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A MODEL OF THE BLACK MARKET FOR DOLLARS

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ABSTRACT

The paper develops an analytical framework to discuss the determinants of the premium in the black market for dollars in Brazil. While the specific details of the model were chosen with the Brazilian case in mind, the structure of the model is quite general and suitable for application to black markets for currency elsewhere.

The building blocks of the model are three. A capital asset pricing approach is used to derive an asset demand for dollars, or equivalently a real yield premium in market equilibrium. The current account of the black market is specified in terms of the sources and uses in the flow market for dollars, mainly smuggling proceeds and flows associated with tourism. The model is closed by a model of official exchange rate policy and the assumption of rational expectations.

In comparative static applications the model has the properties of current account oriented models of the exchange rate. Unanticipated current account improvements due, for example, to increased export taxes that promote smuggling, lead to a decline in the premium. Asset market disturbances, such as increased inflation uncertainty or increased variability in the official real exchange rate policy are shown to have ambiguous effects on the premium. In applying the distinction between anticipated and unanticipated disturbances it is shown that the current expectation of a future maxi-devaluation leads to an immediate rise in the premium, with a subsequent decline when the maxi actually takes place. The paper concludes with a discussion of seasonal patterns in the premium. It is shown that for "always" anticipated disturbances there is no jump in the premium, but a gradual adjustment that precedes the actual seasonal in the current account.

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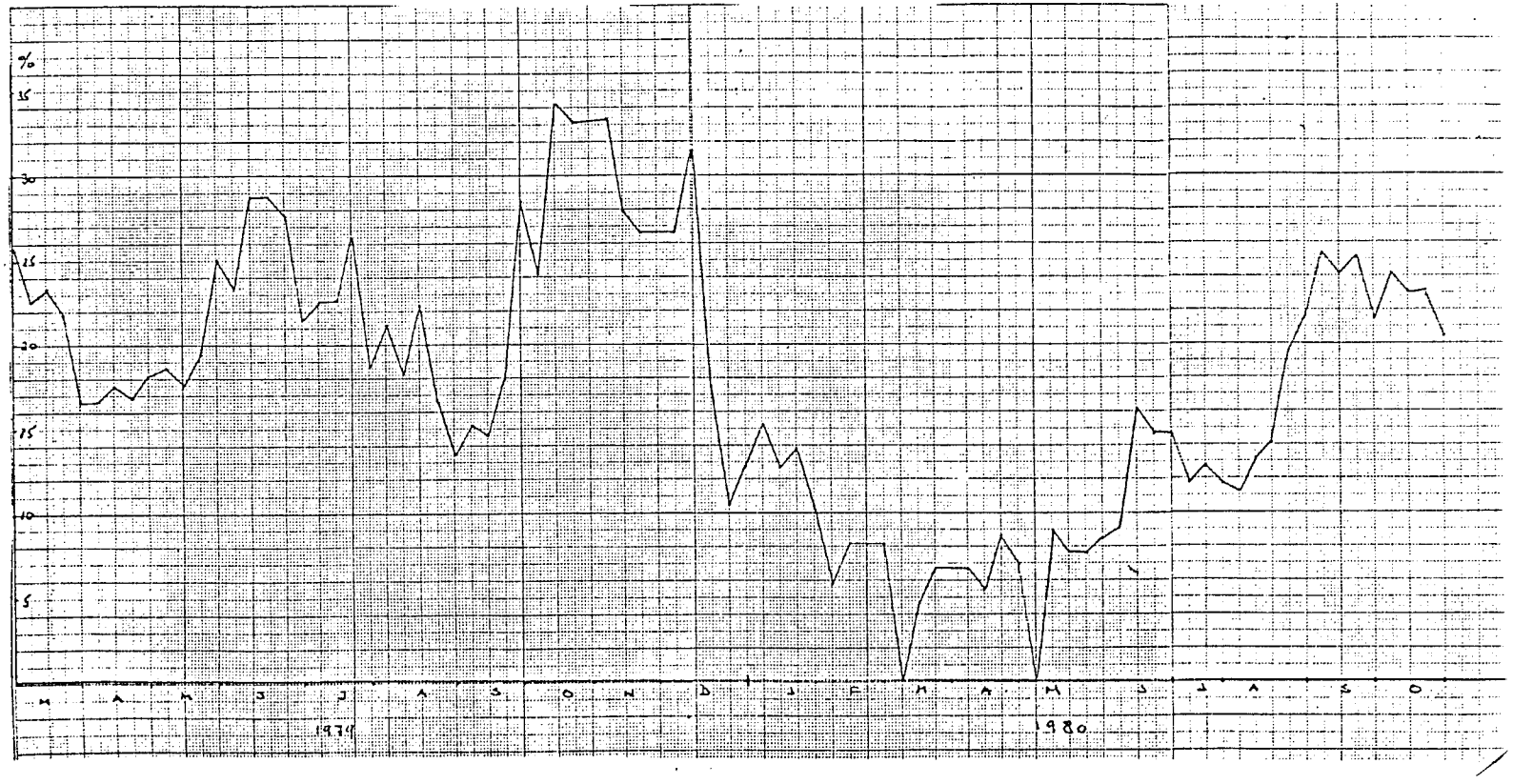
This paper develops a model of the principal determinants of the black market premium on dollars. Chart 1 shows the premium at different points in time between 1978-80 in the Brazilian black market. It is apparent that there is substantial volatility in the premium, but that there is also a sharp run-up in October 1979 with a subsequent decline.

