

NBER Working Paper Series

A MODEL OF SOCIAL SECURITY
AND RETIREMENT DECISIONS

Eytan Sheshinski

Working Paper No. 187

CENTER FOR ECONOMIC ANALYSIS OF HUMAN BEHAVIOR
AND SOCIAL INSTITUTIONS

National Bureau of Economic Research, Inc.
204 Junipero Serra Boulevard, Stanford, CA 94305

July 1977

Preliminary; not for quotation.

NBER working papers are distributed informally and in limited number for comments only. They should not be quoted without written permission of the author.

This report has not undergone the review accorded official NBER publications; in particular, it has not yet been submitted for approval by the Board of Directors.

Partial financial support for this research was provided by a contract with NBER from the Office of Income Security Policy, Department of Health, Education, and Welfare (No. HEW-100-76-0170); the research was also partially supported by The Project on Efficiency of Decision-making in Economic Systems at Harvard University. The author wishes to thank Peter Diamond and Shlomo Yitzhaki for helpful discussions.

A MODEL OF SOCIAL SECURITY AND RETIREMENT DECISIONS

Eytan Sheshinski *

1. Introduction

One of the primary objectives of social security is to replace income during retirement. In so doing, social security benefits supplement and partially substitute for prior savings. The presence of these benefits is therefore expected to affect individuals' decisions concerning consumption, savings and labor supply, including the choice of retirement age. The purpose of the present paper is to focus on the potential inducement to retire earlier in the presence of social security and on the implied effects on lifetime savings.

This problem is analyzed within the framework of a model of intertemporal utility maximization. It is assumed that individuals can either work full time or not work at all. During their working phase, individuals pay a social security tax each period. After retirement they are eligible to receive each period a pension from social security which, in general, may depend on their retirement age and on their

* The paper is prepared for presentation at the NBER Conference on Social Insurance at Stanford in January 1977.

prior earnings. A consumption path and a retirement age are chosen so as to maximize lifetime utility.

Aggregate behavior is examined against two alternative hypotheses. Initially it is assumed that each generation's present discounted value of payments to and benefits from social security are equal. This assumption is intended to separate the substitution effects from the intergenerational transfer aspects of a social security program. As is well known, a program which is based on a 'pay-as-you-go' principle generates intergenerational transfers when the long-run growth rate of population is different from the rate of interest used to discount incomes. The second part of the paper incorporates these transfers into the analysis.

An examination of the equilibrium conditions for the economy reveals the possibility for the existence of multiple equilibria in the presence of social security. Dynamic considerations are then suggested to identify which equilibria are locally stable or unstable. Subsequent comparative static analysis focuses on the stable equilibria. In particular, we evaluate the effects of balanced changes in social security benefits and taxes on the equilibrium retirement age and on the individuals' wealth-income ratio at retirement, under alternative assumptions concerning the dependence of the benefits formula on the retirement age.

The results in this part pertain to a simple case of the underlying model and can be summarized as follows:

- (1) Social security benefits have a very pronounced effect in inducing earlier retirement. For example, when the system

is balanced for each generation, a replacement ratio of twenty percent reduces the retirement age by more than fifty percent relative to retirement in the absence of social security.

(2) The effect on accelerated retirement can be significantly mitigated by allowing benefits to depend positively upon the retirement age. For example, when benefits provide a return on postponement of retirement equal to the rate of interest, then a replacement ratio of twenty percent reduces retirement age only by ten percent relative to retirement in the absence of social security.

(3) The effect of social security benefits on an individual's wealth-income ratio at retirement is uncertain. While increased benefits reduce the need for one's own savings during retirement, induced earlier retirement may lead to more savings during the working phase so as to partially offset the loss in earnings. The results suggest that for a relatively long time horizon the former effect always dominates the latter.

Again, the reduction in the wealth-income ratio can be mitigated by allowing the benefits formula to depend upon retirement age.

Some of these conclusions have to be modified when a "wealth-effect" via intergenerational transfers is allowed. Increases in the population growth rate enable, for given levels of benefits, a reduction in tax rates, thereby leading to a reduction in the equilibrium retirement age. The magnitude of this effect is positively related to the level of the replacement ratio and negatively to the rate of population growth.

The results suggest that the effects on earlier retirement of increased taxes cannot be neglected when, as currently in the U.S., a decrease in the population growth rate requires higher tax rates in order to preserve existing replacement ratios.

An important feature of the post-war U.S. economy has been the rapid decrease in the labor force participation of the elderly. How much of this decrease can be attributed to the emergence of social security, private pension funds or just "poor health" is a matter of debate. Recent results suggest, however, that certain aspects of the social security program, such as the income guarantee and the earnings test, have been a major factor in inducing earlier retirement.¹ Thus, while the idealized life-time planning model described in this paper may be inappropriate for the behavior of a certain fraction of the population (Diamond [1976]), it reveals the potential distortions created by a social security program for individuals that behave rationally. Obviously, these distortions should be evaluated against the redistributive and other objectives of the social security program, not analyzed in this paper.

The organization of this work is as follows. Section 2 presents the model of individual optimization and of the market equilibrium. Sections 3 through 5 present the comparative statics analysis. Section 3 evaluates the effects on the equilibrium retirement age, section 4 modifies the benefits formula to depend on retirement age and section 5 examines the wealth-income ratio effect. Section 6 introduces the intergenerational transfer problem. Section 7 presents the general model underlying the previous sections.

2. Individual Optimization and Market Equilibrium: A Simple Model

Our objective is to construct a model of a competitive economy with a system of social security benefits, focusing on the effect of this system on individuals' retirement decisions. It seems most useful to consider initially a simple case that brings out some of the main issues involved, to be followed subsequently by a more complete model. Such a detailed model, which rigorously justifies the analysis in this section and includes elements neglected here, is presented in section 7.

Consider first a single individual's problem of choosing jointly an optimum consumption and retirement plan. Suppose that the individual has a given life horizon of T , and that he decides to have a fixed level of consumption, c , over his entire lifetime. As is well known, the choice of a constant consumption level is optimal for a utility maximizing individual provided his subjective time preference is equal to the rate of interest.

When working, the individual is assumed to receive a fixed wage, w , independent of age. The amount of labor supplied while working cannot be varied. He may, however, decide to retire from work before the age of T , in which case he is entitled thereafter to social security benefits, at a given level b .

In a perfectly competitive capital market, with free lending and borrowing at a fixed rate of interest, r , the individual's budget constraint equalizes the present values of consumption and income. Treating age, t , as a continuous variable this constraint is written

$$c \int_0^T e^{-rt} dt = w \int_0^R e^{-rt} dt + b \int_R^T e^{-rt} dt$$

or

$$c(1-e^{-rT}) - w(1-e^{-rR}) - b(e^{-rR} - e^{-rT}) = 0 \quad (2.1)$$

where R ($T \geq R \geq 0$) denotes the retirement age.

The individual's optimum retirement is chosen to be the age where the benefits and costs of retirement balance. Assume that he has a fixed utility from retirement (or leisure), v , independent of age. Upon retirement he loses an income of $w-b$. Assume further that his marginal utility of consumption is equal to the inverse of the level of consumption.² Then his loss in terms of utility is equal to $c^{-1}(w-b)$. Thus, the individual's net marginal utility of postponing retirement, $\frac{dU}{dR}$, is equal to $\frac{dU}{dR} = c^{-1}(w-b) - v$. The first-order condition that determines the optimum R is therefore

$$\frac{dU}{dR} = c^{-1}(w-b) - v = 0 \quad (2.2)$$

An interior solution satisfying (2.2) requires, of course, that $w > b$. Equations (2.1) and (2.2) simultaneously determine the individual's optimum R and c . It is easy to verify that the second-order condition $\frac{d^2U}{dR^2} < 0$ is satisfied everywhere, and hence that when an optimum exists, it is unique.

