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INVENTORIES, RATIONAL EXPECTATIONS, AND THE BUSINESS CYCLE

Alan S. Blinder

Stanley Fischer

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ABSTRACT

The simplest macroeconomic models in which markets clear instantaneously, and expectations are rational preclude the existence of "business cycles," that is, of serially correlated deviations of output from trend. This paper studies one of several mechanisms that can be used to make these so-called "new-classical" models produce business cycles; the mechanism is the gradual adjustment of inventory stocks.

Two microeconomic models of inventory holdings are formulated. Both imply, first, that current output should be a decreasing function of the stock of inventories and, second, that inventories, once perturbed from equilibrium levels, should adjust only gradually. These two features are then embedded into an otherwise standard macroeconomic model in which markets clear instantaneously and expectations are rational. Two principal conclusions are reached. First, disturbances such as unanticipated changes in money will set in motion serially correlated deviations of output from trend. Second, if desired inventories are sensitive to the real interest rate, then even fully anticipated changes in money can affect real variables.

Professor Alan S. Blinder
Department of Economics
207 Dickinson Hall
Princeton University
Princeton, NJ 08544

609/452-4010

Professor Stanley Fischer
Department of Economics
Massachusetts Institute of Technology
Cambridge, MA 02139

617/253-6666

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1. Introduction.

There are doubtless many mechanisms that co-operate in producing the serial correlation of deviations of output from trend that we know as "the business cycle." This paper studies the role of inventories in the propagation of the business cycle in a model with rational expectations.[1]

Even a cursory look at the data indicates the importance of inventory fluctuations in the short-run dynamics of output. Table 1 shows peak to trough changes in both real GNP and its most volatile component--inventory investment--during the postwar recessions. The significance of inventory change is evident. Table 2 focusses on the most recent recession and recovery. To cite only the most dramatic figures, almost the entire decline in GNP during the worst quarter of the downturn (1975:1) came from a swing in inventory investment; and about two-thirds of the GNP change in the strongest quarter of the recovery (1975:3) resulted from the end of inventory decumulation. This is not, of course, meant to imply that autonomous movements in inventories cause business cycles, but only to suggest that inventory dynamics play a fundamental role in their propagation.

Recent work on business cycles and monetary policy has been greatly influenced by the approach to aggregate supply due to Lucas (1972, 1973). Basing his argument on intertemporal substitution effects, and on the inability of agents to distinguish between absolute and relative price movements in the short run, Lucas has posited the aggregate supply function:

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[1]For a more complete, but similar, analysis of inventory behavior in a conventional nonstochastic macro model without rational expectations, see Blinder (1978a).

TABLE 2

Changes in Real GNP and Real Inventory Investment, 1973-1976*

	1973:4	1974:1	1974:2	1974:3	1974:4	1975:1	1975:2	1975:3	1975:4	1976:1
Real GNP	+ 6.3	-12.4	-5.7	-7.6	-17.2	-28.1	+18.3	+30.1	+7.9	+27.6
Real Inventory Investment	+11.3	-11.5	-4.7	-7.2	+ 4.8	-26.2	+ 2.7	+18.8	-7.3	+12.7

*Change from previous quarter at annual rate, in billions of 1972 dollars.

TABLE 1

Changes in GNP and in Inventory Investment

in the Postwar Recessions

(1) Dates of Contraction		(2) Decline in Real GNP[a]	(3) Decline in Inventory Investment	Column (3) As a Percentage of Column (2)
Peak	Trough			
1948:4	1949:4	$ 6.7	$13.0	194%
1953:2	1954:2	20.6	10.2	50
1957:3	1958:1	22.2	10.5	47
1960:1	1960:4	8.8	10.5	119
1969:3	1970:4	12.0	10.1	84
1973:4	1975:1	71.0	44.8	63

[a] In billions of 1972 dollars.

$$(1.1) \qquad y_t = k_t + \gamma(p_t - {}_{t-1}p_t) + e_t \ ,$$

where y is (the log of) real output, k is (the log of) the natural rate of output, p is (the log of) the price level, and the notation ${}_{t-1}X_t$ denotes the expectation that is formed at time (t-1) of the variable X_t. In Lucas' work, and in this paper, these expectations will be assumed to be formed rationally. Finally, the error term, e_t, is assumed to be independently and identically distributed.

If the natural rate of output, the k_t term in (1.1), is exogenous, two strong conclusions follow from coupling this supply function with the assumptions that prices always move to clear markets within the period and that expectations are rational. The first is that deviations of output from its natural rate are pure white noise--there is no business cycle. The second is that no feedback rule for monetary policy (or equivalently, no anticipated change in the money stock) can affect deviations of output from the natural rate.

That both these conclusions follow from rational expectations, price flexibility, and (1.1), can be shown simply. The basic implication of rational expectations is that errors in predicting the (logarithm of the) price level must be uncorrelated with any variable that is known as of time t-1, including, in particular, the previous prediction error. Letting u_t (unanticipated inflation) denote these prediction errors, equation (1.1) can be written

$$y_t - k_t = \gamma u_t + e_t \ ,$$

which is just white noise. By like reasoning, no known monetary rule can cause unanticipated inflation; only monetary surprises can do that. So, if markets clear, equation (1.1) allows for no real effects of anticipated money, given the assumed fixed natural rate of output.

Explanations for the business cycle that build on the supply function (1.1) focus on the determinants of the k_t term. Lucas (1975) has shown that the inclusion of capital in the model will produce serial correlation of output, as unanticipated inflation affects current output and thereby future capital stocks. A similar mechanism has been explored in Fischer (1979). Sargent (1979, Ch. 16) has studied a model in which serial correlation of the natural rate of output follows from gradual adjustment of the labor stock by firms faced with adjustment costs.

Modifications of the basic model to allow for serial correlation of output do not necessarily modify the second conclusion--that anticipated policy actions have no real effects. However, if capital is explicitly included in the model, and it is assumed that the rate of accumulation of capital is directly or indirectly a function of the anticipated rate of inflation, then the behavior of the k_t term in (1.1) can be affected by anticipated monetary changes.[2] Alternatively, a role for monetary policy in affecting cyclical behavior may be found by dropping the market clearing assumption, which changes the form of the aggregate supply function.[3]

In this paper we study how the inclusion of storable output affects the two basic conclusions arising from the combination of the supply function (1.1) and rational expectations. First, we show that adding inventories to the model

[2] Fischer (1979).

[3] Fischer (1977), Phelps and Taylor (1977) and Taylor (forthcoming); however, see also McCallum (1977) for a demonstration that some types of non-market clearing still do not permit any role for monetary policy in affecting output.

makes shocks persist. Second, we show that if the demand for inventories is interest elastic, anticipated monetary changes can have real effects. Of the two roles of inventories, we have no doubt that the propagation of disturbances caused by unanticipated events is much the more important. Nonetheless, it is interesting to note that the inclusion of inventories opens a potential channel for even fully anticipated monetary policy to have real effects.

In the next two sections of the paper we show that, in the presence of storable output, the aggregate supply function is modified to a form like:

$$(1.2) \qquad y_t = k_t + \gamma(p_t - {}_{t-1}p_t) + \lambda(N^*_{t+1} - N_t) + e_t$$

where N_t is the stock of inventories at the beginning of the period, and N^*_t is the optimal or desired stock. Section 2 derives a supply function like (1.2) based on utility maximization by a yeoman farmer working in a competitive market, the case that seems closest in spirit to Lucas' analysis.[4] Section 3 derives a similar function in a different setting: that of a profit-maximizing firm with some degree of monopoly power.

The following two sections offer proofs of the assertions we have just made. In Section 4 we show that, even in the most stripped-down macro model with inventories that we can set up, shocks lead to persistent deviations of y_t from its natural level, i.e., to business cycles. And Section 5 demonstrates that, if N^*_t depends on the real interest rate, then anticipated changes in the money stock can have real effects through inventory changes. Section 6 contains conclusions.

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[4]Lucas (1977, p. 18) discusses the way in which the aggregate supply function (1.1) should be modified to take account of inventory behavior, without, however, embodying the modified function in a full model.

2. The Lucas Supply Function Revisited: The Case of the Yeoman Farmer

The Lucas supply function (1.1) is most conveniently thought of as arising from the behavior of individuals selling their own labor in isolated markets (Phelpsian islands).[5] Each individual's supply of labor is an increasing function of the real price (wage) he perceives. However, by virtue of an assumed one-period information lag, he does not know the current aggregate price level. Instead, when he decides how much labor to supply he knows only the nominal price (wage) he receives in his isolated market. His inference problem is to decide what real wage is represented by the nominal wage being offered in his market. Since the price he receives for his services varies from period to period both because the general price level varies and because there are changes in relative prices, he typically believes that part of any unanticipated increase in the <u>nominal</u> price he faces represents a change in the <u>relative</u> price of his services. Accordingly, an unanticipated increase in the general price level is misinterpreted as being in part an increase in relative price, and output is therefore increased. Aggregating over markets, Lucas derives the aggregate supply function (1.1).

