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# TAX ANALYSIS IN AN OLIGOPOLY MODEL

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## ABSTRACT

In this paper we analyze taxation using the conjectural variations model of oligopoly. We demonstrate the way in which the incidence of a tax depends upon the pattern of firm interaction. The results obtained have important implications for the controversy surrounding the question of whether a tax on corporate income can be over-shifted. We also study normative aspects of taxation. The focus here is on the errors that can arise in excess burden calculations when incorrect assumptions on market structure are made.

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## 1. Introduction

Taxation usually is studied in models that postulate a perfectly competitive market structure.[1] Analyses that deviate from this rule tend to focus on the opposite polar case of monopoly. Given that the "in-between" situation--oligopoly--is of major importance in western industrial countries, it might appear surprising that oligopoly has received such scant attention. Of course, at least as far back as Musgrave [1959], it has been recognized that the impact of a tax may depend upon market structure. There is no definitive model of oligopolistic behavior, however, and different stories can have quite different implications for tax shifting. The tendency has been to ignore oligopoly on the grounds that "anything can happen."

Several models of oligopolistic behavior recently have received considerable attention in the industrial organization literature. In this paper we analyze taxation within the framework provided by one of the best-known of these, the conjectural variations model.[2] Although this approach certainly is not the unique solution to "the" oligopoly problem, it is simple and encompasses a wide range of models, including monopolistic and competitive behavior as special cases. In the framework of the conjectural variations model, we are able to demonstrate rigorously how tax incidence depends upon market structure. The results obtained have important implications for the controversy surrounding the question of whether a tax on corporate income can be over-shifted.

In Section 2, we review the essential aspects of the conjectural variations model. Section 3 shows how an industry's output, price and profits are affected by the presence of a factor or output tax. It is demonstrated that, under quite reasonable conditions, the imposition of a tax can lead to an increase in industry profits. The normative analysis of taxation in a conjectural variations model is discussed in Section 4. Our focus is on the errors that can arise in excess burden calculations when incorrect assumptions on market structure are made. In Section 5, we summarize our results and discuss some implications for future research.

## 2. A Conjectural Variations Model of Oligopoly

Consider an industry comprising $n$ firms producing a homogeneous product with a market inverse demand function $P[\cdot]$. Firm $i$ produces $x_i$ units of output and incurs costs $C[x_i,t]$, where $t$ is a tax parameter. The firm's profits are

$$(2.1)\quad \Pi^i[x_i] = P[x_i + \sum_{j\neq i} x_j]x_i - C[x_i,t] .$$

A given firm's output decision will depend upon its expectations concerning the response of its rivals to any change in the firm's level of production. We assume that all firms have identical "conjectural variations" equal to $\delta$. That is, each firm believes that when it raises its output by $dx_i$, the other firms will raise their output by a total of $\delta\, dx_i$:

$$(2.2)\quad \left[\frac{d \sum_{j\neq i} x_j}{dx_i}\right]^{con} = \delta ,$$

where the superscript "con" denotes that it is the conjectured rather than actual response.

Suppose that firm i's current level of output is $x_i^o$ , and the rest of the industry produces $\sum_{j \neq i} x_j^o$ . Firm i's conjectured inverse demand function, $P^{con}[\cdot]$, gives the price that the firm perceives will be associated with each level of its output, conditional on the current levels of its output and the total output of the other firms:

$$P^{con}[x_i; x_i^o, \sum_{j \neq i} x_j^o] \equiv P[x_i + \sum_{j \neq i} x_j^o + \delta(x_i - x_i^o)].$$

Each firm sets its level of output to maximize its profits, taking $P^{con}[\cdot]$ as the inverse demand function. In equilibrium, for each firm the conjectured change in profits due to a change in its output is equal to zero:

$$(2.3) \quad 0 = \left[ \frac{d\Pi^i}{dx_i} \right]^{con} = P[x_i + \sum_{j \neq i} x_j] + (1+\delta)P'[x_i + \sum_{j \neq i} x_j]x_i - C_x[x_i,t].$$

Because all firms have identical cost functions, we will restrict our attention to symmetric equilibria (i.e., those equilibria in which $x_i = x$ for all firms).[3] The equilibrium condition, eq. (2.3), may be written as

$$(2.4) \quad P[nx] + (1+\delta)P'[nx]x - C_x[x,t] = 0 .$$

The second order necessary condition for a firm's optimization problem is

$$2(1+\delta)P'[nx] + (1+\delta)^2 P''[nx]x - C_{xx}[x,t] \leq 0 .$$

The equilibrium level of output depends on the number of firms, the conjectured variation, and the level of taxes.

