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# INFLATION, INCOME TAXES AND OWNER-OCCUPIED HOUSING

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## ABSTRACT

Owner-occupied housing receives favorable treatment under current tax law for several reasons. A homeowner's imputed rent is not taxed, and mortgage interest payments are tax deductible. Many past studies have analyzed the effects of these provisions. Inflation's importance in determining the implicit subsidy to owner-occupied housing has received less attention. Since home-owners can deduct their nominal mortgage payments, they do not bear the full cost of higher interest rates. They also receive essentially untaxed capital gains on their homes during periods of high inflation. The after-tax capital gains outweigh the higher after-tax interest payments, so inflation reduces the effective cost of homeownership.

This paper develops a simple model to estimate the effect of higher expected inflation rates on the real price of houses and the equilibrium housing stock. Simulation results suggest that the inflation-tax interactions can have a substantial impact on the housing market. The increases in expected inflation during the 1970s could have accounted for as much as a thirty percent increase in real house prices. Over time, builders should respond to higher home prices and increase the amount of new construction. The persistence of current inflation rates could lead ultimately to a twenty percent increase in the housing stock.

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Inflation, Income Taxes and Owner-Occupied Housing

by

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The favorable treatment granted to owner-occupied housing under current tax law is widely recognized. Many past studies[1] have explored the effects of not taxing homeowners' imputed rents and allowing mortgage interest payment deductability. The inflation rate plays a crucial role in determining the subsidy to owner occupants. An increase in the rate of overall inflation has two effects: it increases the nominal mortgage interest rate and it provides larger nominal capital gains for homeowners. The tax rate on housing gains is essentially zero, so owner-occupants receive the full capital gain but bear only a fraction of the higher interest costs. Expected inflation therefore reduces the effective cost of homeownership.

During the 1970s, real house prices increased by thirty percent. Prior to the 1979 credit crunch, new home building was at record-breaking levels. The interaction between inflation and the tax system may provide a partial explanation for the housing market's observed behavior. The rising inflation rate during the past decade should have increased the demand for housing services by reducing their effective cost. In the short run, this should have resulted in a bidding up of real house prices. Homebuilders should have responded to higher real prices by increasing the amount of new construction.

This paper presents a partial equilibrium model of the housing market which can be used to analyze the interactions of inflation and the tax code,

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extending past work by Kearl (1975), Buckley and Ermisch (1979), and Hendershott and Hu (1979). I consider the consequences of inflation in the short run, the steady state, and during the transition between the two. Section I outlines the basic model. In section II, I present an analytic treatment of the steady state and transition path consequences of higher inflation rates. Estimates of the housing construction supply function are presented in Section III. The last section reports simulation results which examine the effect of an increase in the overall inflation rate on the housing sector. There is a brief conclusion.

## I. The Theoretical Framework

The housing sector must be considered as two markets: a market for existing houses and a market for new construction. The actors in the first are the individuals who consume housing services. There are two equilibrium conditions for this market. First, the real rental price of the housing services from a unit of residential capital must clear the market for these services. Second, the marginal cost of housing services must equal the services' real rental value.

The desired quantity of housing services at any moment, $HS^d$, depends upon the real rental price of these services, R, and a measure of permanent income, Y.

(1) $$HS^d = f(R,Y) \qquad f_R < 0 \quad f_Y > 0.$$

The flow supply of services, $HS^s$, is assumed to bear a fixed relationship to the stock of houses, H: $HS^s = \alpha H$, where $\alpha$ is a constant parameter which transforms stocks into flows. The housing service market is always in equilibrium, $HS^s = HS^d$, and real rental prices fluctuate to enforce this condition. The equilibrium condition can be solved explicitly for the real rental price which clears the service market, yielding an inverse demand function

(2) $$R = R(H,Y), \qquad R_H < 0 \quad R_Y > 0$$

for housing services. The income variable will be suppressed below and I shall write R(H) for the inverse demand function.

Individuals consume housing services until the marginal value of these services equals their cost. Several simplifying assumptions are employed in formalizing this relationship: i) all houses depreciate at a constant rate $\delta$ and require maintenance and repair costs equal to a fraction $\kappa$ of current value; ii) houses are assessed at full value for property taxes and taxed at a constant rate $\mu$; iii) property tax payments are deductible from income tax liabilities; iv) income taxes are paid at marginal rate $\theta$; v) individuals may borrow or lend at an interest rate i. The one-period cost of housing services from a "unit house" with real price Q is the sum of depreciation, repair costs, property taxes, mortgage payments, and the opportunity cost of housing equity,[2] minus the capital gain on the house: $[\delta + \kappa + (1-\theta)(i+\mu) - \pi_H]Q$. The nominal house price inflation rate is $\pi_H$. The cost expression can be simplified by defining the user cost of housing capital, $\omega$, as the ratio of one-period costs to the house's real price. The condition that marginal cost equal marginal benefit may be written

(3) $$\frac{R(H)}{Q} = \delta + \kappa + (1-\theta)(i+\mu) - \pi_H \equiv \omega.$$

This will be referred to as the asset market equilibrium condition.

