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## THE ROLES OF MONETARY, FINANCIAL AND FISCAL POLICY UNDER RATIONAL EXPECTATIONS

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#### ABSTRACT

The implications for the effectiveness of monetary, fiscal and financial policy of the "rational expectations revolution" are evaluated. The general conclusion is that to anticipate policy is not to neutralize it. This is obviously the case for <u>structural policies</u> that alter the level and composition of full employment output. It also holds for stabilization policies that influence deviations of real variables from their "natural" values. Substitution of bond financing for tax financing of a given real spending program reduces saving and lowers the capital-labor ratio, even when allowance is made for private intergenerational gifts. Anticipated monetary policy affects the cyclical and equilibrium behavior of real variables except in implausible special cases.

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## The Roles of Monetary, Financial and Fiscal Policy under Rational Expectations

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#### Summary

The paper evaluates the implications for the conduct of monetary, fiscal and financial policy of the "rational expectations revolution" in macroeconomics. The general conclusion is that most of the policy conclusions derived from conventional eclectic Neo-Keynesian models remain valid when rational expectations are introduced: to anticipate policy is not to neutralize it. Both <u>structural</u> policies aimed at altering the level and composition of full employment output in the short run and in the long run and <u>stabilization</u> policies aimed at influencing the cyclical departures of output and employment from their full employment levels are considered.

The consequences of structural fiscal policy--changes in non-lump-sum tax rates and in public spending--for full employment output and employment are non-controversial. The confirmation of familiar Neo-Keynesian results about financial and monetary policy may be more surprising. Here the major policy conclusions are the following:

1) <u>The financing of government spending</u>. For a given level and composition of real government spending on goods and services, the substitution of bond financing for current (lump-sum) taxes reduces saving in the short run and lowers the capital-labor ratio in the long run. This crowding out result persists even if each economic agent allows fully for the future taxes "required" to service the stock of privately held interest-bearing public debt. It even holds when allowance is made for the possibility of bequests and other private intergenerational gifts. In a situation of Keynesian unemployment the crowding out of saving in the short run may be an appropriate policy to achieve a higher degree of resource utilization. The substitution of money financing for tax financing is also not a matter of indifference. The "inflation tax" is not equivalent to explicit current taxes. In the simple models considered in the paper, the substitution of money financing for tax financing "crowds in" saving and raises the capital-labor ratio in the long run. 2) <u>Monetary policy and stabilization</u>. Anticipated as well as unanticipated monetary policy will affect the cyclical behavior of remained economic variables. To obtain the result that anticipated monetary policy does not matter for the behavior of real output, employment etc., it is necessarry to assume that all prices always and instantaneously assume their competitive market-clearing values, in addition to having rational expectations based on these same information as is available to the monetary authorities. Even then anticipated monetary policy will not be neutral unless the structural channel through withich monetary policy can operate is also assumed to be ineffective. This structuraral channel is the effect of changes in the money supply process on the anticipated rate of inflation and thus on the rate of return to holding money balances and other nominal financial claims. Anticipated monetary policy therefore affects remained through three channels.1) Sticky nominal prices. 2) Non-identicical information available to the public and private sectors. 3) The effect of monetary growth on inflation and thus on the portfolio choice between real and nomineal assets.

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There are no grounds, in view of these conclusions., for believing that constraining the conduct of fiscal, monetary and financmial policy by very simple, inflexible rules such as a constant growth rate for the money supply, a balanced budget or a constant share of public spending in GNP will be optimal or even sensible.

## The Roles of Monetary, Financial and Fiscal Policy under Rational Expectations

#### I. Introduction

Neo-Keynesian accounts of the roles of monetary, financial, and fiscal policy have recently been challenged by a revival of classical macroeconomic thinking--an approach to theory and policy that many had considered forever buried under the economic ruins of the Great Depression. This New Classical Macroeconomics is associated originally with Milton Friedman (1968) and more recently with Phelps (1970), Lucas (1972a, b, 1975, 1976) Sargent and Wallace (1975, 1976) Barro (1974, 1976, 1979), and a host of others. The main implications of this approach for macroeconomic policy can be summarized in two"neutrality or worse" propositions.

- Deterministic (and known) monetary policy rules can have no effect on the joint probability distribution functions of real economic variables-neutrality--but stochastic monetary policy behavior can increase the variability of real variables relative to their full information values--or worse. This proposition has at times been extended to encompass not only monetary policy but stabilization policy in general. (McCallum, 1977.)

- Debt neutrality: the full real impact of the government sector is measured by the magnitude and composition of its real spending program. The financing mode chosen, current taxes, borrowing or currency creation, is of no consequence. This proposition, first analyzed by Ricardo, has recently been restated by Barro (1974).

In a recent paper, "The Macroeconomics of Dr. Pangloss" (Buiter, 1980 forthcoming), I have presented a nontechnical survey of the two theoretical cornerstones of the New Classical Macroeconomics. These are rational expectations in the sense of Muth and what I have called the "surprise" supply function--the proposition that only unanticipated policy changes (or other unanticipated, exogenous disturbances) can cause the real economic system to depart from an otherwise policy-invariant equilibrium trajectory. In this paper I shall say nothing further about the rational expectations assumption. It, or its deterministic counterpart, perfect foresight, will be incorporated in every model considered below. My purpose is to provide a more formal statement of the propositions advanced in my "Pangloss" paper concerning the scope for monetary policy, debt management policy, and fiscal policy under rational expectations. The setup is as follows. Section II presents a fairly standard, small deterministic macromodel with a number of classical features. All markets clear instantaneously, there is no money illusion and perfect foresight The effects of monetary, financial, and fiscal policy in this rules. model are analyzed. A number of non-neutrality propositions are stated. The drawback of this model is that it is ad hoc in the sense that private behavioral relationships have not been derived from explicit optimizing behavior. Section III therefore studies debt neutrality and monetary superneutrality in a "fully rational" overlapping generations model. This leads to the conclusion that the ad hoc model of Section II is not a bad parable for such fully rational models. Section IV abandons the assumption of universal instantaneous Walrasian equilibrium and considers the consequences of price and wage stickiness for the scope for stabilization policy; stochastic models are analyzed here, which also permits the consideration of some of the interesting issues associated with incomplete information.

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There are two major limitations on the scope of this critique of the New Classical Macroeconomics: it is mainly theoretical and it is limited to models of a closed economy.

## II. Monetary, Financial, and Fiscal Policy in an Ad Hoc Equilibrium Model with Rational Expectations

#### Description of the model

The model represented in equations (1) through (17) is the full employment version of the textbook closed economy IS-LM model. The nominal interest rate, the price level and the money wage rate always instantaneously assume the values required to equilibrate the asset market, the output market and the labor market.

Notation			
cP	:	private consumption	
IP	:	private investment	
c <sup>G</sup>	:	government consumption	
Y	:	real income	
K	:	stock of capital	
N	:	employment	
L	:	demand for real money balances	
М	:	nominal stock of money	
В	:	nominal value of the stock of government bonds	
Α	:	private non-human wealth	
D	:	real government deficit	
Т	:	real taxes net of transfers, excluding debt service	
р	:	price of output	
q	:	price of installed capital in terms of current output	
W	:	money wage	
R	:	nominal interest rate	
r <sub>K</sub>	:	required real rate of return on capital	
<sup>t</sup> R	:	proportional tax on interest income	
t <sub>π</sub>	:	proportional tax on profits	
tc	:	proportional tax on capital gains	
tw	:	proportional tax on labor income	
t <sub>e</sub>	:	proportional payroll tax on employers	
τ	;	lump-sum tax on households	
x <sup>p</sup>	:	expected rate of inflation	
xq	:	expected proportional rated change of q	
δ	:	fraction of the public sector deficit financed by borrowing	

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- α : fraction of private holdings of public sector interest-bearing debt
   perceived as net worth by the private sector
- β
- : fraction of public spending perceived as real private income-inkind

$$(II.1) \quad C^{P} + C^{G} + I^{P} = f(K, N) ; f_{K}, f_{N} > o ; f_{KK} < o ; f_{NN} < o; \\ (II.2) \quad L = \frac{M}{p} \\ (II.3) \quad C^{P} = C(C^{G}, R(1 - t_{R}) - x^{P}, Y, A); -1 \leq C_{1} \leq 1; C_{2} \leq o; o < C_{3} < 1; C_{4} > o \\ (II.4) \quad I^{P} = I(q); I > o; I(1) = o \\ (II.5) \quad q = \frac{f_{K}(K,N)(1 - t_{R}) - t_{e} x^{q}}{r_{K} - x^{q}} \\ (II.6) \quad r_{K} = R(1 - t_{R}) - x^{P} \\ (II.7) \quad L = L \left( R(1 - t_{R}), Y, A \right), L_{1} \leq o; L_{2} > o; o < L_{3} < 1 \\ (II.8) \quad Y = f(K, N) + \frac{RB}{p} - T - (1 - \alpha)(1 - \delta)D + \beta C^{G} - x^{P} \left[ \frac{M + \alpha B}{p} \right] + x^{q} qK \\ (II.9) \quad T = t_{WP}^{M} N + t_{eP}^{M} N + t_{\pi}f_{K}^{K} + t_{c} x^{q} qK + t_{R}\frac{RB}{p} + \tau \\ (II.10) \quad D = C^{G} + \frac{RB}{p} - T \\ (II.11) \quad A = \frac{M + \alpha B}{p} + qK \\ (II.12) \quad \left( \frac{W}{p} \right)^{d} = f_{N}(K, N) \quad (1 - t_{e}) \\ (II.13) \quad \left( \frac{W}{p} \right)^{d} = h(N, C^{G}, R(1 - t_{R}) - x^{P}, A) \quad (1 + t_{w}); h_{1} > o; h_{2} \leq o; h_{3} \leq o \\ (II.14) \quad \left( \frac{W}{p} \right)^{d} = \left( \frac{W}{p} \right)^{S} \\ (II.15) \quad x^{q} = \Delta q/q \\ (II.15) \quad x^{q} = \Delta q/q \\ (II.16) \quad \Delta K = I/q \end{cases}$$

(II.17a) 
$$\Delta(\frac{M}{p}) = \delta D - \frac{\Delta p}{p} \frac{M}{p}$$
  
(II.17b)  $\Delta(\frac{B}{p}) = (1 - \delta)D - \frac{\Delta p}{p} \frac{B}{p}$ 

The steady-state equilibrium of the model is obtained by setting  $\Delta K = \Delta(\frac{M}{p}) = \Delta(\frac{B}{p}) = 0.$  Thus II.16, II.17a and II.176 are replaced by (II.18) q = 1, i.e.  $r_{K} = f_{K}(K, N) (1 - t_{\pi})$ (II.19a)  $\delta D = \frac{\Delta p}{p} \frac{M}{p}$ (II.19b)  $(1 - \delta)D = \frac{\Delta p}{p} \frac{B}{p}$ 

Equation II.1 is the IS equation equating effective demand for output to full employment production. 1/ Full employment production is given by a linear homogenous production function, f, in capital and labor with positive and diminishing marginal products. Equation II.2 is the LM or portfolio balance equation. Private consumption depends on the real interest rate, real income and real private net worth. It also may depend directly on government consumption. 2/ This will be the case, e.g., if government consumption is a substitute for private consumption (free school milk, etc.). Private capital formation is an increasing function of the price of installed capital goods relative to the cost of producing new investment goods. The current price of capital equals the discounted value of current after-tax cash flow plus next period's expected price of capital (equation II.5). Due allowance has to be made for profits taxes (at rate  $t_{\pi}$ ) and capital gains taxation (at rate  $t_{c}$ ). The required rate of return on capital,  $r_{\kappa}$ , equals the real rate of return on bonds (equation II.6), a reflection of the Keynesian assumption that bonds and claims on existing capital are perfect substitutes in private portfolios. Interest income is

taxed at a rate t<sub>p</sub>. Inflationary capital gains or losses on money and bonds are not taxed. The demand for money depends on the real after-tax rate of return differential between bonds-cum-capital and money, on real income and real nonhuman wealth, Real income, equation II.8, consists of disposable private income,  $f + \frac{RB}{p} - T$ , adjusted in a number of ways. First, expected capital gains (or losses) due to expected inflation and expected changes in q are added  $(-x^{p}(\underbrace{M + \alpha B}_{p}) + x^{q}qK)$ . Then we allow for the fact that a fraction  $1 - \alpha$  of the stock of government bonds held by the private sector may not be viewed as private net worth because they are offset by the present value of the future taxes "required" to service this debt. The "flow" counterpart of this wealth adjustment is the subtraction of a fraction  $1 - \alpha$  of the bond-financed part of the government deficit in II.8. That part of the deficit is viewed as equivalent to current taxes. Finally, if government spending on real goods and services is perceived as incomein-kind, this has to be added in. Equation II.9 defines total taxes in terms of its constituent components. Equation II.10 is the public sector deficit, Real non-human wealth, in equation II.11, includes the adjustment for bonds discussed earlier. Equations II.12 and II.13 give the real demand price and real supply price of labor, respectively. The demand price equals the marginal product of labor net of payroll taxes, Labor supply depends on the real interest rate, real net worth and government spending. Labor market equilibrium is assumed in equation II.14. Equations II.15a and II.15b impose rational expectations for the general price level and the price of capital. Equations II.16, II.17a and II.17b are the dynamic equations describing the behavior over time of the capital stock, 3/ of the stock of real money balances and of the real stock of bonds.

