

NBER WORKING PAPER SERIES

THE RELATION BETWEEN VOCATIONAL TRAINING
IN HIGH SCHOOL AND ECONOMIC OUTCOMES

Alan L. Gustman

Thomas L. Steinmeier

Working Paper No. 642

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge MA 02138

March 1981

This paper has been supported by the Division of Technical Systems in the Office of Technical and Analytic Systems, Office of the Assistant Secretary for Planning and Budget, U.S. Department of Education. We would like to thank Lawrence Brown, John Grasso, Daniel Hamermesh and Robert Meyer for their helpful comments, and Jonathan Weed for programming assistance. The research reported here is part of the NBER's research program in Labor Studies and project in Productivity. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

The Relation Between Vocational Training
in High School and Economic Outcomes

ABSTRACT

This paper examines estimated relationships between economic outcomes and vocational training in high school. We find that these relationships are relatively robust with respect to variation in the way that type and quality of vocational training is measured as well as with respect to a number of other variations in specification. None of the evidence we have developed supports a view that vocational training for male students in high school produces any special skills that are valued by firms beyond those that are produced by a general high school education. The evidence does suggest that in the early 70's commercial-business programs taken by young women did produce such valued skills. But the evidence pertaining to later years is inconclusive.

Alan L. Gustman
Department of Economics
Dartmouth College
Hanover, New Hampshire 03755

(603) 646-2641

Thomas L. Steinmeier
Department of Economics
Dartmouth College
Hanover, New Hampshire 03755

(603) 646-2531

This paper is concerned with analyzing empirically the labor market effects of vocational programs in high schools. Such an analysis is of interest because it is an important part of any evaluation of this major program. It is also timely because the Vocational Education Act is to be considered for renewal this year.

There have been a number of studies analyzing the labor market effects of high school vocational training. Many of these studies use data from a single labor market, but recently studies based on a national data sample have appeared.¹ Notable among these are the studies by Grasso and Shea (1979b) and Meyer and Wise (1979).² Despite the availability of these studies, however, several important questions remain unanswered. This paper will attempt to shed some light on two of these questions.

First, there is the issue of the robustness of previous estimates. Are these estimates sensitive to the way the high school program is measured, to the sample used, to the group of control variables included in the regressions, to the time period studied, to the particular cohort or cohorts chosen, etc.? An awareness of the reliability of estimates is important if empirical analyses are to be used responsibly in evaluating vocational training in high schools.

¹For surveys of this literature, see Stromsdorfer (1972) and Grasso and Shea (1979a).

²Other such studies include Creech, et. al. (1977) and Wolfi, et. al. (1978).

The second issue concerns the relationship between vocational training programs and the labor markets. If vocational programs are large, the supply of vocational training graduates may reduce the returns to vocational training in relation to those of non-vocational graduates, even though the vocational education is producing useful skills. Under such circumstances, simple comparisons between the labor market experience of vocational program graduates and others may provide misleading information not only as to the size, but even as to the existence of skills production in high school vocational programs.¹ The problem with the simple comparisons is that they reflect not the total contribution of vocational training, but the contribution of a marginal increment of the amount of that training.

These issues will be addressed using data from two large national longitudinal surveys -- the National Longitudinal Survey conducted at Ohio State University (the Parnes data) and the National Longitudinal Survey of the High School Class of 1972. We will attempt to determine whether findings gleaned from least squares multiple regression analysis tell a consistent story about differences in outcomes for those who terminate their education after high school, and if not, why not.

More specifically, to determine the reliability of parameter estimates, we estimate equations which are designed to answer the following questions: (1) Are similar results obtained when the relation of market outcomes to high school program is estimated with the Parnes data as when it is estimated with data from the Class of

¹This is shown in Gustman and Steinmeier (1980).

1972 Survey? (2) Do the findings vary with the date of the survey, the cohort analyzed, and the age at the time of the survey? (3) How are these findings affected by the inclusion or exclusion of **several** variables (e.g., union membership or job tenure) which may not be strictly exogenous with regard to labor market experience? (4) Does better measurement of vocational training (e.g., number of vocational courses taken, detailed field of study, and type of high school attended) affect estimated parameters? (5) What is the effect of standardizing for interarea differences in the wage level and in youth unemployment rates?

In order to explore possible influences of the size of vocational programs on the measured returns to vocational education, we employ a model in which the relative supplies of vocational and non-vocational graduates influence the wage differential between them. The model we use is described in detail in our earlier paper. If vocational programs do produce useful skills, this model contains implications for equations explaining not only economic outcomes, but also enrollment rates in the programs. These implications are tested by comparing the predictions from the model with results obtained when reduced form equations are estimated. We then conduct a further test by applying a maximum likelihood estimating technique in an effort to estimate the structural supply and demand equations underlying the equations explaining economic outcomes and enrollment patterns.

It should be noted that although we hope to shed some light on two important issues regarding vocational education, the remaining gaps in our knowledge about this subject are quite large. There has been very

little behavioral analysis and virtually no formal modeling of the role of high school vocational training. We simply do not know how vocational training in high school fits into a standard labor market model. One possibility, noted in our previous paper, is that vocational training substitutes for some type of general training that would otherwise be provided by employers on the job and paid for by a reduction in wages. Alternatively, vocational training may reduce the cost of specific training on the job. We know so little about the skills produced by high school vocational training that we cannot tell whether the range of skills effectively augmented by the high school program is very narrow in scope, proving useful in only a limited number of occupations, or whether it is quite broad, improving productivity and/or reducing training costs in a wide variety of later jobs.¹ It is possible that learning of vocational skills takes place at the expense of learning basic skills. Thus a vocational program student may have augmented productivity in some areas, but diminished productivity in others. Perhaps high school vocational training provides information about the job market that otherwise could only be gained through the kind of search and trial and error process that many think is responsible for much of youth unemployment. If this is so, one might find that vocational program graduates make better risks for firm investments in specific training and that for this reason they might exhibit both lower turnover rates and age-earnings profiles that differ

¹A related discussion is contained in Grubb (1979).

from those of general program graduates.¹ We are still a long way from understanding fully the way in which vocational training affects labor market outcomes, if it does at all, and for this reason some amount of caution should still be exercised in interpreting the results of estimates of differentials between vocational and non-vocational graduates.

I. Robustness with Respect to Year, Cohort and Survey

The evidence gleaned from the Parnes data by Grasso and Shea (1979b) suggests that women graduates of commercial (business and office) programs in the high school have higher earnings and a more favorable employment experience than women graduates of other programs with comparable years of schooling. However, those who attended vocational programs that are not commercial have earnings that are no different from those of women graduates of other educational programs. In the case of men, there appears to be no discernible difference in earnings between male graduates of vocational and of other programs. Using data for males from the High School Class of 1972 Survey, Meyer and Wise (1979) have found no evidence of an association between the student having taken any job training in high school, and later earnings.²

¹For an analysis of the relation of job search to firm specific training and labor turnover, see Leighton and Mincer (1979).

²Using data for October 1972, Nolfi et. al. (p. 83) found positive but insignificant differences in earnings between females who enrolled in vocational, as opposed to general or academic programs in high school. For males, earnings of vocational program graduates were lower than earnings of academic or general program graduates, but again the difference was not statistically significant. These regressions standardized for background, ability scores, and other characteristics. Observations for blacks and whites were pooled and a dummy variable for race was included, constraining the coefficients on the indicators of high school program to be the same for both race groups.

To gain some insight about the robustness of these results to changes in the year, the cohort, and the survey data employed, we analyzed data for two different groups of individuals, both of whom were about the same age at the time the data were collected. One group was the Parnes cohort that was 17 years old at the time of the initial Parnes survey. At the time of the 1972 survey, from which our data are taken, this group would have been 21 years old. This year was chosen because it was the year used by Grasso and Shea in deriving their results. The other group came from the High School Class of 1972 Survey. Data from this group was taken from 1976, four years after graduation. For both groups, we include only individuals who did not attain any formal education beyond high school, and who were not enrolled during the year of the survey.¹ The differences between the two groups which are of interest to us are that they are in different cohorts, that the information refers to two different calendar years, and that they are interviewed in two different surveys.

Table 1 presents means of usual weekly earnings and weeks employed last year according to the type of high school program completed, sex, and race for each of the two groups.² The figures in parentheses

¹We also eliminated from the sample all those who did not work in the week preceding the survey, either because they chose not to participate in the labor force, or because they could not find a job, and those who did not report a value for usual weekly earnings or reported a zero value. The resulting possibility of self-selection bias is discussed at the end of this section.

²Figures on weeks worked are not strictly comparable between the samples of women and men in the Parnes survey or between the two surveys. In the Class of 1972 Survey, weeks worked refers to the previous calendar year, while in the Parnes survey this variable refers to the time since the last survey, which may have been taken slightly more or less than an exact calendar year after the previous survey.

Table 1

Means of Usual Weekly Earnings and Weeks Employed Last Year for Those With 12 Years of Schooling and Out of School 4 Years, by High School Program*

	Usual Earnings				Weeks Employed			
	Vocational other than Bus-Comm'l	Bus-Comm'l	General	Academic	Vocational other than Bus-Comm'l	Bus-Comm'l	General	Academic
1972 data from the Parnes Survey								
White Males	\$143.06 (17)	\$127.29 (6)	\$129.19 (43)	\$139.20 (17)	43.5 (17)	39.8 (6)	43.7 (43)	43.1 (17)
Black Males	96.48 (2)	-- ^a	123.13 (13)	104.20 (3)	41.5 (2)	-- ^a	43.7 (13)	33.3 (3)
White Females	86.67 (3)	111.93 (11)	84.52 (23)	77.66 (15)	49.3 (3)	50.9 (11)	46.3 (23)	46.0 (15)
Black Females	-- ^a	118.04 (5)	89.63 (8)	110.00 (1)	-- ^a	53.0 (5)	41.8 (8)	48.0 (1)
1976 data from the Class of 1972 Survey								
White Males	\$230.07 (97)	\$233.53 (38)	\$207.72 (249)	\$219.35 (82)	43.3 (97)	42.5 (38)	42.7 (249)	38.7 (82)
Black Males	152.77 (17)	176.57 (7)	174.00 (24)	182.17 (12)	41.9 (17)	44.0 (7)	40.9 (24)	43.3 (12)
White Females	120.39 (23)	122.17 (113)	134.59 (121)	108.85 (54)	38.2 (23)	37.2 (113)	39.7 (121)	39.3 (54)
Black Females	102.23 (13)	117.94 (17)	123.00 (25)	131.08 (12)	38.8 (13)	31.2 (17)	41.3 (25)	35.8 (12)

* The relevant number of observations associated with each mean is reported in parentheses.

