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ABSTRACT

The tontine, which is an interesting mixture of group annuity, group life insurance, and lottery, has a peculiar place in economic history. In the seventeenth and eighteenth centuries it played a major role in raising funds to finance public goods in Europe, but today it is rarely encountered outside of murder mysteries. This study provides a formal model of individual contribution decisions under a tontine mechanism. We analyze the performance of tontines and compare them to another popular fundraising scheme used today by both government and charitable fundraisers: lotteries. Our major theoretical results are that (i) the optimal tontine for agents with identical valuations of the public good consists of all agents receiving a fixed "prize" amount in the first period equal to a percentage of their total contribution, (ii) contribution levels in the optimal tontine are identical to those of risk-neutral agents in an equivalently valued single prize lottery, (iii) contribution levels for the optimal tontine are independent of risk-aversion, and thereby outperform lotteries when agents are risk-averse, (iv) if agents are sufficiently asymmetric in their valuation of the public good, equilibrium contribution levels are larger under tontines than any lottery. In particular, one can obtain full participation in the tontine mechanism compared to only partial participation in a lottery. These insights highlight that the tontine institution can be a useful tool for fundraisers in the future.

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tontine: *An annuity scheme wherein participants share certain benefits and on the death of any participant his benefits are redistributed among the remaining participants; can run for a fixed period of time or until the death of all but one participant.* Webster's Online Dictionary

I. Introduction

The oldest standing bridge in London (Richmond Bridge), numerous public buildings and other municipality projects throughout the U.S., Britain, the Netherlands, Ireland, and France, and several wars, including the Nine Years' War, all share a common thread: they were wholly, or partially, funded by tontines. The idea of the tontine is believed to have originated in 1652, when an expatriate banker, Lorenzo Tonti, proposed a new mechanism for raising public funds to Cardinal Mazarin of France.[1] Tonti advertised his idea as "A gold mine for the king….a treasure hidden away from the realm." The salesmanship of Tonti coupled with the difficulties associated with raising taxes in seventeenth century France led to an enthusiastic endorsement from King Louis XIV. While the idea, and many affiliated derivatives, prospered as major tools for financing public goods for several decades, tontines have since been banned in Britain and the United States due to the potential incentive for investors to kill one another in order to increase their shares.[2]

In essence, a tontine is a mixture of group annuity, group life insurance, and lottery. While the use and economic operation of each of these components is understood as a vehicle for individual investment/leisure, as a means to fund public goods, the

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[1] Similar mechanisms are believed to have been employed in the Roman Empire several centuries earlier. Tonti's mechanism should not be confused with the *tontines* in Western Africa, which are small, informal savings and loan associations similar to ROSCAs (Rotating Savings and Credit Associations).

[2] As an aside, this allure of the tontine has led to a fantastic plot device for detective story writers (the interested reader should see, e.g., *The Wrong Box* by Robert Louis Stevenson, which was made into a film in 1966 starring Peter Cook, Dudley Moore, Ralph Richardson, Michael Caine, and Tony Hancock).

tontine itself has largely been ignored. It is well established that relying upon voluntary contributions for the provision of public goods generally results in the under provision of such goods relative to first-best levels. Numerous mechanisms have been proposed to alleviate the tendency of agents to free-ride (see e.g., Groves and Ledyard 1977; Walker 1981; Bagnoli and McKee 1991; Varian 1994; Falkinger 1996).

This study adds to the literature on voluntary provision of public goods by formally investigating the performance and optimal design of the tontine. In this spirit, we provide information about the history and modeling results of tontines in order to encourage usage of the best characteristics of the institution in the future. We begin by outlining the conditions that define an optimal tontine—one that maximizes total group contribution levels—when symmetric risk-neutral agents have quasi-linear preferences. Properties of tontines are also explored upon relaxation of symmetry and risk neutrality. We then compare the performance of the tontine to a popular fundraising scheme used today: lotteries (see, e.g., Morgan 2000 and Lange et al. 2004).[3]

Our main findings are as follows: (i) the optimal for tontine for agents with identical valuations of the public good consists of all agents receiving a fixed "prize" amount in the first period equal to a percentage of their total contribution, (ii) contribution levels in this optimal tontine are identical to those of risk-neutral agents in an equivalently valued single prize lottery, (iii) contribution levels for the optimal tontine are independent of risk-aversion, and (iv) with sufficient, and plausible, risk-aversion or asymmetry in individual valuations of the public good, tontines yield higher contributions

[3] Relatedly, Engers and McManus (2002) and Goeree et al. (2004) explore the use of auctions to raise money to finance public goods, and Andreoni (1998) and List and Lucking-Reiley (2002) explore the voluntary contributions mechanism with and without announcements of "seed" money.

than the optimal lottery. Further, one can obtain full participation in the tontine mechanism compared to partial participation in the lottery mechanism. These results have clear implications for empiricists and practitioners in the design of fundraising campaigns. Further, they provide useful avenues for future theoretical work on voluntary provisioning of public goods.