The model developed here treats the black market as a "side-show", assuming that it has no feedback effect on the real economy and on financial markets. It takes a portfolio balance approach to the stock demand for dollars. In combination with a model of the net flow supply of dollars and the assumption of rational expectations, the portfolio model generates a dynamics for the premium and for the stock of black dollars. This part of our analysis resembles closely the modelling of exchange rate dynamics in Kouri (1976) or Dornbusch and Fischer (1980). In the comparative static analysis of the premium we distinguish the role of changes in financial markets-real interest rates, relative asset supplies, inflation uncertainty— from changes in flow markets such as increases in coffee export taxes. A distinction is also drawn between anticipated and unanticipated disturbances and that distinction is applied to investigate the impact of an anticipated maxi on the path of the premium. Finally, in connection with tourism, we investigate the role of seasonal factors in a rational expectations, dynamic model of the black market .

CHART 1 : THE BLACK MARKET PREMIUM

(Thursday quotations, percent of official sale rate)



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1. The Model

The model of the black market developed in this section comprises three parts. A first develops a portfolio demand for dollars derived from the capital asset pricing model. The CAPM model specifies the required rate of return on dollars as a function of the return on the cruzeiro portfolio, the relative supplies of assets and the stochastic structure of returns. The second part is the flow model that specifies additions to and depletion of the dollar pool. Among the main determinants of the black market flows are tourism and the proceeds from smuggling, which in turn are governed by the official real exchange rate, various fiscal measures and the black market premium. In a last part the model is closed by the assumption of rational expectations which allows us to derive the time path of the black market premium.

i. The portfolio demand for dollars

The CAPM model gives rise to a real yield differential, in market equilibrium, that specifies relative security returns as a function of the market coefficient of risk aversion, θ, the variability of relative returns, s^2 and the deviation of the relative supply of assets, $V^*E/(V^*E+V)$, from the minimum variance portfolio share α.

$$(1) \qquad r^* - r = \theta s^2 \left(\frac{EV^*}{EV^*+V} - \alpha \right)$$

In (1) r and r* are the real returns, in terms of the Brazilian consumption basket, on Brazilian securities and on dollars.[1] E denotes the black market cruzeiro price of dollars and V* and V are respectively the nominal stocks of dollars and cruzeiro assets. Equation (1) shows that when the relative supply of dollars exceeds the share dollars would occupy in a minimum variance portfolio, α then dollar assets must carry a real yield premium or a risk premium. The size of the risk premium depends on the deviation of relative asset supplies from the minimum variance portfolio composition as well as on the market coefficient of risk aversion and the variability of the yield differential. For a given stochastic structure both s^2 and α are given, and hence the real yield premium depends only on the relative asset supply.

The real yield differential equation can be rewritten slightly to focus on the black market premium, $x \equiv E/\bar{E}$, where $\bar{E}$ is the official exchange rate. To do so we divide and mutiply the first term in brackets by $\bar{E}$ to obtain:

$$(1)' \qquad r^* - r = \Theta s^2 \left(\frac{xV^*}{xV^* + V/\bar{E}} - \alpha \right)$$

The term $V/\bar{E}$ represents the dollar value of cruzeiro assets measured at the official exchange rate. We treat that variable as exogeneous and governed by exchange rate policy and financial policy.

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[1] The portfolio model draws on Kouri (1978) and Dornbusch (1980). For details see the appendix.

In (1)' the real yield premium on dollars is higher the higher the black market premium and the higher the stock of dollar assets. It is reduced by an increase in the dollar value of cruzeiro assets. The next step is to go from the real yield differential to an expression involving nominal returns and the rate of depreciation of the black market. For that purpose we define the real returns on dollar and cruzeiro assets in terms of the Brazilian consumer price index:

$$(2) \qquad r^* = i^* + d - \dot{p} \quad ; \quad r = i - \dot{p}$$

where i^* and i are the nominal returns on dollar and cruzeiro assets, d is the rate of depreciation of the cruzeiro in the black market and $\dot{p}$ is the Brazilian inflation rate. We note at this point that the black market dollars are held in currency so that i^* may be zero for the major part of holders. Furthermore the penalties associated with illegal holdings of dollars may well imply that the expected nominal return i^* is negative.

From (2) we obtain an equation for the real yield diffferential r^*-r which after substitution in (1)' yields:

$$(3) \qquad d = i - i^* + \theta s^2 \left(\frac{xV^*}{xV^* + V/\bar{E}} - \alpha \right)$$

The equilibrium rate of depreciation of the cruzeiro in the black market derived in (3) thus depends on the nominal yield differential and on the relative supply of assets.

ii. Official Exchange Rate Policy and Premium Dynamics

The official exchange rate policy is assumed to be a purchasing power parity rule with a random component that has zero mean and is serially uncorrelated:

$$(4) \qquad \bar{d} = \dot{p}' - \dot{p}^* + u$$

With the expectation of the random term u equal to zero the expected rate of change of the black market premium is given by $\dot{x} = d-\bar{d}$, or; using (2), (3) and (4),[1]

$$(5) \qquad \dot{x} = d-\bar{d} = (\bar{r} - \bar{r}^*) + \theta s^2 \left(\frac{xV^*}{xV^*+V/\bar{E}} - \alpha \right)$$

For given real returns, r and $\bar{r}^*$, and a given stochastic structure we now have a relation between the premium, its rate of change and the supplies of dollars and the dollar value of cruzeiro assets.

In Figure 1 we show the schedule $\dot{x}=0$ along which the black market premium is constant. The schedule is drawn for given $r-\bar{r}^*$ and $V/\bar{E}$. It is downward sloping since a rise in the premium raises the relative supply of dollar assets and must therefore, given the real yields, be accompanied by an offsetting reduction in the stock of dollar assets, V^*. Points above and to the right of $\dot{x}=0$ correspond to a rising premium and conversely for points below and to the left. When the relative

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[1] The real return on dollar assets in terms of dollar prices is defined as $\bar{r}^* = i^* - \dot{p}^*$ and $\bar{r} = i - \dot{p}'$ is the real return on cruzeiro assets measured in terms of producer prices.

supply of dollars is large, as is the case to the right of $\dot{x}=0$, dollars have to carry a risk premium in the form of an appreciating premium. Conversely, to the left of $\dot{x}=0$ with low relative supply of dollars and a high relative supply of cruzeiros it is the latter that require a risk premium via a depreciating black market dollar premium.

iii. The Flow Market for Dollars

The existing stock of dollar assets in the hands of Brazilian residents rises to the extent that export revenues are withheld from the official market and rather channeled through the black market. The main sources here are the"argentinians" a synonym for tourists, and revenue from coffee and sugar smuggling. The dollar pool is depleted, on the other hand whenever import expenditures are financed through black market resources rather than official dollars. The main sources of this depletion are Brazilian tourists, overinvoicing of exports in the context of coffee exports, and import smuggling.