Although the Lucas supply function is usually thought of as arising in markets in which services are sold by individuals, much the same derivation applies when nonstorable output is supplied by firms which hire labor for the purpose of production. Labor may be regarded as distributed randomly to Phelpsian islands each period; workers are immobile between islands within the period, but mobile between periods. Firms on each island are competitive, and demand labor as a

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[5] See Phelps (1970), Introduction.

function of the wage relative to the price of the good they produce. Workers, however, are concerned with the wage relative to the average price level.[6] An increase in the price of output in a particular market, observed by both firms and workers, shifts up the labor demand curve (with the nominal wage on the vertical axis) proportionately. However, workers interpret any increase in price in this market as only in part an increase in the general price level. The labor supply curve accordingly moves up less than the demand curve, the nominal wage rises less than proportionately to the increase in the price in this market, and output increases. As before, an unanticipated increase in the general price level will lead to an increase in aggregate output. This is almost precisely the story told by Friedman (1968) in explaining the short-run Phillips curve.

In this section we use a similar framework to examine optimal behavior for a yeoman farmer, working without any cooperating factors, who sells his output in a competitive market. Since output is assumed to be storable, he can obtain goods to sell in two ways: by working or by drawing down his inventory stocks. At first we assume that the individual knows both the aggregate price level (the average of the prices of things he buys) and the relative price of his own output. Later we follow Lucas and Phelps in allowing for confusion between the two.

We start with this model not for its realism, but because it is so close in spirit to Lucas' and Phelps' work. As will become clear, however, inventories work in this model mainly through wealth effects--which is not how we imagine they work in a modern industrial economy. Further, the utility analysis to follow is plagued by the usual ambiguities arising from income and substitution effects.

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[6] Both the individual suppliers in the first paragraph and the workers in the second must be assumed to be distributed randomly to other islands to do their shopping after work.

Little beyond the list of arguments for each demand function can in general be derived; meaningful qualitative restrictions on demand and supply functions must generally be assumed--either directly or by restricting the class of utility functions. For both these reasons, we deal briefly with this model, and then turn our attention to a model of the firm.

Consider an individual living and working for two periods,[7] whose output is identical to his labor input:

$$Y_t = L_t \qquad t = 0, 1.$$

He is endowed with beginning-of-period stocks of the good, N_0, and of money, M_0, and must decide how much to produce, how much to consume, and how to carry over his wealth in the two assets available to him: N_1 and M_1. The prices of his own good and of goods in general are W_t and P_t, respectively, where we assume initially that both W_0 and P_0 are known, but W_1 and P_1 are random. It is convenient to work with transformations of W_t and P_t, namely, the individual's relative price, $w_1 = W_1/P_1$, and the purchasing power of money, $q_t = 1/P_t$.

Assuming that exogenous transfers of money and goods are received only by the young, budget constraints for periods 0 and 1 are:

$$C_0 = w_0(L_0 + N_0 - N_1) + q_0(M_0 - M_1) \tag{2.1}$$

$$C_1 = w_1(L_1 + N_1) + q_1 M_1 \quad , \tag{2.2}$$

where C_t is real consumption of goods in period t (t = 0, 1). In period 0, the

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[7] It is straightforward to embed these individuals in an overlapping generations model.

yeoman farmer decides how much to produce, and how much to carry over to period 1 in the form of the two assets available to him: inventories and money. He does this without knowing w_1 or q_1. Thus the demand for money derives only from portfolio considerations in this model, not from any special role of money as the medium of exchange. In period 1, w_1 and q_1 are announced, the yeoman farmer decides how much to produce, and then consumes what this output plus his accumulated wealth allow him to. He is assumed to maximize a separable utility function:

$$J = U(C_0, \bar{L} - L_0) + EV(C_1, \bar{L} - L_1) ,$$

where both U(.) and V(.) are strictly concave; any time discounting is embodied in the functional form of V(.).[8]

The period 1 problem is quite simple. With the carry-over stocks predetermined and the two prices known, the problem is one of certainty, with only labor supply to be chosen. That is, the yeoman farmer maximizes:

$$V(w_1L_1 + (w_1N_1 + q_1M_1), \bar{L} - L_1).$$

Assuming an interior maximum, the first-order condition is the usual one:

$$(2.3) \qquad w_1V_1 - V_2 = 0,$$

where V_i denotes the derivative of V with respect to its i-th argument. Obviously, (2.3) implies a labor supply function with the real wage and real wealth as arguments:

$$(2.4) \qquad L_1 = F(w_1, w_1N_1 + q_1M_1).$$

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[8]Note that this set-up is consistent with multi-period optimization.

F_1 will be positive if the substitution effect dominates the income effect, and F_2 will be negative if leisure is a normal good.

Now, using the budget constraints (2.1) and (2.2) to substitute out for C_0 and C_1, the maximand for the period 0 problem can be written as

$$\max_{\{L_0,\ q_0M_1,\ N_1\}} U(w_0(L_0 + N_0 - N_1) + q_0(M_0 - M_1),\ \bar{L} - L_0) + EV(w_1F(.) + w_1N_1 + q_1M_1,\ \bar{L} - F(.)) \tag{2.5}$$

subject to $N_1 \geq 0$, $M_1 \geq 0$. Here it is convenient to treat real balances (q_0M_1) rather than nominal balances (M_1) as the choice variable. Notice that the second argument of $F(.)$ in (2.4) is:

$$w_1N_1 + q_1M_1 = \frac{w_1}{w_0} w_0N_1 + \frac{q_1}{q_0} q_0M_1 .$$

By examining (2.4) and (2.5), and observing that $w_1F(.) = w_0(w_1/w_0)F(.)$, it is clear that the arguments of the demand functions for L_0, q_0M_1, N_1 (and therefore also for C_0) must be:

(i) w_0, the current wage or relative price;

(ii) $w_0N_0 + q_0M_0$, real wealth;

(iii) the distributions of the returns on money, q_1/q_0, and on inventory holdings, w_1/w_0.

The absolute price level is not an independent argument in the behavioral functions, entering only to deflate the nominal value of money balances.

For subsequent use, we are interested particularly in the supply function for labor (which is also the supply function for output) and the demand function for inventories. We write these as:

$$L_0 = L(w_0, w_0N_0 + q_0M_0; \phi(R_N, R_M)) \tag{2.6}$$

$$N_1 = N(w_0, w_0N_0 + q_0M_0; \phi(R_N, R_M)) \tag{2.7}$$

where the $\phi(.)$ notation indicates that the functions depend on the joint distribution of the two rates of return: $R_N \equiv w_1/w_0$ and $R_M \equiv q_1/q_0$. (The consumption and real balance demand functions that are also implied by the maximization process are not of interest here.)

The first-order conditions for an interior maximum in the first period are:

$$w_0U_1 - U_2 = 0 \tag{2.8'}$$

$$-U_1 + E\{V_1[w_1F_2R_M + R_M] - V_2F_2R_M\} = 0 \tag{2.9'}$$

$$-U_1 + E\{V_1[w_1F_2R_N + R_N] - V_2F_2R_N\} = 0 \tag{2.10'}$$

Using (2.3), the first-order conditions can be simplified to:

$$w_0U_1 - U_2 = 0 \tag{2.8}$$

$$U_1 = E\{V_1R_M\} \tag{2.9}$$

$$U_1 = E\{V_1R_N\} \quad . \tag{2.10}$$

These conditions have simple interpretations. The first is the marginal condition for optimal labor supply in period 0. The next two equations are optimal intertemporal allocation conditions. Equation (2.9) is the standard condition for consumption decisions. And, combining (2.9) and (2.10), we obtain the usual portfolio-allocation condition for a two-asset problem, which equates the marginal-utility-weighted rates of return on the two assets.

While we could now proceed to undertake comparative static exercises to derive the properties of the functions (2.6) and (2.7), paying particular attention to the effects of N_0 on labor supply and inventory demand, we know in advance that conflicting income and substitution effects will render most derivatives ambiguous. So instead let us ask what are the natural assumptions to make about (2.6) and (2.7).

First consider a change in wealth which, we note, is the only way that N_0 has effects in this simple model. If leisure is a normal good, and if demand for both assets rises when wealth increases, then it will be the case that $\frac{\partial L_0}{\partial N_0} < 0$, $0 < \frac{\partial N_1}{\partial N_0} < 1$. In words, an increase in inventories leads to a reduction in production, and to an increase in inventory carry-over which is less than the increase in the initial inventory holdings. The latter amounts to assuming that any increment to wealth will be divided among current consumption, investment in inventories, and investment in money, with positive shares for each. Both of these results will play critical roles in the macro model. Specifically, the notion that if inventories become too large (small) for some reason, the individual will eliminate the excess (shortfall) only gradually over time, constitutes the basic source of the serial correlation of output in the macro model.