The remainder of our study is crafted as follows. Section II provides a brief historical overview of tontines. Section III describes a theoretical model of the tontine and compares the performance of an optimal tontine with that of lotteries. Section IV concludes.

II. Tontines throughout History

Lorenzo Tonti was a Neopolitan of little distinction until his sponsor, Cardinal Mazarin of France, who was responsible for the financial health of France, supported his position in the court of the French King in the 1650s. In this position, Tonti proposed a form of a life contingent annuity with survivorship benefits, whereby subscribers, who were grouped into different age classes, would make a one-time payment of 300 livres to the government. Each year, the government would make a payment to each group equaling five percent of the total capital contributed by that group. These payments would be distributed among the surviving group members based upon each agent's share of total group contributions. The government's debt obligation would cease with the death of the last member of each group. Although the plan was supported enthusiastically by Louis XIV, Tonti's plan was rejected by the French Parliament for two reasons: (i) the uncertain nature of total government debt obligations and (ii) the proposed rate of return was low in comparison with rates on life annuities (Weir, 1989).

While the Netherlands started a successful tontine in 1670, it was not until 1689, when France was engaged in the Nine Years' War, that France offered its first national tontine. The design was quite similar to that originally proposed by Tonti. Later offerings in France coincided with peaks in national capital demand during periods of war and were generally successful in raising the sought-after capital. During France's four major wars of this period, national tontine offerings raised approximately 110 million livres from around 110,000 individuals.

Contrary to the relative success enjoyed by France, tontine offerings in England often failed to raise the desired capital. England provided its first national tontine in 1693; this initial tontine generated but a tenth of the one million pounds set as its goal. Yet England did successfully use the tontine to fund many public projects, including construction of the Richmond Bridge, claimed to be the oldest standing "London" Bridge. Unlike many of the early French tontines, English tontines frequently allowed agents to purchase numerous shares.

While the use of tontines to finance government projects was predominately a European endeavour, the notion that tontines could be used as a means to finance national debt has a historical basis in the U.S as well. Faced with growing principal liability on national debt, Alexander Hamilton proposed a national tontine in the U.S. in his 1790 Report Relative to a Provision for the Support of Public Credit (Jennings et al., 1988). Hamilton's proposal was to reduce principal repayments on national debt by converting old debt with principal that was repayable at the discretion of the government into debt demanding no return on principal.

The structure of the tontine that Hamilton proposed was inspired by a tontine originally proposed by William Pitt in 1789. The proposed tontine included six age classes, and shares in the tontine would be sold for $200 with no limit on the number of shares that any agent could purchase. Individuals could subscribe on their own lives or on the lives of others nominated by them. However, Hamilton proposed a freeze component on debt repayment: the annuities of subscribers who passed away would be divided among living subscribers until only twenty percent of the original subscribers remained. Once this threshold was reached, the payments to remaining survivors would be frozen for the duration of their lives (Dunbar, 1888).

Tontine Insurance in the United States

While tontines proper were not used after the eighteenth century, an adaptation of the tontine was implemented in the U.S. life insurance market in 1868. Tontine insurance was introduced in 1868 by the Equitable Life Assurance Society of the U.S. Under tontine insurance, premiums served two distinct purposes: (i) provision of standard life insurance benefits and (ii) creation of an individual investment fund. Under tontine insurance, policyholders deferred receipt of the dividend payments of standard premium insurance policies. The deferred dividends were pooled and invested by the insurance company on behalf of the policyholders for a specified time period. At the end of this period, the fund plus the investment earnings were divided proportionately among the entire active, surviving policyholders. Investment earnings could be received as either cash or as a fully paid life annuity. Beneficiaries of policyholders that passed away before the end of the tontine period received the specified death benefits, but had no claim on the tontine fund money (Ransom and Sutch, 1987).

Conceptually, tontine insurance had several advantages relative to a standard life insurance policy. Policyholders were able to secure life insurance plus create a retirement fund. Survivors could receive a generous rate of return on these investments if a large proportion of other group members were to pass away or allow their policy to lapse. Tontine insurance provided an opportunity for young individuals to save for retirement by providing a low-risk, high-yield investment fund available on an installment plan. Unfortunately, corruption by the insurance companies led to the prohibition of tontine insurance sales by 1906 (Ransom and Sutch, 1987).