This framework affords great flexibility in modelling firm behavior and the degree of competition within the industry. If $\delta = -1$, for example, each firm will perceive its demand curve to be flat. Eq. (2.4) becomes

$$P[nx] - C_x[x,t] = 0 ,$$

and each firm will behave competitively, setting its output at a level where price is equal to marginal cost. At the other extreme, if $\delta = n-1$, then each firm has a conjectured inverse demand curve that is 1/n of the industry curve. The equilibrium is characterized by

$$P[nx] + nxP'[nx] - C_x[x,t] = 0 ,$$

the monopoly condition that market marginal revenue equal marginal cost. The firms will behave like a monopolist or joint profit maximizing cartel.[4]

## 3. The Output and Profit Effects of Tax Changes

A shift in the tax parameter that affects marginal costs will in general induce changes in the equilibrium levels of price, output, and profits. In this section, we derive comparative statics results for these changes.

Consider an infinitesimal change in the tax parameter. Totally differentiating the equilibrium condition, eq. (2.4), we obtain

$$\{(n+1+\delta)P' + (1+\delta)P'' - C_{xx}\} \; dx - C_{xt}dt = 0 ,$$

or

$$(3.1) \quad \frac{dx}{dt} = \frac{C_{xt}}{(n+1+\delta)P' + (1+\delta)P'' - C_{xx}} .$$ [5]

Marginal costs are assumed to be a nondecreasing function of the level of taxes; $C_{xt} > 0$ . The uniqueness of the equilibrium implies that the denominator of eq. (3.1) is negative. Thus, $\frac{dx}{dt}$ is negative. An increase in $t$ leads to a fall in output and an increase in the market price.

The tax change will alter profits through two effects: (1) the level of per-firm tax payments will change by $C_t$; and (2) the tax-induced shifts in price and output will change before-tax profits by

$$(3.2)\quad (P - C_x)\frac{dx_i}{dt} + x_i\frac{dP}{dt} .$$

Recalling that the equilibrium is symmetric, eq. (3.2) may be rewritten as

$$(3.3)\quad \{P - C_x + xnP'\}\frac{dx}{dt} .$$

From the first order condition for profit maximization, eq. (2.4), we see that the change in before-tax profits, eq. (3.3), is zero for $\delta = n-1$ and positive for all $\delta < n-1$. Intuitively, this result is clear. When $\delta = n-1$, the oligopolists behave like a monopolist and $x$ is set at the joint profit maximizing level. Hence, the tax-induced change in output has no first-order effect on profits. When $\delta < n-1$, the firms' output level is greater than the one at which joint profits are maximized. A tax increase raises marginal costs and induces a reduction in the level of output towards the joint profit maximizing level. The tax has the effect of enforcing a collusive output restriction, and the tax increase leads to higher <u>before</u>-tax profits.

The net effect of a tax change on profits is determined by comparing the increase in before-tax profits with the increase in tax payments:

