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Raj Chetty
Adam Szeidl

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Consumption Commitments and Habit Formation
Raj Chetty and Adam Szeidl
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ABSTRACT

We analyze the implications of household-level adjustment costs for the dynamics of aggregate consumption. We show that an economy in which agents have "consumption commitments" is approximately equivalent to a habit formation model in which the habit stock is a weighted average of past consumption if idiosyncratic risk is large relative to aggregate risk. Consumption commitments can thus explain the empirical regularity that consumption is excessively sensitive and excessively smooth, findings that are typically attributed to habit formation. Unlike habit formation and other theories, but consistent with empirical evidence, the consumption commitments model predicts that excess sensitivity and smoothness vanish for large shocks. These results suggest that behavior previously attributed to habit formation may be better explained by adjustment costs. We develop additional testable predictions to further distinguish the commitment and habit models and show that the two models have different welfare implications despite generating similar aggregate consumption patterns in many environments.

Raj Chetty
Department of Economics
Harvard University
1805 Cambridge St.
Cambridge, MA 02138
and NBER
chetty@fas.harvard.edu

Adam Szeidl
Department of Economics
Central European University
Nador u. 11
Budapest, Hungary
and NBER
szeidla@ceu.hu

1 Introduction

Many households have "consumption commitments" such as housing that are costly to adjust in response to fluctuations in income. Chetty and Szeidl (2007) document that more than 50% of the average U.S. household's budget remains fixed when households face moderate income shocks such as unemployment. Olney (1999) gives historical evidence on the importance of households' installment finance commitments during the Great Depression. Such consumption commitments can amplify the welfare costs of shocks because—for shocks that are not large enough to induce a change in commitments—households are forced to concentrate all reductions in wealth on changes in adjustable (e.g., food) consumption. Through this mechanism, consumption commitments have been cited as an explanation for microeconomic evidence in domains ranging from wage rigidities (Postlewaite, Samuelson and Silverman 2008) to added-worker effects (Chetty and Szeidl 2007), housing choices of couples (Shore and Sinai 2009), and portfolio choice (Chetty and Szeidl 2012).

In this note, we show that household-level consumption commitments also have important implications at the macroeconomic level, especially for the dynamics of aggregate consumption. We show that when idiosyncratic risk is large relative to aggregate risk, nonlinear dynamics due to commitments at the household level aggregate into approximately linear dynamics for larger groups, producing patterns that are approximately identical to representative-agent habit formation.[1] In particular, commitments can explain the key facts—often attributed to habit formation—that consumption exhibits excess sensitivity and excess smoothness. But the commitments model also explains empirical regularities that are not consistent with standard habit formation models and their variants. For instance, the commitments model predicts that excess sensitivity and smoothness vanish for large shocks, providing foundations for an empirical phenomenon termed the "magnitude hypothesis" (Japelli and Pistaferri 2010). Hence, our results suggest that some of the behavior previously attributed to habit formation may be due to adjustment costs in consumption. The distinction between the two models matters because they generate different comparative statics and yield different welfare implications.

We begin our analysis in Section 2 with a household-level model in which changing the consumption of certain goods is costly. These costs could reflect either transaction costs or mental costs such

[1] Beginning with Ryder and Heal (1973), models in which habit is an average of past consumption are widely used in economics. Sundaresan (1989), Constantinides (1990), Campbell and Cochrane (1999) and Boldrin, Christiano and Fisher (2001) use variants of this model in macro-finance, while Carroll, Overland and Weil (2000), Fuhrer (2000), Christiano, Eichenbaum, and Evans (2003) and a literature building on this work uses variants in macroeconomics and monetary policy.

as the effort required for changing plans (Grossman and Laroque 1990, Chetty and Szeidl 2007). We show that in a partial equilibrium economy populated by many agents, aggregate dynamics can be represented by the preferences of a representative agent, whose utility function involves a state variable corresponding to aggregate commitments. This state variable is endogenous: each household chooses commitments to maximize expected utility, and hence aggregate commitments are shaped by the expectations agents hold on the dates on which they update.

In Section 3, we characterize the aggregate dynamics of consumption commitments. Our main result is a precise characterization showing that when the ratio of idiosyncratic to aggregate consumption risk is large, aggregate commitments are well approximated by a (linear) weighted average of past consumption with fixed weights. As a result, the commitments economy is closely approximated by a representative-agent habit model in which the habit stock is a weighted average of past consumption.[2] To understand this result, note that the impulse response to aggregate shocks in the commitments model depends on the distribution of agents in the inaction region for commitment consumption. Aggregate shocks perturb this distribution, while idiosyncratic shocks push it back towards its steady state. When idiosyncratic risk is large, the second effect dominates, and hence on most dates the distribution remains close to its steady state. As a result, impulse-responses are approximately state-independent, which in turn can be generated in a habit model with fixed weights. Since in practice idiosyncratic risk is much larger than economy-wide risk (e.g., Deaton, 1991, Carroll, Hall, and Zeldes 1992), we interpret this result as showing that consumption commitments and habit formation generate similar aggregate consumption dynamics in typical environments.

While the commitments model matches the predictions of habit models in a commonly-studied domain, it yields new predictions in other settings. In Section 4, we illustrate the similarities and differences between the two models using three applications. We first consider the consumption response to income shocks. Two well-documented empirical regularities are that consumption does not respond fully to contemporaneous shocks ("excess smoothness," Deaton 1987) and that anticipated changes affect current consumption ("excess sensitivity," Flavin 1981). Fuhrer (2000) argues that both of these facts can be explained by a habit formation model in which habit responds sluggishly to shocks, which is one reason that habit models have been influential in macroeconomics. Our equivalence result implies that the commitments model also produces sluggish responses in most

[2] Our characterization is analytical. Previous studies of aggregate consumption with adjustment costs use numerical techniques (Marshall and Parekh 1999), or time-dependent adjustment (Lynch 1996, Gabaix and Laibson 2001, Reis 2006). These studies focus on a model with a single illiquid good, as in Grossman and Laroque (1990).

periods and therefore also explains excess sensitivity and smoothness.

However, a key prediction of the commitments model—but not the habit model—is that excess sensitivity of consumption vanishes for large shocks. When such shocks occur, households adjust their commitments and thus behave more in line with the permanent income model. This prediction helps explain a large body of micro evidence about consumption responses to shocks termed the "magnitude hypothesis" by Japelli and Pistaferri (2010). For example, Hsieh (2003) shows that Alaskan households' consumption is excessively sensitive to tax refunds (a small income change), but not to payments from the Alaska Permanent Fund (a large income change). Similarly, Parker (1999), Souleles (1999) and Souleles (2002) find excess sensitivity to small income changes associated with tax and social security payments, but Browning and Collado (2001) and Souleles (2000) find no excess sensitivity to large changes in disposable income coming from bonus salary payments and college tuition. Such facts are difficult to explain with standard habit models, in which the impulse response to income shocks does not depend on shock size. They can, however, be explained by the commitments model, suggesting that a significant part of consumption behavior attributed to habits in preferences may be due to adjustment costs in consumption.[3]

In our second application, we explore how consumption dynamics are affected by changes in the environment. Because commitments are chosen by the consumer, they respond endogenously to such changes. In contrast, the parameters determining reduced-form habit are exogenous and do not vary with the environment. We show that reductions in risk or in expected growth increase sluggishness of consumption in the commitment economy because they reduce the frequency of adjustment. This result yields a new prediction about excess sensitivity: consumption should respond more quickly to shocks in high-growth and high-risk environments. At the macroeconomic level, this logic suggests that recessions may be shorter lived in rapidly growing economies, in which agents reorganize their arrangements frequently. Similarly, recessions may last longer in welfare states that have large social safety nets than in economies with higher levels of uninsured idiosyncratic risk. Evidence on these predictions, which to our knowledge have not yet been studied, would help further distinguish between the commitments and habit models.

In our final application, we turn to the welfare implications of the two models. An advantage of the commitment model is that it yields a natural welfare measure based on expected utility. To compare welfare across models, we make the (somewhat arbitrary) assumption that the habit

[3] Under the assumption of perfect capital markets neither model can explain excess sensitivity to the *timing* of income, as documented for example in Johnson, Parker and Souleles (2006). As Agarwal, Liu and Souleles (2006) show, those results are likely explained by credit constraints.

consumer derives utility only from surplus consumption rather than the habit stock itself. The welfare cost of shocks is generally amplified in both the commitments and the habit models relative to the neoclassical model. However, the endogenous evolution of commitments modifies this prediction for large shocks. Because agents can abandon their commitments but not their habits in extreme events, the welfare cost of large shocks is smaller in the commitments model than in the habit model. This result implies that the optimal size of social insurance programs that insure large shocks such as disability or job displacement may be smaller than predicted by analyses using habit models such as Ljungquist and Uhlig (2000). We also find that reducing idiosyncratic risk—e.g., by expanding social insurance programs—can increase the welfare cost of aggregate shocks by slowing the rate of adjustment.

Our results build on two strands of prior research. First, several papers have pointed out the qualitative similarity between the commitment and habit models. Dybvig (1995) examines ratcheting consumption demand under extreme habit persistence and motivates these preferences by pre-commitment in consumption. Flavin and Nakagawa (2008) study asset pricing in a two-good adjustment cost model and note the similarity to habit. Postlewaite, Samuelson and Silverman (2008), Fratantoni (2001), and Li (2003) also study two-good models and note this similarity in other contexts. We contribute to this literature by analyzing aggregate dynamics, presenting formal conditions under which commitments and habit formation are similar, and deriving new behavioral and welfare predictions that distinguish the two models (summarized in Section 5).

Second, our results also build on an earlier literature on industry dynamics, including Bertola and Caballero (1990), Caballero (1993), and Caballero and Engel (1993, 1999). Our main innovation relative to this literature is to develop a theory of state-dependent impulse responses, which we then use to derive an analytical characterization of aggregate dynamics. Our habit equivalence result is also related to Khan and Thomas (2008), who establish approximate linearity in a general equilibrium production setting computationally. We establish approximate linearity—emerging through a different mechanism—in a partial equilibrium consumption setting analytically.

2 A Model of Consumption Commitments

In this section, we present our model, characterize household behavior, and show the existence of a representative consumer in our setting. We present a map of all proofs in the Appendix, and full technical details in a Supplementary Appendix.

2.1 Setup

We study a continuous-time partial equilibrium economy with a unit mass of consumers. We index agents by $i \in [0,1]$, but suppress the index in notation for simplicity when it does not cause confusion. Each agent maximizes expected lifetime utility given by

$$\mathrm{E} \int_0^\infty e^{-\rho t} \left(\kappa \frac{{a_t}^{1-\gamma}}{1-\gamma} + \frac{{x_t}^{1-\gamma}}{1-\gamma} \right) \mathrm{d}t \tag{1}$$

where ρ is the discount rate. Each agent consumes two goods: a_t and x_t measure the service flows from adjustable (e.g., food) and commitment (e.g., housing) consumption, and κ measures the relative preference for adjustables. Adjusting commitment consumption x_t involves a fixed monetary cost, which may depend both on the pre-existing and new service flow from commitment consumption. Formally, denoting $x_{t-} = \limsup_{s \nearrow t} x_s$, if on date t the agent sets $x_t \neq x_{t-}$, he must pay a monetary cost of $\lambda_1 x_{t-} + \lambda_2 x_t$ where $\lambda_1, \lambda_2 \geq 0$ and at least one of them is positive.[4]

We are interested in characterizing how individual heterogeneity translates into aggregate dynamics in the presence of consumption commitments. We therefore study an economy in which agents are exposed to both idiosyncratic and aggregate risk. We introduce these risks by assuming that agents have access to a variety of financial assets. All of these assets pay out in the adjustable good. Each agent can invest in a bond with a constant instantaneous riskfree return r, so that the face value of the bond evolves as

$$dB_t/B_t = rdt. \tag{2}$$

We also allow two types of risky investments, both with i.i.d. returns. The source of aggregate risk is the stock market, with instantaneous return

$$dS_t/S_t = (r+\pi)dt + \sigma dz_t \tag{3}$$

where z_t is a standard Brownian motion that generates a filtration $\{\mathcal{F}_t, 0 \leq t < \infty\}$, π is the expected excess return, and σ is the standard deviation of asset returns. Households also face idiosyncratic risk in the form of a household-specific risky investment opportunity. This background risk can be thought of as entrepreneurial investment or labor income risk (where "investment" is

[4] Similar utility and adjustment cost specifications have been used by Flavin and Nakagawa (2008), Fratantoni (2001), Li (2003), and Postelwaite, Samuelson and Silverman (2008).

investment in human capital). The return of household i's entrepreneurial investment is given by

$$dS_t^{E,i}/S_t^{E,i} = (r + \pi_E)dt + \sigma_E dz_t^i$$

where the z^is are standard Brownian motions uncorrelated across households.[5] Each household is free to invest or disinvest an arbitrary amount into his private asset at any time. For simplicity, we ignore imperfections in financial markets: the agent can go long or short in all of these assets.

We consider a partial equilibrium framework in which the relative price of adjustable and commitment consumption services is exogenous and normalized to one. We also assume that the agent pays for the commitment consumption service every period (e.g., as with rental housing).[6] Denoting total wealth by w_t^i, the wealth share invested in the stock market by α_t^i, and the wealth share invested in the entrepreneurial asset and $\alpha_t^{E,i}$, the dynamic budget constraint of agent i is

$$dw_t^i = w_t^i \left[\alpha_t^i \frac{dS_t}{S_t} + \alpha_t^{E,i} \frac{dS_t^{E,i}}{S_t^{E,i}} + \left(1 - \alpha_t^i - \alpha_t^{E,i}\right) \frac{dB_t}{B_t} \right] - \left(a_t + x_t\right) dt - 1_{\{x_{t-} \neq x_t\}} \left(\lambda_1 x_{t-} + \lambda_2 x_t\right). \tag{4}$$

We make the standard assumption that $\rho > (1-\gamma) r + \left[\pi^2 / \left(2\sigma^2\right) + \pi_I^2 / \left(2\sigma_I^2\right)\right] (1-\gamma)/\gamma$, which ensures that with zero adjustment costs, expected consumption utility grows at a smaller rate than the discount rate in the optimum, generating finite lifetime utility.

2.2 Discussion of Modelling Choices

Consumption commitments. As a benchmark, we interpret the adjustment cost as the physical transaction cost inherent in changing consumption of illiquid durables such as houses, cars, or appliances, or the cost of renegotiating service contracts (Attanasio 2000, Eberly 1994, Grossman and Laroque 1990).[7] However, the adjustment cost may also represent costs required to respond to new circumstances and make new choices (Browning and Collado 2001, Ergin 2003), and may arise from attention costs or computing costs (Ameriks, Caplin and Leahy, 2003, Reis, 2006).

Partial equilibrium. We focus on a partial equilibrium setting in which prices are exogenous. This is appropriate when—as in our application on excess sensitivity and smoothness—we study

[5] Sun (1998) develops the mathematical foundations for working with a "large number" of independent stochastic processes, and derives exact laws of large numbers in these settings.

[6] Because the agent is free to borrow, this model is equivalent to one in which the agent buys and sells the capitalized service flow at price x/r on every adustment date.

[7] More recently, several studies have examined state-dependent models with two consumption components, one freely adjustable and one that is costly to adjust (Flavin and Nakagawa 2008, Fratantoni 2003, Li 2003).

groups of individuals who are small from the perspective of the aggregate economy. In a parallel literature on adjustment costs in firm investment, Khan and Thomas (2008) have shown that even though dynamics at the firm level are highly nonlinear, general equilibrium price effects create aggregate dynamics which are approximately linear. Our habit equivalence result below is connected to this finding. It establishes that, in our partial-equilibrium consumption setting, a high ratio of idiosyncratic risk to aggregate risk can also create aggregate dynamics which are approximately linear In the Khan and Thomas model, the nonlinearities generated by simultaneous adjustment of many firms are infrequent because relative prices adjust such that the benefits of adjustment are limited. In our model, simultaneous adjustment by many agents is infrequent because idiosyncratic risk keeps the cross-sectional distribution near its steady-state shape. The fact that price effects can push in the same direction as idiosyncratic risk suggests that even when commitments are incorporated in a fully general equilibrium model—an important issue we leave for future research—the results on approximate linearity are not likely to be overturned.[8]

Preferences. When $\kappa \to \infty$, our model converges to a neoclassical model without adjustment costs, and when $\kappa = 0$ we obtain a model with only commitment consumption, as in Grossman and Laroque (1990). Because utility is time-separable, γ measures the elasticity of intertemporal substitution as well as relative risk aversion for an individual who is free to adjust both x and a. We use this functional form to make the evolution of commitments tractable. However, we believe that the intuitions underlying our main results apply more generally to other specifications as well.[9]

2.3 Household Behavior

Optimal choice of commitment and adjustable consumption. The following proposition characterizes the choice of commitment consumption using an (S,s) band. This result has been previously established for a class of models that nests our model as a special case (Flavin and Nakagawa 2008). We state the proposition here as a reference.

Proposition 1 *[Household behavior] There exist $s < s^* < S$ such that x_t^i is not adjusted as long as $x_t^i / w_t^i \in (s, S)$, but adjusted otherwise; and when it is adjusted, the household sets $x_t^i = s^* w_t^i$.*

[8] Our model can be reinterpreted as a general equilibrium setting in which the financial investments represent technologies, and each unit of adjustable consumption can be converted into a unit of commitment consumption, as in Constantinides (1990).

[9] For example, we have shown that Cobb-Douglas preferences also permit a habit representation result. In that case, the representative consumer has proportional habit utility (as in Abel, 1990) over adjustable consumption.

The behavior of adjustable consumption a_t^i can be characterized directly from the Euler equation. The appendix shows that $\log a_t$ is a random walk with drift that satisfies

$$d \log a_t^i = \mu_a \cdot dt + \frac{\pi}{\gamma\sigma} \cdot dz_t + \frac{\pi_I}{\gamma\sigma_I} \cdot dz_t^i. \tag{5}$$

Here, μ_a is the constant mean growth rate, while the second and third terms measure how a_t responds to aggregate shocks dz_t and idiosyncratic shocks dz_t^i. Motivated by (5), we define $\sigma_A = \pi/(\gamma\sigma)$ and $\sigma_I = \pi_E/(\gamma\sigma_E)$, which measure the standard deviation of adjustable consumption due to aggregate respectively idiosyncratic risk. Let $\sigma_T^2 = \sigma_A^2 + \sigma_I^2$ measure total consumption risk.

Characterizing consumption dynamics. Proposition 1 and equation (5) do not constitute a full characterization of consumption dynamics because the (S,s) rule involves the commitments-to-wealth ratio, and, by equation (4), the evolution of wealth depends on portfolio decisions. We obtain a full characterization of optimal consumption dynamics by specifying the household's choice of x_t^i as a function of a_t^i instead of w_t^i. Define $y_t^i = \log(x_t^i/a_t^i)$. It then follows from Proposition 1 that there exist numbers $L < M < U$ such that for $y_t^i \in (L, U)$, the household does not adjust x_t^i from its prior level; but as soon as y_t^i reaches L or U, the household resets x_t^i so that $y_t^i = M$.[10] This rule characterizes the choice of x_t^i with an inaction region over y_t^i. Importantly, because y_t^i depends on the endogenous variable a_t^i, this rule is a *description* of optimal behavior. However, in combination with (5), which characterizes the evolution of a_t^i, this description yields a complete characterization of consumption dynamics. In particular, given initial values for wealth w_0^i and commitment x_0^i, the household chooses the initial level of adjustable consumption a_0^i based on the long-run budget constraint, and the evolution of a_t^i and x_t^i are then completely pinned down.

