NBER WORKING PAPER SERIES

BUSINESS CYCLE ACCOUNTING

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Working Paper 10351
http://www.nber.org/papers/w10351

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 2004

The authors thank the National Science Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.

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JEL No. E1, E12, E13, E27, E3, E32, E37, E44

ABSTRACT

We propose and demonstrate a simple method for guiding researchers in developing quantitative models of economic fluctuations. We show that a large class of models are equivalent to a prototype growth model with time-varying wedges that resemble time-varying productivity, labor taxes, and capital income taxes. We use data to measure these wedges, called efficiency, labor, and investment wedges, and then feed their measured values back into the model. We assess the fraction of fluctuations in output, employment, and investment accounted for by these wedges during the Great Depression and the 1982 recession. For the Depression, the efficiency and labor wedges together account for essentially all of the fluctuations; investment wedges play no role. For the recession, the efficiency wedge plays the most important role; the other two, minor roles. These results are not sensitive to alternative measures of capital utilization or alternative labor supply elasticities.

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We propose and demonstrate the use of a simple method for guiding researchers in developing quantitative models of economic fluctuations. Our method has two components: an equivalence result and an accounting procedure.

The equivalence result is that a large class of models, including models with various frictions, are equivalent to a prototype growth model with time-varying wedges which, at least at face value, look like time-varying productivity, labor taxes, and capital income taxes. For example, we show that an economy in which the technology is constant but input-financing frictions vary over time is equivalent to a growth model with time-varying productivity. We show that models with sticky wages and monetary shocks or unions and antitrust policy shocks are equivalent to a growth model with time-varying labor taxes, and a model with investment-financing frictions and wealth redistribution shocks is equivalent to a growth model with time-varying capital income taxes. These examples lead us to label the time-varying wedges *efficiency wedges*, *labor wedges*, and *investment wedges*.

Our accounting procedure begins by using data together with the equilibrium conditions of a prototype growth model to measure the wedges. We then feed the values of these wedges back into the growth model, one at a time and in combinations, to assess what fraction of the output movements can be attributed to each wedge separately and in combinations. By construction, all three wedges account for all of the observed movements in output. In this sense, our method is an accounting procedure.

We demonstrate the usefulness of our method by applying it to two actual U.S. business cycle episodes: the most extreme in U.S. history—the Great Depression—and a downturn less severe and more like those seen often since World War II–the 1982 recession. During the Great Depression, output, labor, and investment declined dramatically in the early 1930s. The ensuing recovery was slow, so that even by 1939, output was well below trend. The slowness of the recovery was especially marked for labor, which in 1939 was still at its 1933 level. Our accounting shows that the efficiency wedge alone accounts for roughly two-thirds of the decline in output and about one-third of the decline in labor from 1929 to 1933, but this wedge cannot account for the sluggish recovery in either output or labor. The labor wedge alone accounts for much of the fall in labor but can only account for about one-half of the fall in output from 1929 to 1933. In terms of the recovery, the labor wedge accounts for essentially all the sluggishness in labor and the failure of output to return to trend. In combination, the efficiency and labor wedges account for all of the fall in output, labor, and investment from 1929 to 1933 and the behavior of these variables in the recovery. The investment wedge actually drives output the wrong way, that is, it leads to an increase in output during much of the 1930s. Thus, this wedge cannot account for either the downturn or the slow recovery.

For the more typical U.S. recession in 1982, we find that the efficiency wedge alone accounts for most of the decline and recovery in output, but misses some of the downturn in labor. The labor wedge alone produces hardly any fluctuations in output, but captures some of the downturn in labor. Together these two wedges capture the downturn in output well, though they produce a sharper recovery than in the data. The investment wedge is unchanged early in this episode and then steadily worsens, even through the recovery. Relative to the Great Depression, we find that the labor wedge plays a much smaller role in the 1982 recession, and the worsening of the investment wedge helps account for the modest nature of the recovery. The investment wedge plays only a bit larger role here than in the Depression.

We ask whether our results are sensitive to our assumptions about capital utilization rates and labor supply elasticities. In our benchmark model, we assume that the capital utilization rate is fixed, and we use labor supply elasticities similar to those in the business cycle literature. We then investigate what happens when we allow for either variable capital utilization or less elastic labor supply. We find that the size of our measured wedges changes substantially, but not the equilibrium responses to the wedges. The lesson we draw from this finding is that focusing on the size of the measured wedges rather than the equilibrium responses can mislead researchers about the quantitative importance of competing mechanisms of business cycles.

This application of our accounting procedure decomposes business cycle movements along a given realization. We also investigate a complementary spectral decomposition based on the population properties of the model's stochastic process. The results with this spectral decomposition match those of the initial decomposition: the investment wedge plays a minor role in the prewar period and a modest role in the postwar period.

The goal of this business cycle accounting is to guide researchers in developing detailed models with the kinds of frictions that can deliver the quantitatively relevant types of observed wedges in the prototype economy. For example, our method suggests that both the sticky wage and cartelization theories are promising explanations of the observed labor wedges, while the simplest models of investment financing frictions are not. Theorists attempting to develop models of particular channels through which shocks cause large fluctuations in output will benefit from asking whether those channels are consistent with the fluctuations in wedges that we document.

We emphasize that we view our method as a useful first step in guiding the construction of detailed models. In building detailed models, theorists face hard choices on where to introduce frictions into markets. Our method is intended to help make those choices; it is not a way to test particular detailed models. If a detailed model is already at hand, then it makes sense to confront that model directly with the data.

We also emphasize that our method is not well suited to identify the source of primitive shocks. It is intended to help understand the mechanisms through which such shocks lead to economic fluctuations. For example, many economists think that monetary shocks drove the U.S. Great Depression, but economists disagree about the details of the mechanism. Bernanke (1983) argues that financial frictions play a central role, and in the Bernanke and Gertler (1989) model, these frictions show up as investment wedges. In the model of Bordo, Erceg, and Evans (2000), sticky nominal wages play a central role, and these frictions show up as labor wedges. In our work here, we develop a model entirely consistent with the views of Bernanke (1983), but in which financial frictions show up as efficiency wedges. The model could be extended to have monetary shocks as the primitive source of fluctuations in these frictions. Our findings for the Great Depression suggest that, to the extent that monetary shocks drove the Depression, either the sticky wage mechanism of Bordo, Erceg, and Evans (2000) or a monetary version of the financial friction mechanism that we develop is more promising than the mechanism of Bernanke and Gertler (1989).

Other economists, like Cole and Ohanian (1999) and Prescott (1999), argue that non-monetary government policies played an important role in the Great Depression, especially in the slow recovery. Cole and Ohanian (2001b) develop a model in which government-sanctioned increases in the power of unions and cartels lead to labor wedges. Alternative models can easily be developed in which poor government policies lead to efficiency or investment wedges. However, our findings suggest that only models which emphasize the role of efficiency and labor wedges are potentially promising.

Our work is related to the vast business cycle literature that we discuss in detail near the end of this study. Here we highlight some of this literature. In terms of measuring the efficiency wedge, we follow Solow (1957). In terms of measuring the labor wedge, we follow Rotemberg and Woodford (1992), Hall (1997), and Mulligan (2002b). In particular, Hall (1997) plots the measured labor wedge for U.S. postwar data, and Mulligan (2002b) plots this wedge for the entire 20th century. In the business cycle literature, the basic idea of feeding back measured wedges into models to assess their quantitative importance stems from Prescott (1986).

1. Equivalence Results

Here we show how various detailed models with underlying distortions can be viewed as equivalent to a prototype economy with one or more wedges. We choose simple models to illustrate how the detailed models map into the prototypes. Since many models map into the same configuration of wedges, identifying one particular configuration does not uniquely identify a model; rather, it identifies a whole class of models consistent with that configuration. In this sense, our method does not uniquely determine the model most promising to analyze business cycle fluctuations; rather, it guides researchers to focus on the key margins that need to be distorted in order to capture the nature of the fluctuations.

1.1. The Benchmark Prototype Economy

The *benchmark* prototype economy that we use later in our accounting procedure is a growth model with three stochastic variables: the *efficiency wedge* A_t, the *labor wedge* $1-\tau_{lt}$, and the *investment wedge* $1/(1+\tau_{xt})$. Consumers maximize expected utility over consumption c_t and labor l_t,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to the budget constraint

$$c_t + (1 + \tau_{xt})[k_{t+1} - (1 - \delta)k_t] = (1 - \tau_{lt})w_t l_t + r_t k_t + T_t$$

where k_t denotes the capital stock, w_t the wage rate, r_t the rental rate on capital, β the discount factor, δ the depreciation rate of capital, and T_t lump-sum taxes.

The firms' production function is $F(k_t, \gamma^t l_t)$, where γ^t is labor-augmenting technical progress that is assumed to grow at a constant rate. Firms maximize $A_t F(k_t, \gamma^t l_t) - r_t k_t - w_t l_t$. The equilibrium is summarized by the resource constraint,

$$c_t + g_t + k_{t+1} = y_t + (1 - \delta)k_t \tag{1}$$

where y_t and g_t denote aggregate output and government consumption, together with

$$y_t = A_t F(k_t, \gamma^t l_t) \tag{2}$$

$$-\frac{U_{lt}}{U_{ct}} = (1 - \tau_{lt})A_t \gamma^t F_{lt} \tag{3}$$

$$U_{ct}(1 + \tau_{xt}) = \beta E_t U_{ct+1}[A_{t+1}F_{kt+1} + (1 - \delta)(1 + \tau_{xt+1})] \tag{4}$$

where, here and throughout, we use notation like U_{ct}, U_{lt}, F_{lt} and F_{kt} to denote the derivatives of the utility function and the production function with respect to their arguments. We assume that g_t fluctuates around a trend of γ^t.

Notice that the efficiency wedge resembles the productivity parameter and that the labor wedge and the investment wedge resemble tax rates on labor income and investment, respectively. One could consider more elaborate models with other kinds of frictions that look like taxes on consumption or on capital income. Consumption taxes induce a wedge between the consumption-leisure marginal rate of substitution and the marginal product of labor in exactly the same way as do labor taxes. Such taxes, if time-varying, also distort the intertemporal margins in (4). Capital income taxes induce a wedge between the intertemporal marginal rate of substitution and the marginal product of capital which is only slightly different from that induced by a tax on investment.

We illustrate the map between detailed economies and prototype economies for efficiency wedges and labor wedges in the next two sections. For the labor wedges we focus on an economy with sticky wages and in the appendix we demonstrate a similar map for an economy with unions. In the appendix we also demonstrate the map for investment wedges for an economy with financial frictions.

1.2. Efficiency Wedges

Here we develop a detailed economy with input-financing frictions and show that it maps into a prototype economy with an efficiency wedge. In the detailed economy, financing frictions lead to some firms having to finance working capital requirements at higher interest rates than other firms. These frictions lead to a misallocation of inputs across firms. We show that this misallocation of inputs can manifest itself in the prototype economy as an efficiency wedge. For some related work on how frictions can manifest themselves as efficiency wedges see Lagos (2001).

We focus on a stripped-down example which illustrates a more general point. In many economies, underlying frictions either within firms or across firms cause factor inputs to be utilized in an inefficient manner. These frictions in an underlying economy often show up as aggregate productivity shocks in a prototype economy similar to our benchmark. Schmitz (2001) presents an interesting example of within-firm frictions resulting from work rules that lower measured productivity at the firm level.

a. A Detailed Economy With Input-Financing Frictions

Consider a simple detailed economy with distortions in the allocation of intermediate inputs across two types of firms arising from financing frictions. Both types of firms must borrow in order to pay for an intermediate input, before they can produce. One type of firm is financially constrained in the sense that it pays a higher price for borrowing than the other type. We think of these frictions as capturing the idea that some firms, namely, small firms, find it difficult to finance borrowing. One motivation for the higher price paid by the financially constrained firms is that moral hazard problems are more severe for small firms.

Specifically, consider the following economy. Aggregate gross output q_t is made from combining the gross output q_{it} from two sectors, indexed $i = 1, 2$, according to

$$q_t = q_{1t}^{\phi} q_{2t}^{1-\phi}. \tag{5}$$

The representative producer of the gross output q_t chooses q_{1t} and q_{2t} to solve

$$\max \; q_t - p_{1t}q_{1t} - p_{2t}q_{2t}$$

subject to (5), where p_{it} is the price of the output of sector i.

The resource constraint for gross output is

$$c_t + k_{t+1} + m_{1t} + m_{2t} = q_t + (1-\delta)k_t \tag{6}$$

where c_t is consumption, k_t is the capital stock, and m_{1t} and m_{2t} are intermediate goods used in sectors 1 and 2, respectively. Final output, given by $y_t = q_t - m_{1t} - m_{2t}$, is gross output less the use of intermediate goods.

The gross output of sector i, q_{it}, is made from intermediate goods m_{it} and a composite value-added good z_{it} according to

$$q_{it} = m_{it}^{\theta} z_{it}^{1-\theta} \tag{7}$$

where the composite value-added good is produced from capital k_t and labor l_t according to

$$z_{1t} + z_{2t} = z_t = F(k_t, l_t). \tag{8}$$

The producer of gross output of sector i chooses the composite good z_{it} and the intermediate good m_{it} to solve

$$\max \; p_{it}q_{it} - v_t z_{it} - R_{it}m_{it}$$

subject to (7). Here v_t is the price of the composite good and R_{it} is the gross *within-period* interest rate paid on borrowing by firms in sector i. We imagine that firms in sector 1 are more financially constrained than those in sector 2 in that $R_{1t} > R_{2t}$. Let $R_{it} = R_t(1 + \tau_{it})$, where R_t is the rate savers earn within period t and τ_{it} measures the within-period spread between the rate paid to savers and the rate paid by borrowers in sector i induced by financing constraints. Since consumers do not discount utility within the period, $R_t = 1$.

The producer of the composite good z_t chooses k_t and l_t to solve

$$\max \; v_t z_t - w_t l_t - r_t k_t$$

subject to (8), where w_t is the wage rate and r_t is the rental rate on capital.

Consumers solve

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) \tag{9}$$

subject to

$$c_t + k_{t+1} = r_t k_t + w_t l_t + (1 - \delta)k_t + T_t$$

where $l_t = l_{1t} + l_{2t}$ is labor supply and $T_t = R_t \sum_i \tau_{it} m_{it}$ are lump-sum transfers. Here we assume that the financing frictions act like distorting taxes and the proceeds are rebated to consumers. If instead we assumed that the financing frictions represent, say, lost gross output, then we would adjust the resource constraint (6) accordingly.

b. The Associated Prototype Economy With Efficiency Wedges

Now consider a version of the benchmark prototype economy that will have the same aggregate allocations as our input-financing frictions economy. This prototype economy is identical to our benchmark prototype except that we have taxes on capital income rather than taxes on investment and we set government consumption to zero. Here the consumer's budget constraint is

$$c_t + k_{t+1} = (1-\tau_{kt})r_t k_t + (1-\tau_{lt})w_t l_t + (1-\delta)k_t + T_t \tag{10}$$

and the efficiency wedge is given by

$$A_t = \kappa(a_{1t}^{1-\phi}a_{2t}^{\phi})^{\frac{\theta}{1-\theta}}\big(1-\theta(a_{1t}+a_{2t})\big) \tag{11}$$

where $a_{1t} = \phi/(1+\tau_{1t})$, $a_{2t} = (1-\phi)/(1+\tau_{2t})$, $\kappa = \phi^{\phi}(1-\phi)^{1-\phi}\theta^{\frac{\theta}{1-\theta}}$, and τ_{1t} and τ_{2t} are the interest rate spreads in the detailed economy. The following proposition follows immediately from comparing the first-order conditions in the detailed economy with input-financing frictions to those of the associated prototype economy with efficiency wedges.

Proposition 1. Consider the prototype economy with resource constraint (**??**) and budget constraint (10) with exogenous processes the efficiency wedge A_t given in (11),

$$\frac{1}{1-\tau_{lt}} = \frac{1}{1-\theta}\left[1-\theta\left(\frac{\phi}{1+\tau_{1t}}+\frac{1-\phi}{1+\tau_{2t}}\right)\right] \tag{12}$$

and $\tau_{kt} = \tau_{lt}$. Then the equilibrium allocations in this prototype economy coincide with those in the detailed economy with input-financing frictions.

Imagine that in the economy with input-financing frictions, τ_{1t} and τ_{2t} fluctuate over time but in such a way that the weighted average of the interest rate spreads

$$a_{1t} + a_{2t} = \frac{\phi}{1+\tau_{1t}} + \frac{1-\phi}{1+\tau_{2t}} \tag{13}$$

is constant but $a_{1t}^{1-\phi}a_{2t}^{\phi}$ fluctuates. Then from (12) we see that the labor and investment wedges are constant, and from (11) we see that the efficiency wedge fluctuates. Thus, on average, financing frictions are unchanged, but relative frictions fluctuate. An outside observer who attempted to fit the data generated by the detailed economy with input-financing frictions using the prototype economy would identify the fluctuations in relative distortions with fluctuations in technology and would see no fluctuations in either the labor wedge $1 - \tau_{lt}$ or the investment wedge τ_{kt}. In particular, periods in which the relative distortions increase would be misinterpreted as periods of technological regress. This observation leads us to label A_t the *efficiency wedge* in the prototype economy.

