\hat{P} + \tau\hat{P} . \tag{2.8}$$

The boundary conditions (2.7) and (2.8) uniquely determine the constants A and B and we obtain equation (8).

REFERENCES

Banz, R. W., 1981, "The Relationship Between Return and Market Value of Common Stocks," Journal of Financial Economics 9, March, 3-18.

Beebower, G. L. and R. J. Surz, 1980, "Analysis of Equity Trading Execution Costs," Proceedings of the Seminar on the Analysis of Security Prices, University of Chicago, November, 149-163.

Berges, A., J. J. McConnell and G. G. Schlarbaum, 1982, "An Investigation of the Turn-of-the-Year Effect, the Small Firm Effect and the Tax-Loss-Selling-Pressure Hypothesis in Canadian Stock Returns," manuscript, July.

Black, F. and M. S. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," Journal of Political Economy 81, May/June, 637-659.

Branch, B., 1977, "A Tax Loss Trading Rule," Journal of Business 50, April, 198-207.

Breeden, D. T., 1979, "An Intertermporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," Journal of Financial Economics 7, September, 265-296.

Brennan, M. J., 1973, "Taxes, Market Valuation and Corporate Financial Policy," National Tax Journal 23, 417-427.

Brown, P., A. W. Kleidon and T. A. Marsh, 1983, "New Evidence on the Nature of Size Related Anomalies in Stock Prices," Journal of Financial Economics (forthcoming).

Brown, P., D. B. Keim, A. W. Kleidon and T. A. Marsh, 1983, "Stock Return Seasonalities and the 'Tax-Loss Selling' Hypothesis: Analysis of the Arguments and Australian Evidence," Journal of Financial Economics (forthcoming).

Chan, C., N. Chen and D. Hsieh, 1983, "An Exploratory Investigation of the Firm Size Effect," Center for Research in Security Prices, Graduate School of Business, University of Chicago, W.P. No. 99, June.

Chen, N., 1983, "Some Empirical Tests of the Theory of Arbitrage Pricing," Journal of Finance (forthcoming).

Constantinides, G. M., 1980, "Short Term versus Long Term Capital Gains and Losses," manuscript, Graduate School of Business, University of Chicago, November.

Constantinides, G. M., 1983, "Capital Market Equilibrium with Personal Tax," Econometrica 51, May, 611-636.

Constantinides, G. M. and J. E. Ingersoll, Jr., 1983, "Optimal Bond Trading with Personal Taxes: Implications for Bond Prices and Estimated Tax Brackets and Yield Curves," Center for Research in Security Prices, Graduate School of Business, University of Chicago, W. P. No. 70, May.

Constantinides, G. M. and M. S. Scholes, 1980, "Optimal Liquidation of Assets in the Presence of Personal Taxes: Implications for Asset Pricing," Journal of Finance, May, 439-449.

Cox, J. C., S. A. Ross and M. Rubinstein, 1979, "Option Pricing: A Simplified Approach," Journal of Financial Economics 7, September, 229-263.

Dyl, E. A., 1973, "The Effect of Capital Gains Taxation on the Stock Market," unpublished doctoral dissertation, Graduate School of Business, Stanford University, August.

Dyl, E. A., 1977, "Capital Gains Taxation and Year-End Stock Market Behavior," Journal of Finance 32, March, 165-175.

Givoly, D. and A. Ovadia, 1983, "Year-End Tax-Induced Sales and Stock Market Seasonality," Journal of Finance 38, March, 171-185.

Glosten, L. R. and P. R. Milgrom, 1982, "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," manuscript, Northwestern University, October.

Gultekin, M. N. and N. B. Gultekin, 1982, "Stock Market Seasonality and End of the Tax Year Effect," manuscript, University of Pennsylvania, March.

Holt, C. C. and J. P. Shelton, 1962, "The Lock-in Effect of the Capital Gains Tax," National Tax Journal 15, December, 337-351.

Keim, D. B., 1983a, "Size-Related Anomalies and Stock Return Seasonality: Further Empirical Evidence," Journal of Financial Economics (forthcoming).

Keim, D. B., 1983b, "The Interrelation between Dividend Yields, Equity Values and Stock Returns: Implications of Abnormal January Returns," unpublished doctoral dissertation, Graduate School of Business, University of Chicago.

Korajczyk, R., 1982, "Seasonality in Stock Returns: Some International Evidence," manuscript, Graduate School of Business, University of Chicago, April.

Merton, R. C., 1973a, "An Intertemporal Capital Asset Pricing Model," Econometrica 41, September, 867-887.

Merton, R. C., 1973b, "Theory of Rational Option Pricing," The Bell Journal of Economics and Management Science 4, Spring, 141-183.

Reinganum, M. R., 1981, "Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings Yields and Market Values," Journal of Financial Economics, 9, March, 19-46.

Reinganum, M. R., 1983, "The Anomalous Stock Market Behavior of Small Firms in January: Empirical Tests for Year-End Tax Effects," Journal of Financial Economics (forthcoming).

Roll, R., 1983, "The Turn-of-the-Year Effect and the Return Premia of Small Firms," The Journal of Portfolio Management, 18-28.

Rozeff, M. S. and W. R. Kinney, Jr., 1976, "Capital Market Seasonality: The Case of Stock Returns," Journal of Financial Economics 3, October, 379-402.

Smidt, S., 1982, "Capital Gains Taxes and the Seasonal Behavior of Stock Prices and Trading Volumes," manuscript, Cornell University, October.

Stiglitz, J. E., 1983, "Some Aspects of Taxation of Capital Gains," National Bureau of Economic Research, W.P. No. 1094, March.

Stoll, H. R. and R. E. Whaley, 1983, "Transaction Costs and the Small Firm Effect," Journal of Financial Economics (forthcoming).

Varian, H. R., 1981, "Optimal Wash Sales," manuscript, University of Michigan, September.

Wachtel, S. B., 1942, "Certain Observations on Seasonal Movements in Stock Prices," Journal of Business 15, 184-193.

Williams, J., 1981, "Trading Depreciable Assets," manuscript, Graduate School of Business, New York University, October.