A key implication of this characterization is that household consumption $c_t^i = a_t^i + x_t^i$ jumps on adjustment dates. Chetty and Szeidl (2007) document evidence consistent with this prediction and with the (S,s) policy predicted by Proposition 1. Using data from the Panel Study of Income Dynamics, they show that following "small" unemployment shocks that generate a wage income loss of less than 33 percent, most households cut food consumption significantly, while 31 percent of them move out of their house and adjust housing consumption discretely. In response to larger shocks (wage loss greater than 33 percent), households are more likely to adjust on both margins, and in particular 40 percent of them move and change housing consumption discontinuously.

[10] The existence of an inaction region representation with x/a follows from the fact that the consumption function $a_t^i = a(w_t^i, x_t^i)$ is strictly increasing in w_t^i and homogenous of degree one. As a result it can be used to map the (S,s) band over wealth into a band over adjustable consumption: for example $L = 1/a(1/s, 1)$.

Interpreting a_t as permanent income. As shown by equation (5), $\log a_t$ follows a random walk: it adjusts immediately and fully to both aggregate and idiosyncratic shocks. In fact, for an agent facing no adjustment costs ($\lambda_1 = \lambda_2 = 0$) equation (5) would also characterize the dynamics of total consumption, and hence a_t is proportional to what consumption (equivalently, permanent income) would be in the absence of adjustment costs. Thus a_t^i is usefully thought of as a measure of the permanent income of agent i. Given the equivalent characterization of the optimal policy described above, we often take the perspective that a_t^i, defined by (5), measures fluctuations in permanent income, and that x_t^i evolves in response to these fluctuations.

Initial conditions. We assume that at $t = 0$ initial wealth and commitment consumption levels are such that households are all inside their inaction region, that $a_0^i = A_0$ is the same for all households, and that the distribution of y_0^i inside the (L, U) is given by $F_0(y)$.

2.4 Existence of a Representative Consumer

We now show that aggregate dynamics in the adjustment cost model coincide with those of a single-agent economy in which aggregate commitments act as a habit-like reference point for the representative consumer. Let $X_t = \int_i x_t^i di$, $A_t = \int_i a_t^i di$, and $C_t = X_t + A_t$ denote aggregate commitment, adjustable, and total consumption at time t.

Proposition 2 *Assume that $\delta = \rho - \frac{\pi_I^2}{2\sigma_I^2}\left(1 + \frac{1}{\gamma}\right) > 0$. Then the aggregate dynamics of consumption are the optimal policy of a representative consumer with external habit formation utility*

$$E\int_0^\infty e^{-\delta t}\frac{(C_t - X_t)^{1-\gamma}}{1-\gamma}dt \tag{6}$$

where X_t follow the dynamics of aggregate commitments.

The intuition for the existence of a representative consumer is that—as in Grossman and Shiller (1982)—idiosyncratic shocks cancel in the aggregation. The presence of idiosyncratic risk also increases mean consumption growth, and to compensate for this, the representative consumer must be more patient than the individual households. An implication of Proposition 2 is that the functional form for the utility of the representative consumer is identical to the commonly used "additive habit" specification (e.g., Constantinides, 1990, Campbell and Cochrane, 1999). In this framework, the only observational difference in the aggregate between the commitment model and habit formation models comes from the dynamics of X_t.

3 Dynamics of Aggregate Commitments

We now turn to characterize the evolution of the aggregate commitments X_t. We set the stage in Section 3.1 by adapting existing results about the cross-sectional distribution to our setting. The new contribution is in the remainder of the section. In Section 3.2 we present our key idea: we represent X_t as a moving-average of past shocks, in which the weights are state-dependent impulse responses determined by the cross-sectional distribution at the time of the shock. In Section 3.3 we show that habit models admit an analogous representation in which the weights are state-independent. Finally, in Section 3.4 we identify conditions under which the weights of the commitments model are approximately state-independent, establishing approximate linearity and an equivalence with habit formation.

3.1 The Cross-Sectional Distribution

We begin with preliminary results which build on the literature on firm dynamics. Because they have identical preferences, the numbers $\{L, M, U\}$ are the same for all households in the economy. However households face different idiosyncratic shocks and as a result are in general in different locations in the (L, U) region. Characterizing the dynamics of X_t thus requires keeping track of the distribution of households. The main object we use for this purpose is the adjustable-consumption-weighted cross-sectional distribution of y, defined as $F(y,t) = (1/A_t)\int_{\{i:y_i(t)<y\}} a_t^i di$. This quantity equals the share of total adjustable consumption at date t which is consumed by households i whose y_t^i is below y in the inaction region (L, U). Given our discussion in Section 2.3 that a_t reflects lifetime resources, $F(y,t)$ can be intuitively thought of as measuring how permanent income is distributed inside the inaction region. Note that because at $t = 0$ we have $a_0^i = A_0$ for all households, $F(y,0) = F_0(y)$. Let μ_A denote the instantaneous drift of A_t and $f(y,t)$ denote the density of $F(y,t)$, the existence and dynamics of which are characterized by the following result.

Proposition 3 *$f(y,t)$ exists for all $t > 0$ and satisfies the stochastic partial differential equation for $t > 0$ and $y \in (L, U)$*

$$df(y,t) = \left[\left(\mu_A + \frac{\sigma_I^2}{2}\right)\frac{\partial f(y,t)}{\partial y} + \frac{\sigma_T^2}{2}\frac{\partial^2 f(y,t)}{\partial y^2}\right]dt + \sigma_A \frac{\partial f(y,t)}{\partial y}dz \tag{7}$$

together with the following boundary conditions:

$$\frac{\partial f(M,t)}{\partial y}^{+} - \frac{\partial f(M,t)}{\partial y}^{-} = \frac{\partial f(U,t)}{\partial y}^{-} - \frac{\partial f(L,t)}{\partial y}^{+}$$
$$f(U,t) = f(L,t) = 0 \textit{ and } f(M,t)^{+} = f(M,t)^{-}.$$

Aggregate commitments follow the dynamics

$$dX_t = A_t \frac{\sigma_T^2}{2} \cdot \left(f_y(L,t)(e^M - e^L) + f_y(U,t)(e^U - e^M) \right) dt. \tag{8}$$

This result is based on Propositions 1 and 2 in Caballero (1993) combined with Girsanov's theorem to account for a change in drift. Equation (8) shows that the evolution of commitments is "smooth" in the aggregate in the sense that it is a bounded variation process (has no dz term). This follows because the cross-sectional densities go to zero near the boundary of the (S,s) band. As a result the total mass of agents who adjust in response to an aggregate shock of size dz is small: it is proportional to the area under the density at the boundaries, which is of order $(dz)^2 = dt$.

To understand the intuition for equation (7), first consider the case with no aggregate risk ($dz = 0$). Then the final term on the right hand side vanishes, and the resulting partial differential equation has a unique time-invariant solution f^*. This density f^* can be thought of as the "unperturbed" steady state of the economy. In the presence of aggregate shocks, the actual cross-sectional density f is constantly perturbed relative to f^*, as represented by the dz term in (7); but in the long term the system returns to f^* in expectation.

Figure 1 illustrates these results. The top panels plot the steady-state distribution f^* in two environments: one with high aggregate and low idiosyncratic risk and the other with low aggregate and high idiosyncratic risk. The bottom panels show the actual cross-sectional distribution sampled twenty times from simulating the two environments. The actual distributions are more similar to the steady state distribution when idiosyncratic risk is high relative to aggregate risk. This observation—which follows because idiosyncratic risk forces the distribution to converge towards f^*, while aggregate risk pushes it away from f^*—plays a key role in our approximation result below.

3.2 State-Dependent Impulse Responses and a Moving-Average Representation

To connect the dynamics of X_t to exogenous habit models, we develop a moving average (MA) representation for X_t. This representation summarizes the dynamic response of X_t to past aggregate

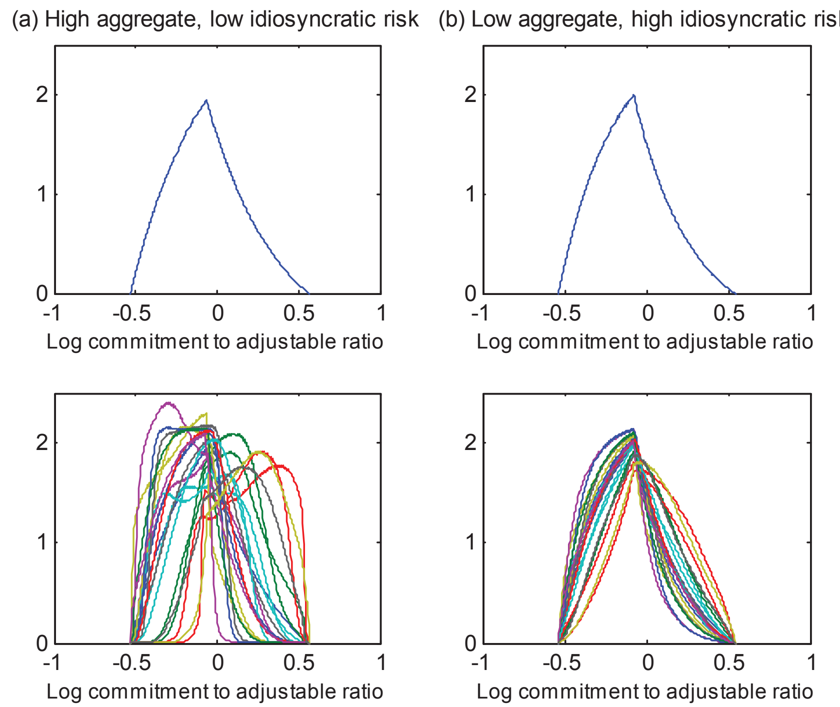

Figure 1: Cross-sectional densities of the log commitment to adjustable consumption ratio. Top panel shows the long run steady state f^*, bottom panel shows twenty realizations. Environment (a) has high aggregate risk ($\sigma_A = .1$) and low idiosyncratic risk ($\sigma_I = .05$), environment (b) has low aggregate risk ($\sigma_A = .05$) and high idiosyncratic risk ($\sigma_I = .1$).

shocks. Because our interest is in fluctuations, we focus on the de-trended processes $\overline{A}_t = e^{-\mu_A t} A_t$, which is a martingale, and $\overline{X}_t = e^{-\mu_A t} X_t$. It is useful to think of $\overline{A}_t$ as summarizing aggregate shocks up to date t.

The next definition introduces the impulse response of commitments to an aggregate shock at $t = 0$. Specifically, we consider a small change in A_0 relative to an initial value A_0^*, holding fixed the initial distribution of commitment consumption. Given the initial value A_0^*, the commitment consumption of agent i is $x_0^{i*} = a_0^i \exp y_0^i = A_0^* \exp y_0^i$. Hence—given that the initial distribution of y_0^i is F_0—the initial cross-sectional distribution of commitment consumption is $F^x(x_0|A_0^*) = F_0[\log x_0 - \log A_0^*]$. We let $\overline{X}_t(A_0, F^x(x_0|A_0^*))$ denote normalized aggregate commitments at date t when $a_0^i = A_0$ may differ from A_0^*, but the initial distribution of commitments is fixed at $F^x(x_0|A_0^*)$.

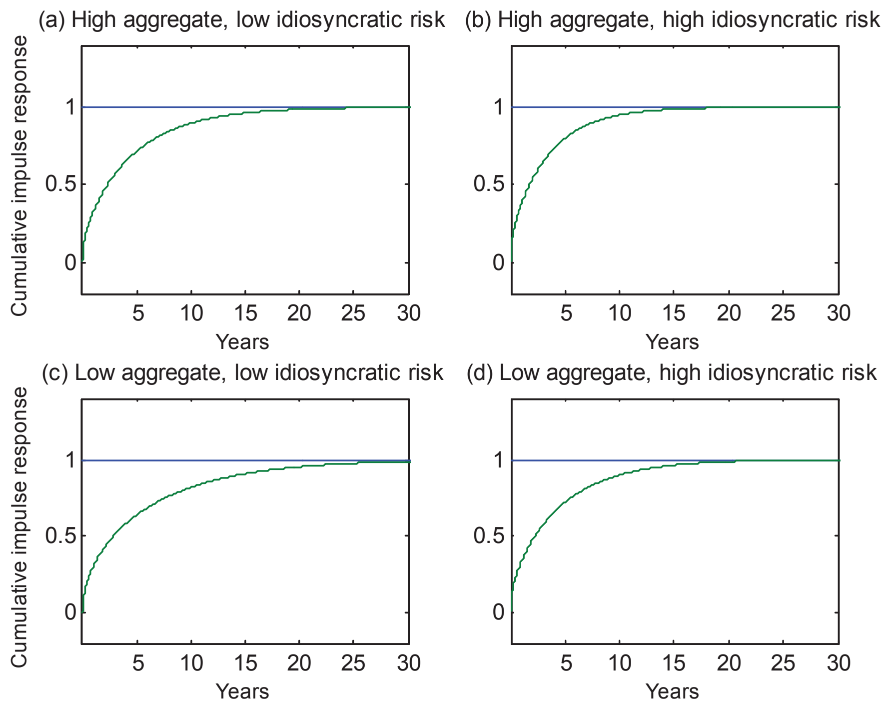

Figure 2: Normalized cumulative impulse response function of aggregate commitment consumption in four environments with high (.1) and low (.05) aggregate and idiosyncratic risk.

Definition 1 *The impulse response function of the commitments model in state F is the function*

$$\xi(t|F) = \left. \frac{\partial E_0 \left[\overline{X}_t \left(A_0, F^x \left(.|A_0^* \right) \right) | \right]}{\partial A_0} \right|_{F_0 = F,\ A_0 = A_0^*}.$$

This is just the derivative of $E_0 \overline{X}_t$ with respect to a uniform change in a_0 for all households, holding fixed initial commitments. The Appendix shows that $\xi(t|F)$ is well-defined and independent of A_0^*. Because we usually work with cross-sectional distributions that have density, we often write $\xi(t|f)$ where f is the density of F, or $\xi(t|f(s))$ when $f(s) = f(y,s)$ is the adjustable-consumption-weighted cross-sectional density at date s.. It is intuitive that impulse responses should depend on the initial distribution: when many households are on the verge of downsizing, a negative aggregate shock will reduce commitments at a faster rate. Figure 2 plots impulse-responses in our model in four environments (assuming $f = f^*$). As $t \to \infty$, these impulse responses gradually converge to a limit (normalized to one in the figures), which corresponds to full adjustment to the initial shock. Higher risk leads to more rapid convergence, as commitments are updated more quickly.

We use $\xi(t|f)$ to make explicit the dependence of X_t on past aggregate shocks.

Proposition 4 *De-trended aggregate commitments admit the moving average representation*

$$\overline{X}_t = \int_0^t \xi\left(t-s, f\left(s\right)\right) d\overline{A}_s + E_0 \overline{X}_t. \tag{9}$$

As we show below, this MA representation is the key diagnostic in analyzing the dynamics of X_t. The result is intuitive: the current level of $\overline{X}_t$ equals it's ex ante expectation plus the sum of the effects of aggregate shocks between date 0 and date t, accounting for partial adjustment to shocks using the impulse response function. We interpret (9) as a "state-dependent MA representation" for commitments, where the coefficients $\xi\left(t-s, f\left(s\right)\right)$ depend on the state of the economy at date s through $f\left(y, s\right)$.

3.3 Habit Models and a State-Independent MA Representation

A leading special case of the moving-average representation in (9) is where the weights ξ are state-*independent*, i.e., do not depend on history. We now show that this special case *coincides* with reduced-form habit models in which X_t is specified as an average of past consumption with weights that only depend on the time lag. Intuitively, if habit is a linear function of past consumption, it should be expressible as a linear function of shocks to past consumption as well.

Habit model. Consider a representative agent economy in which external habit preferences are given by (6), and the habit stock is exogenously determined as

$$X_t^h = o^h(t) X_0^h + \int_0^t \zeta^h(t-s) C_s^h ds \tag{10}$$

with weights ζ^h and o^h which are exogenous locally integrable functions asymptoting to zero. Throughout, we follow the convention that the superscript h refers to the representative agent habit model. We assume that the habit consumer has access to the same stock and bond investment opportunities given in equations (3) and (2). Our habit model is therefore a variant of Constantinides (1990). Since the shock processes are identical, we can think of the habit and commitment models as being defined on the same probability space. It is a direct consequence of the Euler equation that in the optimum, the "surplus" consumption $C_t^h - X_t^h$ for the habit agent follows the same path as A_t in the commitments model. Thus, A_t keeps track of aggregate shocks to marginal utility in both economies.

Moving average representation. Lemma 5 in the appendix shows that we can rewrite (10) into a representation in which X_t^h is a weighted average of past values of A_s, rather than past values of

C_s. This follows essentially because C, X and A are linked by an accounting identity, and hence any linear representation of X in terms of C can also be written as a linear representation in terms of A. From that representation, integration by parts yields

$$\overline{X}_t^h = \int_0^t \xi^h (t-s) \cdot d\overline{A}_s + E_0 \overline{X}_t^h \tag{11}$$

where $\xi^h(u)$ is absolutely continuous with respect to the Lebesgue measure. Equation (11) is an MA representation for the detrended habit stock. The fact that the weights in this MA representation are state-independent is a consequence of starting from a habit model in which the consumption weights are state-independent.

3.4 Equivalence Result: A Fixed-Weight Representation in the Commitments Model

The results above imply that the central difference between the fixed-weight habit and the commitment models comes from the state-dependent nature of impulse-responses in the latter case. We now show that when the ratio of idiosyncratic to aggregate risk is high, aggregate commitments evolve *approximately* according to a fixed weight specification.

We begin by introducing a fixed-weight habit model that generates dynamics which match the evolution of commitments on average.

Definition 2 *A fixed-weight habit model X_t^h matches the steady state impulse response of commitments if $\xi^h(t) = \xi(t, f^*)$ for all t.*

In words, we focus on the habit model that has the same impulse responses as the commitment model in its "unperturbed" steady state f^*. This definition pins down all MA coefficients in (11). We denote the impulse-response weights by $\xi^*(u) = \xi(u, f^*)$ and the habit model by X_t^{h*}.

Main result. Our equivalence result holds when the ratio of idiosyncratic (σ_I) to aggregate (σ_A) consumption risk is large. Since both of these parameters are endogenous, we study sequences of exogenous parameters such that the implied ratio σ_I/σ_A goes to infinity. We explain why σ_I/σ_A drives the result in the discussion below. Consider a sequence of models Θ^n such that, as $n \to \infty$, the following properties hold: 1) $\sigma_I^n/\sigma_A^n \to \infty$; 2) γ, κ and $\overline{\lambda}_i$ remain fixed; 3) r^n stays bounded away from zero; 4) μ_A^n remains bounded; and 5) r^n/ρ^n is bounded away from zero and infinity. An example of such a sequence is when $\pi^n = 1/n$, while all other exogenous parameters stay constant. In this sequence, $\sigma_A^n \to 0$.

