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INVENTORY FLUCTUATIONS, TEMPORARY LAYOFFS AND THE BUSINESS CYCLE

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SUMMARY

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Firms respond to fluctuations in demand by changing their inventories and their levels of production. The relative magnitudes of the inventory and production responses have important implications for the overall cyclical behavior of the economy. Government policies that affect the costs of holding inventories and the costs of the temporary layoffs that accompany reductions in the level of output can therefore have significant effects on the magnitude of aggregate fluctuations. The current paper presents new econometric evidence on the nature of inventory adjustments and then examines how changes in inventory behavior affect the overall business cycle.

The analysis in this paper was motivated by our discovery that the parameter estimates of the traditional productional adjustment model are not consistent with the observed magnitudes of inventory change and the production. We have shown here that this production adjustment model is a special case of a more general two-speed adjustment process in which both production and inventory targets adjust slowly. Our estimates of the two-speed model clearly reject the production adjustment model in favor of the target adjustment model in which the inventory target adjusts slowly to changes in sales but production adjusts rapidly to changes in the desired inventory.

Our analysis of the spectral properties of a simple macroeconomic model show that the production adjustment model and the target adjustment model can imply quite different cyclical behavior of the economy as a whole. Depending on the autocorrelation of the disturbance, government policies that reduce the speed with which production responds to changes in desired inventories and that place greater reliance on inventory adjustment may stabilize national income. Further analysis of these questions with more realistic models would clearly be desirable.

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Inventory Fluctuations, Temporary Layoffs and the Business Cycle

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Firms respond to fluctuations in demand by changing their inventories and their levels of production. The relative magnitudes of the inventory and production responses have important implications for the overall cyclical behavior of the economy. Government policies that affect the costs of holding inventories and the costs of the temporary layoffs that accompany reductions in the level of output can therefore have significant effects on the magnitude of aggregate fluctuations. The current paper presents new econometric evidence on the nature of inventory adjustments and then examines how changes in inventory behavior affect the overall bysiness cycle.

Traditional econometric models of the behavior of inventories of finished goods assume that firms adjust production to eliminate gradually the difference between actual inventories and "desired" or target" inventories.[1] The usual parameter estimates imply that this adjustment occurs very slowly; typically, more than a year is required for half of the adjustment to be completed. In a previous paper (Feldstein and Auerbach, 1976) we showed that the sizes of the total inventory changes that are observed are much too small for such a slow

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[1] See in particular Lovell (1961, 1964), Darling and Lovell (1965), Childs (1967), Belsley (1969) and Hay (1970).

adjustment to be plausible.[1] In place of the traditional model of inventory adjustment we suggested a specification which we called the "target adjustment model": firms adjust their desired or "target" level of inventories to sales with a substantial lag but then adjust production to make actual inventories equal desired inventories within a single quarter. This target adjustment model is consistent with the data and does not involve an implausible contrast between the size of the production adjustment and the time required for it to occur.

The traditional "production adjustment model" and our "target adjustment model" are both extreme cases of a more general process in which the inventory target adjusts with a lag to the level of sales and production then adjusts with a lag to achieve this target. The current paper presents this more general "two speed adjustment model" and discusses the parameter values that we have estimated with new data for both durable goods and nondurable goods manufacturing industries. These estimates coincide almost exactly with the implications of the target adjustment model and clearly contradict the traditional production adjustment model.

The current paper also investigates how changing the parameter values of this adjustment process would alter the cyclical characteristics of the economy. For this purpose we specify a very simple model of the economy with stochastic consumption and then calculate how changes in the inventory adjustment process will alter the stochastic properties of total production. The effect of the inventory process on cyclical stability is summarized by the

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[1] During the period since the end of 1958, the largest contraction of finished goods inventories in the durable goods manufacturing sector occurred between the first and last quarters of 1971, when inventories fell by $727 million. This was equivalent to less than one day's production. Such a correction could obviously occur much more rapidly than the traditional models imply. Similarly, the largest one-year increase in the finished goods inventories in durable goods manufacturing was less than two days of production.

ratio of the variance of the final output to that of the exogenous stochastic element as well as by the transfer function value at selected frequencies. We then discuss how the changes in the inventory adjustment process that we examine could be brought about by a policy of inventory subsidies or by changes in the employer tax that finances unemployment insurance benefits.

1. The Production Adjustment Model

It is useful to begin by presenting and estimating a basic model of production adjustment. The desired end-of-period level of finished goods inventories (I_t^*) is specified most simply as a linear function of sales (S_t):[1]

$$(1.1) \quad I_t^* = \alpha S_t + \beta.$$

For the moment, we assume that sales during the period are known with certainty at the beginning of the period.