Suppose further that the economy consists of numerous identical individuals, and can thus be represented by a single individual. At the outset, we wish to disregard the question of transfers between different generations. This issue is treated in section 6. We therefore postulate

that the present value of social security benefits is equal to the present value of deductions for each generation.

Assume that the individual receives before tax a wage of \bar{w} from which a fraction $1 > \theta \geq 0$ is deducted for social security. Thus, his net wage, w , is equal to $w = \bar{w}(1-\theta)$. The social security's budget constraint is given by

$$\theta \bar{w} \int_0^R e^{-rt} dt = b \int_R^T e^{-rt} dt$$

or

$$\theta \bar{w} (1 - e^{-rR}) - b(e^{-rR} - e^{-rT}) = 0 \quad (2.3)$$

Substituting (2.3) into (2.1) and (2.2) we obtain two equilibrium conditions for the economy, denoted by ψ and φ ,

$$\psi(R, c) \equiv c\phi - \bar{w} = 0 \quad (2.4)$$

and

$$\varphi(R, c) \equiv c^{-1}(\bar{w} - b\phi) - v = 0 \quad (2.5)$$

where $\phi \equiv \frac{1 - e^{-rT}}{1 - e^{-rR}}$ is uniquely related to R . Equations (2.4) and (2.5) determine the economy's equilibrium R and c for any given benefit level b . Denote such an equilibrium by (R^*, c^*) . It turns out that the solution to these equations is in general non-unique. Specifically, substituting (2.4) into (2.5) yields a quadratic equation in ϕ ,

$$\beta\phi^2 - \phi + v = 0 \quad (2.6)$$

where $\beta \equiv \frac{b}{\bar{w}}$ is the "replacement ratio," i.e., the ratio of social security benefits to before-tax income. Notice that the solution depends

only on β and not separately on the level of benefits and income.³ Now, for any positive β , equation (2.6) has generally two solutions:

$$\phi = \frac{1}{2\beta} \pm \frac{\sqrt{1 - 4\beta v}}{2\beta} \quad (2.7)$$

The solutions (2.7) are real-valued provided $1 - 4\beta v > 0$. This imposes an upper bound on the size of social security benefits. The function ϕ is monotone, strictly decreasing in R , with $\phi = 1$ when $R = T$. From (2.6), $\phi = v$ when $\beta = 0$. Hence, to ensure that individuals choose to retire before T in the absence of social security, we assume that $v > 1$.

Denote the two solutions to (2.6) by (R_1^*, c_1^*) and (R_2^*, c_2^*) , illustrated by points A and B in Figure 1. It is easy to verify that the curves $\psi(R, c) = 0$ and $\varphi(R, c) = 0$ intersect at these points as described. Let the equilibrium in the absence of social security be denoted by (\bar{R}, \bar{c}) . The level of \bar{R} is determined by (2.6) when $\phi(\bar{R}) = v$. One can show that $\phi = \frac{1}{2\beta} - \frac{\sqrt{1 - 4\beta v}}{2\beta} > v$ for any $\beta > 0$. It follows that $\bar{R} > R_1^* > R_2^*$. That is, in the presence of a social security program the economy has in general two equilibrium points, each having a lower optimum retirement age than in the absence of such a program.

The existence of multiple equilibria, familiar in "second-best" theory, has a straightforward explanation. Given a certain level of benefits, b , the movement from A to B (in Figure 1) is obtained by an increase in social security taxes, θ , along with a decrease in the optimum retirement age. Individuals choose to retire earlier because the opportunity costs of retirement, $w - b$, decrease due to the reduction in the net wage $w = \bar{w}(1 - \theta)$. The existence of multiple equilibria for the economy is consistent with the optimality of each equilibrium configura-

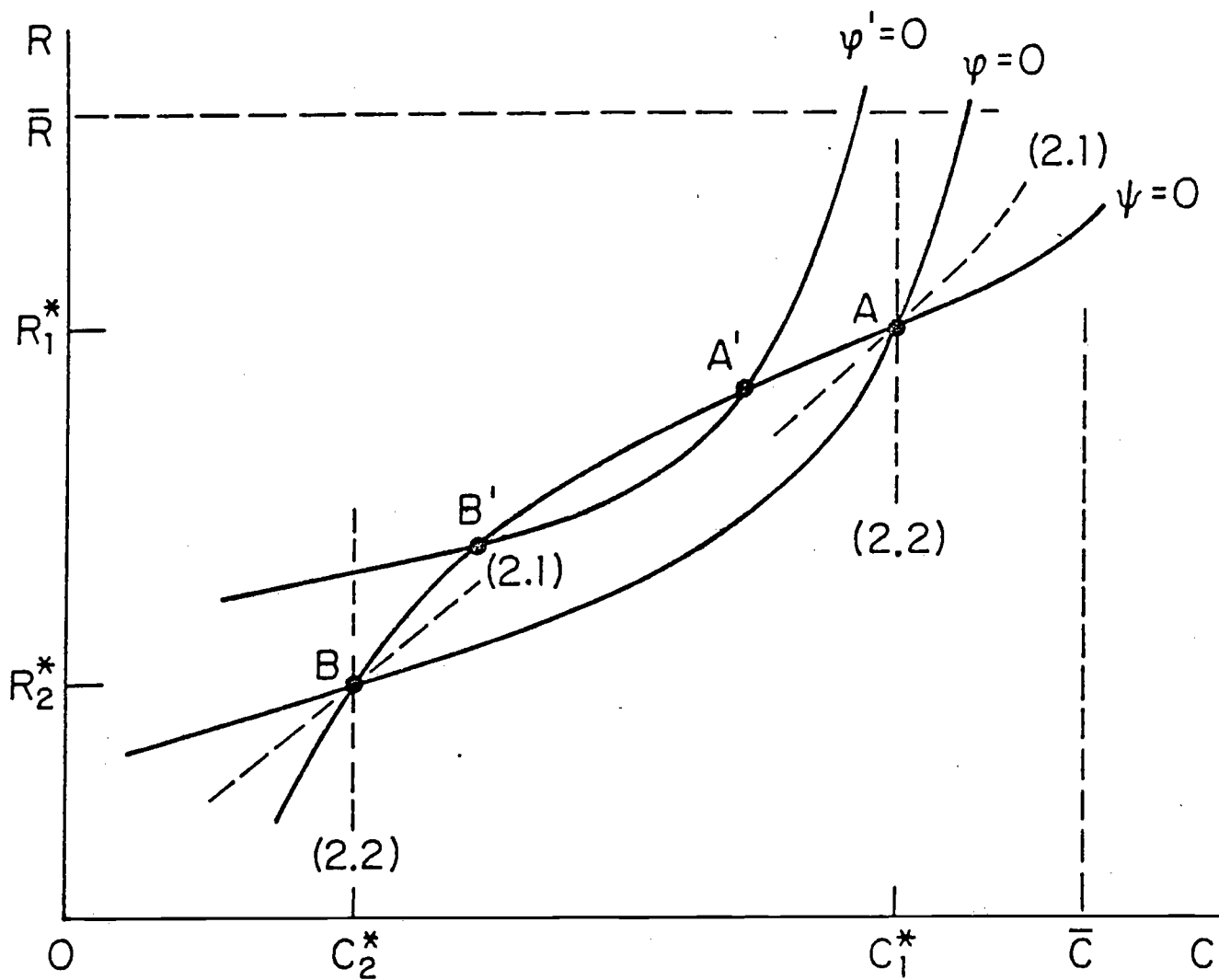


Figure 1

ation from the individual's point of view (Figure 1), since he considers his net wage as given, thereby disregarding the effect of his retirement decisions on the tax rate via the "macro" constraint, (2.3).