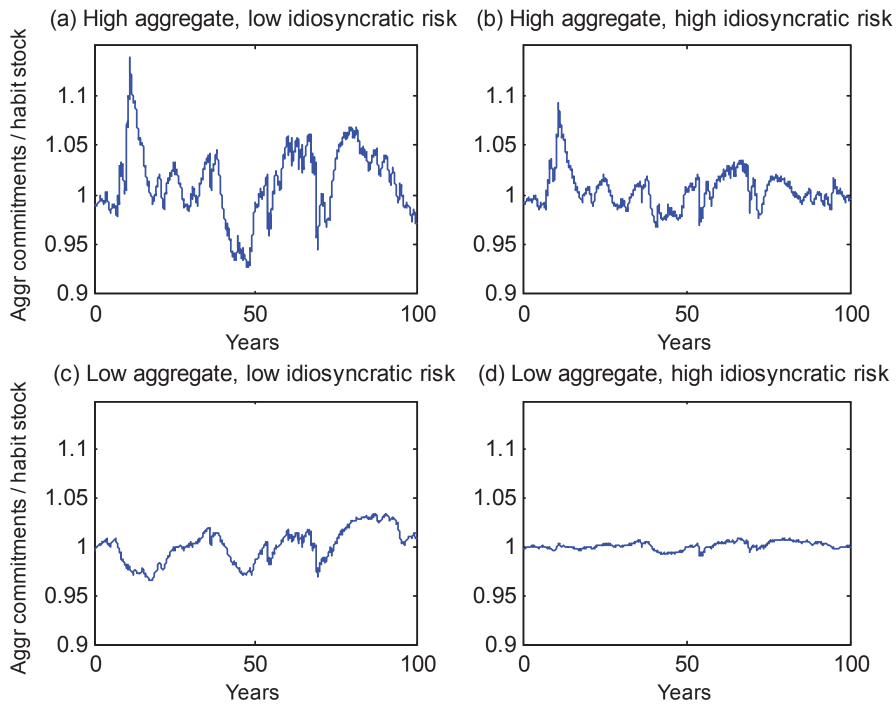

Figure 3: Ratio of aggregate commitments and habit in four environments.

Theorem 1 *For any sequence of models Θ_n specified above and any $p \geq 1$,*

$$\limsup_{t} \left\| \frac{X_t - X_t^{h*}}{A_t} \right\|_p = o\left(\frac{\sigma_A}{\sigma_I}\right).$$

The left hand side of the expression measures the distance between aggregate commitments X_t and habit in the matching fixed-weight model X_t^{h*}, rescaled by a measure of the aggregate economy A_t. Since these quantities are stochastic, we use the L_p norm to measure distance, defined as $\|Y\|_p = [\mathrm{E}Y^p]^{1/p}$ for any random variable Y. The small order $o(.)$ on the right hand side shows the accuracy of the approximation: the distance between the two models becomes an arbitrarily small *share* of σ_A/σ_I when this ratio goes to zero. The interpretation is that fixed-weight habit provides a highly accurate, "better than first-order" approximation. For example, along a sequence where $\sigma_A \to 0$, the difference between commitments and the fixed-weight model goes to zero even *relative to* σ_A: when the size of aggregate shocks shrinks, the approximation error becomes small compared to these shocks. Similarly, when the magnitude of idiosyncratic risk grows, the distance between the two models goes to zero at a faster rate than the growth in σ_I.

Simulations presented in Figure 3 illustrate the theorem. The figure uses a calibration to plot the evolution of X_t^{h*}/X_t in four environments, in which σ_I and σ_A equal either 5% or 10%. The figure shows that the ratio is close to one in most periods, particularly when idiosyncratic risk is high (right panels) and when aggregate risk is low (bottom panels).

The intuition underlying Theorem 1 is that when most of the uncertainty comes from idiosyncratic risk, the cross-sectional distribution is usually close to its steady state. Hence aggregate shocks generate the same pattern of adjustment in most periods, resulting in impulse response weights that are almost constant over time. The proof of the theorem involves several technical steps, but the basic logic is intuitive. The key is to analyze both models using their MA representations. Differencing (9) and (11) yields

$$\begin{aligned}\overline{X}_t - \overline{X}_t^{h*} - E_0\left[\overline{X}_t - \overline{X}_t^{h*}\right] &= \int_0^t \left[\xi^*(t-s) - \xi(t-s, f(s))\right] \cdot d\overline{A}_s \\ &= \int_0^t \left[\xi^*(t-s) - \xi(t-s, f(s))\right] \cdot \sigma_A \cdot \overline{A}_s dz_s\end{aligned}$$

where we use $d\overline{A}_s = \overline{A}_s \sigma_A dz_s$. Focusing on the final integral, consider a sequence of models Θ_n along which the level of aggregate risk $\sigma_A \to 0$. Since the integrand involves σ_A, its value goes to zero as $\sigma_A \to 0$: as aggregate shocks become small, both models will stay close to their unconditional expectation. But the equation also reveals an additional effect. As σ_A/σ_I becomes small, much of the shock each household experiences is idiosyncratic. This pushes the cross-sectional distribution f close to its unperturbed steady state f^*, because the force pushing for convergence, determined by σ_I, becomes stronger relative to the force of divergence, determined by σ_A.[11] As result, f and f^* are usually close. This in turn implies that $\xi^*(t-s) - \xi(t-s, f(s))$ is typically small: when the system is close to the steady state, its impulse response is also close to the steady state impulse response. Thus $\overline{X} - \overline{X}^h$ is on average small even relative to σ_A.

The mechanism described here is illustrated in the bottom panel of Figure 1. As noted above, there is much more "variance" in the evolution of the cross-sectional distribution in the left panel (low σ_I/σ_A), because the forces of divergence are stronger. This creates fluctuations in the impulse-response across periods, producing behavior that diverges from a fixed-weight habit model. In contrast, the cross-sectional density varies much less in the right panel. As a result, the impulse-responses are approximately constant, creating approximately linear aggregate dynamics.

The case where σ_I/σ_A is large is the most empirically relevant scenario, since idiosyncratic consumption risk is generally much larger than economy-wide risk (e.g., Deaton, 1991, Carroll,

[11]This mechanism is labeled the "attractor effect" by Caballero (1993).

Hall, and Zeldes 1992). This suggests commitments can potentially account for behavior typically attributed to habit formation.

4 Comparing Consumption Commitments and Habit Formation

In addition to replicating the patterns previously attributed to habit formation models in a commonly-studied environment, the commitments model also yields new predictions in other settings. In this section, we illustrate these predictions using three applications. We discuss how existing evidence and future empirical work can distinguish between the commtiment and habit models and derive a set of welfare implications which show why distinguishing between the two models is important despite their equivalence in some settings.

4.1 Consumption Dynamics

Two well-documented features of consumption behavior—both in the aggregate and at the micro level—are excess sensitivity and excess smoothness to shocks (see Japelli and Pistaferri (2010) for a review). One major reason for using habit preferences in applied macroeconomic models is that they generate such delayed consumption responses (Fuhrer 2000). In this subsection, we show that the commitments model not only produces these patterns but also matches additional microeconometric evidence on how excess sensitivity depends on the size of the shock and varies across types of consumption.

Fix a date t_0 and history up to t_0. For any $t_1 > t_0$ consider the following regression specification for consumption growth:

$$\log\left(C_{t_1}\right) - \log\left(C_{t_0}\right) = \alpha_1 + \beta_1 \cdot \left[\log A_{t_1} - \log A_{t_0}\right] + \varepsilon. \tag{12}$$

This regression builds on the interpretation developed in Section 2.3 that adjustable consumption—because it immediately and fully responds to shocks—can be thought of as a measure of permanent income for an individual or a group of households. Thus the regression evaluates the extent to which consumption responds to contemporaneous shocks affecting lifetime income. To make explicit its dependence on t_1, we denote the regression coefficient by $\beta_1(t_1)$. The neoclassical permanent income model predicts $\beta_1(t_1) = 1$ for all $t_1 > t_0$. Following Flavin (1981), we say that consumption is *excessively smooth* if $\beta_1(t_1) < 1$ for some $t_1 > t_0$, i.e., if consumption does not fully respond to contemporaneous shocks.

Next, let $t_3 > t_2 > t_1$ and consider the regression

$$\log C_{t_3} - \log C_{t_2} = \alpha_2 + \beta_2 \left[\log A_{t_1} - \log A_{t_0}\right] + \varepsilon. \tag{13}$$

This regression evaluates the extent to which consumption adjusts to income shocks with a delay. Using the notation $(t_1, t_2, t_3) = \bar{t}$, we denote the regression coefficient by $\beta_2\left(\bar{t}\right)$. The neoclassical permanent income model implies $\beta_2\left(\bar{t}\right) = 0$ for all $\bar{t}$ because consumption responds fully at the time of the shock. We say that consumption is *excessively sensitive* if current consumption does respond to past shocks to permanent income, i.e., if there exists a sequence of dates $\bar{t}$ such that $\beta_2\left(\bar{t}\right) > 0$.

Proposition 5 *(Excess smoothness and sensitivity) In the commitments model, consumption is both excessively smooth and excessively sensitive.*

Excess smoothness follows because commitments respond slowly to the shock. Therefore initially (for t_1 close to t_0) $\beta_1(t_1) \approx A_{t_0}/C_{t_0} < 1$ in regression (12). Excess sensitivity is an implication of the fact that eventually, households do adjust their commitments, and hence $\beta_2\left(\bar{t}\right)$ approximates $X_{t_0}/C_{t_0} > 0$ when $t_2 \to t_0$ and $t_3 \to \infty$. The shape of delayed adjustment is illustrated in Figure 2, which plots the normalized steady-state impulse response of commitments. Our model suggests that both the sluggishness and sensitivity of consumption may be consequences of adjustment costs that delay updating.

Large shocks. We now show that excess sensitivity and smoothness vanish for large shocks in the commitments model, but not in the habit model. We first introduce a notion of large shocks. Because our model does not feature jumps, we focus on the (unlikely) events in which A_t changes rapidly during a short interval after t_0. Formally, consider the events in which $\log \overline{A}_{t_1}$ reaches either $\log \overline{A}_{t_0} + \Delta$ or $\log \overline{A}_{t_0} - \Delta$ by date t_1. These events correspond to a positive (respectively negative) shock, and Δ measures the size of the shock, i.e., the percentage change in $\overline{A}_t$. We denote the former event by $S(+, t_1, \Delta)$, the latter event by $S(-, t_1, \Delta)$, and their union by $S(t_1, \Delta)$.[12]

We now compare the commitment model with its matching habit specification introduced in Definition 2 during and after these large shocks. Consider estimating the regression (12) conditional on the shock event $S(t_1, \Delta)$. We denote the regression coefficients by $\beta_1(t_1, \Delta)$ in the commitments model and $\beta_1^h(t_1, \Delta)$ in the habit model. Note that because these coefficients are

[12] The formal way to model these events is to assume that a Brownian bridge drives $\log \overline{A}_t$ between t_0 and t_1.

estimated conditional on the low-probability shock events, they need not match the unconditional coefficients $\beta_1(t_1)$ and $\beta_1^h(t_1)$ introduced earlier.

Proposition 6 *(Excess smoothness for large shocks) The following statements hold:*

(i) In the commitments model excess smoothness vanishes for large shocks. Formally, there exists $K > 0$ such that for all $t_1 > t_0$, $\beta_1(t_1, \Delta) > 1 - K/\Delta$.

(ii) In the habit model excess smoothness remains for large shocks. Formally, there exists $K' < 1$ such that for all Δ large enough, we can find t_1 for which $\beta_1^h(t_1, \Delta) < K'$.

Part (i) shows that in the commitments model the correlation between consumption and permanent income increases in extreme events. Because large shocks force people to adjust their commitments, $\beta_1(t_1, \Delta)$ approaches 1 as $\Delta \to \infty$. Part (ii) shows that this result does not extend to the habit model: because in that setting impulse responses do not depend on the size of the shock, $\beta_1^h(t_1, \Delta)$ remains bounded below 1 even for Δ large.

We now turn to excess sensitivity. Consider estimating the regression (13) conditional on the shock event $S(t_1, \Delta)$. We denote the regression coefficients by $\beta_2\left(\bar{t}, \Delta\right)$ in the commitments model, and by $\beta_2^h\left(\bar{t}, \Delta\right)$ in the habit model. To explore the impact of a *sudden* large shock, we focus on the limit in which, holding fixed Δ the size of the shock, $t_1 \to t_0$. We define the lim sup and lim inf of the regression coefficients to be $\overline{\beta}_2(t_2, t_3, \Delta) = \lim_{t_1 \to t_0} \sup \beta_2\left(\bar{t}, \Delta\right)$ and $\underline{\beta}_2(t_2, t_3, \Delta) = \lim_{t_1 \to t_0} \inf \beta_2\left(\bar{t}, \Delta\right)$ in the commitments model, and define $\overline{\beta}_2^h(t_2, t_3, \Delta)$ and $\underline{\beta}_2^h(t_2, t_3, \Delta)$ analogously for the habit model.

We consider a sequence of models Θ_n as defined in Section 3. The following result is stated for the case when n is large enough, that is, when σ_A/σ_I is small enough. We focus on this case for the technical reason that it ensures that X_t^h/A_t^h remains bounded in L_p norm uniformly in t.

Proposition 7 *(Excess sensitivity for large shocks.) Suppose that n is large enough. Then:*

(i) In the commitments model, excess sensitivity vanishes for large shocks. Formally, there exists $K > 0$ such that for any $t_3 > t_2$, we have $\overline{\beta}_2(t_2, t_3, \Delta) < K/\Delta$.

(ii) In the habit model excess sensitivity remains for large shocks as well. Formally, there exists $K' > 0$ such that for all large enough Δ, we can find t_2 and t_3 for which $\underline{\beta}_2^h(t_2, t_3, \Delta) > K'$.

Part (i) shows that the commitments model does not generate delayed adjustment for large shocks. As more and more households are pushed over the boundary of their (S,s) bands, fewer and fewer of them will adjust to the shock with a lag. As a result, $\overline{\beta}_2(t_2, t_3, \Delta)$ becomes arbitrarily small as Δ grows. Conversely, part (ii) shows that—because impulse responses are state-independent—the habit model produces delayed responses for large shocks as well.

The challenging part of the proof is claim (ii). To establish that result, we need to characterize $X^h_{t_3}/A_{t_3}$ as $t_3 \to \infty$. Since $X^h_{t_3}$ is essentially a weighted sum in which the number of terms grows with t_3, to obtain a characterization we need to make sure that terms corresponding to the distant past, even when normalized by A_{t_3}, remain bounded. Because σ_A governs the variance of the normalizing term $A^h_{t_3}$, while σ_I affects the rate with which the weights in the weighted sum approach zero, this is ensured when σ_A/σ_I is small.

Microeconometric evidence for the "magnitude hypothesis." Summarizing the empirical literature on consumption, Japelli and Pistaferri (2010) write that consumers "tend to smooth consumption and follow the [neoclassical] theory when expected income changes are large, but are less likely to do so when the changes are small and the cost of adjusting consumption is not trivial." Japelli and Pistaferri term this pattern the "magnitude hypothesis." In what follows, we briefly summarize this body of empirical evidence and discuss how it is naturally explained by the commitments model.

Several empirical studies have found that the degree of excess sensitivity in consumption—typically measured in this literature as a consumption response to anticipated income shocks—depends on the size of the shock. Hsieh (2003) shows that Alaskan households increase consumption in the quarter in which they receive their tax refunds (a small income change), but do not increase consumption in the quarter in which they receive payments from the Alaska Permanent Fund (a large income change). In the same spirit, Browning and Crossley (2001) note that Parker (1999) finds excess consumption sensitivity to the income change associated with US households reaching the Social Security payroll cap (a small income change) while Browning and Collado (2001) find that no excess consumption sensitivity of Spanish workers to anticipated bonus salary amounting to two months' wages (a large income change). In support of the idea that the magnitude of the shock may drive these differences, Browning and Crossley (2001) estimate that the welfare cost of ignoring the Spanish bonus system is equivalent to an annual loss of a month's consumption, that of ignoring the Alaska Permanent Fund schedule is equivalent to a week of consumption, and that of the Social Security cap is equivalent to an afternoon's consumption. Similarly, Souleles (1999) finds excess sensitivity to tax refunds and Souleles (2002) to the Reagan tax cuts, but Souleles (2000) finds no excess sensitivity to college expenditures, which are typically larger in magnitude. More recently, Scholnick (2013) presents a more formal test by showing that the anticipated income increase associated with a household's final mortgage payment has a positive effect on contemporaneous consumption, but the effect is decreasing in the size of the mortgage payment.

The commitments model can explain this body of evidence through Propositions 6 and 7, which together imply that the delay with which consumption responds to income shocks is smaller for large shocks.[13] This result can explain Hsieh's findings through the logic that consumers respond slowly to information on tax refunds, because those payments are small. But the same consumers respond quickly to news about the payment of the Alaska Permanent fund because those payments are large. In particular, through Proposition 6 the commitments model predicts that consumers should not increase consumption when the actual payment of the Alaska Permanent Fund is made; instead, they should increase consumption earlier, right after the announcement. This prediction is consistent with Hsieh's finding that the growth in expenditures on durables is lower when the Alaska Permanent Fund payment is higher, suggesting that consumers purchase durables *before* they receive the Fund payment. Similarly, the commitments model predicts slow adjustment to the small income change associated with the relatively small tax refunds and with reaching the social security cap; but like the permanent income model, early adjustment—when the worker is hired, or when a decision is made that the child will attend college—to the wage bonuses and to college expenditures. The habit model does not match these predictions because it produces a state-independent impulse response, as shown in Propositions 6 and 7.[14]

An important caveat is that both the commitment and habit models predict that consumption should be unaffected by the *timing* of income conditional on the announcement date. Both models simply predict gradual adjustment after the announcement, which results in comovement between income and consumption. In particular, neither model can explain the findings of Johnson, Parker and Souleles (2006) that consumption responds to variation in the *timing* of income tax rebates. Other factors likely play a role in explaining this behavior. For instance, Agarwal, Liu and Souleles (2007) document the importance of credit constraints; salience effects (see Bordalo et al. 2012 and Koszegi and Szeidl 2013) may also play a role. Despite these caveats, it is clear that important elements of the evidence on excess sensitivity are more consistent with a model of adjustment costs than with habits, suggesting that at least part of the behavior previously attributed to habit

[13] Here we use the aggregated commitments model to match micro evidence. The interpretation is that the theoretical aggregate corresponds to the group of households who experience the shock.

[14] In the commitments model, total consumption C_t exhibits excess sensitivity and smoothness, while adjustable consumption A_t does not. Since most consumption goods have both adjustable and fixed components, the more general empirical prediction is that more adjustable goods exhibit less excess sensitivity and smoothness. This prediction also accords with empirical evidence. For instance, Chetty and Szeidl (2007) find that consumption of housing responds much more sluggishly to unemployment shocks than consumption of food.

Aggregate risk	Idiosyncratic risk	Riskfree rate	Individ cons growth	How many years till X adjusts $\widetilde{p}$? $\widetilde{p} = 0.25$	$\widetilde{p} = 0.5$	$\widetilde{p} = 0.75$
Adjustment cost= 1* annual consumption						
10%	10%	1%	0.87%	0.44	1.73	4.24
5%	10%	2.5%	0.87%	0.55	2.24	5.6
10%	5%	2.5%	0.87%	0.6	2.34	5.73
5%	5%	4%	0.87%	0.84	3.4	8.64
10%	10%	4%	2.37%	0.4	1.63	4.15
Adjustment cost= 5* annual consumption						
10%	10%	1%	0.87%	1.06	4.15	10.26
5%	10%	2.5%	0.87%	1.15	4.84	12.62
10%	5%	2.5%	0.87%	1.46	5.67	13.79
10%	10%	4%	2.37%	0.73	3.1	8.39

Table 1: Speed of adjustment of consumption commitments

formation may in fact be due to consumption commitments.