For the moment we make the simplifying assumption that the rate of increase of the dollar pool, $\dot{V}^*$, depends positively on the official real exchange rate, $\bar{e}$, and on the black market premium. A more depreciated real exchange rate raises the rate of increase of the pool as it discourages Brazilians going abroad while increasing the flow of tourists from abroad. We reserve for later the discussion of the effects of

(6) $\dot{V}^* = J(\bar{e}, x)$ $\qquad J_{\bar{e}} > 0,\ J_x > 0$

various fiscal measures. In (6) we assume that dollars are held in the form of currency. Otherwise the black market's current account would also include interest earnings and thus would be a function of the stock of dollar holdings, V^*.

In Figure 1 we show the schedule $\dot{V}^*=0$ along which the stock of dollars held by the black market is stationary. The schedule is drawn for a given official real exchange rate and fiscal measures. Points to the right imply a premium such that accretions into the market exceed the outflow so that the stock is rising; conversely points to the left imply a premium so low that depletion exceeds inflows.

iv. The Perfect Foresight Path

The dynamics of Figure 1 readily suggest that there is a unique stable trajectory FF along which the black market converges to a stationary premium and dollar stock. Any other path is a rational expectations path and satisfies the asset market equilibrium condition (3) but does not converge. Following the tradition in exchange rate modelling (see Dornbusch and Fischer 1980 and the references given there) we confine our attention to the convergent path.

The model is closed by the assumption that, for any given initial stock of dollars, the economy moves to the corresponding premium on the perfect foresight path and evolves from there along FF to the steady state. It is interesting to note that in the transition a current account surplus of the black market, to the right of $\dot{V}^*=0$, implies growing dollar assets

and a declinig premium. This result replicates, of course, the link between the current account and the exchange rate in the recent literature.[1]

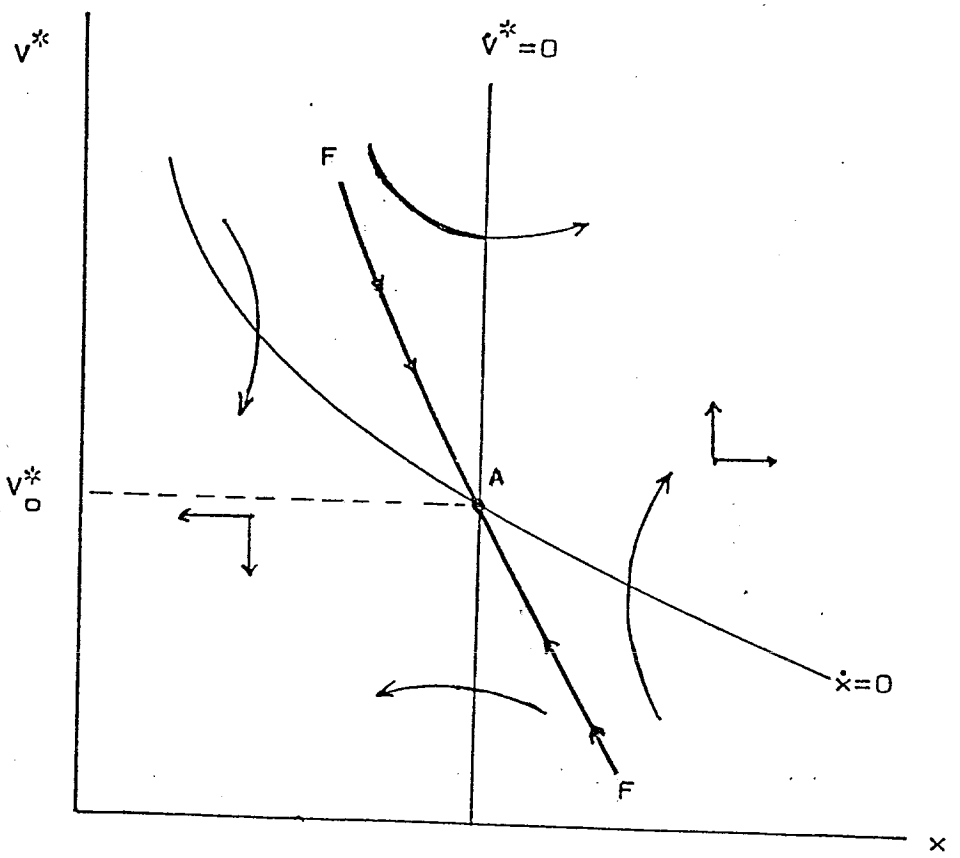

FIGURE 1

[1] For references see Dornbusch (1980a)

2. Financial Policies and The Black Market Premium

In this section we consider the effects of changes in financial policies on the black market premium. Financial policies affect the black market via equation (5) that shows the evolution of the premium as determined by the conditions of portfolio equilibrium and the official exchange rate policy. The prime examples of changes in financial policies are three: changes in the stochastic structure of returns, s^2, changes in the home real interest rate and lastly changes in the dollar value of home nominal assets, $V/\bar{E}$.

