NBER TECHNICAL PAPER SERIES

FROM EACH ACCORDING TO HIS SURPLUS: EQUI-PROPORTIONATE SHARING OF COMMODITY TAX BURDENS

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Technical Paper No. 138

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 1993

We thank Kathryn Dominguez for helpful comments on an earlier draft. This paper is part of NBER's research program in Public Economics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

This paper examines the incidence of commodity taxes, finding that, when demand and marginal cost schedules are linear, the burden of commodity taxation is distributed between buyers and sellers so that each suffers the same *percentage* reduction on pre-tax surplus. This equi-proportionate reduction in surplus is the outcome of commodity taxes set at any rate, and is unaffected by relative demand and supply elasticities. Hence, when demand and marginal cost schedules are linear, commodity taxes resemble flat-rate taxes imposed on market surplus. Similar results apply to nonlinear schedules with certain ranges.

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1. Introduction.

One of the most important features of a tax is the distribution of its burdens. The function of tax incidence theory is to identify the cumulative effect of complex market interactions that transform a tax imposed on one party into a series of price and income changes affecting many parties. The theory usually begins by considering the simple partial-equilibrium effects of per-unit taxes imposed on sales of commodities.[1] Here, matters are relatively straightforward: buyers of a good bear the burden of a tax imposed on its sale insofar as the aggregate demand curve for the good is inelastic, or the supply curve is elastic; the converse holds for sellers, who feel taxes most keenly when demand curves are elastic or the supply curve is inelastic. From this it is a short step to conclude that buyers or sellers of a taxed good are better off if their net demands are elastic.[2]

The purpose of this paper is to reconsider such an interpretation of tax incidence, by calling attention to an elementary, but overlooked, property of the distribution of tax burdens between buyers and sellers. We find that, when demand and marginal cost schedules are linear, the burden of commodity taxation is distributed between buyers and sellers so that each suffers the same *percentage* reduction in pre-tax surplus. This equi-proportionate reduction in surplus is the consequence of commodity taxes set at any rate, and is unaffected by relative demand and supply elasticities. The result is valid for both competitive and monopolistic markets.

The source of the equi-proportionate surplus reduction is most easily illustrated by considering an extremely high and onerous per-unit tax, one that completely shuts down what was previously a functioning market. The burden of this tax is borne by buyers, who lose 100% of their consumer surplus, and by sellers, who lose 100% of their producer surplus; the equality of these

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[1]See, for example, Atkinson and Stiglitz (1980, pp. 162 ff.), Tresch (1981, pp. 373 ff.), Kotlikoff and Summers (1987, pp. 1045 ff.), or Rosen (1991, pp. 283 ff.).

[2]See, for example, Seligman (1927, pp. 232-233), who summarizes his discussion with, "In other words, the greater the elasticity of the demand, the more favorable - other things being equal - will be the situation of the consumer."

percentages is unaffected by demand and supply elasticities. Relative tax burdens are also equal at lower tax rates, provided that demand and marginal cost curves are linear, since the linearity of the schedules ensures that a given tax change always has the same effect on price, regardless of the prior tax level.

Relative demand and supply elasticities not only determine the incidence of commodity taxes, but they also determine pre-tax consumer surplus and producer surplus. Consequently, if demand and supply curves are linear and the demand curve is relatively inelastic, a tax on a commodity imposes a burden on consumers that is larger than its burden on sellers, but not any larger as a fraction of original surplus. Linear commodity taxes have an effect that is similar to the effect of flat-rate income taxes, which impose a much larger absolute burden on high-income taxpayers than on low-income taxpayers, but impose the same average burden on all income classes. Thus, it may be misleading to conclude that tax burdens are borne excessively by those with inelastic net demand schedules.

2. Tax Incidence in Competitive Markets with Linear Demand and Linear Supply.

Figure one illustrates the partial-equilibrium consequences of a specific commodity tax in a competitive equilibrium when demand and supply curves are linear.[3] In the figure, the schedule ACE denotes the relevant portion of the demand curve, and the schedule HGE the relevant portion of the supply curve. Prior to the imposition of the tax, quantity DE is sold at price OD. After

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[3]The first use of demand and supply diagrams to analyze commodity tax incidence appears in Jenkin (1871/1872). Jenkin notes that equilibrium quantities and net-of-tax prices are the same whether taxes are paid by buyers or sellers of a good, and he is careful to define tax burdens as lost consumer surplus and lost producer surplus (though he does not use this terminology, which postdates the 1870s). Jenkin analyzes an example (p. 114) in which demand and supply schedules are linear, and apparently just misses the observation that tax burdens are proportional to original surpluses. In his survey of the history of public finance theory, Musgrave (1985, p. 36) concludes that "the substance of Jenkin's analysis was essentially the same as may be found in textbooks of today."

imposition of a per-unit tax of CG, buyers face a net price of OB, sellers a net price of OF; at these prices, quantity BC is bought and sold.

