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ABSTRACT

This paper uses contemporaneous monetary data to carry out econometric tests of the "equilibrium" approach to modelling the relation between monetary disturbances and macroeconomic fluctuations. The theoretical analysis introduces into an equilibrium macroeconomic model the availability of preliminary data on current monetary aggregates and the process of accumulation of revised monetary data. The econometric analysis tests two hypotheses derived from this extended model. One hypothesis concerns the neutrality of perceived monetary policy. The other hypothesis concerns the nonneutrality of errors in preliminary monetary data. The econometric results imply rejection of both of these hypotheses. These tests provide strong evidence against the reality of the equilibrium approach.

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This paper uses contemporaneous monetary data to carry out econometric tests of the "equilibrium" approach to modelling the relation between monetary disturbances and macroeconomic fluctuations. The adjective "equilibrium," in our terminology, denotes the class of macroeconomic models that assume that expectations are rational, in the sense that private agents behave as if they know the economy's relevant stochastic structure, and that markets clear, in the sense that transactions realize all perceived gains from trade, and that focus on incomplete information about monetary disturbances as the main explanation for the association of fluctuations in real macroeconomic variables with monetary disturbances. The essence of incomplete information about monetary disturbances in equilibrium models is that private agents cannot perfectly anticipate the behavior of monetary aggregates and, also, do not observe contemporaneously either the actual values of monetary aggregates or the values of other macroeconomic variables such as average prices and aggregate output.

With regard to direct monetary information, the classic equilibrium models--for example, Lucas (1972; 1973) and Barro (1976)--assume, specifically, that monetary policy is partly stochastic and that available data provide no information on current monetary policy, but provide full and accurate information on past monetary policy. The starting point of the present paper is the observation that this specification of monetary information is unsatisfactory for at least two reasons.

First, it is an unrealistic abstraction that seems contrary to the strategy of equilibrium modelling. Since the early 1950's, the Federal Reserve Board has issued preliminary monetary data with a lag of no more than one or two months. Since 1965, this lag has been only eight days. Revisions of this data, however, appear over a period of many months or years. These revisions result from such factors as computational corrections, benchmark changes reflecting fuller reporting, and conceptual changes reflecting financial innovations. The classic equilibrium models abstract from both the existence of contemporaneous preliminary monetary data and the process of gradual accumulation of revised monetary data. The neglect of contemporaneous data implies that private agents act as if they ignore readily available and apparently relevant information, an implication that seems inconsistent with the idea of rational expectations. The neglect of the process of data revision implies, in contrast, that private agents act as if they have an unrealistically large amount of information.

Second, abstracting from information about current monetary policy causes the analysis to focus on the predictability of monetary policy. Specifically, the main testable hypothesis that emerges from the classic equilibrium models is that only unanticipated monetary policy affects real variables. Although some existing econometric results suggest that the evidence is consistent with this hypothesis--see, for example, Barro (1977) and Barro and Rush (1980)--the hypothesis itself does not provide a strong test of equilibrium models. Specifically, as some authors, such as Barro and Hercowitz (1980) and Fischer (1980), have recognized, this hypothesis does not discriminate between equilibrium models and an alternative class of models that also assume rational expectations, but allow markets to fail to clear.

The theoretical analysis in the present paper introduces into an equilibrium macroeconomic model both the availability of preliminary data on current monetary aggregates and the process of accumulation of revised monetary data. This generalization permits the derivation of a set of readily testable hypotheses that are specifically associated with the equilibrium approach to macroeconomic modelling. The econometric analysis in the present paper tests two of these hypotheses.

One hypothesis concerns the neutrality of perceived monetary policy. Specifically, the model implies that the innovation in aggregate output and employment is uncorrelated both with the contemporaneous measure of money growth implied by the difference between the currently available estimates of current and past money stocks and with lagged values of this measure. The econometric results imply rejection of this hypothesis.

The other hypothesis concerns the nonneutrality of errors in preliminary monetary data. Specifically, the model implies that the innovation in aggregate output and employment is positively correlated either with the revision in the current measure of money growth implied by the difference between the preliminary contemporaneous measure and the finally reported measure or with such revisions in past measures of money growth. The econometric results fail to reject the contrary of this hypothesis. Each of these two tests provides strong evidence against the reality of the equilibrium approach to modelling macroeconomic fluctuations.

The theoretical analysis underlying these tests builds on earlier work reported in King (1981) and Boschen and Grossman (1980). Our models, like King's model, include a contemporaneous estimate of the money stock, but instead of King's assumption that this estimate is corrected in the next period, we assume that developing the finally reported value of the current money stock involves more than one revision and takes more than one period. The present model extends our earlier work by allowing for positive correlation in the subsequent revisions of current estimates of current and past money stocks. This correlation provides a source of persistence in the effects of monetary disturbances on real variables, because it enables private agents to use information about past monetary policy to draw inferences about current monetary policy. Random factors in past monetary policy influence this inference process and, consequently, affect current real variables.

Our models also extend King's model by allowing explicitly for systematic monetary policy in the form of a target monetary growth rate that responds to the past behavior of aggregate output. The present model introduces as well a production technology that includes a direct effect of past levels of aggregate output on the current level of aggregate output. This extension sharpens the analysis by focusing attention on the relation between contemporaneous measures of money growth and the innovation in aggregate output, rather than, as in earlier work, on aggregate output itself. Taken together, the effect of past output on current output through the production technology and the effect of past output on current monetary policy can create a spurious correlation between aggregate output and contemporaneous monetary data.

In what follows, Sections 1-3 set up the theoretical model, solve the model, and interpret the solution. Section 4 derives the two hypotheses to be tested. Section 5 sets up and reports the results of the econometric tests of these hypotheses. Section 6 discusses general conclusions.

1. Setup of the Model

In the existing literature, the development of equilibrium macroeconomic models has involved various, but mutually consistent, stories about information. The following setup is based on the story told by Friedman [1968] in which the representative producer infrequently purchases many of the items that he consumes and, hence, infrequently observes their prices. Consequently, he does not know precisely the extent to which a change in the nominal value of his product involves a change in his terms of trade between leisure and consumption. His subjective belief about consumption prices and, hence, about the relevant real value of his productive services is the critical expectational variable in the model. The assumption of rational expectations means that this subjective belief is equal to a true mathematical expectation conditional on available information. The structural equations of the model describe the supply and demand for a representative good, the market-clearing condition that determines the output and price of this good, the behavioral pattern of the monetary authority, the nature of available monetary data, and the formation of rational expectations about average prices.

The production technology for representative good z makes current output of the good an increasing function of productive services currently provided by producers of this good. The current supply of these productive services depends on the subjective belief of the representative producer about the current relative price of this good and, because of adjustment costs, on past levels of employment of these productive services. Specifically, we assume the log-linear form

$$y_t^s(z) = \alpha\,[p_t(z) - E_t(z)p_t] + a(L)y_{t-1}(z), \tag{1}$$

where $y_t^s(z)$ is the log of the current supply of good z,

$p_t(z)$ is the log of the current money price of good z,

$E_t(z)p_t$ is the current subjective belief of the representative producer of good z about the average of the logs of money prices,

α is the positive and constant elasticity of supply with respect to the difference, $p_t(z) - E_t(z)p_t$,

$a(L)$ is a polynomial in L such that $a(L) = a_o + a_1L + a_2L^2 + \ldots.$,

L is a lag operator such that $L^j y_{t-1} = y_{t-1-j}$, and

$y_{t-1}(z)$ is the log of output of good z in period t-1.

The current demand for good z depends on the value of aggregate money balances deflated by $p_t(z)$ and on random disturbances to aggregate demand and to the relative demands for the various goods. Specifically, we assume the log-linear form

$$y_t^d(z) = M_t - p_t(z) + v_t + \varepsilon_t(z), \tag{2}$$

where $y_t^d(z)$ is the log of the current demand for good z,

M_t is the log of the current money stock,

v_t is a random variable distributed according to $v \sim N(0,\sigma_v^2)$, uncorrelated serially and uncorrelated with the other random variables in the model, and

$\varepsilon_t(z)$ is a random variable distributed according to $\varepsilon(z) \sim N(0,\sigma_\varepsilon^2)$, uncorrelated serially, uncorrelated with the other random variables in the model, and summing to zero across all goods, i.e., $\sum_z \varepsilon_t(z) = 0$.