$$(3.4)\quad \frac{d\Pi^i}{dt} = \{P - C_x + xnP'\}\frac{dx}{dt} - C_t .$$

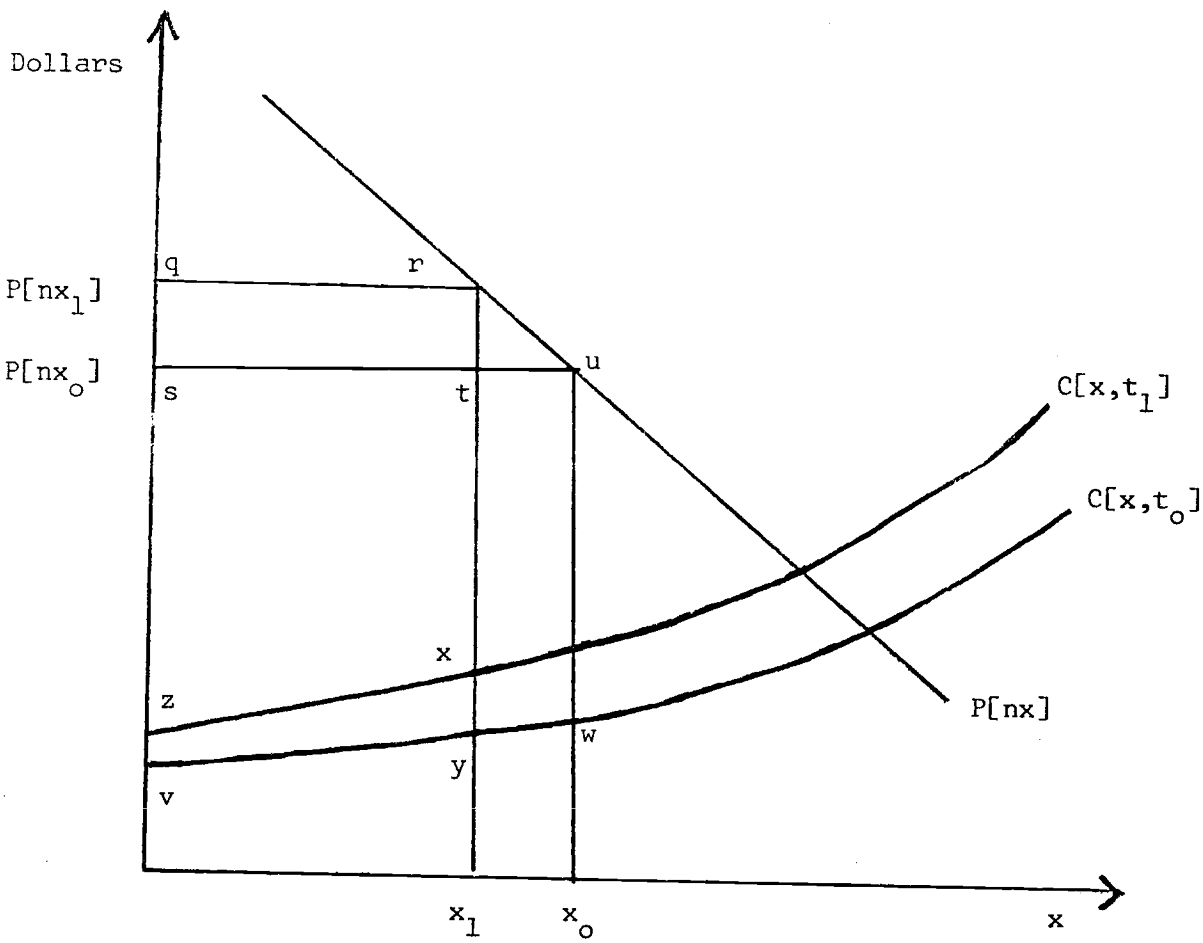
Depending on the patterns of taxes, tastes, and technology, a tax increase may lead to either a fall or rise in after-tax profits. The net effect cannot be known a priori.

To illustrate this point, consider Figure 1. Initially, the tax is set at $t_o$ and firm output is $x_o$. The tax is raised to $t_1$. As a result, the marginal cost curve shifts upward, and the equilibrium level of output declines by $(x_o-x_1)$, the amount given by eq. (3.1). Area tuwy represents the decline in profits due to the reduction of profitable sales, $\Delta x(P-C_x)$. Area qrts represents the increase in profits due to the higher price charged for the $x_1$ units sold after the tax is imposed, $x_1 \Delta P$. Note that ceteris paribus, area qrts increases with $C_{xt}$. By eq. (3.1), higher values of $C_{xt}$ imply greater output contractions and therefore greater values of $\Delta P$.

Area xyvz represents the loss in profits due to the increase in tax payments, $C_t$, which is equal to $x_1$ multiplied by the change in average tax per unit of output. Given that area tuwy is positive, we see that $\frac{d\Pi^i}{dt} > 0$ only if the tax-induced increase in price, $\Delta P$, is greater than the increase in the average per-unit tax, $C_t/x_1$ (i.e., area qrts must be greater than area xyvz). Because qrts moves with $C_{xt}$, we can conclude that the greater the extent to which $C_{xt}$ exceeds $C_t/x_1$, the more likely is $\frac{d\Pi^i}{dt} > 0$.