III. Tontine Theory

To model a tontine as an instrument to fund public goods, we must define the utility structure of agents and their probability of survival in a particular period. For the former, we consider n agents $i = 1,...,n$ whose utility is assumed additively separable in monetary wealth and the benefits from the public good:

$$u_i = y_i + h_i(G),$$

where y_i is a numeraire and G the provision level of the public good. We assume $h_i(G)$ to be increasing and concave ($h_i'(\bullet) > 0$, $h_i''(\bullet) \le 0$).[4] We make the standard public good assumption—that it is socially desirable to provide a positive amount of the public good, i.e., $\sum_i h_i'(0) > 1$

Given an initial endowment w of wealth (income), the choice facing the agent is to determine the amount b_i of wealth to invest in the tontine. Investment b_i in the

[4] For studies that relax the assumption of utility being dependent upon only the level of the public good see Sugden (1982; 1984) and Andreoni (1990); these theories suggest that if one were to rewrite utility such that it is a function of both the level of the funds raised and own individual contributions, then the standard result of free-riding behavior can be reversed.

tontine provides the agent with an uncertain monetary return x_i that is dependent upon her own contributions and those of all other members of a group:

$$u_i = w + x_i - b_i + h_i(G).$$

We assume that the tontine pays $P_t \geq 0$ in period *t* with a total of $\sum_{t=0}^{T-1} P_t = P$. Payments are covered by the players' contributions, i.e., the level of public good provision equals the total contribution minus the aggregate prize level:

$$G = B - P = \sum_{i=1}^{n} b_i - \sum_{t=0}^{n-1} P_t$$

In each period *t*, some individuals might die (exit the game). All survivors receive a payment that is determined by their relative contribution level. That is, for a total tontine payment P_t in period *t*, a surviving player *i* receives a payment $\frac{b_i}{B_t} P_t$ where B_t is the sum of the contributions made by the remaining players in period *t*.

We assume that each agent has a perish probability in period *t* given by μ_t where $\sum_{t=1}^{T} \mu_t = 1$. The probability that an agent will die no later than period *t* is denoted by M_t where $M_t = \sum_{s=1}^{t} \mu_t$. The probability of agents' deaths is i.i.d. Finally we assume for simplicity that agents are risk-neutral and payments are perfectly substitutable across periods. Denoting the set of $k \leq n$ participating agents (with positive contributions) by S_0 ($k = \# S_0$),[5] the *ex ante* expected utility of a player *i* is given by

$$EU_i = w - b_i + h_i(B - P) + \sum_{t=0}^{T-1} P_t \sum_{l=0}^{k-1} M_t^{\,l} (1 - M_t)^{k-l} \left[\sum_{S \subseteq S_0 \setminus i, \# S = l} \frac{b_i}{B - B(S)} \right].$$

[5] We will later show that all agents participate: $k = n$ if there is (at least) one *t* for which $P_t > 0$ and $0 < M_t < 1$.

We immediately obtain the following equilibrium conditions:

$$1-h_i'(B-P)=\sum_{t=0}^{T-1}P_t\sum_{l=0}^{k-1}M_t^{\,l}(1-M_t)^{k-l}\left[\sum_{S\subseteq S_0\setminus i,\#S=l}\frac{B-B(S)-b_i}{(B-B(S))^2}\right]\quad \text{for}\quad i\in S_0$$
$$1-h_i'(B-P)\geq\sum_{t=0}^{T-1}P_t\sum_{l=0}^{k-1}M_t^{\,l}(1-M_t)^{k-l}\left[\sum_{S\subseteq S_0\setminus i,\#S=l}\frac{1}{B-B(S)}\right]\quad \text{for}\quad i\notin S_0$$
$$k-\sum_{i\in S_0}h_i'(B-P)=\sum_{t=0}^{T-1}P_t\sum_{l=0}^{k-1}M_t^{\,l}(1-M_t)^{k-l}\left[\sum_{S\subseteq S_0,\#S=l}\frac{k-l-1}{B-B(S)}\right] \tag{1}$$

In the following we will first consider the case of symmetric risk-neutral agents. Both assumptions are relaxed in later sections.

III.1 Tontines for symmetric risk-neutral agents

If all agents value the public good identically ($h_i(G)=h(G)$), we can concentrate on symmetric equilibria. Here, all *n* agents contribute at a level *b* such that total contributions *B=nb* is given by the symmetric version of first-order condition (1):

$$B(1-h'(B-P))=\sum_{t=0}^{T-1}P_t\left[\sum_{l=0}^{n-1}\binom{n}{l}M_t^{\,l}(1-M_t)^{n-l}\,\frac{n-l-1}{n-l}\right]. \tag{2}$$

We now consider the optimal design of a tontine. In particular, we address the question of how an organization—government or private charitable fundraiser—with a fixed prize budget, $P=\sum_{t=0}^{T-1}P_t$, should allocate this prize money across $t\geq 0$ distinct time periods so as to maximize total contributions. We obtain the following result:

Proposition 1 (Optimal tontine—Symmetric risk-neutral agents)

If agents are symmetric and risk-neutral, contributions to the public good using a tontine are maximal if all the payments are made in the first period, i.e., before anybody has passed away.