The specification of the relevant subjective belief, $E_t(z)p_t$, in equation (1) as involving current, rather than future, consumption prices represents an abstraction from the intertemporal considerations that are implicit in the story about infrequent purchase of consumption goods. The specification of the monetary aggregate, M_t, in equation (2) abstracts from the distinction between the finally reported value of the current money stock and the true value of the current money stock--for more on this issue, see Boschen and Grossman (1980). A more general formulation of the supply and demand functions would include the terms, $p_t(z) - E_t(z)p_t$ and $M_t - p_t(z)$, in both of the functions and would allow for random disturbances to supply. These and other possible generalizations would complicate the algebraic analysis of the model without changing the conclusions regarding the role of monetary information.

The market-clearing condition for good z is that $p_t(z)$ adjusts to satisfy the equality,

(3) $$y_t(z) = y_t^s(z) = y_t^d(z),$$

The rest of the model involves specification of available information, of the determination of M_t, and of the formation of $E_t(z)p_t$.

Currently available monetary data include a series of preliminary and revised estimates of finally reported values of monetary aggregates. These estimates are obtained by sampling from the balance sheets of the Federal Reserve Banks and of other financial institutions. We assume log-linear estimating relations. For the current money stock, we have the preliminary estimate,

(4.1) $${}_t\hat{M}_t = M_t + \delta_t,$$

where ${}_t\hat{M}_t$ is the estimate of M_t reported in period t and δ_t is a random variable distributed according to $\delta \sim N(0,\sigma_\delta^2)$.

For the last period's money stock, we have one revised estimate,

(4.2) $${}_t\hat{M}_{t-1} = M_{t-1} + \eta_t$$

where ${}_t\hat{M}_{t-1}$ is the estimate of M_{t-1} reported in period t and η_t is a random variable distributed according to $\eta \sim N(0,\sigma_\eta^2)$,

in addition to the preliminary estimate,

(4.3) $${}_{t-1}\hat{M}_{t-1} = M_{t-1} + \delta_{t-1},$$

where ${}_{t-1}\hat{M}_{t-1}$ is the estimate of M_{t-1} reported in period t-1. For M_{t-2}, we have two revised estimates in addition to the preliminary estimate, and so forth for earlier periods.

The data on preliminary and finally reported money stocks used in the econometric analysis below indicate that errors in contemporaneous estimates of money stocks are correlated. Specifically, the covariance between δ_t and η_t, denoted $\sigma_{\delta\eta}$, is highly positive. The data also indicate that δ_t and η_t are not correlated with the difference, given by ${}_t\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-1} = \eta_t - \delta_{t-1}$, between the preliminary and revised estimates of M_{t-1} nor with the differences between preliminary and revised estimates of the money stocks of period $t-2$ and earlier. Specifically, none of the correlation coefficients for such pairs of variables is larger than 0.2. Finally, to simplify the calculations necessary to obtain an explicit solution of the model, we assume that δ_t and η_t are uncorrelated with the errors in current estimates of the money stocks of period $t-2$ and earlier. It is easy to relax this last assumption in the implementation of the econometric tests of the model.

Monetary policy involves a target monetary growth rate, which reflects a systematic response to past aggregate output, and a random factor. Specifically, we assume a log-linear relation of the form,

$$M_t = {}_t\hat{M}_{t-1} + b(L)y_{t-1} + g_t, \tag{5}$$

where $b(L)$ is a polynomial in the lag operator, L,

y_{t-1} is the aggregate across all goods of the logs of output last period, i.e., $y_{t-1} = \sum_z y_{t-1}(z)$, and

g_t is a random variable distributed according to $g \sim N(0,\sigma_g^2)$, uncorrelated serially, and uncorrelated with the other random variables in the model.

Within the context of equation (5), the random variable, g_t, has at least two possible interpretations, corresponding to different monetary policy processes. One possible process is that M_t results from adding the term, $b(L)y_{t-1}$, and a random variable, x_t, directly to ${}_t\hat{M}_{t-1}$--that is,

$$M_t = {}_t\hat{M}_{t-1} + b(L)y_{t-1} + x_t.$$

In this case, g_t is equivalent to x_t. A second possible process is that M_t results from adding $b(L)y_{t-1}$ and x_t to M_{t-1}--that is,

$$M_t = M_{t-1} + b(L)y_{t-1} + x_t = {}_t\hat{M}_{t-1} + b(L)y_{t-1} + x_t - \eta_t.$$

In this case, g_t is equivalent to the difference, $x_t - \eta_t$. In general, these two processes have different quantitative implications for the behavior of y_t. These two processes, however, have the same implication for the relation between y_t and contemporaneous monetary data.

The assumed rationality of expectations prescribes that the subjective belief, $E_t(z)p_t$, is equal to the true mathematical expectation of p_t conditional on the information currently known to producers of good z. Specifically,

(6) $$E_t(z)p_t = E\ [P_t \mid I_t(z)],$$

where $I_t(z)$ is the assumed information set. This set contains useful knowledge about the structure of the economy that includes the form of the structural equations (1) - (6), the values of relevant parameters, and the joint distribution of the stochastic variables. The information set also contains useful data that includes the current price of good z, the past levels of average prices and aggregate output, and the available monetary data. The potentially useful information that is not in $I_t(z)$ includes the current average of prices, the current level of aggregate output, the finally reported values of current and recent past money stocks, and the realizations of current and past stochastic variables.

2. Solution of the Model

The explicit derivation of testable hypotheses from the model specified by equations (1) - (6) requires a solution for aggregate output that satisfies the market-clearing condition, given by equation (3), subject to the assumption of rational expectations, given by equation (6). We employ the method of undetermined coefficients to calculate this solution. The first step is to combine equations (1), (2), and (3) to obtain useful expressions for $p_t(z)$ and y_t. Equating supply to actual output of good z gives

$$y_t(z) = \alpha\ [p_t(z) - E_t(z)p_t] + a(L)y_{t-1}(z).$$

Equating demand to actual output of good z gives

$$y_t(z) = M_t - p_t(z) + v_t + \varepsilon_t(z).$$

Equating these two expressions for $y_t(z)$ and solving for $p_t(z)$ gives

$$p_t(z) = (\alpha+1)^{-1}[\alpha E_t(z)p_t + M_t + v_t + \varepsilon_t(z) - a(L)y_{t-1}(z)]. \tag{7}$$

Aggregating the equation of demand and output across all goods gives

$$y_t = M_t - p_t + v_t. \tag{8}$$

The second step is to conjecture a solution for $p_t(z)$ that is a linear combination of a constant term, which allows for known variables, and each of the relevant stochastic disturbances. Taken together, equations (4.1), (5), and (7) imply that $p_t(z)$ is related to $E_t(z)p_t$, other known variables, and the unknown stochastic disturbances, g_t, v_t, δ_t, and $\varepsilon_t(z)$. The rational expectation, $E_t(z)p_t$, in turn should involve joint inferences about the values of these stochastic variables. These joint inferences, as shown below, depend partly on known linear combinations of these variables. In addition, because δ_t is correlated with the unknown stochastic variable, η_t, these joint inferences are related to an inference about η_t. This inference, as also shown below, depends, jointly with inferences about the unknown stochastic variables, g_{t-1} and v_{t-1}, on known linear combinations of g_{t-1}, v_{t-1}, and η_t. As indicated above, in calculating an explicit solution of the model, we ignore possible correlations between δ_t or η_t and errors in estimates of the money stocks of period t-2 and earlier. A fully general solution would have to include these errors as well as other stochastic disturbances in period t-2 and earlier.

