CONSUMPTION, ASSET MARKETS
AND MACROECONOMIC FLUCTUATIONS

Robert J. Shiller

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Abstract

A broad exploratory data analysis is conducted to assess the promise of a kind of model in which long-term asset prices change through time primarily due to consumption related changes in the rate of discount. Aggregate consumption data are used to infer ex-post marginal rates of substitution. Prices of stocks, bonds, short debt, land and housing are examined for the period 1890 to 1980. Methods are explored of evaluating this kind of model in the absence of accurate data on consumption.

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I. Introduction

How much do real discount rates move through time? By real discount rate I mean the interest rates implicit in asset prices, i.e. such that the expected present value with these discount rates of future dividends is today's price. Most people feel they have some idea how variable through time such discount rates are, and generally they feel discount rates are not highly variable. For example, most people feel that stock price changes are due primarily to changing expectations about future dividends rather than changing rates of discount. It is important to find out if this widespread feeling is based on some solid evidence. From a personal point of view, big movements in discount rates actually seem very plausible. In a recession, say, when output and consumption are some percent below their expected level a few years hence, it seems plausible that people might not be deterred by very high real annual rates (10%, 20% or even more) from borrowing to continue consuming at their usual level. Perhaps they cannot actually borrow at these rates due to institutional, legal or moral hazard reasons but they can easily sell their assets. Doesn't it seem plausible that long-term asset prices might drop 30%, 50% or even more in a deep recession, creating an expectation of a 10% or 20% return per year over the next few years as the price returns to a normal level? Selling stock in a recession to consume the proceeds (thereby foregoing the profit opportunity) is the equivalent of borrowing at these rates. If this seems plausible, then we might attribute most of the variability of stock prices to such discount rate changes.

What is meant by the above will be clearer when the theoretical framework is discussed below. The theoretical framework that I shall use here is simply that of maximization of an expected utility function of a form that is widely used in theoretical finance (for example, Merton [1973], Lucas [1978] and Breeden [1979]).

It is the same theoretical framework as that which inspired the model Sanford

Grossman and I used [1981] in a paper on the variability of stock prices, and which

Hall [1981], Hansen and Singleton [1981a], [1981b] and Mankiw [1981] also used to

study the behavior of stock market returns. This framework relates asset returns to

aggregate consumption. Grossman and I suggested that most of the variability of stock

prices might be attributed to information about consumption.

The bulk of this paper will be an exploratory data analysis of the kind advocated

by Tukey [1962] or Simon [1968]. Thus, I will try to try to present in a way useful to

the reader the broadest possible array of evidence relevant to judging the plausibility

of the model. This analysis should be of very general interest, i.e., of interest

from the standpoint of other models as well as the one considered here. Such

exploratory techniques seem especially appropriate here, since the way to convert

the basic theoretical notion into testable hypotheses about actual data is not at

all well established. I will thus try to portray in what ways the data seem to suggest

that real discount rates move a lot and in what ways the data do not seem to

suggest this, without reaching any final verdict. Thus, we will be interested in

empirical regularities which seem to support or weaken support for the model,

even if they apply only to certain time periods or to certain markets, and

even if the presence or absence of the empirical regularity is not proof or

disproof of the basic theoretical notion. This exploratory data analysis is an

adjunct to a more rigorous and more narrowly focussed study of the theory that

Grossman and I are currently producing.

Three substantive questions which I have distilled from numerous dis-

cussions about the model will be considered here in the course of study of

the model: whether the business cycle behavior of real short-term interest

rates, i.e., real returns on short-term debt, is in accordance with the model, whether the model can be evaluated if consumption data are not accurate or are not representative of the consumption of the wealthy minority who hold stocks, and whether prices of other long-term assets behave in accordance with the model, i.e., whether there is an appropriate correlation between price movements and whether the volatility of stock prices is too high relative to the volatility of other long-term assets.

In Section II below, the motivation for our work which emerged from previous work on the volatility of stock prices is briefly described. In Section III the model and some of its implications are reviewed. Data on stock prices as well as short-term interest rates are considered. In Section IV, tests of the model along lines suggested by Breeden [1979] and pursued by Hall [1981], Hansen and Singleton [1981a], [1981b] and Mankiw [1981b] are considered. It is shown to what extent the model can be evaluated even in the absence of data on consumption of stockholders. In Section V data on land prices, housing prices and long-term bond prices are considered. A summary of the findings (but, unfortunately, no definitive conclusion on the merits of the model) appears in Section VI.

II. Security Price Volatility

Some of my earlier work [1979], [1981a], [1981b] suggests that security prices are far too volatile to be accounted for by new information about future dividends alone (an analogous claim was made by LeRoy and Porter [1981]). That is, a model which makes the real price of a share equal to the present value of expected real dividends discounted by a constant real discount rate

would predict a much smaller variance for changes in price. Stock prices show

enormous volatility. Over the last century the standard deviation of the real

annual return on the Standard and Poor Stock Price Index was about 20 percentage

points. Roughly speaking, in a "typical" year the real value of the stock market

changes 20% one way or the other. What is it that's 20 percent different from

one year to the next that accounts for the price change? One way I used to show

graphically the potential importance of dividends in determining price was to

plot for the last century the perfect foresight or ex-post rational real price

per share P_{0t}^{*} : the present value in each year with a constant discount rate

of actual subsequent real Standard and Poor dividends, and of terminal price

at the end of the sample. If actual price P_t is the present value with the

constant discount rate of the mathematical expectation of dividends and terminal

price then $P_t = E_t(P_{0t}^{*})$, i.e., actual price is the mathematical expectation of

P_{0t}^{*} conditional on information available at time t. The real discount rate used

to compute P_{0t}^{*} was taken as the average real Standard and Poor return over the

sample. The P_{0t}^{*} so computed looks very much like a simple trend, and P oscillates

wildly around it. Both P_t and P_{0t}^{*} computed for the shorter sample period used in

Grossman and Shiller (1981) are shown in figure I. Here $P_{1980} = P_{0,1980}^{*}$ by

construction, In my paper (1981a) I tried to formalize in what sense the stock

prices were too volatile by showing that the standard deviations of detrended

stock price changes and dividends appear to violate an inequality implied by the

model. Since the detrending is a possible source of problems, I later showed that

the sample standard deviations of differenced stock prices and differenced dividend

series violate an inequality implied by the model (1981b). The volatility inequal-

ities are more robust to data errors, e.g., small errors in the consumption price

index used to deflate stock prices, than are regression tests of the fore-

castability of real stock returns. Although the use of the volatility inequalities

remain controversial at this date, I do not wish to get into the details of these

inequalities here, nor into the methodological issues raised by such tests, which
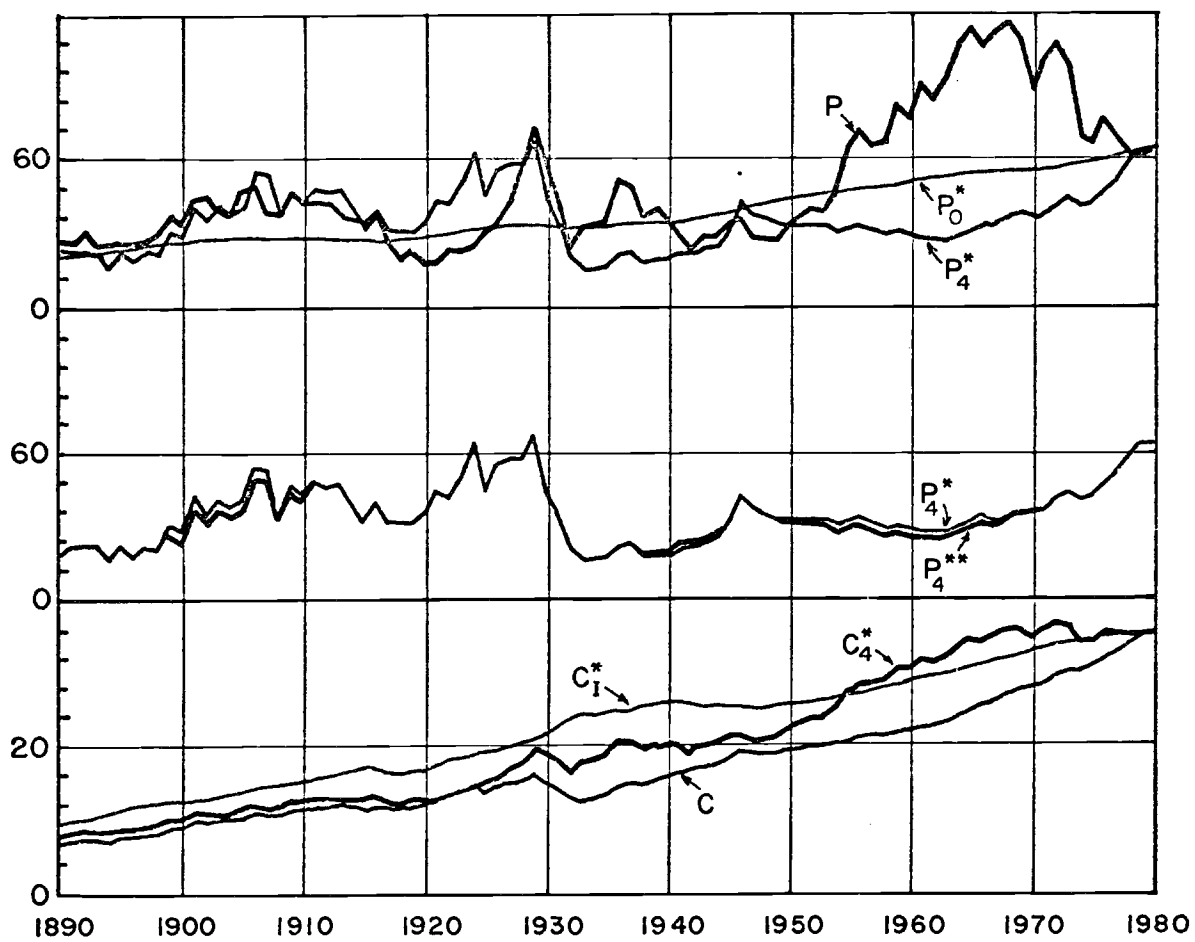
Figure 1 Upper plot: P: Real Standard & Poor Composite Stock Price Index,
    Annual Average in 1972 dollars; P^*_0: Present value with 6.5% discount rate
    of actual subsequent Standard & Poor real dividends and of actual price in
    1980; P^*_4: Perfect foresight or ex-post rational price assuming coefficient
    of relative risk aversion equals 4 where the tax rate assumed is that in the
    municipal corporate bond yield spread.