Consider first a rise in the home real interest rate. This implies a rise in the return on home securities. For the premium to remain unchanging the higher yield calls for a rise in the relative supply of home assets — a fall in the premium or the level of dollar holdings. In terms of Figure 2 the higher home real interest rate shifts the $\dot{x}=0$ schedule down and to the left. From an initial steady state at point A we have an immediate decline in the premium to point A'. This immediate jump of the premium is required to place the economy on the new perfect foresight path and thus ensure convergence to the new steady state. As is apparent from the dynamics, failure to jump to A' would imply that the economy would move from A in a northeasterly direction on an aberrant path. At A' the fall in the premium implies depletion of the stock of dollar holdings. The depletion continues with a falling relative supply of dollars, until point A" is reached. At A" we have a new stockflow equilibrium, where the higher relative yield on home securities is matched by a higher relative supply of these assets. There

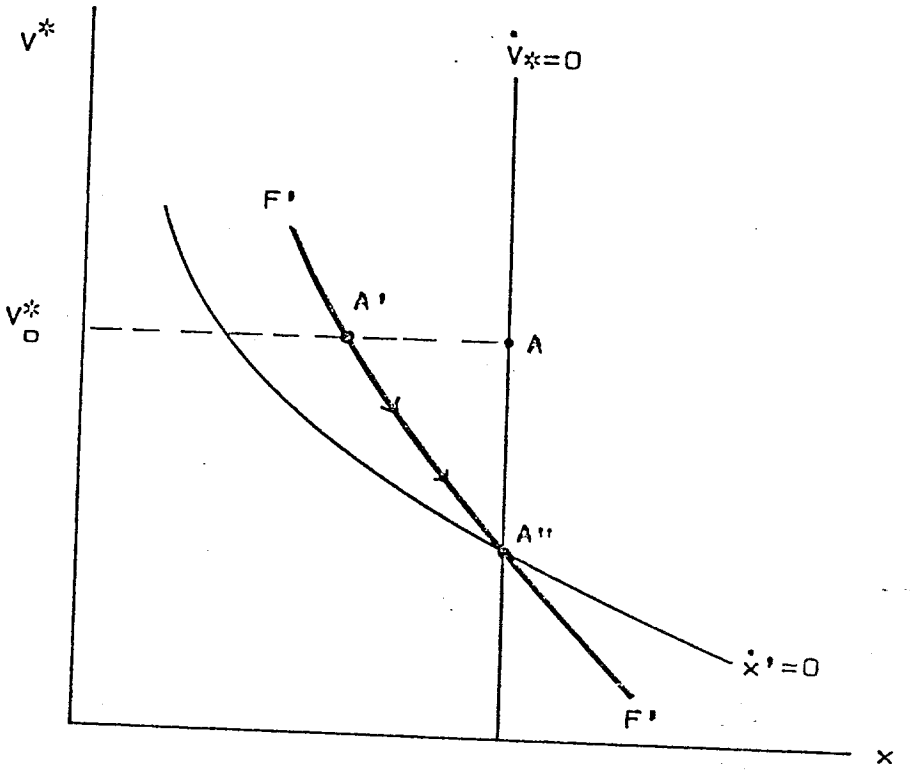

FIGURE 2

is no longrun effect on the premium which returns to its initial steady state level.

The analysis immediately suggests that a rise in the relative supply of home assets $V/\bar{E}$ shifts the $\dot{x}=0$ schedule up and to the right. There will be an immediate increase in

the premium followed by a subsequent decline as the stock of dollar assets rises. Again, in the longrun there is no change in the premium but rather an offsetting increase in the holding of dollar assets.

Consider now a change in the stochastic structure. Specifically assume that at home the variability of inflation rises and hence the variability of the real return on cruzeiro assets increases. The effect of that increased inflation variance on the black market premium depends on whether there is a net shift toward dollars or cruzeiro assets. That shift in turn depends on two factors. On one hand the increased inflation variance raises the variability of relative returns and therefore reduces speculation. On the other hand there is a change in the minimum variance portfolio which now contains relatively less cruzeiro assets. Thus if we start off in a circumstance where the relative supplies of assets exactly match the initial minimum variance portfolio, the net speculative position being zero, then increased inflation uncertainty shifts portfolios toward dollars and thus, in terms of Figure 2, shifts $\dot{x}=0$ out and to the right. In the shortrun there is a rise in the premium, in the longrun there is an increase in the stock of black dollars.

If the initial condition, however, had been one where there was a substantial net speculative position in dollars then it is possible that the reduction in the speculative demand for dollars would more than offset the increased hedging demand. There would accordingly be a net decline in dollar demand, a leftward shift of the $\dot{x}=0$ schedule and thus a fall in the premium.

Increased variability of the real exchange rate can be analyzed in much the same manner. A rise in the variability of u in equation (4) raises the variability of relative returns thereby reducing speculation and reshuffling the minimum variance to come more in line with the shares of imports and domestic goods in the price index. Suppose again that the initial situation is one where relative asset supplies match the minimum variance portfolio. Increased real exchange rate variability now raises the share of dollars in the minimum variance portfolio provided

$$(1-a) > \frac{s_{\dot{p}}^2}{s_{\dot{p}}^2 + s_{\dot{p}^*}^2}$$

that is the share of world prices in the price index exceeds the ratio of the home variance of inflation to the sum of inflation variances. The lesson then is that, if imports have a small cost share and if the home variance of inflation is relatively high, increased real exchange rate variability can readily shift demand toward domestic securities and thus reduce the "dollar premium.

3. Disturbances in the Flow Market

Disturbances originating in the flow market give rise to shifts in the $\dot{V}^*=0$ schedule. Circumstances that raise the net flow supply of dollars shift the schedule to the left while disturbances that imply a higher rate of depletion give rise to a rightward shift.