Production during the period (P_t) is the sum of sales and the change in inventories:

$$(1.2) \quad P_t = S_t + I_t - I_{t-1}.$$

Desired production can therefore be written as

$$(1.3) \quad P_t^* = S_t + I_t^* - I_{t-1}.$$

The production adjustment model emphasizes that actual production adjusts to the desired level of production with a lag. The extent of this production smoothing depends on the cost of changing the rate of production, the cost of holding inventories, and the loss that would result from inadequate inventories (see Holt, et al, P.160). A simple proportional adjustment of the production rate implies:

$$(1.4) \quad P_t - P_{t-1} = \lambda \, (P_t^* - P_{t-1})$$

Using equations 1.1 and 1.3 to eliminate P_t^* yields the estimable equation

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[1]A linear inventory target function of this form is derived explicitly from a quadratic loss function in Holt et.al. (1960).

(1.5) $P_t = \lambda\beta + \lambda(1+\alpha)S_t - \lambda I_{t-1} + (1-\lambda)P_{t-1}$.[1]

Because the actual sales in a period cannot be known precisely at the beginning of the period, it is customary to assume that inventory targets are initially based on the sales expected before the period begins (S_t^e) and then revised as information on actual sales accumulates. We can extend the model of equations 1.1 through 1.5 by specifying that the inventory and production targets are based on a weighted average of the initial sales expectation and the ultimate level of actual sales: $\theta S_t^e + (1-\theta)S_t$. This implies

(1.6) $I_t^* = \alpha S_t + \alpha\theta(S_t^e - S_t) + \beta$

and

(1.7) $P_t^* = S_t + \theta(S_t^e - S_t) + I_t^* - I_{t-1}$.

The estimable production equation is therefore:

(1.8) $P_t = \lambda\beta + \lambda(1+\alpha)S_t - \lambda I_{t-1} + \lambda\theta(1+\alpha)(S_t^e - S_t) + (1-\lambda)P_{t-1}$

We can convert this production equation into an inventory investment equation by using equation 1.2 to substitute sales plus inventory investment for

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[1] The production adjustment model thus differs somewhat from the model of inventory adjustment developed and estimated by Lovell. In Lovell's formulation, the lagged adjustment refers to the correction of inventories rather than production: equation 1.3 is thus $I_t - I_{t-1} = \lambda'(I_t^* - I_{t-1})$. We believe that there is no reason for this assumption that production adjusts fully to changes in sales but only partially to changes in desired inventory. We regard the production adjustment process of equation 1.3, in which production adjusts at the same rate to all changes in desired production (whether due to changes in sales or to changes in desired inventory) as a more appropriate description; see Holt, et.al. (1960) and Hay (1970). In Feldstein and Auerbach (1976), we examined Lovell's inventory adjustment model and found that it had the same implausible parameter values as the current production adjustment model.

production:[1]

$$(1.9)\quad I_t - I_{t-1} = \lambda\beta + \lambda\alpha S_t - \lambda I_{t-1} + \lambda\theta(1+\alpha)(S_t^e - S_t) + (1-\lambda)\ (I_{t-1} - I_{t-2} + S_t - S_{t-1}).$$

We have estimated equation 1.9 with quarterly data for durable goods manufacturing industries and for nondurable goods manufacturing industries. The sample period extends from the first quarter of 1960 through the third quarter of 1976.[2] All of the variables are expressed in constant dollars. Expected sales are represented by the predictions generated by an ARIMA process.[3] We are grateful to the Department of Commerce for providing newly-constructed unpublished estimates of constant dollar inventories.

Ordinary least squares estimates of the parameters of equation 1.9 will be biased if the random disturbances are serially correlated. We can eliminate the effect of first order serial conclation by an autoregressive transformation of equation 1.9; we use nonlinear least squares to obtain an efficient estimate of this autocorrelation coefficient together with the other parameters of equation 1.9. Since higher order serial correlation could still bias the parameter estimates, we also used an alternative instrumental variable method which provides consistent parameter estimates that are robust with

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[1]Note that this equation differs from Lovell's inventory adjustment model by the presence of the final term; see previous footnote.

[2]The inventory series used here is only available from the fourth quarter of 1958. We start with a later period because we require lagged values of the dependent variable.

[3]Although a sales expectation series based on survey data is available, we have substantial reservations about its quality. Our ARIMA estimates are based on a fourth order autoregressive process fitted to percentage changes in sales. In Feldstein and Auerbach (1976) we used the survey data on expectations and found similar results for the target adjustment model.

respect to the degree of serial correlation.[1] The results of this instrumental variable procedure were similar to the results of the autoregressive transformation and will not be presented separately.

The parameter estimates of equation 1.9 for durable goods implies an implausibly slow speed of adjustment: $\hat{\lambda} = 0.057$ with a standard error of 0.024. This implies that only 5.7 percent of the total adjustment between desired and actual production occurs with one-quarter. During the sample period, the maximum peak-to-trough production adjustment, which occurred over a period of six quarters, was equivalent to less than three weeks of production.[2] It is inconceivable that production smoothing would induce such a slow adjustment when the entire amount of the change was equivalent to so few days of production.