The above discussion suggests that an appropriate form for the conjectured solution is

$$(9) \qquad p_t(z) = \Pi_0 + \Pi_1 g_t + \Pi_2 v_t + \Pi_3 \delta_t + \Pi_4 g_{t-1} + \Pi_5 v_{t-1} + \Pi_6 \eta_t + \Pi_7 \varepsilon_t(z),$$

where $\Pi_0, \ldots, \Pi_7$ are the coefficients to be determined. Averaging equation (9) across all goods, given $\sum_z \varepsilon_t(z) = 0$, yields a solution for the average price in the form,

$$(10) \qquad p_t = \Pi_0 + \Pi_1 g_t + \Pi_2 v_t + \Pi_3 \delta_t + \Pi_4 g_{t-1} + \Pi_5 v_{t-1} + \Pi_6 \eta_t.$$

The assumed rationality of expectations means that the subjective belief, $E_t(z)p_t$, is equal to the true mathematical expectation of equation (10) conditional on $I_t(z)$. This expectation is given by

$$(11) \qquad E_t(z)p_t = \Pi_0 + \Pi_1 E_t(z)g_t + \Pi_2 E_t(z)v_t + \Pi_3 E_t(z)\delta_t + \Pi_4 E_t(z)g_{t-1} + \Pi_5 E_t(z)v_{t-1} + \Pi_6 E_t(z)\eta_t,$$

where the terms of on the right-hand side are true mathematical expectations of the respective stochastic disturbances conditional on $I_t(z)$.

The third step is to organize the information that is relevant for inferring these expectations. This process involves combining equations (7) and (8), derived from the market-clearing conditions, with the known structural equations describing monetary information and monetary policy. Starting with the current period, substituting equations (4.1) and (5) separately into equation (7) to eliminate M_t, after rearranging, yields two equations between linear combinations of g_t, v_t, δ_t, and $\varepsilon_t(z)$ and linear combinations of known variables,

$$(12) \qquad -\delta_t + v_t + \varepsilon_t(z) = (1+\alpha)p_t(z) - \alpha E_t(z)p_t - {}_t\hat{M}_t + a(L)y_{t-1}(z) \quad \text{and}$$

$$(13) \qquad g_t + v_t + \varepsilon_t(z) = (1+\alpha)p_t(z) - \alpha E_t(z)p_t - {}_t\hat{M}_{t-1} - b(L)y_{t-1} + a(L)y_{t-1}(z).$$

Equations (12) and (13) indicate that producers of good z know the values of the sums, $-\delta_t + v_t + \varepsilon_t(z)$ and $g_t + v_t + \varepsilon_t(z)$.

Turning to last period, substituting equations (4.2) and (4.3) and equation (5), applied to period t-1, separately into equation (8), applied to period t-1, to eliminate M_{t-1}, after rearranging, yields three equations between linear combinations of g_{t-1}, v_{t-1}, δ_{t-1}, and η_t and linear combinations of known variables,

(14) $$- \eta_t + v_{t-1} = p_{t-1} + y_{t-1} - {}_t\hat{M}_{t-1},$$

(15) $$- \delta_{t-1} + v_{t-1} = p_{t-1} + y_{t-1} - {}_{t-1}\hat{M}_{t-1}, \quad \text{and}$$

(16) $$g_{t-1} + v_{t-1} = p_{t-1} + y_{t-1} - {}_{t-1}\hat{M}_{t-2} - b(L)y_{t-2}.$$

Equations (14) - (16) indicate that producers of good z know the values of the sums, $\eta_t + v_{t-1}$, $\delta_{t-1} + v_{t-1}$, and $g_{t-1} + v_{t-1}$. Equation (14) involves relevant information because η_t is correlated with δ_t, and equation (16) involves relevant information because $g_{t-1} + v_{t-1}$ is correlated with $\eta_t + v_{t-1}$. Equation (15), however, is not useful, because δ_t and η_t are uncorrelated with the difference, $\eta_t - \delta_{t-1}$. Subtracting equation (14) from equation (15) shows that the addition information contained in equation (15) amounts to

$$\eta_t - \delta_{t-1} = {}_t\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-1}.$$

The fourth step is to calculate the expectations that appear on the right-hand side of equation (11). Given the linear normal structure of the model, the relations between the conditional expectations and the known linear combinations of stochastic variables have the form of regression equations,

(17) $$\begin{bmatrix} E_t(z)g_t \\ E_t(z)v_t \\ E_t(z)\delta_t \\ E_t(z)g_{t-1} \\ E_t(z)v_{t-1} \\ E_t(z)\eta_t \end{bmatrix} = [R] \begin{bmatrix} -\delta_t + v_t + \varepsilon_t(z) \\ g_t + v_t + \varepsilon_t(z) \\ -\eta_t + v_{t-1} \\ g_{t-1} + v_{t-1} \end{bmatrix},$$

where [R] is a 6x4 matrix of population regression coefficients given by

$$[R] = \begin{bmatrix} 0 & \sigma_g^2 & 0 & 0 \\ \sigma_v^2 & \sigma_v^2 & 0 & 0 \\ -\sigma_\delta^2 & 0 & -\sigma_{\delta\eta} & 0 \\ 0 & 0 & 0 & \sigma_g^2 \\ 0 & 0 & \sigma_v^2 & \sigma_v^2 \\ -\sigma_{\delta\eta} & 0 & -\sigma_\eta^2 & 0 \end{bmatrix} \begin{bmatrix} \sigma_\delta^2 + \sigma_\varepsilon^2 + \sigma_v^2 & \sigma_\varepsilon^2 + \sigma_v^2 & \sigma_{\delta\eta} & 0 \\ \sigma_\varepsilon^2 + \sigma_v^2 & \sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2 & 0 & 0 \\ \sigma_{\delta\eta} & 0 & \sigma_\eta^2 + \sigma_v^2 & \sigma_v^2 \\ 0 & 0 & \sigma_v^2 & \sigma_g^2 + \sigma_v^2 \end{bmatrix}^{-1} .$$

Appendix A lists the calculated elements of the matrix, [R].

The fifth step is to determine the coefficients, $\Pi_0, \ldots, \Pi_7$. The procedure is to substitute into equation (11) for $E_t(z)p_t$ the values of the expectations given by equation (17). Then, substitute into equation (7) for $p_t(z)$ the value of $E_t(z)p_t$ given by equation (11) and the value of M_t given by equation (5). (An alternative would be to use equation (4.1) to eliminate M_t.) Equation (7) then gives an expression for $p_t(z)$ that is a linear combination of the predetermined and exogenous variables, where the weights involve the undetermined coefficients, $\Pi_0, \ldots, \Pi_7$, and the variances of the stochastic variables. Equating each of these weights to the corresponding coefficient in the trial solution given by equation (9) yields a system of eight simultaneous equations that we can solve for $\Pi_0, \ldots, \Pi_7$. Appendix B lists these equations. The solutions to these equations are

$$\Pi_0 = {}_t\hat{M}_{t-1} + b(L)y_{t-1} - a(L)y_{t-1}(z),$$

$$\Pi_1 = 1-\alpha\sigma_\varepsilon^2[\sigma_\delta^2(\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2) - \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_v^2)]\ \Pi_2\ \Delta^{-1},$$

$$\Pi_2 = \Pi_7 = \{1+\alpha\sigma_\varepsilon^2[(\sigma_\delta^2 + \sigma_g^2)(\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2) - \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_v^2)]\ \Delta^{-1}\}^{-1},$$

$$\Pi_3 = \alpha\sigma_\varepsilon^2\sigma_g^2(\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2)\ \Pi_2\ \Delta^{-1},$$

$$\Pi_4 = -\alpha\sigma_{\delta\eta}\sigma_\varepsilon^2\sigma_g^2\sigma_v^2\ \Pi_2\ \Delta^{-1},$$

$$\Pi_5 = \alpha\sigma_{\delta\eta}\sigma_\varepsilon^2\sigma_g^2\sigma_g^2\ \Pi_2\ \Delta^{-1}, \text{ and}$$

$$\Pi_6 = -\alpha\sigma_{\delta\eta}\sigma_\varepsilon^2\sigma_g^2(\sigma_g^2 + \sigma_v^2)\ \Pi_2\ \Delta^{-1},$$

where $\Delta = (\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2)[\sigma_g^2(\sigma_\delta^2 + \sigma_\varepsilon^2 + \sigma_v^2) + \sigma_\delta^2(\sigma_\varepsilon^2 + \sigma_v^2)]$

$$- \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_v^2)(\sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2).$$

The final step is to write out a solution for current aggregate output in terms of predetermined and exogenous variables. Substituting into equation (8) for y_t the value of M_t given by equation (5) and the value of p_t given by equation (10) yields

(18) $$y_t = a(L)y_{t-1} + (1-\Pi_1)g_t + (1-\Pi_2)v_t - \Pi_3\delta_t - \Pi_4 g_{t-1} - \Pi_5 v_{t-1} - \Pi_6\eta_t,$$

where the values of $\Pi_1, \ldots, \Pi_6$ are as calculated above. Straightforward algebraic manipulation reveals that the coefficients of g_t, v_t, g_{t-1}, and η_t are all positive and that the coefficients of δ_t and v_{t-1} are both negative. All of these coefficients are less than unity in absolute value.