More generally, fluctuations in the interest rate spreads τ_{1t} and τ_{2t} which lead to fluctuations in τ_{lt} and τ_{kt} show up in the prototype economy as fluctuations in all of the wedges.

1.3. Labor Wedges

We turn now to economies with distortions in the labor market. Here we will show that a sticky-wage economy will map into the prototype economy with labor wedges.

Consider a monetary economy populated by a large number of identical, infinitely lived consumers. In each period t, the economy experiences one of finitely many events s_t, which index the shocks. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t. The probability, as of period 0, of any particular history s^t is $\pi(s^t)$. The initial realization s_0 is given. The economy consists of a competitive final goods producer and a continuum of monopolistically competitive unions that set their nominal wages in advance of the realization of the shocks. Each union represents all consumers with a specific type of labor.

In each period t, the commodities in this economy are a consumption-capital good, money, and a continuum of differentiated types of labor indexed by $j \in [0, 1]$. The technology for producing final goods from capital and a labor aggregate at history s^t is constant returns to scale and is given by

$$y(s^t) = F\big(k(s^{t-1}), l(s^t)\big) \tag{14}$$

where $y(s^t)$ is output of the final good, $k(s^{t-1})$ is capital, and

$$l(s^t) = \left[\int l(j, s^t)^v \, dj\right]^{\frac{1}{v}} \tag{15}$$

is an aggregate of the differentiated types of labor $l(j, s^t)$.

The final goods producer behaves competitively. This producer has some initial capital stock $k(s^{-1})$ and accumulates capital according to

$$k(s^t) = (1 - \delta)k(s^{t-1}) + x(s^t) \tag{16}$$

where $x(s^t)$ is investment. The present discounted value of profits for this producer is

$$\sum_{t=0}^{\infty} Q(s^t) \left[P(s^t)y(s^t) - P(s^t)x(s^t) - W(s^{t-1})l(s^t)\right] \tag{17}$$

where $Q(s^t)$ is the price of a dollar at s^t in an abstract unit of account, $P(s^t)$ is the dollar price of final goods at s^t, and $W(s^{t-1})$ is the aggregate nominal wage at s^t which only depends on s^{t-1} because of wage stickiness. The producer's problem can be stated in two parts. First, the producer chooses sequences for capital $k(s^{t-1})$, investment $x(s^t)$, and aggregate labor $l(s^t)$, to maximize (17) subject to (14) and (16). The first-order conditions can be summarized by

$$P(s^t)F_l(s^t) = W(s^{t-1}) \tag{18}$$

$$Q(s^t)P(s^t) = \sum_{s_{t+1}} Q(s^{t+1})P(s^{t+1})[F_k(s^{t+1}) + 1 - \delta]. \tag{19}$$

Second, for any given amount of aggregate labor $l(s^t)$, the demand for each type of differentiated labor is given by the solution to

$$\min_{\{l(j,s^t)\}, j \in [0,1]} \int W(j, s^{t-1})l(j, s^t) \, dj \tag{20}$$

subject to (15), where $W(j, s^{t-1})$ is the nominal wage for differentiated labor of type j. Nominal wages are set by unions before the realization of the event in period t; thus, they can depend on, at most, s^{t-1}. The demand for labor of type j by the final goods producer is

$$l^d(j, s^t) = \left(\frac{W(s^{t-1})}{W(j, s^{t-1})}\right)^{\frac{1}{1-v}} l(s^t) \tag{21}$$

where $W(s^{t-1}) \equiv \left[\int W(j,s^{t-1})^{\frac{v}{v-1}}\,dj\right]^{\frac{v-1}{v}}$ is the aggregate nominal wage. The minimized value in (20) is thus $W(s^{t-1})l(s^t)$.

Consumers can be thought of as being organized into a continuum of unions indexed by j. Each union consists of all the consumers in the economy with labor of type j. Each union realizes that it faces a downward-sloping demand for its type of labor, given by (21). In each period, these new wages are set before the realization of the current shocks.

The preferences of a representative consumer in the jth union is

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)\,U\left(c(j,s^t),l(j,s^t),M(j,s^t)/P(s^t)\right) \tag{22}$$

where $c(j,s^t), l(j,s^t), M(j,s^t)$ are the consumption, labor supply, and money holdings of this consumer and $P(s^t)$ the price level. This economy has complete markets for state-contingent nominal claims. We represent the asset structure by having complete, contingent, one-period nominal bonds. We let $B(j,s^{t+1})$ denote the consumers' holdings of such a bond purchased in period t and with history s^t with payoffs contingent on some particular event s_{t+1} in $t+1$, where $s^{t+1} = (s^t, s_{t+1})$. One unit of this bond pays one dollar in period $t+1$ if the particular event s_{t+1} occurs and 0 otherwise. Let $Q(s^{t+1}|s^t)$ denote the dollar price of this bond in period t and at history s^t. Clearly, $Q(s^{t+1}|s^t) = Q(s^{t+1})/Q(s^t)$.

The problem of the jth union is to maximize (22) subject to the budget constraints

$$\begin{aligned} P(s^t)c(j,s^t) + M(j,s^t) + \sum_{s_{t+1}} Q(s^{t+1}|s^t)B(j,s^{t+1}) \\ \leq W(j,s^{t-1})l^d(j,s^t) + M(j,s^{t-1}) + B(j,s^t) + T(s^t) + D(s^t) \end{aligned}$$

and the borrowing constraint $B(s^{t+1}) \geq -P(s^t)\bar{b}$, where $l^d(j,s^t)$ is given by (21). Here $T(s^t)$ is transfers and the positive constant $\bar{b}$ constrains the amount of real borrowing of the consumer. Also, $D(s^t) = P(s^t)y(s^t) - P(s^t)x(s^t) - W(s^{t-1})l(s^t)$ are the dividends paid by the firms. The initial conditions $M(j,s^{-1})$ and $B(j,s^0)$ are given and assumed to be the same for all j. Notice that in this problem, the union chooses the wage and agrees to supply whatever is demanded at that wage.

The first-order conditions for this problem can be summarized by

$$\frac{U_m(j,s^t)}{P(s^t)} - \frac{U_c(j,s^t)}{P(s^t)} + \beta \sum_{s_{t+1}} \pi(s^{t+1}|s^t) \frac{U_c(j,s^{t+1})}{P(s^{t+1})} = 0 \tag{23}$$

$$Q(s^t|s^{t-1}) = \beta\pi(s^t|s^{t-1}) \frac{U_c(j,s^t)}{U_c(j,s^{t-1})} \frac{P(s^{t-1})}{P(s^t)} \tag{24}$$

$$W(j,s^{t-1}) = -\frac{\sum_{s^t} Q(s^t)P(s^t)U_l(j,s^t)/U_c(j,s^t)l^d(j,s^t)}{v\sum_{s^t} Q(s^t)l^d(j,s^t)}. \tag{25}$$

Here $\pi(s^{t+1}|s^t) = \pi(s^{t+1})/\pi(s^t)$ is the conditional probability of s^{t+1} given. Notice that in a steady state, this condition reduces to $W/P = (1/v)(-U_l/U_c)$, so that real wages are set as a markup over the marginal rate of substitution between labor and consumption. Clearly, given the symmetry among the unions, we know that all of them choose the same consumption, labor, money balances, bond holdings, and wages, which we denote simply by $c(s^t)$, $l(s^t)$, $M(s^t)$, $B(s^{t+1})$, and $W(s^{t-1})$.

Consider next the specification of the money supply process and the market-clearing conditions. The nominal money supply process is given by $M(s^t) = \mu(s^t)M(s^{t-1})$, where $\mu(s^t)$ is a stochastic process. New money balances are distributed to consumers in a lump-sum fashion by having nominal transfers satisfy $T(s^t) = M(s^t) - M(s^{t-1})$. The resource constraint for this economy is

$$c(s^t) + k(s^t) = y(s^t) + (1-\delta)k(s^{t-1}). \tag{26}$$

Bond market–clearing requires that $B(s^{t+1}) = 0$.

a. The Associated Prototype Economy With Labor Wedges

Consider now a prototype economy with money and labor wedges and a technology given by (14). The representative firm maximizes (17) subject to (16). The first-order conditions can be summarized by (18) and (19). The representative consumer maximizes

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)\, U\left(c(s^t), l(s^t), M(s^t)/P(s^t)\right) \tag{27}$$

subject to the budget constraint

$$P(s^t)c(s^t) + M(s^t) + \sum_{s_{t+1}} Q(s^{t+1}|s^t)B(s^{t+1})$$
$$\le W(s^t)[1 - \tau_l(s^t)]l(s^t) + M(s^{t-1}) + B(s^t) + T(s^t) + D(s^t)$$

and a bound on bond holdings, where the lump-sum transfer $T(s^t) = M(s^t) - M(s^{t-1}) + \tau_l(s^t)l(s^t)$ and the dividends $D(s^t) = P(s^t)y(s^t) - P(s^t)x(s^t) - W(s^{t-1})l(s^t)$. Here the first-order conditions for money and bonds are identical to those in (23) and (24) once symmetry has been imposed in them. The first-order condition for labor is given by

$$-\frac{U_l(s^t)}{U_c(s^t)} = [1 - \tau_l(s^t)]\frac{W(s^t)}{P(s^t)}.$$

Consider an equilibrium of the sticky wage economy for some given stochastic process $M^*(s^t)$ on money growth. Denote all of the allocations and prices in this equilibrium with asterisks. Then we can easily establish this proposition:

Proposition 2. Consider the prototype economy just described with a given stochastic process for money growth $M(s^t) = M^*(s^t)$ and labor wedges given by

$$1 - \tau_l(s^t) = -\frac{U_l^*(s^t)}{U_c^*(s^t)} F_l^*(s^t) \tag{28}$$

where $U_l^*(s^t), U_c^*(s^t)$, and $F_l^*(s^t)$ are evaluated at the equilibrium of the sticky wage economy. Then the equilibrium allocations and prices in the sticky wage economy coincide with those in the prototype economy.

The proof of this proposition is immediate from comparing the first-order conditions, the budget constraints, and the resource constraints for the prototype economy with money and labor wedges to those of the sticky wage economy. The key idea is that distortions between the marginal rate of substitution between leisure and consumption and the marginal product of labor implicit in (25) for the sticky wage economy are perfectly captured by the labor wedge (28) in the prototype economy.

Suppose next that the utility function of consumers in the sticky wage economy is additively separable in money, so that $U(c,l,m) = u(c,l) + v(m)$. Consider a real version of the prototype economy with labor wedges. Let the utility function be

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)\, u\left(c(s^t), l(s^t)\right) \tag{29}$$

and the technology be the same as in the monetary prototype economy. Define the rest of the economy in the obvious way. The following is immediate:

Corollary 1. Consider the real prototype economy just described with a given stochastic process for labor wedges

$$1 - \tau_l(s^t) = -\frac{u_l^*(s^t)}{u_c^*(s^t)} F_l^*(s^t)$$

where $u_l^*(s^t), u_c^*(s^t)$, and $F_l^*(s^t)$ are evaluated at the equilibrium of the sticky wage economy with preferences of the form (29). Then the equilibrium allocations in the sticky wage economy coincide with those in the real prototype economy.

2. Applying the Accounting Procedure

We now describe our accounting procedure and demonstrate how to use it for the Great Depression and the postwar recession of 1982.

2.1. The Procedure

Our accounting procedure works as follows. We choose our benchmark prototype model's parameters of preferences and technology in standard ways, as in the quantitative business cycle literature, and then use the equilibrium conditions of our prototype economy to estimate the parameters of a stochastic process for the wedges and government consumption. This collection of parameters implies decision rules for output, labor, and investment which can be used with the data to uncover both a stochastic process for the wedges as well as the wedges realized in the data.

We then ask, What fraction of output fluctuations can be accounted for by each of the wedges separately and in various combinations? To answer this question, we simulate our prototype model using the realized sequence of wedges in the data to assess separately and in combinations the contribution of the wedges to fluctuations in output, labor, and investment. The contribution of these wedges is measured by comparing the realizations of variables like output, labor, and investment from the model to the data on these variables. Our approach is an accounting procedure since, by construction, the three wedges together, along with government consumption, account for all of the movements in the variables.

a. Measuring the Wedges

Our process for measuring the wedges has two steps. We use both the data and the models to first estimate the stochastic process for the wedges and then to measure the realized wedges. Throughout, we use annual U.S. data (for 1901–40 and 1955–2000, excluding the war years). Given data on investment x_t and an initial choice of capital stock k_0, we construct a series for the capital stock using the capital accumulation equation $k_{t+1} = (1-\delta)k_t + x_t$. We also adjust output and its components to remove sales taxes and military compensation and to add the service flow for consumer durables. (In a technical appendix, available on our website, we describe our data sources, computational methods, and estimation procedures in detail.)

□ *Estimating the Stochastic Process for the Wedges*

The first step in the measurement process is to estimate the stochastic process for the wedges. To do that, we use functional forms and parameter values familiar from the business cycle literature. We assume that the production function has the form $F(k,l) = k^{\alpha}l^{1-\alpha}$ and the utility function has the form $U(c,l) = \log c + \psi \log(\bar{l} - l)$. We choose the capital share $\alpha = .35$, the depreciation rate $\delta = .046$, the discount factor $\beta = .97$, the time allocation parameter $\psi = 2.24$, and the endowment of time $\bar{l} = 5,000$ hours per year.

Equations (1)–(4) summarize the equilibrium of the benchmark prototype economy. We substitute for consumption c_t in (3) and (4) using the resource constraint (1) and then log-linearize (2)–(4) to obtain three linear equations. We specify a vector autoregressive (AR1) process for the (demeaned) four wedges $s_t = (\log A_t, \tau_{lt}, \tau_{xt}, \log g_t)$ of the form

$$s_{t+1} = P_0 + P s_t + Q\eta_{t+1} \tag{30}$$

where η_t is standard normal and i.i.d. and Q is a lower-triangular matrix. Here and throughout we refer to government consumption as a *wedge*. We then have seven equations, three from the equilibrium and four from (30).

We then use the maximum likelihood procedure described in McGrattan (1994) to estimate the parameters P_0, P, and Q of the vector AR1 process for the wedges using data on output, labor, investment, and government consumption. We estimate separate sets of parameters for the two periods we analyze. The parameters for the Great Depression analysis are estimated using data for 1901–1940; those for postwar analysis, using data for 1955–2000. In the Great Depression analysis, we impose the additional restriction that the covariance between the innovations to government consumption and to the other wedges is zero. We impose this restriction to avoid having the large movements in government consumption associated with World War I dominate the estimation of the stochastic process.

Table 1 displays the resulting estimated parameter values for P and Q and the associated standard errors for our two periods. The resulting stochastic process (30) will be used by agents in our economy to form their expectations about future wedges.

□ *Measuring the Realized Wedges*

The second step in our measurement procedure is to measure the realized wedges. We take the government consumption wedge directly from the data. To obtain the values of the other three wedges, we use the data for y_t, l_t, x_t, and g_t (together with a series on k_t constructed from x_t) and the model's decision rules. That is, with y_t^d, l_t^d, x_t^d, and k_0^d denoting the data, and $y(s_t, k_t),\ l(s_t, k_t),\ x(s_t, k_t)$ denoting the nonlinear decision rules of the model, the realized wedge series s_t^d solves

$$y_t^d = y(s_t^d, k_t), l_t^d = l(s_t^d, k_t), x_t^d = x(s_t^d, k_t) \tag{31}$$

where $k_{t+1} = (1-\delta)k_t + x_t^d$ and $k_0 = k_0^d$. In effect, we solve for the three unknown elements of the vector s_t using the three equations in (2)–(4). The nonlinear solution method is described in McGrattan (1996). We use these values for the wedges in our experiments.

Note that, in order to measure the efficiency and labor wedges, we do not need to compute the decision rules. These wedges can be directly calculated from (2) and (3). The investment wedge cannot be directly calculated from (4) because that requires specifying expectations over future values of consumption, the capital stock, the wedges, and so on. The decision rules from our model implicitly depend on these expectations and therefore on the stochastic process driving the wedges. Thus, the estimated stochastic process plays a role in measuring only the investment wedge.

b. The Decomposition

We use the model's measured realizations to decompose movements in variables from an initial date (either 1929 or 1979), with an initial capital stock into the four components consisting of movements driven by each of the four wedges away from their values at the initial date. We construct these components as follows.

Define the *efficiency component* of the wedges by letting $s_{1t} = (\log A_t, \tau_{l0}, \tau_{x0}, \log g_0)$ be the vector of wedges in which, in period t, the efficiency wedge takes on its period t value while the other wedges take on their initial values. Define the other components of the wedges—the *labor component* s_{2t}, the *investment component* s_{3t}, and the *government consumption component* s_{4t}—analogously.

Define the capital stock due to component i, for $i = 1, \ldots, 4$, by $k_{it+1} = k(s_{it}, k_{it})$. Given the capital stock components, we define the output components due to wedge i by $y_{it} = y(k_{it}, s_{it})$ for $i = 1, \ldots, 4$, and we construct the labor and investment components similarly.

We also construct joint components. Define the *efficiency plus labor component* by letting $s_{5t} = (\log A_t, \tau_{lt}, \tau_{x0}, \log g_0)$, and define the other joint components similarly.