The parameter estimates of equation 1.9 for the nondurable goods inventories were also implausible but for a different reason. The long-run inventory-sales ratio (α in equation 1.9) has an estimated value of only 0.15 for less than one-third of the average ratio of inventories to sales during the sample period.

It is clear from these estimates that the traditional production adjustment model is inadequate. We turn therefore to the richer two-speed adjustment model.

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[1] More specifically, we used instrumental variables for the lagged investment terms in 1.9 and then used a nonlinear procedure to obtain constrained estimates of the other parameters; see Amemiya (1974).

[2] Production fell from $38.16 billion in 1973:4 to $31.69 billion in 1975:2, a drop of $6.47 billion.

2. The Two-Speed Adjustment Process

In our previous study, we challenged the assumption that the desired level of inventories adjusts immediately to changes in the level of current sales. Firms may adjust their target or desired inventory slowly for a variety of different reasons: inventory guidelines are often established in multiyear plans and revised only slowly; inventory targets depend on the company's warehousing facilities and personnel, which can adjust only slowly; learning and adjustment may be slow because excess inventories involve relatively little cost; etc. We therefore extend equation 1.1 and specify that the inventory target adjusts according to:

$$(2.1)\quad I_t^* - I_{t-1}^* = \mu(\beta + \alpha S_t - I_{t-1}^*).$$

Since actual sales are not known at the beginning of each quarter, it may be preferable to regard equation 2.1 as a model of the adjustment of the inventory target as of the end of each quarter and to assume that production planning is based on an inventory target that depends on a weighted average of the sales anticipated at the beginning and the actual sales during the period. Letting I_t^{*a} denote this anticipated or tentative inventory target, we may write:

$$(2.2)\quad I_t^{*a} - I_{t-1}^* = \mu[(\theta S_t^e + (1-\theta)\, S_t) + \beta - I_{t-1}^*].$$

Desired production is then given by:

$$(2.3)\quad P_t^* = \theta S_t^e + (1-\theta)S_t + I_t^{*a} - I_{t-1}.$$

Note that this depends on I_t^{*a} and not on I_t^*; we therefore combine equations 2.1 and 2.2 to eliminate I_t^* and obtain:

$$(2.4)\quad I_t^{*a} = \frac{\mu}{1-(1-\mu)L}\,(\alpha S_t + \beta) + \mu\theta\alpha(S_t^e - S_t).$$

The two-speed adjustment model combines this adjustment of the inventory target with the partial production adjustment of equation 1.4. Rewriting this production adjustment as

$$(2.5)\quad P_t = \frac{\lambda}{1-(1-\lambda)L}\,P_t^*$$

and using 2.3 and 2.4 to define P_t^* yields the two-speed production adjustment equation:

$$(2.6)\quad P_t = \frac{\lambda}{1-(1-\lambda)L}\left[S_t - I_{t-1} + (1+\alpha\mu)\,\theta\,(S_t^e - S_t)\right] + \frac{\lambda\mu}{[1-(1-\lambda)L]\,[1-(1-\mu)L]}\left[\alpha S_t + \beta\right].$$

Since $P_t = S_t + I_t - I_{t-1}$, equation 2.6 can be rewritten as the corresponding two-speed inventory adjustment model:

$$(2.7)\quad I_t - I_{t-1} = \frac{\lambda\mu}{[1-(1-\mu)L]}\,(\alpha S_t + \beta) + \lambda(1+\alpha\mu)\,\theta\,(S_t^e - S_t) + (1-\lambda)\,(I_{t-1} - I_{t-2}) - (1-\lambda)\,(S_t - S_{t-1}) - \lambda I_{t-1}.$$

In contrast to equation 2.7, the simpler target adjustment model assumes that the evolution of the inventory target is the only source of lags in the production process, i.e., $\lambda = 1$. This implies:[1]

$$(2.8)\quad I_t = \frac{\mu}{1-(1-\mu)L}\,(\alpha S_t + \beta) + (1+\alpha\mu)\,\theta\,(S_t^e - S_t).$$

We have estimated equation 2.7 and tested whether the parameters are consistent with the pure target adjustment model. Our basic conclusion is that the target adjustment model is a very good approximation: firms adjust actual inventories to the changes in the target level of inventories within

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[1] Note that this differs slightly from the original target adjustment model discussed in Feldstein and Auerbach (1976) but that the two are equivalent when $\theta = 0$.

the same quarter but the target itself adjusts only slowly.