Next, consider the effects of an increase in w_0, the current relative price, on labor supply (output) and inventory carry-over. From the first argument in (2.6), production will rise if substitution effects dominate. The effects on inventory demand of an increase in w_0 are more difficult to predict. An increase in w_0, given a fixed distribution of w_1, reduces the expected return to inventory holding. This would be likely to depress inventory demand. (Production might also be depressed by rate-of-return effects, but we assume that the direct wage effect dominates.) Under these circumstances high w_0 encourages current production, and reduces inventory demand, as inventories are sold off to take advantage of a currently high relative price.

If this yeoman farmer is placed in a standard Lucas-Phelps world in which there is imperfect information about the current price level, q_0, he will react in the manner described by equations (2.6) and (2.7) to any disturbance that he believes to be an increase in the relative price of his own good. Apart from real balance effects and adjustments in his nominal money holdings, he will not react to changes in the aggregate price level. Thus, if he is located on a Phelpsian island, he will react to any change in the nominal price of his own output, $W_0 = w_0/q_0$, as if it were partly a relative and partly an absolute price change. That is, his reactions to an increase in the nominal price in his isolated market will be qualitatively the same as his reactions to an increase in w_0. This is nothing but a restatement of Lucas' analysis with respect to price changes; the novelty here is in the analysis of inventory holding behavior.

3. Inventories and the Supply Function of Firms

In the utility maximization model, we derived a demand for inventories even with perfect competition, a linear production function, and no adjustment costs. This will not be possible in a model of the firm; accordingly we assume a convex cost structure, i.e., increasing marginal production costs. But, for reasons explained more fully in Blinder (1978b), this too turns out not to be sufficient to yield a well-defined inventory policy, and certainly not enough to justify an effect of the inventory stock on production decisions at the micro level.

The nonexistence of a well-defined inventory policy at the level of the competitive firm does not, of course, mean that output is independent of inventory stocks at the level of the market, but only that the effects of inventories are indirect: high inventory stocks lead to low market prices, and low prices lead to low production. However, in order to examine the role of inventories at the firm level, we turn next to a model of a firm with some (at least transitory) monopoly power, that is, with a downward sloping demand curve.[9]

Consider a firm with a demand curve that shifts randomly from period to period:

$$p_t = v_t P_t D(X_t), \tag{3.1}$$

where p_t is the firm's own absolute price, v_t is an identically and independently distributed disturbance in relative price, P_t is the aggregate price level (also random), and $D(X_t)$ is a downward-sloping function of the amount that the firm sells, X_t.[10] We will assume initially that the firm can observe both v_0 and P_0 before making its current output and sales decisions, while v_t, P_t $(t = 1, 2, \ldots)$ are random variables. Later we shall comment on what happens if the firm cannot distinguish between v_0 and P_0.

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[9]Blinder (1978b) shows, in a certainty context, that the implications of this model are basically identical to those of a perfect competitor with a sales constraint. It is possible but tedious, to work out a model of the competitive <u>industry</u> which has the same basic implications.

[10]D(.) and the other functions introduced below do not have a time index only to economize on notation. Nothing in the nature of this problem requires that D(.) or production costs or inventory holding costs be the same in each period; however, the firm's expected revenues cannot be growing too fast if an optimum is to exist.

Nominal production costs are assumed to be (a) homogeneous of degree one in the absolute price level, and (b) a convex function of output, Y_t. Specifically:

$$(3.2) \qquad C_t = P_t c(Y_t), \qquad c' > 0, \qquad c'' > 0.$$

We assume that $\lim_{Y \to 0} c'(Y) = 0$ and $\lim_{Y \to \infty} c'(Y) = \infty$, so that the firm will always select an interior maximum for Y_t.

In the current period, the firm must decide how much to produce and how much to sell. These jointly determine its inventory carry over according to:

$$(3.3) \qquad N_{t+1} = N_t + Y_t - X_t ,$$

where N_t is the beginning-of-period inventory stock. N_0 is exogenous. Inventory carrying costs are given by an increasing and convex function, $B(N_t)$.

The firm wants to maximize the expected discounted present value of its real profits. Thus it wants to find:

$$(3.4) \qquad J_0 = \max_{\{X_t, Y_t\}} E_0 \sum_{t=0}^{\infty} \left\{ \frac{R(X_t, v_t)}{\prod_{s=1}^{t} (1+r_s)} - \frac{c(Y_t)}{\prod_{s=1}^{t} (1+r_s)} - \frac{B(N_{t+1})}{\prod_{s=1}^{t+1} (1+r_s)} \right\}$$

where R(.) is the real revenue function, defined as:

$$R(X_t, v_t) = p_t X_t / P_t = v_t D(X_t) X_t ,$$

which we assume has the following properties:

$$R_X \geq 0, \; R_{XX} < 0, \; \lim_{X \to 0} R_X(X, v) = +\infty , \; D(X_t)X_t \text{ is bounded above.}$$

The latter assumptions assure us that X_t will always achieve an interior maximum. The variable r_i is the one period real interest rate in period i. The notation is understood to imply $\prod_{s=\tau}^{t}(1+r_s) = 1$ if $t < \tau$

The problem is set up in dynamic programming form by defining:

$$\pi_t = R(X_t,v_t) - c(Y_t) - \frac{B(N_{t+1})}{1+r_{t+1}}$$

$$J_1 = \max_{\{X_t,Y_t\}} E_1 \sum_{t=1}^{\infty} \left\{ \frac{R(X_t,v_t)}{\prod_{s=2}^{t}(1+r_s)} - \frac{c(Y_t,U_t)}{\prod_{s=2}^{t}(1+r_s)} - \frac{B(N_{t+1})}{\prod_{s=2}^{t+1}(1+r_s)} \right\},$$

so that (3.4) may be rewritten:

$$(3.5) \qquad J_0 = \max_{\{X_0,Y_0\}} \pi_0 + \frac{E_0 J_1}{1+r_1}.$$

It is clear from the set-up of the problem that the J_t functions depend on the initial inventory stock, N_t; the initial realizations of the two random variables, v_t and P_t; the joint distribution of all the stochastic variables; the path of real interest rates; and the functional forms of all the R, c, and B functions. Since the inventory stock is the only state variable of the firm, we shall simply write $J_t = J_t(N_t)$. Our assumptions imply that the $\pi_t(.)$ are concave, continuous, bounded functions; accordingly the J_t are concave and continuous and, given $r_s > 0$ for all s and the assumed stationarity of v_t, bounded. An optimal policy therefore exists.[11]

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[11]See Foley and Hellwig (1975).

To solve the problem for the first-period solution, it is easiest to use (3.3) to eliminate Y_0, and treat X_0 and N_1 as the firm's decision variables. First-order conditions for an interior maximum, on which we concentrate, are then:

$$(3.6) \qquad R_X(X_0,v_0) - c'(X_0 + N_1 - N_0) = 0$$

$$(3.7) \qquad \frac{J_1'(N_1) - B'(N_1)}{1 + r_1} - c'(X_0 + N_1 - N_0) = 0 \quad .$$

The first condition equates marginal revenue with marginal cost as usual. The second says that the marginal value of adding one unit to inventories must be equal to the sum of the costs of producing that unit and carrying it over to the next period. Henceforth, to simplify the notation, we will work with the composite function:

$$G(N_1) = J_1'(N_1) - B'(N_1) \quad .$$

Since $J_1'' < 0$ and $B'' > 0$, $G'(N_1) < 0$.

These first-order conditions imply optimal decision rules for current sales and inventory carry-over, and therefore production, of the form:

$$(3.8) \qquad X_0 = X(N_0,\ v_0,\ 1+r_1)$$

$$(3.9) \qquad N_1 = N(N_0,\ v_0,\ 1+r_1)$$

$$(3.10) \qquad Y_0 = S(N_0,\ v_0,\ 1+r_1) = X(.) + N(.) - N_0 \ ,$$

where future interest rates and the probability distribution of future v's are embodied in the functional forms. The derivatives of these functions can be worked out by the usual comparative statics technique. The following summarizes their relevant properties:

$$
(3.11)\qquad
\begin{aligned}
&0 < \frac{\partial X_0}{\partial N_0} < 1, \quad \frac{\partial X_0}{\partial v_0} > 0, \quad \frac{\partial X_0}{\partial(1+r_1)} > 0 \\
&0 < \frac{\partial N_1}{\partial N_0} < 1, \quad \frac{\partial N_1}{\partial v_0} < 0, \quad \frac{\partial N_1}{\partial(1+r_1)} < 0 \\
&-1 < \frac{\partial Y_1}{\partial N_0} < 0, \quad \frac{\partial Y_0}{\partial v_0} > 0, \quad \frac{\partial Y_0}{\partial(1+r_1)} < 0
\end{aligned}
$$

We are most interested in the effects of the initial stock of inventories. An increase in N_0 leads to a drop in current production, an increase in current sales (i.e., a cut in relative price), and an increase in next period's inventories, but by less than the increase in current inventories. Thus the apparent partial adjustment feature appears here just as it did in the yeoman farmer model.