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#### Effects of 'structural' fiscal and monetary policy

Fiscal and monetary policy will have important effects in this equilibrium model. The level and composition of real full employment output can be influenced in the short run (identified with the unit period of analysis), in the long run (the steady-state equilibrium) and in the real-time intermediate run in which policy decisions are actually made. These real output effects are not due to the elimination of involuntary Keynesian excess supply but to policyinduced changes in important relative prices. This alters the labor-leisure trade-off, the intertemporal consumption trade-off, the marginal cost of labor, etc. Standard comparative statics yield the following policy conclusions:

Taxation. The non lump sum tax changes are constant revenue changes with a.  $\tau$  adjusting endogenously. Consider the special case of the labor supply function where it is independent of the real interest rate and of net worth. If the labor supply schedule is upward-sloping, an increase in the tax rate on labor income, t,, will shift it to the left, reducing full employment output and employment for a given stock of capital. An increase in the payroll tax, or in employers' national insurance contributions, t, reduces employment and output by shifting the labor demand schedule to the left, for a given stock of capital. An increase in  $t_{\pi}$ , the profit tax, will affect-presumably lower---the rate of private capital formation in the short run and the steady-state capital-labor ratio in the long run. An increase in capital gains taxation,  $t_c$ , will have similar effects in the short run. In steady-state equilibrium there are no real capital gains so changes in capital gains taxation will not affect the long-run real equilibrium. An increase in the tax rate on interest income, t<sub>p</sub>, will lower the required rate of return on capital. This will tend to stimulate capital formation in the short run and raise the capital-labor ratio in the long run. An

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increase in lump-sum taxes,  $\tau$ , will reduce real income unless, given the government's real spending program, money- and bond-financed deficits are exactly equivalent to explicit lump-sum taxation. For this to be the case, it is necessary that government bonds do not constitute net private sector wealth to any extent, i.e., that  $\alpha = 0$ . With exclusively bondfinanced deficits,  $\delta = 0$ , the appropriate real income concept then becomes, using II.8

(II.8')  $Y = f(K, N) + (\beta - 1) C^{G} - \frac{\Delta p}{p} \frac{M}{p} + \frac{\Delta q}{q} qK$ 

A substitution of lump-sum tax financing for borrowing will have no real effects. With money-financed deficits,  $\delta = 1$ , equation II.8 can be written as:

(II.8'')  $Y = f(K, N) + (\beta - 1)C^{G} + \frac{\Delta M}{p} - \frac{\Delta p}{p}(\frac{M}{p}) + \frac{\Delta q}{q} qK$ A once-and-for all increase in the <u>level</u> of the stock of money will be strictly neutral in this model only if  $\alpha = 0$ .  $\frac{4}{7}$  This means that  $\frac{\Delta M}{p} - \frac{\Delta p}{p} \frac{M}{p}$  is zero when such a level shift in the money stock path occurs, and that  $\frac{\Delta q}{q}$  is independent of such a level shift. Thus a one-period unexpected shift from money financing to lump-sum tax financing will not have any real effects.

b. <u>Government spending</u>. In this classical equilibrium model, changes in government spending on real goods and services will have powerful shortrun and long-run effects. All tax rates are assumed constant. Any changes in the public sector deficit or surplus are financed according to II.17a and II.17b. If there is no direct crowding out (Buiter, 1977, 1979), i.e., if government consumption is not a substitute or complement for private consumption and leisure ( $C_1 = 0$ ,  $\beta = 0$ ,  $h_2 = 0$ ), government spending will "crowd out" private consumption and investment spending on a one-for-one basis by raising the price level and the interest rate. <u>5</u>/ In the long run, a higher level of public spending is likely to "crowd out" real capital, although this need not be the case. If higher public spending leads to steady state deficits, these could be associated with higher rates of inflation. This would make real capital a more attractive asset compared to money and creates the possibility of long-run "crowding in." Getting rid of all real effects of government spending requires some very strong assumptions. (i) No direct effect of public consumption on the supply of labor:  $h_2 = o$ . (ii) Public consumption is a perfect substitute for private consumption and thus constitutes income in kind on a one-for-one basis:  $C_1 = -1$  and  $\beta = 1$ . (iii) Debt neutrality prevails:  $\alpha = 0$ . (iv) If the higher level of public spending is associated with increased deficits, these either are financed by borrowing, or, to the extent that they are financed by money creation, only involve a once-and-for all step increase in the , money supply. Since most of these assumptions are rather silly, the real trajectory of the economy will certainly not be invariant under alternative paths for real government spending.

c. <u>Money neutrality</u>. A once-and-for all step increase in the nominal stock of money will be neutral, i.e., have no real effects only if  $\alpha = 0$ :

debt neutrality prevails. A given percentage change in M, if it were associated with an equal percentage change in p and W, would reduce the real value of a given nominal stock of bonds. This will only fail to have real consequences if bonds do not matter, i.e., if  $\alpha = 0$ . Debt neutrality is discussed at length in Section III.

d. <u>Monetary superneutrality</u>. This is the property that the real trajectory of the economic system is invariant under alternative proportional rates of growth of the nominal money supply. It is easily seen that if monetary B ≠ o, debt neutrality is a necessary condition for/superneutrality. As can be

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seen from equations II.19a and II.19b, only a common proportional rate of growth of the nominal stocks of money and bonds is consistent with steady state equilibrium if  $\alpha \neq \alpha$ . Even if debt neutrality prevails, superneutrality will be negated by the fact that no market-determined interest rate is paid on money balances. The real rate of return on money is minus the expected rate of inflation. Higher proportional rates of money growth are, across steady states, associated with higher proportional rates of inflation. These lower real returns from holding money will lead to a portfolio shift from money to real capital, thus destroying superneutrality, unless the demand for money is completely interest-inelastic. Any change in fiscal parameters that changes the rate of inflation(because it leads to changes in the growth path of the money supply)will therefore have real effects, even if it were to have no direct structural effects or displacement effects. The next section considers debt neutrality and monetary superneutrality in an utterly classical overlapping generations world.

### III, Debt Neutrality and Monetary Superneutrality in Fully Optimizing Models

#### a. Debt neutrality in an optimizing macromodel

In two recent studies (Buiter and Tobin, 1980, Tobin and Buiter, 1980), James Tobin and I concluded that debt neutrality--the property that the real trajectory of the economic system is invariant under changes in the financing mix, for a given level and composition of real government spending--is a theoretical curiosum. The assumptions required for it to be valid can easily be shown to be contradicted by practical experience. In this section of the paper, I shall restate the case against debt neutrality in the context of a model constructed expressly to be as favorable as possible to classical invariance theorems. The model is a generalization of Diamond's overlapping generations model (Diamond, 1965), and allows for voluntary intergenerational gifts and bequests. (See Barro, 1974, and Buiter, 1980a). A comprehensive treatment of the subject can be found in Carmichael (1979). Except for some minor changes, the treatment of the case of agents with "two-sided intergenerational caring" replicates the original work of Carmichael.

The overlapping generations model used to develop the non-neutrality theories is "classical" in the sense that private actions are derived from explicit optimizing behavior, perfect foresight prevails and all markets are in equilibrium all of the time. All private agents act as price takers. I shall study the behavior of this decentralized, competitive economy when a given government spending program is financed by different combinations of lump-sum taxation or current borrowing. Without loss of generality the level of government spending is assumed to equal zero, which allows us to rephrase the argument in terms of the real effects of alternative debt issue-taxation programs. The restriction to lump-sum taxes is necessary to give the neutrality proposition a chance. Non-lumpsum taxes on labor income, profits, wealth or any other base, will introduce real distortions, impose excess burdens and, except in uninteresting special cases, have real effects. Private, voluntary intergenerational gifts--from parents to children (bequests) or from children to parents -- are essential for the debt neutrality property to prevail. Briefly, the argument for neutrality goes as follows. The stock of real government interest-bearing debt has no effect on private behavior because corresponding to every dollar's worth of income on these bonds is a dollar's worth of tax payments

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to finance the bond income. The value of the government bonds on the asset side of private portfolios is the present discounted value of these future income payments. The value of these bonds is therefore exactly matched by the present discounted value of the future tax payments required to service them. Even if we grant that the future payments stream and the future tax payments stream are identical and that both are discounted in the same manner, a shift from tax financing to borrowing could cause non-neutrality because of an intergenerational redistribution of resources. If the bonds are one-period bonds and each individual is supposed to live for two periods, the intergenerational redistribution that can be associated with such issues is immediately apparent. Let an extra Pound's worth of bonds be issued in period t. It is bought by the then young members of generation t. Next period interest and repayment of principal The tax revenue required for the debt servicing could be levied on the occur. then young members of generation t + 1. In that case, real resources have been redistributed from the young to the old. Consumption and capital formation will be affected. An unfunded social security program will have broadly similar effects. Longer maturity bonds can be incorporated in the analysis without materially altering it. Voluntary intergenerational gifts can remove the real consequences of involuntary intergenerational redistribution through the borrowing-taxation mechanism. Provided the taxes are lump-sum, such private intergenerational transfers will restore the original consumption-investment equilibrium as long as such private actions do not violate the non-negativity constraints on these voluntary intergenerational transfers. If, before the extra Pound's worth of public debt is issued the members of the older generation were all leaving positive bequests to their descendants, the

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option of redistributing resources from the young to the old through a cut in bequests was already open to the older generation . Their decision not to exercise this option reflects that, at the margin, they receive greater utility from the well-being of their heirs than from their own consumption. The government's attempt to redistribute "gross resources" from the young to the old will in that case by met by increased bequests from the old to the young, leaving the "net resources" available to each generation unchanged. If, on the other hand, the older generations were initially at a "zero bequest corner," i.e., if in order to increase their own life-time resources they would gladly have left their children a negative legacy, had this not been ruled out by law, the involuntary intergenerational redistribution would not have been neutralized by an exactly matching voluntary transfer in the opposite direction.

Within the bounds set by the non-negativity constraints on gifts and bequests, lump-sum redistribution through borrowing or unfunded social security schemes will be neutralized by voluntary intergenerational transfers, if bequest or gift motives are present. Private non-market transactions are required to neutralize public non-market transactions. A formal analysis follows below.

Notation

- c<sup>1</sup>t : consumption while young by a member of generation t
  c<sup>2</sup>t : consumption while old by a member of generation t
  Kt : capital stock in existence at the beginning of period t
  Lt : size of generation t
- $k_{t} : K_{t}/L_{t}$

B : saving by a member of generation t while old (i.e., his bequest to young members of generation t + 1)

$G_{t}$	:	gift by a young member of generation t to old members of generation t - 1.
D <sub>t</sub>	:	stock of real one-period government debt in existence at the beginning of period $\mathbf{t}$
d <sub>t</sub>	Ξ	<sup>D</sup> t <sup>/L</sup> t
<sup>T</sup> t	:	lump-sum tax levied on members of generation t while young
τ <sub>t</sub>	Ξ	<sup>T</sup> t <sup>/L</sup> t
rt	:	interest rate on savings carried from period t - 1 into period t
n	:	one-period proportional rate of growth of population
δ	:	one-period discount rate applied to the utility of one's immediate descendant
ρ		one-period discount rate applied to the utility of one's immediate forebear
Ъ.		vernment financing in an overlapping generations model without fts or bequests
	Ea	ch generation consists of identical households that live for two

t

periods. During the first period of their lives each household works a fixed amount, 1. Income earned in the first period is either consumed or saved. These savings, plus accumulated interest, are the only source of income in the second period of a household's life when it is retired. Households are also identical across generations. Initially there is no government borrowing or lending and no taxation. On the output side, the model has a single commodity that can either be consumed or used as a capital good. Until government bonds are introduced, real capital is the only store of value. The model is "real": there are no money balances. The dual role to be performed by durable output--that of being an input in the production function and of being the only store of value may lead to inefficiencies in a decentralized, competitive economy. (See Diamond, 1965; Buiter, 1980a; and Carmichael, 1979.)

#### A competitive economy without government debt

In the absence of government borrowing and lending, the utility maximization program faced by a representative household of generation

t is given by:  
max u(
$$c_t^1$$
,  $c_t^2$ ) 6/  
 $c_t^1$ ,  $c_t^2$ 

subject to

c.