    Center plot: P^*_4 as in upper plot; P^{**}_4 the perfect foresight price
    computed not from actual real dividends but from the exponential trend of
    real dividends.

    Lower plot: C: Per capita real consumption on nondurables and
    services in thousands of 1972 dollars, C^*_4: Perfect foresight optimal con-
    sumption for the utility function assuming perfect knowledge of future stock
    prices and dividends; C^*_I: Perfect foresight optimal consumption assuming
    perfect knowledge of future real short term interest rates.

I have recently discussed elsewhere (1981b). I think it is possible to impress on the reader one basic outcome of this research which should not be controversial. It is quite clear from the values of P_t and P_{0t}^* alone as shown in figure 1 that over the last century high real stock prices did not tend to be followed by correspondingly high real dividends over the relevant horizon, and low real stock prices did not tend to be followed by correspondingly low real dividends. Thus, there is really no evidence in nearly a century of data to support the view that aggregate stock price movements represent evidence of future real dividend movements. It is still possible that real price is equal to the expected value of discounted real dividends if stock price movements reflect changing information about a disaster with low probability (e.g. nationalization) which did not occur in the sample. I discussed such a model elsewhere (1981b). The point is, there is no statistical evidence which would encourage us regarding this model.

I also showed (1981a) that while a model with time varying real discount rates could in principle account for the variability of stock market prices, these expected discount rate movements would (if standard deviations have been correctly measured) have to be very large. The calculations I made reflect earlier work on the volatility of long-term interest rates (1979). If real dividends are very stable, then corporate stock resembles an 'index-consol', and the one year return on stock resembles the one year holding period return on such a bond. If the holding period return has a standard deviation of about 20 percentage points and this standard deviation is to be attributed entirely to new information about one-period expected real interest rate then according to the analysis in that paper these expected one-year real interest rates would have to have a standard deviation of at least four or five percentage points. This would suggest a minimal plus or minus two standard deviation range for one-year exptected real interest rates of, say, from minus five percent to plus fifteen percent, or roughly in the range which I argued above seems plausible.

III Consumption and Utility Maximization

The assumption that individuals choose financial assets so as to maximize the expected value of an additively separable utility function in consumption throughout their lives has played an important role in recent literature on optimal portfolio composition in a dynamic setting (Merton [1973], Lucas [1978], Breeden [1979] and others). The assumption as it will be used here may also be consistent with Keynesian macroeconomic notions. While Keynes called his consumption function a "psychological law", subsequent literature has in some cases reinterpreted his theory in terms of utility maximizing behavior (e.g. Modigliani and Brumberg [1954]) though without the framework of rational expectations.

The assumption here (as in Grossman and Shiller [1981]) is that individuals choose to invest in freely tradable assets with the objective of smoothing their consumption, i.e. that individuals maximize the expected utility function of the conventional form:

$$E_t U_t = E_t \sum_{k=0}^{\infty} \delta^k u(C_{t+k}) \tag{1}$$

where $\delta = 1/(1 +\rho)$ is the subjective discount factor and ρ is the subjective real interest rate or rate of impatience, and $u(C_{t+k})$ is the utility of consumption at time $t+k$. The utility function depends on consumption from t to infinity, although individuals have finite lives. One might interpret the infinite utility function as a household utility function rather than an individual utility function, and thus that individuals have the utility of subsequent generations as an argument in their utility. Individuals may prefer something other than a smooth consumption profile over their lives, an important consideration with regard to studies of individual life-cycle saving behavior as in Modigliani and Ando [1963]. Our aggregate consumption data may be regarded as representing the consumption of a representative household whose average age is unchanging. Kotlikoff and Summers [1980] have established the importance of intergenerational transfers in saving behavior. Data on the changing age structure of the population may yet be incorporated into

the analysis in future research, without moving to the no-bequest life-cycle model which Kotlikoff and Summers criticized.

A first order condition for expected utility maximization is:

$$P_{j,t} u'(C_t) = E_t(\delta u'(C_{t+1})(P_{j,t+1} + D_{j,t+1})) \tag{2}$$

where $P_{j,t}$ is the ex-dividend real price of the jth freely tradable asset and $D_{j,t+1}$ is the real dividend. This says that the utility lost by forgoing consumption to buy a share at time t should, at the margin, equal the expected utility to be gained by selling the share next period and consuming the proceeds. In a world with income taxes, $P_{j,t+1} + D_{j,t+1}$ should be replaced with the after tax value at time t+1 of the investment in one share made at time t. Dividing both sides of (2) by $P_{j,t} u'(C_t)$ and taking this inside the expectation operator (a legitimate operation since $P_{j,t}$ and $u'(C_t)$ are known at time t), we find:

$$E_t(R_{j,t} S_t) = 1 \tag{3}$$

where $R_{j,t} = (P_{jt+1} + D_{jt+1})/P_{jt}$ is the return on the asset (if there are taxes, the after-tax return) and $S_t = \delta u'(C_{t+1})/u'(C_t)$ is the marginal rate of substitution between consumption at time t and consumption at time t+1. This expression (which may be regarded as the cornerstone of the consumption beta model of Breeden [1979], and Rubinstein [1976]), ought to be regarded as a "no profit opportunity" condition where "profits" are defined as an increase in utility. It thus ought to hold for all assets and for all individuals, even small investors who hold very little stock. Because this expression ought to hold for everyone, Breeden showed that we can aggregate over individuals and derive a relation between returns and aggregate consumption, and this aggregation will generally be valid even if individuals have heterogeneous information so long as aggregate S is common information (Grossman and Shiller [1981].) (These papers were couched in continuous time and the results hold only approximately in discrete time.) Since neither R_{jt} nor S is generally known at time t, we cannot express $E(R_{jt})$ in terms of $E(S_t)$. In the case of a one-period index bond, however, R_{jt} is known at time t and hence for such a bond

$R_{jt} = E(S_t)^{-1}$. With other assets whose real return is not known in advance, the covariance at time t between the return and S_t also influences the expected return. In fact, it follows immediately from (3) that:

$$E_t(R_{jt}) = E_t(S_t)^{-1} (1-COV_t(R_{jt},S_t)) \tag{4}$$

If one divides expression (2) by $u'(C_t)$ one gets (dropping the j subscript for brevity) a recursive expression for P_t:

$$P_t = E_t ((\delta u'(C_{t+1})/u'(C_t)) (P_{t+1} + D_{t+1})) \tag{5}$$

Here, $u'(C_t)$ is taken inside the expectations operator, which is a legitimate operation since C_t is known at time t. This is a first-order linear rational expectation model in P_t with a time varying coefficient (i.e. $\delta u'(C_{t+1})/u'(C_t)$ depends on t). It may be solved by recursive substitution. One merely substitutes the same expression led one period in place of P_{t+1}, which yields an expression in D_{t+1}, D_{t+2} and P_{t+2}. Since $E_t E_{t+1} = E_t$, we can dispense with E_{t+1} in the resulting expression. One then substitutes (5) led two periods in place of P_{t+2}, and so on. Under a terminal condition assumption that P does not explode through time we find that:

$$P_t = E_t P_t^* \tag{6}$$

where

$$P_t^* \equiv \sum_{k=1}^{\infty} S_t^{(k)} D_{t+k}$$

and

$$S_t^{(k)} \equiv \delta^k u'(C_{t+k})/u'(C_t)$$

which is the fundamental valuation equation in the Grossman-Shiller papers. Here P_t^* is the "perfect foresight price" which would be the price our theory would predict if both future consumption and future dividends were perfectly known. This P^* reduces to the P_0^* discussed above if people are risk neutral, i.e. the coefficient of relative risk aversion is zero and u(C) does not depend on C. Otherwise, P^* varies with consumption. $S_t^{(k)}$ is the marginal rate of sub-stitution between consumption at time t and consumption at time t+k. Since, in (6) for

P_{jt}, $S_t^{(k)}$ does not depend on j, the discount factors are the same for all securities, i.e. there is no risk premium in them.$\underline{2/}$ Since neither $S_t^{(k)}$ nor D_{t+k} is known at time t, the expectation operator E_t operates on products of random variables. Because expectations operators cannot be brought inside nonlinear functions, price cannot be written as the present value of expected dividends discounted by a vector of discount rates which is invariant across securities. In particular, even if a whole term structure of yields on index bonds were available, the price of stocks whose future real dividends are uncertain would not be the present value of expected real dividends discounted by these market real interest rates. Nor is nominal price the present value of nominal dividends discounted by the nominal interest rates of various horizons implicit in the nominal term structure of interest rates. Of course, it is always possible to represent price as the present value of expected dividends discounted by some discount rate series. One could in fact describe the equation (6) as asserting that price is the present value of expected dividends discounted by market real interest rates adjusted for a risk premium that is specific to a particular stock. The kth term in the summation in (6) for the jth asset can be written as:

$$E_t(S_t^{(k)} D_{j,t+k}) = E_t(S_t^k) E_t(D_{j,t+k}) + COV_t(S_t^{(k)}, D_{j,t+k}) = J_{j,t}^{(k)} E_t(D_{j,t+k}) \quad (7)$$

where

$$J_{j,t}^{(k)} \equiv E_t(S_t^{(k)}) + \phi_{j,t}^{(k)}$$

and

$$\phi_{j,t}^{(k)} \equiv COV_t(S_t^{(k)}, D_{j,t+k}/E_t(D_{j,t+k}))$$

thus, the appropriate risk premium $\Phi_{j,t}^{k}$ to be applied to the expected marginal

rate or substitution $S_t^{(k)}$ in arriving at the discount factor $J_{j,t}^{(k)}$ at time t for

$D_{j,t+k}$ is the covariance between $S_t^{(k)}$ and $D_{j,t+k}$ expressed as a proportion of its

mean. The simpler expression (6) is, however, probably more useful than (7).