3. Interpretation of the Solution

Equation (18) indicates that current aggregate output equals a linear combination of past levels of aggregate output, $\alpha(L)y_{t-1}$, and the realizations of the exogenous random variables that represent the unanticipated part of current monetary policy, g_t, the current disturbance to aggregate demand, v_t, the currently unperceived part of current monetary policy, δ_t, the unanticipated part of past monetary policy, g_{t-1}, the past disturbance to aggregate demand, v_{t-1}, and the currently unperceived part of past monetary policy, η_t. The coefficients of these random variables are functions of their variances and covariances and the variance of the random disturbance to relative demands.

The interpretation of the coefficients of g_t, v_t, and δ_t is familiar from the discussions in King (1981) and Boschen and Grossman (1980). Current aggregate output is positively related to g_t and to v_t because producers of good z mistake some of the increase in the price of good z that results from positive values of g_t or v_t to be an increase in the relative price of good z. Current aggregate output is negatively related to δ_t because a high preliminary estimate of the current money stock causes the expectation of producers of good z about average prices to be too high and their expectations about the relative price of good z to be correspondingly too low.

The solutions for the coefficients, $\Pi_0, \ldots, \Pi_7$, indicate that, if the covariance, $\sigma_{\delta\eta}$, between the errors in the current estimates of current and past money stocks were equal to zero, equation (18) would reduce to the same expression for y_t derived in King (1981) and Boschen and Grossman (1980). Most importantly, in this case, the coefficients of g_{t-1}, v_{t-1}, and η_t would be zero. This result brings out the point that, as suggested above, g_{t-1}, v_{t-1}, and η_t affect y_t in equation (18) only because η_t is correlated with δ_t, and, hence, the known linear combinations, $-\eta_t + v_{t-1}$ and $g_{t-1} + v_{t-1}$, convey information about η_t and δ_t. Current aggregate output is positively related to η_t because a current estimate of the past money stock that is high relative to the known past values of aggregate output and average prices causes producers of good z to raise their expectations about δ_t and, hence, to lower their expectation about current average prices. Current aggregate output is positively related to g_{t-1} and negatively related to v_{t-1} because a high value of g_{t-1} or a low value of v_{t-1} causes the expectations of producers of good z about η_t to be too high.

The presence of the term, g_{t-1}, in equation (18) means that unanticipated monetary policy affects aggregate output over more than one period. This channel of persistence, which results from the structure of monetary information, operates in addition to the separate persistence effect that results from the production technology and causes the term, $a(L)y_{t-1}$ to appear in equation (18). As discussed above, a more general version of the present model would allow for additional correlations of the errors in monetary data. For example, the plausible assumption that δ_t is correlated with errors in estimates of the money stocks of period t-2 and earlier would imply a solution for current aggregate output that includes these errors as well as unanticipated monetary policy and aggregate demand disturbances from period t-2 and earlier.

The dependence of aggregate output on both monetary policy and aggregate demand disturbances described by equation (18) requires that producers of good z be unable to distinguish these aggregate factors from disturbances to relative demands. To see this conclusion, suppose that $\varepsilon_t(z)$ were a deterministic and, hence, known variable, rather than an unknown random variable, and, accordingly, set σ_ε^2 equal to zero in the solutions for the coefficients, $\Pi_1, \ldots, \Pi_7$. In this case, Π_1 and Π_2 would equal unity and Π_3, Π_4, Π_5, and Π_6 would equal zero. Consequently, y_t would simply equal $a(L)y_{t-1}$.

Of more immediate interest is the crucial role played by incomplete monetary information in determining the relation between monetary policy and aggregate output. Consider how equation (18) would change if δ and η were deterministic and, hence, known variables, rather than unknown random variables. Referring back to the information summarized by equations (12) - (16), we observe that knowledge of δ and η would enable producers of good z to calculate exactly the values of g_t, g_{t-1}, and v_{t-1}. Using these five known values, instead of the expectations given by equations (17), to calculate the coefficients, $\Pi_0, \ldots, \Pi_7$, we would obtain the results that Π_1 equals unity and that Π_3, Π_4, Π_5, and Π_6 all equal zero. (The coefficient, Π_2, would still be less than unity.) This result indicates that aggregate output depends on monetary policy if and only if monetary policy is at least partly unperceived. This observation underlies the testable hypotheses that are explicitly derived in the next section.

4. Derivation of Testable Hypotheses

The first hypothesis to be tested involves the neutrality of perceived monetary policy. The innovation in the output of representative good z and, hence, the innovation in aggregate output depend only on the subjective belief of the representative producer about the current relative price of good z. The model implies that this belief is uncorrelated both with the contemporaneous measure of money growth implied by the difference between the currently available estimates of current and past money stocks and with lagged values of this measure. This implication is not trivial, because, as we have seen, these monetary data-- ${}_t\hat{M}_t$, ${}_t\hat{M}_{t-1}$, ${}_{t-1}\hat{M}_{t-1}$, ${}_{t-1}\hat{M}_{t-2}$, etc.--play a critical role in the model as part of the relevant information on which producer decisions are based. An essential result of the assumptions of market clearing and rational expectations, however, is that the behavioral response of producers to this information neutralizes its effect on beliefs about relative prices and, hence, on aggregate output. The general principle involved is that these assumptions imply neutrality for the known part of any disturbance that would be neutral under complete information.

Derivation of the hypothesis that perceived money growth is neutral involves demonstrating that the covariances between $y_t - a(L)y_{t-1}$ and ${}_t\hat{M}_t - {}_t\hat{M}_{t-1}$ and between $y_t - a(L)y_{t-1}$ and ${}_{t-1}\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-2}$ are zero. Rearranging equation (18) gives the following expression for the innovation in aggregate output:

$$y_t - a(L)y_{t-1} = (1-\Pi_1)g_t + (1-\Pi_2)v_t - \Pi_3\delta_t - \Pi_4 g_{t-1} - \Pi_5 v_{t-1} - \Pi_6\eta_t.$$

Combining equations (4.1) and (5) gives the following expression for the contemporaneous measure of money growth:

$${}_t\hat{M}_t - {}_t\hat{M}_{t-1} = \delta_t + g_t + b(L)y_{t-1}.$$

To handle the term, $b(L)y_{t-1}$, apply equation (18) to period $t-1$ to obtain

$$b(L)y_{t-1} = b(L)[a(L)y_{t-2} + (1-\Pi_1)g_{t-1} + (1-\Pi_2)v_{t-1} - \Pi_3\delta_{t-1} - \Pi_4 g_{t-2} - \Pi_5 v_{t-2} - \Pi_6\eta_{t-1}],$$

which can also be written as:

$$b(L)y_{t-1} = b(L)\{a(L)y_{t-2} + (1-\Pi_1)g_{t-1} + (1-\Pi_2)v_{t-1} + \Pi_3[(\eta_t - \delta_{t-1}) - \eta_t] - \Pi_4 g_{t-2} - \Pi_5 v_{t-2} - \Pi_6\eta_{t-1}\}.$$

Finally, combining equation (4.3) and equation (5) applied to period $t-1$ gives the following expression for the contemporaneous measure of past money growth:

$$_{t-1}\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-2} = -(\eta_t - \delta_{t-1}) + \eta_t + g_{t-1} + b(L)y_{t-2}.$$