(III.1) 
$$c_t^1 + \frac{c_t^2}{1 + r_{t+1}} \leq w_t$$
  
 $c_t^1, c_t^2 \geq 0$ 

Equation III.1 states that the present discounted value of lifetime consumption cannot exceed that of labor income. Given our assumptions about the utility function, u, the budget constraint will hold with equality and all solutions for  $c_t^1$  and  $c_t^2$  will be interior. Utility is a function of own lifetime consumption only. There is no gift or bequest motive. The model is completed by adding the economywide constraints, III.2, III.3, and III.4.

(III.2)  $w_t = f(k_t) - k_t f'(k_t)$ (III.3)  $r_t = f'(k_t)$ (III.4)  $w_t - c_t^1 = k_{t+1}(1+n)$ 

Output is produced by a well-behaved neoclassical production function which is linear homogenous in capital and labor. In intensive form it can be written as:  $f(k_t)$  with f(o) = o; f '> o; f ''< o. 7/ Equation III.2 states that the labor market clears and is competitive. The real wage equals the marginal product of labor. Equation III.3 states that the capital rental market clears and is competitive with the rental rate (which in the one-commodity model also equals the interest rate) equal to the marginal product of capital. Equation III.4 is the economywide capital market equilibrium condition. The stock of capital in existence at the beginning of period t,  $K_t$ , is equal to the savings of the previous period. Only the young save in this model without bequests, so saving in t - 1 is given by  $(w_t - 1 - c_t^1 - 1)L_t - 1$ . Our conditions on the production function imply  $k_t > 0$ .

The interior first-order condition for an optimum is:

(III.5) 
$$\frac{u_1(c_t^1, c_t^2)}{u_2(c_t^1, c_t^2)} = (1 + r_{t+1})$$

Its interpretation in terms of a tangency between an indifference curve in  $c_t^1$ ,  $c_t^2$  space and the intertemporal budget constraint is familiar. From the first-order condition and the budget constraint, III.1, we can solve for  $c_t^1$  (and  $c_t^2$ ) as a function of  $w_t$  and  $r_{t+1}$ . Substituting the solution for  $c_t^1$  into the capital market equilibrium condition III.4 and using III.2 and III.3 to substitute for  $w_t$  and  $r_{t+1}$  we obtain a first-order difference equation in  $k_t$  describing the evolution over time of this economy from any arbitrary set of initial conditions.  $\underline{8}/$ 

(III.6) 
$$f(k_t) - k_t f'(k_t) - c^1 \left( f(k_t) - k_t f'(k_t), f'(k_t+1) \right) = k_t + 1^{(1+n)}$$

This system will be locally stable and converge to a steady state equilibrium i.f.f.  $\left| \frac{\partial k_t + 1}{\partial k_t} \right| < 1$ , i.e., when  $\left| \frac{\left( \frac{\partial c}{\partial w}^1 - 1 \right) k_f''}{1 + n + \frac{\partial c}{\partial r}^1 f''} \right| < 1$ , In what follows we shall assume existence, uniqueness and stability and proceed to analyze steady-state equilibria only.

In steady-state equilibrium, the capital-labor ratio and through that all real stock-stock and stock-flow ratios are constant. We solve for it by setting  $k_t = k_{t+1}$  in equations III.1 through III.5. By substituting the marginal productivity conditions III.2 and III.3 into III.1 and III.4 we obtain equations III.7 and III.8, the stationary private budget constraint and aggregate capital market equilibrium condition.

(III.7) 
$$c^{1} + \frac{c^{2}}{1 + f'(k)} = f(k) - kf'(k)$$

(III.8)  $f(k) - kf'(k) - c^{1} = k(1 + n)$ 

From III.7 and III.8 we can solve for the stationary decentralized consumption possibility locus, as in III.9a and III.9b. (III.9a)  $c^2 = \psi(c^1)$ 

(III.9b)  $\psi' = -(1 + f') \left[ 1 + \frac{k(n - f')}{1 + f'} f''(1 + n + kf'')^{-1} \right]$ If the production function is Cobb-Douglas, with  $f(k) = k^{\alpha}$ 

$$o < \alpha < 1, \psi' = \frac{(1 + n) (1 + \alpha^2 k^{(\alpha - 1)})}{(1 - \alpha) \alpha k^{(\alpha - 1)} - (1 + n)}$$

The stationary decentralized consumption possibility locus for the Cobb-Douglas case is graphed in Figure 1. At the origin its slope is  $\frac{\alpha}{1-\alpha}(1+n)$ . k increases monotonically as we move up from 0 towards A. As k approaches infinity (which would be beyond A in the infeasible region) the slope of the consumption possibility locus becomes -1. The locus is strictly concave towards the origin. For large k and more general



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constant returns production functions than the Cobb-Douglas,  $\frac{\partial c^2}{\partial c^1}$  can even become positive again, a case of extreme overaccumulation. With the Cobb-Douglas this is not possible. At the golden rule capital-labor ratio, when f'(k) = n,  $\psi' = \frac{-\partial c^2}{\partial c^1} = -(1 + f') = -(1 + n)$ .

The steady-state equilibrium of a decentralized competitive economy could be achieved anywhere on this locus. Steady-state equilibria like  $E_3$ , corresponding to a capital-labor ratio below the golden rule capital-labor ratio k\*, defined by f'(k\*) = n, are possible as are those like  $E_2$  corresponding to a capital-labor ratio in excess of k\*. The golden rule capitallabor ratio k\* could be achieved by a competitive equilibrium at  $E_1$ , but this is not more likely than any other point on the locus. A competitive stationary equilibrium satisfies two criteria: it lies on the stationary consumption possibility locus and it has a tangency between an indifference curve and a private budget constraint with slope -(1 + r) = -(1 + f'(k)). The private budget constraint will always cut the stationary consumption possibility locus in the manner indicated at  $E_3$  and  $E_2$ . Only at the golden rule ( $E_1$ ) will the private budget constraint be tangent to the locus.

It is instructive to contrast the private decentralized solution with the solution achieved by an omnipotent social planner. The latter is only subject to the aggregate resource constraint:

$$c_{t}^{1}L_{t} + c_{t}^{2} - 1L_{t} - 1 = L_{t}f(k_{t}) - L_{t}(k_{t} + 1(1 + n) - k_{t}) \text{ or}$$
(III.10)  $c_{t}^{1} + \frac{c_{t}^{2} - 1}{1 + n} = f(k_{t}) - k_{t} + 1(1 + n) + k_{t}$ 

The stationary aggregate resource constraint is

(III.11) 
$$c^{1} + \frac{c^{2}}{1+n} = f(k) - nk$$

In order to maximize the stationary per capita amount of resources available for consumption, the social planner selects the golden rule capital-labor ratio k\*. The stationary social consumption possibility locus is the straight line  $\gamma\gamma$  with slope -(1 + n). By distribution through administrative fiat, any point on this  $\gamma\gamma$  locus is available to the social planner. A decentralized competitive equilibrium with a capital-labor ratio below k\*, as at  $E_3$ , is not inefficient. During any transition from  $E_3$  to  $E_1$ , say, capital deepening has to occur, requiring the sacrifice of consumption during the transition in exchange for a permanently higher consumption path after E, has been achieved. A capital-labor ratio in excess of the golden rule is inefficient because it is possible to reduce the capital-labor ratio and thus to have a temporary consumption binge while enjoying a permanently higher path of consumption after k\* has been achieved. This inefficiency is due to capital's dual role as a store of value and a factor of production. In an attempt to shift consumption towards retirement, private agents save by accumulating capital. This depresses the rate of interest. By making available a store of value that has no additional intrinsic use, either as a consumption good or a capital good, government borrowing can alleviate and even eliminate any such inefficiency due to overaccumulation.

#### d. A competitive economy with government debt

Now consider the case in which the government issues real-valued oneperiod bonds. Bonds floated during period t are repaid with interest at a rate  $r_{t+1}$  in period t + 1. Government bonds and real capital are perfect substitutes in private sector portfolios.  $D_t$  can be negative, in which case the public sector lends to the private sector. Such public sector lending to the public sector consists of public purchases of private sector bonds (which are also perfect substitutes for public bonds), not of real capital.  $T_t$  is the total lump-sum tax bill paid by the younger generation in period t. It can be negative in which case it constitutes transfer payments to the young. With government debt and taxes the economy we are considering can be represented as follows:

 $\begin{array}{c} \max \quad u(c_{t}^{1}, c_{t}^{2}) \\ c_{t}^{1}, c_{t}^{2} \\ \text{subject to} \quad c_{t}^{1}, c_{t}^{2} \ge 0. \\ (\text{III.12}) \quad (w_{t} - c_{t}^{1} - \tau_{t}) \quad (1 + r_{t} + 1) = c_{t}^{2} \\ (\text{III.13}) \quad (1 + r_{t}) \quad D_{t} - 1 = D_{t} + T_{t} \\ (\text{III.14}) \quad (w_{t} - c_{t}^{1} - \tau_{t}) L_{t} = D_{t} + K_{t} + 1 \\ w_{t} = f(k_{t}) - k_{t} f'(k_{t}) \\ r_{t} + 1 = f'(k_{t} + 1) \\ k_{t} \ge 0 \end{array}$ 

Equation III.12 is the modified household budget constraint, allowing for taxes while young. III.13 is the government budget constraint. III.14 is the modified capital market equilibrium condition. Total saving has to be equal to the total stock of assets consisting of government bonds and real capital. Private life-cycle optimizing behavior yields a consumption function  $c_t^1 = c^1(w_t - \tau_t, r_{t+1})$ . We again assume o <  $c_1^1(.)$  <1 and  $c_2^1(.) \leq o$ . The complete solution of the model is:

III. 15  

$$c_{t}^{1} = c^{1}(w_{t} - \tau_{t}, r_{t+1})$$
III. 12  

$$(w_{t} - c_{t}^{1} - \tau_{t}) (1 + r_{t+1}) = c_{t}^{2}$$
III. 16  

$$w_{t} - c_{t}^{1} - \tau_{t} = d_{t} + k_{t+1}(1 + n)$$
III. 17  

$$(1 + r_{t}) d_{t-1}^{=d_{t}} (1 + n) + \tau_{t}(1 + n)$$
III. 2  

$$w_{t} = f(k_{t}) - k_{t} f'(k_{t})$$
III. 3  

$$r_{t+1} = f'(k_{t+1})$$

$$c_{t}^{1}, c_{t}^{2}, k_{t} \ge 0$$

At each point in time, t, this system of six equations determines the values of  $c_t^1$ ,  $c_t^2$ ,  $w_t$ ,  $k_{t+1}$  and two of the three government instruments  $\tau_t$ ,  $d_t$  and  $r_{t+1}$ , given the value assigned to the remaining government instrument and the values of the predetermined variables  $r_t$ ,  $d_{t-1}$  and  $k_t$ . I shall, through the rest of this section on debt neutrality, consider the case in which  $d_t$ , the per capita stock of real government debt is kept at a constant value  $d_t = d$ . In that case III. 17 simplifies to:

III. 17' 
$$\tau_t = \frac{(r_t - n)d}{1+n}$$

The model is stable if  $\frac{\partial k_{t+1}}{\partial k_t} < 1$  in equation III.18.

III. 18 
$$f(k_t) - k_t f'(k_t) - c^1 (f(k_t) - k_t f'(k_t) - \frac{(f'(k_t) - n)d}{1 + n}, f'(k_{t+1}))$$
  
-  $\frac{(1 + f'(k_t))d}{1 + n} = k_{t+1}(1 + n).$ 

stability requires

III. 19 
$$\left| \frac{(c_1^1 - 1)f''(k + \frac{d}{1+n})}{1+n+c_2^1 f''} \right| < 1$$

Since  $k + \frac{d}{1+n}$  is non-negative (because  $c^2$  is non-negative), this

stability condition is qualitatively the same as that for the model without government debt, given in III.6).

The steady state equilibrium of the model with public debt is given in equations III. 20 - III. 23.

III. 20 
$$c^{1} = c^{1} (f(k)-kf'(k) - \frac{(f'(k)-n)d}{1+n}, f'(k))$$

III. 21 
$$(f(k)-kf'(k) - \frac{(f'(k)-n)d}{1+n} - c^1)(1+f'(k)) = c^2$$

III. 22 
$$f(k)-kf'(k) - (\frac{1+f'(k)}{1+n})d - c^1 = k(1+n)$$

III. 23 
$$\tau = \frac{(f'(k)-n)}{1+n} d$$

By substituting III. 20 into III. 22 we can derive the steady state effect of an increase in the per capita stock of public debt on the capital-labor ratio:

III. 24 
$$\frac{\partial k}{\partial d} = \frac{(1+c_1^{\perp}n+(1-c_1^{\perp})f')(1+n)^{-\perp}}{(c_1^{\perp}-1)f''(k+\frac{d}{1+n})-[1+n+c_2^{\perp}f'']}$$

If the model is stable (equation III.19) and if  $c_2^1 \leq 0$  and  $0 < c_1^1 < 1$ nator of III. 24 is negative. The denominator is positive. We therefore obtain the familiar result that, comparing steady states, government debt issues reduce the capital-labor ratio, i.e., crowd out real capital. This "crowding out" result also obtains in the short run, as can be checked from equation III. 18. Given  $k_t$ , the effect on  $k_{t+1}$  of an increase in d

III. 25 
$$\frac{\partial k_{t+1}}{\partial d} = \frac{(1+c_1^{1}n+(1-c_1^{1})f')(1+n)^{-1}}{-[1+n+c_2^{1}f'']}$$

is

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This is negative if  $c_2^1 \leq 0$ .