An arbitrage equation for real house prices can be derived from (3). The nominal house price inflation rate, $\pi_H$, equals the sum of overall inflation, $\pi$, and real house price inflation, $\pi_Q$. By definition, $\pi_Q = \dot{Q}/Q$. Thus, (3) can be rewritten as

(4) $$\dot{Q} = -R(H) + \nu Q$$

where $\nu = \delta + \kappa + (1-\theta)(i+\mu) - \pi$. For each value of Q and H, (4) determines the real capital gain on houses, $\dot{Q}$, which is required to induce individuals to hold the entire existing housing stock. The special case $\dot{Q} = 0$ determines the values f Q at any moment which are consistent with the full ownership of the existing stock and constant real house prices.[3]

A house's real price at each moment equals the present value of its net service flow. Define the net service value, S(t), as the sum of the unit's real rental service value minus its depreciation, tax, and maintenance costs.

$$S(t) = R(t) - [(1-\theta)\mu + \delta + \kappa]Q(t). \tag{5}$$

Now from (4), $\dot{Q}(t) = -S(t) + [(1-\theta)i - \pi]\,Q(t)$. This ordinary differential equation for Q is solved by

$$Q(t) = \int_t^{\infty} S(w)e^{-[(1-\theta)i - \pi](w-t)}\,dw \tag{6}$$

Asset market equilibrium as defined in (3) implies that a house's real price equals the present value of the house's future service flow discounted at the real after-tax interest rate, $(1-\theta)i - \pi$.[4]

The second important part of the housing sector is the market for new construction, where the amount of residential investment is determined. The homebuilding industry is assumed to be perfectly competitive.[5] If building firms profit-maximize, then the industry's output can be written as a function of the real price of houses. Since the industry's output is the flow of gross investment, I, this means $I = \psi(Q)$. This approach to investment[6] can be explained graphically using a production possibility frontier diagram. Figure 1 shows the effect of a an increase in the real price of houses on the production of houses and all other goods.

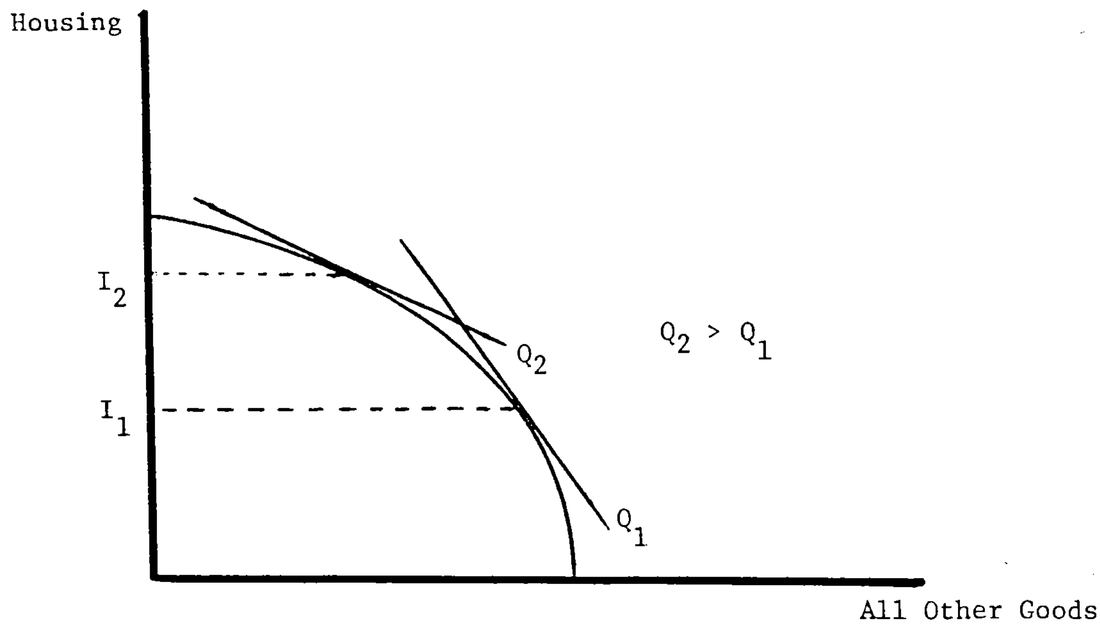


Figure One

A differential equation for the housing capital stock, H, can be derived from the $\psi(Q)$ investment function. The net change in the housing stock during any period, H, equals gross investment in that period minus the depreciation on the existing stock.

(7) $$\dot{H} = I - \delta H = \psi(Q) - \delta H \ .$$

The case of $\dot{H} = 0$ corresponds to the long-run steady state behavior of the housing market.[7] Setting $\dot{H} = 0$, equilibrium Q is determined by $Q = \psi^{-1}(\delta H)$.

## II. Inflationary Consequences

A steady state obtains when both the market for existing houses, a stock, and the market for new construction, a flow, are in equilibrium. The stock-flow equilibrium is characterized by $\dot{Q} = 0$ and $\dot{H} = 0$. This simple framework can be used to consider the steady state consequences of a permanent increase in the expected inflation rate. Higher inflation rates reduce the user cost of homeownership because interest payments are tax deductible, but the capital gain from home appreciation is essentially untaxed.[8] Differentiating the user cost expression yields $\frac{d\omega}{d\pi} = (1-\theta)\frac{di}{d\pi} - \frac{d\pi_H}{d\pi}$ . As an empirical phenomenon in the post-war United States, the nominal interest rate has risen one-for-one with expected overall inflation.[9] This means $\frac{di}{d\pi} = 1$ and along with the fact that $\pi_H = \pi$ in the steady state, implies $\frac{d\omega}{d\pi} = -\theta$. The marginal income tax rate is the key parameter in determining the change in the user cost which results from a given increase in the rate of expected inflation.

