EXCHANGE RATES IN THE 1920'S:
A MONETARY APPROACH

Jacob A. Frenkel

Kenneth W. Clements

Summary of

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Current views about flexible exchange rate systems are based, to a
large extent, on the lessons from the period of the 1920's during which
many exchange rates were flexible. This paper re-examines the evidence from
the perspective of the recently revived monetary approach (or more generally,
asset-market approach) to the exchange rate. The analysis starts by developing
a simple monetary model of exchange rate determination. The key characteristic
of the model lies in the notion that, being a relative price of two monies,
the equilibrium exchange rate is attained when the existing stocks of the
two monies are willingly held. The equilibrium exchange rate is shown to
depend on both real and monetary factors which operate through their influence
on the relative demands and supplies of monies. The analysis then proceeds to
examine the relationship between spot and forward rates for the Franc/Pound,
Dollar/Pound and Franc/Dollar exchange rates and the results are shown to be
consistent with the efficient market hypothesis.

The monetary model is then estimated using monthly data and using the
forward premium on foreign exchange as a measure of expectations. In addition
to the single-equation ordinary-least-squares estimates, the various exchange
rates are also estimated as a system using the mixed-estimation procedure which
combines the sample information with prior information which derives from the
homogeneity postulate and from known properties of the demand for money. The
various results are shown to be consistent with the predictions of the monetary
model.

Professor Jacob A. Frenkel
Department of Economics
University of Chicago
1129 E 59th Street
Chicago, Illinois 60637

(312) 753-4516

Professor Kenneth W. Clements
Graduate School of Business
University of Chicago
5836 Greenwood Avenue
Chicago, Illinois 60637

(312) 753-3616

## Introduction

Current views about flexible exchange rate systems are based, to a large extent, on the lessons from the period of the 1920's during which many exchange rates were flexible. The experience of that period has proven to be extremely important in shaping current thinking about, and understanding of, the operation of flexible exchange rates. It led to an examination of various issues that are associated with a regime of flexible exchange rates like speculation, the efficiency of the foreign exchange markets, the purchasing power parity doctrine, the factors which determine equilibrium exchange rates, and the like.[1] This paper reexamines some of these issues from the perspective of the recently revived monetary approach (or more generally, asset-market approach) to the exchange rate. In Section I we develop a simple monetary model of exchange rate determination. The key characteristic of the model lies in the notion that, being a relative price of two monies, the equilibrium exchange rate is attained when the existing stocks of the two monies are willingly held. The equilibrium exchange rate is shown to depend on both real and monetary factors which operate through their influence on the relative demands and supplies of monies. In Section II we examine some of the efficiency properties of the foreign exchange market where we analyze the relationship between spot and forward rates for the Franc/Pound, Dollar/Pound and Franc/Dollar exchange rates. Section III

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[1]See, for example, Aliber (1962), Dulles (1929), Farag and Ott (1964), Frenkel (1978 ), Hodgson (1972), Stolper (1948) and Tsiang (1959-60).

contains the results of the empirical test of the monetary model of exchange

rate determination. Using monthly data we estimate various versions of the

model and allow for interrelationships between the various exchange rates;

we then use the estimated parameters for dynamic simulations. Section IV

contains some concluding remarks.


I. A Model of Exchange Rate Determination

In this section we outline a simple model of exchange rate determina-

tion which reflects the recent revival of the monetary approach (or more

generally, the asset-market approach) to the analysis of exchange rates.[1]

The monetary view of exchange rate determination emphasizes that the exchange

rate, being the relative price of two national monies, is determined primarily

by factors which affect the relative supplies and demands for these monies.

Since the demands for the various national monies depend on expectations,

incomes, rates of return and other considrations which are relevant for

portfolio choice, the approach is referred to as an asset-market approach to

the determination of exchange rates. The major building blocks of the theory

are hypotheses concerning (i) the properties of the demand for money (ii) the

purchasing power parity condition and (iii) the interest-parity theory.

Consider first the equilibrium in the money markets. The supplies

of domestic and foreign real balances are $M/P$ and $M^*/P^*$ where $M$ and $P$ denote

the nominal money supply and the price level, respectively, and where variables

pertaining to the foreign country are indicated by an asterisk. Denoting the

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[1]For theoretical developments and applications of the approach see,
for example, Dornbusch (1976a,1976b), Kouri (1976), Mussa (1976), Frenkel
(1976), Frenkel and Johnson (1978), Bilson (1978) and Hodrick (1978).

demands for real balances by L and L* (both of which are functions which will be specified below), equilibrium in the money markets is attained when

(1)     $L = M/P$ and

(2)     $L* = M*/P*.$

From equations (1)-(2), equilibrium in the money markets implies that the ratio of the two price levels is:

(3)     $$\frac{P}{P*} = \frac{M}{M*} \frac{L*}{L} .$$

The second building block links domestic and foreign prices through the purchasing power parity condition according to which,

(4)     $P = SP*$

where S denotes the spot exchange rate--the price of foreign exchange in terms of domestic currency.[1] Using equation (4) in (3) yields

(5)     $$S = \frac{M}{M*} \frac{L*}{L}$$

which expresses the exchange rate in terms of domestic and foreign supplies and demands for money. To gain further insight into the determinants of the exchange rate, assume that the demand for money depends on real income (y) and the rate of interest (i) according to:

(6)     $L = ay^{\eta}e^{-\alpha i}$

(7)     $L* = b*y*^{\eta *}e^{-\alpha *i*}$

Using equations (6)-(7) in (5) and assuming for simplicity of exposition that foreign and domestic parameters of the demand for money are the same, i.e.,

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[1]For a discussion of the choice of the relevant price index to be used in equation (4) see Frenkel (1978 ). To the extent that purchasing power parity pertains to traded goods only, the exchange rate equation would also contain terms which relate to the relative prices of traded to non-traded goods; for a formulation

that $\alpha = \alpha*$, and that $\eta = \eta*$, we obtain:

$$(8) \qquad \ln S = C + \ln \frac{M}{M*} + \eta \ln \frac{y*}{y} + \alpha(i - i*)$$

where $C \equiv \ln(b*/a)$.

Denoting a percentage change in a variable by a circumflex (e.g.,
$\hat{x} = \Delta x/x$), the percentage change in the exchange rate is:

$$(9) \qquad \hat{S} = (\hat{M} - \hat{M*}) + \eta(\hat{y*} - \hat{y}) + \alpha\Delta(i - i*).$$

Equation (9) relates the percentage change in the exchange rate to the
differences between domestic and foreign (i) rates of monetary expansion,
(ii) rates of growth of real incomes, and (iii) changes in interest rate
differentials.

The third building block is the interest parity theory according to
which, in equilibrium, the premium on a forward contract for foreign exchange
for a given maturity is (approximately) equal to the interest rate differential:

$$(10) \qquad i - i* = \pi; \; \pi \equiv \ln (F/S)$$

where $\pi$ denotes the forward premium on foreign exchange and where F denotes
the forward exchange rate. Substituting (10) into (8) and (9) yields:

$$(11) \qquad \ln S = C + \ln \frac{M}{M*} + \eta \ln \frac{y*}{y} + \alpha\pi$$

$$(12) \qquad \hat{S} = (\hat{M} - \hat{M*}) + \eta(\hat{y*} - \hat{y}) + \alpha\Delta\pi$$

The role of expectations in the determination of exchange rates is summarized
by the forward premium or discount on foreign exchange. The monetary approach
that is summarized in equations (11)-(12) differs from previous approaches to
exchange rate determination in that concepts like exports, imports, tariffs and

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along these lines see Dornbusch (1976b). A more refined specification would allow
for the effects of tariffs on the relationship between domestic and foreign prices
as well as for short-run effects of unanticipated money on output rather than only
on prices and the exchange rate.

the like, do not appear as being fundamentally relevant for the understanding

of the evolution of the exchange rate. Rather, the relevant concepts

relate to two groups of variables: first are those which are determined by

the monetary authorities and second, are those which affect the demands

for domestic and foreign monies.

The implications of the model are that, ceteris paribus, (i) a rise

in the supply of domestic money will depreciate the home currency (i.e.,

raise S) while a rise in the supply of foreign money will appreciate the

home currency; (ii) a rise in domestic income will appreciate the currency

(lower S), while a rise in foreign income will depreciate the currency,

and (iii) a rise in the forward premium on foreign exchange will depreciate

the currency (raise S). This dependence of the current exchange rate on

expectations concerning the future rate (as summarized by the forward premium) is

a typical characteristic of price determination in asset markets. Thus,

an expected future depreciation of the currency is reflected immediately

in the current value of the currency. The model yields the following predic-

tions concerning the values of the various elasticities (i) the elasticity

of the exchange rate with respect to the domestic money supply is unity

and the elasticity with respect to the foreign money supply is minus unity; (ii) the

elasticity with respect to domestic and foreign real incomes should approxi-

mate the income elasticities of the demand for money $\eta$ (positive for foreign

income and negative for domestic income), and (iii) the (semi) elasticity

with respect to the forward premium should approximate (in absolute value)

the interest (semi) elasticity of the demand for money. These predictions

are examined in Section III below.

In the above model we have not drawn the distinction between "the

demand for domestic money" and "the domestic demand for money." Implicitly

it has been assumed that domestic money is demanded only by domestic

residents while foreign money is demanded only by foreign residents.

Furthermore, the formulation of the demands for real cash balances (in

equations (6)-(7)) included the domestic interest rate in the domestic

demand, and the foreign interest rate in the foreign demand; it has been

implicitly assumed that the only relevant alternative for holding domestic

money is domestic securities while the only relevant alternative for

holding foreign money is foreign securities. In principle, however, the

alternatives to holding domestic money are domestic securities as well as

foreign securities and foreign exchange. It follows that a richer formula-

tion of the demand for money would recognize that, as an analytical matter,

the spectrum of alternative assets and rates of return that are relevant

for the specification of the demand for money is rather broad, including

both rates of interest, $i$ and $i*$, as well as the forward premium on foreign

exchange $\pi$. Furthermore, to the extent that under a flexible exchange rate

system individuals might wish to diversify their currency holdings, the

demand for domestic money would include a foreign component which depends

on foreign income, while the demand for foreign money would include a domestic

component which depends on domestic income.[1] These characteristics reflect

the phenomenon of currency substitution which is likely to arise when the

exchange rate is not pegged.[2]

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[1]For a discussion of the specification of the demand for money under
a flexible exchange rate regime see Frenkel (1977, 1979 ) and Abel, Dornbusch,
Huizinga and Marcus (1979).