Note that the steady-state value of d can be chosen to be negative or positive. Irrespective of d, if the economy is at the golden rule, no net taxes or transfers are required (III.23). The growth in the total demand for debt required to keep the per capita stock of debt constant just suffices to repay the debt held by the old generation, plus interest at the rate of population growth. Positive d requires positive  $\tau$  at capital labor ratios below the golden rule ratio k\* (at interest rates above n), negative  $\tau$  in the inefficient region when f'(k) < n.

The steady-state effect of government debt issue can be illustrated

using a generalization of the stationary competitive consumption possibility locus of Figure 1. The effect of an increase in d on the stationary competitive consumption possibility locus is to shift it up at a rate -(1+n). From III. 21 and 22 it is easily seen that, at any given k,  $\frac{\partial c^1}{\partial d} = -\frac{(1+f')}{(1+n)}$  while  $\frac{\partial c^2}{\partial d} = 1+f'$ . Thus the rate at which, for any given k,  $c^2$  is traded off for  $c^1$  when d increases is -(1+n). Figure 2 shows the general nature of the shift in the locus while Figure 3 focuses on a particular capital-labor ratio,  $\bar{k}$  below the golden rule ratio k\*. A budget line with a common slope  $-(1+f'(\bar{k}))$  passes through  $E_1$  and  $E_2$  in Figure 3.

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Stationary competitive consumption possibility loci with low d (solid line) and with high d (dashed line).





#### Bequests

With bequests, the utility function, the budget constraint and the capital market equilibrium condition are altered.  $B_t$  is the bequest left in the second period of his life by a member of generation t to the members of generation t+1. The bequests are received by members of generation t+1 at the end of the first period of their lives. The value of the bequest to members of generation t+1 at the beginning of their second period is  $B_t(1+r_{t+2})$ . When the rate of population growth is nonzero bequests shared equally among all descendants. Note that bequests must be non-negative, a useful institutional constraint.

III.26  $B_t \ge 0$  for all t.

The utility function of a member of generation t is  $W_t = v (c_t^1, c_t^2, W_{t+1}^*)$ . The utility of a member of generation t depends on his own life-time consumption,  $c_t^1, c_t^2$  and on the maximum utility level attainable by a member of the next generation. For simplicity I shall consider the additively separable function:

III.27  $W_t = u(c_t^1, c_t^2) + (1+\delta)^{-1} W_{t+1}^*$ 

u has all the properties attributed to the utility function of the household without a bequest motive. This ensures interior solutions for  $c_t^1$ and  $c_t^2$  and strict satisfaction of the household budget constraint.  $\delta$  is the "generational" discount rate; it is not to be confused with the individual's pure rate of time preference. Convergence, i.e., boundedness of  $W_t$ requires  $\delta > 0$ . The optimization problem solved by a representative member of generation t is given in equations III.28 and III.29. The new economywide capital market equilibrium condition is given in equation III.30.

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III. 28 
$$W_t^* = \max_{t} W_t = \max_{t} [u(c_t^1, c_t^2) + (1+\delta)^{-1} W_{t+1}^*]; c_t^1, c_t^2, B_t \ge 0.$$

subject to

III. 29 
$$B_t \leq \frac{B_{t-1}(1+r_{t+1})}{1+n} + (w_t - c_t^1)(1+r_{t+1}) - c_t^2$$

III. 30 
$$w_t - c_t^1 + \frac{b_{t-1}}{1+n} = (1+n)k_{t+1}$$
.

The individual's budget constraint now contains the bequest he receives and the bequest he leaves. The capital market equilibrium condition recognizes that now both the young and the old generation can save. As before we have  $r_t = f'(k_t)$  and  $w_t = f(k_t) - k_t f'(k_t)$ 

The first order conditions for an optimum are:

III. 31a 
$$u_1(c_t^1, c_t^2) = (1+r_{t+1}) u_2(c_t^1, c_t^2)$$
  
III. 31b  $u_2(c_t^1, c_t^2) \ge \frac{(1+r_{t+2})u_2(c_{t+1}^1, c_{t+1}^2)}{(1+n)(1+\delta)}$ 

If  $B_t > 0$ , i.e., if there is an interior solution for bequests, III.31b holds with equality. If there is a corner solution for bequests, i.e., if B = 0 is a binding constraint, III.31b holds with strict inequality. The interpretation of these first order conditions is straightforward. III.31a says that the discounted marginal utility of consumption in the second period of one's life should equal the marginal utility of consumption in the first period of one's life. III.31b states that if bequests are positive, the marginal utility of own consumption should equal the marginal utility of leaving a bequest. A marginal unit of income saved by an old member of generation t yields resources  $(1+r_{t+2})$  times larger to generation t+1. The marginal utility to a member of generation t of bequests can be expressed as the discounted value of the marginal utility of consumption of a member of generation t+1. The appropriate discount rate is the generational discount rate  $\delta$ . Finally, since it is the utility of a representative member of generation t+1 that was assumed to enter into the utility function of generation t, rather than the utility of all 1+n descendants, the population growth factor 1+n also discounts the marginal utility of consumption of generation t+1. If the marginal utility of own consumption exceeds the marginal utility of bequests, there will be a corner solution with B=0.

The steady state equilibrium of the model with bequests is given in equations III.32.

- III.32a  $u_1(c^1,c^2) = (1+r) u_2(c^1,c^2)$
- III.32b  $(1+n)(1+\delta) \ge 1+r$ if B > 0,  $(1+n)(1+\delta) = 1+r$ if B = 0 and the zero bequest constraint is binding, $(1+n)(1+\delta)>1+r$
- III.32c  $(\frac{n-r}{1+n})B = (w-c^{1})(1+r) c^{2}$ III.32d  $(1+n)k = \frac{B}{1+n} + w - c^{1}$ r = f'(k)w = f(k)-kf'(k)

The stationary competitive consumption possibility locus with bequests is drawn in Figure 4.  $OA_2A_1$  is the no bequest locus. At capital-labor ratios so high that  $(1+n)(1+\delta) > 1+f'(k)$ , B=0 and the no-bequest locus is again the relevant one. This critical capital-labor ratio,  $\bar{k}$  is at  $A_2$ . Since  $\delta > 0$ ,  $\bar{k} \leq k^*$ , the golden rule capital-labor ratio. Considering equations III.32c and III.32d, we can draw a consumption possibility locus for each value of B. A higher value of B shifts the locus down

and to the right at a rate -(1+n). Thus all steady-state equilibria with interior (positive) solutions for bequests lie on the line segment A2A3A4. All interior bequest solutions have the same capital-labor ratio, defined by  $f'(\bar{k}) = (1+n)(1+\delta)$  which is below the golden rule capital-labor ratio. One such interior solution for bequests is drawn at A3, where an indifference curve is tangent to a budget constraint with slope  $-(1+f'(\overline{k}))$ on the line segment  $A_2A_3A_4$ . The stationary consumption possibility locus for the appropriate positive value of B is represented by the dashed curve through  $A_3$ . The complete stationary locus with bequests is given by the no-bequest locus above  $A_2$  and the line segment  $A_2A_3A_4$ . If the stationary competitive equilibrium is on  $A_1A_2$ , i.e., if there is a corner solution for bequests, the effect of government lending and borrowing is as in the no-bequest model. If the model is stable, the introduction of government borrowing (d > 0) will reduce the equilibrium capital labor-ratio, while the introduction of government lending (d < 0) will increase it. However, government borrowing can never reduce the capital-labor ratio below k. Once k falls to  $\bar{k}$ , any further increase in government borrowing (which represents an involuntary redistribution of income from the young to the old) will be matched by exactly offsetting bequests, voluntary transfers from the old to the young. This is most easily appreciated if we consider the effect of government lending. Start from an initial equilibrium, without government lending, with positive bequests as at A3. With bequests, bonds and taxes the private budget constraint III.29 and the capital market equilibrium condition III.30 are replaced by III.29' and III.30' respectively.



An interior solution with bequests.

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III.29' 
$$B_t = \frac{B_{t-1}(1+r_{t+1})}{1+n} + (w_t - c_t^1 - \tau_t)(1+r_{t+1}) - c_t^2$$
  
III.30'  $w_t - c_t^1 - \tau_t + \frac{B_{t-1}}{1+n} = d + k_{t+1}(1+n)$ 

We also have the budget constraint.

III.17' 
$$(r_{-n})d = \tau_{+}(1+n)$$

The stationary constraints are:

III.33a 
$$\frac{(n-r)}{1+n}$$
 (B-d(1+r)) = (w-c<sup>1</sup>)(1+r) - c<sup>2</sup>

III.33b 
$$\frac{1}{1+n}$$
 (B-d(1+r)) + w-c<sup>1</sup> = k(1+n)

If, with d=0, a stationary solution obtains with  $B=B^{O} > 0$ , a negative value of d=d<sup>O</sup> will still permit the same consumption-capital stock equilibrium to obtain as long as  $B^{O} \ge |d^{O}(1+r)|$ , i.e., as long as bequests can be reduced by an amount equal, in present value, to the amount of government lending. Then the involuntary government redistribution from the old to the young will be neutralized as regards its effect on the lifetime resources of the two generations alive at any one moment, by the reduction in voluntary private redistribution from the old to the young. Given any initial value of bequests, however, there always exists a volume of government lending large enough to put private agents in a zero-bequest corner. Thus, with bequests, the government can always raise the capital-labor ratio above  $\bar{k}$ . It can never bring it down below  $\bar{k}$ .

# Gifts from the Young to the Old

With gifts from the young to the old, the utility function is  $W_t = v(c_t^1, c_t^2, W_{t-1}^*)$ .  $W_{t-1}^*$  is the maximum level of utility attained by a member of generation t-1. We again adopt the additively separable form:

$$W_t = u(c_t^1, c_t^2) + (1+\rho)^{-1} W_{t-1}^*$$

We note that unlike standard time discounting, the utility of a member of the earlier generation is not compounded, but discounted. Convergence requires that the discount rate applied to parents' utility be positive,  $\rho > 0$ . Gifts of course cannot be negative.  $G_{t} \ge 0$ . The behavior of the competitive economy with gifts is summarized below

III.34 
$$W_t^* = \max_{\substack{c_t^1, c_t^2, G_t}} [u(c_t^1, c_t^2) + (1 + \rho)^{-1} W_{t-1}^*]; c_t^1, c_t^2, G_t \ge 0.$$

subject to

III.35 
$$G_{t+1}(1+n) + w_t(1+r_{t+1}) \ge (c_t^1+G_t)(1+r_{t+1}) + c_t^2$$

with

III.36 
$$w_t - c_t^1 - G_t = k_{t+1}(1+n)$$
  
and, as before  $r_t = f'(k_t)$   
 $w_t = f(k_t) - k_t f'(k_t)$ 

The private budget constraint allows for gifts handed out and received. The capital market equilibrium condition reflects the fact that resources given by the young to the old, who do not save, are no longer available for capital formation.

Ø

The first order conditions of the private optimization problem are:

III.37a 
$$u_1(c_t^1, c_t^2) = (1+r_{t+1}) u_2(c_t^1, c_t^2)$$
  
III.37b  $\frac{u_2(c_{t-1}^1, c_{t-1}^2)}{(1+\rho)(1+r_{t+1})}(1+n) \le u_2(c_t^1, c_t^2).$ 

If  $G_t > 0$ , i.e., if there is an interior solution for gifts, III.37b holds with equality. If there is a corner solution for gifts, i.e., if G=0 is a binding constraint, III37b holds with strict inequality. Equation III.37a is the condition for the optimal allocation of consumption for a member of generation t between the two periods of his life.

III.37b states that if gifts are given from generation t to generation t-1, the marginal utility of own consumption should equal the marginal utility of gifts. The marginal utility of gifts is then expressed in terms of the marginal utility of own consumption of a member of generation t-1. This marginal utility of own consumption of generation t-1 is discounted at the generational discount factor  $(1+\rho)$ . Second-period consumption of members of generation t-1 takes place one period before second-period consumption of members of generation t, so interest is foregone and further discounting by  $(1+r_{t+1})$  is required. Finally, there are more members of generation t than of generation t-1. A member of generation t-1 therefore receives  $G_t(1+n)$  for  $G_t$  given up by a member of generation t. If the marginal utility of own consumption exceeds the marginal utility of gifts,  $G_t=0$ .