While it is possible that a tax increase will lead to an increase in after-tax profits, a natural question is whether such an outcome is plausible or is merely a theoretical curiosity. Economists often take linear demand curves and quadratic cost functions as approximations of actual demands and costs. When demands and costs have these forms, $\frac{d\Pi^i}{dt}$ may be positive for reasonable values of the parameters.

FIGURE 1



Suppose, for example, that the market inverse demand function is

$$P[nx] = 200 - 8(nx),$$

and each firm has a cost function

$$C[x,t] = w(1+t)x^2 ,$$

where $w$ is an index of factor prices. Such a cost function arises when the tax is levied proportionately on all factor prices and production is homothetic and homogeneous of degree 1/2.[6] For the calculations, w was taken to be equal to 1 and t equal to zero.

Using eqs. (2.4), (3.1), and (3.4), the tax-induced changes in per-firm output and after-tax profits were calculated for a duopoly under several different conjectural variations. The results are presented in Figure 2. When $\delta = -1$, each firm sets its price equal to marginal cost; output is pushed beyond the joint profit maximizing level. Here, the increase in before-tax profits due to the output restriction dominates the increase in taxes; $\frac{d\Pi^i}{dt} > 0$ . In this example, when firms have Cournot conjectures ($\delta = 0$), the direct effect of increased tax payments dominates; $\frac{d\Pi^i}{dt} < 0$ . At $\delta = 1$, the duopolists act to maximize joint profits, and the only effect of a tax rise on profits is the decrease due to the increase in tax payments.

Recently, several authors have analyzed the notion of "consistent" or "rational" conjectures. Firms are said to have consistent conjectures when, in equilibrium, the conjectured local responses are equal to the true responses. For the case of duopolists with quadratic costs and a linear demand curve, Bresnahan [1981] has developed a closed-form expression for the consistent value of $\delta$ . Applying his formula to our example the consistent conjecture is approximately -0.61. For this conjecture

FIGURE 2

| | $\delta$ | | | |
|---|---|---|---|---|
| | -1 | -.61 | 0 | 1 |
| $X=2x$ | 22.2 | 18.9 | 14.4 | 11.8 |
| $P[2x]$ | 22.2 | 48.5 | 76.9 | 105.9 |
| $C[x,t]$ | 123.5 | 89.6 | 59.2 | 34.6 |
| $\Pi^i$ | 123.5 | 369.7 | 532.6 | 588.2 |
| $\frac{dX}{dt} = 2\frac{dx}{dt}$ | -2.5 | -1.8 | -1.2 | -0.7 |
| $\frac{dP}{dt}$ | 19.7 | 14.3 | 9.5 | 5.5 |
| $\frac{dC}{dt}$ | 123.5 | 89.6 | 59.2 | 34.6 |
| $\frac{d\Pi^i}{dt}$ | 96.0 | 19.7 | -22.8 | -34.6 |

$\frac{d\Pi^i}{dt} = 19.7 > 0$ . Thus, the result that a tax increase can increase profits does not rely on irrationality on the part of firms, at least in this narrowly defined sense.

Our example illustrates the shortcomings of attempting to analyze oligopoly by looking at the polar cases of monopoly and competition. As noted above, for monopoly, before-tax profits are not increased by the tax-induced reduction in output, and $\frac{d\Pi^i}{dt} = -C_t < 0$. At the other pole, under perfect competition free entry leads to $\frac{d\Pi^i}{dt} = 0$ . In some oligopolistic markets the change in profits due to a change in taxes is positive and does not fall between the values of $\frac{d\Pi^i}{dt}$ for the cases of monopoly and perfect competition. Thus, it may be misleading to analyze taxation in an oligopolistic market by interpolating the results for the two polar cases.

This discussion of competition raises the question of entry in the present model. In our analysis, we have taken the number of firms to be fixed exogenously. One may think of the model in two ways. First, it can be viewed as a short run analysis of a market in which capital stocks are fixed. Second, it can be viewed as a long run analysis of a market in which existing firms can adjust the levels of all productive inputs but sufficiently high barriers exist to preclude the entry of new firms. Krzyzaniak and Musgrave [1963 , p. 2] suggested that a positive value of $\frac{d\Pi^i}{dt}$ was likely to be a short run phenomenon, in the sense that it depended on the inability of firms to adjust their capital stocks. Our analysis indicates that "over-shifting" can occur in the long run as well.