[2]For an analysis of the phenomenon of currency substitution see
Boyer (1973), Chen (1973), Chrystal (1977), Girton and Roper (1976), Miles
(1976), Stockman (1976) and Calvo and Rodriguez (1977). On the extent of
currency substitution during the German hyperinflation see Frenkel (1977).

The above discussion suggests the need for a more detailed specification of the demand for money. Let domestic demand for domestic money depend on domestic income and on the three alternative rates of return according to:

(13) $\quad L_1 = ay^{\eta_1} \exp(-\alpha_1 i - \beta_1 i^* - \gamma_1 \pi)$

and let the foreign demand for domestic money be

(14) $\quad L_1^* = a^* y^{* \eta_1^*} \exp(-\alpha_1^* i - \beta_1^* i^* - \gamma_1^* \pi)$

where the total demand for domestic money is $L_1 + L_1^* = L$. Analogously, the demand for foreign money is also composed of domestic and foreign components: domestic demand for foreign money $L_2$ and foreign demand for foreign money $L_2^*$ according to:

(15) $\quad L_2 = by^{\eta_2} \exp(-\alpha_2 i - \beta_2 i^* + \gamma_2 \pi)$

(16) $\quad L_2^* = b^* y^{* \eta_2^*} \exp(-\alpha_2^* i - \beta_2^* i^* + \gamma_2^* \pi)$

where the total demand for foreign money is $L_2 + L_2^* = L^*$.

Using the above relationships, the domestic and the foreign demands for money are $L_1 + L_2$ and $L_1^* + L_2^*$, respectively. Substituting equations (13)-(16) into equation (5) yields the more general relationship between the exchange rate and the various components of the supplies and demands for the two monies. For expositional simplicity, assume that all the various income elasticities of money demands are equal to $\eta$, i.e., that $\eta_1 = \eta_1^* = \eta_2 = \eta_2^* = \eta$, that the semi-elasticities of the demands for money with respect to both interest rates are equal, i.e., that $\alpha_i = \beta_i = \alpha_i^* = \beta_i^*$ $(i = 1,2)$, and that the semi-elasticities of the demands for money with respect to the forward premium on foreign exchange are equal to $\gamma$, i.e., that

$\gamma_1 = \gamma_1^* = \gamma_2 = \gamma_2^* = \gamma$. Under these assumptions the exchange rate can be written as:

$$(17) \qquad S = \frac{M}{M^*} \frac{by^\eta + b^*y^{*\eta}}{ay^\eta + a^*y^{*\eta}} e^{\beta\pi}$$

where $\beta \equiv 2\gamma$.

From (17), the percentage change in the exchange rate is

$$(18) \qquad \hat{S} = (\hat{M} - \hat{M}^*) + \eta(\lambda^* + \lambda - 1)(\hat{y}^* - \hat{y}) + \beta\Delta\pi$$

where

$\lambda^* \equiv L_2^*/L^*$ and

$\lambda \equiv L_1/L$.

The implications of equation (18) are similar to those of equation (12) except for the inference concerning the elasticity of the exchange rate with respect to real incomes. When individuals hold diversified portfolios of currencies, the effects of income growth on the exchange rate depend on the income elasticity of the demand for money as well as on the parameters $\lambda$ and $\lambda^*$ indicating the currency mix of money holdings. If domestic and foreign residents hold portfolios with identical currency mixes, changes in incomes will not affect the exchange rate. In general, it is expected that the typical portfolio of currencies will be intensive in the local currency and thus, that both $\lambda$ and $\lambda^*$ are each larger than one-half and, therefore, that $\lambda + \lambda^* > 1$. Under these circumstances it is expected that the elasticity of the exchange rate with respect to domestic income would be negative while the elasticity with respect to foreign income would be positive but somewhat less than the corresponding income elasticities of the demands for money since $(\lambda + \lambda^* - 1)$ is less than unity. In the special case where currency holdings

are not diverisifed, individuals hold only local currency, $\lambda = \lambda* = 1$, and equation (18) reduces to equation (12).

In this section we outlined a simple model of the determinants of the relative price of two monies. The major determinants of the exchange rate and its evolution were described in terms of the relative supplies and demands for domestic and foreign monies. The prime determinants of the relative demands were shown to be relative incomes, the currency mix of portfolios and expectations concerning the future evolution of the exchange rate as measured by the forward premium on foreign exchange. The special role that is played by expectations concerning future course of events is a typical characteristic of the determinants of prices in asset markets. From the policy perspective, the model highlights the unique role played by the monetary authorities in affecting the rate of exchange.

Prior to concluding this section it should be emphasized that the monetary (or the asset-market) approach to the exchange rate does not claim that the exchange rate is "determined" only in the money or in the asset-markets and that only stock rather than flow considerations are relevant for determining the equilibrium exchange rate. Obviously, general equilibrium relationships which are relevant for the determination of exchange rates include both stock and flow variables. In this respect, the asset market equilibrium relationship that has been used, may be viewed as a reduced form relationship. Furthermore, the fact that the analysis of the exchange rate has been carried out in terms of the supplies and the demands for monies, does not imply that "only money matters"; on the contrary, the demand for money depends on real variables like real income as well as on other real variables which underlie expectations. The rationale for concentrating on

the relative supplies and demands for money is that they provide a convenient

and a natural framework for organizing thoughts concerning the determinants

of the relative price of monies.  It is the same principle which has been

used by proponents of the monetary approach to the balance of payments in

justifying the use of the money demand-money supply framework for the

analysis of the money account of the balance of payments under a pegged exchange

rate system.[1]

## II.  Efficiency of Foreign Exchange Markets

In the previous section we outlined the basic building blocks of an

asset market approach to the determination of exchange rates.  We have argued

that since the exchange rate is the relative price of two assets, it

depends (like other asset prices) on expectations concerning the future course

of events.  We have suggested to measure these expectations by using data

from the forward market for foreign exchange.

The assumption underlying the use of data from the forward market

for foreign exchange is that this market is indeed useful in conveying

information concerning expectations held by market participants.  Therefore,

prior to incorporating such data in the empirical analysis of exchange

rate determination, it is important to study some of the characteristics

of the market.  In this section we examine some of the efficiency properties

of the market for foreign exchange.[2]

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[1] See for example Mussa (1974) and Frenkel and Johnson (1976); in
the context of flexible exchange rates the same argument is made by Dornbusch
(1976b) and Mussa (1976).

[2] For an application of the same methodology in analyzing the efficiency
properties of the foreign exchange market during the German hyperinflation
(1921-1923) see Frenkel (1976, 1977, 1979).  For an application to the 1920's
and the 1970's see Krugman (1977) and for a survey see Levich (1978).

If the foreign exchange market is efficient and if the exchange rate is determined in a fashion similar to other asset prices, we should expect that current prices reflect all available information. Expectations concerning future exchange rates should be incorporated and reflected in forward exchange rates. To examine the efficiency of the market we first regress the logarithm of the current spot exchange rate, $\ln S_t$, on the logarithm of the one-month forward exchange rate prevailing at the previous month, $\ln F_{t-1}$, as in equation (19). Similar regressions are also computed using the levels of the exchange rates rather than their logarithms, as in equation (19').

(19) $$\ln S_t = a + b\ln F_{t-1} + u_t$$

(19') $$S_t = a' + b'F_{t-1} + v_t$$

If the market for foreign exchange is efficient and if the forward exchange rate is an unbiased forecast of the future spot exchange rate, then we expect the following three properties: (i) the constant terms in equations (19) and (19') should not differ significantly from zero, (ii) the slope coefficients should not differ significantly from unity and, (iii) the residuals should be serially uncorrelated. We examine three exchange rates: the Franc/Pound, the Dollar/Pound and the Franc/Dollar. Obviously, only two of the three rates are independent due to triangular arbitrage. Equations (19)-(19') were estimated using monthly data for 51 months over the period February 1921-May 1925 (for details on the data and on data sources see Appendix B). The resulting ordinary-least-squares estimates are reported in Tables 1 and 2. Also reported in these Tables are regression results which include $F_{t-2}$ as an additional explanatory variable.

## TABLE 1

### EFFICIENCY OF FOREIGN EXCHANGE MARKETS

### MONTHLY DATA:    FEBRUARY 1921 – MAY 1925

| Dependent Variable | Constant | $\ln F_{t-1}$ | $\ln F_{t-2}$ | $R^2$ | s.e. | D.W. |
|---|---|---|---|---|---|---|
| $\ln S_t$ (Franc/Pound) | .169 (.179) | .962 (.042) | -- | .91 | .07 | 1.92 |
| $\ln S_t$ (Franc/Pound) | .177 (.187) | .992 (.144) | -.032 (.147) | .91 | .07 | 1.97 |
| $\ln S_t$ (Dollar/Pound) | .057 (.056) | .964 (.038) | -- | .93 | .02 | 1.54 |
| $\ln S_t$ (Dollar/Pound) | .073 (.057) | 1.181 (.143) | -.229 (.142) | .93 | .02 | 2.11 |
| $\ln S_t$ (Franc/Dollar) | .203 (.149) | .928 (.054) | -- | .85 | .08 | 1.95 |
| $\ln S_t$ (Franc/Dollar) | .206 (.156) | .945 (.145) | -.018 (.146) | .85 | .08 | 1.98 |

Standard errors are in parentheses below each coefficient; s.e. is the standard error of the equation.

TABLE 2

EFFICIENCY OF FOREIGN EXCHANGE MARKETS

MONTHLY DATA:   FEBRUARY 1921 – MAY 1925

| Dependent Variable | Constant | $F_{t-1}$ | $F_{t-2}$ | $R^2$ | s.e. | D.W. |
|---|---|---|---|---|---|---|
| $S_t$ (Franc/Pound) | 3.801 (3.335) | .955 (.047) | -- | .89 | 5.43 | 1.95 |
| $S_t$ (Franc/Pound) | 3.898 (3.500) | .966 (.145) | -.012 (.148) | .89 | 5.54 | 1.96 |
| $S_t$ (Dollar/Pound) | .159 (.163) | .967 (.037) | -- | .93 | .08 | 1.56 |
| $S_t$ (Dollar/Pound) | .205 (.167) | 1.173 (.143) | -.217 (.143) | .93 | .08 | 2.11 |
| $S_t$ (Franc/Dollar) | 1.470 (.942) | .913 (.059) | -- | .83 | 1.32 | 1.91 |
| $S_t$ (Franc/Dollar) | 1.520 (.990) | .943 (.145) | -.034 (.147) | .83 | 1.34 | 1.96 |

Standard errors are in parentheses below each coefficient; s.e. is the
standard error of the equation.