^aNo observations for these groups are available.

indicate the number of individuals for which the means are computed. Table 2 presents analogous regression coefficients for the various classifications. In these regressions, dummy variables were entered for the various possible high school programs, and additional variables were entered to standardize for the effects of parent's socioeconomic status, achievement on a test score, and residence in an SMSA. In these regressions, the dummy variable for a general high school program is omitted, so the coefficients in Table 2 measure differences relative to general program graduates in the same sex-race classification. A quick inspection of Tables 1 and 2 reveals that the estimated coefficients in the regression equations of Table 2 reflect fairly closely differences in the sample means of Table 1.

For men, the two surveys appear to tell much the same story. The data in Table 1 indicate that for this group, vocational programs other than business and commercial are more common than business and commercial programs. White male graduates of vocational programs other than business and commercial have earnings estimated in Table 2 to be around \$20 per week greater than comparable graduates of general programs, although the difference is not statistically significant. For black males, the figure for graduates of programs other than commercial and business is about \$20 less than the figure for general graduates, while business and commercial graduates of both races have a small positive earnings advantage over general program graduates. None of these differences are statistically significant either. Much the same can be said of the effect of vocational programs on work experience, as measured by weeks employed in the previous year. Vocational programs

Table 2

Estimated Coefficients for Regressions of Labor Market Outcome on High School Program^a

1972 data from the Parnes Survey (21-year olds only)	Usual Weekly Earnings			Weeks Employed			Number of Observation
	Vocational other than Bus-Comm'l	Bus-Comm'l	Academic	Vocational other than Bus-Comm'l	Bus-Comm'l	Academic	
White Males	16.57 (1.05)	3.95 (.15)	11.50 (.71)	.58 (.14)	1.07 (.16)	2.54 (.58)	83
Black Males	-19.78 (-.64)	-- ^b	-9.91 (-.37)	-1.42 (-.12)	-- ^b	-5.75 (-.56)	18
White Females	10.82 (.62)	23.44 ^{**} (2.18)	-10.50 (-1.01)	4.03 (.69)	4.27 (1.18)	-.87 (-.25)	52
Black Females	-- ^b	18.37 (1.60)	16.18 (.85)	-- ^b	6.95 (1.57)	9.55 (1.30)	14
1976 data from the Class of 1972 Survey							
White Males	21.86 (1.35)	13.33 (.56)	14.50 (.82)	.54 (.38)	.01 (.01)	-3.93 ^{**} (-2.54)	466
Black Males	-22.16 (-.97)	3.72 (.09)	13.77 (.54)	2.03 (.52)	5.50 (1.02)	2.47 (.56)	60
White Females	-14.66 (-.83)	-13.14 (-1.28)	-25.92 [*] (-1.96)	-1.09 (-.34)	-2.27 (-1.22)	-.82 (-.34)	311
Black Females	-21.05 (-.99)	-6.26 (-.32)	4.58 (.21)	-2.55 (-.53)	-9.53 ^{**} (-2.14)	-4.15 (-.83)	67

*Significant at the 90% level.

**Significant at the 95% level.

^aOther variables included in the regressions are parents' socioeconomic status, achievement on a test score, and residence in SMSA. See appendix 1 for complete regression results.^bNo observations for these groups.

of both types influence weeks worked in a generally positive direction, but again none of the differences emerge as statistically significant.

Among females, the effects of vocational programs is fairly uniform between whites and blacks, but there are sharp differences in the results of the two surveys. For the Parnes group, vocational programs have a general positive impact on both usual weekly earnings and for weeks employed. The effect of business and commercial programs (the most common vocational programs among women) on usual weekly earnings is statistically significant for white females. The result matches the findings of Grasso and Shea, who did much the same regression except that they included several cohorts in their analysis and added an experience variable which presumably captured the age-wage profile.¹

Using data from the Class of 1972, the results for women are almost exactly reversed. For this group, vocational programs seem to depress both usual weekly earnings and weeks employed for both white and black women, though the only place where this effect is statistically significant is for weeks employed by black female graduates of business and commercial programs. The differences between the results using the Parnes data and the Class of 1972 data cannot be attributed to small sample sizes. For the Parnes data, business and office training improved weekly earnings by \$23, a figure which is not only statistically significant but also represents over 25 percent of the average weekly earnings for the sample. The Class of 1972 data failed to confirm this result in spite of the fact that the sample size was almost six times as large.

¹ The above regression used only a single cohort because we wanted to have comparable regressions for the Parnes group and the Class of 1972 group, and only a single cohort is available for the latter group.

The discrepancy between the results for the Parnes 21-year-old group in 1972 and the Class of 1972 group in 1976 may arise from one of two sources. Either there was something which changed between 1972 and 1976 which caused the returns to business and commercial training to drop between the two years, or there was something about the way in which the two surveys were conducted which contributed to this result. To gain some insight into which of these explanations is correct, we tried two approaches.

One approach was to take a group from the Parnes data who were 21 years old in 1976. For a couple of reasons, this could be done only approximately. First, the Parnes study conducted surveys in 1975 and 1977, but not in 1976. The 1975 survey was used because by 1977 the youngest cohort in the Parnes study was 23 years old. Even in the 1975 survey, there was but one 21 year old working graduate of a commercial and business program in the sample. To include a reasonable number of these graduates in the regressions, the sample was extended to include women who were either 21 or 22 years old in 1975.

The results of these regressions are reported in Table 3.¹ By comparing the two lines of Table 3 with the third and fourth lines of Table 1, it is seen that the significant positive coefficient in 1972 for the effect of business and commercial programs on the weekly earnings of white females appears to have largely vanished by 1975. The positive, significant, estimated effect of other vocational programs for the sample of white females reflects the earnings of the single

¹An additional variable reflecting years of work experience was added to these regressions, as was done by Grasso and Shea in their regressions which included more than a single cohort.

Table 3

Estimated Coefficients for Regressions of Labor Market Outcome on High School Programs^a

1975 Data from the Parnes Survey (21+ 22-years olds)	Usual Weekly Earnings		Weeks Employed		Number of Observations		
	Vocational other than Bus-Comm'l	Bus-Comm'l	Academic	Vocational other than Bus-Comm'l		Bus-Comm'l	Academic
White Females	107.84** (2.47)	-1.08 (-.06)	-6.65 (-.32)	-11.58 (-.87)	1.17 (.22)	-2.24 (-.36)	38
Black Females	--b	3.02 (.10)	22.86 (.77)	--b	3.99 (.48)	9.25 (1.14)	11

** Significant at the 95% level.

^a Other variables included in the regression are parents' socioeconomic status, achievement on a test score, residence in an SMSA, and years of work experience. See Appendix 2 for complete regression results.

^b No observations for these groups.

white female who enrolled in a vocational program other than office and business. Since there is no evidence in any of the other sample of strong, positive returns for women to vocational **training** outside of the business and office program, this result should be treated with due skepticism.

Another approach was to estimate regressions using the full data set in the Parnes study, and to introduce dummy variables to allow for differences in survey years and age groups. The Parnes data set includes eight survey years from 1968 to 1977, and it uses eleven age cohorts who were 23 to 33 years old by 1977. In these regressions, we separated the constant into roughly 80 dummy variables for various combinations of age and survey years for which we had observations on women who were working high school graduates. The dummy variables for both types of vocational education were similarly divided into separate variables for the different age-survey year combinations. This allows the estimated impact of both types of vocational education to vary from survey year to survey year and among the various age groups.

Table 4 reports the estimated coefficients for the dummy variables which represent business and commercial training in the weekly earnings equation.¹ The coefficient for 21-year-olds in 1972 is \$18.74 and is significant. This is close to the results that we found in Table 2 for regressions using this cohort and survey year along. Similarly, the coefficient for 22-year-olds in 1975 is slightly negative and insignificant, corresponding well to the results for that group shown in Table 3.

¹Weekly earnings were deflated to 1967 using a general index of average hourly earnings.