Equation 2.7 was estimated by non-linear least squares after a first-order autoregressive transformation.[1] The sample period extends from the first quarter of 1960 through the third quarter of 1976. The equation was estimated separately for durable goods and nondurable goods manufacturing industries. Table 1 presents the parameter estimates and the sum of squared residuals corresponding to the full two-speed adjustment process, to several restricted cases of the two-speed adjustment process, and to the target adjustment process.

Consider first the estimates relating to nondurable goods. Equation N1 shows that the production adjustment parameter (λ) is almost exactly one ($\hat{\lambda} = 1.01$), implying that production adjusts fully within the quarter to changes in desired production. In contrast, the inventory target adjustment parameter (μ) is only about one-tenth ($\hat{\mu} = 0.11$), indicating that only about 10 percent of the full adjustment of the inventory target to changes in sales occurs within the first quarter. More generally, $\lambda = 0.11$ implies that half of the target adjustment occurs in six quarters and that three years is required for 75 percent of the full target adjustment to occur. These values of λ and μ clearly indicate that the target adjustment model is a more appropriate description than the traditional production adjustment model.

The estimated value of $\hat{\alpha} = 0.53$ the equilibrium ratio of inventory stock to sales, is only slightly below the average inventory-sales ratio of .67 that prevailed during the same period and clearly more than the value implied by

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[1] The nonlinear least squares procedure constrains the coefficients to provide efficient estimates of the parameters. Relaxing the constraint does reduce the sum of squared residuals by a statistically significant amount. This suggests that a more general model than the two-speed adjustment process might be more appropriate. Our analysis of several possible generalizations did not produce such an alternative. We therefore limit our analysis to the general two-speed adjustment model and to restrictions upon it.

Table 1

Estimated Parameters of the Two-Speed Adjustment Model

Equation	λ	μ	α	β	θ	ν	ρ	DμS	SSR
					Durable manufacturing				
D1	0.94	0.045	0.53	4.87	-.05		0.35	1.77	2.75
D2	0.98	0.044	0.54	4.84	(0.00)	-	0.41	1.78	2.82
D3	(1.00)	0.042	0.53	5.16	0.01	-	0.43	1.72	2.85
D4	(1.00)	0.042	0.53	5.16	(0.00)	-	0.43	1.72	2.85
D5	(1.00)	0.058	0.57	3.17	(0.00)	0.61	0.10	1.84	2.77
					Nondurable manufacturing				
N1	1.01	0.111	0.53	4.39	0.06	-	0.49	2.06	2.44
N2	0.97	0.097	0.51	4.87	(0.00)	-	0.41	2.06	2.49
N3	(1.00)	0.104	0.52	4.57	0.05	-	0.47	2.07	2.45
N4	(1.00)	0.107	0.52	4.50	(0.00)	-	0.46	2.06	2.51
N5	(1.00)	0.142	0.54	3.83	(0.00)	1.17	0.63	2.07	2.48

Estimates of equation 2.7 are based on quarterly data from the first quarter of 1960 through the third quarter of 1976.

Figures in parentheses are constrained values rather than sample estimates. The value of ρ is the estimated first order serial correlation of the disturbances.

the production adjustment model. The value of θ is very low ($\hat{\theta} = 0.06$), implying that the inventory target adjusts to actual sales rather than a weighted average of actual and predicted sales. This is quite consistent with the slow speed of target adjustment.[1] Equation N2 constrains θ to be zero; this changes the other coefficients only slightly and raises the sum of squared residuals by an insignificant amount.[2]

Equation N3 is the target adjustment model with λ constrained to equal one. Since the unconstrained estimate in equation N1 was so close to one, imposing the constraint has almost no effect on the sum of squared residuals. The same is true in equation N4 when θ is also constrained to be zero.

Equation N5 presents a more general form of the target adjustment model; the one-parameter geometric adjustment process is replaced by the more general two-parameter Pascal distribution. The point estimates of the two parameters are $\mu = 0.142$ and $\nu = 1.17$; ν is thus quite close to the geometric value of $\nu = 1$ and μ is quite close to the previous estimates. Comparison of the sums of squared residuals of N5 and N4 shows that the two-parameter Pascal distribution is not statistically superior to the simpler geometric proportional adjustment model.

The results for the durable goods industries are very similar to the nondurable results. Equation D1 shows a value of λ that is close to one and a value of μ that is quite low. When the insignificant (and negative)

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[1] The small coefficient on the difference between actual and expected sales was always a paradox in the context of the traditional production adjustment model: production appeared to react slowly to changes in the target but rapidly to the gap between actual and expected sales. This apparent paradox is eliminated by the target adjustment model.

[2] The SSR rises from 2.44 to 2.49. The likelihood ratio for a sample of 55 observations is .572; using the large-sample approximation that minus twice the logarithm of this ratio (1.116) is distributed as chi-square under the hypothesis implies a test statistic of 2.706 at the 10 percent probability level; we therefore do not reject the null hypothesis that $\theta = 0$.

value of $\hat{\theta}$ is constrained to be zero (equation D2), the estimate of λ is 0.98. The target adjustment model is thus favored by the data over the more general two-speed process. The Pascal lag process shown in D5 is again not statistically better than the simple geometric response lag of equation D4.