## 4. Welfare Analysis

It is well-known that the presence of a pre-existing distortion complicates the welfare analysis of a tax. Typically, monopoly is the only distortion induced by market structure that public finance economists study.[7] In this section we discuss the errors that might be made in estimating the excess burden of a tax if one erroneously assumes that the firms behave as a monopolist ($\delta = n-1$) when in fact the oligopolists do not maximize joint profits ($\delta < n-1$).

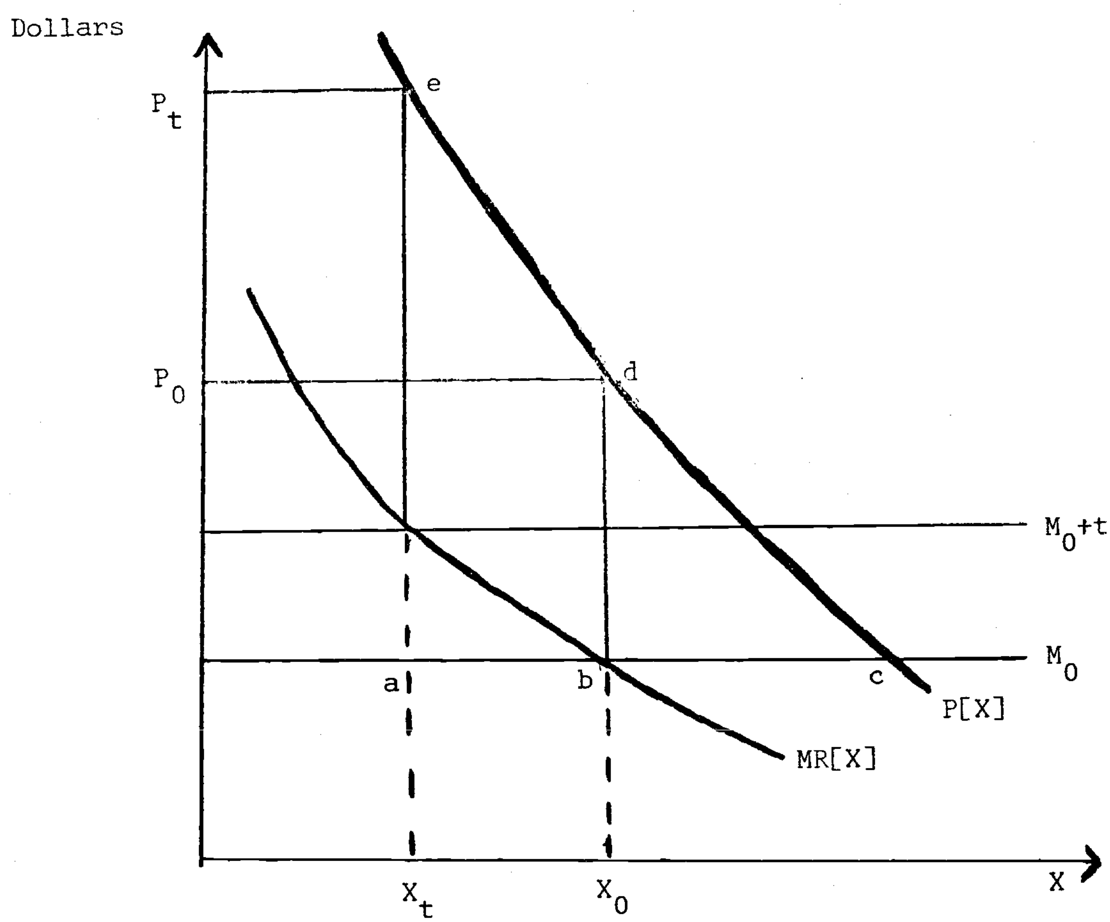
Consider an investigator who has the following information about an industry: it has constant marginal costs $C_x \equiv M_0$, price $P_0$, and market output $X_0$. The investigator notes that the industry is composed of several large firms, so that the competitive assumption is untenable. Instead, he assumes that the firms behave like a monopolist. A unit tax of $t$ is imposed on the output of the industry. What is the excess burden of the tax?

There are a number of ways to proceed. Our investigator might begin by using a diagram like Figure 3. The market demand and market marginal revenue curves are denoted by P[X] and MR[X], respectively, where $X \equiv nx$ is industry output. The curves are sketched so that the marginal revenue curve intersects the marginal cost curve at $X_0$ and the associated price is $P_0$. Prior to imposition of the tax, there is a monopoly excess burden of cbd which is a consequence of the fact that price exceeds marginal cost.[8] After the tax is imposed, price and output are $P_t$ and $X_t$, respectively, and the incremental excess burden is abde.