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The stationary solution with gifts is given by:

III.38a 
$$u_1(c^1,c^2) = u_2(c^1,c^2)(1+r)$$
  
III.38b  $\frac{(1+n)}{1+\rho} \le 1+r$ 

G > 0 implies that III.38b holds with equality. A corner solution with G=0 binding implies that III.38b holds with strict inequality. Stationary equilibrium is given by:

III.38c 
$$G(r-n) = (w-c^{1})(1+r) - c^{2}$$
  
III.38d  $k(1+n) + G = w - c^{1}$   
 $r = f'(k)$   
 $w = f(k)-kf'(k)$ 

The interesting equation is III.38b. Since  $\rho$  is positive, G > 0 implies r < n. An interior solution for gifts implies that the economy is dynamically inefficient, at a capital-labor ratio  $\overline{k}$  above the golden rule capital-labor ratio  $k^*$ . In models with inifite-lived households with a constant pure rate of time preference  $\Omega$ , such an inefficiency can never arise. Steady state equilibrium is characterized by  $(1+n)(1+\Omega) = 1+r$ . With  $\Omega > 0$  this implies r > n. Earlier consumption is cet. par. valued more than later consumption. This is not true when we have a child-parent gift motive. Own earlier consumption may well be valued more than own later consumption. The pure rate of time preference for own consumption,  $\Omega(c) = \frac{u_1(c,c)}{u_2(c,c)}$  however, is cet. par. valued less than later consumption by parents, however, is cet. par. valued less than later consumption by oneself. Parental utility is discounted, even though it "accrues" earlier. Thus child-parent gifts do not make a private decentralized economy with

finite-lived agents equivalent to an economy with infinite-lived agents. It also does not rule out the possibility of dynamic inefficiency through overaccumulation. Quite the contrary, if gifts are positive in the steady state, the steady state is necessarily inefficient. An operative gift motive is indeed a reflection of a very strong desire to shift resources away from early consumption towards later consumption.

The effect of gifts on the steady state consumption possibility locus is indicated in Figure 5.  $0A_2A_1$  is the locus without gifts. For capitallabor ratios below  $\overline{k}$ , defined by  $f'(\overline{k}) = \frac{1+n}{1+\rho}$ , the locus with gifts is

identical with the locus without gifts because the equilibrium solution for G is zero. The stationary capital-labor ratio can never be above  $\overline{k}$ when there is a gift motive. All solutions with G > 0 lie on the line segment  $A_4 A_3 A_2$  with slope -(1+n). Starting at  $A_2$  where G=0 and  $k=\overline{k}$ , an increase in G shifts the stationary consumption possibility locus up and to the left at a rate -(1+n). A typical interior solution for G is drawn at  $A_3$ . An indifference curve is tangent to a budget constraint with slope -(1+f'( $\overline{k}$ )) on the line segment  $A_4 A_3 A_2$ . The stationary consumption possibility locus for the appropriate positive value of G is represented by the dashed curve through  $A_3$ . The entire stationary consumption possibility locus with gifts is given by the segment of the no-gift locus 0  $A_2$  and the straight line  $A_4 A_3 A_2$ .

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The effect of government borrowing and lending in the presence of a gift motive is easily analyzed. As long as the economy stays in the range of capital-labor ratios below  $\overline{k}$ , government lending and borrowing will have the same effect as in the model without gifts and bequests. If  $k=\overline{k}$  initially (with d=0), government lending (d < 0) will not have any effect on the steady-state consumption path and capital-labor ratio. Involuntary government redistribution from the old to the young will be neutralized immediately by matching voluntary gifts from the young to the old. Government borrowing, d > 0, will also be neutralized by matching reductions in gifts from the young to the old, up to the point that the constraint  $G \ge 0$  becomes binding. The private budget constraint, capital market equilibrium condition and government budget constraint with gifts, borrowing and taxes are

 $G_{t+1}$  (1+n) + ( $w_t - \tau_t$ ) (1+r<sub>t+1</sub>) = ( $c_t^1 + G_t$ ) (1+r<sub>t+1</sub>) +  $c_t^2$ and

$$w_t^{-\tau} - c_t^{\perp} - G_t = d + k_{t+1}(1+n)$$
  
 $(r_t^{-n}) d = \tau_t(1+n)$ 

The stationary equations are:

III.39a (n-r)(G +  $\frac{d(1+r)}{1+n}$ ) + (w-c<sup>1</sup>)(1+r) - c<sup>2</sup> = 0

III.39b 
$$G + \frac{d(1+r)}{1+n} + k(1+n) = w - c^{1}$$

Equations III.39a and III.39b show that G and d(1+r) are "perfect substi-1+n

tutes" as long as the constraint  $G \ge 0$  is not violated. Thus as long as, with d=0, the initial G, (G<sup>0</sup>, say) is larger than  $d^{0}(1+r)$ , where  $d^{0}$  is 1+n

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An interior solution for child-parent gifts.

the size of the real per capita government bond issue, the private sector can and will undo the effects of the government action on consumption and the capital-labor ratio by reducing voluntary gifts from the young to the old. For any initial G, however, there always exists a government borrowing program large enough to make  $G \ge 0$  a binding constraint. Such actions will move the economy from  $A_4 A_3 A_2$  onto  $A_2 0$ , lowering the capital-labor ratio. In view of the inefficiency of the private, decentralized competitive solution with positive gifts, such borrowing will always constitute a Pareto improvement as long as it does not lower k below k\*.

#### Gifts and bequests

I now consider the case of "two-sided caring". Each generation cares about the welfare of its immediate ancestors and its immediate descendants. The utility function is:

$$W_{t} = v(c_{t}^{1}, c_{t}^{2}, W_{t-1}^{*}, W_{t+1}^{*}).$$

The special case of the additively separable function is again considered:

$$W_{t} = u(c_{t}^{1}, c_{t}^{2}) + (1+\delta)^{-1} W_{t+1}^{*} + (1+\rho)^{-1} W_{t-1}^{*}.$$

Convergence now requires not only  $\delta > 0$  and  $\rho > 0$  but  $\delta_\rho > 1.$ 

It might be thought that the solution to the problem with both gifts and bequests is in some way a simple combination of the solutions to the cases with just gifts and just bequests. This is not so. With "one-sided caring" (either gift or bequest motives but not both) the private agent's optimization problem is a standard problem in dynamic programming. With a bequest motive, each agent in generation t cares potentially for all his descendants. <u>Directly</u> as regards his immediate heir, <u>indirectly</u> through the dependence of the utility of his immediate descendant on the utility of generation t+2, etc. In the same way, with a gift motive, each agent potentially cares for all his ancestors. In either case utility chains stretch out in one direction only. With both gift and bequest motives, this unidirectional simplicity no longer applies. An agent in generation t cares directly about generations t-1 and t+1. These generations both care directly about generation t. Generation t-1 also cares directly about t-2 and generation t+1 about t+2. Immediately, utility chains can be seen to be running in both directions. These issues were discussed for the first time in Carmichael (1979). A particularly simple solution emerges when the following assumptions are made about the "game" played by a member of generation t with past and future generations:

1. A member of generation t acts competitively in his labor and capital markets, i.e. he takes  $w_t$  and  $r_{t+1}$  as parametric. He also assumes that all past and future generations have acted or will act competitively in their factor markets.

2. A member of generation t, in formulating his consumption giftbequest plan, knows the utility levels and actions of all past generations and correctly anticipates utility levels and actions of all future generations (rational expectations or perfect foresight).

3. A member of generation t plays a <u>non-cooperative</u> gift and bequest game with past and future generations. He rationally believes that all past and future generations play the same game. This strategy is closed-loop as

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regards the utility and actions of the two generations with which he overlaps (generations t-1 and t+1). This means that when evaluating the alternative actions open to him at the beginning of period t, he believes that he can affect the total utility and the actions of his immediate descendants and his immediate forebears.

His strategy is <u>open-loop</u> as regards the utility and actions of all other generations (t-i, i>2 and t+j, j>2). Thus, when evaluating the effects of marginal changes in his actions, he ignores the impact on the actions and utility of generations that are already dead when he is born or that are born after his lifetime. $\frac{9}{}$ 

4. When formulating his closed-loop strategy vis-a-vis generations t-1 and t+1, he believes that he can alter the behavior of these generations (i.e. their consumption, gift and bequest choices) only by altering the total resources available to them, i.e., only through direct transfers.

5. Each generations acts so as to maximize its utility.

6. Each generation views the world and plays the game in the same way as the member of generation t just described.

The resulting equilibrium in this differential game is a Nash equilibrium.

The behavior of this economy can be summarized as follows:

III.40  $W_{t}^{*} = \max_{\substack{c_{t}^{1}, c_{t}^{2}, B_{t}, G_{t}}} W_{t} = \max_{\substack{c_{t}^{1}, c_{t}^{2}, B_{t}, G_{t}}} [u(c_{t}^{1}, c_{t}^{2}) + (1+\delta)^{-1} W_{t+1}^{*} + (1+\rho)^{-1} W_{t-1}^{*}]$ 

subject to:

III.41 
$$B_t - G_{t+1} (1+n) \le (\frac{B_{t-1}}{1+n} - G_t) (1+r_{t+1}) + (w_t - c_t^1) (1+r_{t+1}) - c_t^2$$
  
 $B_t, G_t, c_t^1, c_t^2 \ge 0$ 

with economy-wide constraints:

III.42 
$$w_t - c_t^1 + \frac{b_{t-1}}{1+n} - G_t = k_{t+1}(1+n)$$
  
 $r_{t+1} = f'(k_{t+1})$ 

and

$$w_t = f(k_t) - k_t f'(k_t)$$

The first order conditions are:  $\begin{array}{l} \frac{\partial W_{t}^{*}}{t} \\ 111.43a \end{array} \quad \frac{\partial W_{t}^{*}}{\partial c_{t}} = (1+r_{t+1}) \quad \frac{\partial W_{t}^{*}}{\partial c_{t}} \end{array}$ 

III.43b 
$$\frac{\partial W_t^*}{\partial B_t} \leq \frac{\partial W_t^*}{\partial c_t^2}$$

If  $B_t > 0$ , III.43b holds with strict equality. If  $B_t=0$  is a binding constraint, III.43b holds with strict inequality.

III.43c 
$$\frac{\partial W_{t}^{*}}{\partial G_{t}} \leq \frac{\partial W_{t}^{*}}{\partial c_{t}^{1}}$$

If  $G_t > 0$ , III.43c holds with strict equality. If  $G_t=0$  is a binding constraint, III.43c holds with strict inequality. Using assumptions (1) through (6), the first order conditions and the private budget constraints of current, past and future generations, we can express generation t's marginal utility from bequests in terms of the marginal utility of own consumption of generation t+1.

III.44 
$$\frac{\partial W_{t}^{*}}{\partial B_{t}} = \frac{(1+\rho)(1+r_{t+2})}{[(1+\rho)(1+\delta)-1](1+n)} \frac{\partial W_{t+1}^{*}}{\partial c_{t+1}^{2}}$$

Combining III.44 and III.43b we get

III.45 
$$\frac{(1+\rho)(1+r_{t+2})}{[(1+\rho)(1+\delta)-1](1+n)} \frac{\partial W_{t+1}^{*}}{\partial c_{t+1}^{2}} \leq \frac{\partial W_{t}^{*}}{\partial c_{t}^{2}}$$

If  $B_t > 0$ , then III.45 holds with strict equality. If  $B_t=0$  is a binding constraint, III.45 holds with strict inequality.

In an exactly analogous manner we can express generation t's marginal utility from gifts in terms of the marginal utility of own consumption of generation t-1.

III.46 
$$\frac{\partial W_{t}^{*}}{\partial G_{t}} = \frac{(1+n)(1+\delta)}{(1+\rho)(1+\delta)-1} \frac{\partial W_{t-1}^{*}}{\partial c_{t-1}^{2}}$$

Combining III.46 and III.43c and using III.43a, we obtain:

III.47 
$$\frac{(1+n)(1+\delta)}{[(1+\rho)(1+\delta)-1]} \frac{\partial W_{t-1}^{*}}{\partial c_{t-1}^{2}} \leq \frac{\partial W_{t}^{*}}{\partial c_{t}^{2}} (1+r_{t+1}).$$

If  $G_t > 0$ , then III.47 holds with strict equality. If  $G_t = 0$  is a binding constraint, III.47 holds with strict inequality.

The steady state conditions are:

III.48a 
$$u_1(c^1, c^2) = (1+r) u_2(c^1, c^2) \frac{10}{4}$$
  
III.48b  $1+r \leq [\frac{(1+\rho)(1+\delta)-1}{1+\rho}] (1+n) = \frac{1}{\rho} \beta_t 70.$   
( $\beta = 0 \longrightarrow R^{\frac{1}{2}}$ )

If B > 0 then III.48b holds with equality. If B = 0 is a binding constraint then III.48b holds with strict inequality.

