NBER TECHNICAL WORKING PAPER SERIES

THE SIZE AND POWER OF THE VARIANCE RATIO TEST IN FINITE SAMPLES: A MONTE CARLO INVESTIGATION

Andrew W. Lo

A. Craig MacKinlay

Technical Working Paper No. 66

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 1988

This paper has benefitted considerably from the comments of the Associate Editor and two referees. We thank Chris Cavanagh, John Huizinga, Whitney K. Newey, Ken Singleton, Mark Watson and seminar participants at M.I.T., Northwestern University, Princeton University, Stanford University, UCLA, University of Chicago, University of Michigan, and the University of Pennsylvania for comments on an earlier draft. We are grateful to Stephanie Hogue, Elizabeth Schmidt, and Madhavi Vinjamuri for preparing the manuscript. Research support from the National Science Foundation (Grant No. SES-8520054) and the University of Pennsylvania Research Fund is gratefully acknowledged. Any errors are of course our own. This reseach is part of NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Technical Working Paper #66
June 1988

THE SIZE AND POWER OF THE VARIANCE RATIO TEST IN FINITE SAMPLES: A MONTE CARLO INVESTIGATION

ABSTRACT

We examine the finite sample properties of the variance ratio test of the random walk hypothesis via Monte Carlo simulations under two null and three alternative hypotheses. These results are compared to the performance of the Dickey-Fuller t and the Box-Pierce Q statistics. Under the null hypothesis of a random walk with independent and identically distributed Gaussian increments, the empirical size of all three tests are comparable. Under a heteroscedastic random walk null, the variance ratio test is more reliable than either the Dickey-Fuller or Box-Pierce tests. We compute the power of these three tests against three alternatives of recent empirical interest: a stationary AR(1), the sum of this AR(1) and a random walk, and an integrated AR(1). By choosing the sampling frequency appropriately, the variance ratio test is shown to be as powerful as the Dickey-Fuller and Box-Pierce tests against the stationary alternative, and is more powerful than either of the two tests against the two unit-root alternatives.

Andrew W. Lo
Department of Finance
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104-6367

A. Craig MacKinlay
Department of Finance
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104-6367

1. INTRODUCTION.

Whether or not an economic time series follows a random walk has long been a question of great interest to economists. Although its origins lie in the modelling of games of chance, the random walk hypothesis is also an implication of many diverse models of rational economic behavior.[1] Several recent studies have tested the random walk theory by exploiting the fact that the variance of random walk increments is linear in the sampling interval.[2] Therefore the variance of, for example, quarterly increments must be three times as large as the variance of monthly differences. Comparing the [per unit time] variance estimates from quarterly to monthly data will then yield an indication of the random walk's plausibility. Such a comparison may be formed quantitatively along the lines of the Hausman [1978] specification test and is developed in Lo and MacKinlay [1987]. Due to intractable nonlinearities, the sampling theory of Lo and MacKinlay [1987] is based on standard asymptotic approximations.

In this paper, we investigate the quality of those approximations under the two most commonly advanced null hypotheses: the random walk with independently and identically distributed Gaussian increments, and with uncorrelated but heteroscedastic increments. Under both null hypotheses, the variance ratio test is shown to yield reliable inferences even for moderate sample sizes. Indeed, under a specific heteroscedastic null the variance ratio test is somewhat more reliable than both the Dickey-Fuller t and Box-Pierce portmanteau tests.

We also compare the power of these tests against three empirically interesting alternative hypotheses: a stationary AR(1) which has been advanced as a model of stock market fads, the sum of this AR(1) and a pure random walk, and an ARIMA(1,1,0) which is more consistent with stock market data. Although the Dickey-Fuller t test is more powerful than the Box-Pierce Q against the first alternative and vice-versa against the second, the variance ratio test is comparable to the most powerful of the two tests against the first alternative, and more powerful against the second two alternatives when the variance ratio's sampling intervals are chosen appropriately.

Since the random walk is closely related to what has come to be known as a "unit root" process, a few comments concerning the variance ratio test's place in the unit root literature are appropriate. It is obvious that the random walk possesses a unit root. In addition, random walk increments are required to be uncorrelated. Although earlier studies of unit root tests [e.g. Dickey and Fuller [1979, 1981]] also assumed uncorrelated increments, Phillips [1986, 1987], Phillips and Perron [1986], and Perron [1986] show that much of those results obtain asymptotically even when increments are weakly dependent.[3] Therefore, the random walk model is a proper subset of the unit root null hypothesis. This implies that the power of a consistent unit root test against the random walk hypothesis will converge to the size of the test asymptotically.

The focus of random walk tests also differs from that of the unit root tests. This is best illustrated in the context of Beveridge and Nelson's [1981] decomposition of a unit root process into the sum of a random walk and a stationary process.[4] Recent applications of unit root tests propose the null hypothesis that the random walk component does not exist, whereas tests of the random walk have as their null hypothesis that the stationary component does not exist.[5]

Since there are some important departures from the random walk that unit root tests cannot detect, the variance ratio test is preferred when the attribute of interest is the uncorrelatedness of increments. Moreover, in contrast to the dependence of the unit root test statistics' distributions on nuisance parameters, the variance ratio's limiting distribution is Gaussian and independent of any nuisance parameters.[6] Although we report simulation results for the Dickey-Fuller t and the Box-Pierce Q tests for comparison with the performance of the variance ratio test, we emphasize that these three tests are not direct competitors since they have been designed with different null hypotheses in mind.

The paper is organized as follows. In Section 2 we define the variance ratio statistic, summarize its asymptotic sampling theory, and define the Dickey-Fuller and Box-Pierce tests. Section 3 presents Monte Carlo results for the three tests under two null hypotheses and Section 4 contains the power results for the three alternative hypotheses. We summarize and conclude in Section 5.

2. THE VARIANCE RATIO TEST.

Since the asymptotic sampling theory for the variance ratio statistic is fully developed in Lo and MacKinlay [1987], we present only a brief summary here. Let X_t denote a stochastic process satisfying the following recursive relation:

$$X_t = \mu + X_{t-1} + \epsilon_t \,, \quad E[\epsilon_t] = 0 \text{ for all } t \,. \tag{1a}$$

or

$$\Delta X_t = \mu + \epsilon_t \,, \qquad \Delta X_t \equiv X_t - X_{t-1} \tag{1b}$$

where the drift μ is an arbitrary parameter. The essence of the random walk hypothesis is the restriction that the disturbances ϵ_t are serially uncorrelated, or that innovations are unforecastable from past innovations. We develop our test under two null hypotheses which capture this aspect of the random walk: independently and identically distributed Gaussian increments, and the more general case of uncorrelated but weakly dependent and possibly heteroscedastic increments.

2.1 THE I.I.D. GAUSSIAN NULL HYPOTHESIS.

Let the null hypothesis H_1 denote the case where the ϵ_t's are i.i.d. normal random variables with variance σ^2 hence:

$$H_1 : \epsilon_t \quad \text{i.i.d.} \quad N(0, \sigma^2) \; . \tag{2}$$

In addition to homoscedasticity, we have made the assumption of independent <u>Gaussian</u> increments as in Dickey and Fuller [1979, 1981] and Evans and Savin [1981a,b, 1984].[7] Suppose we obtain nq + 1 observations $X_0, X_1, \ldots, X_{nq}$ of X_t where both n and q are arbitrary integers greater than one. Considering the following estimators for the unknown parameters μ and σ^2:

$$\hat{\mu} \equiv \frac{1}{nq} \sum_{k=1}^{nq} [X_k - X_{k-1}] \equiv \frac{1}{nq} [X_{nq} - X_0] \; . \tag{3}$$

$$\hat{\sigma}_a^2 \equiv \frac{1}{nq} \sum_{k=1}^{nq} [X_k - X_{k-1} - \hat{\mu}]^2 \tag{4}$$

The estimator $\hat{\sigma}_a^2$ is simply the sample variance of the first-difference of X_t; it corresponds to the maximum likelihood estimator of the parameter σ^2 and therefore possesses the usual consistency, asymptotic normality and efficiency properties.

Consider the variance of q-th differences of X_t which, under H_1, is q times the variance of first-differences. By dividing by q, we obtain the estimator $\hat{\sigma}_b^2(q)$ which also converges to σ^2 under H_1, where:

$$\hat{\sigma}_b^2(q) \equiv \frac{1}{nq^2} \sum_{k=q}^{nq} [X_k - X_{k-q} - q\hat{\mu}]^2 \; . \tag{5}$$

We have written $\hat{\sigma}_b^2(q)$ as a function of q [which we term the <u>aggregation value</u>] to emphasize the fact that a distinct alternative estimator of σ^2 may be formed for <u>each</u> q.[8] Under the null hypothesis of a Gaussian random walk, the two estimators $\hat{\sigma}_a^2$ and $\hat{\sigma}_b^2(q)$ should be "close", therefore a test of the random walk may be constructed by computing the difference $M_d(q) = \hat{\sigma}_b^2(q) - \hat{\sigma}_a^2$ and checking its proximity to zero. Alternatively, a test may also be based upon the dimensionless centered variance ratio $M_r(q) \equiv \frac{\hat{\sigma}_b^2(q)}{\hat{\sigma}_a^2} - 1$, which converges in probability to zero as well.[9] It is shown in Lo and MacKinlay [1987] that $M_d(q)$ and $M_r(q)$ possess the following limiting distributions under the null hypothesis H_1:

$$\sqrt{nq}\ M_d(q) \overset{a}{\sim} N\left(0,\ \frac{2(2q-1)(q-1)}{3q}\ \sigma^4\right) \tag{6a}$$

$$\sqrt{nq}\ M_r(q) \overset{a}{\sim} N\left(0,\ \frac{2(2q-1)(q-1)}{3q}\right) . \tag{6b}$$

An additional adjustment which may improve the finite-sample behavior of the test statistics is to use unbiased estimators $\bar{\sigma}_a^2$ and $\bar{\sigma}_b^2(q)$ in computing $M_d(q)$ and $M_r(q)$, where:

$$\bar{\sigma}_a^2 \equiv \frac{1}{(nq-1)} \sum_{k=1}^{nq} (X_k - X_{k-1} - \hat{\mu})^2 \tag{7a}$$

$$\bar{\sigma}_b^2(q) \equiv \frac{1}{m} \sum_{k=q}^{nq} (X_k - X_{k-q} - q\hat{\mu})^2 , \qquad m \equiv q(nq - q + 1)(1 - \frac{q}{nq}) . \tag{7b}$$

We denote the resulting adjusted specification test statistics $\bar{M}_d(q)$ and $\bar{M}_r(q)$. Of course, although the variance estimators $\bar{\sigma}_a^2$ and $\bar{\sigma}_b^2(q)$ are unbiased, only $\bar{M}_d(q)$ is unbiased; $\bar{M}_r(q)$ is not.

2.2 THE HETEROSCEDASTIC NULL HYPOTHESIS.

Since there is already a growing concensus that many economic time series possess time-varying volatilities, we derive a version of our specification test of the random walk model which is robust to heteroscedasticity. As long as the increments are uncorrelated, the variance ratio must still converge to one in probability even with heteroscedastic disturbances. Heuristically, this is simply because the variance of the sum of uncorrelated increments must still equal the sum of the variances. Of course, the asymptotic variance of the variance ratios will depend on the type and degree of heteroscedasticity present. By controlling the degree of heterogeneity and dependence of the process, it is possible to obtain consistent estimators of this asymptotic variance. To relax the i.i.d. Gaussian restriction of the ϵ_t's, we follow White's [1980] and White and Domowitz's [1984] use of mixing and moment conditions to derive heteroscedasticity-consistent estimators of our variance ratio's asymptotic variance. We require the following assumptions on $\{\epsilon_t\}$, which form our second null hypothesis H_2:

H_2: (A1) For all t, $E[\epsilon_t] = 0$, $E[\epsilon_t \epsilon_{t-\tau}] = 0$ for any $\tau \neq 0$.

(A2) $\{\epsilon_t\}$ is ϕ-mixing with coefficients $\phi(m)$ of size $r/(2r-1)$ or is α-mixing with coefficients $\alpha(m)$ of size $r/(r-1)$, $r > 1$ such that for all t and for any $\tau \geq 0$, there exists some $\delta > 0$ for which:

$$E|\epsilon_t \epsilon_{t-\tau}|^{2(r+\delta)} < \Delta < \infty . \tag{8}$$

(A3) $\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} E[\epsilon_t^2] = \sigma_0^2 < \infty$.

(A4) For all t, $E[\epsilon_t \epsilon_{t-j} \epsilon_t \epsilon_{t-k}] = 0$ for any non-zero j, k where $j \neq k$.