Consider now, as an example, a real appreciation of the Argentinian currency as has occurred in the past years on an extraordinary scale. The appreciation would lower the real cost of tourism abroad, including Brazil. An increased flow of tourists would correspond to a higher rate of black market dollar inflows and hence to a leftward shift of the $\dot{V}^*=0$ schedule in Figure 3. The initial effect of the (permanent) real appreciation abroad is to reduce the premium from the initial equilibrium point A to A'. Over time the increased inflow, the reduction in the premium notwithstanding, causes the dollar pool to grow. The premium keeps decling until a new longrun equilibrium at point A" is reached.

The analysis of increased tourist inflows shows the same effects as would be observed for some other disturbances, in particular the imposition of a domestic tourist tax, increased real cost of airtravel, coffee or sugar export taxes and hence increased smuggling. All of these policies, by generating a current account surplus for the black market, lead to a declining premium and an increase in the longrun stock of black dollars.

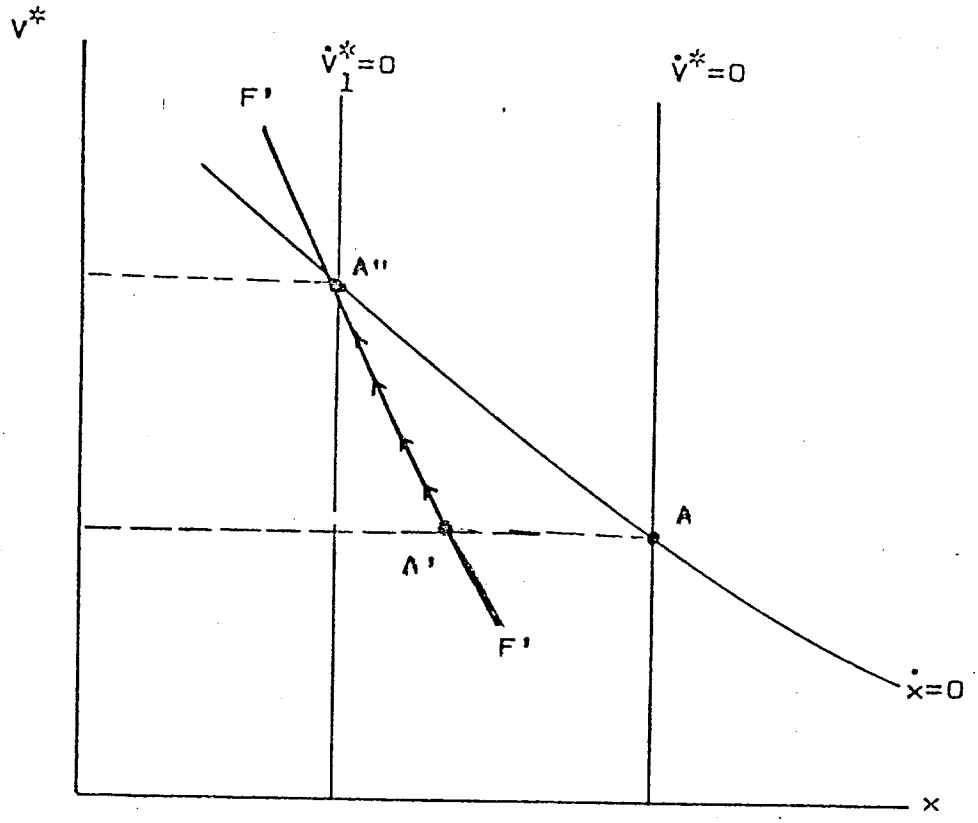

FIGURE 3

4. A Maxi-Devaluation

In this section we discuss a current, unanticipated maxi-devaluation. We leave to the next section the discussion of the current anticipation of a future maxi. Suppose then that an actual maxi occurs. It is important to distinguish between a purely nominal maxi—neither the real exchange rate nor the dollar value of cruzeiro assets change — and a real maxi. It is immediately apparent that a purely nominal maxi has no effect on the premium or on the dollar pool. There will simply be an equiproportionate increase in the black market price of dollars.

Consider next a real maxi. There will be two sides to a real depreciation. On one hand we have an incease in the real rate that raises the net inflow of dollars thus displacing $\dot{V}^*=0$ to the left. A real maxi tends to discourage inward smuggling and raises net tourist inflows, thus giving rise to a black market surplus.

A real maxi may also have a counterpart in financial markets. Policies supporting the maxi through tight money may raise the real interest rate and a failure to expand the nominal asset supply will automatically lead to a decline in the dollar value of cruzeiro assets, $V/\bar{E}$. Either or both events lead to a relative excess supply of dollars and thus cause the $\dot{x}=0$ schedule to shift down and to the left. Figure 4 shows that the net effect of the maxi is an immediate decline from the initial equilibrium at A to point A'. The longerrun adjustment of the dollar pool and the premium de-

depend on the relative shifts of the two schedules. If the current account is very insensitive to the real exchange rate we might get a case as shown in Figure 4. Here in the longrun there is a decline in the premium and in the dollar pool. The alternative case arises if risk aversion is large or the variance of relative returns is large. In that case the $\dot{x}=0$ schedule will shift very little — the higher real interest rate induces relatively little in terms of speculative demand — and accordingly a decline in the premium is accompanied by a rise in the dollar pool.