Table 4

Estimated Difference in Weekly Earnings between White Female
Commercial and General Program Graduates
By Age and Survey Year^a

Age at time of Survey	Survey Year							
	68	69	70	71	72	73	75	77
18	5.68 (.80)	4.09 (.43)	8.11 (1.04)	2.55 (.33)	b			
19	1.46 (.24)	1.10 (.19)	10.44 (1.27)	6.57 (1.02)	11.61 (1.73)	b		
20	4.89 (.79)	11.89 (1.98)	8.12 (1.45)	8.66 (1.09)	2.96 (.41)	6.31 (.84)		
21	10.85 (1.72)	6.70 (1.05)	-.78 (-.13)	7.58 (1.31)	18.74 (2.17)	.97 (.41)	b	
22	17.39 (1.80)	6.50 (.58)	6.80 (1.02)	3.88 (.59)	9.62 (1.57)	10.88 (1.26)	-.13 (-.02)	
23	2.33 (.29)	4.68 (.72)	12.63 (1.75)	4.68 (.66)	-1.29 (-.19)	8.09 (1.33)	4.69 (.65)	b
24	11.12 (1.23)	4.42 (.50)	-4.28 (-.52)	21.12 (2.70)	13.30 (1.77)	-2.33 (-.31)	3.20 (.33)	-1.17 (-.13)
25		11.82 (1.32)	-5.35 (-.59)	1.43 (.17)	18.80 (2.31)	13.81 (1.68)	14.66 (2.02)	5.59 (.66)
26			-12.68 (-1.35)	9.36 (.90)	-26.65 (-2.96)	28.50 (3.37)	-1.80 (-.21)	21.88 (1.84)
27				-8.92 (-.91)	8.80 (.90)	-20.65 (-2.11)	10.14 (1.34)	8.32 (1.15)
28					17.08 (1.78)	-5.49 (-.59)	7.67 (.93)	7.43 (.93)
29						-9.03 (-.99)	7.71 (.71)	11.11 (1.32)
30							-2.86 (-.32)	-10.34 (1.17)
31							-11.71 (-1.17)	-17.78 (-1.77)
32								17.60 (2.10)
33								-13.69 (-1.43)

Simple average of coef-
ficients for those 22-24
during survey year

$$R^2 = .17$$

Number of Observations = 3878

^at-statistics are in parentheses

^bResults not reported because cell contained fewer than 10 observations.
For all reported cells, there were at least 25 observations.

The aspect of Table 4 that we are most interested in is the change in the impact of commercial and business training over time for people in their early 20's. The only age bracket for which we have results in all the survey years is the 24-year-old bracket, and for this bracket there does appear to be some evidence for a decline in the returns to business and commercial training between the 1971-1972 period and the later surveys. If we consider the 22-24-year-old age bracket, we can look at the average effect during the years up to 1975. These average effects which are simply the mean of the three coefficient estimates for these ages, are computed and shown in the bottom row of Table 4. Again, the figures suggest that the returns to commercial and business training declined between the early 1970's and later years, although the evidence on the time trend of these returns cannot be regarded as conclusive. In any case, one thing does seem clear: The high coefficient found for 21-year-olds in 1972 does not seem to be typical of other ages and cohorts. Of the 74 coefficients reported in Table 4, only three are numerically greater than the coefficient for 21-year-olds in 1972.

Why might the returns to business and commercial training have declined for women between 1971-72 and 1975-76? The different time periods are characterized by differences in the state of the overall labor market. In 1972, the unemployment rate was 5.6 percent, while in 1976 it was 7.7 percent. However, one cannot explain a decline in the earnings differential between female vocational and female general program graduates on the basis of cyclical variation of the skilled-unskilled wage differential. The reason is that skill differentials are more likely to narrow in tight labor markets as unskilled workers

are upgraded into skilled jobs. On these grounds, one might have expected smaller returns to vocational training in the tighter market of 1972, not larger ones.¹

Another possible explanation for the declining differential is that basic supply conditions changed between the two periods. Specifically, according to the survey of the class of 1972, and also according to data from the latest relevant cohort obtainable from the Parnes survey consisting of those who terminated their education after graduating from high school in 1970-71, 36 percent of white females were graduating from commercial-business programs.² In contrast, for the Parnes sample of graduates in 1968 who similarly terminated their education after high school, only 21 percent of the white females were enrolled in business and office programs.³ The additional supply of female business and commercial graduates may have eroded their wage advantage, a proposition which is discussed in more detail in Section III.

¹For a general discussion of the phenomenon and an analysis of the exception provided by the construction industry, see Gustman and Segal (1974).

²One must be careful in comparing means for program enrollment across surveys. The Program definitions in the Parnes survey are based on the individual's response, while the class of 1972 data are based on school records.

Corresponding figures for black females are 18 percent of the 1968 graduates and 36 percent of the 1970-71 graduates, but these percentages are based on sample sizes so small as to make them of questionable reliability.

³Consistent with Grasso and Shea, for the sample of females who report they completed exactly twelve years of education, the year of graduation from high school is taken to be the year in which the youth was 17 years old during the January survey month.

Before closing this section, a few words are in order about our reliance on ordinary least squares regression analysis, since the nature of the problem might appear to present econometric difficulties requiring more sophisticated techniques. One such difficulty is the fact that there is self-selection into the groups of vocational and non-vocational graduates we observe. This self-selection goes on at two levels: First, the decision as to the kind of program in which to enroll during high school, and then the decision about whether or not to work after graduation. Self-selection bias occurs if people with unobserved characteristics leading to greater wages tend to end up preferentially in one group or the other. In a comparison between vocational and general program graduates, however, it is not clear a priori which direction this bias should take, and attempts by Meyer and Wise to detect this kind of problem did not yield any evidence that it affected estimated wage equations (although it did seem to play a role in equations explaining employment after graduation).

Another difficulty is that a model explaining the determination of the employment experiences of vocational and non-vocational graduates may not yield the kind of simple equations that are estimated with ordinary least squares. More complicated specifications may, in general, be estimated by maximum likelihood, but only at the cost of introducing much more specific assumptions about the structure of the model. If these assumptions are correct, parameter estimates may be improved, but if they are not, the estimates may well deteriorate with the more complicated technique. This is the case with the maximum likelihood estimation of the model discussed in Section II' of this paper, or with

any other attempts to estimate structural parameters underlying wage or earnings equations.

A third difficulty pertains only to the weeks worked equation. Weeks worked is bounded by zero from below and, more importantly, by 52 weeks from above. This suggests that this equation is more properly estimated by the "Tobit" technique, which allows for truncated dependent variables. Given the costs of implementing this technique, and the lack of serious discrepancies between the stories told by usual weekly earnings equations (which are not subject to this problem) and weeks worked equations, we have not done so.

II. Other Aspects of Robustness

In this section we will investigate the sensitivity of coefficient estimates to several modifications not considered in the last section. Specifically, we will discuss the robustness of the estimates with respect to the following changes: (i) alternative measures of economic outcomes, (ii) changes in the set of variables used to standardize the regressions, (iii) inclusion of variables which are really intervening variables in the effect of vocational training on labor market outcomes, (iv) alternative measures of participation in vocational training programs, and (v) inclusion of measures to standardize for interarea differences in wage level and in unemployment rates. The general conclusion of this section is that parameter estimates are fairly robust with respect to these kinds of changes.

Alternative Measures of Economic Outcomes.

Two alternative measures of economic outcomes were considered in addition to weekly earnings and weeks worked. These were the hourly wage

and yearly earnings from labor. Hourly wage equations analogous to the equations presented in the last section were run for both the Parnes groups and the Class of 1972 group, and yearly earnings equations were run for the Parnes groups. Unfortunately, the Class of 1972 questionnaires did not contain a separate question specifically asking about yearly earnings from labor; the closest question inquired about "wages, salaries, commissions, or net income from a business or farm" and was in some instances clearly contaminated by non-labor income.

With two exceptions, the patterns of signs and levels of significance of the coefficient estimates using these alternative dependent variables closely matches the pattern with weekly earnings. One difference is that for black females in the 1972 Parnes Survey, the coefficient of business and commercial training in the hourly wage equation is significant at the 90% level, whereas the analogous coefficient in the weekly earnings equation in Table 2 was slightly below significance, with a t-statistic of 1.60. The second difference is that the anomolous results in Table 3 for white females in vocational programs other than business and commercial (which we noted previously was based on a single individual) does not carry over into the hourly wage or yearly earnings equations. Both of these differences would, if anything, strengthen the general conclusion of the last section, namely, that there was a significant impact of business and commercial training on the labor market experience of women in 1972 that was weakened by 1975-76.¹

¹We note explicitly that tests of statistical significance are not independent unless separate samples are used. We experiment here with different specifications estimated with the same samples. Nevertheless, if estimated coefficients or t-statistics fluctuate widely with reasonable changes in specification, this should constitute a warning that the estimated coefficients are of questionable reliability.

Modifications of Standardizing Variables

We also experimented to see whether the results were sensitive to changes in the variables used to standardize the regressions. These changes included: (i) the substitution of the "socioeconomic origins" measure in the Parnes data set for the one used in Table 2, (ii) the substitution of a measure of family income for the socioeconomic status of the family for the Class of 1972 Survey, and (iii) the use of dummy variables for measures of the individual's ability and for the socioeconomic status of the parents.¹ None of these changes in specification had any important effect on the reported coefficients and t-statistics of the variables measuring participation in high school vocational programs.

Inclusion of Intervening Variables.

The set of explanatory variables included in the regression which are reported in Tables 2 and 3 is designed to standardize only for the youth's ability, family background, and whether the youth lived in a city. Thus, the estimated impact of vocational training includes all differences in outcome variables which emerged over time, both those operating through such intervening variables as job tenure and unionization, as well as those operating directly on the outcomes. We estimated a variety of equations which included these intervening variables, and in several of the equations we included additional

¹Table 3 used a socioeconomic status variable constructed from the educational levels of the father and the mother and the occupation of the father, so that comparable variables could be constructed for both data sets. The Parnes variable uses the two education levels, the occupation of the father when the respondent was age 14, the education of the oldest sibling if applicable, and an index of the availability of reading materials at home when the individual was age 14.

standardizing variables as well. For example, one such set which was estimated for women included as explanatory variables job tenure, a measure of whether the job is full time, and a measure of collective bargaining coverage. For men, additional explanatory variables include past military service, southern residence, past on-the-job training, and an indicator of collective bargaining coverage.

A comparison of the expanded regressions with those in Table 3 indicates that in most cases the impact of vocational training is not very sensitive to the set of control variables included in the regression equation.¹ The exceptions were the regressions for black women in the Parnes study, for which the small sample size (14 individuals) makes the results particularly sensitive to changes in specification.