Assumption (A1) is the essential property of the random walk that we wish to test. Assumptions (A2) and (A3) are restrictions on the degree of dependence and heterogeneity which are allowed and yet still permit some form of law of large numbers and central limit theorem to obtain. This allows for a variety of forms of heteroscedasticity including deterministic changes in the variance [due, for example, to seasonal components] as well as Engle's [1982] ARCH processes [in which the conditional variance depends upon past information].[10] Assumption (A4) implies that the sample autocorrelations of ϵ_t are asymptotically uncorrelated.[11] Under the null hypothesis H_2, we may obtain heteroscedasticity-consistent estimators $\hat{\delta}(j)$ of the asymptotic variance $\delta(j)$ of the autocorrelations $\hat{\rho}(j)$ of ΔX_t. Using the fact that the variance ratio may be written as an approximate linear combination of autocorrelations [see (12) below] yields the following limiting distribution for $\bar{M}_r(q)$:[12]

$$\bar{M}_r(q) \overset{a}{\sim} N[0,\ V(q)] \tag{9a}$$

where

$$V(q) = \sum_{j=1}^{q-1} \left[\frac{2(q-j)}{q}\right]^2 \cdot \delta(j)\ , \quad \hat{V}(q) = \sum_{j=1}^{q-1} \left[\frac{2(q-j)}{q}\right]^2 \cdot \hat{\delta}(j) \tag{9b}$$

$$\hat{\delta}(j) = \frac{\sum_{k=j+1}^{nq} (X_k - X_{k-1} - \hat{\mu})^2 \cdot (X_{k-j} - X_{k-j-1} - \hat{\mu})^2}{\left[\sum_{k=1}^{nq} (X_k - X_{k-1} - \hat{\mu})^2\right]^2} . \tag{9c}$$

Tests of H_1 and H_2 may then be based on the normalized variance ratios $z_1(q)$ and $z_2(q)$ respectively where:

$$z_1(q) \equiv \sqrt{nq}\ \bar{M}_r(q) \cdot \left(\frac{2(2q-1)(q-1)}{3q}\right)^{-\frac{1}{2}} \quad \overset{a}{\sim} \ N(0,\ 1) . \tag{10a}$$

$$z_2(q) \equiv \sqrt{nq}\ \bar{M}_r(q) \cdot \hat{V}^{-\frac{1}{2}}(q) \quad \overset{a}{\sim} \ N(0,\ 1) . \tag{10b}$$

2.3 VARIANCE RATIOS AND AUTOCORRELATIONS.

To develop some intuition for the variance ratio, observe that for an aggregation value q of 2, the $M_r(q)$ statistic may be re-expressed as:

$$M_r(2) = \hat{\rho}(1) - \frac{1}{4n\hat{\sigma}_a^2}\left[(X_1 - X_0 - \hat{\mu})^2 + (X_{2n} - X_{2n-1} - \hat{\mu})^2\right] \tag{11}$$

hence for q = 2 the $M_r(q)$ statistic is approximately the first-order autocorrelation coefficient estimator $\hat{\rho}(1)$ of the differences of X. More generally, we have the following relation for $q \geq 2$:

$$M_r(q) = \frac{2(q-1)}{q}\hat{\rho}(1) + \frac{2(q-2)}{q}\hat{\rho}(2) + \ldots + \frac{2}{q}\hat{\rho}(q-1) + o_p(n^{-\frac{1}{2}}) \tag{12}$$

where $o_p(n^{-\frac{1}{2}})$ denotes terms which are of order smaller than $n^{-\frac{1}{2}}$ in probability. Equation (12) provides a simple interpretation for the variance ratio computed with an aggregation value q: it is [approximately] a linear combination of the first q-1 autocorrelation coefficient estimators of the first differences with arithmetically declining weights. Note the similarity between this and the Box-Pierce [1970] Q-statistic of order q-1:

$$Q_1(q-1) = T\sum_{k=1}^{q-1}\hat{\rho}^2(k) \tag{13}$$

which is asymptotically distributed as χ^2 with q-1 degrees of freedom.[13] Using (9c) we can also construct a heteroscedasticity-robust Box-Pierce statistic in the obvious way, which we denote by $Q_2(q-1)$. Since the Box-Pierce Q-statistics give equal weighting to the autocorrelations, and are computed by squaring the autocorrelations, their properties will differ from those of the variance ratio test statistics.

For comparison, we also employ the Dickey-Fuller t test. This involves computing the usual t-statistic under the hypothesis $\beta = 1$ in the regression:

$$X_t = \mu + \omega t + \beta X_{t-1} + \nu_t \qquad (14)$$

and using the exact finite sample distribution tabulated by Fuller [1976], Dickey and Fuller [1979, 1981], and Nankervis and Savin [1985].[14,15]

3. PROPERTIES OF THE TEST STATISTIC UNDER THE NULL HYPOTHESES.

To gauge the quality of the asymptotic approximations in Section 2, we perform simulation experiments for the $\bar{M}_r(q)$ statistic under both the Gaussian i.i.d. null hypothesis and a simple heterscedastic null. More extensive simulation experiments indicate that tests based upon the unadjusted statistic $M_r(q)$ generally yield less reliable inferences hence, in the interest of brevity, we only report the results for $\bar{M}_r(q)$. For comparison, we also report the results of Monte Carlo experiments performed for the Box-Pierce Q-statistics and the Dickey-Fuller t-statistic. All simulations are based on 20,000 replications.[16]

3.1 THE GAUSSIAN I.I.D. NULL HYPOTHESIS.

Tables 1a and 1b report the results of simulation experiments conducted under the independent and identically distributed Gaussian random walk null H_1. The results show that the empirical sizes of two-sided 5 percent variance ratio tests based on either the $z_1(q)$ or $z_2(q)$ statistics are close to their nominal values for sample sizes greater than 32. Not surprisingly, for an aggregation value q of 2 the behavior of the variance ratio is comparable to that of the Box-Pierce Q-statistic since $\bar{M}_r(2)$ is approximately equal to the first-order serial correlation coefficient. However, for larger aggregation values the behavior of the two statistics differ.

Table 1a shows that as the aggregation value q increases to one-half the sample size, the empirical size of the Box-Pierce Q_1-test generally declines well below its nominal value, whereas the size of the variance ratio's z_1-test seems to first increase slightly above and then fall back to its nominal value. For example, with a sample size of 1024, the size of the 5 percent Q_1-test falls monotonically from 5.1 to 0.0 percent as q goes from 2 to 512; the size of the 5 percent z_1-test starts at 5.2 percent when q = 2, increases to 6.2 percent at q = 256, and settles at 5.1 percent when q = 512.

Although the size of the variance ratio test is closer to its nominal value for larger q, this does not necessarily imply that large values of q are generally more desirable. To examine this issue, Table 1a separates the size of the variance ratio test into rejection rates of the lower and upper tails of the 1, 5, and 10 percent tests. When q becomes large relative to the sample size, the rejections of the variance ratio test are almost wholly due to the upper tail. One reason for this positive skewness of the $z_1(q)$-statistic is that the variance ratio is bounded below by zero, hence a related lower bound obtains for the test statistic.[17] Although this is of less consequence for the size of the variance ratio test, it has serious power implications and will be discussed more fully in Section 4.1.

Table 1b reports similar results for the heteroscedasticity-robust test statistics $z_2(q)$ and Q_2. For sample sizes greater than 32, the size of the variance ratio test is close to its nominal value when q is small relative to the sample size. As q increases for a given sample, the size increases and then declines, as in Table 1a. Again, the variance ratio rejections are primarily due to its upper tail as q increases relative to the sample size. In contrast to the Q_1-test, the heteroscedasticity-robust Box-Pierce test Q_2-test increases in size as more autocorrelations are used. For example, in samples of 1024 observations the size of the 5 percent Q_2-test increases from 5.1 to 11.3 percent as q ranges from 2 to 512. In contrast, the size of the variance ratio test starts at 5.2 percent when $q = 2$, increases to 6.6 percent at $q = 256$, and falls to 5.8 percent at $q = 512$.

Tables 1a and b indicate that the empirical size of the variance ratio tests is reasonable even for moderate sample sizes, and is closer to its nominal value than the Box-Pierce tests when the aggregation value becomes large relative to the sample size. However, in such cases most of the variance ratio's rejections are from its upper tail; power considerations will need to be weighed against the variance ratio test's reliability under the null.

Since the sampling theory for the Q- and z-statistics obtain only asymptotically, the actual size of any test based on these statistics will of course differ from their nominal values in finite samples. Although Table 1 indicates that such differences may not be large for reasonable aggregation values, it may nevertheless seem more desirable to base tests upon the regression t-statistic for which Fuller [1976], Dickey and Fuller [1979, 1981], and Nankervis and Savin [1985] have tabulated the exact finite sample distribution. Due to the dependence of the t-statistic's distribution on the drift μ, an additional nuisance parameter (a time-trend coefficient) must be estimated to yield a sampling distribution that is independent of the drift. Although it has been demonstrated that the t-statistic from such a regression converges in distribution to that of Dickey and Fuller, there may be some discrepancies in finite samples. Table 2 presents the empirical quantiles of the distribution of the t-statistic associated with the hypothesis $\beta = 1$ in the regression (14). A comparison of these quantiles with those given in Fuller [1976, Table 8.5.2] suggests that there may be some significant differences for small samples, but for sample sizes of 500 or greater the quantiles in Table 2 are almost identical to those of Dickey and Fuller.

3.2 A HETEROSCEDASTIC NULL HYPOTHESIS.

To assess the reliability of the heteroscedasticity-robust statistic $z_2(q)$, we perform simulation experiments under the null hypothesis that the disturbance ϵ_t in (1) is serially uncorrelated but heteroscedastic in the following manner. Let the random walk disturbance ϵ_t satisfy the relation $\epsilon_t \equiv \sigma_t \lambda_t$ where λ_t is i.i.d. N(0, 1) and σ_t satisfies:

$$\ln \sigma_t^2 = \psi \cdot \ln \sigma_{t-1}^2 + \zeta_t \qquad \zeta_t \sim N(0,1) \ . \tag{15}$$

λ_t and ζ_t are assumed to be independent. The empirical studies of French, Schwert, and Stambaugh [1987] and Poterba and Summers [1986] posit such a process for the variance. Note that σ_t^2 cannot be interpreted as the unconditional variance of the random walk disturbance ϵ_t since σ_t^2 is itself stochastic and does not correspond to the unconditional expectation of any random variable. Rather, <u>conditional upon</u> σ_t^2, ϵ_t is normally distributed with expectation 0 and variance σ_t^2. If, in place of (15), the variance σ_t^2 were re-parameterized to depend only upon exogenous variables in the time t-1 information set, this would correspond exactly to Engle's [1982] ARCH process.

The unconditional moments of ϵ_t may be readily deduced by expressing the process explicitly as a function of all the disturbances:

$$\epsilon_t = \lambda_t \sigma_0^{\psi^t} \cdot \prod_{k=1}^{t} \exp\left[\frac{1}{2} \psi^{t-k} \zeta_k\right] \tag{16}$$

Since σ_0, λ_t, and ζ_k are assumed to be mutually independent, it is apparent that ϵ_t is serially uncorrelated at all leads and lags [hence Assumption (A1) is satisfied] but is non-stationary and temporally dependent. Moreover, it is evident that $E[\epsilon_t^2 \epsilon_{t-j} \epsilon_{t-k}] = 0$ for all t and for $j \neq k$ hence Assumption (A4) is also satisfied. A straightforward calculation yields the moments of ϵ_t:

$$E[\epsilon_t^{2p}] = E[\sigma_0^{2p\psi^t}] \cdot \frac{(2p)!}{p!2^p} \exp[\frac{p}{2} \frac{1 - \psi^{2t}}{1 - \psi^2}] \tag{17a}$$

$$E[\epsilon_t^{2p+1}] = 0 , \quad p = 0, 1, 2, \ldots \tag{17b}$$

From these expressions it is apparent that, for $\psi \in (0,1)$, ϵ_t possesses bounded moments of any order and is unconditionally heteroscedastic; similar calculations for the cross-moments verify Assumption (A2). Finally, the following inequality is easily deduced:

$$\frac{1}{n} \sum_{k=1}^{n} E[\epsilon_t^2] < \exp[\frac{5}{2(1 - \psi^2)}] < \infty \tag{18}$$

thus Assumption (A3) is verified. Note that the kurtosis of ϵ_t is:

$$\frac{E[\epsilon_t^4]}{(E[\epsilon_t^2])^2} = 3 \cdot \frac{E[\sigma_0^{4\psi^t}]}{(E[\sigma_0^{2\psi^t}])^2} \geq 3 \tag{19}$$

by Jensen's inequality. This implies that, as for Engle's [1982] stationary ARCH process, the distribution of ϵ_t is more peaked and possesses fatter tails than that of a normal random variate. However, when $\psi = 0$ or as t increases without bound, the kurtosis of ϵ_t is equal to that of a Gaussian process.