3. Cyclical Properties of a Simple Macroeconomic Model

The regression results presented above indicate that inventory behavior in U.S. durable and nondurable goods manufacturing is well described by what we have referred to as the "target adjustment" model, with slow movements in inventory targets and rapid production adjustments to meet these targets and correct for errors in sales prediction. This characterization stands in marked contrast to the theory introduced by Metzler (1941) and empirically developed by Lovell and several others which stresses the importance of the unexpected component of sales in generating inventory cycles as firms slowly adjust production to replenish buffer stocks and approach new target inventory levels.

It is important to emphasize that the difference between our target adjustment model and the traditional production adjustment model is not merely a matter of interpretation but has potentially important implications about the stability of the economy. To understand these characteristics of the business cycle and the implications with respect to the design of stabilization policy, we incorporate a two-speed adjustment model into a small macroeconomic model. Using the techniques of spectral analysis, we analyze the transmission of a stochastic element in the consumption function and how it is influenced by the relative importance of the lags in adjustment of inventory targets and output.

Firm production in our model follows the two-speed adjustment process. Desired, or "target" end-of-quarter inventories, I_t^*, partially adjust to a long run level, determined by sales, S_t:

$$(3.1)\quad I_t^* - I_{t-1}^* = \mu(\alpha S_t - I_{t-1}^*)$$

(3.1) is a simpler version of the model empirically examined above in that the constant, β and the the sales prediction error are omitted. The desired level of production, P_t^*, is that level necessary to meet sales demand and desired inventory accumulation, $I_t^* - I_{t-1}$:

(3.2) $P_t^* = I_t^* - I_{t-1} + S_t$

As before, the relation between actual and desired changes in production follows a partial adjustment model:

(3.3) $P_t - P_{t-1} = \lambda(P_t^* - P_{t-1})$

We complete our characterization of production behavior with the requirement that production must equal sales plus inventory accumulation:

(3.4) $P_t = I_t - I_{t-1} + S_t$

All output in this economy takes the form of consumption goods, either sold immediately or accumulated as inventories. Consumption demand, S_t, is stochastic and follows the permanent income hypothesis. Permanent income, Y_t^*, adjusts slowly to actual income, Y_t:

(3.5) $Y_t^* - Y_{t-1}^* = \mu(Y_t - Y_{t-1}^*)$

Consumption is a constant fraction of permanent income, plus a stochastic element, ε_t:

(3.6) $S_t = mY_t^* + \varepsilon_t$

We assume that the random term, ε_t, is generated by a Markov process:

(3.7) $\varepsilon_t = \rho\varepsilon_{t-1} + \nu_t$

where the underlying disturbances, ν_t, are serially independent and stationary, with zero mean.

To close our model, we include the income-production identity:

(3.8) $Y_t = P_t$

Before analyzing the model consisting of equations (3.1) to (3.8), we must briefly review certain elements of spectral theory.[1]

It is a fundamental result that any real time series sequence, $\{z_t\}$, generated by a covariance stationary stochastic process may be exactly represented by an aggregation of mutually uncorrelated random periodic functions, varying in frequency from zero to π radians (½ cycle) per unit time. The variance of the component of frequency ω, written $f_z(\omega)$, is referred to as the power spectrum at ω. Since the random components are mutually uncorrelated, the variance σ_z^2 of z is just the integral over ω from 0 to π of the power spectrum:

(3.9) $\sigma_z^2 = \int_0^\pi f_z(\omega)d\omega$

Thus, we may view the time series as being composed of a continuum of periodic elements, each contributing to the overall variance according to the magnitude of $f_z(\omega)$, the power spectrum.

If the random variables in the sequence are serially uncorrelated, the process is referred to as white noise, and has a flat spectrum; that is, the

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[1] A more complete exposition of the application of spectral analysis to economic problems may be found in Granger and Hatanaka (1962), Nerlove (1964), or Fishman (1968).

spectrum of a white noise series is:

$$(3.10)\quad f_z(\omega) = \sigma^2/\pi \qquad 0 \leq \omega \leq \pi$$

A linear combination of different elements of a time series is known as a filter. The spectral characteristics of the resulting series are neatly related to those of the original series.