4.2 Comparative Dynamics

In this subsection, we compare the effects of changes in the environment in the commitment and habit models. In the habit model, the weights that determine the speed of adjustment are exogenous and unaffected by environmental changes. In contrast, in the commitments model changes in the environment – such as the level of risk or the trend growth rate – have significant impacts on responses to shocks, because household adjustment behavior depends upon the environment.

To characterize how responses to shocks vary with the environment, let $T(\widetilde{p}, f) = \inf_t \{\xi(t|f) \geq \widetilde{p} \cdot \overline{x}\}$ denote the time required for commitments or habit to adjust, in expectation, a share $\widetilde{p}$ to a unit shock to permanent income.[15] This quantity can be interpreted as a measure of excess sensitivity of consumption. By definition, in a fixed-weight habit model, $T(\widetilde{p})$ is pinned down by the exogenous weights and hence remains constant when other parameters are varied.

We begin with some numerical examples to illustrate the comparative dynamics of the commitments model. Table 1 reports $T(\widetilde{p}|f^*)$ for the commitments model when $\widetilde{p} = 0.25$, 0.5 and 0.75 for various parameters. In the top panel, the adjustment cost equals one year's consumption value of the commitment good, or 1% of its capitalized value with a riskfree rate of 1%. The first row shows that when $\sigma_A = \sigma_I = 10\%$ and $r_f = 1\%$, it takes on average 1.7 years for 50% of full adjustment to occur. The next three rows illustrate the effect of reducing σ_A or σ_I, changing r_f so that expected consumption growth remains unchanged in these comparisons. The table shows that

[15] Here, $\overline{x}$ denotes the steady state ratio of commitments to adjustables, so that $\xi(t)/\overline{x}$ asymptotes to one.

reducing either idiosyncratic or aggregate risk results in a slower response to shocks. The intuition is that higher risk forces consumers to update their commitments more frequently, allowing aggregate shocks to get absorbed into the level of commitment consumption more quickly. Comparing the first and last rows in the top panel shows the effect of higher consumption growth generated by a higher safe return. Faster growth also leads to faster adjustment to shocks, as agents update commitments more frequently in a growing economy. The bottom panel of the table shows that for a higher adjustment cost (5% of the capitalized value of the commitment good), adjustment is more sluggish, but the effects of risk and growth remain similar.

To demonstrate that these results are driven by the intuition we describe, we now establish a formal analog of the preceding numerical examples in a special case of the model. Consider a sequence of economies $\overline{\Theta}^n$ with $n = 1, 2, ...$ in which $\pi^n = \pi^n_E = 1/n$ and $r = \rho$. This sequence is a special case of the Θ_n sequence introduced earlier in which σ_I, σ_A, μ_a and μ_A all go to zero at a rate of $1/n$. When n grows large, this economy converges to an environment in which households face no risk and have zero consumption growth, which we denote by $\overline{\Theta}^*$. Clearly, in that limit economy agents either adjust commitments immediately at $t = 0$ or never do so. The habit model that matches the consumption pattern of $\overline{\Theta}^*$ (as given by Definition 2) is one in which the habit stock remains unchanged at the initial level of commitment x_0 forever.

Proposition 8 *Fix $\widetilde{p} > 0$. In the commitments model, $T^n(\widetilde{p}|x_0)$ is finite but $\lim_{n\to\infty} T^n(\widetilde{p}|x_0) = \infty$. In the habit model, $T^{h,n}(\widetilde{p}|x_0) = \infty$ for all n.*

In the commitments model, adjustment occurs with positive risk and growth (n finite), but as $n \to \infty$, it occurs at a vanishingly small rate, so that the expected time to adjustment converges to infinity. In contrast, in the habit model, the presence of risk and growth does not affect adjustment of the habit stock, which remains constant permanently.

At the macroeconomic level, Proposition 8 suggests that recessions may be shorter in rapidly developing economies, in which households change their arrangements frequently because of high trend growth. Conversely, recessions may be longer in economies with substantial social insurance against idiosyncratic risk (such as European welfare states) because people have weaker incentives to change their commitments. We are not aware of any empirical evidence on these predictions to date. Testing these predictions in future research would allow researchers to further distinguish between the commitments and the habit model as drivers of excess consumption sensitivity.

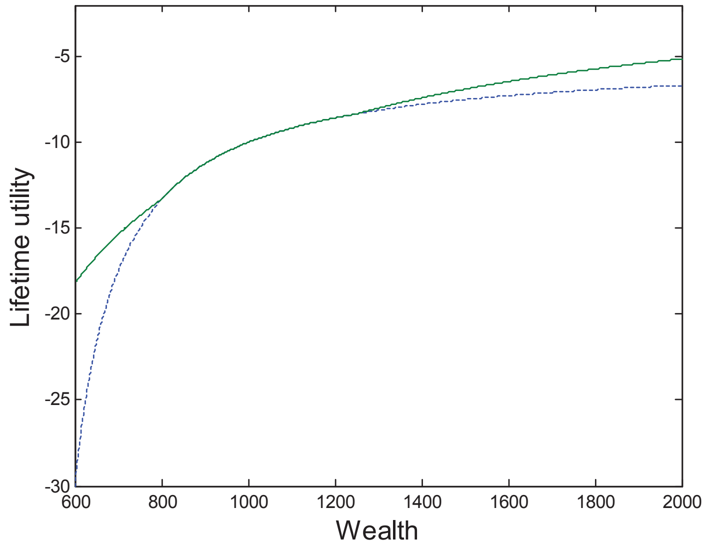

Figure 4: Value as a function of wealth of a commitment agent (solid line) and the matching habit agent (dashed line) in an economy with zero consumption risk and no growth. The value function of the habit agent is shifted vertically to account for the utility value of commitments.

4.3 Welfare Costs of Shocks

In our final application, we show why distinguishing between the commitments and habit models is important by comparing their welfare implications. One benefit of the commitments model is that it offers a natural welfare measure. In contrast, in the habit model the appropriate welfare measure is open to debate – in particular, should habit consumption be included in welfare calculations?

In the Arrow-Pratt tradition, we explore the welfare cost of one-time wealth shocks. To build intuition, we first focus on the economy $\overline{\Theta}^*$ defined earlier, in which there is no aggregate or idiosyncratic risk and no consumption growth ($\pi = \pi_E = 0$ and $r = \rho$ which imply $\mu_a = \sigma_T = 0$). To compare the welfare cost of unanticipated wealth shocks in these two models, consider Figure 4 which plots the value functions of the commitment and habit agents in this environment. As long as it remains optimal for the commitment agent not to move, the two value functions are completely parallel.[16] In this range, all changes in wealth are absorbed by adjustable consumption, and hence the welfare implications of the two models are identical. However, for large shocks, the commitment agent adjusts on both consumption margins, while adjustment of the habit stock is not permitted. As a result, large shocks have a higher welfare cost with habits than with commitments. Intuitively,

[16] There is a difference in the *level* of utility because here we assume that the habit agent does not derive utility from commitment consumption. The figure abstracts away from this effect by shifting the value function of the habit agent vertically.

commitment-based habits absorb large shocks, dampening their welfare cost.

To establish this intuition in a more general setting, we consider an unanticipated wealth shock at time t that hits with probability q and reduces total wealth by a share b, and compare the premium agents in the two models are willing to pay to avoid this risk. Consider the commitment economy in its unperturbed steady state in which all agents face this shock, and contrast it with the matching habit model where the shock affects the representative agent. Define the risk premium to be dollar amount that agents are collectively willing to give up in excess of the expected value to avoid this risk. The proportional risk premium $\Pi(q, b)$ is the risk premium normalized by total wealth in the economy.

Proposition 9 *Assume that $\lambda_1 = 0$ but $\lambda_2 > 0$. Then:*

(i) As $b \to 1$, the proportional risk premium in the fixed-weight habit economy exceeds that in the corresponding commitment economy: $\Pi_h(q, b) > \Pi(q, b)$.

(ii) Consider the sequence of economies $\overline{\Theta}_n$. For $b > 0$ sufficiently small, in the commitment model the risk premium $\Pi^n(q, b) < \Pi^*(q, b)$, while in the habit model $\Pi^{h,n}(q, b) = \Pi^{h*}(q, b)$.

Part (i) implies that habit agents are more averse to large shocks than are commitment agents. Commitments adjust immediately to a big shock, mitigating its impact. In contrast, reduced-form habits adjust sluggishly for all shocks, hence agents suffer relatively more from a large shock.[17]

Part (ii) explores comparative statics of the welfare cost as risk and growth vanish. With commitments, risk and growth reduce the risk premium $\Pi(q, b)$: since agents adjust for other reasons, a shock can be partly absorbed by commitments. Because this possibility is absent in the reduced-form habit model, there the risk premium is unaffected by changes in risk or growth.

A policy lesson from (i) is that a reduced-form habit model that matches observed dynamics of consumption well may nevertheless yield misleading conclusions about the welfare costs of large shocks. Even if consumption is highly persistent for typical shocks, agents may not be extremely averse to big fluctuations because they can make adjustments by paying a fixed cost. Thus the optimal size of social insurance programs that insure large, long-term shocks such as disability or job displacement may be smaller than predicted by analyses using habit models such as Ljungqvist and Uhlig (2000). Result (ii) implies that policies which increase social insurance or reduce growth can

[17]The assumption that $\lambda_1 = 0$ guarantees that when moving, the commitment agents can get rid of all pre-commitments. Otherwise, even when moving they would still have promised expenditures of $\lambda_1 X_{t-}$, which behave like sluggish habits. In simulations, we find that unless λ_1 is very high, the conclusion of the proposition is unaffected. Intuitively, moving costs are much smaller than habit expenditures.

	commitments	habit
Behavioral Predictions		
1. household consumption jumps on adjustment dates	yes	no
2. consumption excessively sensitive/smooth for small shocks	yes	yes
3. excess sensitivity/smoothness vanishes for large shocks	yes	no
4. less excess sensitivity/smoothness for adjustable goods	yes	no
5. reducing long-term growth can increase excess sensitivity	yes	no
6. insuring idiosyncratic risk can increase excess sensitivity	yes	no
Welfare Implications		
1. unambiguous welfare measure	yes	no
2. welfare cost of small shocks amplified	yes	yes
3. welfare cost of large shocks amplified less	yes	no
4. reducing trend growth can increase cost of aggregate shocks	yes	no
5. insuring idiosyncratic risk can increase cost of aggregate shocks	yes	no

Table 2: The main predictions of the commitments and habit models

make aggregate fluctuations more costly by slowing households' response to changing circumstances. These results illustrate the potential importance of distinguishing between the commitments and habit models in analyzing the welfare consequences of economic policies.

5 Conclusion

A large literature in macroeconomics has used habit formation in preferences as an explanation for important properties of macroeconomic consumption dynamics, such as the excess sensitivity and smoothness of consumption with respect to income shocks. In this note, we showed that many of these properties can equivalently be explained by aggregating a model with adjustment costs at the microeconomic level. In addition to replicating the predictions of habit models in a commonly studied domain, the commitments model also yields new predictions in other domains. We conclude by summarizing the key similarities and differences between the two models in Table 2, and suggest directions for future empirical work to test between them.

The first four predictions in Table 2, on the dynamics of consumption and its response to shocks, have been studied in prior empirical research. As discussed above, available evidence on the predictions where the two models differ aligns more closely with the commitments model. However, it would be very useful to have more evidence on these predictions that is specifically directed at distinguishing between the key mechanisms underlying the two models. For example, the commitments model predicts that excess sensitivity should be greater for small wealth shocks (such as lottery winnings) than large wealth shocks, particularly for less adjustable goods like housing or durables, whereas standard habit models do not predict such heterogeneity.

Predictions 5 and 6 on the impacts of changes in the economic environment have not yet been tested and offer new ways to distinguish between the two models. One way to test prediction 6 at the microeconomic level would be to compare the effect of tax rebates on households who have vs. have not recently experienced a positive income shock, such as a promotion. The commitments model predicts that excess sensitivity of consumption to tax rebates should be lower for those who also had another positive income shock, because they are more likely to adjust for that reason. At the macroeconomic level, prediction 6 suggests that countries with more generous welfare systems, such as those in Northern Europe, should have relatively longer business cycles.[18]

Differentiating between the commitment and habit models is important because the two models generate different welfare implications, listed in the second part of Table 2. If commitments are the root cause of habit-like behavior, then the welfare gains from insuring small or moderate shocks may be larger than the gains from insuring large shocks, especially in economies with low trend growth and idiosyncratic risk. In contrast, if consumers have habit formation preferences, then insuring the largest shocks is most important, regardless of the underlying rate of growth and risk in the economy. More broadly, revisiting existing results on optimal policy in models featuring consumption commitments would be a useful direction for future research.

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[18] Naturally, this prediction is more speculative because of potential general equilibrium effects and other factors that may vary across countries that are missing from our stylized model.

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Appendix A: Proof Map

We present a series of Lemmas and arguments that build up to the proof of Theorem 1 and to the applications. Additional proofs are contained in the online Supplementary Appendix.

A.1 Preliminaries

Two convenient probability measures. Let Q be the probabilty measure which weighs the sample paths of y_t by their share in aggregate adjustable consumption. Then $F(y,t) = \Pr_Q \left[y_t^i < y | A_{[0,t]} \right]$. It follows from the proof (in the Supplementary Appendix) of Proposition 2 that the probability density associated with Q is

$$\frac{dQ}{dP}|_t = \frac{a_t^i}{A_t} = \exp\left[\frac{\pi_I}{\gamma\sigma_I} z_t^i - \frac{\pi_I^2}{2\gamma^2\sigma_I^2} t\right]$$

which is an exponential martingale. By the Cameron-Martin-Girsanov theorem, under Q, the process $d\bar{z}_t^i = dz_t^i - \pi_I / (\gamma\sigma_I)\, t$ is a Brownian motion.

For our second probability measure note that—as shown in the proof of Proposition 2—A_t is an exponential random walk, and hence $\overline{A}_t = e^{-\mu_A t} A_t$ is an exponential martingale. We define a measure R by letting, for any random variable Z_t measurable with respect to $\mathcal{F}_t$, $E^R[Z_t] = E\left[Z_t \overline{A}_t\right]$. By the Girsanov theorem, under R, the process $d\overline{z}_t = dz_t - \sigma_A t$ is a martingale. The advantage of this measure is that $E_0 \overline{X}_t = E_0^R\left[\overline{X}_t / \overline{A}_t\right]$. This makes it easier to compute the mean and the impulse response of $\overline{X}_t$, because $\overline{X}_t / \overline{A}_t$ is a bounded process. We can also write $E_0 \overline{X}_t = E_0^R\left[\overline{X}_t / \overline{A}_t\right] = E_0^{QR}[x_t / a_t]$ where the superscript QR means that we first apply the transformation associated with R and then the transformation associated with Q. Because the densities associated with these transformations are driven by independent Brownian motions, QR is also a probability measure. By applying R, we move to using the mean dynamics of $\overline{X}/\overline{A}$; and then, by also applying Q, we can focus on the mean dynamics of a single agent, albeit under a driving process with different drift.

Limits of models. Theorem 1 takes a sequence of models Θ_n. Below we focus on a sequence along which $\sigma_A \to 0$. At the end of the proof we show how to convert this result—using a clock change—to a sequence where $\sigma_I \to \infty$. Along the sequence Θ_n, endogenous parameters of the model, such as U and L, also change. While we do not always indicate it in notation, we always understand those changes to be taking place.

A.2 Auxiliary results about the commitments model

We begin with a technical lemma that establishes the smoothness of conditional expectations of y_t. Consider a new process $\widetilde{w}_t$, which is a Brownian motion with some drift μ_w and variance σ_w reborn at some interior point M_w when hitting the boundaries of the interval $[L_w, U_w]$. With appropriate choice of parameters $\widetilde{w}_t$ will have the same distribution as y_t under QR. We let $h(y, t, \sigma_w, \mu_w, L_w, M_w, U_w) = E\left[e^{\widetilde{w}_t} | \widetilde{w}_0 = y\right]$. Often we just write $h(y, t)$, in which case we assume that the other arguments are given by the optimal policy of the commitments model, so that $h(y, t) = E^{QR} E\left[e^{y_t} | y_0 = y\right]$. Let $L_1 < L_2 < M_1 < M_2 < U_1 < U_2$.

Lemma 1 *$h(y, t, \sigma_w, \mu_w, L_w, U_w, M_w)$ is infinitely many times differentiable in $[L_w, U_w] \times (0, \infty) \times (0, \infty) \times [L_1, L_2] \times [M_1, M_2] \times [U_1, U_2]$.*

Thus h and it's various derivatives in y and t are all continuous and therefore locally bounded in $(\mu_w, \sigma_w, L_w, M_w, U_w)$. This is useful because when we take σ_A to zero as $n \to \infty$, optimal behavior changes, and hence the endogenous parameters $\left(\mu_y, \sigma_y, L, M, U\right)$ vary. But these parameters will all stay in some bounded open set, and due to positive idiosyncratic risk σ_y stays bounded away from zero. Thus along this sequence $h(y, t)$ and its derivatives exist and are all bounded.

Our next Lemma expresses X_t as a moving average with weights determined by h.

Lemma 2 *Let $\xi(u, y) = h(u, y) - h_y(u, y)$ and $\xi(u, f(s)) = \int_L^U \xi(u, y) f(y, s)\, dy$. Then*

$$\overline{X}_t = \int_0^t \xi(t - s, f(s))\, \sigma \overline{A}_s dz_s + E_0\left[\overline{X}_t\right]. \tag{14}$$

Proposition 4 follows from this result. We next show that $\xi(t, y)$ converges exponentially fast.

Lemma 3 *There exists* $\overline{x}$ *such that* $\lim_{t\to\infty} E_0\left[\overline{X}_t\right] = \lim_{t\to\infty}\xi(t,y) = \overline{x}$. *There exist* $K_1, K_2 > 0$ *independent of* y *and* σ_A *so that* $|\xi(t,y) - \overline{x}| < K_1 e^{-K_2 t}$ *and* $\left|E_0\left[\overline{X}_t\right] - \overline{x}\right| < K_1 e^{-K_2 t}$ *for all* $(y, \sigma_A) \in [L, U] \times [0, \overline{\sigma}_A]$.

The next result will be used in the proof of Theorem 1 to show that for σ_A small, the impulse responses of the two models are typically close. Let F^* denote the invariant distribution of y under Q, which is also the long run average cross-sectional distribution of the commitments model.

Lemma 4 $\limsup_{t\to\infty} \left\|\sup_y |F(y,t) - F^*(y)|\right\|_p$ *converges to zero as* $\sigma_A \to 0$.

A.3 Auxiliary results about the habit model

We first show the link between C-weighted and A-weighted habit models.

