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ABSTRACT

In an influential paper, Baily (1978) showed that the optimal level of unemployment insurance (UI) in a stylized static model depends on only three parameters: risk aversion, the consumption-smoothing benefit of UI, and the elasticity of unemployment durations with respect to the benefit rate. This paper examines the key economic assumptions under which these parameters determine the optimal level of social insurance. A Baily-type expression, with an adjustment for precautionary saving motives, holds in a very general class of dynamic models subject to weak regularity conditions. For example, the simple reduced-form formula derived here applies with arbitrary borrowing constraints, endogenous insurance markets, and search and leisure benefits of unemployment. A counterintuitive aspect of this result is that the optimal benefit rate appears not to depend on (1) any benefit of UI besides consumption-smoothing or (2) the relative magnitudes of income and substitution effects in the link between UI benefits and durations. However, these parameters enter implicitly in the optimal benefit calculation, and estimating them can be useful in testing whether the values of the primary inputs are consistent with observed behavior.

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1 Introduction

As social insurance programs grow rapidly in developed economies, a large literature assessing the economic costs and benefits of programs such as unemployment and disability insurance has emerged. The canonical normative analysis of social insurance is due to Baily (1978). Baily analyzes a stylized model of unemployment and obtains a simple inverse-elasticity formula for the optimal unemployment insurance (UI) level in terms of three parameters: (1) the elasticity of unemployment durations with respect to benefits, which captures the moral hazard cost of benefit provision due to behavioral response; (2) the drop in consumption as a function of UI benefits, which quantifies the consumption-smoothing benefits; and (3) the coefficient of relative risk aversion (γ), which reflects the value of having a smoother consumption path. Guided by the intuition that these parameters are central in assessing optimal unemployment insurance, many papers have estimated the effect of UI benefits on durations (see e.g., Moffitt (1985), Meyer (1990)) and consumption (e.g. Gruber (1997), Browning and Crossley (2001)).

Since Baily's contribution, several studies have observed that his framework is restrictive and argued that the optimal level of social insurance differs under alternative assumptions. Examples include borrowing constraints (Flemming 1978; Crossley and Low 2005), more general search technologies (Lentz 2004), and human capital accumulation effects (Brown and Kaufold 1988). These papers seek formulas for the optimal benefit level in terms of the primitive structure of the model and show that these primitives have quantitatively large effects on optimal benefit rates in simulations. More recently, Golosov and Tsyvinski (2005) show that if private endogenous insurance markets exist, the welfare gain from government intervention is greatly reduced. Other studies have remarked on the limits of Baily's results less formally. Feldstein (2005) notes that calculations of optimal UI based on Baily's formula could be misleading because they do not adequately account for savings responses, while Gruber (1997) calibrates Baily's formula and cautions that the introduction of leisure benefits of unemployment could potentially change his results.

While these papers have identified several important factors in the analysis of optimal social insurance, they have not attempted to obtain a reduced-form expression for the optimal benefit level based on observable elasticities (rather than underlying primitives) in the more general setting that they consider. This paper investigates the key economic assumptions necessary to obtain such a formula. I study a dynamic partial-equilibrium model where agents choose consumption, unemployment durations, and M other behaviors (e.g. spousal labor supply or private insurance purchases) that enter a general time-separable utility function.[1] Agents face a budget constraint and N other inequality constraints when choosing these behaviors (e.g., borrowing constraints or hours constraints). An arbitrary stochastic process determines the agent's employment status at each time.

The main result is that Baily-type expressions for both the optimal benefit level and the marginal welfare gain from an increase in social insurance apply much more generally than suggested by the existing literature.[2] In particular, suppose each constraint on consumption while unemployed can be loosened by raising benefits, and each constraint on consumption while employed can be loosened by reducing the UI tax. As discussed below, virtually any economically plausible constraint in a model where income streams are fungible would satisfy this requirement. Then, under some weak regularity conditions that make the government's optimization problem well-behaved, the optimal benefit rate is approximately determined by the same three parameters described above, along with the coefficient of relative prudence. The approximation requires that fourth-order terms of utility over consumption are small; calibrations with power utility functions indicate that the error associated with this approx-

[1] The model is "partial equilibrium" in the sense that production technologies and wages are not endogenous to the level of UI benefits. Hence, the formula derived in this paper does not apply to the recent general equilibrium models of UI analyzed by Acemoglu and Shimer (1999) and others.

[2] Though the model analyzed here refers to an unemployment shock, with a change of notation, the general case can be used to model social insurance against other shocks such as injury or disability. In this sense, the formula derived here is informative about optimal state-contingent redestributive policies in general and not just unemployment insurance.

imation is on the order of 2-4%. When the third-order terms of utility are small as well (i.e., when agents do not have precautionary savings motives), Baily's three-parameter formula carries over directly to the general case.

It follows that the reduced-form formulas proposed here apply even with arbitrary borrowing constraints, endogenous insurance markets, leisure benefits of unemployment, portfolio choice and human capital decisions. An additional implication is that one needs to estimate the responsiveness of only a single nondurable consumption good to UI benefits (e.g. food) to determine the optimal benefit rate, provided that the corresponding measure of risk aversion (e.g., curvature of utility over food) is used. Variations in the model do not affect the envelope argument that is central to deriving the formula because their effects are captured in the primary inputs in the optimal UI calculation.

A more surprising point is perhaps that the optimal benefit rate does *not* appear to depend on several other parameters that one expects should matter. In contrast with the literature on optimal taxation, which emphasizes the distinction between income and substitution effects, the relative magnitudes of these effects in the link between UI and durations appear to be irrelevant. In addition, other factors such as the leisure benefits of unemployment or the potential role of UI in improving job matches by subsidizing search seem to play no role in the calculation of the optimal benefit level. On the surface, the large empirical literature on the effects of social insurance on outcomes such as job match quality, savings rates, etc. thus seems irrelevant in calculating the optimal rate of social insurance.

The second part of this paper explores why the formula exhibits these features. The basic reason is that the elasticities that enter the formula are all functions of other aspects of the agent's behavior and preferences. For instance, if unemployment has large leisure or search benefits, agents would elect to have a longer duration and therefore a larger consumption drop, ultimately leading to a higher optimal benefit rate, exactly as one would expect. To illustrate the importance of understanding these implicit effects, I analyze how income and substitution effects relate to the optimal benefit level in greater detail. I show that the income and substitution elasticities of unemployment durations pin down the coefficient of relative risk aversion. In particular, large income effects imply a highly curved underlying utility over consumption. Intuitively, if an individual shortens his duration significantly to recoup lost income when unemployed, that lost income must have raised the marginal utility of consumption significantly, implying that his utility over consumption is highly curved. Consequently, the optimal benefit rate does in fact depend positively on the size of the income elasticity and negatively on the size of the substitution elasticity, as one would expect. However, *conditional* on the values of the four primary inputs, the magnitudes of these elasticities and all other effects are irrelevant.