Prior to the imposition of the tax, consumers enjoy surplus equal to the area of the triangle ADE; consumer surplus in the post-tax regime equals the area of the triangle ABC.[4] The ratio of consumer surplus after the tax to consumer surplus before the tax is:

$$\frac{\text{area (ABC)}}{\text{area (ADE)}} = \frac{(1/2)(BC)(AB)}{(1/2)(DE)(AD)} = \frac{(BC)}{(DE)} \cdot \frac{(AB)}{(AD)} \tag{1}$$

Equation (1) can be further simplified by noting that ABC and ADE are similar triangles: both share the angle formed by BAC, and both have adjacent 90 degree angles on the vertical axis. The ratios of the lengths of corresponding sides of similar triangles are always equal, so (AB)/(AD) = (BC)/(DE). Hence (1) implies that:

$$\frac{\text{area (ABC)}}{\text{area (ADE)}} = [(BC)/(DE)]^2 \tag{2}$$

An analogous line of reasoning can be used to evaluate the ratio of post-tax producer

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[4]The use of consumer surplus as a measure of consumer welfare offers the advantage of simplicity at the cost of some well-known limitations. Specifically, Marshallian consumer surplus does not generally correspond to a well-defined welfare measure, though in certain circumstances it can represent a useful approximation to compensated measures such as equivalent variation (see, for example, Willig (1976, 1979), McKenzie (1979), Hausman (1981), Vartia (1983), and Auerbach (1985)). Additional limitations, such as path-dependence (see Chipman and Moore (1980)) illustrate the incompleteness of consumer surplus as a welfare measure. If the price of only one good changes, and income effects are zero (which would be the case, for example, if buyers were firms), then consumer surplus equals equivalent variation and compensating variation. Alternatively, consumer surplus as used in the text accurately represents the required compensation if a tax were imposed and consumers *were* compensated for the induced price change, taking the demand schedule to be the *compensated* demand curve.

surplus, area (HFG) in figure one, to pre-tax producer surplus, area (HDE). These two triangles are similar, since they share the angle GHF and they both have 90 degree angles along the vertical axis. Hence the ratio of their areas equals:

$$\frac{\text{area (HFG)}}{\text{area (HDE)}} = \frac{(1/2)(FG)(HF)}{(1/2)(DE)(HD)} = \frac{(FG)}{(DE)} \cdot \frac{(HF)}{(HD)} = [(FG)/(DE)]^2 \qquad (3)$$

The final step is to observe that (BC) = (FG), so the left sides of (2) and (3) equal each other. After the tax is imposed, consumer surplus equals the same fraction of pre-tax consumer surplus that after-tax producer surplus equals of pre-tax producer surplus. Hence the *lost* consumer surplus represents the same fraction of original consumer surplus that lost producer surplus represents of original producer surplus.

This result is fully consistent with the logic that those with inelastic demand or supply schedules bear the entire tax burden. Consumers with perfectly inelastic and linear demand curves have infinite consumer surplus; sellers with perfectly inelastic and linear supply curves have infinite producer surplus.[5] In either case, a finite tax rate reduces the surplus by a finite amount, which represents zero percent of original surplus. Alternatively, if the demand curve is linear and infinitely elastic, then zero consumer surplus is lost as a consequence of the tax; but pre-tax consumer surplus is also zero, so the ratio on the left side of (1) is undefined. Sellers bear the full burden of the tax, but, in the limit as the demand elasticity approaches infinity, the equality of (2) and (3) is still valid. The same argument applies to markets with linear and infinitely elastic supply schedules.

[5]Note that, in order to remain linear, a supply curve that intersects the horizontal axis at a positive quantity must have a region in which the seller's desired supply is positive at a negative price. It is, of course, difficult to construct an example in which such behavior is likely to be observed. Most textbook examples of linear inelastic supply curves are really backward L-shaped supply curves.