As can be seen the results are consistent with the three hypotheses outlined above. In all cases the constant terms do not differ significantly from zero at the 95 percent confidence level. Furthermore, the Durbin-Watson statistics are consistent with the hypothesis of the absence of first-order autocorrelated residuals.[1] In all cases, at the 95 percent confidence level, we cannot reject the joint hypothesis that the constant terms are zero and that the slope coefficients are unity.

We have argued above that in an efficient market, expectations concerning future exchange rates are reflected in forward rates and, that spot exchange rates reflect all available information. If forward exchange rates prevailing at period t-1 summarize all relevant information available at that period, they should also contain the information that is summarized in data corresponding to period t-2. It thus follows that including additional lagged values of the forward rates in equations (19) and (19') should not greatly affect the coefficients of determination and should not yield coefficients that differ significantly from zero. The results reported in Tables 1 and 2 are consistent with this hypothesis; in all cases the coeffcients of $F_{t-2}$ do not differ significantly from zero and the inclusion of the additional lagged variables has not improved the fit.[2]

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[1] With n = 50 and with one explanatory variable and a constant, the lower and upper bounds of the 5 percent points of the Durbin-Watson test statistic are $d_L$ = 1.50 and $d_u$ = 1.59. The corresponding bounds of the one percent points are $d_L$ = 1.32 and $d_u$ = 1.40. Thus in all cases, at the one percent confidence level, we cannot reject the hypothesis that successive residuals are not correlated. At the 5 percent confidence level we reach the same conclusion in all cases except for the Dollar/Pound exchange rate for which the value of the Durbin-Watson statistic falls in the inconclusive range. We have also examined higher order correlation up to 12 lags; no correlation of any order was significant.

[2] To test whether $F_{t-1}$ contains all available information, we have also followed the procedure used by Fama (1975) and have included the lagged dependent variable $S_{t-1}$ instead of the additional lagged independent variable $F_{t-2}$. Since, however, $S_{t-1}$ and $F_{t-1}$ are highly correlated, the resulting point estimates are imprecise. However, as expected the sums of the coefficients of $F_{t-1}$ and $S_{t-1}$ do not differ significantly from unity. It might be noted that since $F_{t-1}$ and $S_{t-1}$ summarize information concerning the same period, they are expected to be highly correlated. In this sense the use of the pair $F_{t-1}$ and $F_{t-2}$ seems preferable. On the relationship between forward and future spot rates, see Fama (1976).

The results reported in this section are consistent with the hypotheses that during the period under consideration the markets for foreign exchange seem to have been efficient, and the various forward exchange rates seem to have been unbiased forecasts of the future spot exchange rates. These results provide support to the notion that data from the forward market for foreign exchange provide useful information concerning expectations held by market participants. In the following section we will use these data as measures of expectations in the analysis of the determinants of exchange rates.

### III. Empirical Tests of the Monetary Approach to the Exchange Rate

In this section we incorporate the information about expectations into the estimation of the monetary model of exchange rate determination. We start with an analysis of the sample information.

### III.1 Estimates of the Model: Sample Information

The analysis in Section I implies that the exchange rate between two currencies can be expressed in terms of domestic and foreign monies, incomes, and the forward premium on foreign exchange which reflects expectations. Thus, for estimation purposes, the exchange rate between currencies i and j at time t can be written as:

$$(20) \quad \ell n \; S_{ijt} = \alpha_{ij}^0 + \alpha_{ij}^1 \; \ell n \; M_{it} + \alpha_{ij}^2 \; \ell n \; M_{jt} + \alpha_{ij}^3 \ell n \; y_{it}$$

$$+ \alpha_{ij}^4 \; \ell n \; y_{jt} + \alpha_{ij}^5 \; \pi_{ijt} + u_{ijt}$$

where $u_{ijt}$ is a stochastic disturbance term.

In terms of our previous notations, if i denotes the home country and j the foreign country then $\ell n \; M_i$ refers to $\ell n \; M$, $\ell n \; M_j$ refers to $\ell n \; M*$, with analogous

notations pertaining to real incomes. As indicated above, the prior expectations are that the elasticity of the exchange rate with respect to domestic money supply is unity while the corresponding elasticity with respect to foreign money supply is minus unity. The discussion concerning currency substitution suggests that we do not have clear prior expectations concerning the magnitude of the income elasticities; we expect, however, that the elasticity with respect to domestic income is negative while the elasticity with respect to foreign income is positive. Finally, we expect the elasticity of the exchange rate with respect to the forward premium to be positive, and related to the magnitude of   the interest semi-elasticity of the demand for money.

We have examined three pairs of currencies:  the Franc/Pound, the Franc/Dollar and the Dollar/Pound exchange rates. Since, however, only two of these rates are independent due to triangular arbitrage, we analyze in detail only the Franc/Pound and the Franc/Dollar exchange rates.[1] Using monthly data on exchange rates, monies, incomes and the forward premia on foreign exchange, we have estimated equation (20) in first differences for the two rates.[2] These results are reported in Table 3. As is apparent, due to the high degree of collinearity, the individual parameter estimates are extremely imprecise; we will deal with this problem below. It should be noted, however, that in terms of the overall fit the regression equations as a whole are reasonably satisfactory, particulary in view of the fact that the model is estimated using monthly data. Figures 1 and 2 illustrate the overall fit

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[1]It should be noted that some of the results obtained for the Dollar/Pound exchange rate are not in full agreement with the implications of the other two rates. We intend to elaborate on the analysis of the Dollar/Pound rate in a separate paper.

[2]Details on the data are provided in Appendix B.

TABLE 3

SINGLE EQUATION LEAST SQUARES ESTIMATES
MONTHLY DATA: FEBRUARY 1921-MAY 1925[a]

| Equation | Constant | $\ell n\,M$ | $\ell n\,M^*$ | $\ell n\,y$ | $\ell n\,y^*$ | $\pi$ | D.W. | RMSE | Quasi-$R^2$ |
|---|---|---|---|---|---|---|---|---|---|
| Franc/Pound | 0.004 (0.011) | 0.952 (0.809) | -0.011 (0.587) | 0.100 (0.287) | 0.974 (0.521) | 3.194 (4.038) | 2.03 | 0.067 | 0.923 |
| Franc/Dollar | 0.004 (0.013) | 0.448 (0.928) | 0.021 (1.429) | 0.235 (0.328) | -0.456 (0.384) | 4.647 (4.491) | 1.83 | 0.074 | 0.865 |

[a] For the exchange rate between the currencies of countries $i$ and $j$, the estimating equation is

$$\ell n\,S_{ijt} = \alpha_{ij}^0 + \alpha_{ij}^1\,\ell n\,M_{it} + \alpha_{ij}^2\,\ell n\,M_{jt} + \alpha_{ij}^3\,\ell n\,y_{it} + \alpha_{ij}^4\,\ell n\,y_{jt} + \alpha_{ij}^5\,\pi_{ijt} + u_{ijt},$$

where $u_{ijt}$ is a stochastic disturbance term. In the table $\ell n\,M$ and $\ell n\,M^*$ refer to $\ell n\,M_i$ and $\ell n\,M_j$, respectively; similarly, $\ell n\,y$ and $\ell n\,y^*$ denote $\ell n\,y_i$ and $\ell n\,y_j$, respectively.

To eliminate residual autocorrelation, we have transformed the data by taking first differences. For equation $i$, RMSE $= \sqrt{\mathrm{var}(\hat{u}_i)}$, where $\hat{u}_i$ is the residual of that equation, Quasi-$R^2 = 1 - \mathrm{var}(\hat{u}_i)/\mathrm{var}(z_i)$, where $z_i$ is the untransformed dependent variable and D.W. is the Durbin-Watson statistic. Standard errors are given in parentheses below each estimated parameter.
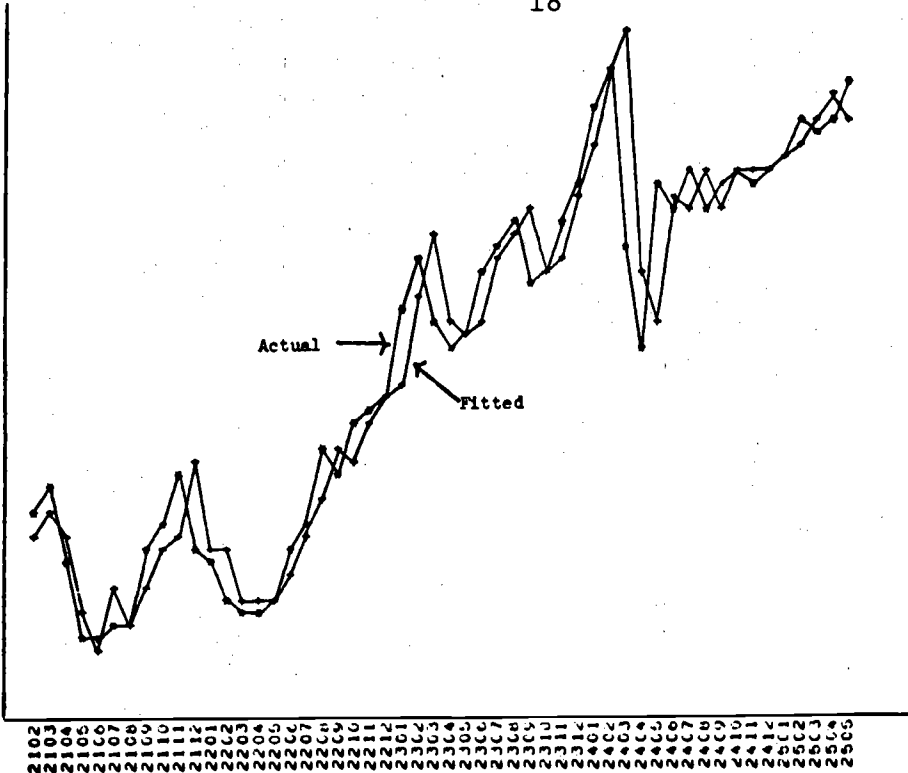
Fig. 1. Franc/Pound spot exchange rate: Actual and fitted
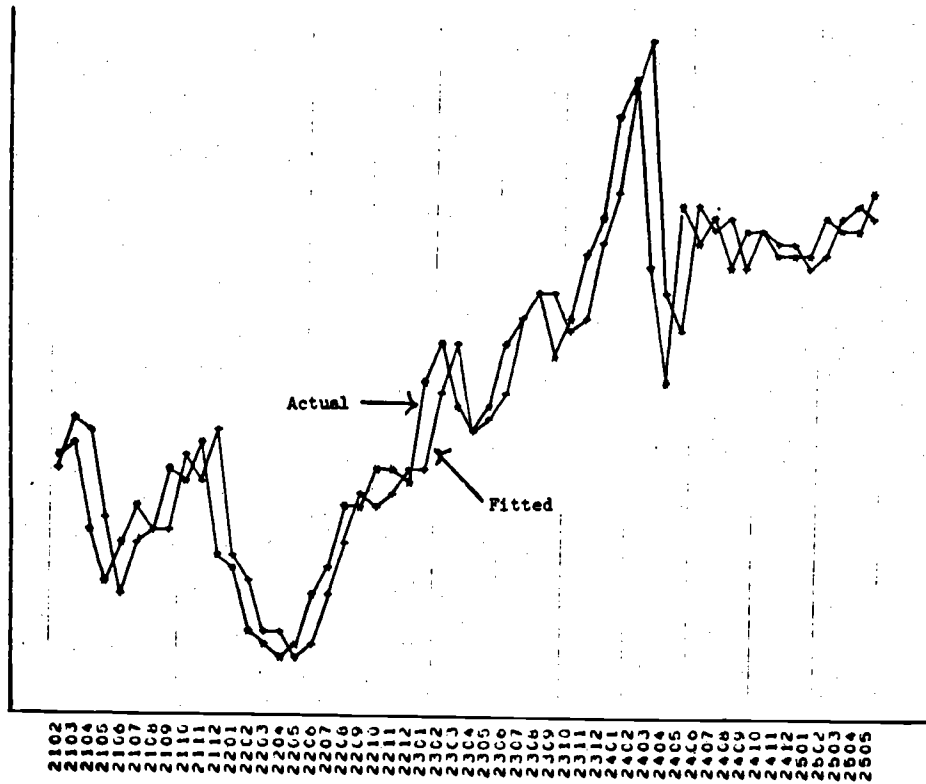(single equation least squares estimates).