Turning next to the relative price shock (shift in the demand curve), the profit maximizing firm will respond by raising both sales and output. But the sales response is greater so that inventory carry-over falls.

The intuition behind these results is straightforward once we keep in mind that the firm is operating on two margins: it is deciding how much to produce for inventories, and it is deciding how much to withdraw from inventories for sale. When the firm's relative price increases, the rewards for selling today (rather than tomorrow) are increased. But neither production costs nor the rewards for selling tomorrow (assuming that v_1 and v_0 are independent) are affected. So the incentive to raise sales is greater than the incentive to raise output, and inventory stocks get depleted.

Naturally, an equiproportionate change in all prices will elicit no behavioral response from the firm. But what if the firm cannot distinguish between a relative price shock (v_0) and an absolute price shock? For the same reasons as before, its reactions will be a muted version of its responses to a known increase in v_0.

Thus the models of a yeoman farmer and of a monopolist have almost identical predictions. Both current production and inventory carry-over depend on unanticipated inflation (interpreted as an increase in relative price), on the current relative price, on the initial stock of inventories, and on expected rates of return. The next two sections embed the conclusions from the micro models into an otherwise standard macro structure, and show that they lead to the two main results mentioned in the introduction: that unanticipated shocks have persistent effects, and that fully anticipated money can have real effects.

4. Inventories and Persistence

The micro models of the previous two sections imply that production, Y_t, should react negatively (though less than unit-for-unit) to the start-of-period stock of inventories, N_t, and positively to the current price-level surprise. Thus we write the supply function:

$$(4.1) \qquad Y_t = K_t + \gamma(P_t - {}_{t-1}P_t) + \lambda(N^*_{t+1} - N_t) + e_{1t}\ ; \quad \gamma > 0,\ 0 < \lambda < 1.$$

Here for convenience Y_t and N_t are levels, while P_t is the log of the current price level. K_t is trend output and N^*_{t+1} are steady-state desired inventories. In terms of the micro models, N^*_{t+1} should be interpreted as the value of N_0 such that, given the values of the other variables in (3.9), N_1=N_0. The supply function

To close the model it is now necessary only to add a specification of N^*_{t+1} and an aggregate demand sector. The micro models imply that N* depends on current and future interest rates and on the probability distribution of all the shocks.[12] To keep things as simple as possible, we write:

$$N^*_{t+1} = N^* - \delta r_t \, , \tag{4.3}$$

where N* and δ are constants and r_t is the current real interest rate.

The aggregate demand sector is almost totally conventional, and so we describe it very briefly. The equations are:

$$M_t - P_t = a_1 X_t - a_2 i_t + e_{3t} \tag{4.4}$$

$$X_t = c_1 Y_t + c_2 (M_t - P_t) - c_3 r_t + e_{4t} \tag{4.5}$$

$$N_{t+1} = N_t + Y_t - X_t \tag{4.6}$$

$$i_t = r_t + {}_tP_{t+1} - P_t \, . \tag{4.7}$$

Equation (4.4) is a standard LM curve except that final sales, X_t, is used instead of output as the transactions variable. Though nothing important hinges on this choice, our reason is as follows. It seems logical (and the micro models imply) that a higher initial inventory stock, N_t, should lead to a lower current price level, P_t. With X_t on the righthand side of (4.4) this obtains since higher inventories lead to higher sales and hence to greater demand for money. With the money stock fixed, the price level must decline. By contrast, had Y_t appeared instead of X_t in (4.4), a higher level of initial inventories, by depressing Y_t,

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[12] In the yeoman farmer model, wealth is also relevant. We ignore that here.

would have led to a reduction in the demand for money and hence to an increase in the price level.

Equation (4.5) defines aggregate demand as a function of production (= income), real balances, and the real interest rate. Equation (4.6), which appears to be an accounting identity, tacitly brings the assumption of market clearance into the model by stating that the amount that firms sell in (4.6) is identical to the amount that consumers demand in (4.5). Notice that equation (4.6) implies that a certain linear combination of e_{1t}, e_{2t}, e_{3t}, and e_{4t} must be zero each period.

Finally, equation (4.7) just relates the nominal and real rates of interest.[13]

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[13] In using this definition for the real interest rate, we depart slightly from the Phelpsian island paradigm. The island paradigm does not allow individuals to know the current aggregate price level with certainty, while equation (4.7) assumes they do know the current price level. A relatively simple way of avoiding this difficulty would appear to be to define anticipated inflation as ${}_{t-1}P_{t+1} - {}_{t-1}P_t$, a device adopted by Sargent and Wallace (1975). However, this too is inconsistent with the island story, since the absolute price in each island gives each individual some information about the current price level. We should actually write ${}_tP_t$ instead of P_t in (4.7), where ${}_tP_t$ is the current estimate of the price level conditional on information available currently. We know that ${}_tP_t$ is a weighted average of the actual aggregate price level and the expectation of P_t conditional on knowledge of the aggregate price level and all other history up to and including t-1. Thus any effects captured in the present version would be present in the more accurate--and considerably more difficult--consistent island paradigm, so long as knowledge of the current nominal interest rate does not serve to identify the current aggregate price level--as it does not, in the present model, in which the money demand and other disturbances prevent identification.

Solution of a rational expectations model of this complexity is a formidable task. Fortunately, it is not necessary in order to make the theoretical points we wish to establish. Our strategy is as follows. The conclusion that output disturbances are serially correlated is straightforward and very robust. So, in the remainder of this section, we demonstrate this central result in a stripped-down version of the model that removes all interest-rate effects. This model, however, leaves no room for fully anticipated money to have real effects. So, in the next section, we restore interest-rate effects, but concentrate on a version of the model in which there is no uncertainty.

Turning to the demonstration of persistence, assume that $\delta = a_2 = 0$. Equations (4.5) and (4.7) are now superfluous to the model, and it is easy to express current output (Y_t) as a function of current and past unanticipated inflation, which we denote by u_t:

$$(4.8) \qquad u_t = P_t - {}_{t-1}P_t \quad .$$

First, from (4.2) with N* a constant, the level of inventories is seen to be a function only of unanticipated inflation, and the stochastic term in the inventory demand function, e_{2t}. Then solve the difference equation (4.2), assuming the economy has an infinite past so that initial conditions can be ignored (given that the model is stable), to obtain:

$$(4.9) \qquad N_t = N^* - \phi \sum_0^\infty (1 - \Theta)^i u_{t-1-i} + \sum_0^\infty (1 - \Theta)^i e_{2,t-1-i} \quad .$$

Equation (4.9) repeats what we already know--that an unanticipated increase in the price level leads inventories to be drawn down, and then only gradually built back to their original level, so that the effect of any burst of unanticipated inflation on the current stock of inventories is smaller the further in the past the inflation surprise occurred.

Now substitute (4.9) into (4.1) to obtain the desired expression for output:

$$(4.10) \qquad Y_t - K_t = \gamma u_t + \lambda\phi \sum_0^\infty (1 - \Theta)^i u_{t-1-i} + e_{1t} - \lambda \sum_0^\infty (1 - \Theta)^i e_{2,t-1-i} .$$

Equation (4.10) shows that output disturbances are positively serially correlated, since unanticipated inflation in the current period pushes output above trend in the current period and in all subsequent periods. Depending on the relative magnitudes of γ and $\lambda\phi$, unanticipated inflation may have its maximal effect on output in the period it occurs, or one period later, and thereafter the effects decline geometrically. If unanticipated inflation has a small direct effect on output, so that γ is small, but leads to a large reduction in inventories, so that ϕ is large, then the inventory rebuilding effects of unanticipated inflation on output will predominate, and the maximum impact on output will occur in the period following a given unanticipated increase in the price level.

Figure 4.1 shows how the stock of inventories and level of output are affected by unanticipated inflation in this case. First, unanticipated inflation reduces the stock of inventories, as sales are increased in response to what firms regard in part as an increase in the relative price of output. Then inventories are gradually built back up; the $(1 - \Theta)^i$ terms in (4.9) result from the partial adjustment of inventories. Equation (4.10) shows that output is increased by current unanticipated inflation. Then in subsequent periods output is higher than it would otherwise have been, as a result of the need to rebuild depleted inventories.