3. Tax Incidence in Competitive Markets: A Generalization.

Consider a market in which the demand for a commodity, with market price p and per-unit tax rate τ imposed on sellers, is given by the function D(p); the supply function is $S(p-\tau)$. We restrict attention to cases in which market price is a continuous function of τ, and in which a unique market equilibrium is associated with each tax rate, denoting the quantity corresponding to equilibrium as $q(\tau)$.[6] For a small change in the tax rate, the envelope theorem guarantees that the change in consumer surplus equals $-q(\tau)dp/d\tau$, while the change in producer surplus equals $q(\tau)(dp/d\tau - 1)$. Hence the ratio of the change in consumer surplus to the change in producer surplus equals:

$$\frac{d(\text{Consumer Surplus})/d\tau}{d(\text{Producer Surplus})/d\tau} = (dp/d\tau)/(1 - dp/d\tau) \qquad (4)$$

It is possible to draw two conclusions from (4): (i) constancy of $dp/d\tau$ is a sufficient condition for tax burdens to be borne in proportion to original surpluses; and (ii) constancy of $dp/d\tau$ is a necessary condition for burdens to be borne equi-proportionately at all rates of taxation.[7] The

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[6]By restricting attention to a single market, we ignore the general equilibrium aspects of tax incidence. General equilibrium considerations have an important place in the history of tax incidence analysis, dating at least as far back as Walras (1874, pp. 457-459), and include important questions such as the incidence of the corporate income tax (Harberger (1962) and McLure (1975)). It is worth noting that interactions with just a second market for related goods can completely reverse some of the most basic results in the partial equilibrium theory of tax incidence (see, for example, the cases identified by Edgeworth (1897) and Hotelling (1933), in which higher commodity taxes reduce after-tax prices faced by consumers, or the apparently paradoxical income effects illustrated in Feldstein (1977), Zeckhauser (1978), and Fane (1984)). Homma (1977) and Diamond (1978) offer more general analyses. The value of partial equilibrium analysis lies in its simple conclusions and ability to approximate more general situations. Black (1939, pp. 125-126) explains that "limitations of intellect" make partial equilibrium analysis the only feasible alternative, though he adds that "However defective the logic of the [partial equilibrium] method as compared with that [general equilibrium method] of Lausanne, experience of it has shown it to be often the more suitable method for obtaining information about the real world."

[7]It will then come as no surprise that constancy of $dp/d\tau$ is one implication of linear demand and supply curves. This can be verified by setting $S(p-\tau) = a_1 + a_2(p-\tau)$, and $D(p) = b_1 - b_2p$. Equality of

first conclusion follows from the observation that, if the left side of (4) is constant for all values of τ, then lost producer surplus from any tax is always the same fraction of lost consumer surplus. Since a very high tax reduces producer surplus by all of its original value, and reduces consumer surplus by all of *its* original value, it follows that smaller taxes reduce producer and consumer surpluses in a ratio that equals the ratio of the original surpluses. Equation (4) also shows that constant $dp/d\tau$ is a necessary condition for burdens at all rates of taxation to be borne in proportion to original surpluses, since nonconstant $dp/d\tau$ in (4) implies that the ratio of lost consumer surplus to lost producer surplus must also be nonconstant, and therefore cannot always equal the ratio of original surpluses.

Hence, (4) implies that linear demand and supply schedules represent special cases in the set of conditions that guarantees that commodity tax burdens are shared in equal proportion to original surpluses. Any configuration of demand and supply schedules implying that $dp/d\tau$ is a constant will carry the same implication; figure two illustrates one nonlinear example.

4. Tax Incidence in Monopolized Markets with Linear Demand and Linear Supply.

This section and the next consider the incidence of commodity taxes in markets with monopolistic sellers, finding that monopolized markets have the equi-proportionate burden sharing features of competitive markets identified in sections two and three. Figure three illustrates the partial-equilibrium consequences of a specific commodity tax in a monopolized market when demand and marginal cost curves are linear. In the figure, the schedule ACE denotes the relevant portion of the demand curve, and the schedule KI the relevant portion of the marginal cost curve. Because the demand curve is linear, the marginal revenue curve is also linear. Schedule AGI denotes the relevant portion of the marginal revenue curve. Prior to the imposition of the tax, the monopolist produces quantity HI, which is sold for price OD. After imposition of a per-unit tax of JK, the monopolist

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supply and demand implies that $p = (b_1-a_1)/(a_2+b_2) + a_2\tau/(a_2+b_2)$, so $dp/d\tau = a_2/(a_2+b_2)$, a constant.

faces an effective marginal cost curve of JG, and chooses to sell FG units of the good at price OB.