This point reveals an important tradeoff in evaluating policies using the formula proposed here. The power of this reduced-form approach is that it does not require complete specification of the underlying model, permitting an analysis that is not sensitive to specific modelling choices. The danger is that one might choose elasticities that are inconsistent with each other or with other behavioral responses. In the income effects example, the problem is that one might calibrate the formula with a low risk aversion parameter, failing to recognize that this could contradict empirical studies that have identified large income effects on labor supply for the unemployed (see e.g., Mincer 1962, Cullen and Gruber 1998, Chetty 2005). This inconsistency may not be immediately apparent because the set of primitives generating these elasticities is never explicitly identified. Hence, while the formula for optimal social insurance derived here is widely applicable, it ideally should be implemented with support from empirical estimates of other behavioral responses coupled with structural tests for consistency of the various parameters.

The remainder of the paper is organized as follows. The next section derives a formula for the optimal benefit level and the welfare gain from raising benefits first using a simple example and then in the general case. Section 3 turns to the counterintuitive features of the result, demonstrating in particular how the size of income and substitution effects matter. The final section offers concluding remarks.

2 A General Formula for the Optimal Benefit Level

I consider the optimal benefits problem in a pay-as-you-go unemployment insurance system where taxes collected in a given period are used to finance benefits in that period. Agents receive a constant unemployment benefit of b while unemployed.[3] The government finances the benefits by levying a lump-sum tax of τ on employed agents. The lump-sum tax assumption simplifies the algebra and also has the virtue of describing actual practice. In the United States, UI benefits are financed by a payroll tax applied only to the first $10,000 of income, and is thus inframarginal (and effectively a lump-sum tax) for most workers.

Throughout the analysis, I take wages as fixed, effectively ignoring the fact that UI benefits could have general-equilibrium effects by changing the supply and demand for jobs with different risk characteristics. I also abstract from distortions to firm behavior (e.g., those caused by imperfect experience rating) by assuming that the expected amount of time spent unemployed is determined by workers who take their tax burden as fixed.[4] Finally, I assume that agents' choices have no externalities (e.g., there are no spillovers to search).

2.1 A Special Case

I begin with a stylized model where the derivation of the optimal benefit rate (b^*) is most transparent. A representative agent lives for one unit of time and arrives at time 0 with assets A_0. At this point, the agent either has a job that pays a wage of w (probability $1-p$) or is unemployed (probability p). Assume that p is exogenous and does not vary with the benefit level; this assumption will be relaxed in the general case below. In the state where the agent has a job, there is no risk of job loss until death, and the agent makes

[3] The question of the optimal path or duration of benefits, which has attracted much attention in recent work on optimal UI (see e.g., Davidson and Woodbury (1997), Hopenhayn and Nicollini (1998), Werning (2004)), is outside the scope of this paper. An interesting direction for future work would be to derive an elasticity-based formula for the optimal path of benefits.

[4] In practice, UI is imperfectly experience rated, and appears to significantly distort firms' layoff decisions (see e.g., Feldstein (1978) and Topel (1983)).

no labor supply choices. In the state where he is unemployed, the agent must search for a job. Assume that the agent can control his unemployment duration, d, deterministically by varying search effort. Search costs, the leisure value of unemployment, and the benefits of additional search via improved job matches are captured by a concave, increasing function $\psi(d)$.

The only constraints are the budget constraint in each state. Assume that the UI tax τ is collected only in the employed state, so that the agent has to pay no taxes while working if he was originally unemployed (this assumption will be relaxed in the general case). Normalize the interest rate and discount rate at 0. Since there is no uncertainty or discounting and no income growth both when unemployed and employed, the optimal consumption path is flat in both states. Let c_e and c_u denote the consumption levels in each state and $u(c)$ denote utility over consumption, which we assume is strictly concave. The agent's problem at time 0 is to choose c_e, c_u, and d to

$$\begin{aligned} &\max(1-p)u(c_e) + p\{u(c_u) + \psi(d)\} \\ &\text{s.t. } A_0 + (w - \tau) - c_e \geq 0 \\ &\qquad A_0 + bd + w(1-d) - c_u \geq 0 \end{aligned}$$

Let $V(b)$ denote the solution to this problem for a given unemployment benefit b. The benevolent social planner's problem is to choose the benefit rate b that maximizes the agent's indirect utility subject to the balanced-budget constraint for the UI system (taxes collected equal benefits paid in expectation):

$$\begin{aligned} &\max_b V(b) \\ &\text{s.t. } (1-p)\tau = pbd \end{aligned}$$

The following proposition gives two approximate solutions to this problem. Note that this and subsequent results about the optimal benefit rate characterize b^* when it is positive. When this condition has a positive solution, that solution is a global maximum. When there is no solution to the equation that defines b^* at an interior optimum, it follows that $b^* = 0$ under the regularity conditions used to ensure strict concavity of $V(b)$.

Proposition 1 *If the third-order terms of $u(c)$ are small ($u'''(c) \approx 0$), the optimal benefit rate b^* is implicitly defined by*

$$\gamma \frac{\Delta c}{c}(b^*) \approx \varepsilon_{d,b} \tag{1}$$

If the fourth-order terms of $u(c)$ are small ($u''''(c) \approx 0$), b^* is defined by

$$\gamma \frac{\Delta c}{c}(b^*)[1 + \frac{1}{2}\rho \frac{\Delta c}{c}(b^*)] \approx \varepsilon_{d,b} \tag{2}$$

where

$$\begin{aligned}
\frac{\Delta c}{c} &= \frac{c_e - c_u}{c_e} = \textit{consumption drop during unemployment} \\
\gamma &= -\frac{u''(c_e)}{u'(c_e)} c_e = \textit{coefficient of relative risk aversion} \\
\rho &= -\frac{u'''(c_e)}{u''(c_e)} c_e = \textit{coefficient of relative prudence} \\
\varepsilon_{d,b} &= \frac{\partial \log d}{\partial \log b} = \textit{ elasticity of duration w.r.t. benefits}
\end{aligned}$$

Proof. At an interior optimum, the optimal benefit rate must satisfy

$$\partial V / \partial b(b^*) = 0$$

To calculate this partial derivative, note that $V(b)$ can be written as

$$V(b) = \max_{c_e, c_u, d, \lambda_e, \lambda_u} (1-p)u(c_e) + p\{u(c_u) + \psi(d)\} + \lambda_e[A_0 + (w-\tau) - c_e] + \lambda_u[A_0 + bd + w(1-d) - c_u]$$

where λ_e and λ_u are the LaGrange multipliers that give the marginal value of relaxing the budget constraint while employed and unemployed. Since this function has already been optimized over $\{c_e, c_u, d, \lambda_e, \lambda_u\}$, changes in these variables will not have first-order effects on V (an application of the Envelope Theorem). Hence,

$$\partial V/\partial b(b^*) = -\lambda_e \frac{\partial \tau}{\partial b} + \lambda_u d = 0 \quad (3)$$

$$\Rightarrow \lambda_e \frac{\partial \tau}{\partial b} = \lambda_u d \quad (4)$$

Agent optimization implies that the multipliers are equal to the marginal utility of consumption in each state:

$$\lambda_e = (1-p)u'(c_e) \quad (5)$$

$$\lambda_u = pu'(c_u) \quad (6)$$

The government's UI budget constraint implies

$$\frac{\partial \tau}{\partial b} = \frac{p}{1-p}[d + b\frac{\partial d}{\partial b}]$$

and plugging these expressions into (3) and simplifying yields

$$u'(c_e)[1 + \frac{b}{d}\frac{\partial d}{\partial b}] = u'(c_u) \quad (7)$$

This optimality condition captures a basic intuition that carries over to the general case:

The optimal level of benefits offsets the marginal benefit of raising consumption by \$1 in the unemployed state (RHS of (7)) against the marginal cost of raising the UI tax in the employed state to cover the added benefits (LHS of (7)). The marginal cost of raising the UI tax to finance a \$1 increase in benefits is given by the direct cost $u'(c_e)$ plus an added term arising from the agent's behavioral response of extending his unemployment duration, which further reduces his consumption while unemployed.