Lemma 5 *Consider two habit models* $X_t = \int_0^t j(t-s)A_s ds + k(t)X_0$ *and* $X_t = o(t)X_0 + \int_0^t \zeta(t-s)C_s ds$ *where the weight functions* j, k, o *and* ζ *are locally integrable. Then there is a one-to-one correspondence between these representations, and the weights are linked to each other through the Volterra integral equations* $\zeta(u) = j(u) - \int_0^u \zeta(v)j(u-v)dv$ *and* $o(t) = k(t) - \int_0^t \zeta(t-s)k(s)ds$ *with initial conditions* $\zeta(0) = j(0)$, $o(0) = k(0)$. *In particular, each* C*-average representation has a unique equivalent* A*-average representation.*

We next construct the best-fit habit model.

Lemma 6 *Let* $\theta(u) = \xi^{*\prime}(u)\cdot e^{\mu_A u}$ *and* $\theta_0(u) = (\overline{x} - \xi^*(u))\cdot e^{\mu_A u}$, *then the habit model* $X_t^h = \int_0^t \theta(t-s)A_s ds + \theta_0(t)A_0$ *generates the impulse response* ξ^*.

A.4 Proof of Theorem 1 when aggregate risk vanishes

We require a technical Lemma bounding the tail of the MA representation in both models.

Lemma 7 *Let* $g(u,s)$ *be progressively measurable with respect to* $\mathcal{F}_s$ *satisfying* $|g(u,s)| \le K_1 e^{-K_2 u}$ *for all* u, s, *and let* $G_t = \left(1/\overline{A}_t\right)\int_0^t g(t-s,s)\overline{A}_s dz_s$. *For any* $1 \le p < \infty$, *for* σ_A *small enough, there exists* $M(p)$ *such that* $\|G_t\|_p \le M(p)$.

Consider a sequence along which $\sigma_A \to 0$. We can write

$$\frac{X_t - X_t^h}{\sigma_A A_t} = \frac{1}{\overline{A}_t}\int_0^t \left[\xi(t-s, f(s)) - \xi^*(t-s)\right]\overline{A}_s dz_s + \frac{E_0\overline{X}_t - \overline{x}}{\overline{A}_t \sigma_A}.$$

We now break this expression into three pieces. Fix some $\varepsilon > 0$, let $k > 0$, and consider

$$\begin{aligned}
&\left\| \frac{1}{\overline{A}_t} \int_0^{t-k} \left[\xi\left(t-s, f(s)\right) - \xi^*\left(t-s\right) \right] \overline{A}_s dz_s \right\|_p \\
&\quad \leq \left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_{2p} \cdot \left\| \frac{1}{\overline{A}_{t-k}} \int_{t-k}^{t} \left[\xi\left(t-s, f(s)\right) - \xi^*\left(t-s\right) \right] \overline{A}_s dz_s \right\|_{2p} \leq K_{2p}\left(k, \sigma_A\right) \cdot M\left(2p\right) \cdot e^{-K_2 k}
\end{aligned}$$

where we used Lemma 7. We can chose k large enough so that this entire term is less than $\varepsilon/3$.

Given this k, we next bound the term

$$\begin{aligned}
\left\| \frac{1}{\overline{A}_t} \int_{t-k}^{t} \left[\xi\left(t-s, f(s)\right) - \xi^*\left(t-s\right) \right] \overline{A}_s dz_s \right\|_p &\leq \left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_{2p} \cdot \left\| \int_{t-k}^{t} \left[\xi\left(t-s, f(s)\right) - \xi^*\left(t-s\right) \right] \frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s \right\|_{2p} \\
&\leq K_{2p}\left(k, \sigma_A\right) \cdot K_{2p}\left(k\right) \cdot \left[E \int_{t-k}^{t} \left[\xi\left(t-s, f(s)\right) - \xi^*\left(t-s\right) \right]^{2p} \left| \frac{\overline{A}_s}{\overline{A}_{t-k}} \right|^{2p} ds \right]^{1/2p} \\
&\leq K_{2p}\left(k, \sigma_A\right) \cdot K_{2p}\left(k\right) \cdot \left[E \int_{t-k}^{t} \left[\xi\left(t-s, f(s)\right) - \xi^*\left(t-s\right) \right]^{4p} ds \right]^{1/4p} \cdot \left[E \int_{t-k}^{t} \left| \frac{\overline{A}_s}{\overline{A}_{t-k}} \right|^{4p} ds \right]^{1/4p} \\
&\leq K_{2p}\left(k, \sigma_A\right) \cdot K_{2p}\left(k\right) \cdot K_{4p}\left(k, \sigma_A\right) \cdot \left[E \int_{t-k}^{t} \left[\xi\left(t-s, f(s)\right) - \xi^*\left(t-s\right) \right]^{4p} ds \right]^{1/4p}
\end{aligned}$$

where we repeatedly used the Cauchy-Schwarz inequality and a martingale moment bound, and where all constants are bounded as σ_A goes to zero. Next note that

$$\begin{aligned}
\xi\left(t-s, f(s)\right) - \xi^*\left(t-s\right) &= \int_L^U \xi\left(t-s, y\right) \cdot \left[f\left(t-s, y\right) - f^*\left(y\right) \right] dy \\
&= -\int_L^U \frac{\partial}{\partial y} \xi\left(t-s, y\right) \cdot \left[F\left(t-s, y\right) - F^*\left(y\right) \right] dy.
\end{aligned}$$

Here, for any fixed k, by Lemma 1, $\partial \xi\left(t-s, y\right)/\partial y$ is uniformly bounded in $\left(y, \sigma_A\right) \in [L, U] \times [0, \overline{\sigma}_A]$. Denoting this bound by $K\left(k\right)$, we have

$$E\left[\xi\left(t-s, f(s)\right) - \xi^*\left(t-s\right) \right]^{4p} < K^{4p}\left(k\right) \cdot E \sup_y \left| F\left(t-s, y\right) - F^*\left(y\right) \right|^{4p}.$$

Lemma 4 shows that the limsup over t of the last term goes to zero as $\sigma_A \to 0$. Thus given k and $\varepsilon > 0$, for all σ_A small enough to make the entire term bounded above by $\varepsilon/3$. Finally, consider

$$\frac{1}{\sigma_A} \cdot \left\| \frac{E_0 \overline{X}_t - \overline{x}}{\overline{A}_t} \right\|_p \leq \frac{1}{\sigma_A} \cdot \left\| \frac{1}{\overline{A}_t} \right\|_p \cdot K_1 e^{-K_2 t} \leq \frac{1}{\sigma_A} \cdot e^{K_3(p) \cdot \sigma_A^2 t} \cdot K_1 e^{-K_2 t}.$$

If σ_A is small enough, then the limsup of this as $t \to \infty$ is zero.

A.5 Proof of Theorem 1 when idiosyncratic risk grows large

We next consider a sequence where $\sigma_I \to \infty$. Here the key is to change the "clock," i.e., the speed with which we go through the Brownian sample paths. This effectively reduces both σ_I and σ_A at the same rate, converting our sequence of models into one in which $\sigma_A \to 0$.

Lemma 8 *Fix $\tau > 0$, and let $\left(\widetilde{a}_t^i, \widetilde{x}_t^i\right)$ denote the optimal solution of a model with deep parameters $\tau \cdot \left(\rho, r, \pi, \sigma^2, \pi_I, \sigma_i^2\right)$, fixed costs $\overline{\lambda} = \left(\overline{\lambda}_1, \overline{\lambda}_2\right)$, curvature γ and relative preference κ. Then the process $\left(\widetilde{a}_t^i, \widetilde{x}_t^i\right)$ has the same distribution as $\tau \cdot \left(a_{\tau t}^i, x_{\tau t}^i\right)$: rescaling the time dimension acts the same way as rescaling the parameters of the model.*

Consider a sequence of models where $\sigma_I \to \infty$ and let $\tau = (\sigma_I)^{-2}$. Changing the clock, dynamics will be identical to a model with parameters $\left(\tau\sigma_I^2, \tau\sigma_A^2, \tau r, \tau\mu_A, \gamma, \overline{\lambda}_1, \overline{\lambda}_2, \kappa\right) = \left(1, \tau\sigma_A^2, \tau r, \tau\mu_A, \overline{\lambda}_1, \overline{\lambda}_2, \kappa\right)$. Along this sequence aggregate risk goes to zero while other parameters remain bounded. Hence this model is close to its habit representation; but then so is the original model.

A.6 Proof map for Section 4

These proofs—which build on the ideas described above—are in the Supplementary Appendix.

Appendix B: Simulations

Details are in the Supplementary Appendix. Our strategy is to choose deep parameters to generate variation in the consumption risk parameters σ_I and σ_A while holding fixed consumption growth. In all environments of Figures 1-3, the parameters $(\gamma, \kappa, \lambda_1, \lambda_2, \delta) = (2, 1, 1, 0, .0326)$ are held fixed. Other parameters and the implied values of σ_A, σ_I, μ_a and μ_A are given below.

	π_M/σ_M	π_E/σ_E	r	σ_A	σ_I	μ_a	μ_A
(a) High aggr, low idiosyncr risk	20%	10%	3.24%	10%	5%	1.24%	1.37%
(b) High aggr, high idiosyncr risk	20%	20%	1%	10%	10%	.87%	1.37%
(c) Low aggr, low idiosyncr risk	10%	10%	4.74%	5%	5%	1.24%	1.37%
(d) Low aggr, high idiosyncr risk	10%	20%	2.5%	5%	10%	.87%	1.37%

Supplementary Material to "Consumption Commitments and Habit Formation"[1]

Raj Chetty
Harvard University and NBER

Adam Szeidl
Central European University and CEPR

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This material supplements the paper "Consumption Commitments and Habit Formation. We provide missing proofs for results stated in the main paper and we explain the numerical methods used to simulate the model.

A-1 Proofs of Propositions *2* and *3*

Proof of Proposition 2. Since the only risky assets for household i are S and S^i, there exists a unique state price density associated with the household-specific private market. The following dynamics for adjustable consumption generates a state price density that prices both risky assets as well as the safe asset

$$a_t^i = a_0^i \exp\left\{\frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho\right)t + \frac{\pi}{\gamma\sigma}z_t + \frac{\pi_I}{\gamma\sigma_I}z_t^i\right\} \tag{15}$$

and hence must describe the optimal choice of household i. Because $a_0^i = A_0$ for all i, aggregating across i yields, by the strong law of large numbers for a continuum of agents (Sun, 1998)

$$\begin{aligned}A_t &= A_0 \exp\left\{\frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho\right)t + \frac{\pi}{\gamma\sigma}z_t\right\}\int_i \exp\left\{\frac{\pi_I}{\gamma\sigma_I}z_t^i\right\}di \\ &= A_0 \exp\left\{\frac{1}{\gamma}\left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2}\left(1+\frac{1}{\gamma}\right) + r - \rho\right)t + \frac{\pi}{\gamma\sigma}z_t\right\}.\end{aligned}$$

[1] E-mails: chetty@fas.harvard.edu, szeidla@ceu.hu .

Define a new discount rate $\delta = \rho - \left(1 + \frac{1}{\gamma}\right) \pi_I^2/(2\sigma_I^2)$. Then the dynamics of aggregate adjustable consumption is given by

$$A_t = A_0 \exp\left\{ \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + r - \delta \right) t + \frac{\pi}{\gamma\sigma} z_t \right\}.$$

This is exactly the dynamics of adjustable consumption that would obtain for a representative consumer with power utility over A_t and discount rate δ who can invest in the publicly traded risky and safe assets.

Proof of Proposition 3. We are interested in characterizing the evolution of the conditional distribution of y_t^i given a realization of the path of A under Q. Using (15) we obtain

$$d \log a_t^i = \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho \right) dt + \frac{\pi}{\gamma\sigma} dz + \frac{\pi_I}{\gamma\sigma_I} dz^i = \theta dt + \frac{\pi}{\gamma\sigma} dz + \frac{\pi_I}{\gamma\sigma_I} d\overline{z}^i$$

where

$$\theta = \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho \right) + \frac{\pi_I^2}{\gamma^2\sigma_I^2} \tag{16}$$

is the drift under Q. We first show that $F(y,t)$ is absolutely continuous for all $t > 0$ for almost all realizations of the path of aggregate shocks. We do this assuming that the initial condition is $a_0^i = A_0$ and $x_0^i = X_0$ for all agents i, i.e., that the initial distribution $F_0(y)$ is concentrated on a single point. For other initial distributions the density $f(y,t)$ can simply be computed as an integral of these densities with respect to $F_0(y)$.

Throughout the argument we work with the probability measure Q. Our proof logic is to fix $t = T$ and the realization of A_t for $t \in [0,T]$, pick a collection of intervals $I \subset [L,U]$, compute an upper bound on the probability that $y_T \in I$, and then establish that the upper bound goes to zero as the total length of these intervals, denoted $|I|$, goes to zero. Our upper bound is obtained by separately bounding the probabilities of two events.

(1) Reaching I through paths that do not involve "too many" adjustments. Let $\widetilde{y}_0^i = y_0$ and

$$d\widetilde{y}_t^i = -\theta \cdot dt - \sigma_A \cdot dz_t - \sigma_I \cdot d\overline{z}_t^i.$$

Given the dynamics of $\log a_t^i$, this specification implies that the evolution of $\widetilde{y}$ is the same as that of y except for the discrete adjustments. In particular, $\widetilde{y}_t^i = y_t^i$ before the first adjustment occurs. More generally, if y^i experiences n_U upward and n_D downward adjustments in the interval $[0,t]$, then $y_t^i = \widetilde{y}_t^i + n_D (M - L) - n_U (U - M)$. Because $\widetilde{y}_t^i$ is a Brownian motion with a drift, its density is bounded from above by some constant which depends on the parameters of the process, which we denote by $K(\mu_a, \sigma_A, \sigma_I, T)$. As a result, for any given $n \geq 1$, the total probability of paths which involve $n_U < n$ upward and $n_D < n$ downward adjustments such that $y_T^i \in I$ is at most $K(\mu_a, \sigma_A, \sigma_I, T) \cdot n^2 \cdot |I|$.

(2) The total probability of paths that involve at least n adjustments. Let $\widetilde{y}_0^A = y_0$ and $d\widetilde{y}_t^A = -\theta \cdot dt - \sigma_A \cdot dz_t$ so that $\widetilde{y}_t^A$ represents the aggregate shocks and trend in $\widetilde{y}_t$, and let $\widetilde{y}_0^{I,i} = 0$ and $d\widetilde{y}_t^{I,i} = \sigma_I \cdot d\overline{z}_t^i$ so that $\widetilde{y}_t^{I,i}$ represents the idiosyncratic shocks. Then $\widetilde{y}_t^i = \widetilde{y}_t^A + \widetilde{y}_t^{I,i}$. The path of $\widetilde{y}_t^A$ contains the same information as the path of aggregate shocks A_t, hence we are effectively conditioning on the realization of the path of $\widetilde{y}_t^A$. Set $\Delta_y = \min(U - M, M - L)/2$. We say that a process u_t moves Δ_y between s and t if $|u_t - u_s| = \Delta_y$. Suppose that $s_1 < s_2$ are two consecutive adjustment dates for household i. Then either $\widetilde{y}_t^A$ or $\widetilde{y}_t^{I,i}$ must move at least Δ_y between s_1 and s_2. Because almost surely the path of $\widetilde{y}_t^A$ is continuous, one can straightforwardly verify that there is an upper bound $K\left(\widetilde{y}_{[0,T]}^A\right)$ on the number of non-overlapping time intervals in $[0, T]$ over which y_t^A moves at least Δ_y. For ease of notation, in the rest of this proof we will simply denote $K\left(\widetilde{y}_{[0,T]}^A\right) = K$. Then, if household i adjusts at least n times in $[0, T]$, there must exist at least $n - K$ non-overlapping intervals in $[0, T]$ over which $\widetilde{y}_t^{I,i}$ moves at least Δ_y. Assume now that $n > 2K + 1$. At least one of these intervals—denote it by $[s_1, s_2]$—cannot be longer than $T/(n - K)$. Now cover the $[0, T]$ interval with subintervals of length $2T/(n - K)$ starting at zero, and by another set starting at $T/(n - K)$. It is clear that an interval in one of these covers, say $[s_0, s_3]$ must fully contain $[s_1, s_2]$.

The probability that $\widetilde{y}_t^{I,i}$ moves at least Δ_y over $[s_1, s_2]$ is bounded by the probability that the difference between the minimum and the maximum of $\widetilde{y}_t^{I,i}$ in $[s_0, s_3]$ is at least Δ_y. Given that the density of the running maximum of a standard Brownian motion is $(2/(\pi t))^{1/2} e^{-m^2/(2t)}$, this probability is bounded above by a universal constant times $\left((n - K)/\left(\pi T \sigma_I^2\right)\right)^{1/2} \exp\left[-\Delta^2 (n - K)/\left(2T\sigma_I^2\right)\right]$. Because the total number of intervals in the two covers we introduced is at most $2(n - K)$, the probability that $\widetilde{y}_t^{I,i}$ moves at least Δ_y over an interval of length at most $T/(n - K)$ is bounded from above by a constant (which depends on T and σ_I) times $(n - K)^{3/2} \exp\left[-\Delta^2 (n - K)/\left(2T\sigma_I^2\right)\right]$. Recalling the assumption that $n > 2K + 1$, the last expression can be bounded above by a different constant (which depends on T and σ_I^2) times $\exp\left[-\Delta^2 n/\left(8T\sigma_I^2\right)\right]$.

We now combine these bounds. Given K, which is determined by the path of $\widetilde{y}_t^A$, and maintaining $n > 2K+1$, the total probability that $y_T^i \in I$ is at most

$$K\left(\mu_a, \sigma_A, \sigma_I, T\right) \cdot n^2 \cdot |I| + K\left(\sigma_I^2, T\right) \cdot \exp\left[-\Delta^2 n / \left(8T\sigma_I^2\right)\right].$$

Setting $n = |I|^{-1/4}$, for small enough $|I|$ such that $n > 2K+1$ is satisfied, the bound becomes

$$K\left(\mu_a, \sigma_A, \sigma_I, T\right) \cdot |I|^{1/2} + K\left(\sigma_I^2, T\right) \cdot \exp\left[-\Delta^2 |I|^{-1/4} / \left(8T\sigma_I^2\right)\right]$$

which goes to zero as $|I|$ goes to zero.

We now turn to the stochastic partial differential equation. Proposition 1 in Caballero (1993) derives a stochastic partial differential equation, given the path of aggregate shocks, for the conditional density of a double-barrier Brownian motion with rebirth. Caballero's equation is

$$df(y,t) = \left[\theta \frac{\partial f(y,t)}{\partial y} + \frac{\sigma_T^2}{2} \frac{\partial^2 f(y,t)}{\partial y^2}\right] dt + \sigma_A \frac{\partial f(y,t)}{\partial y} dz.$$

Substituting in (16) yields the equation in the text. The boundary conditions follow directly from Caballero's proposition.