Algebraically, the area abde can be approximated by

$$(4.1)\quad \text{abde} \cong (P_0 - M_0)\frac{dX}{dt} + \frac{1}{2}\frac{dX}{dt}\frac{dP}{dt} = (P_0 - M_0)\frac{dX}{dP}\frac{dP}{dt} + \frac{1}{2}\frac{dX}{dP}\left(\frac{dP}{dt}\right)^2 .$$

FIGURE 3



Computation of the differentials $\frac{dX}{dt}$ and $\frac{dP}{dt}$ clearly requires some knowledge of the demand curve's shape. A typical approach is to take advantage of the identity that characterizes monopoly equilibrium,

(4.2) $P(1 + \frac{1}{\eta}) = M$ ,

where P is price, M is marginal cost, and $\eta$ is the price elasticity of market demand. Substituting $P_0$ and $M_0$ into eq. (4.2) gives $\eta = P_0/(M_0 - P_0)$. Assuming that $\eta$ is locally constant, eq. (4.2) implies that $dP(1 + \frac{1}{\eta}) = dM$. However, when a unit tax is imposed, dM is just dt, so

(4.3) $\frac{dP}{dt} = \frac{1}{(1 + \frac{1}{\eta})}$

The only additional information required now is an estimate of $\frac{dX}{dt}$. Given the assumption of a locally constant elasticity of demand, $X = P^{\eta}$, and

(4.4) $\frac{dX}{dP} = \eta P^{\eta-1}$

Substituting eqs. (4.3) and (4.4) along with values for $P_0$ and $M_0$ into eq. (4.1) yields an estimate of the excess burden of the tax.

The investigator's belief that the industry behaves as a monopoly when actually it is a conjectural variations oligopoly leads to two errors: (i) an "econometric" error which arises by estimating $\eta$ from eq. (4.2),[9] and (ii) a "behavioral" error which is a consequence of using eq. (4.3) to estimate how the price will respond to the tax. Note that even if error (i) were eliminated, problems would still arise due to the use of eq. (4.3).

To illustrate these points, consider the following example: an industry consists of four identical firms which have constant marginal costs of \$2. Each firm has a conjectural variation of $\delta = 2$. The market demand curve is $X = P^{-2}$. Using the first order condition, eq. (2.4), the equilibrium price is \$3.22, and each firm's output is 0.024, so that market output is 0.096.

As before, the problem is to estimate the excess burden of a "small" per unit tax on industry output. The only data available to the investigator are price, market output, and marginal costs. On the assumption that the four firms can be modelled as a monopoly, our investigator computes the price elasticity of demand by substituting P = \$3.22 and M = \$2.00 into eq. (4.2), and finds $\eta = -2.639$. Assuming an isoelastic demand curve, at least locally, this value of $\eta$ can be substituted into eq. (4.3) to find $\frac{dP}{dt} = 1.610$. Similarly, by substituting into eq. (4.4), $\frac{dX}{dP}$ is -0.0374. There is now enough information to evaluate the excess burden formula, eq. (4.1). Specifically, the welfare loss generated by the tax is \$0.1219 $= (3.22-2)(.0374)(1.610) + \frac{1}{2}(.0374)(1.610)^2$.

Contrast this with a calculation made on the basis of the true model. Because the welfare loss depends upon areas under the <u>market</u> demand curve, Figure 3 still provides the appropriate framework. Using a value of $\eta = -2.0$ in eq. (4.4) yields dX/dP = -0.0599. $\frac{dP}{dt}$ is found by the chain rule: $\frac{dP}{dt} = \frac{dP}{dX}\frac{dX}{dt}$. $\frac{dP}{dX}$ is the reciprocal of $\frac{dX}{dP}$, or 1/-.0599. $\frac{dX}{dt}$ is found by first finding $\frac{dx}{dt}$ from eq. (3.1), and then multiplying by the number of firms in the market (four), giving $\frac{dX}{dt} = -0.196$. Substituting these values in the excess burden formula, eq. (4.1), yields a value of \$0.5598 $= (3.22-2)\ (.196) + \frac{1}{2}\ (.0599)(3.272)^2$ . The true value is about five times the value estimated on the basis of the incorrect assumption of monopolistic behavior (\$0.1219).