Using some assumption about δ, the function u(C), and a single terminal

value for P_t^* one can observe historical values of P^* based on historical

dividend and consumption series. Let us adopt the assumption that u(C) equals

$C_t^{(1-A)}/(1-A)$ where A is the Arrow-Pratt coefficient of relative risk aversion, so

that $S_t^{(k)} = \delta^k (C_t/C_{t+k})^A$. 3/ Then, for a given A, P_{At}^* can be computed recursively

backwards from a terminal value by $P_{At}^* = \delta(C_t/C_{t+1})^A (P_{At+1}^* + D_{t+1})$. Grossman

and I [1981] plotted this P_{At}^* series for years since 1889 using the Standard

and Poor dividend series for D and the U.S. national income accounts/Kuznets real

consumption on nondurables and services per capita for C. We chose, arbitrarily, A=4

and then chose δ so that (3) held for sample mean. An analogous plot for A=4

appears in figure 1. This P_{4t}^* differs from that in our earlier paper in that it

is an after-tax P^*. If P^* were used in place of P_t to compute an after-tax return

R_t, then $R_t S_t$ would equal exactly one at all times. The tax rate used to compute

P_{4t}^* was the marginal tax rate implicit in the municipal corporate yield spread.

The tax rate used was one minus the ratio of the Bond-Buyer municipal Bond yield

average to a corporate bond yield average based on Durand-Homer and Moody data,

except for a few years at the beginning of the sample when, since the implied

tax rate would be negative, the tax rate was set to zero. The implied tax rate

was generally around 20% to 30% in the postwar period. The nominal capital gains

were assumed taxed at the then current effective long-term captial gains rate for

the marginal income tax rate. For most of the period this works out to half the

the income tax rate, after 1978 at .4 times the income tax rate. In practice,

the P_{4t}^* plotted does not look much different from the P^* computed assuming

no taxes which appears in the Grossman Shiller paper (1981b). The income tax

rates used here were 10% or less until the 1930's and did not reach 20% until

World War II. In the postwar period the P_t^* in the absence of taxes in nominal

terms is fairly smooth, so no big year to year movements in P_{4t}^* are induced

by capital gains taxation. The main effect of taxation on P^* is to cause the P_{4t}^*

series to drift down relative to a P_{4t}^* computed without taxes, so that taxes

cause P_{4t}^* to be about 25% lower in the 1950's and early 60's than it would be in

the absence of taxes.

The perfect foresight price P_{4t}^* resembles P_t fairly closely, and, with

this value of A, is about equally volatile i.e. short-run movements were of

about the same magnitude. Note that P_{4t}^* with A=4 is much more volatile than

P_0^* with A=0 (the constant real discount rate case), and thus new information

about discount rate movements would seem to serve as a more likely candidate as

a source of stock price movements than new information about dividends.

The motivation for this analysis was to answer the question: if the real value

of the stock market is 20% different from one year to the next, what other variable

is changing enough to cause this change? Apparently, if the value of A=4 is

reasonable, consumption has been such a variable.[4/]

What is surprising about these series is that there is also a substantial

similarity in the pattern of movements of P_{4t}^* and P, at least until the period after

World War II. We would have expected P_{4t}^* to be much more volatile than P and

not to show a close resemblance to P. The close resemblance suggests that there

is a sense in which a perfect foresight model has some explanatory power.

This similarity arises almost entirely due to the behavior of the consumption

related discount rate rather than the behavior of dividends. To highlight this fact, we also computed P_{4t}^* using, not the actual dividend series but in place of actual dividends a long-run exponential trend fitted to the dividend series. This series computed for A=4 and denoted P_{4t}^{**} is shown in figure 1 middle panel. It appears virtually identical to the P_{4t}^* computed from the actual dividend series, and thus we say that it is consumption and not dividends which accounts for the co-movement of P_t and P_{4t}^*.

We might elaborate on the similarity between P and P_4^* before the recent period. The 1891-2 market rally and 1892-94 market collapse are matched by corresponding movements in P^*, the 1899 market peak is matched by a peak in P_4^*. The sharply rising market between 1900 and 1901 is matched by a corresponding rise in P_4^*. The 1906 market peak is also a peak in P_4^*, as is the 1909 peak. The 1916-17 drop in the market is matched by a drop in P_4^*. In the period of the 20's the short run movements do not match (although the trend in both series is upward) and the P_4^* series shows an anomolous drop from 1924 to 1925 caused by a movement in the real consumption series. The 1925 drop in real consumption does not correspond to a decline as measured by the NBER reference dates, which made September 1924 a trough and October 1926 a peak. The drop is not in evidence in other measures of aggregate economic activity, such as industrial production or unemployment, and thus, may reflect an error in the Kuznets data. Both P and P_4^* reach the major peak in 1929 and drop very dramtically, although they do not bottom out quite together. The next major peak in P is 1936, (and P is fairly level into 1937) while P^* peaks in 1937, then both drop onto the recession of 1938. Here, while P_4^* corresponds to P in overall pattern the amplitude of the movement in P^* is smaller than that of P, a harbinger of the relatively stable behavior of P^* in subsequent years. The 1946 market peak is matched

pretty well by a peak in P_4^*.

The period since the early 1950's does not seem to reveal much similarity between P and P_4^*. One is struck by the dramatic hump shape in P and u shape for P_4^*. There are still some similarities in the short run movements. The first major postwar market peak, in 1956, is matched by a faint peak in P_4^* in the same year. The 1959 market peak is also matched by a faint peak in P_4^*. However, the 1961, 65, 68, and 76 market peaks show no counterpart in P^*. The only recent stock price movement which is predicted by P_4^* is the dramatic market drop from 1973 to 1974, however the actual market drop is 12 times larger than the drop in P_4^*.

An impressionistic description such as this of the resemblance between two series may sometimes see spurious patterns in the data. Some simple check of the significance of the correlation is in order. A simple measure of the short-run correspondence of the two series is the squared coherence between the series which is a sort of R^2 between the series as a function of frequency. The coherence was computed for the period 1889 to 1950 using periodogram averaging (computed without padding series with zeros) with a wrap-triangular filter of width 12 using the TROLL CROSPECT package. The coherence squared between P_4^* and P (both detrended) was above .47, the critical coherence squared at the five percent level, in the range of four to seven years and peaked at .74 for cycles of length six years. The same coherence pattern is found between C and P, since P_4^* is basically just a filtered version of C. If the entire sample is used from 1889 to 1980, the coherence is not significant anywhere.

A resemblance between P_{4t}^* and P_t may not seem altogether surprising, since P_{4t}^* is a funcion of aggregate consumption and since a correlation between the stock market and aggregate economic activity has long been part of the conventional wisdom. However, the resemblance between P_t and P_{4t}^* is much stronger than the resemblance between P_t and C_t. In order to make this clear, the

aggregate consumption per capita on nondurables and services is plotted in
figure 1 below the price series. The resemblance between C_t and P_t is indeed far
less obvious. The P_t^* series looks like a detrended version of C_t which has
been multiplied by a scale factor which makes its fluctuations as a percent of
P look A times bigger than the fluctuations in C_t as a percent of C_t. Linearizing
(6) around $C_t = C_{t+1} = \overline{C}$ and $D_t = D_{t+1} = D_{t+2} = \ldots \ldots \overline{D}$ and assuming $0 < \delta < 1$, we
find that in the absence of taxes:

$$P_t^* \simeq -\frac{A\overline{D}}{(1-\delta)\,\overline{C}} \sum_{k=1}^{\infty} \delta^k \Delta C_{t+k} + \sum_{k=1}^{\infty} \delta^k D_{t+k} \qquad (8)$$

So that P is approximately a moving average with exponentially declining weights
of future Δ C plus a moving average with exponentially declining weights of future
D. The gain from C to P^* is given by:

$$g(\omega) = \frac{A\overline{D}\delta}{(1-\delta)\,\overline{C}} \left\{ \frac{2 - 2\cos(\omega)}{(1+\delta^2) - 2\delta\cos(\omega)} \right\}^{0.5} \qquad -\pi < \omega < \pi \qquad (9)$$

and the phase angle is :[5]

$$\phi(\omega) = \text{Arctan} \left\{ -\frac{(1-\delta)\,\sin(\omega)}{(1+\delta)\,(1 - \cos(\omega))} \right\} \qquad 0 < \omega < \pi, \ -\pi/2 \leq \phi \leq 0 \qquad (10)$$

For δ very close to one, the gain is approximately constant and the phase is
approximately zero for all frequencies except those very close to the zero
frequency (the trend) where gain is approximately zero and the phase approximately
$- \pi/2$.