Equation (4.10) also shows that systematic monetary feedback rules have no impact on the behavior of output under rational expectations. Output is

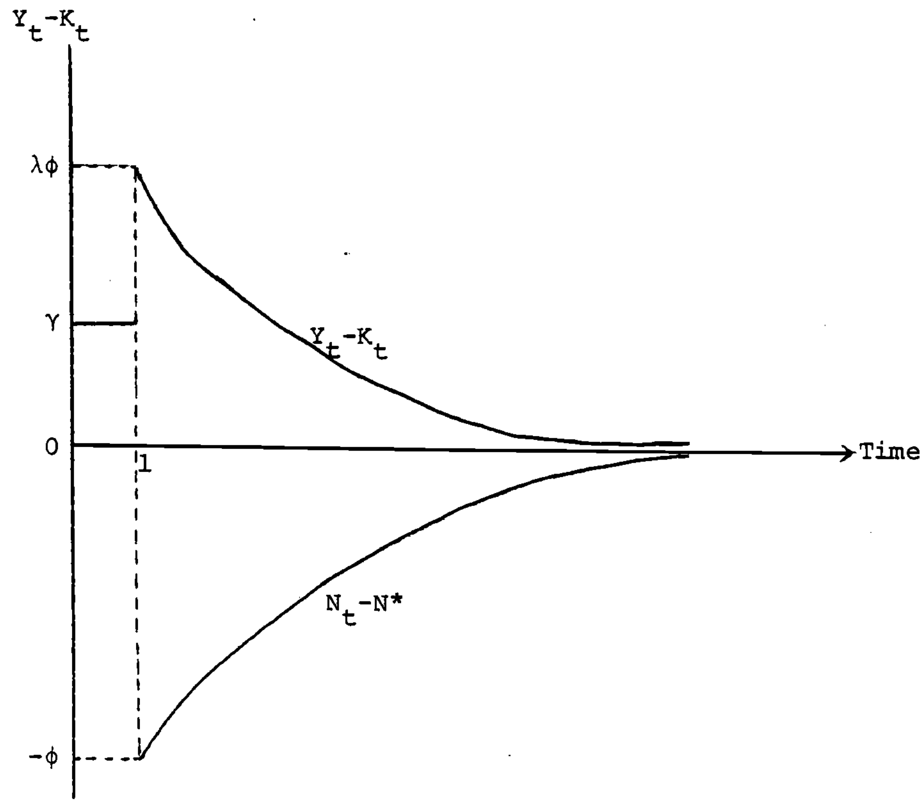

Figure 4.1: Dynamic adjustment of output and inventories to an unanticipated increase in the money stock.

affected only by stochastic disturbances and unanticipated inflation. While systematic feedback rules can produce anticipated inflation, they cannot produce unanticipated inflation, if the feedback rule depends only on information that is available at the time expectations are formed and expectations are rational, as we assume to be the case.

For completeness, we examine also the determinants of the current price level. Combining (4.1), (4.2), (4.6) and (4.4) with $a_2 = 0$, we obtain

$$P_t = M_t - a_1[K_t + (\phi + \gamma)u_t + (\lambda - \Theta)(N^* - N_t) + e_{1t} - e_{2t}] - e_{3t} . \tag{4.11}$$

The price level is accordingly proportional to perfectly anticipated increases in the money stock, and is a decreasing function of the stock of inventories since $\Theta > \lambda$.

It is worth noting that the price equation (4.11), derived from an equilibrium model, bears a striking resemblance to standard price adjustment equations in which the price level is reduced below its equilibrium level (which is $M_t - a_1K_t$) in response to excess holdings of inventories.

In concluding this section, it is worthwhile emphasizing once more the basic source for the serial correlation of output. An unanticipated increase in the price level in this model leads firms to sell out of inventories at the same time as they increase production to take advantage of what is (incorrectly) perceived as an increase in relative price. Then in subsequent periods production remains high as stocks of goods are rebuilt. The serial correlation of ourput does not, however, imply that anticipated monetary policy has real effects.

5. Inventories and Monetary Policy

We turn now to our second objective: to show that, if desired inventory holdings are sensitive to the rate of interest, even fully anticipated changes in money will have real effects.[14] Since interest here focusses on fully anticipated money, nothing substantive is lost, and considerable simplification of the model is achieved, if we assume that u_t and all the e_{it} are always zero.

The source of the result is fairly transparent, and can hardly be surprising to anyone familiar with the seminal papers of Tobin (1965) and Mundell (1963). Fully anticipated changes in money cause changes in the (fully anticipated)

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[14] By "fully anticipated" we mean that the money changes being discussed have always been known about. For a more precise definition, see Fischer (1979).

inflation rate which, under conditions to be spelled out shortly, affect the real interest rate. Desired inventories then adjust according to equation (4.3), and output is (transitorily) affected by equation (4.1). We proceed now to the argument.

Using the notation L for the lag operator, it can be shown that the level of inventories, N_t, the level of output, Y_t, and the real interest rate, r_t, respectively, are given by:

$$(5.1) \qquad N_t = N^* - \frac{\Theta\delta L r_t}{1 - (1-\Theta)L}$$

$$(5.2) \qquad Y_t = K_t - \frac{\lambda\delta(1-L)}{1 - (1-\Theta)L} r_t$$

$$(5.3) \qquad r_t = \text{constant} - \frac{c_2 a_2 ({}_tP_{t+1} - P_t)}{c_3 + c_2 a_2 + \dfrac{\left[\Theta(1 - c_2 a_1) + \lambda(c_1 - 1 + c_2 a_1)\right]\delta(1-L)}{1 - (1-\Theta)L}} .$$

Equation (5.3) displays the basic source of the nonneutrality of anticipated money in this model: anticipated inflation reduces the real rate of interest.[15] By looking at (5.3), we see that there are two necessary conditions for the nonneutrality of money in this model, namely that both c_2 and a_2 be nonzero. The parameter c_2 reflects the role of the real balance effect in the goods market, and a_2 reflects the interest elasticity of the demand for money.

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[15]This statement assumes that the denominator is positive. Since $\Theta > \lambda$, a sufficient condition is $c_2 a_1 < 1$, which means that a \$1 increase in money <u>supply</u> raises money <u>demand</u> by less than \$1.

Equation (5.1) shows that the stock of inventories is negatively related to past real rates of interest and (5.2) shows that the level of output is related to the change of the real interest rate. The coefficient δ that appears in (5.1) and (5.2) is likely to be small. If it were zero, neither Y_t nor N_t would be affected by fully anticipated money.

The remaining task is to study the determination of the price level and the rate of inflation. Working with the model (4.1) through (4.7) with the stochastic terms set to zero, we obtain the following equation for the price level:

$$P_t = b_0 + b_1 P_{t-1} + b_2\, {}_tP_{t+1} + b_3\, {}_{t-1}P_t + b_4 M_t + b_5 M_{t-1} \tag{5.4}$$

with b_1, b_2, $b_4 > 0$ and b_3, $b_5 < 0$. The coefficients b_1 through b_5 are defined in the appendix. They satisfy:

$$\sum_1^5 b_i = 1, \; b_2 + b_4 = 1, \; b_1 + b_3 + b_5 = 0 \quad .$$

The form in which (5.4) is written emphasizes that the current price level P_t is a function of the anticipated price level, ${}_tP_{t+1}$, as well as of the lagged price level, lagged expectations, and current and lagged money stocks. Since we are working with the assumptions that all stochastic terms are zero and that there is perfect foresight about the behavior of the money stock, the expectations in (5.4) will in fact be equal to the actual values of the price level. Nonetheless, (5.4) is a convenient form to write the price equation because it enables us to exploit a solution previously worked out in Fischer (1979).

As shown by Blanchard (1979), there are a variety of solutions for equations of the form of (5.4), some of which make the price level a function only of lagged money stocks. We choose to work with a solution that makes the current price level a function of both lagged and future money stocks, since we believe it reasonable that individuals will take into account the expected evolution of the money stock in forming their expectations of future price levels. The general form of a solution which takes both lagged and future behavior of the money stock into account was studied in some detail in Fischer (1979). In the case where there is no uncertainty this solution is:

$$(5.5) \qquad P_t = \text{constant} + \sum_{0}^{\infty} \pi_i M_{t+i} + \alpha M_{t-1} + \mu P_{t-1}$$

where μ is the root that is less than unity to:

$$(5.6) \qquad b_2\mu^2 + (b_3 - 1)\mu + b_1 = 0 \ ,$$

and

$$(5.7) \qquad \pi_0 = \left[1 - \frac{b_2\mu}{b_1}\right]\left[\frac{\mu - b_2\mu^2}{b_1}\right] > 0$$

$$\pi_i = \left[\frac{b_2\mu}{b_1}\right]^i \pi_0 \qquad i = 1, 2, 3, \ldots$$

$$\alpha = \frac{b_5\mu}{b_1} < 0 \ ,$$

and where

$$0 < \frac{b_2\mu}{b_1} < 1 \ .$$

Equipped with the solution for the price level, (5.5), and equations (5.1) through (5.3), we can now study the effects of monetary changes on the economy. In particular, we first discuss the effects of a perfectly anticipated one time change in the <u>level</u> of the money stock, and then discuss the effects of a one time change in the <u>growth rate</u> of the money stock. In each case we shall assume that the change takes place in period τ, and that it has always been anticipated that the change would occur.