Under certain assumptions concerning the adjustments made by individuals in disequilibrium situations, one equilibrium point can be shown to be locally stable and the other to be locally unstable. Thus, suppose that tax rates are adjusted instantaneously so as to preserve the social security's constraint (2.3). On the other hand, individuals are assumed to adjust their retirement age upwards when the net benefits of postponing retirement, $c^{-1}(w-b) - v$, are positive and vice versa. Similarly, the consumption level is assumed to be adjusted downwards when the present value of consumption exceeds lifetime earning, and vice versa. Using (2.3), these assumptions can be expressed formally by the differential equations

$$\dot{c} = G((\bar{w} - c\phi)(1 - e^{-rR})) \quad (2.8)$$

and

$$\dot{R} = F(c^{-1}(\bar{w} - b\phi) - v) \quad (2.9)$$

where \dot{R} and \dot{c} are time derivatives of the respective variables, and F and G are sign-preserving, monotone-increasing functions.

Linearizing the system (2.8)-(2.9) around the equilibrium point $\dot{R} = \dot{c} = 0$, it is easy to verify that (R_1^*, c_1^*) (point A in Figure 1) is a locally stable equilibrium and (R_2^*, c_2^*) (point B) is a locally unstable equilibrium.

To summarize the present discussion, it has been demonstrated that in the presence of social security the competitive economy has two

equilibrium points. Under certain dynamic assumptions it can be shown that one equilibrium is locally stable and the other is locally unstable. It has further been shown that any equilibrium associated with a positive level of social security benefits has a lower equilibrium retirement age and a lower consumption level than these equilibrium values in the absence of social security.

3. Comparative Statics of the Simple Model

Differentiating (2.4) and (2.5) totally w. r. t. β , we find that

$$\frac{dR^*}{d\beta} = - \frac{(1 - e^{-rT})e^{rR^*}}{r(1 - 2\beta\phi)} \quad (3.1)$$

and

$$\frac{dc^*}{d\beta} = - \frac{\bar{w}}{1 - 2\beta\phi} \quad (3.2)$$

The effect of a change in the level of social security benefits on the equilibrium configuration clearly depends on the initial equilibrium considered.

The sign of the denominator in (3.1) and (3.2) is positive at the stable solution and negative at the unstable solution to (2.7). Hence, at the stable point, $\frac{dR^*}{d\beta} < 0$ and $\frac{dc^*}{d\beta} < 0$ (shift from point A to point A' in Figure 1). Opposite results obtain at the unstable equilibrium.

Some levels of R^* for alternative values of β are presented in Table 1. These levels pertain to the stable solutions of (2.7). It is seen that increases in social security benefits have a substantial effect on reducing the optimum retirement age. For example, a replacement ratio of twenty percent more than halves the chosen retirement age, compared to retirement in the absence of social security.

The large reduction in the retirement age is reflected in the correspondingly large increase in the tax rate, θ , required to finance these replacement ratios. For example, a twenty percent ratio already requires a twelve percent tax rate.

Table 1

Optimum Retirement Age and Tax Rate for
Alternative Replacement Ratios^a

β	T = 70	T = ∞	θ
0	50	63.40	0
.01	48.46	60.41	.001
.05	42.28	50.63	.008
.10	35.51	41.06	.024
.15	29.15	32.92	.055
.20	22.29	24.68	.119

^a Calculated from (2.3), (2.4) and (2.5), with $r = .04$ and $v = .1086$. The differences in the values of θ for $T = 70$ and $T = \infty$ were insignificant.

The unrealistically large response of the optimum retirement age could be mitigated by either one of two assumptions. First, assuming that the utility of retirement, v , is age-dependent. Specifically, if v were an increasing function of R , the effect of an increase in β on R^* can be expected to be smaller. Second, allowing the benefit function to depend positively upon R would also work to reduce the response of retirement to changes in β . The latter possibility is pursued in the next section.

4. Allowing Benefits to Depend Upon Retirement Age

The simplest way to make benefits depend upon retirement age is to postulate that they are paid-out to a retired individual provided his age exceeds a minimum level, say \hat{R} . If retired before the age of \hat{R} , benefits nevertheless are paid only beyond the minimum age.

Under this stipulation, the individual's budget constraint, (2.1), becomes

$$c(1-e^{-rT}) = w(1-e^{-rR}) + b(e^{-r\tilde{R}} - e^{-rT}) \quad (4.1)$$

where $\tilde{R} = \text{Max} [R, \hat{R}]$. Furthermore, by (4.1), the marginal utility of retirement, $\frac{dU}{dR}$, is now given by

$$\frac{dU}{dR} = \begin{cases} c^{-1}w - v = \frac{w(1-e^{-rT})}{w(1-e^{-rR}) + b(e^{-r\hat{R}} - e^{-rT})} - v & R < \hat{R} \\ c^{-1}(w-b) - v = \frac{(w-b)(1-e^{-rT})}{w(1-e^{-rR}) + b(e^{-rR} - e^{-rT})} - v & R \geq \hat{R} \end{cases} \quad (4.2)$$

Equation (4.2) is a decreasing function of R , having a (negative) discontinuity at \hat{R} (Figure 2). As b increases, $\frac{dU}{dR}$ shifts downwards. Eventually, the two parts of (4.2) will be positive and negative respectively, implying that the optimum retirement age is \hat{R} . Thus, over a certain range of values of b , the optimum retirement age remains \hat{R} . That is, in this range R^* is inelastic with respect to β . This conforms, perhaps, to the "clustering" of observed retirement ages in the U.S. around 62-65, the ages specified in the social security benefits formula.

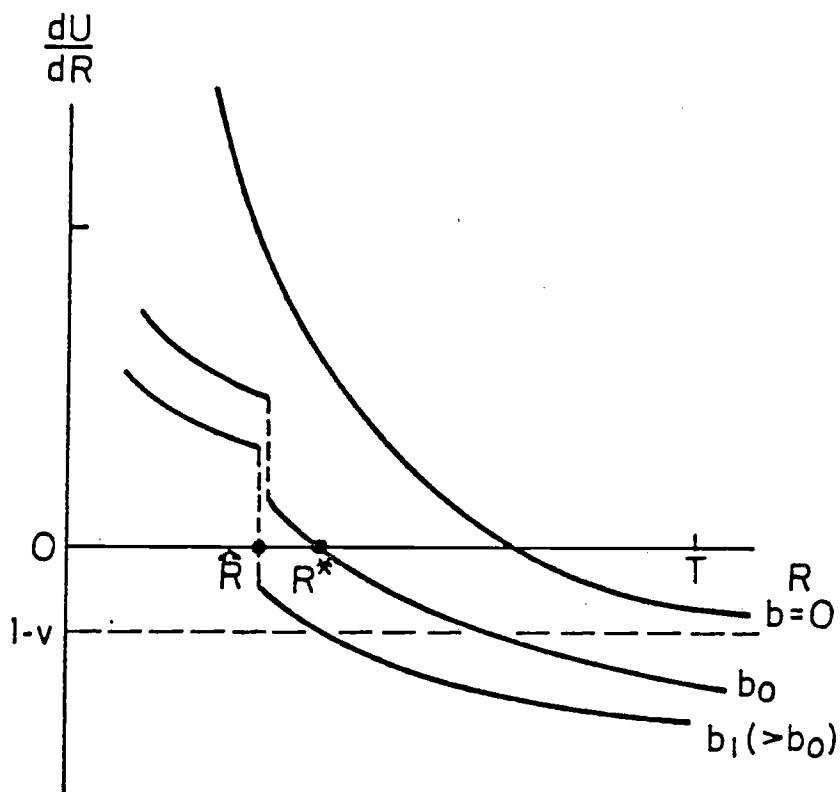


Figure 2

Suppose, alternatively, that benefits are allowed to depend continuously upon the retirement age, $b = b(R)$. The income benefits from a marginal postponement of retirement are now given by