2.2. Accounting Findings

Now we describe the results of applying our accounting procedure to our two selected historical periods. In the data we remove a trend of 1.6% from output, investment, and government consumption. Both output and labor are normalized to equal 100 in the base years: 1929 for the Great Depression and 1979 for the 1982 recession. In both episodes, investment (detrended) is divided by the base year level of output. We have determined (and shown in the technical appendix) that in both episodes, the government consumption component accounts for an insignificant fraction of the fluctuations in output, labor, and investment. Thus, here we focus on the fractions due to the efficiency, labor, and investment wedges.

a. The Great Depression

We begin with our findings for the period 1929–1939, which includes the Great Depression.

In Figure 1, we display actual output and the three measured wedges for that period: the efficiency wedge A, the labor wedge $(1-\tau_l)$, and the investment wedge $1/(1+\tau_x)$. We see in the figure that in 1933 output is 36% below trend and by 1939 is still 22% below. From 1929 to 1933, the efficiency wedge falls 19%, but by 1939 it is back to trend. The labor wedge worsens 26% from 1929 to 1933, and by 1939 it is still 29% below its 1929 level. The investment wedge fluctuates somewhat, but note that throughout the period from 1929 to 1939, it is either essentially at or above its 1929 level.

The underlying distortions that the three wedges reveal thus have different patterns. The distortions that manifest themselves as efficiency and labor wedges became substantially worse between 1929 and 1933. By 1939, the efficiency wedge has returned to trend level, but the labor wedge has worsened. Over the period, the investment wedge fluctuates, but investment decisions are generally less distorted between 1933 and 1939 than in 1929.

In Figure 2, we plot, among other variables, output, labor, and investment in the data. We see that labor declines 27% from 1929 to 1933 and stays relatively low for the rest of the decade. Investment also declines sharply from 1929 to 1933, but partially recovers by the end of the decade. Interestingly, in an algebraic sense, about half of output's 36% fall from 1929 to 1933 is due to the decline in investment.

In terms of the models, we start by assessing the separate contributions of the three wedges. In Figure 2, in addition to the data, we plot output, labor, and investment due to the efficiency wedge and the labor wedge. That is, we plot these variables when the efficiency component s_{1t} and the labor component s_{2t} are used for the wedges.

Consider the contribution of the efficiency wedge. In Figure 2, we see that predicted output declines less than the data and recovers more rapidly. For example, by 1933, predicted output falls about 25% while output itself falls about 36%. Thus, the efficiency wedge accounts for about two-thirds of the decline of output in the data. By 1939, predicted output is only 3% below trend rather than the observed 22%. As can also be seen in Figure 2, the reason for this rapid recovery in predicted is that predicted labor completely misses the continued sluggishness in labor in the data from 1933 onward. Predicted investment shows a fall similar to that in the data, but a faster recovery.

Consider next the contributions of the labor wedge. In this figure, we also see that by 1933, the predicted output due to the labor wedge falls only about half as much output falls in the data: 18% vs. 36%. By 1939, this predicted output completely captures the slow recovery: it falls 22%, exactly as output does in the data. The reason for this slow recovery is that predicted labor due to the labor wedge captures the sluggishness in labor after 1933 remarkably well. The associated prediction for labor completely misses the sharp decline in investment from 1929 to 1933.

Summarizing Figure 2, the efficiency wedge accounts for about two-thirds of the downturn but misses the slow recovery, while the labor wedge accounts for about one-half of the downturn and accounts for essentially all of the slow recovery.

Finally, consider the investment wedge. In Figure 3, we plot the contributions for output, labor, and investment due to the investment wedge along with the data. This figure demonstrates that the contributions from the investment wedge completely miss the observed movements in output, labor, and investment.

These figures suggest that the efficiency and labor wedges account for essentially all of the movements of output, labor, and investment in the Depression period and that the investment wedge accounts for almost none. In Figure 4, we confirm this suggestion. We plot the sum of the contributions from the efficiency, labor, and (insignificant) government consumption wedges (labeled *Model with No Investment Wedge*). As can be seen from the figure, essentially all the fluctuations in output, labor, and investment can be accounted for by movements in these wedges. For comparison, we also plot the sum of the contributions due to the labor, investment, and government consumption wedges (labeled *Model with No Efficiency Wedge*). Comparing Figures 2 and 4, we see that this sum is further from the data than the labor wedge component alone. These findings lead us to conclude that investment distortions played essentially no role in the Great Depression of the United States.

b. The 1982 Recession

Now we apply our accounting procedure to a more typical U.S. business cycle: the recession of 1982. We start by displaying the actual U.S. output over the business cycle period—here, 1979–85—along with the three measured wedges for that period. In Figure 5, we see that output falls 8% relative to trend from 1979 to 1982 and is still 2% below trend in 1985. We also see that the efficiency wedge falls from 1979 to 1982 and returns to trend by 1985. The labor wedge worsens slightly from 1979 to 1982 and improves substantially by 1985. The investment wedge, meanwhile, is essentially unchanged until 1981 and then steadily worsens. Note that this investment wedge pattern does not square with models of business cycles in which financial frictions worsen in downturns and improve in recoveries.

An analysis of the wedges separately for the 1979–85 period is in Figures 6 and 7. In Figure 6, we see that the efficiency wedge accounts for roughly three-quarters of the decline in output from 1979 to 1982, 6% vs. 8%, and accounts for much of the recovery as well. In contrast, the labor wedge accounts for little of the fluctuations. In Figure 7, we see that the investment wedge accounts for little of the decline in output from 1979 to 1982 and actually produces a continued decline in output after 1982 rather than the recovery seen in the data.

Now we examine how well a combination of wedges reproduces the data for the 1982 recession period. In Figure 8, we plot the sum of the efficiency, labor, and (insignificant) government consumption components of the movements in output, labor, and investment during 1979–85 (labeled *Model with No Investment Wedge*). In the output data this sum declines almost 7% by 1982 compared to 8%, but by 1985 shows a sharper recovery than the data. The sum of the labor, investment, and government components (labeled *Model with No Investment Wedge*) comes close to generating the observed values in the data in 1985, but fails to generate the dynamic patterns of recession and recovery. These findings suggest that distortions in investment played a modest role in the 1982 U.S. recession, primarily by slowing down the recovery.

c. In Sum

Overall, we find that the efficiency wedge plays a central role in both the historical business cycles we have examined. The labor wedge plays a major role in the slow recovery from the Great Depression, but little role in the 1982 recession period. The investment wedge plays no role in the Great Depression and only a modest role in the postwar period.

3. Alternative Specifications

Here we ask whether our results are substantially changed with some alternative specifications. We find they are not.

One question is the extent to which our findings are affected by the stochastic process driving the wedges. We have attempted a number of alternative specifications of that process, including perfect foresight. Our substantive findings were essentially unaffected by those changes.

In our accounting exercise, we have made two assumptions that could reasonably be conjectured as being important for our results. We assumed that the capital utilization rate is fixed and that preferences have a particular functional form, that is, logarithmic in both consumption and leisure. Some researchers have argued that capital utilization rates fluctuate systematically over the business cycle while others have argued that labor supply is less elastic than in our specification. If either of those arguments are correct, then our procedure mismeasures the wedges. If capital utilization rates fluctuate systematically, then our procedure mismeasures the efficiency wedge; if labor supply is less elastic than we have assumed, then our procedure mismeasures the labor wedge. In this section, we demonstrate that changing these assumptions—allowing for either variable capital utilization or less elastic labor supply—has little effect on our findings.

We establish these results both quantitatively and analytically. Both changes turn out to produce offsetting effects, leaving our results unchanged. Allowing for variable capital utilization reduces the variability of the efficiency wedge and increases that of the labor wedge. This change in the relative variability of these two wedges does change the relative amounts of the business cycle movements separately accounted for by these wedges. However, this change in relative variability has almost no effect on the sum of the contributions due to these two wedges and, thus, it also has essentially no effect on the amount of fluctuations accounted for by the investment wedge. As such allowing for variable capital utilization does not alter our conclusion that investment wedges play almost no role in the Great Depression or the 1982 recession.

Similarly, reducing the elasticity of the labor supply increases the variability of the labor wedge. But that increased variability of labor is offset by the reduced responsiveness to it, and the overall effect is minimal.

These findings suggest that the size of the measured wedges alone are not very informative for assessing competing business cycle models. The two examples in this section show that the equilibrium responses can be very similar even though the size of the wedges are very different. It should be easy to construct examples in which two models have similar-sized wedges but have very different equilibrium responses. The lesson we draw from these findings is that competing business cycle models should be assessed by the equilibrium responses to the wedges, not by the wedges alone.

3.1. Variable Capital Utilization

In considering an alternative specification of the technology which allows for variable capital utilization, we follow Kydland and Prescott (1988) and Hornstein and Prescott (1993) and assume that the production function is

$$y = A(kh)^{\alpha}(nh)^{1-\alpha} \tag{32}$$

where n is the number of workers employed and h, the length (or *hours*) of the workweek. The labor input is, then, $l = nh$.

In the data, we measure only the labor input l and the capital stock k. We do not directly measure h or n. One interpretation of the benchmark specification for the production function used earlier is that by using it we have assumed that all of the observed variation in measured labor input l is in the number of workers and that the workweek h is constant. Under this interpretation, our *fixed capital utilization* specification correctly measures the efficiency wedge (up to the constant h).

Here we investigate the opposite extreme: we assume that the number of workers n is constant and that all the variation in labor is from the workweek h. Under this *variable capital utilization* specification, the services of capital kh are proportional to the product of the stock k and labor input l, so that variations in labor input induce variations in the flow of capital services. Thus, the capital utilization rate is proportional to labor input l, and the efficiency wedge is proportional to y/k^{α}.

In Figure 9, we plot the efficiency wedges for the two specifications during the Great Depression period. Clearly, the efficiency wedge falls less and recovers to a higher level by 1939 when capital utilization is variable than when it is fixed. We do not plot either the labor wedge or the investment wedge because they are identical, up to a scale factor, in the two specifications.

In Figure 10, we plot the data and the efficiency and labor components for the 1930s. Comparing Figures 10 and 2, we see that with the remeasured efficiency wedge, the labor wedge plays a much larger role in accounting for the downturn and the slow recovery and the efficiency wedge plays a much smaller role. In Figure 11, we plot the three data series and the predictions of the model with just the investment wedge. We see that the investment wedge still accounts for none of the movements in the data. In Figure 12, we compare the contributions of the sum of the efficiency and labor wedges for the two specifications of capital utilization (fixed and variable). The figure shows that these contributions are very similar. We see that while remeasuring the efficiency wedge as we have changes the relative contributions of the two wedges, it has little effect on their combined contribution. Taking account of variable capital utilization thus does not change the basic result that in the Great Depression period efficiency and labor wedges played a central role and the investment wedge a minor role, at best.

This exercise suggests a more general result: allowing for variable capital utilization changes the size of the measured efficiency wedge but does not change equilibrium outcomes. Consider an economy which is identical to a deterministic version of our benchmark model except that the production function is given by $y = Ak^{\alpha}l^{\gamma}$. Note that setting $\gamma = 1 - \alpha$ yields our benchmark model, while setting $\gamma = 1$ yields the variable capital utilization model.

Now consider two economies $i = 1, 2$ with γ equal to γ_1 and γ_2, respectively, and the same initial capital stocks. For some given sequence of wedges $(A_{1t}, \tau_{l1t}, \tau_{x1t})$, let y_{1t}, c_{1t}, l_{1t}, and x_{1t} denote the resulting equilibrium outcomes in the economy with $\gamma = \gamma_1$. We then have the following proposition:

Proposition 3. If the sequence of wedges for economy 2 is given by $A_{2t} = A_{1t}l_{1t}^{(\gamma_1-\gamma_2)}$, $1-\tau_{l2t} = \gamma_1(1-\tau_{l1t})/\gamma_2$, and $\tau_{x2t} = \tau_{x1t}$, then the equilibrium outcomes y_{2t}, c_{2t}, l_{2t}, and x_{2t} for this economy coincide with the equilibrium outcomes y_{1t}, c_{1t}, l_{1t}, and x_{1t} for economy 1.

Proof. We prove this proposition by showing that the equilibrium conditions of economy 2 are satisfied at the equilibrium outcomes of economy 1. Since $y_{1t} = A_{1t}k_{1t}^{\alpha}l_{1t}^{\gamma_1}$, using the definition of A_{2t}, we have that $y_{1t} = A_{2t} k_{1t}^{\alpha} l_{1t}^{\gamma_2}$. The first-order condition for labor in economy 1 is

$$-\frac{U_{lt}(c_{1t}, l_{1t})}{U_{ct}(c_{1t}, l_{1t})} = (1 - \tau_{l1t}) \frac{\gamma_1 y_{1t}}{l_{1t}}.$$

Using the definition of τ_{l2t}, we have that

$$-\frac{U_{lt}(c_{1t}, l_{1t})}{U_{ct}(c_{1t}, l_{1t})} = (1 - \tau_{l2t}) \frac{\gamma_2 y_{1t}}{l_{1t}}.$$

The rest of the equations governing the equilibrium are unaffected. *Q.E.D.*

It is simply a matter of notation to extend this proposition to a stochastic environment.

Notice from Proposition 3 that the size of the measured wedges will be very different when the labor exponents, γ_1 and γ_2, are very different but the outcomes will be the same. To understand this proposition, consider the following thought experiment. Generate data from economy 1 and measure the wedges using the parameter values from economy 2. If these measured wedges are fed back into economy 2, then the data generated from economy 1 will be recovered.

Note that our quantitative exercise above involves a different thought experiment. In this exercise we did not measure the wedges for the variable capital utilization economy using the data generated from the benchmark economy. If we had the results in the two economies would coincide exactly, as Proposition 3 dictates. Instead, we used the U.S. data to measure wedges for the fixed and variable capital utilization economies. We then fed these wedges back into the model economies and analyzed the results. The data generated by the two model economies turned out to be very close to each other because the benchmark economy without the investment wedge is close to the U.S. data.

3.2. Labor Supply Elasticities

It is easy to show that for two economies with differing labor supply elasticities, an analogous result to that in Proposition 3 holds: allowing for different labor supply elasticities changes the size of the measured labor wedge but does not change equilibrium outcomes.

To see that, consider two economies which are identical to a deterministic version of our benchmark model except that the utility function is given by

$$U(c) + V_i(1 - l)$$

for $i = 1, 2$. In our benchmark model, both U and V_i are logarithmic. Clearly, by varying the function V_i, we can generate a wide range of alternative labor supply elasticities.

For some given sequence of wedges $(A_{1t}, \tau_{l1t}, \tau_{x1t})$, let y_{1t}, c_{1t}, l_{1t}, and x_{1t} denote the resulting equilibrium outcomes in economy 1. Let the initial capital stocks be the same in economies 1 and 2. We then have the following proposition:

Proposition 4. If the sequence of wedges for economy 2 is given by

$$1 - \tau_{l2t} = (1 - \tau_{l1t}) \frac{V_2'(1 - l_{1t})}{V_1'(1 - l_{1t})}$$

and $A_{2t} = A_{1t}$ and $\tau_{x2t} = \tau_{x1t}$, then the equilibrium outcomes for economy 2 coincide with those of economy 1.

Proof. We prove this proposition by showing that the equilibrium conditions of economy 2 are satisfied at the equilibrium outcomes of economy 1. The first-order condition for labor input in economy 1 is

$$-\frac{V_1'(1 - l_{1t})}{U'(c_{1t})} = (1 - \tau_{l1t}) \frac{(1 - \alpha) y_{1t}}{l_{1t}}.$$

Using the definition of τ_{l2t}, we have that

$$-\frac{V_2'(1 - l_{1t})}{U'(c_{1t})} = (1 - \tau_{l2t}) \frac{(1 - \alpha) y_{1t}}{l_{1t}}$$

so that the first-order condition for labor in economy 2 is satisfied. The rest of the equations governing the equilibrium are unaffected. *Q.E.D.*

Extending this proposition to a stochastic environment is simply a matter of notation. And for similar reasons as in the variable capital utilization exercise, allowing for differing labor supply elasticities does not change our quantitative results.

4. Spectral Decomposition of Variance

So far we have developed a decomposition of the movements in the data based on the realizations measured using the model. We now develop a decomposition based on the population properties of the stochastic process generated by the model. In this spectral method, we begin by orthogonalizing the innovations to the wedges. At each frequency, we then decompose the variance of output into the variance induced by each orthogonalized innovation. Our results are similar to the realization-based decomposition.

4.1. The Spectral Method

The spectral method is complementary to the episodic method described above. The spectral method has the advantage that it is based on the population properties of the model. It thus captures not just the behavior of a single episode that actually occurred, but also the behavior in other episodes that could have occurred but did not. The disadvantage of this method is that it requires orthogonalizing the innovations to the wedges. The difficulty in interpreting these orthogonalized innovations makes drawing sharp lessons about underlying models harder with this method than with the episodic method.

We orthogonalize the innovations to the wedges as follows. We choose one of 12 possible orderings of the wedges. Consider, for example, this one: the efficiency wedge first, followed in sequence by the labor, investment, and government consumption wedges. Given this ordering, we rewrite (30) as

$$s_{t+1} = Ps_t + Q\tilde{\varepsilon}_{t+1}$$

where Q is the lower triangular matrix that solves $QQ' = V$ and the covariance matrix of $\tilde{\varepsilon}_t$ is the identity matrix. With this ordering, the innovation to the efficiency wedge affects all the other wedges contemporaneously, while the innovation to the labor wedge affects only the labor, investment, and government consumption wedges, and so on.