Table 3a reports simulation results for the z-, Q-, and Dickey-Fuller t-statistics under the heteroscedastic null hypothesis with parameter ψ = 0.50. It is apparent that both the z_1- and Q_1-statistics are unreliable in the presence of heteroscedasticity. Even in samples of 512 observations, the empirical size of the 5 percent variance ratio test with q = 2 is 14.7 percent; the corresponding Box-Pierce 5 percent test has an empirical size of 14.6 percent. In contrast, the Dickey-Fuller t-test's empirical size of 4.9 percent is much closer to its nominal value. This is not surprising since Phillips [1987] and Phillips and Perron [1986] have shown that the Dickey-

Fuller t-test is robust to heteroscedasticity [and weak dependence] whereas the z_1- and Q_1-statistics are not. However, once the heteroscedasticity-robust z_2- and Q_2-statistics are used, both tests compare favorably with the Dickey-Fuller t-test. In fact, for the more severe case of heteroscedasticity considered in Table 3b [where $\psi = 0.95$], the variance ratio and Box-Pierce tests using z_2 and Q_2 are both considerably more reliable than the Dickey-Fuller test.[18] For example, when q/T is ½ in sample sizes of 512 observations the sizes of 5 percent tests using z_2 and Q_2 are 4.7 and 5.7 percent respectively; the size of the 5 percent Dickey-Fuller test is 21.6 percent.

4. POWER.

Since a frequent application of the random walk has been in modelling stock market returns, it is natural to examine the power of the variance ratio test against alternative models of asset price behavior. We consider three specific alternative hypotheses. The first two are specifications of the stock price process that have received the most recent attention: the stationary AR(1) process [as in Shiller [1981] and Shiller and Perron [1985]], and the sum of this process and a random walk [as in Fama and French [1987] and Poterba and Summers [1987]].[19] The third alternative is an integrated AR(1) process which is suggested by the empirical evidence in Lo and MacKinlay [1987].

Before presenting the simulation results, we consider an important limitation of the variance ratio test in Section 4.1. In Section 4.2 we compare the power of the variance ratio test with that of the Dickey-Fuller and Box-Pierce tests against the stationary AR(1) alternative. Section 4.3 reports similar power comparisons for the remaining two alternatives.

4.1 THE VARIANCE RATIO TEST FOR LARGE q.

Although it will become apparent in Sections 4.2 and 4.3 that choosing an appropriate aggregation value q for the variance ratio test depends intimately on the alternative hypothesis of interest, several authors have suggested using large values of q generally.[20] But because the variance ratio test statistic is bounded below, when q is large relative to T the test may have little power. To see this, let the [asymptotic] variance of the test statistic $\bar{M}_r(q)$ be denoted by V, where we have from (6b):

$$V \equiv \frac{2(2q-1)(q-1)}{3nq^2} = \frac{4}{3n} \cdot \left[\frac{q^2 - \frac{3}{2}q + 1}{q^2}\right] . \tag{20}$$

Note that for all natural numbers q, the bracketed function in (20) is bounded between ½ and 1, and is monotonically increasing in q. Therefore, for fixed n, this implies upper and lower bounds $V_U \equiv \frac{4}{3n}$ and $V_L \equiv \frac{2}{3n}$ for the variance V. Since variances must be nonnegative, the lower bound for $\bar{M}_r(q)$ is -1 [since we have defined $\bar{M}_r(q)$ to be the variance ratio minus 1]. Using these two facts, we have the following lower bound on the [asymptotically] standard normal test statistic $z_1(q) \equiv \bar{M}_r(q)/\sqrt{V}$:

$$\inf[z_1(q)] = \frac{-1}{\inf[\sqrt{V}]} = -\frac{1}{\sqrt{V_L}} = -\left[\frac{3n}{2}\right]^{\frac{1}{2}} . \tag{21}$$

Note that n is <u>not</u> the sample size [which is given by nq], but is the number of non-overlapping <u>coarse</u> increments [increments of aggregation value q] available in the sample, and is given by $\frac{q}{T}$.

If q is large relative to the sample size T, this implies a small value for n. For example, if $\frac{q}{T} = \frac{1}{2}$, then the lower bound on the standard normal test statistic $z_1(q)$ is -1.73; the test will never reject draws from the left tail at the 95 percent level of significance!

Of course, there is no corresponding upper bound on the test statistic so in principle it may still reject via draws in the right tail of the distribution. However, for many alternative hypotheses of interest the population values of their variance ratios are less than unity,[21] implying that for those alternatives rejections are more likely to come from large negative rather than large positive draws of $z_1(q)$. For this reason, and because of the unreliability of large-sample theory under the null when q/T is large, we have chosen q to be no more than one-half the total sample size throughout this study.

4.2 POWER AGAINST A STATIONARY AR(1) ALTERNATIVE.

As a model of stock market fads, Shiller [1981] has suggested the following AR(1) specification for the log-price process X_t:

$$X_t = \alpha + \phi\cdot[X_{t-1} - \alpha] + \epsilon_t \qquad \epsilon_t \sim N(0,\sigma_\epsilon^2) \tag{20}$$

where ϕ is positive and less than unity. To determine the power of the variance ratio test against this alternative, we choose values of the parameters (ϕ, σ_ϵ^2, σ_Y^2) that yield an interesting range of power across sample sizes and aggregation values. Since the power does not depend on α, we set it to zero without loss of generality. Table 5a reports the power of the variance ratio, Dickey-Fuller t, and Box-Pierce Q tests at the 1, 5 and 10 percent levels against the AR(1) alternative with parameters $(\phi, \sigma_\epsilon^2) = (0.96, 1)$. The critical values of all three test statistics were empirically determined by simulation under the i.i.d. Gaussian null. In the interest of brevity, we report the empirical critical values in Table 4 for the variance ratio test only.[22]

For a fixed number of observations, the power of the variance ratio test first increases and then declines with the aggregation value q. The increase can be considerable; as the case of 1024 observations demonstrates, the power is 9.2 percent when q = 2 but jumps to 98.3 percent when q = 256. The explanation for the increase in power lies in the behavior of the AR(1) alternative over different sampling intervals: the first-order autocorrelation coefficient of AR(1) increments grows in absolute value (becomes more negative) as the increment interval increases. This implies that, although X_t may have a root close to unity (0.96), its first-differences behave less like random walk increments as the time interval of the increments grows. It is therefore easier to detect an AR(1) departure from the random walk by comparing longer first-difference variances to shorter ones, which is precisely what the variance ratio does for larger q. However, as q is increased further the power declines. This may be attributed to the imprecision with which higher-order autocorrelations are estimated for a fixed sample size. Since the variance ratio with aggregation value q is approximately a linear combination of the first q-1 autocorrelations, a larger value of q/T entails estimating higher-order autocorrelations with a fixed sample size. The increased sampling variation of these additional autocorrelations leads to the decline in power.[23]

Although the most powerful variance ratio test is more powerful than the Dickey-Fuller t-test, the difference is generally not large. However, the variance ratio test clearly dominates the Box-Pierce Q-test. With a sample of 512 observations the power of a 5 percent variance ratio test is 51.4 percent [q = 128] whereas the power of the corresponding Q-test is only 7.1 percent. However, with an aggregation value of q = 2 the variance ratio has comparable power to the Box-Pierce test. Again, this is as expected since they are quite similar statistics when q = 2 [the variance ratio is approximately one plus the first-order autocorrelation coefficient and the Box-Pierce statistic is the first-order autocorrelation squared].

We conclude that, against the stationary AR(1) alternative, the variance ratio test is comparable to the Dickey-Fuller t test in power and both are considerably more powerful than the Box-Pierce test.

4.3. TWO UNIT ROOT ALTERNATIVES TO THE RANDOM WALK.

Several recent studies have suggested the following specification for the log-price process X_t:

$$X_t = Y_t + Z_t \tag{21}$$

where Y_t is a stationary process and Z_t is a Gaussian random walk independent of Y_t.[24] To be specific, let Y_t be an AR(1), thus:

$$Y_t = \alpha + \phi\cdot[Y_{t-1} - \alpha] + \varepsilon_t \qquad \varepsilon_t \ \text{ i.i.d. } \ N(0, \sigma_\varepsilon^2) \tag{22a}$$

$$Z_t = Z_{t-1} + \gamma_t \qquad \gamma_t \ \text{ i.i.d. } \ N(0, \sigma_\gamma^2) \tag{22b}$$

Again, without loss of generality we set α to 0; ρ is set to 0.96; σ_ε^2 is normalized to unity, and σ_γ^2 takes on the values 0.50, 1.00, and 2.00 so that the conditional variability of the random walk relative to the stationary component is two, one, and one-half respectively. Tables 5b-d report the power of the variance ratio, Dickey-Fuller t, and Box-Pierce Q-tests against this alternative. Note that this specification contains a unit root [it is an ARIMA (1,1,1)] hence, asymptotically, the power of the Dickey-Fuller t-test should equal its size.[25] However, since Schwert [1987a,b] has shown the finite-sample behavior of the Dickey-Fuller test to be quite erratic, we report its power for comparison.

Table 5b gives the power results for the z_1-, Q_1-, and t-statistics against this ARIMA(1,1,1) alternative where the variance of the random walk innovation is twice the variance of the AR(1) disturbance. Although none of the tests are especially powerful under these parameter values, the variance ratio test seems to dominate the other two. For a sample size of 1024, the power of the variance ratio test is 24.1 percent for $q = 32$ whereas the corresponding power of the Dickey-Fuller and Box-Pierce tests are 10.4 and 7.9 percent respectively.

As in the case of the stationary AR(1) alternative, the power of the variance ratio test also rises and falls with q against the ARIMA(1,1,1) alternative. In addition to the factors discussed in Section 4.2, there is an added explanation for this pattern of power. For small to medium differencing intervals the increments of X_t behave much like increments of an AR(1), hence power increases with q in this range. For longer differencing intervals the random walk component dominates, hence the power declines beyond some aggregation value q.

As the variance of the random walk's disturbance declines relative to the variance of the stationary component's, the power of the variance ratio test increases. Table 5c reports power results for the case where the variances of the two components' innovations are equal, and in Table 5d the variance of the random walk innovation is half the variance of the AR(1) innovation. In the latter case, the 5 percent variance ratio test has 89.8 percent power for $q = 32$ and $T = 1024$ compared to 41.7 percent and 18.4 percent power for the Dickey-Fuller and Box-Pierce tests respectively. Although the qualitative behavior of the three tests are the same in Tables 5b-d, the variance ratio test is considerably more powerful than the other two when the variance of the stationary component is larger than that of the random walk. Moreover, the pattern of power as a function of q clearly demonstrates that against this alternative, it is not optimal to set q as large as possible.[26]

Since both the stationary AR(1) and the AR(1) plus random walk are not empirically supported by Lo and MacKinlay's [1987] results for weekly stock returns, we consider the power of the variance ratio test against a more relevant alternative hypothesis suggested by their empirical findings: an integrated AR(1), i.e., an ARIMA(1,1,0). Specifically, if X_t is the log-price process then we assume:

$$(X_t - X_{t-1}) = \kappa \cdot (X_{t-1} - X_{t-2}) + \zeta_t \,, \quad \zeta_t \;\; \text{i.i.d.} \;\; N(0, \sigma_\zeta^2) \,. \tag{23}$$

where $|\kappa| < 1$. Since this alternative obviously possesses a unit root, we expect the standard unit root tests to have poor power against it. Nevertheless for comparison we report the power of the Dickey-Fuller t-test along with the power of the variance ratio and Box-Pierce tests. The parameters (κ, σ_ζ^2) are set to (0.20, 1) for all the simulations in Table 5e. Unlike its behavior under the stationary AR(1) alternative, against this integrated process the variance ratio's power declines as q increases. With a sample size of 1024, the power of a 5 percent test is 100 percent when q = 2 but falls to 9.3 percent when q = 512. In contrast to the AR(1), the behavior of the integrated process's increments is farthest from a random walk for short differencing intervals [since the increments follow a stationary AR(1) by construction]. As the differencing interval increases, the autocorrelation of the increments decreases and it becomes more difficult to distinguish between this process and the random walk.

Observe that for smaller aggregation values the variance ratio test is more powerful than the Q test, but the Q test dominates when q is large. This result is due to the fact that the Box-Pierce Q does not distinguish between the upper and lower tails of the null distribution [since Q is the sum of squared autocorrelations] whereas the variance ratio test does.

5. CONCLUSION.

Our simulations indicate that the variance ratio test of the random walk hypothesis generally yields reliable inferences under both the i.i.d. Gaussian and the heteroscedastic null hypotheses. By selecting the aggregation value q appropriately, the power of the variance ratio test is comparable to that of the Box-Pierce and Dickey-Fuller tests against the stationary AR(1) alternative, and is more powerful than either of the two tests against the two unit-root alternatives. However, because of the variance ratio's skewed empirical distribution, caution must be exercised when q is large relative to the sample size.

These results emphasize dramatically the obvious fact that the power of any test may differ substantially across alternatives. A sensible testing strategy must consider not only the null hypothesis but also the most relevant alternative. Although the variance ratio test has advantages over other tests under some null and alternative hypotheses, there are of course other situations in which those tests may possess more desirable properties. Nevertheless, the Monte Carlo evidence suggests that the variance ratio test has reasonable power against a wide range of alternatives.[27] The simplicity, reliability, and flexibility of the variance ratio test make it a valuable tool for inference.

FOOTNOTES

[1]See, for example, Gould and Nelson [1974], Hall [1978], Lucas [1978], Shiller [1981], Kleidon [1986], and Marsh and Merton [1986].

[2]See, for example, Campbell and Mankiw [1987], Cochrane [1987a, b], Fama and French [1987], Huizinga [1987], Lo and MacKinlay [1987], and Poterba and Summers [1987].