These expressions imply that the hypothesis involves evaluating the following covariances:

$$\text{cov}\ [y_t - a(L)y_{t-1},\ \delta_t + g_t] = (1-\Pi_1)\sigma_g^2 - \Pi_3\sigma_\delta^2 - \Pi_6\sigma_{\delta\eta},$$

$$\text{cov}\ [y_t - a(L)y_{t-1},\ y_{t-1}] = -(1-\Pi_1)\Pi_4\sigma_g^2 - (1-\Pi_2)\Pi_5\sigma_v^2$$

$$+ \Pi_3(\Pi_3\sigma_{\delta\eta} + \Pi_6\sigma_\eta^2),\ \text{and}$$

$$\text{cov}\ [y_t - a(L)y_{t-1},\ -(\eta_t - \delta_{t-1}) + \eta_t + g_{t-1} + b(L)y_{t-2}]$$

$$= -\Pi_3\sigma_{\delta\eta} - \Pi_6\sigma_\eta^2 - \Pi_4\sigma_g^2.$$

Substitution of the calculated values for $\Pi_1, \ldots, \Pi_6$ into these expressions reveals, after some algebraic manipulation, that each of these covariances equal zero. Using these results, and an immediate generalization to estimates of money growth in period t-2 and earlier, we have the testable hypothesis:

$$\text{(I)}\quad \text{cov}\ [y_t - a(L)y_{t-1},\ {}_{t-i}\hat{M}_{t-i} - {}_{t-i}\hat{M}_{t-1-i}] = 0$$

for all values of i, $i = 0,1,2, \ldots$

It is worth recalling that the models in King (1981) and Boschen and Grossman (1980) imply the stronger hypotheses that aggregate output, rather than the innovation in aggregate output, is uncorrelated with the contemporaneous measures of money growth. To obtain this stronger hypothesis, it is necessary to assume that either $b(L)$ or $a(L)$ are equal to zero. Otherwise, y_t and ${}_t\hat{M}_t - {}_t\hat{M}_{t-1}$ are correlated through the covariance of $a(L)y_{t-1}$ with $b(L)y_{t-1}$. The essential point is that, without a control for the persistence effect on aggregate output that results from the production technology, the effect of past aggregate output on current monetary policy can create a spurious correlation between aggregate output and contemporaneous monetary data.

The second hypothesis to be tested involves the effects of errors in preliminary monetary data. The model implies that the innovation in aggregate output is positively correlated either with the revisions in the current measure of money growth implied by the differences between the preliminary contemporaneous measure and the finally reported measure or with such revisions in past measures of money growth. The essential idea is that these revisions measure the extent to which monetary policy is unperceived.

Derivation of this hypothesis involves calculating the covariances between $y_t - a(L)y_{t-1}$ and $(M_t - M_{t-1}) - ({}_t\hat{M}_t - {}_t\hat{M}_{t-1})$ and between $y_t - a(L)y_{t-1}$ and $(M_{t-1} - M_{t-2}) - ({}_{t-1}\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-2})$. Combining equations (4.1) and (4.2) implies

$$(M_t - M_{t-1}) - ({}_t\hat{M}_t - {}_t\hat{M}_{t-1}) = \eta_t - \delta_t.$$

Combining equation (4.3) and equation (4.2) applied to period t-2 implies

$$(M_{t-1} - M_{t-2}) - ({}_{t-1}\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-2}) = \eta_{t-1} - \delta_{t-1},$$

which can also be written as

$$(M_{t-1} - M_{t-2}) - ({}_{t-1}\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-2}) = \eta_{t-1} + (\eta_t - \delta_{t-1}) - \eta_t.$$

These expressions imply that the hypothesis involves evaluating the following covariances:

$$\text{cov}\,[y_t - a(L)y_{t-1},\ \eta_t - \delta_t] = \Pi_3\sigma_\delta^2 + (\Pi_6 - \Pi_3)\sigma_{\delta\eta} - \Pi_6\sigma_\eta^2 \quad \text{and}$$

$$\text{cov}\,[y_t - a(L)y_{t-1},\ \eta_{t-1} + (\eta_t - \delta_{t-1}) - \eta_t] = \Pi_3\sigma_{\delta\eta} + \Pi_6\sigma_\eta^2.$$

Substitution of the calculated values for Π_3 and Π_6 into these expressions yields, after some algebraic manipulation,

$$\text{cov}\,[y_t - a(L)y_{t-1},\ (M_t - M_{t-1}) - ({}_t\hat{M}_t - {}_t\hat{M}_{t-1})]$$
$$= \alpha\sigma_\varepsilon^2\sigma_g^2\Pi_2\Delta^{-1}[\sigma_\delta^2(\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2) - \sigma_{\delta\eta}(\sigma_{\delta\eta}\sigma_g^2 + \sigma_{\delta\eta}\sigma_v^2 + \sigma_g^2\sigma_v^2)]$$

and

$$\text{cov}\,[y_t - a(L)y_{t-1},\ (M_{t-1} - M_{t-2}) - ({}_{t-1}\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-2})]$$
$$= \alpha\sigma_\varepsilon^2\sigma_g^2\Pi_2\Delta^{-1}\ \sigma_{\delta\eta}\sigma_g^2\sigma_v^2.$$

If, as seems to the case, the term, $\sigma_{\delta\eta}$, is positive, the second of these calculated covariances is positive, but the sign of the first one is ambiguous. If, alternatively, $\sigma_{\delta\eta}$ were not positive, the second covariance would not be positive, but the first one would be unambiguously positive. Using these results and an immediate generalization to errors in preliminary estimates of money growth in period t-2 and earlier, we have the second testable hypothesis:

(II) $\text{cov}\,[y_t - a(L)y_{t-1},\ (M_{t-i} - M_{t-1-i}) - ({}_{t-i}\hat{M}_{t-i} - {}_{t-i}\hat{M}_{t-1-i})] > 0$

for at least one value of i, $i = 0,1,2,\ldots$

These two hypotheses do not exhaust the possible ways to use contemporaneous monetary data to test the equilibrium approach to macroeconomics. However, given the nature of the econometric results discussed below, explicit derivation and examination of additional testable hypotheses does not seem worthwhile.

5. Econometric Analysis

To test Hypothesis I econometrically, we estimate regression equations of y_t on lagged values of y, to control for the effect of $a(L)y_{t-1}$, on current and lagged values of the contemporaneous measure of money growth--${}_t\hat{M}_t - {}_t\hat{M}_{t-1}$, ${}_{t-1}\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-2}$, etc.--and on seasonal dummy variables. To simplify notation, denote ${}_t\hat{M}_t - {}_t\hat{M}_{t-1}$ as $\hat{m}_t$ and ${}_{t-1}\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-2}$ as $\hat{m}_{t-1}$, etc. The zero vector of covariances involving $y_t - a(L)y_{t-1}$ contained in Hypothesis I implies that the covariances of y_t with the variables, $a(L)y_{t-1}$, $\hat{m}_t$, $\hat{m}_{t-1}$, etc., are equal to the respective covariances of $a(L)y_{t-1}$ with these same variables. Consequently, as we can readily confirm by computing the population regression coefficients, Hypothesis I implies that the estimated coefficients of $\hat{m}_t$, $\hat{m}_{t-1}$, etc., in the calculated regression equations should be zero.

The data are for the United States for 1953 through 1978. As measures of y_t we use both the log of the Federal Reserve Board's industrial production index, denoted I_t, and the log of the ratio of employment to population, denoted N_t. In both cases, we take four seasonally unadjusted observations--for the months of February, May, August, and November--for each year.

The contemporaneous measure of money growth, $\hat{m}_t$, corresponding to y_t is the difference between the latest seasonally unadjusted estimate of the money stock, M1, published prior to the month of the observation of y_t and the revised seasonally unadjusted estimate available at the same time of M1 three months earlier. The construction of this time series involves three different procedures because of three different episodes in the publication of monetary data. For the period from the middle of 1965 through 1978, the Federal Reserve Board published in weekly statistical release H.6 preliminary monetary data with an eight-day lag. For this period, our measure for ${}_t\hat{M}_t$ is the last weekly average of daily estimates of M1 issued during the months--either January, April, July, or October--prior to our observation of y_t. For the period from the last quarter of 1960 until the middle of 1965, the Federal Reserve Board published in semi-monthly statistical release J.3 preliminary monetary data with a lag of from one to two weeks. For this period, our measure of ${}_t\hat{M}_t$ is the last semi-monthly average of daily estimates of M1 issued during the month prior to our observation of y_t. For the period from the beginning of 1953 until the last quarter of 1960, the Federal Reserve Board published preliminary monetary data on a monthly basis with lag of one or two months. For this period our measure of ${}_t\hat{M}_t$ is the latest daily estimate of M1 given in the Federal Reserve Bulletin dated the month prior to our observation of y_t. For all three periods, our measure of ${}_t\hat{M}_{t-1}$ is the revision of the previous estimate, ${}_{t-1}\hat{M}_{t-1}$, available contemporaneously with ${}_t\hat{M}_t$. Experimentation suggests that the econometric results are not sensitive to the precise timing of the four annual observations of the variables.