We can write our equilibrium in state-space form as follows. Let $X_t = (\log k_t, s_t)$ denote the state in period t. The state evolves according to

$$X_{t+1} = AX_t + D\varepsilon_{t+1}. \tag{33}$$

The first row of (33) is the transition law for the capital stock, and the associated value of ε_t is identically zero. The rest of the system describes the vector AR1 process for the four wedges. The matrix D is given by

$$D = \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix}.$$

Let $Y_t = (\log y_t, \log l_t, \log x_t, \log g_t)'$ denote the vector of output, labor, investment, and government consumption. Using the linear decision rules from the model, we can rewrite this vector as

$$Y_t = CX_t \tag{34}$$

where C is a matrix. Using standard methods (as, for example, those of Sargent (1987)), we see that the spectral matrix of Y is given by

$$S(\omega) = C(e^{i\omega}I - A)^{-1}DD'(Ie^{-i\omega} - A')^{-1}C' \tag{35}$$

where ω measures frequency and I is the identity matrix. Let $S_{ij}(\omega)$ be the element in the ith row and the jth column of this matrix. Each such element can be decomposed into four pieces that sum up to one at each frequency ω. Define the spectral matrix associated with each innovation k, for $k = 1, \ldots, 4$, by

$$S^k(\omega) = C(e^{i\omega}I - A)^{-1}De_{kk}D'(Ie^{-i\omega} - A')^{-1}C'$$

where e_{kk} is a matrix with a one in the kk element and zeros elsewhere, and let $S^k_{ij}(\omega)$ denote the ij element of $S^k(\omega)$. Since output is the first variable in Y_t, our decomposition of the variance of output is given by

$$\left[\frac{S^1_{11}(\omega)}{S_{11}(\omega)}, \frac{S^2_{11}(\omega)}{S_{11}(\omega)}, \frac{S^3_{11}(\omega)}{S_{11}(\omega)}, \frac{S^4_{11}(\omega)}{S_{11}(\omega)}\right].$$

The term $S^k_{11}(\omega)/S_{11}(\omega)$ is interpreted as the fraction of variance of output at frequency ω attributable to the innovation in wedge k.

So far we have illustrated our procedure using a specific ordering of the wedges. For each of the 12 possible orderings, the same procedure applies.

4.2. The Spectral Method's Results

For each wedge, we compute the average contribution to the output spectrum over the 12 possible orderings. In Figure 13, we plot this average for the efficiency, labor, and investment wedges for the period from 1901 to 1940. We see that at business cycle frequencies (between two and six years), the combined contribution of the efficiency and labor wedges is more than 80% and the contribution of the investment wedge is less than 15%. This result reinforces our basic finding that investment wedges played at best a minor role in the prewar era. In Figure 14 we plot the analog of Figure 13 for the period from 1955 to 2000. Here the combined contribution of the efficiency and labor wedges is roughly 60% while that of the investment wedges is a little more than 30%. This is consistent with our earlier finding that investment wedges played a somewhat more important role in the postwar era.

5. The Related Literature

Our work here is related to the existing literature in terms of methodology and the interpretation of the wedges.

5.1. Related Methodology

Our basic methodology is to use restrictions from economic theory to back out wedges from the data, formulate stochastic processes for these wedges, and then put them back into a quantitative general equilibrium model for an accounting exercise. This basic idea is at the heart of an enormous amount of work in the real business cycle theory literature. For example, Prescott (1986) explicitly asks what fraction of the variance of output can plausibly be attributed to productivity shocks, which we have referred to as the *efficiency wedge*.

Studies in the subsequent literature have expanded this general equilibrium accounting exercise to include a wide variety of other shocks. For example, for shocks to the marginal efficiency of investment, see Greenwood, Hercowitz, and Huffman (1988); for money shocks, Cooley and Hansen (1989); for broadly interpreted preference shocks, Bencivenga (1992) and Stockman and Tesar (1995); for terms of trade shocks, Mendoza (1991); for foreign technology shocks, Backus, Kehoe, and Kydland (1992) and Baxter and Crucini (1995); for shocks to the home production technology, Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991); for government spending shocks, Christiano and Eichenbaum (1992); for shocks to markups, Rotemberg and Woodford (1992); for shocks to taxes, Braun (1994) and McGrattan (1994); and for shocks to financial intermediation, Cooper and Ejarque (2000).

An important difference between our method and many of those in the later real business cycle literature is that we back out the labor wedge and the investment wedge from the combined consumer and firm first-order conditions while most of this later literature uses direct measures of these shocks. One of the most closely related precursors of our method is that of McGrattan (1991), who, for the postwar U.S. data, decomposes the movements in output into the fraction that comes from the efficiency wedge, the labor wedge, and the investment wedge, which she refers to as productivity shocks, taxes on labor income, and taxes on capital income. She uses no data on taxes but instead simply uses the equilibrium to infer the implicit wedges. Ingram, Kocherlakota, and Savin (1994) advocate a similar approach.

5.2. Interpreting and Assessing the Wedges

The three wedges in our model can arise from a variety of detailed economies. In terms of theory, a large number of studies have shown how distortions in economies manifest themselves as at least one of our three wedges. In terms of applications, a large number of studies have used one or more of the wedges to assess aspects of a model.

a. Theory

The idea that taxes of various kinds distort the relation between various marginal rates is the cornerstone of public finance. Specifically, it is well-known that taxes on intermediate goods lead to aggregate production inefficiency and thus produce an efficiency wedge, that taxes on labor income distort the within-period marginal rates of substitution from the within-period marginal rates of transformation and thus produce a labor wedge, and finally, that taxes on capital income or investment distort the intertemporal marginal rates of substitution from the intertemporal marginal rates of transformation and thus produce an investment wedge. (See, for example, Atkinson and Stiglitz 1980.) Taxes are not the only well-known distortions; monopoly power by unions or firms is also commonly thought to produce a labor wedge. And, the idea that a labor wedge is produced by sticky wages or sticky prices is the cornerstone of the New Keynesian approach to business cycles. See, for example, the recent survey by Rotemberg and Woodford (1999).

One contribution of our work here is to show the precise map between these various wedges and general equilibrium models with frictions. Each distortion in the underlying economy does not map into one and only one wedge. For example, input-financing frictions, in general, distort all three wedges simultaneously. And while models with one period of either wage or price stickiness do produce only labor wedges, models with staggered wage- or price-setting produce efficiency wedges as well. (See Chari, Kehoe, and McGrattan (2002).) Finally, as noted by Carlstrom and Fuerst (1997), the investment frictions from costly state verification result in wedges in the capital accumulation equation as well as investment wedges.

b. Applications to Postwar Data

Many studies have plotted and interpreted one or more of the three wedges. In the real business cycle literature, many studies plot the efficiency wedge and try to sort out whether this wedge is due to misspecified production functions (increasing returns instead of constant returns), mismeasured factor inputs (unobserved utilization of capital or labor), or procyclical productivity. See, among others, the study of Burnside, Eichenbaum, and Rebelo (1993) and the survey by Basu and Fernald (2000).

Studies have also plotted the labor wedge for the U.S. postwar data and discussed various interpretations of it. For example, Parkin (1988), Hall (1997), and Gali, Gertler, and Lopez-Salido (2002) all graph and interpret the labor wedge for the postwar data. Parkin (1988) discusses how monetary shocks might drive the wedge. Hall (1997) mostly interprets the wedge as a preference shock, but also discusses a search interpretation. Gali, Gertler, and López-Salido (2002) discuss a variety of interpretations, as do Rotemberg and Woodford (1991, 1995, and 1999).

The investment wedge has also been investigated. In addition to the work of Braun (1994) and McGrattan (1994), see those of Carlstrom and Fuerst (1997) and Cooper and Ejarque (2000).

c. The Neoclassical Approach to the Great Depression

Lately researchers have begun to reinterpret the Great Depression in the United States and elsewhere through the lens of neoclassical theory. Some of this work has been done by Cole and Ohanian (1999 and 2001a) and Prescott (1999). Cole and Ohanian (1999) find that for the United States, the efficiency wedge can account for only a 15% decline, not the observed 38% decline in detrended output from 1929 to 1933. They argue that some force other than the efficiency wedge is needed, especially to account for the slow recovery. As contributing factors, they consider and dismiss fiscal policy shocks and trade shocks and leave open the possibility that monetary shocks, financial intermediation shocks, and sticky wages may have been involved. Bordo, Erceg, and Evans (2000) use a quantitative model to argue that monetary shocks interacting with sticky wages can account for much of the decline and some of the slow recovery in output in the U.S. Great Depression. Crucini and Kahn (1996) find that tariff shocks can account for only about 2% of the decline in U.S. output. Mulligan (2002a and 2002b) plots the labor wedge for the United States for much of the 20th century, including the Great Depression period. He interprets movements in this wedge as arising from changes in labor market institutions and regulation, including features we discuss here.

For recent attempts to assess the neoclassical growth model's performance in accounting for the great depressions of the 1930s in Germany, see Fisher and Hornstein (2002); for those in Canada, see Amaral and MacGee (2002).

6. Conclusion

This study is aimed at applied theorists who are interested in building detailed models of economic fluctuations. Once such theorists have chosen the primitive sources of shocks, they need to choose the mechanisms through which such shocks lead to fluctuations. We have shown that these mechanisms can be summarized by their effects on three wedges. Our accounting procedure can be used to judge which mechanisms are promising and which are not.

Here we have applied our procedure to the Great Depression and to a typical U.S. recession. We have found that efficiency and labor wedges, in combination, account for essentially all of the decline and recovery in these business cycles; investment wedges play, at best, a minor role. These results suggest that future theoretical work should focus on developing models which lead to fluctuations in efficiency and labor wedges. Many existing models produce fluctuations in labor wedges. The challenging task is to develop detailed models in which primitive shocks lead to fluctuations in efficiency wedges as well.

In the macroeconomics literature vector autoregressions have frequently been applied to guide the development of new theory. These autoregressions attempt to use minimal amounts of economic theory to identify patterns in the data that new theories should attempt to reproduce. Our accounting procedure can be viewed as an alternative to this vector autoregression methodology. Here we use all of the economic theory embedded in the growth model to identify patterns in the data. Since some version of the growth model is central studies of aggregate activity, we think of our procedure as a promising theory-intensive alternative to vector autoregressions.

References

Amaral, Pedro and MacGee, James. "The Great Depression in Canada and the United States: A Neoclassical Perspective." *Review of Economic Dynamics*, January 2002, 5(1), pp. 45–72.

Atkinson, A. B. and Stiglitz, Joseph E. *Lectures on Public Economics.* New York: McGraw-Hill, 1980.

Backus, David K.; Kehoe, Patrick J. and Kydland, Finn E. "International Real Business Cycles." *Journal of Political Economy*, August 1992, 100(4), pp. 745–75.

Basu, Susanto and Fernald, John G. "Why is Productivity Procyclical? Why Do We Care?" NBER Working Paper 7940, 2000.

Baxter, Marianne and Crucini, Mario J. "Business Cycles and the Asset Structure of Foreign Trade." *International Economic Review*, November 1995, 36(4), pp. 821–54.

Bencivenga, Valerie R. "An Econometric Study of Hours and Output Variation with Preference Shocks." *International Economic Review*, May 1992, 33(2), pp. 449–71.

Benhabib, Jess; Rogerson, Richard and Wright, Randall. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations." *Journal of Political Economy,* December 1991, 99(6), pp. 1166–87.

Bernanke, Ben S. "Nonmonetary Effects of the Financial Crisis in the Propagation of the Great Depression." *American Economic Review,* June 1983, 73(3), pp. 257–76.

Bernanke, Ben S. and Gertler, Mark. "Agency Costs, Net Worth, and Business Fluctuations." *American Economic Review*, March 1989, 79(1), pp. 14–31.

Bordo, Michael D.; Erceg, Christopher J. and Evans, Charles L. "Money, Sticky Wages, and the Great Depression." *American Economic Review,* December 2000, 90(5), pp. 1447–63.

Braun, R. Anton. "How Large Is the Optimal Inflation Tax?" *Journal of Monetary Economics,* October 1994, 34(2), pp. 201–14.

Burnside, Craig; Eichenbaum, Martin and Rebelo, Sergio. "Labor Hoarding and the Business Cycle." *Journal of Political Economy*, April 1993, 101(2), pp. 245–73.

Carlstrom, Charles T. and Fuerst, Timothy S. "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis." *American Economic Review,* December 1997, 87(5), pp. 893–910.

Chari, V. V.; Kehoe, Patrick J. and McGrattan, Ellen R. "Accounting for the Great Depression." *American Economic Review*, May 2002, 92(2), pp. 22–27.

Christiano, Lawrence J. and Eichenbaum, Martin. "Liquidity Effects and the Monetary Transmission Mechanism." *American Economic Review*, May 1992, 82(2), pp. 346–53.

Cole, Harold L. and Ohanian, Lee E. "The Great Depression in the United States from a Neoclassical Perspective." *Federal Reserve Bank of Minneapolis Quarterly Review*, Winter 1999, 23(1), pp. 2–24.

Cole, Harold L. and Ohanian, Lee E. "Re-Examining the Contributions of Money and Banking Shocks to the U.S. Great Depression." In Ben S. Bernanke and Kenneth Rogoff, eds., NBER Macroeconomics Annual 2000, 2001a, pp. 183–227. Cambridge, Mass.: MIT Press, 2001a.

Cole, Harold L. and Ohanian, Lee E. "New Deal Policies and the Persistence of the Great Depression: A General Equilibrium Analysis." Research Department Working Paper 597, Federal Reserve Bank of Minneapolis, 2001b.

Cooley, Thomas F. and Hansen, Gary D. "The Inflation Tax in a Real Business Cycle Model." *American Economic Review*, September 1989, 79(4), pp. 733–48.

Cooper, Russell and Ejarque, Joao. "Financial Intermediation and Aggregate Fluctuations: A Quantitative Analysis." *Macroeconomic Dynamics*, December 2000, 4(4) pp. 423–47.

Crucini, Mario J. and Kahn, James. "Tariffs and Aggregate Economic Activity: Lessons from the Great Depression." *Journal of Monetary Economics*, December 1996, 38(3), pp. 427–67.

Fisher, Jonas and Hornstein, Andreas. "The Role of Real Wages, Productivity, and Fiscal Policy in Germany's Great Depression, 1928–1937." *Review of Economic Dynamics*, January 2002, 5(1), pp. 100–127.

Gali, Jordí; Gertler, Mark and López-Salido, J. David. "Markups, Gaps, and the Welfare Costs of Business Fluctuations." NBER Working Paper 8850, 2002.

Greenwood, Jeremy and Hercowitz, Zvi. "The Allocation of Capital and Time over the Business Cycle." *Journal of Political Economy*, December 1991, 99(6), pp. 1188–214.

Greenwood, Jeremy; Hercowitz, Zvi and Huffman, Gregory W. "Investment, Capacity Utilization, and the Real Business Cycle." *American Economic Review*, June 1988, 78(3), pp. 402–17.

Hall, Robert E. "Macroeconomic Fluctuations and the Allocation of Time." *Journal of Labor Economics,* January 1997, 15(1), pp. 223–50.

Hornstein, Andreas and Prescott, Edward C. "The Firm and the Plant in General Equilibrium Theory." In Robert Becker et al., eds., *General Equilibrium, Growth, and Trade II: The Legacy of Lionel McKenzie*, pp. 393–410. San Diego: Academic Press, 1993.

Ingram, Beth Fisher; Kocherlakota, Narayana R. and Savin, N. E. "Explaining Business Cycles: A Multiple-Shock Approach." *Journal of Monetary Economics*, December 1994, 34(3), pp. 415–28.

Kydland, Finn E. and Prescott, Edward C. "The Workweek of Capital and Its Cyclical Implications." *Journal of Monetary Economics*, March/May 1988, 21(2/3), pp. 343–60.

Lagos, Ricardo. A Model of TFP. Mimeo. New York University, 2001.

McGrattan, Ellen R. "The Macroeconomic Effects of Distortionary Taxation." Research Department Working Paper 37, Federal Reserve Bank of Minneapolis, 1991.

McGrattan, Ellen R. "The Macroeconomic Effects of Distortionary Taxation." *Journal of Monetary Economics*, June 1994, 33(3), pp. 573–601.

McGrattan, Ellen R. "Solving the Stochastic Growth Model with a Finite Element Method." *Journal of Economic Dynamics and Control*, 1996, 20, pp. 19–42.

Mendoza, Enrique G. "Real Business Cycles in a Small Open Economy." *American Economic Review*, September 1991, 81(4), pp. 797–818.

Mulligan, Casey B. "A Dual Method of Empirically Evaluating Dynamic Competitive Equilibrium Models with Market Distortions, Applied to the Great Depression and World War II." NBER Working Paper 8775, 2002a.

Mulligan, Casey B. "A Century of Labor-Leisure Distortions." NBER Working Paper 8774, 2002b.

Parkin, Michael. "A Method for Determining Whether Parameters in Aggregative Models are Structural." *Carnegie-Rochester Conference Series on Public Policy*, Autumn 1988, 29, pp. 215–52.