$$w - b + \frac{\partial b}{\partial R} \frac{1}{r} (1 - e^{-r(T-R)})$$

where the last expression is the marginal change in benefits due to the postponement of retirement, integrated over the retirement period and discounted to the retirement date. We naturally assume that $\frac{\partial b}{\partial R} \geq 0$. Condition (2.2) that determines the individual's optimum retirement age now becomes

$$\frac{dU}{dR} = c^{-1} [w - b + \frac{\partial b}{\partial R} \frac{1}{r} (1 - e^{-r(T-R)})] - v = 0 \quad (4.3)$$

From (2.1), (2.3) and (4.3), the equilibrium conditions for the economy are now (2.4) and

$$\varphi(R, c) = c^{-1} [\bar{w} - b(\phi - \frac{\eta}{r} (1 - e^{-r(T-R)}))] - v = 0 \quad (4.4)$$

where $\eta = \frac{1}{b} \frac{\partial b}{\partial R}$ is the percentage yield in benefits on postponement of retirement. Substituting from (2.4) into (4.4) provides an equation to determine R^* ,

$$\beta \phi^2 - [1 + \frac{\beta \eta}{r} (1 - e^{-r(T-R)})] \phi + v = 0 \quad (4.5)$$

As in the previous case, equation (4.5) has, in general, multiple solutions. Clearly, when $\eta \neq 0$ and T is finite, the solutions to (4.5) must be found by iterative procedures. Some values of R^* for alternative levels of β , η and T are presented in Table 2. These values pertain to the stable solution of (4.5).

Before we discuss these calculations, consider the possibility of setting η at a level which will make R^* independent of β , and therefore equal to its value in the absence of social security. By (4.4), this condition is satisfied when $\eta = r\phi[1 - e^{-r(T-R)}]^{-1} > r$. In the special case that $T = \infty$, this condition simplifies to the form

$$b = \hat{b}e^{rR + \log(1 - e^{-rR})} \quad (4.6)$$

where \hat{b} is a scalar independent of R . Observe that efficiency requires the yield on postponement of retirement to be larger than the rate of interest, and also to be age-dependent. Specifically, the yield is seen to decrease with age, approaching r from above. The reason is quite clear. Postponement of retirement not only reduces the period over which benefits have to be paid, but also increases the period over which taxes are collected. This interpretation is particularly transparent in the infinite horizon case. The gain to social security from a marginal postponement of retirement is equal to saving b . The cost is equal to the present value of the increased taxes required to finance the additional benefits, $\frac{1}{r}(1 - e^{-rR})\frac{\partial b}{\partial R}$. At an efficient equilibrium these costs and benefits are equal.

Table 2

Optimum Retirement Age for Alternative Replacement Ratios
When the Benefits Function Depends on Retirement^a

β	T = 70		T = ∞	
	$\eta = .02$	$\eta = .04$	$\eta = .02$	$\eta = .04$
0	50	50	63.40	63.40
.01	48.75	49.20	61.85	63.13
.05	44.16	46.18	55.84	62.10
.10	38.85	42.51	49.12	60.64
.15	34.38	39.32	43.20	59.27
.20	29.30	36.45	37.31	57.59

^a Calculated from (4.5), with $r = .04$ and $r = 1.086$.

Comparing the results in Tables 1 and 2, it is seen that the dependence of the benefits function upon retirement age has a significant effect on the equilibrium levels of R^* . For example, when the benefits function provides a four percent yield on retirement postponement, then a replacement ratio of twenty percent reduces the optimum retirement age by approximately ten percent compared to retirement in the absence of social security benefits, while without such a provision the reduction is more than eighty percent. Still, we notice that even with $\eta = .04$ (equal to the rate of interest in these calculations), the elasticity of retirement with respect to the replacement ratio, $\frac{1}{R^*} \frac{\partial R^*}{\partial \beta}$, is of the order of .5, which seems quite large.

We conclude, therefore, that allowing benefits to depend (positively) on retirement age has a potentially large effect on diminishing the

negative effects of social security on retirement age, yet, in the studied range of parameters, the response of retirement age to changes in benefits cannot be disregarded.

5. Social Security and the Optimum Wealth-Income Ratio at Retirement

We now return to the assumption that benefits do not depend on retirement age, and focus on the effect of social security on savings. While working, individuals save $w-c$ each period, until retirement. These savings, compounded at an interest rate of r , amount to

$$S = \int_0^R (w-c)e^{rt} dt = \frac{(w-c)}{r} (e^{rR} - 1) \quad (5.1)$$

at the retirement age R . Substituting from (2.4) into (5.1), the ratio of wealth, S , to income before tax, \bar{w} , at retirement, denoted by s , is given by

$$s = \frac{S}{\bar{w}} = \frac{1}{r} (1 - e^{-r(T-R)}) \left(\frac{1}{\phi} - \beta \right) \quad (5.2)$$

From (2.6) and (2.7) one can verify that in equilibrium, $\frac{1}{\phi} > \beta$ for any $\beta > 0$, and hence that $s > 0$.

Using conditions (2.4) and (2.5), the equilibrium change in s due to an increase in β is found to be

$$\frac{ds}{d\beta} = \frac{1}{r} \left(\frac{1 - \beta\phi}{1 - 2\beta\phi} \right) [e^{-r(T-R^*)} (1 - e^{-rR^*}) - 2(e^{-rR^*} - e^{-rT})] e^{rR^*} \quad (5.3)$$

The sign of (5.3) is generally indeterminate. It depends, in particular, on whether the initial equilibrium point is locally stable or unstable, on the equilibrium value of R^* and on the parameters r and T . By (2.7), at the stable equilibrium (point A in Figure 1), $\frac{1 - \beta\phi}{1 - 2\beta\phi} > 0$ (< 0 at point B). The sign of (5.3) is then the same as the sign of the term in square brackets. Clearly, for large values of T , this sign is

negative. However, for finite values of T , a positive sign is possible. Some calculations of the equilibrium values of R^* and s with typical parameters are presented in Table 3 below.

Table 3
Optimum Wealth-Income Ratio and Retirement Age
for Alternative Replacement Ratios ^a

β	T = 60		T = 70		T = 85		T = ∞	
	R^*	s	R^*	s	R^*	s	R^*	s
0	45.39	10.19	50.00	12.66	55.98	15.81	63.40	23.02
.01	44.02	10.63	48.46	13.05	53.07	16.24	60.41	22.52
.05	38.94	11.64	42.28	13.72	45.67	16.21	50.63	20.45
.10	33.04	11.41	35.51	13.23	37.80	14.99	41.06	17.66
.15	27.39	10.60	29.15	11.72	30.74	12.89	32.92	14.55
.20	21.13	8.43	22.29	9.10	24.10	9.75	24.68	10.68

^a Calculated from (2.7) and (5.2) for $r = .04$ and $v = 1.086$.