Proof of Proposition 1:

Contributions to the public good are clearly increasing in the right-hand side of the equilibrium condition (2). Thus, we obtain:

$$\begin{aligned}
&\sum_{t=0}^{T-1} P_t \left[\sum_{l=0}^{n-1} \binom{n}{l} M_t^{\,l} (1-M_t)^{n-l} \, \frac{n-l-1}{n-l} \right] \\
&= \sum_{t=0}^{T-1} P_t \left[\sum_{l=0}^{n-1} \binom{n}{l} M_t^{\,l} (1-M_t)^{n-l} \, \frac{n-l-1}{n-l} \right] \\
&\le \sum_{t=0}^{T-1} P_t \left[\sum_{l=0}^{n-1} \binom{n}{l} M_t^{\,l} (1-M_t)^{n-l} \, \frac{n-1}{n} \right] \\
&\le \frac{n-1}{n} P
\end{aligned}$$

which coincides with the right-hand side if all payments are made before any agent has perished, i.e., $P = P_0$.■

The optimal tontine for symmetric agents, therefore, has a simple structure: All agents receive a rebate proportional to their contributions relative to those of the total group. This optimal structure implies that agents are not subject to any risk – all subjects receive their payment with certainty. Given the contribution of all other agents, the payoffs for an agent *i* under the tontine are given by $\frac{b_i}{B}P$, where *P* denotes the prize level.

The certain payoff is therefore given by $w - b_i + h(B-P) + \frac{b_i}{B}P$ which can also be interpreted as the expected payoff in Morgan's (2000) risk-neutral one-prize lottery. All of his results therefore apply. In particular, using his δ-financing rule, the tontine will always be carried out and the contributions will increase in the prize level, P, (see Morgan 2000, lemma 5).[6]

Reconsidering the first-order condition for a symmetric equilibrium (2), the individual ($\bar{b}$) and the total ($\bar{B}$) contribution levels for the optimal tontine are given by

$$n\bar{b} = \bar{B} \qquad\qquad \bar{B}(1-h'(\bar{B}-P)) = \frac{n-1}{n}P. \tag{3}$$

Note that the tontine raises a positive amount of money for the public good net of prize payments, as

$$P(1-h'(0)) > \frac{n-1}{n}P \quad \Leftrightarrow \quad nh'(0) > 1,$$

which coincides with the condition for a public good.

We summarize these results as follows:

Proposition 2 (Contribution levels for optimal tontines—Symmetric players)

For symmetric players, the optimal tontine will always be carried out and raises contributions in excess of the prize-level P. The provision level of the public good is increasing in P.

Historically, tontines clearly have not reflected the *optimal* features derived in Proposition 1. In the seventeenth and eighteenth centuries, tontine "prize" payments were made over a long time span. That is, the tontines differed significantly from the optimal tontine in that repayments were made annually to the surviving subscribers instead of making all repayments before anybody died. In the oft-used tontine repayment system, however, subscribers could die in any period s (even before any payment was

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[6] The optimal tontine that we study in this paper provides a rebate (subsidy) on individual contributions to the public good. This feature resembles the study relating government subsidies and contributions to a public good by Andreoni and Bergstrom (1996). In their case, however, subsidies are financed by taxes, whereas in our model the rebates are taken out of the contribution to the public good. The provision of the public good therefore does not depend on the possibility of enforcing tax payments. To balance the budget, subsidy rates in our model are not exogenously fixed but endogenously given by the individual relative to total contributions.

received) and thus would forego payments in all periods $t > s$ with positive probability. To model this aspect of the mechanism, let us assume that the aggregate prize amount P is spread evenly across $\tilde{T} \leq T-1$ periods. In other words, $P_t = P/\tilde{T}$ for $1 \leq t \leq \tilde{T}$.

Then, the contributions in equilibrium are given by the first-order condition:

$$B(1-h'(B-P)) = \frac{P}{\tilde{T}} \sum_{t=1}^{\tilde{T}} \left[\sum_{l=0}^{n-1} \binom{n}{l} M_t^{\,l} (1-M_t)^{n-l} \frac{n-l-1}{n-l} \right] , \qquad (4)$$

for which we obtain the following result:

Proposition 3 (Suboptimal tontines – Effect of $\tilde{T}$ and n)

Contributions to the public good using a tontine that pays a fixed prize-level in $\tilde{T} \leq T-1$ periods are decreasing in $\tilde{T}$. For any given $\tilde{T}$, they converge towards the contributions to an optimal tontine (or lottery) if the number of (potential) participants, n, increases.