Prior to the imposition of the tax, consumers enjoy surplus equal to area (ADE); consumer surplus after the tax equals area (ABC). Producers enjoy pre-tax surplus equal to area (DKIE); producer surplus after the tax equals area (BJGC). It is helpful to divide the pre-tax surplus into triangle HKI and rectangle DHIE, and the after-tax surplus into triangle FJG and rectangle BFGC.

Using reasoning analogous to that of section 2, it can be shown that the ratio of consumer surplus after the tax to consumer surplus before the tax equals $[BC/DE]^2$. Additionally, because FJG and HKI are similar triangles, the ratio of the areas of the two producer surplus triangles can also be shown to equal $[BC/DE]^2$.

The ratio of the areas of the two producer surplus rectangles equals:

$$\frac{\text{area (BFGC)}}{\text{area (DHIE)}} = \frac{(BC)(CG)}{(DE)(EI)} \tag{5}$$

In order to evaluate (5), it is useful to note that triangle AGC is similar to triangle AIE, since angles AGC and AIE are corresponding angles and both triangles share angle GAC. Hence, (CG)/(EI) = (AG)/(AI). Triangles AFG and AHI are also similar, since they share angle FAG and have adjacent 90 degree angles on the vertical axis. Accordingly, (AG)/(AI) = (FG)/(HI). But, FG = BC and HI = DE. Thus, (CG)/(EI) = (BC)/(DE) and:

$$\frac{\text{area (BFGC)}}{\text{area (DHIE)}} = [(BC)/(DE)]^2 \tag{6}$$

These calculations indicate that the ratio of after-tax consumer surplus to pre-tax consumer surplus equals the corresponding ratio for each component of producer surplus; hence commodity taxes reduce consumer and producer surpluses by the same percentage.

5. Tax Incidence in Monopolized Markets: A Generalization.

This section explores the range of conditions that are sufficient to guarantee that buyers and a monopolistic seller share commodity tax burdens in equal proportion to their respective pre-tax surpluses. Again let D(p) represent consumer demand, and C(q) the monopolist's cost function. With a per-unit tax τ imposed on the firm, the monopolist chooses p to maximize profits (π):

$$\pi = pD(p) - C[D(p)] - \tau D(p) \tag{7}$$

The firm's first-order condition for profit maximization is:

$$D(p) + pD'(p) - [C'(q)+\tau]D'(p) = 0 \tag{8}$$

The monopolist's first-order condition guarantees that local tax-induced price changes do not affect profits, so the effect of the tax rate on producer surplus is simply its direct effect, d(Producer Surplus)/dτ = -q(τ). As before, d(Consumer Surplus)/dτ = -q(τ)dp/dτ. Hence:

$$\frac{d(\text{Consumer Surplus})/d\tau}{d(\text{Producer Surplus})/d\tau} = dp/d\tau \tag{9}$$

By the same reasoning as in section 3, constancy of dp/dτ is a necessary and sufficient condition for equi-proportionate burden sharing at all rates of taxation. In order to identify $dp/d\tau$ in the case of a monopolistic seller, we replace q in (8) with D(p) and differentiate with respect to τ; subsequently imposing (8) yields:

$$1/(dp/d\tau) = 2 - C''(q)D'(p) - D''(p)D(p)/[D'(p)]^2 \qquad (10)$$

Examination of (10) reveals that the linearity of demand and marginal cost schedules ($D''(p) = 0$; $D'(p)$ and $C''(q)$ constants) is sufficient (though not necessary) for $dp/d\tau$ to be constant.

The commodity taxes analyzed in (1)-(10) take the form of constant per-unit charges imposed on sellers. It is well understood that *ad valorem* taxes (in which the tax obligation per unit sold is a specified fraction of the sales price) are equivalent to unit taxes in the case of competitive markets, but not equivalent in the case of monopolistic markets.[8] It is easily verified that linear demand and marginal cost schedules do not represent necessary and sufficient conditions for equi-proportionate burden sharing of *ad valorem* taxes in monopolistic markets.