Rearranging (7), we obtain

$$\frac{u'(c_u) - u'(c_e)}{u'(c_e)} = \frac{b}{d}\frac{\partial d}{\partial b} \tag{8}$$

This equation provides an exact definition for the optimal benefit rate, and can be solved for b^* by choosing a function form for u. An approximate solution can be obtained by simplifying the left hand side of this expression using a Taylor expansion to write

$$u'(c_u) - u'(c_e) \approx u''(c_e)(c_u - c_e) + \frac{1}{2}u'''(c_e)(c_e - c_u)^2.$$

Using the definitions of γ and ρ, we obtain

$$\begin{aligned} \frac{u'(c_u) - u'(c_e)}{u'(c_e)} &\approx -\frac{u''}{u'}c_e\frac{\Delta c}{c} + \frac{1}{2}\frac{u'''}{u''}c_e\frac{u''}{u'}c_e(\frac{\Delta c}{c})^2 \\ &= \gamma\frac{\Delta c}{c} + \frac{1}{2}\rho\gamma(\frac{\Delta c}{c})^2 \end{aligned} \tag{9}$$

Plugging this expression into the left hand side of (8) and factoring yields the formula given in (2). Note that $u''' = 0 \Rightarrow \rho = 0$, in which case (2) reduces to (1).

To prove that b^* is a global maximum, one can show that $\frac{\partial^2 V}{\partial b^2} < 0$. This condition is established under certain regularity conditions for the general case below. ■

The first approximate solution for b^* given in Proposition 1 is the same as Baily's (1978) formula. He ignores third-order terms of u in his derivation, effectively assuming that precautionary savings motives are small, in which case utility is well approximated by a quadratic function. Unfortunately, the approximation error induced by ignoring the third-order terms in this case is sometimes large. In particular, using power (CRRA) utility with γ ranging from 1 to 5, $\frac{\Delta c}{c}(b)$ specified as implied by Gruber's (1997) estimates, and $\varepsilon_{d,b} = 0.5$, Baily's approximate solution sometimes underestimates the exact b^* by more than 30%. To obtain a more precise solution, the effects of third-order terms in u must be taken into account. This yields the formula in (2), which has an additional coefficient of relative prudence term. This formula, which assumes that the fourth and higher-order terms of u are small, is a much more successful approximation: the difference between the exact and approximate b^* is always less than 4% for the calibration exercises described above. Hence, using an estimate of the reduced-form relationship between $\frac{\Delta c}{c}$ and b, one can obtain a reasonably good estimate of the optimal b^* by solving (2) for b.[5]

It is helpful to remark on the mechanism underlying Proposition 1 since it carries over to the general case. At a mathematical level, the basic idea is to exploit the envelope condition, which permits us to write the marginal value of raising b purely in terms of the multipliers λ_u and λ_e. Agent optimization then allows us to express λ_u and λ_e in terms of the marginal utilities of consumption in each state, as in (5) and (6). Intuitively, since higher benefits simply relax the budget constraint and the agent has already equated all marginal utilities at the optimum, we can assume that extra benefits are spent on solely on c_u (and that higher taxes are financed solely by reducing c_e) when computing welfare changes. This allows us to ignore other behavioral responses when calculating b^* except the $\varepsilon_{d,b}$ parameter that enters the government's budget constraint. The next section shows that an envelope condition can be applied to obtain a very similar formula for the optimal benefit level in a general setting.

[5] One can of course formulate examples where even the third-order approximation will not work well. If one has strong priors about the fourth-order terms of u, they can be used to obtain a more precise formula for b^* by expanding the Taylor series in (9)by one more term.

2.2 The General Case

Choice variables. Consider a continuous-time dynamic model where agents face persistent unemployment risk. Normalize the length of life to be one unit, so that time $t \in [0, 1]$. Agents choose consumption depending on their current employment state, $c_e(t)$ if employed and $c_u(t)$ if not, at each time t. In addition, agents choose a vector of other behaviors in each state $x_e(t) = (x_e^1(t), ..., x_e^{M_e}(t))$ and $x_u(t) = (x_u^1(t), ..., x_u^{M_u}(t))$. These could include choices such as search effort while unemployed, reservation wage while unemployed, level of work effort (or shirking) while employed, private insurance purchases, takeup of other social programs (e.g. welfare), amount of borrowing from friends, portfolio choice, human capital investments, etc. Assume that utility is time-separable and let $u(c_t, x_t)$ denote the felicity utility of the agent as a function of his choices at any time t. Let $c = \{c_e(t), c_u(t)\}_{t=0}^1$ and $x = \{x_e(t), x_u(t)\}_{t=0}^1$ denote the full program of choices over time.

Let $\theta_t(c, x, t)$ denote an agent's employment status at time t. If $\theta(c, x, t) = 1$, the agent is employed at t, and if $\theta(c, x, t) = 0$, the agent is unemployed. The process that determines $\theta(c, x, t)$ is left unspecified, and can be an arbitrary function of the agent's behavior at time t as well as other times. The trajectory of θ can be stochastic, with a general, time-varying disturbance term (e.g., to accommodate uncertainty in unemployment duration lengths). To reduce notation, all arguments of θ other than t are suppressed below, but θ is always treated as endogenous throughout this section. In addition, I refer below to an economy with a single representative agent, but discuss later how heterogeneous agents (e.g. with different utilities u or employment dynamics θ) can easily be handled with a simple change of notation in the optimal benefit formula.

Let d denote the fraction of his lifetime that the agent spends unemployed:

$$d = \int_0^1 [1 - \theta(t)]dt$$

Let $\overline{c_e}$ and $\overline{c_u}$ denote mean consumption while employed and unemployed:

$$\begin{aligned} \overline{c_e} &= \frac{\int \theta(t)c_e(t)dt}{\int \theta(t)dt} \\ \overline{c_u} &= \frac{\int (1-\theta(t))c_u(t)dt}{\int (1-\theta(t))dt} \end{aligned}$$

Constraints. The agent faces a standard dynamic budget constraint while employed and unemployed as well as a terminal condition on assets:

$$\dot{A}_e(t) = w - \tau - c_e(t) \ \forall t \tag{10}$$

$$\dot{A}_u(t) = b - c_u(t) \ \forall t \tag{11}$$

$$A(1) = A_0 + \int_0^1 [\theta(t)\dot{A}_e(t) + (1-\theta(t))\dot{A}_u(t)]dt \geq A_{term}$$

In addition, the agent faces a set of N additional constraints at each time t

$$g_{it}(c_\theta(t), x_\theta(t); b, \tau) \geq \overline{k}_{it}, i = 1, ..., N; \theta = 0 \text{ or } 1$$

Let $\lambda_{e,t}$ denote the multiplier on the dynamic budget constraint while employed at time t; $\lambda_{u,t}$ the corresponding multiplier if unemployed; λ_T the multiplier on the terminal condition; and $\lambda_{g_i,t}$ the multipliers on the additional constraints. Each of these multipliers equal the marginal value of relaxing the corresponding constraint in the optimal program.