To derive the dynamics of aggregate commitments, note that $X_t = \int_L^U e^y f(y,t) dy \cdot A_t$ and we can use Ito's lemma to write

$$dX_t = A_t \int_L^U e^y \cdot df(y,t) \cdot dy + dA_t \cdot \int_L^U e^y f(y,t) dy + \left\langle \int_L^U e^y \cdot df(y,t) \cdot dy, dA_t \right\rangle.$$

We now evaluate each term on the right hand side. The first term is

$$A_t \int_L^U e^y \cdot \frac{\partial f(y,t)}{\partial y} \left\{ \left(\mu + \frac{\pi_I^2}{2\gamma^2 \sigma_I^2}\right) dt + \frac{\pi}{\gamma\sigma} dz \right\} dy + F_t \int_L^U e^y \cdot \frac{\partial^2 f(y,t)}{\partial y^2} \frac{\sigma_T^2}{2} dt \cdot dy.$$

Integrating by parts, and using the boundary conditions shows that this term equals

$$-X_t \left(\left(\mu + \frac{\pi_I^2}{2\gamma^2\sigma_I^2}\right) dt + \frac{\pi}{\gamma\sigma} dz \right) + A_t \frac{\sigma_T^2}{2} \cdot \left(f_y(L,t)(e^M - e^L) + f_y(U,t)(e^U - e^M)\right) dt + \frac{\sigma_T^2}{2} X_t dt.$$

The second term is

$$X_t \cdot \frac{dA_t}{A_t} = X_t \left(\left(\mu + \frac{\pi^2}{2\gamma^2\sigma^2}\right) dt + \frac{\pi}{\gamma\sigma} dz \right)$$

while the third term is simply $-\pi^2 / (\gamma\sigma)^2 X_t dt$. Collecting terms gives the result of the proposition.

A-2 Proofs of results leading up to Theorem 1

A-2.1 Proofs of auxiliary results about the commitments model including proof of Proposition 4

Proof of Lemma 1. We start with the case where w_t is driven by a standard Brownian motion. Let $\zeta_y = \inf\{t \geq 0 : w_t \notin [L,U], w_0 = y\}$. Set $F_w(t) = \Pr\left[\zeta_y \leq t\right]$ and $\overline{h}(y,t) = E\left[e^{w_t} \cdot 1\left\{\zeta_y > t\right\}\right]$ be $h(y,t)$ killed at the boundary. Let $F_y^{(1)}(t) = F_y(t)$ and $F_y^{(n+1)}(t) = \int_0^t F_{y^*}^{(n)}(t-\tau)\, dF_y(\tau) = \int_0^t F_M(t-\tau)\, dF_y^{(n)}(\tau)$ be the the distribution of the $n+1$st exit time. Then

$$h(y,t) = \overline{h}(y,t) + \sum_{n=1}^{\infty} \int_0^t \overline{h}(M, t-\tau)\, dF_y^{(n)}(\tau) = \overline{h}(y,t) + \int_0^t \overline{h}(M,t-\tau)\, dF_y^*(\tau) \tag{17}$$

where

$$F_y^*(t) = \sum_{n=1}^{\infty} F_y^{(n)}(t) = F_y(t) + \int_0^t F_M^*(t-\tau)\, dF_y(\tau) = F_y(t) + \int_0^t F_M(t-\tau)\, dF_y^*(\tau) \tag{18}$$

is the expected number of boundary hits until t.

The transition density of the killed diffusion $p(y,y',t) = \Pr\left[\zeta_y > t,\, y_t = y'\right]$ can be expressed as an infinite sum of normal densities (Revuz and Yor, 1992, p 106), and in particular, is infinitely many times differentiable in $[L,U] \times [L,U] \times (0,\infty)$. This implies that $\overline{h}(y,t) = \int e^{y'} p(y,y',t)\, dy'$ is infinitely many times differentiable in $[L,U] \times (0,\infty)$. The density of the first hitting time ζ_y can also be expressed in closed form as an infinite sum (Darling and Sieger, 1953), and is infinitely many times differentiable in y and t over $[L,U] \times (0,\infty)$. This, combined with (18) implies that $F_y^*(t)$ is C^∞ in $[L,U] \times (0,\infty)$. Combining these observations with (17) shows that $h(y,t)$ is also C^∞ in the $[L,U] \times (0,\infty)$ domain.[1]

We next show that h is also smooth when driven by any Brownian motion with drift and variance, and that it is smooth in the other parameters. Changing the clock of y_t scales both the mean and the variance, and is obviously a smooth transformation of $h(y,t)$ as it just scales the time argument. Shifting and rescaling the vertical axis are smooth operations that shift and rescale the triple $[L,M,U]$. Thus we only need to show smoothness in the drift and in M. The drift can be dealt with using the Girsanov theorem, which implies that the density of the killed diffusion under drift can be obtained as $p^{\mu w}(y,y',t) = p(y,y',t) \cdot \exp\left[\mu_w(y'-y) - \mu_w^2 t/2\right]$, which

[1] Grigorescu and Kang (2002) compute the transition density of y explicitly.

is clearly C^{∞} in μ_w, and hence so is $\overline{h}(y,t)$. Next, the distribution of the first hitting time is $1-F_y^{\mu_y}(t)=\int p^{\mu_y}(y,y',t)\,dy'$ is also smooth. The smoothness of h in μ_y now follows from (17). Smoothness in M follows easily from (17).

Proof of Lemma 2. We have

$$E_s\left[\overline{X}_t\right]=\overline{A}_s\cdot E_s^R\left[\overline{X}_t/\overline{A}_t\right]=\overline{A}_s\cdot E_s^{QR}\left[x_t/a_t\right]=\overline{A}_s\cdot\int_L^U h(t-s,y)\,f(y,s)\,dy$$

which is a martingale in s. Computing the Ito-differential

$$d_sE_s\left[\overline{X}_t\right]=d\overline{A}_s\cdot E_s^{QR}\left[x_t/a_t\right]+\overline{A}_s\cdot\int_L^U h(t-s,y)\,f_y(y,s)\,\sigma_Adz_s\cdot dy$$

where we used (7) for the evolution of $f(y,s)$ and collected only the dz terms, since the ds terms must cancel by the martingale property. Equivalently,

$$d_sE_s\left[\overline{X}_t\right]=d\overline{A}_s\cdot\left(E_s^{QR}\left[x_t/a_t\right]+\int_L^U h(t-s,y)\,f_y(y,s)\,dy\right)=d\overline{A}_s\cdot\int_L^U\left(h(u,y)-h_y(u,y)\right)f(y,s)\,dy$$

where we integrated by parts. This equation shows the existence of ξ as well as the desired representation.

Proof of Lemma 3. Ben-Ari and Pinsky (2009) show that $y_t=\log\left[x_t/a_t\right]$ converges exponentially fast to a unique invariant distribution. it follows from Ben-Ari and Pinsky (2007) that the rate of convergence is uniformly bounded if the drift is from a bounded interval. This implies uniform convergence for all $\sigma_A\in[0,\overline{\sigma}_A]$ through a clock-change argument. Since

$$E_0\left[\overline{X}_t\right]=E_0^R\left[\overline{X}_t/\overline{A}_t\right]=E_0^{QR}\left[x_t/a_t\right],$$

it follows that $E_0\left[\overline{X}_t\right]$ converges exponentially fast to the mean $\overline{x}$ of x/a under the invariant distribution, and that this is uniform in σ_A. Recalling that $h(u,y)=E^{QR}\left[x_u/a_u|x_0/a_0=e^y\right]$, we also have $h(u,y)$ converge at the same rate to $\overline{x}$ as $u\to\infty$, uniformly in y and σ_A. Letting $F_t^{QR}\left[y|y_0\right]$ denote the cross-sectional distribution of y_t given initial value y_0, fixing some $s<u$, we

can write

$$h_{y_0}(u, y_0) = \frac{\partial}{\partial y_0}\int_L^U h(u-s, y)\ dF_t^{QR}[y|y_0] = \int_L^U h(u-s, y)\ \frac{\partial^2 F_t^{QR}[y|y_0]}{\partial y_0 \partial y} dy$$
$$= \int_L^U (h(u-s, y) - \overline{x})\ \frac{\partial^2 F_t^{QR}[y|y_0]}{\partial y_0 \partial y} dy$$

where at the last step we used that $\partial^2 F_t^{QR}[y|y_0]/\partial y_0 \partial y$ integrates to zero in y. By the arguments of Lemma 1, $\partial^2 F_t^{QR}[y|y_0]/\partial y_0 \partial y$ is bounded, while $h(u-s, y) - \overline{x}$ converges exponentially fast to zero; hence so does the integral.

Proof of Proposition 4. We show that $\xi(u, f)$ equals the impulse response of Definition 1. Let $\overline{A}_0^*$ be the point at which we want to differentiate $E_0\left[\overline{X}_t\left(A_0, F^x(x_0|A_0^*)\right)\right]$. We can write

$$\begin{aligned} E_0\left[\overline{X}_t\left(A_0, F^x(x_0|A_0^*)\right)\right] &= A_0 \cdot E_0^R\left[\overline{X}_t\left(A_0, F^x(x_0|A_0^*)\right)_t / \overline{A}_t\right] \\ &= A_0 \cdot \int_L^U h\left(t, y - (\log A_0 - \log A_0^*)\right) dF_0(y). \end{aligned}$$

This is because when $\overline{A}_0 = \overline{A}_0^*$, the mass of people at any point y is given by $dF_0(y)$, and the conditional expectation given y is summarized by h. When $\overline{A}_0$ changes, the mass of these people is unaffected, and hence $dF_0(y)$ is unchanged; but—because commitments are held fixed while A_0 changes—their y shifts. Hence we must evaluate h at a point which recognizes this change.

Differentiating this expression in A_0 gives

$$\frac{E_0\left[\overline{X}_t\left(A_0, F^x(x_0|A_0^*)\right)\right]}{\partial A_0} = \int_L^U h(t, y)\, dF_0(y) - \int_L^U h_y(t, y)\, dF_0(y) = \int_L^U \left[h(t, y) - h_y(t, y)\right] dF_0(y)$$

which is exactly the definition of ξ given above when $F_0(y)$ has a density. This confirms that the impulse response is well defined, that it is independent of A_0^*, and that the MA representation claimed in the proposition holds.

Proof of Lemma 4. We know that EF converges to F^* uniformly in y. Fix $\varepsilon > 0$ and pick s so that for all $t > s$, $|EF_t - F^*| < \varepsilon/8$ for all initial conditions and for all σ small enough. Consider the rectangular set $[-\kappa, \kappa] \times [t-s, t]$, and let G_κ denote the event when the realization of $\log \overline{A}_u - \log \overline{A}_{t-s}$ for $u \in [t-s, t]$ is in this set. Let $F\left(y, t, \overline{A}_{[t-s,t]}, y_s\right)$ denote the distribution of y_t under Q when started at y_s in s, and when the realization of aggregate shocks is given by $\overline{A}_{[t-s,t]}$. We then have that $\left\{\sup_{y_t, y_s}\left|F\left(y, t, \overline{A}_{[t-s,t]}, y_s\right) - F\left(y, t, \overline{A}'_{[t-s,t]}, y_s\right)\ \right|\ \overline{A}_{[t-s,t]}, \overline{A}'_{[t-s,t]} \in G_\kappa\right\}$ goes to zero as $\kappa \to 0$: two sufficiently close paths of aggregate consumption generate cross-sectional distributions that are themselves close. This is because the share of people for whom the two aggregate paths result in sufficiently different behavior goes to zero. Take κ small enough so that this quantity is less than $\varepsilon/8$. For any fixed κ we can pick σ small enough so that $\Pr\left[\overline{A}_{[t-s,t]} \in G_\kappa\right] > 1-\varepsilon/8$. This implies that $|E_s F_t - E\left[F_t | f(s), G_\kappa\right]| < \varepsilon/4$. Combining these bounds, for $\overline{A}_{[t-s,t]} \in G_\kappa$ we have

$$\left|F\left(y,t,\overline{A}_{[t-s,t]},f(s)\right) - F^*(y)\right| \leq$$
$$\left|F\left(y,t,\overline{A}_{[t-s,t]},f(s)\right) - E\left[F_t|f(s),G_\kappa\right]\right| + \left|E\left[F_t|f(s),G_\kappa\right] - E_s F_t\right| + \left|E_s F_t - F^*(y)\right| < \frac{\varepsilon}{8}+\frac{\varepsilon}{4}+\frac{\varepsilon}{8} = \frac{\varepsilon}{2}.$$

Using this, we have

$$\left\|\sup_y |F(y,t) - F^*(y)|\right\|_p^p =$$
$$\Pr[G_\kappa]\cdot E\left[\sup_y (F(y,t)-F^*(y))^p | G_\kappa\right] + (1-\Pr[G_\kappa])\cdot E\left[\sup_y (F(y,t)-F^*(y))^p \mid \text{not } G_\kappa\right] \leq$$
$$\left[\left(\frac{\varepsilon}{2}\right)^p + 2^p\frac{\varepsilon}{8}\right] < 2^p\varepsilon.$$

Since this is true for all $t > s$, it is also true for the lim sup. But ε was arbitrary, and the bound applies for all σ small enough given ε; hence the desired result follows.

A-2.2 Proofs of auxiliary results about ther habit model

Proof of Lemma 5. Starting with the A-weighted habit model, consider the unique solution of the integral equations for ζ and o (see Lew, 1972 for existence and uniqueness) and define

$$\widetilde{X}_t = o(t)X_0 + \int_0^t \zeta(t-s)C_s ds.$$

We will show that $\widetilde{X}_t = X_t$ for all $t \geq 0$. First note that

$$\begin{aligned}
\widetilde{X}_t &= o(t)X_0 + \int_0^t \zeta(t-s)\left[A_s + X_s\right] ds \\
&= o(t)X_0 + \int_0^t \zeta(t-s)A_s + \zeta(t-s)\left[\int_0^s j(s-u)A_u du \ + k(s)X_0\right] ds \\
&= o(t)X_0 + \int_0^t A_s\left[\zeta(t-s) + \int_0^{t-s} j(u)\zeta(t-s-u)du\right] ds + X_0\int_0^t \zeta(t-s)k(s)ds.
\end{aligned}$$

Equating coefficients, $X_t = \widetilde{X}_t$ holds if

$$j(t-s) = \zeta(t-s) + \int_0^{t-s} j(u)\zeta(t-s-u)\ du$$

or, with $t-s=u$,

$$\zeta(u) = j(u) - \int_0^u \zeta(v)j(u-v)dv$$

and

$$o(u) = k(u) - \int_0^u \zeta(u-v)k(v)dv.$$

Substituting in $u=0$ gives $\zeta(0)=j(0)$ and $o(0)=k(0)$. The integral equation for $\zeta(u)$ then yields a unique solution, which can be used to determine $o(.)$. By the above argument, a pair of functions that solve these equations also give $X_t = \widetilde{X}_t$, which is the desired representation.

Proof of Lemma 6. Detrending both sides and integrating by parts (using that ξ^* is smooth)

$$\begin{aligned} \overline{X}_t^h &= \int_0^t \xi^{*\prime}(t-s)\,\overline{A}_s ds + [\overline{x} - \xi^*(t)]\,A_0 = \left[-\xi^*(t-u)\,\overline{A}_u\right]_0^t + \int_0^t \xi^*(t-s)\,d\overline{A}_s + [\overline{x} - \xi^*(t)]\,A_0 \\ &= \int_0^t \xi^*(t-s)\,d\overline{A}_s + \overline{x}A_0. \end{aligned}$$

A-2.3 Proofs of results used in establishing Theorem 1

Proof of Lemma 7. We proceed by induction on t. Fix some $k>0$. We show that (i) the desired bound holds when $t \le k$, and (ii) if the bound holds for some t, it also holds for $t+k$. We begin by showing (ii), which is the more difficult part.

We can write

$$\|G_t\|_p \le \left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \int_{t-k}^t g(t-s)\,\frac{\overline{A}_s}{\overline{A}_{u-k}} dz_s \right\|_p + \left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_p \cdot \left\| \frac{1}{\overline{A}_{t-k}} \int_0^{t-k} g(t-s)\,\overline{A}_s dz_s \right\|_p$$

where we used independence of the Brownian increments. Denoting $\overline{g}(u,s) = e^{K_2 k} g(u+k,s)$ we can rewrite the final term in brackets as

$$e^{-K_2 k} \cdot \frac{1}{\overline{A}_{t-k}} \int_0^{t-k} \overline{g}(t-k-s,s)\,A_s dz_s$$

where $|\overline{g}(u,s)| \le K_1 e^{-K_2 u}$ by construction. By our induction assumption, this term has p-norm bounded by $e^{-K_2 k} \cdot M(p)$. To bound the remaining terms, first observe that by lognormality

$$\left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_p \leq K_p(\sigma_A, k)$$

for some $K_p(\sigma_A, k)$ that goes to one in σ_A for all k. Next note that

$$\left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \int_{t-k}^{t} g(t-s,s) \frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s \right\|_p \leq \left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_{2p} \cdot \left\| \int_{t-k}^{t} g(t-s,s) \frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s \right\|_{2p}$$

by the Cauchy-Schwarz inequality. Here

$$\left\| \frac{\overline{A}_{t-k}}{\overline{A}_t} \right\|_{2p} \leq K_{2p}(\sigma_A, k)$$

where $K_{2p}(\sigma_A, k)$ also goes to one in σ_A for all k. Finally, using standard bounds (e.,g., Karatzas and Shreve, 2008) for moments of the Ito integral, we obtain

$$\left\| \int_{t-k}^{t} g(t-s,s) \frac{\overline{A}_s}{\overline{A}_{t-k}} dz_s \right\|_{2p} \leq K_{2p} \left(\int_{t-k}^{t} K_1^2 \left\| \left(\frac{\overline{A}_s}{\overline{A}_{u-k}} \right)^2 \right\|_p ds \right)^{1/2}$$

which is bounded by $K_{2p} K_1 k \cdot K_{2p}(\sigma_A, k)$. Combining terms we obtain

$$\|G_t\|_p \leq K_{2p}^2(\sigma_A, k) \cdot K_{2p} K_1 k + K_p(\sigma_A, k) \cdot e^{-K_2 k} \cdot M(p).$$

It is easy to see that if

$$M(p) = \frac{K_{2p}^2(\sigma_A, k) \cdot K_{2p} K_1 k}{1 - K_p(\sigma_A, k) \cdot e^{-K_2 k}}$$

is positive, then the induction step follows. We can make sure that this is the case by first choosing some $k > 0$, and then picking $\overline{\sigma_A}$ small enough so that for all $\sigma_A \leq \overline{\sigma}_A$ we have $K_p(\sigma_A, k) < e^{K_2 k/2}$. With this choice of $M(p)$, the induction step follows; and (i) can be verified easily from the argument of the induction step.

Proof of Lemma 8. We verify directly that changing the clock is equivalent to rescaling the relevant parameters in the setup of the problem. Maximizing the consumer's problem in the original model is equivalent to maximizing

$$E \int_0^\infty e^{-\rho t \tau} \left(\frac{a_{\tau t}^{1-\gamma}}{1-\gamma} + \mu \frac{x_{\tau t}^{1-\gamma}}{1-\gamma} \right) dt$$

which is proportional to the objective function in the model with new parameters. Similarly, the budget constraint of the original model implies

$$dw_{\tau t} = \left[\left(\tau r + \alpha_{\tau t}\tau\pi + \alpha^i_{\tau t}\tau\pi_I\right) w_t - \tau c_t\right] dt + \alpha_{\tau t} w_{\tau t}\sigma\tau^{1/2} dz_{\tau t} + \alpha^i_{\tau t} w_{\tau t}\sigma_i\tau^{1/2} dz^i_{\tau t}$$

on all non-adjustment dates due to the scaling invariance of Brownian motion. Finally, on adjustment dates, $dw = \overline{\lambda}_1 x_{t-}/r + \overline{\lambda}_2 x_t/r = \overline{\lambda}_1 \cdot \tau x_{t-}/\left(\tau r\right) + \overline{\lambda}_2 \cdot \tau x_t/\left(\tau r\right)$. Since the optimal policy is unique, the claim follows.