From this table and the accompanying diagram (Figure 3) it is seen that for finite T in the chosen range, the optimum values of s first rise and then fall as β increases. Furthermore, as T increases, the increasing phase of s diminishes, eventually vanishing when T becomes infinitely large.

The reason for the ambiguous sign of the relation between the optimum wealth-income ratio at retirement and the replacement ratio seems clear. An increase in the replacement ratio directly reduces the need to finance consumption during retirement out of savings. However, the

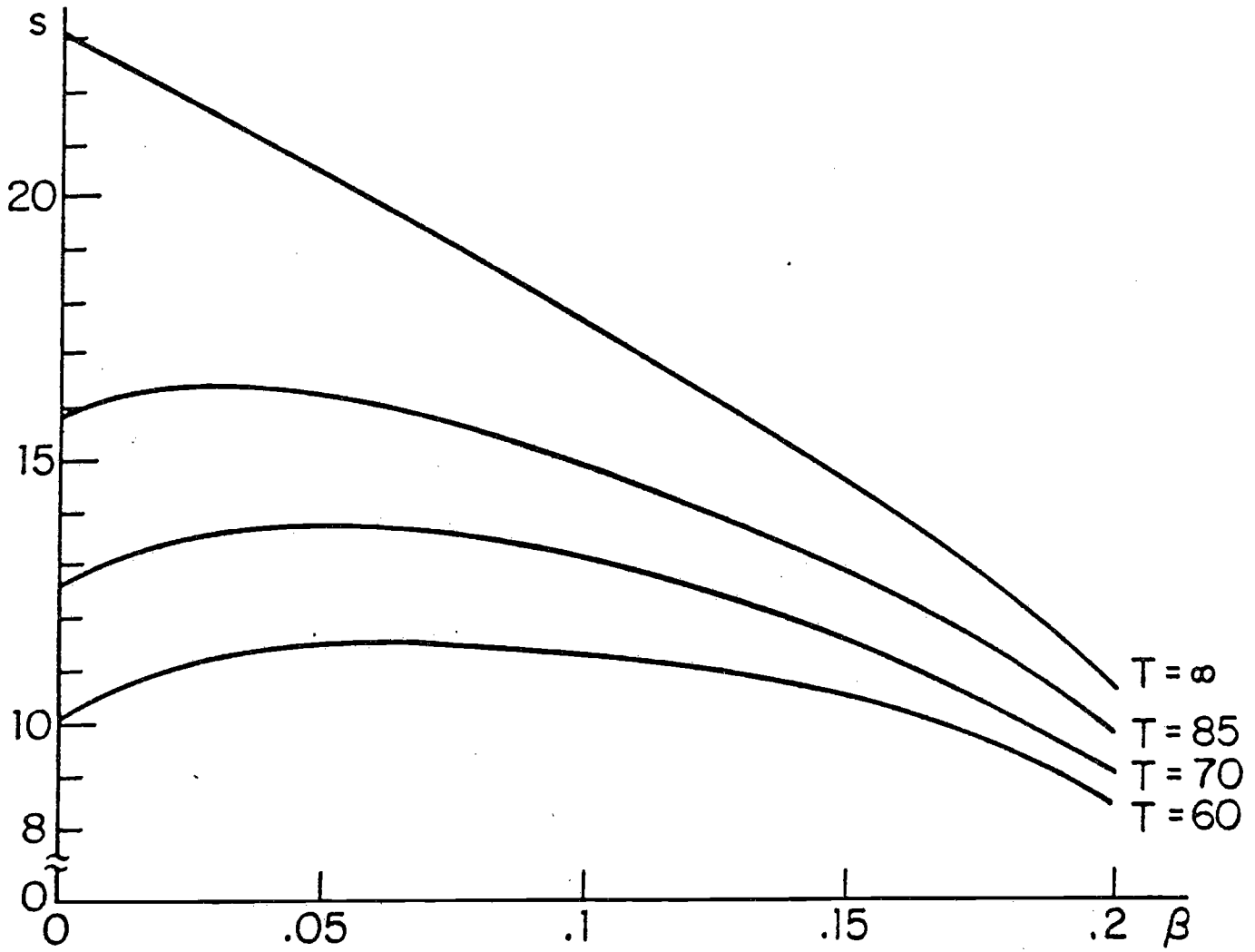


Figure 3

reduction in the chosen retirement age increases the retirement period and this requires, in order to maintain the consumption level, a larger wealth at retirement. These effects work in opposite directions and the net outcome cannot be determined a priori.

We shall make this assumption throughout. It is satisfied, for example,

when $r - \delta \geq 0$, $\frac{\partial w_R}{\partial R} \leq 0$ and $b(R, \bar{w}^{-R})$ is monotone and concave in R :

$$\frac{db}{dR} = \frac{\partial b}{\partial R} + \frac{\partial b}{\partial \bar{w}^{-R}} \frac{\partial \bar{w}^{-R}}{\partial R} \geq 0, \quad \frac{d^2 b}{dR^2} = \frac{\partial^2 b}{\partial R^2} + \frac{\partial}{\partial R} \left(\frac{\partial b}{\partial \bar{w}^{-R}} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \leq 0. \quad \text{The}$$

condition that $r - \delta \geq 0$ implies, by (7.3), that consumption does not decrease with age, while the condition $\frac{\partial w_R}{\partial R} \leq 0$ implies that the wage rate is not increasing at R^* .

Assuming that for each generation the present value of benefits and deductions is equal, the social security's budget constraint is given by

$$\theta \int_0^R \bar{w}_t e^{-rt} dt = b(R, \bar{w}^{-R}) \int_R^T e^{-rt} dt \quad (7.11)$$

where θ is the social security tax rate.

In analogy with standard tax theory, it is assumed that individuals ignore the impact of their decisions on the aggregate constraint (7.11). This is a plausible assumption under competitive conditions with many individuals.

Substitution of (7.11) into (7.4) and (7.5) yields the economy's equilibrium conditions

$$\hat{\phi}(R^*, \lambda^*, b) \equiv \lambda^* e^{(\delta-r)R^*} \left[\bar{w}_R - b\phi + \left(\frac{\partial b}{\partial R} + \frac{\partial b}{\partial \bar{w}^{-R}} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \frac{1}{r} (1 - e^{-r(T-R^*)}) \right] - v(R^*) = 0 \quad (7.12)$$

$$\hat{\psi}(R^*, \lambda^*) \equiv \int_0^T h(\lambda^* e^{(\delta-r)t}) e^{-rt} dt - \int_0^{R^*} \bar{w}_t e^{-rt} dt = 0 \quad (7.13)$$

where

$$\theta \bar{w}(1 - e^{-gr}) - b(e^{-gr} - e^{-gT}) = 0 \quad (6.2)$$

Substituting (6.2) into the individual's first-order conditions (2.1) and (2.2) yields:

$$\psi(R, c) \equiv c\phi_r - \bar{w} + b(\phi_g - \phi_r) = 0 \quad (6.3)$$

$$\varphi(R, c) \equiv c^{-1}(\bar{w} - b\phi_g) - v = 0 \quad (6.4)$$

where $\phi_r \equiv \frac{1 - e^{-rT}}{1 - e^{-rR}}$ and $\phi_g \equiv \frac{1 - e^{-gT}}{1 - e^{-gR}}$. In analogy with (2.4)-(2.5), conditions (6.3)-(6.4) determine the economy's equilibrium levels (R^*, c^*) for given r and g . The former conditions are clearly a special case of the latter when $r = g$.