[3]Dickey and Fuller [1979, 1981] make the stronger assumption of independently and identically distributed Gaussian disturbances.

[4]Also, see Cochrane [1987c] who uses this fact to show that trend-stationarity and difference-stationarity cannot be distinguished with a finite amount of data.

[5]We are grateful to referee 2 for this insight.

[6]The usual regression t-statistic's limiting distribution depends discontinuously on the presence or absence of a non-zero drift [see Nankervis and Savin [1985], and Perron [1986]]. This dependence on the drift may be eliminated by the inclusion of a time trend in the regression, but requires the estimation of an additional parameter and may affect the power of the resulting test. Section 4 reports power comparisons.

[7]The Gaussian assumption may, of course, be weakened considerably. We present results for this simple case only for purposes of comparison to other results in the literature that are derived under identical conditions. In Section 2.2 we relax both the independent and the identically distributed assumptions.

[8]Although we have defined the total number of observatons $T \equiv nq$ to be divisible by the aggregation value q, this is only for expositional convenience and may be easily generalized.

[9]The use of variance ratios is, of course, not new. Most recently, Campbell and Mankiw [1987], Cochrane [1987a, b], Fama and French [1987], French and Roll [1986], and Huizinga [1987] have all computed variance ratios in a variety of contexts. However, those studies do not provide any formal sampling theory for our statistics. Specifically, Cochrane [1987a], Fama and French [1986], and French and Roll [1986] all rely upon Monte Carlo simulations to obtain standard errors for their variance ratios under the null. Campbell and Mankiw [1987] and Cochrane [1987b] do derive the asymptotic variance of the variance ratio but only under the assumption that the aggregation value q grows with [but more slowly than] the sample size T. Specifically, they use Priestley's [1981, p. 463] expression for the asymptotic variance of the estimator of the spectral density of ΔX_t at frequency zero with a Bartlett window as the appropriate asymptotic variance of the variance ratio. But Priestley's result requires [among other things] that $q \to \infty$, $T \to \infty$, and $q/T \to 0$. In this paper, we develop the formal sampling theory of the variance ratio statistics for the more general case.

Our variance ratio may, however, be related to the spectral density estimates in the following way. Letting f(0) denote the spectral density of the increments ΔX_t at frequency zero, we have the following relation:

$$\pi f(0) = \gamma(0) + 2 \cdot \sum_{k=1}^{\infty} \gamma(k)$$

where $\gamma(k)$ is the autocovariance function. Dividing both sides by the variance $\gamma(0)$ then yields:

$$\pi f^*(0) = 1 + 2 \cdot \sum_{k=1}^{\infty} \rho(k)$$

where f^* is the normalized spectral density and $\rho(k)$ is the autocorrelation function. Now in order to estimate the quantity $\pi f^*(0)$, the infinite sum on the right-hand side of the preceding equation must obviously truncated. If, in addition to truncation, the autocorrelations are weighted using Newey and West's [1987] procedure, then the resulting estimator is formally equivalent to our $M_r(q)$ statistic. Although he does not explicitly use this variance ratio, Huizinga [1987] does employ the Newey and West [1987] estimator of the normalized spectral density.

[10]In addition to admitting heteroscedasticity, it should be emphasized that Assumptions (A2) and (A3) also follow for more general heterogeneity and weak dependence. Our reason for focusing on heteroscedasticity is merely its intuitiveness; it is more difficult to produce an interesting example of, for example, an uncorrelated homoscedastic time series which is weakly dependent and heterogeneously distributed.

[11]Although this assumption may be weakened considerably, it would be at the expense of computational simplicity since in that case the asymptotic covariances of the autocorrelations must be estimated. Specifically, since the variance ratio statistic is asymptotically equivalent to a linear combination of autocorrelations, its asymptotic variance is simply the asymptotic variance of the linear combination of autocorrelations. If (A4) obtains, this variance is equal to the weighted sum of the individual autocorrelation variances. If (A4) is violated, then the autocovariances of the autocorrelations must also be estimated. This is readily accomplished using, for example, the approach in Newey and West [1987]. Note that an even more general [and possibly more exact] sampling theory for the variance ratio may be obtained using the results of Dufour and Roy [1985]. Again, this woul sacrifice much of the simplicity of our asymptotic results.

[12]An equivalent and somewhat more intuitive method of arriving at (9c) is to consider the regression of the increments ΔX_t on a constant and the j-th lagged increment ΔX_{t-j}. The estimated slope coefficient is then simply the j-th autocorrelation coefficient and the estimator $\delta(j)$ of its variance is numerically identical to White's [1980] heteroscedasticity-consistent covariance matrix estimator. Note that White [1980] requires independent disturbances whereas White and Domowitz [1984] allow for weak dependence [of which uncorrelated errors is, under suitable regularity conditions, a special case]. Taylor [1984] also obtains this result under the assumption that the multivariate distribution of the sequence of disturbances is symmetric.

[13] Since we include the Box-Pierce test only as an illustrative comparison to the variance ratio test, we have not made any effort to correct for finite-sample biases as in Ljung and Box [1978].

[14] Due to the dependence of the t-statistic's distribution on the drift μ, a time trend t must be included in the regression to yield a sampling theory for the t-statistic which is independent of the nuisance parameter.

[15] Yet another recent test of the random walk hypothesis is the regression test proposed by Fama and French [1987]. Since Monte Carlo experiments by Poterba and Summers [1987] indicate that the variance ratio is more powerful than this regression test against several interesting alternatives, we do not explore its finite-sample properties here.

[16] Null simulations were performed in single-precision FORTRAN on a DEC VAX 8700 using the random number generator GGNML of the IMSL subroutine library. Power simulations were performed on an IBM 3081 and a VAX 8700 also in single-precision FORTRAN using GGNML.

[17] More direct evidence of this skewness is presented in Table 4, in which the fractiles of the variance ratio test statistic are reported. See also the discussion in Section 4.1.

[18] This provides further support for Schwert's [1987b] finding that, although the Dickey-Fuller distribution is still valid asymptotically for a variety of non-i.i.d. disturbances, the t-statistic's rate of convergence may be quite slow.

[19] The latter specification is, of course, not original to the financial economics literature but has its roots in Muth [1960] and, more recently, Beveridge and Nelson [1981].

[20] For example, Campbell and Mankiw's [1987] asymptotic sampling theory requires that q goes to infinity as the sample size T goes to infinity [although q must grow at a slower rate than T]. Also, for a sample size of T Huizinga [1987] sets q to T - 1.

[21] For example, as q increases without bound the variance ratio [population value] of increments any stationary process will converge to 0. For the sum of a random walk and an independent stationary process, the variance ratio of its increments will also converge to a quantity less than unity as q approaches infinity.

[22] Diebold [1987] tabulates the finite sample distributions of actual variance ratios under many other null hypotheses of interest. Although we have not compared each of our empirical quantiles with his, we have spot-checked several for consistency and have found discrepancies only in the extreme tail areas. For example, with a sample size of 1024 and $q = 2$, Diebold's implied value for the upper 0.5 percent quantile of our test statistic z_1 is 2.48 [using his Table 16], whereas our value in Table 4 is 2.63. There are at least two possible causes for this discrepancy. First, Diebold's results are based on 10,000 replications whereas ours use 20,000. Second, we simulated the bias-corrected statistic whereas Diebold employed the unadjusted variance ratio. For larger tail areas, this discrepancy vanishes.

[23] If the variance ratio test were performed using asymptotic critical values against the AR(1) alternative, there is another cause of the power to decline as q increases. Under the AR(1) model, it is apparent that the theoretical values of the variance ratios are all less than unity, implying that the expectations of the z_1 statistics are negative. But it is shown in Section 4.1 that the z_1 statistic is bounded below when the asymptotic variance is used to form z_1, and that the lower bound is an increasing function of the ratio of q to the sample size. Therefore, when the deviation of the alternative from the random walk is in the form of negative draws of z_1 [as in the AR(1) case], the variance ratio test cannot reject the null hypothesis when q is large relative to the number of observations. This is yet another reason we choose q to be less than or equal to one-half the sample size.

[24] See, for example, Summers [1986], Fama and French [1987], and Poterba and Summers [1987].

[25] To see this, observe that (21) has the following ARIMA(1,1,1) representation:

$$(1 - \rho L)(1 - L)X_t = (1 - \lambda L)\nu_t$$

where $\lambda \equiv \rho\sigma_\epsilon^2 + \sigma_\gamma^2$ and $\sigma_\nu^2 \equiv \dfrac{(1 + \rho^2)\sigma_\epsilon^2 + 2\sigma_\gamma^2}{(1 + \lambda^2)}$.

[26] In fact, the q for which the variance test has the most power for a given sample size will depend on the ratio of the stationary component's innovation variance to the variance of the random walk's disturbance. Unfortunately, this fact cannot be observed in our tables because we have set q to be powers of 2 for computational convenience. If the variance ratio test's power were tabulated for q = 2, 3, 4, . . ., T-1, it would be apparent that against this ARIMA(1,1,1) alternative the optimal q changes with the ratio of the innovation variances of the two components.

[27] See Hausman [1988] for further evidence of this.

REFERENCES

Beveridge, S. and C. Nelson, 1981, A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle', Journal of Monetary Economics 7, 151-174.

Box, G. and D. Pierce, 1970, Distribution of residual autocorrelations in autoregressive-integrated moving average time series models, Journal of the American Statistical Association 65, 1509-1526.

Campbell, J. Y. and N. G. Mankiw, 1987, Are output fluctuations transitory?, forthcoming in Quarterly Journal of Economics.

Cochrane, J. H., 1987a, How big is the random walk in GNP, working paper, University of Chicago.

__________, 1987b, Spectral density estimates of unit roots, working paper, University of Chicago.

__________, 1987c, The application of unit roots tests: a critique, working paper, University of Chicago.

Dickey, D. A.,1976, Estimation and Hypothesis Testing for Nonstationary Time Series, Ph.D. Dissertation, Iowa State University, Ames.

Dickey, D. A. and W. A. Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association 74, 427-431.

__________, 1981, Likelihood ratio statistics for autoregressive time series with a unit root, Econometrica 49, 1057-1072.

Diebold, F. 1987, Deviations from random-walk behavior: Tests based on the variance-time function, Special Studies Paper Number 224, Federal Reserve Board, Washington, D.C.

Dufour, J. M. and R. Roy, 1985, Some exact results on sample autocorrelations and tests for randomness, Journal of Econometrics 29, 257-273.

Engle, R., 1982, Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflations, Econometrica 50, 987-1008.

Evans, G. B. A. and N. E. Savin, 1981a, The calculation of the limiting distribution of the least squares estimator of the parameter in a random walk model, Annals of Statistics 9, 1114-1118.

__________, 1981b, Testing for unit roots: 1, Econometrica 49, 753-779.

__________, 1984, Testing for unit roots: 2, Econometrica 52, 1241-1269.

Fama, E. and K. French, 1987, Permanent and temporary components of stock prices, to appear in Journal of Political Economy.

French, K., Schwert, G., and R. Stambaugh, 1985, Expected stock returns and volatility, Journal of Financial Economics 19, 3-30.

Fuller, W., 1976, Introduction to Statistical Time Series. New York: John Wiley and Sons, Inc.

Gould, J. and C. Nelson, 1974, The stochastic structure of the velocity of money, American Economic Review 64, 405-417.

Hall, R., 1978, Stochastic implications of the life cycle-permanent income hypothesis: Theory and evidence, Journal of Political Economy 86, 971-987.

Hausman, J., 1978, Specification tests in econometrics, Econometrica 46, 1251-1272.

__________, 1988, The optimality of autocorrelation-based tests of the random walk hypothesis, Working Paper, Department of Economics, Massachusetts Institute of Technology.

Huizinga, J., 1987, An empirical investigation of the long run behavior of real exchange rates, forthcoming in Carnegie-Rochester Conference Series on Public Policy 27.

Ljung, G. and G. Box, 1978, On a measure of lack of fit in time series models, Biometrika 66, 67-72.

Lo, A. and A. Craig MacKinlay, 1987, Stock market prices do not follow random walks: Evidence from a simple specification test, to appear in Review of Financial Studies.

Lucas, R., 1978, Asset prices in an exchange economy, Econometrica 46, 1429-1446.

Marsh, T. and R. Merton, 1986, Dividend variability and variance bounds tests for the rationality of stock market prices, American Economic Review 76, 483-498.

Muth, J., 1960, Optimal properties of exponentially weighted forecasts, Journal of the American Statistical Association 55, 299-306.

Nankervis, J. C. and N. E. Savin, 1985, Testing the autoregressive parameter with the t statistic, Journal of Econometrics 27, 143-161.

Newey, W. and K. West, 1987, A simple, positive definite, heteroscedasticity and autocorrelation consistent covariance matrix, Econometrica 55, 703-708.

Perron, P., 1986, Tests of joint hypotheses for time series regression with a unit root, University of Montreal C.R.D.E. Working Paper No. 2086, June.

Phillips, P. C. B., 1986, Regression theory for near-integrated time series, to appear in Econometrica.