A-3 Proofs for Section 4.1

A-3.1 Proof of Proposition 5

(1) Excess smoothness. Using a Taylor expression we can write

$$\log C_{t_1} - \log C_{t_0} = \frac{A_{t_0}}{C_{t_0}}\left(\log A_{t_1} - \log A_{t_0}\right) + \varepsilon_{t_1} \tag{19}$$

where, because X_t has bounded variation, there exists K_ε such that $E\varepsilon^2_{t_1} < K_e\left(t_1 - t_0\right)^2$. Thus

$$\beta_1\left(t_1\right) = \frac{\operatorname{cov}\left(\log\left(C_{t_1}/C_{t_0}\right), \log\left(A_{t_1}/A_{t_0}\right)\right)}{\operatorname{var}\left(\log\left(A_{t_1}/A_{t_0}\right)\right)} \leq \frac{A_{t_0}}{C_{t_0}} + \frac{\sigma_A\left(t_1 - t_0\right)^{1/2} K_\varepsilon\left(t_1 - t_0\right)}{\sigma^2_A\left(t_1 - t_0\right)} = \frac{A_{t_0}}{C_{t_0}} + \frac{\left(t_1 - t_0\right)^{1/2} K_\varepsilon}{\sigma_A}$$

and the right-hand side approaches A_{t_0}/C_{t_0} as $t_1 \to t_0$.

(2) Excess sensitivity. Let $t_1 = t_2$. From the proof of Lemma 3 we know that $\log\left[A_{t_3}/C_{t_3}\right]$ converges exponentially fast to an invariant distribution. In particular, $E_{t_1}\left[\log\left[A_{t_3}/C_{t_3}\right]\right]$ converges exponentially fast to the mean of this invariant distribution, which we denote by $\overline{\xi}$, so that we can write $\log C_{t_3} = \log A_{t_3} + \overline{\xi} + \varepsilon_{t_3}$ where $E_{t_1}\left[\varepsilon_{t_3}\right]$ converges to zero at a given exponential rate as $t_3 \to \infty$. Using (19) we can write

$$\begin{aligned}
\log C_{t_3} - \log C_{t_1} &= \log A_{t_3} + \overline{\xi} + \varepsilon_{t_3} - \log C_{t_0} - \frac{A_{t_0}}{C_{t_0}}\left(\log A_{t_1} - \log A_{t_0}\right) - \varepsilon_{t_1} \\
&= \frac{X_{t_0}}{C_{t_0}}\left(\log A_{t_1} - \log A_{t_0}\right) + \left(\log A_{t_3} - \log A_{t_1}\right) + \left(\log A_{t_0} + \overline{\xi}\right) + \left(\varepsilon_{t_3} - \varepsilon_{t_1}\right).
\end{aligned}$$

To compute β_2, we evaluate the covariance of $\log A_{t_1} - \log A_{t_0}$ with each of the terms in this expression. Because $\log A_t$ is a Brownian motion with drift, the covariance with the term in the second parenthesis is zero. Conditional on the history up to t_0, the terms in the third parenthesis are constants, hence their covariance is also zero. The terms in the fourth parenthesis are error terms: just like in the proof of (1), ε_1 can be made arbitrarily small by choosing t_1 small; and ε_{t_3} is approximately orthogonal to events before t_1 for t_3 large. Thus for t_1 small and t_3 large the regression coefficent is determined by the first term, implying that β_2 is approximately $X_{t_0}/C_{t_0} > 0$.

A-3.2 Modeling large shocks

Our approach is to construct, on a single probability space, a set of "shock" processes for each $t_1 > t_0$, such that the distribution of the process for a given t_1 is identical to the distribution of $\overline{A}_t$ conditional on the shock event $S(t_1, \Delta)$. This construct will allow us to take limits while holding fixed the probability space.

Formally, we introduce the auxiliary process $\widetilde{A}_t$, which agrees with $\overline{A}_t$ for $t \leq t_0$, and has the same distribution as $\overline{A}_t$ for $t > t_0$. The idea is that innovations in $\widetilde{A}_t$ will be driving $\overline{A}_t$ after the shock. We also introduce an independent standard Brownian motion $\overline{B}_s$ defined for $s \geq 0$, which will drive the innovations during the shock. We then model the positive shock as a Brownian bridge for $\log \overline{A}_t$ conditioned to start at $\log \overline{A}_{t_0}$ at time t_0, and to reach $\log \overline{A}_{t_0} + \Delta$ at time t_1. We denote this process by $\overline{A}_t(+, t_1, \Delta)$, and construct it as follows: for $t_0 \leq t \leq t_1$, we let log $\overline{A}_t(+, t_1, \Delta) = \sigma_A \left(\overline{B}_{t-t_0} - (t - t_0)\overline{B}_{t_1}\right) + (t - t_0)\Delta$, and for $t \geq t_1$ we let $d\log \overline{A}_t(+, t_1, \Delta) = d\log \widetilde{A}_t$. Although the expression for $t_0 \leq t \leq t_1$ does not make this clear, it is well-known that this Brownian bridge is an Ito-processes. We construct $\overline{A}_t(-, t_1, \Delta)$ analogously. Given that it is a Brownian bridge between $t_0 \leq t \leq t_1$ it follows that log $\overline{A}_t(+, t_1, \Delta)$ has the same distribution as our original process log $\overline{A}_t$ conditional on $S(+, t_1, \Delta)$.

The formulas for the dynamics of X_t, C_t, X_t^h and C_t^h, once we replace $\overline{A}_t$ by $\overline{A}_t(+, t_1, \Delta)$ respectively $\overline{A}_t(-, t_1, \Delta)$, directly extend, and generate the distributions of commitments, habit, and consumption conditional on the shock event. To clarify which process we have in mind, we sometimes use notation such as $\overline{X}_t(+, t_1, \Delta)$ to refer to aggregate commitments (during or after a positive shock) on the probability space just constructed. However, when it does not cause confusion we often just write $\overline{X}_t^h$ and say in words that we work with the "shock" processes.

One key feature of this construction is that instead of considering a sequence of non-overlapping events $S(t_1, \Delta)$, we consider a single probability space and a sequence of processes. The advantage is that we can use the L_p norm on this common probability space when we take various limits over $\bar{t}$. In particular, throughout the analysis below, we use L_p (conditional on the history up to t_0) for all $p \geq 1$ as we take the limits $t_1 \to t_0$ and $t_2 \to t_0$.

A-3.3 Continuity after large shocks

We show that $\overline{X}_t^h$ and $\overline{C}_t^h$ change continuously around t_0 in the limit as $t_1 \to t_0$ and as $t_2 \to t_0$.

Lemma 9 *We have* $\lim_{t_1 \to t_0} \overline{X}_{t_1}^h (+, t_1, \Delta) = \overline{X}_{t_0}^h$ *and* $\lim_{t_1 \to t_0} \overline{X}_{t_1}^h (-, t_1, \Delta) = \overline{X}_{t_0}^h$. *Moreover, even after taking the limit* $t_1 \to t_0$ *the the dynamics of* $\overline{X}_t^h$ *continuous at* t_0*:* $\lim_{t_2 \to t_0} \lim_{t_1 \to t_0} \overline{X}_{t_2}^h (+, t_1, \Delta) = \overline{X}_{t_0}^h$ *and* $\lim_{t_2 \to t_0} \lim_{t_1 \to t_0} \overline{X}_{t_2}^h (-, t_1, \Delta) = \overline{X}_{t_0}^h$.

Proof. Consider the case when the shock is positive. Suppressing in notation that we work with the "shock" processes, according to the representation in Lemma 7, $\overline{X}_{t_2}^h = \int_0^{t_2} \xi^{*\prime} (t_2 - s) \overline{A}_s (+, t_1, \Delta) ds + [\overline{x} - \xi^* (t_2)] A_0$. When $t_2 = t_1$ goes to t_0, this expression converges to $\int_0^{t_0} \xi^{*\prime} (t_2 - s) \overline{A}_s (+, t_1, \Delta) ds + [\overline{x} - \xi^* (t_2)] A_0 = \overline{X}_{t_0}^h$ proving, for a positive shock, the first claim. For the second claim, note that as $t_1 \to t_0$ the last term is constant while the first term converges to $\int_{t_0}^{t_2} \xi^{*\prime} (t_2 - s) \widetilde{A}_s \cdot e^{\Delta} ds + \int_0^{t_0} \xi^{*\prime} (t_2 - s) \overline{A}_s ds$. Here only the first integral depends on t_2 and as $t_2 \to t_0$ it converges to zero. The same logic works when the shock is negative.

Lemma 10 *We have* $\lim_{t_1 \to t_0} \log \left[\overline{C}_{t_1}^h (+, t_1, \Delta)\right] = \log \left[e^{\Delta} \overline{A}_{t_0} + \overline{X}_{t_0}^h\right]$ *and* $\lim_{t_1 \to t_0} \log \left[\overline{C}_{t_1}^h (-, t_1, \Delta)\right] = \log \left[e^{-\Delta} \overline{A}_{t_0} + \overline{X}_{t_0}^h\right]$. *And analogously we have* $\lim_{t_2 \to t_0} \lim_{t_1 \to t_0} \log \left[\overline{C}_{t_2}^h (+, t_1, \Delta)\right] = \log \left[e^{\Delta} \overline{A}_{t_0} + \overline{X}_{t_0}^h\right]$ *and* $\lim_{t_2 \to t_0} \lim_{t_1 \to t_0} \log \left[\overline{C}_{t_2}^h (-, t_1, \Delta)\right] = \log \left[e^{-\Delta} \overline{A}_{t_0} + \overline{X}_{t_0}^h\right]$.

Proof. Suppose the shock is positive. Then, supressing in notation that we work with the "shock" processes, using the fact that $\log (1 + z) \leq z$,

$$
\begin{aligned}
\left|\log \left[\overline{C}_{t_2}^h\right] - \log \left[e^{\Delta} \overline{A}_{t_0} + \overline{X}_{t_0}^h\right]\right| &= \left|\log \left[\frac{\overline{A}_{t_2} + \overline{X}_{t_2}^h}{e^{\Delta} \overline{A}_{t_0} + \overline{X}_{t_0}^h}\right]\right| \\
&\leq \max \left[\frac{\overline{A}_{t_2} + \overline{X}_{t_2}^h}{e^{\Delta} \overline{A}_{t_0} + \overline{X}_{t_0}^h} - 1, \frac{e^{\Delta} \overline{A}_{t_0} + \overline{X}_{t_0}^h}{\overline{A}_{t_2} + \overline{X}_{t_2}^h} - 1\right] \\
&\leq \max \left[\frac{(\overline{A}_{t_2} - e^{\Delta} \overline{A}_{t_0}) + \left(\overline{X}_{t_2}^h - \overline{X}_{t_0}^h\right)}{e^{\Delta} \overline{A}_{t_0} + \overline{X}_{t_0}^h}, \frac{(e^{\Delta} \overline{A}_{t_0} - \overline{A}_{t_2}) + \left(\overline{X}_{t_0}^h - \overline{X}_{t_2}^h\right)}{\overline{A}_{t_2} + \overline{X}_{t_2}^h}\right] \\
&\leq \max \left[\frac{|\overline{A}_{t_2} - e^{\Delta} \overline{A}_{t_0}| + \left|\overline{X}_{t_2}^h - \overline{X}_{t_0}^h\right|}{e^{\Delta} \overline{A}_{t_0}}, \frac{|e^{\Delta} \overline{A}_{t_0} - \overline{A}_{t_2}| + \left|\overline{X}_{t_0}^h - \overline{X}_{t_2}^h\right|}{\overline{A}_{t_2}}\right].
\end{aligned}
$$

For the first set of limits we assume $t_1 = t_2$ and take them to t_0 simultaneously; for the second set of limits we first take $t_1 \to t_0$ and then take $t_2 \to t_0$. In either case, in both terms of the maximum, the numerator converges to zero in L_{2p} while the inverse of the denominator is bounded in L_{2p}. By the Cauchy-Schwarz inequality, the terms themselves converge to zero in L_p, hence so does their maximum. The argument for a negative shock is analogous.

A-3.4 Notation and proof structure

Bounds. We use the notation that $K\left(\bar{t}, \Delta\right)$ refers to a family of random variables which are uniformly bounded independently of Δ, in the limit as $t_1 \to t_0$, when t_2 and t_3 are for appropriately chosen. Formally, we require that there exists a family of constants $K\left(p\right)$, such that given p, for any Δ, we can find $t_2\left(\Delta, p\right)$ small enogh and $t_3\left(\Delta, p\right)$ large enough so that $\lim_{t_1 \to t_0} \sup \left\| K\left(t_1, t_2\left(\Delta, p\right), t_3\left(\Delta, p\right)\right) \right\|_p \leq K\left(p\right)$. Different occurences of $K\left(\bar{t}, \Delta\right)$ may refer to different families of random variables and may have a different $K\left(p\right)$ values associated with them. For example, Lemma 10 implies that $\log\left[\overline{C}^h_{t_2}\left(+, t_1, \Delta\right)\right] = \log\left[e^{\Delta}\overline{A}_{t_0} + \overline{X}^h_{t_0}\right] + K\left(\bar{t}, \Delta\right)$.

Order of limits. The statement of Proposition 7 assumes that n is large enough; this means that σ_A/σ_I is small enough, while other parameters of the model, as described in Section 3.4, remain bounded. We first analyze the case in which σ_A becomes small, and then establish the result when σ_I becomes sufficiently large using a clock change.

A-3.5 Long-term behavior

Lemma 11 *Suppose that n is large enough and σ_A is small enough. Then*

$$\lim_{t_3 \to \infty} \lim_{t_1 \to t_0} \left[\frac{\overline{X}^h_{t_3}\left(-, t_1, \Delta\right)}{\overline{A}_{t_3}\left(-, t_1, \Delta\right)} - \frac{\widetilde{X}^h_{t_3}}{\widetilde{A}_{t_3}}\right] = 0.$$

The intuition for the Lemma is that X_{t_3} is just a weighted sum of past A_s values, with the weights for the distant past going to zero exponentially fast. Thus, if A_s is multiplied by a constant after date t_0, then for t_3 large enough, most of the terms determining X_{t_3} in this weighted sum will also be multiplied by that constant, and hence X_{t_3}/A_{t_3} will be approximately the same as it would be on the no-shock path. The caveat is that the terms in the weighted average corresponding to the distant past, divided by current A_{t_3}, must not blow up. For this we need that $1/A_{t_3}$ does not become big too quickly relative to the rate with which the weights on the past converge to zero. These weights go to zero at a given exponential rate, so if the variance of the A_t process is not too big, we are fine.

Proof of Lemma 11. Suppressing in notation that we work with the "negative shock" processes, we have

$$
\begin{aligned}
\lim_{t_1\to t_0}\frac{\overline{X}^h_{t_3}}{\overline{A}_{t_3}} &= \lim_{t_1\to t_0}\frac{1}{\overline{A}_{t_3}}\int_0^{t_3}\xi^{*\prime}(t_3-s)\,\overline{A}_s ds+\left[\overline{x}-\xi^*(t_3)\right]\frac{\overline{A}_0}{\overline{A}_{t_3}} \\
&= \frac{1}{\overline{A}_{t_3}}\int_{t_0}^{t_3}\xi^{*\prime}(t_3-s)\,\widetilde{A}_s e^{-\Delta}ds+\frac{1}{\overline{A}_{t_3}}\int_0^{t_0}\xi^{*\prime}(t_3-s)\,\overline{A}_s ds+\left[\overline{x}-\xi^*(t_3)\right]\frac{\overline{A}_0}{\overline{A}_{t_3}} \\
&= e^{-\Delta}\frac{\widetilde{X}^h_{t_3}}{\overline{A}_{t_3}}+\left(1-e^{-\Delta}\right)\frac{1}{\overline{A}_{t_3}}\left(\int_0^{t_0}\xi^{*\prime}(t_3-s)\,\overline{A}_s ds+\left[\overline{x}-\xi^*(t_3)\right]\overline{A}_0\right) \\
&= \frac{\widetilde{X}^h_{t_3}}{\widetilde{A}_{t_3}}+\left(1-e^{-\Delta}\right)\frac{1}{\widetilde{A}_{t_3}e^{-\Delta}}\left(\left[-\xi^*(t_3-s)\,\overline{A}_s\right]_0^{t_0}+\int_0^{t_0}\xi^*(t_3-s)\,d\overline{A}_s+\left[\overline{x}-\xi^*(t_3)\right]\overline{A}_0\right) \\
&= \frac{\widetilde{X}^h_{t_3}}{\widetilde{A}_{t_3}}+\left(e^{\Delta}-1\right)\frac{1}{\widetilde{A}_{t_3}}\left(-\xi^*(t_3-t_0)\,\overline{A}_{t_0}+\xi^*(t_3)\,\overline{A}_0+\int_0^{t_0}\xi^*(t_3-s)\,d\overline{A}_s+\left[\overline{x}-\xi^*(t_3)\right]\overline{A}_0\right) \\
&= \frac{\widetilde{X}^h_{t_3}}{\widetilde{A}_{t_3}}+\left(e^{\Delta}-1\right)\frac{1}{\widetilde{A}_{t_3}}\left(\int_0^{t_0}\xi^*(t_3-s)\,d\overline{A}_s+\overline{x}\overline{A}_0-\xi^*(t_3-t_0)\,\overline{A}_{t_0}\right).
\end{aligned}
$$

Here the last term can be written as

$$
\left(e^{\Delta}-1\right)\frac{1}{\widetilde{A}_{t_3}}\left(\int_0^{t_0}\left(\xi^*(t_3-s)-\overline{x}\right)\,d\overline{A}_s+\overline{A}_{t_0}\left[\overline{x}-\xi^*(t_3-t_0)\right]\right).
$$

Because, by Lemma 3, $|\xi^*(t_3-s)-\overline{x}|\le K_1e^{-K_2(t_3-s)}$ for some constants K_1,K_2 independent of n, it follows from Lemma 7 that, for n large enough, the first term here converges to zero as $t_3\to\infty$. Also by Lemma 3 the second term converges to zero as $t_3\to\infty$.

Lemma 12 *Suppose that n is large enough and σ_A is small enough. There exists a constant K_2 such that the following holds. For any Δ, we can find t_2 and t_3 such that for all t_1 close enough to t_0,*

$$
E\left[\log\overline{C}^h_{t_3}-\log\overline{C}^h_{t_2}|\ S(+,t_1,\Delta)\right]-E\left[\log\overline{C}^h_{t_3}-\log\overline{C}^h_{t_2}|\ S(-,t_1,\Delta)\right]\ge\Delta-K_2.
$$

Proof. A key element of the proof is that we bound the left hand side for each realization, that is, without the expectations operator. However, because $S(+,t_1,\Delta)$ and $S(-,t_1,\Delta)$ are disjoint events, we can only do this using the "shock processess", which have the same distribution as the original processes conditioned on the shock events, but are defined on a common probability space.