The effect of inflation can be seen graphically by studying the $\dot{Q} = 0$ and $\dot{H} = 0$ loci in H-Q space. Figure 2 shows an initial steady state at point A, corresponding to $(H^*, Q^*)$. An increase in the inflation rate reduces the user cost and leads to a greater demand for housing services at each real price Q. This corresponds to an outward shift of the $\dot{Q} = 0$ locus as shown. Both the real price of houses and the quantity of housing capital increase in response to higher inflation rates. The equilibrium real price rises because the long-run supply curve for housing services is upward sloping.

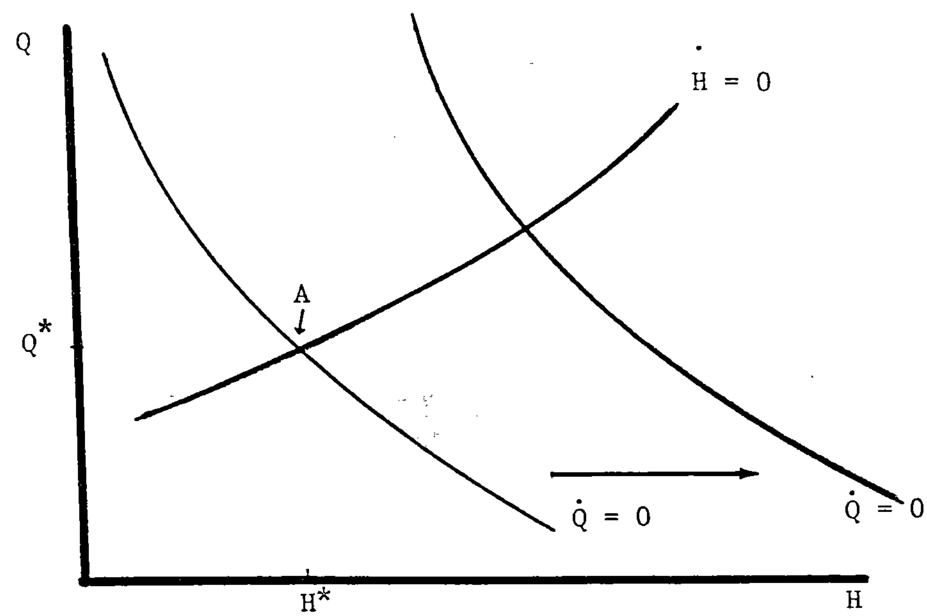


Figure Two

The path by which Q and H move from an initial equilibrium to a new steady state following an exogenous shock can be determined by analyzing the differential equations derived above. The two endogenous variables in this model are the rate of change of real house prices, $\dot{Q}$ , and the rate of net housing capital accummulation, $\dot{H}$. The system of differential equations governing these variables is

(8) $$\dot{H} = \psi(Q) - \delta H$$

$$\dot{Q} = -R(H) + \nu Q.$$

As shown in the Appendix, this system exhibits saddlepoint stability.

The stock and flow equilibrium schedules of Figure 2 can be viewed as defining the phase diagram for the differential equation system. Such a diagram is drawn in Figure 3 and the quadrants are labelled for reference purposes. The "stable arm," the unique stable trajectory, can be found by determining the direction in which real house prices and the capital stock will move from a point in each quadrant. In Quadrants I and III, the real price and housing stock move away from the long-run steady state values. The stable arm cannot be in either of these quadrants; it must therefore pass through quadrants II and IV. The stable arm is labelled as AA in the diagram.

The dynamic behavior of this model can be illustrated in the inflation shock case. When inflation increases, the $\dot{Q} = 0$ locus shifts out as in the steady state analysis above. This is shown in Figure 4 as a shift from $\ell_1$ to $\ell_2$. At the initial equilibrium, the system was described by the $\dot{H} = 0$ locus and $\ell_1$. After the shock, however, $\dot{H} = 0$ and $\ell_2$ constitute the system. This new system possesses a stable arm which is drawn as BB. There is only one trajectory which the system can follow if it is to ultimately reach a new long-run equilibrium at point B. This trajectory requires that Q "jump" at the time of the shock from its initial value at A to a point on the stable arm BB.[10] The unique point which satisfies this condition is labelled as C. After the jump, the system will move along the BB path to the new steady state.

Past work involving saddlepoint stable systems[11] has emphasized that the saddlepoint trajectory corresponds to the case of perfect foresight by economic actors. If actors do not anticipate future capital accumulation, the initial real price change will be larger than in the perfect foresight case. The real