The effects of a permanent change in the stock of money on the price level in previous and subsequent periods are described in Fischer (1979). Figure 5.1 shows the dynamic adjustment of the (log of the) price level to a 1 percent change in the money stock that occurs in period τ. The rate of inflation $(P_t - P_{t-1})$ accelerates up to period τ, and thereafter slows down. Figure 5.1 shows also the implied behavior of the real interest rate, r_t, which falls as the inflation rate accelerates up to time τ, and then starts rising as the inflation rate slows down.

The corresponding behavior of the level of inventories and the level of output are shown in Figure 5.2. Inventories build up as the real interest rate falls, and then, after the increase in the money stock, start being worked off. The behavior of output can be understood by combining (4.1) and (4.2), with all stochastic terms set to zero:

$$Y_t = K_t + \frac{\lambda}{\Theta} (N_{t+1} - N_t) \quad .$$

The rate of production is related to the rate of change of inventories. Accordingly, output is increasing up to the period before the money stock changes; thereafter output actually decreases below its steady state value as the inventory excess is worked off. In the longest of runs, the one time change in the money stock is neutral, resulting only in a proportionately higher price level. But the real economy is affected by the anticipation of the change in the money stock, and continues to be affected after the change has taken place.

We turn our attention next to the effects of a permanent change in the growth rate of the money stock. Before looking at the details, it is worth thinking through the consequences of such a change. Ultimately, we expect the

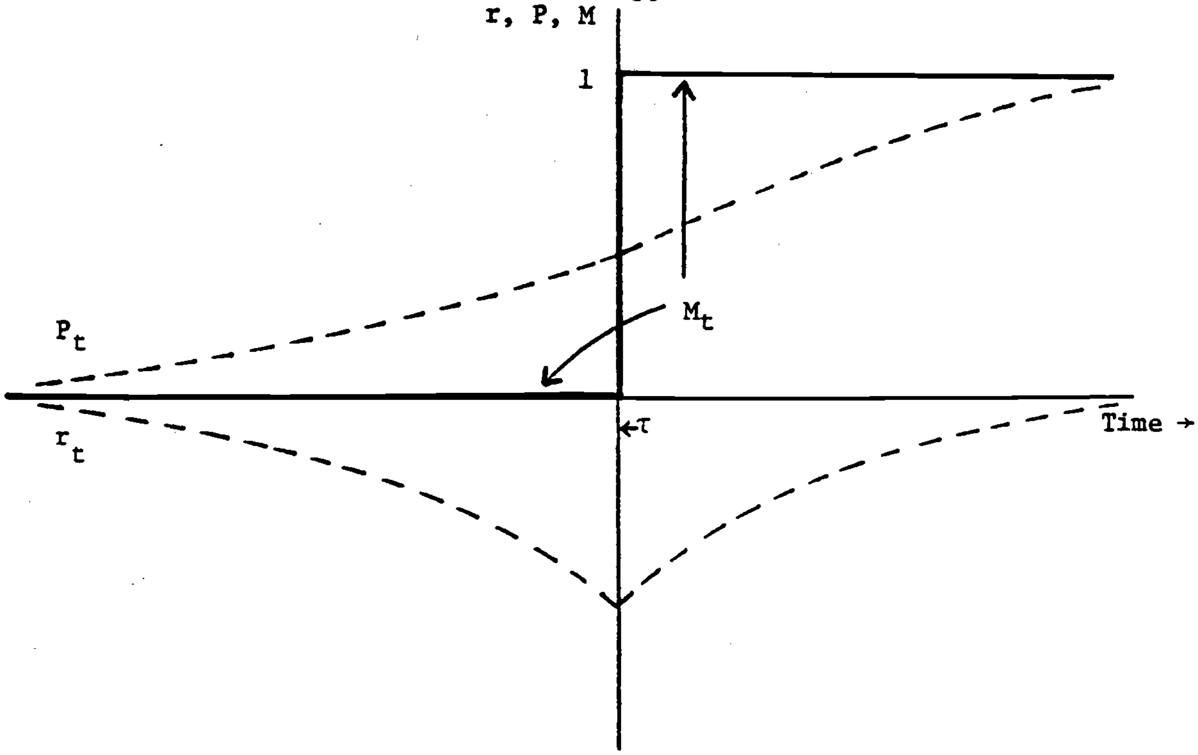

Figure 5.1: Dynamic adjustment of the price level and real interest rate to a permanent change in the money stock.

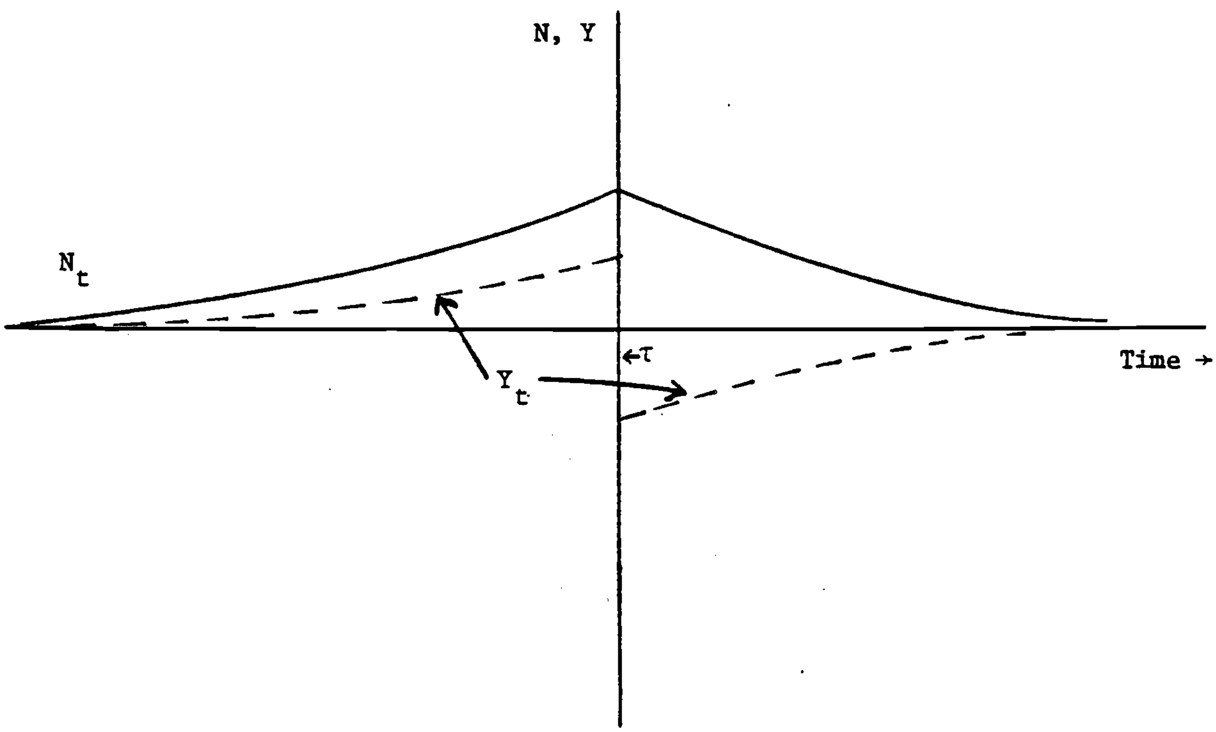

Figure 5.2: Dynamic adjustment of the stock of inventories and level of output to a permanent change in the money stock.

rate of inflation to be equal to the growth rate of money. From (5.3) we see that the real interest rate is reduced by increases in the expected inflation rate, and we should therefore expect a permanent increase in the growth rate of money to reduce the steady state real interest rate. Equation (5.1) shows that, with the new higher rate of growth of money, the level of inventories in the steady state will be higher. From (5.2), however, we note that the level of output is affected only by the first difference of the real interest rate. Therefore, in the steady state, the level of output will be unaffected by the change in the growth rate of money.

Once more, the key to understanding the dynamic adjustment of the economy to the monetary change is the behavior of the price level. This time, we plot the rate of inflation, rather than the price level, in Figure 5.3. The inflation rate increases over the entire period; it accelerates up to the time the growth rate of the money stock changes (between periods τ and $\tau+1$), and then decelerates after the change in the growth rate of money.[16] Precise details are provided in the appendix. Given the continuously increasing rate of inflation, the real rate of interest falls continuously.