The phase angle function does not suggest anything like the conventional
notion that stock prices tend to lead measures of aggregate economic activity by
a few months to a year.[6] Such a notion could be represented by a phase
angle function which is a straight line from the origin with a positive
slope, while the phase angle (10) is negative and does not pass through the
origin. Therefore, the phase angle implies that stock prices slightly lag

behind consumption i.e. that stock prices tend to peak slightly later than consumption prices.[7/] If the component of consumption at ω is considered, then near the peak of the sine waves one can find a unit interval over which the change in the component is zero. Over this interval, return times δ must be unitary. Since price is high, the dividend price ratio is low which must be compensated by an increasing stock price.

The same resemblance between P_t and P_{4t}^* can also be represented in another way. We can compute, at least up to a constant of proportionality, what consumption would be if people knew future returns with certainty, and if stocks were the only asset. Then, the marginal rate of substitution between C_{t+1} and C_t times the after-tax return between t and t+k would equal one exactly. This will be satisfied by perfect foresight consumption C_{At}^* if:

$$C_{At}^* = C_T \prod_{j=0}^{T-1} (\delta \, R_{t+j})^{-1/A} \tag{11}$$

thus, if we computed C_A^* for t_0 to T and substituted the resulting series in place of C_t in (6), then the P_A^* computed would equal P . Conversely, if after-tax returns computed from P_A^* rather than P were substituted into (11) then the C_A^* computed would equal C. The C_A^* we have defined has the property $C_t^{-A} = E_t(C_{At}^{*\,-A})$ as can be verified from (3) by recursive substitution using the fact that R_t and C_{t+1} are known at time t. This property is analogous to that of $P_t = E_t(P_{At}^*)$.[8/]

The C_4^* computed from the after-tax real return on the stock market appears in figure 1, bottom panel, along with actual real per capita consumption C. The same A that was used to compute P_4^* was used to compute C_4^*, and so in effect C_4^* is an attenuated P plus trend while C is an attenuated P_4^* plus trend. Thus, the same short-run correspondence that was noted above between P and P_4^* appears between C and C_4^*, although the correspondence is harder to see since the movements are less conspicuous relative to trend and the trend is strong enough to cancel out some downturns observed in P and P_4^*. The δ that was needed to keep C_4^* with the same trend as C with this A was substantially greater than one ($\delta = 1.02$) and even with this

δ, C_4^* and C show some low frequency divergence; only shorter-run movements correspond.

The similarity between C_t and C_{4t}^* shows that to a substantial extent people behaved as if they knew all future stock returns and were optimizing their consumption given that information. This ought to seem quite remarkable to someone who thought that movements in the stock market make no sense. It would have seemed far more likely that individuals behaved as if they knew all future short-term real interest rates with certainty. Recall however that $C_t^{-A} = E_t(C_t^{*-A})$ where C_t^* is computed using the return on any asset. If C_A^* is computed using the one-year real after-tax return on prime commercial paper in place of the real return on stock (also shown in figure 1 for A=4, denoted C_I^*) one finds that there is little resemblance between this C_I^* and C_t. C_I^* is smoother than actual consumption and thus we cannot justify the magnitude of actual year to year fluctuations without a much larger coefficient of relative risk aversion A. A much larger A would not, however, make C_I^* resemble actual consumption since the short-run movements in C_I^* do not correspond to movements in actual consumption. The overall amplitude of movements in C_I^* is nearly as large as that of C reflecting the strong low-frequency component to real short-term interest rates. Again, however, the low frequency movements of C_I^* do not correspond to those of C.

IV Evaluating the Model Using Returns Data Alone

The model described above, and summarized in expression (6), is a relationship among three stochastic processes: price per share P, dividend per share D and consumption C. One may wish to base testing of the model on only two stochastic processes: the return per share (which is a transformation of the two stochastic processes price and dividend) R and consumption C. Or one may wish to test using returns R alone.

One may wish to test the model using returns and consumption data alone just because the likelihood function may be written in a simple form for these data. Since (6) involves expectations of sums of products, no simple distributional assumptions for D and C yield a simple joint distribution for P, D and C. For example, if D and C are both log normal processes, then the marginal distribution for P is a transcendental function. It is convenient, however, to write a joint lognormal distribution for R and C, although in so doing we lose the information in the separate P and D series, i.e., we lose information that was apparent in the plots of P_{0t}^{*} and P_{4t}^{*}.

As was noted in Grossman and Shiller (1981), with a lognormal assumption and two or more assets the two parameters A and δ are identified even if it is assumed that no information is available that is relevant to the conditional expectation. On the other hand, if other information variables are available, the parameters may be identified using data for returns on only one asset. Grossman and I have been working on estimation of A and δ using such methods.

Hall (1981), Hansen and Singleton (1981a)(1981b), and Mankiw (1981b) have also estimated A and δ using similar methods. Hall found, as did Grossman and I, that postwar data suggest implausibly high estimates of A. Hansen and Singleton, on the other hand, found values of A in the vicinity of one using a monthly series of recent consumption. Mankiw claimed to reject the model, as did Hansen

and Singleton for tests involving more than one asset with their monthly data,
on the basis that returns and consumption were forecastable in a way incon-
sistent with the model, i.e., that, in effect, $R_t S_t$ is forecastable. We can
interpret the negative evidence provided by Mankiw and Hansen and Singleton in
a number of ways without rejecting the basic theoretical model. It could be that
the consumption data are inaccurate. If the reported consumption is a moving
average, say, of actual consumption, consumption will have a spurious forecastability.
It could be that the information set they assumed is bigger than that actually used
monthly by consumers. It is costly for ordinary individuals to process information
on a monthly basis, and thus it seems a priori unlikely that they would do as
well as a vector autoregression in forecasting. Hall (1981) made a similar point
when he claimed that the expected stock market returns by his vector autoregres-
sive model were too variable to be plausible as expectations, and so he considered
a Bayesian alternative to the vector autoregressive model which biases
expected returns to the mean. Moreover, it could be that the model held for
earlier years even if it has broken down for recent years.

The problem with consumption data seems particularly troublesome. The
potential problems are bigger than those suggested just by data collection errors
and interpolation of some components. There are real conceptual problems in
national income accounting. Which expenditures qualify as consumption? Which
as some form of investment? It has been suggested that vacations are really
consumer durables which last a year or longer, in the form of memories. Moreover,
although the expression (3) should in principle hold for all people and thus,
as shown in Breeden (1979) and Grossman and Shiller (1981a), for aggregate consumption,
it may be that only a small percent of the population actually invests in stocks,
perhaps due to the fixed cost of acquiring information. Aggregate consumption may

thus be a poor proxy for the consumption of the generally wealthy minority who hold stocks. It may be influenced substantially be the behavior of a liquidity constrained segment of the population, as argued by Mishkin [1977]. It is worthwhile then to consider testing the model in a manner which is robust to errors in consumption data.

Suppose we do not use data on S_t. What restrictions on the stochastic properties of returns are implied by (3)? One easily sees that the relation implies that there are no riskless arbitrage opportunities. Any asset (or portfolio of assets) whose return R_t is known with certainty must, by (3), have return $R_t = 1/E(S_t)$. It is therefore impossible to earn a sure positive return on a portfolio whose total price is zero.

What, however, does the theory imply about risky profit opportunities? Suppose we have a random vector Z_t whose i^{th} element is the return on the ith asset and for which $\text{Var}(Z_t)$ is nonsingular. Lacking any information at all on the behavior of S_t, the basic first-order condition (3) implies no restrictions whatsoever on the mean or variance of Z.

The restrictions implied by the conventional mean-variance capital asset pricing model (CAPM) are not implied by the model. The CAPM asserts, for example, that any two assets with the same market portfolio covariance will have the same expected return. These two assets, however, may have different covariances with consumption, and it is covariance with consumption that matters. Given this theory, the only way it would make any sense to suppose that the CAPM ought to hold even approximately would be if we had information that the market return was a proxy for consumption.[9]/

The theory does not imply a one-factor model for returns, such as that considered by Ross [1976]. A one-factor model as it is used by psychologists and other social scientists, would assert that $Var(Z) = LL' + D$ where L is a vector and D a diagonal matrix. The essence of such a model is that for each element of the vector the variation not explained by the common factor is entirely specific to the element. We might take consumption as a common factor, but there is nothing in the theory which would suggest that there are not also other common factors which influence returns. There is no reason why there shouldn't also be industry factors, e.g. all railroad stocks tend to rise and fall together. Nor is there any implications from the theory that the first principal component accounts for a large part of the variance of returns, or that a spectral factor analytic model (Sargent and Sims (1977)) obtains.

The reason that there are no restrictions on the mean or variance of Z_t is easily seen. The expression (3) implies a restriction on the mean of each element of Z_t but the restriction involves an unknown parameter, $COV(S_t R_{it})$ which is specific to each element. In the absence of information about S_t this unknown parameter could be anything. It is true, of course, that positive definiteness imposes some restrictions across elements of a variance matrix. Positive definiteness can be described as requiring that all principal minors of $Var(S_t, Z_t)$ are strictly positive. Such restrictions are inequality restrictions. It is easily seen from the definition that all principal minors which involve $Cov(S_t R_{i_t})$ for any i also involve $Var(S_t)$, and that this term multiplies by another principal minor which is strictly positive. If we do not know the variance of S_t, then the inequalities are of no help whatsoever because they all involve an arbitrarily large positive unknown element. The reason why we were able to find testable implications for the case where some assets were perfectly correlated is that in that case some principal minors of $Var(S_t, Z_t)$ involve $COV(R_{i_t}, S_t)$ but do not involve $Var(S_t)$ and thus constitute usable restrictions, i.e. the "no riskless arbitrage" restrictions.