For this group, we find a significant (at the 10% level) impact of business and office training, as do Grasso and Shea, when the hourly wage is utilized as the dependent variable. With yearly earnings as the dependent variable, results are very sensitive to the exact set of independent variables used, with significant results being obtained only with the set of independent variables (excluding years of experience) utilized by Grasso and Shea. Additional regressions using all the cohorts in the Parnes study and including years of work experience, providing 106 observations, continued to exhibit this sensitivity in regressions for yearly earnings.²

¹All regressions for a given sex-race group were estimated from a constant sample so that differences between the regressions could not arise from differential selection into the samples.

²That analogous findings are sensitive to specification in Grasso and Shea can be seen by comparing their Tables (continued on p. 22)

Alternative Measures of Participation in Vocational Programs.

In this part, we consider the effects of using detailed measures of type, intensity and quality of vocational training to determine whether findings are changed by better measurement. These measures are only available for the Class of 1972 survey.

One refinement of the variable measuring vocational education is to separate it into several variables representing the detailed area of specialization within a vocational course of study. These were estimated only for the samples of white males and females and are reported in Table 5. For black males and females, there were empty cells in a number of subfields. The areas of specialization distinguished are agriculture, business, distributive education, health, home economics and trade.

While there is some hint in the estimates of a positive earnings difference for white males who took a program in trade, as compared to general program graduates, and negative earnings differences for white females who took a program in home economics, neither relation is statistically significant. The only significant finding is that white females included in the sample who enrolled in distributive courses work an average of nine fewer weeks than included white females who were in a general course of study.

A second refinement is to measure the intensity of vocational training. One measure of intensity of vocational training is the number

2(Continued from page 21)
 A4.20 and A4.21. They find that a relation between business and office training and yearly earnings which is significant at the 10% level for black females becomes insignificant when 19 observations for those who worked less than 39 weeks are eliminated from the sample.

Table 5

Relation Between Weekly Earnings, Employment and Specialty Area of Vocational Training: Survey of the Class of 1972^a

Dependent Variable	Independent Variable -- Type of Program						Number of Observations
	Agricultural	Business	Distributive Education	Health Fields	Home Economics	Trade	
	<u>White Males</u>						
Usual weekly earnings	-10.20 (-.31)	7.14 (.22)	19.58 (.62)	41.72 (.44)	41.57 (.31)	28.31 (1.60)	466
Weeks worked last year	3.30 (1.13)	-3.89 (-1.34)	3.45 (1.25)	-3.67 (-.44)	-7.95 (-.67)	.12 (.08)	
	<u>White Females</u>						
Usual weekly earnings	8.42 (.18)	-11.83 (-1.12)	-27.46 (-1.07)	12.70 (.45)	-46.84 (-1.63)	-25.13 (-.63)	311
Weeks worked last year	-3.26 (-.39)	-1.65 (-.87)	-9.05* (-1.94)	7.10 (1.39)	-4.87 (-.94)	-9.07 (-1.25)	

* Significant at the 90% level.

^aThe estimated regressions include the same set of independent variables held constant in Table 2: achievement level, parent's socioeconomic status, enrollment in an academic program and residence in an SMSA. t-statistics are in parentheses.

of class periods spent in vocational courses, as indicated in the youth's school record. We estimated the relation of weekly earnings and weeks employed to the total number of class periods in all vocational courses taken (i.e., the total of class meetings for each class summed across the number of vocational classes for each demographic group), holding constant, as in the regressions underlying Table 3, the youth's test score, parent's socioeconomic status, and an indicator of residence in an SMSA. The coefficient for the variable indicating number of periods spent in vocational courses was significant in only one regression, indicating that for white females, each additional vocational class period taken was associated with an additional .046 weeks of employment. The t-statistic was 2.52. Insignificant effects of number of vocational courses taken were also noted when alternative specifications of this regression were estimated. In one set of regressions the intensity variable was separated into two according to whether the major high school program was vocational or commercial-business. In other regressions, the intensity variable was further separated according to narrow high school program (as in Table 5). No systematic relation between the number of class periods spent in vocational courses and economic outcomes was apparent in these regressions.

A final refinement investigates the possibility that type of high school affects learning. Vocational training may occur in a high school that specializes in vocational programs, offering up-to-date training on modern equipment, or it may be simply one of the three tracks in a comprehensive high school where the training may be of relatively low quality. The Class of 1976 Survey provides no direct indicator of the

type of high school, so we employed two alternative measures. One indicator of potential specialization by the school is the proportion of students enrolled in vocational programs. Accordingly, we include in some regressions an explanatory variable which takes on a value of 1 if more than half the students are enrolled in a vocational program. In addition, Andrew Kolstad of the Department of Education identified a subset of schools included in the Class of 1972 Survey which specialize in vocational training. These schools were also identified by a dummy variable. The measures pertaining to type of school were added separately to regressions reported in Table 3. (Give the distribution of responses, the Kolstad measure was added only to the regressions for male youth). In no case were the coefficient estimates for these indicators of type of high school significantly different from zero.

Measures for Interarea Differences in Labor Market Conditions.

The last test for robustness introduces measures for interarea differences in the general wage level and in unemployment rates into the regressions. This test could be done only for the Class of 1972 Survey data, since the Parnes data does not identify specific SMSA's. Using the Class of 1972 data, we confined the sample to those in the largest 98 SMSA's and (i) deflated the youth's wage by a fixed weight index of the average earnings of 25 to 55-year-old high school graduates in the same SMSA, and (ii) included as an explanatory variable a measure of the unemployment rate for 20 to 24-year-old young people in the SMSA.¹

¹The unemployment rate is measured as the ratio of time unemployed to time spent in the labor force in the previous year. Both the unemployment rate and the adult wage rate are fixed-weight indices with national weights based on sex, race, and for the adult wage rate, age. For a further description, see Gustman and Steinmeier, forthcoming.

The data on area wage and youth unemployment were taken from the Survey of Income and Education, which was conducted during the spring and summer of 1976.

With the earnings variables deflated by the local wage index, there were no significant relations between the weekly wage and vocational training programs in the set of regressions specified in Table 3 above. Similarly, with one exception, no significant results were obtained when the SMSA youth unemployment rate was included as an explanatory variable in these regressions. The only exception is in a weeks employed equation for white females. The coefficient, which is significant at a ten percent level, suggests that white females who enrolled in a commercial-business course were employed four weeks per year less than white female general program graduates in comparable circumstances.

III. The Impact of the Capacity of Vocational Programs on Labor Markets

In this section we will consider how the size of the vocational program in an area may affect the measured returns to the program. To indicate the general nature of the argument, it will be helpful first to discuss a simple model of wage determination in the markets for vocational training graduates and for those without such training.

The model distinguishes two kinds of workers: those who have been trained in a high school vocational program or who received equivalent training on the job, and those who have not received such training. For convenience, we will call workers of the first type "trained" and workers of the second type "untrained." The demand curves for the two types of workers are shown in Figure 1 as D_T and D_U . Note that in this

Figure 1

The Labor Market for Vocational and
Non-Vocational Program Graduates

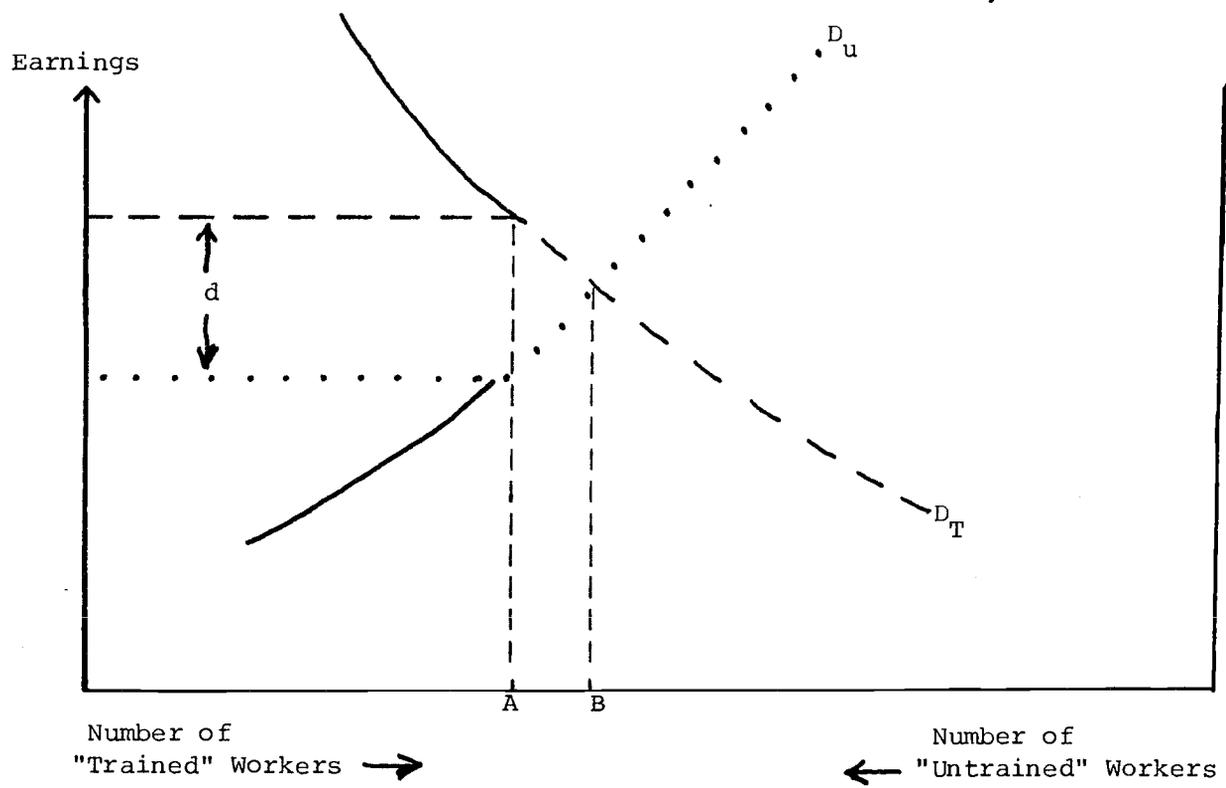


diagram the number of trained workers is measured from the left axis, and the number of untrained workers from the right axis. For any point along the horizontal axis, the sum of the two types of workers will add up to a constant total number of workers.¹

A central feature of this model is that if the worker undergoes on-the-job training at a cost represented by d , he will receive the skills necessary to be considered a trained worker.² If the number of spaces in the vocational program is below A individuals, then in the absence of on-the-job training the wage differential between trained and untrained workers will exceed d . This will induce enough workers to undergo on-the-job training to increase the number of trained workers to A , thus reducing the wage differential to d . If the number of spaces in vocational training programs is between A and B , then it will pay anyone who can to take the vocational training. There will be a positive wage differential between trained and untrained workers, but it will not be large enough to induce anyone to undergo on-the-job training. Finally, if the number of spaces in vocational training programs is B or greater, then it will pay B individuals to accept places in the program. In this case, one would expect no differential between the wages of trained and untrained workers.