Figure 5.4 shows the behavior of inventories and output. The stock of inventories builds up steadily to its new higher level, but the rate of increase of inventories is highest between periods τ and $\tau-1$; thereafter the rate of increase of inventories slows down. Accordingly, the level of output is at a maximum in period $\tau-1$, and gradually slows down thereafter. The change in the growth rate of money has its maximal effect on output in the period before the change, but continues to affect output behavior thereafter. Only asymptotically does output return to its steady state level.

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[16] Note that the overshooting of the inflation rate above the growth rate of money occurs before there is any change in monetary growth.

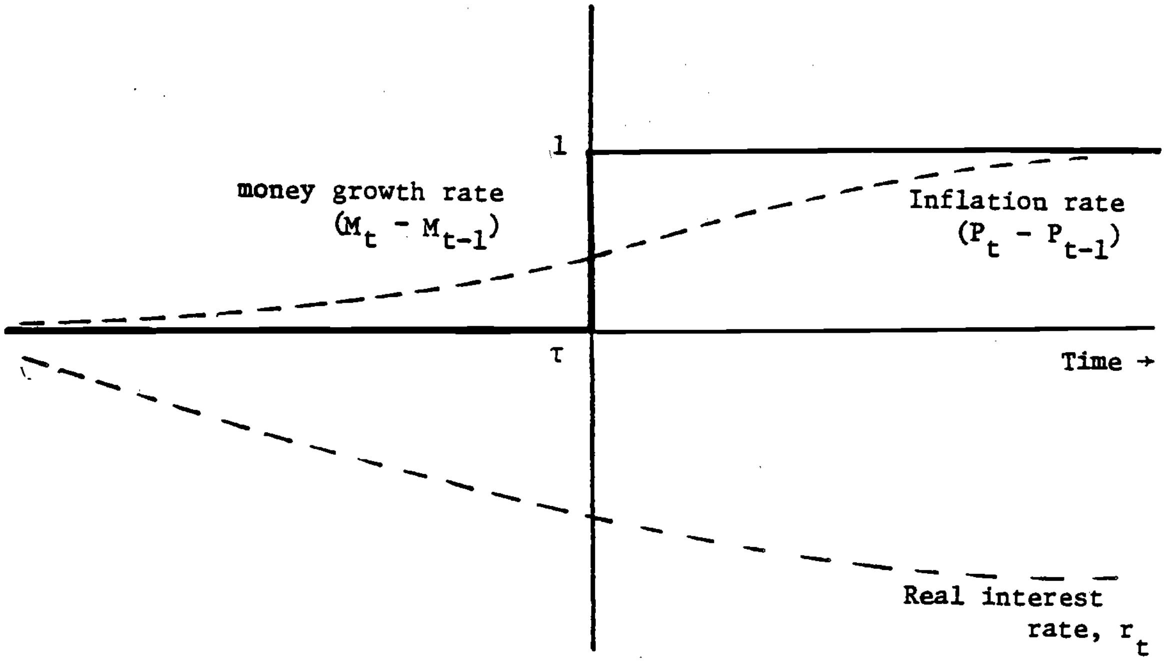

Figure 5.3: Dynamic adjustment of the rate of inflation and real interest rate to an increase in the growth rate of the money stock.

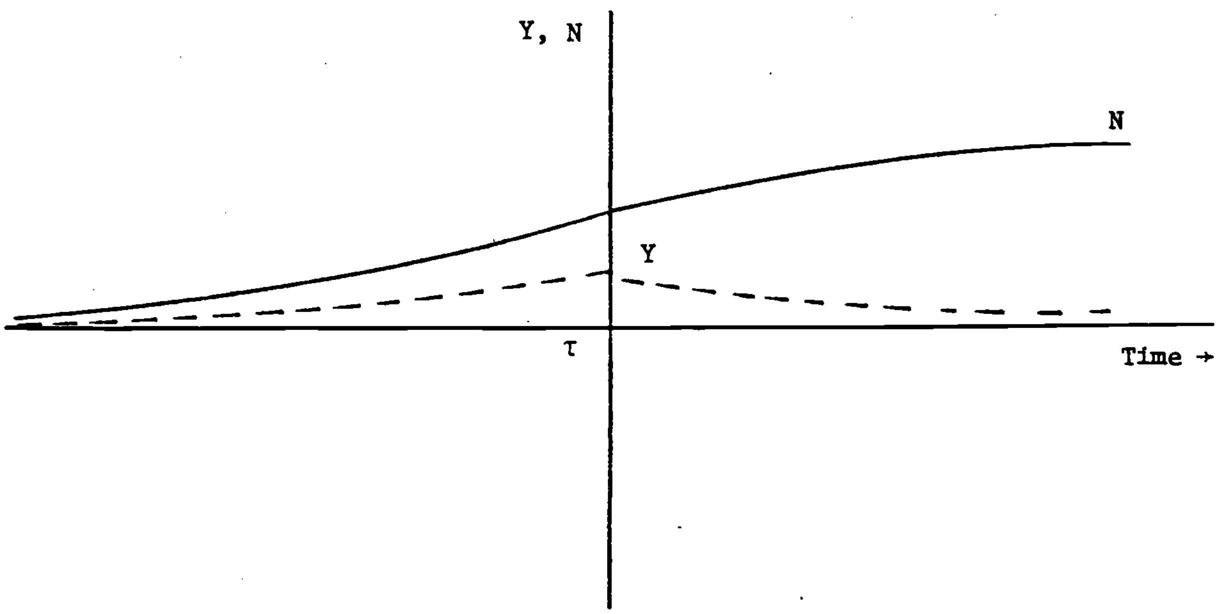

Figure 5.4: Dynamic adjustment of the stock of inventories and level of output to an increase in the growth rate of the money stock.

To sum up, the inclusion of the real interest rate in the demand function for inventories, coupled with the real balance effect on the demand for goods, provides a potential route through which anticipated monetary policy can affect the behavior of output. The behavior of output depends on the change in inventories. In response to a permanent change in the stock of money, inventories build up in anticipation of the change in the money stock, and are then worked off after the change occurs. The proximate cause of the inventory changes in this case is the behavior of the real interest rate, which is in turn fundamentally determined by the expected rate of inflation. Similarly, the response to a permanent increase in the rate of growth of the money stock, which permanently reduces the real rate of interest, is that inventories are built up slowly to a new permanently higher level. Output correspondingly increases above its steady state level, being at its highest level in the period before the growth rate of money changes, and thereafter slowly returns to its steady state level.

6. Conclusions

This paper has examined the way in which the inclusion of storable output modifies the aggregate supply function that is normally used in rational expectations models. The microeconomic foundations examined in Sections 2 and 3 led to a type of "partial adjustment" mechanism for inventories, in which excess inventories are worked off only slowly over time, rather than all in one period. They are worked off in part by reducing the level of output.

Including this sort of inventory behavior changes the dynamics of the macro model substantially. In particular, in a simple rational-expectations model in which output disturbances are otherwise serially uncorrelated, inventory adjustments lead to "business cycles," that is, to long-lived effects on output. This occurs since unanticipated changes in the money stock simultaneously increase current output and decrease inventories, as some inventories are sold off to meet the higher demand. Then, in subsequent periods, output is raised to restore the depleted inventories. This mechanism, which we examined in Section 4, is the most important of this paper in that it provides a very natural vehicle for the propagation of business cycles. A look at the data suggests that this vehicle is probably of great empirical importance.

For example, the data in Table 1 show that a sizeable inventory buildup in 1973:4 (the last quarter before the recession began) preceded three consecutive quarters during which output declined as inventories were being decumulated. Then, in 1974:4, a large negative shock to final demand led to more (presumably unwanted) inventory accumulation--which was reversed in the following quarter, with correspondingly deleterious effects on output.

Finally, in Section 5, we examined the effects of perfectly anticipated changes in the money stock on output in a model in which the desired inventory stock is a function of the real interest rate. In that case, since a permanent change in the stock of money, while ultimately neutral, alters the time path of the real interest rate, it also alters the paths of inventories and output. In particular, inventories and output are raised in anticipation of the change; and a long period of reduced output follows the monetary change, as the excess inventories are worked off. A permanent increase in the growth rate of money leads to a permanent increase in the stock of inventories, and to an output level that remains above the steady state level both before and after the change in the growth rate of money, as inventories are accumulated.