6. Tax Incidence in Competitive Markets with Nonlinear Demand and Supply Schedules.

Properties of models with linear functions often provide insight into the behavior of the same models with nonlinear functions. The purpose of this section is to consider the effect of first-order nonlinearity of the $p(\tau)$ function on the difference between the percentage tax burdens of buyers and sellers in competitive markets. The results suggest that, even with a nonlinear $p(\tau)$ function, the difference is likely to be small.

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[8]See, for example, Suits and Musgrave (1953), who analyze the revenue and incentive effects of unit and *ad valorem* taxes in competitive and monopolistic environments. In addition, important distinctions between the effects of unit and *ad valorem* taxes appear when the government cannot specify precisely the characteristics of the commodity being taxed (a possibility ruled out by assumption in this paper); for an analysis, see Barzel (1976) and Kay and Keen (1987).

Let demand and supply schedules be arbitrary nonlinear functions of price, and denote by CS(t) and PS(t) the amount by which a tax at rate t reduces consumer surplus and producer surplus, respectively. Define τ^* to be the smallest tax rate that completely exhausts consumer and producer surplus; hence $q(\tau^*) = 0$, and $CS(\tau^*)$ and $PS(\tau^*)$ represent pre-tax consumer surplus and producer surplus, respectively. Then consider the absolute value of the difference (Δ) between the buyer's tax burden, expressed as a fraction of the buyer's pre-tax surplus, and the seller's tax burden, expressed as a fraction of pre-tax producer surplus:

$$\Delta \equiv \left| \frac{CS(t)}{CS(\tau^*)} - \frac{PS(t)}{PS(\tau^*)} \right| \tag{11}$$

If tax rates are extremely low, then this difference is negligible, since the tax burdens are very small. If tax rates are extremely high, then this difference is zero, since both buyers and sellers bear burdens equal to 100 percent of their respective pre-tax surpluses. Hence it is only at intermediate rates of taxation that these ratios might differ significantly.

The difference defined in (11) can take any value between zero and one, though the difference is sizable only in those cases in which there is an abrupt nonlinearity in the $p(\tau)$ function, of which figure four illustrates one example. With the demand and supply schedules illustrated in the figure, and at a tax rate equal to T, the value of Δ is one. The analysis in section 3 suggests that large values of Δ are likely to be rare, since any nonlinear function can be approximated by a linear function over a small enough range. The difficulty with such an extrapolation is that consumer surplus and producer surplus are functions of demand and supply schedules over sufficiently broad ranges that a linear approximation - or any other approximation - may be misleading.

We examine the implications of nonlinear $p(\tau)$ functions by considering functions with curvature of the following form:

$$dp(\tau)/d\tau \;=\; a + b\tau \tag{12}$$

in which a and b are constants, and the requirement that $0 \leq dp/d\tau \leq 1$ implies that $0 \leq a \leq 1$. Two considerations motivate the use of (12): the expression is simple, capturing the nonlinearity of $dp/d\tau$ with a single parameter (b), and it represents a linear approximation that is valid for all nonlinear functions in a local neighborhood.[9] The function on the right side of (12) rules out dramatic kinks in the demand schedule (of the type depicted in figure four), while preserving the possibility that the $p(\tau)$ function may exhibit a significant degree of nonlinearity.

A tax at rate t reduces consumer surplus by an amount equal to:

$$CS(t) \;=\; \int_0^t q(\tau)(dp/d\tau)\, d\tau \tag{13}$$

while the tax reduces producer surplus by an amount equal to:

$$PS(t) \;=\; \int_0^t q(\tau)(1 - dp/d\tau)\, d\tau \tag{14}$$

For notational simplicity, we consider cases in which $[CS(t)/CS(\tau^*)] > [PS(t)/PS(\tau^*)]$

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[9]Equality of demand and supply implies that $dp(\tau)/d\tau = S'/(S'-D')$, which in turn implies that $d^2p/d\tau^2 \;=\; [D''(S')^2 - S''(D')^2]/(S'-D')^2$. The closeness of the right side of (12) as an approximation to $dp(\tau)/d\tau$ when demand and supply are nonlinear functions turns on the curvature of the demand and supply derivatives.