Fig. 2. Franc/Dollar spot exchange rate: Actual and fitted
(single equation least squares estimates).

by plotting actual and fitted values of the two exchange rates.[1]

The foregoing analysis has viewed the equation determining the Franc/Pound exchange rate as being completely separated from the corresponding equation determining the Franc/Dollar exchange rate. Implicitly it has been assumed that the right-hand side of equation (20) represents all that there is to know about the specific regression equation. In practice, however, various exchange rates may be subject to common shocks reflecting the fact that markets are interrelated and that in a fundamental sense, the various exchange rates belong to a global system. To allow for the correlation among disturbances we have viewed the two exchange rates as forming a system of equations and have reestimated the system using Zellner's (1962) method for estimating seemingly unrelated regressions.[2] The estimates are reported in Table 4 and, as may be seen, the qualitative results are similar to those of the single equation estimates: the individual parameter estimates are extremely imprecise while the overall fit is reasonably satisfactory. In what follows we attempt to improve the precision of the individual parameter estimates.

### III.2 Combining Sample and Prior Information

Due to the high degree of collinearity, the information that is contained in the sample is not sufficient to provide precise estimates of the various parameters; at best the sample information can provide estimates of the overall fit of the various regression equations. In order to obtain

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[1] In computing the fitted values of the exchange rate, we have added the fitted change in the exchange rate (from Table 3) to the previous periods' actual level of the exchange rate; thus, in effect, we have computed the one period forecast. The dynamic simulations reported in Table 7 below deal with the multiperiod forecast.

[2] This procedure allows for both heteroscedasticity and correlation across equations by applying the Aitken estimator to the model when it is stacked equation by equation.

TABLE 4

SEEMINGLY UNRELATED ESTIMATES
MONTHLY DATA: FEBRUARY 1921–May 1925[a]

| Equation | Constant | $\ln M$ | $\ln M^*$ | $\ln y$ | $\ln y^*$ | $\pi$ | D.W. | RMSE | Quasi-$R^2$ | $\omega$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Franc/Pound | 0.006<br>(0.011) | 1.021<br>(0.808) | -0.121<br>(0.239) | 0.202<br>(0.280) | -0.024<br>(0.216) | 1.163<br>(3.639) | 1.93 | 0.072 | 0.910 | 0.973 |
| Franc/Dollar | 0.005<br>(0.012) | 0.634<br>(0.903) | -0.321<br>(0.594) | 0.166<br>(0.311) | -0.197<br>(0.157) | 2.575<br>(4.005) | 1.91 | 0.074 | 0.863 | |

[a]The model is given in note (a) to Table 3. $\omega$ is the estimated correlation coefficient between the residuals of the two equations. For the meaning of the other notation, see the notes to Table 3. To eliminate residual auto-correlation, we have transformed the data by taking first differences. Standard errors are given in parentheses below each estimated parameter.

estimates that are more precise, a proper procedure is to supplement the information that is contained in the sample with some prior information. In incorporating the prior information, one may follow the Bayesian procedures or, alternatively, apply the mixed-estimation procedure.[1] In what follows we implement the mixed-estimation procedure which, as is shown in Appendix A, may be given a Bayesian interpretation.

We turn now to the specification of the prior information. The homogeneity postulate along with other studies on the relationship among money, prices and the exchange rate yield prior information concerning the elasticity of the exchange rate with respect to domestic and foreign monies, while evidence on the demand for money provide prior information concerning the elasticity of the exchange rate with respect to the forward premium. The prior knowledge is formulated in terms of stochastic restrictions on these elasticities whereby it is assumed that:

$$r_{ij}^1 = \alpha_{ij}^1 + v_{ij}; \quad E(v_{ij}) = 0, \quad E(v_{ij}^2) = .01 \text{ for all } i \text{ and } j,$$

$$E(v_{ij} v_{i'j'}) = 0 \text{ for all } i \neq i', \ j \neq j'.$$

(21)

$$r_{ij}^2 = \alpha_{ij}^2 + w_{ij}; \quad E(w_{ij}) = 0, \quad E(w_{ij}^2) = .01 \text{ for all } i \text{ and } j,$$

$$E(w_{ij} w_{i'j'}) = 0 \text{ for all } i \neq i', \ j \neq j',$$

$$E(v_{ij} \ w_{i'j'}) = 0 \text{ for all } i, i', j, j',$$

where $r_{ij}$ is the random prior estimate of the corresponding elasticity. It is further assumed that, on average, the homogeneity postulate holds and thus,

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[1]For details on the mixed-estimation procedure see Theil and Goldberger (1961), Theil (1963) and Theil (1971, Ch. 7); see also Appendix A. For an application of the mixed-estimation procedure to the analysis of the DM/Pound exchange rate during 1972-76, see Bilson (1978).

consistent with other studies on the relationship between money prices and
the exchange rate, our prior estimates of the elasticities of the exchange
rate with respect to domestic and foreign money supplies are unity and minus
unity, respectively. We allow however for some uncertainty about these
parameter values by including the random terms v and w which are assumed to
be distributed normally with a zero mean and a variance of .01. Thus, using
the 2-sigma range, the approximate 95 percent confidence intervals for $r^1_{ij}$
and $r^2_{ij}$ are (.8, 1.2) and (-.8, -1.2) respectively.

The prior knowledge about the elasticity of the exchange rate with
respect to the forward premium stems from estimates of the demand for money
and is formulated in a similar fashion. It is assumed that

$$r^5_{ij} = \alpha^5_{ij} + s_{ij}; \quad E(s_{ij}) = 0, \quad E(s^2_{ij}) = 1.00 \text{ for all i and j,}$$

(22)
$$E(s_{ij}s_{i'j'}) = 0 \text{ for all } i \neq i', \ j \neq j',$$

$$E(v_{ij}s_{i'j'}) = E(w_{ij}s_{i'j'}) = 0 \text{ for all } i, i', j, j'.$$

Our prior estimate of the coefficient on the forward premium is 4. In terms
of equation (9), with a monthly interest rate of one percent, this amounts to
an interest elasticity of the demand for money of -.04 which is consistent
with Goldfeld's (1973) estimates for the United States. The variance of the
disturbance terms s implies that the 95 percent confidence interval for the
prior estimate of the interest elasticity of the demand for money is approxi-
mately (-.02, -.06).[1]

The basic idea underlying the mixed-estimation method is to combine
the prior information about some of the parameters with the information that
is contained in the sample. The prior information has been summarized in

[1]In terms of the currency substitution model (equation (18)), this
amounts to an interest elasticity of -.02 with a 95 percent confidence
interval of (-.01, -.03).

equations (21)-(22) and the sample information has been summarized in Tables 3 and 4. The mixed-estimation procedure combines these two sources of information. Since neither of these two sources provides complete information about the coefficients, the estimates that are implied by the two sources may differ from each other. Therefore, prior to implementing the mixed-estimation procedure, it is important to verify that the sample and the prior information are compatible with each other. Under the null-hypothesis that the sample and the prior information are compatible, the relevant test is a $\chi^2$ test with degrees of freedom corresponding to the number of restrictions. Performing the compatibility test resulted in $\chi^2$ values of 2.88 and .74 for the Franc/ Pound and the Franc/Dollar exchange rates while the critical value for $\chi^2(3)$ at the .05 level is 7.82. Thus, since we cannot reject the compatibility hypothesis, we proceed with combining the two sources of information by implementing the mixed-estimation procedure to the first differences of the data. Details on the estimation procedure are provided in Appendix A.