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Appendix

1. In this appendix we briefly indicate some of the calculations underlying statements in Section 5 of the paper. First, the values of the coefficients b_1 through b_5 in (5.4) are:

$$b_1 = \xi\left[(1-\Theta)(c_3 + c_2a_2 + a_2c_3) + (1 + a_2)\beta\delta + a_1a_2c_2(\Theta-\lambda)\delta\right] < 0$$

$$b_2 = \xi a_2\left[c_3 + \beta\delta + c_2a_1(\Theta-\lambda)\delta\right] < 0$$

$$b_3 = -\xi a_2\left[(1-\Theta)c_3 + \beta\delta + c_2a_1(\Theta-\lambda)\delta\right] < 0$$

$$b_4 = \xi\left[c_3 + c_2a_2 + \beta\delta\right] > 0$$

$$b_5 = -\xi\left[(1-\Theta)(c_3 + c_2a_2) + \beta\delta\right] < 0$$

$$\xi \equiv \left[c_3 + c_2a_2 + \beta\delta + a_2\{c_3 + \beta\delta + c_2a_1\delta(\Theta-\lambda)\}\right]^{-1} > 0$$

$$\beta \equiv \Theta(1 - c_2a_1) + \lambda(c_1 - 1 + c_2a_1) > 0$$

2. The effects of a fully-anticipated permanent change in the stock of money on the price level are discussed next. It can be shown that in the period, τ, in which the money supply changes:

$$\frac{\partial P_\tau}{\partial M_\tau} = \frac{\mu(1 - b_2\mu)}{b_1 - b_2\mu^2} \leq 1 .$$

In the earlier periods:

$$\frac{\partial P_{\tau-i}}{\partial M_\tau} = \left[\frac{b_2\mu}{b_1}\right]^i \frac{\partial P_\tau}{\partial M_\tau} , \qquad i = 0, 1, 2 \ldots$$

Thus, up to period τ, the inflation rate is given by:

$$\text{(A2.1)} \qquad \frac{\partial P_{\tau-i}}{\partial M_\tau} - \frac{\partial P_{\tau-i-1}}{\partial M_\tau} = \left[\frac{b_2\mu}{b_1}\right]^i \left[1 - \frac{b_2\mu}{b_1}\right] \frac{\partial P_\tau}{\partial M_\tau}, \quad i = 0, 1, \ldots$$

The inflation rate therefore increases up to period τ.

In subsequent periods

$$\frac{\partial P_{\tau+i}}{\partial M_\tau} = 1 - \frac{\mu^i(b_1 - \mu)(1-\mu)}{b_1 - b_2\mu^2} .$$

The inflation rate therefore decreases after period τ.[1]

Finally, we want to show that the maximum inflation rate occurs between periods $(\tau-1)$ and τ. We accordingly have to show that

$$\frac{\partial P_\tau}{\partial M_\tau} - \frac{\partial P_{\tau-1}}{\partial M_\tau} > \frac{\partial P_{\tau+1}}{\partial M_\tau} - \frac{\partial P_\tau}{\partial M_\tau}$$

or

$$\left[1 - \frac{b_2\mu}{b_1}\right](\mu(1 - b_2\mu)) > (b_1 - \mu)(1 - \mu)$$

or

$$\frac{\mu}{b_1}\,\frac{b_1 - b_2\mu}{b_1 - \mu}\,\frac{1 - b_2\mu}{1 - \mu} > 1 .$$

Since $b_2 < 1$, and $\mu < b_1$ (by the assumption noted in the preceding footnote), it will suffice to show that

$$\frac{\mu(1 - b_2\mu)}{b_1(1 - \mu)} > 1 ,$$

[1] This statement requires $b_1 > \mu$, which is guaranteed if $[a_2(\Theta-\lambda)(1-c_1-c_2a_1) + a_2\Theta c_1 - c_3a_1\lambda - c_2a_1a_2\lambda] > 0$, a condition we assume. It is satisfied if Θ is sufficiently greater than λ, for instance.

or $\quad \mu(1 - b_2\mu) > b_1(1 - \mu)$, or $\mu - b_2\mu^2 - b_1 + b_1\mu > 0$.

Now, from (5.6), we can substitute for $-(b_1 + b_2\mu^2)$, so we have to show:

$$\mu + (b_3 - 1)\mu + b_1\mu > 0$$

or $\quad (b_1 + b_3)\mu > 0$.

Since $b_1 + b_3 = -b_5 > 0$, the inflation rate has been shown to be at a maximum between periods $(\tau - 1)$ and τ.

3. To derive the behavior of N_t, r_t and Y_t, we work from (5.1) and (5.3) to obtain

$$\text{(A3.1)} \qquad N_t - N^* = \sum_{1}^{\infty} \Psi_i ({}_{t-i}P_{t-i+1} - P_{t-i})$$

$$\Psi_1 = \frac{c_2 a_2 \Theta\delta}{c_3 + c_2 a_2 + \beta\delta}$$

$$\Psi_i = \left[\frac{(c_3 + c_2 a_2)(1-\Theta) + \beta\delta}{c_3 + c_2 a_2 + \beta\delta}\right]^{i-1} \Psi_1$$

$$= \left[\frac{-b_5}{b_4}\right]^{i-1} \Psi_1$$

$$\text{(A3.2)} \qquad r_t - \text{constant} = \frac{-c_2 a_2}{c_3 + c_2 a_2 + \beta\delta} \sum_{i=0}^{\infty} \xi_i ({}_{t-i}P_{t+1-i} - P_{t-i})$$

$$\xi_0 = 1$$

$$\xi_1 = \frac{\Theta\beta\delta}{c_3 + c_2 a_2 + \beta\delta}$$

$$\xi_i = \left[\frac{(c_3 + c_2a_2)(1-\Theta) + \beta\delta}{c_3 + c_2a_2 + \beta\delta}\right]^{i-1} \xi_1, \; i = 2, \ldots, \infty$$

$$= \left[\frac{-b_5}{b_4}\right]^{i-1} \xi_1$$

We also use

$$\text{(A3.3)} \qquad Y_t = \frac{\lambda}{\Theta}(N_{t+1} - N_t)$$

4. We do not intend giving formulae corresponding to all the figures in Section 5, but note, using (A2.1) and (A3.1) that it can be shown that, in response to a fully anticipated change in the stock of money in period τ:

$$\frac{\partial(N_\tau - N^*)}{\partial M_\tau} = \frac{\Psi_1\mu(1 - b_2)}{b_1 - b_2\mu^2}$$

and

$$\frac{\partial(N_{\tau-i} - N^*)}{\partial M_\tau} = \left[\frac{b_2\mu}{b_1}\right]^i \frac{\partial(N_\tau - N^*)}{\partial M_\tau}$$

$$\frac{\partial(N_{\tau+1} - N^*)}{\partial M_\tau} = \mu \frac{\partial(N_\tau - N^*)}{\partial M_\tau} < \frac{\partial(N_\tau - N^*)}{\partial M_\tau}$$

5. Next we move to the effects of an increase in the <u>growth rate</u> of money. Specifically, we assume

$$M_t - M_{t-1} = 0, \; t = -\infty, \ldots, \tau$$

$$M_t - M_{t-1} = 1, \; t = \tau+1, \ldots, \infty$$

Using (5.5) and (5.7) it is relatively straightforward to show

(A5.1)
$$\frac{\partial(P_{\tau+1} - P_{\tau})}{\partial g} = \frac{\mu - b_2\mu^2}{b_1 - b_2\mu^2} < 1$$

$$\frac{\partial(P_{\tau-i+1} - P_{\tau-i})}{\partial g} = \left[\frac{b_2\mu}{b_1}\right]^i \frac{\partial(P_{\tau+1} - P_{\tau})}{\partial g} \qquad i = 1, 2, \ldots$$

$$\frac{\partial(P_{\tau+1+i} - P_{\tau+i})}{\partial g} = 1 - \frac{\mu^i(b_1 - \mu)}{b_1 - b_2\mu^2} \qquad i = 1, 2, \ldots.$$

where g is the change in the growth rate of money described above.

6. Looking at (A3.1), it is clear that inventories build up as the inflation rate increases; similarly from (A3.2), the real rate of interest falls continuously as the inflation rate increases. To study the behavior of output, use (A3.3); we leave it as an exercise to show, based on (A3.1) and (A5.1) that:

(A6.1)
$$\frac{\partial N_{\tau+1}}{\partial g} = \frac{\Psi_1 \mu b_1(1 - b_2)}{(b_1 - b_2\mu^2)(b_1 - b_2\mu)}$$

$$\frac{\partial N_{\tau+1-i}}{\partial g} = \left[\frac{b_2\mu}{b_1}\right]^i \frac{\partial N_{\tau}}{\partial g} \qquad i = 1, 2, \ldots$$

$$\frac{\partial N_{\tau+1+i}}{\partial g} = \frac{\partial N_{\tau+i}}{\partial g} + \frac{\mu^{i+1}\Psi_1(1 - b_2)}{b_1 - b_2\mu^2} \qquad i = 1, 2, \ldots,$$

Accordingly

$$\frac{\partial(N_{\tau+2} - N_{\tau+1})}{\partial g} < \frac{\partial(N_{\tau+1} - N_{\tau-1})}{\partial g}\ .$$