_________, 1987, Time series regression with a unit root, Econometrica 55, 277-302.

Phillips, P. C. B. and P. Perron, 1986, Testing for a unit root in time series regression, University of Montreal C.R.D.E. Working Paper 2186, June.

Poterba, J. and L Summers, 1986, The persistence of volatility and stock market fluctuations, American Economic Review 76, 1142-1151.

_________, 1987, Mean reversion in stock returns: Evidence and implications, working paper, Department of Economics, Massachusetts Institute of Technology.

Priestley, M., 1981, Spectral Analysis and Time Series. Academic Press: London.

Schwert, G., 1987a, Effects of model specification on tests for unit roots in macroeconomic data, Journal of Monetary Economics 20, 73-104.

_________, 1987b, Tests for unit roots: A Monte Carlo investigation, Simon Graduate School of Business Administration Working Paper GPB87-01, University of Rochester, January.

Shiller, R. J., 1981, The use of volatility measures in assessing market efficiency, Journal of Finance 36, 291-304.

_________, 1984, Stock prices and social dynamics, Brookings Papers on Economic Activity 2, 457-498.

Shiller, R. J. and P. Perron, 1985, Testing the random walk hypothesis: Power versus frequency of observation, Economics Letters 18, 381-386.

Summers, L. H., 1986, Does the stock market rationally reflect fundamental values?, Journal of Finance 41, 591-600.

Taylor, S., 1984, Estimating the variances of autocorrelations calculated from financial time series, Applied Statistics 33, 300-308.

White, H., 1980, A heteroscedasticity-consistent covariance matrix estimator and a direct test for heteroscedasticity, Econometrica 48, 817-838.

White, H. and I. Domowitz, 1984, Nonlinear regression with dependent observations, Econometrica 52, 143-161.

TABLE 1a

Empirical sizes of nominal 1, 5, and 10 percent two-sided variance ratio tests of the random walk null hypothesis with homoscedastic disturbances. The statistic $z_1(q)$ is asymptotically $N(0,1)$ under the i.i.d. random walk. The rejection rates for each of the 1, 5, and 10 percent tests are broken down into upper and lower tail rejections to display the skewness of the z_1-statistic's empirical distribution. For comparison, the empirical sizes of the one-sided Box-Pierce Q-test (Q_1) using q-1 autocorrelations are also reported. Each set of rows with a given sample size forms a separate and independent simulation experiment based on 20,000 replications.

		SIZE OF 1 PERCENT TEST				SIZE OF 5 PERCENT TEST				SIZE OF 10 PERCENT TEST			
Sample Size	q	Lower Tail $z_1(q)$	Upper Tail $z_1(q)$	Size $z_1(q)$	Size Q_1	Lower Tail $z_1(q)$	Upper Tail $z_1(q)$	Size $z_1(q)$	Size Q_1	Lower Tail $z_1(q)$	Upper Tail $z_1(q)$	Size $z_1(q)$	Size Q_1
32	2	0.005	0.006	0.011	0.006	0.027	0.030	0.057	0.044	0.057	0.058	0.115	0.093
32	4	0.000	0.020	0.020	0.007	0.005	0.049	0.054	0.035	0.028	0.078	0.106	0.073
32	8	0.000	0.035	0.035	0.007	0.000	0.065	0.065	0.028	0.000	0.088	0.088	0.056
32	16	0.000	0.027	0.027	0.004	0.000	0.050	0.050	0.016	0.000	0.073	0.073	0.030
64	2	0.005	0.005	0.010	0.008	0.026	0.027	0.053	0.047	0.051	0.053	0.104	0.094
64	4	0.000	0.014	0.014	0.008	0.013	0.039	0.052	0.040	0.037	0.067	0.104	0.084
64	8	0.000	0.023	0.023	0.010	0.002	0.052	0.053	0.039	0.019	0.077	0.096	0.073
64	16	0.000	0.034	0.034	0.011	0.000	0.062	0.062	0.034	0.000	0.084	0.084	0.057
64	32	0.000	0.027	0.027	0.006	0.000	0.050	0.050	0.015	0.000	0.068	0.068	0.025
128	2	0.006	0.005	0.011	0.010	0.028	0.027	0.055	0.051	0.053	0.051	0.104	0.099
128	4	0.001	0.009	0.011	0.009	0.017	0.035	0.052	0.046	0.041	0.061	0.102	0.092
128	8	0.000	0.016	0.016	0.011	0.008	0.044	0.052	0.044	0.032	0.070	0.102	0.087
128	16	0.000	0.025	0.025	0.011	0.001	0.055	0.056	0.041	0.014	0.081	0.095	0.076
128	32	0.000	0.037	0.037	0.011	0.000	0.066	0.066	0.031	0.000	0.087	0.087	0.054
128	64	0.000	0.029	0.029	0.004	0.000	0.054	0.054	0.010	0.000	0.072	0.072	0.017
256	2	0.005	0.004	0.009	0.009	0.026	0.025	0.051	0.049	0.050	0.051	0.101	0.099
256	4	0.002	0.008	0.010	0.010	0.021	0.031	0.052	0.049	0.045	0.056	0.100	0.095
256	8	0.001	0.013	0.014	0.011	0.013	0.038	0.051	0.047	0.036	0.064	0.100	0.092
256	16	0.000	0.016	0.016	0.012	0.006	0.045	0.051	0.048	0.028	0.072	0.099	0.092
256	32	0.000	0.026	0.026	0.011	0.000	0.055	0.055	0.042	0.013	0.080	0.093	0.077
256	64	0.000	0.037	0.037	0.007	0.000	0.065	0.065	0.023	0.000	0.088	0.088	0.043
256	128	0.000	0.029	0.029	0.001	0.000	0.052	0.052	0.004	0.000	0.072	0.072	0.007
512	2	0.005	0.005	0.010	0.010	0.025	0.026	0.051	0.050	0.050	0.051	0.101	0.100
512	4	0.003	0.008	0.011	0.009	0.022	0.030	0.052	0.046	0.046	0.055	0.102	0.093
512	8	0.002	0.010	0.012	0.009	0.018	0.035	0.052	0.046	0.042	0.060	0.102	0.092
512	16	0.001	0.013	0.013	0.010	0.013	0.039	0.052	0.045	0.035	0.063	0.099	0.090
512	32	0.000	0.019	0.019	0.010	0.005	0.047	0.052	0.043	0.027	0.072	0.099	0.083
512	64	0.000	0.028	0.028	0.008	0.000	0.056	0.057	0.032	0.012	0.080	0.092	0.062
512	128	0.000	0.036	0.036	0.004	0.000	0.064	0.064	0.013	0.000	0.086	0.086	0.025
512	256	0.000	0.029	0.029	0.000	0.000	0.052	0.052	0.001	0.000	0.072	0.072	0.001
1024	2	0.004	0.006	0.010	0.010	0.024	0.028	0.052	0.051	0.049	0.052	0.100	0.099
1024	4	0.003	0.007	0.010	0.010	0.020	0.030	0.050	0.050	0.046	0.055	0.101	0.096
1024	8	0.002	0.010	0.012	0.010	0.016	0.032	0.048	0.048	0.041	0.058	0.098	0.096
1024	16	0.001	0.011	0.012	0.010	0.014	0.036	0.051	0.046	0.038	0.062	0.100	0.092
1024	32	0.000	0.015	0.016	0.010	0.010	0.041	0.051	0.045	0.033	0.067	0.100	0.089
1024	64	0.000	0.019	0.019	0.010	0.004	0.045	0.050	0.043	0.026	0.070	0.095	0.081
1024	128	0.000	0.025	0.025	0.006	0.000	0.054	0.055	0.028	0.011	0.078	0.090	0.053
1024	256	0.000	0.034	0.034	0.001	0.000	0.062	0.062	0.006	0.000	0.082	0.082	0.013
1024	512	0.000	0.028	0.028	0.000	0.000	0.051	0.051	0.000	0.000	0.069	0.069	0.000

TABLE 1b

Empirical sizes of nominal 1, 5, and 10 percent two-sided variance ratio tests of the random walk null hypothesis with homoscedastic disturbances. The statistic $z_2(q)$ is asymptotically N(0,1) under the more general conditions of heteroscedastic and weakly dependent (but uncorrelated) random walk increments. The rejection rates for each of the 1, 5, and 10 percent tests are broken down into upper and lower tail rejections to display the skewness of the z_2-statistic's empirical distribution. For comparison, the empirical sizes of the heteroscedasticity-robust one-sided Box-Pierce Q-test (Q_2) using q-1 autocorrelations are also reported. Each set of rows with a given sample size forms a separate and independent simulation experiment based on 20,000 replications.

		SIZE OF 1 PERCENT TEST				SIZE OF 5 PERCENT TEST				SIZE OF 10 PERCENT TEST			
Sample Size	q	Lower Tail $z_2(q)$	Upper Tail $z_2(q)$	Size $z_2(q)$	Size Q_2	Lower Tail $z_2(q)$	Upper Tail $z_2(q)$	Size $z_2(q)$	Size Q_2	Lower Tail $z_2(q)$	Upper Tail $z_2(q)$	Size $z_2(q)$	Size Q_2
32	2	0.005	0.007	0.012	0.006	0.033	0.038	0.071	0.049	0.064	0.070	0.134	0.107
32	4	0.000	0.024	0.025	0.009	0.011	0.061	0.072	0.044	0.038	0.090	0.128	0.097
32	8	0.000	0.042	0.042	0.014	0.000	0.075	0.075	0.054	0.007	0.098	0.105	0.104
32	16	0.000	0.035	0.035	0.025	0.000	0.065	0.065	0.071	0.000	0.087	0.087	0.123
64	2	0.004	0.006	0.010	0.007	0.027	0.030	0.057	0.048	0.056	0.058	0.114	0.102
64	4	0.001	0.014	0.015	0.009	0.016	0.044	0.060	0.046	0.041	0.073	0.114	0.096
64	8	0.000	0.027	0.027	0.013	0.004	0.058	0.061	0.053	0.025	0.084	0.109	0.100
64	16	0.000	0.038	0.038	0.021	0.000	0.069	0.069	0.066	0.001	0.090	0.092	0.116
64	32	0.000	0.034	0.034	0.032	0.000	0.059	0.059	0.084	0.000	0.079	0.079	0.134
128	2	0.006	0.005	0.011	0.009	0.029	0.028	0.057	0.052	0.055	0.054	0.109	0.103
128	4	0.001	0.010	0.012	0.010	0.018	0.038	0.056	0.049	0.043	0.064	0.107	0.101
128	8	0.000	0.017	0.017	0.012	0.010	0.046	0.056	0.053	0.035	0.073	0.109	0.103
128	16	0.000	0.027	0.027	0.017	0.001	0.059	0.060	0.060	0.017	0.085	0.103	0.110
128	32	0.000	0.040	0.040	0.027	0.000	0.071	0.071	0.075	0.000	0.092	0.093	0.124
128	64	0.000	0.036	0.036	0.041	0.000	0.063	0.063	0.095	0.000	0.082	0.082	0.143
256	2	0.005	0.004	0.010	0.009	0.026	0.026	0.052	0.050	0.051	0.052	0.103	0.100
256	4	0.002	0.008	0.010	0.011	0.022	0.032	0.054	0.050	0.045	0.057	0.103	0.098
256	8	0.001	0.014	0.015	0.012	0.014	0.039	0.053	0.050	0.038	0.065	0.104	0.101
256	16	0.000	0.018	0.018	0.015	0.007	0.047	0.054	0.059	0.030	0.074	0.104	0.110
256	32	0.000	0.027	0.027	0.019	0.001	0.057	0.058	0.068	0.016	0.083	0.099	0.119
256	64	0.000	0.040	0.040	0.029	0.000	0.070	0.070	0.083	0.000	0.093	0.093	0.137
256	128	0.000	0.035	0.035	0.047	0.000	0.060	0.060	0.108	0.000	0.081	0.081	0.161
512	2	0.005	0.005	0.010	0.010	0.025	0.026	0.051	0.050	0.050	0.052	0.101	0.100
512	4	0.003	0.008	0.011	0.009	0.022	0.031	0.052	0.047	0.047	0.056	0.103	0.095
512	8	0.001	0.010	0.012	0.011	0.018	0.035	0.053	0.047	0.043	0.061	0.104	0.095
512	16	0.001	0.013	0.014	0.011	0.013	0.039	0.052	0.050	0.037	0.065	0.101	0.100
512	32	0.000	0.020	0.020	0.014	0.006	0.048	0.054	0.056	0.029	0.073	0.103	0.107
512	64	0.000	0.030	0.030	0.018	0.001	0.059	0.059	0.066	0.014	0.083	0.097	0.118
512	128	0.000	0.039	0.039	0.030	0.000	0.068	0.068	0.085	0.000	0.090	0.090	0.138
512	256	0.000	0.034	0.034	0.048	0.000	0.060	0.060	0.110	0.000	0.080	0.080	0.164
1024	2	0.004	0.006	0.010	0.010	0.024	0.028	0.052	0.051	0.049	0.052	0.101	0.100
1024	4	0.003	0.007	0.010	0.010	0.020	0.030	0.050	0.050	0.046	0.056	0.102	0.097
1024	8	0.002	0.010	0.012	0.010	0.017	0.032	0.050	0.049	0.041	0.058	0.099	0.098
1024	16	0.001	0.011	0.012	0.010	0.015	0.036	0.051	0.050	0.038	0.063	0.101	0.098
1024	32	0.001	0.016	0.016	0.012	0.010	0.041	0.052	0.052	0.034	0.067	0.101	0.103
1024	64	0.000	0.020	0.020	0.016	0.005	0.046	0.051	0.062	0.027	0.071	0.099	0.114
1024	128	0.000	0.026	0.026	0.021	0.001	0.056	0.057	0.071	0.014	0.081	0.094	0.123
1024	256	0.000	0.036	0.036	0.032	0.000	0.066	0.066	0.091	0.000	0.087	0.087	0.146
1024	512	0.000	0.033	0.033	0.047	0.000	0.058	0.058	0.113	0.000	0.076	0.076	0.170

Table 2

Empirical quantiles of the (Dickey-Fuller) t-statistic associated with the hypothesis $\beta = 1$ in the regression $X_t = \mu + \omega t + \beta X_{t-1} + \epsilon_t$ where ϵ_t is i.i.d. N(0,1). Each row corresponds to a separate and independent simulation experiment based upon 20,000 replications.