Further substituting (6.3) into (6.4) yields an equation in R analogous to (2.6):

$$\beta\phi_r\phi_g - (1 - v\beta)\phi_r - v\beta\phi_g + v = 0 \quad (6.5)$$

For given β , v , r and g , equation (6.5) has, in general, multiple solutions. This can be seen by expanding the functions ϕ by their linear terms only. Equation (6.5) then becomes a quadratic equation in R , which may generally have two positive solutions. As before, one can infer which solution is locally stable and which is locally unstable. We shall not pursue the characterization of these solutions here. Instead, we proceed to calculate the stable values of R^* and c^* for some alternative levels of β and g . These calculations are presented in Table 4.

Table 4

Optimum Retirement Age and Tax Rate for Alternative
Replacement Ratios and Growth Rates^a

β	$g = .01$		$g = .02$		$g = .04$		$g = .06$	
	R^*	θ	R^*	θ	R^*	θ	R^*	θ
0	50	0	50	0	50	0	50	0
.01	48.46	.001	48.46	.001	48.46	.001	48.47	.001
.05	42.17	.007	42.22	.007	42.28	.007	42.31	.007
.10	34.89	.020	35.19	.020	35.51	.020	35.66	.020
.15	26.84	.045	28.05	.042	29.15	.040	29.62	.039
.20	10.00	.162	17.20	.123	22.24	.074	23.65	.070

^a Calculated from (6.5) and (6.2) for $r = .04$, $v = 1.086$ and $T = 70$.

The main features emerging from these calculations seem to be the following.

The effect of increases in the replacement ratio on reducing the optimum retirement age is larger the smaller is the rate of growth of population. This should be expected since at low rates of growth a given increase in benefits requires a relatively large increase in taxes, which induces the earlier retirement.

For a given replacement ratio, higher population growth rates lead to increases in the equilibrium retirement age. Clearly, higher growth rates imply a shift in the age distribution towards the younger ages and thus enable a reduction in the tax rates required to finance the given level of benefits. This effect is relatively small at low replacement ratios, but very significant at higher replacement ratios. For

example, at a replacement ratio of ten percent an increase in the growth rate from two to four percent raises the retirement age by approximately one percent, while at a replacement ratio of twenty percent the same increase in growth raises retirement by almost thirty percent.

The effect of different growth rates on retirement age is reflected in the implied tax rates. Generally, an increase in the population growth rate enables a reduction of tax rates. These reductions are significant at high replacement ratios and at low population growth rates.

The previous result seems to have an important bearing on current attempts in the U.S. to adjust the level of social security taxes to the projected decrease in the population growth rate. Our analysis suggests that reductions in retirement ages brought about by the contemplated increases in tax rates may substantially aggravate the problem. For example, consider a replacement ratio of twenty percent and a decrease in the population growth rate from 4 to 2 percent. From Table 4, the initial equilibrium tax rate is approximately 7.5 percent. If retirement effects are neglected then, by (6.2), the tax rate should rise to approximately 10.5 percent, compared with the equilibrium value of 12.3 percent. Thus, neglecting the retirement effect leads to an error of approximately sixteen percent in the equilibrium tax rate.

7. A General Model of Individual Optimization and Market Equilibrium

The purpose of this section is to develop the general model of individual optimization that underlies the analysis in the previous sections. It will be shown that the formulation in section 2 is a special case of the model presented below.

Let c_t denote the consumption of an individual at age t . For simplicity, we assume that the individual can either work full time, in which case his utility is $u(c)$, or not work at all, in which case his utility is $\hat{u}(c, t) = u(c) + v(t)$, where $v(t)$ is the utility from retirement at age t . In standard terminology, the utility function is additively separable in consumption and leisure, with the utility of leisure being age-dependent.⁵ We assume that u is twice differentiable, strictly monotone and concave in c : $u' > 0$, $u'' < 0$; and that v is positive and monotone in t : $v > 0$, $v' \geq 0$.

The individual is assumed to have a life horizon of $T > 0$, and to have no bequest motive. Hence, if he retires at age R ($T \geq R \geq 0$), his lifetime utility at age $t = 0$, denoted U , is given by

$$\begin{aligned} U &= \int_0^R u(c_t) e^{-\delta t} dt + \int_R^T \hat{u}(c_t, t) e^{-\delta t} dt \\ &= \int_0^T u(c_t) e^{-\delta t} dt + \int_R^T v(t) e^{-\delta t} dt \end{aligned} \quad (7.1)$$

where $\delta > 0$ is a subjective constant discount rate. If he works, the individual is assumed to receive a wage \bar{w}_t at age t , from which a fixed fraction θ ($1 > \theta > 0$) is deducted for social security. His net wage at age t , w_t , is thus $w_t = \bar{w}_t(1-\theta)$.

After retirement, the individual is eligible for social security benefits. These benefits depend, in general, upon his retirement age and upon certain characteristics of his wage profile up to retirement. Denote this characteristic by \bar{w}^{-R} . For example, benefits may depend on the (arithmetic) average of his wages until retirement, $\bar{w}^{-R} = \frac{1}{R} \int_0^R \bar{w}_t dt$. With minor exceptions, this is the case in the U.S. Another conceivable rule is that benefits be granted according to the maximum earnings obtained prior to retirement, $\bar{w}^{-R} = \text{Max} \{ \bar{w}_t | R \geq t \geq 0 \}$. Notice that if the individual's lifetime earnings have the standard shape, increasing initially and then decreasing, and if the individual retires after passing his income peak, then in the latter case \bar{w}^{-R} is unaffected by R . In general, however, the basis for benefits, \bar{w}^{-R} , may be expected to depend on the individual's retirement date. The benefit function, denoted by $b = b(R, \bar{w}^{-R})$, is assumed to be twice differentiable in R and \bar{w}^{-R} , with $\frac{\partial b}{\partial R} \geq 0$ and $\frac{\partial b}{\partial \bar{w}^{-R}} \geq 0$.

It is assumed that the individual has no income except from wages and social security benefits. In a perfectly competitive capital market, with free lending and borrowing at a given rate of interest r , the individual's budget constraint is given by

$$\int_0^T c_t e^{-rt} dt = \int_0^R w_t e^{-rt} dt + b(R, \bar{w}^{-R}) \int_R^T e^{-rt} dt \quad (7.2)$$

His objective is to maximize (7.1) with respect to (w. r. t.) c_t and R , subject to (7.2). The first-order conditions for an interior solution are

$$u'(c_t) = \lambda e^{(\delta-r)t} \quad \text{or} \quad c_t = h(\lambda e^{(\delta-r)t}) \quad (7.3)$$

and

$$\varphi(R, \lambda, b) \equiv \lambda e^{(\delta-r)R} \left[w_R^{-b} + \left(\frac{\partial b}{\partial R} + \frac{\partial b}{\partial \bar{w}^{-R}} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \frac{1}{r} (1 - e^{-r(T-R)}) \right] - v(R) = 0 \quad (7.4)$$

where $h \equiv u'^{-1}$, and $\lambda > 0$ a constant. An assumption that $u'(0) = \infty$ is sufficient to ensure that (7.3) has an interior solution ($c_t > 0$) for any $\lambda > 0$ and all t .