Proof of Proposition 3:

In order to show that contributions decrease in $\tilde{T}$, it is sufficient to show that the right-hand sides of (4), $\sum_{l=0}^{n-1} \binom{n}{l} M_t^{\,l} (1-M_t)^{n-l} \frac{n-l-1}{n-l}$, are decreasing in *t*. As we know that M_t increases in *t*, we must demonstrate that:

$$\frac{\partial}{\partial M} \sum_{l=0}^{n-1} \frac{n!}{l!(n-l)!} M^l (1-M)^{n-l} \frac{n-l-1}{n-l}$$
$$= \sum_{l=0}^{n-1} \frac{n!}{l!(n-l)!} M^{l-1} (1-M)^{n-l-1} (l-nM) \frac{n-l-1}{n-l} < 0.$$

It is clear that for $(n-1)/n \leq M$ all the summands are negative. For $(n-1)/n > M$, however, we obtain:

$$\sum_{l=0}^{n-1} \frac{n!}{l!(n-l)!} M^{l-1}(1-M)^{n-l-1}(l-nM)\frac{n-l-1}{n-l}$$
$$\leq \frac{n-nM-1}{n-nM} \sum_{l=0}^{n-1} \frac{n!}{l!(n-l)!} M^{l-1}(1-M)^{n-l-1}(l-nM)$$
$$= \frac{n(1-M)-1}{(1-M)^2}\left[\sum_{l=0}^{n-2} \frac{(n-1)!}{l!(n-l-1)!} M^l (1-M)^{n-l-1} - \sum_{l=0}^{n-1} \frac{n!}{l!(n-l)!} M^l (1-M)^{n-l}\right]$$
$$= -\frac{n-1-nM}{1-M} M^{n-1} < 0.$$

To prove the convergence result, we compare the right-hand side of the optimal tontine with the one that pays in all periods $1 \leq t \leq \tilde{T}$:

$$\frac{\frac{P}{\tilde{T}} \sum_{t=1}^{\tilde{T}} \left[\sum_{l=0}^{n-1} \binom{n}{l} M_t^{\,l} (1-M_t)^{n-l} \frac{n-l-1}{n-l}\right]}{P\frac{n-1}{n}}$$
$$= \frac{n}{n-1}\frac{1}{\tilde{T}} \sum_{t=1}^{\tilde{T}} \left[\sum_{l=0}^{n-1} \binom{n}{l} M_t^{\,l} (1-M_t)^{n-l} \frac{n-l-1}{n-l}\right].$$

It is therefore sufficient to show that

$$\sum_{l=0}^{n-1} \binom{n}{l} M^l (1-M)^{n-l} \frac{1}{n-l}$$

converges to zero for all $0 \leq M < 1$ when n goes to infinity. This is easily demonstrated numerically. ■

Proposition 3 highlights that the inefficiency of tontines that pay in later periods is less severe when many participants are expected to participate. As a further feature of such tontines, the expected payments in period t, conditional on agent survival, are clearly small in the beginning (as the likelihood of others' survival is high) but increase rapidly toward the terminal period. As an investment instrument for retirement funds, the tontine therefore provides advantages compared to other instruments. In particular, if one relaxes the assumption of risk-neutrality and perfect substitutability across periods, the tontine is quite practical economically if agents have decreasing external income (salary, pension) and can use the tontine to flatten their temporal payoff streams.

Example 1

We consider contributions to a linear public good when the probability of dying is uniformly distributed: $\mu_t = \frac{1}{T}$ for all $1 \le t \le T$. Assume that there are $n = 50$ symmetric agents and $T = 50$ periods. Figure 1 shows the contribution level to the $\tilde{T}$-tontine relative to the contribution level to the optimal tontine. For the $\tilde{T} = 50$-tontine, Figure 2 illustrates the expected payments in period *t* given survival (payments relative to payment in period 1).

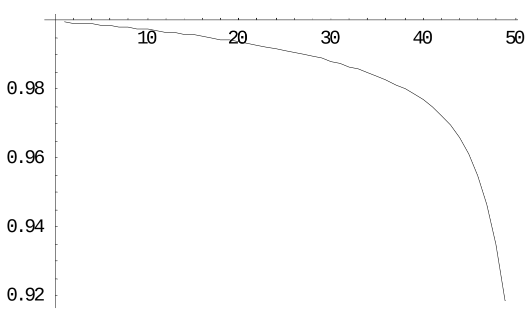

Figure 1:

Total contributions as a function of $\tilde{T}$ (normalized)