The resulting mixed estimates for the two exchange rates are reported in Table 5. As may be seen, the results are consistent with the predictions of the monetary model. For both exchange rates the constant terms do not differ significantly from zero indicating that when we take account of the economic variables, there is not further autonomous trend to the exchange rates. Also, for both exchange rates, the elasticities with respect to domestic and foreign money supplies are unity and minus unity, respectively. These estimates are highly significant; they are about ten times the size of the corresponding standard errors. The semi-elasticities of the exchange rates with respect to the forward premia are about 4. Also these estimates are highly significant; they are about four times the size of the corresponding standard errors. The high level of significance reflects the fact that the sample provides very little

TABLE 5

SINGLE EQUATION MIXED ESTIMATES

MONTHLY DATA: FEBRUARY 1921–MAY 1925[a]

| Equation | Constant | ℓn M | ℓn M* | ℓn y | ℓn y* | π | D.W. | RMSE | Quasi-$R^2$ | $\chi^2$[b] |
|---|---|---|---|---|---|---|---|---|---|---|
| Franc/Pound | 0.001 (0.010) | 0.999 (0.099) | -0.972 (0.099) | 0.188 (0.281) | 0.926 (0.520) | 3.914 (0.970) | 1.86 | 0.069 | 0.917 | 2.88 (7.82) |
| Franc/Dollar | 0.006 (0.011) | 0.995 (0.099) | -0.995 (0.100) | 0.255 (0.327) | -0.369 (0.370) | 3.971 (0.974) | 1.81 | 0.075 | 0.860 | 0.74 (7.82) |

[a]The model is given in note (a) to Table 3. The following stochastic prior information is used in estimation:

$$r^1_{ij} = \alpha^1_{ij} + v_{ij} \quad E(v_{ij}) = 0 \quad E(v^2_{ij}) = 0.01 \quad \forall \; i, j$$

$$E(v_{ij} v_{i'j'}) = 0 \quad \forall \; i \neq i', \; j \neq j'$$

$$r^2_{ij} = \alpha^2_{ij} + w_{ij} \quad E(w_{ij}) = 0 \quad E(w^2_{ij}) = 0.01 \quad \forall \; i, j$$

$$E(w_{ij} w_{i'j'}) = 0 \quad \forall \; i \neq i', \; j \neq j'$$

$$E(w_{ij} v_{i'j'}) = 0 \quad \forall \; i, i', j, j'$$

$$r^5_{ij} = \alpha^5_{ij} + s_{ij} \quad E(s_{ij}) = 0 \quad E(s^2_{ij}) = 1.00 \quad \forall \; i, j$$

$$E(s_{ij} s_{i'j'}) = 0 \quad \forall \; i \neq i', \; j \neq j'$$

$$E(s_{ij} w_{i'j'}) = E(s_{ij} v_{i'j'}) = 0 \quad \forall \; i, i', j, j' \; .$$

The $r^k_{ij}$ are random prior estimates of the $\alpha^k_{ij}$ and the $v_{ij}$, $w_{ij}$, and $s_{ij}$ are stochastic disturbances. Our prior point estimate of $[\alpha^1_{ij} \; \alpha^2_{ij} \; \alpha^5_{ij}]$ is $[1 \; -1 \; 4]$. For further details, see text.

To eliminate residual autocorrelation, we have transformed the data by taking first differences. For the meaning of the notation, see the notes to Table 3. Standard errors are given in parentheses below each estimated parameter.

[b]The $\chi^2$ statistic gives the result of testing the null hypothesis that the sample information as summarized by the single equation least squares estimates given in Table 3 and the stochastic prior information given in the previous note are compatible. Under the null hypothesis, the test statistic is distributed as $\chi^2(q)$, where $q$ is the number of stochastic restrictions. Critical values of the $\chi^2$ distribution at the .05 level are given in parentheses. The null hypothesis is rejected when the value of the test statistic exceeds the critical value. Details of the test are given in Theil (1971, pp. 350-1).

information concerning the magnitude of the individual parameters and, therefore, in computing the various parameter estimates, a high weight is given to the prior information. In Table 5 the income elasticities are not significant at the 95 percent confidence level. In the only case where the income elasticity comes close to being significant (foreign income in the Franc/Pound exchange rate) it has the correct positive sign. The low values of the income elasticities may reflect a poor measure of income[1] as well as some of the implications of diversified portfolios of currencies that were outlined in Section I.

In an anlogous manner to the procedure of the single equation analysis we have also applied the mixed-estimation procedure to the system of equations using the method for estimating seemingly unrelated regressions. The results, which are reported in Table 6, are similar to those of the single equation estimates: the estimates are consistent with the predictions of the monetary model of exchange rate determination. The constant terms do not differ significantly from zero, the elasticities of the exchange rates with respect to domestic and foreign money supplies and the forward premia are significant and have the expected size, and all income elasticities have the expected sign even though, as before, they are not significant. Also reported in Table 6 is the correlation coefficient between the residuals of the two exchange rate equations. The high correlation (.954) indicates the potential gain from the application of the estimation method of seemingly unrelated equations.[2]

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[1]We have also estimated these equations using some measure of permanent income (a distributed lag of the measure of current income) instead of the income variable used here. The resulting income elasticities did not differ significantly from zero.

[2]It might be noted that in this case the $\chi^2$ statistic testing for compatibility of the sample and prior information is 14.72 which exceeds 12.59-- the critical value of $\chi^2(3)$ at the .05 level; it is, however, smaller than 16.81-- the corresponding critical value at the .01 level. Note that the quasi-$R^2$ that is reported for each equation is only suggestive since, strictly speaking, the goodness of fit should be analyzed for the sytem as a whole.

TABLE 6

SYSTEMS MIXED ESTIMATES

MONTHLY DATA:  FEBRUARY 1921-MAY 1925[a]

| Equation | Constant | ℓn M | ℓn M* | ℓn y | ℓn y* | π | D.W. | RMSE | Quasi-$R^2$ | ω | $\chi^2$[b] |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Franc/Pound | 0.003 (0.010) | 1.021 (0.096) | -0.866 (0.092) | 0.271 (0.277) | -0.080 (0.213) | 3.587 (0.876) | 1.81 | 0.071 | 0.913 | | |
| Franc/Dollar | 0.006 (0.011) | 0.979 (0.097) | -0.982 (0.099) | 0.179 (0.309) | -0.129 (0.152) | 4.190 (0.891) | 1.87 | 0.075 | 0.862 | 0.954 | 14.72 (12.59) |

[a]The model and the stochastic prior information used in estimation are given in the notes to Tables 3 and 5.  The systems estimator allows for cross-equation correlation of the disturbances.  To eliminate residual autocorrelation, we have transformed the data by taking first differences.  For the meaning of the notation, see the notes to Tables 3 and 4.  Standard errors are given in parentheses below each estimated parameter.

[b]The $\chi^2$ statistic gives the result of testing the null hypothesis that the sample information as summarized by the seemingly unrelated estimates given in Table 4 and the stochastic prior information given in note (a) to Table 5 are compatible.  The critical value of the $\chi^2$ distribution at the .01 level is 16.81.  For further details of the test, see note (b) to Table 5.

III.3 <u>Dynamic Simulations of the Model</u>

In what follows we investigate how well the estimated models track the (logarithm of the) levels of the exchange rates over the sample period. We simulate the model dynamically by taking only the initial value of the exchange rate as given and taking the simulated value to be the predicted rate of change plus the lagged (logarithm of the) exchange rate as <u>predicted</u> by the model for the previous month. It should be emphasized that this dynamic simulation is a severe test of the predictive ability of the model since simulation errors may accumulate over time.

Table 7 contains the results of these dynamic simulations for the various models using parameter estimates derived from the single equation and the system estimation methods.[1] As may be seen the various models perform reasonably well in tracking the exchange rates over the sample period. The mean error for the Franc/Pound exchange rate ranges from 0.7 percent to -3.4 percent while the mean error for the Franc/Dollar exchange rate ranges from -4.7 percent to -8.0 percent. In general, the simulations of the Franc/Pound rate perform better than those of the Franc/Dollar rate. On the whole there is no marked difference between the performance of simulations based on the sample estimates and those that are based on the mixed estimates. This result is noteworthy since it highlights the fact that the sample does provide satisfactory information on the overall relationship, and that the application of the mixed-estimation procedure is aimed at improving the precision of the individual parameter estimates. Comparison of the performance of the single equation estimates with those of the system estimates show that the simulations based on the single equations perform somewhat better.

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[1]The simulation program we use is PREDIC written by Clifford R. Wymer.

TABLE 7

IN-SAMPLE DYNAMIC SIMULATION OF EXCHANGE RATE LEVELS
MONTHLY DATA: FEBRUARY 1921-MAY 1925[a]

| Estimated Parameters Used in Simulation and Exchange Rate | Squared Correlation Coefficient between Actual and Simulated | Mean Error | Root Mean Square Error | Mean Absolute Error | Theil's Inequality Coefficient |
|---|---|---|---|---|---|
| Single Equation Least Squares Estimates (Table 3) | | | | | |
| Franc/Pound | 0.887 | 0.018 | 0.082 | 0.063 | 0.010 |
| Franc/Dollar | 0.634 | -0.047 | 0.136 | 0.099 | 0.025 |
| Seemingly Unrelated Estimates (Table 4) | | | | | |
| Franc/Pound | 0.854 | -0.023 | 0.099 | 0.079 | 0.012 |
| Franc/Dollar | 0.683 | -0.059 | 0.139 | 0.100 | 0.025 |
| Single Equation Mixed Estimates (Table 5) | | | | | |
| Franc/Pound | 0.893 | 0.007 | 0.079 | 0.058 | 0.009 |
| Franc/Dollar | 0.558 | -0.069 | 0.155 | 0.114 | 0.028 |
| Systems Mixed Estimates (Table 6) | | | | | |
| Franc/Pound | 0.862 | -0.034 | 0.102 | 0.078 | 0.012 |
| Franc/Dollar | 0.616 | -0.080 | 0.159 | 0.116 | 0.029 |

[a]The model is simulated dynamically by computing the predicted value of the exchange rate level as the simulated value of the previous period's level plus the predicted change for the current period. Recall that the model is estimated in first differences. Since the simulations refer to the logarithm of the exchange rate levels, the error statistics are percentages when multiplied by 100.

Finally, we report in Table 7 Theil's inequality coefficient which
is extremely low, indicating the reasonable quality of the predictive ability
of the model.[1]

## IV. Concluding Remarks

The evolution of the international monetary system into a regime of
floating exchange rates has led to a renewed interest in the economics of
exchange rates which resulted in new developments in the theory of exchange rate
determination.  The central insight of the monetary approach to the exchange
rate is that the exchange rate, being a relative price of two monies, is
determined in a manner similar to that of other asset prices.  Equilibrium
is attained when the existing stocks are willingly held and one of the important
determinants of current prices is expectations concerning the future course of
events.  Since current views concerning the operation of flexible exchange
rate systems are based, to a large extent, on the experience of the 1920's,
we have reexamined in this paper the determinants of exchange rates during
the 1920's from the analytical perspective of the monetary approach.  After
developing a simple model of exchange rate determination, we have examined
the efficiency of the foreign exchange markets and have estimated the model
using monthly data.  We have found that the foreign exchange markets were
efficient and that the forward exchange rate was an unbiased forecast of
the future spot rate.  We have then used the forward premium on foreign

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[1]Theil's inequality coefficient measures the quality of forecasts.
Consider a series of prediction $P_1, \ldots, P_n$ and a series of outcomes $A_1, \ldots, A_n$.
The inequality coefficient which is bounded between zero and one is:

$$U = \sqrt{\frac{1}{n} \Sigma (P_i - A_i)^2} \bigg/ \left( \sqrt{\frac{1}{n} \Sigma P_i^2} + \sqrt{\frac{1}{n} \Sigma A_i^2} \right)$$

Thus, in the case of perfect forecast, $P_i = A_i$ and $U = 0$ while on the other
extreme $U = 1$.

exchange as a measure of expectations in estimating the monetary model of the exchange rate. The various results were found to be consistent with the theoretical predictions.