III.48c 
$$1+r \ge \frac{(1+\delta)(1+n)}{(1+\rho)(1+\delta)-1}$$

If G > 0 then III.48c holds with equality. If G = 0 is a binding constraint then III.48c holds with strict inequality.

III.48d 
$$(\frac{n-r}{1+n}) B + (r-n) G = (w-c^{1})(1+r) - c^{2}$$
  
III.48e  $k(1+n) = \frac{B}{1+n} - G + w - c^{1}$ .

and

r = f'(k)w = f(k)-kf'(k)

With n,  $\rho$  and  $\delta$  strictly positive, III.48b and III.48c cannot both hold with equality. This is the commonsense result that there will not be both gifts and bequests in the steady state. Thus, if B > 0, then G = 0, and if G > 0 then B = 0. However, it is possible for III.48b and III.48c to both hold with strict inequality, i.e., for both gifts and bequests to be zero.

Note that if III.48b holds with equality, i.e., if there is an interior solution for bequests, we have r > n: the capital-labor ratio is below the golden rule capital-labor ratio. 11/ Also, if III.48c holds with equality, i.e., if there is an interior solution with gifts, we have r < n: the capital-labor ratio is above its golden level. Figure 6 illustrates the stationary consumption possibility locus when there is both a bequest and a gift motive. For capital labor ratios below that defined by  $(1+f'(k)) = (1+\delta)(1+n)$ but above that defined by (1+f'(k)) = $(1+\rho)(1+\delta)-1$  $[(1+\rho)(1+\delta)-1](1+n)$ , the stationary consumption possibility locus is the 1+0same as it is without gifts and bequests. On the curve segment A2 A5, there are corner solutions for gifts and bequests: G = B = 0. When there is an interior solution for gifts the equilibrium is on the line segment  $A_1 A_2$  with slope -(1+n).  $A_2$  is defined by-(1+f'(k)) = (1+ $\delta$ )(1+n) As  $1-(1+\rho)(1+\delta)$ 

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in the gifts only case, larger positive values of G shift the locus up and to the left at a rate -(1+n). Note that the degree of overaccumulation relative to the golden rule is less when G > 0, if there is both a gift and a bequest motive than if there only is a gift motive. $\frac{12}{12}$  The tendency to "oversave", represented by the gift motive is partly, but not completely, neutralized by the presence of a bequest motive. A3 A4 would be the locus of interior soutions for G if there were only a gift motive. If the bequest motive is operative, i.e., if B > 0, all stationary solutions lie on the line segment  $A_5 A_6$  with slope -(1+n).  $A_5$  is defined by (1+f'(k)) =  $[(1+\rho)(1+\delta)-1]$  (1+n). As in the case of bequests only, larger positive 1 + 0values of B shift the locus down and to the right at a rate -(1+n). A7 A8 would be the locus of interior solutions for B if there were just a bequest motive. The degree of underaccumulation, relative to the golden rule, is less if there is both a bequest and a gift motive than if there is only a gift motive.  $\frac{13}{}$ 

The effects of government lending and borrowing on the steady-state capital-labor ratio are a straightforward combination of the effects of such policies when there was either a gift or a bequest motive but not both. Consider an initial equilibrium without government debt: d = 0. If the initial equilibrium is in the range of k for which there is a corner solution for both B and G, i.e., on  $A_2$   $A_5$ , government borrowing (d > 0) cannot lower k below the value defined by  $1+f'(k) = [(1+\rho)(1+\delta)-1] (1+n)$ nor can government lending (d < 0) raise k above the value defined by  $1+f'(k) = (1+\delta)(1+n) = (1+\delta)(1+\delta)-1$ . If there is an interior solution for gifts, on

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Stationary consumption possibility loci, without gifts and bequests, with either gifts or bequests and with both gifts and bequests.

 $A_1 A_2$ , government borrowing will be offset by a reduction in gifts of equal present value, thus leaving  $c^1, c^2$  and k unchanged, unless the increase in d is larger, in present value, than the original value of G. In other words, there is always a positive value of d large enough to make  $G \ge 0$  a binding constraint. If there is an interior solution for bequests, on  $A_5 A_6$ , government lending (d<0) will be offset by a reduction in bequests of equal present value which leaves  $c^1$ ,  $c^2$  and k unchanged. Again, if the value of lending is larger, in present value, than the original bequest, the constraint  $B \ge 0$  will become binding.

With gifts, bequests, borrowing and taxes the private and public sector budget constraints and the capital market equilibrium conditions are:

III.49a 
$$B_t - \frac{B_{t-1}(1+r_{t+1})}{1+n} - G_{t+1}(1+n) + G_t(1+r_{t+1}) - (w_t - c_t^1 - \tau_t)(1+r_{t+1}) + c_t^2 = 0$$
  
III.49b  $w_t - c_t^1 - \tau_t + \frac{B_{t-1}}{1+n} - G_t = d + k_{t+1}(1+n).$ 

and  $(r_t - n)d = \tau_t(1+n)$ .

The stationary equations are:

III.50a 
$$(n-r)\left[\frac{B}{1+n} - G - \frac{d(1+r)}{1+n}\right] = (w-c^{1})(1+r) -c^{2}$$
  
III.50b  $\frac{B}{1+n} - G - \frac{d(1+r)}{1+n} = k(1+n) - w + c^{1}$ 

We know that if B>0, then G = 0 and if G > 0 then B = 0. Thus if B > 0, a reduction in d by an amount  $\Delta d$  will be neutralized by a reduction in B by the amount  $\Delta d(1+r)$ , as long as this does not violate the constraint  $B \ge 0$ . If G > 0, an increase in d by an amount  $\Delta d$  will be neutralized by a reduction in G by the amount  $\Delta d(1+r)$ , as long as this does not violate the constraint  $G \ge 0$ .

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#### Conclusions

The policy conclusions of this theoretical investigation of debt neutrality are straightforward. While operative intergenerational gift and bequest motives turn finite-lived households into infinite-lived households in a certain sense, there remain essential differences. In particular, if the child-parent gift motive is operative, the decentralized, competitive equilibrium is socially inefficient because it is characterized by a capital-labor ratio above the golden rule. There is therefore a prima facie case for government intervention in the saving-investment process. The second conclusion concerns debt neutrality. Here every neutrality theorem is matched by a non-neutrality theorem. If neither bequests nor gifts are operative, goverment borrowing crowds out capital formation. If child-parent gifts are operative, small increases in government borrowing are neutralized by reductions in gifts. If bequests are operative, small reductions in government borrowing (or increases in lending) are neutralized by reductions in bequests. There always exists an increase in lending or borrowing that will make B=0, respectively G=0 a binding constraint. There always is a government financial strategy that puts the private sector in a zero gift and zero bequest corner solution, where financial policy will affect the capital-labor ratio. In our simple model, all agents are identical, so either everyone is at a corner or no one is. If instead we visualize a distribution of agents, by  $\delta$ , by  $\rho$ , and by the other parameters of their utility functions and the constraints they face, increasing government borrowing can be expected to make the zero gift constraint binding for an increasing number of agents.

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Even in this most classical of models, the conclusion emerges inexorably that the way in which the government finances its real spending program will have major consequences for saving and capital formation. Debt neutrality is not a plausible theoretical proposition. Future research should concentrate on empirical assessments of the extent and nature of non-neutrality.

# b. Monetary Superneutrality in a Fully Optimizing Macromodel

The overlappinggenerations model is also a convenient vehicle for analyzing the issue of superneutrality of money in an explicitly optimizing and fully rational model. Superneutrality is the invariance of the trajectories of the real variables of the economy under different proportional rates of growth of the nominal money stock. Money is defined as a noninterest-bearing (nominally denominated) liability of the government that has no "intrinsic value" in the sense that it is neither used as a consumption good, nor as an input in the productive process. The two functions performed by this financial claim are the medium of exchange function and the store of value function. Carmichael (1979) develops a model in which both functions are incorporated. For simplicity, I shall limit the analysis to a consideration of the store of value function of money. The antisuperneutrality result obtained in the simpler model carries over to the more complex model considered by Carmichael. Consider the introduction of money in a simple overlapping generations model without gifts, bequests or interest-bearing government debt, in which the only other store of value is real reproducible capital. Since money balances are not desired for their own sake but only for their purchasing power over real output, the real rate of return on money balances is the negative of the expected proportional rate of change of the price level  $-\Delta \hat{p}/_{p}$ . In the certainty

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model under consideration, the actual and expected rates of inflation are the same, if expectations are rational.  $\Delta \hat{p} = \Delta p$ . Also, money and p p p real capital are perfect substitutes as stores of value. They will only both be held if their rates of return are equal. If the rate of return on money balances were below that on real capital,  $-\Delta p < f'(k)$ , no money would be held, if the rate of return on money balances were above that on real capital,  $-\Delta p > f'(k)$ , no capital would be held. Considering only those trajectories on which both assets are held, we have:  $\underline{14}/$ 

III.51 
$$-\Delta p = f'(k)$$

Let  $\mu$  denote the proportional rate of growth of the nominal money supply:  $\mu = \Delta M/M$ . M is the nominal stock of money. In steady-state equilibrium, the rate of change of the price level is the rate of growth of the money supply minus the natural rate of growth of the economy:

III.52  $1+\Delta p = 1+\mu$  or, approximately,  $\Delta p \approx \mu$ -n. Equations III.51 p 1+n p pand III.52 are sufficient to refute the superneutrality proposition. Since  $n-\mu = f'(k)$ , an increase in the rate of growth of the money supply will lower the steady-state rate of return on money balances by raising the steady-state rate of inflation. Portfolio balance requires an equal reduction in the rate of return on capital. This is accomplished by a higher capital-labor ratio. Models such as Sidraucki's (1967) which exhibit invariance of steady-state k under different values for  $\mu$  are only superficially rational, optimizing models. By including real money balances as an argument in the <u>direct</u> utility function, on a par with consumption and leisure, the Sidrauski model violates the principle that money is only wanted for what it can buy. It is therefore not an interesting framework for analysing monetary policy. The overlapping generations model with money is summarized in III.53. Money is introduced into the economy via lump-sum transfers. There is no relation between the stock of money held by an economic agent and the size of the transfer. If there were, the monetary injections would be akin to payment of interest on money balances. This would result in superneutrality.  $m_{t} = \frac{M_{t}}{p_{t}L_{t}}$ 

III.53a max  $u(c_t^1, c_t^2)$ 

subject to

- III.53b  $c_{t}^{1} + \frac{c_{t}^{2}}{1+r_{t+1}} w_{t}^{2} \tau_{t}^{\frac{15}{2}}$
- III.53c  $r_{t+1} = -\Delta p_t / p_t$
- III.53d  $w_t c_t^1 \tau_t = k_{t+1}(1+n) + m_t(1+\mu)$
- III.53e  $\mu m_t = -\tau_t$
- III.53f  $m_{t+1} = m_t (1+\mu) (1+n)^{-1} (1 + \frac{\Delta p}{p})^{-1}$

$$r_{t+1} = f'(k_{t+1})$$
  
w<sub>t</sub> = f(k<sub>t</sub>) - k<sub>t</sub>f'(k<sub>t</sub>)

The stationary equilibrium of this economy is characterized by:

III.54a  $u_1(c^1, c^2) = (1+r) u_2(c^1, c^2)$ III.54b  $w + \mu m = c^1 + \frac{c^2}{1+r}$ III.54c  $r = -\frac{\Delta p}{p}$ III.54d  $w-c^1 = k(1+n) + m$ III.54e  $1 + \mu = (1+n)(1 + \frac{\Delta p}{p})$  [since by assumption m > 0] III.54f r = f'(k)III.54g w = f(k) - kf'(k) Equations III.54c, e, and f produce the anti-superneutrality result that  $(1+\mu) = (1+n)(1-f'(k))$ .

The conclusion that "equilibrium" and "rationality" do not imply "neutrality" again emerges forcefully, as it did when debt neutrality was considered. The detailed consideration of the fully rational, optimizing models of this section also leads to the conclusion that the ad hoc model of Section II is in many ways an acceptable parable for the "purer" models of Section III.

#### IV. Stabilization Policy in Stochastic Non-Walrasian Models

A Walrasian economy has two essential features. The first is the existence of a complete set of contingent forward markets. This permits the interpretation of such an economy as a "one shot," static economy. At the beginning of time, t = t, equilibrium prices are established for the current and contingent future delivery of all goods and services. The rest of history then consists solely of the unfolding execution of the contingent forward contracts established at the initial market date. The second essential feature is that prices are competitive, market-clearing prices. This means that at the prevailing set of market prices demand does not exceed supply for any good, when these demands and supplies represent notional plans. Households' notional demands and supplies are derived from utility maximization subject only to the constraint of the households' endowments evaluated at parametric market prices. Firms' notional demands and supplies are derived from profit maximization subject only to the constraint of the production possibility set, with all planned sales and purchases evaluated at parametric market prices. All economic agents act as if, at the prevailing market prices, they can buy or sell any amount of any good or service. The demands and supplies derived under these conditions are consistent.