Agent's and planner's problems. The agent chooses a program c and x to

$$\begin{aligned} \max \int_0^1 & \theta(t)u_t(c_e(t), x_e(t)) + (1-\theta(t))u_t(c_u(t), x_u(t))dt \\ &+ \int_0^1 \lambda_{e,t}(w - \dot{A}_e(t) - \tau - c_e(t))dt + \int_0^1 \lambda_{u,t}(b - \dot{A}_u(t) - c_u(t))dt \\ &+ \lambda_T(A(1) - A_{term}) + \sum_{i=1}^{N} \int_0^1 \lambda_{g_{it}}(g_{it}(c_\theta(t), x_\theta(t)) - \overline{k}_{it})dt \end{aligned}$$

Let $V(b)$ denote the maximal value for this problem for a given unemployment benefit b. The social planner's problem is to find the value b^* that maximizes $V(b)$ subject to the UI budget constraint, which is:

$$\begin{aligned} \tau \int \theta(t)dt &= b \int [1-\theta(t)]dt \\ &\Longrightarrow \tau(1-d) = db \end{aligned}$$

Ensuring that the solution to the social planner's problem can be obtained from first-order conditions requires some regularity assumptions, which are specified below.

Assumption 1. Utility u is smooth, increasing, and strictly quasiconcave in (c, x)

Assumption 2. The set of feasible choices $\{(c(t), x(t) : g_i(c_\theta(t), x_\theta(t)) \geq \overline{k}_i \ \forall i, A(1) \geq A_{term}\}$ is convex

Assumption 3. In the agent's optimal program, the set of binding constraints does not change for a perturbation in b in some open interval $(b - \varepsilon, b + \varepsilon)$.

Assumptions 1-2 guarantee that the agent's problem has a unique global constrained maximizer $(c(t), x(t))$. Together with Assumption 3, these assumptions imply that the Envelope Theorem can be applied to obtain $\frac{\partial V}{\partial b}$ (see the mathematical appendix in Mas-Colell, Whinston, Green (1995) for a proof). Without loss of generality, assume below that all of the auxiliary g constraints are binding; any constraint that is slack can be ignored under the third assumption.

The following set of conditions are sufficient (but not necessary) to establish that $V(b)$ is a strictly concave function, which ensures that any b satisfying the first order condition is a global maximum.

Assumption 4. Consumption while unemployed is weakly increasing in b; consumption while employed is weakly decreasing in τ; and the elasticity of duration with respect to the

UI benefit level is weakly increasing in b:

$$\frac{\partial c_u(t)}{\partial b} \geq 0,\ \frac{\partial c_e(t)}{\partial \tau} \leq 0,\ \frac{\partial \varepsilon_{d,b}}{\partial b} \geq 0$$

The first two parts of this assumption essentially require that the direct effect of changes in the UI tax and benefits are not swamped by behavioral responses in the opposite direction. For the third part, note that the constant-elasticity case ($\frac{\partial \varepsilon_{d,b}}{\partial b} = 0$) is the usual benchmark. Since a 1% increase in benefits constitutes a larger increase in dollar amount when b is large, to the extent that a larger change in amounts induces a larger change in behavior, we may expect an increasing elasticity.

Consumption-UI Constraint Condition. The derivation for the static model shows that we must be able to quantify the costs and benefits of unemployment insurance solely through the marginal utilities of consumption in each state to obtain a simple formula for b^*. Roughly stated, this is feasible if higher benefits relax all constraints on consumption while unemployed and higher taxes tighten all constraints on consumption while employed. The following assumption states the necessary restrictions on the constraints formally.

Assumption 5. The feasible set of choices can be defined using a set of constraints $\{g_{it}\}$ such that $\forall t \forall i$

$$\begin{aligned} \frac{\partial g_{it}}{\partial b} &= \frac{\partial g_{it}}{\partial c_u(t)} \\ \frac{\partial g_{it}}{\partial \tau} &= \frac{\partial g_{it}}{\partial c_e(t)} \\ \frac{\partial g_{it}}{\partial c_\theta(s)} &= 0 \text{ if } t \neq s \end{aligned}$$

Assumption 5 requires that the set of binding constraints can be written so that at all times (a) benefits and consumption while unemployed enter each constraint in the same way, (b) the UI tax and consumption while employed enter each constraint in the same way, and

(c) consumption at two different times s and t do not enter the same constraint together. It is helpful to illustrate when this condition holds with some examples:

(a) Budget constraints. In the simplest model, the only constraints are the budget constraints in each state. To verify that these constraints satisfy assumption 5, recall the definition of the dynamic budget constraints in (10) and (11). Note that $\frac{\partial \dot{A}_u(t)}{\partial b} = \frac{\partial \dot{A}_u(t)}{\partial c_u(t)} = 1$ and $\frac{\partial \dot{A}_e(t)}{\partial b} = \frac{\partial \dot{A}_e(t)}{\partial c_e(t)} = 1$. In addition, only $c_\theta(t)$ appears in each constraint at time t. Hence, assumption 5 is satisfied, explaining why (2) was obtained in the static case.

(b) Borrowing constraint while unemployed at time t:

$$g_{1t} = A(t) + b - c_u(t) \geq 0$$

If this constraint binds, $\frac{\partial g_{1t}}{\partial b} = \frac{\partial g_{1t}}{\partial c_u(t)} = -1$ and $\frac{\partial g_{1t}}{\partial \tau} = \frac{\partial g_{1t}}{\partial c_e(t)} = 0\ \forall t$, so assumption 5 holds.

(c) Private insurance market. Suppose the agent holds a private insurance contract that charges a premium $\rho_e(t)$ in the employed state and has a net payout of $\rho_u(t)$ in the unemployed state at time t. This changes the dynamic budget constraints to:

$$\begin{aligned}\dot{A}_e(t) &= w - \rho_e(t) - \tau - c_e(t)\ \forall t \\ \dot{A}_u(t) &= b + \rho_u(t) - c_u(t)\ \forall t\end{aligned}$$

Then it remains the case that $\frac{\partial \dot{A}_u}{\partial b} = \frac{\partial \dot{A}_u}{\partial c_u(t)} = 1$ and $\frac{\partial \dot{A}_e}{\partial \tau} = \frac{\partial \dot{A}_e}{\partial c_e(t)} = 1$, etc. so assumption 5 still holds.

(d) Hours constraint while employed. Suppose the agent is able to choose labor supply ($l_e(t)$) on the intensive margin while employed but cannot work for more than H hours by law. Then he faces the additional constraint at all times t:

$$g_{2t} = H - l_e(t) \geq 0$$

If this constraint binds, $\frac{\partial g_{2t}}{\partial b} = \frac{\partial g_{2t}}{\partial \tau} = \frac{\partial g_{2t}}{\partial c_\theta(t)} = 0\ \forall t$, so assumption 5 is satisfied.