The choice of seasonally unadjusted data reflects the idea that private agents respond to the unadjusted changes in variables and that the use of seasonally smoothed data may distort the measurement of these responses. In particular, seasonally unadjusted money growth is the appropriate data for proper testing of Hypothesis II, because seasonal adjustment factors are themselves subject to revision and because these revisions introduce differences between preliminary and revised seasonally adjusted monetary data that are not related to changes in information about the money stock.

It remains important, however, to control for possible seasonal correlation between variables that is unrelated to business cycle phenomena. The inclusion of seasonal dummy variables and the length of the dependent variable lags seem to control adequately for the seasonal variation in the dependent variables. The correlograms calculated from the residuals obtained from the regressions of I_t and N_t on their respective lagged values and the seasonal dummies showed the autocorrelation at lag four to be within two standard deviations of zero for both variables. Appendix C lists the time series of monetary data. The specification of the series, M_t, is discussed below.

The main reason for not using all of the monthly observations of the measures of aggregate output involves the problem of constructing corresponding measures of money growth. Experimentation indicated that the measurement of money growth over monthly intervals introduced severe problems of dealing with seasonality and of serial correlation in the residuals. Money growth over short intervals, moreover, might not be large enough to have the potential to produce measurable effects on aggregate output. An alternative procedure of measuring money growth monthly over longer overlapping intervals would create a severe problem of multicollinearity. Moreover, the taking of only four observations annually captures the cyclical movements of aggregate output and money over the sample period and, hence, does not seem to sacrifice any significant information about the relation between these variables.

The reported regression equations involving industrial production, I_t, include four lagged values of the dependent variable, whereas the reported regression equations involving employment, N_t, include six lagged values of the dependent variable. The choice of these specifications reflects experimentation with different lag lengths. The criterion for selecting the number of dependent variable lags was a likelihood ratio test for serial independence of the residuals. The test statistic, distributed asymptotically χ^2--see Geweke (1979)--is

$$\lambda(K) = n \log (\sigma_{\nu}^2/\sigma_r^2),$$

where σ_{ν}^2 is a maximum likelihood estimate of the variance of the residuals from a regression of I_t or N_t on a distributed lag of dependent variables and the seasonal dummy variables,

σ_r^2 is the maximum likelihood estimate of the variance of the residuals from a K^{th} order autoregression using the residuals from the regression that generated σ_ν^2, and ,

n is the number of residuals computed from the K^{th} order autoregression.

The dependent variable lag length in the reported regressions is that number of lags which, based on the likelihood ratio test, rendered a serially independent residual series. For example, regressing N_t on six rather than four lags of itself and the seasonal dummies reduces the value of $\lambda(12)$ from 30.3 to 12.9. The critical levels for these values of $\lambda(12)$ are .003 and .38 respectively. These critical levels indicate that the null hypothesis of independent residuals can be rejected for the four lag N_t equation but not the six lag N_t equation. The value of $\lambda(12)$ for I_t regressed on four lags of itself plus the seasonal dummies was 6.67. The associated critical level of .88 indicates we cannot reject the hypothesis of serially independent residuals.

The selection of the number of independent variable lags is based on the results of experiments using four, six, and ten lags of the independent variable. Since the main conclusions from the econometric analysis did not seem to depend on the number of lagged variables, the results using four lags of the independent variable are shown below.

The results of calculating the two regression equations for testing Hypothesis I are as follows:

$$\text{(i)}\quad I_t = \underset{(3.1)}{.17} + \underset{(9.9)}{1.11 I_{t-1}} - \underset{(-1.8)}{.29 I_{t-2}} - \underset{(-0.9)}{.14 I_{t-3}} + \underset{(0.1)}{.01 I_{t-4}}$$

$$+ \underset{(2.8)}{.65 \hat{m}_t} + \underset{(3.1)}{.70 \hat{m}_{t-1}} + \underset{(1.0)}{.24 \hat{m}_{t-2}} + \underset{(0.6)}{.15 \hat{m}_{t-3}} - \underset{(-0.5)}{.12 \hat{m}_{t-4}}$$

$$- \underset{(-2.7)}{.03 S_1} - \underset{(-1.6)}{.02 S_2} - \underset{(-1.0)}{.01 S_3}$$

$$R^2 = .993 \qquad F^5_{86} = 3.5$$

$$
\begin{aligned}
\text{(ii)}\ N_t = {} & \underset{(-0.7)}{-.01} + \underset{(8.3)}{.93N_{t-1}} - \underset{(-1.0)}{.14N_{t-2}} + \underset{(0.8)}{.09N_{t-3}} + \underset{(3.9)}{.51N_{t-4}} - \underset{(-4.5)}{.61N_{t-5}} \\
& + \underset{(2.0)}{.20N_{t-6}} - \underset{(-0.1)}{.01\hat{m}_t} + \underset{(3.2)}{.22\hat{m}_{t-1}} + \underset{(2.5)}{.18\hat{m}_{t-2}} + \underset{(1.7)}{.13\hat{m}_{t-3}} \\
& + \underset{(0.3)}{.02\hat{m}_{t-4}} - \underset{(-2.1)}{.01S_1} - \underset{(-0.1)}{.001S_2} + \underset{(1.3)}{.01S_3}
\end{aligned}
$$

$$R^2 = .982 \qquad F^5_{83} = 4.2$$

The numbers in parentheses under the coefficients are t-statistics.

The important conclusions from equation (i) for industrial production are that the t-statistics imply rejection at the 99% confidence level of the hypothesis that the coefficients of $\hat{m}_t$ and $\hat{m}_{t-1}$ are zero, and that the F-statistic implies rejection at the 99% confidence level of the hypothesis that the joint effect of the variables, $\hat{m}_t, \ldots, \hat{m}_{t-4}$, is zero. The important conclusions from equation (ii) for employment are that the t-statistics imply rejection at the 96% confidence level of the hypothesis that the coefficients of $\hat{m}_{t-1}$, $\hat{m}_{t-2}$, and $\hat{m}_{t-3}$ are zero, and that the F-statistic again implies rejection at the 99% confidence level of the hypothesis that the joint effect of the variables, $\hat{m}_t, \ldots, \hat{m}_{t-4}$, is zero. In sum, these regression equations indicate that current and lagged values of the contemporaneous measure of money growth have statistically significant effects on industrial production and employment, a result that implies rejection of Hypothesis I.

An appropriate inference from this finding is that the equilibrium model from which Hypothesis I derived is not consistent with the facts--specifically, with the observed relations between preliminary monetary data and industrial production and aggregate employment. Although this conclusion that the equilibrium model is not realistic stands independently of tests of other implications of the model, empirical analysis of Hypothesis II brings to bear additional data--specifically, the finally reported money stock--and, in addition, can generate inferences about the quantitative importance of unperceived money growth as a channel of monetary nonneutrality. Specifically, failure to reject the contrary of Hypothesis II both reinforces the conclusion that the equilibrium model is not realistic and indicates that incomplete monetary information is not a significant source of monetary nonneutrality.

To analyze Hypothesis II econometrically, we estimate regression equations of I_t and N_t on lagged values of the dependent variables, on current and lagged values of revisions on the current measure of money growth--$(M_t - M_{t-1}) - ({}_t\hat{M}_t - {}_t\hat{M}_{t-1})$, $(M_{t-1} - M_{t-2}) - ({}_{t-1}\hat{M}_{t-1} - {}_{t-1}\hat{M}_{t-2})$, etc.--and on seasonal dummy variables. To simplify notation, we denote

$(M_t - M_{t-1}) - ({}_t\hat{M}_t - {}_t\hat{M}_{t-1})$ as R_t, etc. Hypothesis II implies that at least one of the estimated coefficients of the series, R_t, R_{t-1}, ... in these regression equations should be positive.