In this model, a failure to observe any wage differential between graduates of vocational training programs and general programs may arise

¹The essence of the argument is not affected if the total number of workers (high school graduates) is allowed to vary. A more complete version of the model is discussed in Gustman and Steinmeier (1980).

²In the model, the one time costs of on-the-job training are implicitly converted into an annual amount whose present value equals the costs. This annual amount is represented by d .

for one of two reasons. First, it may be that vocational programs are producing no usable skills, so that the amount of on-the-job training necessary to produce an equivalent level of skills is zero ($d = 0$). The other possible explanation is that enrollments in vocational programs are effectively unlimited, so that enough students choose the programs to equalize the wages between graduates of vocational programs and other graduates.

Explanations based either on a decline in d over time or an increase in enrollments in vocational programs are consistent with the evidence presented earlier for female graduates of business and commercial programs. Recall that in 1972, such graduates had higher earnings than did general program graduates, whereas by 1975-76 the evidence pointed to a small positive or zero earnings differential within the Parnes sample, or a zero differential within the Class of 1972 sample. During the same time period female enrollment rates in vocational programs were rising. Hence, the lack of a substantial positive earnings differential in the later years may have occurred either because the program's performance deteriorated over time or because the additional supply of business and commercial graduates caused the earnings differential to narrow. Two different approaches were tried in an attempt to discern which of these explanations is correct.

Evidence from Enrollment Equations.

Suppose that the demand for graduates of vocational programs is higher in some areas than others. In the high-demand areas, the demand curve D_T for trained workers in Figure 1 will be shifted to the right relative to the demand curve D_U for untrained workers. This

will in turn shift the point of intersection between D_T and D_U to the right. In areas where the enrollment in vocational programs is not limited, this intersection gives the number of students who enroll in the program. Hence, if the explanation of a lack of earnings differential between vocational program graduates and general program graduates is that the earnings have been equalized by adjustments equating supply and demand of program graduates, then one would expect to find a positive relation between the demand for vocational program graduates in an area and the number of students enrolled in the program in that area.

To test this proposition, we developed a measure of demand for vocational program graduates. This measure uses the responses to the longitudinal survey of the Class of 1972 to calculate the national percentages of vocational program graduates in each occupation; these percentages are then weighted for each of 100 SMSA's by the fraction of the SMSA's employment in each occupation. A regression was estimated for this group in which the dependent variable was a dummy variable taking on a value of 1 if the youth had enrolled in a vocational program in high school and zero otherwise. Observations were confined to those who terminated their education with a high school degree.¹ There was

¹To be sure that the wage equation estimates discussed in the preceding sections of this paper were not affected by the omission of a measure of demand mix, we reestimated the equation for observations falling in SMSA's where a measure of demand for vocational graduates was available and included the measure of demand, and both that measure and an interaction term between vocational enrollment and the demand measure. The results reported earlier, namely that there is no significant wage difference between vocational and general program graduates, remain unchanged.

no evidence of a significant positive relation between the demand mix variable and the probability of enrolling in a vocational program.¹ This evidence suggests rejecting the hypothesis that supply side adjustments are responsible for the absence of a wage differential between vocational and other program graduates. However, one should be somewhat cautious about rejecting the hypothesis based on this evidence alone. One reason is that the measure of demand may be of too poor a quality to generate significant results. Another is that the Class of 1972 sample was not taken to be representative of enrollments in geographic areas, but to provide information on schools in each area. Thus the measured enrollment probabilities may not provide a reliable indication of actual enrollment probabilities in the given market.

Maximum Likelihood Estimation of the Model.

A second approach to investigate whether the returns to vocational training are affected by supply adjustments involves the maximum likelihood estimation of the model presented in the first part of this section. This is particularly important for women, for whom previously discussed evidence suggests that the returns had diminished but not necessarily disappeared by 1976. If there were positive returns in some cities, then the supply adjustment would not be complete in those cities, and there would be no fixed relationship between enrollments and demand for vocational program graduates. In these circumstances, an enrollment

¹In an equation for black females with 38 observations, the coefficient of the mix variable in an enrollment equation was significant and negative.

equation might fail to find significant results because it applies strictly only to those cities for which the adjustment is complete. To accommodate the possibility that the adjustment may be complete in some cities but incomplete in others, the complete model was estimated with maximum likelihood techniques. Note that this model requires a much more explicit specification of the underlying structural model than do the previous estimates, and the results may be sensitive to the accuracy of this specification. Since individual cities cannot be identified in the Parnes data set, the model was estimated only with the Class of 1972 data.

The empirical specification of the model begins with two demand functions, one each for trained and untrained workers:

$$\ln W_T^* = \alpha_0 + \alpha_1 D_T - \alpha_2 N_T$$

$$\ln W_U^* = \beta_0 + \beta_1 D_T - \beta_2 N_U$$

where W_T^* is the wage rate for trained workers, W_U^* the wage rate for untrained workers, D_T the measure of demand for vocational program graduates described previously, N_T the number of trained workers, and N_U the number of untrained workers. The total number of workers, trained and untrained, in a labor market is given by

$$N = N_T + N_U$$

where N is the number of 20-24 year olds with exactly a high school education expressed as a percentage of the population 20-64 years old.

For a given set of values for the α 's, the β 's, and for a given value of the gap d in Figure 1, the values of W_T^* and W_U^* for a particular labor market depend upon the percentage of young people in that market who have had vocational training. In terms of that figure, the availability of vocational training determines whether the number of trained workers will be A, between A and B, or B. The percentage of high school

graduates with vocational training in each labor market (SMSA) was taken from the October 1979 Educational Supplement to the Current Population Survey.¹ The base wages for trained and untrained workers in a particular SMSA are postulated to be the amounts predicted from this model, plus a random disturbance term:

$$\begin{aligned} \ln W_T &= \ln W_T^* + \epsilon_T \\ \ln W_U &= \ln W_U^* + \epsilon_U \end{aligned} \tag{III.1}$$

where ϵ_T and ϵ_U normally distributed random error terms with variances σ_T and σ_U , respectively.

The wages of individual workers in a labor market are further altered according to whether the worker is currently undergoing or has undergone on-the-job training and by the basic ability level of the individual:

$$\begin{aligned} \ln W_V^i &= \ln W_T + \mu_V^1 d_{CT}^i + \mu_V^2 d_{PT}^i + \mu_V^3 S^i + \epsilon_V^i \\ \ln W_N^j &= \ln W_U + \mu_N^1 d_{CT}^j + \mu_N^2 d_{PT}^j + \mu_N^3 S^j + \epsilon_N^j \end{aligned} \tag{III.2}$$

where W_V^i is the wage of the i^{th} graduate of a vocational program, W_N^j the wage of the j^{th} graduate of a non-vocational program, d_{CT} a dummy (0-1) variable indicating current on-the-job training, d_{PT} a dummy variable indicating past on-the-job training, S the test score used in

¹More specifically, the percentage was calculated as the fraction of high school students in grades 10-12 who were taking vocational training courses and were not taking college preparatory courses such as foreign language, chemistry, physics, or mathematics above beginning algebra.

earlier regressions, and ϵ_V^i and ϵ_N^j individual error terms with variance σ^2 . The coefficients for current on-the-job training are expected to be negative, on the presumption that the worker bears at least some of the cost for the training, and the coefficients for past training are expected to be positive.¹

The likelihood function applicable to this model is derived in an appendix available from the authors on request. It is given by:

$$\begin{aligned} \log L = & \sum_{k=1}^K - \frac{n_V^k + n_N^k}{2} \ln(2\pi) \\ & - \frac{n_V^k + n_N^k - 2}{2} \ln \sigma^2 \\ & - \frac{1}{2} \ln [(n_V^k \sigma_T^2 + \sigma^2) (n_N^k \sigma_U^2 + \sigma^2)] \\ & - \frac{1}{2} \frac{n_V^k}{n_V^k \sigma_T^2 + \sigma^2} (\ln W_T^{*,k} - \ln \tilde{W}_T^{*,k})^2 \\ & - \frac{1}{2} \frac{n_N^k}{n_N^k \sigma_U^2 + \sigma^2} (\ln W_U^{*,k} - \ln \tilde{W}_U^{*,k})^2 \\ & - \frac{1}{2\sigma^2} \sum_{i=1}^{n_V^k} (\ln W_V^{i,k} - \mu_V^1 d_{CT}^{i,k} - \mu_V^2 d_{PT}^{i,k} - \mu_V^3 S^{i,k} - \ln \tilde{W}_T^{*,k})^2 \\ & - \frac{1}{2\sigma^2} \sum_{j=1}^{n_N^k} (\ln W_N^{j,k} - \mu_N^1 d_{CT}^{j,k} - \mu_N^2 d_{PT}^{j,k} - \mu_N^3 S^{j,k} - \ln \tilde{W}_U^{*,k})^2 \end{aligned}$$