(so that the value of Δ in (11) is unaffected by its treatment as an absolute value). In those cases, Δ takes the value:

$$\Delta = \frac{[PS(\tau^*)CS(t) - CS(\tau^*)PS(t)]}{[CS(\tau^*)PS(\tau^*)]} \equiv \frac{\Delta_1}{\Delta_2} \tag{15}$$

Applying (13) and (14), the term Δ_2 equals:

$$\Delta_2 = \{ \int_0^{\tau^*} q(\tau)(dp/d\tau)\, d\tau\} \{ \int_0^{\tau^*} q(\tau)(1 - dp/d\tau)\, d\tau\} \tag{16}$$

Equations (12) and (16) together imply:

$$\Delta_2 = a(1\text{-}a) \{ \int_0^{\tau^*} q(\tau)\, d\tau\}^2 + b(1\text{-}2a) \{ \int_0^{\tau^*} q(\tau)\, d\tau\} \{ \int_0^{\tau^*} q(\tau)\tau\, d\tau\} - b^2 \{ \int_0^{\tau^*} q(\tau)\tau\, d\tau\}^2 \tag{17}$$

In order to evaluate (17), we use a simple linear specification of $q(\tau)$:

$$q(\tau) = q_0 + c\tau \tag{18}$$

in which $q_0 > 0$ and $c < 0$ are constants. Applying (18) to (17) and integrating yields:

$$\Delta_2 = a(1\text{-}a)[q_0\tau^* + c\tau^{*2}/2]^2 + b(1\text{-}2a)[q_0\tau^* + c\tau^{*2}/2][q_0\tau^{*2}/2 + c\tau^{*3}/3] - b^2[q_0\tau^{*2}/2 + c\tau^{*3}/3]^2 \tag{19}$$

The same specification (18) that offers a simplified form of $q(\tau)$ permits a simple substitution for τ^* in (19). Recall that τ^* is the smallest value of τ at which the equilibrium quantity becomes zero; choosing τ^* so that, from (18), $q(\tau^*) = q_0 + c\tau^* = 0$ implies:

$$\tau^* = -q_0/c \tag{20}$$

Then (19) and (20) imply:

$$\Delta_2 = q_0^4[9a(1-a)c^2 - 3(1-2a)bcq_0 - b^2q_0^2]/(36c^4) \tag{21}$$

The specifications (12), (18), and (20) greatly facilitate the evaluation of Δ_1. Applying (12), (13), and (14) to the numerator in (15) yields:

$$\Delta_1 = b[\ \{\int_0^t q(\tau)\tau\, d\tau\}\ \{\int_0^{\tau^*} q(\tau)\, d\tau\}\ -\ \{\int_0^t q(\tau)\, d\tau\}\ \{\int_0^{\tau^*} q(\tau)\tau\, d\tau\}\ ] \tag{22}$$

Applying (18) to (22) and integrating produces:

$$\Delta_1 = b[(q_0t^2/2 + ct^3/3)(q_0\tau^* + c\tau^{*2}/2) - (q_0t + ct^2/2)(q_0\tau^{*2}/2 + c\tau^{*3}/3)] \tag{23}$$

Substituting the value of τ^* from (20), (23) becomes:

$$\Delta_1 = (-bq_0^2/6c^2)[q_0^2t + 2cq_0t^2 + c^2t^3] \tag{24}$$

From (24) it is clear that the values of Δ_1 and Δ depend on t. It is useful to evaluate Δ under two scenarios: one, in which t is chosen to maximize Δ, and a second, in which an average of Δ is constructed over all possible values of t. In both scenarios the values of the critical parameter c are chosen to maximize Δ.

Examination of (24) and (21) indicates that, to find an interior maximum for Δ over the choice of t, all that is required is to differentiate the term in brackets on the right side of (24) with respect to t, producing the condition:

$$q_0^2 + 4cq_0t + 3c^2t^2 = 0 \tag{25}$$

Solving (25) for t:

$$t = \frac{-4cq_0 \pm [16c^2q_0^2 - 12c^2q_0^2]^{1/2}}{6c^2} \tag{26}$$

which, when simplified, implies that $t = -q_0/c$ or $t = -q_0/3c$. The first of these roots clearly does not maximize Δ, since $\tau^* = -q_0/c$, and $\Delta = 0$ when $t = \tau^*$. Hence only the second root, $t = -q_0/3c$ (which implies $t = \tau^*/3$), maximizes Δ. Replacing t in (24) with $-q_0/3c$ yields:

$$\Delta_1 = 2bq_0^5/(81c^3) \tag{27}$$

Since $\Delta \geq 0$ and $\Delta_2 > 0$, it must be the case that $\Delta_1 \geq 0$. Equation (27) is a reminder that the normalization that equates Δ in (11) to Δ in (15) implies that $b < 0$. Equations (27) and (21) yield:

$$\Delta = \frac{8bcq_0}{9[9a(1-a)c^2 - 3(1-2a)bcq_0 - b^2q_0^2]} \tag{28}$$

Differentiating the right side of (28) establishes that $d\Delta/db < 0$ for all parameter values in the feasible range. In choosing the value of b that maximizes Δ, it is necessary to impose the constraint that $0 \leq dp/d\tau \leq 1$, along with (from (12)) $dp/d\tau = a + b\tau$. The smallest constrained value of b satisfies: $a + b\tau^* = 0$, or $b = -a/\tau^*$. Since $\tau^* = -q_0/c$, then the value of b that maximizes (28) is: $b = ac/q_0$. Choosing $b = ac/q_0$, (28) becomes:

$$\Delta = \frac{4}{9(3-2a)} \tag{29}$$

Since the parameter a is constrained to lie between zero and one, it is clear that Δ is maximized when $a = 1$, which implies $\Delta = 4/9$. If the tax rate and curvature parameter b are chosen to maximize Δ, but $a = 1/2$ instead of 1, then (from (29)) $\Delta = 2/9$. The symmetry of the definition of Δ (in (11)) implies that $a = 1/2$ is a critical value, since if $a \geq 1/2$, then Δ is maximized by selecting a, b, and t to maximize the tax burdens of consumers relative to producers, while if $a \leq 1/2$, then Δ is maximized by choosing parameter values to minimize consumers' relative tax burdens.[10] Consequently, when t and b are chosen to maximize Δ, the value of Δ depends on a, and lies between 2/9 and 4/9.

The difference Δ is maximized when $t = \tau^*/3$, but of course t need not always take

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[10]The parameter a is the value of $dp(\tau)/d\tau$ when $\tau = 0$; since the magnitude of Δ depends on the curvature of the $dp(\tau)/d\tau$ function, and this curvature is constrained by the requirement that $0 \leq dp/d\tau \leq 1$, if $a < 1/2$ the magnitude of curvature is maximized by selecting $b > 0$ and making producers' tax burdens relatively large. It is straightforward to show that the solution to the problem is symmetric on either side of $a = 1/2$.

this value. We next consider the average value of Δ over the range of feasible positive values for t. This average value of Δ is defined as $\underline{\Delta}$:

$$\underline{\Delta} \equiv \{ \int_0^{\tau^*} \Delta_1(\tau)\, d\tau \} \,/\, \tau^* \Delta_2 \tag{30}$$

in which $\Delta_1(\tau)$ is as defined in (24). Performing this integration, and substituting $\tau^* = -q_0/c$, yields:

$$\underline{\Delta} = bq_0^5/(72c^3\Delta_2) \tag{31}$$

By comparing (31) with (27), it is clear that $\underline{\Delta}/\Delta = 81/144$. Hence, using (29),

$$\underline{\Delta} = \frac{1}{4(3-2a)} \tag{32}$$

Applying the same reasoning as before, the value of $\underline{\Delta}$ depends on a, and lies between 1/8 and 1/4. The value of $\underline{\Delta}$ represents the average (over all tax rates) of the difference between the tax burden borne by consumers, expressed as a fraction of pre-tax surplus, and the tax burden borne by producers, expressed as a fraction of pre-tax surplus, when the $p(\tau)$ function is restricted to first-degree curvature, $q(\tau)$ is linear in τ, and the curvature of the $p(\tau)$ function is chosen to maximize the difference. The small magnitude of $\underline{\Delta}$ indicates that this class of nonlinear $p(\tau)$ functions carries tax incidence implications that are similar to those of the linear functions examined in sections 2 and 3.

7. Conclusion.

The finding that linear demand and marginal cost schedules imply that tax burdens are borne by buyers and sellers in equal proportion to their pre-tax surpluses suggests a simple interpretation of linear commodity taxes as flat-rate income taxes of a particular, market-specific, variety. The standard interpretation of commodity tax incidence - that burdens are imposed on market participants with inelastic net demands - misses the point that the same inelastic net demands imply that those market participants receive the greatest benefits from the existence of the market. Furthermore, as the results described in section 6 indicate, the connection between tax burdens and market surpluses is not limited to markets with linear demand and marginal cost schedules.