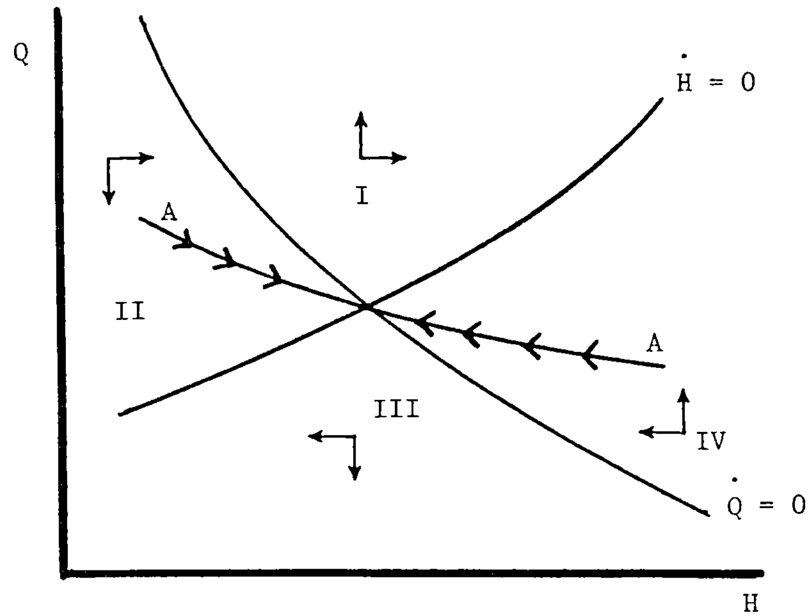


Figure Three

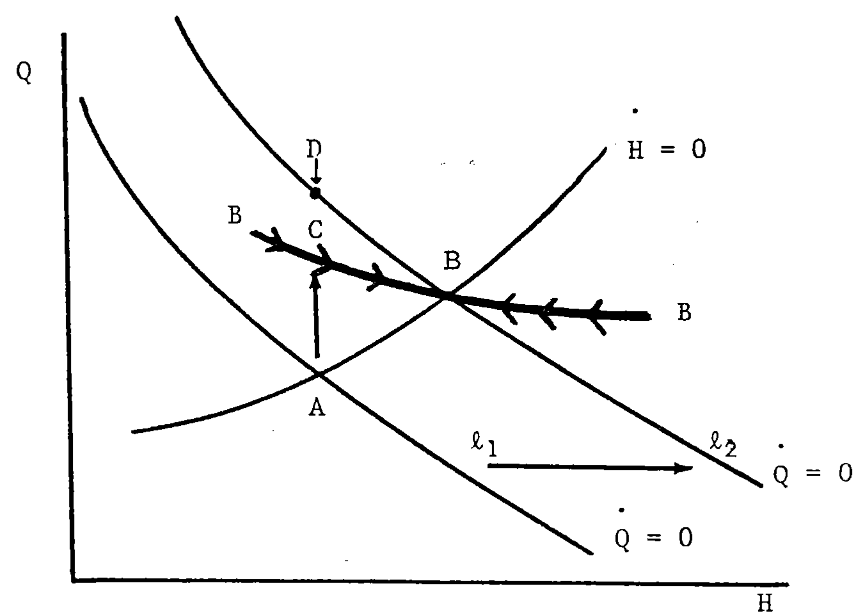


Figure Four

price change when the supply of housing is completely inelastic or actors do not anticipate investment is shown by point D in Figure 4. The simulation results below will contrast this price change, corresponding to "static expectations," with the rational expectations price change.

## III. Estimation

This section describes the choice of parameters for the $\dot{Q}$ and $\dot{H}$ functions. In the next section, the model is simulated to calculate the change in real house prices and the housing stock which result from an inflation shock.

A. The Asset Market Equilibrium Condition:

By taking logarithms and approximating R(H), equation (3) can be written as

(9) $$\alpha_0 + \alpha_1 \log H + \varepsilon = \log Q\omega$$

where $\varepsilon$ represents random factors and omitted variables. This can be rewritten in a more easily interpreted format as

(10) $$\log H = \alpha'_0 + \frac{1}{\alpha_1} \log Q\omega + \varepsilon'$$

where $\alpha'_0 = \alpha_0/\alpha_1$ and $\varepsilon' = \varepsilon/\alpha_1$. The parameter $(1/\alpha_1)$ is now the price elasticity of demand for housing services.

The demand function for housing services has been the subject of a large literature.[12] Micro or panel data probably yield the best estimates. Few existing studies, however, have focused on the real post-tax user cost as a measure of the homeowners' real cost of housing sercies. A noteworthy exception is Rosen's (1979) examination of the Panel Survey of Income Dynamics data set, in which he accounts for variation in both federal income taxes and local property taxes. He finds an income elasticity of .75 and a price elasticity of approximately -1.0 for homeowner housing demand. These parameters are used in the simulations below. I set $\alpha_1 = -1$ and used the income elasticity of .75 to compute a growth-adjusted depreciation rate for housing capital. All of the results below are invariant with respect to $\alpha'_0$. A value of 4.0 was used in the simulations.

B. The New Construction Equation:

The investment supply equation is best estimated from aggregate time-series data. Imposing a constant elasticity functional form on $I = \psi(Q)$ and allowing for the effects of construction costs and credit availability, the investment equation becomes

(11) $$\log I = \beta_0 + \beta_1 + \log Q + \log COSTS + \beta_3 \log CREDIT + \nu.$$

A random error term is represented by $\nu$. Information on the real value of new one-family housing, I, is prepared by the Bureau of Economic Analysis. The real price series, Q, is an unpublished price index for a constant-quality house excluding land, divided by the personal consumption expenditure price deflator.[13] The cost index is the Boeckh index of the price of inputs for a new one-family structure compiled by the American Appraisal Company again deflated by the consumption index. The flow of savings deposits received by savings and loan institutions is used to measure credit conditions. Experiments with other indicators of credit availability, such as federal agency activity or interest rate spreads, produced similar results.

Since the reported investment series is constructed by distributing the value of housing starts in each quarter through the next five quarters to reflect the completion pattern of new starts, independent variables which affect investment should enter both contemporaneously and with several quarter lags. Polynomial distributed lags are used in the estimation; the results are quite insensitive to changes in lag length and order.