¹An alternative way to treat on-the-job training would have been to omit the d_{PT} dummy variables and instead to include anyone with past on-the-job training in the group of "trained" workers. This would have precluded consideration of on-the-job training for vocational program graduates, however, and it would also have assumed that the average amount of on-the-job training actually received is equivalent to the training received in a vocational program. The approach used in the paper avoids these restrictive assumptions.

where the index k ranges over 42 SMSA's, n_V^k and n_N^k are the numbers of vocational and non-vocational graduates from the k^{th} SMSA in the sample, and $\ln \hat{W}_V^{*,k}$ and $\ln \hat{W}_U^{*,k}$ are sample estimates of $\ln W_V^{*,k}$ and $\ln W_N^{*,k}$ and are defined as:

$$\ln \hat{W}_T^{*,k} = \frac{1}{n_V^k} \sum_{i=1}^{n_V^k} (\ln w_V^{i,k} - \mu_V^1 d_{CT}^{i,k} - \mu_V^2 d_{PT}^{i,k} - \mu_V^3 S^{i,k})$$

$$\ln \hat{W}_U^{*,k} = \frac{1}{n_V^k} \sum_{j=1}^{n_V^k} (\ln w_N^{j,k} - \mu_N^1 d_{CT}^{j,k} - \mu_N^2 d_{PT}^{j,k} - \mu_N^3 S^{j,k})$$

The parameter set over which this likelihood function is maximized includes the α 's and the β 's in the demand functions, the μ 's describing wage responses to on-the-job training and test scores, the three variances σ^2 , σ_T^2 , and σ_U^2 , and the gap d . The α 's, β 's and d enter the likelihood function indirectly through their effects on W_T^* and W_U^* .

This likelihood function was maximized separately for white males and white females in the Class of 1972 group. The unrestricted maximum value was compared to the maximum value obtained under the constraint $d = 0$, and for both groups the likelihood ratio test indicated that this constraint is not significant. This means that there is no evidence from the maximum likelihood estimation that the gap d is significantly different from zero, although the point estimates were in both cases positive. Given the rather crude nature of the model, this result alone should probably be used with caution, but it does add further support to the results found using least squares techniques.

IV. Conclusions

The Grasso and Shea analysis of the Parnes data found that in 1972, male vocational program graduates had no higher earnings than general program graduates, but that female graduates of business and office programs did enjoy higher earnings. Fitting analogous equations to observations for 1975 and 1976 using both the Parnes data and the Class of 1972 data, we obtained similar results for men. But we found that business and commercial training for women seemed to have much less of an effect in the mid-1970's than it did in 1972. This result appears to be robust with respect to a number of changes in the estimating equations. It does not vary when additional explanatory variables are included, when attempts are made to refine the measure of participation in vocational programs, or when we standardize for inter-area differences in the general wage level and in youth unemployment rates.

For men, the lack of any measurable impact of vocational training on subsequent earnings may occur either because the programs are not producing useful skills or because supply adjustments have eliminated the earnings advantage for vocational program graduates. The latter would occur if enrollments in vocational training were not limited, i.e., the program could accommodate all who applied, so that enrollments in vocational courses would continue to rise as long as there was any earnings differential. If earnings differentials for men had been eliminated in this manner, however, we would expect in cross-section comparisons between labor markets to find that there are higher percentages of students enrolled in vocational programs in labor markets in which the demand for training is relatively high. Cross-section

regressions testing for a relation between vocational program enrollments and a measure of demand for training by employers in 100 SMSA's did not find significant evidence of this relationship. Thus none of the evidence we have developed for men supports a view that vocational programs in high school produce special skills valued by firms beyond the knowledge gained in a general high school program.

The measured impact of business and commercial training on the earnings of female graduates in 1972 could be considered to be the result of supply side limitations which kept earnings for vocational program graduates higher than those of other program graduates. If this explanation is correct, then the decline in the impact of business and commercial training by the mid 1970's could be attributed to a weakening of the supply limitation.

One cannot test for the existence of supply limitations using cross-sectional data by simply regressing enrollments in vocational programs on measures of the local demand for training, as was done for men. The reason is that if the supply adjustment is not complete, or is complete for some labor markets but not for others, there is no reason to expect a tight relationship between enrollments and demand for training. In fact, the regressions we estimated to explain variation among areas in enrollments by young women in commercial-business programs did not find any relation between an index of demand and commercial-business enrollments. The relationship between the earnings advantage of vocational program graduates and characteristics of the local labor market as described in our discussion, is more complex. This relationship may be estimated by maximum likelihood techniques, albeit at the cost of increased risk of specification error. The results of the maximum

likelihood estimation, however, do not provide further support for supply side adjustments as an explanation of the reduced impact of business and commercial training in cross-section. Thus, in the case of commercial-business programs for women, there is evidence that these programs at one time were productive of skills valued by employers. We cannot be sure whether the skills produced in later years continue to be so valued.

References

- Creech, F. Reid, et al. 1977 "Comparative Analysis of Postsecondary Occupational and Educational Outcomes For the High School Class of 1972." Final Report to the Office of Education.
- Grasso, J. T., and J. R. Shea, 1979a, "Effects of Vocational Education Programs: Research Findings and Issues in National Institute of Education," The Planning Papers for Vocational Education, Vocational Education Study Publication, No. 1, 101-194.
- _____, 1979b, Vocational Education and Training: Impact on Youth, Carnegie Council on Policy Studies, Berkeley, California.
- Grubb, W. Norton, 1979, "The Phoenix of Vocational Education: Implications for Evaluation," in National Institute of Education, The Planning Papers for the Vocational Education Study.
- Gustman, A. L. and Segal, M. 1974, "The Skilled-Unskilled Wage Differential in Construction," Industrial and Labor Relations Review.
- Gustman, A. L., and T. L. Steinmeier, 1979, "The Impact of Wages and Unemployment on Youth Enrollment and Labor Supply," Review of Economics and Statistics, forthcoming.
- _____, 1980, "Labor Markets and Evaluations of Vocational Training Programs in the Public High Schools -- Toward a Framework for Analysis," National Bureau of Economic Research, Working Paper Number 478.

- Leighton, L., and J. Mincer, 1979, "Effects of Minimum Wages on Human Capital Formation," National Bureau of Economic Research Working Paper Number 378.
- Meyer, R. H., and D. A. Wise, 1979, "High School Preparation and Early Labor Force Experience," mimeo, National Bureau of Economic Research.
- Nolfi, George J. et al. 1978 Experiences of Recent High School Graduates, Lexington Books D. C. Heath and Co., Lexington, Mass.
- Stromsdorfer, E. W., 1972, Review and Synthesis of Cost-Effectiveness Studies of Vocational Technical Education (The Ohio State University).

Appendix 1

Regression Equations Underlying the Coefficient Estimates Reported in Table 2^a

1A Results for Weekly Earnings Equations

Independent Variables	Class of 1972				Parnes Data			
	White Males	Black Males	White Females	Black Females	White Males	Black Males	White Females	Black Females
Test score	-1.19 (-.81)	-.07 (-.19)	.06 (.36)	.05 (.11)	.69 (1.30)	.07 (.06)	.003 (.01)	.35 (.68)
Socioeconomic status	1.27 (.37)	6.17 (.91)	-1.83 (-.87)	4.77 (1.09)	-3.46 (-.95)	-5.53 (-.82)	-2.01 (-.92)	-1.51 (-.57)
Enrolled in academic program	14.5 (.82)	13.77 (.54)	-25.92 (-1.96)	4.58 (.21)	11.50 (.71)	-9.91 (-.37)	-10.5 (-1.01)	16.18 (.85)
In SMSA	11.3 (.86)	-11.5 (-.59)	3.57 (.39)	-17.7 (-1.01)	17.8 (1.31)	2.00 (.09)	27.5 (2.98)	31.6 (1.95)
In VOC program (not Com-Bus)	21.9 (1.35)	-22.2 (-.97)	-14.66 (-.83)	-21.06 (-.99)	16.6 (1.05)	-19.8 (-.64)	10.8 (.62)	--
In Com-Bus. program	13.3 (.56)	2.72 (.09)	-13.14 (-1.28)	-6.26 (-.32)	3.95 (.15)	--	23.4 (2.18)	18.4 (1.60)
Con. tant	227 (4.82)	134 (1.44)	141 (3.80)	80.29 (.97)	80.2 (1.35)	158 (1.73)	87.5 (2.09)	49.5 (.90)
R ²	.01	.05	.02	.06	.06	.14	.30	.60
Standard Error	134	69.8	77.5	60.5	54.3	38.11	27.9	17.5
No. of Observations	466	60	311	67	83	18	52	14

a, t-statistics are reported in parentheses.