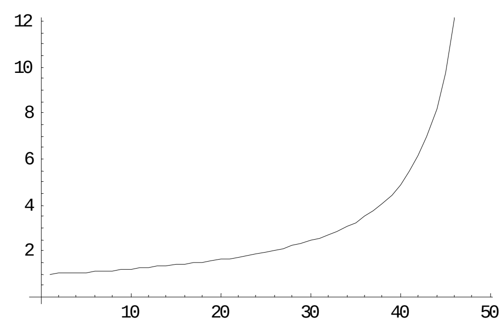

Figure 2:

Expected payments in period t given survival (normalized)

Figure 1 reveals that contributions remain above 90 percent of the optimal levels even if one spreads the tontine payment over the whole potential lifespan of agents. Figure 2 shows that expected payments in period *t*, given that the agent survives until then, increase rapidly toward the end of an agent's lifespan.

III.2 Tontines and risk-aversion

Lange et al. (2004) have shown that contributions to lotteries are decreasing in the level of risk-aversion. For the optimal tontine, however, players are not exposed to any risk. The optimal tontine is therefore a more efficient instrument for fundraising than any lottery.

Proposition 4 (Tontines for risk-averse players)

> *Individual contributions under the tontine that pays only before any agent has died are independent of the risk posture of agents. If agents are risk-averse but symmetric with respect to their valuation of the public good, it dominates any lottery as a fundraising instrument.*

Besides this superiority of tontines for risk-averse agents, a fundraiser does not need any prior beliefs over the risk preference of a potential donor pool when designing the fundraising instrument.

III.3 Tontines with heterogeneous agents

We have seen in the previous section that the optimal tontine for symmetric risk-neutral players coincides with a single-prize lottery or—equivalently—a rebate scheme. In this section, we consider the performance of tontines for agents with heterogeneous valuation of the public good. Conditions are derived under which the rebate scheme, i.e., the degenerate tontine, is optimal.

Reconsidering the individual first-order conditions (1), first observe that if there is (at least) one *t* for which $P_t > 0$ and $0 < M_t < 1$, all players will contribute. The intuition is that there is a chance that in period *t* only one agent will survive. An agent can secure himself this prize $P_t > 0$ by contributing. More formally, looking at the first-order condition for $i \notin S_0$, the right-hand side is clearly infinite (consider $S = S_0 \setminus i$).

Proposition 5 (Participation in tontines):

If there is period t for which $P_t > 0$ and $0 < M_t < 1$, then all players contribute to the tontine.

Even a slight deviation from the degenerate tontine ($P_0 = P$) (alias the rebate scheme) towards $P_t > 0, P_0 < P$ can therefore lead to a discontinuous change in participation and therefore contribution levels. In general, we obtain the following result when a tontine should pay out part of the prizes in later periods:

Proposition 6 (Tontines—Heterogeneous agents):

If agents are sufficiently heterogeneous with respect to their valuation of the public good, the optimal tontine pays $P_t > 0$ for some $t > 0$ with $0 < M_t < 1$. In particular, if a set S_0 of players participates for $P_0 = P$, then contributions can be increased by changing to $P_t > 0$ ($P_t + P_0 = P$)

(i) *if $k < n$, i.e., there is (at least) one agent $i \notin S_0$ who does not contribute if $P_0 = P$:*

$$h_i'(B-P) \leq \frac{1}{k-1}\sum_{j \in S_0} h_j'(B-P) = \frac{H-1}{k-1}$$

(ii) *if $k = n$ for $P_0 = P$ and*

$$\sum_{0\le l\le n-1} M_t^{\,l}(1-M_t)^{n-l}\frac{n-l-1}{n-l}\sum_{S\subseteq S_0,\#S=n-l}\frac{1-H/n}{1-H+(n-1)H(S)/(n-l)}>\frac{n-1}{n}$$

$$\text{where } H(S)=\sum\nolimits_{i\in S} h_i\,'(\bullet) \text{ and } H=\sum\nolimits_{i\in S_0} h_i\,'(\bullet)$$

Proof:

We analyze the tontine that pays $P_t=\varepsilon$ and $P_0=P-\varepsilon$. Here, the first-order conditions (1) are given by:

$$\begin{aligned}1-h_i\,'(B-P)&=(P_0-\varepsilon)\frac{B-b_i}{B^2}+\varepsilon\sum\nolimits_{l=0}^{n-1}M_t^{\,l}(1-M_t)^{n-l}\left[\sum\nolimits_{S:i\notin S,\#S=l}\frac{B-B(S)-b_i}{(B-B(S))^2}\right]\\ n-\sum_i h_i\,'(B-P)&=(P_0-\varepsilon)\frac{n-1}{B}+\varepsilon\sum\nolimits_{l=0}^{n-1}M_t^{\,l}(1-M_t)^{n-l}\left[\sum\nolimits_{S:\#S=l}\frac{n-l-1}{B-B(S)}\right]\end{aligned}\qquad(5)$$