Sample Size	0.005	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990	0.995
32	-4.767	-4.456	-4.043	-3.731	-3.361	-1.222	-0.887	-0.598	-0.246	-0.013
64	-4.449	-4.188	-3.860	-3.570	-3.243	-1.230	-0.906	-0.620	-0.279	-0.019
128	-4.324	-4.087	-3.777	-3.492	-3.186	-1.241	-0.918	-0.635	-0.273	-0.040
256	-4.235	-3.990	-3.684	-3.424	-3.135	-1.241	-0.910	-0.649	-0.276	-0.049
512	-4.173	-3.973	-3.676	-3.424	-3.131	-1.233	-0.903	-0.611	-0.299	-0.032
1024	-4.160	-3.959	-3.663	-3.425	-3.130	-1.252	-0.951	-0.673	-0.319	-0.054

TABLE 3a

Empirical sizes of nominal 1, 5, and 10 percent (two-sided) variance ratio tests of the random walk null hypothesis with heteroscedastic disturbances. The statistic $z_1(q)$ is asymptotically $N(0,1)$ under the i.i.d. random walk; the $z_2(q)$ statistic is asymptotically $N(0,1)$ under the more general conditions of heteroscedastic and weakly dependent (but uncorrelated) increments. For comparison, the empirical sizes of the (two-sided) Dickey-Fuller t-test (D-F), the (one-sided) Box-Pierce Q-test (Q_1) and its heteroscedasticity-consistent counterpart (Q_2) (both using q-1 autocorrelations) are also reported. The specific form of heteroscedasticity is given by $\ln \sigma_t^2 = \psi \ln \sigma_{t-1}^2 + \zeta_t$, ζ_t *i.i.d.* $N(0,1)$ and $\psi = 0.50$. Each set of rows with a given sample size forms a separate and independent simulation experiment based on 20,000 replications.

		1 PERCENT TEST				5 PERCENT TEST				10 PERCENT TEST			
Sample Size	q	$z_1(q)$	Q_1	$z_2(q)$	Q_2	$z_1(q)$	Q_1	$z_2(q)$	Q_2	$z_1(q)$	Q_1	$z_2(q)$	Q_2
32	2	0.024	0.014	0.015	0.003	0.093	0.069	0.071	0.036	0.161	0.133	0.141	0.098
32	4	0.028	0.010	0.030	0.005	0.071	0.047	0.076	0.035	0.132	0.094	0.133	0.080
32	8	0.032	0.008	0.039	0.009	0.064	0.028	0.075	0.042	0.088	0.054	0.112	0.087
32	16	0.023	0.003	0.036	0.017	0.047	0.011	0.065	0.054	0.070	0.020	0.089	0.100
32	D-F	0.023				0.073				0.124			
64	2	0.037	0.029	0.008	0.004	0.107	0.094	0.055	0.039	0.175	0.158	0.118	0.098
64	4	0.029	0.023	0.016	0.006	0.088	0.078	0.061	0.037	0.155	0.134	0.116	0.087
64	8	0.030	0.016	0.028	0.009	0.066	0.053	0.063	0.045	0.119	0.099	0.108	0.092
64	16	0.035	0.012	0.038	0.016	0.065	0.036	0.068	0.056	0.090	0.061	0.097	0.101
64	32	0.026	0.003	0.034	0.025	0.049	0.012	0.061	0.071	0.071	0.020	0.082	0.118
64	D-F	0.019				0.066				0.116			
128	2	0.043	0.039	0.007	0.005	0.123	0.115	0.051	0.043	0.195	0.184	0.109	0.098
128	4	0.033	0.032	0.013	0.007	0.104	0.103	0.053	0.039	0.174	0.169	0.106	0.087
128	8	0.028	0.025	0.018	0.008	0.077	0.080	0.053	0.045	0.138	0.134	0.102	0.090
128	16	0.030	0.020	0.027	0.013	0.063	0.055	0.059	0.052	0.106	0.095	0.097	0.096
128	32	0.036	0.012	0.038	0.021	0.065	0.033	0.067	0.064	0.086	0.056	0.091	0.112
128	64	0.026	0.003	0.032	0.030	0.050	0.008	0.059	0.082	0.070	0.014	0.081	0.132
128	D-F	0.016				0.061				0.111			
256	2	0.053	0.050	0.008	0.007	0.134	0.129	0.047	0.045	0.207	0.200	0.102	0.096
256	4	0.041	0.043	0.010	0.007	0.112	0.122	0.049	0.042	0.183	0.192	0.101	0.089
256	8	0.029	0.033	0.014	0.009	0.087	0.096	0.050	0.044	0.152	0.161	0.099	0.093
256	16	0.025	0.023	0.020	0.010	0.067	0.073	0.052	0.050	0.122	0.127	0.098	0.097
256	32	0.029	0.018	0.027	0.016	0.057	0.053	0.055	0.057	0.096	0.091	0.092	0.106
256	64	0.035	0.010	0.037	0.023	0.063	0.027	0.065	0.073	0.083	0.049	0.087	0.126
256	128	0.027	0.001	0.032	0.035	0.050	0.003	0.058	0.091	0.070	0.005	0.079	0.145
256	D-F	0.012				0.058				0.111			
512	2	0.058	0.056	0.008	0.007	0.147	0.146	0.049	0.047	0.223	0.220	0.105	0.102
512	4	0.046	0.051	0.011	0.008	0.125	0.138	0.053	0.045	0.201	0.218	0.104	0.094
512	8	0.033	0.038	0.013	0.009	0.101	0.113	0.052	0.047	0.169	0.183	0.105	0.097
512	16	0.026	0.029	0.016	0.010	0.076	0.086	0.054	0.050	0.136	0.146	0.103	0.098
512	32	0.024	0.020	0.020	0.012	0.064	0.065	0.056	0.054	0.115	0.116	0.101	0.104
512	64	0.027	0.013	0.026	0.016	0.058	0.044	0.057	0.060	0.097	0.077	0.096	0.111
512	128	0.035	0.005	0.037	0.024	0.062	0.017	0.065	0.075	0.085	0.030	0.089	0.128
512	256	0.027	0.000	0.032	0.036	0.052	0.001	0.059	0.095	0.071	0.002	0.079	0.149
512	D-F	0.010				0.049				0.099			
1024	2	0.059	0.058	0.008	0.008	0.148	0.148	0.046	0.046	0.222	0.222	0.097	0.096
1024	4	0.047	0.057	0.009	0.009	0.128	0.148	0.050	0.049	0.197	0.226	0.100	0.100
1024	8	0.031	0.039	0.010	0.008	0.101	0.116	0.050	0.046	0.167	0.193	0.100	0.095
1024	16	0.021	0.029	0.011	0.010	0.079	0.095	0.050	0.051	0.139	0.160	0.101	0.100
1024	32	0.020	0.022	0.015	0.012	0.063	0.073	0.051	0.053	0.119	0.130	0.100	0.102
1024	64	0.023	0.016	0.021	0.014	0.057	0.058	0.054	0.057	0.106	0.100	0.099	0.107
1024	128	0.030	0.009	0.030	0.019	0.061	0.033	0.061	0.068	0.096	0.061	0.097	0.119
1024	256	0.037	0.002	0.039	0.027	0.064	0.008	0.068	0.083	0.088	0.015	0.093	0.137
1024	512	0.028	0.000	0.034	0.041	0.052	0.000	0.059	0.104	0.070	0.000	0.080	0.161
1024	D-F	0.012				0.054				0.105			

TABLE 3b

Empirical sizes of nominal 1, 5, and 10 percent [two-sided] variance ratio tests of the random walk null hypothesis with heteroscedastic disturbances. The statistic $z_1(q)$ is asymptotically $N(0,1)$ under the i.i.d. random walk; the $z_2(q)$ statistic is asymptotically $N(0,1)$ under the more general conditions of heteroscedastic and weakly dependent [but uncorrelated] increments. For comparison, the empirical sizes of the [two-sided] Dickey-Fuller t-test (D-F), the [one-sided] Box-Pierce Q-test (Q_1) and its heteroscedasticity-consistent counterpart (Q_2) [both using q-1 autocorrelations] are also reported. The specific form of heteroscedasticity is given by $\ln \sigma_t^2 = \psi \ln \sigma_{t-1}^2 + \zeta_t$, ζ_t *i.i.d.* $N(0,1)$ and $\psi = 0.95$. Each set of rows with a given sample size forms a separate and independent simulation experiment based on 20,000 replications.

		1 PERCENT TEST				5 PERCENT TEST				10 PERCENT TEST			
Sample Size	q	$z_1(q)$	Q_1	$z_2(q)$	Q_2	$z_1(q)$	Q_1	$z_2(q)$	Q_2	$z_1(q)$	Q_1	$z_2(q)$	Q_2
32	2	0.087	0.054	0.024	0.002	0.196	0.151	0.080	0.028	0.279	0.231	0.157	0.084
32	4	0.052	0.054	0.033	0.005	0.140	0.142	0.080	0.031	0.229	0.225	0.130	0.068
32	8	0.046	0.031	0.037	0.008	0.077	0.083	0.070	0.037	0.104	0.134	0.097	0.076
32	16	0.028	0.006	0.032	0.012	0.055	0.019	0.060	0.040	0.078	0.032	0.084	0.074
32	D-F	0.088				0.159				0.216			
64	2	0.166	0.142	0.014	0.002	0.288	0.261	0.059	0.031	0.377	0.347	0.126	0.088
64	4	0.127	0.193	0.022	0.005	0.258	0.341	0.068	0.032	0.353	0.439	0.117	0.076
64	8	0.072	0.173	0.032	0.008	0.158	0.312	0.066	0.039	0.262	0.406	0.099	0.081
64	16	0.057	0.084	0.035	0.011	0.089	0.167	0.062	0.045	0.115	0.231	0.085	0.086
64	32	0.037	0.013	0.032	0.016	0.066	0.029	0.059	0.044	0.089	0.044	0.081	0.077
64	D-F	0.106				0.186				0.252			
128	2	0.265	0.252	0.008	0.002	0.391	0.377	0.043	0.031	0.467	0.458	0.106	0.088
128	4	0.231	0.398	0.016	0.005	0.366	0.554	0.055	0.034	0.450	0.638	0.103	0.078
128	8	0.150	0.447	0.026	0.009	0.302	0.596	0.057	0.040	0.400	0.677	0.087	0.084
128	16	0.085	0.361	0.030	0.012	0.176	0.505	0.058	0.044	0.289	0.585	0.083	0.087
128	32	0.060	0.169	0.031	0.014	0.093	0.267	0.056	0.046	0.120	0.329	0.076	0.090
128	64	0.039	0.023	0.029	0.018	0.067	0.041	0.053	0.048	0.088	0.055	0.073	0.083
128	D-F	0.124				0.206				0.273			
256	2	0.367	0.359	0.005	0.003	0.493	0.487	0.036	0.031	0.564	0.557	0.095	0.087
256	4	0.343	0.592	0.013	0.006	0.472	0.717	0.048	0.033	0.544	0.778	0.091	0.076
256	8	0.284	0.691	0.022	0.009	0.429	0.802	0.054	0.040	0.513	0.852	0.087	0.081
256	16	0.177	0.662	0.028	0.011	0.340	0.771	0.058	0.043	0.443	0.824	0.082	0.082
256	32	0.096	0.513	0.031	0.014	0.193	0.623	0.060	0.047	0.314	0.678	0.081	0.088
256	64	0.071	0.241	0.035	0.019	0.102	0.329	0.060	0.053	0.127	0.383	0.079	0.094
256	128	0.043	0.030	0.031	0.022	0.072	0.047	0.055	0.055	0.094	0.060	0.074	0.092
256	D-F	0.134				0.223				0.289			
512	2	0.476	0.474	0.003	0.003	0.582	0.581	0.036	0.034	0.645	0.640	0.093	0.090
512	4	0.453	0.740	0.011	0.005	0.565	0.830	0.045	0.035	0.631	0.873	0.090	0.082
512	8	0.401	0.858	0.021	0.008	0.528	0.921	0.051	0.038	0.600	0.945	0.083	0.083
512	16	0.317	0.868	0.024	0.011	0.461	0.925	0.054	0.045	0.541	0.946	0.080	0.089
512	32	0.188	0.786	0.029	0.011	0.356	0.858	0.056	0.045	0.456	0.890	0.079	0.090
512	64	0.099	0.594	0.033	0.014	0.196	0.678	0.060	0.051	0.315	0.721	0.081	0.094
512	128	0.073	0.301	0.037	0.019	0.104	0.374	0.063	0.058	0.129	0.414	0.081	0.100
512	256	0.044	0.031	0.032	0.024	0.072	0.047	0.055	0.057	0.094	0.060	0.075	0.097
512	D-F	0.134				0.216				0.282			
1024	2	0.576	0.575	0.003	0.003	0.667	0.666	0.035	0.033	0.719	0.718	0.091	0.089
1024	4	0.559	0.851	0.010	0.006	0.651	0.908	0.045	0.036	0.702	0.931	0.096	0.083
1024	8	0.513	0.944	0.019	0.009	0.620	0.971	0.049	0.040	0.680	0.982	0.086	0.084
1024	16	0.445	0.959	0.024	0.010	0.565	0.981	0.053	0.043	0.631	0.988	0.080	0.088
1024	32	0.336	0.931	0.026	0.010	0.483	0.960	0.056	0.042	0.563	0.971	0.081	0.083
1024	64	0.198	0.830	0.029	0.012	0.364	0.885	0.058	0.046	0.464	0.907	0.080	0.089
1024	128	0.100	0.621	0.031	0.015	0.197	0.689	0.060	0.053	0.316	0.724	0.084	0.094
1024	256	0.072	0.320	0.036	0.021	0.103	0.376	0.063	0.059	0.130	0.409	0.085	0.101
1024	512	0.043	0.033	0.031	0.024	0.070	0.045	0.055	0.060	0.090	0.052	0.075	0.101
1024	D-F	0.127				0.214				0.281			