Hansen and Singleton (1981) have performed an interesting test of the model in the absence of both consumption and price index data. They noted that under the assumption that the logs of returns, prices and consumption follow a vector autoregression with the usual homoskedastic residuals, the difference between the log nominal return of the ith asset and the log nominal return of the jth asset is unforecastable. Thus, their maintained hypothesis embodies the assumption that expected returns change through time in an observable way but expected covariances do not. The fact that they were able to forecast these excess returns might mean that the theoretical model is wrong, or that the covariance matrix of residuals changes through time.

The problem we face, of course, is not that we have no information about S_t but instead that we have limited information about S_t. We observe a consumption series which may be inaccurate but we have some idea about how actual consumption behaves from the data. If we really want to form an opinion regarding the value of the model we want to use what prior information we have. It may be that the model can be made consistent with the data only if $\sigma(S)$ is implausibly large. We can derive inequality restrictions on $\sigma(S)$ in terms of stochastic properties of observable variables, and compare these restrictions with our priors on $\sigma(S)$. Another way of describing this approach is that even where no riskless arbitrage opportunities exist there ought to be some way of ascertaining whether "approximate arbitrage" profit opportunities exist. We have no way of deciding whether a profit opportunity is approximately an arbitrage opportunity until we have some way of claiming that certain discrepancies are implausibly large.

One way of putting a lower bound on $\sigma(S)$, in effect, was already noted above which appears in my earlier paper (1981). Using data on dividends and stock prices and using a linearization of the present value relation (6) in terms of dividends and discount rates, I was able to put a lower bound on the standard deviation of the expected real interest rate or marginal rate of substitution in terms of the standard deviation of dividends and the standard deviation of

prices.

It is also possible to arrive at a lower bound on the standard deviation
of the marginal rate of substitution without the linearization and using
data on asset returns alone. From expression (4) and the definition of the correla-
tion coefficient we have for the ith asset:

$$E(R_{i_t}) = E(S)^{-1}(1 - \rho_{is} \ \sigma(R_i)\sigma(S)) \qquad (12)$$

where ρ_{is} is the correlation coefficient between the return on the ith asset and the
marginal rate of substitution. Using this expression for the ith and j^{th} assets,
solving for $\sigma(S)$ and using the fact that correlation coefficients are between
minus one and plus one, one finds that

$$\sigma(S) \geq \frac{E(R_j) - E(R_i)}{\sigma(R_i)E(R_j)+\sigma(R_j)E(R_i)} \qquad (13)$$

This inequality puts a lower bound on the standard deviation of S in terms only
of the means and standard deviations. The inequality holds for $E(R_j) > E(R_i)$ when
$\rho_{js} = -1$ and $\rho_{is} = +1$. A stronger inequality can be derived which makes use of the
covariance between R_i and R_j, but this inequality is simpler. This inequality asserts
that if two assets have very different average returns and their standard deviations
are not sufficiently large, then $\sigma(S)$ must be large if the covariance with S is to
explain the difference in average returns. If one uses the Standard and Poor portfolio
as the jth asset, and prime 4-6 month commercial paper as the ith asset, and sample
means and standard deviations of after-tax real one year returns for 1891 to 1980
in the right hand side of the above inequality, then the lower bound on $\sigma(S)$ is
0.20. Thus, a four standard deviation range for the one year marginal rate of
substituion might be from .6 to 1.4. The large standard deviation for S
arises because of the large difference between the after tax average real
returns on stocks,($E(r_j) = 1.057$ or a rate of return of 5.7% per year for 1891
to 1980) and average after-tax real return on commercial paper ($E(R_i)$=1.014 or

a rate of return of 1.4% per year for 1891 to 1980) while the standard deviations or the real after-tax returns are not sufficiently high (.154 for stocks and .059 for commercial paper) to account for the average return spread unless $\sigma(S)$ is very high. A high $\sigma(S)$ suggest a high coefficient of relative risk aversion A, since $\sigma(S) \simeq A \sigma(\Delta C/C)$. For 1891 to 1980 $\sigma(\Delta C/C)$ was .035, so a lower bound for $\sigma(S)$ of .20 suggests A be over five.

Of course, expected returns and standard deviation of returns are not precisely measured, even in a hundred years of data. An asymptotic standard error for the estimate of the right hand side of the inequality (12) made by substituting sample means and sample standard deviations into the expression, assuming the covariance between $\hat{\sigma}(R_i)$ and $\hat{\sigma}(R_j)$ equals zero, was .078. Thus the estimated lower bound for $\sigma(S)$ is only two and a half standard deviations from zero. Moreover, the asymptotic standard error may not well measure the true standard error in small samples if data are distinctly nonnormal. On the other hand, further research along these lines might put a tighter lower bound on $\sigma(S)$ using a stronger inequality that makes use of the observed covariance between R_i and R_j, or using data on more than two assets or using additional information variables. Even without doing this, however, we see that the conventional notion that stocks have a much higher return than does short debt coupled with the notion that pre-tax stock real returns have a standard deviation in the vicinity of 20 percent per year, commercial paper much less, implies that the standard deviation of S is very high.

V An Exploratory Look at Data on Other Asset Prices

The notion that corporate stock price movements are primarily due to large real discount rate movements suggests that there ought to be other assets whose price movements closely match those of stocks. If there is any asset whose real "dividend" follows a growth path, then its ex-post rational price will be proportional to the P** shown in figure 1. Moreover, any asset whose real dividend shows noisy movements around a trend which are of the same order of magnitude as the movements of real dividends of corporate stock around their trend ought to have an ex post rational price which closely resembles the P_4^{**} shown in figure 1, since as was noted above, the P_4^* for corporate stocks shown in figure 1 closely resembles P_4^{**} . If an asset exists whose P* resembles that of corporate stocks, then we would expect its price should closely resemble that of corporate stocks, since $P_t = E_t(P_t^*)$. The problem we face, however, is that of finding such alternative assets. The nonexistence of index bonds or close substitutes for such assets has long been lamented. In this section, I will review the most obvious candidates for such alternatives, and discuss the adequacy of the data.

Price of Land   A measure of the price of land can be found in the land value series in the year end outstanding table for National Net Worth (Consolidated Domestic Net Assets at Current Cost) of the Balance Sheets for the U.S. Economy provided by the Flow of Funds Section at the Board of Governors of the Federal Reserve System. Since, in contrast to other assets, the quantity of land is fixed a total value series serves also as a price index. The real series (i.e. the series divided by the consumption deflator used in the preceding analysis) is plotted in figure 1 as series $P_{L,FRB}$. The series may give a rough indication of the price of land. Unfortunately, land and structures are not generally marketed