Estimates of (11) from quarterly data, 1964-79, are shown in Table One. Since the two-sector investment theory which underlies the $\psi(Q)$ function implies that the <u>share</u> of total output devoted to construction is determined by Q, equations were estimated using both log I and log (I/CAPACITY GNP) as the dependent variable. The estimated price elasticity of new construction varies between 1.0 and 2.5; when a credit variable is included in the equation, the price term is always statistically significant. By comparison, Huang (1972) estimated elasticities of slightly more than two. The behavior of the construction cost coefficient is mixed: all of the estimates are negative, as expected, but only one is significantly different from zero.[14] Finally, inclusion of the past values of savings deposit inflows has a substantial effect in explaining housing investment variation. The elasticity of new investment with respect to deposit inflows is about 10; at current levels of savings deposits and residential investment, a one billion dollar change in the level (hence a one billion dollar increase in the flow) of savings deposits would lead to a 1.5 billion dollar increase in the value of new construction. For the simulations below, the value of $\beta_1$ is taken to be -2.0 and other variables are assumed constant. The intercept is therefore adjusted to yield $\tilde{\beta}_0 = \beta_0 + \hat{\beta}_2 \overline{\log COSTS} + \hat{\beta}_3 \overline{\log CREDIT} \doteq 2.0$. The investment equation in the simulations is $\log I = \tilde{\beta}_0 + 2.0 \log Q$.

The contrast between the present model and past attempts to estimate equations for new home building should be emphasized. Studies which regard the level of construction as a partial adjustment between the desired housing stock and the existing stock[15] omit the important asset market equilibrium conditions. Studies of this type have often concluded that credit constraint substantially reduces the demanded quantity of housing services. Demand effects, however, should be reflected first in the <u>price</u> of houses and in the quantity of new construction only as a result of this price change.

The hypothesis that rationing affects builders but not the underlying demand for housing services can be tested by examining the change in house

Table One: Investment Supply Equations

| Equation | Constant | Real House Price | Construction Cost | Credit Availability | $\rho_1$ | $\rho_2$ | $\bar{R}^2$ | SSR |
|---|---|---|---|---|---|---|---|---|
| 1. | 3.54<br>(5.69) | 1.39<br>(1.34) | - .10<br>(1.05) | - | 1.35 | -.53 | .92 | 2108 |
| 2. | 5.64<br>(2.00) | 2.52<br>( .53) | - .50<br>(.37) | 10.6<br>(1.6) | .97 | -.44 | .95 | 1363 |
| 3. | 2.68<br>(5.86) | 1.00<br>(1.39) | -1.22<br>(1.07) | - | 1.35 | -.52 | .89 | 2102 |
| 4. | 4.57<br>(2.00) | 2.14<br>( .53) | -1.60<br>( .37) | 10.5<br>(1.6) | .98 | -.45 | .92 | 1369 |

Notes: All equations are estimated quarterly, 1964:2-1979:2, correcting for second order autocorrelation. Coefficients in parenthesis are standard errors. Reported coefficients are the sum of coefficients in a six-quarter, second order polynomial distributed lag with no endpoint constraints. Dependent variable for Equations 1 and 2 is log(I), for 3 and 4 is $\log(\frac{I}{\text{CAPACITY GNP}})$. Data used in performing regressions is available from the author on request.

## TABLE TWO: SUPPLY- VS. DEMAND-EFFECT RATIONING

| Credit Rationing Period | Change in Residential Investment | Change in Real House Prices |
|---|---|---|
| 60:1 - 60:3 | -16.9 | -0.8 |
| 66:3 - 67:2 | -13.1 | -0.5 |
| 69.2 - 70:1 | -24.9 | 0.0 |
| 73:3 - 75:1 | -38.5 | -0.9 |

Notes: Periods of rationing determined by Brayton (1979). Change in residential investment is the largest percentage difference between constant dollar single family investment in the quarter before the rationing period and a quarter during the rationing period. A similar calculation yields the change in real house prices. All values in percentages.

prices and new construction during periods of credit tightness. Brayton (1979) has identified the periods of credit rationing during the past two decades. Table Two shows the percentage change in the value of new construction and the real price of houses from the quarter before the credit crunch began to the minimum point during the period of rationing. While the level of investment falls substantially during each period of restriction, real house prices have never fallen by as much as one percent. These findings support the contention of Meltzer (1974), that credit rationing affects only the flow supply of new construction and not the stock demand for houses.

## IV. Simulation Results

This section uses the parameter estimates from above to conduct simulation experiments about the effect of inflation on the housing market. The two-equation system in (10) and (11) can be rewritten to obtain equations for $\dot{Q}$ and $\dot{H}$. This new system, when interpreted as a nonlinear system of difference equations as in (12), can be used to perform policy simulations. A numerical

$$(12) \qquad H_t = e^{\tilde{\beta}_0} Q_t^{\beta_1} + (1-\delta^*)H_{t-1}$$

$$Q_t = -e^{\alpha_0} H_{t-1}^{\alpha_1} + [1+\delta+\kappa+(1-\theta)(i+\mu) - \pi_{t-1}] \cdot Q_{t-1}$$

algorithm[16] was used to compute the perfect foresight path which H and Q follow after an inflation shock. Both the initial price jump at the time of the shock, as well as the changes in H and Q when the system has reached the new steady state, are reported below. These changes are contrasted with that which would obtain in the static expectations case when actors do not anticipate future investment.