TABLE 4

Empirical quantiles of the [asymptotically] N(0,1) variance ratio test statistic $z_1(q)$ under simulated i.i.d. Gaussian random walk increments, where q is the aggregation value. Each set of rows with a given sample size forms a separate and independent simulation experiment based on 20,000 replications.

T	q	0.005	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990	0.995
32	2	-2.56	-2.33	-1.99	-1.70	-1.35	1.33	1.72	2.04	2.41	2.65
32	4	-1.96	-1.86	-1.67	-1.49	-1.25	1.45	1.95	2.43	3.02	3.34
32	8	-1.52	-1.47	-1.38	-1.28	-1.14	1.50	2.22	2.87	3.72	4.28
32	16	-1.13	-1.11	-1.06	-1.01	-0.92	1.34	1.96	2.63	3.39	3.99
64	2	-2.56	-2.35	-1.97	-1.66	-1.29	1.31	1.67	1.99	2.38	2.61
64	4	-2.16	-2.01	-1.79	-1.54	-1.26	1.37	1.83	2.22	2.72	3.10
64	8	-1.85	-1.75	-1.59	-1.42	-1.21	1.43	1.99	2.51	3.17	3.67
64	16	-1.47	-1.43	-1.35	-1.26	-1.12	1.46	2.18	2.86	3.82	4.48
64	32	-1.10	-1.08	-1.04	-0.99	-0.92	1.29	1.96	2.63	3.50	4.10
128	2	-2.63	-2.36	-2.01	-1.68	-1.29	1.30	1.66	1.98	2.35	2.56
128	4	-2.30	-2.11	-1.81	-1.57	-1.27	1.33	1.76	2.15	2.56	2.85
128	8	-2.05	-1.92	-1.71	-1.50	-1.24	1.39	1.86	2.32	2.88	3.24
128	16	-1.77	-1.69	-1.54	-1.40	-1.20	1.46	2.04	2.59	3.28	3.86
128	32	-1.45	-1.41	-1.33	-1.23	-1.11	1.50	2.25	2.95	3.84	4.60
128	64	-1.10	-1.08	-1.04	-0.99	-0.91	1.33	2.07	2.76	3.67	4.36
256	2	-2.59	-2.34	-1.97	-1.65	-1.29	1.30	1.66	1.96	2.31	2.54
256	4	-2.33	-2.18	-1.89	-1.60	-1.26	1.31	1.70	2.08	2.49	2.81
256	8	-2.20	-2.03	-1.77	-1.53	-1.25	1.34	1.78	2.19	2.72	3.04
256	16	-2.00	-1.87	-1.67	-1.47	-1.22	1.40	1.88	2.33	2.91	3.39
256	32	-1.77	-1.67	-1.53	-1.38	-1.18	1.45	2.03	2.62	3.29	3.76
256	64	-1.45	-1.40	-1.32	-1.23	-1.10	1.51	2.24	2.99	3.91	4.57
256	128	-1.09	-1.07	-1.03	-0.98	-0.91	1.33	2.02	2.72	3.63	4.22
512	2	-2.57	-2.31	-1.96	-1.65	-1.29	1.28	1.65	1.98	2.33	2.58
512	4	-2.46	-2.24	-1.90	-1.61	-1.28	1.32	1.70	2.05	2.46	2.76
512	8	-2.34	-2.13	-1.83	-1.58	-1.26	1.33	1.75	2.11	2.58	2.91
512	16	-2.17	-2.02	-1.76	-1.52	-1.23	1.35	1.79	2.18	2.70	3.09
512	32	-1.97	-1.86	-1.66	-1.47	-1.22	1.39	1.92	2.39	2.93	3.39
512	64	-1.74	-1.67	-1.53	-1.38	-1.19	1.44	2.06	2.68	3.41	3.83
512	128	-1.43	-1.39	-1.31	-1.22	-1.10	1.48	2.22	2.95	3.91	4.64
512	256	-1.09	-1.07	-1.03	-0.98	-0.91	1.30	2.00	2.70	3.58	4.27
1024	2	-2.52	-2.28	-1.94	-1.63	-1.27	1.30	1.66	2.00	2.36	2.63
1024	4	-2.45	-2.19	-1.88	-1.60	-1.27	1.33	1.71	2.04	2.43	2.71
1024	8	-2.35	-2.14	-1.81	-1.56	-1.24	1.33	1.72	2.09	2.55	2.85
1024	16	-2.22	-2.05	-1.80	-1.54	-1.25	1.35	1.77	2.18	2.62	2.97
1024	32	-2.10	-1.96	-1.73	-1.51	-1.23	1.36	1.83	2.27	2.76	3.10
1024	64	-1.94	-1.84	-1.65	-1.46	-1.23	1.40	1.89	2.41	2.95	3.33
1024	28	-1.76	-1.66	-1.53	-1.38	-1.18	1.43	2.02	2.58	3.35	3.93
1024	56	-1.43	-1.39	-1.31	-1.23	-1.10	1.45	2.21	2.92	3.82	4.70
1024	512	-1.09	-1.07	-1.03	-0.98	-0.91	1.27	1.97	2.68	3.56	4.36

Table 5a

Power of the [two-sided] variance ratio test [using the $z_1(q)$ statistic] against the stationary AR(1) alternative $X_t = \phi X_{t-1} + \epsilon_t$, ϵ_t i.i.d. $N(0,1)$ and $\phi = 0.96$. For comparison, the power of the [one-sided] Box-Pierce Q-test (Q_1) and the [two-sided] Dickey-Fuller t-test (D-F) are also reported. Each set of rows with a given sample size forms a separate and independent simulation experiment based on 20,000 replications.

		1 PERCENT TEST		5 PERCENT TEST		10 PERCENT TEST	
Sample Size	q	$z_1(q)$	Q_1	$z_1(q)$	Q_1	$z_1(q)$	Q_1
32	2	0.008	0.009	0.047	0.047	0.093	0.097
32	4	0.009	0.009	0.049	0.045	0.101	0.096
32	8	0.009	0.010	0.048	0.048	0.096	0.098
32	16	0.009	0.010	0.049	0.050	0.101	0.099
32	D-F	0.010		0.050		0.098	
64	2	0.009	0.008	0.048	0.048	0.097	0.100
64	4	0.009	0.009	0.046	0.050	0.093	0.099
64	8	0.008	0.010	0.044	0.051	0.093	0.107
64	16	0.008	0.010	0.043	0.050	0.086	0.101
64	32	0.009	0.010	0.044	0.051	0.088	0.104
64	D-F	0.009		0.042		0.084	
128	2	0.010	0.010	0.047	0.050	0.100	0.106
128	4	0.010	0.011	0.051	0.053	0.102	0.106
128	8	0.011	0.011	0.050	0.054	0.102	0.104
128	16	0.012	0.009	0.053	0.056	0.102	0.112
128	32	0.010	0.009	0.053	0.054	0.103	0.112
128	64	0.010	0.009	0.046	0.053	0.088	0.108
128	D-F	0.008		0.047		0.095	
256	2	0.011	0.012	0.057	0.062	0.111	0.115
256	4	0.017	0.013	0.061	0.062	0.121	0.120
256	8	0.021	0.013	0.079	0.066	0.146	0.123
256	16	0.028	0.013	0.101	0.060	0.180	0.121
256	32	0.030	0.012	0.123	0.059	0.217	0.118
256	64	0.031	0.012	0.130	0.060	0.227	0.114
256	128	0.026	0.011	0.103	0.054	0.189	0.110
256	D-F	0.025		0.118		0.207	
512	2	0.016	0.017	0.066	0.070	0.125	0.131
512	4	0.023	0.019	0.090	0.082	0.165	0.150
512	8	0.038	0.020	0.140	0.087	0.227	0.162
512	16	0.075	0.020	0.225	0.088	0.341	0.161
512	32	0.144	0.019	0.341	0.083	0.491	0.158
512	64	0.203	0.017	0.469	0.079	0.640	0.140
512	128	0.196	0.016	0.514	0.071	0.686	0.130
512	256	0.097	0.014	0.345	0.064	0.517	0.124
512	D-F	0.189		0.478		0.654	
1024	2	0.026	0.025	0.092	0.091	0.159	0.162
1024	4	0.053	0.033	0.165	0.114	0.257	0.206
1024	8	0.124	0.034	0.304	0.136	0.413	0.238
1024	16	0.272	0.038	0.497	0.146	0.632	0.254
1024	32	0.510	0.034	0.755	0.134	0.853	0.235
1024	64	0.769	0.025	0.928	0.107	0.970	0.197
1024	128	0.859	0.023	0.981	0.092	0.995	0.170
1024	256	0.855	0.019	0.983	0.080	0.997	0.155
1024	512	0.530	0.018	0.844	0.075	0.934	0.147
1024	D-F	0.915		0.993		0.999	

Table 5b

Power of the [two-sided] variance ratio test [using the $z_1(q)$ statistic] against the ARIMA(1,1,1) alternative $X_t = Y_t + Z_t$ where $Y_t = 0.95Y_{t-1} + \epsilon_t$, ϵ_t i.i.d. $N(0,1)$ and $Z_t = Z_{t-1} + \gamma_t$, γ_t i.i.d. $N(0, \frac{1}{2})$. For comparison, the power of the [one-sided] Box-Pierce Q-test (Q_1) and the [two-sided] Dickey-Fuller t-test (D-F) are also reported. Each set of rows with a given sample size forms a separate and independent simulation experiment based on 20,000 replications.