Stock Prices

Land Prices

$P_{L,USDA}$

$P_{L,FRB}$

$P_{H,CPI}$

$P_{H,CENSUS}$

$P_{H,GREBLER}$

$P_{H,FISHER}$

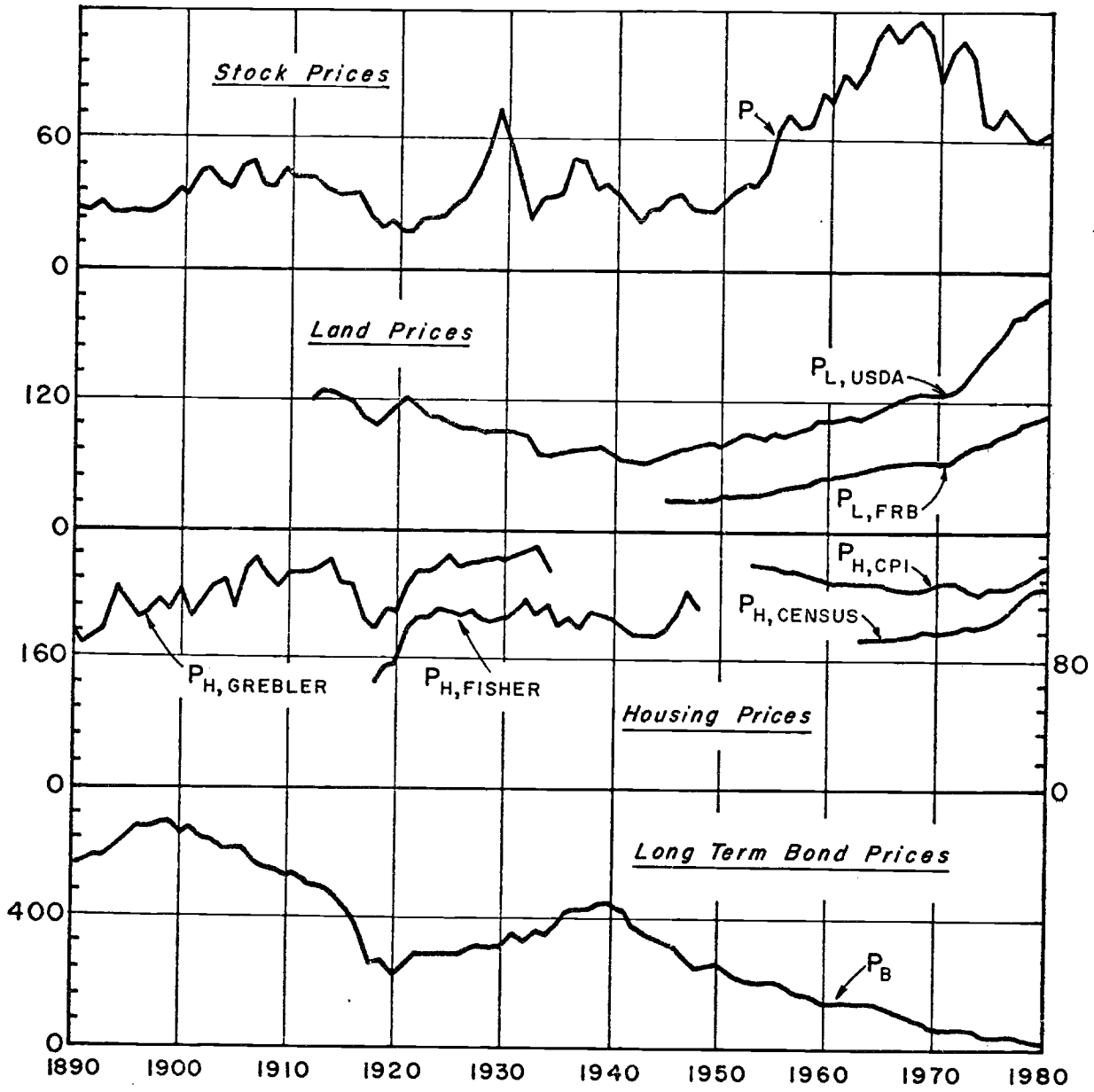Housing Prices

Long Term Bond Prices

P_B

Figure 2: Real prices of corporate stock, land, housing, and long-term bonds.
All series are divided by the consumption deflator for nondurables and services
(1972 = 1.0). Upper panel: P, Standard & Poor Composite Stock Price Index.
Second panel: $P_{L,FRB}$, Federal Reserve Board Land Value series from the Balance
Sheets for the U.S. Economy, year and outstandings, National Net Worth, in ten
billions of dollars; $P_{L,USDA}$, the USDA index number of average value of farm
real estate per acre 1967 = 100; third panel: $P_{H,Grebler}$, Grebler-Blank-Winnick
price index for one-family owner occupied houses 1929 = 100; $P_{H,Fisher}$, Fisher's
median asking price for existing one-family houses, Washington, D.C. in hundreds
of dollars; $P_{H,CPI}$, home purchase component of consumer price index for all urban
consumers, 1967 = 100; $P_{H,Census}$ Census Bureau price index of new
one family houses sold, 1972 = 100 (The scale for $P_{H,CPI}$ and $P_{H,Census}$ is at
right);bottom panel: P_{B}, Price of a 5% 25 year bond whose yield is the Macaulay
Railroad Bond Yield Arithmetic index (1890-1935) and Moody Aaa Corporate Bond
Yield Average (1936-80).

separately, so the price is mostly fictitious. Every five years the censuses

of governments provide estimates of total market value of real estate based on

assessed value and a sample of sales during the census year. This value includes

both structures and land. For these years the nonagricultural component of the

Federal Reserve land value series is based on the total value of real estate

minus the Commerce Department estimate of the total value of structures, i.e.

Fixed Nonresidential Business and Residential Properties. The Commerce Department

figure is computed on a perpetual inventory method based on data on investment in

structures, an assumed depreciation rate, and current sales price data. The

Federal Reserve estimates land values for intercensus years based on an interpolated

ratio of land value to structures value in census years, and on the value of

structures series. While the Federal Reserve Series may be the best we will ever

have for total land value, no such perpetual inventory method can accurately

capture the current market value of structures. Thus, the Federal Reserve total

land value series is useful only as a rough indication of actual land values.

Perhaps there are fewer problems in measuring the price of agricultural land.

The U.S. Department of Agriculture publishes an index of average value of farm real

estate per acre, in a series which extends back to 1912. The series divided by

the price index is plotted in figure 2 as series $P_{L,USDA}$. While this index is

the price per acre of farms rather than of farm land, the structure component

of farm value is likely to be less variable than is the structure component

of aggregate national real estate value. The series is based on reports from

the regular crop reporters, who are asked to estimate the prevailing market value of

farmland in their area (USDA, (1970)).It is reasonable to suppose that the reporters

base their estimates on recent actual sale prices. Since the reporters are
asked to exclude from consideration any land in other than agricultural
uses, but include land whose price may be affected by the prospect of other
uses the series suffers from a sort of data truncation problem. The total amplitude
of the variation in real agricultural land prices is on the same order of
magnitude as that of stock prices. Thus, for example, with the data shown in figure
1, the ratio of the highest real value to the lowest real value in years since
1945 was 3.1 for agricultural land, and 4.2 for corporate stocks, while for
the years before 1945 the corresponding numbers were 2.00 for land and 3.9 for
stocks. The standard deviation of the annual percentage change in the real price of land.
was 8 percentage points in contrast to a standard deviation of percentage change
in real stock prices of 17 percentage points over the same sample period. Land
prices, in contrast to stock prices are serially correlated. The correlation from
1913 to 1980 of the percentage change in real agricultural land prices with the
lagged percentage change was .44, which was significant at the .001 level. The
serial correlation of price change is not prima facie evidence that returns are ser-
ially correlated, since we do not have data on land rents, however it suggests that
the land price series may be artificially smoothed.

Prices of Housing   A number of housing price series are available. I shall
discuss here only those which relate to prices of finished homes rather than
the construction cost indices. The most widely cited series are the average or
median price of new homes. A monthly series (beginning in 1963) produced by
the Federal Home Loan Bank Board appears on the back page of the Wall Street
Journal on Mondays. This series is just the average price of a sample of new
homes financed by conventional mortgages. The Bureau of the Census also publishes
(in Construction Review) an average sale price of new homes sold based on a sample
which is not confined to homes sold with conventional mortgages. Another series by

the National Association of Realtors, which begins in 1966, gives the median

sales price of existing single family homes.

None of the above series controls for quality changes in housing sold. Two

recent series are available, however, which do make a correction for quality

changes. The first is the Census Bureau's price index of new one-family houses sold

which measures the sale price of houses which are the same with respect to ten im-

portant physical characteristics as the houses sold in the base weighting period

1974. This series which is plotted in real terms (i.e. divided by the consumption

deflator in figure 2 and denoted as $P_{H,CENSUS}$) goes back only to 1963. Another

series which purports to measure roughly the same thing is the home purchase com-

ponent of the Consumer Price Index. This series which is plotted in real terms,

denoted $P_{H,CPI}$ in figure 2, is intended to represent the price of both new and

existing homes which are the same with respect to two characteristics; age and

square footage of living space. The series does not reflect costs of financing,

taxes and insurance or maintenance and repairs, which are separate components of

the Consumer Price Index. Unfortunately, the annual average real series, when

converted to real terms, bears virtually no resemblance to the annual average

real Census Bureau series. Notably, $P_{H,CPI}$ declines 3% from 1963 to 1973 while

the $P_{H,CENSUS}$ series rises by 10% over the same period. Moreover, $P_{H,CPI}$ rises

only 4% from 1973 to 1978, while $P_{H,CENSUS}$ rises by 20%. This could be a reflec-

tion of actual relative price changes between the new and existing homes in the

CPI sample and the new homes in the Census sample. John S. Greenlees (1981) has

shown, however, that when a methodology is used which is closer to that of the

Census Bureau to extract an index from the CPI data, the resulting index is

much closer to the Census index. Over the 1973-8 period which he studied, he

resolved the discrepancy between the growth rates. He attributed a large part of

the difference to the fact that the CPI series is derived only from data on

homes sold with FHA mortages, and FHA mortages are subject to a comparatively low

ceiling on principal, a ceiling which is revised from time to time. For example,

the ceiling was raised from $33,000 to $45,000 in 1974, then from $45,000 to

60,000 in 1978. Since mortgage value is closely related to sale price, the CPI data suffer a severe truncation problem which is dealt with only by "linking" out the ceiling change dates.

One prewar housing price series also exists which in effect attempts to control for quality. This series, plotted in real terms in figure 2 as $P_{H, GREBLER}$, constructed by Leo Grebbler, David Blank and Louis Winnick (1956), is derived from data collected in the Financial Survey of Urban Housing conducted by the U.S. Department of Commerce in 1934, which covered 22 cities widely scattered geographically. Each owner of a one-family owner-occupied house was asked the value of his house in 1934 and the year and price of acquisition. The index is based on the median for each year for the homes acquired in that year of the ratio of price of acquisiton to the 1934 price. This median is converted to an index after correcting for a trend rate of depreciation net of improvements. If the trend is correct, then this series should accurately represent the price of a home of constant quality. To the extent that there are systematic intertemporal variations in the aggregate resources expended to maintain and improve homes, the year to year fluctuations in price will also be misrepresented. Other sources of error in the series may come from homeowners' misreporting of data. One suggestion that such errors may be important is the tendency observed by the authors for respondents to report acquisition years (for the earlier years) which are multiples of 5. A more important source of error may be the failure to accurately report the 1934 price. Home owners who did not sell in that year may have only vague impressions as to the current market price of their homes. The authors argued that this causes no bias in the index for years other than 1934 unless "the degree of underestimate or overestimate of value in 1934 were correlated with length of holding." There is reason to suspect however, that the error is correlated with length of holding. If homeowners bias their estimates toward

the acquisition price, the nominal home price series will be less variable

than it should be, and the real price biased in the direction of the reciprocal

of the price index.

Some idea of the importance of these problems can be obtained by looking

at another index of housing prices which, while not quality controlled at all,

does overlap with the $P_{H\ GREBLER}$ series. This series, constructed by Ernest

Fisher (1951) plotted in real terms as $P_{H,\ FISHER,}$ is just the median asking

price of existing one family homes in Washington, D.C. as advertised in newspapers.

The series does not closely resemble the $P_{H\ GREBLER}$ series, but the overall pattern

is similar enough to suggest some confidence in the series. For example, one is led

to suppose that real housing prices did not fall in the Depression as did real

stock prices.[10] However, with real estate, asking prices may in depressions be

upward biased measures of market prices. According to Hoyt (1933 , p. 402),

at this time, "normal sales of real estate have ceased. There is a considerable

number of 'transfers,' but these are mostly conveyances to relatives to avoid

judgments," etc.[11]

The recent housing price indexes look much different from the old, in that

the new price series are much smoother. The standard deviation of the annual

percentage change in the real Census housing price index is only 2.3 percentage points,

compared with 8.2 percentage points in the Grebler Series.