The time series for the finally reported money stock, M_t, is defined to be the log of latest reported weekly average of daily estimates of M1 for the final weeks of January, April, July, and August over the sample period, 1953 through 1978, as of January 1980. Appendix C includes this time series. This constructed time series for M_t differs from the constructed time series for ${}_t\hat{M}_t$ in three ways. First, M_t reflects all of the historical revisions in monetary data, which involve primarily such factors as computational corrections, benchmark changes resulting from fuller reporting, and conceptual changes involving respecification of the components of M1. Computational corrections and benchmark changes clearly represent new information. Conceptual changes, however, represent new information only to the extent that they incorporate previously unavailable data rather than merely rearrange previously available data. As Barro and Hercowitz (1980) point out, revisions in measures of money growth are largely independent of conceptual changes, which mainly alter the overall level of the reported money stock. Second, M_t measures a weekly average of daily estimates of M1, whereas, for the period prior to the last quarter of 1960, ${}_t\hat{M}_t$ is an estimate of M1 for a single day. Third, M_t is a reported value of M1 for the week indicated, whereas ${}_t\hat{M}_t$ is an estimate published during the week indicated of the value of M1 for an earlier week or day. Over the sample period this reporting lag declined from one to two months to eight days.

The results of calculating the two regression equations relating to Hypothesis II were as follows:

(iii)
$$\underset{(1.8)}{I_t = .07} + \underset{(12.4)}{1.27I_{t-1}} - \underset{(-2.5)}{.43I_{t-2}} + \underset{(0.9)}{.16I_{t-3}} - \underset{(-0.2)}{.02I_{t-4}}$$

$$- \underset{(-0.5)}{.13R_t} - \underset{(-2.2)}{.57R_{t-1}} - \underset{(-0.01)}{.002R_{t-2}} + \underset{(0.7)}{.17R_{t-3}} + \underset{(1.5)}{.39R_{t-4}}$$

$$- \underset{(-1.8)}{.02S_1} - \underset{(-2.2)}{.02S_2} - \underset{(-2.5)}{.02S_3}$$

$$R^2 = .994 \qquad F^5_{86} = 2.0$$

(iv)
$$\underset{(1.1)}{N_t = .02} + \underset{(9.9)}{1.09N_{t-1}} - \underset{(-1.4)}{.20N_{t-2}} + \underset{(0.5)}{.06N_{t-3}} + \underset{(4.9)}{.61N_{t-4}}$$

$$- \underset{(-6.0)}{.81N_{t-5}} + \underset{(2.6)}{.27N_{t-6}} + \underset{(0.7)}{.06R_t} - \underset{(-0.9)}{.08R_{t-1}}$$

$$- \underset{(-0.8)}{.07R_{t-2}} - \underset{(-0.2)}{.02R_{t-3}} - \underset{(-1.1)}{.10R_{t-4}}$$

$$- \underset{(-1.9)}{.01S_1} + \underset{(0.9)}{.01S_2} + \underset{(1.5)}{.01S_3}$$

$$R^2 = .978 \qquad F^5_{83} = 0.6$$

The important conclusions from equations (iii) and (iv) are that none of the t-statistics imply rejection of the alternative hypothesis that the coefficients of the variables, $R_t, \ldots, R_{t-4}$, are not positive, and that neither of the F-statistics imply rejection of the alternative hypothesis that the joint effects of these variables are zero. These conclusions accord with the results from similar equations reported by Barro and Hercowitz (1980). Their regression equations used annual averages, used unemployment and GNP as dependent variables, and did not include lagged dependent variables. All of these results support the conclusion that errors in preliminary monetary data do not have statistically significant effects on real variables, and imply that we cannot reject the contrary of Hypothesis II.

6. Conclusions

The basic assumptions of equilibrium macroeconomic models--namely, market clearing and rational expectations--imply that the known part of a monetary disturbance that would be neutral under complete information does not influence aggregate output and employment. In this same context, the assumption of incomplete information about macroeconomic variables implies that such a disturbance is nonneutral if, but only if, it is at least partly unperceived. Hypotheses I and II, thus, seem to be inescapable implications of the equilibrium approach to modelling the relation between monetary disturbances and macroeconomic fluctuations, once the theory takes into account the existence of preliminary data on current monetary aggregates and the process of accumulation of revised monetary data. This extension of the classic equilibrium models provides a rich framework in which current aggregate output depends on both the unanticipated and currently unperceived parts of current monetary policy as well as on both the unanticipated and currently unperceived parts of past monetary policy. The extended model, however, also generates the above econometric tests, which are apparently fatal to the equilibrium approach.

The rejection of Hypothesis I and the failure to reject the contrary of Hypothesis II underscore the unsatisfactory state of the theory of macroeconomic fluctuations. The research program associated with the equilibrium approach has raised essential questions, but has not provided empirically convincing answers. The assumptions of market clearing and rational expectations seem to be compelling elements of a unified theory of economic behavior. Moreover, theory and evidence from a variety of contexts suggest that these assumptions cannot be easily dismissed as unrealistic. Indeed, thanks mainly to equilibrium theorizing, we are no longer satisfied with explanations for the apparent short-run non-neutrality of money that rely on biased expectations or on widespread failure of economic agents to realize perceived gains from trade. The results in this paper, however, indicate that equilibrium theorizing does not provide an alternative explanation of macroeconomic fluctuations whose implications accord with the apparent facts. The business cycle, consequently, seems mysterious. We do not have at present a theory of fluctuations in aggregate output and employment that is consistent both with maximizing behavior and with econometric evidence.

Appendix A

The elements of the matrix, [R], are as follows, where each element has to be divided by the determinant,

$$\Delta = (\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2)[\sigma_g^2(\sigma_\delta^2 + \sigma_\varepsilon^2 + \sigma_v^2) + \sigma_\delta^2(\sigma_\varepsilon^2 + \sigma_v^2)]$$
$$- \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_v^2)(\sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2):$$

$$R_{11} = -\sigma_g^2(\sigma_\varepsilon^2 + \sigma_v^2)(\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2)$$

$$R_{12} = \sigma_g^2[(\sigma_\delta^2 + \sigma_\varepsilon^2 + \sigma_v^2)(\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2) - \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_v^2)$$

$$R_{13} = \sigma_{\delta\eta}\sigma_g^2(\sigma_\varepsilon^2 + \sigma_v^2)(\sigma_g^2 + \sigma_v^2)$$

$$R_{14} = -\sigma_{\delta\eta}\sigma_g^2\sigma_v^2(\sigma_\varepsilon^2 + \sigma_v^2)$$

$$R_{21} = \sigma_g^2\sigma_v^2(\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2)$$

$$R_{22} = \sigma_v^2[\sigma_\delta^2(\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2) - \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_v^2)]$$

$$R_{23} = -\sigma_{\delta\eta}\sigma_g^2\sigma_v^2(\sigma_g^2 + \sigma_v^2)$$

$$R_{24} = \sigma_{\delta\eta}\sigma_g^2\sigma_v^2\sigma_v^2$$

$$R_{31} = -(\sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2)[\sigma_\delta^2(\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2) - \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_v^2)]$$

$$R_{32} = (\sigma_\varepsilon^2 + \sigma_v^2)[\sigma_\delta^2(\sigma_\eta^2\sigma_g^2 + \sigma_\eta^2\sigma_v^2 + \sigma_g^2\sigma_v^2) - \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_v^2)]$$

$$R_{33} = -\sigma_{\delta\eta}\sigma_g^2(\sigma_\varepsilon^2 + \sigma_v^2)(\sigma_g^2 + \sigma_v^2)$$

$$R_{34} = \sigma_{\delta\eta}\sigma_g^2\sigma_v^2(\sigma_\varepsilon^2 + \sigma_v^2)$$

$$R_{41} = \sigma_{\delta\eta}\sigma_g^2\sigma_v^2(\sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2)$$

$$R_{42} = -\sigma_{\delta\eta}\sigma_g^2\sigma_v^2(\sigma_\varepsilon^2 + \sigma_v^2)$$

$$R_{43} = -\sigma_g^2\sigma_v^2[\sigma_g^2(\sigma_\delta^2 + \sigma_\varepsilon^2 + \sigma_v^2) + \sigma_\delta^2(\sigma_\varepsilon^2 + \sigma_v^2)]$$

$$R_{44} = \sigma_g^2\{(\sigma_\eta^2 + \sigma_v^2)[\sigma_g^2(\sigma_\delta^2 + \sigma_\varepsilon^2 + \sigma_v^2) + \sigma_\delta^2(\sigma_\varepsilon^2 + \sigma_v^2)]$$
$$-\sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2)\}$$