From (7.3), the budget constraint (7.2) can be rewritten

$$\psi(R, \lambda, b) \equiv \int_0^T n(\lambda e^{(\delta-r)t}) e^{-rt} dt - \int_0^R w_t e^{-rt} dt - b(R, \bar{w}^{-R}) \int_R^T e^{rt} dt = 0 \quad (7.5)$$

The interpretation of condition (7.4) is straightforward. The direct loss in utility from further postponement of retirement is the utility of retirement, $\hat{u}(c_R, R) - u(c_R) = v(R)$. The gain from such postponement in terms of the present value of receipts is given by the expression in square brackets. Multiplying this gain by $u'(c_R) = \lambda e^{(\delta-r)R}$ converts it into utility units. Condition (7.4) states that at the optimum these gains and losses should be equal.

Notice that when $\partial b / \partial R = 0$ and $\partial \bar{w}^{-R} / \partial R > 0$, an interior solution requires that $w_R > b$, i. e., that social security benefits be smaller than the wage rate at retirement. This is expected in view of the assumed positive utility of retirement, $v(R) > 0$.

Equations (7.4) and (7.5) are two equations to determine the individual's optimum R and λ . We assume that there exists a unique positive solution to these equations, denoted by (R^*, λ^*) .

By (7.1), (7.4) and (7.5), whenever the budget constraint (7.5) is satisfied, $dU/dR = e^{-\delta R} \varphi(R, \lambda)$. Hence, we require that at (R^*, λ^*) , the second-order condition for a maximum be satisfied

$$\frac{d^2U}{dR^2} = -e^{-\delta R^*} \frac{\Delta}{\frac{\partial \psi}{\partial \lambda}} < 0 \quad (7.6)$$

where $\Delta = (\partial \varphi / \partial R)(\partial \psi / \partial \lambda) - (\partial \varphi / \partial \lambda)(\partial \psi / \partial R)$.

From (7.4) and (7.5),

$$\begin{aligned} \frac{\partial \varphi}{\partial R} = & \lambda^* e^{-(\delta-r)R^*} \left[\frac{\partial w_R}{\partial R} - \frac{\partial b}{\partial R} + \frac{1}{r} (1 - e^{-r(T-R^*)}) \frac{\partial}{\partial R} \left(\frac{\partial b}{\partial R} + \frac{\partial b}{\partial w} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \right. \\ & \left. - e^{-r(T-R^*)} \left(\frac{\partial b}{\partial R} + \frac{\partial b}{\partial w} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \right] - v'(R) - (r-\delta)v(R) \end{aligned} \quad (7.7)$$

$$\frac{\partial \varphi}{\partial \lambda} = - \frac{v(R^*)}{\lambda^*} < 0 \quad (7.8)$$

$$\frac{\partial \psi}{\partial R} = - \left[w_R - b + \frac{1}{r} (1 - e^{-r(T-R^*)}) \left(\frac{\partial b}{\partial R} + \frac{\partial b}{\partial w} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \right] e^{-rR^*} \quad (7.9)$$

by (7.4)

$$= - \frac{v(R^*)}{\lambda^*} e^{-\delta R^*} < 0$$

and

$$\frac{\partial \psi}{\partial \lambda} = - \frac{1}{\lambda^*} \int_0^T \frac{h(\lambda e^{(\delta-r)t})}{\sigma} e^{-rt} dt < 0 \quad (7.10)$$

where $\sigma \equiv -u''(c)c/u'(c)$ denotes the elasticity of the marginal utility.

In view of (7.10), condition (7.6) requires that $\Delta < 0$. By (7.7)-(7.10), a sufficient condition for the latter is that $\frac{\partial \varphi}{\partial R} \geq 0$ at (R^*, λ^*) .

We shall make this assumption throughout. It is satisfied, for example,

when $r - \delta \geq 0$, $\frac{\partial w_R}{\partial R} \leq 0$ and $b(R, \bar{w}^{-R})$ is monotone and concave in R :

$$\frac{db}{dR} = \frac{\partial b}{\partial R} + \frac{\partial b}{\partial \bar{w}^{-R}} \frac{\partial \bar{w}^{-R}}{\partial R} \geq 0, \quad \frac{d^2 b}{dR^2} = \frac{\partial^2 b}{\partial R^2} + \frac{\partial}{\partial R} \left(\frac{\partial b}{\partial \bar{w}^{-R}} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \leq 0. \quad \text{The}$$

condition that $r - \delta \geq 0$ implies, by (7.3), that consumption does not decrease with age, while the condition $\frac{\partial w_R}{\partial R} \leq 0$ implies that the wage rate is not increasing at R^* .

Assuming that for each generation the present value of benefits and deductions is equal, the social security's budget constraint is given by

$$\theta \int_0^R \bar{w}_t e^{-rt} dt = b(R, \bar{w}^{-R}) \int_R^T e^{-rt} dt \quad (7.11)$$

where θ is the social security tax rate.

In analogy with standard tax theory, it is assumed that individuals ignore the impact of their decisions on the aggregate constraint (7.11). This is a plausible assumption under competitive conditions with many individuals.

Substitution of (7.11) into (7.4) and (7.5) yields the economy's equilibrium conditions

$$\hat{\phi}(R^*, \lambda^*, b) \equiv \lambda^* e^{(\delta-r)R^*} \left[\bar{w}_R - b\phi + \left(\frac{\partial b}{\partial R} + \frac{\partial b}{\partial \bar{w}^{-R}} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \frac{1}{r} (1 - e^{-r(T-R^*)}) \right] - v(R^*) = 0 \quad (7.12)$$

$$\hat{\psi}(R^*, \lambda^*) \equiv \int_0^T h(\lambda^* e^{(\delta-r)t}) e^{-rt} dt - \int_0^{R^*} \bar{w}_t e^{-rt} dt = 0 \quad (7.13)$$

where

$$\phi \equiv \left(\int_0^{R^*} \bar{w}_t e^{-rt} dt + \bar{w}_{R^*} \int_{R^*}^T e^{-rt} dt \right) \left(\int_0^{R^*} \bar{w}_t e^{-rt} dt \right)^{-1} \quad (7.14)$$

Given the benefit function b , equations (7.12) and (7.13) determine the economy's equilibrium values of R^* and λ^* . By construction, (R^*, λ^*) satisfies the social security's budget constraint (7.11). The effects of policy changes on the equilibrium (R^*, λ^*) should thus be regarded as compensated variations in the individuals' behavior.

We now notice that the equilibrium equations (2.4) and (2.5) in section 2 are a special case of (7.12) and (7.13) when $u'(c) = c^{-1}$, $v'(R) = 0$ (that is, $v(R) = v$), $\bar{w}_t = \bar{w}$ and $r = \delta$. Also note that in that special case the second-order condition $\Delta < 0$ is trivially satisfied since $\frac{\partial \phi}{\partial R} = 0$ in (7.7).

Differentiating (7.12) and (7.13) at (R^*, λ^*) yields

$$\begin{aligned} \frac{\hat{\partial} \phi}{\partial R} &= \lambda^* e^{(\delta-r)R^*} \left[\frac{\partial \bar{w}_R}{\partial R} - \frac{\partial b}{\partial R} (\phi + e^{-r(T-R^*)}) - e^{-r(T-R^*)} \frac{\partial b}{\partial \bar{w}^R} \frac{\partial \bar{w}^R}{\partial R} \right. \\ &\quad \left. - b \frac{\partial \phi}{\partial R} + \frac{1}{r} (1 - e^{-r(T-R^*)}) \frac{\partial}{\partial R} \left(\frac{\partial b}{\partial R} + \frac{\partial b}{\partial \bar{w}^R} \frac{\partial \bar{w}^R}{\partial R} \right) \right] \\ &\quad - v'(R^*) - (r - \delta) v(R^*) \end{aligned} \quad (7.15)$$

$$\frac{\hat{\partial} \phi}{\partial \lambda} = - \frac{v(R^*)}{\lambda^*} < 0 \quad (7.16)$$

$$\frac{\hat{\partial} \psi}{\partial R} = - \bar{w}_R e^{-rR^*} < 0 \quad (7.17)$$

$$\frac{\hat{\partial} \psi}{\partial \lambda} = - \frac{1}{\lambda^*} \int_0^T \frac{h(\lambda^* e^{(\delta-r)t})}{\sigma} e^{-rt} dt < 0 \quad (7.18)$$

Let $\hat{\Delta} = (\hat{\partial} \phi / \partial R)(\hat{\partial} \psi / \partial \lambda) - (\hat{\partial} \phi / \partial \lambda)(\hat{\partial} \psi / \partial R)$. By (7.15)-(7.18), the sign of $\hat{\Delta}$ at (R^*, λ^*) cannot be established unless it can be shown that $\frac{\hat{\partial} \phi}{\partial R} \geq 0$.