Prescott, Edward C. "Theory Ahead of Business Cycle Measurement." *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 1986, 10(4), pp. 9–22.

Prescott, Edward C. "Some Observations on the Great Depression." *Federal Reserve Bank of Minneapolis Quarterly Review*, Winter 1999, 23(1), pp. 25–31.

Rotemberg, Julio J. and Woodford, Michael. "Markups and the Business Cycle." In Olivier Jean Blanchard and Stanley Fischer, eds., *NBER Macroeconomics Annual 1991,* pp. 1003–1026. Cambridge, Mass.: MIT Press, 1991.

Rotemberg, Julio J. and Woodford, Michael. "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity." *Journal of Political Economy*, December 1992, 100(6), pp. 1153–1207.

Rotemberg, Julio J. and Woodford, Michael. "Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets." In Thomas F. Cooley, ed., *Frontiers of Business Cycle Research,* pp. 243–330. Princeton: Princeton University Press, 1995.

Rotemberg, Julio J. and Woodford, Michael. "The Cyclical Behavior of Prices and Costs." In John Taylor and Michael Woodford, eds., *Handbook of Macroeconomics,* Vol.

1B, Chap. 16, pp. 1051–1135. Amsterdam: North-Holland, 1999.

Sargent, Thomas J. *Dynamic Macroeconomic Theory.* Cambridge, Mass.: Harvard University Press, 1987.

Schmitz, James A., Jr. "What Determines Labor Productivity?: Lessons From the Dramatic Recovery of the U.S. and Canadian Iron-ore Industries." Research Department Staff Report 286, Federal Reserve Bank of Minneapolis, 2001.

Solow, Robert M. "Technical Change and the Aggregate Production Function." *Review of Economics and Statistics,* August 1957, 39(3), pp. 312–20.

Stockman, Alan C. and Tesar, Linda L. "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements." *American Economic Review,* March 1995, 85(1), pp. 168–85.

Townsend, Robert M. "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory,* October 1979, 21(2), pp. 265–93.

Appendix: Maps for Wedges Due to Monopoly Unions and Investment Frictions

In this appendix, we demonstrate the mapping from two detailed growth models to two prototype economies with wedges.

A. Labor Wedges Due to Unions

Here we first describe a nonmonetary version of the detailed economy above with sticky wages and monopoly unions. Then we map that model into a prototype economy with labor wedges due to unions.

A Detailed Economy With Unions

Consider the following economy in which fluctuations in policies toward unions show up as fluctuations in labor market distortions in the prototype economy. (See Cole and Ohanian 2001b for a discussion of such policies in the Great Depression.) The economy is a nonmonetary version of the sticky wage economy described above.

The technology for producing final goods in this economy is given by (14) and (15). Capital is accumulated according to (16). The problem faced by the final goods producer is

$$\max \sum_{t=0}^{\infty} q(s^t)\left[y(s^t) - x(s^t) - w(s^t)l(s^t)\right] \tag{36}$$

where $q(s^t)$ is the price of a unit of consumption goods at s^t in an abstract unit of account and $w(s^t)$ is the aggregate real wage at s^t. The producer's problem can be stated in two parts. First, the producer chooses sequences for capital $k(s^{t-1})$, investment $x(s^t)$, and aggregate labor $l(s^t)$ subject to (14) and (16). Second, the demand for labor of type j by the final goods producer is

$$l^d(j, s^t) = \left(\frac{w(s^t)}{w(j, s^t)}\right)^{\frac{1}{1-v}} l(s^t) \tag{37}$$

where $w(s^t) \equiv \left[\int w(j, s^t)^{\frac{v}{v-1}}\, dj\right]^{\frac{v-1}{v}}$ is the aggregate wage.

Analogously to the sticky wage economy, here the representative union faces, in setting its wage, a downward-sloping demand for its type of labor, given by (37). The problem of the jth union is to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)\, u\left(c(j, s^t), l(j, s^t)\right) \tag{38}$$

subject to the budget constraints

$$c(j, s^t) + \sum_{s_{t+1}} q(s^{t+1}|s^t) b(j, s^{t+1}) \leq w(s^t) l^d(j, s^t) + b(j, s^t) + d(s^t)$$

and the borrowing constraint $b(s^{t+1}) \geq -\overline{b}$, where $l^d(j, s^t)$ is given by (37).

Here $b(j, s^t, s_{t+1})$ denotes the consumers' holdings of one-period state contingent bonds purchased in period t and state s^t with payoffs contingent on some particular state s_{t+1} at $t+1$, and $q(s^{t+1}|s^t)$ is the bonds' corresponding price. Clearly, $q(s^{t+1}|s^t) = q(s^{t+1})/q(s^t)$. Also, $d(s^t) = y(s^t) - x(s^t) - w(s^t) l(s^t)$ are the dividends paid by the firms. The initial conditions $b(j, s^0)$ are given and assumed to be the same for all j.

The only distorted first-order condition for this problem is

$$w(j, s^t) = -\frac{1}{v} \frac{U_l(j, s^t)}{U_c(j, s^t)}. \tag{39}$$

Notice that real wages are set as a markup over the marginal rate of substitution between labor and consumption. Clearly, given the symmetry among the consumers, we know that all of them choose the same consumption, labor, bond holdings, and wages, which we denote by $c(s^t)$, $l(s^t)$, $b(s^{t+1})$, and $w(s^t)$, and the resource constraint is as in (26).

We think of government pro-competitive policy as limiting the monopoly power of unions by pressuring them to limit their anti-competitive behavior. We model the government policy as enforcing provisions that make the unions price competitively if the markups exceed, say, $1/\bar{v}(s^t)$, where $\bar{v}(s^t) \leq v$. Under such a policy, then, the markup charged by unions is $1/\bar{v}(s^t)$.

The Associated Prototype Economy With Labor Wedges

Consider next a prototype economy in which the firm maximizes the present discounted value of profits

$$\max \sum_{t=0}^{\infty} q(s^t) \left[F\left(k(s^{t-1}), l(s^t)\right) - x(s^t) - w(s^t) l(s^t)\right] \tag{40}$$

subject to $k(s^t) = (1-\delta)k(s^{t-1}) + x(s^t)$. Consumers maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t)\, u\left(c(s^t), l(s^t)\right) \tag{41}$$

subject to

$$c(s^t) + \sum_{s_{t+1}} q(s^{t+1}|s^t) b(s^{t+1}) \leq [1-\tau(s^t)] w(s^t) l(s^t) + b(s^t) + d(s^t) + T(s^t)$$

where the dividends $d(s^t) = F\big(k(s^{t-1}), l(s^t)\big) - x(s^t) - w(s^t)l(s^t)$ and the lump-sum transfers $T(s^t) = \tau(s^t)w(s^t)l(s^t)$. The resource constraint is as in (26). The only distorted first-order condition is

$$[1-\tau(s^t)]w(s^t) = -\frac{U_l(j, s^t)}{U_c(j, s^t)}.$$

The following proposition is immediate.

Proposition 5. Consider the prototype economy just described with the following stochastic process for labor wedges:

$$1 - \tau(s^t) = \bar{v}(s^t).$$

The equilibrium allocations and prices of this prototype economy coincide with those of the unionized economy.

B. Investment Wedges

A variety of investment frictions affect the economy by raising the cost of investment. These frictions show up in prototype economies as taxes on investment. Some investment frictions also show up as wasted resources in both the resource constraint and the capital accumulation equation. One example of that sort of friction is due to Carlstrom and Fuerst (1997), who exposit a quantitative version of Bernanke and Gertler's (1989) model. Here we show the equivalence between the Carlstrom and Fuerst model and a prototype growth model with adjustment costs.

A Detailed Economy With Investment Frictions

The Carlstrom and Fuerst model has a continuum of risk-neutral entrepreneurs of mass η and a continuum of consumers of mass 1. The timing is as follows. At the beginning of each period, each consumer supplies l_t units of labor, each entrepreneur supplies l_{et} units of labor, and each consumer and each entrepreneur rent capital denoted k_{ct} and k_{et} to firms that produce output according to the technology $F(k_{ct} + \eta k_{et}, l_t, \eta l_{et})$. These firms solve

$$\max\ F(k_{ct} + \eta k_{et}, l_t, \eta l_{et}) - r_t(k_{ct} + \eta k_{et}) - w_t l_t - w_{et} l_{et}$$

where r_t is the rental rate on capital and w_t and w_{et} are the wage rates of consumers and entrepreneurs.

Consumers solve the problem

$$\max\ \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

$$c_t + q_t[k_{ct+1} - (1-\delta)k_{ct}] = w_t l_t + r_t k_{ct} + T_t$$

where q_t is the price of the investment good in units of the consumption good and T_t is a lump-sum transfer. Combining the first-order conditions for the firms and consumers gives

$$-\frac{U_{lt}}{U_{ct}} = F_{lt} \tag{42}$$

$$q_t U_{ct} = \beta U_{ct+1}[q_{t+1}(1-\delta) + F_{kt+1}]. \tag{43}$$

Consumption goods can be transformed into capital goods only by entrepreneurs. Each entrepreneur owns a technology that transforms i_t units of consumption goods at the beginning of any period t into $\omega_t i_t$ units of capital goods at the end of the period, where ω_t is i.i.d. across entrepreneurs and time and has density ϕ and c.d.f. Φ. The realization of ω_t is private information to the entrepreneur. At the beginning of each period, each entrepreneur supplies one unit of labor inelastically, receives labor income w_{et}, receives rental income $r_t k_{et}$, and pays taxes T_{et}. The value of the entrepreneur's capital is $q_t k_{et}(1-\delta)$. Thus, the entrepreneur's net worth in period t is

$$a_t = w_{et} + k_{et}[r_t + q_t(1-\delta)] - T_{et}. \tag{44}$$

Entrepreneurs can use their net worth together with funds borrowed from financial intermediaries to purchase consumption goods and transform them into capital goods. The financial intermediaries can monitor the realized output $\omega_t i_t$ by paying μi_t units of the capital good.

The key restriction on trades is that entrepreneurs are allowed only to enter into within-period deterministic contracts that are made before the realization of ω_t and pay off after that. (In particular, the risk-neutral entrepreneurs are prohibited from entering into contracts that share aggregate risk with the consumers.) With such a restriction, we know from Townsend (1979), the optimal contract is a type of risky debt in which the entrepreneur pays a fixed amount $R_t(i_t - a_t)$ if ω_t is greater than some cutoff level $\bar{\omega}_t$ and $\omega_t i_t$ otherwise, where $R_t(i_t - a_t) = \bar{\omega}_t i_t$. The intermediaries monitor the entrepreneur if and only if $\omega_t < \bar{\omega}_t$.

Under such a contract, the expected income of the entrepreneur is

$$q_t i_t \left[\int_{\bar{\omega}_t}^{\infty} (\omega_t - \bar{\omega}_t)\phi(\omega)\ d\omega \right] \equiv q_t i_t f(\bar{\omega}_t)$$

and the expected income of the financial intermediary is

$$q_t i_t \left[\int_{0}^{\bar{\omega}_t} (\omega_t - \mu)\phi(\omega)\ d\omega + [1 - \Phi(\bar{\omega}_t)]\bar{\omega}_t \right] \equiv q_t i_t g(\bar{\omega}_t).$$

The funds the intermediary lends are from the consumers. The consumers can either store their consumption goods from the beginning until the end of the period at a zero rate of return or lend their goods to the entrepreneur through the financial intermediaries. The mass of entrepreneurs is sufficiently small that the optimal contract maximizes their expected income subject to the constraint that an intermediary's gross return on the investment of $i_t - a_t$ is at least one.

The contract then solves

$$\max_{i_t, \bar{\omega}_t}\ q_t i_t f(\bar{\omega}_t)$$

subject to

$$q_t i_t g(\bar{\omega}_t) \geq i_t - a_t. \tag{45}$$

The first-order conditions imply that

$$\frac{f'(\bar{\omega}_t)}{f(\bar{\omega}_t)} + \frac{q_t g'(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} = 0 \tag{46}$$

and, since (45) holds with equality, the optimal investment level is given by

$$i_t = \frac{a_t}{1 - q_t g(\bar{\omega}_t)}. \tag{47}$$

The expected income of each entrepreneur is thus

$$q_t i_t f(\bar{\omega}_t) = \frac{a_t q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \tag{48}$$

which, by the law of large numbers, is the aggregate income of entrepreneurs.

From (47), we know that investment by each entrepreneur is linear in that entrepreneur's net worth, so that aggregate investment is linear in aggregate net worth. Together the aggregation result and the law of large numbers imply that the aggregate capital held by entrepreneurs has the following law of motion:

$$c_{et} + q_t k_{et+1} = \left[w_{et} + k_{et}\big(r_t + q_t(1-\delta)\big) - T_{et}\right] \frac{q_t f(\bar{\omega}_t)}{1 - q_t g(\bar{\omega}_t)} \tag{49}$$

where the right side is simply $q_t i_t f(\bar{\omega}_t)$ after substitution from (44) and (48).

The entrepreneur's utility function is

$$\sum_{t=0}^{\infty} (\beta\gamma)^t c_{et} \tag{50}$$

where $\gamma < 1$. We assume that entrepreneurs discount the future at a higher rate than consumers. This assumption is needed because the within-period rate of return earned by entrepreneurs is (weakly) greater than the rate of return earned by consumers. If entrepreneurs discounted the future at the same rate as consumers, then the entrepreneurs would postpone consumption indefinitely, and no equilibrium would exist.

Given the risk-neutrality of the entrepreneurs and the aggregation result, it should be clear that the optimal decisions of the entrepreneurs can be obtained by maximizing (50) subject to (49). The lump-sum tax levied on entrepreneurs is redistributed to the consumers, and hence, $T_t = \eta T_{et}$.

The Associated Prototype Economy With Investment Wedges

In the prototype economy associated with the Carlstrom and Fuerst model with investment frictions, the resource constraint is given by $c_t + x_t + g_t = F(k_t, l_t, \eta)$. The firm maximizes $F(k_t, l_t, \eta) - w_t l_t - r_t k_t$ with first-order conditions $F_{kt} = r_t$ and $F_{lt} = w_t$. Consumers maximize $\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$ subject to

$$c_t + (1 + \tau_{xt})x_t = w_t l_t + r_t k_t + T_t + \pi_t$$

$$k_{t+1} = (1 - \delta)k_t + x_t(1 - \theta_t)$$

where π_t denotes profits and the lump-sum transfer T_t in equilibrium is given by $\tau_{xt} x_t$. The first-order conditions are summarized by

$$-\frac{U_{lt}}{U_{ct}} = w_t \tag{51}$$

$$\frac{1 + \tau_{xt}}{1 - \theta_t} U_{ct} = \beta U_{ct+1} \left[r_{t+1} + \frac{1 + \tau_{xt+1}}{1 - \theta_{t+1}} (1 - \delta) \right]. \tag{52}$$

Denoting the equilibrium allocations in the Carlstrom and Fuerst economy with asterisks, we have the following:

Proposition 6. Consider the prototype economy just described with given stochastic processes for adjustment costs $\theta_t = \Phi(\bar{\omega}_t^*)\mu$, capital income taxes $1 + \tau_{xt} = q_t^*(1 - \theta_t)$, and government consumption $g_t = \eta c_{et}$. The aggregate equilibrium allocations for this prototype economy coincide with those of the Carlstrom and Fuerst economy.

In this proposition, we are measuring aggregate consumption by $c_t + \eta_t c_{et}$ in the Carlstrom and Fuerst economy and by $c_t + g_t$ in the associated prototype economy. Proposition 6 is similar to one established by Carlstrom and Fuerst.