$$R_{51} = -\sigma_{\delta\eta}\sigma_g^2\sigma_v^2(\sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2)$$

$$R_{52} = \sigma_{\delta\eta}\sigma_g^2\sigma_v^2(\sigma_\varepsilon^2 + \sigma_v^2)$$

$$R_{53} = \sigma_g^2\sigma_v^2[\sigma_g^2(\sigma_\delta^2 + \sigma_\varepsilon^2 + \sigma_v^2) + \sigma_\delta^2(\sigma_\varepsilon^2 + \sigma_v^2)]$$

$$R_{54} = \sigma_v^2\{\sigma_\eta^2[\sigma_g^2(\sigma_\delta^2 + \sigma_\varepsilon^2 + \sigma_v^2) + \sigma_\delta^2(\sigma_\varepsilon^2 + \sigma_v^2)] - \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2)\}$$

$$R_{61} = -\sigma_{\delta\eta}\sigma_g^2\sigma_v^2(\sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2)$$

$$R_{62} = \sigma_{\delta\eta}\sigma_g^2\sigma_v^2(\sigma_\eta^2 + \sigma_v^2)$$

$$R_{63} = -(\sigma_g^2 + \sigma_v^2)\{\sigma_\eta^2[\sigma_g^2(\sigma_\delta^2 + \sigma_\varepsilon^2 + \sigma_v^2) + \sigma_\delta^2(\sigma_\varepsilon^2 + \sigma_v^2)]$$
$$- \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2)\}$$

$$R_{64} = \sigma_v^2\{\sigma_\eta^2[\sigma_g^2(\sigma_\delta^2 + \sigma_\varepsilon^2 + \sigma_v^2) + \sigma_\delta^2(\sigma_\varepsilon^2 + \sigma_v^2)] - \sigma_{\delta\eta}\sigma_{\delta\eta}(\sigma_g^2 + \sigma_\varepsilon^2 + \sigma_v^2)\}$$

Appendix B

The equations between the weights in equation (7) and the corresponding coefficients in equation (9) are as follows:

known variables: $\Pi_0 = (\alpha+1)^{-1}[\alpha\Pi_0 + {}_t\hat{M}_{t-1} + \phi y_{t-1} - a(L)y_{t-1}(z)]$.

g_t: $\Pi_1 = (1+\alpha)^{-1}[1 + \alpha(\Pi_1 R_{12} + \Pi_2 R_{22} + \Pi_3 R_{32} + \Pi_4 R_{42} + \Pi_5 R_{52}$
$+ \Pi_6 R_{62}]$.

v_t: $\Pi_2 = (1+\alpha)^{-1}\{1 + \alpha[\Pi_1(R_{11}+R_{12}) + \Pi_2(R_{21}+R_{22}) + \Pi_3(R_{31}+R_{32})$
$+ \Pi_4(R_{41}+R_{42}) + \Pi_5(R_{51}+R_{52}) + \Pi_6(R_{61}+R_{62})]\}$.

δ_t: $\Pi_3 = -(1+\alpha)^{-1}\ \alpha(\Pi_1 R_{11} + \Pi_2 R_{21} + \Pi_3 R_{31} + \Pi_4 R_{41} + \Pi_5 R_{51}$
$+ \Pi_6 R_{61})$.

g_{t-1}: $\Pi_4 = (1+\alpha)^{-1}\ \alpha(\Pi_1 R_{14} + \Pi_2 R_{24} + \Pi_3 R_{34} + \Pi_4 R_{44} + \Pi_5 R_{54} + \Pi_6 R_{64})$.

v_{t-1}: $\Pi_5 = (1+\alpha)^{-1}\ \alpha[\Pi_1(R_{13}+R_{14}) + \Pi_2(R_{23}+R_{24}) + \Pi_3(R_{33}+R_{34})$
$+ \Pi_4(R_{43}+R_{44}) + \Pi_5(R_{53}+R_{54}) + \Pi_6(R_{63}+R_{64})]$.

η_t: $\Pi_6 = -(1+\alpha)^{-1}\ \alpha(\Pi_1 R_{13} + \Pi_2 R_{23} + \Pi_3 R_{33} + \Pi_4 R_{43} + \Pi_5 R_{53} + \Pi_6 R_{63})$.

$\varepsilon_t(z)$: $\Pi_7 = \Pi_2$.

Appendix C
Time Series of Monetary Data

	$M1_t$	${}_t\hat{M1}_t$	${}_t\hat{M1}_{t-1}$		$M1_t$	${}_t\hat{M1}_t$	${}_t\hat{M1}_{t-1}$
1953:1	130.5	126.8	122.1	1966:1	175.9	173.4	166.0
2	126.7	125.2	126.8	2	176.9	173.7	173.4
3	127.0	124.5	125.2	3	172.1	168.4	173.9
4	128.8	124.8	124.5	4	175.2	170.3	168.4
1954:1	132.3	128.1	124.8	1967:1	178.4	175.8	170.3
2	127.2	126.6	128.1	2	178.2	174.6	175.7
3	128.8	125.5	126.5	3	180.2	175.6	175.3
4	131.5	126.3	125.5	4	185.5	181.0	176.2
1955:1	136.4	131.7	126.3	1968:1	189.5	190.1	181.1
2	132.8	131.5	131.0	2	192.4	189.7	189.7
3	133.4	130.3	131.4	3	193.6	188.4	189.8
4	135.1	131.2	130.1	4	198.8	189.3	188.5
1956:1	139.1	134.4	131.1	1969:1	203.9	198.4	189.3
2	135.2	132.8	134.8	2	204.1	199.4	198.3
3	134.6	131.6	132.8	3	204.8	192.4	199.4
4	136.2	132.0	131.6	4	207.0	199.2	196.5
1957:1	140.3	136.2	132.0	1970:1	210.8	206.8	199.1
2	136.1	134.1	136.2	2	210.8	205.1	206.8
3	135.6	132.7	134.4	3	211.6	200.9	205.1
4	136.3	132.9	132.7	4	215.9	206.4	200.8
1958:1	138.8	135.7	132.9	1971:1	221.2	221.4	213.3
2	136.4	132.8	135.7	2	224.4	223.3	221.2
3	137.0	133.6	133.0	3	228.5	225.8	223.5
4	140.1	135.6	133.6	4	231.7	226.7	225.8
1959:1	144.6	140.4	135.5	1972:1	236.6	235.9	226.7
2	143.2	137.6	143.1	2	240.8	238.0	236.0
3	143.8	138.9	138.2	3	243.1	238.3	238.0
4	143.8	139.8	139.0	4	249.6	242.9	238.3
1960:1	145.5	144.9	139.9	1973:1	255.8	256.0	242.7
2	143.6	136.9	144.8	2	259.0	262.5	259.2
3	142.6	136.3	136.9	3	262.6	264.6	262.4
4	145.2	137.6	136.1	4	265.6	265.8	264.5
1961:1	147.5	144.0	139.8	1974:1	268.7	273.5	265.7
2	146.5	141.1	144.0	2	275.8	283.0	273.6
3	145.3	140.7	140.9	3	276.0	282.6	284.6
4	148.6	144.0	140.6	4	278.3	281.1	281.4
1962:1	150.8	148.2	142.8	1975:1	278.8	287.2	280.8
2	151.2	146.6	146.8	2	281.1	289.5	287.8
3	148.3	144.4	146.4	3	288.3	292.2	288.6
4	151.2	146.8	144.7	4	289.9	293.8	291.4
1963:1	153.6	152.7	145.8	1976:1	293.0	300.5	292.4
2	154.3	151.1	152.8	2	300.2	307.3	300.5
3	153.4	149.9	148.8	3	302.0	305.0	303.2
4	156.8	152.9	148.8	4	308.0	308.5	305.7
1964:1	159.7	158.6	151.8	1977:1	312.8	320.6	308.5
2	158.4	155.8	158.9	2	321.9	325.6	320.9
3	159.1	155.3	154.5	3	326.4	329.4	324.7
4	163.7	159.1	154.9	4	332.6	333.4	329.5
1965:1	167.0	165.0	158.4	1978:1	340.8	346.0	333.4
2	165.7	161.3	165.4	2	349.0	350.9	345.8
3	165.0	161.1	161.9	3	351.7	353.5	346.5
4	170.0	166.0	161.4	4	355.7	364.6	356.1

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