Two sets of simulations are reported; they differ in their assumed marginal income tax rate, $\theta$. The first assumes a rate of 25 percent, while the second considers inflation's effects in a world with a 35 percent marginal rate. The latter is probably a better approximation to the housing market of the early 1980s.[17]

For simplicity, at first all inflation shocks are assumed to be unanticipated and permanent. The analysis considers four different changes in the inflation rate: a jump from 0 to 2 percent per year, 0 to 5 percent, 0 to 8 percent, and 3 to 9 percent. The magnitude of the last change is roughly comparable to the change in expected inflation rates for the U.S. during the 1970s.[18] Calculations in which the inflation rate was increased by one percentage point per year for each of two, five, or eight years produced price changes quite close to those reported from the one-time shocks considered here. The tendency for inflationary expectations to change slowly in the real world does not, therefore, make the present results irrelevant.

The inflation shock simulations are reported in Table Three. Changes in the overall inflation rate have substantial effects on real house prices. A five percent inflation, introduced into a world with previously stable prices, causes real house prices to jump by 13.6 percent in the twenty-five percent marginal tax rate case and by 21.3 percent in the thirty-five percent case. If inflation rises from 3 to 9 percent, the resulting price changes are 18.7 percent and 32.3 percent, respectively. The steady state real price of houses rises by less than the initial jump; the long-run changes are 10.6 percent and 19.7 percent in the last two cases.

Varying rates of inflation also have implications for the economy's long run housing capital intensity. A five percent inflation rate leads to between a 15 and a 25 percent change in the long-run stock of houses, depending upon the marginal tax rate. The 3 to 9 percent shock induces between 22 and 43 percent equilibrium capital stock growth. If the tax system were indexed for inflation and did not treat inflation-induced increases in the nominal interest rate in the same fashion as changes in the real interest rate, equilibrium housing capital intensity would be unaffected by the rate of overall inflation. The computed transition path provides information about the time required to reach the new equilibrium. In most of the calculations, the capital stock-to-income ratio is within 1 percent of its new long-run equilibrium value within 40 years. The time required for movement halfway to the equilibrium value is about 11 years.

The results in Table Three allow a comparison of the change in real house prices under static expectations and perfect foresight. In the static case with a 25 percent marginal tax rate, the 3 to 9 percent shock leads to a 35.3 percent price increase. The rational expectations jump is only 18.7 percent, about half of the static expectations change. The substantial divergence between these two cases suggests that analyzing markets for reproducible assets without accounting for future investment activity can produce misleading results.

One of the consequences of the large change in the equilibrium capital stock is an increase in the rate of gross investment. From a 3 to 9 percent shock, the gross investment rate rises from an initial equilibrium value of four percent of the housing stock to more than five percent for the ten years after the initial shock. The level of residential investment increases by more than 20 percent for the years after the shock. In 1980 dollars, this would translate to nearly a ten billion dollar increase in the level of gross investment.

Table Three: Unexpected Inflation Shock Simulations

| | Inflation Shock | | | |
|---|---|---|---|---|
| $\theta$ = .25 Case | 0 to .02 | 0 to .05 | 0 to .08 | .03 to .09 |
| Static Expectations Price Change | 8.3 | 23.8 | 44.4 | 35.3 |
| Perfect Foresight Price Change | 5.1 | 13.6 | 23.4 | 18.7 |
| Steady State Price Change | 2.7 | 7.4 | 13.1 | 10.6 |
| Steady State Capital Change | 5.5 | 15.3 | 27.8 | 22.3 |
| $\theta$ = .35 Case | | | | |
| Static Expectation Price Change | 13.0 | 40.2 | 84.8 | 71.2 |
| Perfect Foresight Price Change | 7.7 | 21.3 | 38.7 | 32.3 |
| Steady State Price Change | 4.2 | 12.0 | 22.8 | 19.7 |
| Steady State Capital Change | 8.5 | 25.2 | 50.5 | 43.1 |

All reported changes are percentage movements from initial equilibrium. Assumed exogenous parameter values are $\delta = .015$, $\mu = .02$, $\kappa = .02$, $\delta^* = .04$, real rate of interest $r = .02$.

The approach outlined above can be used to analyze a wide variety of policy shocks. Changes in property taxes, income taxes, or the treatment of imputed rent all affect the housing market by altering the user cost of housing services. In addition, many different expectational paths for the exogenous variables can be studied. An inflation shock need not happen immediately. The consequences of an anticipated increase in the future inflation rate could be studied. A particularly interesting problem concerns the effect of an inflation shock which is known to be temporary.

The temporary inflation simulation assumes that inflation rises from an initial annual rate of three percent to nine percent for a period of fifteen years. At the end of this period, it returns to three percent. At $t_0$, when the new inflation policy is implemented, the user cost of housing falls and the demand curve for housing shifts outward. This is shown in Figure 5 as an outward shift of the $\dot{Q} = 0$ locus. The real price of houses jumps at $t_0$, but it does not move all the way to the stable arm of the new system. Because the system is "below" the stable arm, real house prices thereafter decline. The real price is above its initial level for a few years and there is net investment to meet the higher demand for houses. As the end of the inflationary period approaches, however, the real price of houses falls below its initial value and the rate of net investment becomes negative. The trajectory is an explosive one when viewed from the high-inflation demand curve for housing services. However, at time $t_0 + 15$ the demand curve shifts back toward the origin as the inflation rate falls and the user cost of housing rises. Now there is a new stable arm, and the system has arrived at point C, a point on the new stable path. The real price begins to rise once again and the capital stock falls until the long-run equilibrium is reached at the point from which the system started.