Table 1

Parameters of Vector AR1 Stochastic Processes for Wedges in the Two Periods

Estimated Values (and Standard Errors) Resulting From Maximum Likelihood Procedure
and Data on Output, Labor, Investment, and Government Consumption

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1901–1940

$$P = \begin{bmatrix} \substack{.840 \\ (.088)} & \substack{.055 \\ (.042)} & \substack{-.193 \\ (.229)} & 0 \\ \substack{-.120 \\ (.125)} & \substack{1.032 \\ (.090)} & \substack{.349 \\ (.232)} & 0 \\ \substack{.017 \\ (.315)} & \substack{.000 \\ (.174)} & \substack{.393 \\ (.332)} & 0 \\ 0 & 0 & 0 & \substack{.574 \\ (.310)} \end{bmatrix} \qquad Q = \begin{bmatrix} \substack{-.047 \\ (.007)} & 0 & 0 & 0 \\ \substack{.018 \\ (.014)} & \substack{.045 \\ (.012)} & 0 & 0 \\ \substack{.019 \\ (.016)} & \substack{-.017 \\ (.012)} & \substack{-.029 \\ (.011)} & 0 \\ 0 & 0 & 0 & \substack{.229 \\ (.020)} \end{bmatrix}$$

1955–2000

$$P = \begin{bmatrix} \substack{.695 \\ (.267)} & \substack{.126 \\ (.507)} & \substack{.410 \\ (.616)} & \substack{.104 \\ (.207)} \\ \substack{-.063 \\ (.080)} & \substack{1.074 \\ (.039)} & \substack{.067 \\ (.113)} & \substack{-.001 \\ (.049)} \\ \substack{-.126 \\ (.137)} & \substack{.026 \\ (.204)} & \substack{1.160 \\ (.340)} & \substack{.068 \\ (.101)} \\ \substack{-.036 \\ (.098)} & \substack{.045 \\ (.098)} & \substack{-.004 \\ (.141)} & \substack{1.027 \\ (.072)} \end{bmatrix} \qquad Q = \begin{bmatrix} \substack{.015 \\ (.004)} & 0 & 0 & 0 \\ \substack{-.002 \\ (.005)} & \substack{-.009 \\ (.004)} & 0 & 0 \\ \substack{-.003 \\ (.016)} & \substack{.001 \\ (.011)} & \substack{-.003 \\ (.013)} & 0 \\ \substack{.010 \\ (.007)} & \substack{.003 \\ (.007)} & \substack{-.019 \\ (.019)} & \substack{.000 \\ (186.2)} \end{bmatrix}$$

Figures 1–4
Examining the U.S. Great Depression
Annually, 1929–39; Normalized to Equal 100 in 1929

Figure 1
U.S. Output and Three Measured Wedges

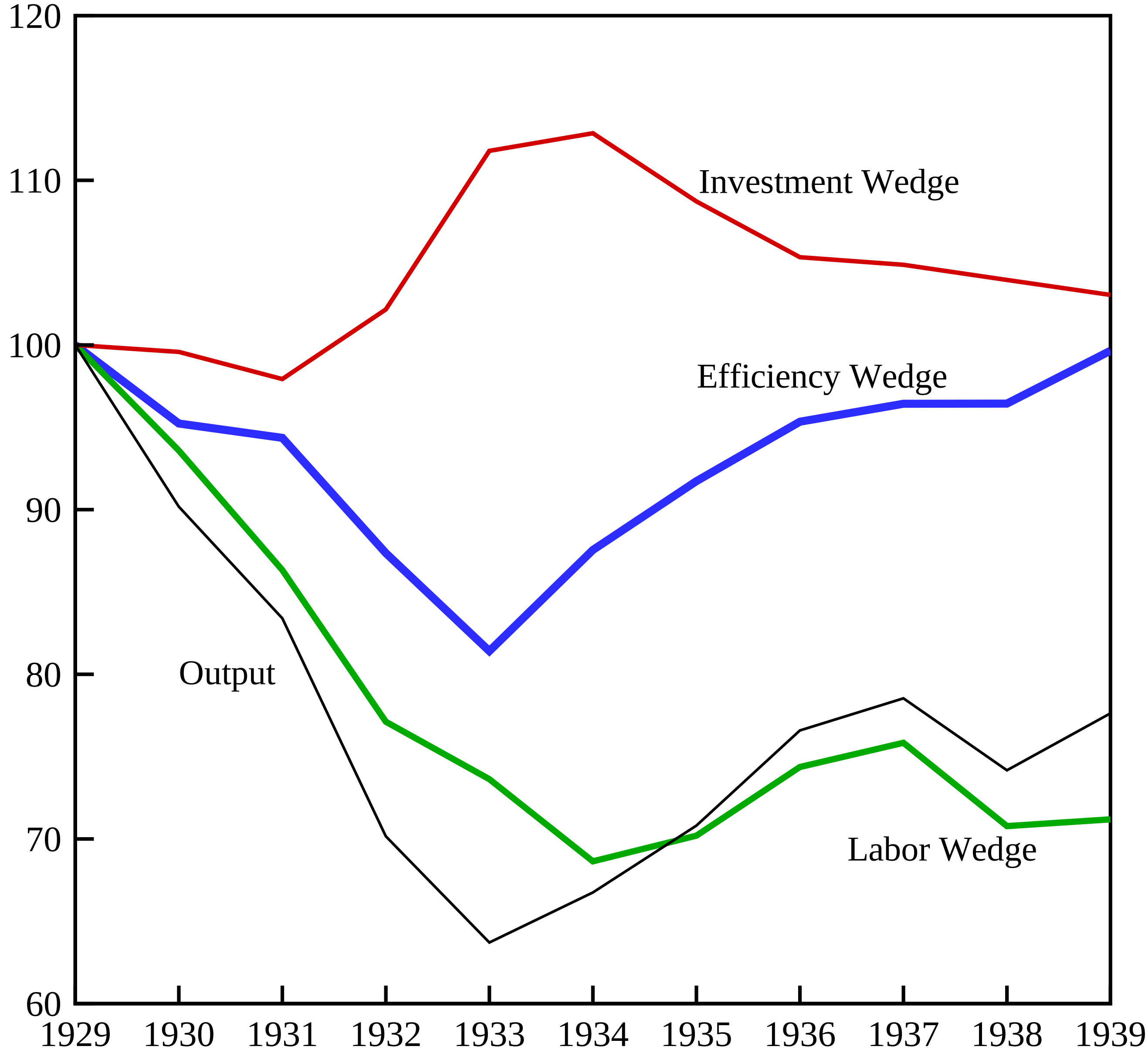


Figure 2
Data and Predictions of Models With Just One Wedge

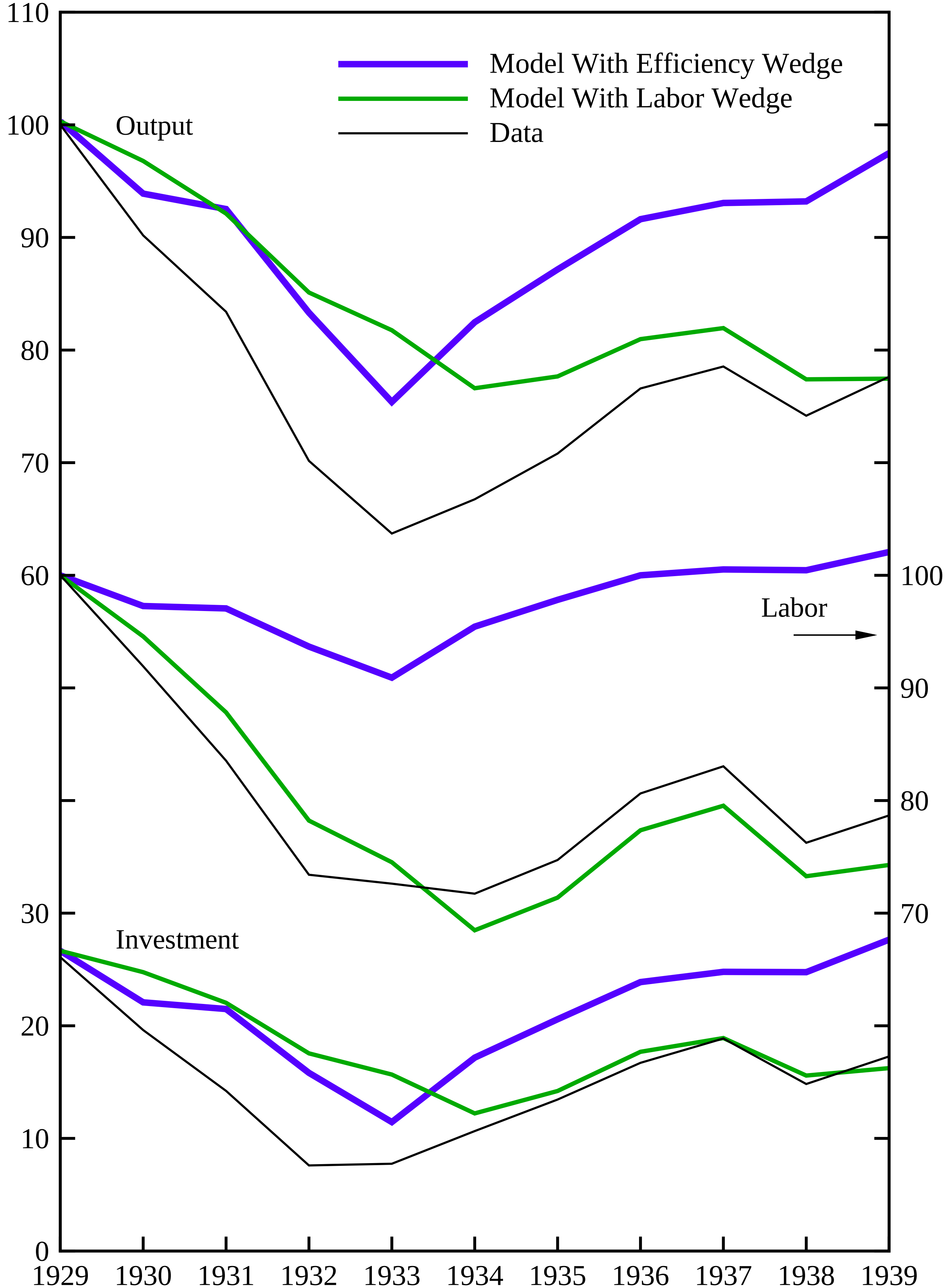


Figure 3
Data and Predictions of Model With Just the Investment Wedge

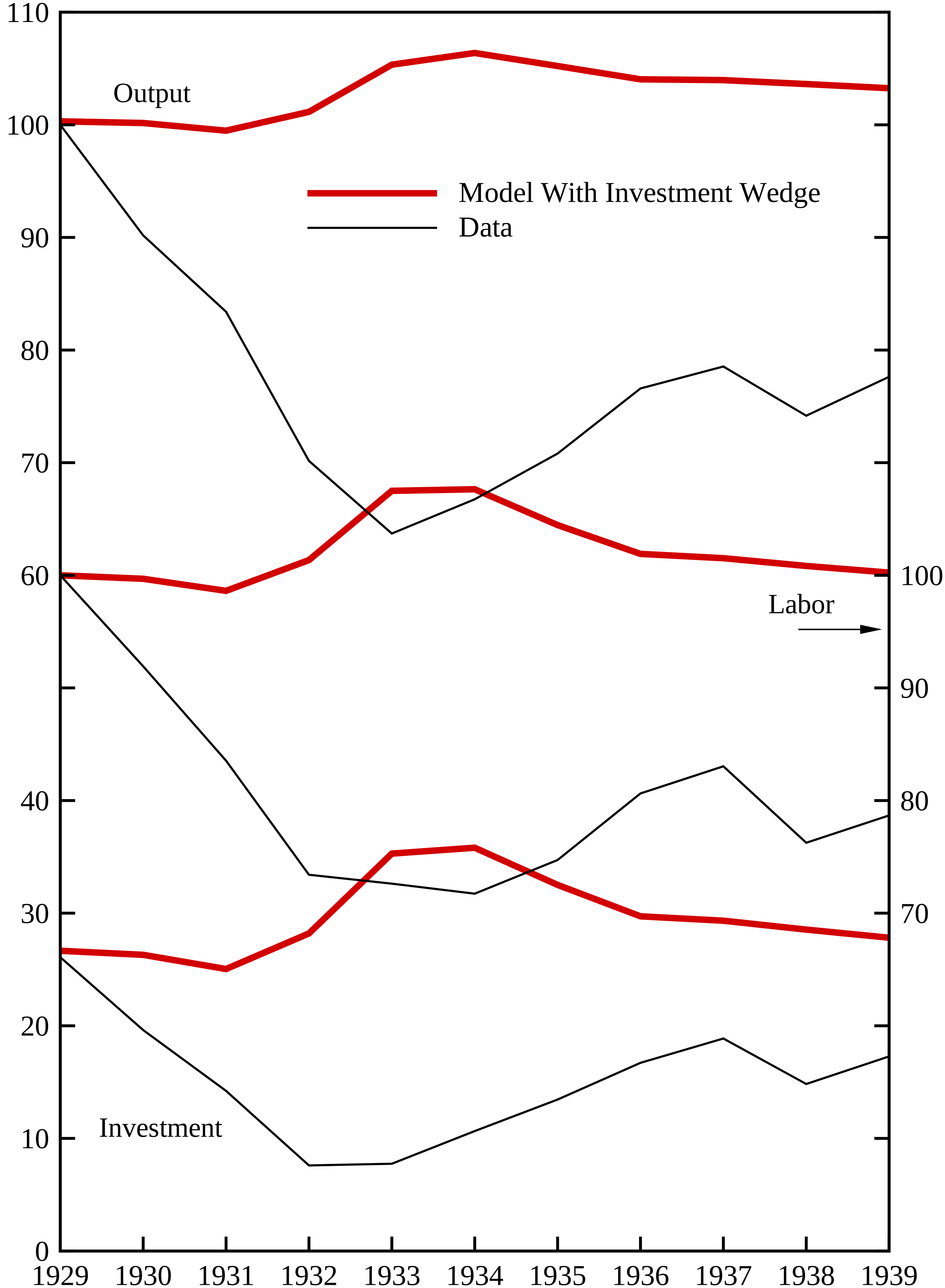
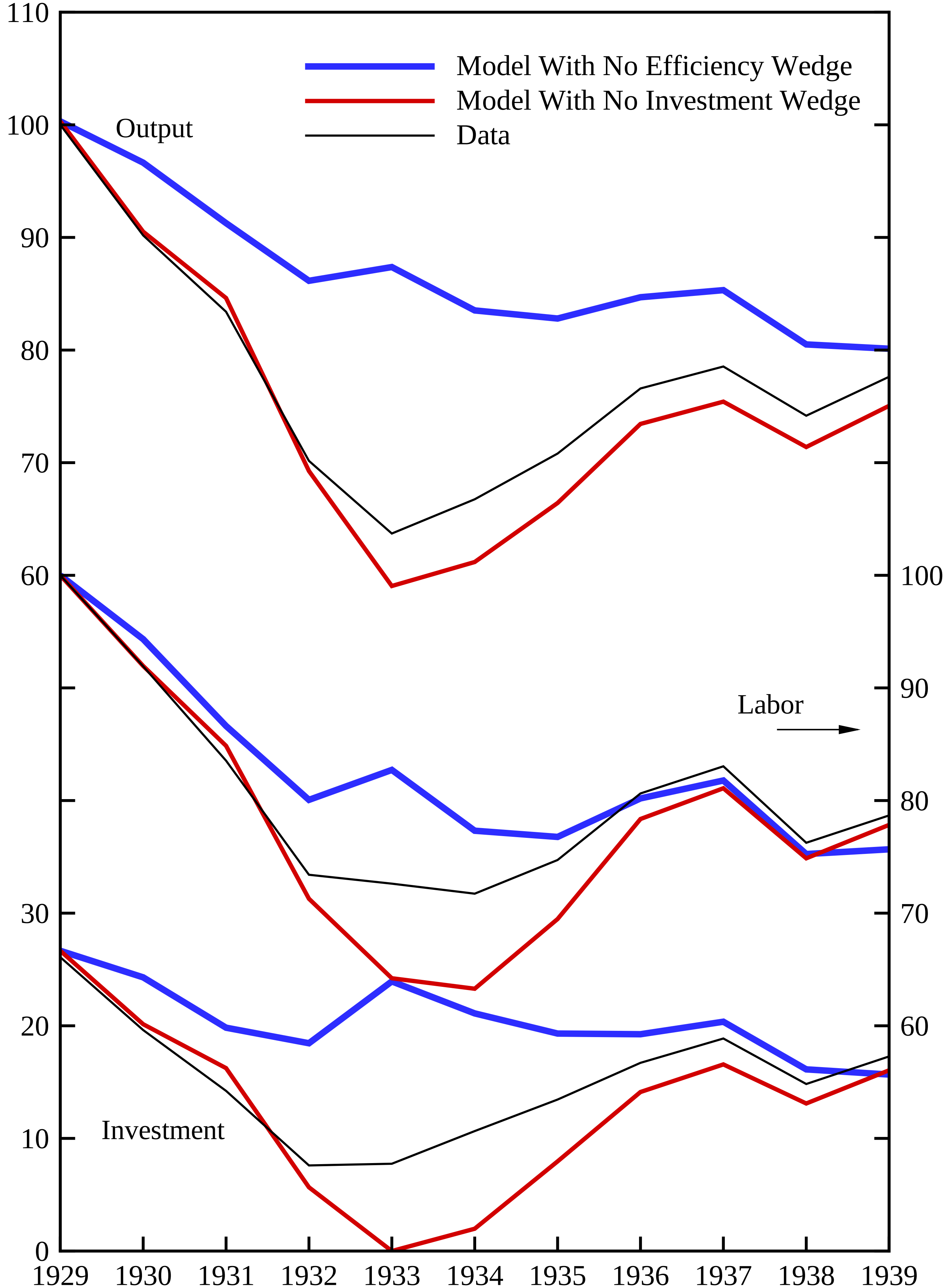


Figure 4
Data and Predictions of Models With All But One Wedge



Figures 5–8
Examining the 1982 U.S. Recession
Annually, 1979–85; Normalized to Equal 100 in 1979