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A non-Walrasian economy, by contrast, is a sequence economy--one in which transactions take place at different dates (see Hahn[1973]). Prices for all contingent future goods and services are not established once-and-for-all, in some initial market period. Markets are incomplete and "reopen," either continuously or at discrete intervals. The overlapping generations model of Section III is therefore, according to this definition, a non-Walrasian economy. Forward contracts between those living today and the unborn are not feasible. Markets are therefore incomplete. The factor markets that do exist reopen each period. In models with uncertainty and imperfect, asymmetric information other "natural" reasons can be found for the nonexistence of certain markets, such as moral hazard and adverse selection. The overlapping generations model maintains the second of the two essential Walrasian features. The incomplete set of markets is always in momentary competitive general equilibrium. No potential buyer or seller is rationed in any market or needs to consider any information other than the known set of parametric market prices and his endowment or technology set.

In this section, I shall briefly review some of the issues surrounding stabilization policy in sequence economies when information is incomplete and when market prices are not automatically and instantaneously established at the values that balance notional demands and supplies. For simplicity I shall limit the analysis to the consideration of monetary policy. All sources of monetary non-neutrality considered in Sections II and III are suppressed. This includes effects on capital formation via the real rate of return on money and wealth effects due to the presence of nominally denominated interest-bearing public debt.

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Sticky nominal prices and real effects of deterministic monetary feedback rules

It is well-known that if a market is inefficient, in the sense that the market price does not fully reflect all available information, pure nominal shocks and disturbances, including anticipated changes in the money stock, will have real effects. The simple textbook IS-LM model with its exogenously fixed money wage and money price level is the best-known example. With the general price level given exogenously, an increase in the nominal stock of money represents an equal proportional increase in the real stock of money balances. By lowering the interest rate and/or via the Pigou effect this will alter real effective demand. It may seem plausible to locate such inefficiences primarily in the goods and factor markets, while treating the financial markets as efficient, with financial asset prices or rates of return always adjusting instantaneously to new information about current or anticipated future events, so as to maintain equilibrium between demand and supply. Interestingly, someof the recent work by Stiglitz on non-market-clearing equilibria and the micro-foundations of price and wage-stickiness dealt with financial markets (Stiglitz[1979]). Imperfect, costly and asymmetric information characterize personal and corporate credit markets and insurance markets as much as the labor market, the housing market or the market in second-hand cars. Work on this subject by Arkerlof [1970], Stiglitz [1979], Grossman [1976], Salop [1978], Wilson [1979] and others shows how privately rational, optimizing behavior can result in socially inefficient, quantity-constrained equilibria in which market prices are "sticky" in the sense that they do not always respond to the existence of excess demand or supply. The simple linear stochastic model described below imposes price rigidity, without explicitly deriving it, as Stiglitz has done, as the [momentary] ecuilibrium outcome of a process generated by optimizing

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economic agents. It is therefore ad hoc. The assumption of instantaneous and continuous competitive equilibrium, however, is equally ad hoc unless the process is specified through which this miracle of coordination is achieved. It is not enough to assert that all feasible trades that are to the perceived mutual advantage of the exchanging parties must be exhausted. What needs to be explained is how the price vector always assumes a value that renders feasible, in the aggregate, actions that appear feasible at the micro-level when each agent acts competitively. The micro-foundations of competitive market clearing are not yet well developed. Essential ingredients are large numbers of potential buyers and sellers, specialized middlemen or brokers, easy identification of the relevant economic characteristics of the commodity in question and other fundamental components of market structure such as the laws, rules and regulations governing the exchange of property rights over the commodity.

The ad hoc linear stochastic disequilibrium model is described in equations IV.1 - IV.6.

IV.1	$p_t^* = \alpha(Y_t - \bar{Y}_t) + \hat{p}_{t-1,t}$	α>0
IV.2	$p_{t}^{-p}_{t-1} = \beta(p_{t}^{*} - p_{t-1})$	0≤β≤1
IV.3	$A_{t} = \gamma(m_{t} - p_{t}) + \varepsilon_{t}^{d}$	γ>0
10.4	$Y_t = A_t$	
10.5	$\bar{Y}_{t} = \bar{Y} + \varepsilon_{t}^{s}$	
IV.6	$x_t = p_t + \varepsilon_t^0$	

 $p_t^*$  is the equilibrium price in period t,  $p_t$  the actual price in period t,  $\hat{p}_{t-1,t}$  the price anticipated, in t-1, to prevail in period t;  $A_t$  is effective

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demand,  $m_t$  the nominal stock of money,  $Y_t$  real output and  $\overline{Y}_t$  capacity output.  $\varepsilon_t^d$  and  $\varepsilon_t^s$  are white noise disturbances. All variables are in logs. Equation IV.1 is a version of the "Lucas supply function." Deviations of output from its natural level are possible if and only if the equilibrium price level is different from the price level expected, last period, to prevail this period. Equation IV.2 models sluggish adjustment of the actual price level to the equilibrium price level. Only if  $\beta=1$ , i.e., only if adjustment is instantaneous, do we have the strict "surprise supply function" according to which the only source of departures of actual output from its natural level is a price forecast error.  $\frac{16}{}$  Equation IV.3 models effective demand as an increasing function of the stock of real money balances and a white noise demand disturbance. Equation IV.5 models capacity output as a constant plus a random supply disturbance. Equation IV.6 asserts that the current price level is not observable. Instead, private agents observe  $x_t$ , which is the true price level plus a white noise disturbance,  $\varepsilon_r^0$ .

If expectations are rational, i.e., if they are minimum mean squared error forecasts, if economic agents know the true structure of the model (the values of  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\overline{Y}$ ), if they know that the disturbances are mutually and serially independent and identically distributed random variables with zero means and if they observe  $x_t$  at time t, the rational expectation of the price level is easily found to be:

IV.6 
$$E(p_t | \phi_{t-1}) = \frac{\alpha \beta \gamma}{1 - \beta + \alpha \beta \gamma} E(m_t | \phi_{t-1}) - \frac{\alpha \beta}{1 - \beta + \alpha \beta \gamma} \overline{Y} + \frac{(1 - \beta)}{1 - \beta + \alpha \beta \gamma} x_{t-1}$$

$$E (p_t | \phi_{t-1}) = \frac{\alpha \beta \gamma}{1 - \beta + \alpha \beta \gamma} E(m_t | \phi_{t-1}) - \frac{\alpha \beta}{1 - \beta + \alpha \beta \gamma} \overline{Y} + \frac{(1 - \beta)}{1 - \beta + \alpha \beta \gamma} p_{t-1} + \frac{(1 - \beta)}{1 - \beta + \alpha \beta \gamma} \varepsilon \frac{o}{t-1}$$

Here E represents the mathematical expectation operator and  $\phi_t$  denotes the private information set, written as a vector, available at time t, conditional on which private expectations are formed. Note that because of the inertia in the price adjustment process, the past price level directly carries information about the current price level via the last term of IV.6 as well as possibly indirectly via  $E(m_t | \phi_{t-1})$ . When price adjustment is instantaneous,  $\beta = 1$ , the past price level has no direct influence on the current price level. IV.6 becomes:

IV.6' 
$$E(p_t | \phi_{t-1}) = E(m_t | \phi_{t-1}) - \frac{1}{\gamma} \overline{\gamma}$$

The reduced form equations for the price level and real output are given in IV.7 and IV.8.

$$IV.7 \quad p_{t} = \frac{(1-\beta)}{1-\beta+\alpha\beta\gamma} \quad p_{t-1} + \frac{\alpha\beta\gamma}{1+\alpha\beta\gamma} m_{t} + \frac{\alpha\beta^{2}\gamma}{(1+\alpha\beta\gamma)(1-\beta+\alpha\beta\gamma)} E(m_{t}|\phi_{t-1}) - \frac{\alpha\beta}{1-\beta+\alpha\beta\gamma} \overline{Y} \\ - \frac{\alpha\beta}{1+\alpha\beta\gamma} \overline{\varepsilon}_{t}^{s} + \frac{\alpha\beta}{1+\alpha\beta\gamma} \varepsilon_{t}^{d} + \frac{\beta(1-\beta)}{(1-\beta+\alpha\beta\gamma)(1+\alpha\beta\gamma)} \varepsilon_{t-1}^{o} \\ IV.8 \quad Y_{t} = \frac{\gamma}{1+\alpha\beta\gamma} m_{t} - \frac{\alpha\beta^{2}\gamma^{2}}{(1+\alpha\beta\gamma)(1-\beta+\alpha\beta\gamma)} E(m_{t}|\phi_{t-1}) + \frac{\alpha\beta\gamma}{1-\beta+\alpha\beta\gamma} \overline{Y} - \frac{(-\beta)\gamma}{1-\beta+\alpha\beta\gamma} p_{t-1}$$

$$.8 \quad Y_{t} = \frac{\gamma}{1+\alpha\beta\gamma} \quad m_{t} - \frac{\alpha\beta\gamma}{(1+\alpha\beta\gamma)(1-\beta+\alpha\beta\gamma)} \quad E(m_{t} \mid \phi_{t-1}) + \frac{\alpha\beta\gamma}{1-\beta+\alpha\beta\gamma} \quad \overline{Y} - \frac{(-\beta)\gamma}{1-\beta+\alpha\beta\gamma} \quad P_{t-1} + \frac{\alpha\beta\gamma}{1+\alpha\beta\gamma} \quad \varepsilon_{t}^{s} + \frac{1}{1+\alpha\beta\gamma} \quad \varepsilon_{t}^{d} - \frac{\beta(1-\beta)\gamma}{(1+\alpha\beta\gamma)(1-\beta+\alpha\beta\gamma)} \quad \varepsilon_{t-1}^{o}$$

The effect of an anticipated increase in the money supply,  $\Delta m^a$ , is given by: IV.9a  $\frac{\Delta Y}{L} = \frac{\gamma(1-\beta)}{1-\beta+\alpha\beta\gamma}$ 

The effect of an unanticipated increase in the money supply, 
$$\Delta m^{u}$$
, is given by:  
IV.9b  $\frac{\Delta Y}{\Delta m^{u}} = \frac{\gamma}{1+\alpha\beta\gamma}$ 

Two propositions emerge. First, as long as the price level is not completely inflexible ( $\beta$ >0), an unanticipated increase in the money supply will have a larger effect on real output than an anticipated increase in the money supply.<sup>17/</sup>

Second, only if the price level is perfectly flexible ( $\beta$ =1), will anticipated money supply changes have no effect on real output. Even then, unanticipated monetary disturbances will continue to have real effects.

Except for the recent papers by Stiglitz <u>et al</u>. most of the theoretical work on wage and price rigidity can be characterized as [implicit] contract theory (Azariadis [1975], Baily [1974], Grossman [1978], Fischer [1977], Phelps and Taylor [1977]). Multi-period employment contracts are viewed as mutually privately rational arrangements for shifting risk from risk-averse workers to risk-neutral or less risk-averse capitalists. Similarly, multiperiod price contracts between well established suppliers and customers can be viewed as transactions and search cost minimizing as well as risk-sharing arrangements (Okun [1975]). What is not clear is why optimal multi-period contracts would set (i.e., predetermine) nominal wages or prices, thus assigning the role of shock absorbers solely to quantities (employment and sales) (Barro [1979]). A recent attempt to overcome some of the shortcomings of contract theory was made by Hall [1979], who rationalizes wage and price stickiness by developing a theory of the role of "prevailing prices and wages" in the efficient organization of markets.

If prices can adjust to clear markets with the same frequency with which new information accrues and with the same speed with which the monetary authority can respond to any new information, monetary policy will enter the reduced form for real output only via a term like  $\sum_{i=0}^{\infty} w^i (m_{t-i} - E(m_{t-i-1} \phi_{t-i-1}))$ , i.e., via a series of one-period ahead forecast errors. With multi-period nominal contracts monetary policy will enter the reduced form for real output via a term like  $\sum_{j=0}^{\infty} i_{j=0}^{\infty} w^i (m_{t-j} - E(m_{t-j} | \phi_{t-j-i-1}))$ , i.e., via a sequence of forecast errors for  $m_{t-i}$  from different dates in the past. Thus, if money wages

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and prices are determined by a series of overlapping multi-period nominal contracts and if the nominal money stock can be adjusted freely each period, the information set available at the time of the current money supply decision will in general be larger than the information sets that were available when most of the currently prevailing money prices and wages were contracted for. The public sector does not have an informational advantage over the private sector at any moment of time, but only the public agent is able to change his control--the money supply--in response to any new information. Private agents are "locked in" by past nominal contracts. Given this differential ability to respond to new information, monetary policy can be used to stabilize (or destabilize) real output.