1B Results for Weeks Employed Equations

Independent Variables	Class of 1972				Parnes Data			
	White Males	Black Males	White Females	Black Females	White Males	Black Males	White Females	Black Females
Test score	-.003 (-.13)	.06 (.90)	.01 (.47)	-.02 (-.20)	-.26 (-1.80)	-.28 (-.68)	-.14 (-1.24)	.34 (1.67)
Socioeconomic status	-.016 (-.05)	-1.82 (-1.56)	.33 (.86)	-1.83 (-1.85)	-1.77 (-1.82)	-2.04 (-.79)	.017 (.02)	-3.31 (-3.24)
Enrolled in academic program	-3.93 (-2.54)	2.47 (.56)	-.82 (-.34)	-4.15 (-.83)	2.54 (.58)	-5.75 (-1.56)	-.87 (-.25)	9.55 (1.30)
In SMSA	-.60 (-.52)	-3.66 (-1.09)	-1.28 (-.78)	6.10 (1.53)	-.75 (-.21)	11.23 (1.30)	5.75 (1.85)	7.34 (1.18)
In VOC program (not Com-Bus)	.53 (.38)	2.03 (.52)	-1.09 (-.34)	-2.55 (-.53)	.58 (.14)	-1.42 (-.12)	4.03 (.69)	--
In Com-Bus program	.13 (.01)	5.50 (1.02)	-2.27 (1.22)	-9.53 (-2.14)	1.07 (.16)	--	4.27 (1.18)	6.95 (1.57)
Constant	43.7 (9.26)	48.6 (3.05)	33.79 (5.01)	58.18 (3.12)	86.4 (5.46)	75.20 (2.15)	56.72 (4.03)	37.95 (1.78)
R ²	.02	.10	.01	.14	.10	.33	.14	.74
Standard Error	11.78	11.98	14.10	13.72	14.5	14.67	9.41	6.75
No. of Observations	466	60	311	67	83	18	52	14

Appendix 2

Regression Equations Underlying the Coefficient
Estimates Reported in Table 3^a

Independent Variables	Usual Weekly Earnings		Weeks Worked	
	White Females	Black Females	White Females	Black Females
Test score	1.01 (1.66)	.14 (.19)	.01 (.06)	-.05 (-.25)
Socioeconomic status	-1.07 (-.02)	-1.19 (-.11)	.29 (.24)	.12 (.04)
Enrolled in academic program	-6.65 (-.32)	22.86 (.77)	-2.19 (-.35)	9.25 (1.14)
Years work experience	3.75 (.53)	9.75 (.80)	2.52 (1.17)	6.00 (1.79)
In SMSA	16.18 (1.06)	40.76 (1.57)	3.27 (.70)	3.83 (.54)
In vocational program (not Com-Bus)	107.84 (2.47)	--	-11.58 (-.87)	--
In Com-Bus Program	-1.08 (-.06)	3.02 (.10)	1.17 (.22)	3.99 (.48)
Constant	-12.87 (-.15)	32.58 (.23)	29.65 (1.17)	24.01 (.62)
R ²	.27	.45	.08	.56
Standard error	41.5	31.3	12.6	8.6
Number of Observations	38	11	38	11

^at-statistics are reported in parentheses.

Means and Standard Deviations of Selected Variables Utilized
in Tables 1-6

Variable	Class of 1972 (1976 data)						Parnes Sample (1972 data)			
	White Males	Black Males	White Females	Black Females	White Males	Black Males	White Females	Black Males	White Females	Black Females
Usual weekly earnings	216 (1.34)	170 (.68)	125 (.77)	119 (.59)	134 (.54)	117 (.35)	88 (.31)	101 (.22)		
Weeks employed	42 (.12)	42 (.12)	39 (.14)	37 (.14)	43 (.15)	42 (.15)	47 (.10)	46 (.10)		
Test score	192 (.28)	156 (.25)	199 (.28)	163 (.20)	101 (.12)	87 (.10)	104 (.12)	88 (.10)		
Parents' socio-economic status	11 (1.9)	9 (1.4)	11 (2.2)	9 (1.9)	10 (1.9)	8 (1.7)	11 (2.0)	9 (2.0)		
Enrolled in academic program	.18 (.38)	.20 (.40)	.17 (.38)	.18 (.39)	.20 (.41)	.17 (.38)	.29 (.46)	.07 (.27)		
In SMSA	.40 (.49)	.42 (.50)	.45 (.50)	.52 (.50)	.66 (.48)	.72 (.46)	.71 (.46)	.86 (.36)		
In vocational program other than Com-Bus ^a	.21 (.41)	.28 (.45)	.07 (.26)	.19 (.40)	.28 (.45)	.11 (.32)	.06 (.24)	---		
In Com-Bus program	.08 (.27)	.12 (.32)	.36 (.48)	.25 (.44)	.07 (.26)	---	.21 (.41)	.36 (.50)		
Military service	.14 (.35)	.17 (.38)	---	---	.34 (.48)	.22 (.43)	---	---		
In South	.28 (.45)	.77 (.43)	---	---	.39 (.49)	.78 (.43)	---	---		

Table continued. . .

Appendix 3 continued

Means and Standard Deviations of Selected Variables Utilized in Tables 1-6

Variable	Class of 1972 (1976 data)				Persons Out of School Four Years				Parnes Sample (1972 data)			
	White Males	Black Males	White Females	Black Females	White Males	Black Males	White Females	Black Females	White Males	Black Males	White Females	Black Females
Training	.53 (.50)	.47 (.50)	.49 (.50)	.45 (.50)	.64 (.48)	.50 (.51)	.56 (.50)	.36 (.50)				
Fraction full-time			.80 (.40)	.83 (.37)	--	--	.83 (.38)	.93 (.27)				
Union	.30 (.46)	.20 (.40)	.11 (.31)	.16 (.37)	.35 (.48)	.28 (.46)	.15 (.36)	.43 (.51)				
Years of job tenure	--	--	1.17 (1.25)	1.51 (1.50)	--	--	1.95 (1.29)	1.67 (1.16)				
Agricultural	.04 (.19)	--	.01 (.10)	--	--	--	--	--				
Business	.04 (.19)	--	.33 (.47)	--	--	--	--	--				
Distributive education	.04 (.20)	--	.03 (.18)	--	--	--	--	--				
Health fields	.004 (.07)	--	.03 (.16)	--	--	--	--	--				
Home economics	.002 (.05)	--	.03 (.16)	--	--	--	--	--				
Trade	.16 (.37)	--	.01 (.11)	--	--	--	--	--				

Table continued...

Means and Standard Deviations of Selected Variables Utilized
in Table 1-6

Variable	Class of 1972 (1972 data)				Parnes Sample (1972 data)			
	White Males	Black Males	White Females	Black Females	White Males	Black Males	White Females	Black Females
Semester hours of vocational courses	28 (.36)	46 (1.07)	37 (.44)	36 (.45)	--	--	--	--
High School has >50% VOC enrollment	.19 (.39)	.20 (.40)	.12 (.32)	.22 (.42)	--	--	--	--
Identified by Kolstad as vocational school	.01 (.11)	.02 (.13)	--	--	--	--	--	--
Usual weekly earnings								Parnes 1975 sample 117 (44)
Weeks employed								48 (8.2)
In vocational program								.026 (.16)
In Commercial-business program								.368 (.49)

^aThe dummy variable measuring participation in a commercial (business or office) course is defined explicitly in the Parnes survey but not in the Class of 1972 Survey. An analogous variable was constructed for the Class of 1972 survey by classifying all those enrolled in a business or distributive program as enrolled in a commercial course of study.

Appendix 4

Derivation of the Likelihood Function

In this appendix, we will derive the likelihood function for the model in Section III. To begin, substitute equations (III.1) into equations (III.2):

$$\begin{aligned} \ln W_V^i &= \ln W_T^* + \mu_V^1 d_{CT}^i + \mu_V^2 d_{PT}^i + \mu_V^3 S^i + \epsilon_T + \epsilon_V^i \\ \ln W_N^j &= \ln W_U^* + \mu_N^1 d_{CT}^j + \mu_N^2 d_{PT}^j + \mu_N^3 S^j + \epsilon_U + \epsilon_N^j \end{aligned} \quad (4.1)$$

Define $\eta_V^i = \epsilon_T + \epsilon_V^i$ and $\eta_N^j = \epsilon_U + \epsilon_N^j$. Under the assumption that ϵ_T and the ϵ_V^i are independent normally distributed random variables, the η_V^i will have variance $\sigma_T^2 + \sigma^2$ and covariance σ_T^2 , where $\text{Var}(\epsilon_T) = \sigma_T^2$ and $\text{Var}(\epsilon_V^i) = \sigma^2$. The variance-covariance matrix Σ_V of the vector $\underline{\eta}_V$ of random variables, may be written as:

$$\begin{aligned} \Sigma_V &= \begin{bmatrix} \sigma_T^2 + \sigma^2 & \sigma_T^2 & \cdot & \cdot & \cdot & \sigma_T^2 & \sigma_T^2 \\ \sigma_T^2 & \sigma_T^2 + \sigma^2 & \cdot & \cdot & \cdot & \sigma_T^2 & \sigma_T^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sigma_T^2 & \sigma_T^2 & \cdot & \cdot & \cdot & \sigma_T^2 + \sigma^2 & \sigma_T^2 \\ \sigma_T^2 & \sigma_T^2 & \cdot & \cdot & \cdot & \sigma_T^2 & \sigma_T^2 + \sigma^2 \end{bmatrix} \\ &= \sigma_T^2 \underline{H}\underline{H}' + \sigma^2 \underline{I} \end{aligned} \quad (4.2)$$

where \underline{H} is a column vector of ones and \underline{I} is the identity matrix. Similarly, the variance-covariance of $\underline{\eta}_N$ is given by $\Sigma_N = \sigma^2 \underline{H}\underline{H}' + \sigma_U^2 \underline{I}$.

Appendix 4 (continued)

The likelihood function for a particular SMSA is simply the joint probability of $\underline{\eta}_V$ and $\underline{\eta}_N$:

$$L = f(\underline{\eta}_V) f(\underline{\eta}_N)$$

where f is the multivariate normal density function.¹ The log of the multivariate density function is given by:

$$\ln L = \ln f(\underline{\eta}_V) + \ln f(\underline{\eta}_N)$$

where

$$\ln f(\underline{\eta}_V) = -\frac{n_V}{2} \ln(2\pi) - \frac{1}{2} \ln |\det \Sigma_V| - \frac{1}{2} \underline{\eta}_V' Z_V^{-1} \underline{\eta}_V$$

and similarly for $\ln f(\underline{\eta}_N)$. The term n_V is the number of observations for vocational graduates in the particular SMSA.