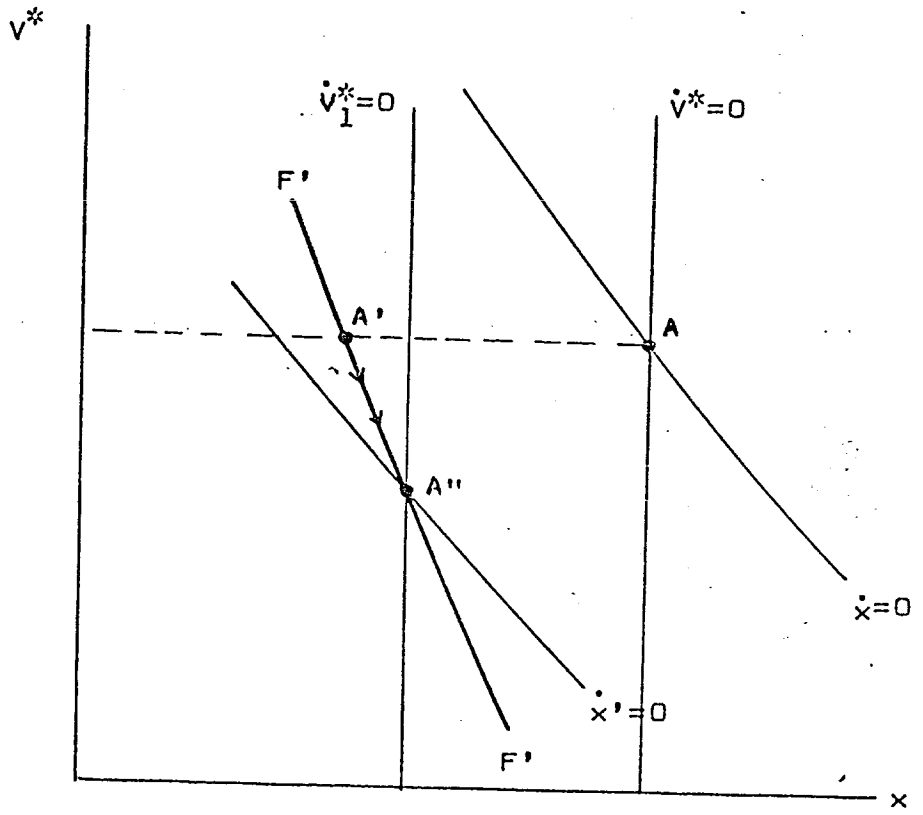

FIGURE 4

5. Anticipations of a Real Maxi

The interaction of flow markets, portfolio balance and expectations comes strongly into its own when we consider the adjustment process to a currently anticipated future real maxi. Wilson (1978), Rogoff (1979) and Dornbusch and Fischer (1980) have studied the adjustment process of exchange rates in the anticipation of change in the determinants at a known future point in time.[1]

In Figure 5 we show the initial equilibrium at point A and the new long run, post maxi steady at A_4. Suppose now that people expect a real maxi to occur at time T in the future. What is the current reaction of the black market to that news? There will be an immediate recognition that once the maxi does take place there will be some increase in the nominal black market rate, the latter being always at least as high as the official rate. The recognition of that future jump and capital gain leads portfolio holders to try and take advantage of that opportunity by holding dollars. The recognition that every market participant will attempt to do so leads immediately to some jump in the nominal black market rate and hence in the premium. This is shown in Figure 5 by the jump from A to A_1.

Now at A_1 the dynamics are still governed by the prevailing real interest rate and real exchange rate. Accordingly at A_1 the premium is rising as is the dollar pool. The economy moves along the path A_1 to A_2 until at point T the maxi actually

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[1] We do not consider here the more complicated question of the "peso-problem"— there is probability distribution attaching to the possibility of a maxi. See Lizindo (1979).