<u>**Case 1**</u>

Consider first the case in which there is $i\notin S_0$ with $h_i\,'(B-P)<\dfrac{H-1}{k-1}$. Then there is a discontinuity in participation and contribution at $\varepsilon=0$ when $P_t=\varepsilon$ and $P_0=P-\varepsilon$. We therefore study the limit of the first-order conditions (5) from above ($\varepsilon\searrow 0$) and get $k(\vec{S}_0)>k(S_0)$, where $\vec{S}_0$ is the set for which $\vec{b}_i=\lim_{\varepsilon\searrow 0} b_i(\varepsilon)>0$. Now we have

$$\begin{aligned}1-h_i\,'(\vec{B}-P)&=P\frac{\vec{B}-\vec{b}_i}{\vec{B}^2}\quad \text{if}\quad \vec{b}_i>0\\ k(\vec{S}_0)-\sum\nolimits_{i\in\vec{S}_0}h_i\,'(\vec{B}-P)&=P\frac{k(\vec{S}_0)-1}{\vec{B}},\end{aligned}$$

from which the claim follows immediately.

<u>**Case 2**</u>

Consider now the case in which there is $h_i\,'(B-P)\ge\dfrac{H-1}{k-1}$ for all i at $P_0=P$. Then, the first-order conditions (5) also hold for $P_0=P$ (as all individual first-order conditions (1) hold with equality). For $P_t = \varepsilon$ and $P_0 = P - \varepsilon$, we study the derivative of B with respect to ε at $\varepsilon = 0$:

$$\frac{\partial B}{\partial \varepsilon}(\varepsilon = 0) = \frac{1}{\frac{n-1}{B^2}P - \sum_i h_i''(\bullet)}\left[-\frac{n-1}{B} + \sum_{l=0}^{n-1} M_t^{\,l}(1-M_t)^{n-l}\left[\sum_{S:\#S=l}\frac{n-l-1}{B-B(S)}\right]\right]$$

Therefore, $\frac{\partial B}{\partial \varepsilon}(\varepsilon = 0) > 0$ iff

$$\sum_{l=0}^{n-1} M_t^{\,l}(1-M_t)^{n-l}\left[\sum_{S:\#S=l}\frac{n-l-1}{B-B(S)}\right] = \sum_{l=0}^{n-1} M_t^{\,l}(1-M_t)^{n-l}\left[\sum_{S:\#S=n-l}\frac{n-l-1}{B(S)}\right] > \frac{n-1}{B}.$$

Using the equilibrium conditions

$$1 - h_i'(B-P) = \frac{B-b_i}{B^2}P \qquad \text{and} \qquad 1 - H = \frac{n-1}{B}P,$$

we obtain the claimed relationship,

$$\sum_{0 \le l \le n-1} M_t^{\,l}(1-M_t)^{n-l}\,\frac{n-l-1}{n-l}\sum_{S \subseteq S_0, \#S=n-l}\frac{1-H/n}{1-H+(n-1)H(S)/(n-l)} > \frac{n-1}{n}$$

and completes the proof. ■

We have demonstrated above that the tontine with $P_0 = P$ coincides with a single-prize lottery. Let us therefore finally compare the conditions in Proposition 5 with those under which one can improve upon the single-prize lottery by offering multiple prizes. As shown by Lange et al. (2004), one can improve upon the single prize lottery by providing (at least) a second prize if:

$$(k-2)\sum_{i \in S_0}\frac{b_i}{B-b_i} > k \quad \Leftrightarrow \quad (1-H/k)\sum_{i \in S_0}\frac{1}{1-h_i} > \frac{(k-1)^2}{k-2}.$$

On the one hand, we immediately see that one can design a tontine that outperforms any lottery if $k < n$ and $(1-H/k)\sum_{i\in S_0}\frac{1}{1-h_i} < \frac{(k-1)^2}{k-2}$. Alternatively, if $k = n$ and $(1-H/n)\sum_{i}\frac{1}{1-h_i} > \frac{(n-1)^2}{n-2}$, one can increase the contributions to the lottery by offering a second prize, but cannot improve upon the degenerate tontine if

$$\sum_{0\le l\le n-1} M_t^{\,l}(1-M_t)^{n-l}\,\frac{n-l-1}{n-l}\sum_{S\subseteq S_0,\#S=n-l}\frac{1-H/n}{1-H+(n-1)H(S)/(n-l)} < \frac{n-1}{n}$$

for all *t*. This, for example, would be the case if M_t is close to one for all *t*. In such cases, the right-hand side of the inequality would be close to zero.