Suppose first that the shock is positive. Supressing in notation that we work with the shock process, we have $\log\overline{C}^h_{t_3}\ge\log\overline{A}_{t_3}=\log\widetilde{A}_{t_3}+\Delta$. Moreover, by Lemma 10, for t_2 close to t_0, we have

$$
\log\overline{C}^h_{t_2}=\log\left[e^{\Delta}\overline{A}_{t_0}+\overline{X}_{t_0}\right]+K\left(\overline{t},\Delta\right)=\log\overline{A}_{t_0}+\Delta+K\left(\overline{t},\Delta\right)
$$

where the second equality follows because, given that we condition on the history up to t_0, $\overline{X}^h_{t_0}/\overline{A}_{t_0}$ is a constant. We can now write, for a positive shock, that

$$\log \overline{C}^h_{t_3} - \log \overline{C}^h_{t_2} \geq \left(\log \widetilde{A}_{t_3} + \Delta\right) - \left(\log \overline{A}_{t_0} + \Delta + K\left(\bar{t}, \Delta\right)\right) = \log \widetilde{A}_{t_3} - \log \overline{A}_{t_0} + K\left(\bar{t}, \Delta\right).$$

Now suppose that the shock is negative. Then, using Lemma 10,

$$\log \overline{C}^h_{t_2} = \log\left[e^{-\Delta}\overline{A}_{t_0} + \overline{X}_{t_0}\right] + K\left(\bar{t}, \Delta\right) = \log \overline{A}_{t_0} + \log\left[e^{-\Delta} + \overline{X}_{t_0}/\overline{A}_{t_0}\right] + K\left(\bar{t}, \Delta\right) \geq \log \overline{A}_{t_0} + K\left(\bar{t}, \Delta\right)$$

because $\overline{X}_{t_0}/\overline{A}_{t_0}$ is a constant. Moreover, using the fact that $\log(1+z) \leq z$,

$$\begin{aligned}
\log \overline{C}^h_{t_3} &= \log\left[\overline{A}_{t_3} + \overline{X}^h_{t_3}\right] = \log \overline{A}_{t_3} + \log\left[1 + \overline{X}^h_{t_3}/\overline{A}_{t_3}\right] \\
&\leq \log \widetilde{A}_{t_3} - \Delta + \overline{X}^h_{t_3}/\overline{A}_{t_3} = \log \widetilde{A}_{t_3} - \Delta + \widetilde{X}^h_{t_3}/\widetilde{A}_{t_3} + K\left(\bar{t}, \Delta\right)
\end{aligned}$$

where at the last step we used Lemma 11. It follows that for a negative shock

$$\log \overline{C}^h_{t_3} - \log \overline{C}^h_{t_2} \leq \log \widetilde{A}_{t_3} - \Delta + \widetilde{X}^h_{t_3}/\widetilde{A}_{t_3} - \log \overline{A}_{t_0} + K\left(\bar{t}, \Delta\right).$$

Combining the inequalities for the positive and the negative shocks yields, for the shock processes, the bound

$$\begin{aligned}
&\left[\log \overline{C}^h_{t_3}(+, t_1, \Delta) - \log \overline{C}^h_{t_2}(+, t_1, \Delta)\right] - \left[\log \overline{C}^h_{t_3}(-, t_1, \Delta) - \log \overline{C}^h_{t_2}(-, t_1, \Delta)\right] \geq \\
&\qquad \log \widetilde{A}_{t_3} - \log \overline{A}_{t_0} - \left(\log \widetilde{A}_{t_3} - \Delta + \widetilde{X}^h_{t_3}/\widetilde{A}_{t_3} - \log \overline{A}_{t_0}\right) + K\left(\bar{t}, \Delta\right) \\
&\qquad = \Delta - \widetilde{X}^h_{t_3}/\widetilde{A}_{t_3} + K\left(\bar{t}, \Delta\right).
\end{aligned}$$

Finally,

$$\frac{\widetilde{X}_{t_3}}{\widetilde{A}_{t_3}} = \frac{1}{\widetilde{A}_{t_3}} \int_0^{t_3} \xi^*\left(t_3 - s\right) d\widetilde{A}_s + \overline{x}\frac{\widetilde{A}_0}{\widetilde{A}_{t_3}} = \overline{x} + \frac{1}{\widetilde{A}_{t_3}} \int_0^{t_3} \left[\xi^*\left(t_3 - s\right) - \overline{x}\right] d\widetilde{A}_s$$

and by Lemma 7 the last term is bounded in L_p for all t_3. Thus the above difference is Δ plus a term bounded in L_p, and the claim of the Lemma follows.

A-3.6 Proofs of Propositions 6 and 7

Proof of Proposition 6. (i) Taking expectations in the regression equation (12) conditional on the shock being positive respectively negative, and differencing, we obtain

$$E\left[\log\overline{C}_{t_1} - \log\overline{C}_{t_0} \middle|\, S(+,t_1,\Delta)\right] - E\left[\log\overline{C}_{t_1} - \log\overline{C}_{t_0} \middle|\, S(-,t_1,\Delta)\right] = 2\beta_1(t_1,\Delta)\cdot\Delta \tag{20}$$

which gives an expression for $\beta_1(t_1,\Delta)$. An analogous formula expresses $\beta_1^h(t_1,\Delta)$. Because X_t/A_t is bounded from below by L and from above by U, we have $\left|\log\left(\overline{C}_{t_1}/\overline{C}_{t_0}\right) - \log\left(\overline{A}_{t_1}/\overline{A}_{t_0}\right)\right| \leq \log(1+U) - \log(1+L) = K_1$ and therefore

$$\begin{aligned} & E\left[\log\overline{C}_{t_1} - \log\overline{C}_{t_0} \middle|\, S(+,t_1,\Delta)\right] - E\left[\log\overline{C}_{t_1} - \log\overline{C}_{t_0} \middle|\, S(-,t_1,\Delta)\right] \\ \geq\; & E\left[\log\left(\overline{A}_{t_1}/\overline{A}_{t_0}\right) \mid S(+,t_1,\Delta)\right] - E\left[\log\left(\overline{A}_{t_1}/\overline{A}_{t_0}\right) \mid S(-,t_1,\Delta)\right] - 2K_1 = 2(\Delta - K_1). \end{aligned}$$

Hence $\beta_1(t_1,\Delta) \geq 1 - K_1/\Delta$.

(ii) Lemma 10 implies that for any positive K_2, we can choose t_1 close enogh to t_0 such that

$$E\left[\log\overline{C}_{t_1} \middle|\, S(+,t_1,\Delta)\right] - E\left[\log\overline{C}_{t_1} \middle|\, S(-,t_1,\Delta)\right] \leq \log\left[e^{\Delta}\overline{A}_{t_0} + \overline{X}_{t_0}^h\right] - \log\left[e^{-\Delta}\overline{A}_{t_0} + \overline{X}_{t_0}^h\right] + K_2.$$

The right-hand side can be bounded as

$$\log\left[\frac{e^{\Delta}\overline{A}_{t_0} + \overline{X}_{t_0}^h}{e^{-\Delta}\overline{A}_{t_0} + \overline{X}_{t_0}^h}\right] = \log\left[\frac{e^{\Delta} + \overline{X}_{t_0}^h/\overline{A}_{t_0}}{e^{-\Delta} + \overline{X}_{t_0}^h/\overline{A}_{t_0}}\right] \leq \log\left[\frac{e^{\Delta} + \overline{X}_{t_0}^h/\overline{A}_{t_0}}{\overline{X}_{t_0}^h/\overline{A}_{t_0}}\right] \leq \Delta + \log\left[\frac{1 + \overline{X}_{t_0}^h/\overline{A}_{t_0}}{\overline{X}_{t_0}^h/\overline{A}_{t_0}}\right] = \Delta + K_3$$

where—given that we condition on the history up to t_0—K_3 is a constant. It then follows from (20) that, for a given Δ, we can choose t_1 close enough to t_0 such that $\beta_1^h(t_1,\Delta) < 1/2 + (K_2 + K_3)/\Delta$.

Proof of Proposition 7. (i) Taking expectations in (13) and differencing, we obtain

$$E\left[\log\overline{C}_{t_3} - \log\overline{C}_{t_2} \middle|\, S(+,t_1,\Delta)\right] - E\left[\log\overline{C}_{t_3} - \log\overline{C}_{t_2} \middle|\, S(-,t_1,\Delta)\right] = 2\beta_2\left(\bar{t},\Delta\right)\cdot\Delta \tag{21}$$

which gives an expression for $\beta_2\left(\bar{t},\Delta\right)$. An analogous formula expresses $\beta_2^h\left(\bar{t},\Delta\right)$. Because X_t/A_t is bounded from below by L and from above by U, we have $\left|\log\left(\overline{C}_{t_3}/\overline{C}_{t_2}\right) - \log\left(\overline{A}_{t_3}/\overline{A}_{t_2}\right)\right| \leq$

$\log(1+U) - \log(1+L) = K_1$ and therefore

$$
\begin{aligned}
& E\left[\log \overline{C}_{t_3} - \log \overline{C}_{t_2} \,|\, S(+, t_1, \Delta)\right] - E\left[\log \overline{C}_{t_3} - \log \overline{C}_{t_2} \,|\, S(-, t_1, \Delta)\right] \\
\leq \quad & E\left[\log\left(\overline{A}_{t_3}/\overline{A}_{t_2}\right) \,|\, S(+, t_1, \Delta)\right] - E\left[\log\left(\overline{A}_{t_3}/\overline{A}_{t_2}\right) \,|\, S(-, t_1, \Delta)\right] + 2K_1 = 2K_1.
\end{aligned}
$$

Using (21) we obtain $\beta_2(t_1, t_2, t_3, \Delta) \leq K_1/\Delta$.

(ii) Using Lemma 12 we can find t_2 and t_3, and t_1 close enough to t_0, such that $\beta_2^h(t_1, t_2, t_3, \Delta) \geq 1 - K_2/\Delta$. This gives the proof along a sequence Θ_n in which $\sigma_A \to 0$. Finally we discuss the case when as $n \to \infty$, we have $\sigma_I \to \infty$. The only step we need to verify is that Lemma 12 also holds for n large enough. To show this, just like in the proof of our main result, we change the clock. Using the transformation introduced in Lemma 8, we let $\tau = 1/\sigma_I^2$ and slow down the model by rescaling deep parameters with τ. In the habit representation of that "rescaled" model, for n large enough Lemma 12 holds, because all the assumptions, in particular, the requirement that σ_A is small enough, are satisfied. And because the habit representation of the model after the clock change is the same as changing the clock in the habit representation of the original model, it follows that—with appropriately unscaled values for t_2 and t_3—Lemma 12 also holds in the original model.

A-4 Proofs for Sections 4.2 and 4.3

Proof of Proposition 8. In $\overline{\Theta}^*$, agents in the interior of the band never adjust, hence $T_*(\widetilde{p}|x_0) = \infty$. For n finite, agents does adjust eventually, but since the drift and variance of y goes to zero, the expected time to adjustment approaches infinity. In the habit model, x never changes, hence $T^{h,n}(\widetilde{p}|x_0) = \infty$.

Proof of Proposition 9. (i) Our first goal is to compute the value function of the habit agent. Let ψ be defined so that the value function of the Merton consumption problem in the environment of the representative habit consumer, but without habit, is $\psi W^{1-\gamma}/(1-\gamma)$. By the envelope theorem, this Merton agent has consumption policy $c = \psi^{-1/\gamma} W$. The surplus consumption of our habit agent is identical to the consumption of a Merton agent, because they solve the same maximization problem. Hence, if the habit consumer sets his initial surplus consumption to be A_0, the dollar cost of his lifetime surplus consumption expenditure is $A_0 \psi^{1/\gamma}$.

To proceed, we now evaluate the lifetime budget constraint of the habit consumer. Each dollar of consumption spending in a period also creates future expenditure in the form of increased habit.

Suppose $1+B$ dollars is the present value of these future expenditures for a dollar of consumption spending today, where $B=0$ with no habits. Then B must satisfy

$$B=\int_{u=0}^{\infty}\theta\left(u\right)e^{-ru}du\cdot\left(1+B\right)$$

because each dollar of consumption creates $\theta\left(u\right)$ habit spending u periods ahead, which has a total cost of $\theta\left(u\right)\left(1+B\right)$ in period u dollars, which we must then discount back at the riskfree rate because these payments are certain. Solving yields

$$B=\frac{1}{1-\int_{u=0}^{\infty}\theta\left(u\right)e^{-ru}du}.$$

At any time t, our habit consumer also has pre-existing habit created by his past consumption. The dollar value of the expenditures generated is

$$Z_t=\left(1+B\right)\cdot\left[\int_{s=0}^{t}C_{t-s}\int_{s}^{\infty}\theta\left(u\right)e^{-r(u-s)}du\;ds+\int_{s=t}^{\infty}\theta_0\left(u\right)X_0e^{-ru}du\right]$$

where the term in parenthesis measures future consumption expenditures created by habits established before t, discounted back at the riskfree rate because these are certain; and the factor $1+B$ is included because each dollar of consumption spending has this total expenditure cost.

The consumer's lifetime budget constraint must then satisfy

$$W_t=A_t\cdot\psi^{1/\gamma}\left(1+B\right)+Z_t$$

and his lifetime utility from surplus consumption, by the Merton value function, is simply $\psi^{1/\gamma}A_t^{1-\gamma}/\left(1-\gamma\right)$. Combining these equations yields

$$V_t^{habit}\left(W_t,X_t\right)=\frac{\psi}{1-\gamma}\left(\frac{W_t-Z_t}{1+B}\right)^{1-\gamma}.$$

The welfare of an individual commitment agent for a move-inducing negative wealth shock is proportional to $\left(w-\lambda_1x\right)^{1-\gamma}/\left(1-\gamma\right)$.

Now compare the welfare cost of shocks in the commitment and the habit economies. As wealth falls to zero, if $Z_t>0$ then the marginal utility of the habit agent will be driven to infinity even with a finite shock. In contrast, when $\lambda_1=0$, the marginal utility of the commitment agent only blows up when all his wealth is taken. It follows that for large finite shocks, $\Pi\left(q,b\right)$ is higher for the habit agent than in the commitment economy.

(ii) Begin with the commitment model. The agent in the limit economy never moves, and hence his value function is proportional to $(W - x/r)^{1-\gamma} / (1-\gamma)$. It follows that the coefficient of relative risk aversion $CRRA^* (W_0, x_0) = \gamma W_0 / (W_0 - x_0/r)$. Now consider an agent in economy n. Let p_0 denote the total dollar value at date zero of his total commitment expenditures on his current home. Given positive risk and growth, this agent does move eventually, implying $p_0 < x_0/r$. One policy available to this consumer at any wealth W is to maintain his spending and moving patterns on current commitments, and adjust spending proportionally on all other goods relative to the optimal policy with initial wealth W_0. Given that $\lambda_1 = 0$, this policy yields lifetime utility $V_n (W_0, x_0) (W - p_0)^{1-\gamma} / (W_0 - p_0)^{1-\gamma}$. This is a lower bound for the agent's true value function, and the both equal $V_n (W_0, x_0)$ at W_0. It follows that the lower bound has higher curvature at W_0. As a result, $CRRA^n (W_0, x_0) \leq \gamma W_0 / (W_0 - p_0)$. Since $p_0 < x_0/r$, we have $CRRA^n (W_0, x_0) < CRRA^* (W_0, x_0)$. Hence for b small, the Arrow-Pratt approximation implies $\Pi^n (q, b) < \Pi^* (q, b)$ uniformly in n.

In the habit model, the value function in every economy is proportional to $(W - x/r)^{1-\gamma} / (1-\gamma)$, and hence $\Pi^{h,n} (q, b) = \Pi^{h*} (q, b)$.

A-5 Simulations

In the simulations we use an ODE characterization of the optimal policy that builds on a similar characterization for the one-good model by Grossman and Laroque. To develop this ODE, we must study the Bellman equation of the commitment agent. Denote the value function by $V (W, x)$, then the Bellman equation between adjustment dates is

$$\rho V (W, x) = \max_{\alpha, \alpha} \left[\kappa \frac{a^{1-\gamma}}{1-\gamma} + \frac{x^{1-\gamma}}{1-\gamma} + V_1 (W, x) EdW + \frac{1}{2} V_{11} (W, x) Var (dW) \right].$$

Following Grossman and Laroque, let $y = W/X - \lambda_1$ and define $h (y) = x^{-1+\gamma} V (W, x) = V (W/x, 1)$. Dividing through by $x^{1-\gamma}$ in the Bellman equation we obtain

$$\rho h (y) = \max_{a, \alpha} \left[\kappa \frac{(a/x)^{1-\gamma}}{1-\gamma} + \frac{1}{1-\gamma} + h' (y) Edy + \frac{1}{2} h'' (y) Var (dy) \right]$$

and the budget constraint yields

$$dy = \left(\left(y+\lambda_1\right)\left(r+\alpha\pi\right) - 1 - a/x\right)dt + \left(y+\lambda_1\right)\alpha\sigma dz.$$

Maximizing in α, the optimal portfolio satisfies

$$\alpha\left(y+\lambda_1\right) = \frac{-h'\left(y\right)}{h''\left(y\right)}\frac{\pi}{\sigma^2}$$

and adjustable consumption is

$$\frac{a}{x} = \left[\frac{h'\left(y\right)}{\kappa}\right]^{-1/\gamma}.$$

Substituting back into the Bellman equation we obtain

$$\rho h\left(y\right) = h'\left(y\right)^{1-1/\gamma}\kappa^{1/\gamma}\frac{\gamma}{1-\gamma} + \frac{1}{1-\gamma} + h'\left(y\right)\left[\left(y+\lambda_1\right)r - 1\right] - \frac{1}{2}\frac{h'\left(y\right)^2}{h''\left(y\right)}\frac{\pi^2}{\sigma^2}.$$

This is an ordinary differential equation for $h\left(y\right)$. To obtain boundary conditions, note that on an adjustment date the value function equals

$$\begin{aligned}
\frac{V\left(W,x\right)}{x^{1-\gamma}} &= \frac{1}{x^{1-\gamma}}\max_{x'} V\left(W-\lambda_1 x-\lambda_2 x', x'\right) \\
&= \left(\frac{W-\lambda_1 x}{x}\right)^{1-\gamma}\cdot\max_{x'}\left(\frac{x'}{W-\lambda_1 x}\right)^{1-\gamma}\cdot V\left(\frac{W-\lambda_1 x}{x'}-\lambda_2, 1\right) \\
&= \left(\frac{W-\lambda_1 x}{x}\right)^{1-\gamma}\cdot\max_{y}\left(y+\lambda_1+\lambda_2\right)^{-1+\gamma}h\left(y\right).
\end{aligned}$$

Define

$$M = \max_y \left(y+\lambda_1+\lambda_2\right)^{-1+\gamma}h\left(y\right)$$

then by the above reasoning, at the edges of the inaction band, denoted y_1 and y_2 we have

$$h\left(y_i\right) = My_i^{1-\gamma}$$

moreover, smooth pasting implies

$$h'\left(y_i\right) = M\left(1-\gamma\right)y_i^{-\gamma}.$$

Finally, the target value of y satisfies

$$y^* = \arg\max \left(y + \lambda_1 + \lambda_2\right)^{-1+\gamma} h\left(y\right).$$

To numerically solve the ODE subject to these conditions, we follow the approach outlined by Grossman and Laroque. We first pick some M, pick y_1, solve the ODE with initial conditions as given above. If there is no y_2 for which the boundary conditions are satisfied, then we start with a different y_1. If the boundary conditions do hold for some y_2, then we check if M satisfies the equation above; if not, we start with a different M.

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