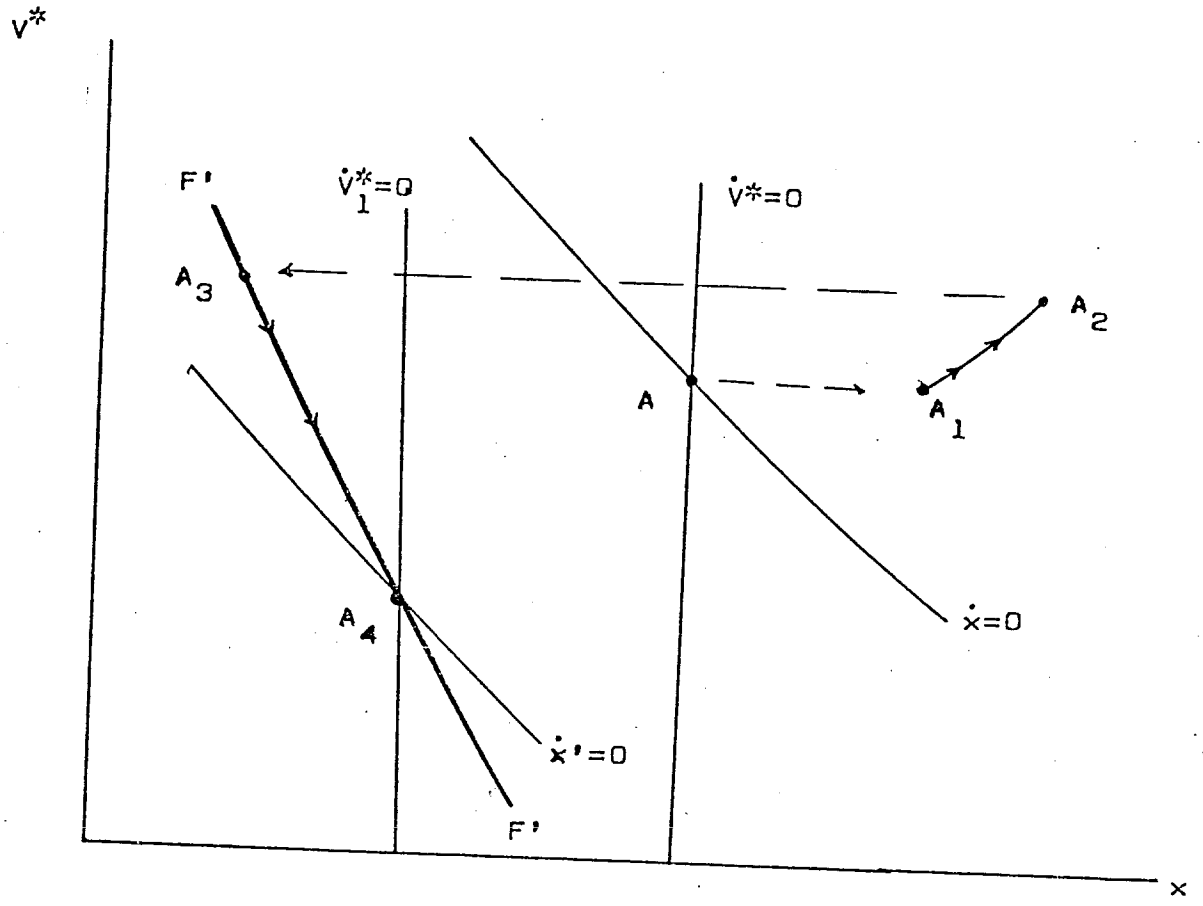

FIGURE 5

does occur. At that point there is a jump in the official rate — no jump in the black rate — and therefore a reduction in the premium that takes us to the new perfect foresight path F'F' at point A_3. The remaining adjustment is one of a depletion in the dollar pool below the intial level accompanied by some increase in the premium.

Figure 6 shows the path of the premium from the initial time where the expectation develops T_o to the time of the actual maxi, T_1, and from there to the steady state. The first jump in the premium is the implication of rational portfolio behavior, the second is due to the actual maxi. The example demonstrates the importance of expectations of future events in any empirical analysis of the premium.

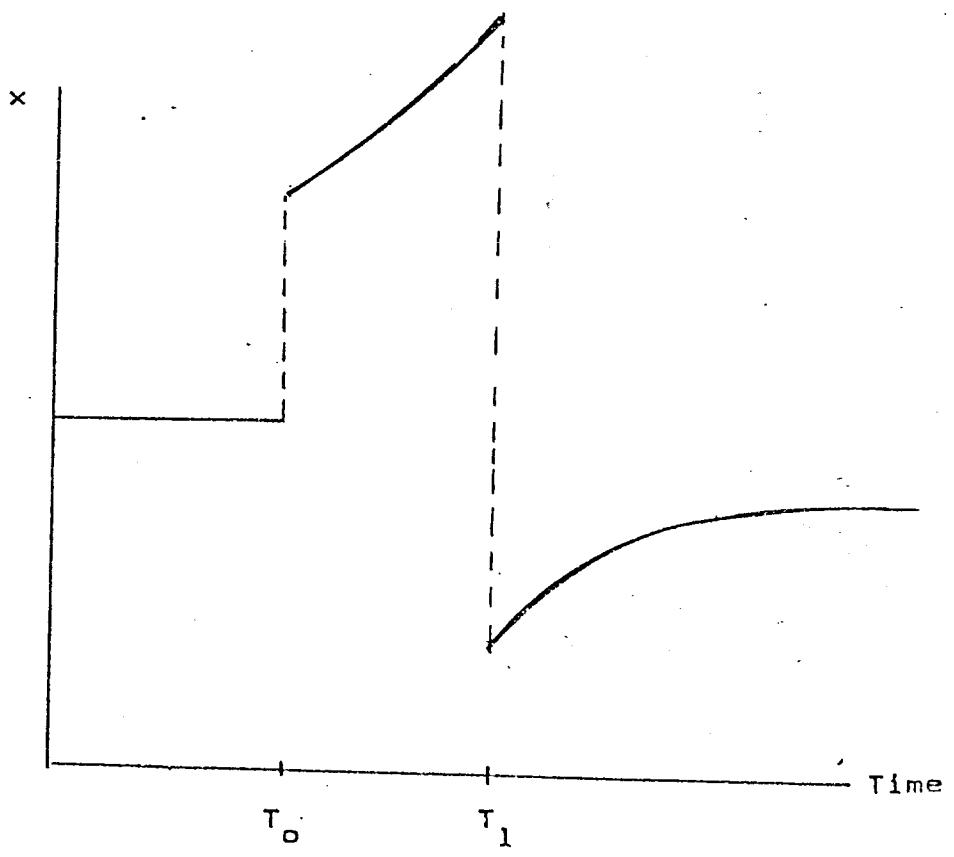

FIGURE 6

6. Seasonals

The next application of our model concerns seasonal factors. In specifying our flow model we implicitly assumed that tourism, smuggling and the like proceed at an even rate over time. This, of course, is not the case. With a bunching of tourist inflows for example, in the summer months there tends to be a seasonally high rate of inflow of dollars. How does the market cope with the recognition of these seasonal factors? Suppose that the seasonal is relatively unpronounced and short so that the economy spends sometime, in between seasonals, in the steady state.

In Figure 7 we show the steady state at point A with the non-seasonal current balance schedule $\dot{V}^*=0$. We also show the portfolio schedule $\dot{x}=0$ that is unaffected by the seasonals. The recognition of the seasonal implies that at some point, prior to the season, the premium starts declining. Unlike in the preceding section there is no jump because is has"always" been known that the seasonal will materialize[1]. The premium starts declining as portfolio holders start diversifying out of dollars and the declining premium actually induces depletion of the dollar stock. As the seasonal starts the economy finds itself at point A' and from here dynamics are governed by the seasonal flow balance shown as the dashed schedule. During the season the dollar pool is uniformly rising. The premium is

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[1] For a discussion of this subtle point see Fischer (1980)

initially falling until the economy crosses $\dot{x}=0$. At the end of the season the economy finds itself at point A" with a seasonally high stock of dollars and a seasonally low premium. From here we return along FF to the steady state.

It is particularly interesting to note that rational speculation implies that the effect of the seasonal on the premium and on the dollar pool are spread beyond the season. The anticipation of the seasonal leads to a prior running down of the dollar pool accompanied by a declining premium, but there is also some carry-over of the seasonally high dollars beyond the season. Thus the interaction of stock and flow markets tends to smooth the premium.