However, even when $\frac{\partial \psi}{\partial R} \geq 0$ in (7.7), so as to satisfy the individual's maximization second-order conditions, it can be seen that the sign of (7.15) is indeterminate. Indeed, as in the special case discussed in section 2, the multiplicity of equilibrium points satisfying (7.12)-(7.13) cannot be ruled out.

In order to examine briefly the effects of changes in the benefits formula, we shall assume that the initial equilibrium point is locally stable, i. e. , $\hat{\Delta} < 0$ at (R^*, λ^*) .

A change in the benefits function can now be represented by a general "shift" parameter α : $b = b(R, \bar{w}^{-R}, \alpha)$. More specific assumptions about the dependence of b on α will be made in the sequel.

Differentiating (7.12) and (7.13) totally w. r. t. α , using (7.14), we obtain

$$\frac{dR^*}{d\alpha} = \frac{\lambda^* e^{(\delta-r)R^*}}{\hat{\Delta}} \frac{\partial \psi}{\partial \lambda} \left[\phi \frac{\partial b}{\partial \alpha} - \frac{1}{r} (1 - e^{-r(T-R^*)}) \frac{\partial}{\partial \alpha} \left(\frac{\partial b}{\partial R} + \frac{\partial b}{\partial \bar{w}^{-R}} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \right] \quad (7.19)$$

$$\frac{d\lambda^*}{d\alpha} = - \frac{\lambda^* e^{(\delta-r)R^*}}{\hat{\Delta}} \frac{\partial \psi}{\partial R} \left[\phi \frac{\partial b}{\partial R} - \frac{1}{r} (1 - e^{-r(T-R^*)}) \frac{\partial}{\partial \alpha} \left(\frac{\partial b}{\partial R} + \frac{\partial b}{\partial \bar{w}^{-R}} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \right] \quad (7.20)$$

In view of (7.17) and the assumption that $\hat{\Delta} < 0$ at (R^*, λ^*) , it is seen that $\frac{dR^*}{d\alpha}$ has the same sign and $\frac{d\lambda^*}{d\alpha}$ the opposite sign of the expression

$$\phi \frac{\partial b}{\partial \alpha} - \frac{1}{r} (1 - e^{-r(T-R^*)}) \frac{\partial}{\partial \alpha} \left(\frac{\partial b}{\partial R} + \frac{\partial b}{\partial \bar{w}^{-R}} \frac{\partial \bar{w}^{-R}}{\partial R} \right) \quad (7.21)$$

Let us consider four special cases of (7.21):

(1) $\frac{\partial b}{\partial \alpha} > 0$ and $\frac{\partial^2 b}{\partial \alpha \partial R} = \frac{\partial^2 b}{\partial \alpha \partial \bar{w}^{-R}} = 0$. This is an additive increase in benefits. Such a change is seen to decrease R^* and to increase λ^* , which implies, by (7.3), a uniform decrease in c_t ;

(2) $\frac{\partial^2 b}{\partial \alpha \partial R} > 0$ and $\frac{\partial b}{\partial \alpha} = \frac{\partial^2 b}{\partial \alpha \partial \bar{w}^{-R}} = 0$. This is a case where the marginal return to postponing retirement, $\frac{\partial b}{\partial R}$, is increased without affecting the level of benefits, b . Such a change is seen to increase R^* and to decrease λ^* (and hence, to increase c_t);

More generally, if

(3) $\frac{\partial}{\partial \alpha} \left(b - \frac{1}{r} \frac{\partial b}{\partial R} \right) > 0$ and $\frac{\partial^2 b}{\partial \alpha \partial \bar{w}^{-R}} = 0$ then (7.21) is non-negative, implying a decrease in R^* and an increase in λ^* . The condition is that the percentage change in benefits due to postponement of retirement, $\frac{1}{b} \frac{\partial b}{\partial R}$, not exceed the rate of interest;

Finally, suppose

(4) $\frac{\partial^2 b}{\partial \alpha \partial \bar{w}^{-R}} > 0$ and $\frac{\partial b}{\partial \alpha} = \frac{\partial^2 b}{\partial \alpha \partial R} = 0$. Since the increase in benefits increases with the variable \bar{w}^{-R} , this may be considered a regressive change in benefits. When $\frac{\partial \bar{w}^{-R}}{\partial R}$ is positive then $\frac{dR^*}{d\alpha} > 0$ and $\frac{d\lambda^*}{d\alpha} < 0$, and vice versa.

Various other comparative statics of the general model can be considered, including the effect of social security benefits on the wealth-income ratio at retirement and the effects of intergenerational transfers. In most cases, one has to impose various restrictions on the benefits function to obtain unambiguous results.

Footnotes

1. See, for example, Steiner and Dorfman (1959), Long (1958), Pechman, Aaron and Taussig (1968), Feldstein (1974) and Boskin (1975).
2. See below, section 7.
3. This is not valid in general. For example, suppose that the marginal utility of consumption is equal to $c^{-\sigma}$, where σ is a positive constant. Equations (2.5) and (2.6) now become $c^{-\sigma}[\bar{w} - b\phi] - v = 0$ and $\bar{w}^{-\sigma}b\phi^{\sigma+1} - \bar{w}^{1-\sigma}\phi^{\sigma} + v = 0$, respectively. Evidently, the latter equation cannot be expressed in terms of $\beta = b/\bar{w}$ alone except when $\sigma = 1$.
4. Due to fluctuations in employment and in the population of eligible recipients, the social security program may occasionally incur losses or gains. Thus, the application of this principle should be interpreted as a long-run or average formula.
5. In the non-separable case, the optimum consumption plan may be discontinuous at the retirement age. Notice also that in a model with uncertain lifetime, $v(t)$ can be interpreted to include the conditional probability of survival at t (Yaari [1964]).

References

- Boskin (1975) "Social Security and Retirement Decisions," National Bureau of Economic Research, Working Paper No. 107, Stanford.
- Diamond, P. A. (1976) "A Framework for Social Security Analysis," Journal of Political Economy (forthcoming).
- Feldstein, M.S. (1974) "Social Security, Induced Retirement and Aggregate Capital Accumulation," Journal of Political Economy, 82, 905-926.
- Long, C. (1958) The Labor Force Under Changing Income and Employment (Princeton University Press).
- Pechman, J. A., H.J. Aaron and M.K. Taussig (1968) Social Security: Perspectives for Reform (The Brookings Institution, Washington, D. C.).
- Steiner, P. and R. Dorfman (1959) The Economic Status of the Aged (University of California Press, Berkeley).
- Yaari, M.E. (1964) "On the Consumer's Lifetime Allocation Process," International Economic Review, 5, 304-317.