		1 PERCENT TEST		5 PERCENT TEST		10 PERCENT TEST	
Sample Size	q	$z_1(q)$	Q_1	$z_1(q)$	Q_1	$z_1(q)$	Q_1
32	2	0.008	0.010	0.045	0.048	0.095	0.098
32	4	0.010	0.010	0.045	0.047	0.098	0.096
32	8	0.010	0.011	0.047	0.049	0.094	0.101
32	16	0.009	0.010	0.046	0.051	0.094	0.100
32	D-F	0.010		0.049		0.094	
64	2	0.009	0.010	0.048	0.048	0.096	0.100
64	4	0.010	0.010	0.046	0.050	0.094	0.102
64	8	0.009	0.009	0.045	0.050	0.092	0.104
64	16	0.009	0.009	0.044	0.052	0.089	0.101
64	32	0.010	0.010	0.047	0.051	0.091	0.104
64	D-F	0.009		0.046		0.094	
128	2	0.009	0.010	0.046	0.051	0.098	0.104
128	4	0.011	0.011	0.052	0.053	0.099	0.104
128	8	0.012	0.011	0.053	0.052	0.104	0.102
128	16	0.011	0.011	0.052	0.054	0.103	0.107
128	32	0.009	0.009	0.047	0.053	0.102	0.105
128	64	0.010	0.009	0.045	0.053	0.087	0.106
128	D-F	0.009		0.048		0.101	
256	2	0.010	0.012	0.054	0.059	0.106	0.111
256	4	0.015	0.012	0.055	0.057	0.113	0.115
256	8	0.015	0.011	0.068	0.059	0.126	0.118
256	16	0.018	0.012	0.075	0.054	0.138	0.106
256	32	0.016	0.013	0.072	0.054	0.131	0.109
256	64	0.014	0.012	0.063	0.056	0.117	0.106
256	128	0.014	0.010	0.055	0.052	0.107	0.104
256	D-F	0.015		0.069		0.129	
512	2	0.014	0.014	0.061	0.065	0.119	0.123
512	4	0.018	0.017	0.077	0.074	0.141	0.139
512	8	0.025	0.016	0.101	0.072	0.178	0.140
512	16	0.034	0.014	0.124	0.071	0.210	0.133
512	32	0.036	0.014	0.120	0.064	0.206	0.129
512	64	0.027	0.014	0.095	0.065	0.170	0.119
512	128	0.020	0.013	0.079	0.064	0.138	0.112
512	256	0.015	0.012	0.063	0.059	0.120	0.115
512	D-F	0.021		0.081		0.147	
1024	2	0.024	0.023	0.085	0.085	0.150	0.153
1024	4	0.040	0.024	0.132	0.091	0.207	0.169
1024	8	0.065	0.023	0.196	0.097	0.290	0.177
1024	16	0.096	0.021	0.236	0.092	0.355	0.163
1024	32	0.094	0.017	0.241	0.079	0.355	0.144
1024	64	0.064	0.014	0.178	0.067	0.277	0.129
1024	128	0.030	0.013	0.118	0.061	0.197	0.120
1024	256	0.025	0.011	0.085	0.057	0.148	0.117
1024	512	0.021	0.012	0.074	0.057	0.132	0.112
1024	D-F	0.032		0.104		0.173	

Table 5c

Power of the [two-sided] variance ratio test [using the $z_1(q)$ statistic] against the ARIMA(1,1,1) alternative $X_t = Y_t + Z_t$ where $Y_t = 0.96Y_{t-1} + \epsilon_t$, ϵ_t i.i.d. N(0,1) and $Z_t = Z_{t-1} + \nu_t$, ν_t i.i.d. N(0,1). For comparison, the power of the [one-sided] Box-Pierce Q-test (Q_1) and the [two-sided] Dickey-Fuller t-test (D-F) are also reported. Each set of rows with a given sample size forms a separate and independent simulation experiment based on 20,000 replications.

		1 PERCENT TEST		5 PERCENT TEST		10 PERCENT TEST	
Sample Size	q	$z_1(q)$	Q_1	$z_1(q)$	Q_1	$z_1(q)$	Q_1
32	2	0.008	0.010	0.049	0.049	0.095	0.099
32	4	0.010	0.010	0.046	0.051	0.093	0.102
32	8	0.009	0.012	0.045	0.052	0.092	0.106
32	16	0.008	0.012	0.045	0.053	0.094	0.102
32	D-F	0.009		0.049		0.096	
64	2	0.010	0.010	0.048	0.049	0.096	0.102
64	4	0.008	0.009	0.048	0.054	0.100	0.105
64	8	0.009	0.009	0.047	0.054	0.099	0.111
64	16	0.010	0.009	0.048	0.054	0.095	0.105
64	32	0.010	0.009	0.047	0.052	0.093	0.107
64	D-F	0.009		0.045		0.092	
128	2	0.010	0.012	0.049	0.056	0.102	0.113
128	4	0.012	0.013	0.062	0.058	0.115	0.113
128	8	0.014	0.013	0.062	0.061	0.122	0.115
128	16	0.016	0.012	0.068	0.060	0.125	0.118
128	32	0.013	0.012	0.060	0.059	0.117	0.115
128	64	0.012	0.012	0.053	0.058	0.098	0.112
128	D-F	0.012		0.056		0.114	
256	2	0.013	0.015	0.060	0.065	0.114	0.122
256	4	0.021	0.015	0.073	0.071	0.142	0.136
256	8	0.027	0.015	0.103	0.072	0.178	0.137
256	16	0.034	0.013	0.120	0.062	0.212	0.122
256	32	0.026	0.013	0.120	0.058	0.207	0.117
256	64	0.023	0.014	0.092	0.062	0.165	0.118
256	128	0.019	0.012	0.072	0.056	0.133	0.112
256	D-F	0.024		0.098		0.175	
512	2	0.022	0.023	0.087	0.092	0.151	0.159
512	4	0.036	0.026	0.129	0.106	0.209	0.188
512	8	0.058	0.024	0.191	0.100	0.294	0.186
512	16	0.088	0.021	0.251	0.093	0.377	0.169
512	32	0.095	0.019	0.257	0.081	0.387	0.153
512	64	0.067	0.018	0.194	0.076	0.311	0.136
512	128	0.044	0.017	0.146	0.070	0.224	0.129
512	256	0.028	0.014	0.106	0.064	0.171	0.124
512	D-F	0.053		0.155		0.241	
1024	2	0.038	0.036	0.122	0.123	0.201	0.206
1024	4	0.085	0.046	0.230	0.156	0.337	0.261
1024	8	0.173	0.043	0.393	0.162	0.513	0.272
1024	16	0.285	0.035	0.513	0.142	0.654	0.245
1024	32	0.305	0.028	0.552	0.116	0.686	0.203
1024	64	0.213	0.021	0.426	0.091	0.571	0.169
1024	128	0.093	0.019	0.259	0.078	0.381	0.148
1024	256	0.062	0.014	0.169	0.068	0.262	0.134
1024	512	0.040	0.013	0.126	0.065	0.200	0.131
1024	D-F	0.078		0.200		0.292	

Table 5d

Power of the [two-sided] variance ratio test [using the $z_1(q)$ statistic] against the ARIMA(1,1,1) alternative $X_t = Y_t + Z_t$ where $Y_t = 0.96Y_{t-1} + \epsilon_t$, ϵ_t i.i.d. N(0,1) and $Z_t = Z_{t-1} + \gamma_t$, γ_t i.i.d. N(0,2). For comparison, the power of the [one-sided] Box-Pierce Q-test (Q_1) and the [two-sided] Dickey-Fuller t-test (D-F) are also reported. Each set of rows with a given sample size forms a separate and independent simulation experiment based on 20,000 replications.

		1 PERCENT TEST		5 PERCENT TEST		10 PERCENT TEST	
Sample Size	q	$z_1(q)$	Q_1	$z_1(q)$	Q_1	$z_1(q)$	Q_1
32	2	0.008	0.010	0.045	0.048	0.091	0.097
32	4	0.010	0.010	0.048	0.050	0.093	0.103
32	8	0.009	0.012	0.046	0.054	0.096	0.110
32	16	0.008	0.012	0.044	0.054	0.093	0.102
32	D-F	0.009		0.048		0.093	
64	2	0.011	0.012	0.050	0.054	0.103	0.112
64	4	0.013	0.012	0.050	0.061	0.104	0.115
64	8	0.011	0.013	0.052	0.059	0.104	0.119
64	16	0.011	0.013	0.047	0.062	0.095	0.116
64	32	0.010	0.013	0.044	0.060	0.089	0.115
64	D-F	0.010		0.047		0.094	
128	2	0.011	0.014	0.054	0.061	0.106	0.117
128	4	0.014	0.014	0.070	0.065	0.127	0.128
128	8	0.019	0.014	0.080	0.068	0.149	0.127
128	16	0.023	0.012	0.089	0.065	0.156	0.124
128	32	0.016	0.011	0.084	0.062	0.155	0.120
128	64	0.014	0.012	0.063	0.057	0.120	0.113
128	D-F	0.015		0.072		0.139	
256	2	0.018	0.021	0.075	0.084	0.139	0.146
256	4	0.035	0.020	0.102	0.088	0.182	0.167
256	8	0.047	0.019	0.155	0.088	0.255	0.166
256	16	0.067	0.016	0.205	0.081	0.324	0.151
256	32	0.060	0.016	0.207	0.072	0.331	0.139
256	64	0.043	0.015	0.160	0.069	0.268	0.128
256	128	0.032	0.012	0.108	0.063	0.196	0.123
256	D-F	0.050		0.170		0.273	
512	2	0.032	0.035	0.113	0.119	0.187	0.196
512	4	0.063	0.040	0.193	0.149	0.299	0.249
512	8	0.121	0.039	0.322	0.145	0.448	0.251
512	16	0.210	0.031	0.463	0.124	0.607	0.220
512	32	0.255	0.025	0.516	0.104	0.669	0.192
512	64	0.178	0.021	0.406	0.091	0.567	0.165
512	128	0.103	0.018	0.280	0.082	0.399	0.150
512	256	0.059	0.017	0.186	0.073	0.283	0.142
512	D-F	0.132		0.306		0.427	
1024	2	0.068	0.065	0.187	0.187	0.282	0.287
1024	4	0.170	0.095	0.374	0.256	0.496	0.391
1024	8	0.371	0.092	0.638	0.292	0.745	0.440
1024	16	0.613	0.074	0.825	0.249	0.904	0.396
1024	32	0.711	0.053	0.898	0.184	0.951	0.304
1024	64	0.576	0.035	0.811	0.134	0.899	0.230
1024	128	0.281	0.028	0.559	0.110	0.699	0.192
1024	256	0.163	0.022	0.344	0.090	0.471	0.169
1024	512	0.100	0.021	0.239	0.086	0.339	0.159
1024	D-F	0.227		0.417		0.525	

Table 5e

Power of the [two-sided] variance ratio test [using the statistic $z_1(q)$] against the ARIMA(1,1,0) alternative $\Delta X_t = \kappa \Delta X_{t-1} + \nu_t$, ν_t i.i.d. $N(0,1)$, $\kappa = 0.20$. For comparison, the power of the [one-sided] Box-Pierce Q-test (Q_1) and the [two-sided] Dickey-Fuller t-test (D-F) are also reported. Each set of rows with a given sample size forms a separate and independent simulation experiment based on 20,000 replications.

		1 PERCENT TEST		5 PERCENT TEST		10 PERCENT TEST	
Sample Size	q	$z_1(q)$	Q_1	$z_1(q)$	Q_1	$z_1(q)$	Q_1
32	2	0.057	0.037	0.176	0.128	0.270	0.213
32	4	0.046	0.021	0.141	0.094	0.226	0.167
32	8	0.029	0.022	0.098	0.086	0.168	0.157
32	16	0.026	0.023	0.095	0.085	0.166	0.147
32	D-F	0.143		0.240		0.301	
64	2	0.148	0.119	0.342	0.292	0.463	0.417
64	4	0.104	0.071	0.263	0.195	0.368	0.298
64	8	0.059	0.050	0.168	0.156	0.254	0.248
64	16	0.035	0.040	0.114	0.135	0.181	0.218
64	32	0.032	0.036	0.097	0.123	0.164	0.209
64	D-F	0.143		0.240		0.308	
128	2	0.377	0.323	0.600	0.564	0.719	0.687
128	4	0.257	0.197	0.455	0.413	0.576	0.542
128	8	0.122	0.126	0.280	0.305	0.388	0.422
128	16	0.059	0.082	0.167	0.231	0.254	0.344
128	32	0.034	0.058	0.108	0.184	0.175	0.290
128	64	0.029	0.050	0.093	0.166	0.153	0.268
128	D-F	0.138		0.235		0.302	
256	2	0.741	0.709	0.887	0.876	0.934	0.928
256	4	0.526	0.529	0.744	0.749	0.836	0.836
256	8	0.276	0.361	0.498	0.612	0.614	0.726
256	16	0.125	0.229	0.298	0.454	0.401	0.588
256	32	0.069	0.160	0.172	0.348	0.261	0.479
256	64	0.036	0.115	0.105	0.285	0.177	0.400
256	128	0.032	0.088	0.095	0.241	0.158	0.362
256	D-F	0.138		0.238		0.299	
512	2	0.972	0.969	0.993	0.993	0.997	0.997
512	4	0.871	0.918	0.957	0.976	0.978	0.988
512	8	0.571	0.813	0.779	0.931	0.855	0.965
512	16	0.290	0.652	0.523	0.845	0.633	0.909
512	32	0.139	0.481	0.300	0.706	0.407	0.809
512	64	0.069	0.320	0.168	0.571	0.263	0.688
512	128	0.035	0.227	0.112	0.455	0.181	0.580
512	256	0.032	0.182	0.097	0.380	0.159	0.519
512	D-F	0.136		0.236		0.304	
1024	2	1.000	1.000	1.000	1.000	1.000	1.000
1024	4	0.996	0.999	0.999	1.000	1.000	1.000
1024	8	0.893	0.995	0.969	0.999	0.985	1.000
1024	16	0.585	0.978	0.783	0.996	0.862	0.998
1024	32	0.301	0.918	0.509	0.976	0.629	0.989
1024	64	0.144	0.767	0.295	0.911	0.411	0.952
1024	128	0.061	0.581	0.176	0.792	0.265	0.874
1024	256	0.035	0.411	0.110	0.663	0.174	0.780
1024	512	0.028	0.334	0.093	0.586	0.157	0.710
1024	D-F	0.142.		0.248		0.314	