# Differential information between the private and public sectors

Equations IV.8 and IV.9 make it clear that sluggishness in the adjustment of prices can be a sufficient reason for anticipated money supply changes to have real effects. Asymmetric information between the private sector and the monetary authorities can be another reason for effective monetary policy, even if the price level is market-clearing (Barro[1976]). It is not necessary for the public sector to have uniformly superior information. All that is required is that different agents have differential access to (and ability to process and assimilate) different kinds of information. Let  $\psi_t$  be the information set of the monetary authority in period t.  $m_t$  will be some function,  $T_t$ , of  $\psi_t$ . For simplicity  $T_t$  is taken to be linear.

IV.10 
$$m_{+} = T_{+}\psi_{+}$$

Consider the equilibrium version of IV.8, where  $\beta=1$ .

IV.8' 
$$Y_t = \frac{\gamma}{1+\alpha\gamma} (m_t - E(m_t | \phi_{t-1})) + \overline{Y} + \frac{1}{1+\alpha\gamma} \varepsilon_t^d + \frac{\alpha\gamma}{1+\alpha\gamma} \varepsilon_t^s$$

Substituting IV.10 into IV.8' gives

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IV.11 
$$Y_t = \frac{\gamma}{1+\alpha\gamma} (T_t \psi_t - E(T_t \psi_t | \phi_{t-1})) + \overline{\gamma} + \frac{1}{1+\alpha\gamma} \varepsilon_t^d + \frac{\alpha\gamma}{1+\alpha\gamma} \varepsilon_t^s$$

The response of real output to monetary policy in the equilibrium model depends on the monetary policy rule,  $T_t$ , on the monetary authority's information set  $\psi_t$  and on the private sector's information set  $\phi_{t-1}$ , as well as on the structural parameters  $\alpha$  and  $\gamma$ . In period t-1, the private sector must forecast the value of the money supply in period t. To do so it must know, or predict, both the policy rule,  $T_{t}$  and the public sector's information set  $\psi_{t}$ . If the policy rule next period is known, and if the public and private sector information sets are identical,  $\psi_t = \phi_t$ , the problem is easily solved. The real effect of monetary policy will be an increasing function of  $T_t \phi_t - E(T_t \phi_t | \phi_{t-1})$ . This expression will be orthogonal to (independent of)  $\phi_{t-1}$ . The conditional distribution function of  $Y_t$ , given  $\phi_{t-1}$ , is therefore independent of the policy function  $T_t$  as long as this function is known and the information sets of the private sector and the monetary authorities coincide. If the policy rule is known but if the private information set and the public sector information set are not identical (and if the latter is not a strict subset of the former),  $T_t[\psi_t - E(\psi_t | \phi_{t-1})]$  will not be orthogonal to  $\phi_{t-1}$  (or  $\psi_{t-1}$ ). The conditional distribution function of  $Y_t$  will therefore not be independent of the known policy rule  $T_t$ . If the policy rule,  $T_t$  is not known to the private sector the conditional distribution function of  $Y_t$  is of course not independent of  $T_t$ , even if the public and private information sets  $\boldsymbol{\psi}_t$  and  $\boldsymbol{\phi}_t$  are otherwise the same. Important and unsettled issues arise when informational asymmetries occur. Can the public sector communicate its "privileged" information to the private sector? If so, are there lags and/or filtering problems as public sector information is disseminated? Is it better to reveal privileged information (assuming this is possible) than to use the informational advantage to influence real private sector behavior?

#### Changing the Structure of Private Sector Information

In a set of recent papers, Laurence Weiss [1979 a,b] has provided another ingeneous argument for a role for active (feedback) monetary policy in rational expectations models. This role exists even if the information sets of the private and public sectors are identical. Let  $\phi_t$ be the original private and public information set, expressed as a finitedimensional vector. Let  $\Omega_t$  be the set of all information that could be relevant.  $\phi_t$  is some subset of  $\Omega_t$ . We will write it as a linear transformation L of  $\Omega_t$ :

 $\phi_t = L\Omega_t$ .

We can, without loss of generality, assume L to be of full rank, although it will not in general be a square matrix. Since the public sector does not hold an informational advantage over the private sector, it cannot add or create additional private information, i.e., it cannot change the <u>rank</u> of the matrix L. It can, however, through feedback policy, change the matrix L to a matrix L' that has the same rank but conveys information that has a different economic impact. An example will illustrate this. Consider a decentralized competitive economy with many separated markets. In any given period a trader is randomly assigned to a given market. His demand or supply decision will depend on the relationship between the current price in his own market and the average price expected to prevail in the next period when he will be reassigned randomly to another market. All he observes in the current period is the price in his market. There are two disturbances, an aggregate nominal disturbance common to all markets,  $n_t$ , and a market-specific real disturbance,  $u_t^i$  in the ith

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market, i=1,...,n. The trader in market i only observes some linear combination of those two disturbances:  $\lambda_1^i \eta_t + \lambda_2^i u_t^i$ ,  $\lambda_1^i$ ,  $\lambda_2^i > 0$ .  $\Omega_t$  is  $[\eta_t, u_t^1, ..., u_t^i, ..., u_t^n]$ . The L matrix of a trader in market i is  $[\lambda_1^i, 0, ..., \lambda_2^i, ..., 0]'$  and has rank one. Only the relative, real shock  $u_t^i$  matters to the ith trader. He will respond to  $\eta_t$  because he cannot filter out the separate contributions of  $\eta_t$  and  $u_t^i$  to the single observed disturbance  $\lambda_1^i \eta_t + \lambda_2 u_t^i$ . Monetary feedback policy may be capable of changing the information structure in such a way as to set  $\lambda_1^i$  equal to zero. The L matrix is  $[0, 0, ..., \lambda_t^i, ..., 0]'$ . The relative, real shock can now be identified. No information is added on balance:  $u_t^i$  has been identified at the expense of the loss of all information on  $\eta_t$ . Since the economic implications of observing  $u_t^i$  are different from those of observing  $\eta_t$ , this change in the structure of information can have real consequences.

## The Insufficiency of Prices in Sequence Economies With Incomplete Markets and rational Expectations

There is an implication of the rational expectations approach that seems to have been overlooked by its advocates. One of the major virtues of a market system is supposed to be the role played by prices determined in efficient markets as effective aggregators of information. Decentralized, atomistic private agents need not worry about the myriad of other agents, about their tastes, technologies and endowments. All information necessary for the optimal planning of consumption, production and sales is contained in current market prices that are parametric to the individual agent. If markets are incomplete and expectations of future spot prices replace currently available prices established in forward markets, rational expectations are a strong reminder of the insufficiency of the price mechanism for efficient resource allocation. Rational expectations are optimal forecasts based on the forecasting agent's knowledge of the true structure of the economy. They require knowledge of the underlying structural parameters of public and private sector preferences, technologies and endowments. The local, partial nature of the knowledge provided by prices in a realworld market economy with incomplete markets, means that additional global, economy-wide informational demands are made on private agents with rational expectations. Such informational demands are often dismissed as unrealistic when central planning is discussed. The informational requirements of central planning are no different from the informational requirements of "decentralized" rational expectations models of sequence economies with incomplete markets.

### V. Conclusion

The "New Classical Macroeconomics" has forced a thorough revaluation of the theoretical and empirical foundations of Neo-Keynesian conventional wisdom. After the rhetoric is stripped away, however, the analytical implications of the New Classical Macroeconomics are surprisingly familar to neo-Keynesians. Substitution of debt financing for tax financing crowds out saving in the short run and lowers the capitallabor ratio in the long run. A higher, fully anticipated rate of growth of the money supply will not be super-neutral and is likely cet. par. to be

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associated with a higher steady state capital-labor ratio. Monetary feedback rules will alter the trajectories of real economic variables. This confirmation of the importance of monetary, fiscal and financial policy for the cyclical and the long-run behavior of the real economy is not necessarily a source of comfort. After all, "policy neutrality" would be most welcome when the conduct of policy is erratic, confusing or incompetent. No such easy escape is available to the policy maker; policy can stabilize and it can destabilize, it can promote growth and prosperity or destroy it. \*This paper was written while I was with the the Financial Studies Division, Research Department, International Monetary Fund. It was presented at the Eighth Money Study Group Oxford Seminar, Brasenose College, Oxford, September 26-28, 1979. I would like to thank George von Furstenberg for helpful comments.

### FOOTNOTES

1/ Government capital formation is ignored for simplicity.

2/ Since labor supply and consumption decisions are jointly determined by the household's intertemporal utility maximization program, the consumption function and labor supply function should have the same arguments. Choosing the more standard (ad hoc) specifications of equations II.3 and II.13 does not affect the argument.

3/ Depreciation is ignored.

4/ This neutrality proposition can be interpreted in one of the two following ways. (a) Compare the solution trajectories for two economies, identical in all respects except that the nominal stock of money in country 2, M<sup>2</sup>(t) exceeds that in country 1, M<sup>1</sup>(t) by a fixed fraction: M<sup>2</sup>(t) = M<sup>1</sup>(t)(1+ $\alpha$ ),  $\alpha > 0$ , for all t. In economy 2 the equilibrium solutions for the price level p<sup>2</sup>(t) and the money wage, w<sup>2</sup>(t), will exceed those in country 1, p<sup>1</sup>(t) and w<sup>1</sup>(t), by the same fraction  $\alpha$ . p<sup>2</sup>(t) = (1+ $\alpha$ ) p<sup>1</sup>(t); w<sup>2</sup>(t) = (1+ $\alpha$ ) w<sup>1</sup>(t). All real variables are the same. (b) Consider the solution trajectory of a given economy before and after an unanticipated change in the money supply at t= $\tau$ . Let the money supply up to  $\tau$  be M<sup>1</sup>(t). Until t= $\tau$ , economic agents expect the money supply to follow M<sup>1</sup>(t) indefintely into the future. At  $\tau$  the actual and anticipated money supply path becomes M<sup>2</sup>(t) = M<sup>1</sup>(t)(1+ $\alpha$ ),  $\alpha > 0$  for t  $\geq \tau$ . If there is this unancipated change at t =  $\tau$  in the money stock process, the price level path after  $\tau$  is p<sup>2</sup>(t) = (1+ $\alpha$ ) p<sup>1</sup>(t), t  $\geq \tau$ . Here P<sup>1</sup>(t) is the price level path that would have prevailed at and beyond t= $\tau$ , had no unanticipated change in the money supply occurred at t= $\tau$ .

 $\frac{5}{1}$  I abstract for simplicity from the wealth effect and real interest rate effect on the labor supply.

6/ u is assumed to be strictly quasiconcave and increasing in  $c^1$  and  $c^2$ .  $u_1(0,c^2) = u_2(c^1,0) = +\infty$ ,  $u_1(\infty,c^2) = u_2(c^1,\infty) = 0$ .

7/ We also assume  $f'(0) = +\infty$ ;  $f'(\infty) = 0$ 

 $\frac{8}{1}$  A solution will exist if  $c^1$  and  $c^2$  are both normal goods and if  $c^1$  does not increase when  $r_{t+1}$  increases. It will be unique if the utility function is homothetic. See Carmichael (1979).

9/ This is different from Carmichael (1979) who considers the case where an individual only ignores the impact of marginal changes in his actions on generations that are already dead, i.e. Carmichael uses a closed-loop strategy vis-a-vis all later generations. The symmetry imposed in my specification considerably simplifies the analysis.

<u>10</u>/ From III.43a we have, in the steady state,  $\frac{\partial W}{\partial c^1} = (1+r) \frac{\partial W}{\partial c^2}$ 

<u>11</u>/ We use the condition  $\rho\delta > 1$ , for the stationary utility function to be bounded.

 $W = \underline{u(c^{1}, c^{2})}_{1-(1+\delta)^{-1}-(1+\rho)^{-1}}$ 

12/ We assume  $\rho$  to be the same in both cases.

13/ We assume  $\delta$  to be the same in both cases.

<u>14</u>/ Since f'(k) > 0, this means that  $\Delta p \ge 0$  is inconsistent with money being held in equilibrium.

15/ We already make the assumption (explicit in III.53c) that both money and capital are held and that the private saver will get the same rate of return on his saving, no matter how he divides it between money and real capital.

<u>16</u>/ Equation IV.2 is only reasonable in an economy without an inflationary or deflationary trend. If there is a trend in the price level (but not in the rate of inflation), IV.2 can be modified to:

 $(p_t - p_{t-1}) - (p_{t-1} - p_{t-2}) = \beta(p_t - p_{t-1} - (p_{t-1} - p_{t-2}))$ 

17/ This is not a general property of rational expectations models. Bailey (1978) provides examples of anticipated fiscal policy having greater "bang per buck" than unanticipated fiscal policy.

<u>18</u>/ Since  $E[T_t\phi_t - E(T_t\phi_t | \phi_{t-1})]\phi_{t-1}] = 0$ 

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