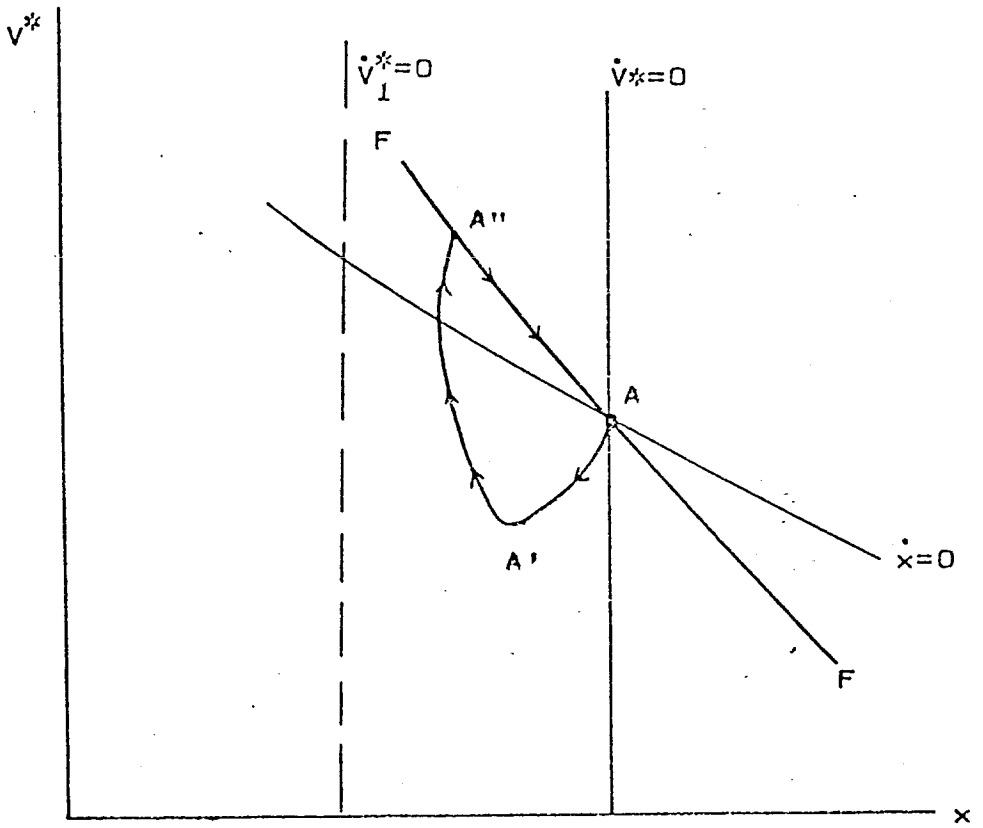

FIGURE 7

7. Seignorage

So far we have only analyzed disturbances originatig in the home country. We now conclude the analysis with an interesting implication of dollar inflation. From the condition of portfolio balance it is apparent that in long run equilibrium the relative supplies of assets are constant. This implies that, in dollar terms, $V/\bar{E}$ and V^* are rising at the dollar rate of inflation. If in long run equilibrium both relative asset supplies and real asset supplies are to be constant then it follows that the black flow market must be running a courrent account surplus so as to generate the additions to nominal dollar holdings that will ensure constancy of real dollar balances.

In terms of our diagram a higher dollar rate of inflation then implies that we require a larger current account surplus so that $(V^*/P^*)^{\cdot} = \dot{V}^*/P^* - (V^*/P^*)\,\dot{p}^* = 0$. Accordingly the analysis must now be recast in terms of real dollar holdings. It is immediately apparent that the inflation adjusted current balance shifts to the right, leading to an increase in the premium and to a redution in the long run real dollar pool.

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APPENDIX

This appendix sketches the financial part of our model, namely the risk premium determination. The appendix follows Dornbusch (1980b). We assyme that portfolio holders maximize a utility function as a function of the mean real return on and variability of their portfolio. The real return on cruzeiro and dollar assets are respectively:

(1) $\quad r = i - \dot{p} \qquad r^* = i^* + d - \dot{p}$

where $\dot{p}$ is the domestic rate of inflation and d is the rate of depreciation of the black market, both of which are random. On average — that is in the steady state — there is a constant premium and the black rate follows the official rate which behaves according to purchasing power parity:

(2) $\quad d = \bar{d} = \dot{p}' - \dot{p}^* + u$

where u is a random term and $\dot{p}'$ is the inflation rate of the domestic output deflator. The home inflation rate is a weighted average of home and import prices:

(3) $\quad \dot{p} = a\dot{p}' + (1-a)\ (\dot{p}^* + \bar{d}) = \dot{p}' + (1-a)u$

with these relations we can calculate the variance and covariance of real returns, assuming that inflation rates are uncorrelated:

$$(4) \qquad s_r^2 = s_{\dot{p}'}^2 + (1-a)^2 s_u^2 \qquad s_{r^*}^2 = s_{\dot{p}^*}^2 + a^2 s_u^2$$

$$s_{rr^*} = -\, a(1-a) s_u^2$$

$$s^2 = s_{\dot{p}'}^2 + s_{\dot{p}^*}^2 + s_u^2$$

where s^2 is the variance of real interest differentials, which is also the variance of the nominal rate of depreciation.

It can be shown that maximisation of utility yields an optimal portfolio shares of dollars, z equal to:

$$(5) \qquad z = \frac{r^* - r}{\theta s^2} + \alpha$$

where θ is the coefficient of relative risk aversion and α is the minimum variance portfolio defined as:

$$(6) \qquad \alpha \equiv \frac{s_{\dot{p}}^2 + (1-a) s_u^2}{s^2}$$

In market equilibrium there is a real yield differential or risk premium, r-r*, that is determined by the deviation of the relative supply of securities from the minimum variance portfolio.