To evaluate $\det \Sigma_V$ in $f(\underline{\eta}_V)$, we note that adding or subtracting one row or column of a matrix to another does not change the value of the determinant. First, subtract the first row from each of the others.

¹The separability of the joint probability function assumes that $\underline{\eta}_V$ and $\underline{\eta}_N$ are independent, which in turn assumes that ϵ_T and ϵ_U are independent. A likelihood function can be constructed for ϵ_T and ϵ_U not independent, but since the maximum likelihood estimate of σ_T^2 was 0, this issue is moot.

Appendix 4 (continued)

$$\det \Sigma_V = \det \begin{bmatrix} \sigma_T^2 + \sigma^2 & \sigma_T^2 & \cdot & \cdot & \cdot & \sigma_T^2 & \sigma_T^2 \\ -\sigma^2 & \sigma^2 & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -\sigma^2 & 0 & \cdot & \cdot & \cdot & \sigma^2 & 0 \\ -\sigma^2 & 0 & \cdot & \cdot & \cdot & 0 & \sigma^2 \end{bmatrix}$$

Next, add columns 2 through n_V to column 1:

$$\det \Sigma_V = \det \begin{bmatrix} n_V \sigma_T^2 + \sigma^2 & \sigma_T^2 & \cdot & \cdot & \cdot & \sigma_T^2 & \sigma_T^2 \\ 0 & \sigma^2 & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & \sigma^2 & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & 0 & \sigma^2 \end{bmatrix}$$

Since the last matrix is triangular, the determinant is simply the product of the diagonal elements, and

$$\det \Sigma_V = (\sigma^2)^{n_V - 1} (n_V \sigma_T^2 + \sigma^2)$$

To evaluate $\eta_V' \Sigma_V^{-1} \eta_V$, three lemmas will be helpful:

Lemma 1. $\Sigma_V^{-1} \underline{H} = \frac{1}{n_V \sigma_T^2 + \sigma^2} \underline{H}$

Pf. By equation (4.2), we have

Appendix 4 (continued)

$$\Sigma_V^{-1} (\sigma_{T_{--}}^2 \mathbf{H}\mathbf{H}' + \sigma^2 \mathbf{I}) = \mathbf{I}$$

Post-multiplying by \mathbf{H} and noting that $\mathbf{H}'\mathbf{H} = n_V$

$$\Sigma_V^{-1} (\sigma_{T_{--}}^2 \mathbf{H}\mathbf{H}'\mathbf{H} + \sigma^2 \mathbf{H}) = \mathbf{H}$$

$$\Sigma_V^{-1} (n_V \sigma_V^2 + \sigma^2) \mathbf{H} = \mathbf{H}$$

Dividing by the scalar quantity $n_V \sigma_V^2 + \sigma^2$ yields the desired result.

Lemma 2.
$$\mathbf{H}' \Sigma_V^{-1} \mathbf{H} = \frac{n_V}{n_V \sigma_V^2 + \sigma^2}$$

Pf. This result follows immediately by pre-multiplying the equation of the previous lemma by \mathbf{H}' and noting that $\mathbf{H}'\mathbf{H} = n_V$.

Lemma 3. If $\underline{\mathbf{X}}$ is a vector such that $\mathbf{H}'\underline{\mathbf{X}} = 0$, then

$$\underline{\mathbf{X}}' \Sigma_V^{-1} \underline{\mathbf{X}} = \frac{1}{\sigma^2} \underline{\mathbf{X}}' \underline{\mathbf{X}}$$

Pf.
$$\underline{\mathbf{X}}' \Sigma_V^{-1} \underline{\mathbf{X}} = \frac{1}{\sigma^2} \underline{\mathbf{X}}' \Sigma_V^{-1} \sigma_{T_{--}}^2 \mathbf{H}\mathbf{H}' \underline{\mathbf{X}} + \underline{\mathbf{X}}' \Sigma_V^{-1} \underline{\mathbf{X}},$$

where the first term is zero because $\mathbf{H}'\underline{\mathbf{X}} = 0$. Continuing,

$$\begin{aligned} \underline{\mathbf{X}}' \Sigma_V^{-1} \underline{\mathbf{X}} &= \frac{1}{\sigma^2} \underline{\mathbf{X}}' \Sigma^{-1} (\sigma_{T_{--}}^2 \mathbf{H}\mathbf{H}' + \sigma^2 \mathbf{I}) \underline{\mathbf{X}} \\ &= \frac{1}{\sigma^2} \underline{\mathbf{X}}' \Sigma^{-1} \Sigma \underline{\mathbf{X}} \\ &= \frac{1}{\sigma^2} \underline{\mathbf{X}}' \underline{\mathbf{X}} \end{aligned}$$

Appendix 4 (continued)

This establishes the validity of Lemma 3.

Define the scalar quantity $\ln \hat{W}_T^*$ as

$$\ln \hat{W}_T^* = \ln W_T^* + \frac{1}{n_V} \sum_{i=1}^{n_V} \eta_V^i \quad (4.3)$$

Note that

$$\sum_{i=1}^{n_V} \eta_V^i = \underline{\eta}_V' \underline{H}$$

so that equation (4.3) may be rearranged as

$$\begin{aligned} 0 &= \underline{\eta}_V' \underline{H} + n_V (\ln W_T^* - \ln \hat{W}_T^*) \\ &= \underline{\eta}_V' \underline{H} + (\ln W_T^* - \ln \hat{W}_T^*) \underline{H}' \underline{H} \\ &= [\underline{\eta}_V + (\ln W_T^* - \ln \hat{W}_T^*) \underline{H}]' \underline{H} \end{aligned} \quad (4.4)$$

Thus the vector $\underline{\eta}_V + (\ln W_T^* - \ln \hat{W}_T^*) \underline{H}$ satisfies the condition of Lemma 3.

The quantity $\underline{\eta}_V' \Sigma_V^{-1} \underline{\eta}_V$ may be rewritten as:

$$\begin{aligned} \underline{\eta}_V' \Sigma_V^{-1} \underline{\eta}_V &= (\ln W_T^* - \ln \hat{W}_T^*)^2 \underline{H}' \Sigma_V^{-1} \underline{H} \\ &\quad - 2 (\ln W_T^* - \ln \hat{W}_T^*) \underline{H}' \Sigma_V^{-1} [\underline{\eta}_V + (\ln W_T^* - \ln \hat{W}_T^*) \underline{H}] \\ &\quad + [\underline{\eta}_V + (\ln W_T^* - \ln \hat{W}_T^*) \underline{H}]' \Sigma_V^{-1} [\underline{\eta}_V + (\ln W_T^* - \ln \hat{W}_T^*) \underline{H}] \end{aligned} \quad (4.5)$$

The first term becomes $\frac{n_V}{n_V \sigma_T^2 + \sigma^2} (\ln W_T^* - \ln \hat{W}_T^*)^2$ by Lemma 2. For the

Appendix 4 (continued)

second term, an application of Lemma 1 yields:

$$\begin{aligned} & (\ln W_T^* - \ln \tilde{W}_T^*) H' \Sigma_V^{-1} [\underline{\eta}_V + (\ln W_T^* - \ln \tilde{W}_T^*) H] \\ &= \frac{1}{n_V \sigma_T^2 + \sigma^2} (\ln W_T^* - \ln \tilde{W}_T^*) H' [\underline{\eta}_V + (\ln W_T^* - \ln \tilde{W}_T^*) H] \end{aligned}$$

Equation (4.4) implies that $H' [\underline{\eta}_V + (\ln W_T^* - \ln \tilde{W}_T^*) H]$, and thereby the entire term, is zero. For the third term of equation (4.5), the condition of Lemma 3 is satisfied, and the term can be rewritten as:

$$\begin{aligned} & [\underline{\eta}_V + (\ln W_T^* - \ln \tilde{W}_T^*) H]' \Sigma_V^{-1} [\underline{\eta}_V + (\ln W_T^* - \ln \tilde{W}_T^*) H] \\ &= \frac{1}{\sigma^2} [\underline{\eta}_V + (\ln W_T^* - \ln \tilde{W}_T^*) H]' [\underline{\eta}_V + (\ln W_T^* - \ln \tilde{W}_T^*) H] \\ &= \frac{1}{\sigma^2} \sum_{i=1}^{n_V} (\eta_V^i + \ln W_T^* - \ln \tilde{W}_T^*)^2 \\ &= \frac{1}{\sigma^2} \sum_{i=1}^{n_V} (\ln W_V^i - \mu_V^i d_{CT}^i - \mu_V^2 d_{PT}^i - \mu_V^3 S^i - \ln \tilde{W}_T^*)^2 \end{aligned}$$

where the last substitution, for $\eta_V^i + \ln W_T^*$, comes from equation (4.1).

Collecting results, the log of the likelihood function for vocational program graduates is given by

$$\begin{aligned} \ln P(\underline{\eta}_V) &= - \frac{n_V}{2} \ln(2\pi) - \frac{(n_V - 1)}{2} \ln \sigma^2 \\ &\quad - \frac{1}{2} \ln(n_V \sigma_T^2 + \sigma^2) \end{aligned}$$

Appendix 4 (continued)

$$\begin{aligned}
& - \frac{1}{2} \frac{n_V}{n_V \sigma_T^2 + \sigma^2} (\ln W_T^* - \ln \tilde{W}_T^*)^2 \\
& - \frac{1}{2\sigma^2} \sum_{i=1}^{n_V} (\ln W_V^i - \mu_V^i d_{CT}^i - \mu_V^2 d_{PT}^i - \mu_V^3 S^i - \ln \tilde{W}_T^*)^2
\end{aligned}$$

An analogous formula may be computed for $\ln f(\underline{\eta}_N)$. The log-likelihood function is then the value of $\ln f(\underline{\eta}_V) + \ln f(\underline{\eta}_N)$ summed over all the SMSA's.