Another possible error in all series might arise since sales contracts

may involve more than just transfer of the house, so that the price reflects

such other factors. Assumable long-term mortgages may be transferred along with

the house. The value of such mortgages depends on the level of interest rates.

Owners may sometimes finance the sale themselves at nonmarket rates thus in

effect subsidizing the purchase of their home. Prices of houses sold at fore-

closure may be bid up to the full amount of the mortgage by the mortgagee to prevent

other creditors of the defaulting owner to secure the property for a smaller

amount (Hoyt (1933)).

Prices of Long-Term Bonds   A bond price series is much easier to find and likely

to be more reliable than the land price or housing price series. To construct

this series, a bond yield average was constructed by joining the Macaulay January

railroad bond yield arithmetic index 1890 to 1935 (Macaulay [1937]) to Moody's Aaa

Corporate bond yield average for the second trading day of the year for 1936 to

1980. These were yields to maturity on long-term bonds. The average time to

maturity in the Moody series ranged from 20 to 35 years. A bond price series was

taken as the price of a 25 year 5% bond with this yield, and is plotted in real

terms in figure 2, bottom panel. The real dividend for this series is of course,

proportional to the inverse of the price level. Thus, the real dividend behaved

as a more or less steady downtrend except for the years 1920 to 1933. An ex post

rational real bond price with a constant discount rate (i.e. a P_0^*) was computed

with the discount rate equal to average real return on bonds. Since the average real

return on bonds was so low, only 1.3%, the moving average was extremely long and thus

P_0^* was virtually a steady downward trend.

Comparison of Stock with the Other Price Series. A comparison of the four major

asset prices: stock prices, land prices, housing prices and bond prices, shown

in figure 2 shows no striking similarities at any frequency in the deviations

of the series from their long-term trends. We have seen that there are serious

problems with the land and housing price series, and one might attribute

the lack of similarity to the data errors. The data problems are perhaps

unlikely to be responsible for such things as the failure of land and housing

prices to fall in the stock market crash of 1929 to 1932, or in the recent

stock market crash of 1968 to 1978. The recent discrepancy between stock prices

and housing prices has been attributed to tax effects (Feldstein (1980a),(1980b),

Summers (1980))however this particular explanation cannot plausibly account for

discrepancies in the pre World War II period. One might also attribute the dis-

crepancies to divergence in the unobserved "dividend" series for land and housing.

For housing, which is a reproducible asset, one might have reason to suspect that the effective dividend was systematically related to the discount rate. In an extreme case where an asset is instantaneously producible and consumable its price should always be constant in terms of consumption. However, housing is hardly instantaneously producible and consumable, and so one would expect that its real price should have fallen in times of sudden economic distress, such as the Great Depression. Perhaps the data are at fault for this period.

The standard deviation of the annual percentage changes of real price in the land and the earlier housing series was a little over half the standard deviation of the percentage change in the real stock price series. The standard deviation of the percentage change of the more recent hedonic housing price indexes in real terms was much lower. These differences in standard deviations are not what one would have expected if one assumes that the "dividends" of land and housing were as trendy as the dividends of stock. The data on long-term bonds, also show a lower percentage real price change from year to year. The standard deviation of a real annual long-term bond return series from 1890 to 1980 is between a third and a half of that on stocks real returns. Since the real dividend on bonds shows a marked downtrend, in contrast to the uptrend in corporate stock real dividends, long-term bonds are in effect a much shorter-term asset than stocks. One might conceivably attribute the smaller stnadard deviation of real returns on bodns relative to stocks to their shorter duration.

VI Summary and Conclusion

I sought in this paper to explore a number of alternative avenues to judging whether there is an element of truth to the notion that large variations in real discount rates are responsible for asset price fluctuations. Some evidence was found which seems encouraging, some of it not so encouraging, none of it clearly decisive.

The most encouraging piece of evidence remains the fact, already noted in my paper with Grossman, that until recently actual stock prices resembled ex-post rational stock prices defined in accordance with the model. The ex-post rational stock price was redone here to take account of taxation but this had no important effect on the conclusion. The same resemblance was also presented here in another way: an ex-post rational consumption was computed which is the utility maximizing consumption given perfect foresight about future stock returns. While the ex-post rational consumption showed some long-term drift relative to actual consumption there were (until recently) a number of corresponding short - term movements in the series. Cross spectral analysis confirmed that, for the period before world War II, there was significant coherence between these short - term movements. Thus, people in fact behaved to some extent as if they knew future stock market returns and were optimizing their consumption pattern. This is not as implausible as it at first seems, nor does it directly imply a counterfactual behavior of short- term real interest rates. Consumption has, in the past, varied a great deal, i.e. sometimes people are apparently substantially worse off than at other times. What is it, other than a substantial expected profit opportunity, which would entice the representative man to hold the existing shares in times of economic distress? Consider, for concreteness, the year 1932. In that year, real aggregate con- sumption on nondurables and services per capita was 18% below the value in 1929. One can try to imagine what it must have been like to suffer such a decline in consumption. The total number of shares per capita outstanding in 1932 was

not much different than it was in 1929, and yet people must have perceived themselves as much worse off in 1932. The price per share must have fallen until the representative man is enticed by the profit opportunity to hold the same number of shares in 1932 as he did in 1929 (disregarding the changes in numbers of shares or in population between 1929 and 1932). Imagine how you might try to justify to your spouse the idea of investing in the stock market in 1932 when the family can't afford ordinary amenities in life which they have grown accustomed to. Would a 20% expected return in one year be enough? Perhaps it would take a 50% expected return in one year before actually holding the same amount of stock as in 1929 would seem like a good idea for the family. These are enormous one-year expected rates of return, but are they implausible? This example is chosen to illustrate the plausibility of large movements in expected return. It should not be inferred that the theory requires that expected one year real interest rates, i.e.,expected real returns on short debt,were high in 1932. The low stock market in 1932 in the face of relatively constant dividends is consistent with any pattern of short-term real interest rate, so long as the long-term real interest rate is high, which in our theory means that the long-run outlook for consumption is upward. What is it reasonable to suppose people actually expected? If we imagine that people use history to make such judgments, then it would seem quite reasonable that they expected the depression to end. The major depressions they remembered - those of the 1870's and 1890's - did come to end after a few years. The fact that P_t resembles P^*_{4t} in Figure I suggests that people had some way of knowing the future path of consumption.

What is not encouraging about the evidence presented in this paper is that if an ex-post rational consumption series is computed using real short-term interest rates rather than using stock returns then we see really no resemblance

with actual consumption. This ex-post rational consuption series is really just

a transformation of an ex-post long-term real rate of return computed from short-

term interest rates. Thus, by comparing this perfect foresight consumption

with actual consumption we are doing the right thing to ascertain whether short-

term interest rates behave in accordance with the perfect foresight model, in

contrast to the wrong inferences about short-term interest rates just warned against

above. It would have been inspiring for us if this perfect foresight consumption

resembled actual consumption at least at some frequencies. Failing that, one must

try to decide whether doing data analysis of a model incorporating uncertainty

about future real short-term interest rates could be more inspiring. Possibly

other considerations are relevant. Individuals cannot borrow at the commercial

paper rate, and because of enforcement problems, short-term credit may be available

only to persons whose consumption is much higher than is suggested by aggregate

data. Credit rationing might prevent actual observed interest rates from rising.

Other institutional factors or "moneyness" might be responsible for the failure

of commercial paper rates to move in such a way as to look encouraging for the

model here.

I regard it as somewhat encouraging for the model that, as shown in Section

IV, the observed difference between average stock returns and average short-term

interest rates suggests the same large movements in marginal rates of substi-

tution which might account for large stock price movements. On the other hand,

the failure of land, housing or bond prices to move with stock prices is discouraging.

Such evidence is hardly convincing, since there are so many problems with the

data. Moreover, we lack data on the "dividends" for land and housing. There may be

other reasons to question the applicability of the model to these series, due

to the very large transactions costs to trading in them, the lumpiness of the

assets, or even special attitudes to farm and home.

Rational expectations models are, of course, best viewed as only a first step or exploratory model which might ultimately suggest more accurate models. Obviously everyone isn't forecasting optimally, and it has been established by experimental psychologists that people in fact systematically violate the axioms of Von Neumann-Morgenstern utility. It is to be expected then, that any rational expectations model will be contradicted in some dimensions even if it appears promising in others.

I am not prepared to advance any well defined psychological model of asset pricing. I do feel that it is conceivable that this might be done in such a way as to preserve the basic notion that stock prices fall in recessions because of consumption smoothing behavior, even if land, housing, short-term or long-term debt do not behave in accordance with the model. Perhaps the very ambiguity in fundamental value which characterizes corporate stocks causes people to regard stock as a psychologically different saving medium, one for which large expected returns ar e less enticing and which seem more dispensable in a recession. Such divergent behavior between stocks and bonds or other assets might suggest a "profit opportunity" for savvy traders, but not an opportunity to get rich quickly or to make money without substantial risk. The number of people who actually do this is limited, and they will consume their wealth or die before they take over the market. One problem with developing such psychological models is that it seems equally plausible, based on casual observation of human behavior in other aspects of life, that asset demand should be influenced by temporary fads or speculative bubbles. Such fads seem especially hard to model econometrically. Moreover, once we get into a phychological theory such as this, there may no longer be a clear distinction between discount rate movements and expected dividend movements. It could be that stock prices fall in a recession partly because of consumption smoothing behavior. Then, because the initial stock price fall creates a bear market psychology, irrational expectations of declining future dividends are engendered.