(e) Subsistence constraint. Suppose the agent must maintain consumption above a level $\underline{c}$ at all times:

$$g_{3t} = c_\theta(t) - \underline{c} \geq 0 \; \forall \theta, t$$

If this constraint binds at some t' for some θ, in that instance $\frac{\partial g_{3t'}}{\partial b} = 0 \neq \frac{\partial g_{3t'}}{\partial c_\theta(t')} = 1$, so assumption 5 is *not* satisfied here.

Though a subsistence constraint can technically violate the consumption-UI constraint condition, it represents a pathological case. Most agents are able to cut consumption when benefits are lowered in practice (Gruber 1997). Moreover, such a constraint is unlikely to literally bind because one would expect the marginal utility of consumption to rise to infinity as consumption falls to $\underline{c}$, preventing agents from reaching this point. More generally, as long as different sources of income are fungible, agents should be able to use higher benefits (or lower taxes) to change their consumption in the relevant state. The only reason this might not be feasible is because of other technological constraints on consumption. Since most economically plausible constraints do not involve such restrictions, they are likely to satisfy the consumption-UI constraint condition.

Assumption 5 essentially guarantees that the marginal value of increasing benefits and raising the UI tax can be read directly from the average marginal utilities of consumption in each state. The following lemma establishes this connection.

Lemma 1 *The marginal value of increasing the UI benefit while balancing the UI budget is*

$$\partial V/\partial b = -\frac{\partial \tau}{\partial b}(1-d)Eu'(c_{e,t}) + dEu'(c_{u,t}) \tag{12}$$

where the average marginal utilities of consumption in each state are

$$Eu'(c_{e,t}) = \frac{\int \theta(t) u'(c_{e,t}) dt}{\int \theta(t) dt}$$

$$Eu'(c_{u,t}) = \frac{\int (1-\theta(t)) u'(c_{u,t}) dt}{\int (1-\theta(t)) dt}$$

Proof. Since behavioral responses to the change in benefits have no first-order effect on V (the envelope condition),

$$\partial V/\partial b = -\frac{\partial \tau}{\partial b}\int[\lambda_{e,t} + \sum \lambda_{g_i,t}\frac{\partial g_{it}}{\partial \tau}]dt + \int[\lambda_{u,t} + \sum \lambda_{g_i,t}\frac{\partial g_{it}}{\partial b}]dt = 0 \qquad (13)$$

Using the third part of assumption 5, agent optimization requires that the marginal utility of consumption in each state can be written as a function of the corresponding multipliers at time t:

$$\begin{aligned} \theta(t)u'(c_{e,t}) &= \lambda_{e,t} + \sum \lambda_{g_i,t}\frac{\partial g_{it}}{\partial c_e(t)} \ \forall t & (14) \\ (1-\theta(t))u'(c_{u,t}) &= \lambda_{u,t} + \sum \lambda_{g_i,t}\frac{\partial g_{it}}{\partial c_u(t)} \ \forall t & (15) \end{aligned}$$

The first two parts of assumption 5 imply that

$$\begin{aligned} \sum \lambda_{g_i,t}\frac{\partial g_{it}}{\partial c_e(t)} &= \sum \lambda_{g_i,t}\frac{\partial g_{it}}{\partial \tau} \ \forall t \\ \sum \lambda_{g_i,t}\frac{\partial g_{it}}{\partial c_u(t)} &= \sum \lambda_{g_i,t}\frac{\partial g_{it}}{\partial b} \ \forall t \end{aligned}$$

Using these expressions, we can substitute (14) and (15) into (13) to obtain

$$\partial V/\partial b = -\frac{\partial \tau}{\partial b}\int \theta(t)u'(c_{e,t})dt + \int(1-\theta(t))u'(c_{u,t})dt \qquad (16)$$

Substituting in the definitions of d and $Eu'(c_{\theta,t})$ yields (12). ■

Lemma 1 reflects the same basic intuition that underlies (7) in the static model. The marginal value of raising benefits by one dollar is the average marginal utility of consumption while unemployed times the amount of time unemployed less the marginal cost of raising those funds from the employed state. This marginal cost is given by the product of the average marginal utility of consumption while employed and $\frac{\partial \tau}{\partial b}$. To see why the consumption-UI constraint condition is needed to establish this result, consider an agent who faces a binding subsistence constraint while unemployed. Note that raising b does *not* loosen this constraint. Consequently, the marginal value of UI benefits cannot be directly inferred from the marginal utility of consumption, since the agent will not be able to equate these two marginal values when optimizing. This type of constraint prevents us from writing the effect of an increase in benefits in terms of marginal utilities of consumption, which is the key step in obtaining a reduced-form expression for b^*.

Approximation for Average Marginal Utilities. It can be shown that the optimal benefit rate depends exactly on the difference in average marginal utilities between the employed and unemployed states, $Eu'(c_{u,t}) - Eu'(c_{e,t})$, under the preceding assumptions. However, it is convenient to identify conditions under which the average marginal utility in each state can be approximated by the marginal utility of average consumption in that state (i.e., when the order of integration can be switched). This is the purpose of the next result.

Lemma 2 *Suppose the third-order terms of u are small ($u''' \approx 0$). Then the average marginal utility of consumption in state θ is approximately the marginal utility of consumption at $\overline{c_\theta}$:*

$$\begin{aligned} Eu'(c_{e,t}) &\approx u'(\overline{c_e}) \\ Eu'(c_{u,t}) &\approx u'(\overline{c_u}) \end{aligned} \tag{17}$$

If fourth-order terms of u are small ($u'''' \approx 0$),

$$\begin{aligned} Eu'(c_{e,t}) &\approx u'(\overline{c_e})(1+\gamma\rho s_e^2) \\ Eu'(c_{u,t}) &\approx u'(\overline{c_u})(1+\gamma\rho s_u^2) \end{aligned} \tag{18}$$

where γ and ρ are defined as in Proposition 1 and $s_\theta = \frac{[E(c_e-\overline{c_e})^2]^{1/2}}{\overline{c_e}}$ is the coefficient of variation of consumption in state θ.

Proof. Consider the employed case. Take a Taylor expansion of u around $\overline{c_e}$:

$$u'(c_{e,t}) \approx u'(\overline{c_e}) + u''(\overline{c_e})(c_e - \overline{c_e}) + \frac{1}{2}u'''(\overline{c_e})(c_e - \overline{c_e})^2$$

Since $Ec_e = \overline{c_e}$ by definition, it follows that

$$Eu'(c_{e,t}) = u'(\overline{c_e}) + \frac{1}{2}u'''(\overline{c_e})E(c_e - \overline{c_e})^2$$

and substituting in the definitions of ρ and γ yields (18). If $u''' = 0$, $\rho = 0$, and (18) reduces to $u'(\overline{c_e})$. Similar reasoning establishes the result for the unemployed case. ■

When utility is well approximated by a quadratic function in the region of consumption fluctuations within an employment state, only the average consumption level is needed to determine average marginal utility. This is a standard certainty equivalence result for quadratic functions. If one wishes to take third order terms into account, the formula also depends on the coefficient of relative prudence ρ and the coefficient of variation of consumption in each state.

Welfare Gain from UI. With these preliminaries taken care of, we can now turn to the welfare gain from an increase in b. I derive an expression for the welfare gain from an increase in b relative to the welfare gain of a permanent one-dollar increase in consumption in the employed state ($\frac{\partial V/\partial b}{(1-d)Eu'(c_{e,t})}$). This formula provides an intuitive money-metric to compute the welfare gain associated with social insurance.