Figure 5
U.S. Output and Three Measured Wedges

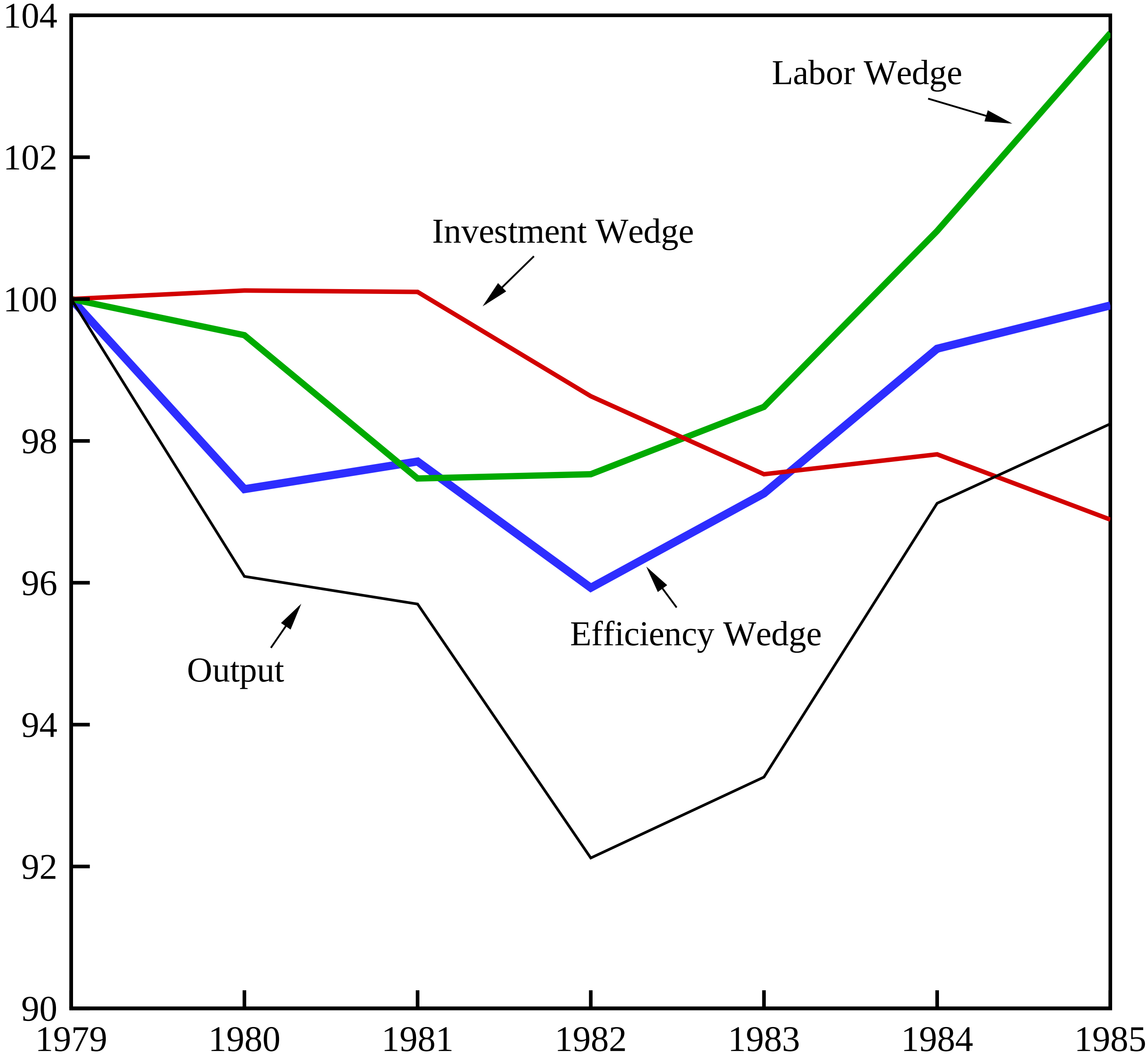


Figure 6
Data and Predictions of Models with Just One Wedge

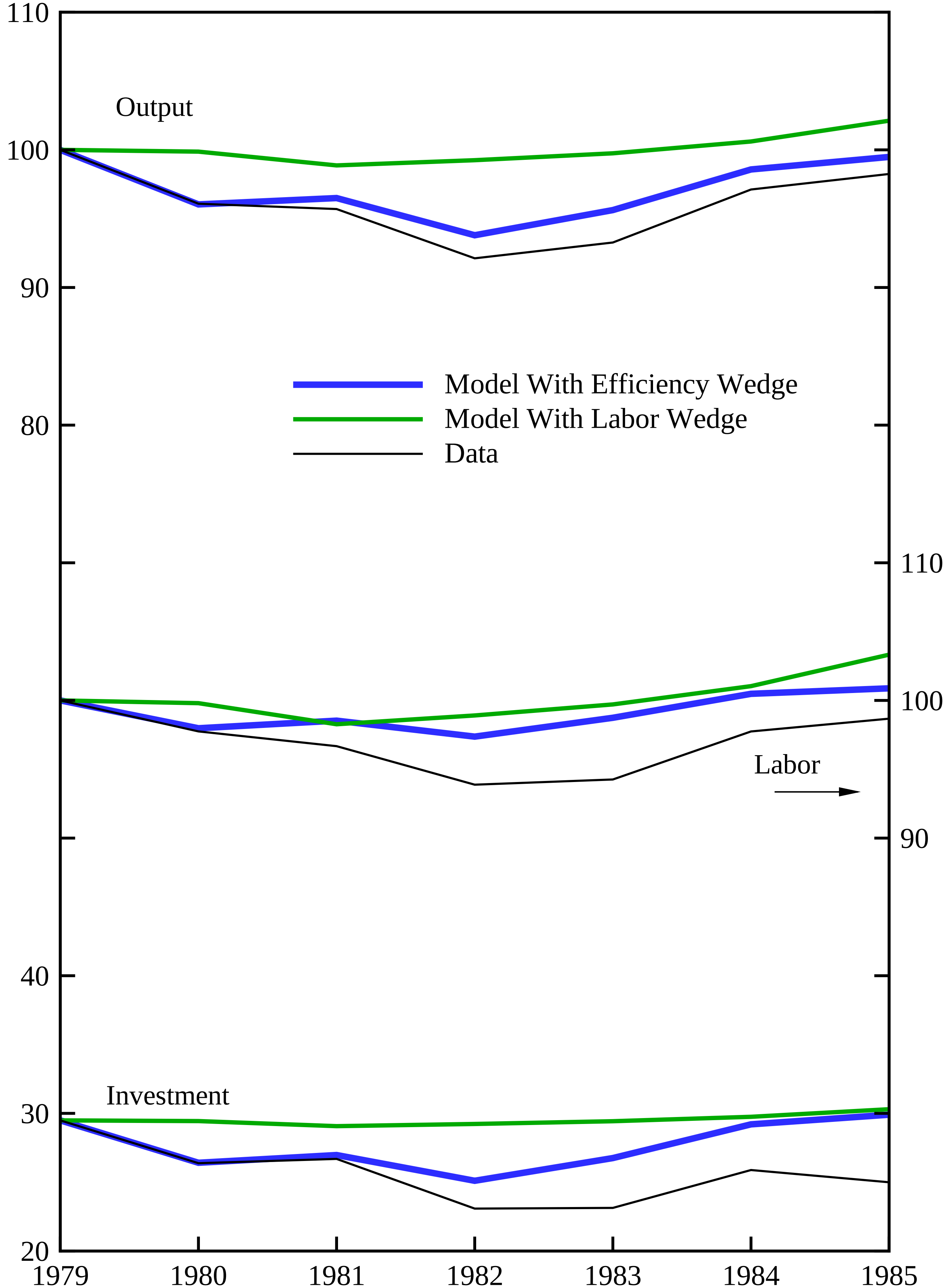


Figure 7
Data and Predictions of Model With Just the Investment Wedge

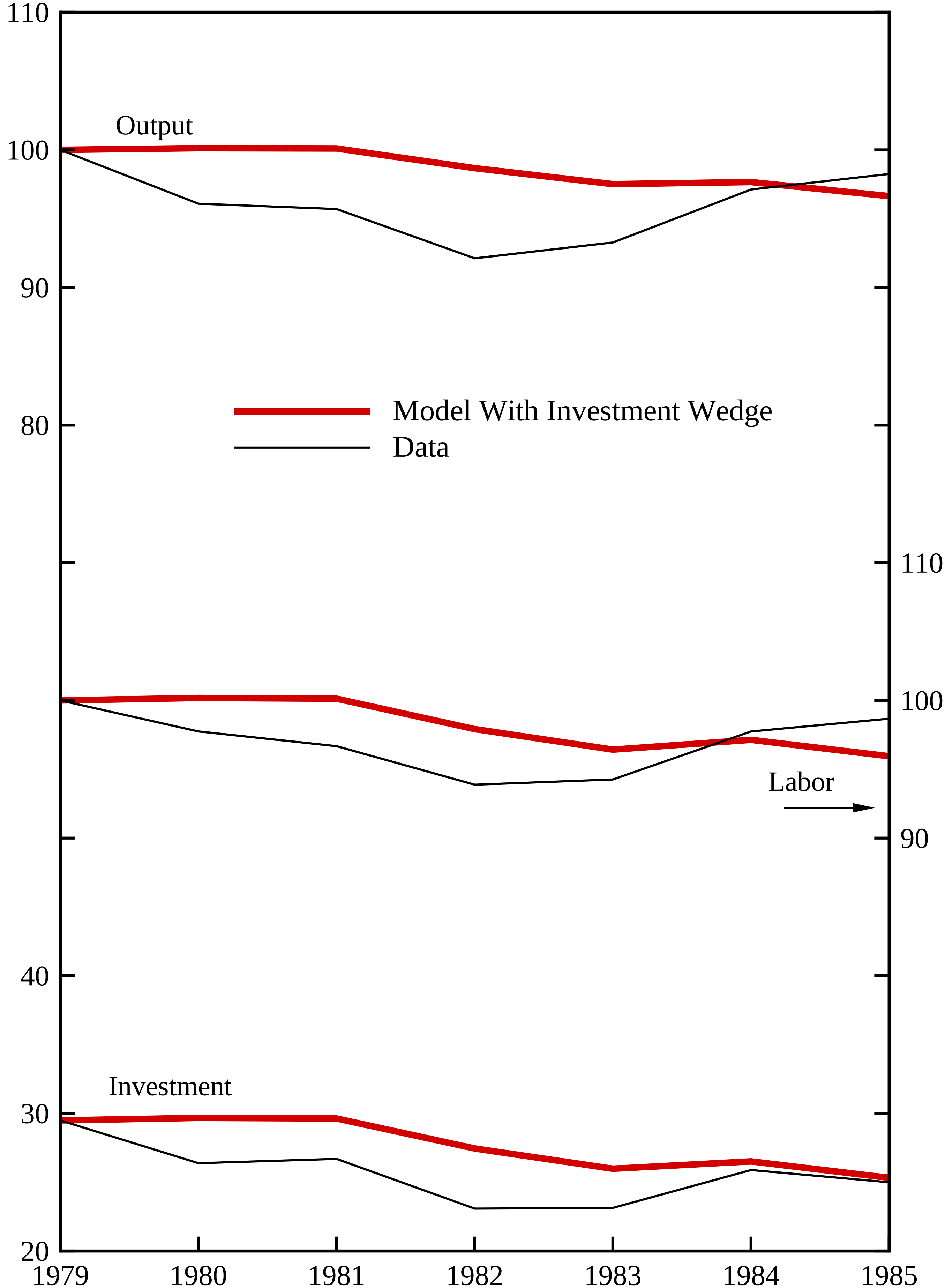
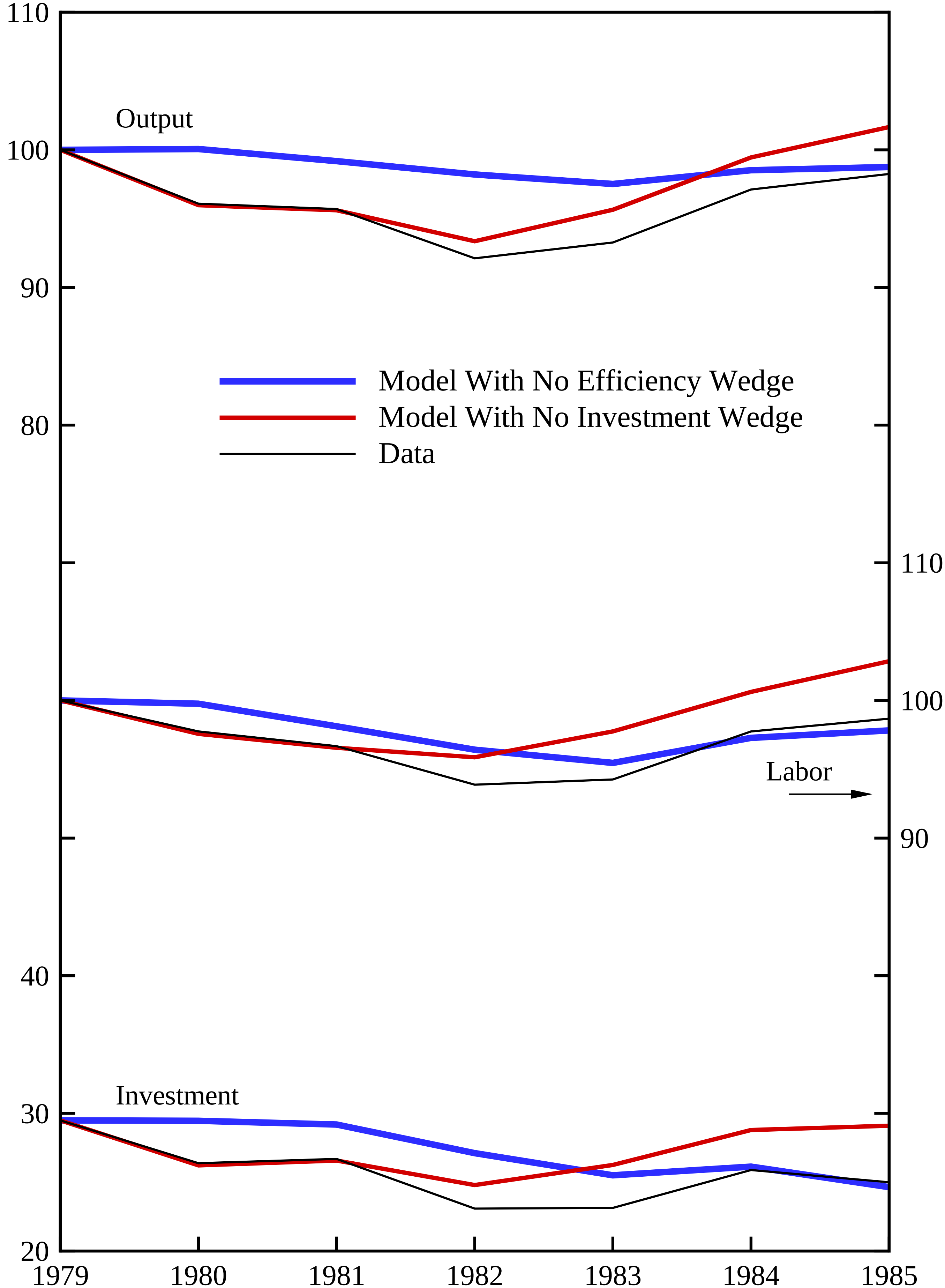


Figure 8
Data and Predictions of Models With All But One Wedge



Figures 9–12
Varying the Capital Utilization Specification
Great Depression Period, 1929–39

Figure 9
Measured Efficiency Wedges for Two Capital Utilization Specifications

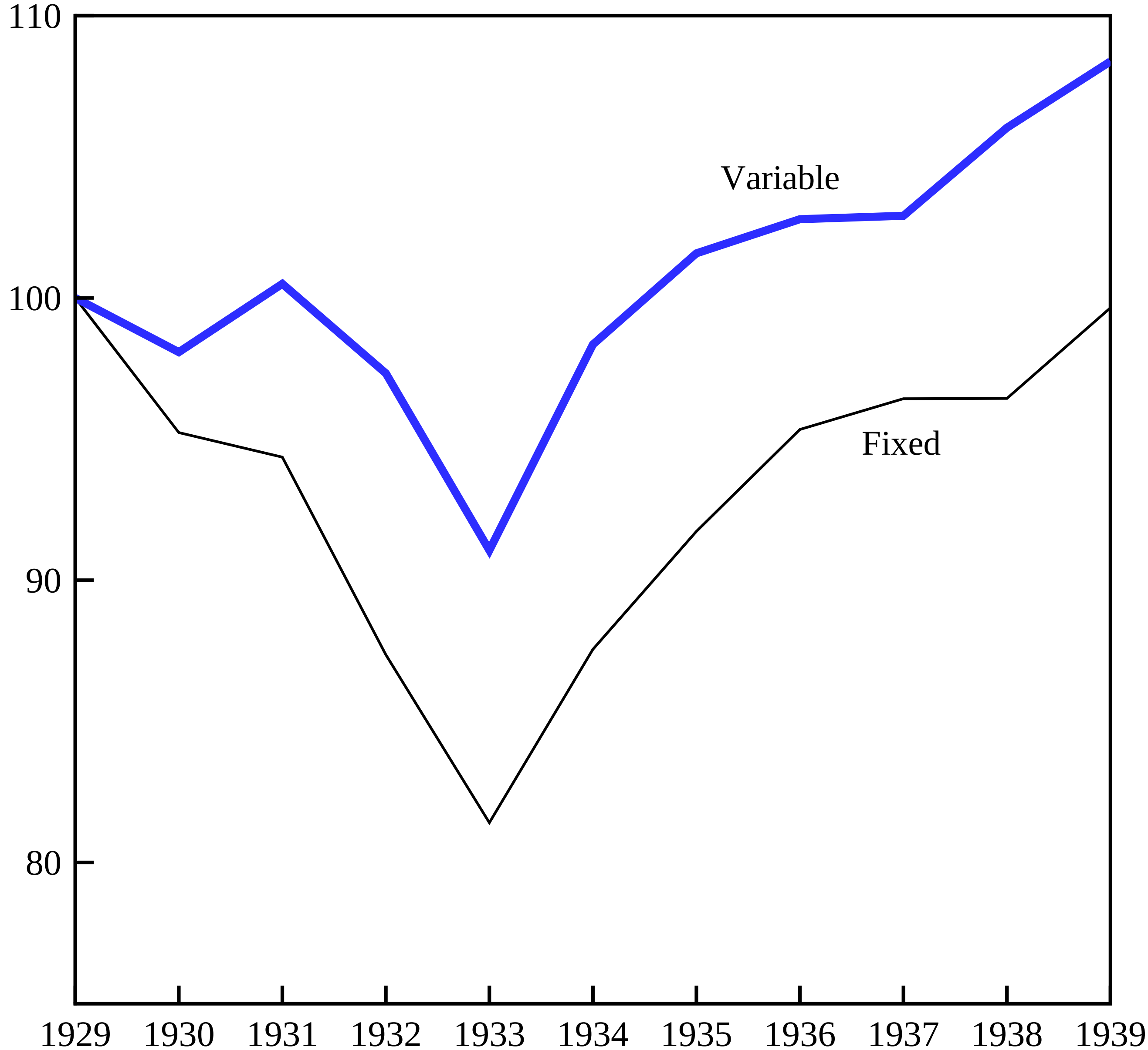


Figure 10
Data and Predictions of Models With
Variable Capital Utilization and Just One Wedge