Rather than summarizing the results, we wish to highlight some of the methodological issues raised by the empirical work. The first involves the use of monthly data. Since it is believed that asset markets clear relatively fast, it is desirable to use frequent observations. We have used monthly data since this is the shortest maturity for which data on forward contracts are available. Second, since the various exchange rates may be subject to common shocks, it might be desirable to supplement single equation estimates with system estimates. Third, since the various determinants of the exchange rate are likely to be highly collinear (like the time series of two national money supplies), estimates might be imprecise. It is desirable in such cases to apply techniques such as the mixed-estimation procedure or Bayesian approaches which complement the sample information with prior information. Using the monetary model, the prior information derives from the homogeneity postulate and from known properties of the demand for money. From the policy perspective, the monetary approach to the exchange rate serves as a reminder that the exchange rate and the conduct of monetary policy are intimately linked to each other and that, as a first approximation, policies which affect the trend of domestic (relative to foreign) monetary growth, also affect the exchange rate in the same manner.

APPENDIX A

ESTIMATION PROCEDURES

In this Appendix we outline some of the details of the estimation
procedures for incorporating prior knowledge with the sample information.
We start with the mixed estimation procedure and then provide a Bayesian
interpretation of the estimator.

Let the T-vector of observations on the dependent variable  y  be
generated according to

(A1) $$y = X\beta + u; \qquad u \sim N(0, \sigma^2 I_T) \quad ,$$

where  X  is a matrix of observations on the  k  independent variables,
$\beta$  is a parameter vector, and  u  is a disturbance vector.  The stochastic
prior information on the parameter values can be written as

(A2) $$r = R\beta + v; \quad E(v) = 0, \; E(vv') = V, \; E(uv') = 0,$$

where  r  is a vector of random prior estimates of  $R\beta$,  R  is a known matrix,
and  v  is an error vector.  Combining (A1) and (A2) yields

(A3) $$\tilde{y} = \tilde{X}\beta + \tilde{u}; \quad E(\tilde{u}) = 0, \; E(\tilde{u}\tilde{u}') = \Omega,$$

in which

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$$\tilde{y} = \left[\begin{array}{c|c} y' & r' \end{array}\right]', \quad \tilde{X} = \left[\begin{array}{c|c} X' & R' \end{array}\right]', \quad \tilde{u} = \left[\begin{array}{c|c} u' & v' \end{array}\right]' \quad \text{and}$$

$$\Omega = \left[\begin{array}{c|c} \sigma^2 I_T & 0 \\ \hline 0' & V \end{array}\right] .$$

Applying the Aitken principle to (A3) gives the mixed estimator of $\beta$:

$$(A4) \qquad \hat{\beta} = (\tilde{X}'\Omega^{-1}\tilde{X})^{-1}\tilde{X}'\Omega^{-1}\tilde{y} = (R'V^{-1}R + X'X/\sigma^2)^{-1}(R'V^{-1}r + X'y/\sigma^2) .$$

The covariance matrix of $\hat{\beta}$ is

$$V(\hat{\beta}) = (R'V^{-1}R + X'X/\sigma^2)^{-1} .$$

To make these expressions operational, the unknown parameter $\sigma^2$ is replaced by an estimate. For further details, see, e.g., Theil (1971, pp. 346-352).

The mixed estimator can also be viewed in a Bayesian framework. The expression given in (A4) is the mean of the conditional (in the sense indicated below) posterior density of the parameter vector when the prior distribution is normal. To show this, let $\bar{\beta}$ and $A^{-1}$ be the prior mean and covariance matrix of $\beta$, and let the prior density be multivariate normal:

$$p(\beta) = (2\pi)^{-k/2}|A|^{1/2}\exp[-\tfrac{1}{2}(\beta - \bar{\beta})'A(\beta - \bar{\beta})]$$

(A5)
$$\propto \exp[-\tfrac{1}{2}(\beta - \bar{\beta})'A(\beta - \bar{\beta})] \quad,$$

where $\propto$ denotes proportionality. Assuming that the disturbance variance in (A1), $\sigma^2$, is known and denoting its known value by $\sigma_0^2$, the likelihood function associated with (A1) is

$$p(y|\beta, \sigma_0^2, X) \propto \exp\left[-\frac{1}{2\sigma_0^2}(y - X\beta)'(y - X\beta)\right]$$

(A6)
$$\propto \exp\left[-\frac{1}{2\sigma_0^2}(\beta - \hat{\beta})'X'X(\beta - \hat{\beta})\right] \quad,$$

where

(A7)
$$\hat{\beta} = (X'X)^{-1}X'y \quad.$$

Using Bayes' theorem to combine the prior density (A5) and the likelihood function (A6), we obtain the following conditional posterior density:

$$p(\beta|\sigma_0^2, y, X) \propto p(\beta)p(y|\beta, \sigma_0^2, X)$$

$$\propto \exp\{-\tfrac{1}{2}[(\beta - \hat{\beta})'X'X(\beta - \hat{\beta})/\sigma_0^2 + (\beta - \bar{\beta})'A(\beta - \bar{\beta})]\}$$

(A8)
$$\propto \exp[-\tfrac{1}{2}(\beta - \bar{\bar{\beta}})'(A + X'X/\sigma_0^2)(\beta - \bar{\bar{\beta}})] \quad,$$

where

(A9)
$$\bar{\bar{\beta}} = (A + X'X/\sigma_0^2)^{-1}(A\bar{\beta} + X'X\hat{\beta}/\sigma_0^2) \quad.$$

Thus from (A8), the conditional posterior distribution of $\beta$ is normal with
the following mean and covariance matrix:

$$E(\beta) = \overline{\overline{\beta}} \qquad V(\beta) = (A + X'X/\sigma_0^2)^{-1} \quad .$$

The conditional posterior mean given in (A9) is a matrix weighted
average of the prior mean $\overline{\beta}$ and the least squares estimator $\hat{\beta}$ defined in
(A7). The weight matrices are the relative precisions of $\overline{\beta}$ and $\hat{\beta}$: the
information contained in the data is given a larger weight the greater is
its relative precision. The weight matrices are both positive definite
and they sum to the identity matrix.

The expression given in (A9) corresponds to the mixed estimator (A4) with
$R = I_k$, $V = A^{-1}$, $\sigma^2 = \sigma_0^2$ and $r = \overline{\beta}$. Hence the Bayesian interpretation.
This analysis of the single-equation case generalizes to multiple-equation
systems (such as in our application).

The Bayesian interpretation can also be extended to the situation in
which $\sigma^2$ is an unknown parameter. Assuming that $\ln \sigma$ has a uniform prior
distribution together with the normal prior for $\beta$, it can be shown that
the quantity given in (A9) is the mean of the leading normal term in an asymp-
totic expansion approximating the posterior distribution of $\beta$. For details,
see Zellner (1971b, Chapter 4). Thus, in this case the previous interpretation
of the mixed estimator applies approximately.

For an analysis of Bayesian and alternative approaches, see Zellner (1971a).

APPENDIX B

THE DATA BASE

In this appendix, we give the detailed definitions of all the variables in the data base, the primary data sources, and a listing of the entire data base. The data base is made up of 53 monthly observations on each variable for the period January, 1921 to May, 1925.

We use the following country subscripting convention: France, the United States, and the United Kingdom are denoted by the subscripts 1, 2, and 3, respectively.

I. Exchange Rates and Forward Premia

The franc/pound and dollar/pound spot rates are taken from Einzig (1937, Appendix 1, pp. 450-458). In that source, weekly rates are given, and we use the rate quoted nearest the end of the month for that month's rate.

The spot franc/dollar rate is computed as the ratio of the spot franc/pound rate to the spot dollar/pound rate. That is, we use the triangular arbitrage condition.

The three spot exchange rates are given in Table B1. Here, we use $S_{ij}$ to denote the spot rate between country $i$'s currency and country $j$'s; $S_{ij}$ is expressed as the cost of a unit of the currency of country $j$ in terms of the currency of country $i$. Country $i$ can be thought of as the home country, and $j$ as the foreign country. For example, using our subscripting convention, $S_{13}$ is the spot franc/pound rate, and a rise in $S_{13}$ means that the franc has depreciated relative to the pound.

Table B1

SPOT EXCHANGE RATES[a]

| Year and Month | | $S_{13}$ | $S_{23}$ | $S_{12}$ |
|---|---|---|---|---|
| 1921 | 1 | 54.3700 | 3.8600 | 14.0855 |
| | 2 | 54.9800 | 3.8700 | 14.2067 |
| | 3 | 56.5800 | 3.9200 | 14.4337 |
| | 4 | 51.1900 | 3.9600 | 12.9268 |
| | 5 | 46.7000 | 3.8950 | 11.9897 |
| | 6 | 46.7400 | 3.7300 | 12.5308 |
| | 7 | 46.9200 | 3.5625 | 13.1705 |
| | 8 | 47.6500 | 3.6875 | 12.9220 |
| | 9 | 52.3500 | 3.7350 | 14.0161 |
| | 10 | 54.0000 | 3.9250 | 13.7580 |
| | 11 | 57.8000 | 3.9850 | 14.5044 |
| | 12 | 51.8800 | 4.2150 | 12.3084 |
| 1922 | 1 | 51.8000 | 4.2500 | 12.1882 |
| | 2 | 49.3000 | 4.3975 | 11.2109 |
| | 3 | 48.5000 | 4.3850 | 11.0604 |
| | 4 | 48.2700 | 4.4225 | 10.9146 |
| | 5 | 48.8700 | 4.4500 | 10.9820 |
| | 6 | 52.1600 | 4.4025 | 11.8478 |
| | 7 | 54.1700 | 4.4475 | 12.1799 |
| | 8 | 59.4000 | 4.4725 | 13.2812 |
| | 9 | 57.7700 | 4.3700 | 13.2197 |
| | 10 | 62.1500 | 4.4625 | 13.9272 |
| | 11 | 62.9700 | 4.5000 | 13.9933 |
| | 12 | 63.5700 | 4.6350 | 13.7152 |
| 1923 | 1 | 72.2500 | 4.6400 | 15.5711 |
| | 2 | 77.6000 | 4.7150 | 16.4581 |
| | 3 | 70.4000 | 4.6750 | 15.0588 |
| | 4 | 68.2200 | 4.6350 | 14.7185 |
| | 5 | 69.8700 | 4.6250 | 15.1070 |
| | 6 | 75.6200 | 4.5725 | 16.5380 |
| | 7 | 77.9000 | 4.5850 | 16.9902 |
| | 8 | 80.3500 | 4.5550 | 17.6400 |
| | 9 | 74.1700 | 4.5525 | 16.2922 |
| | 10 | 75.9200 | 4.5000 | 16.8711 |
| | 11 | 80.8700 | 4.3650 | 18.5269 |
| | 12 | 84.8200 | 4.3375 | 19.5551 |
| 1924 | 1 | 94.2500 | 4.2275 | 22.2945 |
| | 2 | 99.7500 | 4.3125 | 23.1304 |
| | 3 | 78.4500 | 4.3000 | 18.2442 |
| | 4 | 68.7500 | 4.3825 | 15.6874 |
| | 5 | 84.4000 | 4.3050 | 19.6051 |
| | 6 | 81.8000 | 4.3200 | 18.9352 |
| | 7 | 86.1200 | 4.4000 | 19.5727 |
| | 8 | 82.1200 | 4.5025 | 18.2388 |
| | 9 | 84.8000 | 4.4725 | 18.9603 |
| | 10 | 86.2000 | 4.4900 | 19.1982 |
| | 11 | 85.8200 | 4.6225 | 18.5657 |
| | 12 | 87.2700 | 4.7125 | 18.5188 |
| 1925 | 1 | 88.4000 | 4.7950 | 18.4359 |
| | 2 | 92.5700 | 4.7600 | 19.4475 |
| | 3 | 90.6000 | 4.7775 | 18.9639 |
| | 4 | 92.6000 | 4.8125 | 19.2416 |
| | 5 | 96.9200 | 4.8600 | 19.9424 |
| Sample Mean | | 69.2245 | 4.3651 | 15.7676 |
| Standard Deviation | | 16.2619 | 0.3202 | 3.1220 |