Lemma 3 *The change in welfare from an increase in b relative to the change in welfare from a permanent increase in consumption while employed is approximately*

$$\frac{\partial V/\partial b}{(1-d)Eu'(c_{e,t})} \approx \frac{d}{1-d}\left[\frac{\Delta \overline{c}}{c}(b)\gamma\left[1 + \frac{1}{2}\rho\frac{\Delta \overline{c}}{c}(b)\right] - \frac{\varepsilon_{d,b}}{1-d}\right] \tag{19}$$

where

$$\begin{aligned}
\frac{\Delta\overline{c}}{c} &= \frac{\overline{c_e}-\overline{c_u}}{\overline{c_e}} = \textit{mean consumption drop during unemployment} \\
\gamma &= -\frac{u''(\overline{c_e})}{u'(\overline{c_e})}\overline{c_e} = \textit{relative risk aversion} \\
\rho &= -\frac{u'''(\overline{c_e})}{u''(\overline{c_e})}\overline{c_e} = \ \textit{relative prudence} \\
\varepsilon_{d,b} &= \frac{\partial \log d}{\partial \log b} = \ \textit{elasticity of duration w.r.t. benefits}
\end{aligned}$$

Proof. From Lemma 1, we have

$$\frac{\partial V}{\partial b} = -\frac{\partial \tau}{\partial b}(1-d)Eu'(c_{e,t}) + dEu'(c_{u,t}) \tag{20}$$

Differentiating the UI budget constraint implies

$$\frac{\partial \tau}{\partial b} = \frac{d(1-d)+b\frac{\partial d}{\partial b}}{(1-d)^2}$$

and plugging this expression into (20) and simplifying gives:

$$\begin{aligned}
\frac{\partial V}{\partial b} &= -dEu'(c_{e,t})[1+\frac{1}{1-d}\frac{b}{d}\frac{\partial d}{\partial b}] + dEu'(c_{u,t}) &(21)\\
&= dEu'(c_{u,t}) - dEu'(c_{e,t})[1+\frac{\varepsilon_{d,b}}{1-d}] &(22)
\end{aligned}$$

Rearranging (22), it follows that

$$\frac{\partial V/\partial b}{(1-d)Eu'(c_{e,t})} = \frac{d}{1-d}\{\frac{Eu'(c_{u,t})-Eu'(c_{e,t})}{Eu'(c_{e,t})} - \frac{\varepsilon_{d,b}}{1-d}\} \tag{23}$$

This formula gives an exact expression for the welfare gain from increasing b. To simplify this expression, apply the quadratic approximation given in (17) of Lemma 2 for $Eu'(c_{u,t})$

and $Eu'(c_{e,t})$ to obtain

$$\frac{\partial V/\partial b}{(1-d)Eu'(c_{e,t})} = \frac{d}{1-d}\{\frac{u'(\overline{c_u}) - u'(\overline{c_e})}{u'(\overline{c_e})} - \frac{\varepsilon_{d,b}}{1-d}\}$$

The first term in this expression can be approximated using a Taylor expansion analogous to (9) in Proposition 1. Using the definitions of γ, ρ, and $\frac{\Delta\bar{c}}{c}$ yields (19). ■

Lemma 3 shows that the three reduced-form parameters identified by Baily, along with the correction factor ρ, are sufficient to determine the welfare gains from social insurance in a general setting. The result indicates that the welfare gains from social insurance are greater when shocks are more common ($\frac{d}{1-d}$ large). It also confirms the intuition that larger consumption-smoothing benefits and a smaller duration response yield a larger welfare gain.

In Lemma 3, we used the quadratic approximation given in Lemma 2 for $Eu'(c_{\theta,t})$ instead of the cubic approximation given in (18).[6] This is because the quadratic approximation is reasonably accurate for the purpose of computing $\frac{\partial V/\partial b}{(1-d)Eu'(c_{e,t})}$ and b^*. If the third-order approximation were used to simplify (23) instead, we would obtain

$$\frac{\partial V/\partial b}{(1-d)Eu'(c_{e,t})} = \frac{d}{1-d}\{[\frac{\Delta\bar{c}}{c}(b^*)\gamma[1+\frac{1}{2}\rho\frac{\Delta\bar{c}}{c}(b^*)]+1]F - 1 - \frac{\varepsilon_{d,b}}{1-d}\} \tag{24}$$

where $F = \frac{1+\gamma\rho s_u^2}{1+\gamma\rho s_e^2}$ is a correction factor that accounts for differences in the volatility of consumption in the two states. This equation shows that the bias of the quadratic approximation is proportional to the *ratio* of the coefficient of variation of consumption in the unemployed and employed states. A rough estimate from panel data on consumption in the Consumer Expenditure Survey suggests that s_u and s_e are around 20%, with $\frac{s_u}{s_e}$ almost certainly between $\frac{1}{2}$ and 2. In this range, using a power utility function and other parameters chosen as described in the earlier calibration exercise, the exact value of $\frac{\partial V/\partial b}{(1-d)Eu'(c_{e,t})}$ and the

[6] To be clear, note that Lemma 3 still uses a third-order approximation for $u'(\overline{c_u}) - u'(\overline{c_e})$ as in the static model; it is only when approximating $Eu'(c_{\theta,t})$ within a state θ that we are ignoring the u''' terms.

approximate value given by (19) differ by less than 2%. I therefore proceed by assuming $Eu'(c_{\theta,t}) \approx u'(\overline{c_\theta})$ below. However, for those who desire a more precise estimate of b^*, a formula involving the correction factor F can be obtained by setting (24) equal to zero and solving.

Optimal Benefit Level. The generalized formula for the optimal benefit level follows directly from the preceding result on welfare gains.

Proposition 2 *The optimal benefit rate b^* is approximately defined by*

$$\frac{\Delta\overline{c}}{c}(b^*)\gamma[1+\frac{1}{2}\rho\frac{\Delta\overline{c}}{c}(b^*)] \approx \frac{\varepsilon_{d,b}}{1-d} \qquad (25)$$

where $\frac{\Delta\overline{c}}{c}$, γ, ρ, and $\varepsilon_{d,b}$ are defined as in Lemma 3.

Proof. (a) Necessity. The optimal benefit rate must satisfy

$$\partial V/\partial b(b^*) = 0 \qquad (26)$$

Using the expression for $\partial V/\partial b$ in (19) implies

$$\frac{\Delta c}{c}(b^*)\gamma[1+\frac{1}{2}\rho\frac{\Delta\overline{c}}{c}(b^*)] - \frac{\varepsilon_{d,b}}{1-d} \approx 0$$

and rearranging yields (25).