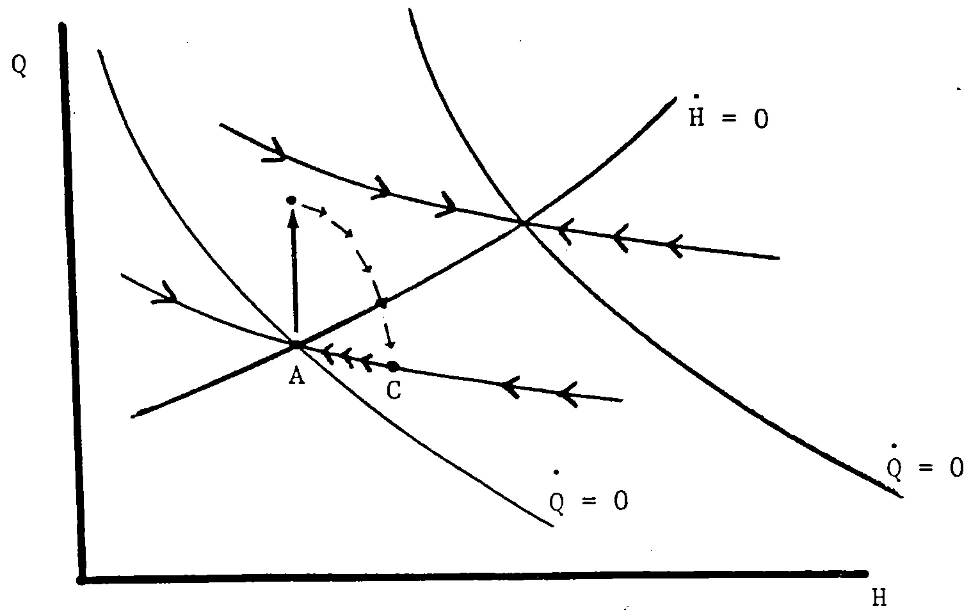


Figure Five

Simulation results illustrate the theoretical predictions. With a 35 percent marginal tax rate, the temporary shock causes a 12.6 percent instantaneous increase in the real price of houses. This should be compared with the 18.7 percent increase if the inflation were expected to persist forever. At the end of the fifteen year period, the real price of houses is 2.2 percent below the level at which it started. These results suggest that even transitory increases in the rate of expected inflation can have large and important real consequences for the housing market.

## V. Conclusion

This paper has developed a simple model of the housing market and used it to investigate the effect of high inflation rates on real house prices and the quantity of housing capital. The present tax treatment of mortgage interest payments and housing capital gains reduces the cost of housing services when the inflation rate rises. Given the change in inflationary expectations which occurred during the 1970s, this effect could have accounted for a thirty percent increase in the real price of houses.

The present work can be extended profitably in several directions. I have not treated the important tenure choice decision between owner-occupancy and renting. The inflation-induced reduction in the user cost of homeownership should increase the desirability of becoming a homeowner. Inflation's effect on the quality of housing services demanded also remains to be analyzed. The jointness of house and land purchases can also be treated more fully. Feldstein (1980) has shown that inflation interacts with the tax code to increase the real price of land relative to corporate capital. These two research themes could be unified to yield a better model of the housing sector.

The partial equilibrium character of the present model could also be altered; a housing sector could be incorporated in a general equilibrium model with corporate capital, for example. Summers (1980b) has begun work along these lines. Finally, as the capital gains component of housing investment becomes more substantial, introducing larger risks, a portfolio model becomes more appropriate. The tax incentives remain unchanged, but a complete analysis must imbed the home purchase decision in a household portfolio model.

Footnotes

1. Laidler (1969) and DeLeeuw and Ozanne (1979) are among those who have studied the treatment of imputed rent. Rosen (1979) and King (1980) emphasize the deductability of mortgage interest payments.

2. If the opportunity cost of funds, $i_0$, is different from the cost of borrowing, $i_\beta$, then the loan-to-value ratio $\ell$ on the housing purchase enters the problem. The user cost in (3) becomes

$$(3') \qquad \omega' = \delta + \kappa + (1-\theta)[\ell i_\beta + (1-\ell) i_0 + \mu] - \pi_H .$$

3. The $\dot{Q} = 0$ locus can be interpreted as the demand curve for housing services when there are no real capital gains associated with homeownership.

4. A more general expression can be obtained by allowing non-constant interest and inflation rates. Defining $\sigma(t) = (1-\theta)i(t) - \pi(t)$, equation (6) becomes

$$(6') \qquad Q(t_0) = \int_{t_0}^{\infty} S(t)\exp(-\int_{t_0}^{t} \rho(x)dx)\, dt.$$

5. This assumption is quite common; see, for example, Mills (1973).

6. Witte (1963) was among the first to suggest this approach to modelling investment.

7. This discussion does not incorporate the effects of economic growth. In a steady state for a growing economy, the ratio of the value of H to real income must be constant. H must therefore rise at a rate $n + \eta_y g$, where n is the rate of population growth, g the rate of growth of real income, and $\eta_y$ the income elasticity of demand for housing services. In Section IV this is incorporated by defining $\delta^* = \delta + n + \eta_y g$ and requiring that $\dot{H} = \psi(Q) - \delta^* H$.