[a] See text for meaning of notation and data sources.

Table B2

ONE MONTH FORWARD EXCHANGE RATES[a]

| Year and Month | | $F_{13}$ | $F_{23}$ | $F_{12}$ |
|---|---|---|---|---|
| 1921 | 1 | 54.0700 | 3.87750 | 13.9446 |
| | 2 | 54.6800 | 3.88250 | 14.0837 |
| | 3 | 56.3100 | 3.92500 | 14.3465 |
| | 4 | 51.0400 | 3.96188 | 12.8828 |
| | 5 | 46.6200 | 3.90375 | 11.9424 |
| | 6 | 46.6600 | 3.73625 | 12.4885 |
| | 7 | 46.9600 | 3.56750 | 13.1633 |
| | 8 | 47.6800 | 3.69125 | 12.9170 |
| | 9 | 52.3600 | 3.73625 | 14.0141 |
| | 10 | 54.0300 | 3.93000 | 13.7481 |
| | 11 | 57.8200 | 3.99000 | 14.4912 |
| | 12 | 51.8900 | 4.21687 | 12.3053 |
| 1922 | 1 | 51.7900 | 4.25062 | 12.1841 |
| | 2 | 49.2950 | 4.39812 | 11.2082 |
| | 3 | 48.5000 | 4.38500 | 11.0604 |
| | 4 | 48.2775 | 4.42281 | 10.9156 |
| | 5 | 48.8800 | 4.45125 | 10.9812 |
| | 6 | 52.1800 | 4.40344 | 11.8498 |
| | 7 | 54.2500 | 4.45062 | 12.1893 |
| | 8 | 59.5100 | 4.47500 | 13.2983 |
| | 9 | 57.8000 | 4.37375 | 13.2152 |
| | 10 | 62.2500 | 4.46875 | 13.9301 |
| | 11 | 63.0700 | 4.51000 | 13.9845 |
| | 12 | 63.6000 | 4.64625 | 13.6885 |
| 1923 | 1 | 72.3200 | 4.65000 | 15.5527 |
| | 2 | 77.7000 | 4.72500 | 16.4444 |
| | 3 | 70.4900 | 4.68500 | 15.0459 |
| | 4 | 68.2500 | 4.64375 | 14.6972 |
| | 5 | 69.9050 | 4.63375 | 15.0861 |
| | 6 | 75.6800 | 4.57875 | 16.5285 |
| | 7 | 77.9400 | 4.58875 | 16.9850 |
| | 8 | 80.3950 | 4.55718 | 17.6414 |
| | 9 | 74.2000 | 4.55625 | 16.2853 |
| | 10 | 75.9650 | 4.50437 | 16.8647 |
| | 11 | 80.9700 | 4.37187 | 18.5207 |
| | 12 | 84.8650 | 4.34250 | 19.5429 |
| 1924 | 1 | 94.5200 | 4.23375 | 22.3254 |
| | 2 | 100.3300 | 4.31625 | 23.2447 |
| | 3 | 79.5700 | 4.30062 | 18.5020 |
| | 4 | 68.9500 | 4.38437 | 15.7263 |
| | 5 | 85.1000 | 4.30625 | 19.7620 |
| | 6 | 82.1000 | 4.31937 | 19.0074 |
| | 7 | 86.1900 | 4.39375 | 19.6165 |
| | 8 | 82.1900 | 4.49687 | 18.2772 |
| | 9 | 84.8900 | 4.46687 | 19.0044 |
| | 10 | 86.3300 | 4.48687 | 19.2406 |
| | 11 | 86.0700 | 4.62219 | 18.6211 |
| | 12 | 87.7400 | 4.71281 | 18.6173 |
| 1925 | 1 | 88.6800 | 4.79562 | 18.4919 |
| | 2 | 93.0500 | 4.76062 | 19.5458 |
| | 3 | 90.9000 | 4.77687 | 19.0292 |
| | 4 | 93.1300 | 4.80937 | 19.3643 |
| | 5 | 97.4700 | 4.85625 | 20.0710 |
| Sample Mean | | 69.3474 | 4.3685 | 15.7825 |
| Standard Deviation | | 16.4215 | 0.3190 | 3.1607 |

[a] See text for meaning of notation and data sources.

Table B3

ONE MONTH FORWARD PREMIA[a]

| Year and Month | | $\pi_{13}$ | $\pi_{23}$ | $\pi_{12}$ |
|---|---|---|---|---|
| 1921 | 1 | -0.553319 | 0.452420 | -1.005740 |
| | 2 | -0.547120 | 0.322437 | -0.869557 |
| | 3 | -0.478362 | 0.127506 | -0.605867 |
| | 4 | -0.293446 | 0.047302 | -0.340748 |
| | 5 | -0.171471 | 0.224400 | -0.395870 |
| | 6 | -0.171280 | 0.167465 | -0.338744 |
| | 7 | 0.085258 | 0.140190 | -0.054932 |
| | 8 | 0.062942 | 0.101662 | -0.038720 |
| | 9 | 0.019074 | 0.033379 | -0.014305 |
| | 10 | 0.055504 | 0.127220 | -0.071716 |
| | 11 | 0.034523 | 0.125313 | -0.090790 |
| | 12 | 0.019264 | 0.044441 | -0.025177 |
| 1922 | 1 | -0.019264 | 0.014687 | -0.033951 |
| | 2 | -0.010109 | 0.014210 | -0.024319 |
| | 3 | 0.000000 | 0.000000 | 0.000000 |
| | 4 | 0.015545 | 0.007057 | 0.008488 |
| | 5 | 0.020504 | 0.028038 | -0.007534 |
| | 6 | 0.038338 | 0.021267 | 0.017071 |
| | 7 | 0.147629 | 0.070190 | 0.077438 |
| | 8 | 0.184917 | 0.055885 | 0.129032 |
| | 9 | 0.051975 | 0.085735 | -0.033760 |
| | 10 | 0.160789 | 0.139999 | 0.020790 |
| | 11 | 0.158691 | 0.221920 | -0.063229 |
| | 12 | 0.047111 | 0.242424 | -0.195312 |
| 1923 | 1 | 0.096798 | 0.215244 | -0.118446 |
| | 2 | 0.128746 | 0.211906 | -0.083160 |
| | 3 | 0.127697 | 0.213623 | -0.085926 |
| | 4 | 0.043964 | 0.188637 | -0.144672 |
| | 5 | 0.050068 | 0.188923 | -0.138855 |
| | 6 | 0.079250 | 0.136566 | -0.057316 |
| | 7 | 0.051308 | 0.081825 | -0.030517 |
| | 8 | 0.055981 | 0.047970 | 0.008011 |
| | 9 | 0.040436 | 0.082397 | -0.041962 |
| | 10 | 0.059318 | 0.097179 | -0.037861 |
| | 11 | 0.123596 | 0.157356 | -0.033760 |
| | 12 | 0.053024 | 0.115204 | -0.062180 |
| 1924 | 1 | 0.286007 | 0.147724 | 0.138283 |
| | 2 | 0.579736 | 0.086879 | 0.492857 |
| | 3 | 1.417540 | 0.014496 | 1.403040 |
| | 4 | 0.290489 | 0.042725 | 0.247764 |
| | 5 | 0.825974 | 0.028992 | 0.796983 |
| | 6 | 0.366116 | -0.014496 | 0.380611 |
| | 7 | 0.081253 | -0.142193 | 0.223446 |
| | 8 | 0.085163 | -0.125027 | 0.210190 |
| | 9 | 0.106049 | -0.125885 | 0.231934 |
| | 10 | 0.150681 | -0.069618 | 0.220299 |
| | 11 | 0.290871 | -0.006771 | 0.297642 |
| | 12 | 0.537108 | 0.006676 | 0.530432 |
| 1925 | 1 | 0.316238 | 0.013065 | 0.303172 |
| | 2 | 0.517178 | 0.013065 | 0.504112 |
| | 3 | 0.330544 | -0.013161 | 0.343704 |
| | 4 | 0.570773 | -0.064945 | 0.635717 |
| | 5 | 0.565812 | -0.077152 | 0.642965 |
| Sample Mean | | 0.133687 | 0.080497 | 0.053190 |
| Standard Deviation | | 0.313671 | 0.112800 | 0.378781 |

[a] See text for meaning of notation and data sources. All entries are to be divided by 100.

The one month forward exchange rate for both the franc/pound and dollar/pound are also from Einzig (1937, Appendix 1, pp. 450-458). We again use the forward rate quoted for the last week in a month for that month's forward rate.

The one month forward franc/dollar rate is computed from the triangular arbitrage relationship, as before.