(b) Sufficiency. To establish that the b^* defined by (25) is a global maximum, we show that $V(b)$ is strictly concave in b. Differentiating the expression for $\partial V/\partial b$ in (22) gives $\partial^2 V/\partial b^2 < 0$ under the conditions of assumption 4, completing the proof. ■

The formula for b^* in the general case (25) coincides with the corresponding condition (2) for the static model, with two exceptions. First, the inputs reflect *average* behavioral responses, because these values need not be constant over time in the more general setting. The consumption drop that is relevant is the percentage difference between average consumption while employed and unemployed. The $\varepsilon_{d,b}$ term is the effect of a 1% increase in b on the fraction of his life the agent spends unemployed. This is equivalent to the effect of an increase in b on the average unemployment duration if the frequency of layoffs is not affected by b.[7] It should be noted that empirical estimates of consumption-smoothing benefits and duration responses in the literature are typically based on analyses of single spells within a lifetime. However, to the extent that the cross-sectional distribution of the individuals in a given sample is representative of average behavior for a given individual over his lifetime, the estimates from these studies can be used in the general formula.[8]

The second difference in the formula for the general case is that it has an added $\frac{1}{1-d}$ term that magnifies the elasticity of durations with respect to benefits. This is because raising consumption while unemployed by \$1 generates not only the added cost of providing benefits during a longer duration, but now also causes a reduction in tax collection because the agent spends less time employed. In practice, this latter effect is likely to be small, especially if the agent is usually employed so that $1-d$ is close to 1. Hence, in approximation, the formula for the static case carries over directly to the general case.

2.3 Implications

Proposition 2 implies that many of the extensions that have followed Baily's analysis do not require a reformulation of the optimal benefit rule proposed here. Some notable cases include:

1. Borrowing constraints. If the consumption-smoothing benefits of UI are estimated using data on consumption rather than simulated based on assumptions about primitives,

[7] When benefits affect the frequency of layoffs, one must take both the average duration effect and the the layoff elasticity into account to compute $\varepsilon_{d,b}$.

[8] This point also suggests that heterogeneity across agents in behavioral responses (as documented e.g., in Crossley and Low, 2005) should not affect the formula for b^* in a universal-benefit program if one uses population averages for $\frac{\Delta c}{c}$ and $\varepsilon_{d,b}$ in (25). If there is also heterogeneity in γ across individuals, aggregation of utilities is more complicated and depends on the structure of the social welfare function. Analysis of this important issue is left to future work.

the particular features of the underlying borrowing constraints that agents face become irrelevant. Tighter borrowing constraints will lead to a larger observed consumption-smoothing effect in the data, and therefore raise the optimal benefit level, consistent with the results of Flemming (1978) and Crossley and Low (2005).

2. Endogenous insurance markets. Golosov and Tsyvinski (2005) show that complete private insurance markets can achieve 97% of the welfare gains from government intervention in a dynamic model of optimal contracts based on calibrations for disability insurance. Since the general case analyzed above allows the agent to purchase insurance contracts with an unspecified load (by introducing a new choice variable in x), (25) is robust to endogenous private insurance markets.[9] Intuitively, these effects are captured through the $\frac{\Delta c}{c}$ parameter, which will be smaller (and therefore imply a lower b^*) if agents have already made informal or formal insurance arrangements.

3. Multiple consumption goods. The proposition shows that it is sufficient to obtain consumption-smoothing estimates for a single good (e.g., food), provided that the appropriate risk aversion parameter (e.g., curvature of utility over food) is used in conjunction with this estimate. This is because all the other consumption goods can be placed in the set of x other choice variables. This point is relevant for two reasons. First, one may be concerned that existing estimates of consumption-smoothing have limited applicability because they only consider a few categories of consumption such as food (see Gruber 1998). The result here suggests that from a normative perspective, it is not critical to have consumption-smoothing estimates for the full consumption bundle. Second, there is a concern that the durability of consumption may affect optimal UI policy. Browning and Crossley (2003) show that postponing expenditures on small durables such as clothes can provide households an additional smoothing channel via an "internal capital market," thereby lowering the optimal

[9] It should be noted that Golosov and Tsyvinski analyze the welfare gains from intervention when the government has a much more general class of instruments than the constant benefit/tax case considered here. Hence, the optimal policy derived here does not replicate their optimal mechanism or welfare calibrations.

level of unemployment insurance. These effects can be captured through additional consumption goods and constraints in the general case analyzed above, and ultimately do not affect b^* conditional on the consumption-smoothing elasticity for food.

4. Search and human capital benefits of UI. Unemployment benefits could affect subsequent wages by subsidizing search and improving job match quality. UI could also increase incentives for risk-averse workers to undertake risky human capital investments (Brown and Kaufold 1988). Under the assumption that UI is financed by a lump-sum tax, the increment in wages from these effects has no effect on UI tax collections, and is therefore fully internalized by the worker. Consequently, these effects can be ignored in calculating the optimal level of benefits; only the consumption-smoothing benefits need to be considered.

5. Leisure value of unemployment. Leisure is also simply another choice variable in the general framework, and thus has no impact on the optimal benefit equation. The intuition that all else held fixed, greater leisure value should raise b^* comes through the $\frac{\Delta c}{c}$ term. Intuitively, if unemployment has higher leisure value (or if there are search benefits), the agent is willing to sacrifice more consumption to take more time off, leading to a larger consumption drop and higher optimal benefit rate. However, *conditional* on knowing $\frac{\Delta c}{c}$ and $\varepsilon_{d,b}$, leisure or search benefits have no additional effect on the optimal benefit rate because they are already taken into account via agent optimization.

6. Dynamic search and savings behavior. Lentz (2004) and others have structurally estimated job search models which permit agents to optimize savings dynamically and allow for potentially complex search dynamics. These models are considerably richer than the static Baily framework, but are nested within the general case considered here. Hence, they should not change conclusions about the optimal benefit rate if it is calculated using (25).

The robustness of (25) to variations in the underlying model suggests that it should provide a reliable estimate of the optimal level of unemployment insurance, as well as other types of social insurance. Unlike the alternative "structural" approach, there is no need to explicitly specify the agent's discount factor, the functional form of u, the stochastic process for θ as a function of search effort, etc. As Gruber (1997) observes, each of these parameters is difficult to estimate, making it hard to implement the structural approach. However, this reduced-form approach also comes with some potential dangers that arise from failing to specify the underlying structure. The next section describes these concerns.

3 The Apparent Irrelevance of Some Parameters

The most surprising feature of the optimal benefit rate formula (25) is perhaps that it does *not* depend on many elasticities that affect the costs and benefits of unemployment insurance. Based on the intuition that estimating the effect of UI on various behaviors and outcomes is helpful in making normative judgements, prior studies have investigated the effect of the benefit level on reemployment wages (Ehrenberg and Oaxaca 1968), reservation wages (Feldstein and Poterba 1984), pre-unemployment savings (Engen and Gruber 1995), spousal labor supply (Cullen and Gruber 2000), and job-match quality (Centeno 2004). According to the formula, none of these empirical results are relevant to the normative analysis of unemployment insurance.[10]

How can the formula be reconciled with the intuition that these other factors should matter for b^*? The key is to recall that the elasticities that enter the formula are all functions of other aspects of the agent's behavior and preferences. The effects identified above affect b^* by altering the values of the main inputs (γ, ρ, $\frac{\Delta c}{c}$, and $\varepsilon_{d,b}$) that enter the formula directly. I now illustrate this point formally by focusing on a specific example – the importance of income vs. substitution effects in determining the optimal benefit level – where the potential pitfalls in applying (25) are apparent.

[10]This point applies only if these other behaviors do not have externalities. For instance, if higher aggregate savings leads to more investment and spillovers that raise growth, it may be important to evaluate the effect of UI benefits on savings rates from a normative perspective.