For those readers who are inclined to conclude squarely against the model, I leave some questions. If stock prices move primarily due to large discount rate movements but these discount rate movements do not correspond to movements in marginal rates of substitutions then what is it that causes the discount rates to move so much? If, on the other hand, the reader goes back to a rational expectations model in which information about potential dividend movements, rather than discount rate movements, causes stock prices to move, then since actual aggregate dividend movements of such magnitude have never been observed, what is the source of information about such potential movements? Can we be satisfied with a model which attributes stock price movements and their business cycle correlation to public rational expectations about movements in a variable which has, in effect, never yet been observed to move?

APPENDIX I

SOURCES OF PRIMARY DATA

The real and nominal consumption series starting in 1929 are the annual average personal consumption expenditure on nondurable goods and services series from the National Income and Product Accounts of the United States. The real and nominal consumption series for 1889 to 1928 are Simon Kuznets flow of goods to consumers (perishables, semi-durables and services), variant III, adjusted to correspond to Commerce Department accounting practices as described by John Kendrick and multiplied by the ratio of the Commerce Department series to the Kuznets-Kendrick series for the year 1929. The resulting series is divided by the population of the United States to arrive at a per capita series. The price index used to deflate the nominal return series into real return series is the consumption deflator implied by the nominal and real consumption series.

The nominal stock price series is the annual average Standard & Poor Monthly Composite Stock Price Index, which is a continuation of the Cowles Commission Common Stock Price Index. The dividend series from 1926 is "dividends per share... 12 month total adjusted to index, fourth quarter, from Standard and Poor statistical service. For 1889 to 1925 total dividends are Cowles' series Da-1 multiplied by .1264 to correct for change in base year.

The Standard and Poor Stock Price Index, which is a continuation of the Cowles Commission Stock Price Index, is, in the words of Cowles, "intended to represent, ignoring the elements of Brokerage charges and taxes, what would have happened to an investor's funds if he had bought at the beginning of 1871 all stocks quoted on the New York Stock Exchange, allocating his purchases among the individual stocks in proportion to their total monetary value, and

each month up to 1937 had by the same criterion redistributed his holdings among all quoted stocks."

The after-tax nominal commercial paper return figure is the product of one plus the after tax commercial paper rate for July and one plus the after-tax commercial paper rate for January of the following year where the rate is for prime 4-6 month commercial paper as reported by the Board of Governors of the Federal Reserve System (1943) and the Federal Reserve Bulletin (Passim).

A printout and description of all data used in this paper is available from the author on request.

NOTES

1. If we include in D dividends on shares exchanged when the company is acquired or merged, and if we include in D any value at time of ultimate liquidation, then we may assume that (4) holds at all times. It follows then that the only exogenous characteristic of a security which influences price and hence which might cause it to have an expected return which is different from that of another security is the stochastic behavior of D. Earnings are relevant only as information useful in predicting dividends.

2. How then may one describe what kinds of dividend processes cause securities to have high expected returns? Formally, one may proceed by substituting expression (6) for P_t into the expression $R_t = (P_{t+1} + D_{t+1})/P_t$ and taking expected values. The resulting expression for $E(R_t^{(1)})$ is, however, difficult to interpret. One might guess that securities whose dividend D_{t+1} is highly correlated with $u'(C_{t+1})/u'(C_t)$ would have high expected returns. It is easily seen, however, that this guess is wrong. Consider an example in which C_{t+1}/C_t is log-normally distributed independently of all information at time t, and $\log(C_{t+1}/C_t)$ has mean μ and variance σ^2. Suppose the dividend at time t+1 is related to C_{t+1}/C_t by $D_{t+1} = \bar{D}(C_{t+1}/C_t)^b$ and it is known with certainty that no dividends will be paid thereafter. Then it is easily established that the expected return $E(R_t^{(1)}) = E(\dfrac{D_{t+1}}{P_t})$ is equal to $(1/\delta)\exp(A\mu - 1/2\ A^2\sigma^2 + bA\sigma^2)$ or just the "sure" return (or expected return on an asset uncorrelated with S_t) times $\exp(bA\sigma^2)$. The higher b the higher the expected return, yet the <u>correlation</u> between D_{t+1} and $u'(C_{t+1})/u'(c_t)$ is maximized (and equals one) if b = -A. The correlation between D_t and $u'(C_{t+1})/u'(C_t)$ in fact approaches zero as b approaches infinity, so that assets with very high expected return would show D_{t+1} with virtually no correlation with $u'(C_{t+1})/u'(C_t)$. Moreover, a strong positive association of $\log(D_{t+1})$ with $\log(C_{t+1}/C_t)$ is not necessary for high expected return. Consider a stock for which it is known with certainty that only one dividend will ever be paid, at time t+2. Suppose the amount of this

Notes, continued

dividend is revealed at time t+1 and equals $\bar{D}(C_{t+1}/C_t)^b$. Clearly its price at time

t+1 will be proportional to the dividend of the stock in the preceding example,

and so its expected return $E(R_t^{(1)}) = E(P_{t+1}/P_t)$ is also the sure rate

times exp $(bA\sigma^2)$. In this example the high expected return arises because

information about <u>future</u> dividends has a high covariance with $u'(C_{t+1})/u'(C_t)$.

3. Hall (1981) has pointed out that the utility function may be generalized

to allow a separate global risk aversion parameter, by taking this utility

function to a power. Consideration of this utility function suggests that

the parameter estimated here might better be described as an intertemporal

substitution parameter rather than a risk aversion parameter.

4. LeRoy and La Civita (1981) have also made the point that consumption

variability may induce stock price variability.

5. Thus, if $C_t = \cos(\omega t)$ and $D_t = \bar{D}$, $P_t^* \simeq g(\omega)\cos(\omega t + \phi(\omega)) + \delta\bar{D}/(1-\delta)$

6. Nor do our data suggest this. A cross spectral analysis between C

and P for 1889 to 1950 shows a phase angle which crosses through zero where

coherence is strongest, i.e., at a wavelength of little over five years.

The phase does, however, show that P leads C by about six months at the

"business cycle" wavelength of 40 months. Since neither the individual spectra

nor the coherences between the individual series are particularly strong in the

vicinity of this frequency, it is hard to see from this cross spectrum why

this lead has been singled out for attention by students of the business cycle.

7. Technically the phase angle does not tell us whether one series leads

another since for cycles of frequency ω a lag of ϕ radians can be described

as a lead of $2\Pi-\phi$ radians. Traditional business cycle theorists clearly

resolved this ambiguity in favor of small ϕ, as is done also here.

8. One may conclude, then, that $Var_t(C_t^{-A}) \leq Var_t(C_{At}^{*-A})$, by analogy to the corres-

ponding inequality for P_t and P_{At}^*. However, if P_t and D_t are stationary indetermin-

istic processes then C_{At}^* will not be stationary, and hence $Var_t(C_{At}^{*-A})$ depends

Notes, continued

on t and does not have the same interpretation. Moreover, C_t^{-A} does not

generally equal $E_t(\overset{\backsim *}{C}_{At}^{-A})$ where $\overset{\backsim *}{C}_{At}$ $t_0 < t < T$ is the consumption that an

individual would choose if he were told all future returns R_{t_0+j} at t_0.

Instead, $\overset{\backsim *}{C}_{At}$ would equal a constant times C_{At}^*. Consider for example a

perfect foresight world in which all income is derived from the initial

endowment W at t_0 of the single asset. The individual's optimal consumption

if he knows all future returns at $t_0 < T$ is:

$$\overset{\backsim *}{C}_{At} = \overset{\backsim *}{C}_{AT} (\prod_{j=0}^{T-1} \delta R_{t+j})^{-1/A} \quad \text{where} \quad \overset{\backsim *}{C}_{At} = W_{t_0} (\prod_{j=t_0}^{T-1} (R_j \delta)^{1/A}) / (1 + \delta^{1/A} R_{t_0}^{1/A-1} +$$

$$\delta^{2/A} (R_{t_0} R_{t_0+1})^{1/A-1} + \ldots).$$

In the same world in which the individual is not told future returns and in

which returns are unforecastable and R_t is independent and identically

distributed as R_{t+j} $j \neq 0$, then $C_t = \gamma W_t$ where $\gamma = 1 - (\delta E(R^{1-A}))^{1/A}$. Then

$$C_T = \gamma W_{t_0} (1-\gamma)^{(T-t_0)} \prod_{j=t_0}^{T-1} R_j. \quad \text{Clearly,} \quad C_t^{-A} \text{ does not equal } E_t(\overset{\backsim *}{C}_{At}^{-A}) \text{ while}$$

it is equal to $E_t(C_{At}^{*-A})$ as defined in the text.

9.   Hansen, Richard and Singleton [1981] have shown some of the connection

between the traditional CAPM and this model. They consider the case in which

there exists a portfolio whose return, the "benchmark return" R_b, is proportional

at time t to S_t: $R_{bt} = a_t S_t$. Then from (3) $a_t = 1/E_t(S_t^2)$. They show that

the benchmark return is the return on the portfolio with the smallest conditional

second moment. If one computes the conditional efficient portfolio frontier

one can always find the point on the frontier with minimum second moment, and

it then follows that one can derive a beta relation using this portfolio which

obtains even conditional on subsets of information. In contrast, the market

Notes, continued

portfolio does not generally lie on the frontier. Still, the model has no

testable implications in the absence of data on S, because the model restrictions

are just sufficient to identify R_b.

10. Of course, nominal housing prices fell then. The large number of fore-

closures then is partly due to the incentive to default created by the lower

prices.

11. Fisher [1951] reports that in 1932 deeds recorded were about one-third

fewer in number than in 1929 for a sample of 9 counties chosen to be

representative of the country. He notes, however, that "there is some evi-

dence that the number of deeds representing bona fide sales is not a constant

proportion of deed recordations." His estimated number of foreclosures

was 80% higher in 1932 than in 1929; foreclosures accounted for 9% of

all deed recordings in a small sample in 1938.

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