The three one month forward rates are given in Table B2. Here, $F_{ij}$ is used to denote the one month forward rate between country $i$'s currency and country $j$'s. The interpretation of $F_{ij}$ is exactly analogous to that for $S_{ij}$ given previously.

The one month forward premium, $\pi_{ij}$, is computed as

$$\pi_{ij} = \ln(F_{ij}/S_{ij}) \qquad j > i \; ; \; i, j = 1, 2, 3 \;.$$

Hence, $\pi_{ij}$ is the proportion by which the one month forward rate is above the current spot rate. A positive value of $\pi_{ij}$ means that the currency of country $j$ for delivery in one month's time is selling at a premium relative to country $i$'s currency--the home country's currency is expected to depreciate relative to that of country $j$.

The forward premia are given in Table B3. Notice that for each month, apart from rounding errors, the franc/dollar forward premium $(\pi_{12})$ is exactly the difference between the franc/pound and the dollar/pound premia $(\pi_{13}$ and $\pi_{23})$. This is because the triangular arbitrage relation is used to construct both the spot and forward rates for the franc/dollar.

## II. Money Supplies

For the French money supply, we use the Bank of France note circulation, which is given in Tinbergen (1934, pp. 66-67, column 17). The units of the French money supply are milliards of francs.

Table B4

MONEY SUPPLIES[a]

| Year and Month | | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|---|
| 1921 | 1 | 37.90 | 34.21 | 179.40 |
| | 2 | 37.80 | 33.93 | 174.10 |
| | 3 | 38.40 | 33.37 | 169.40 |
| | 4 | 38.20 | 33.01 | 169.70 |
| | 5 | 38.20 | 32.92 | 171.50 |
| | 6 | 37.40 | 32.46 | 176.40 |
| | 7 | 36.90 | 32.15 | 176.80 |
| | 8 | 36.80 | 32.24 | 174.30 |
| | 9 | 37.10 | 31.98 | 175.40 |
| | 10 | 37.20 | 32.19 | 179.30 |
| | 11 | 36.30 | 32.24 | 176.50 |
| | 12 | 36.50 | 32.16 | 180.80 |
| 1922 | 1 | 36.40 | 31.93 | 180.10 |
| | 2 | 36.20 | 32.30 | 177.80 |
| | 3 | 35.50 | 32.44 | 171.60 |
| | 4 | 35.80 | 33.13 | 172.10 |
| | 5 | 35.70 | 33.40 | 172.30 |
| | 6 | 36.00 | 33.91 | 174.30 |
| | 7 | 36.00 | 34.28 | 171.00 |
| | 8 | 36.40 | 34.47 | 166.60 |
| | 9 | 36.60 | 34.76 | 163.80 |
| | 10 | 36.70 | 35.00 | 166.60 |
| | 11 | 36.10 | 35.00 | 165.10 |
| | 12 | 36.40 | 35.97 | 167.10 |
| 1923 | 1 | 36.80 | 35.87 | 167.00 |
| | 2 | 37.10 | 36.11 | 162.20 |
| | 3 | 37.20 | 36.08 | 157.90 |
| | 4 | 36.50 | 36.47 | 159.40 |
| | 5 | 36.70 | 36.73 | 158.90 |
| | 6 | 36.70 | 36.69 | 162.80 |
| | 7 | 36.90 | 36.66 | 161.80 |
| | 8 | 37.40 | 36.71 | 158.90 |
| | 9 | 37.60 | 36.90 | 159.10 |
| | 10 | 37.70 | 37.08 | 161.00 |
| | 11 | 37.30 | 37.21 | 160.80 |
| | 12 | 37.90 | 37.38 | 166.40 |
| 1924 | 1 | 38.80 | 37.22 | 165.20 |
| | 2 | 39.80 | 37.36 | 160.70 |
| | 3 | 39.90 | 37.52 | 158.90 |
| | 4 | 39.80 | 37.75 | 160.00 |
| | 5 | 39.60 | 37.97 | 160.20 |
| | 6 | 39.70 | 38.33 | 164.70 |
| | 7 | 40.30 | 38.78 | 162.10 |
| | 8 | 40.00 | 39.21 | 159.30 |
| | 9 | 40.30 | 39.66 | 160.40 |
| | 10 | 40.50 | 39.93 | 160.90 |
| | 11 | 40.40 | 40.42 | 160.60 |
| | 12 | 40.60 | 40.30 | 165.40 |
| 1925 | 1 | 40.80 | 40.77 | 162.40 |
| | 2 | 40.80 | 41.09 | 161.20 |
| | 3 | 40.90 | 41.11 | 158.40 |
| | 4 | 43.00 | 41.30 | 158.80 |
| | 5 | 42.70 | 41.65 | 157.20 |
| Sample Mean | | 38.04 | 36.03 | 166.50 |
| Standard Deviation | | 1.88 | 2.94 | 7.07 |

[a] See text for meaning of notation and data sources.

The U.S. money supply variable is currency held by the public plus demand and time deposits at commercial banks (i.e., it is "$M_2$"), from Friedman and Schwartz (1970, pp. 19-23, column 9).  The units of this variable are thousands of millions of dollars.

Our U.K. money supply variable is the total deposits of the London Clearing Banks.  The source for this is the Committee on Finance and Industry (1931, pp. 285-287).  Here, the units are tens of millions of pounds.

The three money supplies are given in Table B4.  Here, $M_i$ denotes the money supply of country $i$.

III.  Real Incomes

For real income in France, we use the General Index of Production from Tinbergen (1934, pp. 75-76, column 52).

The U.S. real income variable is the Index of Volume of Manufactured Output, also from Tinbergen (1934, pp. 213-14, column 42).

Real income data for the U.K. are readily available only on an annual basis.  We use the monthly unemployment series to interpolate the annual industrial production index as follows.  First, we regress annual industrial production on a time trend and the annual unemployment rate.  This yields (standard errors are given in parentheses below each estimated parameter):

$$y = 98.46 + 0.4277t - 0.0007674t^2 - 1.2799u$$
$$(3.59)\ (0.1035)\ \ (0.0007591)\ \ \ (0.2023)$$

in which
$$R^2 = 0.9651,$$

  $y$  is the annual U.K. Total Industrial Production Index,

  $t = 6, 18, 30, \ldots, 126$, an annual time trend, and

  $u$  is the U.K. unemployment rate in June expressed as a percentage

  (e.g., $u = 3.0$  if the unemployment rate is 3 percent).

Table B5

REAL INCOMES[a]

| Year and Month | | $y_1$ | $y_2$ | $y_3$ |
|---|---|---|---|---|
| 1921 | 1 | 63.00 | 81.00 | 89.80 |
| | 2 | 60.00 | 81.00 | 88.18 |
| | 3 | 57.00 | 76.00 | 86.68 |
| | 4 | 54.00 | 75.00 | 80.83 |
| | 5 | 52.00 | 72.00 | 75.11 |
| | 6 | 53.00 | 74.00 | 74.63 |
| | 7 | 50.00 | 73.00 | 79.79 |
| | 8 | 51.00 | 78.00 | 80.59 |
| | 9 | 50.00 | 79.00 | 83.05 |
| | 10 | 52.00 | 80.00 | 82.57 |
| | 11 | 56.00 | 81.00 | 82.47 |
| | 12 | 62.00 | 84.00 | 82.75 |
| 1922 | 1 | 65.00 | 84.00 | 82.77 |
| | 2 | 68.00 | 89.00 | 83.56 |
| | 3 | 74.00 | 92.00 | 83.97 |
| | 4 | 73.00 | 91.00 | 83.60 |
| | 5 | 77.00 | 95.00 | 84.77 |
| | 6 | 78.00 | 98.00 | 86.07 |
| | 7 | 78.00 | 100.00 | 87.75 |
| | 8 | 81.00 | 97.00 | 88.66 |
| | 9 | 82.00 | 96.00 | 88.67 |
| | 10 | 86.00 | 99.00 | 89.58 |
| | 11 | 88.00 | 106.00 | 89.72 |
| | 12 | 88.00 | 108.00 | 90.62 |
| 1923 | 1 | 88.00 | 106.00 | 91.27 |
| | 2 | 82.00 | 108.00 | 92.55 |
| | 3 | 83.00 | 110.00 | 93.83 |
| | 4 | 82.00 | 113.00 | 95.50 |
| | 5 | 82.00 | 111.00 | 95.88 |
| | 6 | 87.00 | 111.00 | 96.52 |
| | 7 | 87.00 | 108.00 | 97.03 |
| | 8 | 89.00 | 104.00 | 97.15 |
| | 9 | 90.00 | 106.00 | 97.79 |
| | 10 | 93.00 | 104.00 | 98.68 |
| | 11 | 95.00 | 103.00 | 99.43 |
| | 12 | 98.00 | 105.00 | 100.96 |
| 1924 | 1 | 104.00 | 106.00 | 102.35 |
| | 2 | 105.00 | 108.00 | 103.11 |
| | 3 | 106.00 | 106.00 | 103.99 |
| | 4 | 104.00 | 100.00 | 104.74 |
| | 5 | 105.00 | 92.00 | 105.75 |
| | 6 | 106.00 | 89.00 | 105.85 |
| | 7 | 108.00 | 88.00 | 105.96 |
| | 8 | 111.00 | 92.00 | 105.68 |
| | 9 | 114.00 | 96.00 | 105.15 |
| | 10 | 116.00 | 97.00 | 105.37 |
| | 11 | 112.00 | 100.00 | 105.86 |
| | 12 | 113.00 | 103.00 | 105.45 |
| 1925 | 1 | 111.00 | 105.00 | 106.05 |
| | 2 | 109.00 | 105.00 | 105.90 |
| | 3 | 108.00 | 104.00 | 106.76 |
| | 4 | 107.00 | 104.00 | 106.59 |
| | 5 | 106.00 | 102.00 | 106.04 |
| Sample Mean | | 84.89 | 95.75 | 93.84 |
| Standard Deviation | | 20.68 | 11.77 | 9.80 |

[a]See text for meaning of notation and data sources.

This equation is estimated with data for the period 1920 to 1930. The source for the Total Industrial Production Index for the U.K. is Feinstein (1976, p. T112, column 1).

Our measure of u is the Trade Union and National Unemployment Insurance "Unemployed" Percentages, given in International Industrial Relations Institute (1932, pp. 259-260). We then compute a monthly real income series by using a monthly time trend and the monthly unemployment rate in the estimated relationship. Hence, we use the interpolated monthly industrial production index for real income in the U.K.

The three income series are given in Table B5. Here, $y_i$ is the real income for country i.

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