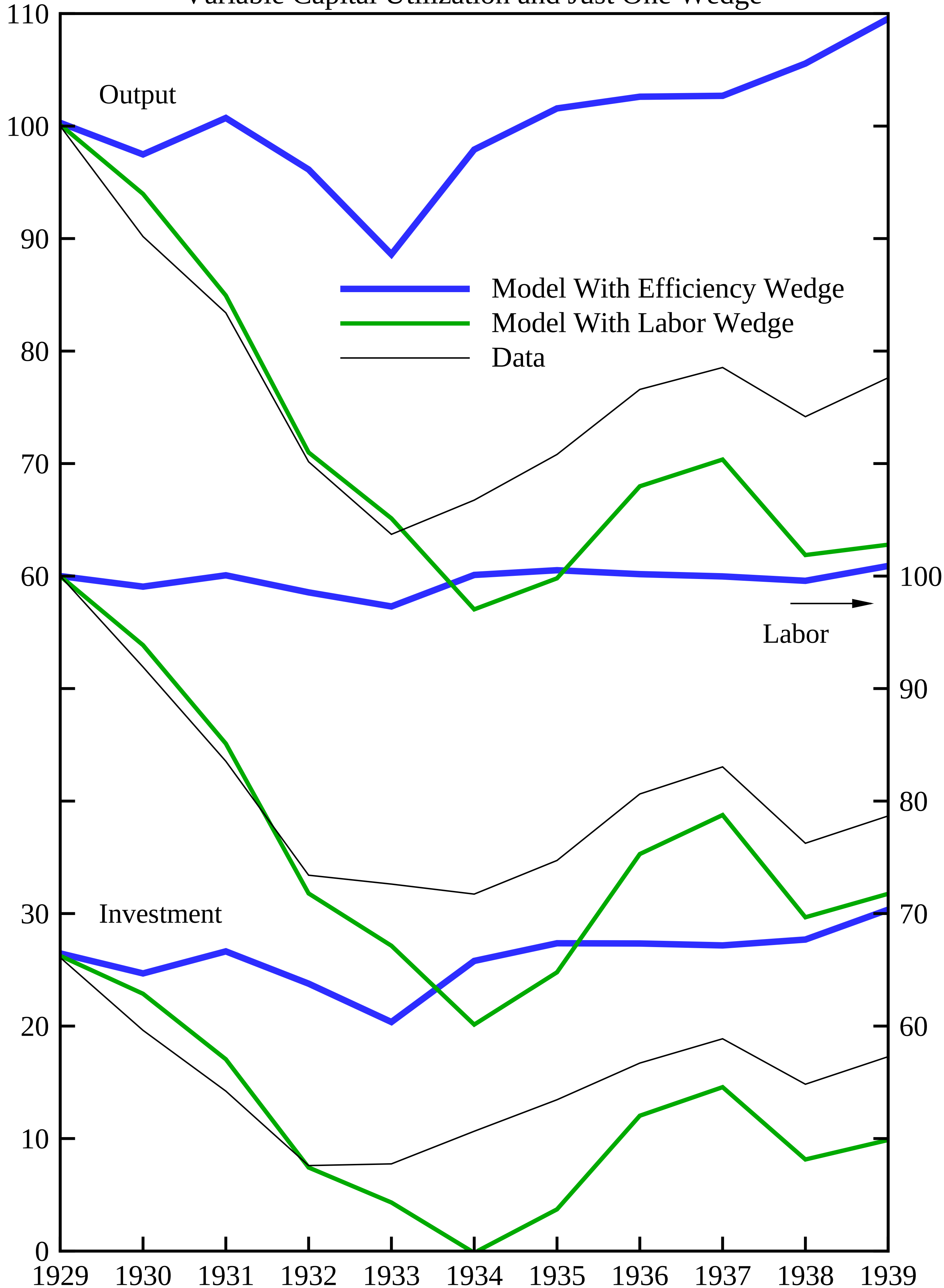


Figure 11
Data and Predictions of Model With
Variable Capital Utilization and Just the Investment Wedge

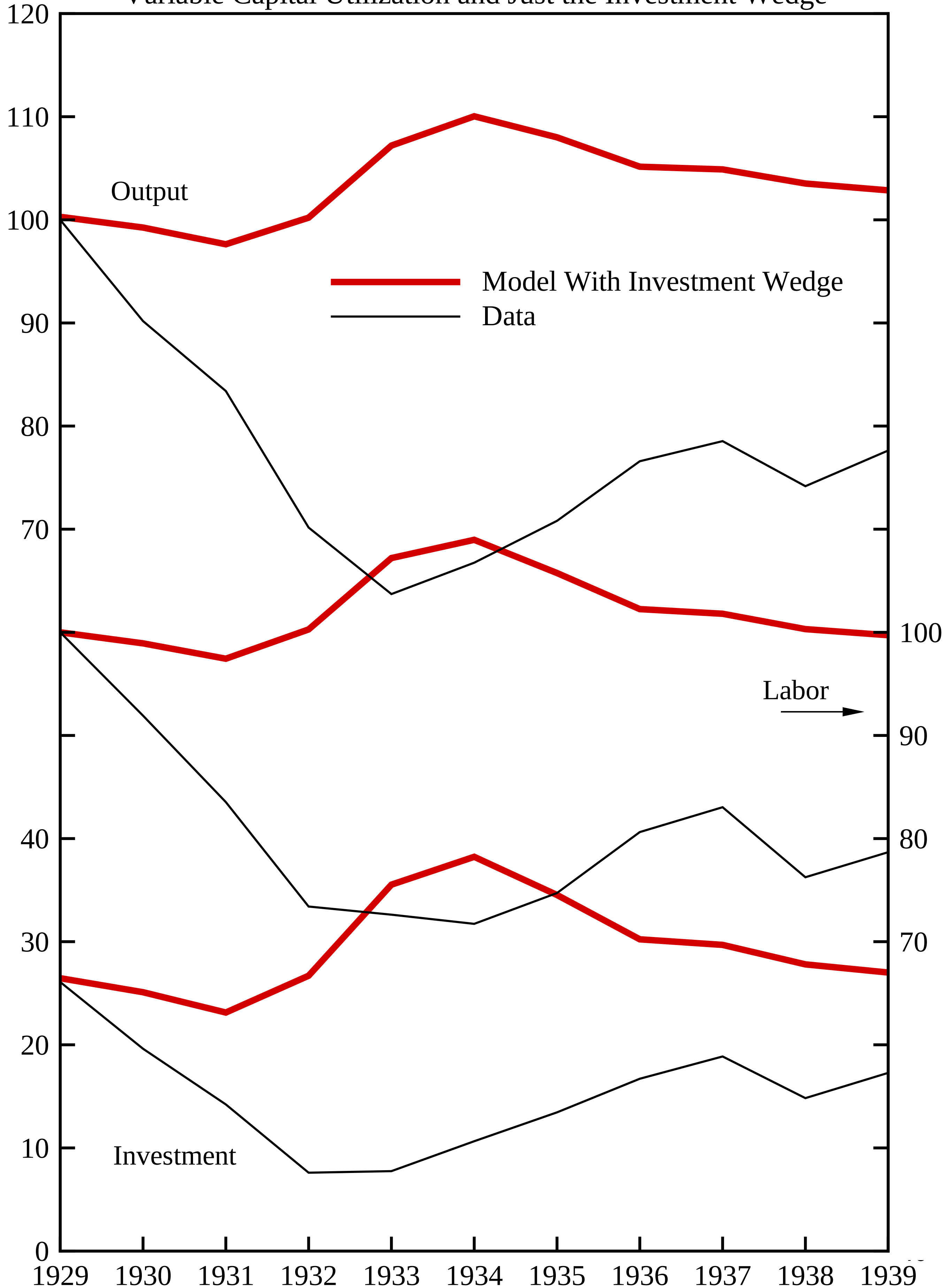
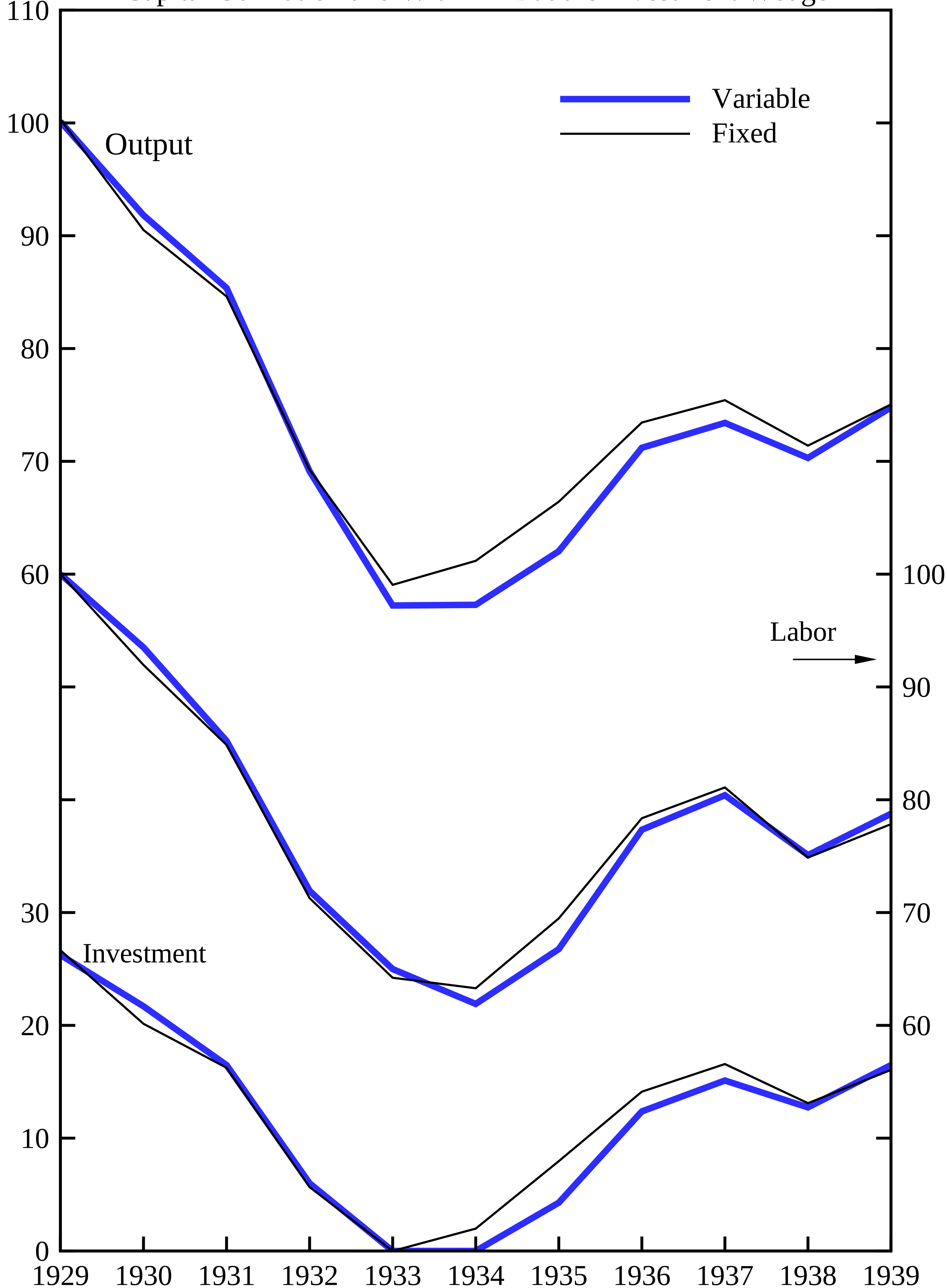


Figure 12
Predictions of Models with Fixed and Variable
Capital Utilization and With All But the Investment Wedge



Figures 13–14
U.S. Spectral Decompositions
in Pre- and Postwar Periods

Figure 13
Each Wedge's Average Contribution to the Output Spectrum, 1901-40

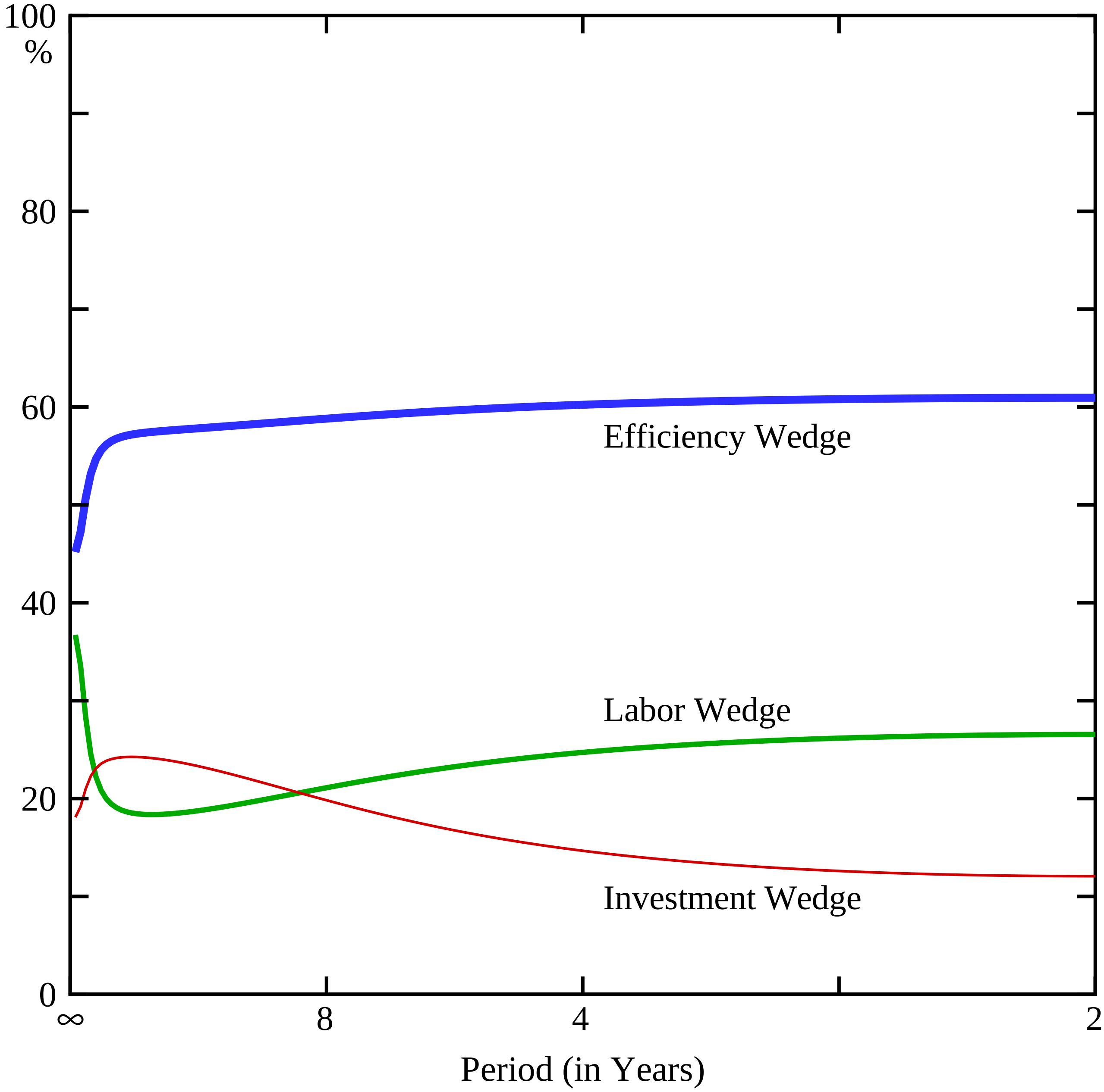


Figure 14
Each Wedge's Average Contribution to the Output Spectrum, 1955-2000