3.1 Income and Substitution Effects

A central result of the optimal tax literature is that the efficiency consequences of taxation, and hence optimal tax rates, are determined by substitution elasticities (and not uncompensated elasticities).[11] Since a UI program is abstractly a particular type of redistributive tax across two types, it may be surprising that the optimal benefit rate appears to depend on the *total* (uncompensated) elasticity of unemployment durations with respect to benefits and not just the substitution elasticity. Understanding this issue is particularly relevant because there is accumulating evidence indicating that unemployment and UI benefits have substantial income effects. Mincer (1962) found that married women's labor supply responds 2-3 times as much to transitory fluctuations in husbands' incomes due to unemployment as it does to permanent differences in husbands' incomes. Cullen and Gruber (1998) exploit variation in UI benefit levels to estimate an income elasticity for wives' labor supply between -0.49 and -1.07. More recently, Chetty (2005) finds that lump-sum severance payments (which have pure income effects) significantly raise average durations.

Although the magnitudes of income and substitution effects do not enter the formula for directly, they affect b^* through the coefficient of relative risk aversion, γ. To see this, let us return to the static model of section 2.1 for simplicity. To consider income effects, suppose agents receive a lump sum income payment b_0 when unemployed (e.g., a severance payment). The first order condition that determines the agent's choice of d in the unemployed state is

$$(w - b)u_c(c_u) = \psi_d(d) \tag{27}$$

where $c_u = A_0 + b_0 + bd + w(1 - d)$ is consumption in the unemployed state. Intuitively, the agent equates the marginal benefit of extending his duration by one day, ψ_d, with the

[11] The need to distinguish between income and substitution effects has been known since at least Ramsey's analysis of optimal commodity taxation; for a more recent example in the context of income taxation, see Saez (2001).

marginal consumption cost of doing so, which is the foregone wage $(w-b)$ times the marginal utility of consumption.

The result follows from the comparative statics implied by this first order condition. Implicit differentiation of (**??**) with respect to b_0 and b yields

$$\begin{aligned} \frac{\partial d}{\partial b_0} &= \frac{(w-b)u_{cc}}{(w-b)^2 u_{cc} + \psi_{dd}} \\ \frac{\partial d}{\partial b} &= \frac{d(w-b)u_{cc} - u_c}{(w-b)^2 u_{cc} + \psi_{dd}} \end{aligned} \tag{28}$$

Using a Slutsky decomposition, the pure price (substitution) effect $\frac{\partial d^c}{\partial b}$ for duration (which is one minus labor supply here) is given by

$$\frac{\partial d^c}{\partial b} = \frac{\partial d}{\partial b} - d\frac{\partial d}{\partial b_0} \tag{29}$$

This implies

$$\begin{aligned} \frac{\partial d/\partial b_0}{\partial d/\partial b - d\partial d/\partial b_0} &= \frac{\partial d/\partial b_0}{\partial d^c/\partial b} = -(w-b)\frac{u_{cc}}{u_c} \\ \Rightarrow \gamma = -\frac{u_{cc}}{u_c}c_u &= \frac{\partial d/\partial b_0}{\partial d^c/\partial b}\frac{c_u}{w-b} \end{aligned} \tag{30}$$

Equation (30) shows that γ is related to the ratio of the income and substitution effects of UI benefits on unemployment durations. This connection between risk aversion and duration elasticities is a special case of a calibration theorem proved in Chetty (2004), which shows that labor supply elasticities place tight bounds on risk aversion in a general model with arbitrary non-separable utility. To see the rough intuition for this result, consider the effects of lump-sum and proportional benefit reductions on duration. An agent's duration response to a proportional benefit (b) reduction is directly related to u_c, the marginal utility of consumption: The larger the magnitude of u_c, the greater the benefit of an additional dollar of income, and the more the agent will work when his effective wage $(w-b)$ goes up.

The duration response to an increase in the severance payment (b_0) is related to how much the marginal utility of consumption changes as consumption changes, u_{cc}. If u_{cc} is large, the marginal utility of consumption rises sharply as income falls, so the agent will shorten his duration a lot to earn more money when his severance pay falls. Since γ is proportional to u_{cc}/u_c, it follows that there is a connection between γ and the ratio of income and price elasticities of benefits.[12]

The preceding derivation establishes that large income effects do indeed generate a higher optimal benefit rate, by raising the risk aversion parameter. Yet, *conditional* on the value of γ, $\partial d/\partial b_0$ and $\partial d^c/\partial b$ play no role in determining b^*. This point shows why the reduced-form formula should be used cautiously. When (25) is calibrated with a low value of γ – as in some of the cases considered by Gruber, 1997 – one is at risk of contradicting the evidence of large income effects described above. Put differently, (25) is only one representation of the formula for optimal benefits; another representation would involve income and substitution elasticities and the consumption drop, but not γ. Since this alternative representation might yield different conclusions about b^*, it is important that the inputs used to calculate b^* are consistent with other estimates of behavioral responses.[13]

The income effects case is just one example of the danger in applying the reduced-form formula without carefully considering the many moment restrictions implied by a fully specified structural model. The broader point is that while only a small set of parameters need to be estimated to draw normative conclusions about social insurance, there can be considerable value in estimating other behavioral responses to perform "overidentification" tests of the validity of the main estimates. The structural and reduced-form approaches to evaluating social insurance are thus quite complementary.

[12] The derivation above assumes that there is no complementarity between durations (leisure) and consumption. In the general case, γ is a function of the ratio of income and price elasticities as well as the degree of complementarity. See Chetty (2004) for details.

[13] This point applies equally to the optimal tax literature. There, optimal tax rates depend on income and substitution elasticites, but could equivalently be written in terms of γ instead. Hence, one should ideally test whether the different representations yield similar predictions for optimal taxation as well.

4 Conclusion

This paper has shown that a simple, empirically implementable formula can be used to compute the welfare gains and optimal level of social insurance in a wide class of stochastic dynamic models. Though the analysis focused on unemployment, this formula can also be applied to analyze other policies (e.g. disability insurance or welfare programs) if one restricts attention to the optimal policy in a two-state model with constant benefits in one state and a constant tax in the other. Hence, empirical estimates of reduced-form behavioral responses can be fruitfully applied to obtain fairly robust estimates of the optimal size of many large government expenditure programs.

This result also helps define the types of departures from standard models that could shed further light on optimal social insurance. Some possibilities include:

1. Endogenous takeup decision. We assumed that all agents receive benefits upon unemployment automatically, so that at the margin, raising benefits had the same welfare consequences as raising consumption while unemployed by an equivalent amount. In practice, takeup rates for social insurance programs are far below 100% and are sensitive to the level of benefits (Andersen and Meyer 1997), suggesting that this departure could have quantitatively significant impacts on the optimal benefit level.

2. Myopic agents. The envelope arguments exploited above rely on the assumption that agents are optimizing. If agents are unable to smooth consumption themselves because they are myopic and do not save enough, the formula for the optimal benefit level may differ.

3. General equilibrium effects. The analysis of this paper assumed that all behavioral responses to UI were solely determined by the agent. This assumption was important because the envelope conditions used to obtain the formula for optimal benefits relied on the idea that all endogenous variables in the model are chosen to maximize the agent's utility. Obtaining a reduced-form formula that takes equilibrium responses by firms into account would be very useful.

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