The results described in this paper suggest that it may be possible to simplify some applied investigations of tax incidence, since empirical studies need only establish that demand and supply schedules are linear, or that $dp/d\tau$ is constant, in order to conclude that a commodity tax imposes burdens in equal proportion to pre-tax surpluses. Of course, empirical regularities of this kind are difficult to identify, since the required constancy of $dp/d\tau$ includes ranges (particularly those near zero quantity) that are seldom observed. Nevertheless, it may be helpful, for certain applications, to consider tax incidence in relation to consumer and producer surplus, and to measure tax burdens accordingly.

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Figure 1
Tax Incidence with Linear Demand and Supply Curves

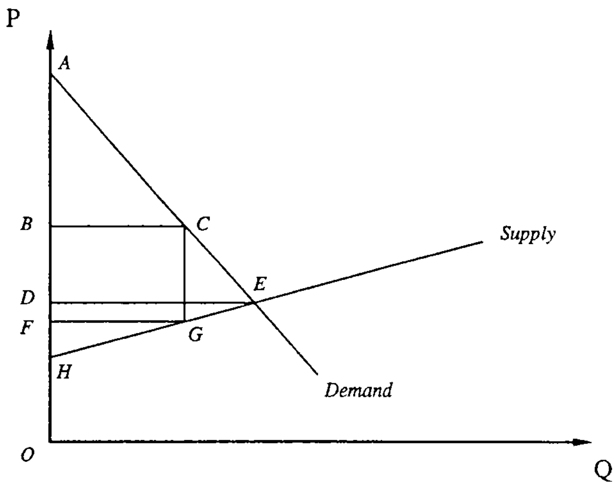

Figure 2
Nonlinear Demand and Supply Curves
with Constant dp/dτ

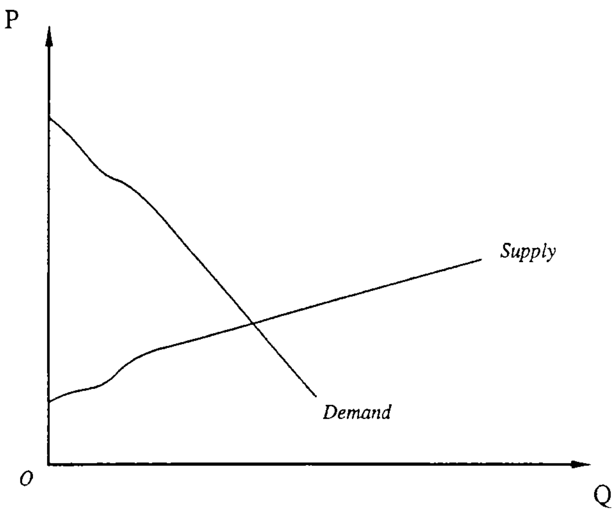

Figure 3
Tax Incidence in a Monopolized Market with Linear Demand and Supply Curves

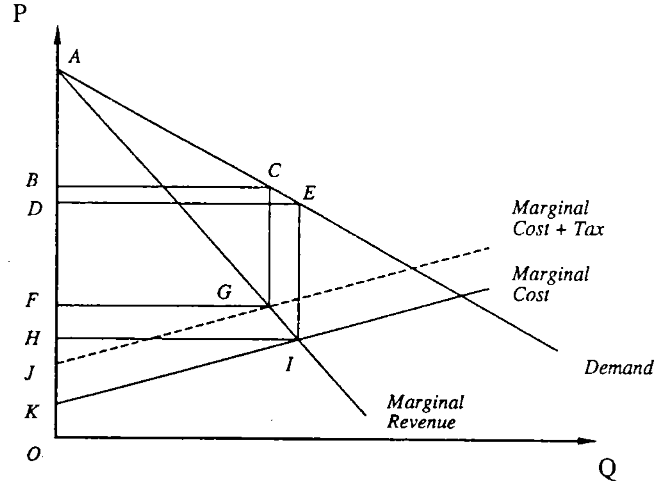

Figure 4
Maximal Difference in Tax Burden Ratios