We therefore can summarize our findings in the following Proposition:

Proposition 7 (Tontines vs. lotteries—Heterogeneous agents):

If agents are risk-neutral and heterogeneous with respect to their valuation of the public good, then there exist situations in which appropriately designed tontines outperform lotteries and vice versa.

Note that in real-world applications there will always be agents who have no valuation, or only a below average valuation, for specific public goods. In such cases, one can always improve upon a single prize lottery by using a tontine with $P_t > 0$ *and* $0 < M_t < 1$; in this case all agents will contribute under the tontine.

III.4 *Tontine as a fundraising instrument*

A charity that seeks to fundraise using a literal version of the historical tontine to replace lotteries might find the simulation of the "probabilities to die" problematic since in each round one must have a random draw for all survivors. The structure of the tontine can be used, however, to design a fundraising instrument (which we also call tontine) whose implementation is quite simple.

For this, we abstract from the independent and identical probabilities of dying considered in the previous section. Instead, sequentially draw one of the *k* participating persons which must leave the game. That is, in period *t* the number of players is *k-t*. For the payments, the sequence of "dying" is decisive.[7] Each sequence has the same probability given by $1/k!$ if *k* players contribute. As in the previous section, a certain amount of money is distributed among the remaining players according to their share in each period (i.e. before the next person leaves).

Compared to the preceding analysis, we only have to change the probability of a certain set *S* of players having passed away until period *t* from $M_t^{\#S}(1-M_t)^{n-\#S}$ to $1/\binom{k}{\#S}$. The first-order conditions (1) therefore convert to

$$1-h_i'(B-P)=\sum\nolimits_{t=0}^{T-1}P_t\sum\nolimits_{l=0}^{k-1}\left[\sum\nolimits_{S\subseteq S_0\backslash i,\#S=l}\frac{B-B(S)-b_i}{(B-B(S))^2}\right]/\binom{k}{l}\quad \text{for}\quad i\in S_0$$

$$1-h_i'(B-P)\geq\sum\nolimits_{t=0}^{T-1}P_t\sum\nolimits_{l=0}^{k-1}\left[\sum\nolimits_{S\subseteq S_0\backslash i,\#S=l}\frac{1}{B-B(S)}\right]/\binom{k}{l}\quad \text{for}\quad i\notin S_0$$

$$k-\sum_{i\in S_0}h_i'(B-P)=\sum\nolimits_{t=0}^{T-1}P_t\sum\nolimits_{l=0}^{k-1}\left[\sum\nolimits_{S\subseteq S_0,\#S=l}\frac{k-l-1}{B-B(S)}\right]/\binom{k}{l}$$

while all the qualitative results remain valid. In particular, payments should be made before anybody leaves the game ($P_0=P$) if agents have similar valuation of the public good. If agents are sufficiently heterogeneous, one can improve upon this degenerate tontine—and possibly upon any lottery—by choosing $P_0<P$.

[7] For example, given identical contributions, a person who leaves last gets the highest payment, the person who leaves first receives the lowest payment.

IV. Concluding Remarks

This article provides a theoretical exploration of tontines, a popular method of financing public goods that was introduced more than three centuries ago. Even though tontines were once quite popular—the name "tontine" remains prominently displayed on several publicly funded projects around the world—little is known about their formal structure and whether it would be apropos to reintroduce tontines today.

In this study, we highlight the best characteristics of the tontine that might be utilized in future fundraising drives by deriving the optimal tontine and formally linking the tontine to a popular modern fundraising scheme used by both government and charitable fundraisers: lotteries. We show that the optimal tontine generates contributions that are equivalent to those under a single prize lottery when agents are symmetric and risk neutral. For symmetric risk-averse agents, contributions under the optimal tontine strictly dominate contributions raised under any lottery type. Further, the design of an optimal tontine is independent of underlying risk posture and generates contributions that weakly dominate those of *any* lottery. If agents are sufficiently asymmetric, tontines yield higher contribution levels than the optimal lottery—having a chance of being the only survivor in a period with positive payment provides incentives for *all* players to contribute. If a fundraiser also seeks a high participation rate in order to collect the names of potential contributors for future fundraising drives, then the tontine has an additional "hidden" advantage in that it maximizes participation rates.

While this article has addressed the performance of tontines as a fundraising mechanism, there are a number of outstanding issues. For example, under the optimal tontine each agent receives a positive monetary payment with certainty. The *ex post* allocation of wealth is thus more equitable than that which results from any *k*-prize lottery. Given that inequality-averse preferences have been found to be prevalent among agents in laboratory experiments (see, e.g., Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000), there are reasons to suspect that contribution levels under a tontine would exceed even those predicted by our model. We hope that future work examines this issue in greater detail and evaluates the performance of tontines in the laboratory and in the field.

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