Consider the filter:

$$(3.11)\quad r_t = \sum_{s=0}^{\infty} a_s z_{t-s}$$

Letting L be the lag operator, we may alternatively describe the filter in (3.11) in terms of

$$(3.12)\quad r_t = a(L) z_t$$

where:

$$(3.13)\quad a(L) = \sum_{s=0}^{\infty} a_s L^s$$

If we define the filter's <u>transfer</u> <u>function</u> at frequency ω by:

$$(3.14)\quad T_{rz}(\omega) = \left(\sum_{s=0}^{\infty} a_s \cos\omega s\right)^2 + \left(\sum_{s=0}^{\infty} a_s \sin\omega s\right)^2$$

then the spectrum of r obeys:

$$(3.15)\quad f_r(\omega) = T_{rz}(\omega)\, f_z(\omega)$$

Thus, the transfer function determines the extent to which the filter magnifies or attenuates the variance component at each frequency.

It is a simple extention of the above result that the application of a second filter, with transfer function T_{qr}, to series r_t will result in a series q_t with power spectrum:

(3.16) $f_q(\omega) = T_{qr}(\omega)T_{rz}(\omega)f_z(\omega)$

The model outlined in equations (3.1) - (3.8) describes a series of filters through which white noise, represented by ν_t, is transmitted to the output series, P_t. Through manipulation of these equations, we obtain the "final form" of output expressed in terms of the exogenous random shocks:

$$(3.17)\quad P_t = \left\{\frac{\phi_0(L)}{\phi_1(L)} - \frac{xm}{[1-(1-x)L]}\right\}^{-1} \varepsilon_t$$

where the lag polynomials $\phi_0(L)$ and $\phi_1(L)$ are:

$$(3.18)\quad \begin{aligned} \phi_0(L) &= 1 + (-3+2\lambda+\mu)L + (1-\lambda)\ (3-2\mu)L^2 - \mu - \lambda)\ (1-\mu)L^3 \\ \phi_1(L) &= (1+\mu\alpha)\lambda + [\mu(1-\alpha)-1]\ \lambda L \end{aligned}$$

Similarly, we obtain from (3.7)

$$(3.19)\quad \varepsilon_t = [1-\rho L]^{-1}\ \nu_t$$

Both (3.17) and (3.19) define linear filters with infinite lags, which we will represent by the lag polynomials $a^P(L)$ and $a^\varepsilon(L)$, respectively.

Returning to our original objective, we are interested in determining the relationship between ε_t, the stochastic demand component, and output, P_t, and how it is influenced by the values of the production parameters μ, λ and α. Of particular interest are the magnitude and spectral shape of the transfer function, $T_{p\varepsilon}$, of the filter $a^P(L)$, and the relative size of the overall variances, σ_p^2 of P and σ_ε^2 of ε.

Letting $T_{\varepsilon\nu}$ be the transfer function of the filter $a^\varepsilon(L)$, the variance of output is:

$$(3.20)\quad \sigma_p^2 = \int_0^\pi f_p(\omega)d\omega = \frac{\sigma_\nu^2}{\pi}\int_0^\pi T_{p\varepsilon}\ (\omega)\ T_{\varepsilon\nu}\ (\omega)\ d\omega$$

It is easy to demonstrate that

$$(3.21)\quad \sigma_\varepsilon^2 = \sigma_\nu^2 / (1 - \rho^2)$$

Thus, the ratio $\sigma_p^2/\sigma_\varepsilon^2$, which we shall denote R, is:

$$(3.22)\quad R = \frac{(1-\rho^2)}{\pi} \int_0^\pi T_{p\varepsilon}(\omega) T_{\varepsilon\nu}(\omega)\, d\omega$$

A computer algorithm was used to calculate the filter coefficients of $a^p(L)$ and $a^\varepsilon(L)$; these were then used to calculate the transfer functions. Since both filters have an infinite number of coefficients, truncation was necessary. Starting from the zero lag, coefficients were calculated successively until stringent partial sum convergence criteria were satisfied. Experiments with different criteria indicated that the truncation error imparted to $T_{p\varepsilon}$ and $T_{p\nu}$ was negligible. The actual number of coefficients calculated tended to be about 280 for $a^p(L)$ and fewer for $a^\varepsilon(L)$.

To calculate the variance ratio R, the integral in (3.22) was approximated by a summation over a grid of size π/N:

$$(3.23)\quad \hat{R} = \frac{(1-\rho^2)}{N} \sum_{s=1}^{N} T_{\rho\varepsilon}(s\,\tfrac{\pi}{N})\, T_{\varepsilon\nu}(s\tfrac{\pi}{N})$$

After experimentation with different values of N, a grid size of $\pi/360$ radians, or $\frac{1}{2}^{\circ}$, was used. There was virtually no change in $\hat{R}$ caused by using a finer grid.

In the simulation runs, the unit of time was taken to be one quarter, since our empirical findings were based on quarterly data. The values of m and μ were kept fixed at .9 and .15, respectively, the latter value indicating an adjustment of permanent to actual income of a little less than 50 percent in the first year.