8. Several factors make the choice of zero as an effective capital gains tax rate quite reasonable. First, the capital gain on a house is untaxed whenever the proceeds are invested in another home. The U.S. Savings League (1977)

found that 78% of all home sellers bought another house immediately. Unfortunately, the percentage of people who traded down to smaller houses and were unable to reinvest the full capital gain on their earlier home is unknown. Second, for those who do not purchase new homes but are over sixty-five, the first one hundred thousand dollars of capital gains is tax exempt. Finally, for the small fraction of sellers who are required to pay capital gains taxes, the taxes are paid on realization and not on accrual. This reduces the effective tax rate still further.

9. Feldstein and Summers (1978) show that while in a world with taxes $\frac{di}{d\pi}$ need not be unity, this value cannot be rejected for the United States in the postwar period.

10. If the real price of houses in the period when the shock occurs lies above the stable arm, there will be an infinite future path of capital gains on houses and the housing stock will eventually become infinite. Alternatively, for points below the stable arm, house prices will deflate forever and the housing stock will shrink toward zero.

11. Discussions of saddlepoint stable systems may be found in, for example, Dornbusch (1976), Blanchard (1978), and Summers (1980a).

12. DeLeeuw (1971) surveyed some the early work on housing demand. More recently, Polinsky and Ellwood (1979) have attempted to reconcile the disparate results of micro and metro studies.

13. This unpublished data was provided by the U.S. Census Department.

14. Construction costs are probably endogenous to the construction sector and a more complete analysis might address the issue of simultaneous equation bias.

15. Examples of such studies include Kearl and Mishkin (1977) and Muth (1960).

16. A full discussion of the algorithm used for solving the system of non-linear difference equations is contained in Lipton, Poterba, Sachs and Summers (1980).

17. The mean annual income of FHA-backed homebuyers in late 1979 was above $28,000 and the current marginal tax rate on a family of four earning $30,000 is 37 percent. See FHA (1979). Since FHA buyers are generally of lower income than average homebuyers, even this may be an underestimate.

18. This was computed from data on the National Bureau of Economic Research-American Statistical Association Business Outlook Survey. Past data provided courtesy of ASA.

## Appendix: Stable Path Analytics

The mathematical properties of the differential equation system in (8) are now considered in greater detail. First, the equation for the stable arm is solved for analytically. Then, by computing the roots of the differential equations, a comment is made about the speed of convergence to the new equilibrium.

The system given by (8) in the text can be linearized around an initial equilibrium ($H_0$, $Q_0$) to yield

$$\text{(A1)} \qquad \begin{bmatrix} \dot{H} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} -\delta & \psi' \\ -R_H & \nu \end{bmatrix} \begin{bmatrix} H - H_0 \\ Q - Q_0 \end{bmatrix}$$

The solutions to this differential equation system are of the form

$$\text{(A2)} \qquad \begin{bmatrix} H \\ Q \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} e^{\lambda_1 t} \\ e^{\lambda_2 t} \end{bmatrix}$$

where $\lambda_1$ and $\lambda_2$ are the eigenvalues of the coefficient matrix.

The constants $c_{\cdot j}$ are the elements of an eigenvector of the coefficient matrix corresponding to eigenvalue $\lambda_j$. The eigenvalues of the matrix in (A1) are

$$\text{(A3)} \qquad \lambda_1 = \frac{\nu - \delta + [(\delta-\nu)^2 - 4(R_H\psi' - \delta\nu)]^{\frac{1}{2}}}{2} > 0$$

and

$$\text{(A4)} \qquad \lambda_2 = \frac{\nu - \delta - [(\delta-\nu)^2 - 4(R_H\psi' - \delta\nu)]^{\frac{1}{2}}}{2} < 0 .$$

One positive and one negative root indicate that the system is saddlepoint stable.

The stable arm is found by setting the weight on the $e^{\lambda_i t}$ for which $\lambda_i > 0$ equal to zero. The solution is therefore given parametrically by

$$\text{(A5)} \qquad (H - H_0) = \alpha Z_1 e^{\lambda_2 t} = c_{12} e^{\lambda_2 t}$$

$$(Q - Q_0) = \alpha Z_2 e^{\lambda_2 t} = c_{22} e^{\lambda_2 t} \quad .$$

In this expression, $\alpha$ is a constant depending on the initial conditions and $Z_1$ and $Z_2$ are the elements in the eigenvector corresponding to $\lambda_2$. $Z_1$ and $Z_2$ satisfy the proportionality equation

$$\text{(A6)} \qquad \frac{Z_1}{Z_2} = \frac{\psi'}{\lambda_2 + \delta} = \frac{\nu - \lambda_2}{R_H} \quad .$$

Thus, the equations in (A5) can be rewritten

$$\text{(A7)} \qquad (H - H_0) = \alpha(\nu - \lambda_2) e^{\lambda_2 t}$$

$$(Q - Q_0) = \alpha R_H e^{\lambda_2 t} \quad .$$

The slope of the stable arm is just $\frac{dQ}{dH} = \frac{\nu - \lambda_2}{R_H} < 0$ .

The size of the system's negative eigenvalue determines the speed of convergence along the stable arm. Larger negative numbers imply faster convergence. The only important point to be drawn from this is that the responsiveness of the investment rate to changes in the real price of houses, $\psi'$, is a key parameter for the convergence speed. Differentiating (A4) with respect to $\psi'$ yields $\frac{d\lambda_2}{d\psi'} = 2R_H[(\delta - \nu)^2 - 4(R_H\psi' - \delta\nu)]^{-\frac{1}{2}} < 0$ .

Thus as $\psi'$ increases, $\lambda_2$ becomes a larger negative number and the system moves to the new equilibrium more rapidly.

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