It might be argued that the comparison is unfair because the investigator who assumes monopoly should also be allowed to start with the correct value of the price elasticity of demand. Assume, then, that the investigator obtains independently the true value of $\eta$.[10] Setting $\eta = -2.0$ in eq. (4.3) gives $\frac{dP}{dt} = 2.0$. $\frac{dX}{dP}$, found by substituting into eq. (4.4), is $-0.0599$. Substituting these values into eq. (4.1) yields an excess burden of \$0.2659 $= (3.22-2)\ (.0599)\ (2.0) + \frac{1}{2}\ (0.0599)2^2$. Although larger than the earlier calculation of \$0.1219, it is still less than half the true value of \$0.5598.

As we emphasized at the outset, these calculations are intended merely to illustrate the formulas presented earlier in this section, and there is no reason to take the specific quantitative results seriously. They do show, however, that making incorrect assumptions on market structure potentially can lead to major errors in welfare cost estimates.

## V. Conclusion

We have discussed some positive and normative aspects of taxation using a conjectural variations model of oligopoly. The assumption that an oligopolistic industry acts as if it is competitive or monopolistic can produce misleading results. For example, it is quite possible that a tax on a factor used by oligopolists will raise their economic profits, although this result never could arise in the polar cases. More generally, we have shown that impacts of a tax upon an oligopolistic industry need not lie between those of monopoly and competition.

In their famous econometric study published almost twenty years ago, Krzyzaniak and Musgrave [1963] asserted that there was a positive relationship between the corporate income tax rate and corporate profits. The finding was roundly attacked. While our concern here is not the merits of their particular statistical procedure, other economists' comments on the theoretical plausibility of the result are of some interest. The critics Cragg, Harberger and Mieszkowski [1967] observed:

> Not only does this result run counter to most economists' judgments of plausibility, it also opens questions concerning the pricing behavior of corporations which have wide ramifications beyond the specific issue of corporation tax incidence. Indeed, it is certainly not far from the truth to say that if we accept the Krzyzaniak-Musgrave results at face value, we must also accept the task of rebuilding the foundations of the theory of the behavior of the firm. (pp. 811-812)

We have shown that far from being outside the pale of economic theory, the Krzyzaniak-Musgrave result can be rationalized using fairly conventional neoclassical tools.

Since the time of the debate over the Krzyzaniak-Musgrave study, virtually all the work on taxation has assumed perfect competition. Within this framework, authors have studied the effects of alternative assumptions concerning production technologies and demand structures. General equilibrium responses have been carefully taken into account, as have been dynamic considerations.[11] Our results suggest that there might be a high payoff to analyzing models that are perhaps simpler along these dimensions, but include a more realistic description of market behavior. For example, it could be instructive to fit a basic conjectural variations model to industry data.[12] The estimated coefficients then could be used to conduct positive and normative analyses of taxation along the lines suggested here. "Anything can happen" is not an excuse for ignoring the empirically important case of oligopoly in the study of tax policy.

## Footnotes

[1]See, for example, the discussion of tax theory in Tresch [1981].

[2]The conjectural variations model is discussed by Bresnahan [1981] and Seade [1980].

[3]Asymmetric equilibria may arise in cases where a firm's profit function achieves its maximum at two distinct output levels. Hence, under the conventional assumption that each firm's profit function is strictly concave, only symmetric equilibria will exist.

[4]Assuming that costs are convex.

[5]Hereafter for the sake of clarity, we will suppress the arguments of the inverse demand and cost functions where there is no ambiguity.

[6]A similar cost function can arise when only a subset of factors is taxed.

[7]See, e.g., Harberger [1974, pp. 160-162].

[8]It is assumed throughout that the structure of demand is such that consumer surplus measures provide good approximations to welfare changes.

[9]Even more sophisticated methods for estimating the elasticity of the market demand curve can be expected to lead to incorrect estimates if the underlying theoretical model is misspecified.

[10]Of course, this means that he must ignore the fact that eq. (4.2) is no longer satisfied.

[11] For some examples of the former, see Fullerton, Shoven, and Whalley [1978]. For the latter, see Feldstein [1974].

[12] See, for example, Gollop and Roberts [1979].

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