The regression results of the previous two sections have supported the view that it is inventory targets, rather than production levels, which adjust slowly, and that production itself adjusts quite rapidly. The cyclical behavior of output implied by this "target adjustment" model (TAM) is compared in Table 2 with that which results from the more traditional "production adjustment" model (PAM). For each case, the parameter α is set at .5, a value close to those empirically estimated above. For the production adjustment model, we set $\lambda = .05$ and $\mu = 1$, while for the target adjustment model, $\lambda = 1$ and $\mu = .05$.

The two models imply clearly different cyclical characteristics. The transfer function from the stochastic element ε to output P, which measures the relative variance of periodic components of P and ε at particular frequencies, is more uniform across different frequencies for the TAM than for the PAM. The maximum value of the TAM transfer function is lower and its minimum value is higher. While, for the TAM, the transfer function falls between 1 and 5 for all but 3% of the periodic frequencies, the corresponding values for the PAM lie <u>outside</u> this range for 96% of all frequencies, falling both below unity and above 5. Thus, the stability of output under the production adjustment model depends more on the particular "spectral shape" of the random element, ε. When the autocorrelation coefficient, ρ equals zero, ε is white noise and has a flat spectrum. In this case, under PAM, output has a variance which is smaller than that of ε ($R = \sigma_p^2/\sigma_\varepsilon^2 = 0.99$), while for TAM the variance of output is almost twice that of ε ($R = 1.94$). As ρ increases, the spectral shape of ε changes and leads to increases in the volatility of output under both regimes. Although the impact of this change in the spectral shape of ε is greater in the PAM, only as ρ approaches 0.9 does output become more volatile under PAM than under TAM.

Table 2

	Production Adjustment Model $\mu=1,\ \lambda=.05,\ \alpha=.5$	Target Adjustment Model $\mu=.05,\ \lambda=1,\ \alpha=.5$
$\sigma_p^2\ /\ \sigma_\varepsilon^2$		
$\rho=0$	0.99	1.94
$\rho=.5$	2.90	3.31
$\rho=.9$	13.90	12.27
Transfer Function		
Maximum value	71.3689	69.3114
Minimum value	0.0007	1.2278
% below 1.00	91%	0%
% above 5.00	5%	3%

In short, our analysis of the simple stochastic model suggests that an economy in which inventory investment follows the target adjustment model will on average experience greater variance of production than an economy that follows the production adjustment model. Although this is true on average, the PAM economy will have greater cyclical volatility in response to disturbances of particular frequencies combined with much less sensitivity over a broad range of frequencies. Since the model used in this section is obviously over-simplified, these implications must be regarded as only suggestive. They do however show the importance of distinguishing the two types of inventory behavior and indicate the desirability of examining their stability properties in more realistic models.

In concluding this section, it is useful to examine how tax policies that alter the costs of different inventory and production policies affect the stability of the economy. When demand for a firm's output fluctuates cyclically, firms may react to such shifts by initiating changes in the level of output, by running up or down the level of inventories, or by the use of some intermediate policy. Shifting production and employment levels involves costs for the firm. However, relying on inventories to respond to fluctuations is also costly, since a greater level of inventories must be kept on hand, on average. The relative magnitude of the costs of shifting employment and holding inventories will determine the pattern of production. Feldstein (1976) has suggested that the current structure of unemployment compensation in the U.S. reduces the cost to the firm of temporarily laying workers off in response to cyclical drops in demand. Hence, enactment of various proposed reforms could be expected to lead to a decline in the use of temporary layoffs to respond to changes in demand, with a concommitant increase in average inventory holdings. In our model, this corresponds to a decline in the production adjustment parameter λ, and a rise in the long-run inventory-sales ratio, α. The application of a subsidy to the holding of inventories would have a similar effect.

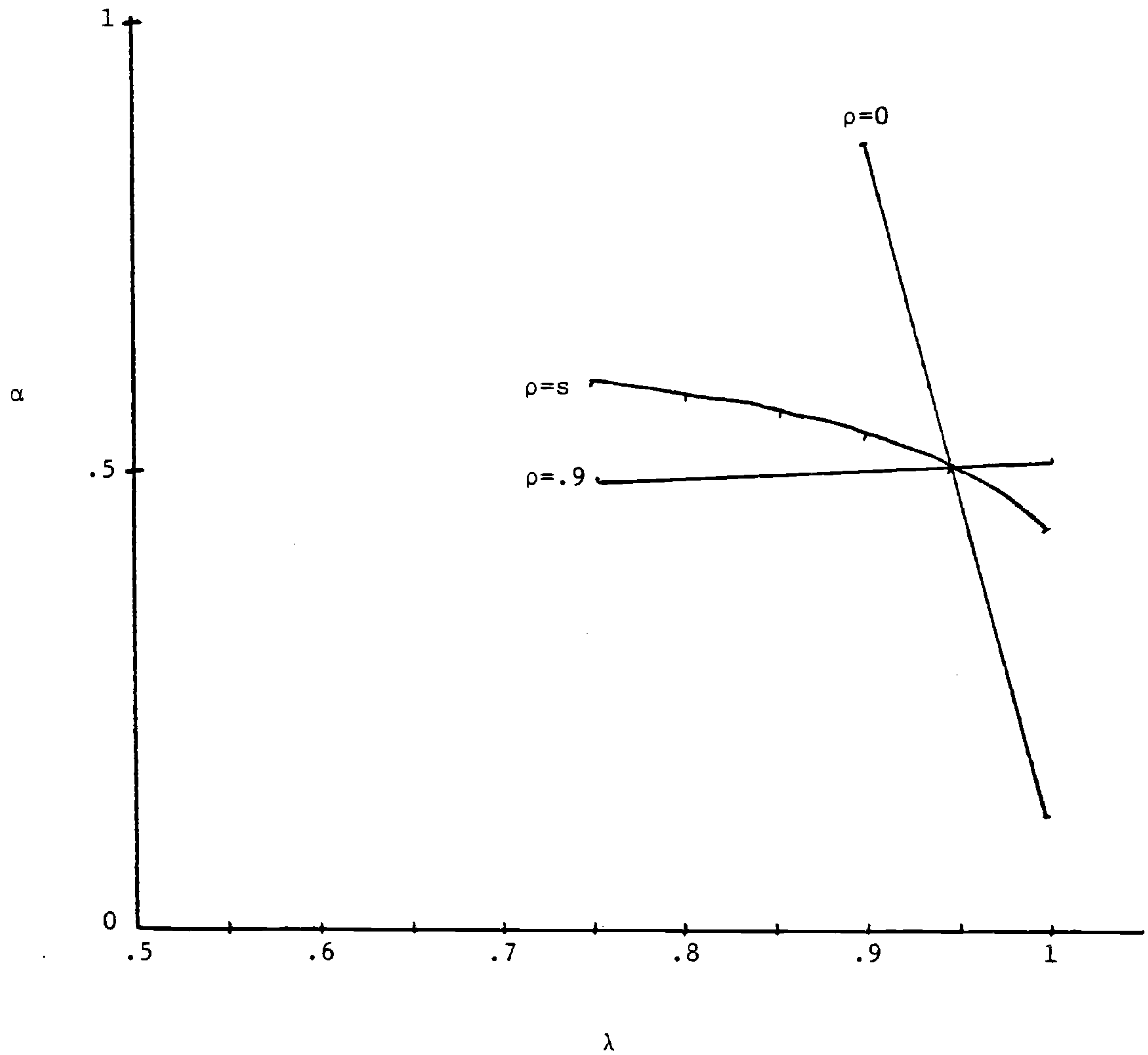
We may use our target adjustment model to analyze the net impact on the cyclical stability of production of policies which lower λ and raise α. Figure 1 shows those combinations of α and λ which yield the same variance of output as occurs when $\lambda = 0.95$ and $\alpha = 0.50$. These values of λ and α are representative of the empirical findings presented above. Each curve corresponds to a different autocorrelation coefficient of the random comport ε.

In all cases examined, increases in the inventory-sales ratio α lead to an increase in the variance of output. However, for low values of the error autocorrelation, small decreases in λ make output much less volatile. For example, with $\rho = 0$ a decrease in λ from 0.95 to 0.90 would be sufficient to offset an increase in α from 0.50 to 0.85. In this situation, a policy that reduced the speed of production adjustment even slightly would be stabilizing even if the policy also raised the inventory-sales ratio substantially.

However, as ρ increases, the stabilizing impact of a decline in λ diminishes to the point where, at $\rho = 0.9$, a slower speed of production adjustment would be destabilizing. For very high values of ρ, the adoption of policies that raise α and lower λ would therefore be destabilizing.

The very simple character of the model that we have studied implies that these results must be regarded with caution. They do suggest however that policies that change the current incentives for rapid production adjustment with correspondingly low inventories may increase overall stability.

Figure 1



4. Conclusion

The analysis in this paper was motivated by our discovery that the parameter estimates of the traditional productional adjustment model are not consistent with the observed magnitudes of inventory change and production. We have shown here that this production adjustment model is a special case of a more general two-speed adjustment process in which both production and inventory targets adjust slowly. Our estimates of the two-speed model clearly reject the production adjustment model in favor of the target adjustment model in which the inventory target adjusts slowly to changes in sales but production adjusts rapidly to changes in the desired inventory.

Our analysis of the spectral properties of a simple macroeconomic model show that the production adjustment model and the target adjustment model can imply quite different cyclical behavior of the economy as a whole. Depending on the autocorrelation of the disturbance, government policies that reduce the speed with which production responds to changes in desired inventories and that place greater reliance on inventory adjustment may stabilize national income. Further anlaysis of these questions with more realistic models would clearly be desirable.

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